OPTIMAL COLLECTION POLICIES FOR RETURNED PRODUCTS IN THE REVERSE SUPPLY CHAIN

A Thesis Presented

By

Moulik D. Kapadia

to
The Department of Mechanical and Industrial Engineering

in partial fulfillment of the requirements for the degree of

Master of Science

in the field of
Industrial Engineering

Northeastern University
Boston, Massachusetts

December 2018
ABSTRACT

With the emergence of e-commerce and increasing customer service expectations, a growing number of firms are leveraging liberal return policies to drive a strategic competitive advantage. In 2017, U.S. return delivery costs amounted to 381 billion U.S. dollars and are expected to reach 550 billion U.S. dollars by 2020, which is 75% more than just 4 years prior. This surge of total merchandise returns is due to the ‘try-before-you-buy’ model that most retailers are starting to adopt. According to a survey, 79% of consumers want their stores to offer free return shipping and 92% will make a repeat purchase if returns are simple. As a result, almost 49% retailers offer free return shipping. Unfortunately, the increase in return delivery costs is not the only complication that a reverse supply chain has to deal with. In a reverse supply chain, the source node is the customer, which creates a time lag and an uncertainty that makes this a reactive process; there is little if any proactive decision-making approaches of resource allocation, forecasting or planning. Another complication of the reverse logistics network is that it involves a many-to-one distribution/collection relationship unlike the traditional forward logistics, which combined with the uncertainty of the source, makes reverse logistics a very difficult problem to solve. This thesis focuses on the development and comparison of different collection models that reduce the impact of product returns on a firm’s inventory and transportation costs by leveraging economies of scale and optimizing the collection period across multiple initial collection points (ICP) in a three-echelon network before transshipping the returned products to a centralized return center (CRC). First, an optimal collection policy for the case of a single product and a single ICP is described, which is then extended to the case with multiple products and a single ICP. Then, determination of the optimal collection policies for the case of multiple ICPs and a single product is presented: 1) individual shipment policy, 2) combined shipment policy and 3) hybrid shipment policy. Mathematical models in terms of collection periods are developed for calculating the combined inventory and transportation costs of all policies, and an optimization approach is designed to minimize the number of computations required to reach the optimal collection period. Finally, the results are presented with an experimental dataset.
First and foremost, I would like to express my sincere gratitude to my advisors and thesis committee members, Prof. Emanuel Melachrinoudis and Prof. Nizar Zaarour. Prof. Manny, as everybody knows him, was the first one to motivate me to take on the thesis work amongst other options. They always encouraged me to express my ideas and showed me how to research a problem and achieve my goals. They spent endless time amidst their busy schedules to proofread my work, asking me questions that enabled me to think harder and supported me during the difficult times in my research. Our weekly progress meetings were our means of communication and a way to stay in touch that motivated me to work harder and put in the best effort possible. Without their work, encouragement, continuous guidance and insights, I could not have finished this thesis.

I also want to thank Prof. Ozlem Ergun and Prof. Onder Ondemir for their teaching and piquing my interest in the fields of optimization and supply chain. Prof. Ergun was also my program advisor during my time at Northeastern and I also worked with Prof. Ondemir as his teaching assistant for the courses of Operations Research. Their time, help and advice were invaluable.

Special thanks go to the faculty and staff of the departments of Mechanical and Industrial Engineering at the College of Engineering, and Supply Chain Management at the D’Amore McKim School of Business. And also, to the staff of the Office of Global Services and Graduate School of Engineering. I want to thank them for their constant support and help in processing my paperwork and solving my problems. They made my experience in Boston and Northeastern very unique, smooth and enriching.

Lastly and most importantly, I want to express my deepest gratitude to my friends and family, specially my parents Mr. Dharmesh Kapadia and Ms. Paru Kapadia, for their understanding and support during my academic time overseas for almost three years. Without their encouragement and assistance, I would not have been here, and this thesis would not have been possible.
# LIST OF CONTENTS

**ABSTRACT** ........................................................................................................... ii

**LIST OF FIGURES** .................................................................................................. v

**LIST OF TABLES** ..................................................................................................... vi

**CHAPTER 1: INTRODUCTION** .................................................................................. 1

1.1 BACKGROUND ....................................................................................................... 2

1.2 OBJECTIVE ........................................................................................................... 4

**CHAPTER 2: LITERATURE REVIEW** ........................................................................ 6

2.1 CLOSED-LOOP SUPPLY CHAIN ............................................................................ 6

2.2 OPTIMIZATION IN REVERSE LOGISTICS .......................................................... 7

**CHAPTER 3: DETERMINING THE OPTIMAL COLLECTION PERIOD** .............. 13

3.1 INTRODUCTION ..................................................................................................... 13

3.2 ASSUMPTIONS ...................................................................................................... 15

3.3 DEFINITIONS ........................................................................................................ 15

3.4 THE CASE OF A SINGLE PRODUCT AND A SINGLE COLLECTION POINT .......... 17

3.4.1 MATHEMATICAL FORMULATION ..................................................................... 17

3.4.2 SOLUTION ......................................................................................................... 19

3.5 THE CASE OF MULTIPLE PRODUCTS AND A SINGLE COLLECTION POINT ....... 21

3.5.1 MATHEMATICAL FORMULATION ..................................................................... 22

3.5.2 SOLUTION ......................................................................................................... 23

3.6 THE CASE OF A SINGLE PRODUCT AND MULTIPLE COLLECTION POINTS ....... 25

3.6.1 INDIVIDUAL SHIPMENT POLICY .................................................................... 26

3.6.2 COMBINED SHIPMENT POLICY .................................................................... 27

3.6.3 HYBRID SHIPMENT POLICY ............................................................................ 29

3.6.3.1 MATHEMATICAL FORMULATION ................................................................ 31

3.6.3.2 SOLUTION .................................................................................................... 33

**CHAPTER 4: EXPERIMENTAL RESULTS** ............................................................... 41

**CHAPTER 5: CONCLUDING REMARKS AND FUTURE SCOPE** ......................... 50

**REFERENCES** ......................................................................................................... 51
LIST OF FIGURES

Figure 1: A typical reverse logistics flow of the returned products ...................... 14
Figure 2: A typical behavior of the transportation and inventory costs with respect to holding time ......................................................................................... 14
Figure 3: Preselected shipment volume breakpoints .................................................. 16
Figure 4: Sub-problem with one ICP and one CRC .................................................... 17
Figure 5: The number of products held at the ICP each day ..................................... 18
Figure 6: The number of products k held at the ICP each day ................................... 22
Figure 7: Individual shipment policy, products are collected from each ICP individually ........................................................................................................... 26
Figure 8: Combined shipment policy, products are collected via milk runs from every ICP in one shipment ........................................................................... 28
Figure 9: Hybrid shipment policy .............................................................................. 30
Figure 10: Products collected from only ICP 1 at $T_1 + t - 1T_2$ and ICP 1 & ICP 2 at $tT_2$, where, $t = 1, 2, ...$ ................................................................................. 30
Figure 11: Total costs with respect to holding times for the combined shipment policy .............................................................................................................. 42
Figure 12: Transportation cost function with respect to $T_2$ for $T_1 = 1$ to 10 ......... 45
Figure 13: Inventory cost function with respect to $T_2$ for $T_1 = 1$ to 10 ................. 46
Figure 14: Total cost function with respect to $T_2$ for $T_1 = 1$ to 10 ...................... 47
LIST OF TABLES

Table 1: Total costs with respect to holding times for the combined shipment policy ........................................................................................................................................................................................................................................................................42

Table 2: Total costs of the individual shipment policy v/s combined shipment policy ........................................................................................................................................................................................................................................................................42

Table 3: Potential values of $T_1$ and $T_2$ for the hybrid shipment policy with $T_1 = 5$ and $T_2 = 10$ as optimal ........................................................................................................................................................................................................................................................................43

Table 4: Total costs for various values of $T_1$ and $T_2$ for the hybrid shipment policy ........................................................................................................................................................................................................................................................................44

Table 5: Comparison of the costs for all the three policies ........................................48
CHAPTER 1: INTRODUCTION

In this chapter the reader is introduced to the fundamental ideas of this thesis and its importance in the current state of reverse supply chain management. First, the concepts of reverse logistics are introduced and its growing importance both in general and to the retail industry. The following subsections provide some additional background and objectives of this thesis. Finally, the research structure and methodology are explained.

The definition of reverse logistics as published in a magazine by the Reverse Logistics Association is “The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal. More precisely, reverse logistics is the process of moving goods from their typical final destination for the purpose of capturing value, or proper disposal. Remanufacturing and refurbishing activities also may be included in the definition of reverse logistics.” The key differentiator here for reverse logistics when compared to the forward logistics, is the movement of products and information from “the point of consumption to the point of origin.” Understanding the differences between the forward and reverse supply chains is of high importance, however this is not realized in many contexts and industries. Also, reverse logistics is more than recycling containers and packaging material or reducing pollution and consumption of energy from transportation, these are important activities, but they might be secondary to the real importance of overall reverse logistics. Products can be returned for many reasons: product defects; damages during transit; order fulfilment errors; unwanted/unneeded products resulting from impulse purchases; product exchanges; warranty returns; product recalls; and end of life recovery. Reverse logistics involving product returns often presents unique logistics challenges (Zaarour et al., 2014a).

In the retail industry, customer satisfaction is extremely important to having a convenient and efficient e-commerce solution. When customers want to return unwanted or faulty goods, a retailer should be able to quickly remedy the situation so that the customer will continue to purchase products from them. Unfortunately, handling customer
product returns in e-commerce can be a difficult task. Returned products come in all different sizes, shapes, and conditions. Many of them are received damaged, without original packages, and mixed up with other products. As such, returned products are more difficult and costly to handle than original products. Indeed, the logistics of handling returned products accounts for nearly 1% of the total U.S. gross domestic product (Gecker, 2007). According to a report on Consumer returns in the Retail Industry published by Appriss Retail, total merchandise returns account for more than 351 billion U.S. dollars in lost sales for US retailers in 2017 which is almost 10% of total sales. This size is overwhelming, and more than half of the 2017 federal budget deficit of 666 billion U.S. dollars. These return delivery costs are estimated to amount to 550 U.S. billion dollars by 2020, 75.2% more than just four years prior. During the 2015 holiday season, almost one quarter (24%) of consumers returned or exchanged the gifts they received, and this rate of return represented an increase over the 21% from the previous year (Taylor, 2016). In particular, Winkler (2018) reported that nearly one third of online purchases were returned in 2015, in some cases (e.g. expensive items) the return rate of online purchases reached 50% and 77% of returns come from repeat customers.

1.1 BACKGROUND

Despite high returned value on products, it is becoming a daily routine for a growing number of firms to offer “hassle-free” returns and liberal return policies as a tool to attract and retain more customers. And thus, product returns show no sign of abatement and has become the necessary evil for doing business. Generally, product returns stemmed from two phenomena: (1) consumer returns of products to the retailer due to defects, damages during transit, product recalls, impulse purchases, incomplete product description, customer frauds, and inaccurate order fulfillments; and (2) supplier returns of overstocked or unsold items to the manufacturer as part of the “buyback” policy (Zaarour et al., 2014a). Reverse logistics involving product returns often presents unique logistics challenges. One of these challenges includes the complexity of handling returned products not in pristine conditions which cannot be put straight back on the shelf and thus some of those should be disposed of or sold at discounted prices. Another challenge includes a
limited volume of returned products which makes the reverse logistics process more expensive than the forward logistics process (Min et al., 2006a)\(^40\). As a matter of fact, product returns are typically three to four times more expensive than forward outbound logistics and thus require careful logistics planning (Norek, 2002\(^{44}\); Shear et al., 2003\(^{53}\)). Most retailers have viewed this series of costs resulting from product returns as unavoidable items. However, few of them get a clear understanding of the composition of product return losses and cannot master an effective method to avoid the losses (Wang et al., 2017\(^{59}\)). To better manage product return problems and then reduce the adverse impact of product returns on the company’s overall profitability, the company needs to develop a reverse logistics strategy that can coordinate and smooth out the reverse flows of products. The typical objective of such a reverse logistics strategy is to maximize the value of all returned products and minimize the cost of handling these products.

