ADHESION MEASUREMENT IN PHOTOVOLTAIC SOLAR SYSTEMS

USING THE BLISTER TEST

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ABSTRACT

As a low-carbon, renewable energy source, photovoltaic (PV) solar technology has the potential to contribute greatly to reduction in greenhouse gas emissions and mitigation of climate change over the next several decades. Over the past two decades, the technology has undergone exponential growth. Since 1992, the total global deployment, in electrical output, has increased, on average, 29% per year, and doubled every 2 ½ years. By 2050, it is predicted that PV electricity generation will comprise up to 16% of total global electricity generation. Reduction in cost of the technology has been a major driving force behind this growth. Therefore, extending the longevity and further lowering the cost of the materials and manufacturing methods is key to its continuation. Adequate measurement science and a fundamental understanding of the performance of PV technology is, therefore, essential.

PV modules—the ubiquitous units of the technology, nowadays visible on many rooftops—are layered structures comprised of a glass front-plate, an array of photovoltaic cells, and a polymer backsheet—adhered together with two intermediate layers of polymer encapsulant. After prolonged exposure to environmental stressors such as high temperature and humidity, as well as UV radiation, the polymeric materials undergo chemical and mechanical degradation—including adhesive degradation. The latter often manifests itself as delamination at material interfaces, and can result in reduced heat dissipation and exposure of electrical components to moisture and corrosion. The PV industry is currently lacking in adequate methods for assessing the adhesive strength of these material interfaces.

In this thesis, we apply a well-known adhesion testing method—the blister test—to this task. In particular, we investigate the application of several variants of the method to PV
material systems—both real, field-weathered PV modules, as well as laboratory-made material samples. Our results both demonstrate the applicability of these methods to these systems, as well as illuminate some intriguing insights into the nature of adhesive debonding in PV modules. Our findings show much promise in these methods for contributing to the body of adhesion testing standards in the PV industry, as well as to the body of fundamental understanding of PV module adhesive performance.
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DISCLAIMER

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1. INTRODUCTION

1.1. Recent Photovoltaic Technology Growth

Photovoltaic (PV) solar technology is the well-known electrical energy generation technology, wherein sunlight is directly converted to electricity using semiconductor material. It is often manifested as the dark blue modules that are becoming ever more ubiquitous on the rooftops of homes and businesses, and in arrays in open fields (Figure 1). As a residential, commercial, and industrial power generation technology, PV technology is advantageous in that it is highly modular and scalable, and it generates no greenhouse gases, once manufactured and installed.

Figure 1. Example of a PV array, mounted in a field (from [1]).

Residential and industrial PV power generation technology has undergone a significant reduction in cost over the past decade, and, in some areas of the globe, is approaching economic competitiveness with other conventional power generation technologies, such as petroleum, coal, and gas [2]. As a result, the growth in deployment of PV power generation over the past decade has been tremendous, and it is predicted to play a key role in reducing
global greenhouse gas emissions in the coming decades. Since 1992, for example, global PV power generation capacity has grown from just 105 megawatts, to 230 gigawatts in 2015, and has occurred at an exponential rate averaging 29% growth per year [3, 4, 5]. In 2015, the International Energy Agency (IEA)—an intergovernmental agency that promotes energy security and sustainability through policy and research—predicted that, combined with wind energy, solar energy will account for one-third of net global power capacity growth from 2015 to 2020 [6]. Furthermore, they predict that account for as much as 16% of global electricity production by 2050 [2], up from 1.3% in 2015 [7]. Key to sustaining this growth is, therefore, the continued reduction in cost of PV technology. This requires the continued development of lower-cost materials and manufacturing methods, and the extension of the life-span and long-term reliability of PV modules. This, in turn, depends upon the existence of an adequate body of testing and measurement methods, available for assessing the performance of PV modules during their service life. Our focus, in this thesis, is on developing an adhesion testing method for the material interfaces in a module—particularly in the backsheet—using the well-known thin film adhesion testing method: the blister test.

1.2. PV Module Construction

PV modules are multi-layered structures that typically consist of the following layering scheme (from top to bottom): a glass top-plate, a layer of clear polymer encapsulant, the PV wafer and electrical interconnects, another layer of clear polymer encapsulant, and a protective polymer backsheet (see Figure 2). The glass top-plate provides rigidity to the module and protection from direct contact with mechanical and environmental (i.e. moisture
and soiling) elements. The encapsulant serves as an adhesive between the adjoining layers, as well as a sealant for the PV wafer and electrical interconnects. The backsheet provides protection of the backside of the module from contact with mechanical and environmental elements, as well as electrical insulation of the wafer and interconnects.

