ANALYSIS OF THE COMBINED MODE RESONANCE IN ALUMINUM NITRIDE MEMS RESONATORS

A Thesis Presented

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Aluminum Nitride (AlN) contour-extensional mode resonators (CMRs) have shown great promise for use in a wide range of technologies due to their scalability and the ability to combine multiple resonant frequencies on a single chip. However, their low coupling, and hence figure of merit, is a significant downside to their use compared to other resonator designs such as film bulk acoustic wave resonators (FBARs) and surface acoustic wave resonators. Recently, researchers have discovered a theoretical new mode of resonance that would combine $d_{31}$ and $d_{33}$ piezoelectric coefficients. By combining both modes, the resonator achieves notably improved $k^2$, without reducing the quality factor. This thesis is a thorough analysis of this new mode, looking closely at how and why it occurs, as well as the primary underlying design variables that control it. Through extensive use of finite element model (FEM) simulation, it is first proven that the new mode in fact combines $d_{31}$ and $d_{33}$. This thesis also examines how the coupling behavior alters the resonant frequency and how it affects, and is affected by, spurious modes. The results of this analysis show how to design improved combined mode resonators without the extensive use of time consuming FEM simulations. To achieve this goal, models are adapted from FBAR design to enable the prediction of the combined mode’s behavior. As a result, all significant underlying design variables are examined for their individual effects on this new resonance mode. When properly implemented, the combined mode effectively adds the coupling of its two underlying modes, leading to a
considerable increase in $k t^2$, allowing it to possibly surpass both FBARs and standard CMRs in figure of merit.
2. Introduction

In the past few years, Micro and Nano Electro Mechanical Systems (MEMS/NEMS) resonators have been widely adopted for a variety of sensing applications due to a combination of extremely high sensitivity to external perturbations and ultra-low noise performance. They are further improved not only by their small footprint and IC integration capability, but also by a relatively unique scaling capability as high quality ultra-thin (~10 nm) AlN films can be directly deposited on silicon via sputtering. This has led to AlN nano plate resonant sensor (NPR-S) technology [1], which involves high frequency (100 MHz to 10 GHz) bulk acoustic waves in piezoelectric nano plates (thickness < 1 μm) made out of AlN, a nano-scale form of the contour-extensional mode resonator (CMR). The result has been a promising array of solutions for the creation of extremely sensitive, miniaturized, low power sensors [2-4]. AlN NPR-S achieves amazingly improved values for the detection limit and detection speed due to the high quality factor ($Q$) and power handling ability combined with low mass and a high operating frequency. The main competition in this design is from film bulk acoustic wave resonators (FBARs), which tend to have a significantly better electromechanical coupling factor ($k_t^2$) due to a stronger piezoelectric response, but are limited to one frequency per chip unlike AlN NPR-S.

Recently, some research has been done concerning combined mode resonators (XMRs), which theoretically attempt to take advantage of the FBAR’s strong piezoelectric response and combine it with the CMR design [5]. Much has been done in the micro resonator area but is clearly scalable to the NPR-S technology. In order to combine the two modes, all that would need to be done, in theory, is to set the pitch and
thickness of the resonator such that the two modes will resonate at the same frequency. When done properly, this results in a significantly improved $k_t^2$, and thus an improved figure of merit (FOM). This improvement however is limited to a single frequency per chip, as the function of the combined mode is dependent on the thickness, much in the same way as FBARs are limited to one frequency per chip. However, unlike FBARs, CMRs can be combined on the same chip for different frequencies. While this could be done with the combined mode resonator, secondary frequencies would not actually be in the combined mode, and behave like standard CMRs. The resulting improved FOM would make AlN lamb-wave resonators and AlN NPR-S even more competitive with current FBAR or Quartz designs.

The work presented in this thesis involves a thorough analysis of the coupling that occurs between the two modes of resonance, using a previously designed XMR and its resulting paper [5] as a jumping off point. While there are several known bodies of work concerning XMRs, thus far a more comprehensive analysis of the combined mode resonance has not been done. 2D COMSOL simulations are used widely to test and compare the wide array of designs used to probe the behavior of the combined mode resonance. Therefore the significance of [5] cannot be understated, as it provides an essential connection between reality and simulation, showing that the COMSOL simulation results of combined mode resonators closely match reality. Methods for modeling the combined mode behavior outside of finite element model (FEM) simulations, using modified Mason models, is also developed and utilized. The thesis is organized into the following chapters:
In Chapter 3, the mechanism and fundamental parameters of AlN CMRs are briefly reviewed, followed by a review of the previously designed XMR. A comprehensive examination of how and under what circumstances the two resonance modes combine follows. A close examination of exactly which resonance modes are combining is presented.

In Chapter 4, modeling and design methods are presented that will identify the conditions under which one can achieve combined mode resonance. Each significant design variable of the resonator that can affect the outcome is analyzed and discussed. Finally, issues with, and methods to mitigate spurious, interfering peaks is presented.

In Chapter 5, the significant accomplishments of this research work are summarized and discussed, as well as a view of how others works could use this as another stepping stone. Finally, some of the issues that have been encountered, and will be encountered in future works on XMRs, are presented.

3. Analysis of Resonant Modes in AlN Combined Mode Resonators

3.1 AlN Resonators

Aluminum Nitride is a dielectric material, belonging to the dihexagonal polar class of crystals, which also exhibits direct piezoelectric effect. To effectively describe this piezoelectric action in the form of equations, electromechanical coupling can be expressed in the d-form piezoelectric coefficient matrix (1):

$$S = sT + d^T E$$

$$D = dT + \varepsilon E$$

(1)
Where $S$ is the strain ($6 \times 1$ matrix), $T$ is the stress ($6 \times 1$ matrix), $E$ is the electric field ($3 \times 1$ matrix), $D$ is the electric displacement ($3 \times 1$ matrix), $s$ is the compliance ($6 \times 6$ matrix), $\varepsilon$ is the permittivity ($3 \times 3$ matrix) and $dT$ is the transpose of the strain-charge form ($d$-form) piezoelectric coefficient ($3 \times 6$ matrix). For AlN, the $d$ matrix is given by Eq. (2)

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 3.84 & 0 \\
0 & 0 & 0 & -3.84 & 0 & 0 \\
-1.73 & -2.78 & 5.49 & 0 & 0 & 0
\end{bmatrix} [pCN^{-1}]
$$

(2)

The $d33$ coefficient is used in FBARs for duplexer applications [6-7], but since resonant frequency is set by the thin film thickness, it is not suitable for single chip multi-frequency operation. The $d33$ piezoelectric coefficient can result in the excitation of a thickness displacement or vertical vibration when an AC electric field is applied in the thickness direction. If instead the $d31$ piezoelectric coefficient is used with the same AC electric field applied, in-plane displacement or lateral vibration can be excited in the MEMS structure (Figure 3.1). This lateral-extensional mode of vibration is the basis for the piezoelectric AlN CMRs analyzed in this work, but the $d33$ piezoelectric coefficient plays a prominent role as well.
Figure 3.1 Schematic representation of the dihexagonal structure of AlN. The fundamental X(1), Y(2) and Z(3) directions are indicated. As shown, the anisotropic nature of the film permits the excitation of contour mode shapes through the $d_{31}$ coefficient.