Product returns, as an integral part of a reverse logistics strategy, involve the collection of returned products at the initial collection center (e.g. designated regional distribution centers of retail outlets, designated takeback sites including local convenience stores), the transfer and consolidation of returned products at centralized return center (CRCs), the asset recovery of returned products through repairing, refurbishing, and remanufacturing, and the disposal of returned products with no commercial value (Zhou and Min, 2011\(^{63}\)). In this typical product return process, however, products are often sent back to initial collection points (ICPs) in small quantities. This limited volume of returned products at the initial stage of collection has become the main source of increased per unit shipping cost due to a lack of freight consolidation opportunities resulting from individual piece-by-piece returns (Guide et al., 2003\(^{19}\); Min et al., 2006\(^{b42}\)). To create a greater volume, some of those returned products need to be aggregated into a larger shipment. But, such aggregation increases product holding time at the ICP or a CRC which in turn increases inventory carrying costs. And thus, addressing this trade-off between inventory and transportation costs of returning products from the ICP to a CRC is the main goal of this thesis.
1.2 OBJECTIVE

The objectives of this thesis can be stated as follows:

1) Develop collection models under a special structure that allows the decision maker to make the optimal trade-off between inventory carrying costs and transportation costs

2) Determine the optimal collection policies in terms of holding times at the initial collection points before the returned products are transshipped to the centralized return center

3) Examine the sensitivity of the optimal model solutions by varying the product return rates, product volumes and holding costs, and then assessing their impacts on multiple collection periods and total reverse logistics costs

The purpose of this thesis is to study, understand and identify the cost driving factors that affect the collection of returned products in the reverse supply chain. Then, models that calculate the total reverse logistics costs need to be developed and optimized that take into consideration these factors in the form of input parameters. The reader is introduced to the special structure of a three-echelon collection network under consideration and relevant concepts and optimization approaches are discussed.

The increasing impact of reverse logistics costs on a firm’s base line requires a reverse logistics strategy that can utilize and coordinate its resources to smooth out the flow of products in the reverse logistics network. As mentioned earlier, the typical objective of such a reverse logistics strategy is to maximize the value of all returned products and minimize the cost of handling these products. There are many researches and theories that focus on maximizing the value of returned products, however, the research on minimizing the handling costs is relatively sparse. The existing literature on strategies and theories for the same are also discussed in the following sections.

This work also helps the readers understand the bigger picture when there are multiple operational costs in play for a firm as a whole. And, how reducing one these costs (e.g. inventory cost) may cause an increase in others (e.g. transportation cost) is explained, and the importance of considering these trade-offs are understood. Zaarour et al. (2014a) first developed a model to address this trade-off specific to a single product single ICP-CRC
connection in the three-echelon network. However, in the real world, firms are required to deal with a variety of products for customers spread all over the geographical locations. This in turn also requires locating many collection points across these locations.

This thesis takes into consideration these complications and presents models to solve first, the multiple products case and then, the multiple ICP-CRC connections case. The difference with having multiple products is the added complexity in dealing with different volumes and holding costs of different products. The problem of collecting from multiple ICPs and shipping to a CRC is that, since a reverse logistics network is a many to one distribution relationship, the increase in an ICP increases the possible collection strategies from these ICPs exponentially. This work demonstrates the added complexity of dealing with two ICPs and develops mathematical models for the resulting collection policies. These models are optimized, and the results are then compared with the objective of identifying the optimal collection periods and performing sensitivity analyses of the parameters while minimizing inventory costs and transportation costs by leveraging economies of scale.
CHAPTER 2: LITERATURE REVIEW

The purpose of this chapter is to review the existing literature in the field of closed-loop supply chain, especially in reverse logistics. The focus and conclusions of these existing literature are covered while describing the motivation that led to the development of this thesis.

2.1 CLOSED-LOOP SUPPLY CHAIN

A closed-loop supply chain is mainly concerned with the management of raw materials, parts/components, work-in-process inventories, and finished goods from the source of their supply (e.g. supplier) to the point of consumption in such a way that their adverse social, environmental, and economic impacts can be minimized. The closed-loop supply chain often involves product returns/recoveries, source reduction/conservation, cleaner energy use, pollution prevention, recycling, salvage, substitution, reuse, disposal, disassembly, refurbishment, repair and remanufacturing (Stock, 1992; Guide and Van Wassenhove, 2003; Min et al., 2006; Seuring and Muller, 2008; Min and Kim, 2012).

Many researchers in the field of supply chain management have focused on the forward movement (the transformation of the materials from the suppliers to the end consumer, or the forward logistic). Much less attention, however, has been devoted to the field of reverse logistics (Al-Anzi et al., 2007). Owing to an increasing awareness of environmental concerns, industries need to give more attention toward the management of waste streams and reduction of non-renewable resources usage (Gungor and Gupta, 1999; Kaebernick et al., 2003; Govindan et al., 2012; Kannan et al., 2012; Kannan et al., 2009a). Due to the above reasons, industries have started to implement the reverse logistics concept (Diabat et al., 2013). Reverse logistics is the process of planning, implementing and controlling the efficient, effective inbound flow and storage of secondary goods and related information opposite to the traditional supply chain directions for the purpose of recovering value and proper disposal (Rogers and Lembke, 1999). The
main motive for a firm’s environmental efforts and implementation of Reverse Logistics systems are the intense competition and stringent environmental regulations forcing the firms to collect and reuse the products that they produce (Kwak and Kim, 2015).

Most of the research in the field of closed-loop supply chain concerns managing the reverse flows of products that are at their end of use or end of life. The focus for these types of return flows is on cost-efficient recovery and meeting environmental standards. With consumer returns, however, the focus is on maximizing asset recovery which generally requires flexible and responsive reverse supply chains (Ruiz-Benítez et al., 2014). As such, a closed-loop supply chain requires the systematic co-ordination and synchronization of interrelated multi-echelon business processes involving both forward and backward flow of products from a cradle to the grave. Reflecting the world-wide attention to eco-friendly, closed loop supply chain operations, there is a growing body of the literature dealing with diverse reverse logistics problems (Carter and Ellram, 1998; Fleischmann et al., 2000; Fleischmann and Kuik, 2003; Mead et al., 2007; Srivastava, 2007; Channtrakul et al., 2009; Pochampally et al., 2009; Pokharel and Mutha, 2009; Min and Kim, 2012; Agrawal et al., 2015; Govindan et al., 2015).

2.2 OPTIMIZATION IN REVERSE LOGISTICS

As mentioned earlier, in contrast with forward logistics studies, prior research dealing with the economic impacts of reverse logistics is relatively sparse. Some of the earlier pioneering studies on reverse logistics include: Min (1989), Kroon and Vrijens (1995), Melachrinoudis et al. (1995), and Del Castillo and Cochran (1996).

To elaborate, Min (1989) developed a multiple objective mixed integer program that was designed to select the most desirable shipping options (direct versus consolidated) and transportation modes for reverse logistics involving recalled products. Although he considered a tradeoff between transportation time and cost associated with reverse logistics, his model did not take into account inventory carrying cost. Kroon and Vrijens (1995) conducted a case study that designed a mixed integer program to determine the number of reusable containers as well as the number and location of container depots within the reverse logistics channel. Considering a multiple planning horizon,
Melachrinoudis et al. (1995)\textsuperscript{37} developed a multiple objective integer program for the dynamic location of capacitated sanitary landfills. Del Castillo and Cochran (1996)\textsuperscript{9} presented a pair of mathematical models (one aggregated and another disaggregated) and a simulation model to optimally configure the reverse logistics network involving the return of reusable containers so that the number of reusable containers was maximized. However, they did not consider transportation issues related to reverse logistics. In an effort to recycle construction waste as sieved sand, Barros et al. (1998)\textsuperscript{4} proposed a mixed integer program which determined the location of regional depots for receiving the flow of sieved sand and treatment facilities for cleaning and storing polluted sand. Unlike other previous models, they considered two-echelon location problems with capacity constraints. Similarly, Louwers et al. (1999)\textsuperscript{35} proposed a nonlinear program and the linear approximation solution procedure to determine the location and size of regional recycling centers within a reverse logistics network involving carpet waste.

Following suit, Jayaraman et al. (1999)\textsuperscript{22} presented a mixed integer program to determine the optimal number and location of remanufacturing facilities for electronic equipment. Jayaraman et al. (2003)\textsuperscript{23} extended their prior work to solve the two-level hierarchical location problem involving the reverse logistics operations of hazardous products. They also developed heuristic concentration procedures combined with heuristic expansion components to handle relatively large problems with up to 40 collection sites and 30 refurbishment sites. Schultmann et al. (2003)\textsuperscript{51} combined a two-stage location model with a simulation model for planning a reverse-supply network involving the recycling of spent batteries in the steelmaking industry. Despite their success in solving practical reverse logistics problems, none of these prior studies dealt with the possibility of making trade-offs between freight rate discounts and inventory cost savings resulting from consolidation of returned products.

Later, Min et al. (2006b)\textsuperscript{42} presented a nonlinear integer program for solving the multi-echelon reverse logistics problem involving product returns. To overcome inherent computational complexity involved in the non-linear program structure, they utilized genetic algorithm (GA). Their contributions include the consideration of freight consolidation possibilities across geographical areas and holding time. Especially, they explored a possibility that returned products will be aggregated at the ICPs and then held
up for a few days for freight consolidation before those returned products will be transshipped from the ICPs to the CRCs for asset recovery, remanufacturing or disposal. Their models and solution procedures were designed to determine the optimal locations of ICPs and CRCs from sets of candidate locations, the optimal collection period at the ICPs, and the most desirable shipping volumes from the ICPs to the CRCs.

Since then, a lot of research has come up in dealing with the same strategic decision-making problem of reverse logistics network design. Li et al. (2006)\textsuperscript{34} addressed the problem of determining the number and locations of CRCs where returned products from retailers or end-customers were collected, sorted and consolidated into a large shipment. They formulated a stochastic nonlinear mixed integer programming model and used Monte Carlo method to solve the problem. Salema et al. (2007)\textsuperscript{50} designed the reverse distribution network by developing a generalized mixed integer model where returns of multi-commodities in an uncertain environment were taken into considerations. Srivastava (2008)\textsuperscript{55} further conceptualized a product return process within the reverse logistics network that consists of collection centers and two types of rework facilities set up by original equipment manufacturers (OEMs) or their consortia for a few categories of product returns under various strategic, operational and customer service constraints. Tan and Kumar (2008)\textsuperscript{57} approached a reverse logistics model from a profit maximization perspective rather than a typical cost minimization perspective. Min et al. (2008)\textsuperscript{41} employed a mixed-integer program and GA to solve the reverse logistics problem involving the consolidation of product returns in e-tail environments. Chandiran and Rao (2008)\textsuperscript{6} and Kannan et al. (2009b)\textsuperscript{28} went a step further from these prior studies by considering a two-way flow coordination/integration of both forward and reverse logistics activities. Also, Du and Evans (2008)\textsuperscript{11} considered two conflicting objectives of reverse logistics: costs and tardiness of cycle time.

Improving on the work of Min et al. (2006b)\textsuperscript{42}, Diabat et al. (2013)\textsuperscript{10} used the algorithms, GA and AIS, to optimize the problem and got a better result from AIS. The purpose is again network design but here they have used sensitivity analyses on various parameters and studied the variation in the model solution, this also includes maximizing the holding time at the initial collection points. Recently, Zaarour et al. (2014b)\textsuperscript{62}, Ghezavati and Nia (2015)\textsuperscript{16}, and Wang et al. (2017)\textsuperscript{59} made modifications on the model of
Min et al. Zaarour et al. transformed it to a linear form; Ghezavati and Nia added a new dimension concerning product type to the model and Wang et al. use a mixed integer non-linear programming model, specifically, Modified Plant Growth Simulation Algorithm (MPGSA), to optimize the 3-echelon product returns network. In these studies, the portrayals of product returns network were detailed focusing on just collecting returns in a cost-effective network. And the managerial suggestions generated in the models were crystal clear and easy-controlled for company managers. To the returns management of the time-sensitive perishable commodity, goods accumulation and bulk transport would reduce the shipping cost, but simultaneously the delay would result in the decrease of market value. For this reason, Ruiz-Benítez et al. (2014) looked for the optimal collection period of perishable products and explored the value of information sharing between the collection points and the central processing facilities., the focus on the value of information sharing is interesting but lacks the direct economic issue of collection times with respect to transportation and inventory costs.

Given the complexity and effort required, the collection of used/returned products is very difficult and needs a well-estimated structure in reverse logistics (Lee and Dong, 2009). Other works have tried to approach the reverse logistics problem, Ellili et al. (2016) developed multi-product and multi-period mathematical models regarding some end-of-life products with respect to five related application areas, namely: location, life cycle assessment, production planning, inventory management, along with the establishment of the most appropriate product-collection routes, however, the end-of-life product aspect does not take into account the time sensitivity required in the case of consumer returns for maximum asset recovery.

