Novel Pen Designs to Reduce the Effects of Hand Tremors to Writing

A Thesis Presented
by
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to

The Department of Mechanical and Industrial Engineering

in partial fulfillment of the requirements
for the degree of

Master of Science

in the field of

Mechanical Engineering

Northeastern University
Boston, Massachusetts

May 2015
First, I would like to express my sincere gratitude and appreciation to my thesis adviser Professor Rifat Sipahi for his selfless help, guidance, encouragement and trust. It would be impossible to finish the work without him. Thanks for sharing his wisdom and mind opening ideas.

Moreover, I want to thank Professor Andrew Gouldstone and Professor Beverly Kris Jaeger for their invaluable advice and suggestions, and Mr. Abate De May for building the prototype used in his independent study for this thesis.

In addition, I would like to thank Melda Ulusoy, Payam Mahmoudi Nia, Payam Parsinejad, Tingting Zhu, and many other friends who helped and encouraged me a lot during my study. I would also like thank my family for offering me this precious opportunity to study in the USA, for their trust and their love.

Finally, this research was supported in part by the US National Science Foundation Award CBET 1133992. Any opinions presented in this thesis are those of the author and not of the funding agency. In addition, some of the ideas presented in this thesis are under intellectual property protection at Northeastern University.
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ABSTRACT

Tremor is an involuntary, rhythmic, oscillatory movement of a body part, which is the most common human movement disorder. Over eleven million Americans have either Essential Tremor or Parkinson’s disease induced tremor. Tremors affect human upper limbs and challenge patients daily tasks, could embarrass them in social settings, and may even lead to depression. Writing is one of the most commonly performed daily tasks that is strongly deteriorated by hand tremors.

This thesis first presents a theoretical study on an actively controlled pendulum pen model with derivation of the pen dynamics and control design. The pen comprises a pendulum-like pen-rod that swings inside a tubular pen casing. An active actuation employed in between those two parts can compensate perturbations, while minimizing involuntary pen-tip oscillations. Since arbitrary pole placement is not possible in this control model, a constrained nonlinear optimization algorithm is constructed to minimize the deviation on the pen-tip and to also achieve fast system response, while keeping control effort at practical level. This thesis then presents an experimental study on a magnetically controlled tapping pen prototype to investigate the deviation of pen markings with the tapping motion. The prototype consists of electromagnet that generates cyclic electromagnetic forces to drive the pen-tip mounted to a permanent magnet. When the frequency of the pen-tip oscillation matches to that of the hand tremors, the pen-tip contacts the paper at the same point. Continuous dots appear if the user slowly moves the pen. Mechanically generated oscillations are used to emulate the tremors. Experimental results demonstrate the increase of pen markings at higher frequencies and decrease of pen markings with correct phase lag selection in the prototype control signal.
Chapter 1

Introduction

1.1 Tremor Definition

Tremor defined as involuntary, rhythmic, oscillatory movement of a body part is the most common human movement disorder [7]. Among many different types of tremor characteristics, two of them, action and rest tremor are often encountered in people with neurological disorders. Action tremor which can be subcategorized as postural, isometric, or kinetic tremor is observed when a patient voluntarily contracts his/her muscle [7]. Rest tremor, on the other hand, is involuntary, and occurs when a patient is relaxed and supported against gravity [7].

1.2 Common Forms of Tremors

The most common action tremor condition is Essential Tremor (ET), which affects patients from hands and wrists (95%) to head (34%), lower limbs (20%), voice (12%), face and trunk (2%), and oscillates between 4 and 12 Hz. It is estimated that up to 10 million American population is affected by ET and a large portion of them are over 65 years old [8, 9].

The most common form of rest tremor is caused by Parkinson’s disease (PD), a condition impacting about one million people in the US [10]. Research shows that more than 70 percent of patients with PD have rest tremor typically in their upper limbs with tremor frequency ranging from 4 to 6 Hz. A combination of both rest and action tremor is also observed in clinical practice in PD-induced tremors [7, 10].
1.3 Impacts of Tremors in Life

In general, tremors affect patients’ upper limbs which happen to be the dominant body part in performing daily tasks. Although tremors in patients’ upper limbs are no fatal, they impact daily tasks involving precision movements, such as feeding, drinking, typing, buttoning and writing. This could cause embarrassment in social settings, and can even lead to lack of self-sufficiency, and depression \[10, 11\]. Evidence of this is found in \[8\], where it is reported that up to 25 percent of ET patients retire prematurely and 60 percent of patients shift their career paths due to involuntary oscillations in their hands \[8\].

1.4 Treatments

Treatment of tremor varies from pharmacological treatment to gamma-knife procedures, lesioning surgery, and deep brain stimulation (DBS). In terms of pharmacological treatment, Levodopa is mainly used for PD-induced tremors, while its efficacy declines in years and users may have severe side effects called “long-term Levodopa syndrome” \[12\]. For ETs, Primidone has a superior efficacy in reducing magnitudes of tremors. This tremor reduction is reported to improve patient’s writing and speaking functions \[12\]. The most promising non-pharmaceutical treatment is DBS that reduces tremors by about 90%. To perform the stimulation, electrodes, which can be removed without irreversible damage, are implanted into the brain. The DBS is adjustable based on each patient, and is well-tolerated by elder patients \[12\]. However, safety and efficacy of the different DBS target areas are unknown that requires more controlled trials \[13\].

When treatments fail or patient is anxious about invasive brain surgeries, suppressing hand tremors in a less invasive, more effective, and more affordable way would be valuable for the target population. Studies along these lines include designing assistive devices to suppress or manage hand tremors in daily tasks. Next, let us review some of these designs.

A painting tool is designed using Microsoft Visual C++ language that allows users with hand tremors to draw smoothly on a computer. The design applies a moving average method to create blurring effects during drawing and uses compulsory elimination to compensate sudden movements \[14\].

Drinking cups are designed to prevent water from spilling out using damping fluid \[15\] or a gyroscopic stabilizer \[16\]. Cited work \[16\] implements ”Schlick stabilizer”, which
consists of a high speed rotating disk, to generate gyroscopic torque in the opposite direction of the incoming disturbance torques. On the other hand, in [15], a fully passive cup stabilizer is designed using damping fluid. User could put the drinking cup into the cup holder filled with damping fluid to balance the cup motion provoked by the user’s tremors.

A meal-assist robot in [17] is designed, which is able to estimate tremor frequencies through an adaptive filter and suppress hand tremors based on the estimation, while insulating the voluntary motion in the feeding process.

In addition to assistive devices, orthotic devices are developed to suppress hand tremors in a musculoskeletal level [18, 19]. The device is able to actively suppress tremors in human upper limbs up to 99.2% by employing pneumatic cylinders to generate suppressive forces.

Also, technologies outside of the rehabilitation field are found in microsurgery where active controlled tools are needed to stabilize tremors in surgeons’ hands [20]. The key component which is a three degrees of freedom piezoelectric actuator, minimizes the undesired hand motion while preserving eye-hand coordination. The device is reported to have a reduction in position error between 32% and 52%.

