Supergravity Unification, Dark Matter and LHC Signatures Post Higgs Boson Discovery

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Abstract

In this work, we analyze the landscape of sparticle mass hierarchies generated from supergravity unified models consistent with the Higgs boson mass measurement, flavor constraints and cosmological data. Both universal and nonuniversal supergravity models are analyzed, including the mSUGRA model with universal boundary conditions, and nuSUGRA models with nonuniversalities in the SU(2)_L gaugino mass sector, the SU(3)_C gaugino mass sector, the Higgs boson mass sector, and the third generation sfermion mass sector. The relation of hierarchical models to simplified models is discussed, which opens the way to connect simplified models to a UV-complete theory. Supergravity model signatures are explored and favorable benchmark models are analyzed for discovery at the LHC Run-II. Further, implications of hierarchical models for the direct detection of dark matter are discussed through spin-independent neutralino-proton cross sections and LSP neutralino mass. The analysis establishes a complementarity between LHC data and dark matter detectors in the identification of the nature of symmetry breaking in high scale models. Consequently, this work serves as a tool to help delineate the nature of high scale boundary conditions for the underlying supergravity grand unification model.
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Introduction

The standard model of elementary particle physics has successfully described a vast amount of phenomena concerning the electroweak and strong interactions. The standard model is based on the gauge group product $SU(3)_C \times SU(2)_L \times U(1)_Y$, where $SU(3)_C$ is the strong color gauge group, $SU(2)_L$ is the left-chiral weak gauge group, and $U(1)_Y$ is the weak hypercharge gauge group. The SM requires that the electroweak symmetry is spontaneously broken into the electromagnetic gauge group, $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$, via the Higgs mechanism. Namely, an $SU(2)_L$ Higgs boson doublet acquires a nonzero vacuum expectation value and interacts with the electroweak gauge group fields, mixing into the mass eigenstates that compose the massive electroweak gauge bosons $W^\pm$ and $Z$, as well as leaving the photon massless [1–12]. The discovery in 2012 [13,14] of the Higgs boson [15–17] marked the final fundamental particle to be discovered by the theoretical predictions of the SM, completing the proposed particle spectrum.

Although the SM has been highly successful in explaining a large amount of experimental data, the model fails to explain many known phenomena such as the existence of dark matter and the excess of matter over anti-matter in the universe. Thus, the SM can explain only about 5% of the matter in the universe which constitutes the visible parts of the galaxies and intergalactic gas. The remaining mass of the universe, which arises from cold dark matter ($\sim 25\%$) and dark energy ($\sim 70\%$), remains unexplained. Additionally, the standard model appears theoretically incomplete. This is due to the large number of arbitrary parameters, such as the values of quark and lepton masses, gauge coupling constraints, Yukawa couplings, and $CP$-violating parameters which must be inserted by hand. Further, there exists the hierarchy problem in that the Higgs boson can receive a large correction to the mass at the quantum level, which requires a fine tuning of $1 \text{ part in } 10^{28}$ in order to evolve its mass to the $\sim 100 \text{ GeV}$ scale. Finally, gravity does not enter into the unification scheme of the SM while it is one of the central interactions of nature.

A promising framework for physics beyond the standard model is supersymmetry and one of the leading candidate theories is the supergravity grand unified model. Supergravity unified models exhibit 31 extra particles beyond those that appear in the standard model. One of the interesting predictions of supergravity unified models [18–21] is that the lightest $CP$-even Higgs boson should have a mass which lies below $130 \text{ GeV}$ [22–29]. It is
noteworthy that the observed Higgs boson mass [13,14] lies below but close to the upper limit on the Higgs boson mass predicted by supergravity grand unified models [18–21].

The focus of this work is to explore physics beyond the standard model based on supergravity unified models as well as the implications of the Higgs boson discovery and the measurement of its mass at \( \sim 126 \text{ GeV} \) for the discovery of supersymmetry in Run-II of the Large Hadron Collider (LHC) and in dark matter searches. One of the aims of the investigation is to correlate the pattern of the sparticle spectrum that will be observed at the LHC with the nature of symmetry breaking that generates sparticle masses. Significant information can be gained by the study of sparticle mass hierarchies. In this work, we analyze a variety of boundary conditions at the grand unification scale to determine the type of sparticle mass hierarchies which can be generated. The hierarchical patterns are severely constrained by electroweak symmetry breaking as well as by the astrophysical and particle physics data. They are further constrained by the Higgs boson mass measurement. The sparticle mass hierarchies can be used to generate simplified models. Namely, the mass hierarchies and their truncated versions significantly enlarge the list of simplified models currently being used in the analysis of LHC data. Regarding this analysis, a comparison of simplified models and high scale models is explored. We also analyze spin-independent neutralino-proton cross sections exhibiting the Higgs boson mass dependence and their sensitivity to the hierarchical patterns. Specifically, we investigate the complementarity of dark matter detectors and the LHC in the identification of the nature of symmetry breaking in high scale models.

The organization of this paper is as follows: Chapter 1 discusses the standard model. Chapter 2 discusses supersymmetry, which is one of the best candidates for physics beyond the standard model and produces phenomenologically viable results which can be experimentally tested. Chapter 3 discusses supergravity as a local theory of supersymmetry, naturally giving rise to gravity. In Chapter 4, we use the predictive powers of supergravity unified models to discuss sparticle mass hierarchies for both universal and nonuniversal boundary conditions. In Chapter 5, we investigate the classes of simplified models that can arise from truncation of sparticle mass hierarchies generated from high scale models and compare them to simplified models which are not manifestly UV-complete. In Chapter 6, we discuss supergravity model signatures for the LHC Run-II and how they will be strongly correlated with the type of mass hierarchical pattern considered. In Chapter 7, we give benchmark models which pass a variety of collider, flavor,
and cosmological constraints, which serve as illustrative examples of signature analyses that can lead to new discovery channels at the LHC. Further, separate sections in this chapter focus on a detailed analysis of the discovery potential for both a universal and nonuniversal supergravity benchmark model. In Chapter 8, we explore dark matter and analyze spin-independent neutralino-proton cross sections as a function of dark matter mass for supergravity models generated from an assortment of high scale boundary conditions. The dark matter cross sections and mass profiles exhibit both Higgs boson mass dependence as well as their sensitivity to the hierarchical mass patterns. Separate sections within this chapter focus on explaining the extreme smallness of certain dark matter cross sections as well as discussing the LSP content (bino, wino, Higgsino fractions) for various supergravity models. We explore the role dark matter content plays in identifying the sparticle mass hierarchies and consequently the high energy boundary conditions. Conclusions are given in Chapter 9.
Chapter 1

The Standard Model

The standard model (SM) describes the electroweak and strong interactions of elementary particles and is based on the gauge group

$$SU(3)_C \times SU(2)_L \times U(1)_Y,$$  

(1.1)

where $SU(3)_C$ is the color gauge group which describes the strong interactions among quarks and gluons, and $SU(2)_L \times U(1)_Y$ describes the electroweak interactions for quarks, leptons and electroweak gauge bosons $W^\pm, Z$ and $\gamma$ [1–12]. Here $SU(2)_L$ is the gauge group which acts on left-handed chiral quark and lepton multiplets and $U(1)_Y$ is the hypercharge gauge group. The quarks consist of left-handed $SU(2)_L$ doublets and right-handed singlets

$$Q_{iL} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, \quad u_{iR}, \quad d_{iR}. \quad (1.2)$$

Likewise, the leptons consist of left-handed $SU(2)_L$ doublets and right-handed singlets

$$L_{iL} = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L, \quad e_{iR}. \quad (1.3)$$

There does not exist a right-handed neutrino observed in nature.
where $i = 1, 2, 3$ denotes the generation or family. In addition, there is a scalar Higgs boson doublet which will be discussed later. In brief, the standard model particles consist of the quarks, leptons, gauge and Higgs bosons, as detailed in Table 1.1. The quarks and leptons are spin-$\frac{1}{2}$ fermions, the gauge bosons are spin-1 vector bosons, and the only scalar particle in the SM is the spin-0 Higgs boson.

The electroweak interactions do not treat left and right-handed chiral components of a field equally. In general, a quantum field $\psi$, which satisfies the Dirac equation, can be decomposed into left and right-handed chiral components

$$\psi = \psi_L + \psi_R , \quad (1.4)$$

where each component can be expressed as

$$\psi_L = P_L \psi , \quad \psi_R = P_R \psi . \quad (1.5)$$

Thus, the chirality of the field is defined in terms of the projection operator

$$P_{L,R} = \frac{1}{2}(1 \pm \gamma_5) , \quad (1.6)$$

where $\gamma_5$ is given as\(^2\)

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 . \quad (1.8)$$

### 1.1 Gauge Invariance

The standard model includes both abelian and nonabelian gauge fields. Quantum electrodynamics (QED) is based on a local U(1) symmetry, which explains the interaction between a charged spin-$\frac{1}{2}$ field $\psi$, and a spin-1 gauge field $A_\mu$. The QED Lagrangian for

\(^2\) The gamma matrices satisfy the Clifford algebra $C\ell_{1,3}(\mathbb{R})$ with anticommutation relations

$$\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu} , \quad (1.7)$$

where $\eta^{\mu\nu}$ is the Minkowski spacetime metric, with chosen signature \{-, +, +, +\}.  

5
Table 1.1: An overview of the particle content of the SM. The last column shows the dimension of the gauge group representations of SU(3)_C and SU(2)_L, followed by the U(1)_Y hypercharge generator \( T_Y = Y/2 \), as shown in Equation (1.32).

<table>
<thead>
<tr>
<th>Name</th>
<th>Spin</th>
<th>Details</th>
<th>Symbol</th>
<th>SU(3)_C \times SU(2)_L \times U(1)_Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarks</td>
<td>( \frac{1}{2} )</td>
<td>L-handed quarks</td>
<td>( (u_i, d_i)<em>L = Q</em>{iL} )</td>
<td>( (3, 2, \frac{1}{6}) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R-handed u-type quarks</td>
<td>( u_{iR} )</td>
<td>( (3, 1, \frac{2}{3}) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R-handed d-type quarks</td>
<td>( d_{iR} )</td>
<td>( (3, 1, -\frac{1}{3}) )</td>
</tr>
<tr>
<td>Leptons</td>
<td>( \frac{1}{2} )</td>
<td>L-handed leptons</td>
<td>( (\nu_{e_i}, e_i)<em>L = L</em>{iL} )</td>
<td>( (1, 2, -\frac{1}{2}) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R-handed charged leptons</td>
<td>( e_{iR} )</td>
<td>( (1, 1, -1) )</td>
</tr>
<tr>
<td>Gauge Bosons</td>
<td>1</td>
<td>Gluons</td>
<td>( g )</td>
<td>( (8, 1, 0) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>W and Z bosons</td>
<td>( W^\pm, Z )</td>
<td>( (1, 3, 0) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Photon</td>
<td>( \gamma )</td>
<td>( (1, 1, 0) )</td>
</tr>
<tr>
<td>Scalars</td>
<td>0</td>
<td>Higgs boson</td>
<td>( (\phi^+, \phi^0) = H )</td>
<td>( (1, 2, \frac{1}{2}) )</td>
</tr>
</tbody>
</table>

A Dirac fermion \( \psi \), with mass \( m \), coupled to a massless gauge boson \( A_\mu \) is

\[
\mathcal{L}_{\text{QED}} = \bar{\psi} (i \slashed{D} - m) \psi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu}, \tag{1.9}
\]

where \( \bar{\psi} = \psi^\dagger \gamma^0 \) and \( \slashed{D} = \gamma^\mu D_\mu \). Equation (1.9) can be decomposed into three terms,

\[
\mathcal{L}_{\text{QED}} = \mathcal{L}_{\psi}^{(0)} + \mathcal{L}_A^{(0)} + \mathcal{L}_{\text{int}} \tag{1.10}
= \bar{\psi} (i \slashed{D} - m) \psi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + g_1 \bar{\psi} \gamma^\mu \psi A_\mu , \tag{1.11}
\]

where \( \mathcal{L}_{\psi}^{(0)} \) is the free\(^3 \) Dirac Lagrangian, \( \mathcal{L}_A^{(0)} \) is the free Maxwell Lagrangian, and \( \mathcal{L}_{\text{int}} \) is the interaction Lagrangian between the fields. For a local U(1) symmetry with a coupling constant \( g_1 \), the fields transform as

\[
\psi \to e^{i \lambda(x)} \psi , \tag{1.12}
\]

\[
A_\mu \to A_\mu + \frac{1}{g_1} \partial_\mu \lambda(x) , \tag{1.13}
\]

\(^{3}\)Note that a free Lagrangian will be denoted by \( \mathcal{L}^{(0)} \), while a general Lagrangian is simply \( \mathcal{L} \), which is not limited to non-interactions.
where $\lambda(x)$ is the spacetime-dependent transformation parameter. Equation (1.12) is the result of the unitary group action, whereas Equation (1.13) is chosen such that the gauge covariant derivative

$$D_\mu = \partial_\mu - ig_1 A_\mu ,$$

(1.14)

satisfies the commutator

$$[D_\mu, \delta U(1)] = 0 ,$$

(1.15)

where $\delta U(1)$ is the infinitesimal deviation$^4$ of the symmetry transformation. The gauge covariant derivative gives rise to the kinetic terms for the vector field $A_\mu$ as well as the interactions between the fields. Notice that the commutator of two covariant derivatives gives rise to the electromagnetic field tensor $F_{\mu\nu}$,

$$[D_\mu, D_\nu] = -ig_1 (\partial_\mu A_\nu - \partial_\nu A_\mu)$$

(1.16)

$$= -ig_1 F_{\mu\nu} .$$

(1.17)

For nonabelian gauge theories, such as quantum chromodynamics (QCD), we simply generalize the approach given above. The gauge group being $SU(n)$, we introduce the Yang-Mill’s Lagrangian,

$$\mathcal{L}_{YM} = \bar{\psi} (i \not{D} - m) \psi - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} ,$$

(1.18)

where

$$D_\mu = \partial_\mu - ig_n T^a A^a_\mu ,$$

(1.19)

and

$$F^a_{\mu\nu} = F_{\mu\nu}^{a(1)} + F_{\mu\nu}^{a(2)}$$

(1.20)

$$= (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) + g_n f^{abc} A^b_\mu A^c_\nu .$$

$^4$ Note that the infinitesimal $U(1)$ transformation of $\psi$ is given by $\psi \rightarrow \psi + \delta \psi$, where $\delta \psi = i\lambda \psi$. 

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Under this symmetry group action, the fields transform infinitesimally as

$$\psi_\alpha \to \psi_\alpha + i\lambda^a(T^a)_{\alpha\beta}\psi_\beta ,$$  \hspace{1cm} (1.21)

$$A^a_\mu \to A^a_\mu + \frac{1}{g_n}\partial_\mu \lambda^a - f^{abc}\lambda^b A^c_\mu ,$$  \hspace{1cm} (1.22)

where $T^a$ are the generators of SU($n$), $f^{abc}$ are the structure constants of the symmetry group, $A^a_\mu$ are the gauge fields, and $g_n$ is the coupling constant. Note that although $\mu, \nu$ are Lorentz indices and require the Minkowski metric to be raised or lowered, the indices $a, b$ are Euclidean in nature, so raising and lowering is trivial. Thus, summation over these indices does not require one upper and one lower index.

### 1.2 The Higgs Mechanism

Mass generation in the SM occurs through spontaneous breaking of the SU(2)$_L \times$ U(1)$_Y$ gauge symmetry via the Higgs mechanism [15–17] where an SU(2)$_L$ Higgs boson doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} ,$$  \hspace{1cm} (1.23)

acquires a nonzero vacuum expectation value (VEV)

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} ,$$  \hspace{1cm} (1.24)

where $v \neq 0$. Namely, the VEV is the first order expansion of $\phi$, such that

$$\phi = \langle \phi \rangle + h ,$$  \hspace{1cm} (1.25)

and $h$ is the quantum field. For a complex scalar field $\phi$, the most general Lagrangian which is both gauge invariant and renormalizable is

$$L_\phi = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi) ,$$  \hspace{1cm} (1.26)
where the Higgs potential is of the form

\[ V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2. \]  

(1.27)

Spontaneous symmetry breaking implies that the parameters are constrained as follows,

\[ \mu^2 > 0, \quad \lambda > 0. \]  

(1.28)

The \( \mu \) constraint ensures that the minimum of the potential is nonzero, while the \( \lambda \) constraint keeps \( V(\phi) \) bounded from below. Note that the gauge-covariant derivative in Equation (1.26) can be expressed in its most general form\(^5\) as

\[ D_\mu = \partial_\mu - i \left( g_3 G^a T_C^a + g_2 W^i T_L^i + g_1 B_\mu T_Y \right), \]  

(1.29)

where the generators\(^6\) of each respective gauge group are

\[ T_C^a = \frac{\lambda^a}{2}, \quad \text{SU}(3)_C \implies a = 1-8 \]  

(1.30)

\[ T_L^i = \frac{\tau^i}{2}, \quad \text{SU}(2)_L \implies i = 1-3 \]  

(1.31)

\[ T_Y = \frac{Y}{2}, \quad \text{U}(1)_Y. \]  

(1.32)

In Equations (1.30)-(1.32), we identify \( \lambda^a \) as the Gell-Mann matrices, \( \tau^i \) as the Pauli matrices, and \( Y \) as the hypercharge. The electric charge, hypercharge, and the third component of isospin are related via the Gell-Mann-Nishijima relation,

\[ Q = T_L^3 + \frac{Y}{2}. \]  

(1.33)

\(^5\)The gauge-covariant derivative appearing in Equation (1.26) is not acting on fields with color, which implies that the strong terms will vanish.

\(^6\)The generators of each nonabelian gauge group satisfy the commutation relations \([T^a, T^b] = i f^{abc} T^c\), where \( f^{abc} \) are the structure constants for the particular Lie group and its associated Lie algebra.
The gauge boson masses are generated due to the first term in Equation (1.26). Expanding out the kinetic term at $\phi = \langle \phi \rangle$, we find that

$$\left. (D^\mu \phi)^\dagger (D_\mu \phi) \right|_{\phi = \langle \phi \rangle} = \frac{v^2}{8} \left[ g_2^2 \left( |W_\mu^1|^2 + |W_\mu^2|^2 \right) + \left( g_1 W_\mu^3 - g_2 B_\mu \right)^2 \right] ,$$

(1.34)

which can be rewritten as

$$\frac{1}{2} V_\mu M^2 V^\mu ,$$

(1.35)

where we define, $V_\mu \equiv (W_\mu^1, W_\mu^2, W_\mu^3, B_\mu)$. Consequently, the squared mass matrix is

$$M^2 = \left( \frac{v}{2} \right)^2 \begin{pmatrix} g_2^2 & 0 & 0 & 0 \\ 0 & g_2^2 & 0 & 0 \\ 0 & 0 & g_2^2 & -g_1 g_2 \\ 0 & 0 & -g_1 g_2 & g_1^2 \end{pmatrix} .$$

(1.36)

By diagonalizing $M^2$, we obtain the mass eigenstates,

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left( W_\mu^1 \pm i W_\mu^2 \right) ,$$

(1.37)

$$Z_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_2 W_\mu^3 - g_1 B_\mu) ,$$

(1.38)

$$W_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_1 W_\mu^3 + g_2 B_\mu) ,$$

(1.39)

with mass eigenvalues,

$$m_W = \frac{v}{2} g_2 ,$$

(1.40)

$$m_Z = \frac{v}{2} \sqrt{g_1^2 + g_2^2} ,$$

(1.41)

$$m_A = 0 .$$

(1.42)

In this way, the Higgs mechanism gives mass to the $W^\pm$ and $Z$ bosons by absorbing the spontaneously broken components of the Higgs fields, while the photon remains massless, and we are left with a residual neutral Higgs field. Now, if we expand the gauge-covariant
derivative using the mass eigenstate fields, we note that the coefficient of the photon coupling is what we define as the electric charge,
\[ e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}. \quad (1.43) \]
We introduce the weak mixing angle, \( \theta_w \), as the rotation angle which transforms between the gauge fields \( \{ W_\mu^3, B_\mu \} \) and the physical mass eigenstates \( \{ Z_\mu, A_\mu \} \),
\[
\begin{pmatrix}
Z_\mu \\
A_\mu
\end{pmatrix} =
\begin{pmatrix}
\cos \theta_w & -\sin \theta_w \\
\sin \theta_w & \cos \theta_w
\end{pmatrix}
\begin{pmatrix}
W_\mu^3 \\
B_\mu
\end{pmatrix},
\quad (1.44)
\]
where we define
\[
\cos \theta_w = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}, \quad \sin \theta_w = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}. \quad (1.45)
\]
Thus, the SU(2)_L \times U(1)_Y gauge couplings depend on the electric charge and the weak mixing angle,
\[ g = \frac{e}{\sin \theta_w}. \quad (1.46) \]
Additionally, the Higgs mechanism also gives mass to the quarks and leptons via the Yukawa interactions. Let us define
\[
\mathcal{L}_{\text{Yukawa}} = y_{ij}^{(u)} \bar{Q}_L \phi u_{jR} + y_{ij}^{(d)} \bar{Q}_L \phi d_{jR} + y_{ij}^{(e)} \bar{L}_L \phi e_{jR} + \text{h.c.}, \quad (1.47)
\]
with an implicit sum over the generation indices \( i \) and \( j \). Here, we introduce the Yukawa couplings \( y_{ij} \) between one Higgs doublet and two fermions, in the form \( \bar{\psi}_L \phi \psi_R \). To maintain zero hypercharge for these terms in the Lagrangian, we define \( \tilde{\phi} = \epsilon_{ij} \phi_j^* \) such that the hypercharge satisfies \( Y(\tilde{\phi}) = -Y(\phi) \).

The residual neutral Higgs boson field is an important signature for the validity of the standard model. Thus, its recent discovery at the Large Hadron Collider [13, 14] is a true milestone as the Higgs boson was the last missing piece of the particle spectrum predicted by the standard model. The particle content for the SM is summarized in Table 1.1.
SM has been highly successful in correlating a vast amount of data that has come forth from LEP, the Tevatron, and the first run of the LHC.

Although the SM has repeatedly identified itself as a successful theory in describing nature to a great extent, the theory unfortunately falls short since it cannot correctly predict all known phenomena, nor can it answer many of the unsolved questions in physics. A few problems concerning the SM include the following: gravity cannot be described by the SM since its inclusion leads to a non-renormalizable theory, the tree-level masses and coupling constants are not explained, the Higgs boson mass is not protected from high-scale corrections which leads to the so-called hierarchy problem, and the SM can only explain less than 5% of the universe since dark matter and dark energy have yet to rest firmly on an accepted theoretical framework.
Chapter 2

Supersymmetry

In order to reconcile many of the unsolved problems of the SM, we turn to theories beyond the Standard Model. One of the best known candidates is supersymmetry (SUSY) [31–36].

2.1 Motivation for Additional Symmetries

Before we introduce SUSY, recall that there exist many symmetries in the SM which then give rise to conserved symmetry charges. A $U(1)$ symmetry generates the familiar electromagnetic charge,

$$ Q = e \int d^3x \, \psi^\dagger \psi, $$

where $\psi$ is a spinor. These charges are Lorentz scalars as noted by the fact that they do not carry any uncontracted Lorentz indices.

For symmetry charges with Lorentz indices, we turn to spacetime symmetries, which are symmetries of the Poincaré group, $\text{ISO}(1, 3) \simeq \text{SO}(1, 3) \times \mathbb{R}^{(1,3)}$. Recall that global
Poincaré transformations are of the form

\[ x^\mu \rightarrow \Lambda^\mu_\nu x^\nu + a^\mu, \tag{2.3} \]

where \( \Lambda^\mu_\nu \in \text{SO}(1,3) \) is a spacetime rotation and \( a^\mu \in \mathbb{R}^{(1,3)} \) is a spacetime translation. There exist Lorentz vector charges, known as momentum operators, \( P_\mu \), which are the generators of spacetime translations. Also, there exist antisymmetric tensor charges, \( M_{\mu\nu} \), which are the generators of spacetime rotations. The Coleman-Mandula theorem \([37]\) states that there does not exist a conserved symmetric tensor charge, \( Q_{\mu\nu} \), that would allow nontrivial scattering. That is, the symmetry operators of spacetime translations and rotations do not allow for any additional conservation laws generated from operators which have uncontracted Lorentz indices, since this would overconstrain the system. However, conserved operators which transform under Lorentz transformation as spinors are allowed. Hence, we introduce the Weyl spinor charges \( Q_\alpha \) and \( Q_\dagger_\dot{\alpha} \), which serve as the fermionic generators of a new symmetry called supersymmetry, where \( \alpha = 1, 2 \) and \( \dot{\alpha} = 1, 2 \) are distinct spinor indices.