Ramos et al. (2009) presented a multi-depot, multi-trip Vehicle Routing Problem in Reverse Logistics such that three types of recyclable waste products (multi-product) amongst with glass, paper, and plastic/metal are collected. It was assumed that there are some vehicles at each depot and if a vehicle starts its route, it is compulsory to come back to the same depot. They proposed two mathematical programs and developed a heuristic algorithm and compared them with the solutions of two formulations.

Kheirkhah and Rezaei (2016) focused on the strategic part of Reverse Logistics by introducing crossdocking centers and studied a multi-echelon Reverse Logistics
network design with cross-docking as well as the transshipment problem, in which the model should decide how used products are delivered to recycling centers through cross-docking centers. In the model, all returning products are collected from customers to be unloaded into cross-docking centers. All operations including collection, inspection, and separation are done within cross-docking centers and then, these products are transported to different recycling centers such as remanufacturing, repairing, and renovating centers.

Cross-dock is a consolidation strategy used mainly to leverage the economies of scale with the purpose of minimizing transportation costs and achieving efficient delivery times. In such cases, several less-than-truck-load orders from different suppliers are transferred to the cross-dock and are consolidated and sent to the customer according to their destinations and due dates (Cóccola et al., 2015)\(^8\). Nikolopoulou et al. (2017)\(^{43}\) compare the movement of products via cross-docking versus direct-shipping and note that, for problems with a many-to-many relation between suppliers and customers, the cross-docking strategy outperforms the direct-shipping one, when suppliers and customers are densely connected.

Kaboudani et al. (2018)\(^{24}\) developed mathematical models for vehicle routing strategies using cross docks for both forward and reverse logistics with the objective of finding the best allocation of customers to vehicles and the best route for each vehicle in order to minimize the total transportation cost. This is one of the first works on using cross-docks with the purpose of integrating forward and reverse logistics.

Khorshidian et al. (2016)\(^{30}\) developed a bi-objective mathematical model to optimize (1) the total cost and (2) operation time. More specifically, the total cost is the summation of: (1) the earliness and tardiness penalty costs, (2) the fixed cost of using inbound and outbound trucks, and (3) the cost of heterogeneous cargo and less than truck loads. They discuss inbound/outbound truck scheduling using cross-docks for perishable products in forward/reverse logistics networks but do not discuss the impact of price discount on the products collection in the optimization model.

In these studies, the consolidation of products using cross-docks do not take into account the inventory carrying costs. Also, none of these prior studies examined the dynamic interplay between shipping volume (i.e. return rates) and the collection period for returned products. Zaarour et al. (2014a)\(^{61}\) were the first to do so, but as mentioned earlier,
they do not consider the real-life complication of having a variety of products or multiple collection points, which is accounted for in this thesis.
CHAPTER 3: DETERMINING THE OPTIMAL COLLECTION PERIOD

This chapter describes the structure and methodology used in this thesis to reach the objective of minimizing the sum of inventory and transportation costs by identifying the optimal collection times in a reverse logistics network. First, the reader is introduced to the special structure of a three-echelon collection network followed by the assumptions made and definitions used while formulating the models that follow. The following subsections are the different cases of multiple products/multiple collection points and their collection policies. The mathematical formulations are developed for the policies in these subsections and an optimization approach is presented to reach the optimal collection period with minimum number of computations.

3.1 INTRODUCTION

The three-echelon network is a special structure for the collection of returned products in reverse logistics that leverages economies of scale by holding and consolidating the shipments at the initial collection points before transshipping the products to the centralized return centers. The first node in this network are the customers who are returning the products, the second node are the initial collection points (ICP) where these returned products are held and consolidated to create larger shipments (e.g. full truckload) so as to benefit from economies of scale, and the third node are the centralized return centers (CRC) where the products are shipped to for the purposes of sorting, repair, refurbishment, remanufacturing, redistribution or disposal. Figure 1 depicts the typical reverse logistics flow of the returned products in a three-echelon network.

Let us consider multiple customers returning products individually to multiple ICPs and then from these ICPs products are being transshipped to CRCs. However, customers are divided into clusters where each cluster is within the geographical area of a single ICP. Alternatively, it may be assumed that customers have already been assigned to ICPs and ICPs have been assigned to CRCs. In this scenario, the challenging decision that needs to be made is to determine the finite collection time of holding the returned products at each
ICP to create economies of scale for reduction of per unit shipping cost. Figure 2 depicts a typical behavior of transportation and inventory costs and their sum with respect to the collection period.

**Figure 1:** A typical reverse logistics flow of the returned products

**Figure 2:** A typical behavior of the transportation and inventory costs with respect to holding time
3.2 ASSUMPTIONS

Prior to formulating the models for various cases and collection policies, the following assumptions are made:

1) All customer locations are known and fixed a priori
2) The transportation cost between customers and their designated ICP is negligible due to short distances between customer and ICP locations
3) Given a small volume of individual returns, an ICP has adequate capacity to hold returned products during the collection period
4) For the single product cases (Sections 3.4 and 3.6), returned products are of the same kind
5) For the multiple ICP case (Section 3.6), the distance between the multiple ICPs is negligible compared to the distance between these ICPs and the CRC. An example of this scenario, all ICPs are located in and around Boston while the CRC is in Phoenix
6) No capacity limit is imposed on the CRC
7) The collection period (in days) is discrete, which means that the collection of returned products at the ICP is assumed to occur once on the collection day (e.g. at the end of the day)

3.3 DEFINITIONS

The decision variables of collection periods and shipment volumes are defined individually for each section at the beginning of the section.

Indices

\[ i = \text{Index for customers; } i \in Z \]
\[ j = \text{Index for ICPs; } j \in Z \]
\[ k = \text{Index for type of products; } k \in Z \]
\[ l = \text{Index for pre-selected shipment volume ranges associated with freight rate discounts} \]

Model parameters
\(c_j = \text{Number of customers at ICP } j\)

\(p = \text{Number of product types}\)

\(s = \text{Number of ICPs}\)

\(w = \text{Annual working days}\)

\(r_{ijk} = \text{units of product } k \text{ returned by customer } i \text{ per day to ICP } j, \text{ where, } i = 1, 2, \ldots, c_j \text{ and } j = 1, 2, \ldots, s\)

\(R_{jk} = \sum_{i=1}^{c_j} r_{ij} = \text{units of product } k \text{ returned per day at ICP } j, \text{ where, } j = 1, 2, \ldots, s\)

\(v_k = \text{volume of product } k \ (\text{ft}^3 / \text{unit})\)

\(b_k = \text{unit holding cost for product } k \ ($ / \text{unit / day})\)

\(d = \text{distance from the ICPs to CRC}\)

\(E = \text{freight rate per unit volume for a given distance } d \text{ from ICPs to CRC } ($ / \text{ft}^3)\)

\(n = \text{number of shipment volume breakpoints}\)

\(V_l = \text{preselected shipment volume breakpoints where } l = 0, 1, 2, \ldots, n\)

\(\alpha_l = \text{volume discount rate dependent on shipment volume, } l = 0, 1, 2, \ldots, n \text{ and } \alpha_0 = 1\)

Let the shipment volume be \(X\), then,

\[
f(X, d) = \begin{cases} 
E\alpha_{l-1} & \text{for } V_l < X \leq V_{l+1}, \ l = 0, 1, 2, \ldots, n - 1 \\
E\alpha_n & \text{for } V_n < X 
\end{cases}
\]

is the unit transportation cost for a given distance \(d\) between the ICPs and the CRC. See the shipment volume quadrants in Figure 3 below.

![Figure 3: Preselected shipment volume breakpoints](image)
3.4 THE CASE OF A SINGLE PRODUCT AND A SINGLE COLLECTION POINT

In this section we develop and optimize the model for the sub-problem of a single type of product and a single ICP. This model is based on the work done by Zaarour et al. (2014a)^61, with a slight modification where we consider the volume of a product and the freight rates are based on shipment volumes (ft³). Figure 4 depicts this sub-problem.

For this case,
- Types of product, \( p = 1 \), and thus, \( k = 1 \)
- Number of ICPs, \( s = 1 \), and thus, \( j = 1 \)
- And we say for notational simplicity,
  - \( r_i = r_{i11} \), is the units of product returned by customer \( i \) at the ICP
  - \( R = R_{11} = \sum_{i=1}^{c_1} r_i \), is the sum of units of product returned per day at the ICP
  - \( v = v_1 \), is the volume of the product (ft³ / unit)
  - \( b = b_1 \), is the holding cost per unit of the product ($ / unit / day)

**Decision variables**
- \( T \) = length of a collection period (in days) at the ICP
- \( X \) = shipment volume of products returned from the ICP to the CRC at collection period \( T \)

![Figure 4: Sub-problem with one ICP and one CRC](image)

3.4.1 MATHEMATICAL FORMULATION

We are interested in determining the optimal collection period \( T \) of returned products at an ICP, so that, the sum of annual inventory and transportation costs are minimized. The collection period \( T \) has been assumed to be discrete, \( T = 1, 2, ..., \), with the unit being one day. The model can be readily adjusted to accommodate other units. The shipment volume, \( X \) is the total volume collected during the collection period \( T \).
\[ X = vRT \]

The transportation cost for a shipment is the shipment volume multiplied by the corresponding discounted freight rates, which is based on where the shipment volume lies on the volume breakpoints illustrated by Figure 3.

Transportation cost per shipment = \( E \alpha_t X = E \alpha_t vRT \)

The annual transportation cost is the transportation cost per shipment multiplied by the number of shipments from the ICP to CRC in a year \((w/T)\). Therefore,

Annual transportation cost = \( E \alpha_t vRw \)

The inventory cost is calculated based on the number of units of returned products being held at the ICP during the collection period. This is illustrated by Figure 5. And the total number of units held at the ICP is the sum, \( R + 2R + 3R + \ldots + TR \)

![Image of graph](image)

**Figure 5: The number of products held at the ICP each day**

Inventory cost per collection period \( T = b(R + 2R + 3R + \ldots + TR) = \frac{T}{2} (bR(T + 1)) \)

The annual inventory cost is the inventory cost per collection period multiplied by the number of collection periods in a year \((w/T)\). Therefore,

Annual inventory cost = \( bwR \frac{(T+1)}{2} \)

And hence, total annual cost = annual transportation cost + annual inventory cost

\[ = E \alpha_t vRw + bwR \frac{(T+1)}{2} \]
3.4.2 SOLUTION

Clearly, the transportation cost depends on the total volume of the products collected within period $T$ at the ICP, which is proportional to $vR$. In the analysis below, we consider several cases for the range of values of $vR$ as they relate to the shipment volume breakpoints, $V_l$, where, $l = 0, 1, 2, ..., n$.

Let us consider the case of very high daily return volume, $vR \geq V_n$. Then the shipment discount rate is fixed at $\alpha_n$. And thus, the annual transportation cost is fixed at $E\alpha_n vRw$, while the annual inventory cost is increasing with $T$, $bwR \frac{(T+1)}{2}$. The sum of the transportation and inventory costs is then minimized for the lowest possible value of $T$. Therefore, if the total daily return volume is greater than the largest shipment volume breakpoint, $vR > V_n$, then the optimal collection period is $T = 1$. i.e. a daily transfer of returned products from the ICP to the CRC should take place.

Without loss of generality, let us assume that the total daily return volume is less than the first shipment volume breakpoint, $0 \leq vR < V_1$. The following analysis for determining the optimal collection period can apply for any $vR < V_n$. The minimum length of the collection period $T$ is 1. If $T = 1$, the annual transportation is $EvRw$ and the annual inventory carrying cost is $bwR$. Consider now, $T = 2$ and assume that the collected volume in this period at the ICP does not exceed the first shipment volume breakpoint, $V_1$, i.e. $2vR < V_1$. Then, the annual transportation cost remains unchanged at $EwvR$ while the annual inventory carrying cost increases to $\frac{3}{2} bwR$. Therefore, shipping to the CRC every day is better than shipping every other day. The smallest value of $T$ at which we benefit from shipping discounts is at $T = \lceil \frac{V_1}{vR} \rceil$, where $\lceil . \rceil$ is the ceiling function. For values of $T$ such that $1 < T < \lceil \frac{V_1}{vR} \rceil$, the transportation cost remains unchanged at $EwvR$ while the inventory cost, $bwR \frac{(T+1)}{2}$, increases with $T$. Since the total cost is an increasing function of $T$, collection periods $1 < T < \lceil \frac{V_1}{vR} \rceil$ are not optimal. The next value of interest after $T = 1$ is $T = \lceil \frac{V_1}{vR} \rceil$. 
In general, the collection period at \( T = \left\lfloor \frac{V_l}{vR} \right\rfloor \) is better than collection periods \( T \) such that \( \left\lfloor \frac{V_l}{vR} \right\rfloor < T < \left\lfloor \frac{V_{l+1}}{vR} \right\rfloor \), \( l = 1, \ldots, n - 1 \), and the collection period at \( T = \left\lfloor \frac{V_n}{vR} \right\rfloor \) is better than the ones for \( T > \left\lfloor \frac{V_n}{vR} \right\rfloor \) because the total cost is an increasing function of \( T \) within each shipment discount range, \( bwR(T + \frac{1}{2}) + E\alpha_l wR, \ l = 1, 2, \ldots, n - 1 \). Therefore, the following theorem holds:

**THEOREM 1**: The optimal collection period can be either at 1 or at \( \left\lfloor \frac{V_l}{vR} \right\rfloor \), where \( l = 1, 2, \ldots, n \).