Figure 2. Typical construction of a PV module (from [8]).

Of particular interest to us is the module backsheet. Several common types of backsheet exist in the industry, with three of the most common depicted in Figure 3. Backsheets are typically multi-layered, with each layer serving a particular function. For example, the very common PVF/PET/EVA backsheet (Fig. 3a) is comprised of a polyvinyl fluoride (PVF) outer
layer, a polyethylene terephthalate (PET) core layer, and an ethylene co-vinyl acetate (EVA) inner layer. PVF is tough and very chemically stable, and can therefore tolerate mechanical stresses (e.g. scratching or gouging) and prolonged exposure to UV light and high temperature/humidity. It serves as a protective layer for the PET core layer, which is less stable under these environmental stressors, but is mechanically strong and comparatively less expensive. The PET layer provides the bulk of the mechanical strength of the backsheet. Finally, the EVA inner layer serves as a “seed” layer, for facilitating bonding between the backsheet and the module encapsulant (typically EVA). The other varieties of backsheet layering—such as AAA and PPE—provide similar combinations of advantageous properties among the different layers.

![Diagram of three common types of PV backsheets](image)

Figure 3. Three common types of PV backsheets. PVF stands for polyvinyl fluoride, PET for polyethylene terephthalate, WPET for white-PET, EVA for ethylene co-vinyl acetate, and PA for polyamide.

### 1.3. Field Degradation of PV Materials

In the field, PV modules are exposed to such environmental factors as—among others—mechanical impact, such as hail and handling during installation; high and low temperatures,
and temperature cycling; high and low relative humidity and moisture; prolonged UV radiation exposure; and liquid water and ice. In particular, the polymer components of a module—e.g. the backsheet and encapsulant—undergo gradual chemical changes due, especially, to high temperature, high relative humidity, and UV exposure. Such changes can degrade the performance of a module, in a number of different ways: among others, electrically, optically, mechanically, and adhesively. For example, the common encapsulant, ethylene co-vinyl acetate (EVA), has been reported to undergo a yellowing process, in some cases, in the layer between the front-glass and PV wafer [9]. The resulting decreased optical transparency has often resulted in decreased electrical output of the module. With regards to adhesion, delamination on the front side of the module—at the glass/encapsulant or encapsulant/PV-cell interfaces—can result in optical decoupling and decreased solar transmittance [10, 11] (c.f. Figure 4). In backsheets, degradation has been reported in the adhesives between the material layers, after exposure to long-term heat and humidity [10, 12]. Such delamination can result in decreased heat dissipation through the backside of the module, which can lead to overheating of the PV cells [11]. Furthermore, the resulting void can be a preferential location for the accumulation of moisture, which can accelerate further delamination and enable corrosion of the electrical components [13].
1.4. Existing Adhesion Testing Methods

So as to design module materials and manufacturing methods that can avoid adhesive failures, it is necessary for the industry to have adhesion testing standards that allow the adhesive strength of the module materials and interfaces to be assessed. Currently, the industry has several widely-adopted qualification standards for evaluating the field-readiness of new module designs. They focus on such factors as durability and safety [14, 15]. With regards to the mechanical durability, it is assessed by such tests as: exposure to high temperature (85°C) and high relative humidity (85%) for a prolonged period of time (1000 hrs); thermal cycling between high (85°C) and low (-40°C) temperatures, for several hundred cycles; post-exposure visual inspection for damage, such as cracking of the backsheet; static mechanical loading of the module; and impact-testing by ice balls, to simulate hail impact [14]. However, there is currently no qualification test for assessing the strength of the adhesives used in a module, nor any requirement that the adhesive strength—as determined by outside methods—meet some minimum level.