SUSY is a global spacetime symmetry which promotes the Poincaré group to the super-Poincaré group, endowed with a \( \mathbb{Z}_2 \)-graded supersymmetry algebra. In this algebra, the even component is comprised of the bosonic Poincaré algebra and the odd component of a new fermionic algebra, with generators \( Q_\alpha \) and \( Q_\dagger_\dot{\alpha} \). The fermionic (spinor) algebra has anticommutation relations which relate the even and odd components of the superalgebra, such that

\[
\{ Q_\alpha, Q_\dagger_\dot{\beta} \} = -2 (\sigma^\mu)_{\alpha\dot{\beta}} P_\mu, \tag{2.4}
\]

\[
\{ Q_\alpha, Q_\beta \} = \{ Q_\alpha, Q_\dagger_\dot{\alpha} \} = 0. \tag{2.5}
\]

Thus, SUSY is a symmetry between bosons and fermions. This notion extends our spacetime coordinates to superspace coordinates which incorporates both bosonic (commuting) and fermionic (anticommuting) degrees of freedom. SUSY exhibits many important phenomenological implications. All known particles can now be represented as part of a
supermultiplet, pairing each particle with a supersymmetric particle (sparticle) and an auxiliary field. Since the number of bosonic and fermionic degrees of freedom need to be equal within a supermultiplet, then none of the SM particles can be each others superpartners, which predicts the existence of at least twice as many particles as the SM. Further, since $Q_\alpha$ and $Q^\dagger_\dot{\alpha}$ commute with both $P^2$ and the gauge symmetries, particles occupying the same multiplet must also have the same mass and gauge charges. Since we have yet to discover any light sparticles, then SUSY must be broken; however, the mechanism has yet to be understood. Moreover, SUSY solves the hierarchy problem by adding symmetric contributions to the high-scale quantum loop corrections to the Higgs mass, canceling the quadratic divergences. Lastly, with $R$-parity conservation, the lightest supersymmetric particle (LSP) can be a candidate for dark matter, provided that it is charge neutral.

In SUSY, often a two-component spinor representation is more useful than a four-component one, so we will express the Lagrangian using this notation. Firstly, a four-component Dirac spinor is expressed as a two-component Weyl spinor as

$$
\psi_D = \begin{pmatrix} \xi_\alpha \\ \chi^{\dot{\alpha}} \end{pmatrix}, \quad \bar{\psi}_D = \begin{pmatrix} \chi^\alpha \\ \xi_\dot{\alpha} \end{pmatrix},
$$ (2.6)

where $\xi$ and $\chi^\dagger$ are known as left-handed and right-handed Weyl spinors, respectively. Hermitian conjugation transforms between the handedness of an arbitrary spinor. The spinor indices are raised and lowered using the antisymmetric tensor $\epsilon_{ij}$. Suppression of indices are implicitly defined as

$$
\xi \chi = \xi^\alpha \chi_\alpha, \quad \xi^{\dagger} \chi^{\dagger} = \xi^{\dagger}_\dot{\alpha} \chi^{\dagger}_{\dot{\alpha}}.
$$ (2.7)
Thus, the free Dirac Lagrangian for a Dirac fermion $\psi_D$, with mass $m$, can be expressed more naturally with regards to chirality using Weyl fermions,

$$\mathcal{L}^{(0)}_{\psi} = \bar{\psi}_D i\gamma^\mu \partial_\mu \psi_D - m\bar{\psi}_D \psi_D$$

$$= i\xi^\dagger \bar{\sigma}^\mu \partial_\mu \xi + i\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi - m(\xi\chi + \xi^\dagger\chi^\dagger).$$

(2.8) (2.9)

By separating the components in this way, the Weyl representation emphasizes the chirality of nature, as we know that left and right-handed quantities are not treated equally under the weak interactions.

As with the symmetries introduced earlier in this section, the conserved fermionic supersymmetry charges can be expressed as the zero component of an integrated current density,

$$Q_\alpha = \sqrt{2} \int d^3x \, J_\alpha^0,$$

(2.10)

where the conserved supercurrent density$^1$ for a free chiral supermultiplet is of the form

$$J_\alpha^\mu = (\sigma^\nu \bar{\sigma}^\mu \psi)_\alpha \partial_\nu \phi^*.$$

(2.11)

### 2.2 SUSY Lagrangians

The simplest form of supersymmetry is built from two supermultiplets, chiral and gauge. A chiral supermultiplet combines a Weyl fermion $\psi_\alpha$ with a complex scalar $\phi$, whereas a gauge supermultiplet combines a gauge boson $A_\mu^a$ with a Weyl fermion $\lambda_\alpha^a$. Note that $\alpha$ is a spinor index, $\mu$ is a Lorentz index, and $a$ is a gauge group index running over the dimension of the adjoint irreducible representation of the gauge group. For brevity, we have removed the flavor indices from the chiral supermultiplets.

Within each supermultiplet, the two components must have equal degrees of freedom. Further, each supermultiplet must be closed under the SUSY algebra. To remedy both

$^1$ See Equation (2.41) for further details regarding a conserved supercurrent density for the general theory, including both chiral and gauge supermultiplets.
the number of degrees of freedom and ensure off-shell SUSY algebra closure, we introduce a complex scalar auxiliary field $F$, to the chiral supermultiplet and a real scalar auxiliary field $D$, to the gauge supermultiplet. Thus, the chiral supermultiplets are composed of the fields $\{\phi, \psi_\alpha, F\}$ and the gauge supermultiplets are comprised of the fields $\{A_\mu^a, \lambda^a_\alpha, D^a\}$.

### 2.2.1 Chiral Supermultiplets

The SUSY-invariant Lagrangian for a free chiral supermultiplet\(^2\) is of the form

$$L^{(0)}_{\text{chiral}} = L^{(0)}_\phi + L^{(0)}_\psi + L_F = -\partial^\mu \phi^* \partial_\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + F^* F , \quad (2.12)$$

where the SUSY transformations of the fields are

$$\delta \phi = \epsilon \psi , \quad (2.13)$$
$$\delta \psi_\alpha = -i(\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi + \epsilon_\alpha F , \quad (2.14)$$
$$\delta F = -i \epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \psi , \quad (2.15)$$

where $\epsilon^\alpha$ is a small, anticommuting, constant Weyl fermion parameterizing the global SUSY transformation with mass dimension $[\epsilon] = -\frac{1}{2}$. Supersymmetry algebra closure for the fields of a chiral supermultiplet implies that the transformations satisfy

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] X = i(-\epsilon_1 \sigma^\mu \epsilon^\dagger_2 + \epsilon_2 \sigma^\mu \epsilon^\dagger_1) \partial_\mu X , \quad (2.16)$$

for both on and off-shell fields $X \in \{\phi, \phi^*, \psi, \psi^\dagger, F, F^*\}$. Recall that for this relation to hold off-shell, we require the addition of the auxiliary field $F$. Under these transformations, the total chiral action is left invariant,

$$\delta A_{\text{chiral}} = \int d^4x \ [\delta L_\phi + \delta L_\psi + \delta L_F] = 0 . \quad (2.17)$$

\(^2\)The first two terms comprise what is known as the massless, non-interacting Wess-Zumino model [38].
The chiral supermultiplet interactions give rise to a supersymmetric theory which incorporates masses and non-gauge couplings. The most general renormalizable non-gauge interactions for chiral supermultiplets are of the form

$$L_{\text{int}} = \left( -\frac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i \right) + c.c,$$

(2.18)

where $W$ is the superpotential, an analytic function of the scalar fields $\phi_i$, defined as

$$W = L^i \phi_i + \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k .$$

(2.19)

We further define $W^i = \frac{\partial W}{\partial \phi_i}$ and subsequently $W^{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$.

### 2.2.2 Gauge Supermultiplets

The SUSY-invariant Lagrangian for a gauge supermultiplet takes the form

$$L_{\text{gauge}} = L_A + L_\lambda + L_D = -\frac{1}{4} F^{a \mu \nu} F^{a \mu \nu} + i \lambda^{a} \bar{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2} D^a D^a ,$$

(2.20)

where $F^{a \mu \nu}$ is previously defined in Equation (1.20), and the gauge covariant derivative $\nabla_\mu$ acts on the fermion field $\lambda^a$ such that it couples the fermion to the boson field $A^a_\mu$:

$$\nabla_\mu \lambda^a = \partial_\mu \lambda^a + g f^{abc} A^b_\mu \lambda^c .$$

(2.21)

The infinitesimal gauge transformations are

$$A^a_\mu \rightarrow A^a_\mu + \partial_\mu \Lambda^a + g f^{abc} A^b_\mu \Lambda^c ,$$

(2.22)

$$\lambda^a \rightarrow \lambda^a + g f^{abc} \lambda^b \Lambda^c ,$$

(2.23)
whereas, the SUSY transformations of the fields appearing in $\mathcal{L}_{\text{gauge}}$ are

$$
\delta A_{\mu}^a = -\frac{1}{\sqrt{2}}(\epsilon_{\mu}^\dagger \bar{\sigma} \lambda^a + \lambda^{\dagger a} \bar{\sigma} \mu \epsilon) ,
$$

(2.24)

$$
\delta \lambda_{\alpha}^a = \frac{i}{2\sqrt{2}}(\sigma^\mu \bar{\sigma}^\nu)_{\alpha} F_{\mu\nu}^a + \frac{1}{\sqrt{2}} \epsilon_\alpha D^a ,
$$

(2.25)

$$
\delta D^a = \frac{i}{\sqrt{2}}(-\epsilon_{\mu}^\dagger \bar{\sigma} \ni \lambda^a + \nabla_{\mu} \lambda^{\dagger a} \bar{\sigma} \mu \epsilon) .
$$

(2.26)

Supersymmetry algebra closure implies that the transformations satisfy

$$
[\delta_1, \delta_2] X = i(-\epsilon_1 \sigma^\mu \epsilon_2^\dagger + \epsilon_2 \sigma^\mu \epsilon_1^\dagger) \nabla_\mu X ,
$$

(2.27)

for both on and off-shell fields $X \in \{F_{\mu\nu}^a, \lambda^a, \lambda^\dagger a, D^a\}$. Recall that for this relation to hold off-shell, we require the addition of the auxiliary field $D^a$. Similar to the chiral case, under these SUSY transformations, the total gauge action remains invariant,

$$
\delta A_{\text{gauge}} = \int d^4x \left[ \delta \mathcal{L}_A + \delta \mathcal{L}_\lambda + \delta \mathcal{L}_D \right] = 0 .
$$

(2.28)

Notice that we have used the gauge covariant derivative in this section since we have the gauge field in this vector supermultiplet. In order to construct a general Lagrangian for a supersymmetric theory which includes both chiral and gauge supermultiplets, we must let the chiral supermultiplets transform under an irreducible representation of the gauge group. Since SUSY and gauge transformations commute, then each field which constitutes the chiral supermultiplet must also be in the same irreducible representation of the gauge group. Namely, the infinitesimal gauge transformations are

$$
X_i \rightarrow X_i + ig\Lambda^a(T^a X)_i ,
$$

(2.29)
where $X_i \in \{ \phi_i, \psi_i, F_i \}$ and $\Lambda^a$ parametrizes the gauge transformation. Now, since the Lagrangian is gauge invariant, then we make the replacement $\partial_{\mu} \rightarrow \nabla_{\mu}$ such that

\[
\nabla_{\mu} \phi_i = \partial_{\mu} \phi_i - ig A_{\mu}^a (T^a \phi)_i ,
\n\nabla_{\mu} \psi_i = \partial_{\mu} \psi_i - ig A_{\mu}^a (T^a \psi)_i ,
\]

(2.30)

(2.31)

which couples the gauge field $A_{\mu}^a$ to the chiral supermultiplet fields $\phi_i$ and $\psi_i$. In order to couple $A_{\mu}^a$ to $\lambda^a$ and $D^a$, we have three possibilities,

\[
(\phi^* T^a \psi) \lambda^a , \quad \lambda^i (\psi^* T^a \phi) , \quad (\phi^* T^a \phi) D^a .
\]

(2.32)

Once we add these terms to the sum of $L_{\text{chiral}}$ and $L_{\text{gauge}}$ and demand SUSY invariance, we will have to modify the chiral supermultiplet SUSY transformations such that $\partial_{\mu} \rightarrow \nabla_{\mu}$ as well as add a term to $\delta F_i$,

\[
\delta \phi_i = \epsilon \psi_i ,
\]

(2.33)

\[
\delta \psi_{i\alpha} = -i (\sigma^\mu \epsilon^{\dagger})_{\alpha} \nabla_{\mu} \phi_i + \epsilon_{\alpha} F_i ,
\]

(2.34)

\[
\delta F_i = -i \epsilon^{\dagger} \bar{\sigma}^\mu \nabla_{\mu} \psi_i + \sqrt{2} g (T^a \phi)_i \epsilon^{\dagger} \lambda^i ,
\]

(2.35)

In effect, we have constrained the coefficients of the three terms in Equation (2.32). Thus, we have the following Lagrangian,

\[
L = L_{\text{chiral}} + L_{\text{gauge}} - \sqrt{2} g (\phi^* T^a \psi) \lambda^a - \sqrt{2} g \lambda^i (\psi^* T^a \phi) + g (\phi^* T^a \phi) D^a .
\]

(2.36)

Notice that the auxiliary fields enter the Lagrangian with terms of the form

\[
L_F = F^* F , \quad L_D = \frac{1}{2} D^a D^a .
\]

(2.37)
Since these fields do not propagate, they are fixed by their equations of motion. Applying the Euler-Lagrange equation to the auxiliary fields

$$\frac{\partial L}{\partial \chi} - \partial_\mu \frac{\partial L}{\partial (\partial_\mu \chi)} = 0,$$

(2.38)

where $\chi = F$ or $D^a$, we find that

$$F_i = -W_i^*, \quad F^* = -W^* \quad D^a = -g\phi^* T^a \phi .$$

(2.39)

The equations of motion for $D^a$ are nontrivial; however, they would be trivial if the chiral fields were not coupled to the gauge fields. If we plug the expressions for the auxiliary fields terms back into the Lagrangian, we find that

$$V(\phi, \phi^*) = F^* F_i + \frac{1}{2} \sum_a D^a D^a$$

$$= W_i^* W^* + \frac{1}{2} \sum_a g_a^2 (\phi^* T^a \phi)^2 ,$$

(2.40)

where the first term is the $F$-term and the second is the $D$-term. It is important to remember that the Lagrangian contains $V(\phi, \phi^*)$ within it. Thus, the $F$ and $D$-term contributions given in (2.40) show that in SUSY, the scalar potential is completely fixed by the other interactions. Namely, the $F$-term is fixed by the Yukawa couplings and the fermion mass terms, while the $D$-term is fixed by the gauge couplings.

The conserved supercurrent density for a theory which includes chiral and gauge supermultiplets as well as their interactions is

$$J_\mu^\alpha = (\sigma^\nu \bar{\sigma}^\mu \phi)_\alpha \nabla_\nu \phi^* + i(\sigma^\mu \psi^\dagger)_\alpha W^*$$

$$- \frac{1}{2\sqrt{2}} (\sigma^\nu \bar{\sigma}^\mu \sigma^\lambda \lambda^\dagger)_\alpha F^\mu_\nu + \frac{i}{\sqrt{2}} g_a \phi^* T^a \phi (\sigma^\mu \lambda^\dagger)_\alpha .$$

(2.41)
<table>
<thead>
<tr>
<th>Particle Names</th>
<th>Spin-0</th>
<th>Spin-$\frac{1}{2}$</th>
<th>$\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squarks, Quarks</td>
<td>$Q_i$</td>
<td>$(\tilde{u}_i, \tilde{d}<em>i)</em>{L}$</td>
<td>$(u_i, d_i)_{L}$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{u}_i$</td>
<td>$\tilde{u}<em>i^{\dagger}</em>{R}$</td>
<td>$u_{iR}^{\dagger}$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{d}_i$</td>
<td>$\tilde{d}<em>i^{\dagger}</em>{R}$</td>
<td>$d_{iR}^{\dagger}$</td>
</tr>
<tr>
<td>Sleptons, Leptons</td>
<td>$L_i$</td>
<td>$(\tilde{\nu}<em>{e_i}, \tilde{\nu}</em>{i})_{L}$</td>
<td>$(\nu_{e_i}, e_{i})_{L}$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{e}_i$</td>
<td>$\tilde{e}<em>i^{\dagger}</em>{R}$</td>
<td>$e_{iR}^{\dagger}$</td>
</tr>
<tr>
<td>Higgs, Higgsinos</td>
<td>$H_u$</td>
<td>$(H_u^+, H_u^0)$</td>
<td>$(H_u^+, H_u^0)$</td>
</tr>
<tr>
<td></td>
<td>$H_d$</td>
<td>$(H_d^0, H_d^-)$</td>
<td>$(H_d^0, H_d^-)$</td>
</tr>
</tbody>
</table>

Table 2.1: An overview of the chiral supermultiplets of the MSSM. The last column shows the dimension of the gauge group representations of $\text{SU}(3)_C$ and $\text{SU}(2)_L$, followed by the $\text{U}(1)_Y$ hypercharge generator $T_Y = Y/2$, as shown in Equation (1.32).

<table>
<thead>
<tr>
<th>Particle Names</th>
<th>Spin-$\frac{1}{2}$</th>
<th>Spin-1</th>
<th>$\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gluino, Gluon</td>
<td>$\tilde{g}$</td>
<td>$g$</td>
<td>$(8, 1, 0)$</td>
</tr>
<tr>
<td>Winos, W Bosons</td>
<td>$\tilde{W}^\pm, \tilde{W}^0$</td>
<td>$W^\pm, W^0$</td>
<td>$(1, 3, 0)$</td>
</tr>
<tr>
<td>Bino, B Boson</td>
<td>$\tilde{B}^0$</td>
<td>$B^0$</td>
<td>$(1, 1, 0)$</td>
</tr>
</tbody>
</table>

Table 2.2: An overview of the gauge supermultiplets of the MSSM. The last column shows the dimension of the gauge group representations of $\text{SU}(3)_C$ and $\text{SU}(2)_L$, followed by the $\text{U}(1)_Y$ hypercharge generator $T_Y = Y/2$, as shown in Equation (1.32).

### 2.3 The Minimal Supersymmetric Standard Model

The simplest supersymmetric extension of the SM is the minimal supersymmetric standard model (MSSM), which is a choice of superpotential, such that

$$W_{\text{MSSM}} = \bar{u}_u y_u Q H_u - \bar{d}_u y_d Q H_d - \bar{e}_e y_e L H_d + \mu H_u H_d .$$  \hspace{1cm} (2.42)

There exist two Higgs doublets in $W_{\text{MSSM}}$ so that the up quarks can generate mass via $H_u$, while the down quarks and the leptons can generate mass via $H_d$. Further, two Higgs doublets are needed for anomaly cancellations. The existence of two Higgs doublets implies eight degrees of freedom. As in the SM, the gauge bosons absorb three degrees of
Table 2.3: An overview of the sparticle mass spectrum of the MSSM, showing the mixing of gauge eigenstates to form mass eigenstates. The mixing of the first two families of sfermions are assumed to be negligible. Also shown is the $R$-Parity for each particle, as defined in Equation (2.43).

<table>
<thead>
<tr>
<th>Particle Names</th>
<th>Spin</th>
<th>$R$</th>
<th>Gauge Eigenstates</th>
<th>Mass Eigenstates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higgs Bosons</td>
<td>0</td>
<td>+1</td>
<td>$H_u^0, H_d^0, H_u^+, H_d^-$</td>
<td>$h^0, H^0, A^0, H^\pm$</td>
</tr>
<tr>
<td>Squarks</td>
<td>0</td>
<td>-1</td>
<td>$\tilde{u}_L, \tilde{u}_R, \tilde{d}_L, \tilde{d}_R$</td>
<td>$\tilde{u}_L, \tilde{u}_R, \tilde{d}_L, \tilde{d}_R$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\tilde{c}_L, \tilde{c}_R, \tilde{s}_L, \tilde{s}_R$</td>
<td>$\tilde{c}_L, \tilde{c}_R, \tilde{s}_L, \tilde{s}_R$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\tilde{t}_L, \tilde{t}_R, \tilde{b}_L, \tilde{b}_R$</td>
<td>$\tilde{t}_1, \tilde{t}_2, \tilde{b}_1, \tilde{b}_2$</td>
</tr>
<tr>
<td>Sleptons</td>
<td>0</td>
<td>-1</td>
<td>$\tilde{\epsilon}_L, \tilde{\epsilon}<em>R, \tilde{\nu}</em>{\epsilon_L}$</td>
<td>$\tilde{\epsilon}_L, \tilde{\epsilon}<em>R, \tilde{\nu}</em>{\epsilon_L}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\tilde{\mu}_L, \tilde{\mu}<em>R, \tilde{\nu}</em>{\mu_L}$</td>
<td>$\tilde{\mu}_L, \tilde{\mu}<em>R, \tilde{\nu}</em>{\mu_L}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\tilde{\tau}_L, \tilde{\tau}<em>R, \tilde{\nu}</em>{\tau_L}$</td>
<td>$\tilde{\tau}_1, \tilde{\tau}<em>2, \tilde{\nu}</em>{\tau_L}$</td>
</tr>
<tr>
<td>Neutralinos</td>
<td>1/2</td>
<td>-1</td>
<td>$\tilde{B}^0, \tilde{W}^0, \tilde{H}_u^0, \tilde{H}_d^0$</td>
<td>$\tilde{\chi}_0^0, \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0$</td>
</tr>
<tr>
<td>Charginos</td>
<td>1/2</td>
<td>-1</td>
<td>$\tilde{W}^\pm, \tilde{H}_u^0, \tilde{H}_d^0$</td>
<td>$\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$</td>
</tr>
<tr>
<td>Gluino</td>
<td>1/2</td>
<td>-1</td>
<td>$\tilde{g}$</td>
<td>$\tilde{g}$</td>
</tr>
</tbody>
</table>

This MSSM superpotential conserves $R$-parity, which is a symmetry defined by the multiplicative quantum number

$$R = (-1)^{3(B-L)-2s},$$

(2.43)

where $B$ is the baryon number, $L$ is the lepton number, and $s$ is the spin of the particle. Since supersymmetric particles have spin which differs by $1/2$, the $R$-parity will differ as well. Thus, SM particles have $R = 1$, while SUSY particles have $R = -1$. An exciting consequence of this symmetry is that SUSY particles are always produced in pairs at colliders, since the initial states are SM particles. Further, conservation of $R$ implies freedom from the Higgs, now leaving five residual Higgs bosons instead of one. The Higgs mass eigenstates are now $h^0, H^0, H^\pm$, and $A^0$, where $A^0$ is the only $CP$-odd particle. The MSSM introduces 32 additional particles to the spectrum. The superpartners for each SM particle are outlined in Tables 2.1-2.2. The sparticle spectrum mass eigenstates are detailed in Table 2.3.
that the LSP is stable and all supersymmetric particles will eventually decay into a state with an odd number of LSPs. Within the framework of many SUSY models, the LSP is the lightest neutralino, $\tilde{\chi}_1^0$. Since the LSP is absolutely stable, has zero charge, and interacts very weakly with ordinary matter, this makes the LSP an excellent dark matter candidate.

2.4 Soft Supersymmetry Breaking

Supersymmetry must be a broken symmetry, since unbroken supersymmetry predicts that the particles composing each supermultiplet must be equal in mass. Namely, each SM particle would have a superpartner which has identical mass; however, we have yet to discover any such particles at these known mass energies. The theory resists breaking, as there is no phenomenologically viable way to break global supersymmetry spontaneously. Moreover, the breaking of a global symmetry introduces a Goldstone-like particle. Since SUSY is a fermionic symmetry, then this particle is a massless spin-$\frac{1}{2}$ Goldstino. Unfortunately, there does not exist a physically good candidate for this particle in nature. Because of these issues, we can alternatively insert supersymmetry breaking by hand, being careful not to introduce new divergences and ensuring that the Lagrangian remains renormalizable, which is called soft breaking. The soft supersymmetry breaking Lagrangian terms for the MSSM are

$$\mathcal{L}_{\text{soft}} = -\frac{1}{2} \left( M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + \text{c.c.} \right)$$

$$- \tilde{Q}^\dagger m_Q^2 \tilde{Q} - \tilde{L}^\dagger m_L^2 \tilde{L} - \tilde{u}^\dagger m_u^2 \tilde{u}^\dagger - \tilde{d}^\dagger m_d^2 \tilde{d}^\dagger - \tilde{e}^\dagger m_e^2 \tilde{e}^\dagger$$

$$- m_{H_u}^2 H_u^+ H_u - m_{H_d}^2 H_d^+ H_d - (b H_u H_d + \text{c.c.})$$

$$- \left( \tilde{u}^a_a \tilde{Q} H_u - \tilde{d}^a_a \tilde{Q} H_d - \tilde{e}^a_a \tilde{L} H_d + \text{c.c.} \right), \quad (2.44)$$

where the first line of Equation (2.44) gives the gaugino masses, the second line gives the squark and slepton masses, the third line gives the Higgs masses and their mixings, and the fourth line gives the trilinear scalar couplings. Here, $M_i$ and $b$ are complex-valued, $m_{H_i}^2$ are real-valued, the squared mass matrices are $3 \times 3$ hermitian matrices in family
space, and the scalar couplings are $3 \times 3$ complex matrices in family space. Note that these newly added terms maintain $R$-parity conservation.