The above theorem suggests a straightforward procedure for finding the optimal collection period by comparing the total cost at only \( n + 1 \) values of \( T \). Experimental results indicate that the optimal collection period often occurs in the two extreme candidate values of \( T \), i.e. 1 and \( \left\lfloor \frac{V_n}{vR} \right\rfloor \). Below we find the range of values of the daily collected volume \( vR \) that will yield as optimal either of the two extreme values of \( T \) by comparing the respective total costs. Based on Theorem 1, the optimality condition for \( T = 1 \) is

\[
EwR + bwR \leq \frac{bwR}{2} \left( \left\lfloor \frac{V_l}{vR} \right\rfloor + 1 \right) + E\alpha_l wR, \ l = 1, 2, \ldots, n.
\]

\[
\Rightarrow \left\lfloor \frac{V_l}{vR} \right\rfloor \geq 2Ev\left( \frac{1-a_l}{b} \right) + 1, \ where, \ l = 1, 2, \ldots, n.
\]

Noting that \( \left\lfloor \frac{V_l}{vR} \right\rfloor \) is a positive integer,

\[
\left\lfloor \frac{V_l}{vR} \right\rfloor \geq \left\lfloor \frac{2Ev(1-a_l)}{b} + 1 \right\rfloor, \ where, \ l = 1, 2, \ldots, n
\]

and solving for \( vR \) we obtain,

\[
vR \leq R_L, \ where,
\]

\[
R_L = \min_{l=1,2,\ldots,n} \frac{V_l}{2Ev(1-a_l)+1}.
\]

(1)

Similarly, the optimality conditions for \( T = \left\lfloor \frac{V_n}{vR} \right\rfloor \) are,

\[
\frac{bwR}{2} \left( \left\lfloor \frac{V_n}{vR} \right\rfloor + 1 \right) + E\alpha_n wR \leq \frac{bwR}{2} \left( \left\lfloor \frac{V_l}{vR} \right\rfloor + 1 \right) + E\alpha_l wR, \ where, \ l = 1, 2, \ldots, n - 1 and
\]

\[
vR \leq R_L, \ where,
\]

\[
R_L = \min_{l=1,2,\ldots,n-1} \frac{V_l}{2Ev(1-a_l)+1}.
\]
\[
\frac{bwR}{2} \left( \left\lceil \frac{V_n}{vR} \right\rceil + 1 \right) + Ea_n wvR \leq EwvR + bwR
\]

These reduce to \( vR \geq R_U \), where \( R_U \) is the minimum value of \( vR \) satisfying

\[
\left\lceil \frac{V_n}{vR} \right\rceil \leq \left\lceil 2Ev \frac{(\alpha_l - \alpha_n)}{b} \right\rceil + \left\lceil \frac{V_l}{vR} \right\rceil, \quad \text{where, } l = 1, 2, \ldots, n - 1 \quad \text{and}
\]

\[
vR \geq \frac{V_n}{2Ev \left( \frac{1 - \alpha_n}{b} \right)}.
\]

This gives us the following corollary for finding the optimal collection period:

**COROLLARY 1:** The optimal collection period is given by,

\[
T^* = \begin{cases} 
1 & \text{if } vR \leq R_L, \\
\left\lceil \frac{V_n}{vR} \right\rceil & \text{if } vR \geq R_U, \\
\left\lceil \frac{V_l}{vR} \right\rceil & \text{if } R_L < vR < R_U,
\end{cases}
\]

where \( R_L \) and \( R_U \) are given by the set of conditions (1) and (2), respectively.

### 3.5 THE CASE OF MULTIPLE PRODUCTS AND A SINGLE COLLECTION POINT

In this section we develop and optimize the model for the sub-problem of multiple types of products and a single ICP, as depicted in Figure 4. This model is an extension to the previous model with different product volumes and holding costs.

For this case, the number of ICPs, \( s = 1 \), and thus, \( j = 1 \)

For notational simplicity, we say,

\( r_{ik} = r_{1ik} \), is the units of product \( k \) returned by customer \( i \) at the ICP

\( R_k = R_{1k} = \sum_{i=1}^{c_1} r_{ik} \), is the sum of units of product \( k \) returned per day at the ICP

**Decision variables**

\( T = \) length of a collection period (in days) at the ICP

\( X = \) shipment volume of products returned from the ICP to the CRC at collection period \( T \)
3.5.1 MATHEMATICAL FORMULATION

Similar to the previous case, we want to minimize the sum of inventory and transportation costs by determining the optimal collection period $T$ of returned products at the ICP, where the collection period (in days) is assumed to be discrete. Here, the shipment volume, $X$ is the total volume of all the products collected in the period $T$.

$$X = v_1 R_1 T + v_2 R_2 T + v_3 R_3 T + \cdots + v_p R_p T = (\sum_{k=1}^{p} v_k R_k) T$$

The transportation cost is based on where the shipment volume lies on the volume breakpoints illustrated by Figure 3.

Transportation cost per shipment $= E\alpha_l X = E\alpha_l (\sum_{k=1}^{p} v_k R_k) T$

The annual transportation cost is the transportation cost per shipment multiplied by the number of shipments from the ICP to CRC in a year ($w/T$). Therefore,

Annual transportation cost $= E\alpha_l (\sum_{k=1}^{p} v_k R_k) w$

The inventory cost is calculated based on the number of units of each type of returned products being held at the ICP during the collection period. This is illustrated by Figure 6 for the type of product $k$. And the total number of units of that type of product held at the ICP is the sum, $R_k + 2R_k + 3R_k + \cdots + TR_k$

Figure 6: The number of products $k$ held at the ICP each day
Inventory cost of product $k$ per collection period $T = b_k(R_k + 2R_k + 3R_k + \ldots + TR_k) = \frac{T}{2}(b_k R_k(T + 1))$

The annual inventory cost is the inventory cost per collection period multiplied by the number of collection periods in a year ($w/T$). Therefore,

Annual inventory cost of product $k = wb_kR_k\left(\frac{T+1}{2}\right)$

The total annual inventory cost of all $p$ products $= wb_1R_1\left(\frac{T+1}{2}\right) + wb_2R_2\left(\frac{T+1}{2}\right) + \cdots + wb_pR_p\left(\frac{T+1}{2}\right) = w(\Sigma_{k=1}^{p}b_kR_k)\left(\frac{T+1}{2}\right)$

And hence, total annual cost $= \text{annual transportation cost} + \text{annual inventory cost}$

$$= E\alpha_l(\Sigma_{k=1}^{p}v_k R_k)w + w(\Sigma_{k=1}^{p}b_kR_k)\left(\frac{T+1}{2}\right)$$

### 3.5.2 SOLUTION

As we can see that the only differences in the total cost function for the case of multiple products when compared to the case of a single product is that we have combined volumes ($\Sigma_{k=1}^{p}v_k R_k$) instead of $vR$ and combined inventory costs ($\Sigma_{k=1}^{p}b_kR_k$) instead of $bR$. Hence, the determination of the optimal collection period is similar to the one described above. The inventory cost increases linearly with the collection period $T$ and the transportation cost is constant for a shipment volume between two volume breakpoints, i.e. $V_l \leq \Sigma_{k=1}^{p}v_k R_k < V_{l+1}$, where, $l = 0, 1, 2, \ldots, n - 1$. The lowest value of the sum of the inventory and transportation costs is then obtained at the leftmost point, $T = \left\lfloor \frac{V_l}{\Sigma_{k=1}^{p}v_k R_k} \right\rfloor$, where, $l = 1, 2, \ldots, n$. Therefore, the following theorem holds:

**THEOREM 2:** For $p$ types of product returned at the ICP, the optimal collection period is either at $T = 1$ or $T = \left\lfloor \frac{V_l}{\Sigma_{k=1}^{p}v_k R_k} \right\rfloor$, where, $l = 1, 2, \ldots, n$.

This theorem, like the previous one, suggests a straightforward procedure for finding the optimal collection period by comparing the total costs at only $n + 1$ values of $T$. And experimental results indicate that the optimal collection periods often occur in the
two extreme candidate values of $T$, i.e. 1 and $\left[ \frac{V_n}{\sum_{k=1}^{p} v_k R_k} \right]$. Below, we find the conditions for which either of these extreme values will yield the optimal collection period by comparing the total costs.

Based on Theorem 2, the optimality condition for $T = 1$ is,

\[
Ew(\Sigma_{k=1}^{p} v_k R_k) + w(\Sigma_{k=1}^{p} b_k R_k) \leq E\alpha w(\Sigma_{k=1}^{p} v_k R_k) + w \left( \frac{\sum_{k=1}^{p} b_k R_k}{2} \left[ \frac{V_l}{\sum_{k=1}^{p} v_k R_k} \right] \right) + 1,
\]

where, $l = 1, 2, \ldots, n$.

\[\Rightarrow \left[ \frac{V_l}{\sum_{k=1}^{p} v_k R_k} \right] \geq 2E \left( \frac{\sum_{k=1}^{p} v_k R_k}{\sum_{k=1}^{p} b_k R_k} \right)(1 - \alpha_l) + 1, \text{ where, } l = 1, 2, \ldots, n.\]

Since \(\left[ \frac{V_l}{\sum_{k=1}^{p} v_k R_k} \right]\) is a positive integer,

\[\left[ \frac{V_l}{\sum_{k=1}^{p} v_k R_k} \right] \geq \left[ 2E \left( \frac{\sum_{k=1}^{p} v_k R_k}{\sum_{k=1}^{p} b_k R_k} \right)(1 - \alpha_l) + 1 \right], \text{ where, } l = 1, 2, \ldots, n.\]

Therefore, we get the following condition,

\[
\sum_{k=1}^{p} v_k R_k \leq \frac{V_l}{2E \left( \frac{\sum_{k=1}^{p} v_k R_k}{\sum_{k=1}^{p} b_k R_k} \right)(1 - \alpha_l) + 1}, \text{ where, } l = 1, 2, \ldots, n. \quad (3)
\]

which when satisfied for every shipment volume breakpoint, $l = 1, 2, \ldots, n$, gives us $T = 1$, as the optimal collection period.

Similarly, the optimality conditions for $\left[ \frac{V_n}{\sum_{k=1}^{p} v_k R_k} \right]$ are,

\[
E\alpha w(\Sigma_{k=1}^{p} v_k R_k) + w \left( \frac{\sum_{k=1}^{p} b_k R_k}{2} \left[ \frac{V_n}{\sum_{k=1}^{p} v_k R_k} \right] \right) + 1 \leq E\alpha w(\Sigma_{k=1}^{p} v_k R_k) + w \left( \frac{\sum_{k=1}^{p} b_k R_k}{2} \left[ \frac{V_l}{\sum_{k=1}^{p} v_k R_k} \right] \right) + 1, \text{ where, } l = 1, 2, \ldots, n \text{ and}
\]

\[E\alpha w(\Sigma_{k=1}^{p} v_k R_k) + w \left( \frac{\sum_{k=1}^{p} b_k R_k}{2} \left[ \frac{V_n}{\sum_{k=1}^{p} v_k R_k} \right] \right) + 1 \leq Ew(\Sigma_{k=1}^{p} v_k R_k) + w(\Sigma_{k=1}^{p} b_k R_k)
\]

Upon simplification, we get the conditions,
\[
\left\lfloor \frac{V_n}{\sum_{k=1}^{p} v_k R_k} \right\rfloor \leq 2E \left( \frac{\sum_{k=1}^{p} v_k R_k}{\sum_{k=1}^{b} b_k R_k} \right)(\alpha_l - \alpha_n) + \left\lfloor \frac{V_l}{\sum_{k=1}^{p} v_k R_k} \right\rfloor, \text{ where, } l = 1, 2, \ldots, n - 1
\]

and

\[
\sum_{k=1}^{p} v_k R_k \geq \frac{V_n}{2E \left( \frac{\sum_{k=1}^{p} v_k R_k}{\sum_{k=1}^{b} b_k R_k} \right)(1 - \alpha_n) + 1}.
\]

This gives us the following corollary for finding the optimal collection period:

**COROLLARY 2:** The optimal collection period is given by,

\[
T^* = \begin{cases} 
1 & \text{if condition (3) is satisfied}, \\
\left\lfloor \frac{V_n}{\sum_{k=1}^{p} v_k R_k} \right\rfloor & \text{if conditions (4) are satisfied}, \\
\left\lfloor \frac{V_l}{\sum_{k=1}^{p} v_k R_k} \right\rfloor & \text{otherwise}.
\end{cases}
\]

### 3.6 THE CASE OF A SINGLE PRODUCT AND MULTIPLE COLLECTION POINTS

In this section, we discuss about the various policies that can be adopted to collect the returned products when we have more than one ICP. The problem of collecting from multiple ICPs increases the number of possible ways in which the returned products can be shipped from these ICPs. As demonstrated later, these possible policies increase exponentially with the increase in number of ICPs. The shipments through which returned products are collected from the ICPs can be categorized into three main types of policies. Namely, the individual shipment policy, combined shipment policy and, hybrid shipment policy. The first two can be considered as the extensions of the models described previously, while the hybrid shipment policy, as described later, adds some complexity into the model and may need dealing with each additional ICP separately. For the purpose of this thesis we develop and optimize the hybrid shipment policy model for two ICPs and hence, demonstrate the added complexity of adding just one additional ICP. In the following subsections we discuss these policies in detail.
3.6.1 INDIVIDUAL SHIPMENT POLICY

As mentioned above, this is an extension of the single product and single collection point model described earlier in the section 3.4. Specifically, in this policy treat every ICP individually and make shipments based on the volume of returned products collected at each of the ICPs irrespective of the volume collected at the other ICPs. Such a policy is illustrated in Figure 7.