1.5 Problem Statement

Writing, which is strongly deteriorated due to tremors, is one of the most important daily task. Though one may argue that it can be replaced by a computer in many situations, computers are not always desirable: the target population does not naturally use computers, and typing is problematic due to double tapping. Moreover, a legible signature produced by hand is always required in legal documents. On the other hand, there are not many innovations on pen designs to improve the handwriting of tremor patients. This presents an opportunity to design novel pens that can suppress effects of hand tremors on writing.

1.6 Literature Review: Pen Design for Tremor Reduction

There exist several products and publications related to pen designs for suppressing hand tremors effect in writing. The existing designs can be categorized as passively controlled and actively controlled devices. While most of the designs have not yet been val-
idated by human subject studies, these devices reveal promising results in terms of either customer feedback or theoretical results.

On Amazon retail website, there are two major types of passively controlled assistive writing devices. The first type employs ergonomic grips that can decrease the pressure needed for fingers to hold the pen and so to relax the muscles in upper limbs. The second type suppresses hand tremors by adding extra weights to the pen. See Figure 1.1 for related products. Although there are no publications on mathematical modeling of these designs, the human upper limb dynamics studied in [21] reveals their potential for use. The cited study indicates the similarity between the human upper limbs and the model of a spring-mass dynamics, where the natural frequency of the human upper limbs can be calculated as:

\[ \omega_n = \sqrt{\frac{K}{I_r}} \]  

(1.1)

where \( I_r \) denotes the musculoskeletal inertia, and \( K \) denotes the stiffness of the muscle. It is self-evident that the “Ergonomic pen” decreases the tremor frequency by lowering the stiffness on user’s muscles, while the “Weighted pen” suppresses the hand tremors by inertia increment.

Customer ratings for these two kinds of pens vary from 3 to 4 stars with sample size of less than 40 customers. Negative reviews focus on efficacy of the pen, the unnatural feeling during writing, and the fatigue caused in long term usage.

Other passively controlled designs are proposed in studies [4] and [5] from California State University (CSU), where Naritomi proposed three pen designs and Dessau proposed one pen design. All the models in the cited work are based on similar models with certain modifications. The pen models have a spring-damper element sandwiched between the pen-shell and the pen-rod to passively isolate the tremor-induced involuntary movement of the pen in the dominant direction (perpendicular to the arm) while preserving the writing action, see Figure 1.2 - 1.3.

To be able to compensate for hand tremors during writing, the system parameters in [4] and [5] are designed based on the minimum transmissibility equation:

\[ TR = \frac{X}{Y} = \sqrt{\frac{1 + 4 \zeta^2 r^2}{(1 - r^2)^2 + 4 \zeta^2 r^2}} \]  

(1.2)
Figure 1.1: a) Weighted pen with ergonomic grip [1], b) Ergonomic Y shaped pen [2], c) Weighted glove [3].

Figure 1.2: Passively controlled hand tremor pen [4] in 2012.
where $\zeta$ is the damping ratio, and $r$ is the frequency ratio defined as:

$$r = \frac{\omega}{\omega_n}$$

(1.3)

where $\omega$ is the excitation frequency determined by the user’s tremor frequency, and $\omega_n$ is the natural frequency of the pen. While the relation of transmission ratio and frequency ratio [22] proves that $\zeta$ is semi-independent of frequency ratio, the system parameters can be designed based on user’s tremor frequency.

Different from [4], in the work of [5], studies are conducted with 15 human subjects with either ET or PD-induced tremor. The study tested four assistive pens, and a regular ball-point pen. Four assistive pens include an “Ergonomic pen”, a “Weighted pen”, a prototype from [4], and a prototype from [5]. Participants are asked to test selected devices and rate the effectiveness of each device. Interestingly, among all the assistive pens, the “Weighted pen” has the highest rating and Prototype from [4] is rated the lowest. The researcher mentions the existence of a learning curve which may be the cause of low ratings for some of the prototypes. Finally, note that an orthotic drafting arm was also tested in the cited work, where a 60% of effectiveness was rated by the subjects, nevertheless this device is an orthotic device, no an actual pen. In addition, the human subject study in the cited work can be expanded to further allow all subjects to try all the devices. This would enhance the statistical results, and reduce bias in user feedback since feedback is based on subject’s perception of the devices tested.
The advantage of passively controlled designs are their zero energy consumption and portability based on the size. They are more durable and easier for manufacturing since they have fewer components. On the other hand, no optimized performance is guaranteed, since the tremor frequency has variability [8, 10].

Generally speaking, higher performance may be guaranteed with actively controlled devices in which the actuation element could compensate the system’s natural frequency to avoid the resonance [23]. In the application of tremor cancellation, an active controller could address the changes in tremor frequency from patient to patient, and with respect to time.

A tremor pen design along these lines is introduced in [6]. A Linear Voice Coil Actuator is mounted to the tail of an assistive pen which is able to generate displacement in vertical directions. Then with an axis translation mechanism, the vertical displacement can be converted to a horizontal displacement for oscillation compensation, see Figure 1.4.

![Figure 1.4: Proportional controlled pen](image)

While there is no human subject study on the pen design to validate system performance in [6], a testing platform, Figure 1.5 is built to mechanically generate tremor based on hand tremor data from a PD patient. It is stated that the proposed pen design is able to suppress the mechanically generated tremor by 56.57%. In the cited study, however, analysis on the pen dynamics as well as controller design are not presented, see Figure 1.5.
1.7 Benchmarking

In order to compare the design features in each pen presented above, we compared them in terms control method, mathematical modeling study, control design study and the validation on the performance. For each feature the design has, we will grant it with a check mark, see Table 1.1.

Table 1.1: Benchmarking of pen design features

<table>
<thead>
<tr>
<th>Pen type/Terminology</th>
<th>Passive control</th>
<th>Active control</th>
<th>Math. model</th>
<th>Control design</th>
<th>Performance validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ergonomic/weighted pen</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CSU pen</td>
<td>✓</td>
<td>✓ (1D)</td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>LVCA pen</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The ergonomic/weighted pen is a passively controlled pen design. To the best of our knowledge, no studies on its mathematical model nor performance has been published. The CSU pen is also a passively controlled pen design based on only single degree of freedom pen dynamics. The LVCA pen is an actively controlled pen, but is does not have any model derivation nor control design study.
Therefore, we can conclude that, to our best knowledge, no actively controlled pen design has been studied in terms of complete mathematical model derivation and control algorithm design.

1.8 Objective

The objectives of this thesis are determined based on the observations made in the previous sections. It is to design an actively controlled pen with a complete dynamic modeling and controller design, and to experimentally verify the performance of an existing prototype.

In the following sections, we will first theoretically study the detailed dynamic model derivation of an actively controlled pen dynamics expanding on [4, 5], which we will call Actively-controlled pendulum pen (ACPP). The control algorithms are designed based on constrained nonlinear optimization and relative stability conditions, since the pen dynamics is found to be state uncontrollable. Then we will report results from the experimental studies on an actively controlled pen prototype which we call Magnetically controlled tapping pen (MCTP), in which we will present how to imitate hand tremors using a IP02 cart with position control provided by Quanser.
Chapter 2

Actively Controlled Pendulum Pen (ACPP)

2.1 Design Overview

The ACPP model proposed here is analogous to a pendulum-cart system and its very first version was conceived by our group at Northeastern University in the spring of 2010 as a part of an NSF grant application. The design includes a pen-shell (orange), a pen-rod (blue), a pen-weight (grey), and a spring-damper-actuator element (red) sandwiched between the pen-rod and pen-shell, see Figure 2.1 for a CAD drawing. In some sense, it expands on the work of [4, 5].

