Probing Coannihilation Regions of Supergravity Unification at the LHC

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Dedication

For my son.
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Abstract of Dissertation

This work addresses two of the main open problems of particle physics: the discovery of supersymmetry and the identification of the constituents of dark matter. Supersymmetry is a proposed symmetry of nature whose extension, the supergravity theory, is widely believed to govern the nature of physics beyond the standard model. One of the predictions of supersymmetry and supergravity theories is the existence of new particles, called sparticles, which are counterparts of the existing matter in the universe. Their discovery is essential for the confirmation of supersymmetry and supergravity as the governing frameworks for physics beyond the standard model. The work of this thesis focuses on the exploration of supersymmetry and supergravity models which can simultaneously explain—through the phenomenon of coannihilation—the observed amount of dark matter, i.e., about 27% of the total mass and energy of the universe. This work describes models for which the properties of sparticles are consistent with the observed properties of dark matter in the universe and discusses strategies for the discovery of sparticles at the Large Hadron Collider, which is currently collecting data at a center of mass energy of 13 TeV. The possibility of direct detection of dark matter in deep underground experiments such as LUX-ZEPLIN is also explored.
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Introduction

In the summer of 2012, ATLAS and CMS, the primary experimental collaborations at the Large Hadron Collider (LHC), announced the discovery of the Higgs boson [1–3], capping off an era in particle physics marked by the systematic search and discovery of the fundamental particles that make up the standard model of particle physics [4–9]. In the years since, the LHC has been upgraded to higher energy and luminosity capacity and is now probing for hints of new particles and new physics. The possibility of such a discovery has high energy physicists everywhere waiting for their first glimpse of what lies beyond the standard model. So far, however, all is quiet on the energy frontier.

The expectation that there are more particles to be discovered is more than a blind hope; the standard model, as well tested as it is within its domain, falls silent regarding many
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of the modern era’s most vexing fundamental physics questions. Empirically, there exists

data it cannot explain, such as the existence of dark matter. Theoretically, it presents

numerous issues, including its ad hoc nature, severe hierarchy problem, lack of unification

with gravity, and observed Higgs boson mass that seems to imply the instability of the

universe. Thus a search for physics beyond the standard model is imperative.

In this search, supersymmetry [10–15] occupies a privileged status as a leading framework

for physics beyond the standard model. By positing a fundamental symmetry between

bosons and fermions, supersymmetry adds to the standard model a spectrum of super-

symmetric particles (sparticles) which naturally solves many standard model problems.

Supersymmetry is an intrinsically high-scale symmetry connected rigidly to the spacetime

Poincaré symmetry and thus can come into play at the Planck scale. However, because

breaking a rigid symmetry is difficult, it is necessary to gauge supersymmetry, which leads

to supergravity unification [16]. The algebra of the extended theory contains Majorana

spinor generators in addition to the Poincaré generators, and models based on $N = 1$

supergravity (where $N$ is the number of spinor generators) allow for the possibility of

breaking supersymmetry in a way that yields the phenomena observed at and below the

weak scale [17].

Models based on $N = 1$ supergravity resolve several significant problems inherent in the

standard model. For example, while the standard model is based on the assumption of

da tachyonic Higgs boson mass, there is no mechanism in the standard model to generate

such a tachyonic mass. A remarkable aspect of supergravity unified models is that a
tachyonic Higgs mass is generated in a natural way in renormalization group evolution of high-scale parameters to the electroweak scale. In another problem, gauge coupling constants fail to unify with the mass spectrum of the standard model. This unification is achieved within supersymmetry and supergravity unified models. For more discussion of the standard model, its shortcomings, and the ways that supersymmetry and supergravity unified models can solve them, see Chapters 2 to 4.

Since 2012, one more piece of evidence has emerged that strengthens the case for supersymmetry. As mentioned above, in 2012 the ATLAS [2] and CMS [1] collaborations, using the combined 7 TeV and 8 TeV data sets, discovered the Higgs boson [3] with a mass of 125 GeV. Such a mass would make the universe unstable, since the standard model Higgs boson mass needed for stability up to the Planck scale must be greater than 129 GeV. With a Higgs boson mass of 125 GeV, the vacuum can be stable only up to about $10^{10} - 10^{11}$ GeV in the standard model [18]. In models based on supersymmetry, however, the vacuum remains stable up to the Planck scale even with a Higgs mass of 125 GeV.

Another important implication of the Higgs mass of 125 GeV concerns proton stability. Even with assumption of $R$-parity symmetry, proton decay can proceed via baryon- and lepton-number-violating dimension-five operators in supersymmetry (for a review see [19]). This decay depends on the scale of weak-scale supersymmetry. The 125 GeV Higgs mass requires that this scale be in the TeV region to boost the value of the Higgs mass, which lies below $M_Z$ at the tree level, to its experimentally observed value [20].
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Thus high values of weak-scale supersymmetry also tend to stabilize the proton against decay [21].

The Higgs mass of 125 GeV requires a significant loop correction in supersymmetry. However, the weak-scale supersymmetry mass that enters is the geometric average of the two stop masses, allowing for the possibility that one eigenvalue of the stop mass-squared matrix could lie in the few hundred GeV range, while the other eigenvalue is in several TeV range. More generally, there can be many types of split supersymmetry spectra, with some light sparticles and some heavy ones, provided the stop loop correction remains large. The sparticle landscape of supergravity unified models is itself rather large [22, 23], so it is expected that this sort of splitting can be achieved in multiple ways.

An important element of a large class of string and supergravity unified models is the prediction that under $R$-parity conservation, the lightest odd $R$-parity particle is a possible candidate for dark matter [24]. Renormalization group analyses show this to be the case [25]. The relic density of such a particle can be analyzed within the Big Bang cosmology. The relevant equation that governs this decay is the Boltzmann equation derived by Lee and Weinberg [26]. This equation involves the Hubble parameter $H$ of the Friedman-Robertson-Walker geometry as well as the thermally averaged cross section for annihilation of the dark sparticles into standard model particles and its back reaction (for more detail on this process see Chapter 5). For the case of the neutralinos, the annihilation cross section depends sharply on the composition of the lightest neutralino,
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assumed to be the lightest supersymmetric particle (LSP). In the minimal supersymmetric standard model, the neutralino has four components: a $U(1)$ gaugino $\tilde{\lambda}_1$, a neutral $SU(2)$ gaugino $\tilde{\lambda}_3$, and the up and down higgsinos $\tilde{H}_1$ and $\tilde{H}_2$.

In significant regions of the parameter space of supergravity unified models, the neutralino is mostly a $U(1)$ gaugino, and its annihilation cross section is therefore small enough that sufficient annihilation of dark matter into standard model particles is not possible. In such cases, to further reduce the relic density, a phenomenon known as coannihilation is needed. In coannihilation two or more particles enter in the annihilation process, enhancing annihilation of the dark matter and bringing its relic density down to a value at or below the observed limits. However, by construction, coannihilation involves two sparticles ($A$ and $B$) where $A$ is the dark sparticle and $B$ is the coannihilating sparticle. At the LHC, if such a sparticle $B$ is produced, it will decay according to $B \to A + X$, where $X$ consists of standard model states. Conventional supersymmetry signatures rely heavily on the size of the mass gap $\Delta \equiv (m_B - m_A)$, since a large $\Delta$ will lead to large kinetic energy of the visible state $X$, allowing discrimination of such events. However, in the coannihilation region, $\Delta$ is small such that $\Delta \ll (m_A + m_B)/2$, and the available kinetic energy for final states $X$ is similarly small, making the conventional identification of such processes difficult.

It was pointed out by Goodman and Witten [27] that dark matter could be observed in direct detection experiments where a dark matter particle scatters from a quark inside a nucleus, producing detectable signal from the nucleus recoil. It turns out that in the
coannihilation region the cross section for dark matter–proton scattering is in the range of $10^{-46}$–$10^{-48}$ cm$^2$, which, while small, is above the neutrino floor [28]. With increased sensitivity such cross sections could in principle be detected in future experiments [29, 30].

Thus the focus of this work is further exploration of the coannihilation regions in the context of high-scale models. Specifically we will use supergravity unified models with gravity-mediated breaking of supersymmetry. Such models also include a variety of string-based models, so the analysis here may have a broader applicability. It is to be noted that supergravity unified models with gravity mediation allow for both universal as well as non-universal soft breaking at high scales. Thus the low energy theory below the grand unification scale, after spontaneous breaking of supersymmetry, will contain soft parameters which in general could be non-universal across generations and also have non-trivial dependence on phases that violate charge conjugation (C) and parity (P), i.e. violate CP invariance. In the analysis of this work we will work on the real manifold of soft parameters but will allow for non-universalities. Such non-universalities are often needed to achieve the appropriate coannihilation regions with the desired properties outlined above.

We will discuss three coannihilation regions in some detail: stop coannihilation, gluino coannihilation, and stau coannihilation, including multi-particle coannihilation between the stau, chargino, and neutralino. For the case of stop coannihilation we use minimal supergravity unification with universal boundary conditions at the grand unification scale to generate a large mass gap between the two eigenvalues of the third generation up squark
mass-squared matrix. A detailed study indicates that regions in the parameter space exist where it is possible to achieve a large loop correction to the Higgs boson tree mass as well as coannihilate enough dark matter to have consistency with the current data. For the stop coannihilation analysis the optimal LHC signal regions are expected to be those based upon low-multiplicity jet events. Thus, the signal regions used were versions of the ATLAS monojet and low-multiplicity jet signal regions for 8 TeV LHC beam energy [31, 32]. Of these searches, the signal region for two jets with loose background rejection (2jI, where I stands for loose background rejection in the ATLAS nomenclature) was the optimal region. Furthermore, this region could be optimized for $\sim 30\%$ better performance for the stop coannihilation region in particular by changing the cuts on $\Delta \phi_{j1}, E_{T}^{\text{miss}}$, the angle between the leading jet transverse momentum and the missing transverse energy. The analysis found that coannihilation region stops as light as 375 GeV could still be awaiting discovery, and that stops in this region as heavy as 600 GeV could be discovered with the full integrated luminosity of 3000 fb$^{-1}$ of data expected to be achieved at the LHC. These results are thus very promising for the discovery of supersymmetry in the stop coannihilation region at LHC13. The results of this analysis are presented in [33] and discussed in Chapter 7.

Next we discuss gluino coannihilation. Since the gluino carries a large color factor it is expected to have a relatively high production cross section at a hadron collider, which is helpful in signature analysis. To achieve gluino coannihilation we need to have non-universality in the soft parameters in the gaugino sector. The decay topologies for events
in which gluinos are produced are very similar to those of the stop model because both the stop and the gluino carry color. Thus, the low-multiplicity jet signal regions are also the most appropriate signal regions for the gluino coannihilation region. The signal regions used are the low-multiplicity jet ATLAS signal regions, updated for 13 TeV LHC beam energies [34]. In close agreement with the results for the stop case, the signal region found to be best for the gluino coannihilation region is the region for two jets with medium background rejection (2jm, where m stands for medium background rejection). Based upon these results, coannihilation region gluinos with a mass as low as 700 GeV would have yet evaded discovery, and the LHC can expect to be able to observe coannihilation region gluinos up to masses around 1300 GeV. Some optimization on the 2jm signal region was found to be possible by manipulating the cuts on the quantity $E_T^{\text{miss}}/\sqrt{\sum P_T}$, the ratio of the missing transverse energy to the square root of the scalar sum of the transverse momenta of all jets. The results of this analysis are presented in [35] and discussed in Chapter 8.

Finally, we investigate the stau coannihilation region. To achieve stau coannihilation consistent with the Higgs boson mass and dark matter limit we utilize the $\tilde{g}$SUGRA model [36]. Here the universal scalar mass is low but the gluino mass is high, lying in the several TeV region. The large gluino mass splits the squark masses from the slepton masses, driving the squark masses high while allowing the slepton masses to remain low. Because staus are color singlets, the decay topologies differ greatly from those of the stop and the gluino coannihilation models. Two different search strategies are employed, one
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searching directly for initial state radiation and tau leptons which arise from the decay of the staus [37], and the other searching for the signatures of the electroweak gauginos, namely the second neutrino and the chargino, which can produce both multileptonic signals and jets [38–40]. In addition, it is possible through additional nonuniversalities to lower the chargino mass close to the stau mass, allowing multi-particle coannihilation to be the mechanism that decreases the relic density. The stau coannihilation model is presented in [41] and discussed in Chapter 9.

A number of computational tools are employed in the analyses describe above. These include MadGraph and MadEvent [42–44], which includes Pythia [45] for hadronization of decays and Delphes [46] for detector simulation and event reconstruction within the geometry and triggering environment of a detector. Signal region analyses are run using scripts in ROOT [47]. To compare to the standard model background, pre-generated SNOWMASS backgrounds [48] are run against the same signal region scripts. Other tools utilized include SOFTSUSY [49] for the mass spectrum calculations, SUSY-HIT [50], for decay widths and branching ratios, and micrOMEGAS [51] for the relic density and other standard model calculations. Processing of data files is facilitated with PySLHA [52]. With these tools, it is possible to describe not only the high-scale input parameters that characterize the coannihilation regions of interest, but also the nature of the resulting decays if production occurs at the LHC, and the types of signal regions most likely to uncover the existence of sparticles belonging to such a parameter region.

The outline of the rest of this thesis is as follows: in Chapter 2 we give a brief overview of
Chapter 1. Introduction

the structure of the standard model, including the theoretical and experimental successes and deficiencies. Chapter 3 introduces the theoretical framework of supersymmetry and the minimal supersymmetric standard model (MSSM), detailing the motivation for such a model, in particular focusing on the ways that supersymmetry provides answers to some of the issues discussed in Chapter 2. In Chapter 4 we discuss supergravity unification, which provides the framework for high-scale models, known as supergravity unified models, on which the work of later chapters is based. Chapter 5 outlines key elements of dark matter, in particular describing the coannihilation processes studied in this work, as well as the reasons that these processes are important for generating realistic dark matter. Chapter 6 describes the simulation tools and techniques used to analyze and predict the discovery potential of coannihilation region models at the LHC. An analysis of the parameter space of supergravity unified models consistent with stop coannihilation is given in Chapter 7. A similar analysis but for gluino coannihilation is given in Chapter 8. Another coannihilation region, the stau coannihilation region, is investigated in Chapter 9. Future prospects are discussed and conclusions are drawn from the work as a whole in Chapter 10.
Chapter 2

The Standard Model

2.1 Introduction to the Standard Model

The standard model of particle physics is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, where the $SU(2)_L \times U(1)_Y$ factor describes the electroweak interactions [4–6, 9] and the $SU(3)_C$ factor describes the strong force interactions [8]. Particles that are charged under a gauge group belong to a nontrivial representation of that group, while uncharged particles belong to the trivial singlet representation. The particles that make up the standard model consist of three generations of quarks and leptons, which are
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represented by the left-handed $SU(2)_L$ doublets

$$Q_{Li} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, \quad L_{Li} = \begin{pmatrix} \nu_i \\ e_i^- \end{pmatrix}_L, \quad (2.1)$$

and the right-handed $SU(2)_L$ singlets

$$u_{Ri}^1, \quad d_{Ri}^1, \quad e_{Ri}^1, \quad (2.2)$$

where $i = 1, 2, 3$ is the generation index. In addition there is an $SU(2)_L$ doublet of scalar Higgs bosons

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}. \quad (2.3)$$

The quarks carry color, while the leptons are color singlets. The quarks $u_{Li}$ and $d_{Li}$ belong to the $3$ representation of $SU(3)_C$, while $u_{Ri}^1$ and $d_{Ri}^1$ belong to the $\bar{3}$ representation of $SU(3)_C$.

Interactions in the standard model are mediated by gauge bosons in the adjoint representation of the relevant group. Electroweak interactions between fermions are mediated by three $SU(2)_L$ gauge bosons $A^a_\mu$ ($a = 1, 2, 3$) and the $U(1)_Y$ gauge boson $B_\mu$. The color interactions are mediated by the $SU(3)_C$ gauge bosons $g^a_\mu$, ($a = 1 \ldots 8$), known as gluons. Table 2.1 summarizes the standard model particles and their representations in $SU(3)_C \times SU(2)_L \times U(1)_Y$. 

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<table>
<thead>
<tr>
<th>Field</th>
<th>Particle</th>
<th>$SU(3)_C$</th>
<th>$SU(2)_L$</th>
<th>$U(1)_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{iL}$</td>
<td>$(u_{Li}, d_{Li})$</td>
<td>3</td>
<td>2</td>
<td>1/3</td>
</tr>
<tr>
<td>$u_{Ri}^\dagger$</td>
<td>$u_{Ri}^\dagger$</td>
<td>3</td>
<td>1</td>
<td>4/3</td>
</tr>
<tr>
<td>$d_{Ri}^\dagger$</td>
<td>$d_{Ri}^\dagger$</td>
<td>3</td>
<td>1</td>
<td>−2/3</td>
</tr>
<tr>
<td>$L_{Li}$</td>
<td>$(\nu_{Li}, e_{Li}^−)$</td>
<td>1</td>
<td>2</td>
<td>−1</td>
</tr>
<tr>
<td>$e_{Ri}^\dagger$</td>
<td>$e_{Ri}^\dagger$</td>
<td>1</td>
<td>1</td>
<td>−2</td>
</tr>
<tr>
<td>$G$</td>
<td>$g_\mu^a$</td>
<td>Adj.</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$A$</td>
<td>$A_\mu^a$</td>
<td>1</td>
<td>Adj.</td>
<td>0</td>
</tr>
<tr>
<td>$B$</td>
<td>$B_\mu$</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$H$</td>
<td>$(H^+, H^0)$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2.1: Table of standard model particles based upon their representations in $SU(3)_C \times SU(2)_L \times U(1)_Y$, where $i$ labels fermion generation and Adj. indicates the adjoint representation. Hypercharge values are assigned according to the convention $Q = T_3 + \frac{Y}{2}$.

2.2 The Standard Model Lagrangian

The basic Lagrangian that governs the standard model can be written as

$$ L_{SM} = L_{EW} + L_{SU(3)_C}, $$

(2.4)

making the electroweak and color parts explicit. The Lagrangian for the electroweak interactions $L_{EW}$ can be further divided into the following parts:

$$ L_{EW} = L_{gauge} + L_{gauge-matter} + L_{Higgs} + L_{Yukawa}, $$

(2.5)
where the various terms are as follows:

\begin{align*}
    L_{\text{gauge}} &= -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \\
    F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\epsilon^{a}_{bc} A^b_\mu A^c_\nu, \\
    B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu; \\
\end{align*}

\begin{equation}
    (2.6)
\end{equation}

\begin{align*}
    L_{\text{gauge-matter}} &= \bar{L}_i \gamma^\mu D_\mu L_i + \bar{e}_{Ri} \gamma^\mu D_\mu e_{Ri} + \bar{Q}_i \gamma^\mu D_\mu Q_i \\
    &+ \bar{u}_{Ri} \gamma^\mu D_\mu u_{Ri} + \bar{d}_{Ri} \gamma^\mu D_\mu d_{Ri}, \\
\end{align*}

\begin{equation}
    (2.7)
\end{equation}

where \( D_\mu \) is the gauge covariant derivative given by

\begin{equation}
    D_\mu = (\partial_\mu + ig T_a A^a_\mu + ig' \frac{Y}{2} B_\mu),
\end{equation}

\begin{equation}
    (2.8)
\end{equation}

with \( g \) the coupling to \( SU(2)_L \), \( g' \) the coupling to \( U(1)_Y \), and \( T_a \) the generators of \( SU(2)_L \);

\begin{equation}
    L_{\text{Higgs}} = -(D_\mu H)^\dagger D^\mu H - V(H),
\end{equation}

\begin{equation}
    (2.9)
\end{equation}

where \( V(H) \) is the Higgs boson potential (given by (2.14) below); and finally

\begin{equation}
    L_{\text{Yukawa}} = y_u^{ij} Q^i_i H u_{Rj} + y_d^{ij} Q^i_i H^* d_{Rj} + y_e^{ij} L^i_i H^* e_{Rj},
\end{equation}

\begin{equation}
    (2.10)
\end{equation}

where the \( y_u^{ij}, y_d^{ij}, y_e^{ij} \) are the Yukawa coupling matrices for the up and down type quarks and the leptons, respectively.

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For the $SU(3)_C$ sector the color interactions are given by

$$L_{SU(3)_C} = -\frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a - \sum_i \bar{Q}_i \gamma^\mu (\partial_\mu - ig_3 G_{\mu}^a T_a) Q_i,$$

(2.11)

$$G_{\mu\nu}^a = \partial_\mu g_\nu^a - \partial_\nu g_\mu^a - g_3 f_{abc} g_\mu^b g_\nu^c,$$

with $g_3$ giving the coupling to $SU(3)_C$ and where $T_a$ ($a = 1...8$) are the generators and $f_{abc}$ are the structure constants of the group $SU(3)_C$.

For a theory to be an acceptable quantum field theory, there must be cancellation of anomalies. In general the anomalies involving gauge fields consist of

$$U(1)_Y^3, \quad U(1)_Y \times (SU(2)_L)^2, \quad U(1)_Y \times (SU(3)_c)^2, \quad (SU(3)_C)^3.$$

(2.12)

It is easily checked that these anomalies cancel using the fermionic spectrum of the standard model. In addition the gauge–gravitational anomalies also vanish since here the sum

$$\sum_i Y_i = 0$$

(2.13)

appears, which vanishes.

2.3 The Higgs Mechanism

The gauge bosons of the standard model as discussed above are all massless. Specifically, in addition to the massless gluons of $SU(3)_C$ found in nature, in the unbroken electroweak
sector we have $3 + 1$ massless gauge bosons. Unlike the case for the gluons, in nature we find that in the electroweak sector only the photon is massless. Further, the fermions of the unbroken standard model are also all massless, which is clearly unphysical. Both problems can be overcome if the model has spontaneous symmetry breaking. The basic mechanism for this was worked out in the pioneering papers [3, 53, 54], where a vacuum expectation value is given to the Higgs boson by the spontaneous symmetry breaking of the Higgs potential $V(H)$. To accomplish this, we consider the following form of the Higgs boson potential:

$$V(H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2.$$  \hspace{1cm} (2.14)

Since it is necessary to preserve charge, only the neutral component $H^0$ can enter into the potential. In addition, we must have $\lambda > 0$ for vacuum stability. Under these constraints the potential can have a minimum away from the symmetric point only if $\mu^2$ is negative—that is, if the Higgs boson is taken to be tachyonic. In this case, spontaneous symmetry breaking occurs and we may write

$$H^0 = \frac{1}{2} (v + h(x)).$$  \hspace{1cm} (2.15)

It is then seen that three Goldstone bosons arise from the spontaneous symmetry breaking

$$SU(2)_L \times U(1)_Y \to U(1)_{em},$$  \hspace{1cm} (2.16)
which are absorbed by the gauge bosons corresponding to the broken generators. These gain mass, becoming the $W^\pm$ and $Z$ bosons.

The entire theory can now be written around the new vacuum of the spontaneously broken phase of the theory. The $W^\pm$ and the $Z$ can be related to the original fields as follows:

$$W^\pm \equiv \frac{A^1 \mp iA^2}{\sqrt{2}}, \quad Z \equiv \frac{-g'B + gA^3}{\sqrt{g^2 + g'^2}}.$$  \hfill (2.17)

Defining the weak angle $\theta_W$ so that

$$\tan \theta_W \equiv \frac{g}{g'},$$  \hfill (2.18)

the masses of the $W^\pm$ and the $Z$ bosons are given by

$$M_W = \frac{g v}{2},$$

$$M_Z = \frac{M_W}{\cos \theta_W}.$$  \hfill (2.19)
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The field of the $Z$ boson can then be more simply expressed as

$$Z = -\sin \theta_W B + \cos \theta_W A^3$$  \hspace{1cm} (2.20)

and the orthogonal combination is the photon field

$$A = \cos \theta_W B + \sin \theta_W A^3,$$  \hspace{1cm} (2.21)

which remains massless. In addition to the two massive vector bosons $W^\pm$ and $Z$, there remains a real scalar field $h(x)$ which is also massive. This mass is given by

$$m_h = \sqrt{2\lambda v}.$$  \hspace{1cm} (2.22)

$h(x)$ is the field of the Higgs boson, which was discovered in experiments at CERN in 2012 [1, 2].

The $W^\pm$, $Z$, and Higgs bosons are not the only particles which gain mass in the presence of a nonzero Higgs vacuum expectation value. The previously massless fermions can gain mass after the spontaneous breaking of electroweak symmetry through Yukawa
interactions with the Higgs field. In this case, starting from (2.10), we have

\[ L_{\text{Yukawa}} = y^i_u Q_i^\dagger H u_{Rj} + y^i_d Q_i^\dagger H^* d_{Rj} + y^i_e L_i^\dagger H^* e_{Rj} \]

\[ = m_{ui} \bar{u}_i(x) u_i(x) + m_{di} \bar{d}_i(x) d_i(x) + m_{ei} \bar{e}_i(x) e_i(x) \]

\[ + 2^{1/4} \sqrt{G_F} m_{ui} \bar{u}_i(x) u_i(x) h(x) + 2^{1/4} \sqrt{G_F} m_{di} \bar{d}_i(x) d_i(x) h(x) \]

\[ + 2^{1/4} \sqrt{G_F} m_{ei} \bar{e}_i(x) e_i(x) h(x), \]

where \( G_F \) is the Fermi constant

\[ G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}. \]

From (2.23) above, a key prediction of the standard model is that the couplings of the Higgs boson to the fermions are proportional to the respective fermion masses. Because of the small scale of the Fermi constant, the couplings of the Higgs boson to the fermions are themselves small, with the largest coupling occurring for the most massive fermion, the top quark. The data from colliders thus far confirms this prediction, and more accurate tests are proposed at future \( e^+e^- \) colliders such as the ILC, CEPC, and FCC-ee(TLC). Colliders of this type are known as Higgs factories.
2.4 Shortcomings of the Standard Model

Despite its impressive agreement with electroweak data, the standard model has a number of empirically and theoretically unsatisfactory elements, some of which are listed below. We will see in subsequent chapters how supersymmetry and supergravity unification can remedy many of these deficiencies.

(i) As discussed above, a tachyonic mass term ($\mu^2$ in (2.14)) is central to achieving the spontaneous breaking of electroweak symmetry and to giving mass to the vector bosons and the fermions. However, the standard model does not provide an explanation for why this mass term is tachyonic; this feature is inserted into the Lagrangian by hand.

(ii) At the quantum level, the Higgs boson mass squared receives corrections $\delta m_H^2$ which are quadratically divergent and must be controlled by a cutoff $\Lambda$. For example, the contribution arising from the exchange of a $q\bar{q}$ in a loop, as shown in Figure 2.2, gives the result

$$\delta m_H^2 = c \left( \frac{g^2}{16\pi^2} \right) \Lambda^2 + \cdots$$

where $g$ is a coupling constant, $c$ is a constant of order 1, and $\Lambda$ is the cutoff to control the quadratically divergent integral. In a unified theory, $\Lambda$ can be as high as the grand unification scale $M_G \sim 10^{16}$ GeV or even the Planck scale $M_P \sim 10^{18}$ GeV. With $\Lambda = M_G$, the correction $\delta m_H^2$ approaches $10^{32}$ GeV$^2$ and a cancellation of one
part in $10^{28}$ would be necessary to bring the Higgs mass squared down to the $10^4$ GeV$^2$ scale observed in nature.

![Diagram](image)

**Figure 2.2:** Diagram for the correction to the Higgs mass squared arising from $q\bar{q}$ in a loop.

(iii) In the standard model, vacuum stability is not guaranteed up to the grand unification or Planck scales. An analysis using the next-leading order as well as the next-to-next-leading order corrections to the quartic coupling of the Higgs potential shows that requiring vacuum stability up to the Planck scale means that the Higgs boson mass must obey the condition [55]

$$m_h > 129.4 + 1.4 \left( \frac{m_t - 173.1}{0.7} \right) - 0.5 \left( \frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.0, \quad (2.26)$$

where $m_h$, $m_t$, and $M_Z$ are given in GeV. When including the theoretical error in the evaluation of $m_h$, estimated at $\pm 1.0$ GeV, and the experimental error on the top mass and $\alpha_s$, the analysis of [55] finds that the Higgs boson mass in the standard model must obey the constraint

$$m_h > 129.4 \pm 1.8 \text{ GeV} \quad (2.27)$$
in order to guarantee that the vacuum will be absolutely stable in the standard model up to the Planck scale. Since the measured value of the Higgs boson mass is 125 GeV, the standard model does not guarantee vacuum stability up to the Planck scale. Indeed, for the Higgs boson mass of 125 GeV, vacuum stability is guaranteed only up to scales between $10^9 - 10^{10}$ GeV.

(iv) The visible amount of matter constitutes only about 5% of the total mass and energy of the universe. Of the remainder, 68% is dark energy and 27% is dark matter. The standard model does not provide any candidates for either dark matter or dark energy.

(v) The standard model also does not explain the excess of baryons over anti-baryons in the universe. A parameter that quantifies the baryon excess in the universe is the baryon asymmetry parameter $\eta_B$, where

$$\eta_B = (n_B - \bar{n}_B)/n_{\gamma} \simeq 6 \times 10^{-10}.$$  \hspace{1cm} (2.28)

Here $n_B$ ($\bar{n}_B$) is the baryon (anti-baryon) number density and $n_{\gamma}$ is the number density of photons in the cosmic background radiation. The mechanism for the generation of a baryon excess is well known and consists of three conditions according to [56]: there must be interactions that violate baryon number, there must be charge and parity (CP) symmetry violation, and there must be out-of-equilibrium processes. The prediction for $\eta_B$ in the standard model is too small and thus the standard model
cannot explain the observed baryon asymmetry in the universe.

(vi) Because $SU(3)_C \times SU(2)_L \times U(1)_Y$ is a product group, there is no unification of the
electroweak and strong interactions in the standard model.

(vii) The parameters that enter in the standard model consist of 3 lepton masses, 6 quark
masses, 4 CKM parameters which include three mixing angles and a CP phase, 3
gauge coupling constants, 2 Higgs potential parameters $\mu$ and $\lambda$, and the strong CP
violating parameter $\theta_{QCD}$. Together, this makes 19 parameters in the theory. It is
hoped that a fundamental theory would not have such a large number of arbitrary
parameters, but would instead fix these parameters as a consequence of a deeper
relationship.

In theories based on supersymmetry and supergravity unification, items (i)-(iii) listed
above are fully resolved. Regarding (iv), supersymmetry and supergravity unification
theories provide a candidate for dark matter. In addition, supersymmetric theories ad-
dress item (v) by providing additional sources of CP violation and thus helping to generate
a baryon excess which can be compatible with experiment. The lack of unification dis-
cussed in item (vi) can be resolved by embedding the standard model gauge group in a
simple group such as $SU(5)$, $SO(10)$, or $E_6$, which, while not explicitly involving super-
symmetry, is compatible with it. A solution to item (vii) is more difficult and requires
in part an understanding of the particle generation and flavor puzzle, which again may
include supersymmetry.
Chapter 3

Supersymmetry

3.1 Introduction to Supersymmetry

Supersymmetry [10–14, 57–59] is a hypothesized extension to the standard model that proposes a symmetry between bosons and fermions: that bosons and fermions exist in paired supermultiplets such that every observed fermionic helicity state has a bosonic counterpart and vice versa. This can be realized by including alongside the Poincaré group a supersymmetry operator $Q_\alpha$ such that

\[
\{Q_\alpha, Q_\beta^\dagger\} = -2(P_L \gamma^\mu \gamma^0)_{\alpha\beta} P_\mu, \quad (3.1)
\]

\[
\{Q_\alpha, Q_\beta\} = [Q_\alpha, P_\mu] = [P_\mu, P_\nu] = 0, \quad (3.2)
\]
where $Q_\alpha$ is a Majorana spinor, $P^\mu$ is the energy-momentum four vector, and

$$P_L = \frac{1 - \gamma^5}{2} \quad (3.3)$$

is the Left-projection operator. The algebra represented by (3.1) and (3.2) is an $N = 1$ supersymmetry algebra, where $N$ denotes the number of Majorana spinor generators. Supersymmetry is an example of a graded Lie algebra and has the property that it is the only possible graded extension of a Lorentz covariant field theory. It is therefore unique as an extension to relativistic quantum field theories [60].

