Automated Target Detection for Geophysical Applications

A Dissertation Presented

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To my wife Yifat, whose constant support made this work possible.
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List of Acronyms

**BP** Band Pass In reference to filters, a Band Pass filter is a filter whose pass band contains a limited range of frequencies.

**DCT** Discrete Cosine Transform A Fourier-related transform similar to the discrete Fourier transform (DFT), but using only real numbers. The transformed signal is being expressed by sum of cosine functions.

**GPR** Ground Penetrating Radar. An imaging device that works by emitting UWB signals and recording the reflected echoes. Usually the transmitting signal is a non-modulated short pulse.

**GSSI** Geophysical Survey Systems, Inc. A GPR manufacturer. Situated in New Hampshire, USA.

**IIR** Infinite Impulse Response A type of linear filter, whose response to an impulse input does not nullify after a limited number of steps.

**QoF** Quality of Fit A performance index used to choose the best physical model which describes the data. This is typically $l_p$ norm for some $p$.

**RF** Radio Frequency Electro-Magnetic wave frequencies that lie in the range extending from around 3 kHz to 300 GHz.

**UWB** Ultra Wide Band. The FCC \[1\] defines UWB devise as “any device where the fractional bandwidth is greater than 0.25 or occupies 1.5 GHz or more of spectrum. The formula proposed by the Commission for calculating fractional bandwidth is $\frac{f_H - f_L}{f_H + f_L}$ where $f_H$ is the upper frequency of the 10 dB emission point and $f_L$ is the lower frequency of the 10 dB emission point.”

**SNR** Signal to Noise Ratio. The ratio of the power of the signal and the noise variance at a given sample. This is a performance metric, used in classic detection scheme to determine the performance of a detection system.

**SUT** Scan Under Test. The scan for which a decision on the existence of target need to be made.

**SVM** Support Vector Machine. Supervised learning method. Invented by Vapnik \[2\] in 1995, is an efficient binary classifier. It find the optimal hyperplane that separates the two classes.
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Abstract of the Dissertation

Automated Target Detection for Geophysical Applications

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In many geophysical surveys there is a pre-defined goal - to detect and locate very specific anomalies, ones which correspond to buried objects (targets). The types of targets range from various types of pipes (metallic or not), to re-bars or wires in walls to land mines. This work presents a novel unsupervised method for automatically detecting targets, and extracting information about them and the medium in which they reside. It does so by efficiently analyzing strips of the B-Scan, and detecting the geometrical signature of a target in the image.

Most existing detection methods are supervised, which means that one has to provide a training set (which can be labor expensive) in order to train a classifier. By contrast, the method presented here is unsupervised and is model based, which alleviates the need to manually annotate a training set. Another drawback of many existing methods is the underlying assumption of a homogeneous medium. This assumption is greatly relaxed for this method, since it assumes no prior knowledge of the medium. Instead, it learns the medium’s properties from the targets themselves. Furthermore, this method is designed to be computationally efficient, applicable in real time applications. The current work presents two version of this algorithm. The first version was designed to detect locally isolated targets (i.e. - without having cross targets interferences in the B-Scan). The second version generalizes the first, and is able to locate targets in complex scenarios, at the cost of increasing computational complexity. Both versions were implemented on a commercial Ground Penetrating Radar (GPR) system (GSSI’s StructureScan™ Mini XT system) and were tested using multiple systems on real life scenarios. The algorithm was designed to have an extremely high probability of detection - above 95% for the first version, and 98% for the second. The processing time, however, was increased from 20 to 600 µSec per scan. The experimental results show that both methods are able to detect the targets with high positioning accuracy and sufficiently low false detection rate.
Chapter 1

Introduction

1.1 Motivation

Despite its name, a GPR system is not a RADAR system - it is missing a fundamental component - target detection. Without detection, GPR is a sub-surface imaging system. To this date, GPR manufacturers rely on skilled operator to analyze the resulting B-scan (also referred in this work as the GPR image or the radargram), and to manually detect the anomalies that might be in it. There are two main reasons for this missing capability: a lack of willingness to shoulder the liability for detection (or mis-detection) among the manufacturers, and the fundamental problem of defining what a target is.

The liability issue is an obvious one. GPR equipment is being used to locate many different things: from cracks in ice sheets and dams to sewage or utility pipes to metallic re-bars. Failing to detect one of the latter targets could cost a small fortune to a construction company, for example, and might place people in real danger if one of the former targets is missed. For this reason, GPR manufacturers sold their equipment to service providers - who then sold their expertise to those who needed the information. However, it is the author’s personal belief that in this day and age, given that a properly worded legal disclaimer is presented and signed by the user, this issue could be mitigated, if not overcome completely. For this reason, this work will ignore the the legal issues that might be involved. It does mean that when coming to design an algorithm which will be able to automatically detect targets, it has to have a very high probability of detection, probably at the price of having a high false detection rate.

GPR is a versatile tool, used to detect many things. It works by emitting a Radio Frequency (RF) signal, and recording the reflected echoes. These echoes will be refracted by discontinuities
in the permittivity of the medium through which the signal is traveling. Discontinuities could be caused by many factors, but the discontinuities of interest to the vast majority of GPR users are those which originate from having a material which is different than its surrounding. Generally, these could be classified into two families: discontinuities (anomalies) which are small relative to the scanned distance, and discontinuities that are not. The former anomalies will be referred to as targets, and this work will focus on attempting to detect and locate them in real time. The latter are layers in the medium. Examples for targets include mines in the ground, re–bars and PVC pipes in concrete or sewage/utility pipes/tunnels in the ground. In these cases, typically the users are would primarily like to detect and locate the targets. Additional information of interest might include the size of the targets and the material from which they are made of (determining whether an object is metallic or air filled, for example). Examples for layers of interest would be asphalt layer on top of a packed gravel layer in a paved road, and ice sheets on top of a river or a pond. In these examples, the required data is typically the layers’ thickness and their permittivity.

This work focuses on detecting targets. This family of anomalies is wide enough to be of interest to many of the current GPR users, and indeed, if a good enough detection method for these targets exists - might alleviate the need for skilled GPR operator to analyze the image in order to find these targets. Automatic target detection will result in making the data more accessible and easier for interpretation, and this in turn might lead to more people who will use the system.

### 1.2 Related Work

GPR operates by emitting (“firing”) an Ultra Wide Band (UWB) signal, recording the echoes and then being moved to a different location and starting again. The echoes from a single signal form an A-scan. Collection of A-scans form the B-scan, that is the resultant GPR image. Numerous works were conducted on understanding the signal as it travels in the ground. Richard Yelf [3] and Stanley Radzevicius [4], for example, discuss the subject of finding the true time zero in the A-scans. This time zero is the precise moment in which the energy was emitted by the system, and hence it is the reference to the measurements of the time in which the signal was received after its round trip to the target and back. Since the system can only sample the received signal, finding the precise timing of transmission is non–trivial. It is crucial, however, for depth and permittivity calculations. This work follows Yelf’s approach. In his method, the system is placed at a known height above a metal plate reflector. Since the height is known, and the propagation speed in the medium (which is air, making it the speed of light) is known, the expected (two way) flight time
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of the signal could be calculated. By comparing the expected time to the measured, one can back calculate time zero.

GPR is a versatile tool, and its images are used to detect a wide range of sub-surface anomalies. Many studies were performed on detecting different type of anomalies: mine detection (5–8), sinkholes [9], subterranean voids and tunnel detection [10–11], buried utilities [12], cracks in ice sheets [13] as well as for bridges and roads assessment [14]. The vast majority of detection methods involve some pre-processing stage (for example: Qin et Al. [10] used Discrete Cosine Transform (DCT) on the image) followed by a classifier (usually Support Vector Machine (SVM) [15] or neural network [16]). These methods, however, are computationally intensive, rendering them unsuitable for real time decision making. In addition they need a training set to train the classifier - which means that the datagram needs to be manually annotated to let the classifier know where the targets in the training set are. This work presents a model based detection scheme which mitigates the above shortcomings - being computationally efficient and minimizing the need for expert assistance in the initial (training) stage. Another contribution is its ability to learn the model parameters - target depth and dielectric constant - from the targets.

A point target creates a distinct shape (a hyperbola) in the datagram. This fact is being used extensively in geophysical data processing, such as the various types of migration and move out analysis [17], [18], as well as recent works which combined compressive sensing with GPR [19], [20]. Radzecius [4], Al-Naumi et al. [12] and Kang et al. [21] have presented simple bi-static model which produces this shape. Khan et al. [7] uses the more accurate model than the bi-static one. Their model incorporated some antenna distance off the ground - but ignored the fact that introducing a layer of air above the medium will bend the energy rays as they pass from one layer to another. Gurbuz [22], [20] uses the same model as Khan et al., but takes into account this bending. Martens et al. [23] presented an interesting variation of the basic model, incorporating knowledge of the targets’ diameter. In practice, however, it turned out that their model does not describe the data well - probably due to the low ratio of the target size to the system’ wavelength. This shall be explored and elaborated in Section 2.2.

Notable previous works are the works of Martens et al. [23], Wang et al. [24] and Pasolli et al. [25] and Lee et al. [18]. These works do not relay on expert knowledge of the current medium’ properties (i.e. dielectric constant) but rather attempt to learn it from the data itself. However, their pre-processing stage is computationally intensive (template matching for Wang et al., histogram calculation of the entire image, in order to get the optimal thresholding value in Pasolli et al.). The method proposed by Martens et al. could possibly be adapted to a real time algorithm, though their
current runtime was measured in minutes for a full B-Scan, as compared to few milliseconds for that the algorithm presented in this work would require. Three other notable differences between those works and the one presented here are the detection accuracy (the ability to pin-point the location of the target), detection performance (as measured both in the probability of detection and false detection rate in real field data) and the number of data sets used to calculate the detection performance. For example, Pasolli et al. did not use a single field data file, but rather relied on a simulator to show their performance. Even so, they reported a probability of detection of 62%, with no mention of the number of false detections. Wang et al. showed a single file (albeit a long one) but do not mention the performance of their algorithm at all. Martens et al., in contrast, did an extensive study of the performance of their algorithm, and report a probability of detection which ranges from 72% to 92% with some false detection. However, many of the detections were inaccurate (wrong sample was chosen for target location) which reduces the overall performance of the algorithm. The algorithm proposed in this work showed a probability of detection greater than 95.3%. In 50 different data sets, collected by GSSI’s StructureScan™ Mini XT system, only 8 occurrences of false detections were observed, as shall be described extensively in Section 3.3. Since those numbers were insufficient for a commercial system, a second version of the algorithm was developed. Using 361 different data sets, the probability of detection was shown to be 98.65%. This came at the price of increasing the run time of the algorithm (from 20 to 600 µSec) and increasing the false detection rate. This is described in Section 4.4.
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1.3 Structure of This Work

The current work relies on the shape (hyperbola) created by point targets for detection. It analyzes the GPR image to locate this shape and compare it to the expected, theoretical shape. The first part of the work will focus on finding the best physical model that describes the true shape of the target. Review of the different propagation models is covered in Chapter 2 as well as comparison to the actual data collected by a GPR system. A detailed explanation of how this comparison was carried out is found on Appendix A. This comparison will allow us to select the optimal physics based model to be used later in the detection algorithm. Appendix B completes the analysis by presenting a sensitivity analysis of that model.

The model is used in Chapter 3 to create a simple and effective target detection algorithm, which is suited for real time applications and works well in most of the cases. Its shortcoming, however, are the fact that it assumes a simple enough scenario - a single target in a locally homogeneous medium. Chapter 4 will remove those assumptions and present a method to generalize the algorithm to more complex (and more realistic) scenarios - where multiple layers and targets interfere with one another. It does so by modifying the pre-processing stage and the decision making stage - while retaining the core idea of the first algorithm - that of comparing the collected data to an expected, theoretical curve.
Chapter 2

Propagation Models of GPR Signals

The propagation model which will be used throughout the work shall be developed in this chapter. It shall be used to predict the geometric shape that a target is expected to produce on the datagram (B-Scan). The method for efficiently detect this shape will be described in the next chapter. This chapter is organized as follows: Section 2.1 begins by detailing this model and clearly stating the main assumptions underlying it. Since obtaining analytical solution to the model might be challenging, three approximation to the model are derived. By comparing the predicted time-of-flight that each model yields with data collected by a 2.6 GHz antenna, the optimal propagation model to be used in this work is chosen in Section 2.2.

2.1 Review of the Target Models

A model-based approach was used in this work to target detection. This section provides details of the theoretical model used. The terminology and definitions, used by the various propagation models are presented in Section 2.1.1. The assumptions which the model is based upon are clearly and explicitly stated in Section 2.1.2. The system of equations which mathematically describes the full propagation model are presented in Section 2.1.3. This system is a set of non-linear equations. Section 2.1.4 and Section 2.1.5 provide two alternative approximations to the full propagation model – a mono-static and bi-static solution, respectively. Another approximation is provided in Section 2.1.7 – a polynomial approximation to the bi-static solution. By equating the coefficients of this polynomial with those obtained with a regression analysis, the detection method will be able to efficiently determine the parameters of the model - the (unknown) target depth and the permittivity of the bulk medium in the neighborhood above the target. For completeness, two of the approximations,
CHAPTER 2. PROPAGATION MODELS OF GPR SIGNALS

explored in Sections 2.1.4 and 2.1.5 are combined to yield to simplest propagation model, described in Section 2.1.8. Indeed, it is this simplified model that one finds in most of the geophysical literature.

2.1.1 Definitions and Problem Description

The creation of the curve (the synthetic target) will be described in this section as well as the variables and definitions used by the theoretical models which will be presented in the next sections. By comparing the curves, created by the theoretical target model with data collected from a GPR device, the best model could be chosen.

This work uses the following terminology and definitions:

- **GPR/Imaging System** - An imaging system is transmitting signals every \( dx \) meters. This imaging system has a spatial separation of \( 2\Delta[m] \) between the transmitter and the receiver, and an elevation (sometime referred as “air gap”) of \( g \) meters from the surface of the medium. Possible sources for this air gap include the antenna housing (radom), and the sledge or the cart in which the antenna lies. The magnitudes of the reflecting echoes are being recorded at each location, and are used to generate the resulting images A and B-scans of the system. Examples for these type of images are provided in Figure 3.1 (A-scan) and Figures 3.5 – 3.8 (B-scans).

- **Medium** - The half space in which the targets resides. This could be ice, drywall, concrete, soil or any other material that might contains the target of interest. This work defines the vertical (depth) axis as starting from the surface of the medium, with the positive direction going into the medium.