For this policy,

Types of products, \( p = 1 \), and thus, \( k = 1 \)

And for notational simplicity, we say,

\[
\tau_{ij} = r_{ij1}, \text{ is the units of product returned by customer } i \text{ at ICP } j, \text{ where, } j = 1, 2, ..., s
\]

\[
R_j = R_{j1} = \sum_{i=1}^{s} r_{ij}, \text{ is the sum of units of product returned per day at ICP } j, \text{ where, } j = 1, 2, ..., s
\]

\[
v = v_1, \text{ is the volume of the product (ft}^3 / \text{ unit})
\]

\[
b = b_1, \text{ is the holding cost per unit of the product ($ / \text{ unit} / \text{ day})}
\]
Decision variables

\( T_j \) = length of collection period (in days) when a shipment is made from ICP \( j \), where \( j = 1, 2, ..., s \)

\( X_j \) = shipment volume of products returned from ICP \( j \) to the CRC at collection period \( T_j \), where, \( j = 1, 2, ..., s \)

\( \alpha^l_j \) = discount rate at ICP \( j \) dependent on shipment volume \( X_j \), is based on the shipment volume breakpoints illustrated earlier in Figure 3, where, \( j = 1, 2, ..., s \) and \( l = 1, 2, ..., n \) and \( X_j = vR_jT_j \).

Using the model in section 3.4, the sum of the annual transportation and inventory costs for an ICP can be written as follows,

Total annual cost for ICP \( j \) for shipments made at collection period \( T_j \) = \( E \alpha^l_j vRw + bwR \frac{(T_j+1)}{2} \)

To solve for the optimal collection periods, say \( T_j^* \), corollary 1 can be applied to each ICP \( j \) individually, and hence for a total of \( s \) times as \( j = 1, 2, ..., s \).

Therefore, the optimal total annual cost can be written as follows,

Optimal total annual cost for all ICPs = \( E \alpha^1 vRw + bwR \frac{(T_1^*+1)}{2} + E \alpha^2 vRw + bwR \frac{(T_2^*+1)}{2} + E \alpha^3 vRw + bwR \frac{(T_3^*+1)}{2} + ... + E \alpha^s vRw + bwR \frac{(T_s^*+1)}{2} \), where \( \alpha^l \) is the discount rate for the optimal shipment volume at ICP \( j \) and \( j = 1, 2, ..., s \).

3.6.2 COMBINED SHIPMENT POLICY

This policy again is an extension of the model described in section 3.4. However, the difference is that instead of making shipments from each ICP individually at different collection periods we combine all the returned products from all ICPs for every shipment at one collection period. Here, at the collection period a milk run is performed through all ICPs and the returned products are collected from every ICP. This policy is illustrated by the Figure 8.
Here,

Type of products, \( p = 1 \), and thus, \( k = 1 \)

And for notational simplicity, we say,

\[ r_{ij} = r_{ij1}, \] is the units of product returned by customer \( i \) at ICP \( j \), where, \( j = 1, 2, ..., s \)

\[ R = \sum_{j=1}^{s} R_{j1} = \sum_{j=1}^{s} \sum_{i=1}^{c_j} r_{ij}, \] is the cumulative sum of units of product returned per day at all \( s \) ICPs

\( v = v_1 \), is the volume of the product (ft\(^3\) / unit)

\( b = b_1 \), is the holding cost per unit of the product ($ / unit / day)

**Decision variables**

\( T \) = length of a collection period (in days) when the milk run is performed to collect the returned products from all ICPs

\( X = vRT \); is the shipment volume of products returned from the ICPs to the CRC at the collection period \( T \)

Again, using the model in section 3.4, the sum of the annual transportation and inventory costs for an ICP can be written as follows,
Total annual cost for shipments made at collection period \( T = Eα_tvRw + bwR \frac{(T+1)}{2} \)

Using corollary 1, the optimal collection period \( T \) can be determined.

### 3.6.3 HYBRID SHIPMENT POLICY

The shipments in a hybrid shipment policy can be considered as the partial combinations of the individual shipment policy and the combined shipment policy. The complication here, as previously stated, is that the number of possible combinations in which the shipments can be made increase exponentially with the increase in number of ICPs. To illustrate, let us consider a case of two ICPs, then the shipments can be made from ICP 1, ICP 2 and both ICP 1 & ICP 2, i.e. three ways to make the shipments. For the case of three ICPs, the shipments can be made from ICP 1, ICP 2, ICP 3, ICP 1 and ICP 2, ICP 1 and ICP 3, ICP 2 and ICP 3, ICP 1 and ICP 2 and ICP 3, i.e. seven ways to make the shipments. Thus, to generalize, for the case of \( s \) ICPs, the number of ways the shipments can made is \( 2^s - 1 \), i.e. increases exponentially with the number of ICPs, \( s \).

In the rest of this section we consider the case of two ICPs. We develop and optimize the model of the hybrid shipment policy with two ICPs and a single product, and thus, demonstrate the added complexity in the model by increasing just one ICP.

For this case,

Number of ICPs, \( s = 2 \), and thus, \( j = 1, 2 \)

Number of products, \( p = 1 \), and thus, \( k = 1 \)

And we say for notational simplicity,

\( r_{ij} = r_{ij1} \) is the units of product returned by customer \( i \) at the ICP \( j \)

\( R_j = R_{j1} = \sum_{i=1}^{c_j} r_{ij} \) is the sum of units of product returned per day at ICP \( j \), where, \( j = 1, 2 \)

Here, we assume that the units of product returned per day at ICP 1 is more than ICP 2, i.e. \( R_1 > R_2 \). This assumption is justified because, if \( R_1 = R_2 \), then the ICPs can be treated as one big ICP with twice the returned volume and the model becomes that of a
single ICP. In any other case, the volume of the returned products in one of the ICPs will always be more than the other. And the ICP with higher return rate can be indexed ICP 1 while the other ICP 2. A similar assumption can be extended to the cases of more than two ICPs where the shipments from the ICP with the lowest volume of returned products can be made only while making the final combined shipment*. The number of possible ways to make the shipments per cycle would then reduce from $2^s - 1$ to $2^s - 2^{s-1}$ (or $\frac{2^s}{2}$).

$v = v_1$, is the volume of the product ($ft^3 / unit$)

$b = b_1$, is the holding cost per unit of the product ($\$ / unit / day$)

A hybrid shipment policy with two ICPs is illustrated by Figure 9 and Figure 10.

* For the case of three ICPs, the possible shipments reduce to ICP 1, ICP 2, ICP 1 & ICP 2, and ICP 1 & ICP 2 & ICP 3, ($2^3 - 2^2 = 8 - 4 = 4$).
**Decision variables**

\( T_1 \) = collection period after which a shipment is made from ICP 1 to CRC

\( T_2 \) = collection period after which a shipment is made from (ICP 1 and ICP 2) to CRC

\( X_{T_1} \) = volume of products returned per journey from ICP 1 to CRC (\( \text{ft}^3 \))

\( X_{T_2} \) = volume of products returned per journey from (ICP 1 and ICP 2) to CRC (\( \text{ft}^3 \))

Since \( R_1 > R_2 \), it can also be assumed that, \( T_1 < T_2 \), i.e. the ICP with higher rate of returned products will require more frequent shipments.

**3.6.3.1 MATHEMATICAL FORMULATION**

Like all other cases, we want to minimize the total annual inventory and transportation costs with respect to the collection periods, \( T_1 \) and \( T_2 \). The shipment volumes \( X_{T_1} \) and \( X_{T_2} \), for the collection periods \( T_1 \) and \( T_2 \) respectively, can be defined as,

\[
X_{T_1} = vR_1T_1
\]

\[
X_{T_2} = vR_1(T_2 - T_1\left(\left\lceil \frac{T_2}{T_1}\right\rceil - 1\right)) + vR_2T_2
\]

\( \alpha_y \) = discount rate at \( X_{T_1} \)

\( \alpha_z \) = discount rate at \( X_{T_2} \)

where, \( (\alpha_y, \alpha_z) \in \alpha_l \), where, \( l = 0, 1, 2, ..., n \)

It is important to note here that the cycle of making shipments from ICP 1 at every \( T_1 \) days and then shipments from both ICPs at every \( T_2 \) days, repeats every \( T_2 \) days. The factor \( \left(\left\lceil \frac{T_2}{T_1}\right\rceil - 1\right) \) is the number of times the shorter collection period \( T_1 \) occurs in one longer collection period \( T_2 \). Therefore, \( T_2 - T_1\left(\left\lceil \frac{T_2}{T_1}\right\rceil - 1\right) \) is the number of days for which the returned products are held at ICP 1 when making the shipment at \( T_2 \), after making \( \left(\left\lceil \frac{T_2}{T_1}\right\rceil - 1\right) \) shipments from ICP 1 at collection periods \( T_1 \).
The transportation cost per individual shipment from ICP 1 at collection periods $T_1$ is given by, $E\alpha_yX_{T_1}$ and the transportation cost per combined shipment from ICP 1 and ICP 2 at collection period $T_2$ is given by, $E\alpha_zX_{T_2}$. The number of individual shipments from ICP 1 annually are $(\left\lceil \frac{T_2}{T_1} \right\rceil - 1) \frac{w}{T_2}$, and the number of combined shipments from both ICPs in collection period $T_2$ is $\frac{w}{T_2}$.

Therefore, the total annual transportation cost is given as follows,

$$E\alpha_yvR_1w\left(\frac{T_2}{T_1} - 1\right)\frac{T_1}{T_2} + E\alpha_z[vR_1\left(T_2 - T_1\left(\left\lceil \frac{T_2}{T_1} \right\rceil - 1\right)\right] + vR_2T_2] \frac{w}{T_2}$$

$$\Rightarrow Evw\left[\alpha_yR_1\left(\frac{T_1}{T_2}\left(\frac{T_2}{T_1} - 1\right)\right)\right] + \alpha_z\left[R_1\left(1 - \frac{T_1}{T_2}\left(\left\lceil \frac{T_2}{T_1} \right\rceil - 1\right)\right) + R_2\right].$$

The inventory cost is calculated by identifying the number of days the products are held at each ICP during each of the collection periods. At ICP 1 the products are held until a shipment is made at the collection period $T_1$, then they are held again at ICP 1 for $(T_2 - T_1\left(\left\lceil \frac{T_2}{T_1} \right\rceil - 1\right))$ days until the shipment at $T_2$ occurs, and at ICP 2 the products are held until the collection period $T_2$.