Figure 4. Adhesive failure at the cell/top-EVA interface of a module, shown as small blisters (from [10]).
Despite a lack of a method for qualification testing, several adhesion testing methods have been used in the industry, for a number of years. The most common are the $90^\circ$, $180^\circ$, and T-peel tests [16, 17] (see Figure 5). Here, the free end of a strip (or pair of strips) is gripped and pulled, and the peel force and peel distance are measured. Figure 6 depicts a generalized peel test, conducted at an arbitrary angle, $\theta$, with respect to a rigid substrate [18]. The peel force is $F$, the width of the strip of material is $b$, and $da$ is an incremental length of peeling from the substrate. Assuming the strip is inextensible, one can, theoretically, obtain an “adhesion energy” of the interface—a measure of the fundamental strength of adhesive bonding between the strip and the substrate. This is done as follows: equation (1) gives the “strain energy release rate” (SERR), $G$—the amount of mechanical energy released by the peel for an incremental amount of peeling—as a function of $F$, $b$, and $\theta$. During peeling, this SERR represents the adhesion energy of the interface. (Note that the adhesion energy is sometimes also referred to as the “adhesion toughness”, “critical SERR”, “debond energy”, “debond toughness”, or “interfacial toughness”.) This test then takes the form of the $90^\circ$ or $180^\circ$ peel tests by simply setting $\theta$ to one or the other, respectively. It takes the form of the T-peel test by a similar approach.
Figure 5. The 90°, 180°, and T-peel tests (from [19]).

Figure 6. Schematic of a generalized peel test, conducted at an arbitrary angle, $\theta$ (from [18]). The applied force is $F$, the width of the peel is $b$, the thickness of the peel is $h$, and an incremental length of peeling from the surface is $da$. 
Although, in theory, it is possible to obtain an interfacial adhesion energy from the peel test, in practice, it is difficult. Typically, the strip is not inextensible, but, rather—for a system with strong adhesion, such as that often found in PV modules—undergoes extensive stretching—often both elastic and plastic—during the test. This additional store or release of energy means that one no longer measures only the adhesion energy of the interface, but a combination of stretching and adhesion energy. Furthermore, in order to reduce this effect, one must reduce the applied force by maximizing $\theta$. The $180^0$ peel test is, therefore, most attractive from this standpoint. However, at large angles, the root of the free peel undergoes a large degree of bending, which results in significant elastic—and often plastic—strain at this spot. This confounds the measurement of the adhesion energy in a manner analogous to the influence of peel extensibility. Finally, the peel test suffers from an edge effect, whereby this presence of the lateral edges of the peel leads to a transition from plane stress to plane strain behavior between the center of the peel and the edges, respectively. For interfaces with strong adhesion and relatively compliant peels, this can lead to the edges of the peel “curling” at the root, and a resulting curved crack front. A sufficiently curved crack front can no longer be modeled by equation (1).

In addition to the peel test several shear tests have been used to characterize the strength of adhesives in PV material systems. These include the overlap shear [20], compressive shear [21], and the rotational torque [22] tests. The overlap shear test is typically used to test adhesives, and consists of two overlapping rigid plates (a.k.a. “handles”), bonded together by
a layer of adhesive (c.f. Figure 7). Theoretically, the test measures the mode II (shearing mode) toughness of the adhesive or adhesive/handle interface. The plates are pulled apart in shear, and the force required to pull them apart, $P$, is divided by the adhesive area, $A$, to give the overlap shear strength, in units of force/area.

$$\text{Lap Shear Strength} = \frac{P}{A}$$

Figure 7. Lap shear test (from [20]). Two overlapping plates are bonded together with adhesive, then subsequently pulled apart, in shear.

In the compressive shear test, two plates are bonded together by the adhesive, then subjected to a combination of compression and shear, as shown in Figure 8. The shear stress in the adhesive joint between the two plates is then expressed as

$$\tau = \frac{F \cos \theta}{a \ b}$$

where $F$, is the force applied by the vertical crosshead of the testing apparatus, $\theta$ is the inclination angle between the joint and the vertical, and $a$ and $b$ are the length and width of the adhesive patch in the joint, respectively. The shear strain in the joint can be computed by
\[
\gamma = \frac{l}{b}
\]

where \( l \) is the small amount of relative sliding of the two plates with respect to one another, as measured by the crosshead. The adhesive energy of the bond, \( U_{ad} \), given in units of energy/volume of adhesive, is therefore the elastic energy dissipated during rupture of the joint, given by the area under the shear stress-shear strain curve during the test, from start to final rupture [21]

\[
U_{ad} = \int \tau(\gamma) d\gamma
\]

Figure 8. Compressive shear test. Two plates are bonded together, then subjected to a combination of compression and shear, as the rigs are pulled apart, as shown (from [21]).

In the rotational torque test, a coring drillbit is used to core an annular hole through several layers of the material (c.f. Figure 9). A torque “handle” is then epoxied to the top of the core, and a torque is applied until the weakest interface twists apart. The recorded torque, \( T \), vs. twist-angle, \( \theta \), data is used to compute the “toughness” of the interface—the area under the curve—in units of energy/angle of twist:

\[
U_{ad} = \int T(\theta) d\theta
\]
Figure 9. Rotational torque test. A coring drillbit is used to core an annular hole through several layers of the material. A torque “handle” is then epoxied to the top of the core, and a torque is applied until the peak torque—at which the weakest interface twists apart—is reached (from [22]).