A conventional AlN CMR is composed of an AlN film sandwiched between two metal electrodes (Figure 3.2). When AC voltage is applied to the interdigital electrode, a contour-extensional mode of vibration can be excited via the equivalent $d_{31}$ piezoelectric coefficient of the AlN [8].

Figure 3.2 Schematic representation of a conventional AlN CMR. The inset shows a FEM simulation of the device mode of vibration.
Given the equivalent mass density \( (\rho_{eq}) \), and Young’s modulus \( (E_{eq}) \), of the overall structure (AlN and electrodes) that forms the resonator, the resonant frequency \( (f_r) \) of this laterally vibrating mechanical structure is defined by the pitch \( (W_0) \) of the metal electrode patterned on the AlN plate. The resonance frequency of the device can be approximately defined by Eq. (3) [9].

\[
f_r = \frac{1}{2W_0} \sqrt{\frac{E_{eq}}{\rho_{eq}}}
\]

Several of these unitary cells of width, \( W_0 \) (known as fingers), are arrayed together and excited with opposing phases (two adjacent fingers are excited 180° out of phase with respect to each other) in order to form an equivalent symmetric contour-extension vibration wave [10-12] in the AlN plate (Figure 3.3).

![Figure 3.3 Schematic representation of a three-finger AlN CMR](image)

The remaining geometrical dimensions, length \( (L) \) and thickness \( (T) \), define the equivalent electrical impedance of the resonator and can be designed relatively
independently of the desired resonance frequency. The film thickness \((T)\), the number of fingers \((n)\), and their length, \(L\), (also known as aperture of the transducer) can be used to the equivalent resonator electrical capacitance, \(C0\), and its motional resistance, \(Rm\), [9] which can be approximately defined as expressed in Eq. (4).

\[
R_m \propto \frac{T}{nL}
\]

\[
C_0 \propto \frac{nLW_0}{T}
\]

(4)

A conventional AlN FBAR is very similar to the conventional AlN CMR in terms of its basic structure. Both are simply a film of AlN sandwiched between two electrodes (Figure 3.4). However, when an AC voltage is applied across the inter-digital fingers, the \(d_{33}\) piezoelectric coefficient will make the result a thickness vibration.

**Figure 3.4** Schematic representation of a Film Bulk Acoustic Wave Resonator (FBAR). The inset shows a FEM simulation of the device mode of vibration.
If the equivalent mass density ($\rho_{eq}$), and Young’s modulus ($E_{eq}$), of the overall structure (AlN and electrodes) that forms the resonator, the anti-resonant frequency ($f_0$) of this laterally vibrating mechanical structure is defined by the thickness (T) of the AlN plate. This anti-resonance frequency of the device can be approximately defined by Eq. (5) [13]

$$f_a = \frac{1}{2T_0} \sqrt{\frac{E_{eq}}{\rho_{eq}}}$$

(5)

FBARs can be composed of multiple unitary cells of width, known as fingers, to be defined by the designer, however it is more common to have FBARs fabricated with only a single unit. The fingers are excited, in a manner similar to the previous example, 180° out of phase from each other, resulting in a thickness vibrational mode occurring throughout the structure.

The remaining geometrical dimensions, width (W) and length (L), set the equivalent electrical impedance of the resonator and can be defined relatively independently of anti-resonance frequency. The length (L), width (W), number of fingers (n), and, if there is more than one finger, pitch can be used to define the equivalent resonator capacitance ($C_0$) and it’s motional resistance ($R_m$) as shown in Eq. (6) [5]

$$R_m \propto \frac{W_0}{nL}$$

$$C_0 \propto \frac{nLW_0}{T}$$

(6)
As the combined mode resonator borrows aspects of both FBAR design and AlN CMR design, all these parameters and equations will be used to guide the analysis and design in the following section.

3.2 AlN Combined Mode Resonator Basis

The design from which a large portion of this analysis is sourced is a variation of the CMR design discussed in the previous section. Instead of having fingers on the top and bottom of the AlN plate, this design has fingers only on top, which significantly reduces the fabrication complexity (Figure 3.5) and will be referred to as a top electrode only contour mode resonator (TEO-CMR). The major downside of this design is that it also results in a loss in the electromechanical coupling constant, as the resulting electric fields aren’t purely vertical.

![Figure 3.5 2-mask microfabrication process of the AlN combined mode MEMS resonator: (a) AlN thin film was sputter-deposited on top of Si substrate; (b) Al/TiN film was deposited on top of AlN film and patterned by lift-off process; (c) AlN film was](Image 122x788 to 527x1108)
etched by ICP in Cl2 based chemistry; (d) Si substrate was released by XeF2 isotropic etching.

One researcher designed, built and tested an AlN XMR using that base design [5]. His methodology was to simulate seven different thickness values around where a combined mode was expected and had the result around the highest $k t^2$ fabricated and tested. (Figure 3.6)

**Figure 3.6** FEM simulated $k t^2$ dependence on $T/\lambda$ for an AlN LFE resonator employing a single top inter-digital electrode ($\lambda=2W$).

The resulting design had a thickness of 1.5 μm, a length of 57.4 μm and the number of fingers set at 37, each with a pitch of 1.6 μm (Figure 3.7). The electrodes themselves were composed of 0.2 μm of aluminum and .05 μm of titanium nitride (TiN). When compared, the results from testing of the actual resonator closely match those of simulation (Figure 3.8) [5]. The minor differences can be easily explained by the fact the COMSOL, like all simulators, is a representation of the ideal case while in reality there are minor deviations due to real-world design factors. This shows that COMSOL, while not perfect, is an accurate predictor of the behavior of AlN XMR designs.
Figure 3.7 Scanning Electron Microscope (SEM) image of the fabricated AlN combined mode MEMS resonator.

![SEM image of AlN MEMS resonator](image)

Figure 3.8 (a) Measured admittance curve and BVD fitting of the fabricated LFE combined mode AlN MEMS resonator. (b) FEM simulated admittance curve and 2D mode shape of vibration of the LFE combined mode AlN resonator. A spurious mode at a slightly lower frequency was predicted and also observed in the experimental data.