Unfortunately, supersymmetry breaking introduces over 100 new parameters in the MSSM Lagrangian, including masses, phases, and mixing angles which cannot be reduced. Consequently, the theory has lost much of its predictive power. In order to minimize the number of free parameters and solve the Goldstino problem, we must turn to a grand unifying model which extends global supersymmetry to local supersymmetry, called supergravity. Supergravity grand unifying models give rise to the MSSM and its soft breaking terms naturally and constrain the parameter space heavily by applying high energy boundary conditions.
Chapter 3

Supergravity

Since supersymmetry is a global symmetry, then spontaneous symmetry breaking does not lead to a phenomenologically viable model. As discussed in the previous section, one problem that arises in spontaneous breaking of SUSY is the appearance of a massless Goldstino. This problem can be solved by gauging supersymmetry, which incorporates gravity and leads to supergravity [39–42]. Supergravity (SUGRA) incorporates gravity as a spin-2 graviton along with the spin-$\frac{3}{2}$ gravitino as its superpartner, which can be viewed as the ‘gauge field’ of a local supersymmetry transformation. The soft SUSY-breaking terms are naturally generated by supergravity models by extending the Higgs mechanism to the super-Higgs mechanism. This super-Higgs mechanism allows the massless gravitino to absorb the massless Goldstino and become massive, solving the goldstino problem of global supersymmetry. To generate soft breaking terms, supergravity grand unification [18,19] creates two sectors: one hidden and one visible. There is no direct interaction at the level of the superpotential between the hidden and the visible sectors. Instead, supersymmetry is broken in the hidden sector and then communicates by gravity to the visible sector. This communication generates soft terms in the visible sector which are of the size $(m^2/M_{Pl})$. Thus, with $m \sim 10^{10}$ GeV and $M_{Pl} = 1.2 \times 10^{18}$ GeV, soft terms of the size $\mathcal{O}$( TeV) can be generated. These soft terms in turn induce radiative electroweak
symmetry breaking driven by quantum corrections. Supergravity models with both universal and nonuniversal boundary conditions for soft parameters at the unification scale are of interest, since these high energy parameters have the capability to generate distinct particle spectra that can be tested at colliders and elsewhere.

3.1 Localizing Supersymmetry

To make supersymmetry local, we need to gauge supersymmetry by introducing a gauge field multiplet. Recall that in a local \( U(1) \) gauge symmetry, where the group action is of the form \( U = e^{i\lambda(x)} \), the gauge field \( A_\mu \) allowed us to write the gauge covariant derivative as \( D_\mu = \partial_\mu - ig_1 A_\mu \), which transforms under group action as a global symmetry. This is achieved by having the gauge field transform under infinitesimal gauge transformation as \( \delta A_\mu = \frac{1}{g_1} \partial_\mu \lambda(x) \). Consequently, the total Lagrangian remains invariant under the action of a local gauge group transformation.

Now, in local supersymmetry, the analogous ‘gauge field’ would be the gravitino, \( \Psi^\alpha_\mu(x) \), which is the superpartner to the graviton, a massless spin-2 particle. The gravitino carries spin-\( \frac{3}{2} \) and is equipped with both a vector index \( \mu \), and spinor index \( \alpha \). Thus, the supergravity supermultiplet is composed of the graviton and the gravitino. To see how supergravity automatically arises as one tries to make an action invariant under local supersymmetry transformations, we consider the simplest case. The Lagrangian of a free chiral supermultiplet \( \{\phi(x), \psi(x), F(x)\} \), which under global SUSY takes the form

\[
\mathcal{L}^{(0)} = \mathcal{L}^{(0)}_\phi + \mathcal{L}^{(0)}_\psi + \mathcal{L}_F \\
= -\partial_\mu \phi^\dagger \partial^\mu \phi + i \bar{\psi} \gamma^\mu \partial_\mu \psi + F^* F .
\] (3.1)

Ignoring the auxiliary field \( F \), Equation (3.1) is invariant under the global SUSY transformations

\[
\delta \phi = \bar{\epsilon} \psi , \\
\delta \psi = -i \gamma^\mu \partial_\mu \phi \epsilon ,
\] (3.2, 3.3)
where $\epsilon$ is a small, anticommuting, constant spinor parameterizing the global SUSY transformation. To make $\mathcal{L}^{(0)}$ invariant under local SUSY transformations, we must demand that

$$\epsilon \to \epsilon(x),$$  

(3.4)

and introduce additional fields with transformation rules that exactly cancel the additional terms that spoil the symmetry. Here the gravitino $\Psi^\alpha_\mu(x)$ is the compensating field which plays the role of our gauge field in local SUSY. Suppressing spinor indices, the gravitino field transforms as

$$\delta \Psi_\mu(x) = \frac{1}{\kappa} \partial_\mu \epsilon(x),$$  

(3.5)

where $\kappa$ has mass dimension $[\kappa] = -1$. We find that the appropriate choice for $\kappa$ is

$$\kappa = \frac{1}{M_{\text{Pl}}} = \sqrt{8\pi G},$$  

(3.6)

where $M_{\text{Pl}}$ is the reduced Planck mass and $G$ is Newton's gravitational constant. The spacetime dependence of $\epsilon(x)$ implies that we need to cancel the $\partial_\mu \epsilon(x)$ term with an interaction term

$$\mathcal{L}^{(1)} = -\kappa \bar{\Psi}_\mu \gamma^\mu \gamma^\nu \partial_\nu \phi \psi.$$  

(3.7)

The total Lagrangian is not yet supersymmetric. This is because we still need to cancel the leftover term

$$\delta(\mathcal{L}^{(0)} + \mathcal{L}^{(1)}) = \kappa \bar{\Psi}_\nu \gamma_\mu T^{\mu\nu} \epsilon(x),$$  

(3.8)

where $T^{\mu\nu}$ is the stress-energy tensor of the scalar field $\phi(x)$. In order to cancel this leftover term, we look at the transformation of the metric $g_{\mu\nu}$ under supersymmetry so that

$$\delta g_{\mu\nu} = \kappa \bar{\Psi}_\mu \gamma_\nu \epsilon(x).$$  

(3.9)

To cancel the term in Equation (3.8), we add the following coupling to the total Lagrangian,

$$\mathcal{L}^{(2)} = -g_{\mu\nu} T^{\mu\nu}.$$  

(3.10)
Thus, in the end, we have a supergravity supermultiplet $\{\Psi^\mu, g_{\mu\nu}\}$ which is the consequence of localizing supersymmetry. We would have came to the same conclusion if we had started with gravity and supersymmetrized the theory. With this supermultiplet, we can write the free supergravity Lagrangian, which ignores coupling to matter fields. For $N = 1$ supergravity, we have the kinetic term for the graviton (Einstein-Hilbert Lagrangian) plus the kinetic term for the gravitino (Rarita-Schwinger Lagrangian)

$$L_{SG}^{(0)} = -\frac{1}{2\kappa^2}eR - \frac{1}{2}e^{\mu\rho\lambda}\tilde{\Psi}^\mu \gamma^5 \gamma_\nu D_\rho \Psi_\lambda ,$$  \hspace{1cm} (3.11)

where $R$ is the Ricci curvature scalar, $e = \det[e^a_\mu] = \sqrt{g}$, and $e^a_\mu$ is the vielbein, defined by

$$g_{\mu\nu}(x) = e^a_\mu(x)e^b_\nu(x)\eta_{ab} ,$$  \hspace{1cm} (3.12)

which relates the generally curved metric and the flat Minkowski metric and therefore the curved and flat indices. Here, the supergravity transformations are

$$\delta e^a_\mu = \frac{\kappa}{2} \tilde{\epsilon} \gamma^a \Psi_\mu ,$$  \hspace{1cm} (3.13)

$$\delta \Psi_\mu = \frac{1}{\kappa} D_\mu \epsilon ,$$  \hspace{1cm} (3.14)

where the covariant derivative $D_\mu$ takes the form,

$$D_\mu = \partial_\mu + \frac{1}{2}\omega^a_\mu \sigma_{ab} ,$$  \hspace{1cm} (3.15)

where $\sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$ and $\omega^a_\mu$ is the spin connection ($\delta \omega^a_\mu = 0$ above). The supergravity transformations above involving the covariant derivative ensure general coordinate invariance, which is the central symmetry introduced by general relativity\(^1\).

\(^1\) See Appendix A for more detail regarding general relativity.
3.2 Supergravity Coupling to Matter

To build a viable grand unification model, we must couple the supergravity supermultiplet \( \{ \Psi_\mu, g_{\mu\nu} \} \) to the matter fields. Namely, the fields which need to be considered are within the chiral supermultiplets \( \{ \phi, \psi \} \) and the vector supermultiplets \( \{ \lambda, V_\mu \} \), where \( \phi \) are the spin-0 squarks, sleptons, and Higgs fields, \( \psi \) are the spin-1/2 quarks, leptons, and Higgsinos, \( \lambda \) are the spin-1/2 gauginos, and \( V_\mu \) are the spin-1 gauge bosons. The Lagrangian of interest for grand unified model building depends on three functions: the superpotential \( W(\phi_i) \), the Kähler potential \( K(\phi_i, \phi^*_i) \), and the gauge kinetic function \( f_{ab}(\phi_i) \), where \( \phi_i \) are the spin-0 fields of the left handed chiral supermultiplets and \( a, b \) are gauge indices in the adjoint representation of the gauge group. Let us define the Kähler function \( G = G(W, K) \), such that

\[
G = \kappa^2 K + \ln[\kappa^6|W|^2] .
\]  

(3.16)

The Kähler function \( G \) is a real, gauge invariant function; whereas, the superpotential \( W \) and the gauge kinetic function \( f \) are complex analytic. The chiral supergravity Lagrangian depends only on the Kähler function of the scalar fields. Thus, the scalar-field space can be viewed as a Kähler manifold, with Kähler metric

\[
g_{ij} = K_{ij^*} \equiv \frac{\partial^2 K}{\partial \phi_i \partial \phi^*_j} ,
\]

(3.17)

which implies Kähler invariance. Namely, the system is invariant under the Kähler transformation

\[
K \rightarrow K + F(\phi) + F^*(\phi^*) ,
\]

(3.18)

\[
W \rightarrow e^{-\kappa^2 F(\phi)} W ,
\]

(3.19)

where \( F \) is an arbitrary function of the scalar fields. Note that Equations (3.16) and (3.17) imply that \( K_{ij^*} = \kappa^2 G_{ij^*} \). The simplest possibility for \( K \) is the choice

\[
K = \phi_i \phi^{*i} ,
\]

(3.20)
which leads to a flat metric

$$g_{ij} = K_{ij} = \delta_{ij}.$$  \hfill (3.21)

To obtain the complete $D = 4, N = 1$ supergravity Lagrangian we need to include both chiral and vector supermultiplet couplings to supergravity. Let us define the total tree-level Lagrangian as

$$\mathcal{L} = \mathcal{L}_1^{(C)} + \mathcal{L}_2^{(C)} + \mathcal{L}_3^{(C)} + \mathcal{L}_1^{(V)} + \mathcal{L}_2^{(V)} + \mathcal{L}_3^{(V)},$$  \hfill (3.22)

where (C) and (V) denote chiral and vector terms, respectively. Regarding the subscripts for both chiral and vector terms, $\mathcal{L}_1$ contains bosonic fields, $\mathcal{L}_2$ contains fermionic fields and covariant derivatives, and $\mathcal{L}_3$ contains fermionic fields without covariant derivatives.

As the Lagrangian terms are listed throughout this section, the gravitational coupling factor $\kappa$ has been set to unity for notational convenience, but can be reinserted using dimensional analysis if needed. The chiral Lagrangian terms are

$$e^{-1} \mathcal{L}_1^{(C)} = -\frac{1}{2} R - G_{ij} D_\mu \phi^i D^{\mu} \phi^j - e^G \left( G_i (G^{-1})^{ij} G_j \right)^* + 3,$$  \hfill (3.23)

$$e^{-1} \mathcal{L}_2^{(C)} = \frac{1}{2e^G} \bar{\Psi}_L \gamma^\mu \gamma^\nu \Psi_R \gamma^\rho \Psi_L \gamma_L \gamma_R \Psi_R + \frac{1}{\sqrt{2}} \bar{\Psi}_L \gamma^\mu \phi^j \gamma^\nu \Psi_R + \text{h.c.},$$  \hfill (3.24)

$$e^{-1} \mathcal{L}_3^{(C)} = e^{G/2} \bar{\Psi}_L \gamma^\mu \gamma^\nu \Psi_R + \bar{\Psi}_L \gamma^\mu \gamma^\nu \Psi_R + \frac{1}{2} e^{G/2} \left( G_{ij} + G_{jk} - G_{ik} (G^{-1})^{jl} G_l \right) \bar{\Psi}_L \gamma^\mu \gamma^\nu \Psi_R$$

$$+ \text{h.c.} + \left( \frac{1}{2} G_{ij} \phi^j \gamma^k \gamma^l \Psi_L \gamma^\mu \gamma^\nu \Psi_R \right) \bar{\Psi}_L \gamma^\mu \gamma^\nu \Psi_R$$

$$- \frac{1}{2} G_{ik} \phi^j \gamma^k \gamma^l \Psi_L \gamma^\mu \gamma^\nu \Psi_R - \frac{1}{4} G_{ik} \phi^j \gamma^k \gamma^l \Psi_L \gamma^\mu \gamma^\nu \Psi_R$$

$$- \frac{1}{8} G_{ik} \phi^j \gamma^k \gamma^l \Psi_L \gamma^\mu \gamma^\nu \Psi_R.$$  \hfill (3.25)

Note that the third term of $\mathcal{L}_1^{(C)}$ is due to the chiral supermultiplet auxiliary fields $F_i$ being eliminated using their equations of motion,

$$F_i = e^{G/2} (G^{-1})^{ij} G_j - (G^{-1})^{ik} G_{jk} \phi^j \gamma^l \Psi_L \gamma^\mu \gamma^\nu \Psi_R + \frac{1}{2} \psi_i (G_j \phi^j).$$  \hfill (3.26)
Now, the vector Lagrangian terms are

\[
e^{-1} \mathcal{L}_1^{(V)} = -\frac{1}{4} \text{Re}(f_{ab})(F^a)_{\mu\nu}(F^b)^{\mu\nu} + \frac{i}{4} \text{Im}(f_{ab})(F^a)_{\mu\nu}(\tilde{F}^b)^{\mu\nu}
\]

\[
- \frac{1}{2} g^2 \text{Re}(f^{-1})^{ab} \mathcal{G}_i(T_a)^{ij} \phi_j \mathcal{G}_k(T_b)^{kl} \phi_l ,
\]

(3.27)

\[
e^{-1} \mathcal{L}_2^{(V)} = \frac{1}{2} \text{Re}(f_{ab}) \left( \frac{1}{2} \lambda^a \slashed{\partial} \lambda^b + \frac{1}{2} \tilde{\lambda}^a \gamma^\mu \sigma^{\nu\rho} \Psi_{i}^{(F^b)_{\nu\rho}} + \frac{1}{2} \mathcal{G}_i D^i \phi^i \tilde{\lambda}_L^a \gamma_{L} \lambda_L^b \right)
\]

\[ - \frac{i}{2} \text{Im}(f_{ab}) D_{\mu} \left( \bar{e} \lambda_{L}^a \gamma_{L} \lambda_{L}^b \right) - \frac{1}{2} \frac{\partial f_{ab}}{\partial \phi_i} \tilde{\Psi}_{R} \sigma^{\mu\nu} (F^a)_{\mu\nu} \lambda_L^b + \text{h.c.} ,
\]

(3.28)

\[
e^{-1} \mathcal{L}_3^{(V)} = \frac{1}{4} \phi^{ij} \frac{\partial f_{ab}}{\partial \phi_k} (G^{-1})^{ik} k \mathcal{G}_k \lambda^a \lambda^b - \frac{i}{2} g \mathcal{G}_i (T^a)^{ij} \phi_j \bar{\psi}_{kR} \lambda_c L
\]

\[ + 2 i g \mathcal{G}_{ij} \lambda^a \lambda^b \bar{\psi}_{kR} L - \frac{i}{2} g \text{Re}(f^{-1})^{ab} \frac{\partial f_{ac}}{\partial \phi_k} \mathcal{G}_i (T_a)^{ij} \phi_j \bar{\psi}_{kR} \lambda_c L
\]

\[ - \frac{1}{32} (G^{-1})^{ik} \frac{\partial f_{ab}}{\partial \phi_i} \frac{\partial f_{cd}}{\partial \phi_k} \tilde{\lambda}_L^a \tilde{\lambda}_L^b \lambda_{R}^c \lambda_R^d + \frac{3}{32} \left( \text{Re}(f_{ab}) \tilde{\lambda}_L^c \gamma_{L} \lambda_1^b \right)^2
\]

\[ + \frac{1}{8} \text{Re}(f_{ab}) \tilde{\lambda}_L^a \gamma_{L} \sigma^{\mu\nu} \Psi_{i}^{(F^a)_{\mu\nu}} \lambda^b + \frac{1}{2} \frac{\partial f_{ab}}{\partial \phi_i} \left( \bar{\psi}_{Li} \sigma^{\mu\nu} \lambda_L^a \Psi_{i}^{(F^a)_{\mu\nu}} \lambda^b + \frac{1}{4} \bar{\psi}_{Ri} \gamma_{L} \lambda_L^a \lambda_L^b \right)
\]

\[ + \frac{1}{16} \bar{\psi}_{Li} \lambda^a \lambda^b \lambda_L^c \lambda_L^d \left( 2 \mathcal{G}_{ij} \lambda^a \lambda^b \right) (G^{-1})^{ik} \frac{\partial f_{cd}}{\partial \phi_i} \frac{\partial f_{cd}}{\partial \phi_j} - 4 \frac{\partial^2 f_{cd}}{\partial \phi_i \partial \phi_j} + \text{Re}(f^{-1})^{ab} \frac{\partial f_{ac}}{\partial \phi_i} \frac{\partial f_{bd}}{\partial \phi_j} + \text{h.c.} ,
\]

(3.29)

Likewise, the third term of \( \mathcal{L}_1^{(V)} \) is due to the vector supermultiplet auxiliary fields \( D_a \) being eliminated using their equations of motion,

\[
D_a = i \text{Re}(f^{-1})^{ab} \left( g \mathcal{G}_i (T_b)^{ij} \phi_j + \frac{i}{2} \frac{\partial f_{ac}}{\partial \phi_i} \bar{\psi}_L \lambda^c - \frac{i}{2} \frac{\partial f_{ac}^*}{\partial \phi^*_i} \psi^{iL} \lambda^c \right) + \frac{1}{2} \lambda_a (\mathcal{G}_{ij} \psi_i) .
\]

(3.30)

where \( f_{ab}(\phi_i) \) must transform as a symmetric product of adjoint representations of the gauge group. Here, we can see that \( f \) is called the gauge kinetic function because it multiplies the gauge kinetic terms. The supergravity scalar potential is

\[
V = e^\mathcal{G}_i (G^{-1})^{ij} \mathcal{G}_j - 3 + \frac{1}{2} g^2 \text{Re}(f^{-1})^{ab} \mathcal{G}_i (T_a)^{ij} \phi_j \mathcal{G}_k (T_b)^{kl} \phi_l ,
\]

(3.31)

which is due to the \( F \)-term contribution from \( \mathcal{L}_1^{(C)} \) and the \( D \)-term contribution from
$\mathcal{L}^{(V)}_1$. Note that this scalar potential is not positive definite, as opposed to the positive (semi-)definite scalar potential in global supersymmetry, as shown in Equation (2.40). This will be an important aspect of radiative electroweak symmetry breaking. Remarkably, the entire $D = 4$, $N = 1$ supergravity Lagrangian\(^2\) is specified by only two functions of the scalar fields, $\mathcal{G}(\phi_i, \phi_i^*)$ and $f_{ab}(\phi_i)$.

### 3.3 Spontaneous Breaking of Supergravity

The spontaneous breaking of the electroweak symmetry $\text{SU}(2)_L \times \text{U}(1)_Y$ in the standard model is a consequence of inserting a negative squared mass term in the Higgs potential. This ad hoc approach is not desirable, as we would prefer a model which naturally produces the negative squared mass term dynamically. Fortunately, the theory of supergravity offers precisely this solution. Namely, the spontaneous breaking of local supersymmetry at the Planck scale triggers the breaking of the electroweak symmetry due to radiative corrections. This is known as radiative electroweak symmetry breaking (REWSB).

The high scale breaking of supersymmetry is a consequence of the super-Higgs mechanism, a process by which the scalar fields acquire vacuum expectation values dynamically, spontaneously breaking supergravity. The spontaneous breaking of this symmetry produces a Goldstone fermion, the Goldstino, which is a linear combination of the superpartners to the scalar fields. The massless gravitino then swallows the Goldstino, which gives it mass

$$m_{3/2} = \kappa^2 e^{\kappa/2} |W|,$$

by adding two degrees of freedom ($\pm \frac{1}{2}$ helicity states), resulting in helicity $\pm \frac{3}{2}$, $\pm \frac{1}{2}$ states. The scalar fields responsible for the high scale breaking of supersymmetry are the auxiliary fields $F_i$ ($F$-term breaking) and/or $D_a$ ($D$-term breaking).

\(^2\) For a more elegant formulation of both global and local supersymmetry, I recommend using the superspace formalism [34].
Unfortunately, the breaking of supersymmetry using supergravity theory leads to a new hierarchy problem. The super-Higgs VEV is on the order of the Planck mass, which may lead to physical particles inheriting Planck-size masses due to supergravity interactions. In order to prevent this, we introduce the following solution. The super-Higgs field must couple very weakly to the matter fields via gravitational interactions only. For this to happen, let us write the superpotential in the form

\[ W(\phi_i, \phi_j^{(h)}) = W_v(\phi_i) + W_h(\phi_j^{(h)}) , \]  

(3.33)

where \( \phi_i \) are the scalar fields of the MSSM (squarks, sleptons, Higgs bosons), which populate the visible (observable, or physical) sector. The fields \( \phi_j^{(h)} \) are the additional scalar fields responsible for supersymmetry breaking, belonging to a hidden sector, which is shielded from the visible sector by gravity. The super-Higgs must be a singlet under the visible gauge group. The hidden sector fields lack gauge interactions with the visible sector. Most importantly, other couplings between \( \phi_i \) and \( \phi_j^{(h)} \) will be gravitationally suppressed, i.e. scaled down by powers of \( \kappa \). This process will produce VEVs and superheavy masses on the order of the grand unified theory (GUT) scale, \( M_G \). Integrating out these superheavy fields, we find an effective SU(3)_C × SU(2)_L × U(1)_Y theory just below \( M_G \) which is consistent with the standard model.

When supergravity is broken in the hidden sector, then the soft Lagrangian terms discussed in Section 2.4 are generated naturally, instead of inserting them by hand in the global SUSY case. The effective potential for the minimal supergravity case is

\[ V = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 + m_0^2 \sum_i \phi_i^\dagger \phi_i + \left[ A_0 W^{(3)} + B_0 W^{(2)} + \text{h.c.} \right] + m_{1/2} \sum_a \lambda_a^\dagger \lambda_a , \]  

(3.34)

with effective superpotential quadratic and cubic parts,

\[ W^{(2)} = \mu_0 H_u H_d , \]  

(3.35)

\[ W^{(3)} = \bar{u} y_u Q H_u - \bar{d} y_d Q H_d - \bar{e} y_e LH_d . \]  

(3.36)
Note that $W = W^{(2)} + W^{(3)}$ is the MSSM superpotential, implying that the model is $R$-parity invariant. Thus, the four soft breaking terms of mSUGRA are specified by the four soft parameters

$$m_0, \ m_{1/2}, \ A_0, \ B_0,$$  

where $m_0$ is the universal scalar mass, $m_{1/2}$ is the universal gaugino mass, $A_0$ is the universal trilinear coupling constant, and $B_0$ is the universal quadratic coupling constant.