All three of these tests have limitations. Foremost is the fact that all three measure pure mode II (shearing mode) toughness of the material or interface. However, debonding in PV modules—which often takes the shape of blisters (c.f. Figure 4)—likely includes a combination of mode I (opening) and mode II components. Furthermore, in the overlap shear and compressive shear tests, the shear strength that the tests yield is more a measure of a bulk material property than an interfacial property. The adhesive strength measured is not a true, fundamental adhesion energy. Rather, it is simply and “adhesive strength” that is particular to the type of testing method being utilized. In addition to this, the lap shear and compressive shear methods are not good at targeting a particular interface between dissimilar material layers—let alone a multi-layered composite structure like a PV module. The rotational torque test does overcome this limitation, however, by allowing targeting of a specific interface through selectively drilling down to it.
Figure 10. Modes of loading in a crack system (from [23]). Blisters in PV modules are likely a combination of mode I (opening) and mode II (in-plane shear).

The double-cantilever beam (DCB) method has been used to measure the adhesion between layers of a freestanding film or film-stack, such as a PV cell [24]. The method involves sandwiching the freestanding film between two semi-rigid, elastic beams, using an adhesive (see Figure 11). The beams are then subjected to a pulling force at one end, which causes a debond tip to form at the weakest interface in the sample material. The applied force, $P$, lever-arm length, $a$, beam width, $b$, beam plane-strain elastic modulus, $E' = \frac{E}{1-\nu^2}$, and beam height, $h$, are used to calculate the SERR, $G$:

$$G = \frac{12P^2 a^2}{b^2 E' h^3} \left( 1 + 0.64 \frac{h}{a} \right)^2$$

The adhesive strength of the debonding interface is the critical value of the SERR, $G_C$, over which debonding occurs.
Figure 11. Double-cantilever beam (DCB) test. The applied separation force, $P$, debond tip propagation distance, $a$, and beam opening separation, $c$ (not shown), are measured as the debond tip propagates, and used to compute the debond toughness, $G_C$, of the debonded interface. (from [24]).

A single-cantilever beam (SCB) method has also been developed for measuring interfacial toughness in PV materials [25, 26]. This method is applicable to materials wherein the thin film or film stack is adhered, on one side, to a rigid substrate—such as a PV module, itself, or coupons made of backsheet or encapsulant laminated to glass. On such a sample, a cantilever beam of semi-rigid elastic material is bonded on the film side (see Figure 12). The substrate is then held fixed, while a force is applied to one end of the beam, causing a debond tip to form at the weakest interface in the film stack, and propagate along the direction of the beam. The SERR, $G$, is calculated from:

$$G = \frac{6P^2a^2}{b^2Eh^3}$$  \hspace{1cm} (3)

where the variable definitions are the same as in (2).
Both the DCB and SCB methods are advantageous in that their low debond angle and is more representative of the delamination mode observed in PV modules. In addition, they are simple in geometry, which results in simple relationships between the measured test variables and the interfacial debond toughness. Furthermore, because it is based on fracture mechanics fundamentals, the interfacial debond toughness that they produce is a true interfacial property, independent of the testing configuration used.

However, despite these advantages, both methods have several drawbacks. Firstly, their geometry, in general, is unrepresentative of the geometry of debonding in PV modules, which is often characterized by blister-like delaminated regions [10]. Secondly, their pure mode I mode of debonding is unrepresentative of the mixed-mode (modes I and II) debonding that likely occurs in PV modules.

We propose an alternative to all of the above testing methods: the one- and two-dimensional blister tests. This blister test is a thin-film adhesion testing method, wherein a thin film—or film-stack—bonded to a rigid substrate experiences a delamination driving force due to a force applied by a shaft or an internal fluid pressure at the interface of interest.
Representative systems are shown in Figures 13 and 14, and are described in more detail, below. Several variations of the two-dimensional (2-D) blister test exist, including the shaft-loaded blister test (SLBT), pressurized blister test (PBT), and thermal blister test (TBT). A variant of the one-dimensional (1-D) blister test is the v-peel (or double-peel) test (VPT).