The pen-rod which is fixed to the pen-shell is designed with one end pinned to the pen-shell, around which it can perform small rotations. The spring-damper-actuator element is used to actively isolate the pen-rod from the vibrations on the pen-shell subjected to tremor-induced periodic forcing. While the pen-rod is pinned in this study and thus can cancel hand tremors in its dominant direction perpendicular to the forearm, future versions will consider a ball joint to allow the pen to be controlled on the plane of paper using actuators placed orthogonal with respect to each other.

Due to the nature of hand tremor, tremor frequency may vary with time and from subject to subject. To render optimized performance from the pen, a sensing element should be added onboard the pen to detect user’s tremor frequency in real-time, and a micro-controller is needed to fine tune the controller actions continuously on the pen-rod in order to render the pen-tip as insensitive as possible to such tremor-induced vibrations.
As shown in the schematic demonstration of the pen design in Figure 2.2, the system has two degrees of freedom. The pen-shell with mass $m_1$ is able to move in $x$ direction when subjected to tremor-induced force $F$. Pen-rod with mass $m_2$ is able to move in $x$ direction and rotate in $\theta$ direction. The length of the pen-rod from the pin to the tip is $L$ and the spring-damper-actuator element is placed $L_1$ units away from the pin. The element has a stiffness of $k_1$, and a damping coefficient of $c_2$ and is able to generate controller force $u(t)$. The friction between the pen-tip and the paper is modeled as velocity-dependent viscous friction with a constant of $c_1$.

Assuming for the moment that the tremor-induced force disturbance is applied at the center of mass, we can ignore any rotary motion on the pen-shell. Moreover, since the distance the pen-tip travels should not be stabilized due to the nature of writing, the pen-tip speed $v(t)$ is the control variable. In other words, we aim to suppress the effects of hand tremor in writing by means of minimizing the magnitudes of speed at the pen-tip.
2.2 Derivation of Equations of Motion

Based on the proposed system, it is valid to assign the independent set of generalized coordinates as $\xi = (\xi_1, \xi_2) = (x, \theta)$, and the independent set of associated admissible variations as $\delta \xi = (\delta x, \delta \theta)$. Since the number of generalized coordinates equals the number of degrees of freedom, the system is holonomic. Thus, one can use the Lagrangian formulation to develop the equations of motion [24].

First, before we proceed to any further derivations on the equations of motion, the displacement and speed at point $A$, $B$, and $C$ in Figure 2.2 need to be found. Assuming that point $A$ and $B$ have no vertical displacement, we have displacement of $A$, $B$ and $C$ in $x$ direction denoted by $x_A$, $x_B$, and $x_C$, respectively, and the displacement of point $C$ in $y$
direction by \( y_C \). Based on the geometry, the following relationships hold:

\[
\begin{align*}
    x_A &= L_1 \sin(\theta) + x \\
    x_B &= L \sin(\theta) + x \\
    x_C &= \frac{1}{2} L \sin(\theta) + x \\
    y_C &= -\frac{1}{2} L \cos(\theta) + \frac{1}{2} L
\end{align*}
\]

By taking the derivatives in Eqns. (2.1) - (2.4) with respect to time, the velocity of points \( A \), \( B \), and \( C \) in each direction are found as:

\[
\begin{align*}
    \dot{x}_A &= L_1 \dot{\theta} \cos(\theta) + \dot{x} \\
    \dot{x}_B &= L \dot{\theta} \cos(\theta) + \dot{x} \\
    \dot{x}_C &= \frac{1}{2} L \dot{\theta} \cos(\theta) + \dot{x} \\
    \dot{y}_C &= \frac{1}{2} L \dot{\theta} \sin(\theta)
\end{align*}
\]

Next, we find the generalized forces using the relation of variational work:

\[
\delta W^{nc} = \sum_{i=1}^{N} f_i^{nc} \cdot \delta R_i = \sum_{j=1}^{n} \Xi_j \delta \xi_j
\]

where \( f_i^{nc} \) are nonconservative forces acting through admissible variations \( \delta R_i \), \( N \) is the number of \( f_i^{nc} \) as well as \( \delta R_i \), \( \Xi_j \) are the generalized forces doing work under variations \( \delta \xi_j \) in each independent generalized coordinate \( \xi_j \), and \( n \) is the number of \( \Xi_j \) as well as \( \delta \xi_j \).

Given the dynamic system at hand, the variational work is done by the tremor-induced force \( F(t) \), actuation force \( u(t) \), and dissipative forces due to dashpot and friction effects. Hence, the term \( \sum_{i=1}^{N} f_i^{nc} \cdot \delta R_i \) in Eqn. (2.9) becomes:

\[
\sum_{i=1}^{N} f_i^{nc} \cdot \delta R_i = F(t) \delta(x) + u(t) \delta(x_A - x) - c_2 (\dot{x}_A - \dot{x}) \delta(x_A - x) - c_1 \dot{x}_{x,B} \delta(x_B)
\]

Substituting Eqn. (2.10) into Eqn. (2.9) results in:

\[
F(t) \delta(x) + u(t) \delta(x_A - x) - c_2 (\dot{x}_A - \dot{x}) \delta(x_A - x) - c_1 \dot{x}_{x,B} \delta(x_B)
= \Xi_x \delta(x) + \Xi_\theta \delta(\theta)
\]
Substitute next Eqns. (2.1) - (2.8) into Eqn. (2.11). This rewrites the left hand side of Eqn. (2.11) as:

\[ F(t) \delta x + u(t) \delta (L_1 \sin(\theta)) - c_2 \left( L_1 \dot{\theta} \cos(\theta) \right) \delta (L_1 \sin(\theta)) - c_1 \left( L \dot{\theta} \cos(\theta) + \dot{x} \right) \delta (L \sin(\theta) + x) \tag{2.12} \]

Thus, the generalized forces for \( x \) and \( \theta \) coordinates are extracted from Eqn. (2.11) as:

\[ \Xi_x = F(t) - c_1 \left( \dot{x} + L \dot{\theta} \cos(\theta) \right) \tag{2.13} \]

\[ \Xi_\theta = -c_1 \dot{x} L \cos(\theta) - \left( c_1 L^2 + c_2 L_1^2 \right) \dot{\theta} (\cos(\theta))^2 + u(t)L \cos(\theta) \tag{2.14} \]

Next, the kinetic energy of the pen-rod and pen-shell is calculated as:

\[ T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} m_2 \left( \dot{x}_C^2 + \dot{y}_C^2 \right) \tag{2.15} \]

Substituting Eqns. (2.7) - (2.8) into Eqn. (2.15), we get:

\[ T = \frac{1}{2} m_2 \left( \dot{x}^2 + L \dot{\theta} \cos(\theta) + \frac{1}{4} L^2 \dot{\theta}^2 \right) + \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} I \dot{\theta}^2 \tag{2.16} \]

Similarly, the potential energy of the system, which is only a function of pen-rod motion, is expressed as:

\[ V = \frac{1}{2} k_1 (x_A - x)^2 + mg y_C \tag{2.17} \]