- **Target** - In this work, targets are considered to be dimensionless (point targets). The origin of the horizontal (distance) axis is defined to be the location of the target. The target is located \( h \) [m] below the surface, making its coordinates \((x, z) = (0, h) \) meters in the medium.
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2.1.2 Model Assumptions

The assumptions listed below holds true regardless of the propagation model used.

1. The model is noise free.

2. The model assumes far field propagation.

3. The medium is locally homogeneous - it could be characterized as having a signal propagation speed of $v_p(x)$, though the dependence on the system location $x$ will be omitted in this work.

4. The medium has no special magnetic properties - the relative permeability $K_r$ of the medium is known, and equal to 1, throughout the (2 way) path.

5. The medium’s surface is flat as compared to the system wavelengths, in the small neighborhood above the target.

6. The antennas are a point source and a point receiver.

7. The (2 way) propagation time $t$ is known precisely.

It should be emphasized that this model deals with a single target. If multiple targets exists in the medium, it is assumed that they are sufficiently separated spatially, so that only a single target could be considered at a given moment.

The assumptions of locally homogeneous medium, combined with the assumption of non magnetic medium, gives us the ability to use the dielectric constant $\epsilon_r$ of the medium is locally constant throughout the (2 way) path:

$$\epsilon_r = \left( \frac{c}{v_p} \right)^2$$  \hspace{1cm} (2.1)

Where $c$ is the speed of light in vacuum.

2.1.3 The system of equations for the full propagation model

Combining the definitions and assumptions above, one gets the following ray tracing scheme for the Full Propagation Model, which will be described next. The signal is traveling from the transmitter in the air mainly via the ray path $r_1$, with an angle of $\Theta_1$ between the wavefront and the surface, and gets to the target via the ray path $r_2$ at an angle of $\Theta_2$. When the signal interacts with the target, it is reflected off (and is refracted by) the target in all directions. The main energy component that will eventually arrive at the receiver travels through the medium via path $r_3$, with an
angle of $\Theta_3$ between the wavefront and the medium’s surface and arrives at the receiver mainly via path $r_4$. (See Figure 2.1 below).

Figure 2.1: Ray Tracing schematics. A Bi-Static system is located at (x,g), and the target at (0,-h).

The equations controlling the ray paths could be logically divided into three groups:

1. Equations which originate from the geometry of the problem:

   From Figure 2.1 one can write the following set of equations:

   $$r_1 \cos \Theta_1 = g$$  \hspace{1cm} (2.2)

   $$r_4 \cos \Theta_4 = g$$  \hspace{1cm} (2.3)

   $$r_2 \cos \Theta_2 = h$$  \hspace{1cm} (2.4)

   $$r_3 \cos \Theta_3 = h$$  \hspace{1cm} (2.5)

   $$r_1 \sin \Theta_1 + r_2 \sin \Theta_2 = x - \Delta$$  \hspace{1cm} (2.6)

   $$r_3 \sin \Theta_3 + r_4 \sin \Theta_4 = x + \Delta$$  \hspace{1cm} (2.7)
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2. Equations which originate from the boundary crossing:

Since far field propagation is assumed, one may use Snell’s law to obtain the following connections between the angles:

\[
\frac{\sin \Theta_1}{\sin \Theta_2} = \frac{c}{v_p} \Delta = n
\]  
(2.8)

\[
\frac{\sin \Theta_4}{\sin \Theta_3} = \frac{c}{v_p} \Delta = n
\]  
(2.9)

Where \( n \) is the refraction coefficient of the medium. Using Equation (2.1) one get that \( n = \sqrt{\varepsilon_r} \).

3. Time Measurement: The transmitted signal hits the target and is reflected back. The (two way) travel time of the signal, along the path of \( r_1 \rightarrow r_2 \rightarrow r_3 \rightarrow r_4 \) is given by:

\[
\frac{r_1 + r_4}{c} + \frac{r_2 + r_3}{v_p} = t(x, h, \Delta, g, v_p)
\]  
(2.10)

In this work \( t \)’s dependence on system location \( x \), Bi static offset \( \Delta \), air gap \( g \), target depth \( h \) and signal velocity in the medium \( v_p \) is dropped for brevity.

Equations 2.2 – 2.10 form a (non linear) system of 9 equations and 10 variables. Several simplifying approximations to this system will be presented next. All of these approximations are being used in practice, in different geophysical works. A numerical method for solving them is presented in Appendix A. This method searches for the crossing point of the signal (from the air to the ground or vice versa). The location of this crossing point is described by a fourth-order polynomial, which is solved numerically. Once this point is found, the ray paths and the total time of flight are calculated. A different method for calculating the time of flight for this model was presented by Rappaport [26].
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2.1.4 Approximation 1: Elevated, Mono-Static Model

The elevated, Mono-Static model assumes that \( \Delta \ll 1 \). This assumption is useful for air-launched signals, where the system is high above the ground relative to the antenna separation. Under this assumption, the transmitter and the receiver are located at the same point, which automatically means that \( r_1 = r_4, r_2 = r_3, \Theta_1 = \Theta_4, \Theta_2 = \Theta_3 \). The set of Equations 2.2–2.10 reduces to:

\[
\begin{align*}
    r_1 \sqrt{1 - \sin^2 \Theta_1} &= g \quad (2.11) \\
    r_2 \sqrt{1 - \sin^2 \Theta_2} &= h \quad (2.12) \\
    \sin \Theta_1 &= \frac{\sin \Theta_2}{n} \quad (2.13) \\
    r_1 \sin \Theta_1 + r_2 \sin \Theta_2 &= x \quad (2.14) \\
    r_1 + nr_2 &= \frac{ct}{2} \quad (2.15)
\end{align*}
\]

This, in turn leads to

\[
\begin{align*}
    \frac{ng \sin \Theta_2}{\sqrt{1 - n^2 \sin^2 \Theta_2}} + \frac{h \sin \Theta_2}{\sqrt{1 - \sin^2 \Theta_2}} &= x \quad (2.16) \\
    \frac{g}{\sqrt{1 - n^2 \sin^2 \Theta_2}} + \frac{nh}{\sqrt{1 - \sin^2 \Theta_2}} &= \frac{ct}{2} \quad (2.17)
\end{align*}
\]

Like in the Full Model case, theoretical curve generation requires numerical methods - either the one described in Appendix A (which was used in this work) or by the method described in [26].

2.1.5 Approximation 2: Bi-Static Model

In the Bi Static Model, the air gap \( g \) is neglected \( g \ll 1 \). This approximation is useful if the air gap is small compared to the wavelengths used by the system. Under this assumption, the transmitter and the receiver are located on the ground, which means that \( r_1 = r_4 = 0 \). The set of Equations 2.2–2.10 will be:

\[
\begin{align*}
    r_2 &= \sqrt{h^2 + (x - \Delta)^2} \quad (2.18) \\
    r_3 &= \sqrt{h^2 + (x + \Delta)^2} \quad (2.19) \\
    \Rightarrow n\sqrt{h^2 + (x - \Delta)^2} + n\sqrt{h^2 + (x + \Delta)^2} &= ct \quad (2.20)
\end{align*}
\]
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2.1.6 Approximation 3: Modified Bi-Static Model

In their paper, Laurence Martens [23] and her colleagues presented an interesting modification of the Bi-Static Model: The target was assumed to be an infinitely long cylinder with a radius $R$, rather than a point target. The target depth $h$ is now measured to the topmost point of the cylinder (see Figure 2.2). Their two assumptions, which add to the assumptions already listed in Section 2.1.2 are:

- The energy rays are traveling directly towards the center of the cylinder.
- The time of propagation along the surface of the cylinder (via the form of creeping waves, for example) is negligible compared to the overall flight time of the signal.

This chapter will deviate from the point target assumption whenever dealing with this model. As will be shown, their model does not yield the best description of actual data collected by the StructureScan MiniXT system; for that reason, the rest of this work will continue using the assumption of a point target. Following their path, one starts with the propagation model resulted from the assumptions listed above:

Figure 2.2: Ray Tracing schematics of the Modified BiStatic Model. A Bi-Static system is located at $(x,0)$, and the target at $(0,-(h+R))$. 

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From Figure 2.2 one get the following set of equations:

\[ r_1 = \sqrt{(h + R)^2 + (x + \Delta)^2} - R \]  
(2.21)

\[ r_2 = \sqrt{(h + R)^2 + (x - \Delta)^2} - R \]  
(2.22)

\[ t = \frac{n}{c} (r_1 + r_2) \]  
(2.23)

This model is very interesting, because it allows the determination of the re–bar size from the time of flight curve. Though not fully explored in the current work, the topic of re–bar size assessment is of great importance for structural integrity and strength assessment of bridges, dams and buildings. Re–bars tend to shrink in size due to corrosion, so having a non destructive method to assess their size is very useful. Most existing methods \[ \text{[27–34]} \] uses the amplitude of the signal in order to determine the size of the re–bar. This has the limitation of amplitude variation from one antenna to another, due to variabilities in the components of each antenna. Other methods (Utsi & Utsi \[ \text{[27]} \]) use ratio from cross polarize signal. This requires to double the number of channels. If the proposed model is correct (i.e. low norm of the error when subtracting the theoretical curve from the measured targets), then it means that the re–bar size could be inferred from the data directly (possibly at the cost of having to assume the correct dielectric of the medium). This has several advantages over the existing methods - for the time of flight of the GPR signal is significantly more stable than the reflected amplitude, which allows for a more accurate estimation of the size of the re–bar. In addition, no cross polarization information is needed in order to generate it, so compared to the method which uses cross polarization, the number of transmitter-receiver pairs is halved.

Unfortunately, as will be shown in Section 2.2, this model does not describe the actual data collected by the GPR system (GSSI StructureScan MiniXT system). This is probably due to the fact that the ratio between the re–bar diameter (which, in the targets measured was 1.3 cm) to the wavelength of the signal in the medium (which in a medium which has a dielectric constant of \(\epsilon_r \approx 6\) is about 5 cm) was too low.
2.1.7 A Polynomial Approximation to the Bi-Static Model

Since \( t(x) \) is a symmetric function with respect to the location of the system relative to the target \( t(x) = t(-x) \), when calculating the McLauren polynomial which best describes it, only even derivatives of it will be non-zero. Hence, obtain:

\[
t(x) = t(0) + 0.5t''(0)x^2 + O(x^4)
\]  

(2.24)

The derivatives themselves are given by

\[
c(t) = n\sqrt{h^2 + (x-\Delta)^2} + n\sqrt{h^2 + (x+\Delta)^2} 
\]  

⇒ \( t(0) = \frac{2n}{c} \sqrt{h^2 + \Delta^2} \)  

(2.25)

\[
c(t') = \frac{n(x-\Delta)}{\sqrt{h^2 + (x-\Delta)^2}} + \frac{n(x+\Delta)}{\sqrt{h^2 + (x+\Delta)^2}} 
\]  

⇒ \( t'(0) = 0 \)  

(2.26)

\[
c(t'') = \frac{nh^2}{(h^2 + (x-\Delta)^2)^{3/2}} + \frac{nh^2}{(h^2 + (x+\Delta)^2)^{3/2}} 
\]  

⇒ \( t''(0) = \frac{2n}{c} \frac{h^2}{(h^2 + \Delta^2)^{3/2}} \)  

(2.27)

\[
c(t^(3)) = \frac{-3nh^2(x-\Delta)}{(h^2 + (x-\Delta)^2)^{5/2}} - \frac{3nh^2(x+\Delta)}{(h^2 + (x+\Delta)^2)^{5/2}} 
\]  

⇒ \( t^(3) = 0 \)  

(2.28)

Equations 2.28, 2.26 prove Equation 2.24 and shows that a second order polynomial is actually the third order approximation to the time of flight function, centered above the target. From Equations 2.25 and 2.27 one gets that this parabolic function has to have positive offset \( t(0) > 0 \) and positive curvature \( t''(0) > 0 \). These facts will be exploited in the next chapters of this work.
CHAPTER 2. PROPAGATION MODELS OF GPR SIGNALS

2.1.8 The Naïve Model - Mono-Static, Ground Coupled System

Combine the two approximations to the Full Model - that of no elevation \((g << 1)\) and that of no bi-static offset \((\Delta << 1)\), one gets the wave propagation model which is commonly used in the literature [17]. Under these assumptions, one has that \(r_1 = r_4 = 0\) and \(r_2 = r_3\), and the of Equations 2.2–2.10 becomes:

\[
t = \frac{2n\sqrt{h^2 + x^2}}{c}
\]

(2.29)

This approximation is very useful in practice, and is widely used. If the antenna is above the target \((t(0))\), Equation 2.29 provides a linear relationship between the target depth and the time-of-flight of the signal, and so the vertical scale of the radargram becomes linear. It also gives an easy method of calibrating the scale: if one knows the true depth of a target, and measures the time of flight at the apex (i.e. \(t(0)\)), the relative permittivity of the medium \(\varepsilon_r = n^2\) is easily obtained from Equation Equation 2.29.

2.2 Selecting the Appropriate Physics-Based Model

In order to choose the best physics-based model which describes the data collected by a GPR system, several measurements were taken. 25 different datasets were obtained using a GPR system - Geophysical Survey Systems, Inc. (GSSI) antenna, with a center frequency of 2.6 GHz. The datasets were collected on various blocks of different material (in order to have a range of dielectric constants). The material used were: Styrofoam block with a dielectric of \(\varepsilon_r = 1.07\) (to test the propagation model in extreme value), as well as several concrete slabs with dielectric values of \(\varepsilon_r = 5.9, 6.7, 7.3\) and 9.5, respectively. Re-bars at various (known) depths were placed inside these blocks, at depths which ranged from 1.5 to 10 cm. The re–bar diameter was 1.27 cm. Each block was scanned using several different GPR systems, and the targets were extracted from the resulting B-Scan by tracking the maximal peak of the scan adjacent to the target location (total of 63 points per target). A constant of \(t_{\text{offset}} = 0.035\) nanoseconds was added to the measured flight time of the signal to accommodate for the time it takes for the signal to travel from the transmitter to the receiver.

The depth (and size) of the targets as well as the mediums’ permittivity were known precisely enabling the generation of the theoretical curves, as each of the models, described in the previous section, predicted. By comparing the data collected by the system to the different curves,
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the optimal model will be selected. When comparing the data to a theoretical curve, two performance metrics might be of interest:

- The shape of the curve - the $l_2$ norm of the difference between the data points and the theoretical curve. This metric will predict how well a given model describes the overall shape of a target.

- The difference at the apex (minimal point) - this difference (absolute or in percentage) will describe how well could a model determine the correct depth of a target (given the dielectric).