Irreducible representations of the supersymmetry algebra form supermultiplets which contain fermionic and bosonic states of equal mass. This can be seen in (3.2) as the energy-momentum operator $P^\mu$ commutes with the supersymmetry operator $Q_\alpha$. In fact, $Q_\alpha$ also commutes with the generators of the standard model gauge groups $SU(3)_C \times SU(2)_L \times U(1)_Y$, and therefore the superpartner fields belong to the same representations of $SU(3)_C \times SU(2)_L \times U(1)_Y$ as their standard model counterparts—they have the same charge, isospin, and color degrees of freedom. Since there are manifestly no superpartner fields with masses equal to the standard model particles, supersymmetry must be a broken symmetry (see Section 3.3).

The two simplest types of supermultiplets are the chiral multiplet and the vector multiplet. A chiral multiplet takes as its starting point a single Weyl spinor $\psi(x)$ and adds as a superpartner a complex scalar field $\phi(x)$. The spinor's two helicity states are balanced
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by one state for each of the real and imaginary components of the scalar field, such that the total number of states $n_F = n_B = 2$. The Weyl spinors can be identified with standard model quarks and leptons, in which case the complex scalar represents squarks and leptons.

For a vector multiplet, the starting point is a massless gauge boson $A^\mu(x)$. This is a spin-1 vector boson field, and since it is massless it has two helicity states. These states are matched to a Majorana spinor $\lambda(x)$, which also has two helicity states. Again we have $n_F = n_B = 2$ for this supermultiplet. The bosons of a vector multiplet are standard model gauge bosons, and the spinors are the superpartner gauginos.

### 3.2 Supersymmetry Formalism

The analysis of supersymmetry is made easier by adapting the two-component notation and by using the superfield formalism. The superfield formalism is useful because actions written in terms of superfields acting in superspace leads to inherently supersymmetric actions in ordinary space. We can begin by defining the two-component formalism, which makes use of Weyl spinors. We denote a left-handed Weyl spinor by $\xi_\alpha (\alpha = 1, 2)$ and a right-handed Weyl spinor by $\bar{\chi}^\dot{\alpha}$. Then we have $\xi^\alpha$ and $\bar{\chi}_{\dot{\alpha}}$ such that

$$\xi^\alpha = \epsilon^{\alpha\beta} \xi_\beta, \quad \bar{\chi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}} \bar{\chi}^\dot{\beta}$$  \hspace{1cm} (3.4)
Chapter 3. **Supersymmetry**

with $\epsilon_{\alpha\beta}$ defined by

$$
\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha},
$$

$$
\epsilon^{12} = 1,
$$

$$
\epsilon^{\alpha\gamma}\epsilon_{\gamma\beta} = \delta^\alpha_\beta = \epsilon_{\beta\gamma}\epsilon^{\gamma\alpha}.
$$

In this notation a four-component Dirac field $\psi$ and $\bar{\psi}$ can be written as

$$
\psi = \begin{pmatrix} \xi^\alpha \\ \bar{\chi}^\dot{\alpha} \end{pmatrix},
$$

$$
\bar{\psi} = (\xi^\alpha, \bar{\chi}^\dot{\alpha}),
$$

with

$$
\bar{\psi}\psi = \chi^\alpha \xi_{\alpha} + \bar{\xi}^\dot{\alpha} \bar{\chi}^\dot{\alpha}
$$

We can define the $2 \times 2$ matrices $\sigma^\mu$ and $\bar{\sigma}^\mu$ so that

$$
\sigma^0 = \bar{\sigma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = -\bar{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},
$$

$$
\sigma^2 = -\bar{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = -\bar{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
$$
Chapter 3. **Supersymmetry**

In terms of these matrices, the $\gamma$ matrices that enter into the Dirac equation can be written as

$$
\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}.
$$

(3.10)

As mentioned earlier, when analyzing supersymmetry it is convenient to introduce superspace and superfields. Superfields depend not only on the ordinary spacetime coordinates $x^\mu$ but also on Grassmann coordinates $\theta_\alpha$, which are anticommuting objects such that

$$
\{\theta_\alpha, \theta_\beta\} = 0, \quad \alpha, \beta = 1 \cdots n
$$

(3.11)

where the $\theta_\alpha$ satisfy the properties

$$
\int d\theta_\alpha = 0, \\
\int d\theta_\alpha \theta_\beta = \delta_{\alpha\beta}.
$$

(3.12)

Because the Grassman coordinates anticommute, an expansion of a function $\mathcal{F}$ over the coordinates $\theta_\alpha$ will terminate. For example, in the case of two coordinates $\theta_1$ and $\theta_2$, a function $\mathcal{F}(\theta_1, \theta_2)$ can be expanded as

$$
\mathcal{F}(\theta_1, \theta_2) = \mathcal{F}_0 + \mathcal{F}_1 \theta_1 + \mathcal{F}_2 \theta_2 + \mathcal{F}_{12} \theta_1 \theta_2.
$$

(3.13)
Similarly we can introduce another set of Grassmann coordinates $\bar{\theta}_{\dot{\alpha}}$ where

$$\{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = 0, \quad \dot{\alpha}, \dot{\beta} = \hat{1} \cdots \hat{n}, \quad (3.14)$$

and where $\bar{\theta}_{\dot{\alpha}}$ satisfies properties analogous to $\theta_\alpha$.

With the Grassman coordinates defined, we have an extension of ordinary spacetime $x^\mu$ into superspace $z^A = (x^\mu, \theta^\alpha)$. It is now possible to construct supersymmetric models of particle physics with superfields $\Phi(x, \theta, \bar{\theta})$ defined in superspace. First we discuss chiral multiplets, which can be left or right handed, and which accommodate matter and Higgs fields and their superpartners. Then we consider vector multiplets, which represent the gauge fields and their superpartners.

### 3.2.1 The Left Chiral Multiplet

A chiral superfield can be one of two types, left chiral or right chiral. Let us first consider the left chiral superfield. It is defined so that

$$\bar{D}_{\dot{\alpha}}(x, \theta, \bar{\theta})\Phi_L(x, \theta, \bar{\theta}) = 0, \quad (3.15)$$

with

$$\bar{D}_{\dot{\alpha}}(x, \theta, \bar{\theta}) = -\frac{\partial}{\partial \theta^{\dot{\alpha}}} - i\theta^{\alpha}(\sigma^\mu)_{\alpha\dot{\alpha}} \frac{\partial}{\partial x^\mu}. \quad (3.16)$$
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If we define $y^\mu$ such that

$$y^\mu = x^\mu + i\theta (\sigma^\mu)\bar{\theta} ,$$

(3.17)

it can be seen that

$$\bar{D}_\alpha(x, \theta, \bar{\theta}) y^\mu = 0 .$$

(3.18)

The constraints of (3.15) mean that $\Phi_L = \Phi_L(y, \theta)$ and can be expanded as

$$\Phi_L(y, \theta) = \phi(y) + \sqrt{2} \theta \psi(y) + \theta \theta F(y) .$$

(3.19)

Expanding this in terms of $x$, $\theta$, and $\bar{\theta}$ gives

$$\Phi_L(x, \theta, \bar{\theta}) = \phi(x) + \sqrt{2} \theta \psi(x) + \theta \theta F(x) + i\theta \sigma^\mu \bar{\theta} \partial_\mu \phi(x)$$

$$- \frac{i}{\sqrt{2}} \theta \theta \partial_\mu \psi(x) \sigma^\mu \bar{\theta} + \frac{1}{4} \theta \theta \bar{\theta} \bar{\theta} \Box \phi(x) .$$

(3.20)

The supersymmetry transformation is an operation that will rotate the bosonic states of a supermultiplet into the fermionic states and vice versa. This transformation can be parameterized by an infinitesimal anticommuting parameter $\epsilon_\alpha$. For chiral supermultiplets, the transformations are

$$\delta_\epsilon \phi(x) = \sqrt{2} \epsilon \psi(x) ,$$

$$\delta_\epsilon \psi_\alpha(x) = i \sqrt{2} (\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi(x) + \sqrt{2} \epsilon_\alpha F(x) ,$$

$$\delta_\epsilon F(x) = -i \sqrt{2} \partial_\mu \psi(x) \sigma^\mu \epsilon .$$

(3.21)
Applying the supersymmetry commutator to the bosonic field $\phi(x)$ gives the result

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\phi(x) = 2i(\epsilon_2 \sigma^\mu \epsilon_1^\dagger \sigma^\mu \epsilon_2^\dagger) \partial_\mu \phi(x).$$

Checking this against (3.1) confirms that, as expected, operation of the supersymmetry commutator on $\phi(x)$ gives $i\partial_\mu \phi(x)$, a translation of the field. A similar analysis holds for the remaining fields.

### 3.2.2 The Right Chiral Multiplet

An analysis much like the above can be applied to the right chiral superfield. Here a right chiral superfield $\Phi_R(x, \theta, \bar{\theta})$ is defined so that

$$D_\alpha(x, \theta, \bar{\theta})\Phi_R(x, \theta, \bar{\theta}) = 0,$$

with

$$D_\alpha(x, \theta, \bar{\theta}) = \frac{\partial}{\partial \theta^\alpha} + i(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial x^\mu}.$$  \hspace{1cm} (3.24)

One may check that

$$D_\alpha z^\mu = 0,$$  \hspace{1cm} (3.25)

where

$$z^\mu = x^\mu - i\theta(\sigma^\mu)\bar{\theta}.$$  \hspace{1cm} (3.26)
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Thus a right chiral superfield has the expansion

\[ \Phi_R(z, \bar{\theta}) = \bar{\phi}(z) + \sqrt{2} \bar{\theta} \bar{\psi}(z) + \bar{\theta} \bar{\bar{\theta}} F(z). \]  

(3.27)

Expansion of \( \Phi_R(z, \bar{\theta}) \) using the standard variables gives

\[ \Phi_R(z, \bar{\theta}) = \bar{\phi}(x) + \sqrt{2} \bar{\theta} \bar{\psi}(x) + \bar{\theta} \bar{\bar{\theta}} F(x) - i \theta \sigma^\mu \bar{\theta} \partial_\mu \bar{\phi}(x) \]

\[ + \frac{i}{\sqrt{2}} \bar{\theta} \sigma^\mu \bar{\theta} \partial_\mu \bar{\phi}(x) + \frac{1}{4} \bar{\theta} \bar{\theta} \bar{\bar{\theta}} \bar{\bar{\theta}} F(x). \]  

(3.28)

The product of two or more left chiral superfields gives a left chiral superfield, while the product of two or more right chiral superfields gives a right chiral superfield.

### 3.2.3 The Superpotential and the Kinetic Energy of Chiral Fields

A superpotential \( \hat{W} \) is a function that depends only on superfields of one chirality and is therefore a holomorphic function in those fields. Because of this, the terms of a superpotential involving \( F \) constitute a total divergence, and the action that involves these terms is invariant under supersymmetry transformations. This allows us to write terms in the Lagrangian of the type

\[ \mathcal{L}_{\text{int}} = \int d^2 \theta \ d^2 \bar{\theta} \ W(y, \theta) \delta(\bar{\theta}^2) + \text{h.c.}. \]  

(3.29)

Suppose we have left chiral superfields \( \Phi_i \). The superpotential is then a simple polynomial expansion in the superfields and the powers of terms in the superpotential are
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constrained only by the condition of renormalizability. Typically this permits terms in
the superpotential which are up to cubic in the superfields. We can write

\[ \hat{W} = L^i \Phi_i + \frac{1}{2} M^{ij} \Phi_i \Phi_j + \frac{1}{6} y^{ijk} \Phi_i \Phi_j \Phi_k. \quad (3.30) \]

Here $M^{ij}$ is a mass matrix and $y^{ijk}$ a Yukawa coupling matrix.

The terms in this Lagrangian that survive integration over the Grassmann variables have the form

\[ \mathcal{L}_{\text{int}} = -\frac{1}{2} W^{ij} \bar{\psi}_i \psi_j + W^i F_i + \text{h.c.}, \quad (3.31) \]

where

\[ W^i = \frac{\partial W}{\partial \Phi_i}, \]
\[ W^{ij} = \frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j}. \quad (3.32) \]

and where $W$ indicates the superpotential written only in terms of the chiral scalar fields $\phi_i$, i.e.

\[ W = L^i \phi_i + \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k. \quad (3.33) \]

The kinetic energy terms for the components of the chiral superfield arise from a product of a left chiral superfield and its hermitian conjugate so that

\[ \int \! d^2 \theta \, d^2 \bar{\theta} \, \bar{\Phi}_a^\dagger (z, \bar{\theta}) \Phi_a (y, \theta) = \phi_a^* \Box^2 \phi_a (x) + i \partial_\mu \bar{\psi}_a (x) \bar{\sigma}^\mu \psi_a (x) \]
\[ + F_a^* (x) F_a (x) + \text{total div.} \quad (3.34) \]
Adding the contributions to the Lagrangian from the kinetic terms and the superpotential terms we have

\[ \mathcal{L} = \phi_a^* \Box^2 \phi_a(x) + i \partial_{\mu} \bar{\psi}_a(x) \bar{\sigma}^{\mu} \psi_a(x) + F_a^*(x) F_a(x) \]

\[ + \frac{\partial W[\phi]}{\partial \phi_a} F_a(x) - \frac{1}{2} \frac{\partial^2 W[\phi]}{\partial \phi_a \partial \phi_b} \psi_a \psi_b + \text{h.c.}. \]  

(3.35)

The Lagrangian above contains the auxiliary fields \( F_i \). These can be eliminated by using field equations to define

\[ F_i^* = -W_i = -\frac{\partial W}{\partial \phi_i}. \]  

(3.36)

Inserting this back into the Lagrangian gives

\[ \mathcal{L} = \phi_i^* \Box^2 \phi_i(x) + i \partial_{\mu} \bar{\psi}_i(x) \bar{\sigma}^{\mu} \psi_i(x) - \frac{1}{2} \frac{\partial^2 W[\phi]}{\partial \phi_i \partial \phi_j} \psi_i \psi_j + \text{h.c.} - V(\phi, \phi^*). \]  

(3.37)

Here \( V(\phi, \phi^*) \) is the \( F \)-term scalar potential given by

\[ V(\phi, \phi^*) = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2. \]  

(3.38)

### 3.2.4 The Vector Superfield and Vector Field Strength

A vector superfield is a scalar superfield \( \Phi(x, \theta, \bar{\theta}) \) which obeys the constraint

\[ V = \Phi(x, \theta, \bar{\theta}) = \Phi^\dagger(x, \theta, \bar{\theta}). \]  

(3.39)
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In the Wess-Zumino gauge it is given by [61]

\[
V(x, \theta, \bar{\theta}) = -\theta \sigma^\mu \bar{\theta} v_\mu(x) - i \bar{\theta} \theta \lambda(x) + i \theta \bar{\theta} \bar{\lambda}(x) + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} D(x). \tag{3.40}
\]

Here \( v_\mu \) is a vector field, \( \lambda \) is a gaugino, and \( D \) is an auxiliary field. Similar to the process for the chiral superfields we introduce \( W_\alpha \) and \( \bar{W}_\dot{\alpha} \) so that

\[
W_\alpha = -\frac{1}{4} \bar{D} \bar{D} D_\alpha V(x, \theta, \bar{\theta}),
\]

\[
\bar{W}_{\dot{\alpha}} = -\frac{1}{4} D D \bar{D}_{\dot{\alpha}} V(x, \theta, \bar{\theta}),
\tag{3.41}
\]

where \( D \) and \( \bar{D} \) are as defined earlier in (3.16) and (3.24). The field strengths are then given by

\[
\int d^2 \theta \frac{1}{4} W^\alpha W_\alpha + \int d^2 \bar{\theta} \frac{1}{4} \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}} = -\frac{1}{4} v_{\mu\nu}(x) v^{\mu\nu}(x) + i \partial_{\mu} \lambda(x) \sigma^\mu \bar{\lambda}(x) + \frac{1}{2} D^2(x), \tag{3.42}
\]

where a total divergence has been discarded since it does not contribute to the action. We can directly extend the analysis to non-abelian gauge fields so that

\[
V_a(x, \theta, \bar{\theta}) = -\theta \sigma^\mu \bar{\theta} v_{\mu a}(x) - i \bar{\theta} \theta \lambda_a(x) - i \bar{\theta} \theta \lambda(x) + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} D_a(x), \tag{3.43}
\]
where the gauge multiplet belongs to the adjoint representation of a non-abelian gauge group. Analogous to the abelian case one can introduce objects $W_{\alpha a}$ and $\bar{W}_{\dot{\alpha} a}$ so that

$$W_{\alpha a} = \frac{1}{4} \left[ \bar{D}^2 D_\alpha V_a - \frac{i}{2} g f_{abc} \bar{D}^2 V_b D_\alpha V_c \right],$$

$$\bar{W}_{\dot{\alpha} a} = \frac{1}{4} \left[ -D^2 D_\dot{\alpha} V_a - \frac{i}{2} g f_{abc} D^2 V_b \bar{D}_\dot{\alpha} V_c \right],$$

(3.44)

and therefore

$$W_\alpha = W_{\alpha a} T^a,$$

(3.45)

$$\bar{W}_{\dot{\alpha}} = \bar{W}_{\dot{\alpha} a} T^a.$$

This allows us to compute the kinetic energy terms for the gauge multiplet using

$$\mathcal{L}_G = \frac{1}{4} \left[ \int d^2 \theta W_{\alpha a}^a W_{\alpha a} + \int d^2 \bar{\theta} \bar{W}_{\dot{\alpha} a}^a \bar{W}_{\dot{\alpha} a} \right].$$

(3.46)

An explicit analysis in the Wess-Zumino gauge then gives

$$\mathcal{L}_G = -\frac{1}{4} v_{\mu \nu a} v^{\mu \nu a} - i \lambda_\alpha \sigma^\mu D_\mu \chi_\alpha + \frac{1}{2} D_\alpha D^\alpha,$$

(3.47)

where

$$v_{\mu \nu a} = \partial_\mu v_{\nu a} - \partial_\nu v_{\mu a} - \frac{1}{2} g f_{abc} v_{\mu b} v_{\nu c},$$

$$D_{\mu ab} = \delta_{ab} \partial_\mu + \frac{g}{2} f_{abc} v_\nu^c.$$
### 3.2.5 Coupling Chiral Matter to Gauge Fields

To construct models of particle physics we need to couple chiral matter fields with gauge fields. Such an interaction is given by

\[
\mathcal{L}_{M-G} = \int d^2 \theta d^2 \bar{\theta} \Phi^\dagger e^{\bar{\psi} \tilde{V}} \Phi .
\]  

(3.49)

An explicit computation in the Wess-Zumino gives

\[
\mathcal{L}_{M-G} = \bar{D}_{\mu ij} \phi^*_j D^\mu_{i k} \phi_k - i \psi_i \sigma^\mu \bar{D}_{\mu ij} \bar{\psi}_j
\]

\[
+ \frac{i}{\sqrt{2}} g (\phi^*_i \lambda_{ij} \psi_j - \bar{\psi}_i \bar{\lambda}_{ij} \phi_j)
\]

\[
+ F_i^* F_i + \frac{1}{2} g \phi_i^* D_{ij} \phi_j,
\]  

(3.50)

where \(\lambda_{ij} = \lambda_a (T^a)_{ij}\) and \(D_{ij} = D_a (T^a)_{ij}\). Like the \(F_i\) fields, the fields \(D_a\) are auxiliary fields. They can be eliminated using field equations to get

\[
D_a = -g (\phi^* T_a \phi).
\]  

(3.51)

Eliminating both \(F_i\) and \(D_a\), the full scalar potential is given by

\[
V(\phi, \phi^*) = F^{*i} F_i + \frac{1}{2} D^a D_a = W_i^* W^i + \frac{1}{2} g_a^2 (\phi^* T^a \phi)^2.
\]  

(3.52)
3.3 Supersymmetry Breaking

As mentioned in section 3.1, the fact that the supersymmetry operator $Q_\alpha$ commutes with $P^\mu$ indicates that particles in a supermultiplet must have the same mass in unbroken supersymmetry. Similar to the case for electroweak symmetry breaking (see section 2.3), spontaneous breaking of supersymmetry requires that the energy minimum lie away from the symmetric point of the field, i.e. that the theory has a non-supersymmetric vacuum. Creating such a point is much more difficult in supersymmetry, however, as evidenced by using (3.1) to express the Hamiltonian in terms of the supersymmetry generator. Reverting to the four component notation of (3.1) we have

$$ H \equiv P^0 = \frac{1}{4} (QQ^\dagger + Q^\dagger Q) \geq 0. $$

(3.53)

In unbroken supersymmetry for a vacuum state $|0\rangle$ we have

$$ Q_\alpha |0\rangle = 0, $$

$$ Q^\dagger_\alpha |0\rangle = 0, $$

(3.54)

while in broken supersymmetry we have

$$ Q_\alpha |0\rangle \neq 0, $$

$$ Q^\dagger_\alpha |0\rangle \neq 0. $$

(3.55)
This, coupled with the fact that $H$ is positive semi-definite, indicates that the vacuum in unbroken supersymmetry will always lie below the vacuum in broken supersymmetry. Thus, spontaneous breaking of supersymmetry requires that the scalar potential $V(\phi, \phi^*)$ have a nonvanishing expectation value in the vacuum state $|0\rangle$. Inspection of (3.52) indicates that this is possible if either one of $F(x)$ or $D(x)$ is nonvanishing in the vacuum, i.e. that $\langle 0 | F(x) | 0 \rangle \neq 0$ or $\langle 0 | D(x) | 0 \rangle \neq 0$.

Unfortunately, breaking global supersymmetry, whether through $F$-term or $D$-term methods, does not leave a phenomenologically satisfactory set of fields. In particular, any method will generate a massless chiral fermion known as a Goldstino, which corresponds to the fermionic component of the supermultiplet associated with the auxiliary field responsible for breaking supersymmetry. Because there is no observed massless fermion in nature, any model resulting in a Goldstino must account for that particle in a way consistent with the physical world. We will see in Chapter 4 that this can be solved by gauging supersymmetry, where the Goldstino gives mass to the gravitino via the super-Higgs mechanism.

### 3.3.1 $F$-type Breaking

Global supersymmetry can be broken through the $F$-term via O’Raifeartaigh breaking [62]. Consider a superpotential of the form

$$W = \lambda \phi_1 (\phi_3^2 - \mu^2) + m \phi_2 \phi_3.$$  \hspace{1cm} (3.56)
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Using (3.36) we find that

\[
F_1^* = -\lambda (\phi_3^2 - \mu^2),
\]

\[
F_2^* = -m\phi_3,
\]

\[
F_3^* = -(2\lambda \phi_1 \phi_3 + m\phi_2).
\]

(3.57)

Examining the \( F_i \) fields demonstrates that there is no way to arrange the fields \( \phi_i \) such that all \( F_i \) vanish at the same time; thus supersymmetry is broken by the \( F \)-term in \( V \).

### 3.3.2 \( D \)-type Breaking

A second way to achieve spontaneous breaking of global supersymmetry is through the \( D \)-term. This can be demonstrated by coupling a chiral supermultiplet to a vector multiplet of a \( U(1) \) gauge field as described in Section 3.2.5 and including the linear Fayet-Iliopoulos term \[63\] \( L_{FI} = \xi D \). In this case, the scalar potential (3.52) is given by

\[
V = \xi D - \frac{1}{2}D^2 - gDq_i|\phi^i|^2
\]

(3.58)

and the equation of motion for \( D \) (3.51) becomes

\[
D = \xi - gq_i|\phi^i|^2.
\]

(3.59)
Thus it can be seen that if the scalars have $\langle \phi_i \rangle = 0$ then there must be a nonvanishing scalar potential

$$V = \frac{\xi^2}{2} > 0.$$  \hspace{1cm} (3.60)

### 3.4 The Minimal Supersymmetric Standard Model

The identification in Section 3.1 of chiral multiplets with the fermions and vector multiplets with the gauge bosons of the standard model suggests a simple extension of the standard model into supersymmetry: simply promote standard model fermions to chiral multiplets of supersymmetry and likewise promote standard model gauge bosons to vector multiplets. This scheme, plus some minor additional particle content in the Higgs sector for anomaly cancelation, is known as the minimal supersymmetric standard model, or MSSM.

In the Higgs sector of the MSSM, the standard model Higgs doublet is promoted to a chiral multiplet, giving a superpartner higgsino. Because the Higgs doublet carries hypercharge $Y$, existence of a single higgsino raises the possibility of uncanceled anomalies. As we have seen in (2.13), anomaly cancellation requires that $\sum Y = 0$. Thus, a second Higgs doublet must be introduced, this time with opposite hypercharge. Then with $Y_{H_1} = -1$ and $Y_{H_2} = +1$ anomaly cancellation is guaranteed. In the MSSM, $H_2$ gives mass to the up quarks and $H_1$ gives mass to the down quarks and the leptons.
With these extensions in mind, Table 2.1 can be reproduced including the superpartner fields. Table 3.1 displays the MSSM chiral supermultiplets while Table 3.2 displays the vector supermultiplets of the MSSM.

The MSSM superpotential is given by

$$W_{\text{MSSM}} = Y_d^{ij} H_1 Q_i d_j^c - Y_u^{ij} H_2 Q_i u_j^c + Y_e^{ij} H_1 L_i e_j^c - \mu \epsilon_{ij} H_1^i H_2^j. \quad (3.61)$$

As mentioned earlier, in global supersymmetry there is no phenomenologically viable way to spontaneously break supersymmetry. Instead, for the MSSM we add soft terms
by hand achieve this effect. Unfortunately there are upward of a hundred soft parameters that can be added to the Lagrangian to achieve symmetry breaking. A sample of such parameters is given below [59]:

\[-\mathcal{L}_{\text{soft}} = M_Q^2 [\tilde{u}_L \tilde{u}_L + \tilde{d}_L \tilde{d}_L] + M_U^2 \tilde{u}_R \tilde{u}_R + M_D^2 \tilde{d}_R \tilde{d}_R + M_L^2 [\tilde{\nu}_e \tilde{\nu}_e + \tilde{\nu}_L \tilde{\nu}_L] + M_E^2 \tilde{e}_R \tilde{e}_R + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_1^2 \epsilon_{ij} H_1^j H_2^i + \text{h.c.} + \epsilon_{ij} [\lambda_e A_e H_1^i \tilde{E}^*_L + \lambda_d A_d H_1^i \tilde{d}_L^* \tilde{d}_R - \lambda_u A_u H_2^i \tilde{q}_L^* \tilde{u}_R + \text{h.c.}] + \frac{1}{2} [\tilde{M}_3 \lambda_a \lambda_a + \tilde{M}_2 \lambda_A \lambda_A + \tilde{M}_1 \lambda_0 \lambda_0].\]

The Higgs potential is given by

\[V_H^0 = (\mu^2 + m_{H_1}^2)(|H_1|^2 + |H_1^-|^2)^2 + (\mu^2 + m_{H_2}^2)(|H_2^+|^2 + |H_2^0|^2)^2 - B\mu (H_1^0 H_2^0 - H_1^- H_2^+ + \text{h.c.}) + \frac{g^2}{2} |H_1^0 H_2^+ + H_1^- H_2^0|^2 + \frac{g^2 + g'^2}{8} (|H_1|^2 + |H_1^-|^2 - |H_2^+|^2 - |H_2^0|^2)^2,\]

where \(g\) and \(g'\) are the gauge couplings for the groups \(SU(2)_L\) and \(U(1)_Y\). Further loop corrections to the Higgs potential are important, and at the one-loop level one finds

\[\Delta V = \frac{1}{64\pi^2} \sum_a (-1)^{2J_a} (2J_a + 1) M_a^4 \ln \left[ \frac{M_a^2}{e^{3/2} Q^2} \right],\]

where \(J\) is the spin of the particle in the loop and \(Q\) is the renormalization group scale.

In the spontaneous breaking of electroweak symmetry, this potential is minimized under
the conditions that the charge and color be preserved, which requires that the charged Higgs fields do not get an expectation value. With this restriction the complex Higgs doublets have eight degrees of freedom. After spontaneous breaking, three degrees are absorbed to give masses to the $W^{\pm}$ and the $Z$ boson. This leaves us with five degrees of freedom. Of these, three are neutral fields and one is charged. These fields are called

$$h^0, \quad H^0, \quad A^0, \quad H^{\pm}. \quad (3.65)$$

The fields $h^0$ and $H^0$ are the light and heavy CP-even Higgs, $A^0$ is CP odd, and $H^{\pm}$ is the charged Higgs boson. If supersymmetry is true, one may identify the light CP-even Higgs boson $h_0$ with the recently discovered standard model Higgs field [1, 2, 20, 64–71].
Chapter 4

Supergravity Unification

4.1 Introduction to Supergravity Unification

As discussed in Section 3.3, in order to have a phenomenologically acceptable model of supersymmetry breaking, it is necessary to gauge supersymmetry. That is, it is necessary to promote supersymmetry from a global symmetry to a local symmetry such that $\epsilon_\alpha$ in (3.21) becomes a function of spacetime

$$\epsilon_\alpha \rightarrow \epsilon_\alpha(x). \quad (4.1)$$

Doing so requires inclusion of gravity in the theory, which leads to supergravity [16, 72–74] and supergravity unified models [17, 25, 59, 75–79].
Chapter 4. Supergravity Unification

To see how gauging supersymmetry requires the inclusion of gravity, consider the transformation of the free Lagrangian of a chiral multiplet given by

\[ \mathcal{L} = -\partial_\mu \phi^\dagger \partial^\mu \phi - \bar{\psi} \frac{1}{i} \gamma^\mu \partial_\mu \psi \quad (4.2) \]

under the local transformation of (4.1). Under this transformation one finds that (4.2) is no longer invariant and there is an uncanceled term

\[ \delta \mathcal{L} = \partial_\mu \bar{\epsilon}(x) \gamma^\mu \gamma^\nu \psi(x) \partial_\nu \phi(x) + \text{total div.} \quad (4.3) \]

To restore gauge symmetry, we follow the standard procedure for a gauge theory and introduce a compensating field. In this case, that field \( \psi_\alpha^\mu(x) \) is spin 3/2 and has the interaction

\[ \mathcal{L}_{\text{int}} = -\kappa \bar{\psi}_\mu(x) \gamma^\mu \gamma^\nu \psi(x) \partial_\nu \phi(x), \quad (4.4) \]

with transformation under (4.1) given by

\[ \delta_{\epsilon(x)} \psi_\mu = \kappa^{-1} \partial^\mu \epsilon(x), \quad (4.5) \]

where \( \kappa \) has the dimension of mass\(^{-1}\).