Given that the simulations can be expected to closely mirror real world results, COMSOL can be used to test and examine a variety of designs in order to achieve a clearer view of how the two vibration modes combine and under what circumstances.
3.3 Analysis of Resonant Modes Involved in a Combined Mode Resonator

A common assumption used in the creation and analysis of the combined mode resonator is that it is the combination of the vibration modes stemming from the $d_{33}$ and $d_{31}$ piezoelectric constants, however this has not been conclusively proven. At a glance, this connection makes sense, as the designs generally start by matching the $d_{33}$ resonance frequency and $d_{31}$ resonance frequency using Eqs 3 and 5, then adjust the thickness from there to find the best result. Similarly, the mode of vibration that is output looks like what is expected from the superimposition of the two modes on each other. Unfortunately this level simple examination doesn’t suffice as proof. When the resonator is significantly thinner than would be required for the combined mode to occur, the result looks like a standard $d_{31}$ mode shape (Figure 3.9a,b,c). However, as the thickness increases to combined mode level and beyond, the displacement field results look very similar whether or not the resonator is designed to be a combined mode resonator, due to the effects of Poisson’s ratio (Figures 3.10a,b,c 3.11a,b,c and 3.12a,b,c).
Figure 3.9a,b,c Displacement field results for low thickness \((T=.6 \, \mu m)\) resonator. a, b, and c show overall displacement, \(X\) displacement, and \(Y\) displacement respectively.

Figure 3.10a,b,c Displacement field results for XMR \((T=1.4 \, \mu m)\). a, b, and c show overall displacement, \(X\) displacement, and \(Y\) displacement respectively. Use legend on right for a. For b and c use the left legend.

Figure 3.11a,b,c Displacement field results for high thickness \((T=3.0 \, \mu m)\) XMR. a, b, and c show overall displacement, \(X\) displacement, and \(Y\) displacement respectively.

Figure 3.12a,b,c Displacement field results for high thickness \((T=2.2 \, \mu m)\) resonator. a, b, and c show overall displacement, \(X\) displacement, and \(Y\) displacement respectively.
Therefore, the starting goal of this analysis of the combined mode resonator was to prove, or at least create a preponderance of evidence that shows that this is truly a combination of the effects of $d_{31}$ and $d_{33}$ piezoelectric effects. (For simplicity, “$d_{xx}$ mode” or “$d_{xx}$ resonant frequency” will be used in this paper to refer to resonant frequencies or vibration modes caused by the appropriate piezoelectric constant.)

The very first step was to verify all of the information done previously, and ensure all simulations matched each other. The next step taken was to find a more exact ideal thickness for the combined mode to resonate, and use that as a basis for further study. The result of testing various thicknesses and comparing the results found that the ideal thickness for a pitch of 1.6 µm, while leaving the electrodes unchanged, was 1.40 µm. (Figure 3.13)

![Figure 3.13 FEM simulation results of combined mode with idealized thickness, given all other parameters remain unchanged.](image)

\[
\begin{align*}
    f_0 &= 2.91 \text{ GHz} \\
    \kappa t^2 &= 2.81\% 
\end{align*}
\]

The next step was to simulate how the resonator would behave if one of the piezoelectric constants theoretically involved in the combined mode were removed from the equation. Within Comsol, the properties of AlN can be modified such that it will
simulate AlN as if the $d_{31}$ or $d_{33}$ constant was removed. Both cases were tested and the results were compared to the original combined mode results. (Figures 2.14 and 2.15)

\[ f_0 = 2.898 \text{ GHz} \]
\[ k t^2 = 1.67\% \]

\[ f_0 = 3.626 \text{ GHz} \]
\[ k t^2 = 0.39\% \]

**Figure 3.14** Graph of resonator with the $d_{31}$ piezoelectric constant set to zero. Peak 1 is the $d_{33}$ resonance frequency and peak 2 is where the $d_{31}$ peak would be, as shown in Figure 3.15.

\[ f_0 = 2.877 \text{ GHz} \]
\[ k t^2 = 0.095\% \]

\[ f_0 = 3.634 \text{ GHz} \]
\[ k t^2 = 0.704\% \]

**Figure 3.15** Graph of resonator with the $d_{31}$ piezoelectric constant set to zero. Peak 1 is where the $d_{33}$ peak would be, as shown in Figure 3.14, and peak 2 is the $d_{31}$ resonance frequency.
These results largely matched expectations, wherein it was found that if either mode of resonance was removed, the $k^2_t$ value went down precipitously. This heavily suggests that both modes are necessary for the effect of the combined mode to occur. To ensure these results were not a one-time fluke, the resonator was adjusted to a new pitch, this time 1.85 µm, and a new ideal thickness of 1.6 µm was identified. When the same experiment was run, the same result was seen, where the loss of either mode resulted in a significant loss of $k^2_t$ (Figures 2.16, 2.17, 2.18). With both designs it was noticed that the sum of $k^2_t$ values for the marked peaks in the individual modes is approximately equal to the sum of the $k^2_t$ values for the resonance peaks of the combined mode.

![Admittance graph of XMR with a pitch and thickness of 1.85 and 1.6µm respectively. These settings result in the highest $k^2_t$ measured and therefore is closest to the ideal combined mode. Resonance peaks are marked and the $k^2_t$ that results are labeled.](image)

1. $f_0 = 2.582$ GHz $k^2_t = 2.677\%$
2. $f_0 = 3.197$ GHz $k^2_t = 0.224\%$
Figure 3.17 Admittance graph of XMR with a pitch and thickness of 1.85 and 1.6 µm respectively. The $d_{31}$ resonance mode has been turned off, leaving the $d_{33}$ mode as the primary remaining mode of resonance. The resulting resonance peak is marked and the $kt^2$ that results is labeled.

1. $f_0 = 2.566$ GHz $kt^2 = 1.434\%$
2. $f_0 = 3.193$ GHz $kt^2 = 0.463\%$

1. $f_0 = 2.547$ GHz $kt^2 = 0.194\%$
2. $f_0 = 3.194$ GHz $kt^2 = 0.770\%$
Figure 3.18 Admittance graph of XMR with a pitch and thickness of 1.85 and 1.6 µm respectively. The $d_{33}$ resonance mode has been turned off, leaving the $d_{31}$ mode as the primary remaining mode of resonance. The resulting resonance peak is marked and the $kt^2$ that results is labeled.

To further analyze the behavior, after returning the design with 1.6 µm pitch, the thickness was swept from .2 µm to 3 µm while looking at an extremely wide frequency range. Similarly, using the Mason model, the each mode was tested to see how the resonant frequency of each would vary as the thickness changed along the same values (Figure 3.19).

![Comparison of Individual Mode Behavior Versus Combined Mode](image)

Figure 3.19 Comparison of the individual resonant mode's behavior to the simulated behavior of the resonator as thickness changes. When the $d_{33}$ mode begins to approach the $d_{31}$, the simulated results start showing a much steeper downward curve, one that levels off as the individual modes begin to separate at higher thicknesses.