Therefore, the total annual inventory cost is given as follows,

$$bwR_1\frac{T_1}{T_2}\left[\frac{T_1 + 1}{2}\left(\frac{T_2}{T_1} - 1\right)\right] + bwR_1\left[T_2 - T_1\left(\left\lceil \frac{T_2}{T_1} \right\rceil - 1\right)\right] + 1]$$

$$+ bwR_2\frac{T_2}{T_2}\left[\frac{T_2 + 1}{2}\right]$$

$$\Rightarrow \frac{bw}{2}\left[R_1\frac{T_1}{T_2}\left[\frac{T_1 + 1}{2}\left(\frac{T_2}{T_1} - 1\right)\right] + R_1\left[1 - \frac{T_1}{T_2}\left(\left\lceil \frac{T_2}{T_1} \right\rceil - 1\right)\right] + \left[T_2 - T_1\left(\left\lceil \frac{T_2}{T_1} \right\rceil - 1\right)\right] + 1\right]$$

$$+ R_2[T_2 + 1].$$
3.6.3.2 SOLUTION

For the purpose of simplification, let us substitute the collection periods $T_1$ and $T_2$ with the variables, $T$ and $q$ such that, $T = T_1$ and $q = T_2 - T_1$, where $q = 1, 2, 3, \ldots$ and so on, because $T_1 < T_2$. Upon making this substitution, we get the following set of equations $(I)$.  

\[ X_T = X_{T_1} = vR_1T, \]
\[ X_{T+q} = X_{T_2} = vR_1 \left( T \left( 1 - \left\lfloor \frac{q}{T} \right\rfloor \right) + q \right) + vR_2(T + q), \]

Total annual transportation cost = $Evw\left[ \alpha_z(R_1 + R_2) + \frac{R_1(\alpha_y - \alpha_z)T \left\lfloor \frac{q}{T} \right\rfloor}{T + q} \right]$,

Total annual inventory cost = $bwR_1 \left( \left\lfloor \frac{q}{T} \right\rfloor^2 - \left\lfloor \frac{q}{T} \right\rfloor + 1 \right) T^2 + \left( 2q + 1 - 2q \left\lfloor \frac{q}{T} \right\rfloor \right) T + (q^2 + q) \right] + bwR_2[T + q + 1]$.

Now the objective is to minimize the sum of transportation and inventory costs by identifying the optimal values of $T$ and $q$ as defined by the set of equations $(I)$. To do so, we first need to analyze the change in transportation and inventory costs with respect to the changes in $T$ and $q$.

To that end, it can be observed that the term $\left\lfloor \frac{q}{T} \right\rfloor$ is fixed for values of $q$ between two adjacent multiples of $T$, for some $T$. Therefore, if $mT$ is $m^{th}$ multiple of $T$, then $\left\lfloor \frac{q}{T} \right\rfloor$ is fixed at $\left\lfloor \frac{q}{T} \right\rfloor = m$ for $(m - 1)T < q \leq mT$. Alternatively, the range of $q$, $(m - 1)T < q \leq mT$, can be redefined as $q = (m - 1)T + a$, such that $a = 1, 2, \ldots, T$. This gives us two sub-problems to solve,

1) When $q = (m - 1)T$, where $m = 2, 3, \ldots$ and so on, i.e. $q$ is a multiple of $T$

2) When $q = (m - 1)T + a$, such that $a = 1, 2, \ldots, T$, i.e. $q$ is a value between two adjacent multiples of $T$, for some $T$
We redefine the set of equations (I) for the two sub-problems mentioned above, namely equations (II) and (III) respectively.

For the set of equations (II) for sub-problem 1, \( q = (m - 1)T \), where \( m = 2, 3, ... \) and so on. Also, \( \left\lfloor \frac{q}{T} \right\rfloor = m - 1 \). Upon making these substitutions, we get,

\[
X_T = vR_1T, \text{ and the corresponding discount rate is } \alpha_y.
\]

\[
X_{mT} = vR_1T + mTvR_2, \text{ and the corresponding discount rate is } \alpha_z,
\]

Total annual transportation cost = \( Evw[R_1(\alpha_y + \frac{\alpha_z - \alpha_y}{m}) + R_2\alpha_z] \).

Total annual inventory cost = \( bw[R_1(T + 1) + R_2(mT + 1)] \).

For the set of equations (III) for sub-problem 2, \( q = (m - 1)T + a \), such that, \( a = 1, 2, ..., T \). Also, \( \left\lfloor \frac{q}{T} \right\rfloor = m \). Upon making these substitutions, we get,

\[
X_T = vR_1T, \text{ and the corresponding discount rate is } \alpha_y.
\]

\[
X_{mT+a} = (vR_1 + vR_2)a + mTvR_2, \text{ and the corresponding discount rate is } \alpha_z,
\]

Total annual transportation cost = \( Evw[R_1\left(\frac{mT\alpha_y + aa_z}{mT+a}\right) + R_2\alpha_z] \).

Total annual inventory cost = \( bwR_1\left[\frac{(mT^2 + mT + a^2 + a)}{mT+a}\right] + bwR_2[mT + a + 1] \).

Let us now consider the inventory cost for the sub-problem 2. For this sub-problem, inventory cost is a function of \( [R_1\left(\frac{(mT^2 + mT + a^2 + a)}{(mT+a)}\right) + R_2[mT + a + 1]] \). If \( T \) were continuous, then we could identify the nature of this function with respect to \( T \) by partial differentiation. The partial derivative of this function with respect to \( T \) is given by,

\[
\frac{m}{(mT+a)^2}(mT^2 + a(2T - a))
\]

This partial derivative is always positive since \( a \leq T \). Therefore, we say that the inventory cost for the sub-problem 2 is an increasing function of \( T \). Similarly, the inventory cost for the sub-problem 1 increases linearly with \( T \).

Now, the transportation cost is dependent on the shipment volumes which is also a function of \( T \). Let us consider a case where the shipment volume \( X_T \) is more than the largest volume breakpoint illustrated in Figure 3, i.e. \( X_T > V_n \). In this case, the freight discount
rate $\alpha_y$ is fixed at $\alpha_n$. This occurs when $vR_1 T > V_n$, or $T > \frac{V_n}{vR_1}$, since $T$ is a positive integer, we get, $T = \left\lceil \frac{V_n}{vR_1} \right\rceil$. When $T \geq \left\lceil \frac{V_n}{vR_1} \right\rceil$, we can get some values of $m$ and $a$ such that, $X_mT$ and $X_{mT+a}$ are also greater than $V_n$. When this occurs, we get $\alpha_y = \alpha_z = \alpha_n$, i.e. maximum possible discounts on the shipments. And thus, the transportation cost is minimum at this point. And since the inventory costs are increasing with $T$, the sum of the transportation and inventory costs increase with the increase in $T$ beyond $T = \left\lceil \frac{V_n}{vR_1} \right\rceil$. Therefore, the following proposition holds,

**PROPOSITION 1:** The maximum value of $T$ beyond which we do not benefit when minimizing the inventory and transportation costs is $\left\lceil \frac{V_n}{vR_1} \right\rceil$.

Let us call this value of $T$, $T_{max}$, i.e. $T_{max} = \left\lceil \frac{V_n}{vR_1} \right\rceil$. Then to find the optimal solution for minimum inventory and transportation costs, we look at every value of $T$ up to $T_{max}$. Therefore, in the following analysis we consider that $T$ is known since we reach the optimal solution by solving for every $T = 1, 2, ..., T_{max}$. For a given $T$, the shipment volume $X_T = vR_1T$ is fixed and thus, $\alpha_y$ is constant. For the sub-problem 1 (when $q$ is a multiple of $T$), the inventory cost increases linearly with $m$ for a fixed $T$. And the transportation cost depends on $m$ and the freight discount rate $\alpha_z$ based on the second shipment volume $X_mT$. Also, for the following analysis, it is important to note that $X_mT > X_T$, because $X_mT = vR_1T + mTvR_2$ and $X_T = vR_1T$, where $m = 2, 3, ..., therefore, \alpha_y \geq \alpha_z$. Without loss of generality, let us assume that the shipment volume $X_mT$ is between the breakpoints $V_0$ and $V_1$, i.e. $V_0 \leq X_mT < V_1$ and $\alpha_z = \alpha_y = 1$. If $m = 2^\dagger$, the annual transportation cost is $Evw[R_1(\alpha_y + \frac{1}{2}(\alpha_z - \alpha_y)) + R_2]$ and the inventory cost is $bw[R_1(T + 1) + R_2(2T + 1)]$. Consider now $m = 3$, and assume that the shipment volume $X_mT$ is still such that $V_0 \leq X_mT < V_1$. Then the annual transportation cost is $Evw[R_1(\alpha_y + \frac{1}{3}(\alpha_z - \alpha_y)) + R_2]$ and the inventory cost is $bw[R_1(T + 1) + R_2(3T + 1)]$. Both transportation and inventory costs at $m = 3$ are more than that at

$\dagger m$ starts at 2, because if $m = 1$, then $T_1 = T_2$. 
\( m = 2 \). Therefore, the sum of transportation and inventory costs are minimum at \( m = 2 \), i.e. the leftmost point. Since, \( \alpha_y \geq \alpha_z \), this holds true for any values of \( \alpha_z \). The point at which we benefit from shipping discounts is when \( X_{mT} = V_1 \), i.e. \( m = \left\lceil \frac{V_1 - vR_1T - vR_2T}{vR_2T} \right\rceil \), and values of \( 2 < m < \left\lceil \frac{V_1 - vR_1T - vR_2T}{vR_2T} \right\rceil \) are not optimal. The next value of interest after \( m = 2 \) is \( \left\lceil \frac{V_1 - vR_1T - vR_2T}{vR_2T} \right\rceil \).

In general, the value of \( m = \left\lceil \frac{V_l - vR_1T - vR_2T}{vR_2T} \right\rceil \) is better than \( \left\lceil \frac{V_{l+1} - vR_1T - vR_2T}{vR_2T} \right\rceil \), \( l = 1, 2, ..., n - 1 \), and \( m = \left\lceil \frac{V_n - vR_1T - vR_2T}{vR_2T} \right\rceil \) is better than \( m > \left\lceil \frac{V_n - vR_1T - vR_2T}{vR_2T} \right\rceil \) because the total cost is an increasing function of \( m \) for the sub-problem 1, where \( q = (m - 1)T \). Therefore, the following lemma holds,

**Lemma 1:** When \( q \) is a multiple of \( T \), such that \( q = (m - 1)T \), the optimal values of \( T \) and \( m \) can be determined by comparing the total costs at either the leftmost point of \( m \) or \( m = \left\lceil \frac{V_l - vR_1T - vR_2T}{vR_2T} \right\rceil \), where, \( l = 1, 2, ..., n \). for \( T = 1, 2, ..., T_{\text{max}} \), such that, \( T_{\text{max}} = \left\lfloor \frac{V_n}{vR_1} \right\rfloor \).

We note here that the values of \( m \) determined from the above lemma are better than all the other values of \( m \) for the sub-problem 1. It should also be noted, that when solving for sub-problem 2, the value of \( m = 1 \) is considered arbitrarily since it is not accounted for in the sub-problem 1, and, \( m = 1 \) is also the left-most point. Now for the sub-problem 2, where \( q \) is between two adjacent multiples of \( T \), i.e. \( q = (m - 1)T + a \), where \( a = 1, 2, ..., T \), we analyze the ranges of values of \( a \) as they relate to the transportation and inventory costs for a fixed \( T \) which holds true for every value of \( T \) up to \( T_{\text{max}} \) and a fixed value of \( m \). Therefore, for the following analysis, we say that the values of \( T \) and \( m \) are known.

For the transportation cost for sub-problem 2, we make the below claim and provide the proof for it, followed by the corresponding proposition.
CLAIM 1: When $T$ and $m$ are given and $q = (m - 1)T + a$, where, $a = 1, 2, \ldots, T$, the transportation cost at $a = T$ is always less than the transportation cost at $a = 1, 2, \ldots, T - 1$, i.e. $q = mT$ has lower transportation cost than $q = (m - 1)T + a$, where, $a = 1, 2, \ldots, T - 1$.