Because all fields must be members of supermultiplets, the spin 3/2 compensating field \( \psi_\mu \) must have a scalar superpartner, which in this case is spin 2. The massless spin 2 field is identified with the vierbein \( e^a_\mu \) of gravity. Thus gauging the supersymmetry operator \( Q_\alpha \)
requires gauging the other generators of the extended Poincaré algebra, making gravity appear naturally from the requirement of local supersymmetry.

The kinetic energy terms for the supergravity supermultiplet are given by the Einstein Lagrangian for the vierbein $\mathcal{L}_E$ and the Rarita-Schwinger Lagrangian $\mathcal{L}_{RS}$ [73, 74, 80]

$$\mathcal{L}_{2/3-2} = \mathcal{L}_E + \mathcal{L}_{RS} = -\frac{1}{2\kappa^2} \left[ \det e^a_\mu \right] R - \frac{1}{2} \epsilon^{\mu
u\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma, \quad (4.6)$$

where $R$ is the curvature scalar and $D_\mu$ is the covariant derivative defined by

$$D_\mu = \partial_\mu - \frac{i}{2} \omega_\mu^{ab} s_{ab}. \quad (4.7)$$

Here $\omega_\mu^{ab}$ is the spin connection and $s_{ab} = \sigma_{ab}/2$ where

$$\sigma_{ab} = \frac{i}{2} [\gamma_a, \gamma_b]. \quad (4.8)$$

### 4.2 Coupling Supergravity to Matter and Gauge Fields

The Lagrangian for supergravity coupled to chiral and vector supermultiplet fields depends ultimately on three functions [17, 81–84], given by

$$W(z), \quad K(z, z^\dagger), \quad f_{\alpha\beta}(z), \quad (4.9)$$
where $W$ is the superpotential as discussed in Section 3.2.3; $K$ is the Kähler potential, which depends on both the chiral fields and their conjugates; and $f_{\alpha\beta}$ is the gauge kinetic function, which in general is field dependent, but in the simplest case has the form $f_{\alpha\beta} = \delta_{\alpha\beta}$.

The tree Lagrangian depends on the superpotential $W$ and the Kähler potential $K$ in a specific combination $G$ where

$$G(z, z^\dagger) = -\frac{\kappa^2}{2} K(z, z^\dagger) - \ln \left[ \frac{\kappa^6}{4} |W(z)|^2 \right].$$ (4.10)

The Kähler metric is given by

$$G^a_{\dot{b}} = \frac{\partial^2 G}{\partial z^b \partial z_a},$$ (4.11)

with $G^a_{\dot{b}} = \delta^a_{\dot{b}}$ indicating a flat Kähler metric. The supergravity Lagrangian coupled with matter and gauge fields can be broken into bosonic and fermionic parts, given by

$$\mathcal{L} = \mathcal{L}_{\text{Bose}} + \mathcal{L}_{\text{Fermi}}.$$ (4.12)

The bosonic part of the Lagrangian is given by

$$\mathcal{L}_{\text{Bose}} = -\frac{e}{2\kappa^2} R(e, \omega) + \frac{e}{2} K_{\dot{a}a} \mathcal{D}_\mu z^a \mathcal{D}^\mu z^\dot{a} - \frac{1}{4} e F^a_{\mu\nu} F^\mu_{\alpha} - e^{-1} V.$$ (4.13)
Here $D_\mu z$ is the gauge covariant derivative, $e$ is the determinant of the vierbein, and $V$ is the scalar potential given by [17, 82]

$$e^{-1}V = \frac{1}{2} e^{\frac{\kappa^2}{T^a}} \left[ (K^{-1})^a_b \left( \frac{\partial W}{\partial z^a} + \frac{\kappa^2}{2} K_a W \right) \left( \frac{\partial W^*}{\partial z^b} + \frac{\kappa^2}{2} K^b W^* \right) - \frac{3}{2} \kappa^2 |W|^2 \right]$$

$$+ \frac{1}{8\kappa^4} |g_a G_a (T^a z)^a|^2 .$$

(4.14)

The potential given by (4.14) is invariant under the simultaneous transformations of $W$ and $K$ where

$$W \rightarrow e^{-f(z)} W ,$$

(4.15)

$$K \rightarrow K + f(z) + f^\dagger(z^\dagger) .$$

(4.16)

We note in passing that invariance under (4.16) is valid only at the tree level and is not respected at the quantum level [85–88]

### 4.3 Gravity-Mediated Supersymmetry Breaking

As discussed in Section 3.3, breaking global supersymmetry creates a massless Goldstino, which is problematic if supersymmetry is to be a viable model for physics beyond the standard model. With local supersymmetry, however, the presence of a gravitino solves this problem, as the gravitino absorbs the Goldstino and becomes massive, with mass given by

$$m_{3/2} = \frac{1}{\kappa} e^{-G/2} .$$

(4.17)
Thus, spontaneous symmetry breaking of local supersymmetry solves the problem of a massless fermion present in both unbroken local supersymmetry and broken global supersymmetry.

In gravity-mediated breaking, supersymmetry is broken by a super-Higgs field $Z$ with a general superpotential

$$W_{SH}(z) = \frac{m^2}{\kappa} f_s(\kappa Z),$$

(4.18)

where $f_s$ is a function of $\kappa Z$. The super-Higgs VEV then is

$$\langle Z \rangle \sim O(\kappa^{-1}) \sim O(M_{Pl}).$$

(4.19)

Because the field responsible for breaking local supersymmetry has a Planck-scale VEV, this super-Higgs field cannot couple directly to physical fields, otherwise these fields would themselves acquire Planck-scale masses. To enforce this, the total superpotential $W_{tot}$ is written in a separable form

$$W_{tot} = W(Z_\alpha) + W_{SH}(Z),$$

(4.20)

where $W(Z_\alpha)$ represents the superpotential for the visible fields of the model $Z_\alpha$. Without direct interaction terms in the superpotential, the breaking of supersymmetry is communicated to the physical sector via gravitational interactions. These contribute soft terms to the visible sector of the Lagrangian. The soft terms are typically of size $m^2/M_{Pl}$, where $m \sim 10^{10}$ GeV, resulting in soft terms which are approximately the size of the electroweak
In string theory, supersymmetry breaking can arise via gaugino condensation where \( <\lambda\gamma^0\lambda> \neq 0 \) (see [89, 90]).

Including the same simplifying assumptions as in Section 4.2, namely a flat Kähler metric and that the gauge kinetic function \( f_{\alpha\beta} \) transforms like a singlet, these soft terms in the Lagrangian will take the form, in contrast to (3.62), of

\[
L_{\text{soft}} = -m_0^2 Z_\alpha^\dagger Z_\alpha - \frac{1}{2} m_{1/2} \bar{\lambda}_\alpha \lambda_\alpha - \mu B_0 H_1 H_2 - A_0 W^{(3)}. \tag{4.21}
\]

This is the universal form of \( L_{\text{soft}} \), with the parameters

\[
m_0, \ m_{1/2}, \ \mu, \ B_0, \ A_0. \tag{4.22}\]

Here the scalar masses \( m_0 \), the gaugino masses \( m_{1/2} \), and the trilinear coupling \( A_0 \) are all in the universal case. They can be made nonuniversal via relaxation of the initial simplifying assumptions. For example, the gaugino mass term

\[
L^\lambda_m = -\frac{1}{2} m_{1/2} \bar{\lambda}_\alpha \lambda_\alpha \tag{4.23}
\]

depends on \( f_{\alpha\beta} \) through

\[
L^\lambda_m = -\frac{1}{2} m_{\alpha\beta} \bar{\lambda}_\alpha \lambda_\beta \tag{4.24}
\]

with

\[
m_{\alpha\beta} = -2\kappa^{-1} e^{-G/2} (G^{-1})_j^i G^i f_{\alpha\beta,i}^j. \tag{4.25}\]
Thus a different choice of gauge kinetic function will allow for nonuniversal gaugino mass terms so that

\[ m_{1/2} \rightarrow M_1, M_2, M_3. \]  

(4.26)

Similar arguments apply to \( m_0 \) and \( A_0 \).

### 4.4 Radiative Electroweak Symmetry Breaking

Recall from sections 2.3 and 2.4 that the tachyonic Higgs mass responsible for electroweak symmetry breaking in the standard model is inserted by hand. One of the advantages of supergravity unification is that this phenomenon is a natural consequence of the soft terms introduced due to supersymmetry breaking. In supersymmetry the scalar potential takes the form given in (3.63). As for the standard model Higgs mechanism, in order to preserve charge we must ensure that it is only the neutral Higgs fields which gain a VEV. This constraint reduces the potential to

\[
V^0 = (\mu^2 + m_{H_1}^2)|H_1^0|^2 + (\mu^2 + m_{H_2}^2)|H_2^0|^2 - B\mu(H_1^0 H_2^0 + \text{h.c.}) + \frac{g^2 + g'^2}{8}(|H_1^0|^2 - |H_2^0|^2)^2.
\]  

(4.27)

Including the one-loop correction \( \Delta V \) given in (3.64), the scalar potential \( V = V^0_H + \Delta V \) can be minimized to find the final VEV for \( H_1 \) and \( H_2 \), denoted as \( v_1 \) and \( v_2 \). At the potential minimum we find two relations. Defining

\[
\tan \beta \equiv \frac{v_2}{v_1},
\]  

(4.28)
these relations are given by

\[ \frac{1}{2} M_Z^2 = \frac{\mu_1 - \mu_2 \tan^2 \beta}{\tan^2 \beta - 1}, \]  
\( (4.29) \)

\[ \sin 2\beta = \frac{2|m_3^3|}{\mu^2_1 + \mu^2_2}, \]  
\( (4.30) \)

where

\[ m_{1,2}^2(t) = m_{H_{1,2}}^2(t) + \mu^2(t), \]  
\( (4.31) \)

\[ m_3^2 = -B(t) \mu(t), \]  
\( (4.32) \)

\[ \mu^2_i = m_i^2 + \Sigma_i, \]  
\( (4.33) \)

with \( \Sigma_i \) the loop correction such that

\[ \Sigma_i = \frac{1}{32\pi^2} \Sigma_a (-1)^{2s_a} n_a M^2_a \ln \left[ \frac{M^2_a}{e^{1/2} Q^2} \right] \left( \frac{\partial M^2_a}{\partial v_i} \right). \]  
\( (4.34) \)

Here \( a \) indicates a particle and \( n_a \) its degrees of freedom.

Thus \( (4.29) \) and \( (4.30) \) can be solved to determine the values of \( v_1 \) and \( v_2 \) after renormalization group evolution to some scale \( Q \) in terms of the GUT scale parameters given in \( (4.22) \). In practice, however, \( (4.29) \) is used to determine \( \mu^2 \), and therefore the magnitude (but not sign) of \( \mu \). Then \( (4.30) \) can be used to give \( B_0 \) in terms of \( \tan \beta \). This creates a
new set of GUT scale inputs, given by

\[ m_0, \ m_{1/2}, \ A_0, \ \tan \beta, \ \text{sign}(\mu). \]  (4.35)

The model making use of this input parameter set is known as minimal supergravity, or mSUGRA, and is the starting point for most of the parameter space analyses in this work. In addition, the parameter space of supergravity unified models has been analyzed in a number of previous works (see [22, 91–100]).
Chapter 5

Dark Matter and Coannihilation

As suggested in previous chapters, solving the dark matter puzzle is a crucial part of furthering our understanding of the universe [101]. Its existence, and the fact that it cannot be explained via standard model physics alone, makes including dark matter a challenge for any theory beyond the standard model. Furthermore, interesting connections between LHC physics and dark matter have been noted in previous works (see [102-108]). Thus, as we will see in this chapter, finding a supersymmetry model that satisfies the known properties of dark matter is an important motivation of this work. This chapter will briefly outline the evidence for the existence of dark matter, including reasons that any explanation of dark matter requires physics beyond the standard model. It will then indicate how supersymmetry provides a natural dark matter candidate particle in the form of the neutralino, if that neutralino is the lightest supersymmetric particle (LSP).
Finally it will describe the process of coannihilation, demonstrating why it is a necessary feature of the models considered in this work.

5.1 Evidence for Dark Matter

There is a great deal of evidence for the existence of dark matter, drawn from a wide variety of observations which are primarily cosmological in nature. While an evaluation of the totality of this evidence is outside the scope of this thesis, a few examples will suffice to motivate the need to explain dark matter in a theory of physics beyond the standard model. (For a more detailed review of dark matter, see [109].)

In many cases, evidence for the presence of dark matter takes the form of evidence for additional mass over and above that which is seen using traditional, i.e. photon-based, means of observation. One example of this is galactic rotation curves. In a rotating galaxy, we expect the circular velocity \( v(r) \) to obey the relation

\[
v(r) = \sqrt{\frac{GM(r)}{r}}
\]

where

\[
M(r) \equiv 4\pi \int \rho(r)r^2 \, dr.
\]

When the observed motion of rotating galaxies is compared to the observed density profile, it becomes strikingly clear that additional mass is necessary to reproduce the
velocity curve $v(r)$. In other words, the density profile $\rho(r)$ of visible matter would lead to a velocity curve which falls off with increasing $r$, but the observed velocity curves are actually fairly constant in $r$, demonstrating the presence of additional mass density beyond what is visible. This was initially established in [110].

Additional gravitational evidence comes via the observation of gravitational lensing, the process by which the presence of matter and energy alters the path of incoming light to create observable distortions of visible objects. These distortions can be used to infer the distribution of mass creating the lensing effect, and can therefore be used to indirectly observe the presence of matter that is otherwise undetectable. This method has been used most dramatically to observe the existence of dark matter in 1E0657-558, otherwise known as the Bullet Cluster [111]. The Bullet Cluster is a merging galaxy cluster in which the visible matter is dramatically displaced from the mass distribution measured via gravitational lensing. Furthermore, because this separation is a result of visible matter self-interacting via non-gravitational means (i.e. electromagnetism), the Bullet Cluster also provides evidence that dark matter is collisionless, meaning that its self-interaction cross section as well as its interaction cross sections with standard model particles are very small.

Yet another piece of evidence for dark matter’s existence which is especially relevant to this work comes from the success of cosmological modeling of the early universe. The most widely accepted model is known as the $\Lambda$CDM model. This model assumes the existence of non-baryonic cold dark matter, as it takes as two of its six inputs the baryonic
and cold dark matter relic densities, $\Omega_b h^2$ and $\Omega_c h^2$ respectively. In this notation $h$ is the dimensionless Hubble parameter, such that the Hubble constant is defined as $H = h \times 100$ km/s/Mpc, and $\Omega_i$ represents $\rho_i/\rho_{\text{crit}}$, the density of matter of type $i$ as a fraction of the critical density for a flat universe $\rho_{\text{crit}}$ (for more detail see Section 5.2). The success of this model in describing the observed properties of the cosmic microwave background, for example by the Wilkinson Microwave Anisotropy Probe (WMAP) [112], is strong evidence that non-baryonic cold dark matter is a large component of the matter content of our universe. WMAP’s results are obtained by measuring with high precision the temperature and polarization anisotropy of the cosmic microwave background, and from that, using computational techniques to extract maximum likelihood values for the $\Lambda$CDM input parameters. WMAP gives relic density results of $\Omega_b h^2 = 0.02266 \pm 0.00043$ and $\Omega_c h^2 = 0.1157 \pm 0.0023$. Similar results are achieved by the Planck experiment [113]. The particular value of the dark matter relic density will be very important to the analysis of this work, as any dark matter candidate must give a relic density equal to this value (or less for multi-component dark matter).

5.2 Relic Density of Dark Matter

We begin with the basic equations that govern the evolution of the universe. A homogeneous and isotropic universe is described by the Friedmann-Robertson-Walker line element
\[
ds^2 = -(dt)^2 + R(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right].
\]
Chapter 5. *Dark Matter and Coannihilation*

In the above, \( R(t) \) is the cosmic scale factor and \( k \) controls the geometry of the universe; a closed, flat, or open universe will correspond to \( k = 1, \ 0, \) or \(-1\), respectively. The equations of motion are given by the Einstein equation

\[
\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = -8\pi G N T_{\mu\nu},
\]

(5.4)

where for the stress tensor of the universe \( T_{\mu\nu} \) one assumes

\[
T_{\mu\nu} = p g_{\mu\nu} + (p + \rho) u_\mu u_\nu.
\]

(5.5)

Here \( \rho \) is the energy density, \( p \) is the pressure, and

\[
u^\mu = \frac{dx^\mu}{ds}
\]

(5.6)

is assumed to have the form

\[
u^t = 1, \quad \nu^i = 0, \quad i = 1, 2, 3.
\]

(5.7)

Using this assumed form of the stress tensor one can derive the two Friedman equations

\[
\left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{R^2},
\]

(5.8)

\[
\frac{\ddot{R}}{R} = -\frac{4\pi G}{3} (\rho + 3p).
\]

(5.9)
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Using the flat universe condition \( k = 0 \) in the first Friedman equation one can obtain a relation for the critical relic density

\[
\rho_{\text{crit}} = \frac{3H^2}{8\pi G},
\]

where we have defined the Hubble parameter \( H \) so that

\[
H \equiv \frac{\dot{R}}{R}.
\]

As mentioned in Section 5.1, the relic density of matter for a species labeled by \( i \) is expressed as

\[
\Omega_i = \frac{\rho_i}{\rho_{\text{crit}}}. \tag{5.12}
\]

The particular value of relic density for a particle species is determined by the mass and the interaction cross section for production and annihilation of that particle, and can be computed by considering the conditions of the early universe. At sufficiently high temperature, i.e. where \( T \gg m_\chi \), the dark matter species \( \chi \) is in thermal equilibrium, where the rates of production \( SM \bar{SM} \rightarrow \chi \bar{\chi} \) and annihilation \( \chi \bar{\chi} \rightarrow SM \bar{SM} \) are the same. Thus at thermal equilibrium we have [26]

\[
\frac{dn}{dt} + 3Hn + \langle \sigma v \rangle (n^2 - n_0^2) = 0, \tag{5.13}
\]

where \( n \) is the number density of dark particles, \( \sigma \) is the \( \chi - \chi \) annihilation cross section,
and \( v \) is the relative velocity of the annihilating dark matter particle. \( \langle \sigma v \rangle \) is the thermally averaged cross section given by

\[
\langle \sigma v \rangle = \frac{\int_0^\infty dv v^2 (\sigma v) e^{-mv^2/4T}}{\int_0^\infty dv v^2 e^{-mv^2/4T}} \tag{5.14}
\]

while \( n_0 \) is the equilibrium number density in the nonrelativistic approximation [114]

\[
n_0 = g \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T}, \tag{5.15}
\]

with \( g \) here giving the number of degrees of freedom of \( \chi \).

At a sufficiently high temperature, \( n \) is approximately equal to \( n_0 \), and \( \chi \) remains in thermal equilibrium with the early universe plasma. As the universe expands and \( T \) decreases below the mass of \( \chi \), the value of \( n_0 \) drops sharply in proportion to the Boltzmann factor \( e^{-m/T} \). The production rate quickly reaches the point, known as freeze-out, at which the density of dark matter is too low for additional annihilation to occur, and the dark particle abundance per comoving volume becomes fixed. Thus, below the freeze-out temperature we can neglect the \( n_0 \) term in (5.13) so that it becomes

\[
\frac{dn}{dt} + 3Hn + \langle \sigma v \rangle n^2 = 0, \tag{5.16}
\]
As described in [59], computing the relic density can be simplified by defining a new function $f(t)$ of the number density $n(t)$ where

$$f(t) = \frac{n(t)}{g_s T^3}.$$ (5.17)

Here $g_s$ is defined using the fact that entropy $s(t)$ is conserved as the universe expands and therefore

$$s(t) R^3(t) = g_s T^3 R^3 = C$$ (5.18)

is independent of time. Then from (5.13) and (5.17) we have

$$\frac{df(t)}{dt} = \frac{1}{g_s T^3} \left[ \frac{dn}{dt} + 3Hn \right]$$ (5.19)

and (5.13) can be written in terms of $f$ as

$$\frac{df(t)}{dt} = -g_s T^3 \langle\sigma v\rangle (f^2 - f_0^2).$$ (5.20)

with

$$f_0 = \frac{n_0(t)}{g_s(t) T^3(t)}.$$ (5.21)

Analogous to (5.16), $f(t)$ after freeze-out is given by dropping $f_0$ from (5.20):

$$\frac{df(t)}{dt} = -g_s T^3 \langle\sigma v\rangle f^2.$$ (5.22)
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To determine the freeze-out temperature for $\chi$, we must first obtain an expression for $T(t)$. In a radiation-dominated universe, $H = \frac{1}{2t}$ and therefore (5.10) can be written

$$\rho = \frac{3}{8\pi G} \frac{1}{4t^2}.$$  

(5.23)

Using the fact that such a universe also has

$$\rho = \frac{\pi^2}{30} g_s T^3,$$  

(5.24)

we can write $T(t)$ as

$$T(t) = \left(\frac{45}{16\pi^3 G g_s}\right)^{\frac{1}{2}} \frac{1}{\sqrt{t}}.$$  

(5.25)

Defining the quantity $x$ as

$$x = \frac{T(t)}{m}$$  

(5.26)

gives new expressions for (5.22) and $f_0$:

$$\frac{df}{dx} = m \left(\frac{45}{4\pi^3 G g_s}\right)^{\frac{1}{2}} g_s \langle \sigma v \rangle f^2(x),$$  

(5.27)

$$f_0(x) = \frac{2}{g_s} \left(\frac{1}{2\pi x}\right)^{\frac{3}{2}} e^{-\frac{x}{2}}.$$  

(5.28)

We can denote the freeze-out temperature as $x_f$ and the current temperature as $x_\gamma = T_\gamma/m$. By integrating (5.28) across this range, we obtain the expression

$$\frac{1}{f(x_\gamma)} = \frac{1}{f(x_f)} + I(x_f)$$  

(5.29)
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with the integral $I(x_f)$ given by

$$I(x_f) = m \int_0^{x_f} dx \left( \frac{45}{4\pi^3G} \right)^{\frac{1}{2}} \frac{g_s}{\sqrt{g_s}} \langle \sigma v \rangle .$$  \hspace{1cm} (5.30)

The relic density will be proportional to $1/I(x_f)$. By using the definition of $f$ in (5.17) to return to number density, and the fact that relic density is related to number density by mass, we have

$$\rho_\chi = \left( \frac{4\pi^3G}{45} \right)^{\frac{3}{2}} T^3 \frac{g_s(x_\gamma)}{\bar{J}(x_f)} ,$$  \hspace{1cm} (5.31)

with the integral $\bar{J}$ only containing parameters which vary in $x$:

$$\bar{J}(x_f) = \int_0^{x_f} dx \frac{g_s}{\sqrt{g_s}} \langle \sigma v \rangle .$$  \hspace{1cm} (5.32)

Thus, the final relic density for $\chi$ depends upon the annihilation cross section $\sigma$ according to $\langle \sigma v \rangle^{-1}$. Larger annihilation cross sections will yield larger relic densities and vice versa. As we will find, coannihilation will be required for the supersymmetry models considered here because cross sections are very small, resulting in a relic density which is much larger than the measured value for dark matter.

### 5.3 Neutralino Dark Matter

In supersymmetry there are four neutralinos, each of which is a different linear combination of the $U(1)$ gaugino $\tilde{\lambda}_1$, the neutral $SU(2)$ gaugino $\tilde{\lambda}_2$, and the up and down higgsinos
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$\tilde{H}_1$ and $\tilde{H}_2$. When it is the LSP, the lightest neutralino $\tilde{\chi}_1^0$ is a dark matter candidate particle. Besides satisfying a minimum dark matter criterion of being electrically neutral, the neutralino LSP is an ideal dark matter candidate for two reasons. The first is that, assuming $R$-parity conservation, it is stable; it does not spontaneously decay into some other particle. The second is that while electrically neutral, the neutralino does take part in weak interactions. Thus, it generally has a self-annihilation cross section $\sigma_{\tilde{\chi}_1^0}$ of appropriate order to have a relic density near the expected value. This phenomenon—that the strength of the weak force is such that self-annihilation cross sections are of roughly the right order to give approximately the right dark matter relic density—is known as the WIMP miracle, where WIMP stands for weakly interacting massive particle. The lightest neutralino is a prime candidate for WIMP dark matter.

As mentioned above, for the stability of the LSP we require $R$-parity [115], defined as

$$R = (-1)^{3B + L + 2s}, \quad (5.33)$$

where $B$ represents the baryon number, $L$ is the lepton number, and $s$ is particle spin. This definition has the result that $R$-parity is a multiplicative symmetry where every standard model particle has $R$-parity of +1, while all supersymmetric particles have $R$-parity of −1.

In the types of models studied here, however, the dominant component in the lightest neutralino is that of the $U(1)$ gaugino, sometimes called the bino. In the case of bino-like
dark matter, the only interactions possible are hypercharge interactions, i.e. those based on $U(1)_Y$. Thus, a very low self-annihilation cross section $\sigma$ results in relic densities that are much higher than the observed value. For these models, it is necessary to appeal to a mechanism outside of self-annihilation to bring the relic density down to the value observed for dark matter. This mechanism is coannihilation.

5.4 Coannihilation

Coannihilation [114, 116–119] is a process by which the LSP and other nearby sparticles annihilate together into standard model particles, which results in a lower relic density for the LSP relative to self-annihilation. Because the LSP is protected from direct decay, this process is critical for reducing the relic density of neutralino dark matter to observed levels when that dark matter would be overproduced. For two sparticles $\tilde{\chi}_i$ and $\tilde{\chi}_j$, the processes that affect the number density $n_i$ are

$$\tilde{\chi}_i\tilde{\chi}_j \leftrightarrow XX', \quad \tilde{\chi}_iX \leftrightarrow \tilde{\chi}_jX', \quad \tilde{\chi}_i \leftrightarrow \tilde{\chi}_jXX', \quad (5.34)$$
where \( X \) and \( X' \) stand for standard model states and it is allowed that \( i = j \). In this case, (5.13) for \( n_i \) is given by [114]

\[
\frac{dn_i}{dt} + 3Hn_i + \sum_{j,X} \left[ \langle \sigma_{ij} v \rangle (n_in_j - n_i^0 n_j^0) \\
- \left( \langle \sigma'_{ij} v \rangle n_in_X - \langle \sigma'_{ji} v \rangle n_jn_{X'} \right) \\
- \Gamma_{ij}(n_i - n_i^0) \right] = 0, \tag{5.35}
\]

with

\[
\sigma_{ij} = \sigma(\tilde{\chi}_i\tilde{\chi}_j \rightarrow XX'), \tag{5.36}
\]
\[
\sigma'_{ij} = \sigma(\tilde{\chi}_iX \rightarrow \tilde{\chi}_jX'), \tag{5.37}
\]
\[
\Gamma_{ij} = \Gamma(\tilde{\chi}_i \rightarrow \tilde{\chi}_jXX'). \tag{5.38}
\]

Because we are here considering the case where all sparticles \( \tilde{\chi}_i \) have decayed into the LSP \( \tilde{\chi}_1 \), (5.35) can be simplified considerably to

\[
\frac{dn}{dt} + 3Hn + \sum_j \langle \sigma_{ij} v \rangle (n_in_j - n_i^0 n_j^0) = 0 \tag{5.39}
\]

with \( i = 1 \). We can introduce the quantity \( r_i \) defined such that

\[
r_i \equiv \frac{n_i^0}{n^0} = \frac{g_i(1 + \Delta_i)^{3/2} e^{-\Delta_i/x}}{g_{\text{eff}}}, \tag{5.40}
\]

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where \( g_i \) are the degrees of freedom of \( \tilde{\chi}_i \), \( x \) is defined as in (5.26),

\[
ge_{\text{eff}} = \sum_j g_j (1 + \Delta_j)^{3/2} e^{-\Delta_j/x},
\]

and

\[
\Delta_i = \frac{m_i - m_1}{m_1}.
\]

By making the approximation

\[
\frac{n_i}{n} \approx \frac{n_i^0}{n^0}
\]

we can express (5.39) as

\[
\frac{dn}{dt} + 3Hn + \langle \sigma v \rangle (n^2 - n_0^2) = 0,
\]

where

\[
\sigma_{\text{eff}} = \sum_{ij} \sigma_{ij} r_i r_j
\]

\[
= \sum_{ij} \sigma_{ij} \frac{g_i g_j}{g_{\text{eff}}} (1 + \Delta_i)^{3/2} (1 + \Delta_j)^{3/2} e^{-(\Delta_i + \Delta_j)/x}.
\]

Thus for cases where \( \Delta \) is small and \( \sigma_{\text{eff}} \) is not exponentially suppressed, \( \langle \sigma v \rangle \) in (5.16) is replaced by \( \langle \sigma_{\text{eff}} v \rangle \), which will have a larger value. The analysis of Section 5.2 is then carried out with this new value, resulting in a relic density that depends on the reciprocal of the integral

\[
\bar{J}(x_f) = \int_{0}^{x_f} dx \frac{g_*}{\sqrt{g_*}} \langle \sigma_{\text{eff}} v \rangle.
\]
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and is therefore smaller than the case without coannihilation. Note that the definition of $\sigma_{\text{eff}}$ allows for multi-component coannihilation, i.e. where multiple species $\tilde{\chi}_i$ and $\tilde{\chi}_j$, which are close in mass to the LSP $\tilde{\chi}_1$, contribute to $\sigma_{\text{eff}}$.

5.5 Direct Detection Experiments

Since WIMP dark matter is expected to have a nonzero interaction cross section with standard model matter via the weak force, it is in principle possible to observe neutralino scattering in a direct detection experiment [27]. In such an experiment, a dark matter particle scatters from a quark inside a nucleus, producing a detectable signal from the nucleus recoil. There are two interactions which can contribute to such a cross section: spin-independent and spin-dependent interactions [101]. A spin-dependent interaction involves the coupling of the neutralino to the quark axial current $\bar{q}\gamma_\mu\gamma_5 q$ and corresponds to an effective Lagrangian term of the form

$$\mathcal{L} \supset \alpha_q^A (\bar{\chi}\gamma_\mu\gamma_5 \chi)(\bar{q}\gamma_\mu\gamma_5 q). \quad (5.47)$$

This represents, for example, $Z$-exchange between the neutralino and the nuclear quark and will scale according to the total nuclear angular momentum $(J + 1)/J$. Spin-independent scattering of a neutralino comes from the exchange of a scalar, such as a Higgs or sfermion. The effective Lagrangian term has the form

$$\mathcal{L} \supset \alpha_q^S \bar{\chi}\chi\bar{q} q. \quad (5.48)$$
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This interaction scales with the square of the nucleon count, $A^2$.

Many experiments designed to search for direct interactions of dark matter with heavy nuclei exist, with next-generation designs planned for the future [29, 30]. One such design is the use of large chambers of liquid xenon to observe scintillation as a result of a nuclear recoil event. XENON100 [120] and LUX [121] are two detectors built according to this scheme, both of which have finished their data runs and are preparing for the next-generation experiments LZ [122] and XENON1T [123].