Also, the Higgs mixing parameter $\mu_0$ is another free parameter which must be specified. Remarkably, using supergravity instead of global SUSY, we have reduced the number of free parameters in the theory from over 100 to just five,

$$\{m_0, \ m_{1/2}, \ A_0, \ B_0\}, \ \mu_0.$$  

By examining how each term in Equation (3.34) arises due to terms in the supergravity Lagrangian, we find that all the parameters in (3.38) are of the same scale, which is that of the gravitino,

$$m_{3/2} \sim \kappa^2 \langle W_h \rangle.$$  

Because $m_0 = m_{3/2}$, the gravitino mass sets the overall scale of the soft parameters.

Since $\langle W_h \rangle \sim m_S^2 M_{Pl}$, where $m_S \sim 10^{10}$ GeV is the supersymmetry-breaking scale and $M_{Pl} = \kappa^{-1} \sim 10^{18}$ GeV, then $m_0 \sim 10^2$ GeV. Thus, all the free parameters in mSUGRA are approximately on the order of the electroweak scale.

Now, let us examine the radiative breaking of the electroweak symmetry. REWSB is a powerful constraint on supergravity unified models, since it allows us to eliminate one more parameter from the already small parameter space. The grand unified theory models are defined at the GUT scale $M_G$; however, experiments are done at the electroweak scale.

To bridge this mass gap and link these two scales, we turn to the renormalization group equations (RGEs). The renormalization-group-improved effective potential for the Higgs is of the form

$$V_H = V_0 + \Delta V_1,$$  

35
where $V_0$ is the tree-level potential

$$V_0 = m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2(H_1H_2 + \text{h.c.}) + \frac{1}{8}(g_1^2 + g_2^2)(|H_1|^2 - |H_2|^2)^2 , \quad (3.41)$$

and $\Delta V_1$ is the one-loop correction

$$\Delta V_1 = \frac{1}{64\pi^2} \sum_a (-1)^{2s_a} n_a M_a^4 \ln \left( \frac{M_a^2}{\mu^2} \right) . \quad (3.42)$$

Here, $H_1 = H_d$, $H_2 = H_u$, and $\{m_i(t), g_1(t), g_2(t)\}$ are the running masses and coupling constants as a function of $t = \ln \left( \frac{M^2}{\mu^2} \right)$ at the mass energy scale $Q$. $M_a = M_a(v_1, v_2)$ is the tree-level mass of particle $a$ as a function of $v_i = \langle H_i \rangle$. Also, $s_a$ and $n_a$ are the spin and number of helicity states, respectively, for particle $a$. We define

$$m_i^2(t) = m_{H_i}^2(t) + \mu^2(t) , \quad (3.43)$$
$$m_3^2(t) = -B(t)\mu(t) , \quad (3.44)$$

where $i = 1, 2$ and the boundary conditions at the GUT scale $Q = M_G$ (which implies initial conditions $t = 0$)

$$m_i^2(0) = m_{0_i}^2 + \mu_0^2 , \quad (3.45)$$
$$m_3^2(0) = -B_0\mu_0 , \quad (3.46)$$
$$\alpha_2(0) = \frac{5}{3}\alpha_1(0) = \alpha_G . \quad (3.47)$$

The RGEs allow us to write all parameters as functions of the GUT scale parameters. Each SUGRA model implies a specific choice of the high energy boundary conditions defined at the GUT scale, i.e. $\{m_0, m_{1/2}, A_0, B_0, \mu_0\}$. Starting at $M_G$, all scalar particles have a positive squared mass, since $m_0^2 > 0$. As we integrate the RGEs down to lower energies, the electroweak symmetry breaks when a squared mass turns negative, implying
the existence of a nonzero VEV,

\[ v_i = \langle H_i \rangle \neq 0 \]  \hspace{1cm} (3.48)

A negative squared mass implies that the determinant of the squared mass matrix is negative, thus there exists one negative eigenvalue. Also, the potential must be bounded from below for there to exist a valid minimum. Thus, considering the tree-level potential \( V_0 \) shown in Equation (3.41), we require

\[ D = m_1^2 m_2^2 - m_3^4 < 0 \]  \hspace{1cm} (3.49)
\[ L = m_1^2 + m_2^2 - 2|m_3^2| > 0 \]  \hspace{1cm} (3.50)

Minimization of the full potential, \( \frac{\partial V}{\partial v_i} = 0 \), gives

\[ \frac{1}{2} M_Z^2 = \frac{\mu_i^2 - \mu_2^2 \tan^2 \beta}{\tan^2 \beta - 1} \]  \hspace{1cm} (3.51)
\[ \sin(2\beta) = \frac{2m_3^2}{\mu_1^2 + \mu_2^2} \]  \hspace{1cm} (3.52)

where \( \tan \beta = \frac{v_2}{v_1} \) and \( \mu_i^2 = m_i^2 + \Sigma_i \). We define \( \Sigma_i \) as

\[ \Sigma_i = \frac{1}{32\pi^2} \sum_a (-1)^{a+1} n_a M_a^2 \ln \left[ \frac{M_a^2}{\alpha_{a/2}^2 Q^2} \right] \frac{\partial M_a^2}{\partial v_i}, \]  \hspace{1cm} (3.53)

which is the one-loop correction due to \( \Delta V_1 \) in the Higgs potential. Note that Equation (3.52) also accounts for the stable minimum condition shown in Equation (3.50) since \( \sin(2\beta) \leq 1 \), but as a generalization since it includes radiative corrections. Instead of using the minimization of \( V_H \) to solve for the two Higgs VEVs, we will choose to replace the GUT scale parameter \( B_0 \) with \( \tan \beta \) using Equation (3.52) as well as determine \( \mu \) up to a sign using Equation (3.51). Thus, we have reduced the mSUGRA parameter space to just four free parameters plus a sign,

\[ m_0, \ m_{1/2}, \ A_0, \ \tan \beta, \ \text{sgn}(\mu). \]  \hspace{1cm} (3.54)
The existence of solutions to the minimization of $V_H$ is not guaranteed unless the following conditions are satisfied: there exists at least one nonzero soft breaking parameter \{m_0, m_{1/2}, A_0, B_0\}, the GUT scale parameter $\mu_0$ is nonzero, and the top quark mass is sufficiently heavy (i.e. $m_t \geq 90\text{ GeV}$). The top quark is quite significant for the radiative breakdown of the electroweak symmetry, since the radiative corrections depend most sensitively on the top quark Yukawa coupling to $H_2 = H_u$, which drives the value of $m^2_2 = m^2_{H_u}$ negative as we RG evolve from $M_G$ down to the electroweak scale.
Chapter 4

Mass Hierarchies for SUGRA Unification

The discovery \cite{13,14} of the Higgs boson \cite{15–17} and the measurement of its mass at \( \sim 126 \) GeV have strong implications for discovery of supersymmetry. In the MSSM, one identifies the observed Higgs boson as the lightest \( CP \)-even state \( h^0 \) \cite{22–29}. It is noteworthy that the observed Higgs boson mass lies below but close to the upper limit on the Higgs boson mass predicted in supergravity grand unified models \cite{18–21} with radiative breaking of the electroweak symmetry (for a review see \cite{43}), and this upper limit is well known to be around \( 130 \) GeV \cite{22–29,44,45}. Further, in supersymmetric models and specifically those within supergravity grand unification, one finds that a Higgs mass of \( \sim 126 \) GeV implies the scale of supersymmetry to be large, with the squark masses typically lying in the few TeV region \cite{22–29,44–46}. The largeness of the SUSY scale explains the non-observation of sparticles for searches in Run-I of the LHC. However, the LHC energy is being increased to \( \sqrt{s} = 13 \) TeV for Run-II, which is an energy where one expects some of the light sparticles to be detected\(^1\).

\(^1\) Although the current plan is for the LHC to operate at \( \sqrt{s} = 13 \) TeV for Run-II, we will carry out the analysis at \( \sqrt{s} = 14 \) TeV, using the Snowmass \cite{47} standard model backgrounds.
The nature of the observed sparticles, and more generally the hierarchical mass patterns, hold a key to the nature of symmetry breaking at high scales in unified models. Given that there are 31 additional particles beyond the spectrum of the standard model, there are \( a \ priori \ 31! \sim 8 \times 10^{33} \) ways in which these particles can arrange themselves. This is the landscape of possible mass hierarchies of the new particles\(^2\).\(^3\). The number of allowed possibilities is significantly reduced in supergravity grand unification with radiative breaking of the electroweak symmetry [48–50]. Additionally, the accelerator and dark matter constraints further reduce the allowed number of possibilities. The landscape of supergravity based models was analyzed in a number of works [48–55]; however, such analyses were all before the discovery of the Higgs boson and a measurement of its mass.

Regarding the Higgs boson mass of 126 GeV, there are a limited number of ways in which one can lift its tree mass, which lies below \( M_Z \), to the observed value. These include \( D \)-term contributions from extra U(1)’s, loop contributions from extra matter [56] or large loop corrections from within the MSSM. The latter possibility implies a relatively high scale of supersymmetry, which explains in part the reason for its non-observation thus far.

In this work, we revisit the sparticle landscape analysis taking into account the constraint from the Higgs boson mass measurement on the sparticle landscape. We analyze several different classes of high scale models: these include minimal supergravity (mSUGRA or CMSSM, for some recent works see [85–88]) and supergravity models with nonuniversal boundary conditions at the grand unification scale\(^4\) in the SU(2)\(_L\) and SU(3)\(_C\) gaugino sector, in the Higgs sector and in the third generation sfermion sector. The most dominant hierarchical patterns that emerge are identified. The hierarchical patterns provide a simple way to connect the simplified models [57–65] with grand unified models. Specifically, we consider five particle mass hierarchies where the various combinations of the five lightest particles that originate in supergravity models are investigated. These five

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\(^2\)We would loosely call these the sparticle mass hierarchies even though they contain the Higgs boson states, \( H^0, A^0, H^\pm \), which are \( R \)-parity even.

\(^3\)The landscape is even larger in that the mass gaps among the sparticles can vary continuously, which makes the allowed sparticle landscape larger than even the string landscape which has as many as \( 10^{500} \) possible vacua.

\(^4\)The literature on supergravity models with nonuniversalities is vast. For a sample of works on nonuniversalities, the reader may refer to [66–78] and for a review see [55].
particle mass hierarchies effectively constitute existing and novel simplified models. The hierarchy of five particles can be further truncated to give simplified models with three or four lightest particles, as has been more common. It should be noted that these simplified models are necessarily part of a UV-complete theory since they are obtained by truncation of the spectrum arising from a high scale model.

We consider several different supergravity unifying models of soft breaking [30]. These include supergravity models with [1] universal boundary conditions (mSUGRA) as well as supergravity models with nonuniversal boundary conditions (nuSUGRA) at the grand unification scale. Within mSUGRA, we consider [2] nonuniversalities in the SU(2) \textsubscript{L} gaugino mass sector, [3] nonuniversalities in the SU(3) \textsubscript{C} gaugino mass sector, [4] nonuniversalities in the flavor sector with the squark masses for the third generation being different from the masses in the first two generations, and [5] nonuniversalities in the Higgs sector. The parameter space probed for each of these models is discussed below.

**Model [1]: mSUGRA**

\[
\begin{align*}
  m_0 & \in [0.1, 10] \text{ TeV} , \\
  m_{1/2} & \in [0.1, 1.5] \text{ TeV} , \\
  \frac{A_0}{m_0} & \in [-5, 5] , \\
  \tan \beta & \in [2, 50] .
\end{align*}
\]

(4.1)

**Model [2]: nuSUGRA, light chargino**

\[
\begin{align*}
  m_0 & \in [0.1, 10] \text{ TeV} , \\
  M_1 = M_3 = m_{1/2} & \in [0.1, 1.5] \text{ TeV} , \\
  M_2 & = \alpha m_{1/2} , \quad \alpha \in [1/2, 1] , \\
  \frac{A_0}{m_0} & \in [-5, 5] , \\
  \tan \beta & \in [2, 50] .
\end{align*}
\]

(4.2)
Model [3]: nuSUGRA, light gluino

\[ m_0 \in [0.1, 10] \text{ TeV} , \]
\[ M_1 = M_2 = m_{1/2} \in [0.1, 1.5] \text{ TeV} , \]
\[ M_3 = \alpha m_{1/2} , \ \alpha \in [1/6, 1] , \]
\[ \frac{A_0}{m_0} \in [-5, 5] , \]
\[ \tan \beta \in [2, 50] . \quad (4.3) \]

Model [4]: nuSUGRA, nonuniversal Higgs

\[ m_0 \in [0.1, 10] \text{ TeV} , \]
\[ m_{1/2} \in [0.1, 1.5] \text{ TeV} , \]
\[ \frac{A_0}{m_0} \in [-5, 5] , \]
\[ \tan \beta \in [2, 50] , \]
\[ m_{H_1}(M_G) = m_0(1 + \delta_i) , \ \delta_i \in [-0.9, 1] . \quad (4.4) \]

Model [5]: nuSUGRA, light 3rd generation

\[ m_0^{(1)} = m_0^{(2)} = m_0 \in [0.1, 10] \text{ TeV} , \]
\[ m_0^{(3)} = \frac{m_0^2}{1 \text{ TeV} + m_0} , \]
\[ m_{1/2} \in [0.1, 1.5] \text{ TeV} , \]
\[ \frac{A_0}{m_0} \in [-5, 5] , \]
\[ \tan \beta \in [2, 50] . \quad (4.5) \]

In each of the above cases, \( \mu \) is taken to be positive. An overview of the parameter scan for the five classes of models listed above is given in Table 4.1. The scans were performed using \texttt{SusyKit} \cite{79}, which employs \texttt{SOFTSUSY} \cite{80} for 2-loop RG evolution, including sparticle thresholds and for sparticle mass calculations, \texttt{FeynHiggs} \cite{81,82} for computing the Higgs boson masses, including the recently added resummation to all orders of leading and subleading logs of type \( \log(m_{\tilde{t}}/m_{\tilde{t}}) \) \cite{83}, and \texttt{micrOMEGAS} \cite{84} to calculate the dark
Table 4.1: An overview of the parameter scans, detailing the number of parameter points simulated, the number of NLSP patterns, the number of 4-sparticle and the number of 5-sparticle mass hierarchies for Models [1]-[5].

<table>
<thead>
<tr>
<th>SUGRA Model</th>
<th>Parameter Points</th>
<th>NLSP Patterns</th>
<th>4-sparticle Patterns</th>
<th>5-sparticle Patterns</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1] mSUGRA</td>
<td>5453</td>
<td>5</td>
<td>15</td>
<td>32</td>
</tr>
<tr>
<td>[2] Light Chargino</td>
<td>8000</td>
<td>4</td>
<td>26</td>
<td>89</td>
</tr>
<tr>
<td>[3] Light Gluino</td>
<td>8008</td>
<td>7</td>
<td>44</td>
<td>85</td>
</tr>
<tr>
<td>[4] Nonuniversal Higgs</td>
<td>4738</td>
<td>5</td>
<td>26</td>
<td>59</td>
</tr>
<tr>
<td>[5] Light 3rd Generation</td>
<td>2668</td>
<td>6</td>
<td>26</td>
<td>43</td>
</tr>
</tbody>
</table>

matter relic density and flavor observables. We only considered points where the LSP is the neutralino, the thermal relic density of the LSP is not overabundant and the Higgs boson mass is not too light\(^5\),

\[
\Omega_{\chi} h^2 < 0.12, \quad m_{h^0} > 120 \text{ GeV}.
\] (4.6)

These constraints immediately reduce the available number of sparticle mass hierarchies. Of the large number of model points in the scan of Table 4.1, two models were investigated in detail for their discovery potential at Run-II of LHC. One of these is an mSUGRA model whose parameters are listed in Table 7.2 and the other is an nuSUGRA model whose parameters are listed in Table 7.3. The signatures for these models were investigated using an adaptation from the ATLAS cuts at \(\sqrt{s} = 8\) TeV \([106]\), applied to our analysis at \(\sqrt{s} = 14\) TeV as shown in Table 7.4.

We begin the study of the hierarchies by first identifying the next-to-lightest sparticle

---

\(^5\) We have chosen a relatively generous window on the Higgs boson mass for the following reason: aside from the possible errors in theory computations, which are now reduced due to FeynHiggs, in extended models with extra U(1)'s, corrections to the Higgs mass can arise of the size \(\mathcal{O}(1\text{ GeV})\) due to extra \(D\)-term contributions. Also extra vector-like matter, if it exists, could make a contribution of the same size (see, e.g., \([56]\)). Thus, in order that our analysis also applies to such models, we consider the Higgs mass window as stated; however, the results for the narrower window on the Higgs mass are easily extracted from the analysis presented in the paper. For this reason, we have color-coded Fig. 8.1 for the mSUGRA case and Fig. 8.2 for the nuSUGRA cases with the Higgs mass.
(NLSP). For each NLSP case, we next determine the possible 4-sparticle hierarchies (LSP, NLSP, and two heavier sparticles). Within these, we additionally identify the next lightest sparticle, giving a hierarchy of the 5 lightest sparticles. These hierarchies are labeled by the following scheme: we begin with a symbol for the NLSP, followed by a number for one of the possible 4-sparticle hierarchies for the given NLSP, and lastly append a letter for the 5th lightest sparticle. We note that when the mass gap $\Delta m(H^0, A^0)$ is $\mathcal{O}(1\,\text{GeV})$, $H^0$ and $A^0$ are treated as degenerate in mass in the classification scheme.

In Table 4.2, the hierarchical mass patterns for the mSUGRA case are shown. The sparticle mass hierarchies in the nuSUGRA Models [2]-[5] are exhibited in Tables 4.3 to 4.6. The sparticle mass hierarchies for nuSUGRA Model [2] with nonuniversalities in the SU(2)$_L$ gaugino mass sector are given in Table 4.3, while those for the nuSUGRA Model [3] with nonuniversality in the gluino mass sector are given in Table 4.4. In Table 4.5, we give an analysis of the sparticle mass hierarchies for the nuSUGRA Model [4] with nonuniversality in the Higgs boson mass sector, and in Table 4.6 an analysis is given of the sparticle mass hierarchies for nuSUGRA Model [5] for the case when nonuniversality is in the third generation sfermion sector. In all cases, the exhibited mass hierarchies are those that remain after the constraints in Equation (4.6) have been applied.
<table>
<thead>
<tr>
<th>Pattern Label</th>
<th>Mass Hierarchy</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>mSP[C1a]</td>
<td>$\chi^+_1 &lt; \chi^0_2 &lt; \chi^0_3 &lt; \chi^0_4$</td>
<td>83.8</td>
</tr>
<tr>
<td>mSP[C1b]</td>
<td>$\chi^+_1 &lt; \chi^0_2 &lt; \chi^0_3 &lt; H^0$</td>
<td>2.49</td>
</tr>
<tr>
<td>mSP[C1c]</td>
<td>$\chi^+_1 &lt; \chi^0_2 &lt; \chi^0_3 &lt; \chi^+_2$</td>
<td>1.62</td>
</tr>
<tr>
<td>mSP[C2]</td>
<td>$\chi^+_1 &lt; \chi^0_2 &lt; H^0 &lt; A^0$</td>
<td>0.65</td>
</tr>
<tr>
<td>mSP[C3]</td>
<td>$\chi^+_1 &lt; \chi^0_2 &lt; g &lt; \chi^0_3$</td>
<td>0.04</td>
</tr>
<tr>
<td>mSP[C4]</td>
<td>$\chi^+_1 &lt; \chi^0_2 &lt; A^0 &lt; H^0$</td>
<td>0.02</td>
</tr>
<tr>
<td>mSP[τ1a]</td>
<td>$\tau_1 &lt; \chi^0_2 &lt; \chi^+_1 &lt; H^0$</td>
<td>3.89</td>
</tr>
<tr>
<td>mSP[τ1b]</td>
<td>$\tau_1 &lt; \chi^0_2 &lt; \chi^+<em>1 &lt; \mu</em>\tau$</td>
<td>0.89</td>
</tr>
<tr>
<td>mSP[τ1c]</td>
<td>$\tau_1 &lt; \chi^0_2 &lt; \chi^+<em>1 &lt; \nu</em>\tau$</td>
<td>0.15</td>
</tr>
<tr>
<td>mSP[τ1d]</td>
<td>$\tau_1 &lt; \chi^0_2 &lt; \chi^+_1 &lt; t_1$</td>
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<tr>
<td>mSP[τ2a]</td>
<td>$\tau_1 &lt; \mu_\tau &lt; e_\tau &lt; \chi^0_2$</td>
<td>0.69</td>
</tr>
<tr>
<td>mSP[τ2b]</td>
<td>$\tau_1 &lt; \mu_\tau &lt; e_\tau &lt; \nu_\tau$</td>
<td>0.52</td>
</tr>
<tr>
<td>mSP[τ3a]</td>
<td>$\tau_1 &lt; H^0 &lt; A^0 &lt; \chi^0_2$</td>
<td>0.04</td>
</tr>
<tr>
<td>mSP[τ3b]</td>
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</tr>
<tr>
<td>mSP[τ4]</td>
<td>$\tau_1 &lt; t_1 &lt; \chi^0_2 &lt; \chi^+_1$</td>
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<tr>
<td>mSP[t1a]</td>
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<tr>
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<td>0.06</td>
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<tr>
<td>mSP[t1c]</td>
<td>$t_1 &lt; \chi^0_2 &lt; \chi^+_1 &lt; b_1$</td>
<td>0.02</td>
</tr>
<tr>
<td>mSP[N1a]</td>
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<tr>
<td>mSP[N1c]</td>
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<tr>
<td>mSP[N2a]</td>
<td>$\chi^0_2 &lt; \chi^+_1 &lt; \chi^0_3 &lt; H^0$</td>
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<tr>
<td>mSP[N2b]</td>
<td>$\chi^0_2 &lt; \chi^+_1 &lt; \chi^0_3 &lt; \chi^0_4$</td>
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<td>mSP[N3]</td>
<td>$\chi^0_2 &lt; \chi^+_1 &lt; \tau_1 &lt; H^0$</td>
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<tr>
<td>mSP[N4]</td>
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<td>mSP[N5]</td>
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<tr>
<td>mSP[N6]</td>
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<td>$H^0 &lt; A^0 &lt; H^\pm &lt; \chi^+_1$</td>
<td>0.15</td>
</tr>
<tr>
<td>mSP[H1b]</td>
<td>$H^0 &lt; A^0 &lt; H^\pm &lt; \chi^0_2$</td>
<td>0.06</td>
</tr>
<tr>
<td>mSP[H2]</td>
<td>$H^0 &lt; A^0 &lt; H^\pm &lt; \chi^+_2$</td>
<td>0.15</td>
</tr>
<tr>
<td>mSP[H3]</td>
<td>$H^0 &lt; \chi^0_2 &lt; A^0 &lt; \chi^+_1$</td>
<td>0.02</td>
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<tr>
<td>mSP[H4]</td>
<td>$H^0 &lt; \chi^0_2 &lt; \chi^+_1 &lt; A^0$</td>
<td>0.02</td>
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Table 4.2: Sparticle mass hierarchies for the mSUGRA parameter space (Model [1]), where $\chi^0_1$ is the LSP. The high scale parameters lie in the range $m_0 \in [0.1, 10]$ TeV, $m_{1/2} \in [0.1, 1.5]$ TeV, $\alpha_{\text{em}} \in [-5, 5]$, $\tan \beta \in [2, 50]$, $\mu > 0$, with the constraints $\Omega h^2 < 0.12$, $m_{h^0} > 120$ GeV. Here and in Tables 4.3 to 4.6, the last column gives the percentage with which the patterns appear in the scans detailed in Table 4.1. Please note that the tildes for all sparticles are absent in order to reduce clutter.
<table>
<thead>
<tr>
<th>Pattern Label</th>
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<th>%</th>
<th>Pattern Label</th>
<th>Mass Hierarchy</th>
<th>%</th>
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<tr>
<td>nuSUGRA[2a]</td>
<td>$\chi_0^+ &lt; \chi_0 &lt; \chi_2^0 &lt; \chi_1^0 &lt; \chi_0^-$</td>
<td>35.71</td>
<td>nuSUGRA[2b]</td>
<td>$H^0 &lt; A^0 &lt; H^\pm &lt; H^0$</td>
<td>0.016</td>
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<tr>
<td>nuSUGRA[2c]</td>
<td>$\chi_0^+ &lt; \chi_0^0 &lt; \chi_1^0 &lt; \chi_2^0 &lt; \chi_0^-$</td>
<td>14.57</td>
<td>nuSUGRA[2d]</td>
<td>$H^0 &lt; A^0 &lt; H^\pm &lt; H^0$</td>
<td>0.003</td>
</tr>
<tr>
<td>nuSUGRA[2e]</td>
<td>$\chi_0^+ &lt; \chi_0^0 &lt; \chi_1^0 &lt; \chi_2^0 &lt; \chi_0^-$</td>
<td>0.188</td>
<td>nuSUGRA[2f]</td>
<td>$A^0 &lt; H^0 &lt; H^\pm &lt; H^0$</td>
<td>0.001</td>
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<td>nuSUGRA[2g]</td>
<td>$\chi_0^+ &lt; \chi_0^0 &lt; \chi_1^0 &lt; \chi_2^0 &lt; \chi_0^-$</td>
<td>0.010</td>
<td>nuSUGRA[2h]</td>
<td>$A^0 &lt; H^0 &lt; H^\pm &lt; H^0$</td>
<td>0.001</td>
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<tr>
<td>nuSUGRA[2i]</td>
<td>$\chi_0^+ &lt; \chi_0^0 &lt; \chi_1^0 &lt; \chi_2^0 &lt; \chi_0^-$</td>
<td>0.009</td>
<td>nuSUGRA[2j]</td>
<td>$H^0 &lt; A^0 &lt; \chi_1^0 &lt; \chi_2^0$</td>
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</tr>
<tr>
<td>nuSUGRA[2k]</td>
<td>$\chi_0^+ &lt; \chi_0^0 &lt; \chi_1^0 &lt; \chi_2^0 &lt; \chi_0^-$</td>
<td>0.003</td>
<td>nuSUGRA[2l]</td>
<td>$H^0 &lt; A^0 &lt; \chi_1^0 &lt; \chi_2^0$</td>
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<tr>
<td>nuSUGRA[2m]</td>
<td>$\chi_0^+ &lt; \chi_0^0 &lt; \chi_1^0 &lt; \chi_2^0 &lt; \chi_0^-$</td>
<td>0.001</td>
<td>nuSUGRA[2n]</td>
<td>$H^0 &lt; A^0 &lt; \chi_1^0 &lt; \chi_2^0$</td>
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<td>$\chi_0^+ &lt; \chi_0^0 &lt; \chi_1^0 &lt; \chi_2^0 &lt; \chi_0^-$</td>
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<td>nuSUGRA[2p]</td>
<td>$H^0 &lt; A^0 &lt; \chi_1^0 &lt; \chi_2^0$</td>
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</table>