The result shows that main resonant peak tracked in the COMSOL simulations largely matches the of the $d_{31}$ mode, as one might expect, until the $d_{33}$ resonant mode would get within approximately 25% of the $d_{31}$ mode, at which point the results show the peak starts tracking along with the $d_{33}$. This continues until the two modes begin to
separate, which results in a peak closely matching $d_{33}$ slowly decreasing in significance and $k t^2$ while a new peak shows up around 3.3 GHz, approximately where one would expect for the $d_{31}$ mode. (Figure 3.20)

![Figure 3.20](image)

**Figure 3.20** Admittance response of resonator with pitch of 1.6μm and thickness of 2.2μm. The $d_{33}$ resonance peak and $d_{31}$ resonance peak are each marked.

The analysis of how $k t^2$ is affected by the thickness is similarly unusual. (Figure 3.21) Theoretically, if this examined just the $d_{31}$ mode, then as the thickness increased, the $k t^2$ would rise somewhat at first then settle. The initial rise would be because the coupling is bad at very small thicknesses, but should not vary significantly after it has gotten above .8μm. Instead it rises, peaks, and then falls. Additionally, it should be noted that the $k t^2$ peaks as the resonant frequency is shifting the quickest, when the two resonant modes are overlapping.
Figure 3.21 Graph of $k_t^2$ versus thickness for a resonator with a pitch of 1.6 microns. The highest point is at a thickness of 1.4 microns. Results from two tested resonators also included in blue.

In reality, these results can be shown in actual resonators. As shown in Figure 3.21, when a resonator with a pitch and thickness of 1.6µm and 0.3µm is tested, the resulting $k_t^2$ is significantly less than the original design with a thickness of 1.5µm. These results create a preponderance of evidence suggesting that two modes combine with each other to create a new and improved resonance mode. Furthermore the evidence heavily suggests that it is the $d_{31}$ mode combining with the $d_{33}$ mode to create the new mode.

3.4 Analysis of Combined Mode Behavior

Given the two modes involved are $d_{33}$ and $d_{31}$, there is still further analysis to be done on the behavior of this mode. As was briefly referenced previously, as the thickness changes, the resonant peaks combine and separate from each other. This separation is most easily shown in a 3D plot of resonance frequency (Figure 3.21), where $k_t^2$ is used to
denote the relative strength of each resonant peak. If spurious or otherwise insignificant modes are ignored, then it can easily be shown that the main resonant peak experiences significant decreases in frequency as the ideal thickness is approached. As that value is passed, it decreases in strength, and a new main peak takes its place.

**Figure 3.21** A 3D plot of resonant frequencies versus thickness with strength denoted by $k^2$. The curve in the middle, starting at 2 GHz and ending around 3.5 GHz at the back, is the main combined mode. As its strength decreases, the other spurious modes increase.

At the precipice of the coupling, some unusual behavior can be observed. As the ideal thickness is approached, many of the spurious modes significantly decrease in strength as the primary mode increases (Figure 3.22, 2.23, 2.24, 2.25). These graphs show the how the spurious mode changes around the resonant frequency. Resonant modes that exist in Figure 3.22, then decrease in Figure 3.23. Most have disappeared at the ideal thickness shown in Figure 3.13, start to reappear in Figure 3.24 and show up
more fully again in Figure 3.25. This suggests that the energy that would normally go into these lesser resonant modes gets absorbed into the combined mode, with only minor spurious modes remaining.

**Figure 3.22** Graph of admittance for a resonator with pitch and thickness of 1.6 and 1.0 microns respectively. At this point there is, at most, only a minor amount of coupling between resonant modes, and there are a fair number of spurious modes.
Figure 3.23 Graph of admittance for a resonator with pitch and thickness of 1.6 and 1.2 microns respectively. Compared to Figure 3.22, coupling has increased and most other resonant and spurious modes besides the main one have decreased in strength.
Figure 3.24 Graph of admittance for a resonator with pitch and thickness of 1.6 and 1.6 microns respectively. Compared to Figure 3.13, coupling is beginning to decrease and spurious modes are beginning to return, especially around the main resonance peak.

Figure 3.25 Graph of admittance for a resonator with pitch and thickness of 1.6 and 2.0 microns respectively. Compared to Figure 3.24, the two modes have significantly decreased coupling and many spurious and other resonant modes have reappeared as a result.

For many of these peaks displacement data was taken and another oddity was noted. Possibly related to the TEO-CMR design base of the resonator, and its lack of bottom electrode, the $d_{33}$ mode of vibration never shows up independently, it is always coupled with some other mode. This holds true at low or high frequency and can be proven by examining the displacement field data, especially when looking at separated X and Y data. At the low frequency end, there is a resonance peak around 1.4 GHz which exists in that approximate location so long as the thickness is above 1 µm. (Figure 3.26) This is shown to be a surface acoustic wave type of resonance largely by examining the
displacement data (Figure 3.27a,b,c). This peak is not simply a simulation error, and was also appear in the fabricated XMR with 1.6µm pitch and 1.5µm thickness (Figure 3.28).

**Figure 3.26** Graph of admittance for a resonator with pitch and thickness of 1.6 and 1.3 microns respectively. Primary resonance peak and Mode 1 resonance peak are marked.
Figure 3.27 a,b,c Displacement pattern at M1 resonant frequency marked in Figure 3.26. Figure 3.27a shows the overall displacement, b shows the displacement pattern in the x direction and c show the displacement in the y direction. The legend on the right applies to all three displacement patterns.

However, as the d_{33} resonance peak approaches that mode, the M1 resonance peak becomes smaller and the resulting vibration becomes garbled, while the d_{33} mode, which is beginning to couple with the M1 mode, deforms. Normally, the d_{33} mode would have a line of zero displacement almost exactly in the center of the structure. When the d_{33} mode begins to couple to the M1, the resulting vibration has that line of zero displacement shifted upwards from the center. (Figures 3.29, 3.30a,b,c and 3.31 a,b,c)
Figure 3.29 Graph of admittance for a resonator with pitch and thickness of 1.6 and 3.0 microns respectively. The M1 resonance peak and the $d_{33}$ that is beginning to couple to the M1 are both marked. Also marked is the resonance peak that is the beginning of coupling between the $d_{31}$ and the second harmonic of the $d_{33}$.

Figure 3.30 a,b,c Displacement pattern at coupled $d_{33}$ and M1 resonant frequency marked in Figure 3.29. 3.30a shows the overall displacement, b shows the displacement pattern in the x direction and c shows the displacement in the y direction. The legend on the right applies to all three displacement patterns. The x displacement is insignificant, but the y displacement and total displacement show how the horizontal line of zero displacement, normally located at zero (the center of the AlN), is shifted upwards.
**Figure 3.31 a,b,c** Displacement pattern at M1 resonant frequency marked in Figure 3.29. 3.31a shows the overall displacement, b shows the displacement pattern in the x direction and c shows the displacement in the y direction. The legend on the right applies to all three displacement patterns. When each is compared to its equivalent in Figure 3.27, the deviation from the expected M1 pattern represented there becomes extremely obvious. This muddling is likely a byproduct of the coupling to the d$_{33}$ mode.