A key drawback of these types of experiments is the fact that neutrinos will interact in a similar way. Thus, detection cannot be made for particles whose interaction cross section is smaller than that of the neutrino; this limitation is known as the neutrino floor [28].
Chapter 6

Methodology and LHC Simulation

In order to analyze coannihilation region spectra for supergravity unified models, it is necessary to make use of several existing software tools. The methodology used in analyses for this work has the following general pattern: first, the parameter space of supergravity unified models is analyzed to find a coannihilation region that satisfies all of the necessary constraints. Once a region is found, a representative set of benchmark points is selected. LHC collisions are simulated for each benchmark point, starting with the sparticles produced from $pp$ collisions, decaying those sparticles back to stable standard model final states, and finally reconstructing the event within the geometry and triggering environment of a detector. Once simulation is complete, an analysis is done to determine how many events meet specific values of kinematic variables, called cuts, for a set of signal regions chosen based upon the expected decay topology. The number of events passing
the cuts is compared to the number of background events passing the cuts, with the discovery threshold set to \( S/\sqrt{B} \geq 5 \), where \( S \) is the signal count and \( B \) is the background count.

### 6.1 Finding the Coannihilation Region

The first step in the analysis of a coannihilation region consists of identifying the parameter region of universal or non-universal supergravity unified models that gives the correct Higgs mass as well as a LSP relic density within experimental limits, with the desired NLSP acting as coannihilator. Sparticle spectra are generated using \texttt{SOFTSUSY} [49], which calculates spectra using renormalization group equations for a number of supersymmetry models, including universal and non-universal supergravity unified models. The calculation of the relic density and other dark matter parameters is performed using \texttt{micrOMEGAS} [51].

Since the parameter space is large, scans are targeted by choosing soft parameters at the high scale considered likely to give the desired mass hierarchy for a particular NLSP. Depending on the size of the parameter space under consideration, either a scan over a fixed grid or a Monte Carlo scan inside set limits is used. Once a subset of points is found which satisfies all constraints, the subset is systematically expanded in multiple parameter dimensions to determine the contours of the coannihilation region for that model. Because of the large number of SLHA-format output files used, \texttt{PySLHA} [52] is
6.2 Simulating Decay Events

After the desired coannihilation region is located, several benchmark points are selected for further analysis. These points are chosen to be representative of the entire parameter space of the coannihilation region and are arranged in order of increasing LSP and NLSP mass. This is because by searching over a range of increasing mass scale, the limits for discovery at the LHC can be located at the largest mass scale that gives an integrated luminosity for discovery \( \leq 3000 \text{ fb}^{-1} \), which is the maximum design luminosity of the LHC.

Once the benchmarks are selected, they are simulated using the MadGraph [42–44, 124] software stack, which includes Pythia [45] for hadronization of decays and Delphes [46] for detector simulation and event reconstruction. To ensure that every possible decay is considered, Feynman diagrams for the decays \( pp \to \tilde{\chi}\tilde{\chi} \) are generated in MadGraph, where \( \tilde{\chi} \) represents any supersymmetric particle. Both initial state and final state radiation are included in the analysis. These diagrams are then used by MadEvent to generate 50,000 supersymmetry decay events for each benchmark point, using each point's calculated sparticle mass spectrum from SOFTSUSY, as well as decay widths and branching ratios as calculated\(^1\) by SUSY-HIT [50], which combines SDECAY [126, 127] and HDECAY [128, 129].

\(^1\)Particular to the coannihilation region for stops, the flavor-violating stop decay module of SUSY-HIT [125] is used to obtain more accurate results for the compressed spectrum stop decays. The
For each benchmark point, MadGraph generates production cross section each individual production mode for all supersymmetric particles, and then randomly generates collision events according to those cross sections. After generating each individual event, decay of the resulting sparticles is simulated according to the input branching ratios. The jets produced are hadronized using Pythia. After all 50,000 events are generated, Delphes completes the detector simulation by reconstructing the event according to the geometry, efficiencies, and triggering environment of the ATLAS detector. The results are placed into a ROOT [47] output file which represents the simulated output data from the sparticle production events for that benchmark.

6.3 Signal Region Analysis

To evaluate the discovery potential of the LHC, potential signal regions must be identified that will allow discrimination of the supersymmetry signal over the standard model background. A signal region consists of a series of selection rules, often called cuts, that seek to identify values of kinematic variables and other event properties which will be broadly characteristic of the supersymmetry signal but not of the background.

The first step to performing a signal region analysis is identifying the decay topologies for the model in question, as these will determine the type of signal region most suitable.

One output of the MadGraph event simulation is the sparticle production cross section for results calculated by SUSY-HIT in flavor-violating stop decay mode are inserted back into the general spectrum output given by the vanilla SUSY-HIT code.
all possible processes. By identifying the sparticles most likely to be produced and then
determining the dominant decays based on branching ratio calculations from SUSY-HIT,
the signature properties of a given benchmark point can be determined. These are then
matched to published signal regions.

With the selected set of signal regions, an analysis is performed using ROOT to determine,
event by event, whether each event passes the signal region cuts. The total count of all
signal region events passing the cuts, known as $S$, is then retained for comparison to the
background.

In the analysis here, only statistical errors were considered and no attempt was made to
estimate systematic errors. Such an effort requires detailed and specialized knowledge
of the experimental apparatus. Furthermore, systematic errors at $\sqrt{s} = 14$ TeV are
expected to be very different in form and magnitude than the published systematic errors
at $\sqrt{s} = 8$ TeV and at lower luminosity. Our estimates of the integrated luminosities
given here are based on single-channel analyses, and one expects that combining channels
can potentially result in discovery with even smaller integrated luminosities, even after
accounting for systematic error.

6.4 Background Analysis

To compare to the standard model background, signal region analyses are also performed
on the pre-generated background published by SNOWMASS [48]. The SNOWMASS
background data consists of multiple ROOT files representing different standard model final states across different energy ranges, simulated for a collider center of mass energy of 14 TeV\(^2\). Each final state and energy range is sampled at the necessary luminosity to obtain sufficiently many events for acceptable statistics across the entire set of background processes. As a result, each individual file must be scaled and weighted individually according to its own luminosity. From the total supersymmetry cross section \(\sigma\) calculated by MadGraph and the number of events simulated \(N\), an implied luminosity \(\mathcal{L}\) can also be calculated for each signal point using

\[
\mathcal{L} = \frac{N}{\sigma} . \tag{6.1}
\]

This allows direct comparison to the background after the processes are scaled individually by implied luminosity. The many final states which comprise the SNOWMASS background are weighted and combined in Figure 6.1. Each final state is represented by a different color in these plots. The notation is given by

\[
J = \{ u, \bar{u}, d, \bar{d}, s, \bar{s}, c, \bar{c}, b, \bar{b}\} , \tag{6.2}
\]

\[
L = \{ e^+, e^-, \mu^+, \mu^-, \tau^+, \tau^-, \nu_e, \nu_\mu, \nu_\tau\} ,
\]

\[
B = \{ W^+, W^-, Z, \gamma\} ,
\]

\[
T = \{ t, \bar{t}\} ,
\]

\[
H = \{ h_0 \} .
\]

\(^2\)It is because of this that all event simulations in this work are carried out at 14 TeV, the original LHC Run II design energy, rather than 13 TeV, the actual value used for the current run.
In general, events with gauge bosons and standard model Higgs bosons in the final state are grouped into a single boson (B) category. Thus, for example, the data set Bjj-vbf represents production via vector boson fusion of a gauge boson or a Higgs with at least two additional light quark jets.

Note that a cut of $E_{\text{miss}}^T \geq 100$ GeV is performed on the background prior to any additional signal region cuts. This is to facilitate analysis—the raw background files are sufficiently large that it takes a great deal of time to perform any sort of signal region analysis on them. By cutting on missing transverse energy, the overall file size (and therefore analysis time) is drastically reduced, and since all of the signal regions analyzed in this work include $E_{\text{miss}}^T$ cuts above 100 GeV, no portion of the background relevant to the signal is lost. When signal region cuts are applied to these backgrounds, the individual counts for each process and energy range are weighted and summed before calculating the integrated luminosity for discovery. Since the desired threshold for discovery is $5\sigma$, the integrated luminosity required for discovery is calculated using

$$L = 25 \frac{B_1}{S_1^2},$$

where $S_1$ is the number of signal events which pass a signal region scaled to 1 fb$^{-1}$ and $B_1$ is the summed and weighted count of background events which pass a signal region, also scaled to 1 fb$^{-1}$.

The two panels of Figure 6.1 illustrate two key kinematic quantities, $M_{\text{eff}}$ and $E_{\text{miss}}^T$ for
Figure 6.1: Full SNOWMASS standard model background [48] after triggering cuts and a cut of $E_T^{\text{miss}} \geq 100$ GeV, broken into final states and scaled to 100 fb$^{-1}$. The top panel gives $M_{\text{eff}}(\text{incl.})$ and the bottom panel gives $E_T^{\text{miss}}$. Individual data sets are labeled according to (6.2). Figures taken from [33].

the full background, after minimal cuts for trigger simulation and the $E_T^{\text{miss}} \geq 100$ GeV cut. Here and throughout, $M_{\text{eff}}$ is specifically $M_{\text{eff}}(\text{incl.})$, defined as the scalar sum of $E_T^{\text{miss}}$ and the $p_T$ of all jets with $p_T \geq 40$ GeV.
Chapter 7

The Stop Coannihilation Model

7.1 Introduction

As a first example of coannihilation models, we consider stop coannihilation with the LSP. Here coannihilation is controlled by the mass gap $\Delta m$ between the light stop and the LSP

$$\Delta m \equiv m_{\tilde{t}_1} - m_{\tilde{\chi}_1^0} \ll m_{\tilde{\chi}_1^0}.$$ (7.1)

Satisfaction of the WMAP [112] and Planck [113] relic density constraints in this case requires that the lightest stop mass lies within 20\% of the LSP mass. (For some recent works on light stops see [130–140].) In the analysis of light stops we must take into account the constraint that there be no instability arising from color and charge breaking

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minima. For most recent works on the necessary constraints for stability see [141, 142].

For this class of models the decay of the stop is likely to produce soft jets, making detection difficult. It is found that the lightest stop can have a mass as low as 375 GeV and stop pair production cross sections can be as large as 2 pb. Further, using optimized signal regions it is found that the 375 GeV stop can have a $5\sigma$ discovery with 60 fb$^{-1}$ of integrated luminosity. Additionally it is found that in this model stop masses up to or exceeding 600 GeV with only a $\sim 40$ GeV mass gap between the stop and the neutralino can be discovered with the design parameters of the LHC.

The outline of the rest of this chapter is as follows: In Section 7.2 we discuss the parameter space of interest, examining the stop–neutralino coannihilation region of minimal supergravity unification parameter space under the constraints of the Higgs boson mass, WMAP and Planck relic density, and LHC Run I exclusion plots on the sparticle mass spectra. In Section 7.3 we carry out a signature analysis of a representative set of parameter points for LHC run parameters. Here we analyze various decay channels and signal regions to determine the best avenues for the discovery of this class of models and determine the minimum luminosity needed for the discovery of each of the parameter points discussed. In Section 7.4 we investigate the direct detection of dark matter for these models. Conclusions are given in Section 7.5.
7.2 The Stop Coannihilation Model

As noted in Section 2.4, generating the Higgs mass of 125 GeV in supersymmetry requires a significant loop correction to lift the value of the Higgs boson mass from its tree value, which is \( \leq M_Z \), to the experimentally observed value. The large loop correction implies that the size of weak scale supersymmetry \( M_s \) lies in at least the several-TeV region.

Since the largest contribution to the loop correction arises from stop exchange in a loop, the scale \( M_s \) is typically the geometric mean of the two stop masses, i.e. \( M_s \sim \sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}} \).

Because \( M_s \) is a product of two eigenvalues it is possible that one eigenvalue of the stop mass squared matrix lies in the few hundred GeV region, while the other lies in the several-TeV region. This would allow for the production of light stops and their possible observation at the LHC while still satisfying the Higgs mass constraint.

In models with large trilinear couplings \( A_0 \), this large split between the stop masses is automatic. For this reason, while the general framework of supergravity unification allows for non-universalities in multiple sectors, the stop coannihilation model does not require stepping outside of the universal case. Here we will constrain the parameter space so that the Higgs mass in the model is consistent with the LHC data and the relic density is consistent with the WMAP [112] and Planck [113] data. The resulting parameter space consistent with these constraints requires that the light stops lie in a narrow corridor such that [33]

\[
m_{\tilde{t}} \leq 1.2 \times m_{\tilde{\chi}_1^0} \quad \text{(7.2)}
\]
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In this region the relic density is satisfied by stop–neutralino coannihilation [95, 143–152].

An analysis of the parameter space of minimal supergravity unification was carried out to identify regions with light stops that also satisfy the Higgs boson constraint, the relic density constraint, and the sparticle mass lower limits from LHC Run I [153]. The imposition of these constraints drastically reduces the parameter space of models in a manner discussed below. The sparticle spectrum was generated as described in Chapter 6 with minimal supergravity unification input parameters as described in (4.35).

The parameter region of the stop coannihilation model is exhibited in the two panels of Figure 7.1. The left panel of Figure 7.1 gives a three-dimensional plot of the coannihilation region with $m_0$, $A_0/m_0$, and $m_{\tilde{t}}$ as the three coordinates. The vertical axis on the right gives the color coding for $A_0/m_0$. The analysis shows that the coannihilation region stretches out quite far in $m_0$ and $m_{\tilde{t}}$, with $m_0$ extending past 20,000 GeV and $m_{\tilde{t}}$ getting as large as 6000 GeV. The right panel of Figure 7.1 gives the projection of the left panel in the $m_0 - m_{\tilde{t}}$ plane. The vertical axis on the right gives the color coding for the mass gap between the stop and the neutralino. The analysis of Figure 7.1 was done under the Higgs boson mass constraint $126 \pm 2$ GeV and the mass gap constraint from (7.2)

$$\frac{m_{\tilde{t}} - m_{\tilde{\chi}_1^0}}{m_{\tilde{\chi}_1^0}} \leq 0.2,$$

but with no relic density constraints.
Chapter 7. The Stop Coannihilation Model

Figure 7.1: Left panel: A three-dimensional plot of the stop-neutralino coannihilation region satisfying (7.3) and the Higgs boson mass constraint with three axes chosen as \( m_0, A_0/m_0, \) and \( m_1^2 \) and with \( \tan \beta = 10 \). The image is colored by \( A_0/m_0 \) values in the range \([-2.2, -2.3]\). Right panel: Two-dimensional projection of the left panel in the \( m_1^2 - m_0 \) plane. The allowed regions are colored by the stop-neutralino mass gap \( \Delta m \equiv m_\tilde{\tau} - m_\tilde{\chi}_0 \). Figures taken from [33].

It is possible to satisfy the WMAP and Planck relic density constraints in the coannihilation region shown in Figure 7.1. We consider two sets of relic density constraints: (i) the weak relic density constraint \( \Omega_\tilde{\chi}_0 < 0.12 \), and (ii) the strong relic density constraint, which we take to be \( 0.0946 < \Omega_\tilde{\chi}_0 < 0.1306 \), where the range is \( \pm 5\sigma \) around the mean WMAP result. The weak relic density constraint allows for multi-component dark matter—i.e. that the relic density is made up of one or more dark matter components other than the neutralino—while the strong relic density constraint strictly requires that the dark matter is constituted only of the neutralino LSP. The range \( \pm 5\sigma \) around the mean of WMAP is taken to allow for possible uncertainties in theoretical computations of the relic density. We give now an analysis of the parameter space for these cases.

In the top two panels of Figure 7.2, we investigate the parameter space for the case where we impose the weak relic density constraint (i). The axes in Figure 7.2 are as in Figure 7.1.
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Here we find that the major difference between Figure 7.1 and the top panels of Figure 7.2 is that the coannihilation region has shrunk and the mass gap $\Delta m$ between the stop and the neutralino is reduced to $\leq 45$ GeV. In the bottom two panels of Figure 7.2, we investigate the parameter space for the case where the stronger relic density constraint (ii) is imposed. The two-sided constraint in this case narrows the allowed range of the mass gap between the stop and the neutralino to between 30 and 40 GeV, as exhibited by the vertical bar to the right of the bottom right panel. As in Figure 7.1, the analysis of all panels in Figure 7.2 is done under the Higgs boson mass constraint. Similar plots arise for other $\tan \beta$ values, such as $\tan \beta = 30$ and $\tan \beta = 50$.

Figure 7.2: Top panels: Same as the two panels of Figure 7.1 except that relic density constraint $\Omega_{\tilde{\chi}}^0 < 0.12$ is imposed. Bottom panels: Same as the two panels of Figure 7.1 except that the strong relic density constraint $0.0946 < \Omega_{\tilde{\chi}}^0 < 0.1306$ is imposed. Figures taken from [33].

The analysis of Figures 7.1 and 7.2 allows for a few conclusions. While Figure 7.1 suggests
that the coannihilation region as defined by (7.2) continues in $m_0$ past $\sim 25$ TeV, there is an upper cutoff around $m_0 \sim 8000$ GeV once the relic density constraint is taken into account, as shown in Figure 7.2. Figure 7.3 displays the $\tilde{t}_1 - \tilde{\chi}_1^0$ mass plane and demonstrates that, once the relic density constraint is applied, the allowed mass gap is greatly constrained to be between 30 and 40 GeV. From this analysis we conclude that once relic density is considered, the mass gap $\Delta m$ always lies below the top mass $m_t$ so that the on-shell decay $\tilde{t}_1 \rightarrow t\tilde{\chi}_1^0$ does not occur; the dominant decay for the stop in the coannihilation region is $\tilde{t}_1 \rightarrow c\tilde{\chi}_1^0$. This decay remains dominant in the region where $\Delta m$ has an upper limit of $m_W + m_b \sim 85$ GeV. Of course the off-shell decay $\tilde{t}_1 \rightarrow Wb\tilde{\chi}_1^0$ can still occur, which will produce signatures of the type $\tilde{t}_1 \rightarrow bff'\tilde{\chi}_1^0$, where $ff'$ arises from the decay of the $W$. The jets arising from the decay will be soft, but high $p_T$ jets could arise from initial and final state radiation.

**Figure 7.3:** Left panel: A plot of the neutralino mass versus stop mass for the full coannihilation region defined by (7.2) and displayed in Figure 7.1. The image is colored by the mass gap $\Delta m$ between the stop and neutralino masses. All masses are in GeV. Right panel: Same as left panel but restricted to only those points which satisfy the strict relic density constraint (ii) and which are discoverable with a luminosity of $L \leq 3000$ fb$^{-1}$, according to the analysis in Section 7.3. Figures taken from [33].
7.3 Signal Analysis for Stop Coannihilation Models at the LHC

7.3.1 LHC Production and Signal Definitions

In Table 7.1 we present a set of model points in the minimal supergravity unification parameter space with light stops that give a Higgs boson mass consistent with LHC measurements and have a relic density consistent with WMAP and Planck observations. For the sake of consistency, all points in Table 7.1 assume \( \tan \beta = 10 \), though the results we will discuss are largely insensitive to the precise value of this parameter. Our simulation of these points at \( \sqrt{s} = 8 \) TeV shows that they would have escaped detection using the total integrated luminosity accumulated at LHC Run I [153]. In all cases, for the parameter points listed in Table 7.1 the light stop is the NLSP and its dominant decay mode is to a light chargino and a charm quark, which is a flavor-changing process. The other lightest particles are the second lightest neutralino \( \tilde{\chi}_2^0 \), the light chargino \( \tilde{\chi}_1^\pm \), and the gluino \( \tilde{g} \). Thus in the model points of Table 7.1, the sparticle mass hierarchy is\(^1\)

\[
m_{\tilde{\chi}_1^0} < m_{\tilde{t}_1} < m_{\tilde{\chi}_2^0} < m_{\tilde{\chi}_1^\pm} < m_{\tilde{g}}. \tag{7.4}
\]

Each of the heavier sparticle decays, i.e. the decays of \( \tilde{\chi}_2^0 \), \( \tilde{\chi}_1^\pm \), and \( \tilde{g} \), involves a light stop in their dominant decay. However, the decay of \( \tilde{t}_1 \) in the coannihilation region will

\(^1\)The hierarchy eq. 7.4 is mSP[t1a] in the notation of Table 5 of [23].
yield soft jets because of the small mass gap between its mass and the LSP mass, which makes it difficult to observe directly.

\[ m_0 \quad A_0 \quad m_{\tilde t_1} \quad m_{\tilde\chi^0_1} \quad \Delta m_{\tilde t_1,\tilde\chi^0_1} \quad m_h \quad m_{\tilde\chi^0_2} \quad m_{\tilde\chi^\pm_1} \]

<table>
<thead>
<tr>
<th>Model</th>
<th>(m_0)</th>
<th>(A_0)</th>
<th>(m_{\tilde t_1})</th>
<th>(m_{\tilde\chi^0_1})</th>
<th>(\Delta m_{\tilde t_1,\tilde\chi^0_1})</th>
<th>(m_h)</th>
<th>(m_{\tilde\chi^0_2})</th>
<th>(m_{\tilde\chi^\pm_1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.</td>
<td>2910</td>
<td>-6347</td>
<td>770</td>
<td>374</td>
<td>337</td>
<td>37</td>
<td>124.5</td>
<td>2316</td>
</tr>
<tr>
<td>ii.</td>
<td>2990</td>
<td>-6727</td>
<td>790</td>
<td>383</td>
<td>346</td>
<td>37</td>
<td>125.6</td>
<td>2376</td>
</tr>
<tr>
<td>iii.</td>
<td>3150</td>
<td>-7087</td>
<td>830</td>
<td>403</td>
<td>365</td>
<td>38</td>
<td>127.9</td>
<td>2497</td>
</tr>
<tr>
<td>iv.</td>
<td>3350</td>
<td>-7570</td>
<td>900</td>
<td>436</td>
<td>397</td>
<td>39</td>
<td>124.8</td>
<td>2663</td>
</tr>
<tr>
<td>v.</td>
<td>3470</td>
<td>-7842</td>
<td>930</td>
<td>447</td>
<td>410</td>
<td>37</td>
<td>127.1</td>
<td>2754</td>
</tr>
<tr>
<td>vi.</td>
<td>3730</td>
<td>-8467</td>
<td>1020</td>
<td>490</td>
<td>452</td>
<td>38</td>
<td>124.1</td>
<td>2968</td>
</tr>
<tr>
<td>vii.</td>
<td>3810</td>
<td>-8648</td>
<td>1040</td>
<td>498</td>
<td>461</td>
<td>37</td>
<td>125.3</td>
<td>3029</td>
</tr>
<tr>
<td>viii.</td>
<td>3920</td>
<td>-8898</td>
<td>1070</td>
<td>515</td>
<td>475</td>
<td>40</td>
<td>125.5</td>
<td>3114</td>
</tr>
<tr>
<td>ix.</td>
<td>4150</td>
<td>-9420</td>
<td>1130</td>
<td>543</td>
<td>503</td>
<td>40</td>
<td>127.8</td>
<td>3289</td>
</tr>
<tr>
<td>x.</td>
<td>4350</td>
<td>-9918</td>
<td>1210</td>
<td>579</td>
<td>540</td>
<td>39</td>
<td>124.1</td>
<td>3462</td>
</tr>
<tr>
<td>xi.</td>
<td>4460</td>
<td>-10168</td>
<td>1240</td>
<td>595</td>
<td>554</td>
<td>41</td>
<td>124.4</td>
<td>3547</td>
</tr>
<tr>
<td>xii.</td>
<td>4610</td>
<td>-10510</td>
<td>1280</td>
<td>614</td>
<td>572</td>
<td>42</td>
<td>125.4</td>
<td>3692</td>
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</tbody>
</table>

Table 7.1: The subset of minimal supergravity unification parameter points that satisfy the Higgs boson mass constraint and the relic density constraint in the stop coannihilation region that were chosen for LHC analysis. The minimal supergravity unification parameters are given (all points have \(\tan\beta = 10\) and \(\mu > 0\)), followed by key masses (in GeV). Our simulations show that the stop-neutralino mass combinations arising from Models i-xii lie outside of the exclusion plots of LHC Run I. Table taken from [33].

In Table 7.2 we give the production processes involving \(\tilde t_1\), \(\tilde\chi^0_2\), \(\tilde\chi^\pm_1\), and \(\tilde g\) at the LHC for each of the parameter points listed in Table 7.1. The dominant production modes generally consist of the following set:

\[
\begin{align*}
    gg &\rightarrow \tilde t_1 \tilde t_1, \\
    qq &\rightarrow \tilde t_1 \tilde t_1, \\
    qq &\rightarrow \tilde\chi^0_2 \tilde\chi^\pm_1, \\
    qq &\rightarrow \tilde\chi^\pm_1 \tilde\chi^-_1, \\
    gg &\rightarrow \tilde g \tilde g, \\
    qq &\rightarrow \tilde g \tilde g.
\end{align*}
\]

(7.5)

(7.6)

The largest production cross section is for the stops, followed by the production of the weak gauginos, and then the gluino. For the parameter regime presented in Table 7.1, the typical mass difference \(\Delta m_{\tilde t_1,\tilde\chi^0_1}\) is consistently near 40 GeV, with the flavor-violating
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decay $\tilde{t}_1 \rightarrow c\tilde{\chi}_1^0$ typically representing 60% or more of the total decay width of the lightest stop.

<table>
<thead>
<tr>
<th>Model</th>
<th>$gg \rightarrow \tilde{t}_1\tilde{t}_1$</th>
<th>$gg \rightarrow \tilde{t}_1\tilde{t}_1$</th>
<th>$gg \rightarrow \tilde{t}_1\tilde{t}_1$</th>
<th>$gg \rightarrow \tilde{t}_1\tilde{t}_1$</th>
<th>$gg \rightarrow \tilde{t}_1\tilde{t}_1$</th>
<th>$BR(t_1\tilde{t}_1 \rightarrow c\tilde{\chi}_1^0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.</td>
<td>2.8</td>
<td>0.25</td>
<td>$1.0 \times 10^{-2}$</td>
<td>$4.8 \times 10^{-3}$</td>
<td>$9.0 \times 10^{-4}$</td>
<td>2.1 \times 10^{-4}</td>
</tr>
<tr>
<td>ii.</td>
<td>1.7</td>
<td>0.22</td>
<td>$8.8 \times 10^{-3}$</td>
<td>$4.3 \times 10^{-3}$</td>
<td>$6.9 \times 10^{-4}$</td>
<td>1.8 \times 10^{-4}</td>
</tr>
<tr>
<td>iii.</td>
<td>1.3</td>
<td>0.18</td>
<td>$6.9 \times 10^{-3}$</td>
<td>$3.3 \times 10^{-3}$</td>
<td>$4.1 \times 10^{-4}$</td>
<td>1.2 \times 10^{-4}</td>
</tr>
<tr>
<td>iv.</td>
<td>0.85</td>
<td>0.12</td>
<td>$4.6 \times 10^{-3}$</td>
<td>$2.1 \times 10^{-3}$</td>
<td>$1.8 \times 10^{-4}$</td>
<td>5.9 \times 10^{-5}</td>
</tr>
<tr>
<td>v.</td>
<td>0.74</td>
<td>0.13</td>
<td>$3.9 \times 10^{-3}$</td>
<td>$1.3 \times 10^{-3}$</td>
<td>$4.4 \times 10^{-5}$</td>
<td>4.0 \times 10^{-5}</td>
</tr>
<tr>
<td>vi.</td>
<td>0.44</td>
<td>0.12</td>
<td>$7.1 \times 10^{-2}$</td>
<td>$2.4 \times 10^{-3}$</td>
<td>$1.1 \times 10^{-3}$</td>
<td>$3.8 \times 10^{-5}$</td>
</tr>
<tr>
<td>vii.</td>
<td>0.40</td>
<td>0.17</td>
<td>$6.6 \times 10^{-2}$</td>
<td>$2.4 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$3.2 \times 10^{-5}$</td>
</tr>
<tr>
<td>viii.</td>
<td>0.33</td>
<td>0.12</td>
<td>$5.6 \times 10^{-2}$</td>
<td>$1.9 \times 10^{-3}$</td>
<td>$8.9 \times 10^{-4}$</td>
<td>$2.1 \times 10^{-5}$</td>
</tr>
<tr>
<td>ix.</td>
<td>0.24</td>
<td>0.12</td>
<td>$4.3 \times 10^{-2}$</td>
<td>$1.4 \times 10^{-3}$</td>
<td>$6.4 \times 10^{-4}$</td>
<td>$9.4 \times 10^{-6}$</td>
</tr>
<tr>
<td>x.</td>
<td>0.17</td>
<td>0.12</td>
<td>$3.1 \times 10^{-2}$</td>
<td>$1.0 \times 10^{-4}$</td>
<td>$4.3 \times 10^{-4}$</td>
<td>$3.9 \times 10^{-6}$</td>
</tr>
<tr>
<td>xi.</td>
<td>0.14</td>
<td>0.10</td>
<td>$2.7 \times 10^{-2}$</td>
<td>$7.9 \times 10^{-4}$</td>
<td>$3.7 \times 10^{-4}$</td>
<td>$2.9 \times 10^{-6}$</td>
</tr>
<tr>
<td>xii.</td>
<td>0.12</td>
<td>0.12</td>
<td>$2.3 \times 10^{-2}$</td>
<td>$6.6 \times 10^{-4}$</td>
<td>$3.0 \times 10^{-4}$</td>
<td>$1.6 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Table 7.2: LHC Production cross sections in pb of the dominant supersymmetric modes $gg \rightarrow \tilde{t}_1\tilde{t}_1$, $gg \rightarrow \tilde{t}_1\tilde{t}_1$, $gg \rightarrow \tilde{\chi}^0_{2}\tilde{\chi}_1^+$, $qq \rightarrow \tilde{\chi}_1^+\tilde{\chi}^{-}_1$, $gg \rightarrow \tilde{g}\tilde{g}$, and $qq \rightarrow \tilde{g}\tilde{g}$, as well as the branching ratio for the process $t_1\tilde{t}_1 \rightarrow c\tilde{\chi}_1^0$, for the set of parameter points given in Table 7.1. Table taken from [33].

Given the dominance of stop production followed by $\tilde{t}_1 \rightarrow c\tilde{\chi}_1^0$, our signature analysis concentrates on two ATLAS searches: the low-multiplicity jets plus missing transverse energy $E_T^{\text{miss}}$ search [32], and the dedicated stop search of [31]. The multi-jet search defines fourteen signal regions with jet multiplicities between 2–6 jets, and with varying requirements on the inclusive effective mass $M_{\text{Eff(inc.)}}$, designated as loose (low $M_{\text{Eff(inc.)}}$) to tight (high $M_{\text{Eff(inc.)}}$). The dominant signal for the points in Table 7.1 involves typically 2–4 reconstructed jets with $p_T(j) \geq 20$ GeV and a low effective mass. Examination of all 14 signatures therefore reveals that the two-jet loose signature (2jL) is the most effective.

\footnote{An alternative set of signal regions for supersymmetry discovery are used by the CMS Collaboration, see e.g. [154, 155] which involve razor variables.}
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topology for searching for these models. This signal region has the selection requirements as listed in the left panel of Table 7.3.