Table 4.3: Sparticle mass hierarchies for the nuSUGRA light chargino case (Model 2). The high scale parameters lie in the range $m_0 \in [0.1, 10]$ TeV, $M_1 = M_3 = m_{1/2} \in [0.1, 1.5]$ TeV, $M_2 = \alpha m_{1/2}$, $\alpha \in [12, 1]$, $A_{0} \in [-5, 5]$, $\tan \beta \in [2, 50]$, $\mu > 0$, with the constraints $\Omega h^2 < 0.12$, $m_{H_0} > 120$ GeV. For brevity, tildes are absent for all sparticles.
<table>
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<tr>
<th>Pattern Label</th>
<th>Mass Hierarchy</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>muSP_{3}[C1a]</td>
<td>$\chi_t^0 &lt; \chi_2^0 &lt; \chi_1^0 &lt; \chi_1^\pm$</td>
<td>63.273</td>
</tr>
<tr>
<td>muSP_{3}[C1b]</td>
<td>$\chi_t^0 &lt; \chi_2^0 &lt; \chi_1^0 &lt; \chi_1^\pm$</td>
<td>10.263</td>
</tr>
<tr>
<td>muSP_{3}[C1c]</td>
<td>$\chi_1^\pm &lt; \chi_2^0 &lt; \chi_0^0 &lt; H^0$</td>
<td>4.587</td>
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<tr>
<td>muSP_{3}[C1d]</td>
<td>$\chi_1^\pm &lt; \chi_2^0 &lt; \chi_0^0 &lt; H^0$</td>
<td>4.243</td>
</tr>
<tr>
<td>muSP_{3}[C1e]</td>
<td>$\chi_1^\pm &lt; \chi_2^0 &lt; \chi_0^0 &lt; H^0$</td>
<td>4.549</td>
</tr>
<tr>
<td>muSP_{3}[C1f]</td>
<td>$\chi_1^\pm &lt; \chi_2^0 &lt; \chi_0^0 &lt; H^0$</td>
<td>0.482</td>
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<tr>
<td>muSP_{3}[C2a]</td>
<td>$\chi_1^0 &lt; \chi_2^0 &lt; \chi_0^0 &lt; \mu_r$</td>
<td>0.854</td>
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<tr>
<td>muSP_{3}[C2b]</td>
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<td>0.647</td>
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<td>muSP_{3}[C2c]</td>
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<tr>
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<tr>
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<tr>
<td>muSP_{3}[C3c]</td>
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<td>0.555</td>
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<tr>
<td>muSP_{3}[C3d]</td>
<td>$\chi_1^0 &lt; \chi_2^0 &lt; \chi_0^0 &lt; \mu_r$</td>
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<tr>
<td>muSP_{3}[C4]</td>
<td>$\chi_1^0 &lt; \chi_2^0 &lt; H^0 &lt; A^0$</td>
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<td>$\chi_1^0 &lt; \chi_2^0 &lt; H^0 &lt; A^0$</td>
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</tr>
<tr>
<td>muSP_{3}[C6a]</td>
<td>$\chi_1^0 &lt; \chi_2^0 &lt; \mu_r &lt; \chi_0^0$</td>
<td>0.096</td>
</tr>
<tr>
<td>muSP_{3}[C6b]</td>
<td>$\chi_1^0 &lt; \chi_2^0 &lt; \mu_r &lt; \chi_0^0$</td>
<td>0.096</td>
</tr>
<tr>
<td>muSP_{3}[C6c]</td>
<td>$\chi_1^0 &lt; \chi_2^0 &lt; H^0 &lt; A^0$</td>
<td>0.014</td>
</tr>
<tr>
<td>muSP_{3}[C6d]</td>
<td>$\chi_1^0 &lt; \chi_2^0 &lt; H^0 &lt; A^0$</td>
<td>0.014</td>
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<tr>
<td>muSP_{3}[C7]</td>
<td>$\chi_1^0 &lt; g &lt; \chi_2^0 &lt; \chi_3^0$</td>
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<tr>
<td>muSP_{3}[C8]</td>
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<td>$\chi_2^0 &lt; \chi_3^0 &lt; H^0 &lt; A^0$</td>
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<tr>
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<td>$\chi_2^0 &lt; \chi_3^0 &lt; H^0 &lt; A^0$</td>
<td>0.014</td>
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<tr>
<td>muSP_{3}[N2]</td>
<td>$\chi_3^0 &lt; \chi_4^0 &lt; g &lt; \chi_0^0$</td>
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<td>muSP_{3}[N3a]</td>
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<tr>
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<td>0.028</td>
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<tr>
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<td>$\chi_2^0 &lt; \chi_3^0 &lt; \tau &lt; H^0$</td>
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<tr>
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<td>$\chi_2^0 &lt; \chi_3^0 &lt; \tau &lt; H^0$</td>
<td>0.014</td>
</tr>
<tr>
<td>muSP_{3}[g1a]</td>
<td>$g &lt; \chi_0^0 &lt; \chi_1^0 &lt; \chi_2^0$</td>
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<tr>
<td>muSP_{3}[g1b]</td>
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<td>$g &lt; \chi_1^0 &lt; \chi_2^0 &lt; \chi_3^0$</td>
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<tr>
<td>muSP_{3}[H1]</td>
<td>$H^0 &lt; A^0 &lt; \chi_0^0 &lt; \chi_1^\pm$</td>
<td>0.069</td>
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<td>muSP_{3}[H2]</td>
<td>$H^0 &lt; A^0 &lt; \chi_0^0 &lt; \chi_1^\pm$</td>
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<tr>
<td>muSP_{3}[H3a]</td>
<td>$H^0 &lt; A^0 &lt; \chi_0^0 &lt; \chi_1^\pm$</td>
<td>0.069</td>
</tr>
<tr>
<td>muSP_{3}[H3b]</td>
<td>$H^0 &lt; A^0 &lt; \chi_0^0 &lt; \chi_1^\pm$</td>
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<td>muSP_{3}[H3c]</td>
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<td>$A^0 &lt; H^0 &lt; \chi_1^\pm &lt; \chi_2^0$</td>
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Table 4.4: Particle mass hierarchies for the muSUGRA light gluino case (Model [3]).

The high scale parameters lie in the range $m_0 \in [0.1, 10] \text{TeV}$, $M_1 = M_2 = m_{1/2} \in [0.1, 1.5] \text{TeV}$, $M_3 = am_{1/2}$, $\alpha \in [12, 1]$, $\frac{A_0}{m_0} \in [-5, 5]$, tan $\beta \in [2, 50]$, $\mu > 0$, with the constraints $\Omega h^2 < 0.12$, $m_{h^0} > 120 \text{GeV}$. For brevity, tildes are absent for all sparticles.
<table>
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<th>Pattern Label</th>
<th>Mass Hierarchy</th>
<th>%</th>
<th>Pattern Label</th>
<th>Mass Hierarchy</th>
<th>%</th>
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<tr>
<td>nuSP_C1a</td>
<td>$\chi_1^\pm &lt; \chi_2^0 &lt; \chi_3^0 &lt; \chi_4^0$</td>
<td>55.065</td>
<td>nuSP_N1a</td>
<td>$\chi_2^0 &lt; \chi_3^+ &lt; H^0 &lt; A^0$</td>
<td>2.216</td>
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<td>$\chi_1^+ &lt; \chi_2^0 &lt; \chi_3^+ &lt; \chi_4^0 &lt; \tau_1$</td>
<td>12.727</td>
<td>nuSP_N1b</td>
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<tr>
<td>nuSP_C1e</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; \chi_3^0 &lt; \chi_4^0 &lt; \tau_1$</td>
<td>0.886</td>
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<td>$\chi_2^0 &lt; \chi_3^+ &lt; \tau_1 &lt; \chi_4^0$</td>
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<tr>
<td>nuSP_C1f</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; \chi_3^0 &lt; \chi_4^0 &lt; \tau_1$</td>
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<td>$\chi_1^+ &lt; \chi_2^0 &lt; \chi_3^0 &lt; \chi_4^0 &lt; H^\pm$</td>
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<tr>
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<td>1.224</td>
<td>nuSP_N4b</td>
<td>$\chi_2^0 &lt; \chi_3^+ &lt; \tau_1 &lt; \chi_4^0$</td>
<td>0.042</td>
</tr>
<tr>
<td>nuSP_C2b</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; H^0 &lt; A^0$</td>
<td>0.549</td>
<td>nuSP_N4c</td>
<td>$\chi_2^0 &lt; \chi_3^+ &lt; \chi_4^0 &lt; A^0$</td>
<td>0.042</td>
</tr>
<tr>
<td>nuSP_C3a</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; \tau_1 &lt; \chi_4^0$</td>
<td>1.203</td>
<td>nuSP_N4d</td>
<td>$\chi_2^0 &lt; \chi_3^+ &lt; \tau_1 &lt; H^\pm$</td>
<td>0.021</td>
</tr>
<tr>
<td>nuSP_C3b</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; \tau_1 &lt; H^0$</td>
<td>0.042</td>
<td>nuSP_N5</td>
<td>$\chi_2^0 &lt; \chi_3^+ &lt; \tau_1 &lt; g$</td>
<td>0.021</td>
</tr>
<tr>
<td>nuSP_C3c</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; \tau_1 &lt; A^0$</td>
<td>0.021</td>
<td>nuSP_1a</td>
<td>$t_1 &lt; \chi_3^+ &lt; H^\pm &lt; \chi_4^0$</td>
<td>0.359</td>
</tr>
<tr>
<td>nuSP_C4</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; \tau_1 &lt; \chi_3^0$</td>
<td>0.169</td>
<td>nuSP_1b</td>
<td>$t_1 &lt; \chi_2^0 &lt; \chi_3^+ &lt; \chi_4^0$</td>
<td>0.317</td>
</tr>
<tr>
<td>nuSP_C5</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; A^0 &lt; H^0 &lt; \chi_2^0$</td>
<td>0.084</td>
<td>nuSP_1c</td>
<td>$t_1 &lt; \chi_2^0 &lt; \chi_3^+ &lt; \tau_1$</td>
<td>0.127</td>
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<tr>
<td>nuSP_C6</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; H^0 &lt; \chi_3^0$</td>
<td>0.063</td>
<td>nuSP_1d</td>
<td>$t_1 &lt; \chi_2^0 &lt; \chi_3^+ &lt; H^0$</td>
<td>0.042</td>
</tr>
<tr>
<td>nuSP_C7</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; \tau_1 &lt; \chi_4^0$</td>
<td>0.021</td>
<td>nuSP_1e</td>
<td>$t_1 &lt; \chi_2^0 &lt; \chi_3^+ &lt; \tau_1$</td>
<td>0.021</td>
</tr>
<tr>
<td>nuSP_C8</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; H^\pm &lt; A^0$</td>
<td>0.021</td>
<td>nuSP_2a</td>
<td>$t_1 &lt; \chi_1^+ &lt; \chi_2^0 &lt; \chi_4^0$</td>
<td>0.106</td>
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<tr>
<td>nuSP_1a</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; H^0$</td>
<td>2.617</td>
<td>nuSP_2b</td>
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<td>0.021</td>
</tr>
<tr>
<td>nuSP_1b</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; \chi_3^0 &lt; \chi_4^0 &lt; \mu_\tau$</td>
<td>0.971</td>
<td>nuSP_3a</td>
<td>$t_1 &lt; H^0 &lt; A^0 &lt; \chi^+_1$</td>
<td>0.127</td>
</tr>
<tr>
<td>nuSP_1c</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; \chi_3^0 &lt; \chi_4^0 &lt; \nu_\tau$</td>
<td>0.802</td>
<td>nuSP_3b</td>
<td>$t_1 &lt; H^0 &lt; A^0 &lt; H^\pm &lt; \chi^+_1$</td>
<td>0.021</td>
</tr>
<tr>
<td>nuSP_2a</td>
<td>$\chi_1^+ &lt; \mu_\tau &lt; \chi_1^0 &lt; \chi_2^0 &lt; \chi_3^0 &lt; \chi_4^0$</td>
<td>0.021</td>
<td>nuSP_H1a</td>
<td>$H^0 &lt; A^0 &lt; H^\pm &lt; \chi^+_1$</td>
<td>0.127</td>
</tr>
<tr>
<td>nuSP_2b</td>
<td>$\chi_1^+ &lt; \mu_\tau &lt; \chi_1^0 &lt; \mu_\tau$</td>
<td>2.849</td>
<td>nuSP_H1b</td>
<td>$H^0 &lt; A^0 &lt; H^\pm &lt; \chi^+_1$</td>
<td>0.021</td>
</tr>
<tr>
<td>nuSP_2c</td>
<td>$\chi_1^+ &lt; \mu_\tau &lt; \chi_1^0 &lt; \chi_2^0 &lt; \chi_3^0 &lt; \chi_4^0$</td>
<td>1.224</td>
<td>nuSP_H2a</td>
<td>$H^0 &lt; A^0 &lt; \chi_2^0 &lt; \chi^+_1$</td>
<td>0.084</td>
</tr>
<tr>
<td>nuSP_2d</td>
<td>$\chi_1^+ &lt; \mu_\tau &lt; \chi_2^0 &lt; \chi_3^0 &lt; \chi_4^0 &lt; \mu_\tau$</td>
<td>0.021</td>
<td>nuSP_H2b</td>
<td>$H^0 &lt; H^0 &lt; \chi^+_1$</td>
<td>0.021</td>
</tr>
<tr>
<td>nuSP_3a</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; \chi_3^0 &lt; \chi_4^0 &lt; \mu_\tau$</td>
<td>1.625</td>
<td>nuSP_H3</td>
<td>$A^0 &lt; H^0 &lt; \chi^+_1 &lt; H^\pm$</td>
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<tr>
<td>nuSP_3b</td>
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<td>0.295</td>
<td>nuSP_H4</td>
<td>$H^0 &lt; \chi_2^0 &lt; \chi^+_1 &lt; H^\pm$</td>
<td>0.021</td>
</tr>
<tr>
<td>nuSP_3c</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; \chi_3^0 &lt; \chi_4^0 &lt; \mu_\tau$</td>
<td>0.084</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>nuSP_4a</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; \chi_3^0 &lt; \chi_4^0 &lt; \mu_\tau$</td>
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<td>0.063</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>nuSP_4c</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; \chi_3^0 &lt; \chi_4^0 &lt; \mu_\tau$</td>
<td>0.021</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>nuSP_4d</td>
<td>$\chi_1^+ &lt; \chi_2^0 &lt; \chi_3^0 &lt; \chi_4^0 &lt; \mu_\tau$</td>
<td>0.021</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5: Sparticle mass hierarchies for the muSUGRA nonuniversal Higgs case (Model [4]). The high scale mass parameters lie in the range $m_0 \in [0.1, 1.0]$ TeV, $m_{1/2} \in [0.1, 1.5]$ TeV, $\Delta m^2(\text{m}) \in [-5, 5]$, $\tan \beta \in [2, 50]$, $\mu > 0$, with the constraints $\Omega h^2 < 0.12$, $m_{3/2} > 120$ GeV. However, the Higgs masses at the GUT scale are nonuniversal, $m_{H_i}(M_{G}) = m_0(1 + \delta_i)$, $i = 1, 2$ where $\delta_i \in [-0.9, 1]$. For brevity, tildes are absent for all sparticles.
Table 4.6: Sparticle mass hierarchies for the nuSUGRA light third generation case (Model [5]). The high scale parameters lie in the range \( m_0^{(1)} = m_0^{(2)} = m_0 \in [0.1, 10] \) TeV, \( m_0^{(3)} = \frac{m_0^2}{1 + \tan \beta \tan \gamma} \), \( m_{1/2} \in [0.1, 1.5] \) TeV, \( \frac{A_0}{m_0} \in [-5, 5] \), \( \tan \beta \in [2, 50] \), \( \mu > 0 \), with the constraints \( \Omega h^2 < 0.12 \), \( m_{h^0} > 120 \) GeV. For brevity, tildes are absent for all sparticles.
Chapter 5

Mass Hierarchies & Simplified Models

Recently a new avenue for the exploration of new physics at colliders has been explored via the so-called simplified models \[57–60, 62–65\]. For example, one might consider a system of 3 particles: \(A, B, C\) with masses \(m_A, m_B, m_C\) and the hierarchy

\[ m_A > m_B > m_C . \] \hspace{1cm} \text{(5.1)}

One further assumes that the branching ratio of the decay of \(A\) into the state \(B\) is 100%, and likewise the branching ratio of \(B\) to \(C\) is also 100%. More generally, one could also have \(A\) going directly to \(C\), but often in simplified models the direct decay of \(A\) to \(C\) is ignored. One could also consider simplified models including four particles \(A, B, C, D\) with the mass hierarchy

\[ m_A > m_B > m_C > m_D . \] \hspace{1cm} \text{(5.2)}

Here one has six allowed branchings \(A \rightarrow \{B, C, D\}, B \rightarrow \{C, D\}\) and \(C \rightarrow D\). Again, the simplified assumption would be to consider just a cascade type decay \(A \rightarrow B \rightarrow C \rightarrow D\), which involves only three branchings which can all be assumed to be 100%. Specific
examples of three, four and five particle simplified models are given by the decay chains

\[
\chi^0_2 \rightarrow \chi^\pm_1 \rightarrow \chi^0_1, \quad (5.3)
\]

\[
\tilde{g} \rightarrow \chi^0_2 \rightarrow \chi^\pm_1 \rightarrow \chi^0_1, \quad (5.4)
\]

\[
\chi^0_3 \rightarrow \tilde{g} \rightarrow \chi^0_2 \rightarrow \chi^\pm_1 \rightarrow \chi^0_1. \quad (5.5)
\]

In simplified models, one makes an ad hoc choice of the lightest particles and their decay chains. However, a simplified model with particles chosen in this fashion may not be embeddable in a high scale model. Thus, it is worthwhile to investigate the classes of simplified models that can arise from truncation of sparticle mass hierarchies generated from a high scale model. In this way, one can work with simplified models at the level of a preliminary investigation, keeping in mind that it is a truncation of a UV-complete model, which would eventually replace the simplified model by a more complete one. In this work, we give a fairly exhaustive analysis\(^1\) of the various classes of simplified models that arise in truncation of sparticle mass hierarchies in supergravity unified models, using universal as well as nonuniversal boundary conditions, i.e., mSUGRA as well as nuSUGRA models as discussed in Chapter 4. The results of the mass hierarchies that arise for the lightest five particles (including the LSP) are given in Table 4.2 for the mSUGRA case and in Tables 4.3 to 4.6 for the nuSUGRA cases. Note that one can always truncate these patterns to give mass patterns for just three or four particle mass hierarchies.

An illustration of several simplified models arising from mSUGRA and from nonuniversal SUGRA models is given in Fig. 5.1 where the masses of the particles are in ascending order. One can use each of the columns to generate three, four or five particle simplified models. Most of the current work centers around keeping just three particles in the analysis. Such an approximation is valid if the mass gaps between the first three and the remaining higher ones are sufficiently large such that their inclusion would not radically change the signature calculus. However, often this is not the case. In mSUGRA, often \(\chi^\pm_1\) and \(\chi^0_2\) are essentially degenerate, as are \(H^0, A^0\), and \(H^\pm\); thus, a truncation will lead to

\(^1\) Some of the preliminary results were presented at SUSY2014 [89].
**Figure 5.1:** An illustration of simplified models with five lightest particles arising from SUGRA unified models where the masses are in ascending order. Simplified models with a lower number of particles can be obtained by retaining the appropriate lower number of particles in the mass hierarchy. A more complete set of three, four and five particle simplified models can be obtained by retaining three, four and five particles in Table 4.2 and in Table 4.3 - Table 4.6.

significant errors. Additionally, if there is a strongly interacting particle lying just above the lowest three, a neglect of this particle will also lead to erroneous results.

We discuss further the relative advantages and disadvantages of the simplified models. In the analysis of the simplified models, the coupling and the masses of the particles can be taken to be adjustable parameters. This procedure circumvents carrying out radiative breaking of the electroweak symmetry every time the parameters are varied. Here one already has the masses and the couplings and so the task of fitting the data is made easier. Further, the simplified models can explore the phase space of signatures which may otherwise be forbidden from the point of view of well-motivated models with constraints such as, for example, the constraint of radiative breaking of the electroweak symmetry. However, the disadvantage is that some of the models may not be embeddable in a UV-complete theory and the connection of such models with fundamental physics one is trying to explore becomes tenuous. On the other hand, if one uses the simplified models arising from the supergravity unified models, the connection with a high scale theory is much stronger.