Similarly, at lower thickness values and therefore higher frequencies, the d$_{33}$ mode combines with one or several spurious modes, as can be seen in Figure 3.22. The resulting displacement can look somewhat similar to what one might expect for d$_{33}$ resonance but it is typically very garbled, preventing definitive identification (Figure 3.32a,b,c). This is part of the reason why, when the d$_{33}$ mode couples to the d$_{31}$ mode,
many of the spurious modes see a significant decrease in their significance, as measured by admittance of \( k^2 \). It also makes the \( d_{33} \) mode difficult to isolate on its own without manipulating variables, which leads to other errors and inaccuracies.

Figure 3.32 a,b,c Displacement pattern at resonant frequency marked “Unknown + \( d_{33} \)” in Figure 3.22. 3.32a shows the overall displacement, b shows the displacement pattern in the x direction and c shows the displacement in the y direction. The legend on the right applies to all three displacement patterns. While warped, the y displacement pattern is based on the \( d_{33} \) resonant mode, with the warping due to the spurious mode it is combined with.

The final interesting piece of data noted during this sweep is results from the very high end of the thickness data. As the thickness approaches upper value of 3 \( \mu \text{m} \), the \( d_{31} \) resonance peak starts to decrease in a manner similar to what occurred when coupling was just beginning to occur in the combined mode (Figure 3.29). The most likely explanation of this is that it is the result of coupling, in this case it would be between the \( d_{31} \) mode and the second harmonic of the \( d_{33} \) mode. Given that the \( d_{33} \) mode is difficult to isolate, proving this supposition would require a similar or larger amount of work as was required to come to the conclusion of the current combined mode. However the displacement data taken from the same peak would tend to back this up (Figure 3.33a,b,c). The y displacement data clearly show a second order resonance, occurring
without additional electrodes, the second order resonance would suggest it is a second harmonic. This evidence, in addition to utilizing the simpler proof originally used to suggest that the combined mode was the combination of $d_{33}$ and $d_{31}$ modes, it can be suggested that the resonance frequency of the $d_{31}$ is approximately double the resonance frequency of the $d_{33}$ mode, implying a similar conclusion.

Figure 3.33 a,b,c Displacement pattern at resonant frequency marked “$d_{33} + 2^{nd}$ harmonic of $d_{33}$ ” in Figure 3.29. 3.33a shows the overall displacement, b shows the displacement pattern in the x direction and c shows the displacement in the y direction. The legend on the right applies to all three displacement patterns. The y displacement clearly shows a second order resonance pattern, suggesting a second harmonic.
The resulting x displacement pattern shown in Figure 3.33b is unusual because it behaves as if the electric field were reversed on the bottom half. This exact cause of this behavior is unknown and should be investigated further in future research.

4 Designing an XMR

4.1 Mason Model

To enable widespread design and use of XMRs, the basic process of finding the appropriate thickness to width ratios must be simplified from the current methodology. The current process involves running multiple FEM simulations, which can consume significant amounts of time, in order to come up with a single device. The first step to simplification is creating or utilizing an appropriate model. While Eq 3 can be used to predict the resonance frequency of the d$_{31}$ mode, the same cannot be said for the d$_{33}$ mode. As was briefly mentioned previously, Eq 5 predicts the anti-resonance (or series resonance) frequency of the d$_{33}$, while what is needed is the resonance frequency. Therefore a more in-depth model of resonant behavior is required.

The mason model can be used to predict the resonance frequency, overall admittance curve, and therefore $kt^2$ for the d$_{33}$ resonant mode. The mason model also takes into account the effect of electrodes on the overall resonance, an important characteristic as the d$_{33}$ vibration mode includes the electrodes in its path (Figure 4.1).

[13] All of the information required for the model can be derived from standard material properties. However when being used to predict the d$_{31}$ mode, a slightly altered version of the model is used, in which the negative capacitance is removed from the electrical part of the circuit (Figure 4.2). [13]
**Figure 4.1** Mason model equivalent circuit for FBAR

**Figure 4.2** Mason model equivalent circuit for TEO-CMR

$Z$ is the acoustic impedance of material. $k$ is the wavenumber and $d$ is the path distance of each material. $C_0$ is the intrinsic capacitance in the design and $N$ takes into account the piezoelectric effect. There are all defined by either design parameters, or material parameters, and are solved in Eq 7.

$$Z = \sqrt{c_{33} \cdot \rho} = \rho \cdot v_a$$
\[ v_a = \sqrt{\frac{c_{33}}{\rho}} = \sqrt{\frac{E_{eq}}{\rho_{eq}}} \]

\[ k = \frac{v_a}{\omega} \]

\[ C_0 = \varepsilon_{33} \frac{A_e}{T_p} \]

\[ N = hC_0 = e_{ij} \frac{A_p}{d_p} \]  \hspace{1cm} (7)

The stiffness constant, \( c_{33} \), is also known as Young’s modulus. \( A_e \) is the electrode area while \( T_p \) is the thickness of the piezoelectric material. For calculating \( N \), \( e_{ij} \) is the relevant piezoelectric constant while \( A_p \) and \( d_p \) are the active piezoelectric area and the path distance in the piezoelectric material.

The two most significant differences between the models are electrode sections and the \(-C_0\). The \( d_{31} \) resonance mode does not, in theory, include the electrodes in the acoustic path. In reality, the electrodes have some effect and are accounted for in the equivalent Young’s modulus and equivalent density. Since the vertical vibration of the \( d_{33} \) mode includes the electrodes in its path, they are accounted for in the model itself, so the use of equivalent Young’s modulus and density are unnecessary. The negative capacitance exists a due to the difference in how the two modes are oriented. The model in Figure 4.1 is used when the applied electric field is in the same direction propagation of the acoustic wave. The model in Figure 4.2 is used when the propagation of the acoustic wave is perpendicular to the electric field [13].
4.2 Modified Mason Models

The mason model has two very significant flaws; however one is correctable and the other can be compensated for. The least significant flaw in the mason model is the lack of resistance. In the mason model, at resonance, the admittance can reach infinity, a clearly unrealistic prediction. While converting the mason model to be entered into Matlab and comparing it with similar models and equations, it can be shown that the BVD model is extremely similar to the mason model, but with some simplifications that create minor inaccuracies (Figure 4.3). [17]

\[ R_a = \frac{\pi TL}{8nLv_aE_{eq}d_{31}^2Q} \]

\[ L_a = \frac{WT}{8nLv_a^2E_{eq}d_{31}^2} \]

\[ C_a = \frac{8nWLE_{eq}d_{31}^2}{\pi^2T} \]
If the resistor is ignored, as it has no effect on the resonant frequency, then the
two models simplify down as shown in Eq (9). Therefore any circuit parts used in one
model can be transferred to the other, provided its equivalent is removed to avoid
duplication.