<table>
<thead>
<tr>
<th>Requirement (2jl SR)</th>
<th>Value</th>
<th>Requirement (M1 SR)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_T^{\text{miss}}$ (GeV)</td>
<td>$&gt;160$</td>
<td>$E_T^{\text{miss}}$ (GeV)</td>
<td>$&gt;220$</td>
</tr>
<tr>
<td>$p_T(j_1)$ (GeV)</td>
<td>$&gt;130$</td>
<td>$p_T(j_1)$ (GeV)</td>
<td>$&gt;280$</td>
</tr>
<tr>
<td>$p_T(j_2)$ (GeV)</td>
<td>$&gt;60$</td>
<td>$\Delta\phi(j_1, E_T^{\text{miss}})_{\min}$</td>
<td>$&gt;0.4$</td>
</tr>
<tr>
<td>$\Delta\phi(j_1, E_T^{\text{miss}})_{\min}$</td>
<td>$&gt;0.4$</td>
<td>At most 2 other jets with $p_T$ (GeV)</td>
<td></td>
</tr>
<tr>
<td>$E_T^{\text{miss}}/\sqrt{H_T}$ (GeV)</td>
<td>$&gt;8$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{\text{Eff}}$ (GeV)</td>
<td>$&gt;800$</td>
<td>$p_T$ (GeV)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 7.3:** Left: The selection criteria used for the signal region 2jl, where 2j stands for two jets and l stands for loose in the nomenclature of Table 2 of the ATLAS analysis [32]. This signal region is found to be the most dominant for the parameter set given in Table 7.1. Right: The selection criteria for the mono jet signal region M1 from the ATLAS dedicated stop search [31]. This signal region is subdominant for the parameter set of interest. Tables taken from [33].

The dedicated stop search of [31] involves two topologies. The first is a monojet-like signature, divided into three signal regions—M1, M2, and M3—defined by increasingly stringent requirements on the transverse momentum of the leading jet $p_T(j_1)$ and the missing transverse energy. The second topology also involves a hard leading jet, with $p_T(j_1)$ requirements similar to that of the monojet analysis, but also requires a charm tag on at least one of the sub-leading jets. The multivariate technique that generates the charm tag is not something that is easily reproduced nor modeled in Delphes. We therefore focus only on the monojet searches in this work. The most successful monojet search for this parameter region is M1, which has the selection requirements specified in the right panel of Table 7.3.

In addition, because the tightly constrained parameter space yields a similarly constrained sparticle spectrum and signal, variations of these signal regions were tested in order to
optimize them for sparticle spectra with the particular properties of Tables 7.1 and 7.2 (see Section 7.3.2). The minimum integrated luminosity results of these analyses are presented in Table 7.4. It is found that these model points could be discoverable with integrated luminosities beginning at \( \sim 90 \text{ fb}^{-1} \) using existing searches or \( \sim 60 \text{ fb}^{-1} \) using an optimized search, described in Section 7.3.2.

Results for the M1 signal region are represented in Figure 7.4, which displays the Model (i) signal and the square root of the summed background after M1 cuts. The left panel plots \( M_{\text{Eff}} \) while the right panel plots \( E_T^{\text{miss}} \). These figures are shown at 369 fb\(^{-1}\), the necessary integrated luminosity for 5\( \sigma \) discovery for Model i using signal region M1.
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Figure 7.4: Left panel: Distribution in $M_{\text{Eff}}$ for the M1 signal region for Model (i). Plotted is the number of counts for the signal and the square root of the total standard model background. The analysis is done at 369 fb$^{-1}$ of integrated luminosity, which gives a 5$\sigma$ discovery in this signal region. Right panel: The same analysis as in the left panel but for $E_T^{\text{miss}}$. Figures taken from [33].

Figure 7.5: Left panel: Distribution in $M_{\text{Eff}}$ for the 2jl signal region for Model (i). Plotted is the number of counts for the signal and the square root of the total standard model background. The analysis is done at 88 fb$^{-1}$ of integrated luminosity, which gives a 5$\sigma$ discovery in this signal region. Right panel: The same analysis as in the left panel but for $E_T^{\text{miss}}$. Figures taken from [33].

Likewise, a representative result for the 2jl signal region is shown in Figure 7.5. Like Figure 7.4, this displays the signal for Model (i) superimposed upon the square root of the total summed background, both after cuts, this time in 2jl. The left panel plots $M_{\text{Eff}}$ and the right panel plots $E_T^{\text{miss}}$. The integrated luminosity used is 88 fb$^{-1}$, which gives 5$\sigma$ discovery of Model (i) using signal region 2jl.

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Comparing the results of Table 7.4 for the dominant signal and for the subdominant signals, we find that the integrated luminosity needed for the subdominant signal M1 can be larger by a factor of 3 or more. All other signal regions from [31] and [32] that were analyzed required even higher integrated luminosities for 5σ discovery. This verifies the claim that the 2jl is the dominant signal for discovery of the models listed in Table 7.1. However, because 2jl is a general-purpose search optimized at 8 TeV, it is possible to improve upon its performance for this parameter region.

7.3.2 Optimizing the Signal Regions

As mentioned above, because the leading signal region 2jl is a general-purpose signal region developed for application at 8 TeV, it should be possible to improve on its performance by closely examining the cuts made to certain key parameters. Three primary changes were identified for analysis and comparison to the baseline signal regions: relaxing the kinematic cuts on $M_{\text{Eff}}(\text{inc.})$, $E_T^{\text{miss}}$, or $p_T(j)$; increasing the cut on $\Delta\phi$ between $E_T^{\text{miss}}$ and the two leading jets; and making an additional cut on a new parameter given by the ratio $p_T(j_1)/E_T^{\text{miss}}$. Optimizations on these parameters were performed on the 2jl and M1 signal regions described in Table 7.3, as well as on signal region 3j from [32], which differs from 2jl primarily by requiring a third jet with $p_T(j_3) > 60$ GeV and by imposing a much tighter cut on $M_{\text{Eff}} > 2200$ GeV (as opposed to $M_{\text{Eff}} > 800$ GeV).

With values of $\Delta m_{\tilde{t}_1,\tilde{\chi}_1^0}$ of around 40 GeV, there is only energy available in the $\tilde{t}_1 \rightarrow c\tilde{\chi}_1^0$ to make a soft jet; initial state radiation is generally relied upon to produce high-$p_T$ jets in
the final state. Thus, it is reasonable that relaxing the kinematic cuts on $M_{\text{Eff}}(\text{inc.})$ could yield an improved signal region. For the case of 2jl, reducing the $M_{\text{Eff}}(\text{incl.})$ cut from 800 GeV to 400 GeV improved the result from the base case by almost a factor of 2 for the lightest points in the parameter space. But as parameter points become heavier this advantage decreases, and eventually the modified signal region worsens. This trend is reflected also if the kinematic cuts in M1 are reduced so that both $E_{\text{T}}^{\text{miss}} > 150$ GeV and $p_T(j_1) > 150$ GeV are required; performance of the signal region improves for the lightest points but worsens for higher-mass regions of the parameter space. Reducing the $M_{\text{Eff}}(\text{incl.})$ cut on 3j from 2200 GeV to 800 GeV, a value that matches the cut for 2jl, dramatically improves the 3j performance, leaving it only slightly worse than 2jl. In fact, after this change there is little that distinguishes the two signal regions; the modified 3j requires a third jet with a minimum $p_T(j_3) > 60$ GeV, while 2jl does not look for a third jet. Thus, 3j is dropped from consideration because its optimization evolves it into 2jl.

Since the signal is expected to consist primarily of a single jet recoiling against $E_T^{\text{miss}}$, a higher $\Delta \phi(j, E_T^{\text{miss}})$ is expected in the signal relative to the more uniform background.

Figure 7.6: Left panel: Distribution of $\Delta \phi(j_1, E_T^{\text{miss}})$ against the square root of the full SM background at 100 fb$^{-1}$ for Model i before any cuts are applied. Right panel: The same as in the left panel but for $\Delta \phi(j_2, E_T^{\text{miss}})$. Figures taken from [33].

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This is demonstrated in Figure 7.6, where the left panel shows the shape of $\Delta \phi(j_1, E_T^{\text{miss}})$ against the square root of the summed background for Model (i) before any cuts are applied and the right panel shows the same for $\Delta \phi(j_2, E_T^{\text{miss}})$. Based upon these results, $\Delta \phi$ cuts were increased from 0.4 to $\frac{\pi}{2}$ for $j_1$ and from 0.4 to 1.0 for $j_2$. Making this change improved the 2jl results by $\sim 30\%$, yielding the signal region described in Table 7.4 as 2jl-opt. Making this same change in the M1 signal region yielded a poorer search as the integrated luminosity required for 5$\sigma$ discovery increased.

Finally, motivated by recent suggestions in [156, 157], we considered including an additional kinematic parameter, $r = p_T(j_1)/E_T^{\text{miss}}$. Because this parameter measures the degree to which the recoiling missing energy is concentrated in a single jet, it was expected that it would help distinguish this signal. However, when $r > 0.5$ was applied to 2jl and M1, with and without optimization in $\Delta \phi$, the new signal region was worse in every case.

After these investigations, the final optimal signal region was deemed to be 2jl-opt, which, as described above, is the same as 2jl (see Table 7.3) but with $\Delta \phi$ cuts increased from 0.4 to $\frac{\pi}{2}$ for $j_1$ and from 0.4 to 1.0 for $j_2$. This signal region boasts a consistent 30\% increase in performance as compared to 2jl across the full range of the parameter space.

In Figure 7.7 we exhibit $M_{\text{Eff}}$ and $E_T^{\text{miss}}$ distributions in the optimized signal region 2jl-opt for three selected models (i, vii, and xii). Plotted are the number of counts per 30 GeV energy bins for the signal and the square root of the full standard model background. In
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the left panel of each figure we give the analysis for $M_{Eff}$ and in the right we give the analysis for $E_T^{miss}$. The top panels show the analysis for Model (i). Here the distribution is given at 61 fb$^{-1}$ of integrated luminosity, which is found to be the minimum integrated luminosity required to see a 5σ discovery signal for supersymmetry in 2jl-opt. Identical analyses are given in the middle panels for Model (vii), but at 367 fb$^{-1}$ of integrated luminosity, which gives a 5σ discovery signal for that point, and in the bottom panels for Model (xii). Here one finds that a 5σ discovery requires a minimum of 2140 fb$^{-1}$ of integrated luminosity. Thus Models (i), (vii), and (xii) are all eventually discoverable with the design parameters of the LHC at $\sqrt{s} = 14$ TeV.
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Figure 7.7: Distribution in $M_{\text{Eff}}$ (left panels) and $E_{T}^{\text{miss}}$ (right panels) for the optimized two-jet signal region 2jl-opt. Top panels show Model i normalized to 61 fb$^{-1}$ of integrated luminosity, while the middle and bottom panels give Models vii and xii, normalized to 367 fb$^{-1}$ and 2140 fb$^{-1}$ of integrated luminosity, respectively. These numbers give a 5$\sigma$ discovery in this signal region for the three model points. Plotted in all three cases is the number of counts for the signal and the square root of the total standard model background. Figures taken from [33].
### 7.4 Direct Detection of Dark Matter

<table>
<thead>
<tr>
<th>Model</th>
<th>$\sigma_{SI_{p\chi_0^1}} \times 10^{48}$</th>
<th>$\sigma_{SD_{p\chi_0^1}} \times 10^{46}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.</td>
<td>3.5</td>
<td>9.6</td>
</tr>
<tr>
<td>ii.</td>
<td>3.2</td>
<td>8.6</td>
</tr>
<tr>
<td>iii.</td>
<td>2.6</td>
<td>6.9</td>
</tr>
<tr>
<td>iv.</td>
<td>2.6</td>
<td>5.2</td>
</tr>
<tr>
<td>v.</td>
<td>2.2</td>
<td>4.5</td>
</tr>
<tr>
<td>vi.</td>
<td>2.1</td>
<td>3.2</td>
</tr>
<tr>
<td>vii.</td>
<td>1.9</td>
<td>3.0</td>
</tr>
<tr>
<td>viii.</td>
<td>1.8</td>
<td>2.6</td>
</tr>
<tr>
<td>ix.</td>
<td>1.5</td>
<td>2.1</td>
</tr>
<tr>
<td>x.</td>
<td>1.6</td>
<td>1.7</td>
</tr>
<tr>
<td>xi.</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>xii.</td>
<td>1.4</td>
<td>1.3</td>
</tr>
</tbody>
</table>

#### Table 7.5: Proton-neutralino spin-independent ($\sigma_{SI_{p\chi_0^1}}$) and spin-dependent ($\sigma_{SD_{p\chi_0^1}}$) cross sections in units of $\text{cm}^{-2}$ for the set of model points of Table 7.1. None of the parameter points are ruled out by the current dark matter experiments, in particular by LUX. Table taken from [33].

In the models considered here, the connection between LHC physics and dark matter is very strong because the requirement that dark matter satisfies the WMAP and Planck relic density constraints forces the stop and neutralino masses to lie within $\sim 30$ GeV of each other for the lightest stops, and restricts the allowed parameter space of the model in the $m_0 - m_{1/2}$ plane to a narrow strip. Further, since $A_0/m_0$ is also determined to be in a narrow range centered at $\sim -2.25$ in order to generate the right Higgs boson mass, the model is very predictive. It is of interest then to investigate the implications of these constraints for the direct detection of dark matter. For the parameter space of the model, the neutralino is almost exclusively a bino with extremely small wino and higgsino content. Therefore proton-neutralino scattering cross sections are expected to be small. The analysis of spin-independent and spin-dependent proton-neutralino cross sections

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Chapter 7. The Stop Coannihilation Model

is presented in Table 7.5. The spin-independent cross sections lie in the range between $10^{-47}$ and $10^{-48}$ cm$^{-2}$. While cross sections of this size are decidedly small, they could still be visible in the next generation LUX-ZEPLIN (LZ) dark matter experiment, which is projected to reach a sensitivity of $\sim 10^{-47}$ cm$^{-2}$ [29, 30]. Regarding the spin-dependent proton–neutralino cross section, the LUX-ZEPLIN will have a maximum sensitivity of $10^{-42}$ cm$^{-2}$, which is still about three orders of magnitude smaller in sensitivity than what is needed to observe the spin-dependent proton–neutralino cross section exhibited in Table 7.5. Figure 7.8 displays the cross section $R \times \sigma_{p,\chi_0^0}^{SI}$ as a function of LSP mass against the current- and next-generation direct detection capabilities. Thus the observation of spin-dependent cross sections will be more difficult.

![Figure 7.8: $R \times \sigma_{p,\chi_0^0}^{SI}$ as a function of LSP mass displayed alongside the current and projected range of the XENON and LUX experiments and the neutrino floor for the stop coannihilation models.](image)

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7.5 Summary of Stop Coannihilation Results

In this chapter we have analyzed the lightest stop masses that can arise in high-scale models consistent with the Higgs boson mass constraint, relic density constraints from WMAP and Planck, and constraints on sparticle mass spectra from LHC Run I. The analysis was done within the framework of minimal supergravity unification, where it was found that stop masses as low as 400 GeV or lower can still exist consistent with these constraints. However, in the model space analyzed it was found that the stop must be the NLSP, with a mass $\sim 1.1$ times the LSP mass. This restriction is needed in order to satisfy the relic density constraint via stop-neutralino coannihilation.

Requiring that the lightest neutralino supply all of the thermal relic abundance puts a lower bound of approximately 30 GeV on $\Delta m = m_{\tilde{t}_1} - m_{\tilde{\chi}_1^0}$. Over the range of parameter space accessible in the lifetime of the LHC (i.e. within 3000 fb$^{-1}$ of integrated luminosity), this gap never exceeds 45 GeV. Thus neither the decay $\tilde{t}_1 \rightarrow \tilde{\chi}_1^0 t$ nor on-shell $W$-decays of the form $\tilde{t}_1 \rightarrow W b \tilde{\chi}_1^0$ occur. Rather the dominant decay of the stop in the allowed parameter space is the flavor-violating process $\tilde{t}_1 \rightarrow c \tilde{\chi}_1^0$.

Production of sparticles is dominated by stop pair production, with all other processes reduced by approximately two orders of magnitude. The analysis of this work shows that the discovery of supersymmetry for the class of models with light stops discussed here can occur with an integrated luminosity as low as $\sim 60$ fb$^{-1}$ at LHC Run II, and will
be dominated by searches involving low jet multiplicity, lepton vetoes, and large missing energy. We have carried out a signature analysis for this class of models and find the dominant signature to be 2jl, where the characteristics of the 2jl SR are given in Table 7.3. We have also carried out an analysis of the subdominant signature M1 and an optimized signature 2jl-opt, the characteristics of which are given in Table 7.3. It is shown that the light stop can be discovered in 2jl with $88 \text{ fb}^{-1}$ and in 2jl-opt with $\sim 60 \text{ fb}^{-1}$ of integrated luminosity at the LHC, while a confirmation of the subdominant signal will require three times as much luminosity. The minimum integrated luminosity can no doubt be reduced with further improvement in the efficiency to tag charm jets [34, 158, 159].

We also discussed the dark matter associated with this class of models. It was shown that the spin-independent proton–neutralino cross section could be within reach of the next generation LUX-ZEPLIN dark matter detector while the spin-dependent proton–neutralino cross section will be more difficult to observe. Any observed signal will be comparable to, but slightly larger than, coherent scattering by atmospheric neutrinos [160].
Chapter 8

The Gluino Coannihilation Model

8.1 Introduction

The next example for coannihilation we consider is coannihilation in which the gluino is the coannihilator. In this case a gluino with mass lower than 1 TeV would have escaped detection at LHC Run I and also in the analyses of Run II based on the 2015 accumulated integrated luminosity of 3.2 fb$^{-1}$ [34]. Similar to the stop coannihilation analysis, here we investigate the sparticle spectrum in the gluino coannihilation region under the Higgs boson mass constraint and the relic density constraint. In the analysis of this model class we take the scalar mass to be high, lying in the several TeV-region, and universal, but with a relatively low-lying gluino mass, i.e. 1 TeV or less. In order to achieve this relatively light gluino mass, we utilize non-universal soft breaking in the gaugino sector.
Specifically, we take the $U(1)_Y$ and the $SU(2)_L$ gaugino masses to be equal ($m_1 = m_2$) and relatively high, while the $SU(3)_C$ gaugino mass ($m_3$) is relatively low at the unification scale.

Coannihilation of a gluino with mass in proximity to the LSP mass allows one to achieve dark matter relic density consistent with experimental constraints. The analysis presented below shows that with gluino coannihilation, a gluino mass in the range of 700–1300 GeV can exist consistent with the Higgs mass and the relic density constraints. Furthermore, a gluino in this mass range would have escaped detection so far, but would be accessible at the LHC with the maximum achievable integrated luminosity of 3000 fb$^{-1}$.

For each benchmark point, seven signal regions were analyzed as listed by the ATLAS Collaboration [34] and displayed in Table 8.3. Each signal region is defined by up to 12 different cuts on kinematic variables such as jet $p_T$, $E_T^{miss}$, etc., which are meant to reduce the background and enhance the signal. It is found that over the entire gluino mass region analyzed, the optimum signal regions are 2jl and 2jm (using the nomenclature of ATLAS [34]), where 2jl and 2jm are as defined in Table 8.3.

The outline of the rest of this chapter is as follows: In Section 8.2 we examine the parameter space of models with coannihilation of a light ($< 2$ TeV) gluino with the LSP under the constraints of the Higgs boson mass, WMAP and Planck relic density, and LHC Run I exclusions on the sparticle mass spectra. In Section 8.3 we carry out a signature analysis of a subset of benchmark model points. For these we analyze the
seven signal regions of Table 8.3 and determine the leading and subleading signal regions and the minimum integrated luminosity for LHC discovery for each of the model points analyzed. This allows us to determine the range of gluino masses which would have escaped detection thus far but would be accessible at the LHC with up to 3000 fb$^{-1}$ of integrated luminosity.

We also compare to the latest constraints from ATLAS [34] using simplified models with 3.2 fb$^{-1}$ of integrated luminosity for the 13 TeV data. In Section 8.3.3 we carry out a signal region optimization. In Section 8.4 we investigate the direct detection of dark matter for the benchmark points of Section 8.3. Conclusions are given in Section 8.5.

8.2 The Gluino Coannihilation Model

As mentioned in Chapter 4, nonuniversal supergravity unified models allow for the possibility of nonuniversalities in the gaugino masses. In this work we consider this possibility by assuming that the scalar masses are all universal at high scale, but that the gaugino masses are split, i.e. gaugino masses in the $U(1)$ and $SU(2)$ sectors are equal, while the $SU(3)$ gaugino mass is not. Specifically we assume the following parameter space for the model:

$$m_0, A_0, m_1 = m_2 \neq m_3, \tan \beta, \sign(\mu),$$

(8.1)

where $m_0$, $A_0$, and $\tan \beta$ are defined as in (4.35), $m_1$ and $m_2$ are the gaugino masses for the $U(1)_Y$ and $SU(2)_L$ sectors, and $m_3$ is the $SU(3)_C$ gaugino mass.
Chapter 8. The Gluino Coannihilation Model

We wish to examine the gluino coannihilation region, which requires that the gluino be the NLSP. In high-scale models this can be achieved by letting $m_3$ lie significantly lower than the common scale $m_1 = m_2$, as shown in Table 8.1. Thus in Table 8.1 we list several benchmark points which are chosen as illustrative examples. In generating the parameter set of Table 8.1 we have imposed the constraints that the Higgs boson mass obey $m_h = 125 \pm 2$ GeV and $\Omega h^2 = 0.11 \pm 0.013$, consistent with WMAP and Planck experiments [112, 113] The ranges are chosen to take account of possible errors in theoretical computations given by codes.

Table 8.1 shows that the gluino mass lies close to the neutralino mass, with the mass difference $\Delta m$ lying in the range of $\sim 70 - 100$ GeV and the gluino mass lying around $1.1 \times$ the neutralino mass over essentially the entire gluino mass range. It is also to be noted that the scale of $m_0$ is high, lying in the several-TeV region, and therefore the light stop mass lies in the range of 2–3 TeV. The largeness of the third-generation scalar masses is what provides a large loop correction to the Higgs boson to lift its tree value to the desired experimentally observed value.

<table>
<thead>
<tr>
<th>Model</th>
<th>Gluino</th>
<th>Neutralino</th>
<th>Stop</th>
<th>Higgs</th>
<th>$\Omega h^2$</th>
<th>$m_0$</th>
<th>$A_0$</th>
<th>$m_1 = m_2$</th>
<th>$m_3$</th>
</tr>
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<tbody>
<tr>
<td>(i)</td>
<td>706</td>
<td>634</td>
<td>2124</td>
<td>123.8</td>
<td>0.122</td>
<td>5000</td>
<td>-7000</td>
<td>1400</td>
<td>250</td>
</tr>
<tr>
<td>(ii)</td>
<td>836</td>
<td>755</td>
<td>3497</td>
<td>124.1</td>
<td>0.110</td>
<td>7000</td>
<td>-8000</td>
<td>1650</td>
<td>300</td>
</tr>
<tr>
<td>(iii)</td>
<td>955</td>
<td>868</td>
<td>2367</td>
<td>125.5</td>
<td>0.112</td>
<td>6000</td>
<td>-9000</td>
<td>1900</td>
<td>350</td>
</tr>
<tr>
<td>(iv)</td>
<td>1057</td>
<td>975</td>
<td>2754</td>
<td>123.2</td>
<td>0.102</td>
<td>5500</td>
<td>-5500</td>
<td>2150</td>
<td>400</td>
</tr>
<tr>
<td>(v)</td>
<td>1129</td>
<td>1046</td>
<td>2910</td>
<td>123.5</td>
<td>0.101</td>
<td>5800</td>
<td>-5800</td>
<td>2300</td>
<td>430</td>
</tr>
<tr>
<td>(vi)</td>
<td>1201</td>
<td>1107</td>
<td>2932</td>
<td>126.7</td>
<td>0.110</td>
<td>7500</td>
<td>-11500</td>
<td>2400</td>
<td>450</td>
</tr>
<tr>
<td>(vii)</td>
<td>1252</td>
<td>1167</td>
<td>3459</td>
<td>124.1</td>
<td>0.101</td>
<td>6800</td>
<td>-6800</td>
<td>2550</td>
<td>480</td>
</tr>
</tbody>
</table>

Table 8.1: A list of gluino coannihilation benchmark points which satisfy Higgs boson mass and relic density constraints for the case when $\tan \beta = 10$ and sign($\mu$) is positive. All masses are in GeV. Relic density constraints were determined by taking $\pm 2.5 \times$ the WMAP7 error of $\pm 0.0056$. Table taken from [35].
Chapter 8. The Gluino Coannihilation Model

The reason for the selection of the illustrative points of Table 8.1 is that all of the benchmarks in Table 8.1 would have escaped detection during LHC Run I but would be observable over the lifetime of the LHC, as discussed in Section 8.3. For Model (i) the sparticle mass spectrum is exhibited in Figure 8.1, which shows the mass hierarchy of all the sparticle states. The sparticles lying in the mass region below 2.5 TeV have the mass hierarchy

\[ m_{\tilde{\chi}_1^0} < m_{\tilde{g}} < m_{\tilde{\chi}_1^\pm} \approx m_{\tilde{\chi}_2^0} < m_{\tilde{t}_1} < m_{\tilde{\chi}_2^\pm} \approx m_{\tilde{\chi}_3^0}, \]  

(8.2)

In the sparticle landscape this is the mass hierarchy labeled NUSP14 in the nomenclature of [161]. This hierarchy is a useful guide to what one may expect in this class of models.

Figure 8.1: An exhibition of the sparticle mass hierarchy for gluino coannihilation Model (i). Figure taken from [35].
8.3 Signal Analysis for Gluino Coannihilation Models at the LHC

8.3.1 LHC Production and Signal Definitions

In Table 8.2 we give an analysis of the supersymmetry production cross sections, which for the gluino coannihilation models considered here is essentially limited to gluino pair-production. The main mechanisms for gluino production are gluon fusion $gg \rightarrow \tilde{g}\tilde{g}$ and $q\bar{q} \rightarrow \tilde{g}\tilde{g}$. Because of the small mass gap between the gluino and neutralino masses needed for gluino coannihilation, the decay of the gluino is dominated by three-body decay involving a neutralino and quarks, i.e. by the process $\tilde{g} \rightarrow q\bar{q}\tilde{\chi}^0_1$. The subleading decay is $\tilde{g} \rightarrow g\tilde{\chi}^0_1$ which typically has a branching ratio of only a few percent. The decay mode $\tilde{g} \rightarrow q\bar{q}\tilde{\chi}^0_1$ represents 95% or more over most of the parameter space, as can be seen in Table 8.2 by adding the two columns on the right.

The multi-jet search strategy described in [34] defines seven signal regions with jet multiplicities ranging from 2-6 jets and with cuts on the inclusive effective mass $M_{\text{eff}}(\text{incl.})$ varying from loose (low $M_{\text{eff}}(\text{incl.})$) to tight (high $M_{\text{eff}}(\text{incl.})$). When examining the results for the gluino coannihilation benchmark points, 2jm and 2jl were found to be the dominant and subdominant signal regions for discovery. In Table 8.3, $H_T$ is defined as the scalar sum of the transverse momentum of all jets, $M_{\text{eff}}(\text{incl.})$ is defined as the scalar
Chapter 8. The Gluino Coannihilation Model

<table>
<thead>
<tr>
<th>Model</th>
<th>SUSY Cross Section (pb)</th>
<th>$\tilde{g} \rightarrow q\bar{q}\chi^0_1$, $q \in {u, d}$</th>
<th>$\tilde{g} \rightarrow q\bar{q}\chi^0_1$, $q \in {c, s, b}$</th>
<th>$\tilde{g} \rightarrow g\chi^0_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>2.8</td>
<td>0.42</td>
<td>0.55</td>
<td>0.03</td>
</tr>
<tr>
<td>(ii)</td>
<td>0.95</td>
<td>0.43</td>
<td>0.56</td>
<td>0.02</td>
</tr>
<tr>
<td>(iii)</td>
<td>0.38</td>
<td>0.43</td>
<td>0.56</td>
<td>0.01</td>
</tr>
<tr>
<td>(iv)</td>
<td>0.18</td>
<td>0.38</td>
<td>0.49</td>
<td>0.13</td>
</tr>
<tr>
<td>(v)</td>
<td>0.11</td>
<td>0.38</td>
<td>0.49</td>
<td>0.13</td>
</tr>
<tr>
<td>(vi)</td>
<td>0.071</td>
<td>0.43</td>
<td>0.57</td>
<td>0.00</td>
</tr>
<tr>
<td>(vii)</td>
<td>0.051</td>
<td>0.39</td>
<td>0.51</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 8.2: Production cross sections in pb and the gluino decay branching ratios into light quarks (u,d) and other non-top quarks (c,s,b) for gluino coannihilation Models (i)-(vii) of Table 8.1 at the LHC. Gluinos are pair-produced by gluon fusion via the process $gg \rightarrow \tilde{g}\tilde{g}$ as well as by $q\bar{q} \rightarrow \tilde{g}\tilde{g}$. The gluinos subsequently decay with the leading decay mode being $\tilde{g} \rightarrow q\bar{q}\chi^0_1$. Table taken from [35].

sum of $E_T^{miss}$ and the transverse momentum of all jets with $p_T > 50$ GeV, while $M_{eff}(N_j)$ is the scalar sum of $E_T^{miss}$ and the transverse momentum of the first $N$ jets.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_T^{miss}$ (GeV) &gt;</td>
<td>2j1 2jm 2jt 4jt 5j 6jm 6jt</td>
</tr>
<tr>
<td>$p_T(j_1)$ (GeV) &gt;</td>
<td>200 300 200 200 200 200 200</td>
</tr>
<tr>
<td>$p_T(j_2)$ (GeV) &gt;</td>
<td>200 50 200 100 100 100 100</td>
</tr>
<tr>
<td>$p_T(j_3)$ (GeV) &gt;</td>
<td>100 100 100 100 100</td>
</tr>
<tr>
<td>$p_T(j_4)$ (GeV) &gt;</td>
<td>100 100 100 100 100</td>
</tr>
<tr>
<td>$p_T(j_5)$ (GeV) &gt;</td>
<td>100 100 100 100 100</td>
</tr>
<tr>
<td>$p_T(j_6)$ (GeV) &gt;</td>
<td>100 100 100 100 100</td>
</tr>
<tr>
<td>$\Delta \phi(j_{1,2,3},E_T^{miss})_{min}$ &gt;</td>
<td>0.8 0.4 0.8 0.4 0.4 0.4 0.4</td>
</tr>
<tr>
<td>$\Delta \phi(j_{1,2,3},E_T^{miss})_{min}$ &gt;</td>
<td>0.2 0.2 0.2 0.2 0.2 0.2</td>
</tr>
<tr>
<td>$E_T^{miss}/\sqrt{\mathcal{H}}$ (GeV) &gt;</td>
<td>15 15 20 15 15 15 15</td>
</tr>
<tr>
<td>$E_T^{miss}/M_{eff}(N_j)$ &gt;</td>
<td>0.25 0.25 0.25 0.25 0.25 0.25 0.25</td>
</tr>
<tr>
<td>$M_{eff}(incl.)$ (GeV) &gt;</td>
<td>1200 1600 2000 2200 1600 1600 2000</td>
</tr>
</tbody>
</table>

Table 8.3: The selection criteria used for the signal regions in the nomenclature of Table 2 of the ATLAS analysis [34]. Table taken from [35].