Substituting Eqns. (2.1) - (2.4) into Eqn. (2.17), we get:

\[ V = \frac{1}{2} k_1 L_1^2 (\sin(\theta))^2 + mg (L/2 - 1/2 L \cos(\theta)) \tag{2.18} \]

Consequently, the Lagrangian function \( \mathcal{L} = T - V \) can be formulated using Eqn. (2.16) and Eqn. (2.18). Combining \( \mathcal{L} \) with Eqns. (2.13) - (2.14), we have,

\[ \frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\xi}_j} \right) - \frac{\partial \mathcal{L}}{\partial \xi_j} = \Xi_j \tag{2.19} \]

which gives the equations of motion:

\[ (m_1 + m_2) \ddot{x} + 1/2 m_2 L \ddot{\theta} \cos(\theta) + 1/2 m_2 L \dot{\theta}^2 \sin(\theta) + c_1 \dot{x} + c_1 L \dot{\theta} \cos(\theta) - F(t) = 0 \tag{2.20} \]

\[ (I + 1/4 L^2 m_2) \ddot{\theta} + 1/2 m_2 L \ddot{x} \cos(\theta) - m_2 L \dot{x} \dot{\theta} \sin(\theta) + 1/2 mg L \sin(\theta) + k_1 L_1^2 \sin(\theta) \cos(\theta) + c_1 L \dot{\theta} \cos(\theta) + (c_1 L^2 + c_2 L_1^2) \dot{\theta} (\cos(\theta))^2 - u(t)L \dot{\theta} \cos(\theta) = 0 \tag{2.21} \]
2.3 Linearization

As expected, the equations of motion Eqns. \((2.20) - (2.21)\) represent the nonlinear pen dynamics. Note, however, that the pen-tip should ideally operate around its equilibrium position, \(\theta = 0\), desirably with small motion, \(|\theta| < 5^\circ\). Hence, it makes sense to pursue a linear systems approach. To this end, we linearize Eqns. \((2.20) - (2.21)\) around its equilibrium point \([24]\), where \(\theta(t), x(t), \dot{\theta}(t), \) and \(\dot{x}(t)\) are small quantities such that \(\sin(\theta) \approx \theta, \cos(\theta) \approx 1, \dot{\theta}^2 \approx 0, \ddot{x} \dot{\theta} \approx 0\). This leads to the following linear time-invariant pen dynamics:

\[
\begin{align*}
(m_1 + m_2) \ddot{x} + \frac{1}{2} m_2 L \ddot{\theta} + c_1 \dot{x} + c_1 L \dot{\theta} &= F(t) \\
\left(I + \frac{1}{4} L^2 m_2\right) \ddot{\theta} + \frac{1}{2} m_2 L \ddot{x} + \left(\frac{1}{2} L m_2 g + k_1 L_1^2\right) \theta + \left(c_1 L^2 + c_2 L_1^2\right) \dot{\theta} + c_1 L \dot{x} &= u(t)L_1 
\end{align*}
\]

(2.22) (2.23)

In the latter sections, we derive three cases based on Eqns. \((2.22) - (2.23)\), where each case represents a passively controlled, an actively controlled, and a regular pen model.

2.4 Tremor Force Estimation

Before we proceed to case studies, it would be beneficial to estimate the magnitude of tremor-induced forces arising in realistic handwriting activities in order to better estimate pen-tip deviations and propose remedies for it. While such force estimation to our best knowledge has not been published, it is possible to make this estimation indirectly from handwriting samples. Through our literature review, we came across many such samples, yet most of them did not present any scale. Ref \([25]\) is an exception, where researchers provided a scale along with a patient’s tracing of Archimedes Spirals using a pen. The results borrowed from the cited study are on page 949.

To estimate the tremor force, we first calculate the amplitude of the sample under the untreated condition. For measurement purposes, we imported the figure into SolidWorks using “Sketch Picture” command. This command enables users to define a certain distance in a digital picture and relate the distance based on a reference scale. By using the “Measure Tool” in SolidWorks, we are able to measure the distance between any two points. The average tremor amplitude in the writing sample is found as 0.17 inches (4.3 mm). In the following, two methods are presented for estimating tremor-induced forces based on the calculated average amplitudes in tremor-induced handwriting.
2.4.1 Tremor Force Calculation Method A

Consider that the handwriting sample of a patient is similar to a sine function given by

\[ x = A \sin(\omega t) \] (2.24)

We then obtain velocity and acceleration as:

\[ v(t) = A\omega \cos(\omega t) \] (2.25)
\[ a(t) = -A\omega^2 \sin(\omega t) \] (2.26)

Hence, the amplitude of the tremor-induced force reads \( F = mA\omega^2 \), where we assume the tremor frequency to be 10 Hz (\( \approx 60 \text{ rad/s} \)), the effective mass of the pen 0.1 kg, and the amplitude of oscillations \( 4.3 \cdot 10^{-3} \text{ m} \) based on the measurements from SolidWorks. Plugging the numerical values into \( F \), we get an estimation of tremor-induced force at about 1.55 N.

2.4.2 Tremor Force Calculation Method B

We simulate the behavior of the pen using Eqns. (2.22) - (2.23), when the pen is subject to a tremor-induced cyclic force, and no controller is used \( u(t) = 0 \), and the pen rod is rigidly attached to the pen-shell \((k_1 = \infty, c_2 = 0)\). Simulation shows a displacement amplitude of 2.2 mm when the input force amplitude is 1 N. Thus, to have a 4.3 mm amplitude displacement on the paper as measured from SolidWorks, the input force on the pen needs to be 1.95 N, under the linear system assumption. Since the viscous friction for the pen being used and the precise frequency of oscillations during the handwriting sample collection are unknown, this may cause discrepancy between method B and method A, yet both methods capture the same order of magnitude.

2.5 Case Studies

In the following sections, we will discuss three cases derived from the linearized equations of motion. Consider in Eqn. (2.23) when \( u(t) = 0 \), no controller is available, we recover a passively controlled pen, which is similar to that described in [4, 5]. Next, if we set \( u(t) \neq 0 \), the pen dynamics is actively controlled, which will be mainly discussed in the
later sections. In addition, assuming $u(t) = 0$, $k_1 = +\infty$, and $c_2 = 0$, we obtain a regular writing pen where the pen-rod is fixed to the pen-shell, see Table 2.1.

Table 2.1: Three cases derived from linearized equations of motion

<table>
<thead>
<tr>
<th>Case</th>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: $u(t) = 0$</td>
<td>No controller is available. The pen dynamics similar to the cited works [4, 5] is recovered.</td>
<td></td>
</tr>
<tr>
<td>Case 2: $u(t) \neq 0$</td>
<td>The pen dynamics is actively controlled.</td>
<td></td>
</tr>
<tr>
<td>Case 3: $u(t) = 0$, $k_1 = +\infty$, $c_2 = 0$</td>
<td>We received a regular writing pen model.</td>
<td></td>
</tr>
</tbody>
</table>

In the following sections, we will compare the pen-tip speed magnitudes for each cased study on Table 2.1.