\[
Y = j\omega C_0 \left(1 + \frac{e_{31}^2}{\varepsilon c_{33}} \left(\tan\left(\frac{kd}{2}\right)\right)\right) \approx Y = j\omega C_0 \left(1 + \frac{e_{31}^2}{\varepsilon c_{33}} \left(\frac{8}{\pi^2 - \frac{W^2 \omega^2}{v_a^2}}\right)\right)
\]

(9)

However the BVD model includes something the mason model lacks, motional
resistance \(R_m\), which prevents infinite admittance at resonance. Given the functional
similarity between the two models, this motional resistance was added to the mason
model. This motional resistance, defined by the same equation, is put in series with the
transformer in both models. The quality factor, one of the only variables not intrinsic
from the material, can be assumed to be similar to other design or set arbitrarily.

The other most significant flaw in this model is that it assumes all the voltage
applied causes a mechanical response. In other words, it assumes the electric field is
completely vertical along the entire material. While this can be assume when the design
utilizes electrode on the top and bottom of the AlN, as in thickness excitation (TE), it
cannot be assume for LFE. The net effect is, without compensation, modeling would
suggest a far higher \(kt^2\) than would be achieved with any particular design.
The methodology for compensating for this effect is currently rather simple, but determining how heavy to compensate is currently rather crude. While it is easy to identify this error, given that it results in completely unrealistically high values for $k_t^2$, it is difficult to determine what the correct value should be. The method of compensating involves introducing a loss factor into the equation. In this case the effect of having only top electrodes can be modeled by a loss factor that approximately halves the piezoelectric constant. This loss factor is not static within the TEO-CMR design type, and varies depending on thickness (Figure 4.4).

![Model Prediction of $k_t^2$ Versus Thickness](image)

**Figure 4.4** Graph of $k_t^2$ reported by the modified mason model as thickness changes for the $d_{31}$ resonance mode. At low thicknesses, the reported values clearly become ludicrously high. Fortunately the $k_t^2$ value doesn’t affect the resonant frequency in this model.

This problem is the largest impediment remaining to predicting an XMR using circuit models, as opposed to FEM simulation. This loss factor can be crudely approximated presuming the designer already knows the approximate $k_t^2$ that would be
expected individually for each mode for their given resonator style. With the TEO-CMR design, the $k \tau^2$ typically does not rise above 1% for thicknesses on the order of µm. Using that and FEM simulation results as guidance, the approximate loss factors for the individual modes could be found for several different XMRs. These loss factors are noticed to vary inversely from each other, however no equation was found to adequately describe them.

4.3 Predicting Combined Mode

While the currently modified mason model allows the prediction of the frequency of the individual resonance modes, it does not in and of itself enable the prediction of an actual combined mode. If all the variables were set such that the resonant peaks of both modes occupied the same spot in the model, the fabricated result would not be a combined mode resonator. This is due to the frequency shift that occurs when the two modes couple to each other.

The source of the frequency shift can ultimately be reduced to the effect of Poisson’s ratio. The stress from the $d_{33}$ vertical vibration creates a strain horizontally, which feeds into the $d_{31}$ vibration. Similarly, the stress from the $d_{31}$ vibration creates a vertical strain that feeds into the $d_{33}$ vibration. The net result is that each mode individually has more strain placed upon it than would be suggested by modeling said mode individually. This extra strain can be treated as an effective reduction in acoustic velocity, which enables the prediction of the resulting resonant frequency. (Eq 10)

\[
f_r = \frac{1}{2W_0} \sqrt{\frac{E_{eq}}{(1 + r_{31})\rho_{eq}}} \quad f_a = \frac{1}{2T_0} \sqrt{\frac{E_{eq}}{(1 + r_{33})\rho_{eq}}}
\]  

(10)
This effective stiffness reduction is also a potential source of difficulty for designing XMRs. The effective stiffness reduction (r) for the d_{33} resonance mode is actually approximately half the reduction found in the d_{31} mode. The d_{31} resonance mode’s stiffness is effectively reduced by between 1.2 and 1.24, while the d_{33} mode is reduced by between 1.09 and 1.12. This means r_{31} varies from .2 to .24, effectively Poisson’s ratio for AlN (nominal value of .24), and r_{33} varies from .09 to 1.2, approximately half r_{31}. This reduction must be taken into account when designing an XMR to operate at a specific frequency. Any differences must also be accounted for in modeling.

While the mason model will not model the actual coupling, it can be manipulated to allow a predictions of the an XMR design. To combine the two modes, the two acoustic paths must be treated as separate, but in parallel to each other and to a single parasitic capacitance (C_0). When the two branches are combined such that one fuller overlaps the other, the resulting admittance graph is a rough prediction of the actual outcome (Figure 4.5a,b,c). The resulting k_t^2 can be calculated from the admittance curves, and is roughly the k_t^2 of the two individual modes added together.
4.4 Ideal Electrode Design

The XMR includes significant aspects of both FBAR design and conventional TEO-CMR design. For both cases there are ideal design parameters that often conflict with each other. One significant step is determining which design parameters are most significant and which have to be balanced. Outside the most significant variables, like pitch and thickness, many of the remaining significant variables are in the electrode design.
In regards to designing a TEO-CMR, the electrodes are ideally as thin as possible to reduce effect of mass loading. Mass loading is an effect caused by the mass of electrodes, which reduces important characteristics of a resonator such as coupling and quality. However, for an FBAR, having electrodes as thin as possible is less than ideal. In FBAR design, the ideal electrode thickness for the highest $k^2$ varies depending on the thickness of the piezoelectric material (Figure 4.6) [15].