For each target and for each of the theoretical models explored in the previous sections, the two metrics described above were calculated. In order to choose the best model that describes the data, the results were averaged over the range of target depths and permittivity of the mediums. The averaged results are presented in Table 2.1.

Table 2.1: Averaged values for the two performance indices.

<table>
<thead>
<tr>
<th>Theoretical model</th>
<th>$l_2$ norm of the difference</th>
<th>Absolute difference of the minimal point [percent]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full model</td>
<td>0.4044</td>
<td>3.3091</td>
</tr>
<tr>
<td>Elevated, Mono-static model</td>
<td>0.8623</td>
<td>21.2522</td>
</tr>
<tr>
<td>Bi-static model</td>
<td>0.328</td>
<td>5.577</td>
</tr>
<tr>
<td>Modified Bi-static model</td>
<td>0.3096</td>
<td>5.194</td>
</tr>
<tr>
<td>Polynomial approximation to the Bi-static model</td>
<td>0.2923</td>
<td>5.577</td>
</tr>
<tr>
<td>Naïve model</td>
<td>1.0111</td>
<td>33.8417</td>
</tr>
</tbody>
</table>

The system which was used for data collection (GSSI’s Mini XT system) has wheels which elevates it to 8 mm above the blocks. Its instantaneous bandwidth spans from 1.2 GHz to 3.8 GHz, which means that (in the air) the shortest wavelength is approximately 7.8 cm - almost ten times bigger than the air gap. This can explain the fact that the polynomial approximation to the Bi-Static model outperformed the other models - with an average $l_2$ norm of 0.2923. It should be noted that if one is interested in the other performance metric - which model gives the best prediction for the target depth, given the dielectric - one gets that the Full model outperforms the other models, as the reader might expect. It is interesting to note that though the Modified Bi Static model uses more information (the diameter of the target) than the other models, it does not out perform any other
models - not in predicting the target depth (via the location of the apex of the resulting shape) nor by predicting the target’s shape (hyperbola).

Four examples of targets (real measurements) Vs. the theoretical curves that the various models predicted are presented in Figures 2.7 - 2.6.

Figure 2.3: Example for model comparison - a target was placed in Styrofoam block at depth of 14.4 cm.
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Figure 2.4: Example for model comparison - a target was placed in a concrete block with a permittivity of 5.9.

Figure 2.5: Example for model comparison - a target was placed in a concrete block with a permittivity of 6.7.
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Figure 2.6: Example for model comparison - a target was placed in a concrete block with a permittivity of 7.3.

Figure 2.7: Example for model comparison - a target was placed in Styrofoam block at depth of 5 cm.
Chapter 3

Locally Isolated Target Case

The various propagation models used by different works were discussed in Chapter 2, as well as the decision to use the Polynomial Approximation to the Bi-Static Model as the best model (best in the sense that it describes the measurements with the least square error). A target detection algorithm which utilizes this model will be described in this chapter. In order to use the model, the algorithm will first obtain the data points of what is (potentially) the target. By comparing them to the model (using regression analysis), the model parameters (i.e. the target depth $h$ and the refraction coefficient $n$) are being determined. By verifying that these parameters are within “normal” range, a decision could be made regarding the existence of a target. This approach has several advantages over existing target detection methods:

- It uses only local information. By nature, a target creates a local interference in the datagram. While in some cases using information from different parts of the picture might assist the decision (for example, to obtain statistics about the noise and DC levels in the B-Scan), one cannot know in advance whether these parts of the image are indeed free from targets, layers boundaries or any other inhomogeneity that might result in a strong signal, and will therefore distort the noise estimation. In addition, using local information allows the algorithm to be used in real time applications.

- It does not assume any pre-existing knowledge about the dielectric constant. As the previous chapter showed, all the propagation model depends on the dielectric constant of the medium. As appendix B shows, the selected model is sensitive to the dielectric. Any method that uses a propagation model directly (for example - by performing migration, which is taking the weighted sum of the pixels along the theoretical curve predicted by the model) has to assume
CHAPTER 3. LOCALLY ISOLATED TARGET CASE

the pre-knowledge of the permittivity of the medium. It is possible to assess it from the surface
reflection, but this estimation is not very accurate. In the case of multiple layers (a subject
which will be discussed in the next chapter) the estimation error of the dielectric constant of
the deeper layer is too big to be of any practical use.

This chapter is organized as follows: The proposed algorithm is discussed in Section 3.1. This method is composed of three stages: pre-processing, feature extraction, and target detection. In
the pre-processing stage polynomial regression is performed both as a filter - increasing the resulting Signal to Noise Ratio (SNR) which improves the accuracy of the measurements - and as a method
for reducing the dimension of the problem, lowering the overall number of computations in the
following stages of the algorithm. The feature extraction stage again uses polynomial regression,
in order to obtain the first coefficients of the Taylor’s series of the pulses’ flight time function. By
matching these coefficients to the theoretical model and using the Quality of Fit (QoF) as one of the
detection criteria, one easily determines whether the cell under test is the apex of a hyperbola or not.
Furthermore, from these coefficients one can learn the model parameters - the (locally averaged)
signal velocity in the medium and the target depth. Using this information, the decision of a target
existence is revisited in order to rule out false detections. This step, in which the model parameters
are extracted from the targets, will be explored in Section 3.2. The overall results of the algorithm
will be presented in Section 3.3 with some discussion in Section 3.4.

3.1 Target Detection Algorithm

As was discussed in Section 2.1 and demonstrated in Section 2.2, the existence of a target
in the B-scan is visible through a distinct shape - a hyperbola. These shapes can be seen for example
in Figures 3.5 - 3.8. The location of a hyperbola’s apex and its aperture are being uniquely determined
by the model parameters - the target depth \( h \) and the permittivity of the medium \( \epsilon_r \).

The existence of a target creates an echo of the original pulse transmitted by the system
some time after the first arrival of the energy at the receiver - see Figure 3.1. In order to find
hyperbolas in the data, the pre-processing stage will locate these echoes in each of the scans, and
retain only this information. This produces a single dimensional function (instead of the full image)
- time of flight vs location. Since the influence of a target on this function diminishes the further
the system is from the target, the detection could be performed on localized observation of this
function - only the neighboring scans of the Scan Under Test (SUT) are important for the detection.
CHAPTER 3. LOCALLY ISOLATED TARGET CASE

This enables a real time algorithm, since the algorithm needs to retain only a limited amount of information in order to perform the target detection.

The algorithm parameters are described in Section 3.1.1 as well as some trade-offs for choosing their nominal values. The algorithm itself is detailed in Section 3.1.2: each new scan is processed to determine the location of a possible echo. The timing of these locations are stacked in a buffer, and a regression analysis for finding the best fit parabola is then performed. The model parameters - target depth and propagation speed of the signal in the medium - are being determined by equating the results of this analysis to the theoretical model. If these values are within range, a detection is declared. A detailed example of this process is illustrated in Section 3.1.3.

A GPR system scan rate can be as high as a several KHz. This rate sets an upper bound on the time frame upon which the algorithm has to run in order to be considered operating in real time. This run time typically ranges from a few tens of microseconds up to a full millisecond. The addition of the power consumed for the processing calls for the minimal number of computation required to enable the desired performance of the algorithm. The computational complexity of the algorithm shall be explored in Section 3.1.4 were a detailed review of the main mathematical tool (polynomial regression) will be given. Since the algorithm equates the polynomial resulted from the regression to the one resulting from a truncated Taylors’ series, the relationship between the two will be reviewed in Section 3.1.5.

3.1.1 Definitions and Terminology

The following definitions are used throughout this section:

- $M$ is the size of the buffer used for the time of flight function. This buffer can be viewed as a sliding window over the time of flight function, as measured by the system. With each new scan, this window will slide to the direction in which the system is currently moving, so the corresponding curve could be created in real time.

- SUT - the scan in the middle of the buffer. Since the regression is performed on a full buffer, increasing the buffer size $M$ will cause longer delay (one will have to wait $\lceil M/2 \rceil$ scans to get an answer).

- Out of the full scan, $N$ samples are used at the pre-processing step to determine the precise peak location.

- $t_d$ - the arrival time of the direct coupling signal.
CHAPTER 3. LOCALLY ISOLATED TARGET CASE

- $t_a$ - the arrival time of the signal reflected from the target.

- $t_{\text{offset}}$ - a constant offset added to the difference between $t_a$ and $t_d$ to compute the resulting time of flight to the target. The theoretical models assume that the two way flight time is known precisely. However, by analyzing the A-scan one can only deduce the time in which the signal was received, and is lacking the knowledge of the time of transmission. The measured flight time $t_a - t_d$ is a relative time - the true flight time to the target, minus the propagation time from the transmitter to the receiver. In order to compare the measured and the predicted results, one has to add the constant time of $t_{\text{offset}}$, which is the propagation time of the signal from the transmitter to the receiver. The value used in this work for $t_{\text{offset}}$ was found empirically by Dr. Roger Roberts, in a similar method to the one described by Yelf [3]. This method was detailed in Section 1.2, and it should be noted that this constant will be system dependent.

- $t_{\text{flight}}$ - the total time it took the signal to travel from the transmitter to the target, and back to the receiver. It is given by

$$t_{\text{flight}} = t_a - t_d + t_{\text{offset}}$$

It should be noted that the measurement of $t_d$ will depend on the material of the target. Signals reflected off targets which have low dielectric constants (such as PVC pipes or air filled cavities, for example) will have an opposite polarity compared to a signal reflected of a target with a high dielectric constant (metallic re-bar, for example). This should be taken into account when choosing the point of measurement (maximal or minimal peak).

There are several trade-offs when in choosing values for $M$ and $N$. One of them - the inherent delay of the detection algorithm - was already mentioned. Others include robustness of the algorithm: the larger $M$, the more the algorithm is immune to a noisy single scan. The fact that the “tails” of different hyperbolas created by different targets can cross makes detection harder. Similarly, the larger $N$, the more the algorithm will be immune to noisy samples within a scan. As will be shown, increasing $M$ and $N$ also increases the number of computations required at every step of the algorithm.
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3.1.2 Summary of Detection Steps

The detection algorithm is summarized in Algorithm 1.

Algorithm 1 Summary of the detection algorithm.

1. Initialization step: Fill the buffer - go over the first M scans, and process them (as defined in step 2 below).

2. Given a new scan, do the following:
   (a) Determine the arrival time of the direct signal $t_d$ (to be used as a reference):
      i. Find the sample number $n$ which corresponds to the maximal amplitude in the scan.
      ii. Use polynomial regression to fit the $N$ samples in the neighborhood of sample $n$.
      iii. Determine the precise time, in which the polynomial reaches its maximal point.
   (b) Subtract the background from the scan.
   (c) Determine the arrival time of the target $t_a$. This is done in a similar fashion to $t_d$, but is performed on the modified scan (without the background).
   (d) The (relative) time of flight for that scan is given by Equation 3.1.
   (e) Store this value at the end of the buffer. This will discard the first value in the buffer and will shift all other values one place closer to the beginning of the buffer.

3. Using quadratic regression procedure, find the best parabola which describes the data stored in the buffer.

4. Determine whether the SUT is indeed a valid target.

5. Go back to step 2, and process a new scan.
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The decision which is performed in step 4 is based on several parameters:

- The QoF of the parabola. For example, the $l_2$ or the $l_\infty$ norm of the fitting error.

- The location of the target (the apex of the parabola) has to be no more than half a scan interval away from the SUT.

- By using feature extraction, obtain the model parameters (target depth and permittivity of the medium - see Section 3.2); if these parameters are out of the expected range than the SUT cannot contain a target.

This algorithm will be demonstrated in Section 3.1.3 and its computational load analyzed in Section 3.1.4.

3.1.3 Example of the Intermediate Steps

To demonstrate how the algorithm process a given data set, a block with a permittivity of 6 was scanned using GSSI system with a center frequency of 2.6 GHz. This block has in it a mesh of re-bars which are 10 cm apart. The resulted GPR image is presented in Figure 3.6.

The algorithm begins by processing each individual scan. For this example, scan number 331 was chosen (shown in Figure 3.1). This scan contains a target, which later will be detected by the algorithm. This is displayed in Figure 3.2. The strongest part (maximal amplitude) of the scan is the result of the signal traveling via the direct path, and arriving at the receiver without entering the medium. For that reason, a simple search for the maximal value in the scan yields the sample which corresponds to $t_d$. In order to get a more precise time of arrival of this signal, the algorithm uses cubic polynomial with $N = 15$ (step 2ii in Algorithm 1).
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Figure 3.1: Detection algorithm example - Scan 331. The pulse traveling through the direct path, and the one reflected off a target visible. The arrival time of the peak of these pulses are marked by $t_d$ and $t_a$, respectively.

Figure 3.2: Detection algorithm example - Performing cubic polynomial fitting on the maximal peak of the signal, in order to determine the precise position of the peak.
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The background was obtained by using a running average over the previous scans in the image. By subtracting this background from the SUT the first reflection (of the signal traveling via the direct path) was removed, leaving the reflection from the target as the strongest (maximal) part of the signal. This is demonstrated in Figure 3.3 and it justifies the repeated search for the maximal sample in the (this time - background-removed) scan. This search shall yield the sample corresponding to $t_a$. Cubic polynomial is used again in order to obtain a precise time measurement.

Figure 3.3: Detection algorithm example - Scan 331 after background subtraction. Only the pulse reflected off a target remains.

The result of step 2 is a single value from the 512 samples in the scan. This value is stored in the processing buffer. To detect the shape, a regression analysis is performed on this buffer, as described in step 3 of the algorithm. Continuing the example, 20 more scans were processed, so that the value obtained from processing scan #331 would be in the center point in the buffer. The quadratic polynomial that best fits the buffer, as well as the data in the buffer, are seen in Figure 3.4.

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Figure 3.4: Detection algorithm example - Performing quadratic polynomial fitting on the flight time curve.

Matching the coefficients of this polynomial to Equations 2.25-2.27 (see Section 3.2 below) yields an estimation for the local permittivity of the block as well as the target depth. These values are used for the decision process, and are not required to be accurate. In order to assess their accuracy, the resulted values of 50 different targets, which were located at various depths (1.5 - 10 cm) at different concrete blocks were averaged. The permittivity of the blocks ranged from 5.7 - 9. The results are summarized in Table 3.1.

Table 3.1: Comparing the resulting averaged values from the algorithm to the ground truth.