2.6 ACPP Case One: $u(t) = 0$

When the pen-rod is assumed to be attached rigidly to the pen-shell and no controller is available, we recover a passively controlled pen, which is similar to that described in [4, 5]. This corresponds to $u(t) = 0$ in ACPP model. Taking next the Laplace transformation on Eqns. (2.22) - (2.23) with zero initial conditions, one obtains:

$$
(m_1 + m_2) X(s) + \frac{1}{2} m_2 L \Theta(s) s^2 + (c_1 X(s) + c_1 L \Theta(s)) s = F(s) \quad (2.27)
$$

$$(I + \frac{1}{4} L^2 m_2) s^2 \Theta(s) + \frac{1}{2} m_2 L s^2 X(s) + c_1 L s X(s) + (c_2 L_1^2 + c_1 L^2) s \Theta(s)$$

$$+ \left( \frac{1}{2} L m_2 g + k_1 L_1 \right) \Theta(s) = 0 \quad (2.28)$$

Since the goal here is to study the tip speed of the pen, the open-loop transfer function $G_0(s) = V(s)/F(s)$ needs to be calculated, where $V(s) = s(X(s) + L \Theta(s))$ is the speed at pen-tip. This transfer function can be confirmed to be:

$$G_0(s) = \frac{n_0 s^2 + n_1 s + n_2}{d_0 s^3 + d_1 s^2 + d_2 s + d_3} \quad (2.29)$$
where

\[ n_0 = 4 I - L^2 m_2 \]
\[ n_1 = 4 c_2 L_1^2 \]
\[ n_2 = 2 L m_2 g + 4 k_1 L_1^2 \]
\[ d_0 = L^2 m_2 m_1 + 4 I m_1 + 4 I m_2 \]
\[ d_1 = 4 c_2 L_1^2 m_1 + 4 I c_1 + 4 c_1 L^2 m_1 + 4 c_2 L_1^2 m_2 + L^2 m_2 c_1 \]
\[ d_2 = 2 L m_2^2 g + 4 k_1 L_1^2 m_1 + 4 c_2 L_1^2 c_1 + 2 L m_2 g m_1 + 4 k_1 L_1^2 m_2 \]
\[ d_3 = 2 L m_2 g c_1 + 4 k_1 L_1^2 c_1 \]
\[ L_1 = 2/3 L \]
\[ I = 1/12 L^2 m_2 \]

Numerical values are selected as shown on Table 2.2. One note here is that the viscous friction between the pen-tip and paper is determined from Chigira’s work [26], where the friction acting between a ballpoint pen and a paper was studied based on the force acting on the pen and pen velocity measurements. Assuming \( F = c \dot{x} \), the viscous friction constant can be approximated from the cited study as 5.714 Ns/m. Based on these numerical values,

Table 2.2: Numerical values of the parameters of the pen dynamics

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pen-shell mass ((m_1))</td>
<td>0.02</td>
<td>kg</td>
</tr>
<tr>
<td>Pen-rod mass ((m_2))</td>
<td>0.005</td>
<td>kg</td>
</tr>
<tr>
<td>Pen-rod length ((L))</td>
<td>0.15</td>
<td>m</td>
</tr>
<tr>
<td>Spring stiffness ((k_1))</td>
<td>10</td>
<td>N/m</td>
</tr>
<tr>
<td>Damping coefficient ((c_2))</td>
<td>0.003</td>
<td>Ns/m</td>
</tr>
<tr>
<td>Viscous friction ((c_1))</td>
<td>5.714</td>
<td>Ns/m</td>
</tr>
<tr>
<td>Gravity constant ((g))</td>
<td>9.81</td>
<td>m/sec^2</td>
</tr>
</tbody>
</table>

Eqn. (2.29) reads:

\[
G_0(s) = \frac{-23.529 s^2 + 130106.667 + 37.647 s}{s^3 + 3496.565 s^2 + 3467.782 s + 743429.493} \tag{2.30}
\]
In Figure 2.3, we present the Bode plot of the transfer function $G_0(s)$. We find that for a cyclic force with 1 Newton of amplitude at 8 Hz applied on the pen-shell, the pen-tip velocity magnitude is at -32.6 dB, which is about $2.34 \cdot 10^{-2}$ m/s. This corresponds to speed to force ratio of 1:43. Further, simulation reveals that the pen-tip strokes under cyclic disturbance forces at 8 Hz will produce around 0.95 mm of ink markings while the settling time for the system is about 8 seconds, see Figure 2.4 and Figure 2.5. The Bode plot and simulations are produced by MATLAB and Simulink. In addition, the 1 N cyclic force can be substituted for the estimated tremor-induced force, 1.95 N, calculated in the previous sections, which yields $(1.95) \cdot (0.95) = 1.85$ mm (0.073 in) ink markings on the paper.

Figure 2.3: Bode plot of pen-tip speed for ACPP case one represented by $G_0(s)$ in Eqn. (2.30)

2.7 ACPP Case Two: $u(t) \neq 0$

Here we present the pen dynamics where the controller is available to compensate oscillations caused by tremor-induced cyclic forces. State-space representation of the pen dynamics from Eqns. (2.22) - (2.23) is given by:
Figure 2.4: Simulation of pen-tip displacement versus time in ACPP case one

Figure 2.5: Simulation of pen-tip speed versus time in ACPP case one

\[
\dot{x}(t) = Ax(t) + Bu(t) + EF(t) \quad (2.31)
\]
\[
y(t) = Cx(t) \quad (2.32)
\]

where the state variable matrix \(x(t)\), the state matrix \(A\), control matrix \(B\), input matrix \(E\), cyclic tremor-induced force \(F\), and the output matrix \(C\) are self-evident from Eqns. (2.22)
Here $u(t)$, which is the controller input, needs to be designed. With the parameters selected in Table 2.2, matrices stated above becomes:

$$
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 134.4470588 & 48.79000000 & 20.18117647 \\
0 & 0 & 0 & 1 \\
0 & -24200.47059 & -3252.666667 & -3631.011765
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
0 \\
-47.05882353 \\
0 \\
3137.254902
\end{bmatrix};
E = \begin{bmatrix}
0 \\
47.05882353 \\
0 \\
-470.5882353
\end{bmatrix};
C = \begin{bmatrix}
0 & 1 & 0 & 0.15
\end{bmatrix}
$$

To control the system, we first study its controllability. We find that the rank of the controllability matrix is three. Since the number of states is four, this means there exists one uncontrollable state and hence arbitrary pole placement is impossible. Notice, however, that we aim to control the speed at the pen-tip, not all four states. Thus, output controllability is studied next to investigate the feasibility of controlling only the pen-tip speed [27]. The rank of the output controllability matrix is found to be one, which equals to the number of rows in the output matrix $C$. Hence, the pen dynamic system is output controllable.