![Figure 4.6](image-url)  
**Figure 4.6** Relationship between $k^2$ ($k_{eff}^2$) and the thickness of the top electrode for Aluminum and Molybdenum for a standard FBAR. ($Si_3N_4$ membrane = 0.2 µm, Mo bottom electrode = 0.1 µm, and AlN film = 2.25µm)

In the combined mode resonator, this analysis has found that the ideal electrode thickness behaves more similarly to the FBAR. In many of the designs tested, the ideal electrode thickness was around .2 µm, varying depending on the thickness of the design tested and provided the electrode material was aluminum. This very similar to the results of previous FBAR research, and significantly different from what would be expected if the combined mode followed standard TEO-CMR design idealities (Figures 3.7 and 3.8).
**Figure 4.7** Relationship between $k't^2$ and top electrode thickness for XMR with top and bottom electrodes, both made of aluminum. (Al bottom electrode = .1µm, AlN film = 1.3µm)

**Figure 4.8** Relationship between $k't^2$ and bottom electrode thickness for XMR with top and bottom electrodes, both made of aluminum. This relationship was determined at the ideal top electrode thickness. (Al top electrode = .23µm, AlN film = 1.3µm)
Another significant variable is the electrode width. This variable is often determined relative to the pitch, and in TEO-CMR design, is often chosen to be a significant percentage of the pitch. In FBAR design, the electrode width typically is used to set the impedance, and is not very critical. However, in this analysis, it was found that the ideal electrode width was 50% of the pitch (Figure 4.9). This is true across multiple thicknesses and even when designing for a combined mode with electrodes on the top and bottom.

**Figure 4.9** Graph of how $k^2$ varies with the width of the electrode relative to the pitch. When only top electrodes are used, the ideal was found to be 50%, but when top and bottom were used 54% performed slightly better.

In TEO-CMR design and FBAR design, one primary driver of choice in electrode materials is using a material with as close an acoustic velocity to the piezoelectric material as possible. This helps reduce the effect the electrodes have on the resonator. However in FBAR design, there are also benefits to choosing a material with a high
acoustic impedance, to reduce acoustic loss. As shown in Figure 4.6, for FBAR design, Molybdenum works better than aluminum as an electrode material because, while it has a similar acoustic velocity, resulting in similar mass loading, it has a much higher acoustic impedance. A simple comparison between the two materials was run and found that changing electrode materials from aluminum to molybdenum grants a significant improvement to the resulting \( k_t^2 \). The change resulted in \( k_t^2 \) changing from 5.54% to 6.3%, even though the setup simulated with molybdenum would not be ideal. If idealized the improvement could be even greater, however there was one caveat to this. At lower thicknesses, which FBAR results suggest would be closer to ideal for molybdenum, significant spurious modes are introduced along the resonance peak. These spurious modes do not disperse from the resonance peak unless the thickness is increased (Figure 4.10).

![Admittance curves for XMR with molybdenum electrodes](image)

(a) Figure 4.10a shows the admittance curve for the XMR with molybdenum electrodes of 0.15\( \mu \)m. All throughout the resonance peak and trough, there are large spurious modes that are not easily removed without increasing the thickness. Figure 4.10b shows the admittance curve for the XMR with molybdenum electrode 0.23\( \mu \)m thick. With the exception of the different material, this design is exactly the same as the XMR with electrodes on both sides discussed earlier. At this thickness, the spurious modes are gone.
4.5 Top Electrode Only Contour Mode Versus Conventional Contour Mode

In this analysis, both design styles were tested and compared. Many of the previous conclusions in reference to XMRs based on the TEO-CMR design hold true for the standard CMR with electrodes on the top and bottom. The two most significant differences are very well known. The TEO-CMR design results in a lower fabrication complexity but also results in a much lower $k_t^2$. This holds true when comparing the two during combined mode resonance. With the same pitch of 1.6 µm, the TEO-CMR style achieved a $k_t^2$ of 2.81%, while the more standard design style achieved a $k_t^2$ of 5.54%. This is largely due to the better coupling that occurs for both modes when electrodes are on the top and bottom. There is also more mass loading due to having twice as much metal on the structure. The benefits to $k_t^2$ of having both electrodes far outweigh the damage caused to it by their extra mass. As a result, the main side effect of having the extra metal is primarily related to FBAR design. The extra electrodes in the path, means the path is longer, so to compensate the thickness of the AlN must be reduced.

One of the more interesting differences between the standard CMR and the TEO-CMR design style is from the reduction constant. As discussed previously, the $d_{31}$ reduction is twice that of the $d_{33}$ reduction and relatively equivalent to Poisson’s ratio. However this is only true for the TEO-CMR design style. Comparing the model and results for the more standard design reveals this reduction constant is about equal for both modes, and approximately half of Poisson’s ratio. As a result, the resonance frequencies that would be designed for each individual mode to achieve the combined mode are different depending on the base design being used.
Discussed previously, a loss factor was added to the mason model to compensate for inaccuracies. Although reduced, this loss factor is still necessary when modeling either mode using the standard CMR design. Theoretically, this loss factor should not be needed, as the bottom electrode would make the electric fields across the AlN primary vertical and far closer to the how they are assumed to behave in the model. The continued need for this loss factor therefore implies that there are further sources of loss not already accounted for in the model itself.

4.6 Spurious Modes

A significant impediment to the design and implementation of XMRs is the existence of spurious modes. Many more potentially viable XMR designs were tested than were discussed previously due to spurious modes. Fortunately for design purposes, they are a few methods to avoid or suppress these modes when they would otherwise interfere with the resonance peak of an XMR.

While a concrete relationship has not been established, one factor seems to play a significant role in the existence of interfering spurious modes, the number of fingers. Whether the number of fingers is even or odd can play a significant role in spurious mode location. There were three times as many designs with even numbers of fingers that could not be adequately simulated due to the existence of a spurious mode in the resonant area than designs with odd numbers of fingers.

In dealing with spurious modes, relatively minor changes can result in shifts just large enough to cause a spurious mode to no longer interfere with the measurement of a resonance peak. Therefore it is important to know how much a design can change before other important variables change significantly. In that respect, it is best not to change
electrode materials, however changes in electrode width or thickness can shift the resonant frequency away from a spurious mode, with little damage done to $kt^2$, provided that the changes made are relatively small. As shown in Figures 3.7 and 3.8, there is an area of relative stability in $kt^2$, where minor changes will damage it, but not significantly. This region also happens to be the region that experiences the fastest change in resonant frequency versus thickness, as shown in Figure 3.19. The resulting effects can be enough to remove spurious modes from the resonance area.

The thickness of the piezoelectric material can also be changed to avoid a spurious mode. The effect of a change in thickness on a spurious depends largely on the source of the spurious mode. In some cases, a spurious mode is dependent on the thickness to a similar extent as the XMR, and therefore will not shift out of the resonance area before the $kt^2$ has decreased significantly as a result of the changes. This was very common in the failed designs with an even number of fingers, and prevented any accurate measurements anywhere near the ideal thickness for the combined mode. In other cases, the spurious mode is relatively independent of thickness, unlike the XMR, so minor shifts in the resonant frequency by changing the thickness can alleviate the problem. In Figure 3.22, in a manner similar to the electrodes, the peak is relatively flat near the ideal thickness, so minor changes in thickness produce relatively little change in $kt^2$.