<table>
<thead>
<tr>
<th></th>
<th>Results from the algorithm</th>
<th>Ground truth</th>
<th>Error [percent]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target depth [cm]</td>
<td>4.74</td>
<td>5.55</td>
<td>14.6</td>
</tr>
<tr>
<td>Permittivity value</td>
<td>6.63</td>
<td>5.80</td>
<td>14.29</td>
</tr>
</tbody>
</table>

The averaged error of the depth and permittivity estimation is 14.6 and percent, respectively. Though the latter is too high for most applications, the former is quite good, especially when considering the fact that in reality no target is a true point target. In this work the target which were examined were round metallic re-bars, each with a diameter of 1.27 centimeters. This diameter is
greater than the absolute error (as measured in centimeters from the top most point of the targets).

It should be noted that the error in the estimation of both parameters is biased - the predicted target depth and local permittivity were lower than the ground truth in all the targets tested by the author. This presents the opportunity to decrease the (averaged) error by adding some fixed percentage, so that the error will be unbiased. Since the estimation was used only for setting the decision rules, this work did not pursue this route.

### 3.1.4 Computational Load of the Algorithm

The above algorithm relies on polynomial regression analysis of the data. Three regression procedures are being carried out for each new scan. Therefore, in order to calculate the computational load of the algorithm, it is important to understand how this procedure can be carried out efficiently.

Given a set of K measurements $Y = (y_1 \ldots y_K)^T$ which satisfies $y_k = f(x_k)$ for some function $f$, one searches for the best polynomial $P$ of degree $L$ that fits the function $f$ (optimal in the sense that it minimizes the $l_2$ norm of the difference between $P(x_k)$ and $f(x_k)$ for all $k \in \{1 \ldots K\}$).

To do this, define

$$X^n \triangleq (x_1^n \ldots x_K^n)^T$$

$$H \triangleq [X^L, X^{L-1}, \ldots, X, X^0]$$

The coefficient of $P$ are given by

$$a \triangleq (a_L, a_{L-1}, \ldots, a_1, a_0)^T = H^\dagger Y$$

Where $H^\dagger \triangleq (H^T H)^{-1} H^T$ is the pseudo inverse of the matrix $H$.

This algorithm uses two such matrices: one for the peak arrival time estimation (where cubic $L = 3$ polynomial is fitted to $N = 15$ samples of the scan in order to obtain precise measurement of the arrival time), and another for step 3, (where quadratic $L = 2$ polynomial is fitted to the buffer of length $M = 41$, in order to get the target’s parameters).

Thus, the total computational load of the regression part (obtained by using Equation 3.4 with fixed matrix $H^\dagger$) would be $8N + 3M$ multiplications and $8N + 3M - 11$ additions for each new scan that arrives.

In step (2i) the algorithm finds the sample with the maximal value. This operation is also bounded by the scan length (which is fixed for the runtime of the algorithm). In practice, one can utilize knowledge about the approximate position of the sample in order to speed up the search.
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Indeed, since the emission time \( t_e = t_d - t_{\text{offset}} \) is stable from one scan to the next, one can use the value from previous scans to narrow the search region for the new scan.

The above bound on the number of calculations required for each new scan, and the fact that this bound is constant regardless of the number of scans, make the algorithm suitable for real time applications.

3.1.5 Relationship Between Taylor Expansion and Polynomial Regression

The Stone–Weierstrass theorem states that one can approximate any continuous function by a polynomial function. Limiting the function space to \( C^\infty(\mathbb{R}) \), one get that for every point \( a \in \mathbb{R} \) and every function \( f \in C^\infty(\mathbb{R}) \),

\[
 f(x) = \sum_{n=0}^{L} \frac{f^{(n)}(a)}{n!} (x - a)^n + \sum_{n=L+1}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n
\]

Equation (3.5) gives an upper bound on the distance of the function \( f \) and the result of the \( L \)th order polynomial approximation to this function around the point \( a \).

Polynomial regression was described in Section 3.1.4. The fitting error of the resulting of the \( L \)th order polynomial \( P_L(x) \) to the \( M \) data point \( \{x_i, y_i\}^M_{i=1} \) used is

\[
 E = \sum_{i=1}^{M} \epsilon^2(x_i) = \sum_{i=1}^{M} (y_i - P_L(x_i))^2
\]

Using the vector notation of \( Y, X, P_L(X) \) to be the \( M \times 1 \) vectors, containing the \( \{y_i, x_i, P_L(x_i)\} \) points, respectively, Equation (3.6) becomes

\[
 E = (Y - P_L(X))^t (Y - P_L(X))
\]

Defining the \((L + 1) \times M\) Matrix \( T = \begin{pmatrix} 1 & X & X^2 & \ldots & X^L \end{pmatrix} \) and \( a \) as the vector containing the coefficients of \( P_L \), one get that

\[
 P_L(X) = Ta
\]

\[
 \Rightarrow E = (Y - Ta)^t (Y - Ta)
\]
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To find the optimal polynomial function, one needs to derive Equation 3.9 and equate it to 0. By doing that, one gets:

\[ 0 = 2T^t Y - 2T^t Ta \]
\[ \Rightarrow a = T^t Y = (T^t T)^{-1} T^t Y \]

This work uses a (noiseless) model. Its derivatives, and therefore its Taylors’ approximation, were calculated in Equations 2.25 – 2.28. For the decision process this polynomial is equated the result to a polynomial function obtained from performing polynomial Regression on (noisy) samples, obtained from the signal. It is important to stress that the two projection have absolutely nothing to do one with the other. In fact, though they both have the same image (the space of all polynomial functions of a given order \( L \)), their domain is different, so the two projections do not produce the same polynomials in the general case. Equating the two polynomial is therefore non trivial and requires some justification. This will ultimately come from the bound on the error of the Taylor’s expansion (see Equation 3.5). To clarify this, consider the following case: Let \( f \in C^\infty(\mathbb{R}) \). By using Equation 3.5 one chooses an arbitrary point \( a \) in the domain of \( f \), and approximate \( f \) by a polynomial function of order \( L \) \( P_L \). A polynomial regression process performed on \( M \geq L \) points, sampled from the image of \( P_L \), will reconstruct the polynomial \( P_L \) back without an error. However, adding noise to these samples will result in deviation from \( P_L \), so the reconstruction will not be perfect. To conclude, there are two sources of error in this process: one is fundamental approximation error - the distance between the original function \( f \) and the polynomial function \( P_L \). The other one is the noise in the samples. The former source is bounded by a polynomial function, whose first \( L+1 \) coefficients are zero. To minimize this one must ensure that the sampling points \( \{x_i\}_{i=1}^M \) will be close enough to the expansion point \( a \). The latter source of error could be minimized by increasing the number of points \( M \). These two methods are conflicting, for the larger the number of points the bigger the distance between the points and the center of the buffer.
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3.2 Learning the Model Parameters From the Target

Once a target has been detected, one naturally wishes to extract as much information as possible about the target. This works reverses this logic: the decision of whether a given SUT contains a target or not is based on the information extracted from it. This information are the model parameters - the target depth $h$, and the signal’s speed of propagation in the medium $v_p$ in the neighborhood of the target. Based on the model in use, one expects to obtain different curves from different model parameters; This is demonstrated in Appendix B. Therefore, by matching the two curves (one obtained from the quadratic regression which was described in step 3 of the algorithm, and the other as obtained from the model) one is able to extract the desired parameters.

In general, this is an optimization problem and could easily be solved numerically. In one special case, the approximation to the bi-static model, one can obtain an analytic solution to the curve matching problem. This in turn guarantees that the matching is carried out with the minimum number of computations, which allows the process to be performed in real time. For off-line calculations, the use of a more accurate model would yield better results. Since it was shown that the approximated bi-static model describes best (in terms of MMSE) the shape of the data, it was chosen for the initial target detection stage. If a more precise estimation of the model parameters is required, then possibly curve matching should be performed using a more accurate model on detected targets. To maintain the efficiency of the overall algorithm, this stage will be performed only as a last step - on buffers that are known to contain a target. The decision on the existence of a target in the buffer could then be re-visited.

Step 3 of the algorithm produces a quadratic polynomial function $t = a_2x^2 + a_1x + a_0$. Equation 2.26 shows that $a_1 = 0$. In general, this will not be true for the coefficients of the polynomial, and therefore it needs to be centralized:

$$a_2(x - \frac{a_1}{2a_2})^2 + a_1(x - \frac{a_1}{2a_2}) + a_0 =$$
$$= a_2x^2 + a_0 - \frac{a_1^2}{4a_2}$$

(3.12)
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Using Equations (2.25 – 2.27), one finds that

\[ a_2 = \frac{1}{2} t''(0) = \frac{n h^2}{c (h^2 + \Delta^2)^{3/2}} \]  \hspace{1cm} (3.13)

\[ a_0 - \frac{a_1^2}{4a_2} = t(0) = \frac{2n}{c} \sqrt{h^2 + \Delta^2} \]  \hspace{1cm} (3.14)

And by solving Equations 3.13 – 3.14 for \( h^2 \) and \( n = \sqrt{\epsilon_r} \), one obtains the (approximated) model parameters.

It should be noted that in order to avoid the inherent uncertainty of the pulse firing time \( t_0 \), one can use the second and fourth derivatives of the flight time, thus avoiding the need for \( t_{\text{offset}} \) which was introduced in Equation 3.1. This would require replacing the quadratic polynomial in step 3 with a fourth order one and match the resulting coefficients to the derivatives in a similar fashion to the method that was described above.

3.3 Results

To assess the performance of the algorithm, 50 data sets with 278 targets were collected. The targets were re-bars inserted into 6 inches blocks at known depths. The blocks were measured independently in order to determine their (averaged) permittivity. Each block was scanned using a GSSI system with a centered frequency of 2.6 GHz. Target depth ranged from 1.5 cm to 10 cm and the permittivity of the blocks ranged from 1.07 to 14.

A total of 265 out of the 278 targets were located in those 50 different data sets, making the probability of detection of these algorithm to \( p_d = 0.953 \). In the entire collection of data sets, a total of 8 scans were mis-classified as containing targets. Four examples of these data sets are presented in Figures 3.5 – 3.8. The location of the targets, as were found by the algorithm, are marked on the data.
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Figure 3.5: Detection algorithm performance - Two targets block with permittivity of 5. The algorithm marked the detected scans.

Figure 3.6: Detection algorithm performance - Six targets in depth ranging from 10 to 1.5 cm in a block with permittivity of 5.8.
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Figure 3.7: Detection algorithm performance - Six targets in depth ranging from 10 to 1.5 cm in a block with permittivity of 9.

Figure 3.8: Detection algorithm performance - A block with re-bars, located at depth of 10 cm. The block permittivity is 6.
3.4 Discussion

The core ideas which drives the detection algorithm presented in this work were presented in this chapter. They were developed into a full, working algorithm in itself, which is extremely efficient and shows good performance metrics. Several parameters control this algorithm. The most important one is the regression length - how many data points (scans) are used to perform the regression analysis. This parameter presents some obvious trade-offs. On the one hand, the larger the length is, the more noise reduction will be performed and the better the parameter estimation will be. On the other hand, increasing this length means that the algorithm might miss targets from the edge of the survey. Moreover, if the length is too large, the algorithm might miss all the targets altogether when one target will start to cross another. For this work, this parameter was chosen manually, though future work might consider adapting it in an iterative process to find an optimal value. As will be shown in the next chapter, if multiple targets exists in the medium, in a close proximity to one another, their hyperbole might cross, which will distort the shape of the curve in the buffer. This in turn will lead to the wrong model parameter to be extracted, and the targets might be missed altogether. This situation could easily be avoided if the buffer length is small enough.

The biggest shortcoming of the algorithm is in its focus on the (second) strongest reflection in the signal as the one which contains the time information from the target. In case of multiple layers and/or targets, this will not necessarily be the case. Mitigation to this will be the subject of the next chapter.
Chapter 4

Generalization of the Automated Target Detection Algorithm

The algorithm described in the previous chapter works very well as long as the assumptions on which it was based hold. These are detailed in Section 2.1.2. The two weak assumptions listed there are the lack of interference that one target might introduce to another, and the fact that the medium is locally homogeneous. The results of removing these two assumptions are the subject of In this chapter. The effects created to the signal (and hence - to the buffer) by removing those assumptions are explained in Section 4.1. Since these effects change the data stream at such an early stage of the algorithm, significant modification was required to in order to enable the data handling. While retaining the core ideas which guide the creation of the detection original algorithm, the pre-processing stage underwent significant revision. This revision was done in order to be able to detect the targets in any location they might be in the image - including under layers or other targets. Additional sub-stage was added to the pre-processing stage of the original algorithm. In it, a target trajectory is being assembled from the data stored in the buffer. It turned out that the pre-processing stage was “too strong” - and many areas in the image were detected as targets (i.e. - increasing false detection rate). For that reason, the decision stage was revised to incorporate more knowledge about the targets. The end result of all of these modifications shall be described in Section 4.2. were the algorithm will be re-built to accommodate for a more complex (and more realistic) scenario. The results of the algorithm are presented in Section 4.4
4.1 Analyzing the Weakness of the Detection Algorithm

In a fully homogeneous medium, the only reflection which might arrive at the receiver are those which result from the signal going from one layer to another. The boundary between the two layers - where there is a theoretical discontinuity in the permittivity of the medium - will be the point of refraction for the electromagnetic energy of the signal. In Section 2.1.2 this assumption (of a locally homogeneous medium) was made, and the pre-processing stage was designed accordingly. Step 2 of the algorithm was performed twice for each scan, on the two maximal peaks. The underlying thought behind this was that these peaks were originated from the signal traveling via the direct path (forming the strongest amplitude of the A-Scan) and the signal reflected off the target, forming the second highest amplitudes in the signal. Since it was also assumed that the medium’ surface is flat compared to the wavelength, one expect to detect the maximal amplitude of the signal at (roughly) the same sample number at each A-scan. The design constraints included the overall efficiency and minimal run time of the algorithm. For this reason the fact of the signal arriving at an almost constant sample was fully exploited in the implementation. Detecting the second highest amplitude called for a different approach, since no prior knowledge could be utilized in order to speed up the search. To detect it efficiently, the fact that the maximal amplitude arrives at an (almost) constant sample in the A-scans was used again: by removing the averaged scan (background) the algorithm practically eliminated this part. Signals which were reflected off targets will exists only on few scans out of the entire image, and the effect of such a removal on them will be minimal. This enable the use for a simple search for a maximal amplitude on the background-removed scan, and eliminated the need to detect all the maxima points of the scan, order them and choosing the strongest among them. It should be noted that the permittivity of a “real-life” medium is never spatially constant, and so reflection are generated at each and every time sample. However, the local change within the mediums’ permittivity is typically small, and the amplitude of a reflection is relative to this change, yielding that the reflection from the medium itself are usually weaker than the one coming from a target, and the reflection from the target remains the (second) strongest signal in the A-scan. This ceased to be the case if the medium is composed of multiple layers, as explained in Section 4.1.1.