Set next the controller to be $u(t) = -Kx(t)$ for the ACPP system, assuming all the states are measurable. The substitution of $u(t)$ into Eqn. (2.31) leads to:

$$
\dot{x}(t) = (A - BK)x(t) + EF(t)
$$

$$
y(t) = Cx(t)
$$

Recall that, since the above system at hand is not state-controllable, arbitrary pole placement is impossible, and the design of $K$ needs additional effort. Moreover, another challenge is to minimize pen-tip speed amplitudes against cyclic perturbations due to $F(t)$, while, at the same time, achieving rapid transient response, and keeping the controller forces at acceptable values. Table 2.3 lists the three challenges in the controller design.
Table 2.3: List of challenges in the controller design

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A:</td>
<td>System stability</td>
</tr>
<tr>
<td>B:</td>
<td>Minimized pen-tip speed magnitude under large cyclic perturbations</td>
</tr>
<tr>
<td>C:</td>
<td>Rapid transient response</td>
</tr>
</tbody>
</table>

### 2.7.1 Case Two Controller Design

Here, the objective is to find the controller matrix $K$ satisfying the aforementioned conditions. For this, relative stability of the system from the pole placement point of view needs to be guaranteed within a Bode analysis, and the controller effort will be analyzed via simulations once $K$ is designed. We start by shifting the $s$-plane imaginary axis $\sigma$ units to the left via the linear transformation $s = s' - \sigma$ [27], and applying Routh’s stability criterion on this transformed equation. From Routh’s Array, we obtain the first column elements, which must be all positive as condition of $\sigma$-stability. These conditions will be used below in the design of $K$.

Since $\sigma$-stability analysis is not sufficient as we need to also guarantee certain system performance in frequency response, we need to minimize the amplitude of the closed-loop system within a Bode analysis, while respecting $\sigma$-stability conditions. This can be achieved by performing constrained nonlinear optimization using MATLAB.

### 2.7.2 Constrained Nonlinear Optimization

The command “fmincon” from MATLAB is used to perform the aforementioned constrained nonlinear optimization. This optimization problem is defined as:

$$
\begin{align*}
  & \quad \min \quad f(x) \\
  & \quad \text{subject to} \quad h(x) = 0 \\
  & \quad \quad \quad \quad \quad \quad g(x) \leq 0
\end{align*}
$$

(2.35)

where $f(x)$ represents the function to be minimized, which is the magnitude of the pen-tip speed given by $|G_c(j\omega)|$ with $G_c(s)$ being the closed-loop system transfer function found from Eqns. (2.33) - (2.34) and $\omega$ being the excitation frequency of the tremor-induced force, which is taken as 50 rad/sec (8 Hz); negative $g(x)$ represents the nonlinear inequalities
given by the negative of the first column of Routh’s array in light of $\sigma$-stability; $h(x)$ is the set of nonlinear equalities, which is null in this study.

Here, we use the interior-point algorithm to replace the optimization problem Eqn. (2.35) to a sequence of barrier sub-problems in the form of:

$$\min_{x,s} f(x) - \mu \sum_{i=1}^{m} \ln (s_i)$$

subject to $g(x) + s = 0$  \hspace{1cm} (2.36)

where $\mu$ is the barrier parameter and $s_i$ is a vector of slack variables $s = \{s_i\}$. As $\mu$ decreases to zero, minimum of the optimization problem in Eqn. (2.36) should converge to that in Eqn. (2.35) \cite{28,29}. Figure 2.6 is the process flow for finding the candidate controllers with respect to each $\sigma$.

2.7.3 Case Two Candidate Controllers

We calculated a set of fifteen candidate controllers using command “fmincon” in MATLAB as described above, with different $\sigma$ values ranging from 0 to 14. After the controllers are calculated, system response is analyzed at the frequency of 8 Hz ($\omega \approx 50$ rad/s) to validate the design. Bode diagram is plotted using each controller, to find the magnitude of the pen-tip speed at this frequency. Moreover, simulations are performed to validate the system.
settling time as well as to compute the controller effort. While various conditions can be set for system performance, here the system should be deemed acceptable if its performance is superior over the uncontrolled pen dynamics. For this purpose, we aim to achieve a settling time of less than 1.5 seconds, a controller force of less than 5 Newtons, and pen-tip speed magnitudes attenuated below $-40$ dB.

Results are presented in Figures 2.7 - 2.11. In Figure 2.7, we present how $\sigma$ and the pen-tip speed magnitude are related. Interestingly, we find here a tradeoff between the transient response speed of the system determined by larger $\sigma$ and how the pen-tip speed magnitudes grow undesirably with larger $\sigma$. In Figures 2.8 - 2.9 we see that $\sigma$ is inversely proportional to control effort. This result promises that sufficiently large $\sigma$ and hence high response speed is possible with an actuator force less than 10 N. Finally, we find that the settling time can be reduced, at the same time, to less than 1 sec for $\sigma > 9$, see Figure 2.11.

The acceptable $\sigma$ values are considered to be from 6 to 11, where the pen-tip speed magnitudes range from $-51.55$ dB to $-41.81$ dB, controller forces are from 2.18 N to 1.22 N, and settling times are between 1.3 s to 0.75 s.

![Figure 2.7: Plot of pen-tip speed magnitude in dB versus $\sigma$-shift in ACPP Case Two](image)

Figure 2.7: Plot of pen-tip speed magnitude in dB versus $\sigma$-shift in ACPP Case Two

To help better understanding the system performance with the designed controllers, we next study the Bode diagram and simulation results for the ACPP model, Case Two. Under these settings, the pen-tip speed magnitude for the closed-loop system is $-46.9$ dB ($4.5 \times 10^{-3}$ m/s) which means a speed to force ratio of 1:220. Also, simulations show that
Figure 2.8: The maximum control effort from simulation versus $\sigma$-shift in ACPP Case Two

![Graph showing controller forces vs sigma-shift](image)

Figure 2.9: Figure enlarged from Figure 2.8

for a cyclic force with 1 N of amplitude at 8 Hz as a disturbance, in the closed-loop system, the pen-tip strokes produce around 0.18 mm of ink markings. Using the estimated tremor force from the previous section, the incoming force has 1.95 Newtons of magnitude which yields $(1.95) \cdot (0.18) = 0.35$ mm (0.014 inches) of ink markings on the paper.
Case Three: $u(t) = 0$, $k_1 = +\infty$, $c_2 = 0$

In case three, where a regular pen model is calculated using the open-loop transfer function $G_0(s)$ (Eqn. 2.29) in Section 2.6 and setting $k_1 = +\infty$ and $c_2 = 0$. Following similar procedures as in case one, we use MATLAB and Simulink to produce the Bode plots and system simulations. Under these settings, the pen-tip speed magnitude for the
regular pen model at 8 Hz is -15.3 dB (0.17 m/s) which in terms of speed to force ratio gives 1:6. Also, simulations show that for a cyclic force with 1 N of amplitude at 8 Hz as a perturbation, in the regular pen model, the pen-tip strokes produce around 7 mm of ink markings. Using the tremor-induced forces obtained from the previous sections, 1.95 N of magnitude, yields $(1.95) \cdot 7 = 13.65$ mm (0.54 in) of ink markings on the paper.
2.9 Comparison on Three Cases

As stated above, case one has a speed to force ratio of 1:43, case two has a speed to force ratio of 1:220, and case three has a speed to force ratio of 1:6. Apparently, our actively controlled model (ACPP model case 2) has 5 times better performance than the passively controlled model (ACPP case 1 model) and has 37 times better performance than a regular
pen (ACPP model case 3). In Figure 2.18 to 2.19, we superimpose Bode plots, speed simulations, and displacement simulations of each case in the same figure respectively for comparison.
Figure 2.18: Bode plot for case 1 through case 3

Figure 2.19: Pen-tip displacement simulation for case 1 through case 3
In conclusion, with system performance and control efforts in mind, we are able to select appropriate actuators based on the above design findings. In the next section, we will list some of the actuators that can be potentially used in prototyping.
Chapter 3

Actuator Selection

A voice coil actuator (VCA) is an electromagnetic field driven actuator by placing coils with current into a magnetic field to generate forces. It is able to produce linear motion with characteristics of high power densities, high bandwidths, quiet and smooth motion [30, 31]. One of the disadvantages of VCA can be its inefficiency due to heat losses, and uneven magnetic fields [30]. Even though there may exist better candidate actuators, the actuators listed below do meet the control criteria in terms of stroke, mass, and peak forces as calculated in the previous sections.