The transportation cost for sub-problem 2, for known values of $T$ and $m$ and given by, say $TC(a) = \text{Evw}[R_1 \left( \frac{mT \alpha_y + a \alpha_z}{mT + a} \right) + R_2 \alpha_z]$, is dependent on the value of $a$ and the freight discount rate $\alpha_z$ based on the shipment volume, $X_{mT+a}$. The shipment volume, $X_{mT+a} = (vR_1 + vR_2)a + mTvR_2$, increases linearly with the increase in $a$ from 1 to $T$, where, $X_{mT+1} = (vR_1 + vR_2) + mTvR_2$ and $X_{mT+T} = (vR_1 + vR_2)T + mTvR_2$. Let us now assume that as $a$ approaches $T$, the shipment volume does not cross the next shipment volume breakpoint say, $V_i+1$, i.e. $V_i \leq X_{mT+1}, X_{mT+T} < V_i+1$, and $\alpha_z$ is fixed. Then, the transportation cost is entirely a function of $a$. If $a$ were continuous such that, $0 < a \leq T$, then the derivative of the transportation cost with respect to $a$ would give us an insight into the behavior of, say, $\overline{TC}(a)$, where $a$ is continuous and $\alpha_z$ if fixed. Thus, differentiating $\overline{TC}(a)$ with respect to $a$, gives us,

$$\frac{d(\overline{TC}(a))}{da} = \frac{mT}{(mT + a)^2} (\alpha_z - \alpha_y).$$

Thus, the nature of $\overline{TC}(a)$ depends on the values of $\alpha_y$ and $\alpha_z$. Let us then compare the corresponding shipment volumes, $X_T$ and $X_{mT+a}$. For $X_{mT+a} > X_T$, we get, $(vR_1 + vR_2)a + mTvR_2 > vR_1T$. This reduces to, $a > T \left[ \frac{R_1 - mR_2}{R_1 + R_2} \right]$. Therefore, for values of $a > T \left[ \frac{R_1 - mR_2}{R_1 + R_2} \right]$, the shipment volume $X_{mT+a} > X_T$, and thus, $\alpha_z < \alpha_y$ and $\overline{TC}(a)$ is decreasing for $a > T \left[ \frac{R_1 - mR_2}{R_1 + R_2} \right]$. Now, if $\alpha_z$ is not fixed, then as we move from $a = \left[ T \left[ \frac{R_1 - mR_2}{R_1 + R_2} \right] \right]$ to $a = T$, the shipment volume increases and $\alpha_z$ decreases, thus, if anything, we would reduce the transportation cost even more. We can therefore say that the transportation cost at $a = T$ is less than the transportation cost for those values of $a$ such that, $T \left[ \frac{R_1 - mR_2}{R_1 + R_2} \right] < a \leq T - 1.$
Let us now compare the total annual transportation costs for \( a = T \) and all other values of \( a = 1, 2, ..., T - 1 \). The total annual transportation cost is given by,

\[
TC(a) = Evw[R_1\left(\frac{mTa_y + a\alpha}{mT + a}\right) + R_2\alpha z] \tag{108x104}
\]

And on substituting \( a = T \), we get,

\[
TC(a = T) = Evw[R_1\left(\frac{mTa_y + a\alpha}{m+1}\right) + R_2\alpha z] \tag{108x125}
\]

Let us call the discount rates associated with the costs \( TC(a) \) and \( TC(a = T) \), \( \alpha_z \) and \( \alpha_{z_T} \), respectively. And their corresponding shipment volumes are,

\[
X_{mT+a} = (vR_1 + vR_2)a + mTvR_2 \quad \text{and} \quad X_{mT+T} = (vR_1 + vR_2)T + mTvR_2
\]

Before we get into comparing these costs let us first make the following observations. Since the maximum value of \( a \) in \( TC(a) \) is \( T - 1 \), the shipment volume \( X_{mT+T} \) is always greater than \( X_{mT+a} \), i.e. \( X_{mT+T} > X_{mT+a} \). And therefore, \( \alpha_{z_T} \leq \alpha_z \). Also, the shipment volume \( X_T = vR_1T \) is clearly always less than \( X_{mT+T} \), and thus, \( \alpha_{z_T} \leq \alpha_y \). Now comparing the costs \( TC(a) \) and \( TC(a = T) \), we get,

\[
TC(a = T) - TC(a) = Evw\left[R_1\left(\frac{mTa_y + a\alpha}{m+1}\right) + R_2\alpha_{z_T}\right] - Evw\left[R_1\left(\frac{mTa_y + a\alpha}{mT + a}\right) + R_2\alpha_{z}\right],
\]

which reduces to,

\[
TC(a = T) - TC(a) = Evw\left[R_1\left(\frac{a(m\alpha_y - \alpha_z) + m\alpha\alpha_T - \alpha_y + a\alpha_{z_T} - \alpha_z}{(m+1)(mT + a)}\right) + R_2(\alpha_{z_T} - \alpha_z)\right].
\]

Now based on the observations made earlier, \( \alpha_{z_T} - \alpha_y \leq 0 \) and \( \alpha_{z_T} - \alpha_z \leq 0 \). And for \( (\alpha_y - \alpha_z) \) we compare \( X_T \) and \( X_{mT+a} \), as done earlier. For \( X_{mT+a} \leq X_T \), we get \( a \leq T\left[\frac{R_1 - mR_2}{R_1 + R_2}\right] \). Therefore, \( (\alpha_y - \alpha_z) \leq 0 \), and as a result, \( TC(a = T) - TC(a) \leq 0 \), for all values of \( a \), such that \( 1 \leq a \leq T\left[\frac{R_1 - mR_2}{R_1 + R_2}\right] \). We have already proven above that the transportation cost at \( a = T \) is less than the transportation cost for those values of \( a \) such that, \( T\left[\frac{R_1 - mR_2}{R_1 + R_2}\right] < a \leq T - 1 \). Therefore, the claim made above, and the following proposition hold true.

**PROPOSITION 2:** When \( T \) and \( m \) are given and \( q = (m - 1)T + a \), where, \( a = 1, 2, ..., T \), the transportation cost at \( a = T \) is always less than the transportation cost at \( a = 1, 2, ..., T - 1 \), i.e. \( q = mT \) has lower transportation cost than \( q = (m - 1)T + a \), where, \( a = 1, 2, ..., T - 1 \).
The inventory cost, for the sub-problem 2 when \( T \) and \( m \) are given, is a function of \( a \), such that, say, \( IC(a) = bwR_1 \left[ \frac{(mT^2 + mT + a^2 + a)}{(mT + a)} \right] + bwR_2 [mT + a + 1] \). If \( a \) were continuous such that, \( 0 < a \leq T \), then the derivatives of the inventory cost, \( IC(a) \) with respect to \( a \) would give us an insight into the behavior of \( IC(a) \) with respect to \( a \). For the following analysis, let us assume that \( a \) is continuous. Then the first derivative of \( IC(a) \) is given by,

\[
\frac{d(IC(a))}{da} = bw \left[ R_1 \left( \frac{2amT + a^2 - mT^2}{(m^2T^2 + a^2 + 2amT)} \right) + R_2 \right]
\]

And the second derivative is given by,

\[
\frac{d^2(IC(a))}{da^2} = bw \left[ R_1 \left( \frac{2m^2T^3 + 2am^2T^2 + 2amT^2 + 2am^2 - 2a^2T^2}{(m^2T^2 + a^2 + 2amT)^2} \right) \right]
\]

Since \( T, m, a \) are all positive integers, the second derivative of \( IC(a) \) is greater than 0 and, we say that \( IC(a) \) is convex with a minimum at some \( a = a^* \).

Putting, \( \frac{d(IC(a))}{da} = 0 \), we get, \( a^* = T \left( \sqrt[3]{\frac{m(m+1)R_1}{R_1 + R_2}} - m \right) \).

Therefore, since \( a \) is discrete, the value of \( a = a^* \), for which the inventory cost is minimum for some \( T \) and \( m \), is either at \( a^* = \left[ T(\sqrt[3]{\frac{m(m+1)R_1}{R_1 + R_2}} - m) \right] \) or \( a^* = \left[ T(\sqrt[3]{\frac{m(m+1)R_1}{R_1 + R_2}} - m) \right] \), where \( a = 1, 2, ..., T \).

Based on this, we make the following observation:

- We know that \( a > 0 \), and thus, \( a^* > 0 \). This holds true only when \( m < \frac{R_1}{R_2} \). This implies that for \( m \geq \frac{R_1}{R_2} \) the minimum value of \( IC(a) \) occurs at \( a = a^* \leq 0 \), and thus, the inventory cost function is strictly increasing with respect to \( a \) for \( m \geq \frac{R_1}{R_2} \).

Therefore, the following proposition holds,

**PROPOSITION 3:** We solve for \( a \) in sub-problem 2, where \( q = (m - 1)T + a \) is between two adjacent multiples of \( T \), only for those integer values of \( m \) such that, \( 1 \leq m \leq \frac{R_1}{R_2} \).

Since \( IC(a) \) is a convex function of \( a \), we can find an upper bound on the value of \( a \), say \( \bar{a} \), such that the inventory cost for all values of \( a > \bar{a} \), in \( q = mT - T + a \), is always greater than the inventory cost for \( q = mT - T \). Thus, we compare the inventory costs for
sub-problem 1 and 2. i.e. range of values of $a$ such that inventory cost at $q = mT - T + a$ is less than the inventory cost at $q = mT - T$ is given by,

$$bw\left[R_1\left(\frac{(mT^2+mT+a^2+a)}{(mT+a)}\right) + R_2[mT + a + 1]\right] \leq bw[R_1(T + 1) + R_2[mT + 1],$$

$$\Rightarrow a \leq \frac{(R_1-mR_2)}{R_1+R_2}.$$

Therefore, the range of integer values of $a$ for which the inventory cost for $q = mT - T + a$ is less than that at $q = mT - T$, is $1 \leq a \leq \bar{a}$, where $\bar{a} = \left\lceil T \frac{(R_1-mR_2)}{R_1+R_2} \right\rceil$. And also, based on the claim made earlier that the transportation costs are always minimum at $q = mT$ when compared to the transportation cost at $q = mT - T + a$, the following lemma holds,

**LEMMA 2:** When $q$ is a value between two known adjacent multiples of a given $T$ such that, $q = (m - 1)T + a$, where $a = 1, 2, ..., T$, then the optimal value of $a$ such that the sum of the total annual inventory and transportation costs is minimum can be determined by comparing the costs at either $1 \leq a \leq \left\lceil T \frac{(R_1-mR_2)}{R_1+R_2} \right\rceil$ or $a = T$.

Therefore, to solve for the optimal values of $T$ and $q$, in the set of equations (I) for the minimum sum of inventory and transportation costs, we first identify the optimal range of values $T$ and $m$ by solving for the sub-problem of those values of $q$, such that, $q = (m - 1)T$. And for every value of $T$ and $m$ thus identified we solve, for $a$, the sub-problem of those values of $q$ such that $q = (m - 1)T + a$, where $a = 1, 2, ..., T$. Based on lemma 1, lemma 2 and proposition 3, the following theorem holds,

**THEOREM 3:** The optimal collection periods $T$ and $q$ for the minimum sum of transportation and inventory costs can be determined by solving for costs at the integer values of $a$ such that, $1 \leq a \leq \left\lceil T \frac{(R_1-mR_2)}{R_1+R_2} \right\rceil$ or $a = T$, for positive integer values of $m \leq \left\lfloor \frac{R_1}{R_2} \right\rfloor$, such that either $m = 1$ or $\left\lfloor \frac{V_l - vR_3T - vR_2T}{vR_2T} \right\rfloor$, where, $l = 1, 2, ..., n$, for $T = 1, 2, ..., T_{\text{max}}$, such that, $T_{\text{max}} = \left\lfloor \frac{V_n}{vR_2} \right\rfloor$. 
CHAPTER 4: EXPERIMENTAL RESULTS

This chapter demonstrates the collection policies developed in the previous chapter with an experimental dataset. We determine the optimal collection periods for a case of a single product and two ICPs, and illustrate the results of the corresponding individual shipment, combined shipment and hybrid shipment policies. Finally, we compare the results and show examples of when a particular policy is optimal.

To illustrate the collection policies, we ran the model for clusters of customers returning products to two ICPs. We show the determination of the optimal collection periods for the combined shipment policy and hybrid shipment policy. Since the approach for combined policy is similar to that of single shipment policy applied for every ICP, we directly compare the results for the single shipment policy.

A cluster of customers return the products to ICP 1 at the daily rate of returned units, \( R_1 = 300 \), and another cluster of customers return the products to ICP 2 at the rate, \( R_2 = 50 \). The volume of the product \( v \) is 1, the daily inventory carrying cost \( b \) is 0.1, the unit standard transportation freight rate \( E \) is 1 and the number of working days per year \( w \) is 250. The four shipment volume breakpoints are \( V_1 = 500, V_2 = 1000, V_3 = 1500 \) and \( V_4 = 2000 \), beyond which the freight discount rates are \( \alpha_1 = 0.8, \alpha_2 = 0.75, \alpha_3 = 0.6 \), and \( \alpha_4 = 0.5 \), respectively.