Another assumption which was made in Section 2.1.2 was that of a locally isolated target. when the medium contains multiple targets who are too close to one another, the resulted B-scan is the composition of the reflection from all targets. This makes the separation of them challenging. This subject is detailed in Section 4.1.2.
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4.1.1 Multi Layered Medium

If the assumption of a locally homogeneous medium does not hold (i.e. there is more than a single horizontal layer in the scanned area), one might get a “hall-of-mirrors” effect: the signal will bounce between the boundaries of the two layers, and the reflections will arrive at the receiver not only from the surface of the medium and from the target, but also from multiples of the travel path. The shape of the transmitted signal is Ricker Wavelet [35]. Combining this with the hall of mirrors effect results in a “wavy” signal, where the multiple wavelets are visible in the A-Scan. In addition, when the signal travels from a layer of relatively low permittivity to a layer with a higher one - the polarity of the wavelet reverses. The received signal in this case will be composed of positive and negative reflections, which might interfere with one another.

Multiple horizontal layers are very common in practice. Examples could be found in walls (where the time range of the signal might be longer than the propagation time in the wall, causing the signal to bounce inside the wall), as well as in paved roads (where there is the gravel underneath the asphalt layer, and usually another bedrock underneath the gravel). Indeed, there are very few cases where the assumption of a completely homogeneous medium does hold. There are cases, however, where reflections from layer boundaries occlude the reflections from the target, so the simple pre-processing approach of finding the target in a scan - by removing the background and searching for the maximal peak - will not necessarily work in this case. An example for this is presented in Figure 4.1, where a three layered medium was scanned. Note the bottom layer, which might be an artifact due to energy bouncing from one layer to the next. The maximal sample in the background removed signal in this case will be the reflection from the boundary between the first and second layers and not from the target, leading to a complete miss-detection of the targets in the second layer.
4.1.2 Multiple Targets

Another assumption which was described in Section 2.1.2 and used by the algorithm was that of a locally isolated target. As a result of this assumption, the second strongest sample in the A-Scan must belong to the same target, if one exists in the neighborhood of the A-scan. This is why all the timing were stored in the same buffer and processed together. As is seen in Figure 4.1, for example, there are scenarios were a hyperbola, created by the existence of one target, might cross the hyperbola of a different target. This scenario is very common when scanning a block with a re-bar mesh. As could be seen in Figure 4.1, the targets can be at different depths and of different sizes and material. The reflected signal will be the superposition of all the various reflections coming from the different targets (and different layers, if those exists). Due to this superposition, the shape of the data in the buffer might get distorted - for by taking the maximal peak in the (background removed) scan, the maximal might belong to a different target. This will result in miss detection of both targets. Example for such a complex scenario is presented in Figure 4.2: in the scanned block there are two mesh grids, with targets between them, in a single block of concrete.
Mitigations to the above short coming shall be explored In the next section.

4.2 Detection Algorithm, Re-Visited

The miss detection described in the previous section resulted from information loss incurred in the pre-processing step, described in Section 3.1.2. The problem in that stage was the focus on the strongest reflection in the (background removed) scan, which may not be a stable enough when multiple targets and layers exists. For that reason, the algorithm had to be modified, as will be described next. Since this modification happened at such an early stage of the algorithm (the second step of the pre-processing stage), many steps in the algorithm also had to be redesigned in order to handle the new data stream. The core of the algorithm, however - using polynomial regression process in conjecture with a theoretical model to generate the decision - was kept, since it was shown to be highly efficient and a high performance stage. The guideline for the change was to recover the curve in any location it might occur (rather than at the strongest reflection point). Once a curve was reconstructed, the decision could be made as in the previous chapter. In that way, the curves of the targets are decoupled, allowing for a simplified processing to take place. As it was discovered during the work, the additional information that is being collected for the decoupling purposes, could also be utilized to reduce the false detections.
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The algorithm uses two matrices, T and A (of equal size - $K \times M$), which replace the single buffer ($1 \times M$) which was used previously. With the arrival of a new scan it is being processed (as described in Section 4.2.1) to obtain the information regarding the first K extrema points in it (both negative and positive). The timing and the amplitude information of those points would be stored in the appropriate columns of the T and A matrices, respectively. The algorithm will then loop through the rows of the T and A matrices, and will attempt to recover the curve (as described in Section 4.2.2). Only then could the regression process be performed on the recovered curve, and the detection made on that row. By retaining both the positive and negative extrema points, two goals were achieved. The first is that the algorithm gained the ability to detect both types of targets (metallic and non-metallic). The second is that additional information about each target is gathered. This is demonstrated in Section 4.2.3. The calculation of the model parameters from the time of flight curves is detailed in Section 4.2.4, with a sensitivity analysis given in Section 4.2.5. During the derivation of the equations, decision rules emerge. These rules are being used in order to make the decision on the existence of a target in the SUT as described in Section 4.3.

4.2.1 Analyzing a New Scan

The information that exist in a single scan is stored in the peaks of each scan: the location (time delay) of the peak, the amplitude of the samples and their sign (positive or negative). In order to avoid occlusion of targets (by either layers or other targets), all the relevant peaks from each scan are retained. To achieve that, the derivative of the scan is calculated and the algorithm searches for the zero crossing in the derivative.

As described in Chapter 3, the first peak will correspond to the energy traveling via the direct path. The other peaks might be related to targets, layers or other reflections. Each wavelet is composed of three peaks of alternating polarity - negative, positive and negative for metallic objects and reverse polarity for non metallic objects. Retaining all those peaks gives additional information. This information will assist the decision stage. Obtaining the precise timing of a given peak is done by using a cubic polynomial regression, As in step 2(a)i in Algorithm 1. This yields an accurate estimation for the peak’s delay and amplitude. The delay information of all the peaks are then referenced to timing of the maximal peak in the scan (with the additional constant of $t_{offset}$, as in Equation 3.1). The amplitude is normalized by taking the ratio between the current peak and the first one. As was already stated and exploited in the previous chapter, the first peak would always have the maximal amplitude in the scan, and therefore the normalized amplitude can never be greater
than one (in absolute value). The normalization helps to eliminate differences in the signal amplitude which might be the result of hardware changes (either from one system to another, or possibly due to minute changes due to varied environmental conditions). They also mitigate differences in the amplitude of the signal due to different permittivity values of different blocks. Once the peaks of the scan were discovered and analyzed, they are being stored in the two matrices T and A (one for the timing information and another for the amplitude, respectively). These matrices will generalize the buffer used in Section \ref{sec:3.1.2}, but they serve a similar purpose. This process is summarized in Algorithm \ref{alg:2}.

\begin{algorithm}
\caption{Scan Processing Procedure.}
\label{alg:2}
\begin{algorithmic}
\State Initialize the point counter by $k \leftarrow 0$.
\For{every sample $n \in$ new scan}
\State Check the derivative of $s$ in $n$. If the derivative changed its sign:
\State \hspace{1em} 1. $k \leftarrow k + 1$.
\State \hspace{1em} 2. Use polynomial regression to fit the $N$ samples in the neighborhood of sample $n$.
\State \hspace{1em} 3. On the first point ($k = 0$) set the reference time $t_d$ and reference amplitude $a_d$.
\State \hspace{1em} 4. On any other point ($K > 0$) set the arrival time of the signal $t_a(k)$ and the amplitude $a(k)$.
\State \hspace{1em} 5. The (relative) time of flight for that scan is given by
\begin{align}
    t_{\text{flight}}(k) &= t_a(k) - t_d + t_{\text{offset}} \\
    \text{(4.1)}
\end{align}
\State \hspace{1em} Where $t_{\text{offset}}$ is a constant of the system.
\State \hspace{1em} 6. The normalized amplitude of the signal is given by
\begin{align}
    \tilde{a}(k) &= a(k)/a_d \\
    \text{(4.2)}
\end{align}
\State \hspace{1em} 7. Store the $t_{\text{flight}}(k)$ and the $\tilde{a}(k)$ values at the $k$-th row of the T and A matrices, respectively, at the last column. If needed, discard the first values in that row and shift all other values one place closer to the beginning of the matrices.
\State \hspace{1em} 8. Verify that $k \leq K$. If not - exit the loop.
\EndFor
\end{algorithmic}
\end{algorithm}
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The results of Algorithm 2 are demonstrated in Figure 4.3. In this example, only the first $K = 17$ extrema points were retained, which gives the algorithm the ability to detect targets and the bottom part of the image.

![Figure 4.3: Analysis of a scan. The local extrema points are marks by orange X and are numbered by the algorithm.](image)

Reflections from different targets and layers frequently interact. Such an interaction is presented in Figure 4.3. In it, a shallow target (visible as the fifth peak) is interacting with the signal traveling via the direct path (surface reflection). This is visible by comparing the negative part of the first reflection (second peaks) which interacts with the (leading) negative part of the reflection from the target, to form the third and fourth peaks. Note peak number 9, which is a reflection of the target (which exists in peak number 5). Additional reflections exist in peaks # 11 and 13, where the negative peak at 5.22 nanoseconds is the signal reflected off the bottom part of the concrete slab used.
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4.2.2 Building Trajectories

Once the T and A matrices have been filled, a search is conducted on their rows. A typical example for a row (which does contain a target) is presented in Figure 4.4.

![Time of flight curve](image)

Figure 4.4: The time of flight curve of a target before the trajectory building step. The jumps from one peak to the next are clearly visible.

As seen in Figure 4.4, the time of flight curve is (roughly) continuous in a small neighborhood of the target. Further away from the target, the information is present at other rows. This causes a visible “jump” (strong discontinuity) in the data. This “jump” is due to additional pairs of extrema points which exist in some of the scans (but not all of them). Algorithm [3] was created to alleviate this problem.
Algorithm 3 Trajectory Building Algorithm.

For row $i$ of the $T$ and $A$ matrices, we build the $1 \times M$ time of flight vector $v$ and the amplitude curve $u$, by initializing them to the $i$th row of $T$ and $A$, respectively.

1. Start from the middle column (number $\lfloor M/2 \rfloor$) and go forward:

   For $g = 1$ to $\lfloor M/2 \rfloor$

   (a) Search for all peaks in the $g + \lfloor M/2 \rfloor$ column of the matrix $T$, whose polarity matches that of the time to the target.

   (b) From those peaks, select the entry $t^* = T(g + \lfloor M/2 \rfloor, j^*)$ such that
       
       \[ j^* = \arg \min_j |T(g + \lfloor M/2 \rfloor, j) - v(g - 1 + \lfloor M/2 \rfloor)|. \]

       Define this minimal difference as $dt^* \triangleq |T(g + \lfloor M/2 \rfloor, j^*) - v(g - 1 + \lfloor M/2 \rfloor)|$.

   (c) If $dt^* > G$ for some constant guard value $G$:
       
       i. Calculate the discrete derivative $dt = v(g - 1 + \lfloor M/2 \rfloor) - v(g - 2 + \lfloor M/2 \rfloor)$.

       ii. $v(g + \lfloor M/2 \rfloor) = v(g - 1 + \lfloor M/2 \rfloor) + dt$.

   (d) If $dt^* \leq G$, then $v(g + \lfloor M/2 \rfloor) = T(g + \lfloor M/2 \rfloor, j^*)$.

2. Repeat step 1 for $u(g + \lfloor M/2 \rfloor)$, by using the $A$ matrix instead of $T$.

3. Repeat steps 1 and 2, but for the left side of the $T$ and $A$ matrices - i.e., looking at column $\lfloor M/2 \rfloor - g$ for $g = 1$ to $\lfloor M/2 \rfloor$.

The guard value $G$ is being used by the algorithm to prevent it from jumping to a different curve in the matrices. The output of this algorithm is presented in Figure 4.5.
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Figure 4.5: The time of flight curve of a target after the trajectory building step.

As could be seen in the figure, algorithm 4.5 was able to reconstruct the target trajectory, enabling the next steps in the detection algorithm to take place. The guard value $G$ is found dynamically during the runtime, by taking the time difference of two rows in the middle column of the $T$ matrix.

This algorithm takes $M$ steps, and has to be performed $K$ times for each scan, making the process inefficient. When implemented on the StructureScan Mini XT system, with $M = 63$, $k = 25$, The overall detection algorithm spent about half of the runtime (300 $\mu$Sec per scan) on this section alone.

4.2.3 Utilizing Amplitude Information

The amplitude information is obtained as a by product of the polynomial regression which was performed during the scan processing stage (see Section 4.2.1), in order to obtain the timing information. It could be used in order to assist the decision process, resulting by reduction in the false detection rate. In order to utilize this information, another regression step was added. In this step the best quadrature polynomial that describes the amplitude curve is found. By analyzing the curvature, the apex of the polynomial function, and the fitting error, additional parameters were obtained to be use in the decision process.
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The further away the system is from a target, the larger the attenuation the signal is expected to have. This means that when fitting a quadratic polynomial \( P(x) = b_0 + b_1 x + b_2 x^2 \) to the curve, the curvature has to satisfy

\[
 b_2 < 0
 \]  

(4.3)

And Equation 4.3 could be added to the decision rules which guide the decision stage.

An example for the amplitude curve could be seen in Figure 4.6.

Figure 4.6: The amplitude curve of a target after the trajectory building step.

It should be noted that had the polarity been negative, the curve in Figure 4.6 was convex (rather than concave). This is due to the fact that all values on the curve will be negative. In order to be able to use the same set of rules for both metallic and non metallic targets, the absolute value of the amplitude is taken.

Compared to the time of flight information, the amplitude information is very unreliable. This is due to two main reasons: variations between systems and cross target interference. Every transmitter is a little bit different, and the hardware tends to change over time (as the board ages) and temperature. When two curves cross one another, their amplitudes adds up. The timing of the maximal (or minimal) peaks is affected, but to a significantly lesser degree than the amplitude curve. This makes the amplitude of the signal by itself an unsuitable candidate for detection; However, combined with the timing information, it can assist the decision making.
Other parameters which are used in the decision process are the model parameters - the target depth $h$ and the refraction coefficient of the block $n$. These are useful due to the fact that they have physical meaning, and so their expected range could be pre-determined. Obtaining those parameter from the time of flight curve is the subject of the next subsection.