<table>
<thead>
<tr>
<th>mSP[C2]</th>
<th>mSP[C3]</th>
<th>mSP[τ4]</th>
<th>nuSP3[t6a]</th>
<th>nuSP3[τ4]</th>
<th>nuSP3[g3]</th>
<th>nuSP3[τ4]</th>
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<tbody>
<tr>
<td>$A^0$</td>
<td>$\chi_2^0$</td>
<td>$\chi_1^2$</td>
<td>$\chi_2^2$</td>
<td>$\chi_1^2$</td>
<td>$\chi_3^2$</td>
<td>$H^\pm$</td>
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<tr>
<td>$H^0$</td>
<td>$\tilde{\chi}_2$</td>
<td>$\chi_2^0$</td>
<td>$\chi_1^2$</td>
<td>$\chi_2^0$</td>
<td>$\chi_3^0$</td>
<td>$A^0$</td>
</tr>
<tr>
<td>$\chi_2^0$</td>
<td>$\chi_1^0$</td>
<td>$\tau_1$</td>
<td>$\tilde{\tau}_1$</td>
<td>$\tilde{\chi}_2$</td>
<td>$\tilde{\tau}_1$</td>
<td>$H^u$</td>
</tr>
<tr>
<td>$\chi_3^0$</td>
<td>$\chi_1^0$</td>
<td>$\tilde{\tau}_1$</td>
<td>$\tilde{\tau}_1$</td>
<td>$\tilde{\tau}_1$</td>
<td>$\tilde{\tau}_1$</td>
<td>$\tilde{\tau}_1$</td>
</tr>
<tr>
<td>$\chi_3^0$</td>
<td>$\chi_1^0$</td>
<td>$\tilde{\tau}_1$</td>
<td>$\tilde{\tau}_1$</td>
<td>$\tilde{\tau}_1$</td>
<td>$\tilde{\tau}_1$</td>
<td>$\tilde{\tau}_1$</td>
</tr>
</tbody>
</table>
While the simplified models are much easier to deal with, the approximation of using just a few particles can be very limiting as noted earlier. For example, the case where several particles are clustered, the approximation of just keeping the three lowest ones is not justified. An example of this is given in Fig. 5.2, where the electroweak gauginos are packed closely together. Also as noted above, if there is a strongly interacting particle in the vicinity, then an analysis that ignores this would lead to error-prone results. Additionally, we note that any randomly chosen three light particles would not necessarily arise from the spectrum of a UV-complete theory.

Notice that in Table 4.2 as well in Tables 4.3 to 4.6, the first five particles are mostly electroweak. The colored particles, when present, appear with lower frequency as given by the percentage of occurrence shown in the last column of the tables. The reason for this phenomenon can be traced, in part, to the largeness of the Higgs mass and to the lower experimental limit on the gluino mass of around 1 TeV. Thus, the Higgs boson mass of $\sim 125-126$ GeV requires that the average SUSY scale

$$M_s \sim \sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}},$$

be in the few TeV range to produce the loop correction necessary to raise the Higgs boson mass from the tree value of $\leq M_Z$ to the experimentally observed value. In mSUGRA, a large $M_s$ can arise from a large $m_0$, a large gluino mass, or by a combination of both. For large $m_0$, all the squarks and sleptons, except for the possibility of a light stop, will be above the TeV region. Combined with the current experimental lower limit on the gluino mass of roughly a TeV, these constraints imply that all of the colored particles, except for the possibility of a light stop or gluino, will not populate the lightest set of sparticle patterns. There is, however, an alternate possibility of generating a large $M_s$ needed for producing the large loop correction to lift the tree level Higgs boson mass. Here, one can choose $m_0$ to be in the low $O(100 \text{ GeV})$ region, and choose the gluino mass at the GUT scale to be in the several TeV region [46]. In this case, the RG running drives the squark masses to high values while the slepton masses remain low since they do not receive
Figure 5.2: An exhibition of particle clustering arising from a mSUGRA model, where $m_0 = 6038\, \text{GeV}$, $m_{1/2} = 538\, \text{GeV}$, $A_0 = -2734\, \text{GeV}$, $\tan \beta = 10$ and $\text{sgn}(\mu) > 0$.

Large corrections from RG evolution. In each of these cases, one finds that the low-mass sparticle spectrum will contain mostly the electroweak particles and color particles would be rare, often consisting of just the stop or the gluino consistent with the experimental lower mass limits. Thus, the result of this analysis has important implications regarding sparticle searches. Namely, the focus of the searches should be geared to look for the electroweak particles and in the color sector for the stop and the gluino, which are often the lightest colored particles.
Chapter 6

SUGRA Model Signatures

The most effective signatures for the LHC Run-II will be strongly correlated with the type of mass hierarchical patterns considered. Below we discuss a few illustrative examples and the possible signatures associated with the chosen patterns. First, let us consider the sparticle production for the hierarchical patterns mSP[C1] - mSP[C4]. If we retain only the three lightest particles, then the patterns mSP[C1] - mSP[C4] are indistinguishable. To discriminate among the subcases, for example, between mSP[C1] and mSP[C2], we need to consider at least a four particle mass hierarchy. For both of these patterns, the first three particles have the hierarchy \( \chi_1^0 < \chi_1^+ < \chi_2^0 \), but differ in the placement of the 4th lightest particle. For mSP[C1], it is \( \chi_3^0 \), while for mSP[C2], it is \( H^0 \). A similar situation arises for the hierarchical mass patterns nuSP_2[C1] - nuSP_2[C5]. Here, these five patterns can also only be discriminated by considering more than three particles. Further, the only difference between mSP[C1] - mSP[C4] and nuSP_2[C1] - nuSP_2[C5] relates to the mass gaps between the three sparticle states. If we keep only the first three particles in the mass hierarchies, then the particles likely to be produced at the colliders for the patterns mSP[C1] - mSP[C4] and nuSP_2[C1] - nuSP_2[C5] are as follows,

\[
\text{mSP}[\text{C1-C4}, \text{nuSP}_2[\text{C1-C5}]] : \chi_1^\pm \chi_1^\mp, \chi_1^\pm \chi_1^0, \chi_1^0 \chi_1^0, \chi_1^\pm \chi_2^0, \chi_2^0 \chi_2^0. \tag{6.1}
\]
These sparticle pairs would arise from the parton-level processes $q\bar{q} \rightarrow \chi_i^\pm \chi_j^\mp, \chi_k \chi_\ell^0, u\bar{d} \rightarrow \chi_i^+ \chi_k^0$, etc. The chargino can decay such that $\chi_1^- \rightarrow W^− \chi_1^0$, where $W^− \rightarrow \ell^− \nu$, yielding $\ell^− + E_T^{miss}$ in the decay of the chargino. Thus, $\chi_1^+ \chi_1^0$ will produce a charged lepton and missing energy. Next, $\chi_1^+ \chi_1^-$ will produce two charged leptons and $E_T^{miss}$. Since a chargino can also decay via a $W^*$ into $q_i \bar{q}_2 + \chi_1^0$, we can have, in addition, a single charged lepton, jets and $E_T^{miss}$. Now, $\chi_2^0$ will have decays such as $\ell^+ \ell^- + E_T^{miss}$, which implies that $\chi_1^+ \chi_2^0$ will have trileptonic decays [90–93] $\ell_1^+ \ell_2^+ \ell_2^- + E_T^{miss}$. The production channels $\chi_1^0 \chi_2^0$ will have decays of the type $\ell^+ \ell^- + E_T^{miss}$ and $\chi_2^0 \chi_2^0$ will have 4-lepton decays $\ell_1^+ \ell_1^- \ell_2^+ \ell_2^- + E_T^{miss}$ as well as decays of the type $\ell^+ \ell^- + jets + E_T^{miss}$. For cases where the $\chi_2^0$ mass is sufficiently large, such that $m_{\chi_2^0} > m_{\chi_1^0} + m_{h^0}$, we can have on-shell decays $\chi_2^0 \rightarrow \chi_1^0 h^0$, and $h^0$ predominantly decays via the mode $h^0 \rightarrow b\bar{b}$. This gives rise to important new signatures such as $\chi_1^+ \chi_2^0 \rightarrow \ell^+ b\bar{b} + E_T^{miss}$ and $\chi_2^0 \chi_2^0 \rightarrow \ell^+ \ell^- b\bar{b}$.

Next, we consider the patterns nuSP_2[C6] and nuSP_2[C7], which have the same lowest three-particle pattern, $\chi_1^0, \chi_1^+, \tilde{\tau}_1$ with the mass hierarchy $m_{\chi_1^0} < m_{\chi_1^+} < m_{\tilde{\tau}_1}$. Again, to distinguish between nuSP_2[C6] and nuSP_2[C7], we would need to consider a four-particle mass hierarchy. For these mass patterns we can produce a chargino-neutralino, two charginos or two staus in $pp$ collisions,

$$nuSP_2[C6-C7] : \chi_1^0 \chi_1^+, \chi_1^+ \chi_1^-, \tilde{\tau}_1 \tilde{\tau}_1^+.$$  \hspace{1cm} (6.2)

The signatures arising from $\chi_1^+ \chi_1^0$ and $\chi_1^+ \chi_1^-$ production are as discussed for mSP[C1]-mSP[C5] and nuSP_2[C1]-nuSP_2[C5]. Here, we additionally have $\tilde{\tau}_1 \tilde{\tau}_1^+$ production. Since $\tilde{\tau}_1$ has the decay $\tilde{\tau}_1 \rightarrow \tau \chi_1^0$, we will have a signature of the type $\tau^+ \tau^- + E_T^{miss}$. Additionally, since $\tilde{\tau}_1$ is heavier than the chargino, we will have the decay $\tilde{\tau}_1^- \rightarrow \chi_1^- \nu$, with $\chi_1^- \rightarrow \ell^- \nu$. This will lead to signatures such as $\ell_i^+ \ell_j^- + E_T^{miss}$, where $\ell_i = e, \mu, \tau$. Closely related to the signatures arising from nuSP_2[C6] and nuSP_2[C7] are the signatures arising from nuSP_2[τ2a] and nuSP_2[τ3], where we have the same three particles but the mass hierarchy is inverted for the top two, $m_{\chi_1^0} < m_{\tilde{\tau}_1} < m_{\chi_1^\pm}$. Thus, the final states produced for these patterns will be the same as in nuSP_2[C6] and nuSP_2[C7], except that here the chargino
is heavier than the stau. Consequently, instead of stau decay into a chargino, we have a chargino decaying into a stau, $\chi^-_1 \rightarrow \tilde{\tau}^-_1 \tilde{\nu}_\tau$.

Next, we consider the pattern $\nu SP_2[C9]$ where the three lightest particles are $\chi^0_1$, $\chi^\pm_1$, and $\tilde{t}_1$ with the mass hierarchy $m_{\chi^0_1} < m_{\chi^\pm_1} < m_{\tilde{t}_1}$. The sparticle states produced in $pp$ collisions are as follows,

$$\nu SP_2[C9] : \chi^\pm_1 \chi^0_1, \chi^+_1 \chi^-_1, \tilde{t}_1 \tilde{\nu}_1^*.$$ (6.3)

The new production mode here is $\tilde{t}_1 \tilde{\nu}_1^*$, where $\tilde{t}_1$ has the decay channels

$$\tilde{t}_1 \rightarrow t \chi^0_1, b \chi^\pm_1, c \chi^0_1.$$ (6.4)

Further, the chargino will have the decay $\chi^\pm_1 \rightarrow W^\pm \chi^0_1 \rightarrow \ell \nu \chi^0_1, jj \chi^0_1$ and the top quark has the decays

$$t \rightarrow jjb, \ell \nu b.$$ (6.5)

Thus, $\tilde{t}_1 \tilde{\nu}_1^*$ production will lead to a variety of signals involving leptons, jets and missing energy such as the signals $\ell^+ \ell^- b\bar{b} + E_T^{miss}$, $\ell b \bar{b} +$ jets $+ E_T^{miss}$, etc.

The final illustrative example we give is the mass pattern $\nu SP_3[g1]$, where the lightest three particles are $\chi^0_1$, $\tilde{g}$, and $\chi^0_2$, characterized by the mass hierarchy $m_{\chi^0_1} < m_{\tilde{g}} < m_{\chi^0_2}$. The sparticle states produced in $pp$ collisions consist of the following,

$$\nu SP_3[g1] : \tilde{g}\tilde{g}, \chi^0_2 \chi^0_2.$$ (6.6)

The new production mode here is $\tilde{g}\tilde{g}$, which will dominate the signatures since the gluino interacts strongly and the production cross section for $\tilde{g}\tilde{g}$ will be much greater than that of electroweak gaugino production. The gluino will have the decay $\tilde{g} \rightarrow t \tilde{\nu}_1^*$, where the decays of $\tilde{t}_1$ are given in Equation (6.4) and the decays of the top in Equation (6.5). These will generate a variety of signatures involving leptons, at least 2 $b$-jets, light jets and missing energy.
An overall issue in the analysis of signatures concerns the NLSP and LSP mass difference. This mass difference determines the $p_T$ of the jets, leptons and the $E_T^{\text{miss}}$ in the NLSP decay. A small mass difference between the NLSP and the LSP will lead to softer jets and leptons and a small $E_T^{\text{miss}}$ may not pass the cuts or be distinguishable from the background. However, there are a variety of other signatures that can be investigated. For recent analyses relating to sparticle signature identification at the upgraded LHC, see [94, 95]. An interesting issue relates to how one may discriminate among “mirror patterns”. Consider, for example, the third and the fourth columns of Fig. 5.1. Retaining only the three lowest mass particles, column three has the hierarchy $\chi^0_1 < \tilde{\tau}_1 < \tilde{t}_1$ while column four has the hierarchy $\chi^0_1 < \tilde{t}_1 < \tilde{\tau}_1$. Thus, aside from the LSP, the spectrum for column four is inverted relative to that for column three. In a similar fashion the lightest three particle spectrum arising from column five is $\chi^0_1 < \tilde{t}_1 < \tilde{g}$, while that from column six is $\chi^0_1 < \tilde{g} < \tilde{t}_1$. One may call such pairs mirror patterns. It should be interesting to investigate the characteristic signals that can discriminate between such complementary mirror patterns.

As mentioned, a study of signatures based on the three lowest-lying particles would not lift the degeneracy among those patterns which have the same three lowest-mass particles; therefore, we would need to include higher-lying particles to discriminate among the patterns. Further, as discussed in Chapter 5, keeping just the three lowest-lying particles is inadequate when there is a clustering of particles as illustrated in Fig. 5.2. In this case, all particles within the cluster must be accounted for. Another example where truncation to three or four particles is inadequate is when there is a strongly interacting particle lying closely above. Again, in this case, one may be led to erroneous results by the truncation procedure of constructing a simplified model.
Benchmarks for Discovery at the LHC

Benchmarks are useful as instructive examples of signature analyses that can lead to new discovery channels for superpartner particles. Here we give a few benchmarks which satisfy all the current collider, flavor and cosmological constraints. Specifically, we impose the following set of constraints in choosing the benchmarks,

\[ m_{h^0} \in [123, 127] \text{GeV}, \]  
\[ \Omega_{\chi_0^0} h^2 < 0.12, \]  
\[ Br(B_s^0 \to \mu^+\mu^-) < 6.2 \times 10^{-9}, \]  
\[ Br(B \to X_s\gamma) < 4.27 \times 10^{-4}. \]

Additionally, we only select benchmarks that have sparticle mass hierarchies that are frequently observed in the constrained parameter space. In Table 7.1, we give a set of benchmarks for both universal and nonuniversal SUGRA cases. We also identify the corresponding SUGRA pattern to which they belong. The benchmarks that are displayed satisfy the LHC Run-I exclusion constraint in the \((m_0, m_{1/2})\) plane. These benchmarks should be useful for future SUSY searches at colliders. An interesting feature of Table 7.1 is that most of the benchmarks have relatively small \(\mu\) compared to \(m_0\), which points to
the fact that they lie on the hyperbolic branch of radiative breaking of the electroweak symmetry [96–100] and are thus natural according to the criteria discussed in [96].

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
SUGRA Hierarchy & \(m_0\) (GeV) & \(m_{1/2}\) (GeV) & \(A_0\) (GeV) & \(\text{tan}\beta\) & \(\delta M_2\) & \(\delta M_3\) & \(\delta H_1, \delta H_2\) & \(\delta M_{43}\) & \(\mu\) (GeV) & \(m_{\chi^0_1}\) (GeV) \\
\hline
mS\([C1a]\) & 6183 & 470 & -4469 & 52 & - & - & - & - & 269 & 126.1 \\
mS\([C1a]\) & 3715 & 1080 & 706 & 52 & - & - & - & - & 569 & 123.1 \\
mS\([C1a]\) & 5464 & 1049 & 4845 & 52 & -0.063 & - & - & - & 583 & 124.0 \\
mS\([C1a]\) & 2005 & 1234 & -3105 & 32 & -0.446 & - & - & - & 1999 & 125.4 \\
mS\([C1a]\) & 5446 & 500 & -3940 & 24 & -0.524 & - & - & - & 285 & 125.3 \\
mS\([g1a]\) & 5019 & 846 & 7759 & 15 & -0.819 & - & - & - & 2066 & 123.4 \\
mS\([C1a]\) & 1210 & 848 & -1656 & 26 & - & - & (-0.830, -1.205) & - & 571 & 123.5 \\
mS\([g2a]\) & 591 & 901 & -1746 & 31 & - & - & (-2.089, -6.704) & - & 1419 & 123.6 \\
mS\([C1a]\) & 2007 & 1155 & -989 & 48 & - & - & - & -0.361 & 589 & 123.3 \\
mS\([C2a]\) & 2301 & 1241 & -2185 & 31 & - & - & - & -0.584 & 541 & 126.0 \\
\hline
\end{tabular}
\caption{Benchmarks are given for SUGRA Models [1]-[5]. The sparticle mass hierarchy is specified for each benchmark. These particular model points are chosen due to having a mass pattern which has an especially large percentage of occurrence and also having passed collider, flavor and cosmological constraints. Further, they satisfy \(\frac{m_{\text{NLSP}} + m_{\text{NNLSP}}}{2} < 600\) GeV and have a maximum NLSP-LSP mass gap.}
\end{table}

7.1 mSUGRA Benchmark Analysis

First, we consider a benchmark within mSUGRA. Our model’s GUT scale parameters and observables are given in Table 7.2. The particle mass hierarchy for this model point is \(m_{\chi^0_1} < m_{\chi^0_2} < m_{\chi^0_3} < m_{\chi^0_4}\). This benchmark has a light LSP of mass \(m_{\chi^0_1} = 199\) GeV, a chargino NLSP of mass \(m_{\chi^\pm_1} = 261\) GeV and a second neutralino of mass \(m_{\chi^0_2} = 271\) GeV. The first and the second generation squarks are heavy, \(m_{\tilde{q}} \gtrsim 6\) TeV, the stop and the sbottom are \(m_{\tilde{t}_1} = 3.6\) TeV, \(m_{\tilde{b}_1} = 4\) TeV, and a gluino at \(m_{\tilde{g}} = 1257\) GeV. With the light neutralinos/charginos and a relatively light gluino, the total SUSY cross section is 740 fb at 14 TeV. Since the rest of the spectrum is heavy, \(\chi^\pm_1\) and \(\chi^0_2\) decay through an off-shell \(W\) and \(Z\) generating relatively soft jets and leptons.

To test the visibility of this model at 14 TeV, we study the trilepton final state and follow the ATLAS search [101]. This analysis is based on a simplified model with only \(\chi^0_1, \chi^0_2, \chi^0_3,\)
and $\chi_1^\pm$. It is optimized for 8 TeV and should be re-tuned for 14 TeV, but we use similar cuts as our starting point. The ATLAS analysis defines the signal region SR0$\tau a$, which is most sensitive to $\chi_1^\pm$ and $\chi_2^0$ decays through $W$ and $Z$ bosons. This signal region requires 3 leptons ($e$ or $\mu$) including a same flavor opposite sign (SFOS) pair. The SFOS pair, giving the invariant mass closest to the $Z$ mass, is then identified and the remaining lepton’s momentum is used to calculate the transverse mass defined as

$$m_T^2(p_T^e, p_T^{miss}) = 2p_T^e E_T^{miss} - 2\vec{p}_T^e \cdot \vec{p}_T^{miss}. \quad (7.5)$$

The main irreducible background for this channel is diboson ($WZ$ and $ZZ$), $t\bar{t}V$ and $tZ$ productions. The reducible backgrounds include single and pair production of top quarks. We follow the ATLAS analysis and veto events with $b$-tagged jets to suppress the top quark production.

In a previous work on signature analysis by two of the authors (BA and PN) the standard model backgrounds at LHC energy of $\sqrt{s} = 7$ TeV were generated in-house [102]. Fortunately, for the current analysis this computer intensive process was circumvented by employing the Snowmass standard model background [47] normalized to NLO and generated with five flavor MLM matching and in bins of $S_T$, which is the scalar sum of $p_T$ of all generator-level particles. For signal event generation, we use Pythia [103] for hard scattering and showering/hadronization, and Delphes [104] for detector simulation with the same card used in the Snowmass background that was tuned according to the

<table>
<thead>
<tr>
<th>Model Parameters</th>
<th>Observables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_0$</td>
<td>6183 GeV</td>
</tr>
<tr>
<td>$m_{1/2}$</td>
<td>470 GeV</td>
</tr>
<tr>
<td>$A_0$</td>
<td>$-4469$ GeV</td>
</tr>
<tr>
<td>$\tan \beta$</td>
<td>52.1</td>
</tr>
<tr>
<td>sgn $(\mu)$</td>
<td>+1</td>
</tr>
<tr>
<td>$m_h^0$</td>
<td>126.1 GeV</td>
</tr>
<tr>
<td>$(m_{\tilde{g}}, m_{\tilde{t}_1})$</td>
<td>$(1257, 3601)$ GeV</td>
</tr>
<tr>
<td>$(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_1^\pm})$</td>
<td>$(199, 271, 261)$ GeV</td>
</tr>
<tr>
<td>$\Omega_{\chi_1^0}h^2$</td>
<td>&lt; 0.12</td>
</tr>
<tr>
<td>$\sigma (pp \to \chi_2^0\chi_1^\pm)$</td>
<td>134.7 fb</td>
</tr>
</tbody>
</table>

Table 7.2: Model parameters and observables of our mSUGRA benchmark model.
Figure 7.1: Distribution of $m_{\text{SFOS}}$ in signal region SR0$\tau$ for bins 6 (on the left with cuts $m_T < 80 \text{ GeV}$, $E_T^{\text{miss}} > 75 \text{ GeV}$) and 12 (on the right with cuts $m_T > 110 \text{ GeV}$, $E_T^{\text{miss}} > 75 \text{ GeV}$) prior to the requirements on these variables. Main SM background consisting of $VV$ (green) and $t\bar{t}V + tV$ (blue) are shown. The signal is shown in dashed red. The remaining invariant mass cuts used to define each signal region in the ATLAS analysis are also displayed. Our calculations show that $5\sigma$ significance is obtained with an integrated luminosity of $L \gtrsim 340 \text{ fb}^{-1}$ (left) and $L \gtrsim 135 \text{ fb}^{-1}$ (right).

detector performances in the last run. We normalize our signal cross section to match NLO cross section obtained by PROSPINO [105].

In the ATLAS analysis, the signal region SR0$\tau$ is composed of 20 disjoint bins with varying ranges of $m_{\text{SFOS}}$, $m_T$ and $E_T^{\text{miss}}$. We follow a similar approach and simply study two of those bins that offer the best discrimination of signal from the SM background for our benchmark point. These are bin-6 with cuts $m_T < 80 \text{ GeV}$, $E_T^{\text{miss}} > 75 \text{ GeV}$ and bin-12 with cuts $m_T > 110 \text{ GeV}$, $E_T^{\text{miss}} > 75 \text{ GeV}$. The distributions of $m_{\text{SFOS}}$ in those bins are displayed in Fig. 7.1 prior to the cut on that variable. A further cut to constrain $m_{\text{SFOS}}$ into a mass window following the ATLAS analysis is also displayed. Our calculations show that the SUSY signal produced by our mSUGRA benchmark point will be discoverable at the LHC Run-II at $5\sigma$ significance defined by $S/\sqrt{B} = 5$ with an integrated luminosity of $L \gtrsim 340 \text{ fb}^{-1}$($135 \text{ fb}^{-1}$) by using the cuts of bin 6(12).
7.2 nuSUGRA Benchmark Analysis

Next, we consider a nonuniversal SUGRA benchmark with nonuniversality in the gaugino sector as given in Table 7.3. The particle mass hierarchy for this model point is $m_{\tilde{\chi}_1^\pm} < m_{\tilde{\chi}_2^0} < m_{\tilde{g}} < m_{\tilde{t}_1}$. Our benchmark has a neutralino LSP of mass $m_{\tilde{\chi}_1^0} = 441$ GeV, $\chi_2^0$ and $\chi_1^\pm$ are within 20 GeV of the LSP, with a gluino of mass 1446 GeV and a stop of mass 1481 GeV. These masses are beyond the current limits obtained by ATLAS and CMS experiments with simplified models. With a relatively light LSP and NLSP, this model can have considerable electroweak production of neutralinos and charginos. For example, we obtain $\sigma(pp \to \chi_2^0\chi_1^\pm) = 69.7$ fb and $\sigma(pp \to \chi_1^\pm\chi_1^-) = 33.2$ fb at NLO. But the small mass gap between LSP and NLSP/NNLSP make the final decay products quite soft resulting in some or most of them being missed by the triggers. One possibility of obtaining sufficiently energetic final states to pass the triggers is via a high-$p_T$ ISR jet that will boost the event and provide large momenta to the final decay products. But with all these challenges, it will be quite difficult to observe this model at the LHC solely from electroweak production of neutralinos and charginos.