Using the techniques and signal regions described above, we analyzed each of the benchmark points of Table 8.1 to identify a signal region of minimum required luminosity for $S/\sqrt{B} \geq 5\sigma$ discovery of this point at the LHC. These results can be directly compared to
the results of [34] for simplified models involving gluinos and neutralinos with decoupled squarks. In that analysis it was found that for such a simplified model, using 3.2 fb\(^{-1}\) of data on the same signal regions to detect the same gluino decay channel \((\tilde{g} \rightarrow q\bar{q}\tilde{\chi}_{1}^{0})\), we can establish a gluino mass lower limit of \(\sim 1500\) GeV for neutralino masses up to \(\sim 800\) GeV. For LSP masses above this, no exclusion is established at 3.2 fb\(^{-1}\) for any gluino masses, demonstrating that the gluino coannihilation region remains viable for continued searching.

### 8.3.2 Discussion of Results

Using the signal regions defined in [34], we find (see Table 8.4 and Table 8.5) that the two leading signal regions for the detection of the decay of the gluino for the gluino coannihilation models are 2jl and 2jm over the entire range of gluino masses given in Table 8.1. Specifically, in Table 8.4 we present a grid where the ratio \(S\sqrt{B}\) for each of the Model (i)-(vii) for the seven signal regions is exhibited for an integrated luminosity sufficient for the discovery of the corresponding model. It is seen that for Model (i) 2jl is the discovery signal region while for Models (ii)-(vii), 2jm is the signal region for discovery. All other signal regions lie significantly below the threshold.

Since for each signal region up to 12 kinematical variables with cuts are utilized, it is of interest to analyze in further detail as to the most sensitive of these kinematical quantities that allow us to discriminate the signal over the background. It is found that among the 12 kinematical variables, 3 stand out as the most sensitive for our analysis. These are
Chapter 8. The Gluino Coannihilation Model

Table 8.4: A grid showing $S\sqrt{B}$ for the seven signal regions analyzed for each of the Models (i)-(vii) at the minimum $\mathcal{L}$ needed for discovery. The signal regions reaching the $5\sigma$ threshold at the minimum $\mathcal{L}$ needed for discovery are indicated in bold. Table taken from [35].

<table>
<thead>
<tr>
<th>Model</th>
<th>$L$ (fb$^{-1}$)</th>
<th>$2j\ell$</th>
<th>$2jm$</th>
<th>$2jt$</th>
<th>$4jt$</th>
<th>$5j$</th>
<th>$6jm$</th>
<th>$6jt$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>14</td>
<td>$5.0$</td>
<td>4.1</td>
<td>1.9</td>
<td>1.2</td>
<td>2.0</td>
<td>1.5</td>
<td>0.8</td>
</tr>
<tr>
<td>(ii)</td>
<td>66</td>
<td>4.8</td>
<td>$5.0$</td>
<td>2.5</td>
<td>1.8</td>
<td>2.0</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>(iii)</td>
<td>180</td>
<td>4.2</td>
<td>$5.0$</td>
<td>2.3</td>
<td>1.5</td>
<td>1.7</td>
<td>1.2</td>
<td>0.8</td>
</tr>
<tr>
<td>(iv)</td>
<td>440</td>
<td>3.8</td>
<td>$5.0$</td>
<td>2.7</td>
<td>1.4</td>
<td>1.4</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>(v)</td>
<td>970</td>
<td>3.6</td>
<td>$5.0$</td>
<td>3.0</td>
<td>1.7</td>
<td>1.6</td>
<td>1.0</td>
<td>1.1</td>
</tr>
<tr>
<td>(vi)</td>
<td>2000</td>
<td>3.5</td>
<td>$5.0$</td>
<td>2.8</td>
<td>1.6</td>
<td>1.4</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>(vii)</td>
<td>3400</td>
<td>3.3</td>
<td>$5.0$</td>
<td>3.1</td>
<td>1.9</td>
<td>1.1</td>
<td>0.5</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 8.5: Integrated luminosity for discovery in the leading and subleading signal regions for gluino coannihilation Models (i)-(vii) of Table 8.1 at the LHC. Table taken from [35].

<table>
<thead>
<tr>
<th>Model</th>
<th>Leading SR</th>
<th>$L$ (fb$^{-1}$)</th>
<th>Subleading SR</th>
<th>$L$ (fb$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>$2j\ell$</td>
<td>14</td>
<td>$2jm$</td>
<td>20</td>
</tr>
<tr>
<td>(ii)</td>
<td>$2jm$</td>
<td>66</td>
<td>$2j\ell$</td>
<td>71</td>
</tr>
<tr>
<td>(iii)</td>
<td>$2jm$</td>
<td>180</td>
<td>$2j\ell$</td>
<td>260</td>
</tr>
<tr>
<td>(iv)</td>
<td>$2jm$</td>
<td>440</td>
<td>$2j\ell$</td>
<td>760</td>
</tr>
<tr>
<td>(v)</td>
<td>$2jm$</td>
<td>970</td>
<td>$2j\ell$</td>
<td>1900</td>
</tr>
<tr>
<td>(vi)</td>
<td>$2jm$</td>
<td>2000</td>
<td>$2j\ell$</td>
<td>4200</td>
</tr>
<tr>
<td>(vii)</td>
<td>$2jm$</td>
<td>3400</td>
<td>$2j\ell$</td>
<td>7600</td>
</tr>
</tbody>
</table>

$M_{\text{eff}}$, $E_{T}^{\text{miss}}$, and $E_{T}^{\text{miss}}/\sqrt{H_{T}}$. We illustrate this for the cases (i), (ii), (vi), and (vii) in Figures 8.2 to 8.5.

In the left panel of Figure 8.2 we exhibit the $2j\ell$ signal region for Model (i), where we plot the number of signal events (red) and the square root of the total standard model background (blue) vs. $M_{\text{eff}}$. The analysis is done at an integrated luminosity of 14 fb$^{-1}$. Similar analyses are given in the middle panel of Figure 8.2 for $E_{T}^{\text{miss}}$ and in the right panel of Figure 8.2 for $E_{T}^{\text{miss}}/\sqrt{H_{T}}$. Very similar analyses were carried out for Models (ii), (vi), and (vii) in Figures 8.3 to 8.5. Here, however, the leading signal region is $2jm$ and
Chapter 8. The Gluino Coannihilation Model

Figure 8.2: Left panel: Distribution in $M_{\text{eff}}$ for the 2j signal region for gluino coannihilation Model (i). Plotted is the number of counts for the signal per 30 GeV and the square root of the total standard model background. The analysis is done at 14 fb$^{-1}$ of integrated luminosity at the LHC, which gives a 5$\sigma$ discovery in this signal region. Middle panel: The same analysis as in the left panel but for $E_{T}^{\text{miss}}$. Right panel: The same analysis but for $E_{T}^{\text{miss}}/\sqrt{H_{T}}$. Figures taken from [35].

Figure 8.3: Left panel: Distribution in $M_{\text{eff}}$ for the 2jm signal region for gluino coannihilation Model (ii). Plotted is the number of counts for the signal per 30 GeV and the square root of the total standard model background. The analysis is done at 66 fb$^{-1}$ of integrated luminosity at the LHC, which gives a 5$\sigma$ discovery in this signal region. Middle panel: The same analysis as in the left panel but for $E_{T}^{\text{miss}}$. Right panel: The same analysis but for $E_{T}^{\text{miss}}/\sqrt{H_{T}}$. Figures taken from [35].

Figure 8.4: Left panel: Distribution in $M_{\text{eff}}$ for the 2jm signal region for gluino coannihilation Model (vi). Plotted is the number of counts for the signal per 30 GeV and the square root of the total standard model backgrounds. The analysis is done at 2000 fb$^{-1}$ of integrated luminosity at the LHC, which gives a 5$\sigma$ discovery in this signal region. Middle panel: The same analysis as in the left panel but for $E_{T}^{\text{miss}}$. Right panel: The same analysis but for $E_{T}^{\text{miss}}/\sqrt{H_{T}}$. Figures taken from [35].
Chapter 8. The Gluino Coannihilation Model

Figure 8.5: Left panel: Distribution in $M_{\text{eff}}$ for the 2jm signal region for gluino coannihilation Model (vii). Plotted is the number of counts for the signal per 30 GeV and the square root of the total standard model background. The analysis is done at 3400 fb$^{-1}$ of integrated luminosity at the LHC, which gives a 5$\sigma$ discovery in this signal region. Middle panel: The same analysis as in the left panel but for $E_T^{\text{miss}}$. Left panel: The same analysis but for $E_T^{\text{miss}}/\sqrt{H_T}$. Figures taken from [35].

the analyses are done at an integrated luminosity of 66 fb$^{-1}$ for Model (ii) in Figure 8.3, at 2000 fb$^{-1}$ for Model (vi) Figure 8.4, and at 3400 fb$^{-1}$ for Model (vii) in Figure 8.5. In each of these cases the signal meets the 5$\sigma$ limit needed for discovery. A tabulation of the integrated luminosity needed for the discovery of all the models of Table 8.1 is given in Table 8.5.

The analysis of Figures 8.2 to 8.5 and of Table 8.5 shows the remarkable reduction in
the potential of the LHC for the discovery of gluinos if the gluino mass lies in the gluino
coannihilation region. A further illustration of this result is given in Figure 8.6, where
the largest gluino mass discoverable as a function of the integrated luminosity at the LHC
is given if the gluino mass lies in the gluino coannihilation region. Specifically one finds
that even with $\sim 3000$ fb$^{-1}$ of integrated luminosity the LHC will probe a gluino mass of
only $\sim 1200$ GeV. In contrast, away from the gluino coannihilation region, the discovery
potential of the LHC increases in a dramatic fashion as illustrated by the red dot in the
upper left-hand corner of Figure 8.6, which is taken from the ATLAS analysis [34] using
3.2 fb$^{-1}$ of integrated luminosity at LHC Run II. Thus the analysis of Figure 8.6 shows that if the gluino mass lies in the gluino coannihilation region, its mass could be much smaller than the current lower limits, where the mass gap between the gluino mass and the neutralino mass is large.

![Figure 8.6: Integrated luminosity required at the LHC for 5$\sigma$ discovery of a gluino in the gluino coannihilation region as a function of the gluino mass. For comparison, the ATLAS result from [34] using simplified models is also exhibited. One finds that the exclusion limit for the gluino mass in the coannihilation region is much lower than the ATLAS result. Figure taken from [35].](image)

### 8.3.3 Signal Region Optimization

Because the signal regions as defined in [34] are for generic simplified model light gluinos, it is advantageous to consider whether the signals can be improved upon by altering cuts to one or several of the variables in Table 8.5. It was found that very modest reduction (between 2–10%) in integrated luminosity required for 5$\sigma$ discovery can be achieved in the 2jm signal region by altering the cut on the variable $E_T^{\text{miss}}/\sqrt{H_T}$. To demonstrate this choice, Figure 8.7 shows the results of the 2jm signal region in Models (i), (ii), (vi), and
(vii) prior to any cuts on $E_T^{\text{miss}}/\sqrt{H_T}$ at the required integrated luminosity for discovery for that model.

By analyzing these figures, the original choice to require $E_T^{\text{miss}}/\sqrt{H_T} > 15\sqrt{\text{GeV}}$ (see Table 8.3) is shown to be a very good choice, as this is the point where the SUSY signal (in red) overtakes the square root of the standard model background (in blue). However, some improvement can be made by also requiring $E_T^{\text{miss}}/\sqrt{H_T}$ to be less than a specific critical value, where the signal drops below the square root of the background. This
Chapter 8. The Gluino Coannihilation Model

occurs around the value of 25 \( \sqrt{\text{GeV}} \) for the lighter models (Model (i) and (ii)) and around 30 \( \sqrt{\text{GeV}} \) for the heavier models (Model (vi) and (vii)). For the optimization, a value of 30 \( \sqrt{\text{GeV}} \) was chosen to optimize the fit for the heaviest gluino models, thereby extending the reach of the LHC. We call this signal region 2jm–HT. Table 8.6 indicates the improvement by modifying the cut on \( E_T^{\text{miss}}/\sqrt{H_T} \) in this way, while Figure 8.8 displays the binned count data after the optimized cut. Further improvement may be possible by investigating the substructure of the jets themselves, as discussed in [162].

![Figure 8.8: Upper left panel: Distribution in \( E_T^{\text{miss}}/\sqrt{H_T} \) for the 2jm–HT signal region for gluino coannihilation Model (i) after optimized cuts in \( E_T^{\text{miss}}/\sqrt{H_T} \). The analysis is done at 18 \( \text{fb}^{-1} \) of integrated luminosity at the LHC, which gives a \( \sigma \) discovery in this signal region. Upper right panel: The same analysis but for Model (ii) at 60 \( \text{fb}^{-1} \) of integrated luminosity. Lower left panel: The same analysis but for Model (vi) at 1980 \( \text{fb}^{-1} \) of integrated luminosity. Lower right panel: The same analysis but for Model (vii) at 3400 \( \text{fb}^{-1} \) of integrated luminosity. Figures taken from [35].](image)
Chapter 8. The Gluino Coannihilation Model

<table>
<thead>
<tr>
<th>Model</th>
<th>$\mathcal{L}$ (fb$^{-1}$) in 2jm</th>
<th>$\mathcal{L}$ (fb$^{-1}$) in 2jm-HT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>(ii)</td>
<td>66</td>
<td>60</td>
</tr>
<tr>
<td>(iii)</td>
<td>180</td>
<td>170</td>
</tr>
<tr>
<td>(iv)</td>
<td>440</td>
<td>420</td>
</tr>
<tr>
<td>(v)</td>
<td>970</td>
<td>950</td>
</tr>
<tr>
<td>(vi)</td>
<td>2000</td>
<td>1980</td>
</tr>
<tr>
<td>(vii)</td>
<td>3400</td>
<td>3400</td>
</tr>
</tbody>
</table>

Table 8.6: Integrated luminosity for discovery in 2jm and 2jm-HT, where 2jm-HT requires that $15 < E_T^{\text{miss}} / \sqrt{H_T} < 30$ in units of $\sqrt{\text{GeV}}$. Table taken from [35].

### 8.4 Direct Detection of Dark Matter

Numerical analysis shows that the effective cross section for dark matter annihilation $\sigma_{\text{eff}}$ is dominated by the processes involving the gluino such that a smaller mass gap between the gluino and the LSP leads to more dominant gluino processes. The relic density depends critically on this mass gap, and we wish to keep the relic density constant as the gluino mass increases. However, an increasing gluino mass reduces the cross section for the annihilating gluino. In order to compensate for the reduction in the gluino annihilation cross section so that $\sigma_{\text{eff}}$ remains constant, the mass gap between the gluino and the LSP must decrease [161, 163]. Specifically this requires that

$$\Delta_{\tilde{g}\tilde{\chi}^0} = \left( \frac{m_{\tilde{g}}}{m_{\tilde{\chi}^0}} - 1 \right)$$

must decrease as the LSP mass increases to achieve constant relic density. This is what is seen in the analysis of Table 8.1. Further, from the trend in Table 8.1, one expects that as the gluino mass gets large, $m_{\tilde{g}}/m_{\tilde{\chi}^0} \to 1$ and $\Delta_{\tilde{g}\tilde{\chi}^0} \to 0$ in order to achieve relic
density in the desired range. A similar phenomenon was seen in the analysis of [33].

It is pertinent to ask what the effect of non-perturbative corrections to the annihilation cross section implies regarding the relic density analysis. In general, non-perturbative corrections can be significant near the threshold, since here multiple gluon exchanges can occur producing the Sommerfeld enhancement factor. These effects may be approximated by the function $\mathcal{E}$ [164], where

$$\mathcal{E}_j = \frac{C_j \pi \alpha_s}{\beta} \left[ 1 - \exp \left\{ - \frac{C_j \pi \alpha_s}{\beta} \right\} \right]^{-1}. \tag{8.4}$$

Here $C_{j=g(q)} = 1/2(3/2)$ for $\tilde{g} \tilde{g} \to gg (\tilde{g} \tilde{g} \to q\bar{q})$ and $\beta = \sqrt{1 - 4m_{\tilde{g}}^2/s}$. In [161] an analysis was carried out for the relic density including the effect of Sommerfeld enhancement and results compared to those using micrOMEGAs, which uses only perturbative cross sections. The analysis of [161] shows that the effect of Sommerfeld enhancement is equivalent to an upward shift of the gluino mass by 3–6 GeV without the inclusion of Sommerfeld enhancement. Thus, based on the analysis of [161], including Sommerfeld enhancement would modify the results by only a few percent, and our conclusions are not affected in any significant way.

In addition to Sommerfeld enhancement, there are also higher-order QCD corrections beyond the tree-level prediction given by the code. While the inclusion of the higher-order effects is beyond the scope of this work, we can estimate the possible impact of such effects by enlarging the error corridor of the relic density and determining its impact.
Chapter 8. The Gluino Coannihilation Model

on the discovery potential of the LHC in this case for a given model point. Thus in Table 8.7, the $m_1 = m_2$ parameter of Model (i) is adjusted to achieve the range of $\Omega h^2$ at the 95% confidence interval as given in [165].

<table>
<thead>
<tr>
<th>Model</th>
<th>$m_1 = m_2$</th>
<th>Gluino</th>
<th>Neutralino</th>
<th>$\Omega_{\text{LSP}} h^2$</th>
<th>Leading SR</th>
<th>$\mathcal{L}$ (fb$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>1400</td>
<td>706</td>
<td>634</td>
<td>0.122</td>
<td>2jl</td>
<td>14</td>
</tr>
<tr>
<td>(i-a)</td>
<td>1396</td>
<td>706</td>
<td>632</td>
<td>0.137</td>
<td>2jl</td>
<td>16</td>
</tr>
<tr>
<td>(i-b)</td>
<td>1402</td>
<td>706</td>
<td>635</td>
<td>0.093</td>
<td>2jl</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 8.7: Effect on Model (i) of perturbing inputs to achieve $\Omega h^2$ encompassing the 95% confidence range given in [165]. The change in the integrated luminosity needed to discover Model (i) with an enlarged range of the relic density is given in the last column. The effect is less than 15%, exhibiting that the analysis is robust. All mass parameters are given in GeV. Table taken from [35].

The strong connection between LHC physics and dark matter has been discussed in the literature for some time (for a review see [15]). In the context of the gluino coannihilation models the connection is even stronger. This is due to the close proximity of the neutralino mass to the gluino mass in this class of models. Thus a determination of gluino mass at the LHC would indirectly imply a determination of the neutralino mass, since it lies within 10% of the gluino mass. One can thus make more definitive predictions for the direct detection of dark matter in this case. In general the cross section for the direct detection of neutralino dark matter depends on the higgsino content of the neutralino.

The gaugino and higgsino eigencomponents for Models (i)–(vii) are given in Table 8.8. One may define the higgsino content of the neutralino by the quantity $\sqrt{\gamma^2 + \delta^2}$. From Table 8.8 we see that the higgsino content of the neutralino is typically small ($< 0.05$). In these models a small higgsino content indicates that the neutralino–proton cross sections will be small in the gluino coannihilation region. This is discussed below.
Chapter 8. The Gluino Coannihilation Model

<table>
<thead>
<tr>
<th>Model</th>
<th>$\Delta \times 10^{3}$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>$&lt; 1$</td>
<td>-0.001</td>
<td>0.021</td>
<td>-0.008</td>
</tr>
<tr>
<td>(ii)</td>
<td>$&lt; 1$</td>
<td>-0.001</td>
<td>0.022</td>
<td>-0.009</td>
</tr>
<tr>
<td>(iii)</td>
<td>$&lt; 1$</td>
<td>-0.001</td>
<td>0.017</td>
<td>-0.006</td>
</tr>
<tr>
<td>(iv)</td>
<td>1</td>
<td>-0.003</td>
<td>0.042</td>
<td>-0.027</td>
</tr>
<tr>
<td>(v)</td>
<td>1</td>
<td>-0.003</td>
<td>0.042</td>
<td>-0.028</td>
</tr>
<tr>
<td>(vi)</td>
<td>$&lt; 1$</td>
<td>-0.001</td>
<td>0.013</td>
<td>-0.005</td>
</tr>
<tr>
<td>(vii)</td>
<td>1</td>
<td>-0.002</td>
<td>0.037</td>
<td>-0.024</td>
</tr>
</tbody>
</table>

Table 8.8: Gaugino and higgsino eigencodeponents of the neutralino for gluino coannihilation Models (i)-(vii), where $\Delta = 1 - \alpha$. Table taken from [35].

In Table 8.9 we give a computation of the spin-independent and spin-dependent proton–neutralino cross sections. For the spin-independent case we find $\sigma^{SI}_{p\chi_1^0}$ lying in the range $(1-10) \times 10^{-47}$ cm$^2$. The next generation LUX-ZEPLIN dark matter experiment [29, 30] is projected to reach a sensitivity of $\sim 10^{-47}$ cm$^{-2}$ [29, 30]. Thus the spin-independent proton–neutralino cross section of the gluino coannihilation models lies largely within the sensitivity range of the next-generation LUX-ZEPLIN experiment. A graphical illustration of the spin-independent proton–neutralino cross section is given in Figure 8.9. Here we are using the Models (i)-(vii) except that we allow $\tan \beta$ to vary between 2–50 and retain only those model points that satisfy the constraint $\Omega h^2 < 0.123$. We find that all of the models lie above the neutrino floor, some by an order of magnitude or more. Thus most of the parameter points of Figure 8.9 are discoverable by LUX-ZEPLIN. The neutralino–proton spin-dependent cross section $\sigma^{SI}_{p\chi_1^0}$ given by Table 8.9 lies in the range $(4-36) \times 10^{-45}$ cm$^2$. Here, LUX-ZEPLIN will have a maximum sensitivity of $10^{-42}$ cm$^2$, which is about two orders of magnitude smaller in sensitivity than what is needed to test the model in this sector.
Chapter 8. The Gluino Coannihilation Model

Table 8.9: $R \times \sigma_{SI}^{\tilde{\chi}_0^1}$ and $R \times \sigma_{SD}^{\tilde{\chi}_0^1}$ in units of cm$^2$ for the gluino coannihilation models of Table 8.1, where $R = \rho_{\tilde{\chi}_0^1}/\rho_c$ with $\rho_{\tilde{\chi}_0^1}$ giving the neutralino relic density and $\rho_c$ giving the critical relic density ($R \sim 1$ for the selected gluino coannihilation model points). The higgsino content of the neutralino in each case is also exhibited. Table taken from [35].

<table>
<thead>
<tr>
<th>Model</th>
<th>$R \times \sigma_{SI}^{\tilde{\chi}_0^1} \times 10^{47}$</th>
<th>$R \times \sigma_{SD}^{\tilde{\chi}_0^1} \times 10^{45}$</th>
<th>higgsino content</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>0.86 4.3</td>
<td>0.022</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>0.92 4.9</td>
<td>0.024</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>0.49 1.3</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td>(iv)</td>
<td>7.3 35</td>
<td>0.050</td>
<td></td>
</tr>
<tr>
<td>(v)</td>
<td>7.2 30</td>
<td>0.050</td>
<td></td>
</tr>
<tr>
<td>(vi)</td>
<td>0.29 0.50</td>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td>(vii)</td>
<td>5.5 19</td>
<td>0.044</td>
<td></td>
</tr>
</tbody>
</table>

Figure 8.9: $R \times \sigma_{SI}^{\tilde{\chi}_0^1}$ as a function of the neutralino mass where the vertical scatter points show the dependence on tan$\beta$ in the range of tan$\beta = 2$–50 for a given model. Only those parameter points that satisfy the constraint $\Omega h^2 < 0.123$ are admitted. Also displayed are the current and projected reaches of the XENON and LUX experiments in the relevant mass range and the neutrino floor [28, 29, 166, 167]. Figure taken from [35].

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8.5 Summary of Gluino Coannihilation Results

In this chapter it is shown that in the gluino coannihilation region, gluino masses as low as 700 GeV consistent with the Higgs boson mass constraint and the relic density constraint would have escaped detection at LHC Run I and Run II with 3.2 fb$^{-1}$ of integrated luminosity. Our analysis is done within the framework of high-scale supergravity unified models, allowing for non-universality in the gaugino sector. An illustrative list of models is given in Table 8.1 where the model points all satisfy the Higgs boson mass constraint and the relic density constraint consistent with WMAP and Planck. In the model space analyzed, the gluino is the NLSP, with a mass $\sim 1.1$ times the LSP mass. The small mass gap between the gluino and the neutralino is needed to satisfy the relic density constraint via gluino coannihilation. Requiring that the lightest neutralino supplies all of the thermal relic abundance implies that $\Delta m = m_{\tilde{g}} - m_{\tilde{\chi}_1^0}$ lies in the range 70–100 GeV over the entire region of the parameter space analyzed in Table 8.1. Because of the small mass gap, the gluino decay modes such as $\chi_{1}^\pm q_1 \bar{q}_2$ are suppressed and the decay occurs dominantly to $\chi_{1}^0 q \bar{q}$ with the subdominant decay mode being $\chi_{1}^0 g$. In the analysis of the Models (i)–(vii) we have used the signal regions used by the ATLAS Collaboration where an optimization of signal regions was carried out to determine the best regions for gluino discovery in the gluino coannihilation region. It is found that among the seven signal regions analyzed, the leading regions are 2jl and 2jm. All models listed in Table 8.1 are discoverable at the LHC with up to $\sim 3400$ fb$^{-1}$ of integrated luminosity.
The implications of the gluino coannihilation models for the discovery of dark matter were also discussed. As shown in Table 8.9 it is found that the spin-independent neutralino–proton cross section lies in the range \((1-10) \times 10^{-47}\text{cm}^{-2}\), and this range can be explored in the next-generation experiments on dark matter, e.g. LUX-ZEPLIN [29, 30]. The observed signal from the gluino coannihilation region would be a factor of up to ten times stronger than the one from the neutrino floor [160]. Also, as shown in Table 8.9, the spin-dependent neutralino–proton cross section lies in the range \((0.4 - 4) \times 10^{-44}\text{cm}^2\). This is about two orders of magnitude smaller than the sensitivity of LUX-ZEPLIN and thus will be more difficult to observe.
Chapter 9

The Stau Coannihilation Model

In this chapter we investigate the capability of the LHC for discovery of supersymmetry in the stau coannihilation region. We work within the framework of nonuniversal supergravity unification with nonuniversality in the gaugino sector, specifically the $\tilde{g}$SUGRA model [36]. In the analysis we model our signal regions along those ones used by the ATLAS collaboration and others, optimized for the stau coannihilation region [37, 39, 40]. As in the analysis of stop coannihilation and gluino coannihilation, in the stau coannihilation analysis we impose the relic density constraints as well as constraints of the Higgs boson mass. The range of sparticle masses discoverable at the LHC with up to 3000 fb$^{-1}$ of integrated luminosity is investigated. It is found that the mass difference between the stau and the neutralino does not exceed $\sim 25$ GeV over the entire mass range investigated. Thus discovery of the stau coannihilation region will also provide a measurement of the
neutralino mass. The direct detection of neutralino dark matter is analyzed within the class of stau coannihilation models investigated. It is found that the spin-independent neutralino-proton cross-section lies just out of reach of the next generation LUX-ZEPLIN experiment.

9.1 Introduction

In Chapters 7 and 8 we have investigated the prospects for the discovery of supersymmetry in the stop and gluino coannihilation regions. In this chapter we extend the analysis to the stau coannihilation region.

Stau–neutralino coannihilation has previously been investigated by a number of works [37, 168, 169]. Specifically in [168] an analysis has been carried out for the stau–neutralino coannihilation region at the LHC. However, the analysis of [168] was limited insofar as only neutralino masses below 100 GeV were investigated. In this work we use nonuniversal supergravity unification with nonuniversalities in the gaugino sector to investigate the full range of neutralino and stau mass ranges that are discoverable at the LHC. In our analysis we impose the relic density constraint as well as the constraint of the Higgs boson mass. Specifically we use the $\tilde{g}$SUGRA model [36] where $m_3 >> m_1 = m_2$ (typically $m_1 : m_2 : m_3 = 1 : 1 : 10$) for stau–neutralino coannihilation, as well as a relaxed version of this model with $m_3 >> m_1 > m_2$ for multiparticle coannihilation. We use signal
regions based on those previously published in [37, 39] but optimized for the stau coannihilation region. An analysis of dark matter is also given.

### 9.2 The Stau Coannihilation Model

For this parameter space we consider a nonuniversal supergravity model, allowing nonuniversality in the gaugino sector by assuming that the scalar masses are universal at high scale but the gaugino masses are split, i.e. that gaugino masses in the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ sectors can all be different. Specifically, for the case where it is desired to have a stau with mass very close to the LSP, we use the $\tilde{g}$SUGRA model [36] with the parameter space

$$m_0, \ A_0, \ m_1 = m_2 \ll m_3, \ \tan \beta, \ \text{sign(}\mu\text{)}, \quad (9.1)$$

For the multiparticle coannihilation parameter space, we relax the requirement that $m_1 = m_2$, allowing $m_2$ to be slightly less than $m_1$. This brings the mass of the chargino and second neutralino closer to the stau and LSP so that those particles also contribute to coannihilation. In this case we use the following parameter space for the model:

$$m_0, \ A_0, \ m_1 < m_2 \ll m_3, \ \tan \beta, \ \text{sign(}\mu\text{)} . \quad (9.2)$$

In these models, the stau, neutralino, and chargino masses are all bunched together, as
Chapter 9. *The Stau Coannihilation Model*

can be seen in Figures 9.1 and 9.2. The stop and gluino masses lie in the TeV range and this provides a large loop correction to the Higgs boson to lift its tree value to the desired experimentally observed value.

**Figure 9.1:** An exhibition of the sparticle mass hierarchy for stau coannihilation Model (a). Top panel: Full spectrum. Bottom panel: Only sparticles with mass < 500 GeV. Figures taken from [41].
Figure 9.2: An exhibition of the sparticle mass hierarchy for multiparticle coannihilation Model (iii). Top panel: Full spectrum. Bottom panel: Only sparticles with mass < 500 GeV. Figures taken from [41].
9.2.1 Two-Particle Coannihilation

First we consider parameter regions of the $\tilde{g}$SUGRA model with the Higgs mass in the range $125 \pm 2$ GeV where stau–LSP coannihilation gives rise to an LSP relic density within the known limit $\Omega h^2 < 0.128$. A sample set of such points is given in Tables 9.1 and 9.2, where Table 9.1 gives the $\tilde{g}$SUGRA input parameters and Table 9.2 gives the sparticle masses for those inputs.

<table>
<thead>
<tr>
<th>Model</th>
<th>$m_0$</th>
<th>$A_0$</th>
<th>$m_1 = m_2$</th>
<th>$m_3$</th>
<th>$\tan \beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>286</td>
<td>-523</td>
<td>314</td>
<td>3015</td>
<td>10</td>
</tr>
<tr>
<td>b.</td>
<td>297</td>
<td>-553</td>
<td>343</td>
<td>3246</td>
<td>10</td>
</tr>
<tr>
<td>c.</td>
<td>267</td>
<td>-378</td>
<td>367</td>
<td>2911</td>
<td>10</td>
</tr>
<tr>
<td>d.</td>
<td>295</td>
<td>-491</td>
<td>381</td>
<td>2821</td>
<td>13</td>
</tr>
<tr>
<td>e.</td>
<td>325</td>
<td>-416</td>
<td>412</td>
<td>3156</td>
<td>14</td>
</tr>
<tr>
<td>f.</td>
<td>317</td>
<td>-497</td>
<td>437</td>
<td>3065</td>
<td>14</td>
</tr>
<tr>
<td>g.</td>
<td>364</td>
<td>-587</td>
<td>445</td>
<td>3728</td>
<td>14</td>
</tr>
<tr>
<td>h.</td>
<td>412</td>
<td>-904</td>
<td>503</td>
<td>4688</td>
<td>13</td>
</tr>
<tr>
<td>j.</td>
<td>337</td>
<td>833</td>
<td>593</td>
<td>3626</td>
<td>15</td>
</tr>
<tr>
<td>k.</td>
<td>295</td>
<td>-551</td>
<td>302</td>
<td>3165</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 9.1: Input parameters for representative stau coannihilation benchmark points. All masses are in GeV. Table taken from [41].