### 4.2.4 Obtaining the Model Parameters

By equating a quadratic polynomial $P(x) = a_0 + a_1 x + a_2 x^2$ which describes a time curve to the (second order) Taylor’s polynomial that describes the theoretical model, the model’s parameter could be found (Equations 3.13 – 3.14). For completeness, the process of obtaining them shall be repeated here. This allows to explain some of the decision rules used in the decision step. Since the regression process is noise sensitive, a sensitivity analysis is performed in Section 4.2.5.

\[
a_2 = \frac{1}{2} \frac{d^2}{dx^2} (0) = \frac{n}{c} \frac{h^2}{(h^2 + \Delta^2)^{3/2}}
\]

\[
a_0 - \frac{a_1^2}{4a_2} = t(0) = \frac{2n}{c} \sqrt{h^2 + \Delta^2}
\]

Note that $h \geq 0$, $n \geq 1$, and define

\[
Q = \sqrt{\frac{a_0 - \frac{a_1^2}{4a_2}}{2a_2}} = \frac{h^2 + \Delta^2}{h}
\]

\[
\Rightarrow 0 = h^2 - Qh + \Delta^2
\]

Note that $a_0$, being the time of flight to the target, can never be negative. The curvature $a_2$ likewise has to be strictly positive, since the time of flight has to increase the further the system is from the target. For the SUT one expect the center of the curve ($x = -\frac{a_1}{2a_2}$) to be close enough to the scan, which means that $|a_1|$ is sufficiently small, so that $Q$ is well defined.

For the StructureScan Mini XT system, the desired target range is

\[
0 < h \leq 0.2[m]
\]

Equation 4.7 has two solutions:

\[
h_{1,2} = \frac{Q}{2} \pm \sqrt{\frac{Q^2}{4} - \Delta^2}
\]
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To ensure that the two solutions are real, $Q$ has to satisfy

$$Q \geq 2\Delta \quad (4.10)$$

Equations 4.8 and 4.10 could be added to the decision rules, along with the requirement for $a_2 > 0$ and a small enough center $\left| \frac{a_1}{2a_2} \right| < C$ for some constant $C$.

Using Equations 4.5 and 4.9 one gets a solution to the refraction coefficient $n$:

$$n_{1,2} = \left( a_0 - \frac{a_1^2}{4a_2} \right) \frac{c}{2\sqrt{h_{1,2}^2 + \Delta^2}} =$$

$$= \left( a_0 - \frac{a_1^2}{4a_2} \right) \frac{c}{2\sqrt{Qh_{1,2}}}$$

Where the last equation holds because $h_{1,2}$ are the solutions to Equation 4.7.

Equation 4.11 assists in determining the correct target depth $h$ - for the value of both parameters has to be within normal range. For the StructureScan Mini XT system, this range was set to be

$$\sqrt{5} \leq n = \sqrt{\epsilon_r} \leq \sqrt{8} \quad (4.12)$$

Note that for a large enough value of $Q$, the correct solution to Equations 4.9-4.11 will be

$$\begin{align*}
  h &= 0.5Q - \sqrt{0.25Q^2 - \Delta^2} \\
  n &= \frac{c}{2} \left( a_0 - \frac{a_1^2}{4a_2} \right) (Qh)^{-1/2}
\end{align*} \quad (4.13)$$

Equation 4.13 is well defined, since $Q \geq 2\Delta$ and $h > 0$.

4.2.5 Sensitivity Analysis of the Model Parameters

The regression process is sensitive to noise in the samples, so one naturally expects the coefficients of the resulted polynomial $\{a_0, a_1, a_2\}$ to be slightly off from their nominal value. This section explores the effect of “being slightly off”, to analyze how sensitive will the model parameters are to the variations in the coefficient. Noting that the time of flight curve of a target should be centralized to the scan, this analysis will begin by assuming that $a_1 \approx 0$, and so this coefficient has little (if at all) effect on the model parameters. This section will therefore focus on analyzing the effects of small perturbations in the curve’ offset $a_0$ and curvature $a_2$. 

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1. Sensitivity to perturbation in the curvature. Let $a_2 = \hat{a}_2 + \delta a_2$, where $\delta a_2 \ll \hat{a}_2$. From Equation 4.6:

$$ Q \triangleq \sqrt{\frac{a_0}{2\hat{a}_2 + 2\delta a_2}} = \sqrt{\frac{a_0}{2\hat{a}_2}} \sqrt{1 + \frac{\delta a_2}{\hat{a}_2}} \approx \sqrt{\frac{a_0}{2\hat{a}_2}} \left(1 - \frac{\delta a_2}{\hat{a}_2}\right) $$

$$ \Rightarrow \delta Q = Q - \hat{Q} = \frac{\hat{Q}}{2\hat{a}_2} \delta a_2 $$

(4.14)

2. Sensitivity to perturbation in the curve offset.

$$ Q = \sqrt{\frac{\hat{a}_0 + \delta a_0}{2a_2}} = \sqrt{\frac{\hat{a}_0}{2a_2}} \sqrt{1 + \frac{\delta a_0}{\hat{a}_0}} \approx \hat{Q} \left(1 + \frac{\delta a_0}{\hat{a}_0}\right) $$

(4.15)

$$ \Rightarrow \delta Q = \hat{Q} \frac{\hat{Q}}{2\hat{a}_0} \delta a_0 $$

(4.16)

Typical values for the offset are few nanosecond ($1 < a_0 < 12$ for the StructureScan MiniXT System) where typical values for the curvature range from 30 to 120 \([nSec/m^2]\), making $Q$ more sensitive to the offset than to the curvature. The typical range for $\hat{Q}$ is obtained by combining the ranges mentioned above:

$$ 2\Delta = 0.06 < \hat{Q} < 2[m] $$

(4.17)
The sensitivity of the depth to a perturbation in $Q$ would be:

$$h = 0.5(\hat{Q} + \delta Q) - \sqrt{0.25 (\hat{Q} + \delta Q)^2 - \Delta^2} \approx 0$$ (4.18)

$$= 0.5(\hat{Q} + \delta Q) - \sqrt{0.25 \hat{Q}^2 - \Delta^2 \sqrt{1 + \frac{0.5\hat{Q}\delta Q}{0.25\hat{Q}^2 - \Delta^2}}} \approx 0$$

$$\approx 0.5(\hat{Q} + \delta Q) - \sqrt{0.25\hat{Q}^2 - \Delta^2} \left[1 + \frac{0.25\hat{Q}}{0.25\hat{Q}^2 - \Delta^2}\delta Q\right]$$

$$= \hat{h} + \left(0.5 - \frac{0.25\hat{Q}}{\sqrt{0.25\hat{Q}^2 - \Delta^2}}\right)\delta Q = \hat{h} + \frac{1}{2} \left[1 - \left(1 - \frac{4\Delta^2}{\hat{Q}^2}\right)^{-1/2}\right] \delta Q$$ (4.19)

$$\Rightarrow \delta h = \frac{1}{2} \left[1 - \left(1 - \frac{4\Delta^2}{\hat{Q}^2}\right)^{-1/2}\right] \delta Q$$ (4.20)

By using the range for $\hat{Q}$, one gets that $\delta h \approx 3\delta Q$ - so they are the same order of magnitude.

The sensitivity of the refraction coefficient would be:

$$n = \frac{c}{2} \left(a_0 - \frac{a_1^2}{4a_2} + \delta a\right) \left[(\hat{Q} + \delta Q)(\hat{h} + \delta h)\right]^{-1/2} \approx \frac{c}{2} \left(a_0 - \frac{a_1^2}{4a_2} + \delta a\right) (\hat{Q}\hat{h} + \hat{h}\delta Q + \hat{Q}\delta h)^{-1/2} = \frac{c}{2} \left(a_0 - \frac{a_1^2}{4a_2} + \delta a\right) (\hat{Q}\hat{h} + \left[\frac{\hat{h} + \frac{\hat{Q}}{2} \left[1 - \left(1 - \frac{4\Delta^2}{\hat{Q}^2}\right)^{-1/2}\right]}{2\hat{Q}\hat{h}}\right] \delta Q)^{-1/2} = \frac{c}{2} \left(a_0 - \frac{a_1^2}{4a_2} + \delta a\right) \left(\frac{\hat{Q}\hat{h}}{2\hat{Q}\hat{h}}\right)^{-1/2} \left[1 - \frac{\hat{h} + \frac{\hat{Q}}{2} \left[1 - \left(1 - \frac{4\Delta^2}{\hat{Q}^2}\right)^{-1/2}\right]}{\hat{Q}\hat{h}}\right] \delta Q \approx \hat{n} \left[1 - \frac{\hat{h} + \frac{\hat{Q}}{2} \left[1 - \left(1 - \frac{4\Delta^2}{\hat{Q}^2}\right)^{-1/2}\right]}{2\hat{Q}\hat{h}}\right] \delta Q$$ (4.21)

$$\Rightarrow \delta n = \hat{n} \delta Q + \frac{\delta a}{\sqrt{\hat{Q}\hat{h}}}$$ (4.22)
CHAPTER 4. GENERALIZATION OF THE AUTOMATED TARGET DETECTION ALGORITHM

Where \( \delta a \) is either \( \delta a_0 \) in the case of perturbation in the offset or \( \frac{a_1^2}{\delta a_2} \delta a_2 \) in case of a perturbation in the curvature. Noting that \( \frac{a_1^2}{\delta a_2} \ll 1 \), this part could be neglected for the curvature. Since both \( h \) and \( \hat{Q} \) could be quite small, the refraction coefficient is significantly more sensitive to any error which might be in the measurements. For that reason, when designing the final decision boundaries, one needs to extend them beyond the physical limit of the material. This allows noisy targets to be detected.

4.3 Decision Making Stage

During the previous steps of the algorithm, the GPR image was analyzed and converted into potential targets. When a new scan arrives, the T and A matrices get updated. The pre-processing stage continues by re-ordering the information in the rows of the T and A matrices, as was described in Algorithm 3. The next step is to use quadratic polynomial regression on each (ordered) row, to obtain the polynomial coefficient and the fitting error of the regression. From these coefficients, one can extract the model parameters, as was described in Section 4.2.4. These steps converted the image into potential targets, in a way that preserved the physical information of the signal, thus enabling an efficient detection stage. Since every potential target would consist of three rows in the T and A matrices, one gets that every potential target have 32 parameters which describe it:

- 18 polynomial coefficients and 6 fitting error values - the \( l_{inf} \) norm of the fitting error - from the regression process of each of the six rows.
- Three estimations for the depth and dielectric, as predicted be Equation 4.13 from the time curves.
- The polarity of the target (whether the middle row contains maxima or minima information).
- The sample number in which the target exists.

These 32 parameters define the parameter space, where each potential target is a single point in this space. In order to ensure a high probability of detection while maintaining low false detection rate, one needs the decision boundaries to tightly-fit the true target (i.e., minimizing the volume in which the true targets exists). This could be achieved in different approaches. In this work, the decision rules (which define the decision boundaries) were found manually: first, all the GPR images were analyzed and manually annotated. In this stage, the decision rules were adjusted to ensure that every true target will get detected. The targets were then added to a database. After
CHAPTER 4. GENERALIZATION OF THE AUTOMATED TARGET DETECTION ALGORITHM

all the images were analyzed, the database was used to create new rules and adjust existing ones. This was done by projecting the database into a two dimensional space, and creating a rule in that space - which means some (typically non linear) relationship between two of the parameters has to be maintained. Since the parameters have physical meaning, domain knowledge was applied for building the rules and for choosing which two dimensions should be examined. For example, from the propagation models one expects that the deeper the target is, the flatter it will be. This implies some relation between the target depth $h$ and the curvature of the main time curve $a_2$. Other examples are the expected relations between the curvatures of all six curves, or of that between the position of the centers of each curve to the fitting errors of that curve.

An example for such a relation, along with the derived decision boundary is presented in Figure 4.7. In this Figure, the fitting error of the main (middle row) time curve vs. the position of the minimal point of that curve is presented for all of the targets in the database.

![Figure 4.7](image)

Figure 4.7: Example of a single decision boundary that was created for the detection stage.

In order to minimize the false detection rate, the volume inside the decision boundaries needs to be minimized. In order to avoid over fitting, some margin were manually added between the target points and the boundaries. This is an optimization problem, which could be solved in several different approaches. In this work, it was solved manually, but the guiding reasoning behind this solution is similar to a single class SVM (See for example [36]). Section 4.4.3 compares the performance of a single class SVM and the performance of the manual approach that this work pursued.
CHAPTER 4. GENERALIZATION OF THE AUTOMATED TARGET DETECTION ALGORITHM

4.4 Results

The performance of the algorithm is detailed in this section, as was measured on ground truth data. The test set is described in Section 4.4.1, with the performance metrics detailed in Section 4.4.2. Since the decision rules were created manually for the algorithm, a comparison with an automated classifier (single class SVM [2]) was conducted, and is described in Section 4.4.3.

4.4.1 Testing the Algorithm

The algorithm was tested on 12 different concrete blocks, with targets at different depths in each block. Ground truth was available for these blocks (both dielectric constant, target location, material and size were known). 20 different StructureScan Mini XT Units were used to collect total of 361 data sets on these blocks, and the detection algorithm was performed on those data sets. The datasets include files from all possible time ranges, in order to assure the robustness of the algorithm against arbitrary choice of depth range and dielectric constant by the systems’ user. Examples for the blocks and their detection results are presented in Figures 4.8–4.10.
(a) Image of the block, before the concrete was poured.

(b) Detection on the block. The scan was taken off the center of the block.

Figure 4.8: Block with crossing bars and tension cords.
Figure 4.9: Block with Pan Decking.
4.4.2 Performance of the Algorithm

The StructureScan Mini XT system has two modes of operation – High and Low Scan density modes. The scan density of the system is either 800 or 400 scans/meter, depending on the mode of the system. The performance of the algorithm was evaluated separately for these two modes, as well as for the entire data set.

1. **Probability of Detection** - The total number of targets in those files is 2561, with 35 total misses, which means that the probability of detection for the algorithm is 98.65 percent. This is attributed to the fact that the decision boundaries were set manually, allowing the vast majority of the targets to be detected.

2. **False Detection Rate** - There are a total of 276 false detections in the entire data set. It should be noted that most of those are reflections of real targets but they were still counted as false detections. The data set consists of a total of 347,719 scans. This means that the probability of false detection is \( 7.9374 \times 10^{-4} \), or roughly a single false detection for every 1260 scans. Breaking this into high and low scan density modes, one see that the algorithm performs better.
in high scan density, as is demonstrated in Table 4.1. This comes as no surprise, because in the high density mode the algorithm uses larger T matrix (63 points per curve, as compared to the 41 points for the low density mode).

Table 4.1: Final False detections.