The candidate actuators manufactured by Akribis-systems provide direct drive, zero backlash, low mass and fast system response. There is neither wear nor tear on the components, since the coil and the core do not contact with each other and hence durable [31]. The most important three criteria to select a candidate VCA are its maximum stroke, peak force, and continuous force. The maximum stroke can be found from the simulations of the pen-tip displacement. The peak force is determined by the controller force $u(t)$ given by the simulations, and the continuous force is calculated based on the root mean square (RMS) of the forces at steady state.

Using the simulation results from Section 2.7.3, the maximum stroke required is 0.35 mm, and the maximum force required to stabilize the system is 1.7 N. The RMS force is then determined by the following equation [31]:

$$F_{RMS} = \sqrt{\frac{F_p^2 T_1 + F_p^2 T_2}{T_1 + T_2 + T_3}}$$  \hspace{1cm} (3.1)
where $F_p$ is the peak force, $T_1$ is the acceleration time, $T_2$ is the deceleration time, and $T_3$ is the dwell time. Based on the simulation, the pen dynamics does not have a dwell time during operations and the acceleration time equals to the deceleration time. Hence, the RMS force equals to the peak force.

With these criteria in mind, we are able to find candidate VCAs that have relatively small dimension and mass. From the product catalog [32], AVM24-5 and AVM14-HF-5.4-C19 are ideal candidates. AVM24-5 provides a maximum stroke of 5 mm, peak force of 11.4 N, and continuous force of 2.10 N. The dimension of this VCA is 20 mm in height and 24 mm in diameter. On the other hand, the other candidate VCA provides a maximum stroke of 5.4 mm, peak force of 7.33 N, and continuous force of 2.44 N. The dimension of the product is 25 mm in height and 14 mm in diameter, see Table 3.1.

Table 3.1: AVM24-5 versus AVM14-HF-5.4-C19 tech specs (control criteria)

<table>
<thead>
<tr>
<th>Model</th>
<th>Stroke (mm)</th>
<th>Peak force (N)</th>
<th>Continuous force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVM24-5</td>
<td>5</td>
<td>11.4</td>
<td>2.1</td>
</tr>
<tr>
<td>AVM14-HF-5.4-C19</td>
<td>5.4</td>
<td>7.33</td>
<td>2.44</td>
</tr>
</tbody>
</table>

Table 3.2: AVM24-5 versus AVM14-HF-5.4-C19 tech specs (dimension criteria)

<table>
<thead>
<tr>
<th>Model</th>
<th>Height (mm)</th>
<th>Diameter (mm)</th>
<th>mass (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVM24-5</td>
<td>20</td>
<td>24</td>
<td>40</td>
</tr>
<tr>
<td>AVM14-HF-5.4-C19</td>
<td>25</td>
<td>14</td>
<td>20</td>
</tr>
</tbody>
</table>

Both models have similar performance, yet model AVM14-HF-5.4-C19 has smaller mass compared to the other one. Nonetheless, in terms of dimensions, model AVM24-5 has lower height than the other one. Overall, both VCAs are equally acceptable. The next step would be testing both of them on the prototype and investigate the dominant criteria.
Chapter 4

Magnetically Controlled Tapping Pen (MCTP)

Model MCTP was conceived and prototyped in Northeastern University 3D printing studio by De Mey in spring 2014 [33], yet neither model dynamics study nor experiments have been performed. The following sections will focus on the description of MCTP prototype, equipment set up, and discussion of experimental results.

4.1 MCTP Prototype

The MCTP prototype consists of two pieces of green pen-shell mounted together by screws, an electromagnet, a spring, a permanent magnet, and a pen-rod glued to the permanent magnet. In Figure 4.1, the electromagnet is used to generate cyclic electromagnet forces with maximum input voltage of 3V. Subjected to such forces, the permanent magnet will drive the pen-tip to oscillate. If the oscillation frequency of the pen matches to that of the user’s hand tremor, the pen-tip will always contact the paper at the same point. A series of continuous dots would appear on the paper if the user slowly moves the pen, which will hence form legible handwriting.

4.2 Mechanically Generated Tremor

To test the performance of MCTP prototype, a Linear Motion Servo Plant IP02 (IP02) is used to mechanically generate hand tremor. In Figure 4.2 (left), the black cart driven by
Figure 4.1: Photo of MCTP prototype

a DC motor is capable of moving along the track. An encoder is attached to the cart to feedback displacement signals. With proper position control, the cart is able to oscillate at user defined frequencies and amplitudes, and can hence imitate hand tremors. The position control system of IP02 is provided by the IP02 user package where Simulation and MATLAB files along with QuaRC software by Quansor are used to manipulate the cart position. In addition, a Quanser Q4 Terminal Board (Q4), Figure 4.2 (right), is used to connect IP02 with the control system to communicate with the data acquisition board. Moreover, control signals are amplified by Quanser Universal Power Module (UPM), and then fed to the cart and the MCTP prototype.

Figure 4.2: Photo of IP02 (left) and Q4 Terminal Board (right)
4.3 Equipment Set Up

We first demonstrate experimental set up schematically in Figure 4.3 to layout the relation of each equipment. With user defined system parameters, the computer will send control signals to the cart and MCTP through Q4 and amplified by UPM.

![Experiment Set Up Diagram](image)

Figure 4.3: Experiment Set Up

In the computer segment, Figure 4.4, we used Simulink to control voltage applied to the prototype to regulate the IP02 cart position. Recall that we want to match the pen-tip oscillation frequency to that of the user’s hand tremor, hence the operation frequency of both the prototype and the cart are kept the same. Here, we define the Unit Sine Input block for operation frequency, Delay block for MCTP control signal phase lag, Distance block for cart position, and Voltage block to drive the electromagnet. The Simulink file to drive the cart is provided by IP02 user package with pre-defined blocks and the package also includes m files to calculate position controller gains $K_p$ and $K_v$ for the cart.

The user defined input signals to the IP02 cart and MCTP prototype are defined as:

\[
\begin{align*}
\text{IP02 Cart:} & \quad f_c(t) = A_c \sin(\omega t) & (4.1) \\
\text{MCTP Prototype:} & \quad f_p(t) = A_p \sin(\omega t + \phi_p) & (4.2)
\end{align*}
\]

where $f_c$ is the reference position of the cart, $f_p$ is the input voltage to MCTP, $A_c$ is the desired cart position amplitude, $\omega$ is the operation frequency, $A_p$ is the voltage amplitude.
input to the MCTP prototype which is always kept at 3 volts in this study, and $\phi_p$ is the phase of MCTP prototype control signal.