As demonstrated in Table 9.2, the parameter points in the stau–neutralino coannihilation region have a very small stau–neutralino mass gap $\Delta \sim 20$ GeV. Such a small gap raises many challenges for discovery of these sparticles. In cases with such little energy available for decay jets, initial and final state radiation (ISR and FSR) are often relied upon to produce a more detectible signal in colliders.
Chapter 9. The Stau Coannihilation Model

Table 9.2: The Higgs boson ($h^0$) mass, some relevant sparticle masses, and the relic density for the stau coannihilation benchmark points of Table 9.1. All masses are in GeV. Table taken from [41].

<table>
<thead>
<tr>
<th>Model</th>
<th>$h^0$</th>
<th>$\tilde{\tau}$</th>
<th>$\tilde{\chi}_1^0$</th>
<th>$\tilde{\chi}_1^\pm$</th>
<th>$\tilde{t}$</th>
<th>$\tilde{g}$</th>
<th>$\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>123.2</td>
<td>134.4</td>
<td>112.4</td>
<td>208.4</td>
<td>4522</td>
<td>6168</td>
<td>0.125</td>
</tr>
<tr>
<td>b.</td>
<td>123.4</td>
<td>144.3</td>
<td>123.9</td>
<td>229.7</td>
<td>4842</td>
<td>6608</td>
<td>0.121</td>
</tr>
<tr>
<td>c.</td>
<td>123.1</td>
<td>155.1</td>
<td>136.5</td>
<td>256.0</td>
<td>4376</td>
<td>5961</td>
<td>0.119</td>
</tr>
<tr>
<td>d.</td>
<td>123.1</td>
<td>163.9</td>
<td>143.7</td>
<td>270.7</td>
<td>4244</td>
<td>5787</td>
<td>0.115</td>
</tr>
<tr>
<td>e.</td>
<td>123.2</td>
<td>176.7</td>
<td>155.5</td>
<td>292.2</td>
<td>4720</td>
<td>6428</td>
<td>0.133</td>
</tr>
<tr>
<td>f.</td>
<td>123.3</td>
<td>188.9</td>
<td>167.3</td>
<td>315.5</td>
<td>4584</td>
<td>6251</td>
<td>0.126</td>
</tr>
<tr>
<td>g.</td>
<td>123.4</td>
<td>190.3</td>
<td>167.0</td>
<td>312.0</td>
<td>5506</td>
<td>7517</td>
<td>0.125</td>
</tr>
<tr>
<td>h.</td>
<td>123.9</td>
<td>212.0</td>
<td>187.4</td>
<td>347.6</td>
<td>6775</td>
<td>9287</td>
<td>0.126</td>
</tr>
<tr>
<td>j.</td>
<td>123.7</td>
<td>254.0</td>
<td>232.9</td>
<td>439.5</td>
<td>5422</td>
<td>7308</td>
<td>0.116</td>
</tr>
<tr>
<td>k.</td>
<td>123.2</td>
<td>121.9</td>
<td>106.2</td>
<td>195.3</td>
<td>4732</td>
<td>6456</td>
<td>0.072</td>
</tr>
</tbody>
</table>

9.2.2 Multiparticle Coannihilation

Next we consider a model of multiparticle coannihilation between the neutralino (LSP), the stau (NLSP), and the chargino and the second neutralino, which in this model remain nearly degenerate. The parameter points of Table 9.3 are chosen so as to satisfy the constraints on the Higgs mass, $m_h = 125 \pm 2$ GeV, and the relic density $\Omega h^2 < 0.128$ and in such a way to produce the mass hierarchy $m_{\chi_1^0} < m_{\tilde{\tau}} < m_{\chi_1^\pm}$, i.e. so that the stau–neutralino mass gap is always larger than the chargino–stau mass gap. This is demonstrated in Table 9.4. Bringing the chargino mass closer to the stau will help deplete the relic density but allow for harder stau decay products due to a larger mass gap $\Delta m$ between the LSP and the stau. Decays of the chargino or second neutralino, however, are in this case expected to be much softer.
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<table>
<thead>
<tr>
<th>Model</th>
<th>$m_0$</th>
<th>$A_0$</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>$m_3$</th>
<th>$\tan \beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.</td>
<td>345</td>
<td>68</td>
<td>394</td>
<td>287</td>
<td>3690</td>
<td>10</td>
</tr>
<tr>
<td>ii.</td>
<td>385</td>
<td>152</td>
<td>403</td>
<td>290</td>
<td>3972</td>
<td>12</td>
</tr>
<tr>
<td>iii.</td>
<td>318</td>
<td>248</td>
<td>357</td>
<td>249</td>
<td>2973</td>
<td>12</td>
</tr>
<tr>
<td>iv.</td>
<td>386</td>
<td>-47</td>
<td>401</td>
<td>284</td>
<td>3809</td>
<td>13</td>
</tr>
<tr>
<td>v.</td>
<td>367</td>
<td>78</td>
<td>409</td>
<td>290</td>
<td>3550</td>
<td>13</td>
</tr>
<tr>
<td>vi.</td>
<td>423</td>
<td>-19</td>
<td>431</td>
<td>314</td>
<td>4396</td>
<td>13</td>
</tr>
<tr>
<td>vii.</td>
<td>353</td>
<td>202</td>
<td>427</td>
<td>298</td>
<td>3351</td>
<td>13</td>
</tr>
<tr>
<td>viii.</td>
<td>390</td>
<td>-161</td>
<td>440</td>
<td>308</td>
<td>3864</td>
<td>13</td>
</tr>
<tr>
<td>ix.</td>
<td>321</td>
<td>246</td>
<td>423</td>
<td>296</td>
<td>3328</td>
<td>10</td>
</tr>
<tr>
<td>x.</td>
<td>432</td>
<td>264</td>
<td>494</td>
<td>350</td>
<td>4234</td>
<td>15</td>
</tr>
<tr>
<td>xi.</td>
<td>304</td>
<td>-745</td>
<td>260</td>
<td>221</td>
<td>2793</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 9.3: Input parameters for representative stau-chargino coannihilation benchmark points. All masses are in GeV. Table taken from [41].

The sparticle spectra for these models feature a stau-neutralino mass gap above 20 GeV and a chargino-stau mass gap from 1 to 12 GeV.

<table>
<thead>
<tr>
<th>Model</th>
<th>$h^0$</th>
<th>$\tilde{\tau}$</th>
<th>$\tilde{\chi}_1^0$</th>
<th>$\tilde{\chi}_1^\pm$</th>
<th>$\tilde{\chi}_1^0$</th>
<th>$\tilde{\chi}_2^0$</th>
<th>$\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.</td>
<td>123.8</td>
<td>161.4</td>
<td>142.3</td>
<td>171.7</td>
<td>5511</td>
<td>7468</td>
<td>0.124</td>
</tr>
<tr>
<td>ii.</td>
<td>123.8</td>
<td>166.5</td>
<td>144.6</td>
<td>169.7</td>
<td>5912</td>
<td>8007</td>
<td>0.127</td>
</tr>
<tr>
<td>iii.</td>
<td>123.2</td>
<td>150.2</td>
<td>130.4</td>
<td>151.4</td>
<td>4521</td>
<td>6098</td>
<td>0.114</td>
</tr>
<tr>
<td>iv.</td>
<td>123.6</td>
<td>166.9</td>
<td>145.0</td>
<td>167.9</td>
<td>5677</td>
<td>7698</td>
<td>0.115</td>
</tr>
<tr>
<td>v.</td>
<td>123.6</td>
<td>171.4</td>
<td>150.2</td>
<td>177.8</td>
<td>5320</td>
<td>7201</td>
<td>0.120</td>
</tr>
<tr>
<td>vi.</td>
<td>123.8</td>
<td>176.5</td>
<td>154.7</td>
<td>183.9</td>
<td>6488</td>
<td>8808</td>
<td>0.107</td>
</tr>
<tr>
<td>vii.</td>
<td>123.5</td>
<td>179.6</td>
<td>159.3</td>
<td>188.2</td>
<td>5045</td>
<td>6818</td>
<td>0.117</td>
</tr>
<tr>
<td>viii.</td>
<td>123.8</td>
<td>182.8</td>
<td>162.2</td>
<td>188.5</td>
<td>5742</td>
<td>7797</td>
<td>0.103</td>
</tr>
<tr>
<td>ix.</td>
<td>123.6</td>
<td>175.1</td>
<td>157.3</td>
<td>185.9</td>
<td>5011</td>
<td>6773</td>
<td>0.121</td>
</tr>
<tr>
<td>x.</td>
<td>123.5</td>
<td>206.5</td>
<td>184.0</td>
<td>219.0</td>
<td>6272</td>
<td>8492</td>
<td>0.101</td>
</tr>
<tr>
<td>xi.</td>
<td>123.1</td>
<td>121.6</td>
<td>89.9</td>
<td>131.5</td>
<td>4212</td>
<td>5756</td>
<td>0.125</td>
</tr>
</tbody>
</table>

Table 9.4: The Higgs boson mass, some relevant sparticle masses, and the relic density for the stau-chargino coannihilation benchmark points of Table 9.3. All masses are in GeV. Table taken from [41].
9.3 Signal Analysis for Stau Coannihilation Models at the LHC

After a scan of the nonuniversal supergravity parameter space was performed to select benchmark points for each of the two coannihilation models satisfying the Higgs mass constraint, the relic density, and the desired neutralino, stau, and chargino mass hierarchies discussed in the previous section (Tables 9.1 and 9.3), those points are then used for a Monte Carlo analysis of LHC signal regions. A large set of search analyses were performed on the generated events for each benchmark point using signal regions involving hadronic $\tau$ final states (see Section 9.3.2) and other leptonic final states (see Section 9.3.3).

9.3.1 LHC Production and Signal Definitions

The signal regions considered here comprise two major categories, based upon the sparticle whose decay signatures they are meant to capture. The first category of signal regions includes signatures based on hadronically decaying taus, which are an expected result of stau decay. The second category involves signatures of multiple light leptons, which are meant to search for the decays of charginos and heavy neutralinos. Because both of the coannihilation regions under investigation have light staus and electroweakinos, it is expected that both signal region categories are viable for the stau coannihilation models considered here.
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The first signal region studied in this work involves at most 1 hadronically decaying tau in the final state. The selection criteria for one $\tau_h$ are based on an optimization of those defined in [37]. The second set of signal regions looks for at most 2 hadronically decaying taus in the final state. The selection criteria used are a modification of those in [37], where the first (SC-1) involves cuts on the transverse momenta of $\tau_h$ and the second (SC-2) involves cuts on $M_{\text{Eff}}$, defined as the sum of the missing transverse energy $E_{\text{T}}^\text{miss}$ and the transverse momenta of the two leading hadronic taus. Finally, the set of signal regions using selection criteria SC-3 applies cuts on the transverse mass $m_T$ (see (9.3)) of the taus, following the strategy of [40]. These signal regions are discussed in greater detail in Section 9.3.2.

Next we analyze electron and muon signal regions based on the work of [39]. One set of signal regions requires two leptons in the final state, comprising either a same flavor opposite sign pair, or a different flavor opposite sign pair, with increasing cuts on kinematic variables. The second set requires three leptons in the final state, two of which form a same flavor opposite sign pair. These are discussed further in Section 9.3.3.

Using the techniques and signal regions described above, we analyzed each of the benchmark points in Tables 9.1 and 9.3 to identify a signal region with minimum required luminosity for $5\sigma S/\sqrt{B}$ discovery of that point at the LHC.

In Tables 9.5 and 9.6 we give an analysis of the supersymmetry production cross sections for the models under study. The cross section for all models is dominated by the
production of the neutralino \( \tilde{\chi}_2^0 \) and chargino \( \tilde{\chi}_1^\pm \). In nearly every model point the only decay mode of \( \tilde{\chi}_2^0 \) is via the channel \( \tilde{\chi}_2^0 \rightarrow \tilde{\tau} \tau \), while the primary decay of the chargino is via the channel \( \tilde{\chi}_1^\pm \rightarrow \tilde{\tau} \nu \tau \) (see Tables 9.7 and 9.9). The stau always decays through one channel, \( \tilde{\tau} \rightarrow \tilde{\chi}_1^0 \tau \) (see Tables 9.8 and 9.10), where the available phase space for the emitted tau is small, resulting in a soft tau production making it difficult to observe with low luminosity.

<table>
<thead>
<tr>
<th>Model</th>
<th>full SUSY</th>
<th>( \tilde{\chi}_2^0 \tilde{\chi}_1^\pm )</th>
<th>( \tilde{\chi}_1^+ \tilde{\chi}_1^- )</th>
<th>( \tilde{\tau}^+ \tilde{\tau}^- )</th>
<th>( \tilde{\tau} \nu \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>2.09</td>
<td>2.57</td>
<td>0.62</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>b.</td>
<td>1.48</td>
<td>0.88</td>
<td>0.43</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>c.</td>
<td>1.01</td>
<td>0.58</td>
<td>0.29</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>d.</td>
<td>0.79</td>
<td>0.47</td>
<td>0.23</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>e.</td>
<td>0.59</td>
<td>0.35</td>
<td>0.17</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>f.</td>
<td>0.44</td>
<td>0.26</td>
<td>0.13</td>
<td>0.01</td>
<td>0.007</td>
</tr>
<tr>
<td>g.</td>
<td>0.46</td>
<td>0.28</td>
<td>0.13</td>
<td>0.01</td>
<td>0.008</td>
</tr>
<tr>
<td>h.</td>
<td>0.31</td>
<td>0.18</td>
<td>0.09</td>
<td>0.007</td>
<td>0.006</td>
</tr>
<tr>
<td>j.</td>
<td>0.12</td>
<td>0.07</td>
<td>0.03</td>
<td>0.003</td>
<td>0.002</td>
</tr>
<tr>
<td>k.</td>
<td>2.65</td>
<td>1.61</td>
<td>0.79</td>
<td>0.06</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 9.5: SUSY production cross sections in picobarns for stau coannihilation benchmark points of Table 9.1. Table taken from [41].

<table>
<thead>
<tr>
<th>Model</th>
<th>full SUSY</th>
<th>( \tilde{\chi}_2^0 \tilde{\chi}_1^\pm )</th>
<th>( \tilde{\chi}_1^+ \tilde{\chi}_1^- )</th>
<th>( \tilde{\tau}^+ \tilde{\tau}^- )</th>
<th>( \tilde{\tau} \nu \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.</td>
<td>3.99</td>
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<td>1.26</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>ii.</td>
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<td>2.68</td>
<td>1.32</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>iii.</td>
<td>6.17</td>
<td>4.02</td>
<td>1.98</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>iv.</td>
<td>4.25</td>
<td>2.78</td>
<td>1.37</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>v.</td>
<td>3.48</td>
<td>2.27</td>
<td>1.11</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>vi.</td>
<td>3.08</td>
<td>2.01</td>
<td>0.98</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>vii.</td>
<td>2.84</td>
<td>1.84</td>
<td>0.90</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
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<td>0.90</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>ix.</td>
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<td>1.93</td>
<td>0.94</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>x.</td>
<td>1.63</td>
<td>1.06</td>
<td>0.52</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>xi.</td>
<td>10.19</td>
<td>6.65</td>
<td>3.29</td>
<td>0.06</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 9.6: SUSY production cross sections in picobarns for multiparticle coannihilation benchmark points of Table 9.3. Table taken from [41].
9.3.2 \( \tau \)-Based Signals

We start by discussing the \( 1 \tau \) signature search by applying the selection criteria given in [37]. It turns out that the calculated luminosity necessary for a \( 5 \sigma \) discovery ranges from \( \sim 10^4 - 10^5 \text{ fb}^{-1} \), which is beyond the LHC’s maximum design luminosity. The main problem is the cut on the missing transverse energy. In [37], \( E_T^{\text{miss}} > 230 \text{ GeV} \).
was the choice of cut and by inspecting the left panel of Figure 9.3, one can see that we begin to lose the signal for $E_T^{\text{miss}} > 250$ GeV and hence the signal to background ratio becomes worse. Furthermore, the choice $15 < p_T(\tau_h) < 35$ GeV applied on the hadronic tau transverse momentum is not optimized for the models considered here. The right panel of Figure 9.3 shows that signal is above the background in the range 100–250 GeV, where the signal counts peak at around 40 GeV. Thus increasing the range of cut on
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$p_T(\tau_h)$ will produce better results.

Figure 9.3: Left panel: Distribution in $E_T^{\text{miss}}$ for the 1τ signal region for multiparticle coannihilation Model (iii) prior to any cuts. Plotted is the number of counts for the SUSY signal per 30 GeV and the square root of the total SM SNOWMASS background. The analysis is done at 1550 fb$^{-1}$ of integrated luminosity, which gives a 5σ discovery in this signal region. Right panel: The same analysis as in the left panel but for $p_T(\tau_h)$ with counts per 10 GeV. Figures taken from [41].

The optimized cuts for the 1τ signature are displayed in Table 9.11 including three variations: 1τ-A, 1τ-B and 1τ-C. They correspond to variations of the cut on $p_T(\tau_h)$.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_T^{\text{miss}}$ (GeV) &gt;</td>
<td>130</td>
</tr>
<tr>
<td>$p_T(j_1)$ (GeV) &gt;</td>
<td>100</td>
</tr>
<tr>
<td>$</td>
<td>\eta(j_1)</td>
</tr>
<tr>
<td>$p_T(\tau_h)$ (GeV) &gt;</td>
<td>15</td>
</tr>
<tr>
<td>$p_T(\tau_h)$ (GeV) &lt;</td>
<td>50</td>
</tr>
<tr>
<td>$</td>
<td>\eta(\tau_h)</td>
</tr>
<tr>
<td>$\Delta R(\tau_h, j_1)$ &gt;</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 9.11: The selection criteria used for the signal regions with one hadronically decaying tau in the final state and a veto on electrons, muons and b-jets. The angles are in rad. Table taken from [41].

Table 9.12 gives the minimum integrated luminosity needed for a 5σ discovery using these cuts on each of the 10 benchmark points. The best results are obtained for the cuts of 1τ-A where the luminosity ranges from 1510 to 2650 fb$^{-1}$, which is within reach of the LHC design luminosity of $\sim 3000$ fb$^{-1}$. Results obtained from 1τ-C show, for the most
part, luminosities greater than 3000 fb\(^{-1}\) since the range of the cut on \(p_T(\tau_h)\) extends to 150 GeV, which is above the value at which the signal generally begins dropping below background.

<table>
<thead>
<tr>
<th>Model</th>
<th>(\mathcal{L}) for 5(\sigma) discovery in 1(\tau)-A</th>
<th>(\mathcal{L}) for 5(\sigma) discovery in 1(\tau)-B</th>
<th>(\mathcal{L}) for 5(\sigma) discovery in 1(\tau)-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.</td>
<td>1510</td>
<td>1810</td>
<td>2520</td>
</tr>
<tr>
<td>ii.</td>
<td>1550</td>
<td>1800</td>
<td>2630</td>
</tr>
<tr>
<td>iii.</td>
<td>1550</td>
<td>1910</td>
<td>2730</td>
</tr>
<tr>
<td>iv.</td>
<td>1580</td>
<td>2020</td>
<td>2930</td>
</tr>
<tr>
<td>v.</td>
<td>1800</td>
<td>2260</td>
<td>..</td>
</tr>
<tr>
<td>vi.</td>
<td>2010</td>
<td>2290</td>
<td>..</td>
</tr>
<tr>
<td>vii.</td>
<td>2010</td>
<td>2330</td>
<td>..</td>
</tr>
<tr>
<td>viii.</td>
<td>2090</td>
<td>2340</td>
<td>..</td>
</tr>
<tr>
<td>ix.</td>
<td>2400</td>
<td>2880</td>
<td>..</td>
</tr>
<tr>
<td>x.</td>
<td>2650</td>
<td>..</td>
<td>..</td>
</tr>
<tr>
<td>xi.</td>
<td>1610</td>
<td>1420</td>
<td>1720</td>
</tr>
</tbody>
</table>

Table 9.12: Analysis of the discovery potential for supersymmetry for the parameter space of Table 9.3, using the selection criteria of Table 9.11, where the minimum integrated luminosity needed for 5\(\sigma\) discovery is given in fb\(^{-1}\). Here and in the tables following two dots (\(\cdots\)) indicate that the minimum integrated luminosity needed for 5\(\sigma\) discovery exceeds 3000 fb\(^{-1}\). Table taken from [41].

Another \(\tau\) signature of interest is that of two hadronically decaying taus in the final state. Here we adapt the signal regions of Table 9.11 to the 2\(\tau\) case by considering two selection criteria SC-1 and SC-2 as shown in Table 9.13. The first (SC-1) is a duplication of the cuts from Table 9.11, modified to require a second \(\tau\), while in SC-2 we introduce the variable \(M_{\text{eff}}\), defined as the scalar sum of the missing transverse energy and the transverse momenta of the two leading hadronic taus, \(M_{\text{eff}} = E_{T}^{\text{miss}} + p_{T}^{\tau_{1h}} + p_{T}^{\tau_{2h}}\). For completeness, we apply those cuts also to the 1\(\tau\) signal regions and find that this improves upon our results from Table 9.12.
Thus, the new set of $\tau$ based signal regions after inclusion of additional selection criteria SC-1 and SC-2 is presented in Table 9.13. Here we veto on electrons, muons, and b-jets. In this set, we have removed the cut on the pseudorapidity of the leading jet which was among the cuts for the $1\tau$ signature in Table 9.11. Also, an upper bound has been placed on the $E_T^{\text{miss}}$ cut to suppress values where the signal passed below the background. In Figure 9.4 we exhibit the distributions in $p_T(j_1)$, the transverse momentum of the leading jet, and the effective mass $m_{\text{eff}}$ for Model (xi) at 73 fb$^{-1}$. The signal appears to be above the background for lower $p_T(j_1)$ and $m_{\text{eff}}$ values at which the cuts were applied (Table 9.13).

In Figures 9.4 to 9.6 we exhibit the distributions in different kinematic variables for the multiparticle coannihilation Model (xi) at 73 fb$^{-1}$ for signal region $2\tau$SC2-A, where we plot the number of supersymmetry signal events (red) against the square root of the SM background (blue). In Figure 9.4 the left panel gives the $p_T$ of the leading jet, while the right panel gives $m_{\text{eff}}$. The left panel of Figure 9.5 shows the distribution in the transverse momentum of the leading hadronic tau, $p_T(\tau_{1h})$, while the right panel shows $p_T(\tau_{2h})$, the transverse momentum of the subleading hadronic tau. In Fig. 9.6 the same analysis is done but for $\Delta R(\tau_{2h}, j_1)$ (right panel) and the missing transverse energy $E_T^{\text{miss}}$ (left panel).

Out of the signal regions in Table 9.13, those giving the lowest luminosities for discovery often, but not always, belong to the set SC-1, as shown in Table 9.14 for the stau coannihilation models and Table 9.15 for the multiparticle coannihilation models. Here
### Table 9.13: The selection criteria (SC) used for the signal regions with the 1τ and 2τ signatures. The SRs SC1 and SC2 have a common cut on the missing transverse energy of $100 \text{ GeV} < E^{\text{miss}}_T < 200 \text{ GeV}$, with a veto on electrons, muons and b-jets. The dashes mean that the kinematic variable is not applicable to the corresponding SR. The angles are in rad. Tables taken from [41].

<table>
<thead>
<tr>
<th>Requirement</th>
<th>SC1</th>
<th>SC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_T(j_1)$ (GeV) &gt;</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$p_T(j_1)$ (GeV) &lt;</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>$p_T(\tau_{1h})$ (GeV) &gt;</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$p_T(\tau_{1h})$ (GeV) &lt;</td>
<td>50</td>
<td>70</td>
</tr>
<tr>
<td>$p_T(\tau_{2h})$ (GeV) &gt;</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$p_T(\tau_{2h})$ (GeV) &lt;</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$</td>
<td>\eta(\tau_{1h})</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>\eta(\tau_{2h})</td>
<td>$</td>
</tr>
<tr>
<td>$\Delta R(\tau_{1h}, j_1)$ &gt;</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$\Delta R(\tau_{1h}, j_1)$ &lt;</td>
<td>1.8</td>
<td>1.8</td>
</tr>
<tr>
<td>$\Delta R(\tau_{2h}, j_1)$ &gt;</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta R(\tau_{2h}, j_1)$ &lt;</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$N(\tau_h)$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

- Table 9.13: The selection criteria (SC) used for the signal regions with the 1τ and 2τ signatures. The SRs SC1 and SC2 have a common cut on the missing transverse energy of $100 \text{ GeV} < E^{\text{miss}}_T < 200 \text{ GeV}$, with a veto on electrons, muons and b-jets. The dashes mean that the kinematic variable is not applicable to the corresponding SR. The angles are in rad. Tables taken from [41].
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Figure 9.4: Left panel: Distribution in $p_T(j_1)$ for the $2\tau$SC2-A signal region for multiparticle coannihilation Model (xi) prior to any cuts. Plotted is the number of counts for the SUSY signal per 10 GeV and the square root of the total SM SNOWMASS backgrounds. The analysis is done at 73 fb$^{-1}$ of integrated luminosity, which gives a 5$\sigma$ discovery in this signal region. Right panel: The same analysis as in the left panel but for $m_{\text{eff}}$ with counts for the SUSY signal per 30 GeV. Figures taken from [41].

Figure 9.5: Left panel: Distribution in $p_T(\tau_{1h})$ for the $2\tau$SC2-A signal region for the multiparticle coannihilation Model (xi) prior to any cuts. Plotted is the number of counts for the SUSY signal per 10 GeV and the square root of the total SM SNOWMASS backgrounds. The analysis is done at 73 fb$^{-1}$ of integrated luminosity, which gives a 5$\sigma$ discovery in this signal region. Right panel: The same analysis as in the left panel but for $p_T(\tau_{2h})$. Figures taken from [41].

luminosities as low as 73 fb$^{-1}$ are achieved. For the 1$\tau$ signatures, there is a major improvement in luminosity with values from $\sim 144$ fb$^{-1}$ in comparison to the values obtained in using the cuts in Table 9.11, starting at $\sim 1500$ fb$^{-1}$.

As mentioned above, for the majority of cases in Tables 9.14 and 9.15, SC-1 variations (A, B and C) give better results than SC-2, except in a few cases. This demonstrates that $M_{\text{eff}}$ is not terribly valuable as a kinematic variable for cuts in the coannihilation region.
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Figure 9.6: Left panel: Distribution in $\Delta R(\tau_{2h}, j_1)$ for the $2\tau$SC2-A signal region for the multiparticle coannihilation Model (xi) prior to any cuts. Plotted is the number of counts for the SUSY signal per 0.035 rad and the square root of the total SM SNOWMASS backgrounds. The analysis is done at 73 fb$^{-1}$ of integrated luminosity, which gives a 5$\sigma$ discovery in this signal region. Right panel: The same analysis as in the left panel but for $E_T^{\text{miss}}$ with counts for the SUSY signal per 30 GeV. Figures taken from [41].

In addition, for equivalent kinematic cuts, signal regions demanding two hadronically decaying taus performed better than those demanding a single tau for the multiparticle coannihilation region, while those requiring only one tau perform better for the stau coannihilation region. This is consistent with the multiparticle coannihilation region allowing more energy for stau decay products.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\mathcal{L}$ for 5$\sigma$ discovery in SC1</th>
<th>$\mathcal{L}$ for 5$\sigma$ discovery in SC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>786 487 303</td>
<td>745 383 313</td>
</tr>
<tr>
<td>b.</td>
<td>1310 674 416</td>
<td>1120 621 536</td>
</tr>
<tr>
<td>c.</td>
<td>2760 1280 756</td>
<td>2460 1340 1020</td>
</tr>
<tr>
<td>d.</td>
<td>2960 1490 967</td>
<td>2840 1470 1050</td>
</tr>
<tr>
<td>e.</td>
<td>.. 2860 1700</td>
<td>.. 2170 1660</td>
</tr>
<tr>
<td>f.</td>
<td>.. .. 2210</td>
<td>.. .. 2340</td>
</tr>
<tr>
<td>g.</td>
<td>.. 2460 220</td>
<td>.. .. 2340</td>
</tr>
<tr>
<td>h.</td>
<td>427 279 220</td>
<td>644 349 299</td>
</tr>
</tbody>
</table>

Table 9.14: Analysis of the discovery potential for supersymmetry for the parameter space of Table 9.1, using the selection criteria of Table 9.13, where the minimum integrated luminosity needed for 5$\sigma$ discovery is given in fb$^{-1}$. Models (h) and (j) are not listed because the integrated luminosity for discovery exceeds 3000 fb$^{-1}$. Only 1$\tau$ signal regions are displayed, as those are the signal regions which give luminosities for discovery in the reasonable range. Table taken from [41].
Chapter 9. The Stau Coannihilation Model

<table>
<thead>
<tr>
<th>Model</th>
<th>( \mathcal{L} ) for 5( \sigma ) discovery in SC1</th>
<th>( \mathcal{L} ) for 5( \sigma ) discovery in SC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.</td>
<td>1020 704 625 292 2090 2460 1290</td>
<td>1040 715 694 477 579 582</td>
</tr>
<tr>
<td>ii.</td>
<td>501 380 512 472 458 1030 542</td>
<td>536 370 352 200 243 244</td>
</tr>
<tr>
<td>iii.</td>
<td>677 575 532 1440 1440 1710</td>
<td>827 669 648 200 243 244</td>
</tr>
<tr>
<td>iv.</td>
<td>654 475 411 941 695 666</td>
<td>941 695 666</td>
</tr>
<tr>
<td>v.</td>
<td>898 853 650 1170 743 693</td>
<td>1170 743 693</td>
</tr>
<tr>
<td>vi.</td>
<td>730 605 508 1190 825 870</td>
<td>1190 825 870</td>
</tr>
<tr>
<td>vii.</td>
<td>1040 746 660 2200 1250 889 842 2170 1170 1170</td>
<td></td>
</tr>
<tr>
<td>viii.</td>
<td>1190 713 661 1610 1020 1020 842 575 578</td>
<td></td>
</tr>
<tr>
<td>ix.</td>
<td>1430 1230 1090 1950 1340 1270 2880</td>
<td></td>
</tr>
<tr>
<td>x.</td>
<td>265 169 144 176 143 119 73 89 90</td>
<td></td>
</tr>
<tr>
<td>xi.</td>
<td>1040 746 660 168 378 199 176 143 119</td>
<td></td>
</tr>
</tbody>
</table>

Table 9.15: Analysis of the discovery potential for supersymmetry for the parameter space of Table 9.3, using the selection criteria of Table 9.13, where the minimum integrated luminosity needed for 5\( \sigma \) discovery is given in fb\(^{-1} \). The dashes mean that zero events have passed the applied cuts. Table taken from [41].

Finally, following the general approach of [40] we introduce cuts on the kinematic variable \( m_T \), given by

\[
m_T(p_{T1}, p_{T2}) = \sqrt{2(p_{T1} p_{T2} - p_{T1} \cdot p_{T2})}.
\] (9.3)

This new selection criterion is denoted SC-3 and defines additional signal regions given in Table 9.16. The performance of these signal regions is given in Table 9.17, while Figures 9.7 and 9.8 demonstrate the distributions on key kinematic variables for Models (iii) and (xi).