1.5. Proposed New Testing Methods

1.5.1. Shaft-Loaded Blister Test (SLBT)

The shaft-loaded blister test is shown in Figure 13. A thin film of thickness, $h$,—either singular or composite—is laminated to a rigid substrate. A circular hole is then etched—either mechanically or chemically—through substrate to the film, leaving the film, thereat, free-standing. A shaft—either cylindrically- or spherically-ended—is then driven against the film—with an applied force, $F$, and displacement, $w$—causing deformation of the film and its development into a blister geometry. At a certain threshold, delamination—either between the film and substrate, or between layers within the film—initiates, and the blister radius, $a$, increases in size. During the process, $F$ and $w$ are measured using the testing apparatus, and $a$ is measured using a camera. These three variables are used to calculated the strain energy release rate (SERR), $G$, which can then be used to determine the fracture toughness of the delamination interface.
1.5.2. Pressurized Blister Test (PBT)

In the case of the pressurized blister test (PBT), the sample geometry is the same as in the SLBT, but the delamination driving force is a pressure, $P$, applied to the blister by injecting a fluid. The applied pressure, along with the blister radius, $a$, and blister height, $w$, are measured throughout the experiment, and used to calculate the SERR, $G$, for determination of the fracture toughness of the delamination interface.
1.5.3. Thermal Blister Test (TBT)

The thermal blister test (TBT) is a variant of the PBT, wherein a fixed amount of air is sealed within the “blister cavity” by a plate adhered to the backside of the substrate hole. Subsequent heating of the sample causes the enclosed air to expand, driving delamination. The initial mass of air, \( m \), is computed from the initial volume of the undeformed blister, and combined with the blister radius, \( a \), and blister height, \( w \), to compute the SERR, \( G \). A variation of the test—which we call the thermal cycling test (TCT)—can be performed, whereby the sample is subjected to thermal cycling between two environmental chambers of different temperatures, causing a cyclic expansion and contraction of the internal gas. This thermally-induced cycling is likely the same mode by which blisters in fielded PV modules progressively grow in size, over time.

Figure 15. The thermal blister test (TBT). A fixed amount of air is sealed within the “blister cavity” by a plate adhered to the backside of the substrate hole. Subsequent heating of the sample causes the enclosed air to expand, driving delamination. The initial mass of air, \( m \), is computed from the initial volume of the undeformed blister, and combined with the blister radius, \( a \), and blister height, \( w \), to compute the SERR, \( G \).
1.5.4. V-Peel Test

The v-peel test (VPT) is a one-dimensional variant of the SLBT, wherein, instead of etching a circular hole through the substrate during sample fabrication, a rectangular slot is etched. During testing, a wide, narrow, rectangular shaft is driven against the film, causing delamination to occur by propagation of two straight, parallel crack fronts, in opposite directions, away from the slot. The crack front separation, $2l$, is measured using a sideview camera, and combined with the punch applied force, $F$, and displacement, $w$, to compute the SERR, $G$.

![Diagram of V-Peel Test](image)

Figure 16. The v-peel test (VPT). A rectangular slot is etched through the substrate during sample fabrication. During testing, a wide, narrow, rectangular shaft is driven against the film, causing delamination to occur by propagation of two straight, parallel crack fronts, in opposite directions, away from the slot. The crack front separation, $2l$, is measured using a sideview camera, and combined with the punch applied force, $F$, and displacement, $w$, to compute the SERR, $G$.

The blister tests have several advantages over the methods used in the literature. Foremost is the fact that the blister geometry most closely resembles the geometry of debonded areas in real PV modules (c.f. Figure 4). As such, they, furthermore, provide a combination of modes I and II during debonding, which is representative of the debonding modes in real PV modules. Furthermore, the circular debond front of the 2-D variants is
edgeless, and therefore avoids any edge effects. In addition to this, the low debond angles of all the methods, avoids—in contrast to the peel tests—excessive bending energy stored at the root of the crack front. Finally, the blister tests are based upon fracture mechanics fundamentals, and therefore yield true interfacial adhesive toughesses [27, 28, 29, 30].

1.6. Project Funding

In pursuit of our work on these methods, we have secured funding from two grants: the Department of Energy “Physics of Reliability: Evaluating Design Insights for Component Technologies in Solar 2” (PREDICTS2) project; and the National Institute of Standards and Technology (NIST) “Measurement Science for Service Life Prediction of Polymers Used in Photovoltaic (PV) Systems” project (“NIST project”) (see Acknowledgements for grant numbers). The goal of these two projects is to (i) develop measurement techniques for studying the degradation mechanisms in PV modules, and (ii) compare results between field-weathered and laboratory-exposed module materials for developing predictive models for the service life and reliability of modules. The focus of both projects is on the backsheets, in particular.