<table>
<thead>
<tr>
<th>Scan Density [Scans/Meters]</th>
<th>Number of false detections</th>
<th>Total Number of Scans</th>
<th>$P_{fa}$</th>
<th>Average Number of scans per false detection</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>175</td>
<td>116,317</td>
<td>0.0015</td>
<td>665</td>
</tr>
<tr>
<td>400</td>
<td>101</td>
<td>231,402</td>
<td>$4.3647 \times 10^{-4}$</td>
<td>2290</td>
</tr>
<tr>
<td>Total</td>
<td>276</td>
<td>347,719</td>
<td>$7.9374 \times 10^{-4}$</td>
<td>1260</td>
</tr>
</tbody>
</table>

3. **Total Runtime** - A [GPR] system handles hundreds scans in every second, and this leads for an upper bound on the runtime of the signal processing algorithms for a scan to be no more than 1 millisecond for processing a new scan. The averaged runtime of the algorithm was measured on the StructureScan Mini XT system to be 0.6 mSec per scan of 512 sample. This is significantly slower than the previous version of the algorithm (which was presented in [3]), but still meets the requirements for a real time detection algorithm.

### 4.4.3 Comparing the Algorithm Performance with That of a Single Class SVM

The detection stage of the algorithm is based on multiple decision rules, as was described in Section 4.3. This process follows the logic behind a single class SVM [36]: a new data point will be classified as a target based on its similarity to existing targets. It does have two main advantages when compared to the single class SVM:

- It alleviates the need to choose the kernel (and the parameters thereof) of the SVM.
- It allows for arbitrary boundaries, which might fit the data better than the SVM (thus reducing the false detection rate for the same probability of detection).

The disadvantage is the danger of over fitting the targets, which might degrade the performance of this algorithm when tested on different data sets. To ensure the high performance metrics required by this algorithm, a large and diverse set of files was collected.

In order to compare the performance of the algorithm to one from the literature, a single class SVM (using radial basis functions for the kernel) was trained on the same database of 2651 targets. The classifier was then given the task of classifying the database of the 276 false detections - and it classified all of them as true targets. This means that the performance of the SVM classifier
cannot exceed that of the current detector. In order to get an idea of the performance difference of our algorithm compared to one-class SVM, a new file was processed twice, with different decision rules at the final stage of the algorithm. Though without a ground truth, the file presents a complex scenario, having both multiple layers and crossing targets - both metallic and non metallic. The results of the algorithm are presented in Figure 4.11 and that of the SVM-based detection are presented in Figure 4.12.

Figure 4.11: Detection on the floor, Using Manually created decision rules for the detection stage.
In Figure 4.11, one can see four false detections and one missed target (the one on scan 1000). In contrast, Figure 4.12 shows that all the metallic targets were detected correctly (occasionally multiple times) but the non-metallic target (visible in scan 580 in Figure 4.11) was not. The number of false detection rose significantly. This implies that the volume contained by the automatically created rules of the SVM is greater than that of the volume of the target space contained by the manually created rules. We believe that the manually created rules performed better than the one class SVM because we were able to incorporate domain knowledge in addition to minimizing the volume in the target space in generating the rules.
Chapter 5

Discussion and Future Research

The current algorithm is built in a modular way, where almost each and every component could be replaced and/or improved. The modules shall be described in this chapter, along with a detailed analysis of their strengths and weaknesses, accompanied by suggestions for improving them. It should be noted that the chapter follows the data stream of the system, and not the order of the importance of the modules or the improvements.

5.1 Processing a New Scan

1. As the new scan arrives from the RADAR board, it is being filtered first. This is done for display purposes, but it helps the algorithm by enabling the next step, which uses information about the derivative of the scan. Currently this is a Band Pass (BP) Infinite Impulse Response (IIR) filter, with fixed frequencies which were found to yield good results on a large enough data sets. Another (single pole, HP IIR) filter works horizontally - In order to remove the background. The purpose of these filters are removing undesired features from the image (noise, reverberations of the signal, and layers, to name a few) and increase the contrast (SNR) of the targets. Both filters could be redesigned and re-implement in several ways:

- Combining them into a single, two dimensional filter (mask).
- Replacing them by adaptive filters.
CHAPTER 5. DISCUSSION AND FUTURE RESEARCH

2. Using the derivative of the scan to find the extrema points. This part could be improved in several ways:

- Using knowledge for the location of the extrema points in the previous scans to set a starting point for the search for them in the new scan.
- Parallelize the search.
- For some situations, certain points matter more than others. Examples include the single target case, which was discussed in Chapter 3. Other examples might include the search for voids (were minima points would be proffered) vs. metallic points (were maxima points would be. By limiting the type of targets, the number of relevant points from each scan would be halved, and the overall performance of the algorithm be increased.

5.2 Processing the T and A Matrices

1. The first (and possible the most crucial) step of this stage is going over the buffer and apply the Curve Building algorithm (Algorithm 3), as described in Chapter 4. This algorithm is the most inefficient part of the entire detection algorithm, taking more than half(!) the run time of the entire algorithm. Future research might go into early detection and elimination of undesired parts of the signal (at the scan processing stage), so as to eliminate the need for this part.

2. Using quadratic polynomial regression fitting on the resulted curve. This part is the heart of the detection algorithm, were the shape of the curve is revealed. As a sub-step, the shape is compared to the theoretical model, to solve for the model parameters. It is by no means the only method to do so. Few possible alternatives would be

- Bayesian approach: using the model of

\[
\begin{align*}
p(x|d = 1, h, n) &= t(x, h, n) + \text{noise in presence of a target} \\
p(x|d = 0, h, n) &= \text{noise without the presence of a target}
\end{align*}
\]  

(5.1)

With \(d\) being an indicator for the existence of a target and proper statistics for the noise models as well as priors to the model parameters. An optimal detector could be designed, which will (possibly as a by-product) uncover the model parameters from the data. Though possible, this would not be a simple task, since the noise in the samples (or even the curves) that do not contain a target at the center of them might be heavily influenced by the presence of a target, layer or a reverberation of the signal.

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CHAPTER 5. DISCUSSION AND FUTURE RESEARCH

- Maximal Likelihood approach: similar to the Bayesian approach, assuming that the samples of curves are the result of a model with unknown parameters (with added noise) may lead to finding the optimal parameters which best describe the data. Although more restricted than the Bayesian approach, it may be more practical since it avoids the problems of estimating the prior for the model parameters.

5.3 Decision Making

The algorithm described in this work suffers from a (relatively) high false alarm rate. In order to reduce it, one might use off the shelf algorithms like SVM or Neural Networks to separate targets from non-targets, based on the target parameters. Another related research would be a full analysis of the algorithm, to produce the performance curve of the algorithm and show the trade-off between the false detection rate and the probability of detection.

5.4 Additional topics for research

This work presented a detection algorithm, and the sections above described possible research directions related to this specific algorithm. However, the field of target detection for geophysical purposes has other research directions in it. Several of them will be covered in this section.

- In a given dataset there is typically a low number of targets - which means that it could be treated as a sparse dataset. Advances in the field of Compressed Sensing might enable a different detection algorithm, which would take advantage of this sparsity.

- A different approach might be to attempt reconstructing the volume by placing targets in it - using Probabilistic Graphical Models (PGMs) for example.

- This work focused solely on 2D data. A different problem exists when combining multiple datagrams into a 3D volume cube: there is the question of alignment, similar to the one existing in producing Synthetic Aperture Radar Images. Detecting targets in a volume would require a different algorithm, and possibly different approach altogether, than the one presented here.
Chapter 6

Conclusion

The knowledge of a target’s location might lead GPR users to try to uncover it, and the knowledge of a region free of targets might assist GPR users in selecting a suitable location for drilling. In both these cases, a high probability of detection and a low false detection rate is required. Since at this stage the algorithm is meant to assist in the detection, and not to replace the operator altogether, a stronger effort was put into obtaining the former, probably at the expense of the latter. It is the author’s hope and belief that future works will improve the results presented here. This will allow for creation of detections devices simple enough for the general public usage.

This work presents a good starting point for that goal - an efficient, real time target detection algorithm to be used on geophysical data sets. It was implemented and tested on a single GPR system (GSSI’s StructureScan™ Mini XT system), but it is a general algorithm and could be applied to any geophysical dataset - from both GPR and seismic devices - provided that the basic model assumption (of a Bi Static system) holds true for that system. At the core of the algorithm lies a simple shape detection. Its efficiency is being achieved by preprocessing the image, so that the targets are being searched only at specific regions.

A typical GPR dataset has tens of thousands of pixels in it. In order to handle the vast amount of data, the algorithm performs the following:

- Limit the search window to be of no more than 63 scans. At 512 samples per scan for the StructureScan Mini XT system, this gives us 32,256 pixels.
- By selecting the appropriate pixels from each scan, only 20 pixels out of every 512 are retained (reducing the data by a factor of 25).
CHAPTER 6. CONCLUSION

- By iterating through the resulting layers, only three out of the 20 are being processed (reducing the data by a factor of 7).
- By processing together groups of 63 scans and fitting a parabola to the data points, 63 values are reduced to just 3 (reducing the data again by a factor of 20).

The resulting is a target space with 32 parameters, and it was found to be sufficient to provide us with a good separation between real targets and false detections. The decision boundaries were found empirically, by analyzing datasets containing targets and ensuring that those targets are inside the detection zone (i.e. - designing the detection zone boundaries so that real targets will be detected). Compared to automated boundary setting algorithms, such as SVM or Neural Networks, the fact that this was a manual process has two main advantages:

- It provides an easy was to ensure a high probability of detection, as was specified for the algorithm. Indeed, the final algorithm reached a probability of detection exceeding 98%.
- It provides the freedom to create non linear boundaries, which might be more efficient to compute and fit the targets more tightly than a linear combination of kernel functions. Indeed, it bypass the problem of choosing the kernel functions altogether.

In creating this space, knowledge about how a GPR system operates was utilized. This assisted the detection process by providing criteria for the detection process. In order to do so, several common propagation models were investigated and compared. They were contrasted with real data, providing guidance for future researchers to use the best model appropriate for their needs.

There are several novel items in this work; possibly the most important of them is the relaxation of the homogeneous layers’ assumption. In most of the prior work, it was always assumed that the ground is composed of (possibly several) homogeneous layers - which means that the ground properties are constant within the layer. This work breaks the layer into (approximately vertical) segments, and assumes that each layer is only homogeneous in each segment. Moreover, the segments could overlap, which would mean that the requirement of a homogeneous medium could be relaxed in a slightly different way - by allow the properties of the layer to vary slow enough, compared to the segment size. The relaxed requirement would be that the change in the layer’s properties will be small relative to their average value. Another novel idea was the use of a simple polynomial regression to perform feature extraction and detect targets. Using this allows for a very efficient algorithm - one that works in O(m) operations per scan, where m is the number of samples in each scan. This, in addition to the other known properties of the regression analysis: reduces the overall
noise in the output, and increases the robustness of the overall algorithm. The use of Polynomial Regression combined with Taylor’s polynomial is unconventional. The reason that it works is that the uncertainty of the regression procedure, induced by the noise in the samples, overlaps some of the uncertainty region caused by the approximation error of the Taylor’s polynomial and the initial error of the model itself.

An ancient claim is that “the better is the enemy of good”. Like many other things this algorithm could be improved and enhanced. Several possible directions were already mentioned:

- Modify the pre-process stages. During the work on the algorithm, it was found to have a crucial effect, and so modifying the pre-processing stage will change the overall performance of the algorithm.

- Vary the algorithm parameters automatically, based on the data.

- Replace the “detection engine”- instead of using the regression process, one might use other approaches in order to determine the model parameters. Two likely candidates for that would be a Bayesian, and the Maximal Likelihood approached. The former approaches utilizes (or learn) prior knowledge regarding the statistics of the data. The latter attempts to obtain the optimal parameters which justify the data.

- Enhance the algorithm to detect other features in the datasets: layers, rectangular targets (pan-decking, for example) and other feature that might be of interest.

And so this work by no means provides an absolute and final solution to the problem it set out to resolve. It is the hope of the author that the reader will find this work provides a good enough basis for further research, adaptation and tailoring to his\ hers needs.
Appendix A

Calculating the Theoretical Curves

The different theoretical models which were presented in Chapter 2. In that Chapter, the resulted (predicted) curves were contrasted with the measured curve of different targets in known conditions. Though generating the curves for the Bi Static Model (see Section 2.1.5) and its derivative is relatively simple, the set of Equations 2.2 – 2.10 needs some manipulations in order to easily produce the curves. In this appendix these manipulations are presented. The curves for both the Full Model (See Section 2.1.3) and the Elevated, Mono Static Model (see Section 2.1.4) are produced using the same mechanism: Solving for the (one way) time of flight of the signal, when the transceiver is at a known location \((y, -g)\). For the Elevated, Mono Static Model, \(x = y\), that is the transmitter is at the exact point of the receiver. By doubling the resulted flight time, the two way flight time is obtained. In order to calculate the two way time of flight for the Full model, for each location \(x\) of the system, two (one way) flight times should be obtained: from the transmitter to the target \((y = x - \Delta)\) and from the target to the receiver \((y = x + \Delta)\).

In order to calculate the one way flight time of the signal from an arbitrary point \((y, -g)\) to the target, which is located at \((0, h)\), denote the crossing point of the energy to/from the ground by \((p, 0)\) - see Figure A.1.
APPENDIX A. CALCULATING THE THEORETICAL CURVES

Figure A.1: Ray Tracing schematics. A transceiver is located at (y,-g), and the target at (0,h).