### 9.3.3 \( e \) and \( \mu \)-Based Signals

In addition to direct production of \( \tau \) leptons due to the decay of stau particles, it is expected that decays of charginos and heavy neutrinos will result in detectible light
Chapter 9. The Stau Coannihilation Model

**Table 9.16:** The selection criteria used for the signal regions SR-SC3 with 2 hadronically decaying taus in the final state \((N(\tau_h) = 2)\) and a veto on electrons, muons and b-jets. Table taken from [41].

<table>
<thead>
<tr>
<th>Requirement</th>
<th>SR-SC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_T^{\text{miss}}) (GeV) &gt; 100</td>
<td>2(\tau)-A</td>
</tr>
<tr>
<td>(E_T^{\text{miss}}) (GeV) &lt; 200</td>
<td>2(\tau)-B</td>
</tr>
<tr>
<td>(p_T(j_1)) (GeV) &lt; 180</td>
<td>2(\tau)-C</td>
</tr>
<tr>
<td>(m_{\text{eff}}) (GeV) &gt; 130</td>
<td></td>
</tr>
<tr>
<td>(m_{\text{eff}}) (GeV) &lt; 200</td>
<td></td>
</tr>
<tr>
<td>(m_{T\tau_1} + m_{T\tau_2}) &gt; 100</td>
<td></td>
</tr>
<tr>
<td>(m_{T\tau_1} + m_{T\tau_2}) &lt; 200</td>
<td></td>
</tr>
<tr>
<td>(\Delta R(\tau_h, \tau_h)) &gt; 2.5</td>
<td></td>
</tr>
<tr>
<td>(\Delta R(\tau_h, \tau_h)) &lt; 3.5</td>
<td></td>
</tr>
</tbody>
</table>

**Table 9.17:** Analysis of the discovery potential for supersymmetry for the parameter space of Table 9.3, using the selection criteria of Table 9.16, where the minimum integrated luminosity needed for 5\(\sigma\) discovery is given in fb\(^{-1}\). Models iv, vi, and ix are not listed because the minimum integrated luminosity needed for 5\(\sigma\) discovery exceeded 3000 fb\(^{-1}\). Table taken from [41].

<table>
<thead>
<tr>
<th>Model</th>
<th>(\mathcal{L}) for 5(\sigma) discovery in</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2(\tau)-SC3-A</td>
</tr>
<tr>
<td>i.</td>
<td>1240</td>
</tr>
<tr>
<td>ii.</td>
<td>1820</td>
</tr>
<tr>
<td>iii.</td>
<td>1170</td>
</tr>
<tr>
<td>v.</td>
<td>1640</td>
</tr>
<tr>
<td>vii.</td>
<td>1710</td>
</tr>
<tr>
<td>viii.</td>
<td>2870</td>
</tr>
<tr>
<td>x.</td>
<td>2690</td>
</tr>
<tr>
<td>xi.</td>
<td>670</td>
</tr>
</tbody>
</table>

leptons (electrons and muons) upon which further signal regions can be based. To evaluate the effectiveness of these types of searches in regions of both stau coannihilation and multiparticle coannihilation, benchmark parameters for both cases are evaluated against electroweakino signals designed to search for decays of \(\widetilde{\chi}^+_1\widetilde{\chi}_1^-\) and \(\widetilde{\chi}_1^\pm\widetilde{\chi}_2^0\) [39]. These signal regions are classified according to the number of signal leptons required.
Figure 9.7: Left panel: Distribution in $m_{T\tau_1} + m_{T\tau_2}$ for the $2\tau$SC3-A signal region for the multiparticle coannihilation Model (iii) prior to any cuts. Plotted is the number of counts for the SUSY signal per 30 GeV and the square root of the total SM SNOWMASS backgrounds. The analysis is done at 200 fb$^{-1}$ of integrated luminosity, which gives a $5\sigma$ discovery in this signal region. Right panel: The same analysis as in the left panel but for $E_T^{vis}$ with counts for the SUSY signal per 30 GeV. Figures taken from [41].

Figure 9.8: Left panel: Distribution in $m_{T\tau_1} + m_{T\tau_2}$ for the $2\tau$SC3-A signal region for the multiparticle coannihilation Model (xi) prior to any cuts. Plotted is the number of counts for the SUSY signal per 30 GeV and the square root of the total SM SNOWMASS backgrounds. The analysis is done at 670 fb$^{-1}$ of integrated luminosity, which gives a $5\sigma$ discovery in this signal region. Right panel: The same analysis as in the left panel but for $\Delta R(\tau_h, \tau_h)$ with counts for the SUSY signal per 0.05 rad. Figures taken from [41].

In the two lepton case, six signal regions are defined in two broad categories: signal regions denoted as 2l-SF require that the signal leptons form a same flavor, opposite sign (SFOS) pair, while signal regions denoted as 2l-DF require a different flavor, opposite sign (DFOS) pair. The variations A, B, and C indicate different cuts on the kinematic
variable $m_{T2}$ [170–172], which is defined as

$$m_{T2} = \min \left[ \max (m_T(\ell_1, q_T), m_T(\ell_2, p_T^{\text{miss}} - q_T)) \right] \quad (9.4)$$

where $q_T$ is an arbitrary vector chosen to find the appropriate minimum and $m_T$ is the transverse mass given by (9.3).

<table>
<thead>
<tr>
<th>Requirement</th>
<th>SF</th>
<th>SF</th>
<th>SF</th>
<th>DF</th>
<th>DF</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_T^{\text{miss}}$ (GeV) &gt;</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>light jet $p_T$ (GeV) &lt;</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>$b$-jet $p_T$ (GeV) &lt;</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>forward jet $p_T$ (GeV) &lt;</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>$</td>
<td>m_{\ell\ell} - m_Z</td>
<td>$ (GeV) &gt;</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>$m_{T2}$ (GeV) &gt;</td>
<td>90</td>
<td>120</td>
<td>150</td>
<td>90</td>
<td>120</td>
<td>150</td>
</tr>
</tbody>
</table>

**Table 9.18:** The selection criteria used for the signal regions related to the 2 lepton signature, based upon the 2 lepton signal regions from [39]. Here and in the Tables following SF stands for same flavor opposite sign lepton pair and DF stands for different flavor opposite sign lepton pair. A dash denotes a cut which is not applicable to the given signal region. Table taken from [41].

In addition to cutting on $m_{T2}$ and missing transverse energy $E_T^{\text{miss}}$, the two lepton signal regions contain three jet vetos, requiring that events contain no jets other than very soft jets in three jet categories: $b$-tagged jets ($b$-jet veto), jets which are not $b$-tagged and which have $|\eta| \leq 2.4$ (light jet veto), and jets which are not $b$-tagged and which have $2.4 \leq |\eta| \leq 4.5$ (forward jet veto). Finally, for the 2l-SF signal regions, there is a $Z$ veto which requires that the invariant mass of the SFOS lepton pair not lie within 10 GeV of the $Z$ mass. These signal regions are summarized in Table 9.18.

For the three lepton case, two of the leptons are required to comprise a SFOS pair, with
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the third lepton allowed to have the same or different flavor. For the case where all three leptons are the same flavor, the SFOS pair is chosen to be that whose invariant mass is closest to the $Z$ mass. The three lepton case admits two signal regions A and B, with B representing tighter cuts on relevant kinematic variables. Here, in addition to a veto on

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{T}^{\text{miss}}$ (GeV) $&gt;$</td>
<td>120</td>
</tr>
<tr>
<td>$p_T(\ell_3)$ (GeV) $&gt;$</td>
<td>30</td>
</tr>
<tr>
<td>$m_T$ (GeV) $&lt;$</td>
<td>110</td>
</tr>
<tr>
<td>$m_{\text{SFOS}}$ (GeV) $\notin$ [21.2, 101.2] $&gt;$</td>
<td>101.2</td>
</tr>
<tr>
<td>$N(b\text{-jet})$</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 9.19:** The selection criteria used for the signal regions related to the 3 lepton signature, based upon the 2 lepton signal regions from [39]. Table taken from [41].

$b$-tagged jets, cuts are applied to the missing transverse energy, the transverse momentum of the third lepton, the transverse mass as defined above, and the invariant mass of the SFOS pair. The three lepton signal region criteria are given in Table 9.19.

With these signal regions, it is possible to assess the discovery potential of stau coannihilation region parameter points based on the signal from electroweakino decays. Tables 9.20 to 9.22 below describe the results in terms of the integrated luminosity in fb$^{-1}$ required for a 5$\sigma$ discovery. Results for the three lepton signal regions described in Table 9.19 as well as the DFOS signals from Table 9.18 are not displayed because it was found that in these cases the required luminosity for discovery was much larger than for the two lepton SFOS case, indeed larger than the 3000 fb$^{-1}$ maximum limit. This is due to the fact that the decay events from these coannihilation regions almost never produce three final state leptons nor DFOS pairs.
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#### Table 9.20: Analysis of the discovery potential for supersymmetry for the parameter space of Table 9.1, using the 2 lepton same flavor (SF) selection criteria of Table 9.18, where the minimum integrated luminosity needed for 5σ discovery is given in fb$^{-1}$. The different flavor (DF) signal regions are omitted due to poor performance (i.e. requiring over 3000 fb$^{-1}$ of integrated luminosity for discovery). Table taken from [41].

<table>
<thead>
<tr>
<th>Model</th>
<th>2l-SF-A</th>
<th>2l-SF-B</th>
<th>2l-SF-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>187</td>
<td>266</td>
<td>266</td>
</tr>
<tr>
<td>b.</td>
<td>362</td>
<td>420</td>
<td>441</td>
</tr>
<tr>
<td>c.</td>
<td>165</td>
<td>188</td>
<td>169</td>
</tr>
<tr>
<td>d.</td>
<td>781</td>
<td>953</td>
<td>884</td>
</tr>
<tr>
<td>e.</td>
<td>1480</td>
<td>1630</td>
<td>1700</td>
</tr>
<tr>
<td>f.</td>
<td>1110</td>
<td>1380</td>
<td>1250</td>
</tr>
<tr>
<td>g.</td>
<td>1850</td>
<td>1850</td>
<td>1790</td>
</tr>
<tr>
<td>h.</td>
<td>1860</td>
<td>2050</td>
<td>1660</td>
</tr>
<tr>
<td>j.</td>
<td>2160</td>
<td>2250</td>
<td>1880</td>
</tr>
<tr>
<td>k.</td>
<td>97</td>
<td>185</td>
<td>225</td>
</tr>
</tbody>
</table>

### Table 9.21: Analysis of the discovery potential for supersymmetry for the parameter space of Table 9.3, using the 2 lepton same flavor (SF) selection criteria of Table 9.18, where the minimum integrated luminosity needed for 5σ discovery is given in fb$^{-1}$. The different flavor (DF) signal regions are omitted due to poor performance (i.e. requiring over 3000 fb$^{-1}$ of integrated luminosity for discovery). Model x is not listed because the minimum integrated luminosity needed for 5σ discovery exceeded 3000 fb$^{-1}$ Table taken from [41].

<table>
<thead>
<tr>
<th>Model</th>
<th>2l-SF-A</th>
<th>2l-SF-B</th>
<th>2l-SF-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.</td>
<td>545</td>
<td>623</td>
<td>696</td>
</tr>
<tr>
<td>ii.</td>
<td>315</td>
<td>306</td>
<td>273</td>
</tr>
<tr>
<td>iii.</td>
<td>181</td>
<td>271</td>
<td>238</td>
</tr>
<tr>
<td>iv.</td>
<td>640</td>
<td>843</td>
<td>934</td>
</tr>
<tr>
<td>v.</td>
<td>1410</td>
<td>1460</td>
<td>1690</td>
</tr>
<tr>
<td>vi.</td>
<td>1090</td>
<td>1610</td>
<td>1500</td>
</tr>
<tr>
<td>vii.</td>
<td>944</td>
<td>1450</td>
<td>1510</td>
</tr>
<tr>
<td>viii.</td>
<td>732</td>
<td>1090</td>
<td>1190</td>
</tr>
<tr>
<td>ix.</td>
<td>360</td>
<td>487</td>
<td>624</td>
</tr>
<tr>
<td>xi.</td>
<td>224</td>
<td>450</td>
<td>547</td>
</tr>
</tbody>
</table>
We find that for the leptonic signal regions, as mentioned earlier it is only the SFOS two-lepton signals that give promising results. Thus, this specific signal region topology is found to be the best leptonic signal for the stau and multiparticle coannihilation regions. The remaining variation is upon kinematic cuts, in this case the cut on the variable $m_{T2}$.

As expected for a kinematic cut, the softer cut of 2l-SF-A is optimal for lower mass-scale benchmark points, while the harder cut 2l-SF-C is optimal for higher mass-scale points. The intermediate signal region 2l-SF-B was not optimal for any case studied.

Figures 9.9 and 9.10 display the $M_{T2}$ and $E_T^{\text{miss}}$ kinematic variables for signal and combined background after cuts. Figure 9.9 gives counts in $M_{T2}$ after the 2l-SF-A signal region cuts for Models (k) and (c), models for which that signal region is optimal, displayed at the integrated luminosity calculated as necessary for discovery. Figure 9.10 gives counts in the same signal region and models, this time for $E_T^{\text{miss}}$. As before, this signal region is optimal for the chosen models and the counts are displayed at the integrated luminosity.
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luminosity calculated as necessary for discovery.

Figure 9.9: Left panel: Distribution in $M_{T2}$ for the 2l-SF-A signal region for stau coannihilation Model (a) after cuts in that region. Plotted is the number of counts for the supersymmetry signal per 15 GeV and the square root of the total standard model SNOWMASS background. The analysis is done at 187 fb$^{-1}$ of integrated luminosity, which gives a 5σ discovery in this signal region. Right panel: The same analysis as in the left panel but for Model (b) at 362 fb$^{-1}$. Figures taken from [41].

Figure 9.10: Left panel: Distribution in $M_{T2}$ for the 2l-SF-C signal region for stau coannihilation Model (h) after cuts in that region. Plotted is the number of counts for the supersymmetry signal per 15 GeV and the square root of the total standard model SNOWMASS background. The analysis is done at 1660 fb$^{-1}$ of integrated luminosity, which gives a 5σ discovery in this signal region. Right panel: The same analysis as in the left panel but for Model (j) at 1880 fb$^{-1}$. Figures taken from [41].

9.3.4 Combined Signal Region Results

As an overall view of the considered signal regions and their success in discriminating between signal and background, we list in Tables 9.23 and 9.24 the leading and subleading signal regions (SR) and the corresponding model points for the stau and multiparticle
coannihilation regions, respectively. Model points are listed in ascending order of luminosity.

<table>
<thead>
<tr>
<th>Model</th>
<th>Leading SR</th>
<th>( \mathcal{L} ) (fb(^{-1} ))</th>
<th>Sub-leading SR</th>
<th>( \mathcal{L} ) (fb(^{-1} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>k.</td>
<td>2l-SF-A</td>
<td>97</td>
<td>2l-SF-B</td>
<td>185</td>
</tr>
<tr>
<td>c.</td>
<td>2l-SF-A</td>
<td>165</td>
<td>2l-SF-C</td>
<td>169</td>
</tr>
<tr>
<td>a.</td>
<td>2l-SF-A</td>
<td>187</td>
<td>2l-SF-B</td>
<td>266</td>
</tr>
<tr>
<td>b.</td>
<td>2l-SF-A</td>
<td>362</td>
<td>1( \tau )-SC2-C</td>
<td>416</td>
</tr>
<tr>
<td>d.</td>
<td>2l-SF-A</td>
<td>781</td>
<td>2l-SF-C</td>
<td>884</td>
</tr>
<tr>
<td>f.</td>
<td>2l-SF-A</td>
<td>1110</td>
<td>2l-SF-C</td>
<td>1250</td>
</tr>
<tr>
<td>e.</td>
<td>2l-SF-A</td>
<td>1480</td>
<td>2l-SF-B</td>
<td>1630</td>
</tr>
<tr>
<td>g.</td>
<td>2l-SF-C</td>
<td>1790</td>
<td>2l-SF-A</td>
<td>1850</td>
</tr>
<tr>
<td>h.</td>
<td>2l-SF-C</td>
<td>1660</td>
<td>2l-SF-A</td>
<td>1860</td>
</tr>
<tr>
<td>j.</td>
<td>2l-SF-C</td>
<td>1880</td>
<td>2l-SF-A</td>
<td>2160</td>
</tr>
</tbody>
</table>

**Table 9.23**: The overall minimum integrated luminosities needed for 5\( \sigma \) discovery using the leading and the sub-leading signal regions for stau coannihilation models of Table 9.1, including the \( \tau \) based signal regions discussed in Section 9.3.2 as well as the \( e \) and \( \mu \) based signal regions discussed in Section 9.3.3. Table taken from [41].

<table>
<thead>
<tr>
<th>Model</th>
<th>Leading SR</th>
<th>( \mathcal{L} ) (fb(^{-1} ))</th>
<th>Sub-leading SR</th>
<th>( \mathcal{L} ) (fb(^{-1} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>xi.</td>
<td>2( \tau )-SC2-A</td>
<td>73</td>
<td>2( \tau )-SC2-B</td>
<td>89</td>
</tr>
<tr>
<td>iii.</td>
<td>2l-SF-A</td>
<td>181</td>
<td>2( \tau )-SC2-A</td>
<td>200</td>
</tr>
<tr>
<td>ii.</td>
<td>2l-SF-C</td>
<td>273</td>
<td>2l-SF-B</td>
<td>306</td>
</tr>
<tr>
<td>ix.</td>
<td>2l-SF-A</td>
<td>360</td>
<td>2l-SF-B</td>
<td>487</td>
</tr>
<tr>
<td>v.</td>
<td>1( \tau )-SC1-C</td>
<td>411</td>
<td>1( \tau )-SC1-B</td>
<td>475</td>
</tr>
<tr>
<td>i.</td>
<td>2( \tau )-SC2-A</td>
<td>477</td>
<td>2l-SF-A</td>
<td>545</td>
</tr>
<tr>
<td>vii.</td>
<td>1( \tau )-SC1-C</td>
<td>508</td>
<td>1( \tau )-SC1-B</td>
<td>605</td>
</tr>
<tr>
<td>iv.</td>
<td>1( \tau )-SC1-C</td>
<td>532</td>
<td>1( \tau )-SC1-B</td>
<td>575</td>
</tr>
<tr>
<td>vi.</td>
<td>1( \tau )-SC1-C</td>
<td>650</td>
<td>1( \tau )-SC2-C</td>
<td>693</td>
</tr>
<tr>
<td>viii.</td>
<td>1( \tau )-SC1-C</td>
<td>660</td>
<td>2l-SF-A</td>
<td>732</td>
</tr>
<tr>
<td>x.</td>
<td>1( \tau )-SC1-C</td>
<td>1090</td>
<td>1( \tau )-SC1-B</td>
<td>1230</td>
</tr>
</tbody>
</table>

**Table 9.24**: The overall minimum integrated luminosities needed for 5\( \sigma \) discovery using the leading and sub-leading signal regions for stau coannihilation models of Table 9.3, including the \( \tau \) based signal regions discussed in Section 9.3.2 as well as the \( e \) and \( \mu \) based signal regions discussed in Section 9.3.3. Table taken from [41].
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9.4 Direct Detection of Dark Matter

Because the purpose of seeking a model with coannihilation is to find a parameter space with the appropriate relic density, it is important to consider the properties of the dark matter of such a model over and above the relic density. In particular, it is relevant to ask whether these models are detectible by current or next generation dark matter direct detection experiments. The relevant properties there are the cross sections for nucleon–CDM interaction. These values are displayed for the stau and multiparticle coannihilation models in Table 9.25.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\sigma^\text{SI}_{p,\chi^0_1} \times 10^{48}$</th>
<th>$\sigma^\text{SD}_{p,\chi^0_1} \times 10^{46}$</th>
<th>Model</th>
<th>$\sigma^\text{SI}_{p,\chi^0_1} \times 10^{48}$</th>
<th>$\sigma^\text{SD}_{p,\chi^0_1} \times 10^{45}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>0.92</td>
<td>4.77</td>
<td>i.</td>
<td>1.33</td>
<td>3.02</td>
</tr>
<tr>
<td>b.</td>
<td>0.80</td>
<td>3.67</td>
<td>ii.</td>
<td>1.97</td>
<td>3.54</td>
</tr>
<tr>
<td>c.</td>
<td>1.08</td>
<td>5.60</td>
<td>iii.</td>
<td>0.94</td>
<td>1.03</td>
</tr>
<tr>
<td>d.</td>
<td>0.80</td>
<td>6.35</td>
<td>iv.</td>
<td>1.94</td>
<td>2.87</td>
</tr>
<tr>
<td>e.</td>
<td>0.58</td>
<td>4.43</td>
<td>v.</td>
<td>1.74</td>
<td>2.45</td>
</tr>
<tr>
<td>f.</td>
<td>0.64</td>
<td>4.82</td>
<td>vi.</td>
<td>2.90</td>
<td>5.66</td>
</tr>
<tr>
<td>g.</td>
<td>0.40</td>
<td>2.39</td>
<td>vii.</td>
<td>1.46</td>
<td>1.93</td>
</tr>
<tr>
<td>h.</td>
<td>0.27</td>
<td>1.03</td>
<td>viii.</td>
<td>2.02</td>
<td>3.28</td>
</tr>
<tr>
<td>j.</td>
<td>0.53</td>
<td>3.52</td>
<td>ix.</td>
<td>0.96</td>
<td>1.88</td>
</tr>
<tr>
<td>k.</td>
<td>1.22</td>
<td>0.25</td>
<td>x.</td>
<td>3.01</td>
<td>4.77</td>
</tr>
<tr>
<td>xi.</td>
<td>1.11</td>
<td>1.53</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9.25: Three left columns: Proton–neutralino spin-independent ($\sigma^\text{SI}_{p,\chi^0_1}$) and spin-dependent ($\sigma^\text{SD}_{p,\chi^0_1}$) cross-sections in units of cm$^{-2}$ for the 10 benchmark points of Table 9.1. Three right columns: Proton–neutralino spin-independent ($\sigma^\text{SI}_{p,\chi^0_1}$) and spin-dependent ($\sigma^\text{SD}_{p,\chi^0_1}$) cross-sections in units of cm$^{-2}$ for the 11 benchmark points of Table 9.3. Tables taken from [41].

Based on these values, we find that while the detectability of the LSP in these models lies below the reach of current detectors, it is still above the neutrino floor, which is the
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minimum threshold for detectability. This is demonstrated in figure 9.11, which shows
the proton-LSP cross-section vs LSP mass for the stau and multiparticle coannihilation
models as compared to the limits of current experiments LUX and XENON100, as well
as future experiments LZ and XENON1T.
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Figure 9.11: $R \times \sigma_{p,\chi_1^0}^{\text{SI}}$ as a function of LSP mass displayed alongside the current and projected range of the XENON and LUX experiments and the neutrino floor. Top panel: Data for the stau coannihilation models. Bottom panel: data for the multiparticle coannihilation case. Figures taken from [41].
9.5 Summary of Stau Coannihilation Results

In this chapter we show that staus in the coannihilation region of mass as low as $\sim 120$ GeV through $\sim 250$ GeV or larger would have evaded detection at the LHC so far, and could be detected with additional integrated luminosity. We considered two variations of the coannihilation region, one where the stau is the sole coannihilator with the LSP and another where the stau, the chargino, and the second neutralino all enter into the coannihilation to some extent. It was found that for the stau coannihilation region, signal regions based on hadronic taus and signal regions based on light leptons were both viable, with the optimal signal region trading off depending on model specifics. For the multiparticle region, the tau-based signals generally dominate. We find also that 3 lepton signals are not viable for either coannihilation region. As for the detectability of the LSP dark matter in these models, the proton–LSP cross section is low enough to present a challenge even to next-generation dark matter direct-detection experiments. Because the cross section remains above the neutrino floor, however, detection at an experiment of this type remains possible.
Chapter 10

Conclusions and Future Prospects

The coannihilation region of supergravity unified models presents a challenge for detection at the LHC, but nevertheless remains a compelling possibility for detection of supersymmetry. Because of the experimental observation that the Higgs boson mass is 125 GeV, we know that the scale of supersymmetry must be large. However, in most of the parameter space of supergravity models where this Higgs boson mass constraint is met, the lightest neutralino is mostly a bino, and consequently its annihilation cross section is not large enough to deplete the relic density of neutralinos down to the desired values for dark matter indicated by the WMAP and PLANCK experiments. Thus, in order to achieve the desired relic density, it is necessary to consider models that feature LSP coannihilation with another sparticle. The coannihilating sparticle’s mass must lie close enough to the neutralino to enhance the effective annihilation cross section, which
Chapter 10. Conclusions and Future Prospects

enters in the analysis of the relic density of the neutralinos. Unfortunately, since the mass gap between the LSP and the coannihilating NLSP sparticle must be small, the decay products of the NLSP are necessarily soft, making it difficult to detect these sparticles even when they are produced at a collider. As a result, the lower limit on the sparticle masses may be significantly lower when the coannihilation effects are taken into account relative to generic searches where coannihilation effects are ignored.

In this thesis we have investigated three different types of coannihilation processes: stop coannihilation, gluino coannihilation, and stau coannihilation (including multiparticle coannihilation with light charginos and neutralinos). For the case of stop coannihilation, it was found that stops as low as $\sim 400$ GeV could have evaded detection at the LHC, and that with the full luminosity of the LHC, the stop coannihilation region can be probed to a stop mass of at least $\sim 600$ GeV. For gluinos, these limits lie in the range $\sim 700$ GeV to $\sim 1300$ GeV. Without inclusion of coannihilation, the reported lower limits on the stop mass is $820$ GeV [173] and the lower limit on the gluino mass is $1900$ GeV [174]. For the stau, the lower limit including coannihilation lies in the range $\sim 120$ GeV to $\sim 250$ GeV. Also discussed in this thesis was the prospect of direct detection of the LSP in direct-detection experiments. It was found for all models studied that the spin-independent LSP-nucleon cross section lies above the neutrino floor, but below the detection capability of the current-generation experiments. However, it is found that the next-generation direct detection experiments should be able to achieve the desired sensitivity for discovery of many models. Figure 10.1 displays the combined results for
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direct dark matter detection of the models considered at current and next generation
LUX and XENON experiments.

\[ \text{Figure 10.1: } R \times \sigma_{SI}^{p-X_1^0} \text{ for all coannihilation models as a function of LSP mass alongside the current and projected ranges of the XENON and LUX experiments and the neutrino floor} \]

The work of this thesis lends itself to further extensions, as the coannihilation regions considered in this work do not exhaust the full set of possibilities. Depending on the nature of soft breaking, other sparticles could play a role in coannihilation. One possibility concerns chargino–neutralino coannihilation [175, 176]. In this case, nonuniversality in
the gaugino sector will bring the chargino in close proximity to the neutralino, with other sparticles significantly heavier.

Another line of study for future work is to further investigate the possibility of multi-sparticle coannihilation, where two or more sparticles lie close to the LSP and coannihilate. Chapter 9 investigates one model of multiparticle coannihilation, between the chargino, stau, and neutralino, but other coannihilations could also be viable. This type of coannihilation would allow for greater mass gaps among coannihilating sparticles and the LSP than would be possible with only one coannihilator. This is due to the larger effective annihilation cross section $\sigma_{\text{eff}}$, which would receive contributions from all of the multiple coannihilating sparticles. Increasing the cross section in this way allows the mass gaps for each sparticle to be relatively larger than in the case of a single coannihilator. This is particularly true if the gluino, for example, is the third coannihilator. In this case, because of the larger gluino annihilation cross section, one could have a heavier gluino with a larger mass gap and via other coannihilators still contribute a significant amount to the effective cross section. The larger mass gaps would allow for decays that are less soft, and therefore more likely to be detected at the LHC.

In the analysis presented here, a significant effort was made to cover the full range of existing signal regions relevant for the specific processes analyzed. However, there remain possibilities for devising new techniques and exploring other kinematic variables that might be better discriminators of the coannihilation region and of the compressed sparticle spectrum. Developing better tools for the analysis of these regions remains an open
conclusion and future prospects

Even though the LHC has many more years of data-taking ahead of it, it is not too soon to look ahead and consider the type of analyses that would be needed to explore the signatures of supersymmetry at colliders with different designs and higher energies. There are two main types of high energy colliders being considered for the future: the $e^+e^-$ colliders and the $pp$ colliders. Regarding the $e^+e^-$ colliders, the following possibilities exist:

1. ILC: International Linear Collider (Japan),
2. CEPC: Circular Collider (China),

The $e^+e^-$ colliders are primarily Higgs factories, and they are likely to run at an energy around 240 GeV, which gives the optimal cross section for the Higgsstrahlung process $e^+e^- \rightarrow Zh$. The $Zh$ final state is the preferred mode for study of Higgs properties rather than $e^+e^- \rightarrow hh$ since $Z$ can be efficiently detected via $Z \rightarrow \ell^+\ell^-$. The Higgs factories will study couplings of the Higgs to fermions and the electroweak parameters with great accuracy. However, $\sqrt{s} = 240$ GeV is too small for sparticle production. For that reason, to explore supersymmetry one needs $pp$ colliders with higher energies. The
following possibilities for $pp$ colliders beyond the LHC energy range are being considered:

1. SppC: $70 - 100 \text{ TeV}$ $pp$ collider (China),

2. VLHC: $100 \text{ TeV}$ $pp$ collider (CERN)

If either of those colliders materialize, it becomes feasible to explore supersymmetry in a mass range much larger than the one that can be explored at the LHC, even with optimal integrated luminosity. Specifically of interest is the prospect of further exploration of the coannihilation region and of compressed spectra at these supercolliders.

As the ongoing efforts of the ATLAS and CMS collaborations continue to rule out large segments of generic supersymmetry parameter space, it is crucial to understand as well as possible those models that have the greatest chance of corresponding to the observations that have already been made. These observations, such as the Higgs mass and the dark matter relic density, place great constraints on model parameters. The coannihilation region models satisfy these constraints with the additional advantage that they are based on high-scale models and thus are highly predictive; parameters such as the full sparticle mass hierarchy and the branching ratios can be calculated rather than assumed, leading to a better understanding of what supersymmetry must look like in order to remain undetected. It is unknown what phenomena may be uncovered in the coming years of data taking at the LHC and at dark matter detectors such as LZ, but we hope that this work can help provide a roadmap for a few of the possibilities.
Bibliography


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[34] The ATLAS collaboration. Search for squarks and gluinos in final states with jets and missing transverse momentum at √s =13 TeV with the ATLAS detector. 2015.


[39] The ATLAS collaboration. Search for supersymmetry with two and three leptons and missing transverse momentum in the final state at $\sqrt{s} = 13\text{TeV}$ with the ATLAS detector. 2016.


Bibliography


Bibliography


Bibliography


[173] The ATLAS collaboration. Search for the Supersymmetric Partner of the Top Quark in the Jets+Emiss Final State at sqrt(s) = 13 TeV. 2016.

[174] The ATLAS collaboration. Search for pair production of gluinos decaying via top or bottom squarks in events with b-jets and large missing transverse momentum in pp collisions at sqrt(s) = 13 TeV with the ATLAS detector. 2016.