In the pen-cart segment, the prototype is mounted to the IP02 cart as shown in Figure 4.5 to emulate the condition of a hand tremor patient holding a pen during writing. A piece of scotch tape stuck to a piece of paper is placed under the pen-tip and is moved manually and perpendicular to the cart oscillation direction. The MCTP prototype will write on the scotch tape instead of a piece of paper for smaller viscous friction and ink diffusion. Because of the tapping motion, the distance of the pen-tip to the tape affects the pen markings greatly. Imagine the extreme condition that the pen-tip is always pressed on the paper, then the effects of tapping could not be properly rendered, which make it difficult to compare the results between experiments.

4.4 Results from Experiments

Here, we compare the pen markings under the tremors emulated by the cart displacement. Since the cart displacement may not be perfectly equal to $A_c$ due to the imperfect cart position controllers, we measure the cart displacement based on feedback signals from the encoder onboard the cart by inspecting the Scope block in Simulink. Obviously, the cart displacement is analogous to a sinusoid and we define $M_c$ as two times of the amplitude in the sinusoid. The stroke of pen markings defined as $M_p$ are measured directly from the experimental results. In addition, when MCTP electromagnet is off, we define the pen markings on the paper as $M_a$. 

Figure 4.4: Schematic demonstration of Simulink file used in the experiments
Three experiments have been conducted to test the performance of the prototype. In experiment one, we test how frequency affects the prototype pen markings. In the second experiment, we investigate appropriate phase lag in MCTP control signal. Last, in experiment three, we first keep electromagnet in MCTP off to imitate the handwriting of a hand tremor patient holding a regular pen and compare the pen markings obtained from MCTP with electromagnet on.

It is important to note that due to imperfections and rigidness issues in the mechanical system, frequencies in the range of 4 - 12 Hz could not be tested. To demonstrate the proof of concept, the experiments were conducted in the range of 1 - 4 Hz.

4.4.1 Experiment One

Experiment one presents the influence of tremor frequencies to the pen markings. Experiment one (A) and (B) are two similar experiments with different user defined parameters. In Figure 4.6, the pen is moving from left to right with displacement of 10 mm with respect to paper, while $\omega$ is gradually increasing (0.5 to 1.5 Hz in (A) and 1.0 to 2.0 Hz in (B)). Different from expectations, instead of a series of dots, a series of short lines appears on the scotch tape as pen markings. In addition, the pen markings tend to increase in size as the tremor frequencies increase, which is not a good sign in general.
4.4.2 Experiment Two

The second experiment studies the magnitude of pen markings subjected to various phase lag in MCTP control signals. During the experiment two (A), we keep $A_c$ (5 mm) and $\omega$ (1 Hz) as constants, while changing $\phi_p$ from 0 s to 1.0 s. The pen markings in Figure 4.7 are maximized at $\phi_p$ equals to 0.2 s and 0.8 s, and are minimized around $\phi_p$ equals to 0 s and 1.0 s. Then we investigate, in experiment two (B), the magnitude of pen markings around $\phi_p$ equals to 0 s. We measure $M_p$ using a caliper and compare it with the cart displacement $M_c$ monitored from Simulink. As shown in Figure 4.8 at $\phi_p$ equals to 0.03 s, the pen markings are suppressed to 26.28% of the cart displacement. In experiment three, we will fix $\phi_p$ as 0.03 s in the MCTP control signal phase.

4.4.3 Experiment Three

In the last experiment, we set $\phi_p = 0.03$ s inherited from experiment two (B), $A_c$ equals to 5 mm, and we increase $\omega$ from 1 Hz to 4 Hz. For the first 10 seconds in the experiment,
we turn off the electromagnet to imitate the condition of tremor affected writing with a regular pen and turned on the electromagnet afterwards. By measuring the pen markings shown in Figure 4.9, we are able to compare the performance of the MCTP prototype to a regular pen under similar conditions. At $\omega$ equals to 1 Hz, the pen markings from the MCTP prototype is 1.52 mm. Compared to that of the regular pen, the pen-tip deviations of the prototype is 17.4% of the regular pen. As the tremor frequency increases, the pen markings of the prototype also increase. At $\omega$ equals to 4 Hz, the prototype pen markings are 42.2 percent of the regular pen markings. However, results from operation frequency of 4 Hz are not very accurate for the IP02 system start to vibrate undesirably at such high operation frequency.

### 4.4.4 Discussion

To sum up, we discovered that the larger hand tremor frequency may increase the pen markings, yet this needs to be validated on a more rigid mechanical setup; input signal to the MCTP prototype has a certain phase lag which can help reduce tremor effects; compared
Figure 4.9: Comparison of pen markings between the MCTP prototype and the regular pen with a similar regular pen, the MCTP prototype suppresses pen markings up to 17.4% of the regular pen markings.

We also observed that, in the same time duration, the cart moves for about 10 mm, while the pen-tip moves for a much smaller distance, and hence the cart moves faster than the pen-tip. This may be one of the reasons that the prototype drew short lines instead of dots. In addition, the motion curve of the pen-tip is similar to a sinusoid. However, in Figure 4.1, the pen-shell converge in shape around the pen-tip, which could potentially constrain the pen-tip motion. Meaning there exist the possibility that the pen-tip could not reach its theoretical valley in its motion curve. Thus, the pen-tip would contact the paper for a longer time than desired.
Chapter 5

Conclusion

The actively controlled pen model ACPP promises to reduce the effects of hand tremors on writing, with an improvement on the passively controlled model and superior results over a regular pen model. The theoretical studies in this thesis should provide a strong reference for further studies on the design and realization of such a pen with a potential of helping over eleven million people in the United States.

Future studies should focus on pen-shall material selection, ergonomic design, actuator and sensor selection, and further design improvements. To realize the goal of real-time control, a sensor must be used and it is necessary to design appropriate filters to distinguish tremor motions from voluntary motions intended for writing. In terms of portability in the future, proper control design might decrease the power consumption and hence increase the battery life (or decrease the battery size) of the pen. During the studies, the author also designed controllers using Linear-Quadratic-Regulator (LQR). It was found that the controller designed based on LQR may not render minimized pen-tip speed magnitudes, while only minimizing the control effort to the system.

Regarding MCTP, experimental study demonstrates the possibility of suppressing hand tremors using the “tapping” motion, yet further experiments are needed at higher tremor frequencies. The experimental study also reveals the critical impact of the distance from pen-tip to the paper on the pen markings.

In the future study, to obtain consistent experimental results, it would be beneficial to place the pen-tip on a roller to fix the distance of the pen-tip to the paper. The roller could also provide normal force to realize the “tapping” motion in reality where people naturally tend to put pressure on the paper during writing. In addition, investigation is needed on
the prototype to check whether the pen-tip could reach its theoretical minimum. This may require revisions in the prototype both from mechanical and sensory aspects. Moreover, there should be a study on the behavior of human upper limbs during writing. The tremor frequencies used in this thesis is based on clinical diagnostics rather than the ones during writing, which may not be representative of frequencies observed in writing. Future studies should also focus on analyzing hand motion during writing, in order to distinguish hand tremors from intended writing.
REFERENCES