Let us first consider the combined shipment policy first, in this policy the returned products are collected from both ICPs in a single shipment at the optimal collection period before being transshipped to the CRC. The values for the total costs for \( T = 1 \) to 10 are shown in Table 1 and the corresponding plot in Figure 11. The combined daily rate of the volume of returned products, \( vR = 350 \), and the total cost function increases at the constant rate of \( \frac{bwR}{2} = 4375 \text{$/day} \) and dips at \( \left\lceil \frac{V_l}{vR} \right\rceil \) where \( l = 1, 2, 3, 4 \); these dips at the volume breakpoints are highlighted in Table 1. Using corollary 1, we find the values of \( R_L \) and \( R_U \), as \( R_L = 100 \) and \( R_U = 223 \). Since, \( vR = 350 > R_U = 223 \), the optimal collection period is at \( \left\lceil \frac{V_4}{vR} \right\rceil = 6 \). Similarly, we calculate the optimal collection period for the single shipment policy for two daily rates of returned volume, \( R_1 \) and \( R_2 \) with respect
to ICP 1 and ICP 2 individually. The optimal collection periods, $T_1$ and $T_2$, and the corresponding total cost for the individual shipment policy is compared with that of the combined shipment policy in Table 2.

<table>
<thead>
<tr>
<th>Time</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>96250</td>
</tr>
<tr>
<td>2</td>
<td>83125</td>
</tr>
<tr>
<td>3</td>
<td>83125</td>
</tr>
<tr>
<td>4</td>
<td>87500</td>
</tr>
<tr>
<td>5</td>
<td>78750</td>
</tr>
<tr>
<td>6</td>
<td>74375</td>
</tr>
<tr>
<td>7</td>
<td>78750</td>
</tr>
<tr>
<td>8</td>
<td>83125</td>
</tr>
<tr>
<td>9</td>
<td>87500</td>
</tr>
<tr>
<td>10</td>
<td>91875</td>
</tr>
</tbody>
</table>

Table 1: Total costs with respect to holding times for the combined shipment policy

<table>
<thead>
<tr>
<th>Time</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>83125</td>
</tr>
<tr>
<td>2</td>
<td>83125</td>
</tr>
<tr>
<td>3</td>
<td>78750</td>
</tr>
<tr>
<td>4</td>
<td>74375*</td>
</tr>
</tbody>
</table>

Table 2: Total costs with respect to holding times for the combined shipment policy
For the hybrid shipment policy, the returned products are collected from ICP 1 at time $T_1$ and are collected from both ICP 1 and ICP 2 at time $T_2$. The potential values of $T_1$ and $T_2$ are based on the daily return volumes $R_1$, $R_2$ and the given shipment volume breakpoints, as shown in Table 3 and the total cost values for some values of $T_1$ and $T_2$ are shown in the Table 4, and the potential values of $T_1$ and $T_2$ are also highlighted. It is important to note that the values of freight rate, $E$ and holding cost per unit, $b$ do not change the potential values of $T_1$ and $T_2$, but instead higher values of $E$ push towards larger values of $T_1$ and $T_2$ among the potential values, while higher values of $b$ push towards smaller values of $T_1$ and $T_2$. Based on Theorem 3, we first calculate the maximum value of $T_{max} = T$, then the corresponding potential values of $m$ based on every value of $T$ upto $T_{max}$, and finally we identify the range of values of $a$ for every potential value of $m$ and $T$. Here, $T_{max} = \left\lceil \frac{V_n}{nR_1} \right\rceil = 7$, and the optimal $(T_1, T_2) = (5,10)$. The figures 12, 13 and 14 illustrate the transportation, inventory and total costs as functions of $T_2$ for every value of $T_1$ from 1 to 10, the potential and optimal values are marked.

<table>
<thead>
<tr>
<th>$T_1$</th>
<th>$m$</th>
<th>$a$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1, 2</td>
<td>3, 4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1, 2</td>
<td>5, 6</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1, 2</td>
<td>7, 8</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1, 2, 3</td>
<td>4, 5, 6</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1, 2, 3</td>
<td>10, 11, 12</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1, 2, 3, 4</td>
<td>5, 6, 7, 8</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1, 2, 3, 4</td>
<td>13, 14, 15, 16</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1, 2, 3, 4, 5</td>
<td>6, 7, 8, 9, 10*</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1, 2, 3, 4, 5, 6</td>
<td>7, 8, 9, 10, 11, 12</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1, 2, 3, 4, 5, 7</td>
<td>8, 9, 10, 11, 12, 14</td>
</tr>
</tbody>
</table>

Table 3: Potential values of $T_1$ and $T_2$ for the hybrid shipment policy with $T_1 = 5$ and $T_2 = 10$ as optimal
<table>
<thead>
<tr>
<th>T1 \ T2</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>96875</td>
<td>97500</td>
<td>91875</td>
<td>93250</td>
<td>94375</td>
<td>95357</td>
<td>96250</td>
<td>97083</td>
<td>97875</td>
<td>98636</td>
<td>99375</td>
<td>100096</td>
<td>99911</td>
<td>100625</td>
<td>101328</td>
<td>102022</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>90000</td>
<td>84375</td>
<td>84250</td>
<td>85625</td>
<td>85714</td>
<td>85313</td>
<td>87083</td>
<td>86750</td>
<td>88409</td>
<td>88125</td>
<td>89712</td>
<td>89464</td>
<td>90125</td>
<td>90781</td>
<td>91434</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>86250</td>
<td>87250</td>
<td>86875</td>
<td>88929</td>
<td>88125</td>
<td>89375</td>
<td>91125</td>
<td>90511</td>
<td>86875</td>
<td>93173</td>
<td>92679</td>
<td>89500</td>
<td>94297</td>
<td>94743</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>87250</td>
<td>88125</td>
<td>87768</td>
<td>82500</td>
<td>90417</td>
<td>89750</td>
<td>90852</td>
<td>86875</td>
<td>93173</td>
<td>92679</td>
<td>89500</td>
<td>87188</td>
<td>94963</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td>81875</td>
<td>83571</td>
<td>83906</td>
<td>79583</td>
<td>76875</td>
<td>85000</td>
<td>85000</td>
<td>82019</td>
<td>83304</td>
<td>81250</td>
<td>87266</td>
<td>88125</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td>85714</td>
<td>85313</td>
<td>86875</td>
<td>82625</td>
<td>79886</td>
<td>81875</td>
<td>89712</td>
<td>89464</td>
<td>86500</td>
<td>84375</td>
<td>85441</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>83125</td>
<td>83125</td>
<td>84875</td>
<td>81136</td>
<td>78750</td>
<td>80769</td>
<td>83125</td>
<td>86625</td>
<td>87500</td>
<td>84926</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>87083</td>
<td>86750</td>
<td>88125</td>
<td>84375</td>
<td>81923</td>
<td>83661</td>
<td>85750</td>
<td>88125</td>
<td>91434</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>91125</td>
<td>90511</td>
<td>86875</td>
<td>87788</td>
<td>85268</td>
<td>86750</td>
<td>88594</td>
<td>90735</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>95227</td>
<td>94375</td>
<td>90673</td>
<td>91339</td>
<td>88750</td>
<td>90000</td>
</tr>
</tbody>
</table>

Table 4: Total costs for various values of $T_1$ and $T_2$ for the hybrid shipment policy
Figure 12: Transportation cost function with respect to $T_2$ for $T_1 = 1$ to $10$. 
Figure 13: Inventory cost function with respect to $T_2$ for $T_1 = 1$ to 10.
Figure 14: Total cost function with respect to $T_2$ for $T_1 = 1$ to 10.
It should be noted here that using the optimization approach shown by Theorem 3 above, we calculate the costs in this case for 39 combinations of $T_1$ and $T_2$. If we were to just use Proposition 1 and Proposition 3, this would give us 140 combinations of $T_1$ and $T_2$. Thus, we can say that, the optimization approach developed in this thesis reduces the number of computations by 72%. And this percentage reduction in the number of computations increases as the ratio $\frac{R_1}{R_2}$ increases.

On comparing the total cost values at the optimal collection periods for all the three policies, as illustrated by Table 5, it can be seen, that in this scenario the combined shipment policy turns out to be optimal. Intuitively, this makes sense, since for the combined shipment policy we can have $T_1 = T_2 = T$ and thus, faster rates of returned products which means we can leverage economies of scale sooner than the other policies which results in more savings. However, in a practical scenario it may not always be ideal to make combined shipments. Below, we discuss some practical scenarios under which the combined shipment policy is not the optimal policy when compared to the hybrid shipment policy.

<table>
<thead>
<tr>
<th>Policy</th>
<th>Inventory</th>
<th>Transportation</th>
<th>Total</th>
<th>Optimal Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual shipments from each ICP</td>
<td>31250</td>
<td>50000</td>
<td>81250</td>
<td>$T_1 = 7, T_2 = 1$</td>
</tr>
<tr>
<td>Combined shipment</td>
<td>30625</td>
<td>43750</td>
<td>74375</td>
<td>$T = 6$</td>
</tr>
<tr>
<td>Hybrid shipments</td>
<td>29375</td>
<td>47500</td>
<td>76875</td>
<td>$T_1 = 5, T_2 = 10$</td>
</tr>
</tbody>
</table>

Table 5: Comparison of the costs for all the three policies

Example 1: Let us assume that there is some fixed loading, unloading and administrative costs every time the products are collected at an ICP. Let us say that this fixed cost is $350 per collection process. Then for the combined shipment policy, the shipment cycle is 6 days during which the products are collected twice at the ICPs. For a year with number of working days, say $w = 240$ for simplicity, the number of times the products are collected is $\frac{2}{6} \times 240 = 80$ times. Therefore, the fixed cost is $28000 per year. Similarly, for the hybrid shipment policy, the cycle is repeated every 10 days, during which the products are collected thrice at the ICPs. The total fixed cost of collection for the hybrid shipment policy is $25200 per year. When this cost is added to the sum of inventory and
transportation costs, it can be observed that the hybrid shipment policy is optimal than the combined shipment policy.

Example 2: For the combined shipment policy, the shipment volume at every shipment is $2100 \, ft^3$. And the shipment volumes for the two shipments of the hybrid shipment policy are, $1500 \, ft^3$ and $2000 \, ft^3$. Also, the largest shipment volume breakpoint for maximum discounts is $2000 \, ft^3$, therefore, it can be assumed that $2000 \, ft^3$ is the case of a full truckload and thus, the capacity for every shipment is $2000 \, ft^3$. In such a scenario, the combined shipments cannot be made at $T = 6$ and the next best case is to make the shipments at $T = 5$, for which the total costs are $78750$ (Table 1). This is more than the cost of $76875$ for the hybrid shipment policy, and thus, it is optimal to have hybrid shipments instead of combining them every time.

The above examples for which the hybrid shipment policy turns out to be optimal are defined based on some assumptions. However, let us now consider the real scenario under which we have more than two ICPs, say ten. Then, collecting the returned products from all the ten ICPs at every collection period does not seem practical. It may not be even possible to make combined shipments under some constraints like the ones mentioned in the above examples. Therefore, it becomes necessary to split these ICPs into some sets and then make the hybrid shipments for these different sets of ICPs. We can therefore say that even though the combined shipment policy seems optimal in the case under consideration, but under practical scenarios where there are additional costs and constraints, making hybrid shipments becomes necessary and determining the optimal collection periods for these shipments is required to minimize the total costs.
CHAPTER 5: CONCLUDING REMARKS AND FUTURE SCOPE

The return delivery costs are expected to reach 550 billion U.S. dollars by the year 2020. Despite this enormous cost saving opportunity, a lot of companies still do not consider reverse logistics as their core ‘value-adding’ activity. There has been a lot of progress over the years and companies are now starting to view returns as not only a cost saving opportunity but also as a tool to gain strategic competitive advantage by offering liberal return policies and thus, attracting and retaining more customers. This however, calls for even more attention to the reverse logistics problem.

This thesis makes both practical and theoretical contributions to the field of reverse supply chain. And, is one of the first works to deal with the reverse logistics network problem with more than one collection points. We not only demonstrate the complexity of adding more collection points in the reverse logistics network, but also, propose an approach to consider the various possible collection policies and thus, determine the optimal collection policy. To demonstrate the robustness of the proposed policies, we not only provide mathematical proofs, but also perform sensitivity analyses with varying return rates, product volumes and collection periods. In addition, we also compare the different policies and provide practical scenarios that may affect the decisions of determining the optimal policy. This work can aid supply chain professionals in dealing with the problem of collecting returned products from multiple collection points and making the decisions to determine the optimal ways in which the products can be moved from the initial collection centers to a centralized warehouse.

Despite the proven benefits of the proposed policies, further research work needs to be done. For example,

1) The proposed policies involving two collection points can be extended to consider the cases of more than two collection points, and also for the case of having multiple random product returns and freight rate fluctuations

2) For the cases of more than two collection points, a clustering approach may be developed to group these collection points into two groups so that the proposed policies can be applied

3) The proposed policies can be tested for real-world problems with actual data
REFERENCES