From Figure [A.1] one gets the following set of equations.

\[ r_1 = \sqrt{(y-p)^2 + g^2} \quad (A.1) \]
\[ r_2 = \sqrt{p^2 + h^2} \quad (A.2) \]
\[ n = \frac{\Delta}{v_p} \quad (A.3) \]

Using Snell’s law in a similar way it was used in Section [2.1.3] yields

\[ \sin \Theta_1 = \frac{y-p}{r_1} = \frac{y-p}{\sqrt{(y-p)^2 + g^2}} \quad (A.4) \]
\[ \sin \Theta_2 = \frac{p}{r_2} = \frac{p}{\sqrt{p^2 + h^2}} \quad (A.5) \]
\[ n = \frac{(y-p)r_2}{pr_1} = \frac{(y-p)\sqrt{p^2 + h^2}}{p\sqrt{(y-p)^2 + g^2}} \quad (A.6) \]
\[ \epsilon_r = n^2 = \frac{p^4 - 2yp^3 + (h^2 + y^2)p^2 - 2yh^2p + y^2h^2}{p^4 - 2yp^3 + (y^2 + g^2)p^2} \quad (A.7) \]
\[ \Rightarrow 0 = (1 - \epsilon_r)p^4 - 2y(1 - \epsilon_r)p^3 + (h^2 + (1 - \epsilon_r)g^2)p^2 - 2yh^2p + y^2h^2 \quad (A.8) \]
Defining $\gamma \triangleq (\epsilon_r - 1)$, one gets

$$0 = \gamma p^4 + 2\gamma yp^3 - (h^2 + \gamma y^2 - \epsilon_r g^2)p^2 + 2yh^2p - y^2h^2$$  \hspace{0.5cm} (A.10)

This is a polynomial equation of the fourth order, which could easily be resolved - for example, via numerical methods. In general it will have four solutions. The correct solution has to satisfy that

$$\begin{cases} p \in \mathbb{R} \\ \text{sgn}(p) = \text{sgn}(y) \\ 0 \leq |p| \leq |y| \end{cases}$$  \hspace{0.5cm} (A.11)

Once the crossing point $(p, 0)$ is found, the time of flight for the single sided problem is given by

$$r_1 = \sqrt{(y-p)^2 + g^2}$$  \hspace{0.5cm} (A.12)

$$r_2 = \sqrt{p^2 + h^2}$$  \hspace{0.5cm} (A.13)

$$t = \frac{r_1 + nr_2}{c}$$  \hspace{0.5cm} (A.14)
Appendix B

Sensitivity Analysis of the Propagation Models

As a part of the detection algorithm, the model parameters are obtained, in order to assist the decision making stage. For that reason, it would be beneficial if the theoretical model used to predict the target’s shape was sensitive to the model parameters (target depth $h$ and refraction coefficient $n$), so that different curves would yield different parameters. As will be shown and analyzed here, this is indeed the case for the model used (Polynomial Approximation to the Bi Static Model – see Section 2.1.7). In practice, a target creates a unique shape; this means that the information regarding the parameters should be available by studying its shape. However, due to noise in and the limited precision of our measurements, obtaining those parameters might prove difficult. Using the correct model is therefore crucial.

When examining the sensitivity of a model to its parameters, we shall define two criteria:

- The **curve offset** is defined as the two way propagation time to the target when the system is directly above the target $x = 0$.

- The **effect of the change in curve’s shape** is defined as the change in the curve at $x = 8$ cm from the target.

It should be noted that our choice of parameters is somewhat arbitrary, and other parameters could have been used - for example, the $l_2$ norm of the change in the curve. However, it was found that these two are suffice for a qualitative analysis of the sensitivity of the model.
APPENDIX B. SENSITIVITY ANALYSIS OF THE PROPAGATION MODELS

In this appendix we shall analyze two of the six models presented in Chapter 2 – The full propagation model (which was shown to have the best fit in the sense of minimal offset) and the Polynomial Approximation to the Bi Static Model, which was shown to best describe the overall shape of the curve.

B.1 Sensitivity of the Full Propagation Model to the Parameters

A qualitative sensitivity analysis of the Full propagation model is presented in this section. Several realistic target depths (1 - 17 cm) were chosen for that purpose, along with dielectric constant values typical in concrete slabs (4 - 8). Varying a single parameter at the time and calculating the curves predicted by the Full model (See Appendix A), Figures B.1 – B.2 were obtained.

B.1.1 Effect of Parameter Changing on the Curve Offset

![Graph](image)

Figure B.1: Different Curves Produced by the Full Propagation Model for a target in different types of medium.
APPENDIX B. SENSITIVITY ANALYSIS OF THE PROPAGATION MODELS

![Figure B.2: Different Curves Produced by the Full Propagation Model for a target in different Depth.](image)

As could be observed in the Figures above, varying the dielectric of the medium or the target depth affects the curve offset. The effect on the curve offset is summarized in Table B.1 and in Table B.2. The difference column is calculated as the Curve Offset of the current offset minus the previous one.

Table B.1: Sensitivity analysis of the target depth on the resulting Curve offset in the Full Model.

<table>
<thead>
<tr>
<th>Target Depth</th>
<th>Curve Offset [nSec]</th>
<th>Difference [nSec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3574</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.9646</td>
<td>0.6072</td>
</tr>
<tr>
<td>9</td>
<td>1.5888</td>
<td>0.6242</td>
</tr>
<tr>
<td>13</td>
<td>2.2261</td>
<td>0.6373</td>
</tr>
<tr>
<td>17</td>
<td>2.8698</td>
<td>0.6437</td>
</tr>
</tbody>
</table>

As shown in Table B.1, the target depth on the Curve Offset (for a given dielectric constant of the medium) increases with the target depth. This non-linearity could be explained by the fact that in this model changing the depth $h$ will affect the crossing point $p$ of the signal to/from the medium.
APPENDIX B. SENSITIVITY ANALYSIS OF THE PROPAGATION MODELS

Table B.2: Sensitivity analysis of the medium’s permittivity on the resulting Curve offset in the Full Model.

<table>
<thead>
<tr>
<th>Dielectric Constant $\epsilon_r$</th>
<th>Curve Offset [nSec]</th>
<th>$\frac{\text{Curve Offset}}{\sqrt{\epsilon_r}}$ [nSec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.8039</td>
<td>0.4019</td>
</tr>
<tr>
<td>5</td>
<td>0.8886</td>
<td>0.3974</td>
</tr>
<tr>
<td>6</td>
<td>0.9646</td>
<td>0.3938</td>
</tr>
<tr>
<td>7</td>
<td>1.0340</td>
<td>0.3908</td>
</tr>
<tr>
<td>8</td>
<td>1.0982</td>
<td>0.3883</td>
</tr>
</tbody>
</table>

Changing the dielectric constant of the medium has a lesser effect on the Curve Offset than the target depth - as one might expect. In addition, since the crossing point $p$ is also affected by the dielectric constant of the medium, one gets that the change in the ratio between the Curve Offset and the refraction coefficient $n = \sqrt{\epsilon_r}$ is not constant. This, again, is explained by the fact that the crossing point $p$ moves towards the transceiver as we lower the dielectric.
B.1.2 Effect of Parameter Changing on the change of the curvature

By subtracting the offset of each of the curves from the curves themselves, a better view of the change in the shape of the curves was obtained. The results are presented in Figures B.3 - B.4.

Figure B.3: Different Shape of the Curves Produced by the Full Propagation Model for a target in different types of medium.

As seen in Figure B.3, the shape of the curves remains relatively unchanged when varying the dielectric constant. The maximal change (at \( x = 8 \) cm from the target, when comparing the curves created by \( \varepsilon_r = 8 \) and \( \varepsilon_r = 4 \)) is 8.9 pSec. A typical sampling interval of a GPR signal is 15 - 30 pSec (for a signal with a centered frequency of 2.5 GHz), and therefore this change in the shape of the curve would be negligible compared to the noise in the samples and the inherent limitation of the system.
APPENDIX B. SENSITIVITY ANALYSIS OF THE PROPAGATION MODELS

Figure B.4: Different Shape of the Curves Produced by the Full Propagation Model for a target in different Depth.

In contrast to the dielectric, the target depth does have a visible effect on the shape of the curve - for the aperture of the curve tend to increase with the depth. This is so due to the fact that the change in the flight time decreases with increasing the target depth, and our measuring point is fixed (at \( x = 8 \text{ cm} \)). Another visible change is the overall shape - for the curve produced for a target at \( h = 1 \text{ cm} \) has a relatively flat region, which is not observed in the other curves. It should be noted that such a shallow target would be well within the near field zone, and so the model’ assumptions no longer hold; still, it is interesting to see how the model behaves in extreme cases.

B.2 Sensitivity of the Approximated Bi Static Propagation Model to the Parameters

The Polynomial Approximation to the Bi Static model was shown to have the best fit (in the MMSE sense) to the data. For that reason, the detection algorithm was based on this model. It is therefore important to repeat the sensitivity analysis for this model, and see how accurate could one hope to get the parameters from the data.
APPENDIX B. SENSITIVITY ANALYSIS OF THE PROPAGATION MODELS

B.2.1 Analytical Analysis of the Sensitivity of the Model

The time of flight is given by Equation 2.24, which, along with Equations 2.25 – 2.27 enable a derivation of a full analytical sensitivity analysis.

\[ t(n, h, x) \approx \frac{1}{c} \left[ t(0) + \frac{1}{2} \frac{\partial^2}{\partial x^2}(0) x^2 \right] = \]

\[ = \frac{2n\sqrt{h^2 + \Delta^2}}{c} \left[ 1 + \frac{h^2x^2}{2(h^2 + \Delta^2)^2} \right] \]  \hspace{1cm} (B.1)

From Equation B.1 one gets a nice separation between the Curve Offset and it’s shape.

B.2.2 Effect of Parameter Changing on the Curve Offset

Using Equation B.1 we get the following response to a small perturbation \( \delta h \) in the Curve Offset:

\[ t(n, h + \delta h, 0) = \frac{2n\sqrt{(h + \delta h)^2 + \Delta^2}}{c} = \]

\[ = \frac{2n\sqrt{h^2 + \Delta^2 + 2h\delta h + (\delta h)^2}}{c} \approx \]

\[ \approx \frac{2n\sqrt{h^2 + \Delta^2}}{c} \sqrt{1 + \frac{2h\delta h}{h^2 + \Delta^2}} \approx \]

\[ \approx \frac{2n\sqrt{h^2 + \Delta^2}}{c} \left( 1 + \frac{h\delta h}{h^2 + \Delta^2} \right) = \]

\[ = t(n, h, 0) + \frac{2n}{c\sqrt{1 + (\frac{\Delta}{h})^2}} \delta h \]  \hspace{1cm} (B.2)

And from Equation B.2 one can get an expression for the change in the Curve Offset \( \delta t = t(n, h + \delta h, 0) - t(n, h, 0) \) in response of a change \( \delta h \) to the target depth.

Similarly, given a small perturbation \( \delta n \) in the refraction coefficient, one gets

\[ t(n + \delta n, h, 0) = \frac{2(n + \delta n)\sqrt{h^2 + \Delta^2}}{c} = \]

\[ = t(n + \delta n, h, 0) + \frac{2\sqrt{h^2 + \Delta^2}}{c} \delta n \]  \hspace{1cm} (B.3)

Examples of the changes in The curve Offset are given in Figures B.5 – B.6 and in Tables B.4 – B.4
APPENDIX B. SENSITIVITY ANALYSIS OF THE PROPAGATION MODELS

Figure B.5: Different Curves Produced by the Polynomial Approximation to the Bi Static Propagation Model for a target in different types of medium.

Figure B.6: Different Curves Produced by the Polynomial Approximation to the Bi Static Propagation Model for a target in different Depth.
APPENDIX B. SENSITIVITY ANALYSIS OF THE PROPAGATION MODELS

Table B.3: Sensitivity analysis of the target depth on the resulting Curve offset in the Full Model.

<table>
<thead>
<tr>
<th>Target Depth</th>
<th>Curve Offset [nSec]</th>
<th>Difference [nSec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5168</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.9528</td>
<td>0.4361</td>
</tr>
<tr>
<td>9</td>
<td>1.5503</td>
<td>0.5974</td>
</tr>
<tr>
<td>13</td>
<td>2.1802</td>
<td>0.6299</td>
</tr>
<tr>
<td>17</td>
<td>2.8209</td>
<td>0.6407</td>
</tr>
</tbody>
</table>

As one might expects from Equation [B.2], the change in the Curve Offset varies with the target depth (and the refraction coefficient).

Table B.4: Sensitivity analysis of the medium’s permittivity on the resulting Curve offset in the Approximated Bi Static Model.

<table>
<thead>
<tr>
<th>Dielectric Constant $\epsilon_r$</th>
<th>Curve Offset [nSec]</th>
<th>$\frac{\text{Curve Offset}}{\sqrt{\epsilon_r}}$ [nSec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.7780</td>
<td>0.3890</td>
</tr>
<tr>
<td>5</td>
<td>0.8698</td>
<td>0.3890</td>
</tr>
<tr>
<td>6</td>
<td>0.9528</td>
<td>0.3890</td>
</tr>
<tr>
<td>7</td>
<td>1.0292</td>
<td>0.3890</td>
</tr>
<tr>
<td>8</td>
<td>1.1003</td>
<td>0.3890</td>
</tr>
</tbody>
</table>

As could be seen from Equation [B.3], the change in the curve offset (when the change in the dielectric in constant) depends only on the target depth $h$, which is constant $h = 5$ cm in our example.
B.2.3 Effect of Parameter Changing on the change of the curvature

From Equation B.1 one can get an analytical expression for the sensitivity of the shape of the curve:

\[ t(n + \delta n, h, x)'' = \frac{(n + \delta n)h^2}{c(h^2 + \Delta^2)^{3/2}} = \]

\[ = t(n, h, x)' + \frac{1}{ch(1 + (\frac{\Delta}{h})^2)^{3/2}} \delta n \]

And so the deeper the target the less sensitive is this model to a change in the dielectric (or the refraction coefficient), as is demonstrated in Figure B.7

![The shape of the curve which the Approx. Bi Static Model generates for target depth of 5 cm](image)

Figure B.7: Different Shape of the Curves Produced by the Polynomial Approximation to the Bi Static Propagation Model for a target in different types of medium.
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\[
t(n, h + \delta h, x)'' = \frac{n(h + \delta h)^2}{c ((h + \delta h)^2 + \Delta^2)^{3/2}} = \frac{n(h^2 + 2h\delta h + \delta h^2)}{c (h^2 + 2h\delta h + \delta h^2)^{3/2}} \approx \frac{n(h^2 + 2h\delta h)}{c (h^2 + 2h\delta h + \Delta^2)^{3/2}} = \frac{n}{c (h^2 + \Delta^2)^{3/2}} \left( h^2 + 2h\delta h \right) \left( 1 + \frac{2h\delta h}{h^2 + \Delta^2} \right)^{3/2} \approx \frac{n}{c (h^2 + \Delta^2)^{3/2}} \left( h^2 + \left( 2h - \frac{3h^3}{h^2 + \Delta^2} \right) \delta h \right) = t(n, h, x)'' \left( 1 + \left( \frac{2}{h} - \frac{3h}{h^2 + \Delta^2} \right) \delta h \right) \Rightarrow \delta t = 2 \frac{t(n, h, x)''}{h} \left( \frac{\Delta^2 - h^2}{h^2 + \Delta^2} \right) \delta h
\]

(B.5)

The error is a multiplicative of the current curvature, which suggests a high degree of sensitivity. The curvature itself increases with deeper targets. This leads to the conclusion that the shape predicted by the polynomial approximation to the bi static model is very sensitive to the target depth. This is demonstrated in Figure B.8.
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Figure B.8: Different Shape of the Curves Produced by the Polynomial Approximation to the Bi Static Propagation Model for a target in different Depth.
Bibliography


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