This model, however, has a gluino of mass 1446 GeV which gives rise to a gluino pair production cross section of $\sigma(pp \to \tilde{g}\tilde{g}) = 23.0$ fb at NLO. Although this is smaller than the chargino-neutralino production cross section, the large mass gap provides energetic final states and large missing momentum which can easily get triggered. Since our squarks are all heavy except the stop, the gluino in this benchmark decays mostly into $t\bar{t}\chi_1^0$, producing 4 b-jets in the final state. To check the viability of detecting this benchmark at the LHC Run-II, we considered the ATLAS analysis [106] for gluino pair production. Like the trilepton analysis we used for our mSUGRA benchmark model, this ATLAS analysis is also optimized for 8 TeV, but we use the exact same cuts as the ATLAS analysis [106] as our starting point. The ATLAS analysis introduces 9 signal regions demanding at least 4/6/7 jets with at least 3 of them being tagged as a b-jet and 0 or 1 lepton. The main reducible background for this process is $t\bar{t}$ production where a c-jet or...
Table 7.3: Model parameters and observables of our nuSUGRA benchmark.

<table>
<thead>
<tr>
<th>Model Parameters</th>
<th>Observables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_0$ (4 TeV)</td>
<td>$m_{h^0}$ (124.7 GeV)</td>
</tr>
<tr>
<td>$(M_1, M_2, M_3)$ (980, 520, 550) GeV</td>
<td>$(m_{\tilde{g}}, m_{\tilde{t}})$ (1446, 1481) GeV</td>
</tr>
<tr>
<td>$A_0$ (-7 TeV)</td>
<td>$(m_{\chi_1^0, m_{\chi_2^0, m_{\chi_1^\pm}}}$ (441, 461, 462) GeV</td>
</tr>
<tr>
<td>$\tan \beta$ (30)</td>
<td>$\Omega_{\chi_1^0} h^2$ (0.12)</td>
</tr>
<tr>
<td>sgn ($\mu$) (+1)</td>
<td>$\sigma(pp \to \bar{g}g)$ (23.0 fb)</td>
</tr>
</tbody>
</table>

a hadronically decaying $\tau$ lepton is mis-tagged as a b-jet. The irreducible backgrounds from $t\bar{t} + b/b\bar{b}$ and $t\bar{t} + Z/h(\to b\bar{b})$ are dominant.

The following kinematic variables correlated with the overall mass scale are introduced: $H_T^{4j}$ which is the scalar sum of the transverse momenta of the four leading jets, $m_{4j}^{\text{eff}}$ which is the scalar sum of the $E_T^{\text{miss}}$ and the transverse momenta of the four leading jets, and $m_{\text{incl}}^{\text{eff}}$ which is the scalar sum of $E_T^{\text{miss}}$ and the transverse momenta of all jets with $p_T > 30$ GeV. A cut on the minimum azimuthal separation between any of the four leading jets and the missing transverse momenta is also used to remove the multi-jet events. Transverse mass, defined in Eq. (7.5), computed from the leading lepton and the missing transverse momenta is used to cut down the $t\bar{t}$ events where one of the $W$ bosons decays leptonically. In the analysis, we use the signal regions SR-0$\ell$-4j-[A,B,C], SR-0$\ell$-7j-[A,B,C] and SR-1$\ell$-6j-[A,B,C] as defined by ATLAS in their analysis of SUSY signals at $\sqrt{s} = 8$ TeV [106] and summarized in Table 7.4.

Distributions of $m_{4j}^{\text{eff}}$, $m_{\text{incl}}^{\text{eff}}$ and $E_T^{\text{miss}}$ for all the signal regions SR-0$\ell$-4j-[A,B,C], SR-0$\ell$-7j-[A,B,C] and SR-1$\ell$-6j-[A,B,C] are shown in Figs. 7.2 to 7.4 prior to the requirements on these variables. We follow the ATLAS analysis and apply all the cuts including the ones shown with arrows in the distributions and calculate the minimum integrated luminosities required for a $5\sigma$ discovery. The analysis presented in Figs. 7.2 to 7.4 shows that for our benchmark model, almost all the signal regions are effective for discovery of sparticles at
<table>
<thead>
<tr>
<th>Signal Region</th>
<th>N jets ($p_T$)</th>
<th>$E_T^{\text{miss}}$</th>
<th>$m_{\text{eff}}$</th>
<th>$E_T^{\text{miss}}/\sqrt{H_T^{6j}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR- 0$\ell$ - 4$j$ - A</td>
<td>$\geq 4(50)$</td>
<td>$&gt; 250$</td>
<td>$&gt; 1300$</td>
<td>-</td>
</tr>
<tr>
<td>SR- 0$\ell$ - 4$j$ - B</td>
<td>$\geq 4(50)$</td>
<td>$&gt; 350$</td>
<td>$&gt; 1100$</td>
<td>-</td>
</tr>
<tr>
<td>SR- 0$\ell$ - 4$j$ - C</td>
<td>$\geq 4(30)$</td>
<td>$&gt; 400$</td>
<td>$&gt; 1000$</td>
<td>$&gt; 16$</td>
</tr>
<tr>
<td>Signal Region</td>
<td>N jets ($p_T$)</td>
<td>$E_T^{\text{miss}}$</td>
<td>$m_{\text{inc}}^{\text{eff}}$</td>
<td>$E_T^{\text{miss}}/\sqrt{H_T^{6j}}$</td>
</tr>
<tr>
<td>---------------</td>
<td>----------------</td>
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<td>-------------------</td>
</tr>
<tr>
<td>SR- 0$\ell$ - 7$j$ - A</td>
<td>$\geq 7(30)$</td>
<td>$&gt; 200$</td>
<td>$&gt; 1000$</td>
<td>-</td>
</tr>
<tr>
<td>SR- 0$\ell$ - 7$j$ - B</td>
<td>$\geq 7(30)$</td>
<td>$&gt; 350$</td>
<td>$&gt; 1000$</td>
<td>-</td>
</tr>
<tr>
<td>SR- 0$\ell$ - 7$j$ - C</td>
<td>$\geq 7(30)$</td>
<td>$&gt; 250$</td>
<td>$&gt; 1500$</td>
<td>-</td>
</tr>
<tr>
<td>Signal Region</td>
<td>N jets ($p_T$)</td>
<td>$E_T^{\text{miss}}$</td>
<td>$m_T$</td>
<td>$m_{\text{inc}}^{\text{eff}}$</td>
</tr>
<tr>
<td>---------------</td>
<td>----------------</td>
<td>-----------------</td>
<td>----------</td>
<td>-------------------</td>
</tr>
<tr>
<td>SR- 1$\ell$ - 6$j$ - A</td>
<td>$\geq 6(30)$</td>
<td>$&gt; 175$</td>
<td>$&gt; 140$</td>
<td>$&gt; 700$</td>
</tr>
<tr>
<td>SR- 1$\ell$ - 6$j$ - B</td>
<td>$\geq 6(30)$</td>
<td>$&gt; 225$</td>
<td>$&gt; 140$</td>
<td>$&gt; 800$</td>
</tr>
<tr>
<td>SR- 1$\ell$ - 6$j$ - C</td>
<td>$\geq 6(30)$</td>
<td>$&gt; 275$</td>
<td>$&gt; 160$</td>
<td>$&gt; 900$</td>
</tr>
</tbody>
</table>

Table 7.4: A summary of signal regions (SR) with cuts used in the signature analysis of the model benchmark. The top two blocks in the table have the additional constraints of $p_T(j_1) > 90$ GeV, $E_T^{\text{miss}} > 150$ GeV, $\geq 4$ jets with $p_T > 30$ GeV, $\Delta\phi^{4j}_{\text{min}} > 0.5$, $E_T^{\text{miss}}/m_{\text{eff}}^{4j} > 0.2$, $\geq 3$ b-jets with $p_T > 30$ GeV. The bottom block in the table has the constraints $p_T(j_1) > 90$ GeV, $E_T^{\text{miss}} > 150$, $\geq 4$ jets with $p_T > 30$ GeV, $\geq 3$ b-jets with $p_T > 30$ GeV. These constraints are adopted from the ATLAS analysis at $\sqrt{s} = 8$ TeV [106] and applied to our analysis at $\sqrt{s} = 14$ TeV.

Specifically, we find that the signal regions SR-0$\ell$-4j-[A,B,C] will require an integrated luminosity of $L \gtrsim 45/60/920$ fb$^{-1}$, the signal regions SR-0$\ell$-7j-[A,B,C] will require an integrated luminosity of $L \gtrsim 235/45/25$ fb$^{-1}$, and the signal regions SR-1$\ell$-6j-[A,B,C] will require an integrated luminosity of $L \gtrsim 510/265/160$ fb$^{-1}$.
Figure 7.2: Distribution of $m_{4j}^{\text{eff}}$ and $E_T^{\text{miss}}$ in the signal regions SR-0$\ell$-4j-[A,B,C] prior to the requirements on these variables. Arrows indicate the remaining cuts used to define the signal regions. Main SM background consisting of $t\bar{t} +$ jets (blue) and $t\bar{t} + V/H$ (green) are shown. The signal is shown in dashed red. Our calculations show that $5\sigma$ significance is obtained with an integrated luminosity of $L \gtrsim 45 \text{ fb}^{-1}$ (left), $L \gtrsim 60 \text{ fb}^{-1}$ (right) and $L \gtrsim 920 \text{ fb}^{-1}$ (bottom).
Figure 7.3: Distribution of $m_{\text{incl}}^{\text{eff}}$ and $E_T^{\text{miss}}$ in the signal regions SR-0ℓ-7j-[A,B,C] prior to the requirements on these variables. Arrows indicate the remaining cuts used to define the signal regions. Main SM background consisting of $t\bar{t} + \text{jets}$ (blue) and $t\bar{t} + V/H$ (green) are shown. The signal is shown in dashed red. Our calculations show that $5\sigma$ significance is obtained with an integrated luminosity of $L \gtrsim 235 \text{ fb}^{-1}$ (left), $L \gtrsim 45 \text{ fb}^{-1}$ (right) and $L \gtrsim 25 \text{ fb}^{-1}$ (bottom).
Figure 7.4: Distribution of $n_{\text{eff}}^{\text{incl}}$ and $E_{T}^{\text{miss}}$ in the signal regions SR-1$\ell$-6j-[A,B,C] prior to the requirements on these variables. Arrows indicate the remaining cuts used to define the signal regions. Main SM background consisting of $t\bar{t}$ + jets (blue) and $t\bar{t}$ + V/H (green) are shown. The signal is shown in dashed red. Our calculations show that 5σ significance is obtained with an integrated luminosity of $L \gtrsim 510$ fb$^{-1}$ (left), $L \gtrsim 265$ fb$^{-1}$ (right) and $L \gtrsim 160$ fb$^{-1}$ (bottom).
Chapter 8

Dark Matter

As mentioned in Section 2.3, the lightest supersymmetric particle (LSP) under $R$-parity conservation is absolutely stable and, if the LSP is charge neutral, becomes a possible candidate for cold dark matter (CDM). Remarkably, for a large subset of the SUGRA parameter space, the lightest neutralino $\chi_1^0$ is the LSP and thus the lightest neutralino becomes a candidate for dark matter in SUGRA models. Of course, for the neutralino to be a viable dark matter candidate, its relic density must be consistent with the relic density constraints from WMAP and PLANCK experiments on cold dark matter. We give below a brief discussion of the relic density analyses in SUGRA models. The relic density $\Omega_\chi$, for the neutralino LSP $\chi_1^0$, is defined as

$$\Omega_\chi = \frac{\rho_\chi}{\rho_c}, \quad (8.1)$$

where $\rho_\chi$ is the neutralino matter density and $\rho_c$ is the critical mass density needed to close the universe,

$$\rho_c = \frac{3H_0^2}{\kappa^2} \simeq 1.9h_0^2 \times 10^{-29} \frac{g}{\text{cm}^3}, \quad (8.2)$$

where $\kappa = \sqrt{8\pi G_N}$ and $G_N$ is Newton’s gravitational constant. Here, $H_0$ is the Hubble parameter, defined as $H_0 = h_0 \frac{100 \text{km}}{\text{sMpc}}$, where $h_0 = 0.738 \pm 0.024$. 
Current estimates on the composition of the universe show that $\sim 5\%$ of the total mass of the universe is composed of visible matter, which consists of stars and intergalactic dust, and $\sim 68\%$ is dark energy; therefore, the rest is composed of cold dark matter. More precisely, WMAP [111] gives a constraint on $\Omega_{\text{CDM}}h^2_0$ so that

$$\Omega_{\text{CDM}}h^2_0 \simeq 0.12 . \quad (8.3)$$

The above constraint would hold if dark matter is composed of a single component. However, for the case when dark matter is composed of more than one component, such as the LSP neutralino along with another partner (perhaps a particle from a hidden sector) the above constraint would act as an upper limit on the relic density of the LSP neutralino.

Next, we will analyze the neutralino relic density. In the early universe, particles were in thermal equilibrium. Consequently, the thermal bath production rate of particles was equal to the annihilation rate, $\Gamma$. If the universe were static, then by lowering the temperature below a threshold neutralino mass, the neutralino abundance would freeze out to a specific value, since it is thermally suppressed by factors of $e^{-m_\chi/kT}$. However, we know that the universe is not static, but expanding at a rate dependent on the Hubble parameter $H(t)$, a function of time. Thus, freeze-out occurs when the expansion rate becomes larger than the annihilation rate, $H \gg \Gamma$.

The neutralino relic density calculation begins with the neutralino number density $n$, which satisfies the Boltzmann equation during the time of the early universe,

$$\frac{dn}{dt} = -3Hn - \langle \sigma v \rangle (n^2 - n_0^2) , \quad (8.4)$$

where we define $\sigma = \sigma(\chi_1^0\chi_1^0 \rightarrow X)$ as the neutralino annihilation cross section, $v$ as the neutralino relative velocity, and $n_0$ as the number density at thermal equilibrium. On the right-hand side of Equation (8.4), the three terms account for the universe expansion, the annihilation scattering processes, and the inverse scattering processes, respectively.
Since $R$-parity must be conserved at each vertex, pair annihilation dominantly occurs via s-channel exchange of $h^0, H^0, A^0, Z$ (SM particles have $R = 1$) and t-channel exchange of $\tilde q, \tilde \ell, \chi^\pm$ (SUSY particles have $R = -1$). Annihilation occurs at temperatures when neutralinos are non-relativistic, so we can use the Boltzmann distribution to model the system. The thermal average $\langle \sigma v \rangle$ is given by

$$\langle \sigma v \rangle = \frac{\int_0^\infty dv v^2 (\sigma v)e^{-\frac{v^2}{4x}}}{\int_0^\infty dv v^2 e^{-\frac{v^2}{4x}}},$$

(8.5)

where $x = kT/m_\chi$. Changing variables from $n(t)$ to $f[x(T)] = n/T^3$, the dependent variable is now $T$. At the freeze-out temperature $T_f$, the expansion rate overtakes the annihilation rate, causing the neutralino to decouple from the background. Solving for $x_f$ gives a relatively small number, $x_f \sim \frac{1}{20}$. Solving Equation (8.4) using the change of variables and applying the boundary conditions at freeze-out, one finds that

$$\Omega_{\chi h^2} = 2.5 \times 10^{-11} \left(\frac{T_\chi}{T_\gamma}\right)^3 \left(\frac{T_\gamma}{2.75}\right)^3 \frac{\sqrt{N_f}}{J(x_f)},$$

(8.6)

where $T_\gamma$ is the present cosmic microwave background (CMB) temperature, $N_f$ is the number of degrees of freedom at freeze-out ($N_f \sim 300$), and $(T_\chi/T_\gamma)^3$ is a factor that arises from the reheating of the photon temperature due to annihilation of particles of mass less than $kT_f$. Typically, $(T_\gamma/T_\chi)^3 \simeq 18.5$. Lastly, $J(x_f)$ is defined as

$$J(x_f) = \int_0^{x_f} dx \langle \sigma v \rangle x \text{ GeV}^{-2}.$$
More realistically, relic density analyses require co-annihilation, where more than one channel enter in the annihilation process. Thus, a typical example of co-annihilation is

\[ \chi^0 + \chi^0 \rightarrow \tau + \bar{\tau} , \]  
(8.8)

\[ \chi^0 + \tilde{\tau} \rightarrow \gamma + \tau , \]  
(8.9)

\[ \tilde{\tau} + \tilde{\tau}^* \rightarrow \gamma + \gamma . \]  
(8.10)

The above leads to coupled Boltzmann equations. Such equations are difficult to solve analytically and one employs numerical techniques for analyzing the relic density. In our analysis, we use the program \textsc{micrOMEGAs} \cite{84} to calculate the relic density and flavor observables.

The detection of dark matter in the laboratory poses one of the great challenges to experimental particle physics. Thus, one expects that dark matter particles, which are in the Milky Way, will scatter off quarks in nuclei in detectors in the process

\[ \chi^0 + q \rightarrow \chi^0 + q . \]  
(8.11)

There are a number of ways in which such processes can be detected in sensitive dark matter detectors. Such scatterings lead to both spin-independent neutralino-proton cross sections \( \sigma_{px1}^{SI} \), as well as spin-dependent neutralino-proton cross sections \( \sigma_{px1}^{SD} \). The spin-independent neutralino-proton scattering arises from the four-Fermi interaction \( \bar{\chi}^0_{1} \chi^0_{1} \bar{\chi}^0_{1} \bar{q} q \), while the spin-dependent scattering arises from couplings of the type \( \bar{\chi}^0_{1} \chi^0_{1} \bar{\gamma}_\mu \gamma_5 q \). In this work, we will focus on the spin-independent neutralino-proton scattering. To allow for the possibility that not all of dark matter may be composed of neutralinos, we will consider the quantity \( R \times \sigma_{px1}^{SI} \) in our analyses, where \( R = \rho_{\chi^0_{1}} / \rho_c \) where \( \rho_{\chi^0_{1}} \) is the neutralino relic density and \( \rho_c \) is the critical relic density as shown in Equation (8.2).
8.1 Mass Patterns in the \( (R \times \sigma_{p\chi_1^0}^{\text{SI}}, m_{\chi_1^0}) \) Plane

We discuss now the details of our dark matter analysis. Each of the Models [1]-[5] in Chapter 4 gives rise to spin-independent neutralino-proton cross sections which can have a significant range and, in some cases, span several orders of magnitude depending on the content of the neutralino being a bino, wino, Higgsino, or an admixture. The Higgs boson mass measurement is also a strong constraint on the spin-independent neutralino-proton cross section. In the top panel of Fig. 8.1, we display the \( (R \times \sigma_{p\chi_1^0}^{\text{SI}}, m_{\chi_1^0}) \) profile for mSUGRA Model [1] where \( \sigma_{p\chi_1^0}^{\text{SI}} \) is the spin-independent neutralino-proton cross section and we define \( R = \frac{\rho_{\chi_1^0}}{\rho_c} \), where \( \rho_{\chi_1^0} \) is the neutralino relic density and \( \rho_c \) is the critical relic density as discussed above. The allowed parameter space in the \( (R \times \sigma_{p\chi_1^0}^{\text{SI}}, m_{\chi_1^0}) \) plane is colored according to the Higgs boson mass. In the bottom panel of Fig. 8.1, we give an analysis of the \( (R \times \sigma_{p\chi_1^0}^{\text{SI}}, m_{\chi_1^0}) \) plane for mSUGRA where we exhibit the sparticle mass patterns.

A very similar analysis for nuSUGRA Models is given in Figs. 8.2 to 8.5. Thus, in Fig. 8.2, an analysis is given of the \( (R \times \sigma_{p\chi_1^0}^{\text{SI}}, m_{\chi_1^0}) \) plane for nuSUGRA Models [2]-[5], where the colors specify the Higgs boson mass. In Fig. 8.3, we give a composite of all cases considered, i.e., Models [1]-[5] with the following constraints,

\[
m_{h^0} \in [123, 127] \text{ GeV ,} \\
\Omega_{\chi_1^0} h^2 < 0.12 , \\
\text{Br} \left( B^0_s \rightarrow \mu^+ \mu^- \right) < 6.2 \times 10^{-9} , \\
\text{Br} \left( B \rightarrow X_s \gamma \right) < 4.27 \times 10^{-4} .
\]

Note that the additional constraints consist of a narrower Higgs mass window as well as flavor constraints. In the figures here and also in later figures, we have exhibited the line where the signals from coherent scattering of solar, atmospheric and diffuse supernova neutrinos will begin to appear. The test of the neutrino cross sections is interesting in itself. However, for the purpose of WIMP detection, the neutrino background must be
subtracted or it would require a directional analysis in direct detection experiments to separate the neutrino backgrounds from the dark matter signals [107].

In Fig. 8.4, we give an analysis of the mass patterns for the nuSUGRA Models [2]-[3] and in Fig. 8.5, we give an analysis of the mass patterns for the nuSUGRA Models [4]-[5]. A very interesting phenomenon relates to the fact that dark matter searches can be used as a diagnostic for the type of underlying sparticle pattern, and thus of the sparticle mass hierarchy. From the bottom panel of Fig. 8.1 and from Figs. 8.4 and 8.5, we note that the models which dominate the parameter space mostly have the chargino as NLSP; however, there exist regions where some patterns are more frequent than others. For example, in the bottom panel of Fig. 8.1, all the models that lie close to the neutrino coherent scattering line are the ones where the stop is the NLSP and the ones above those are mostly where the lightest stau is the NLSP. Further, the region between neutralino mass of 60-105 GeV, which lies below the LUX limit, consists exclusively of chargino NLSP patterns. In the analysis of nuSUGRA models, as shown in Figs. 8.4 and 8.5, one finds that there are certain regions where only very specific patterns appear, while in others only a combination of two or three patterns appear. In these cases, a knowledge of the spin-independent cross section along with a knowledge of the neutralino mass, hopefully from collider data, will allow us to narrow down the possibilities for the allowed hierarchical patterns. Consequently, this would help delineate the nature of high scale boundary conditions for the underlying supergravity grand unification model.
Figure 8.1: Top panel: A display of the \((R \times \sigma_{pX_0}^{SI}, m_{X_0})\) plane for mSUGRA (Model [1]), where the Higgs boson mass is indicated by color. Bottom panel: Exhibited are the various hierarchical patterns contributing to the allowed parameter space. The analysis indicates that a knowledge of the spin-independent neutralino-proton cross section along with the neutralino mass will allow one to identify the possible underlying sparticle mass hierarchy. In both panels, the constraints \(\Omega h^2 < 0.12, m_{\nu} > 120\) GeV hold.
Figure 8.2: Top to bottom: A display of the \((R \times \sigma_{pX_1}^{\text{SI}}, m_{\chi_0})\) plane for nuSUGRA Models [2]-[5], with color specifying the Higgs boson mass. The constraints \(\Omega h^2 < 0.12\) and \(m_{h^0} > 120\text{GeV}\) hold.
Figure 8.3: A composite display of the \((R \times \sigma^{SI}_{p\chi_1^0}, m_{\chi_1^0})\) plane for all five SUGRA Models [1]-[5]. Additional constraints are applied, which include

\[ m_{h^0} \in [123, 127] \text{ GeV}, \quad \Omega h^2 < 0.12, \quad B_s^0 \to \mu^+\mu^- < 6.2 \times 10^{-9}, \quad B \to X_s\gamma < 4.27 \times 10^{-4}. \]
Figure 8.4: A display of the \((R \times \sigma_{\text{SI}}^\chi_0, m_{\chi_1^0})\) plane for two nuSUGRA models, including the light chargino model (Model [2]) and the light gluino model (Model [3]). The color and shape specify the particular sparticle mass pattern. The constraints \(\Omega h^2 < 0.12\) and \(m_{\chi_1^0} > 120\,\text{GeV}\) hold.
Figure 8.5: A display of the \((R \times \sigma_{SI}^{P_{X_1}}, m_{\chi_1^0})\) plane for two nuSUGRA models, including the nonuniversal Higgs model (Model [4]) and the light third generation model (Model [5]). The color and shape specify the particular sparticle mass pattern. The constraints \(\Omega h^2 < 0.12\) and \(m_{\mu^0} > 120\)GeV hold.
8.2 Relative Size of Dark Matter Cross Sections