5.0 Conclusions

5.1 Summary

In this thesis, the behavior and underlying variables defining the combined mode of resonance were closely examined. First, the assumption that the two modes that were
combining were the $d_{31}$ and $d_{33}$ modes was closely examined. While it cannot be proven beyond a shadow of a doubt, there is now a preponderance of evidence showing that the two modes are indeed the $d_{33}$ and $d_{31}$ modes. When either mode is shut off, the $kt^2$ decreases as a result, and the curves of $kt^2$ and resonant frequency show significant deviation from the expected when the $d_{33}$ mode is modeled to cross near the $d_{31}$ mode. One factor complicating efforts was the lack of an independent $d_{33}$ mode, which was often combined with some other mode, making it impossible to tightly define and track its resonance peak as the thickness was varied. This issue may be less of a factor in designs with top and bottom electrodes, as having both allows for better coupling.

The analysis done in this thesis was also utilized to develop design principles for XMRs. While equations 3 and 5 can be used to approximately calculate the resonant or anti-resonant frequency for a certain design, equation 5 only finds the anti-resonant frequency and accounting for the electrodes in the acoustic path is difficult. Therefore mason models were utilized and altered to more closely match simulation results. The benefit of modeling was it enabled the prediction of the modes independent of each other, with a decent level of accuracy. While these models do not match the level of precision FEM simulation offers, they are also much faster.

A side effect of the combined mode is a downward frequency shift. This frequency shift can be treated as an effective reduction in the stiffness constant. The amount of reduction was approximated for two different design styles by comparing the independent model results to simulation results. The amount of reduction to the stiffness constant was determined to vary depending on design style and which mode of resonance was being considered.
The electrodes were analyzed as an independent variable because, while electrodes have some effect on standard TEO-CMR resonator behavior due to mass loading effects, they are much more significant in FBAR design as a result of being in the acoustic path. As such, the ideal variables for electrodes tended to match results from previous FBAR results, albeit with a few wrinkles. The ideal thickness of an electrode matches behavior shown for FBARs, where a low thickness is ideal, but the smallest thickness possible results in a more negative impact. Similarly, the electrode material, which plays a large role in FBAR design, also can play a significant role in XMR design. Simply changing electrode materials from aluminum to molybdenum, which has a similar acoustic velocity to aluminum, results a $k t^2$ that is more than 1 percentage point higher. This is despite the fact that this change makes the resonator in a non-ideal setup; when idealized the improvement is even greater. The final variable, electrode width, is typically only important in defining the input resistance for an FBAR, and for TEO-CMR design is typically a very high percentage of the pitch. However this analysis found that the ideal electrode width was actually around 50% of the pitch.

Finally, interfering spurious modes, a big potential issue with XMRs and resonators in general, were analyzed and dealt with. In many cases, shifting the thickness or width of a material a little is enough to remove the interference of a spurious mode, as they typically would not shift along with the resonance frequency. However, it was common to find interfering spurious modes that were significantly harder to remove on designs with an even number of fingers. As a result, it would be recommended to generally utilize an odd number of fingers to avoid this issue.
The XMR is a recently discovered resonance mode that provides a significant advantage over other modes with a larger $kt^2$. While a single chip running with combined mode cannot also run at multiple frequencies, the XMR can be used to provide a higher FOM for a specific important frequency, while other resonators on the same chip run as standard AlN resonators for other frequencies. Given results found in varying electrode materials, this resonance mode has the potential to be a significant improvement over standard FBARs as well, replacing them in designs that only require one on-chip frequency. Many of the methodologies for manipulating the underlying modes also apply to this combined mode, rendering it very open to specific customization.

5.2 Future Work

There were several interesting facets of this thesis that deserve or require a more thorough look. The combination of the $d_{33}$ mode and M1 mode that occurs at higher thicknesses could use further examination, given the potential for a different style of combined mode. Similarly, the combination between what is presumed to be the $d_{31}$ mode and the second harmonic of the $d_{33}$ mode needs a closer examination. First it should be determined as best as possible what two modes are combining. Second, the odd x displacement pattern that occurred, in which the two halves on the resonator were vibrating in opposing directions, needs further examination on how and why it occurred. It may also be worthwhile to examine why spurious modes near the combined mode experience a significant decrease in strength as the ideal thickness is approached.

The mason model, which could be an important tool for designing XMRs in the future, needs improvement. The source of the coupling loss factor needs to be accounted
for in the equation without resorting to approximation for the model to be fully viable. Currently it is relatively accurate at predicting the $d_{31}$ resonance frequency but less so for the $k t^2$ that goes with it and the anti-resonance frequency for the $d_{33}$ and its $k t^2$. To aid with modeling for the XMR in the future, a more precise definition and explanation for the stiffness reduction factor is needed, specifically an explanation for the variation in it amongst resonance modes and design styles. The theory proposed here relates it to the change in thickness that occurs when changing between design styles, but this issue needs a more in depth explanation. Finally, the issue briefly mentioned concerning molybdenum, with the extra spurious modes that result, deserves additional examination. While the spurious modes can be avoided with thicker amounts of molybdenum, FBAR testing data suggests that slightly thinner electrodes would allow for an even greater improvement. Future work would look into methods to incorporate thinner electrodes while avoiding the damaging spurious modes.
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Figure 6.1 Admittance curve of AlN resonator with thickness of .2µm and pitch of 1.6µm.
**Figure 6.2** Admittance curve of AlN resonator with thickness of .4µm and pitch of 1.6µm.

![Graph of admittance curve for AlN resonator with thickness of .4µm and pitch of 1.6µm.]

**Figure 6.3** Admittance curve of AlN resonator with thickness of .6µm and pitch of 1.6µm.

![Graph of admittance curve for AlN resonator with thickness of .6µm and pitch of 1.6µm.]

**Figure 6.4** Admittance curve of AlN resonator with thickness of .8µm and pitch of 1.6µm.

![Graph of admittance curve for AlN resonator with thickness of .8µm and pitch of 1.6µm.]
Figure 6.5 Admittance curve of AlN resonator with thickness of 1.1µm and pitch of 1.6µm.

Figure 6.6 Admittance curve of AlN resonator with thickness of 1.5µm and pitch of 1.6µm.
Figure 6.7 Admittance curve of AlN resonator with thickness of 1.7µm and pitch of 1.6µm.

Figure 6.8 Admittance curve of AlN resonator with thickness of 1.8µm and pitch of 1.6µm.
**Figure 6.9** Admittance curve of AlN resonator with thickness of 2.4µm and pitch of 1.6µm.

**Figure 6.10** Admittance curve of AlN resonator with thickness of 2.6µm and pitch of 1.6µm.
Figure 6.11 Admittance curve of AlN resonator with thickness of 2.8\(\mu\)m and pitch of 1.6\(\mu\)m.

Figure 6.12 Admittance curve of AlN resonator with thickness of 4.0\(\mu\)m and pitch of 1.6\(\mu\)m.