The spin-independent neutralino-proton cross sections span several orders of magnitude, with a large subset of the hierarchical patterns being accessible by dark matter experiments. However, the analysis of Figs. 8.1 to 8.5 shows that certain sparticle patterns often give too small a cross section which are below the reach of XENON1T and other similar size dark matter experiments, while some of the other patterns have cross sections which lie even below the neutrino floor. Specifically, from the lower panel of Fig. 8.1, we see that the stop patterns mSP[t1] have cross sections which cross the neutrino coherent scattering line, and thus the cross sections from these would be extremely challenging to observe. The reason for the extreme smallness of the cross sections is that the neutralino, in these cases, is essentially almost 100% bino-like, and there does not exist light first and second generation squark states, which suppresses the strength of interaction in the scattering off nuclei. Similar observations apply to the analysis of Figs. 8.2 to 8.5. In Table 8.1, we display the bino and Higgsino fractions of such model points along with the corresponding spin-independent neutralino-proton cross sections. To explain the smallness of the spin-independent cross section further for some of the simplified models, we consider the explicit form of the scalar cross section for the neutralino-nucleus scattering,

\[
\sigma^\text{SI}_{\chi^0_1N} = \frac{4\mu_r^2}{\pi} \left[ Z f_p + (A - Z) f_n \right]^2 ,
\]  

(8.16)

where \( Z \) is the total number of protons, \( A \) the total sum of protons and neutrons in the nucleus, and \( \mu_r \) is the neutralino reduced mass. For the case when the squarks are very heavy, the s-channel pole diagrams in neutralino-quark scattering give a relatively small contribution, which is dominated by the t-channel Higgs-boson exchanges. In this circumstance, \( f_{p/n} \) are given by

\[
f_{p/n} = \sum_{q=u,d,s} f_{T_q}^{(p/n)} C_q \frac{m_{p/n}}{m_q} + \frac{2}{2\pi} f_{T_G}^{(p/n)} \sum_{q=c,b,t} C_q \frac{m_{p/n}}{m_q} ,
\]  

(8.17)
where the form factors $f_{T_q}^{(p/n)}$ and $f_{T_G}^{(p/n)}$ are given in [108–110] and the couplings $C_i$ are given by

$$C_q = \frac{g_2 m_q}{4 m_W^2 \delta_3} (g_2 n_{12} - g_1 n_{11}) \left[ \delta_1 \delta_4 \delta_5 \left( -\frac{1}{m_H^2} + \frac{1}{m_h^2} \right) + \delta_2 \left( \frac{\delta_2^2}{m_H^2} + \frac{\delta_5^2}{m_h^2} \right) \right] .$$  

(8.18)

In the above, $n_{1k}$ are defined such that

$$\chi_1^0 = n_{11} \tilde{B} + n_{12} \tilde{W}_3 + n_{13} \tilde{H}_1 + n_{14} \tilde{H}_2 ,$$  

(8.19)

where $\tilde{B}, \tilde{W}_3, \tilde{H}_1, \tilde{H}_2$ are respectively the bino, wino, Higgsino 1 and Higgsino 2 fields. Also, $\delta_i$ are defined

$$\delta_i = \begin{cases} 
(n_{13}, n_{14}, \sin \beta, \sin \alpha, \cos \alpha) & \text{for up quarks} \\
(n_{14}, -n_{13}, \cos \beta, \cos \alpha, -\sin \alpha) & \text{for down quarks} 
\end{cases}$$  

(8.20)

where $i$ runs from 1 to 5 and $\alpha$ is the neutral Higgs mixing parameter. From Equation (8.18), we see that the cross section depends directly on $\delta_1, \delta_2$ and consequently on $n_{13}, n_{14}$ and hence on the Higgsino fraction. From the last column of Table 8.1, we see that the

<table>
<thead>
<tr>
<th>SUGRA Hierarchy</th>
<th>$m_0$ (GeV)</th>
<th>$m_{1/2}$ (GeV)</th>
<th>$A_0$ (GeV)</th>
<th>$\tan \beta$</th>
<th>$\delta_{M2}$</th>
<th>$\delta_{M3}$</th>
<th>$(\delta_{H1}, \delta_{H2})$</th>
<th>$\delta_{M43}$</th>
<th>$R \times 10^{3}$</th>
<th>LSP Bino Fraction</th>
<th>LSP Higgsino Fraction</th>
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</thead>
<tbody>
<tr>
<td>mSPLH[1b]</td>
<td>2221</td>
<td>424</td>
<td>-3862</td>
<td>56</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3.00 x 10^{-4}</td>
<td>0.9997</td>
<td>0.0213</td>
</tr>
<tr>
<td>mSPLH[2]</td>
<td>3762</td>
<td>416</td>
<td>-6727</td>
<td>54</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.76 x 10^{-4}</td>
<td>0.9994</td>
<td>0.0331</td>
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<tr>
<td>mSPLH[3]</td>
<td>3075</td>
<td>1706</td>
<td>13772</td>
<td>18</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>6.33 x 10^{-4}</td>
<td>0.9999</td>
<td>0.0122</td>
</tr>
<tr>
<td>mSPLH[1b]</td>
<td>4032</td>
<td>1808</td>
<td>15322</td>
<td>12</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.17 x 10^{-4}</td>
<td>0.9999</td>
<td>0.0188</td>
</tr>
<tr>
<td>mSPLH[2b]</td>
<td>9874</td>
<td>455</td>
<td>-13049</td>
<td>25</td>
<td>-0.480</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4.70 x 10^{-4}</td>
<td>0.9998</td>
<td>0.0132</td>
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<tr>
<td>mSPLH[3b]</td>
<td>4334</td>
<td>535</td>
<td>-7758</td>
<td>10</td>
<td>-0.458</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5.01 x 10^{-4}</td>
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<td>0.0171</td>
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<tr>
<td>mSPLH[4b]</td>
<td>9072</td>
<td>1963</td>
<td>-11309</td>
<td>27</td>
<td>-0.821</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5.01 x 10^{-4}</td>
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<td>0.0184</td>
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<td>mSPLH[1b]</td>
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<td>1959</td>
<td>-10789</td>
<td>31</td>
<td>-0.832</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4.98 x 10^{-4}</td>
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<td>0.0156</td>
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<tr>
<td>mSPLH[2b]</td>
<td>6983</td>
<td>1665</td>
<td>17544</td>
<td>12</td>
<td>-</td>
<td>-</td>
<td>(1.032, -1.692)</td>
<td>-</td>
<td>9.93 x 10^{-4}</td>
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<tr>
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<td>3204</td>
<td>1585</td>
<td>-7123</td>
<td>53</td>
<td>-</td>
<td>-</td>
<td>(-2.182, -2.439)</td>
<td>-</td>
<td>3.34 x 10^{-4}</td>
<td>0.9999</td>
<td>0.0125</td>
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<td>1077</td>
<td>-3457</td>
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<td>-</td>
<td>-</td>
<td>2.055</td>
<td>2</td>
<td>6.56 x 10^{-4}</td>
<td>0.9997</td>
<td>0.0217</td>
</tr>
</tbody>
</table>

TABLE 8.1: A sample of mSUGRA and nuSUGRA model points with especially low spin-independent neutralino-proton cross section are given, accompanied by the particle mass pattern to which they belong. As displayed, the gaugino-Higgsino content of the LSP is almost entirely bino for these parameter points. Since these points exhibit a nearly 100% bino-like LSP, this gives reason for the smallness of the observed cross sections. The LSP Bino and Higgsino fractions are defined as $|n_{11}|$ and $\sqrt{|n_{13}|^2 + |n_{14}|^2}$.
Higgsino content is indeed very small for the patterns listed in Table 8.1, which explains the smallness of spin-independent cross sections listed in the 10th column of Table 8.1 and further explains the smallness of the cross sections that appear in Figs. 8.1 to 8.5.

8.3 Gaugino-Higgsino Content of the Neutralino LSP

The dark matter content, specified by the bino, wino, and Higgsino fractions, varies greatly depending on the sparticle mass hierarchy characterizing the SUGRA unified model point. Note that any two dark matter content fractions implies the third by the unitary condition,

\[
\sum_{m=1}^{4} |n_{1m}|^2 = |n_{11}|^2 + |n_{12}|^2 + |n_{13}|^2 + |n_{14}|^2 = 1 ,
\]  

(8.21)

where we define the wino, bino and Higgsino fractions as \(|n_{11}|, |n_{12}| \) and \(\sqrt{|n_{13}|^2 + |n_{14}|^2}\), respectively, as shown in Equation (8.19). In Fig. 8.6, we have plotted the bino and wino fractions for mSUGRA (Model [1]) with the following additional constraints,

\[
m_{h^0} \in [123, 127] \text{GeV},
\]

\[
\Omega_{\chi_1^0} h^2 \in [0.0946, 0.1306] ,
\]

\[
Br (B^0_s \rightarrow \mu^+ \mu^-) < 6.2 \times 10^{-9} ,
\]

\[
Br (B \rightarrow X_s \gamma) < 4.27 \times 10^{-4} .
\]

(8.22)  

(8.23)  

(8.24)  

(8.25)

Here, we choose similar constraints as the previous benchmark and dark matter analysis; however, the LSP relic density window is bounded not only from above to limit overabundance, but also from below such that

\[
\Omega_{\chi_1^0} h^2 = \Omega_{\text{CDM}} h^2 \pm 5\sigma ,
\]

(8.26)

where \(\Omega_{\text{CDM}} h^2 = 0.1126\) and \(\sigma = 0.0036\) [111]. In Fig. 8.6, notice that certain model points dominate specific regions of the \((|n_{11}|, |n_{12}|)\) plane. Namely, mSUGRA models with
chargino NLSP mass hierarchies populate most of the space which satisfies $|n_{11}| < 0.95$. Since the $\chi_1^\pm$ NLSP patterns dominate the majority of the plot, we must zoom in to the 100% bino-like LSP region to analyze the convergence of various mass patterns. That is, as we approach $(|n_{11}|, |n_{12}|) \to (1 - \delta, 0)$, where $\delta \ll 1$, we find that there exists mSUGRA models with $\chi_2^0$, $\tilde{\tau}$, and $H^0$ NLSP mass hierarchies. An interesting feature of Fig. 8.6 can be found within the region $|n_{11}| > 0.99, |n_{12}| > 2 \times 10^{-2}$. Here, the only patterns which survive are the $\chi_2^0$ NLSP mass hierarchies. Thus, there exists a complementarity between the dark matter LSP content and the mass hierarchy for a specific SUGRA model. Specifically, a knowledge of the dark matter content limits the space of possible mass hierarchies given a SUGRA model, or conversely, certain mass patterns constrain the space of possible dark matter bino, wino, and Higgsino fractions.

Since our earlier analysis of sparticle mass patterns in Chapter 4 shows that the chargino NLSP mass patterns exhibit the highest percentage of occurrence for Models [1]-[5], let us now focus our attention on such SUGRA models. In Fig. 8.7, we show the $(|n_{11}|, |n_{12}|)$ plane for all five SUGRA models with the constraint that the NLSP is $\chi_1^\pm$, as well as the earlier constraints in Equations (8.22)-(8.25). When we consider SUGRA models with both universal and nonuniversal boundary conditions, we see that the boundary conditions greatly affect the possibilities of dark matter content. Notice that the nuSUGRA model with nonuniversalities in the SU(2)$_L$ gaugino sector (Model [2]) dominates as $|n_{11}| \to 1$, suggesting a higher probability of a 100% bino-like LSP. Further, the bottom panel of Fig. 8.7 shows that mSUGRA (Model [1]) and the other nuSUGRA models (Models [3]-[5]) converge to $|n_{11}| = 0.999$ as $|n_{12}| \to 0$, suggesting that these models require a nonvanishing Higgsino fraction for the LSP. Thus, dark matter content can be used as a tool to help delineate the sparticle mass hierarchies and consequently the high energy boundary conditions necessary for understanding the nature of symmetry breaking at the high energy scale.
Figure 8.6: A display of the bino fraction $|n_{11}|$ to wino fraction $|n_{12}|$ for mSUGRA (Model [1]) where the color exhibits the sparticle mass pattern. The top panel is linear in both axes; however, the bottom panel is semi-logarithmic and zooms in considerably such that $|n_{11}| \in [0.99, 1]$, to differentiate the model points which cluster as $(|n_{11}|, |n_{12}|)$ approaches $(1 - \delta, 0)$, where $\delta \ll 1$. 
Figure 8.7: A display of the bino fraction $|n_{11}|$ to wino fraction $|n_{12}|$ for all five SUGRA models, including both mSUGRA and nuSUGRA. The top and middle panel are linear in both axes; however, the bottom panel is semi-logarithmic and zooms in considerably such that $|n_{11}| \in [0.99, 1]$, to differentiate the various SUGRA model points which cluster as $(|n_{11}|, |n_{12}|)$ approaches $(0.999, 0)$. 

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Chapter 9

Conclusion

Sparticle mass hierarchies are crucial for understanding the nature of symmetry breaking at the high energy scale. In this work, we have analyzed the landscape of sparticle mass hierarchies generated from supergravity unified models consistent with the Higgs boson mass measurement, flavor constraints, and cosmological data. Both universal and nonuniversal supergravity models have been analyzed. As the soft breaking parameters at the grand unification scale will generally be nonuniversal, the different patterns of soft parameters as well as their specific values lead to a variety of different hierarchical patterns for sparticle masses. For 31 particle masses beyond the standard model, one finds that a landscape of $10^{33}$ mass hierarchies arises. A large number of these are eliminated under the constraints of electroweak symmetry breaking, as well as collider and dark matter constraints. However, the residual number of patterns is still quite large. This number is drastically reduced when one considers the hierarchical patterns for the case when the number of particles taken into account is five, including the neutralino LSP.

The analysis of the hierarchical patterns has been applied to several different high scale models. These include [1] the mSUGRA model with universal boundary conditions, [2] the nuSUGRA model with nonuniversality in the SU(2)$_L$ gaugino mass sector, [3] the nuSUGRA model with nonuniversality in the SU(3)$_C$ gaugino mass sector, [4] the nuSUGRA model with nonuniversality in the Higgs boson mass sector, and [5] the
nuSUGRA model with nonuniversality in the third generation sfermion sector. Further, we have produced relatively exhaustive lists of simplified models with 3, 4 or 5 sparticle mass hierarchies along with their relative occurrences that arise from the SUGRA models. These hierarchical patterns will be helpful in establishing the nature of the high scale models which give rise to the hierarchical patterns. In addition, we have provided benchmarks for the SUGRA models, which span the parameter space of models with universal as well as nonuniversal boundary conditions. These benchmarks should be useful for SUSY searches at the LHC Run-II. We have also carried out an explicit signature analysis for a mSUGRA benchmark model and a nuSUGRA benchmark model.

In addition to the analysis of sparticle mass hierarchies, we have analyzed the spin-independent neutralino-proton cross section for the five classes of supergravity models discussed in Chapter 4. It is found that the latest limits from the LUX dark matter experiment probe a significant region of the parameter space of models, and the XENON1T and SuperCDMS will be able to exhaust significantly more of the parameter space of many of the SUGRA models. However, for the case of the SUGRA models with nonuniversalities, especially in the SU(2)_L and SU(3)_C gaugino sectors, the neutralino-proton cross sections extend downwards even past the neutrino floor as shown in Figs. 8.2 to 8.5. Another important aspect of the analysis relates to the diagnostic potential of the spin-independent cross section as a function of the neutralino mass. In the analysis of the patterns in the \( R \times \sigma_{p\chi^0_1}^{\text{SI}}, m_{\chi^0_1} \) plane, one finds that certain regions are populated dominantly by one or two patterns. Consequently, a simultaneous measurement of the spin-independent cross section and a knowledge of the neutralino mass, such as from collider experiments, could isolate the likely sparticle mass hierarchies, providing strong clues to the nature of symmetry breaking in high scale models. Thus, dark matter analyses along with analyses of the LHC Run-II can allow one to delineate the nature of the high scale models leading to the sparticle mass patterns.
References


[98] H. Baer, C. Balazs, A. Belyaev, T. Krupovnickas, and X. Tata, *Updated reach of the CERN LHC and constraints from relic density, b \( \rightarrow s \gamma \) and a(\( \mu \)) in the mSUGRA model*, JHEP 0306 (2003) 054, [hep-ph/0304303].


Appendix A:
General Relativity

Einstein’s general relativity (GR) is the theory of gravity [112] as the curvature of spacetime, modeled as a Lorentzian (pseudo-Riemannian) manifold. Locally, the space is homeomorphic to Minkowski spacetime (pseudo-Euclidean), with chosen signature \{-, +, +, +\}. The general spacetime metric $g_{\mu\nu}$ allows us to calculate distances in curved spacetime,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu . \quad (A.1)$$

Let a curve $x^\mu$ be parametrized by the proper time parameter $\tau$. Then the distance between two points, $A$ and $B$, is

$$s_{AB} = \int_A^B ds = \int_A^B \frac{ds}{d\tau} d\tau = \int_A^B \sqrt{g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} d\tau . \quad (A.2)$$

In curved spacetime, a free particle propagates along a geodesic, which is a path that extremizes (minimizes) the proper spacetime interval. Consequently, the equations of motion characterize the minimal path, which gives the geodesic equation

$$\ddot{x}^\mu + \Gamma^\mu_{\nu\lambda} \dot{x}^\nu \dot{x}^\lambda = 0 , \quad (A.3)$$
where $\dot{x} = \frac{dx}{d\tau}$ and $\Gamma^\mu_{\nu\beta}$ is the Christoffel symbol, or affine connection,

$$\Gamma^\mu_{\nu\lambda} = \frac{1}{2}g^{\mu\alpha}(\partial_\lambda g_{\alpha\nu} + \partial_\nu g_{\alpha\lambda} - \partial_\alpha g_{\nu\lambda}) .$$

(A.4)

Note the symmetry under interchange of $\nu$, $\lambda$, such that $\Gamma^\mu_{\nu\lambda} = \Gamma^\mu_{\lambda\nu}$. Thus, if we know the curvature of spacetime (via the metric) due to the influence of mass energy distributions, then a free particle will move according to this geodesic equation of motion. To solve for the metric, we use the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} ,$$

(A.5)

where the stress-energy tensor $T_{\mu\nu}$ serves as the source for the spacetime curvature. Here, $R$ and $R_{\mu\nu}$ are the Ricci curvature scalar and tensor, respectively. These quantities are contractions of the Riemann curvature tensor $R^\lambda_{\sigma\mu\nu}$. We define

$$R = g_{\mu\nu}R^{\mu\nu}$$

(A.6)

$$R_{\mu\nu} = R^\alpha_{\mu\alpha\nu} = g^{\alpha\beta}R_{\alpha\beta\mu\nu}$$

(A.7)

$$R^\lambda_{\sigma\mu\nu} = \partial_\mu \Gamma^\lambda_{\sigma\nu} - \partial_\nu \Gamma^\lambda_{\sigma\mu} + \Gamma^\lambda_{\mu\rho} \Gamma^\rho_{\nu\sigma} - \Gamma^\lambda_{\nu\rho} \Gamma^\rho_{\mu\sigma} ,$$

(A.8)

Further, the stress-energy tensor $T_{\mu\nu}$ in Equation (A.5) obeys covariant transversality

$$D^\mu T_{\mu\nu} = 0 ,$$

(A.9)

which is a generalization of energy and momentum conservation ($\partial_\mu T_{\mu\nu} = 0$), using the covariant derivative

$$D_\nu = (D_\nu)^\lambda_{\lambda} = \partial_\nu \delta^\lambda_{\lambda} + \Gamma^\mu_{\nu\lambda} .$$

(A.10)

\footnote{To derive the (homogeneous) Einstein field equations, one can solve for the Euler-Lagrange equations of motion by varying the Einstein-Hilbert action $\frac{\delta S}{\delta g_{\mu\nu}} = 0$, where $S = \int d^4x \sqrt{-G}$.}
Thus, the Christoffel symbol plays the role of a gauge field, allowing us to connect all the tangent spaces by defining when a vector \( w^\lambda \) is parallel transported along a curve \( x^\nu \),

\[
v^\nu (D_\nu)^\mu_\lambda w^\lambda = 0 ,
\]

(A.11)

where \( v^\nu = \dot{x}^\nu \). Notice that if \( w^\lambda = v^\lambda = \dot{x}^\lambda \), then we find that a geodesic can equivalently be formulated as a curve whose own velocity vector (tangent to the curve) is always parallel transported,

\[
v^\nu (D_\nu)^\mu_\lambda v^\lambda = 0
\]

(A.12)

\[
v^\nu \left[ \partial_\nu \delta^\mu_\lambda + \Gamma^\mu_\nu\lambda \right] v^\lambda = 0
\]

(A.13)

\[
\left[ \frac{d}{d\tau} \delta^\mu_\lambda + \Gamma^\mu_\nu\lambda \frac{dx^\nu}{d\tau} \right] \frac{dx^\lambda}{d\tau} = 0
\]

(A.14)

\[
\ddot{x}^\mu + \Gamma^\mu_\nu\lambda \dot{x}^\nu \dot{x}^\lambda = 0 .
\]

(A.15)

The operator acting on the parallel transported vector \( v^\lambda \) above is sometimes given as \((D_\tau)^\mu_\lambda = v^\nu (D_\nu)^\mu_\lambda\). The covariant derivative is related to the Riemann curvature tensor by the relation

\[
[D_\mu, D_\nu] v_\sigma = R^\lambda_\sigma\mu\nu v_\lambda ,
\]

(A.16)

for any covariant vector \( v_\lambda \). Thus, if the Christoffel symbol plays the role of the gauge field in gravity, then the Riemann curvature tensor is the field strength. The covariant derivative is the tool we need for GR to obtain the symmetry of general coordinate invariance, or diffeomorphism invariance. Namely, physics is unchanged under a change of coordinates via a diffeomorphism (a smooth bijective map with a smooth inverse).

We will now formulate the same theory in terms of the vielbein \( e^a_\mu \) and the spin connection \( \omega^{ab}_\mu \), which further expresses the analogy of gravity as a (deformed) gauge theory. Since spacetime is a Lorentzian manifold, it is locally flat (Minkowski) and thus locally invariant
under Lorentz transformations. The vielbein is defined such that

\[ g_{\mu\nu}(x) = e^a_\mu(x)e^b_\nu(x)\eta_{ab}, \]

(A.17)

which relates the generally curved metric and the flat Minkowski metric, and therefore the curved and flat indices. Note that \( e^a_\mu(x) \) has the same number of degrees of freedom as \( g_{\mu\nu} \), since the metric has \( d(d+1)/2 \) components (a symmetric tensor) and the vielbein has \( d^2 - d(d-1)/2 = d(d+1)/2 \) components since we can set to zero the number of components which are equal to the total number of Lorentz transformation components, \( \Lambda_{\mu\nu} \), due to local Lorentz invariance. The index \( a \) is in the fundamental representation of the Lorentz group, not the adjoint representation, so we cannot identify the vielbein exactly as a gauge field, but something closely analogous.

The spin connection \( \omega_{\mu}^{ab} \) is the connection for the action of the Lorentz group on spinors. Here, the indices \( a, b \) are in the adjoint representation of the Lorentz group, which makes them antisymmetric. The covariant derivative acting on a spinor \( \psi \) is defined as

\[ D_\mu\psi = \partial_\mu\psi + \frac{1}{2}\omega_{\mu}^{ab}\sigma_{ab}, \]

(A.18)

where \( \sigma_{ab} = \frac{1}{4}[[\gamma_a, \gamma_b] \) specifies the generators of the Lorentz group in the spinor representation. Since the vielbein already has the same number of degrees of freedom as the metric, we require that the spin connection is fixed in terms of the vielbein. This is called the vielbein postulate, which states that the torsion \( T^a \) (somewhat like a field strength of the vielbein) vanishes,

\[ T^a_{[\mu\nu]} = 2D_{[\mu}e^a_{\nu]} = 2\partial_{[\mu}e^a_{\nu]} + 2\omega_{[\mu}^{ab}e^b_{\nu]} = 0. \]

(A.19)
For the spin connection, the object which acts like a field strength is

\[ R^{ab}_{\mu\nu}(\omega) = \partial_\mu \omega^{ab}_\nu - \partial_\nu \omega^{ab}_\mu + \omega^{ab}_\mu \omega^{bc}_\nu - \omega^{ac}_\nu \omega^{cb}_\mu. \] (A.20)

The relationship between \( R(\omega) \) and \( R(\Gamma) \) is

\[ R^{ab}_{\mu\rho\sigma} = e^a_\mu e^{-1,\nu}_b R^{\mu\nu}_{\rho\sigma}(\Gamma(\epsilon)), \] (A.21)

and thus the Ricci scalar can be expressed as

\[ R = R^{ab}_{\mu\nu} e^{-1,\mu}_a e^{-1,\nu}_b. \] (A.22)