The PREDICTS2 project is a collaboration between six different agencies: Case Western Reserve University (Cleveland, OH), Underwriters Laboratories (“UL”; Northbrook, IL), NIST, Arkema, Inc. (King of Prussia, PA), the National Renewable Energy Laboratory (NREL; Golden, CO), and Northeastern University. In this project, field-exposed (FE) modules—covering a variety of different makes and models, including a variety of different backsheet types, and exposed in a variety of different climatic zones around the world—are
obtained for testing. They are subsequently tested, by different members of the group, using various types of optical, chemical, and mechanical tests, so as to assess the field-related degradation of the modules by a variety of measures. The tests include, among others: measuring the degree of yellowing (“yellowness index” or YI) of backsheets that are initially white in color; measuring the degree of “gloss-loss” of initially-glossy backsheets; measuring the chemical composition of the backsheets, using optical techniques (e.g. FTIR and Raman spectroscopy); and measuring the tensile and adhesive strength of the backsheets, using mechanical techniques. Northeastern’s role in this project is to measure the adhesion in the backsheets using the blister test. In addition to performing these tests on FE modules, they are also performed on laboratory-made “accelerated-exposure” (AE) samples (“coupons”) comprised of different types of backsheets laminated to glass. Prior to testing, the coupons are exposed, in-laboratory, to high temperature, high relative humidity, and/or UV radiation, for prolonged periods of time (e.g. 2000 hours), so as to artificially simulate—at an accelerated rate—the environmental stressors experienced by modules in the field. The data collected from the tests on both the FE modules and the AE coupons is, ultimately, gathered together and processed in a statistical engine that forms quantitative models between the exposure conditions (“stressor” variables) and test measurements (“response” variables). The ultimate goal is to develop a statistical model that can predict the service life of modules, based on these “stressor-response” relationships.

The NIST project, is a collaboration between NIST and NEU, wherein the same measurement techniques as the PREDICTS project—yellowing index, gloss-loss, Raman and FTIR spectroscopy, and tensile and adhesion tests—are used, but are applied to only one
specific type of backsheet: PPE. Currently, the bulk of the work is performed on AE 
coupons that have been exposed to UV radiation in NIST’s famous SPHERE—a UV chamber 
that is capable of producing irradiances as high as twenty times that of the Sun, at a variety of 
different wavelengths (see Figure 17). The focus of the project is on applying the various 
testing methods to SPHERE-exposed PPE/EVA/glass backsheet coupons, so as to gain a 
detailed understanding of the mechanism of backsheet degradation under prolonged UV 
exposure. Northeastern’s role in this project is, again, on applying the blister test toward 
measuring the adhesion in these samples—specifically, between the layers of the PPE 
backsheet.

Figure 17. NIST SPHERE (“Simulated Photodegradation using High-Energy Radiant 
Emission”).
1.7. Samples

From our collaborators on both grants, Northeastern has received samples in the form of both field-weathered module seconds, as well as laboratory-made coupons, from the two project groups.

1.7.1. Field-Weathered Module Sections

The PREDICTS2 group has supplied Northeastern with a total of 17 segments of field-retrieved modules. An example is shown in Figure 18. The sections are typically approximately 18 in. x 18 in. in size, and they each have been cut, using a diamond circular saw, from a full-sized module that was retrieved from the field. The modules have been exposed over a range of different exposure periods—from two years to 27 years—and across several different climate zones. They consist of 15 different models from 13 different manufacturers, and consist, collectively, of over eight different backsheet types. Table 3 shows a summary of this information. To protect commercially-sensitive confidential information, make and model information has been omitted.

Figure 18. A field-weathered module section from the PREDICTS2 project.
1.7.2. Laboratory-Made Coupons

The PREDICTS2 group has also supplied Northeastern with laboratory-made coupons of backsheet (BS)/EVA/glass. The coupons are 2 in. x 3 in. in size, and are made with 3-mm annealed glass. Coupons of three different types of backsheet—the three depicted in Figure 3—were supplied. They cover a range of accelerated exposure (AE) conditions. Table 1 summarizes the three different AE programs performed. Two of them consisted of prolonged exposure to UV radiation in a xenon arc lamp UV chamber, at moderate temperature and low relative humidity. One of the two included a periodic water spray, to introduce the effect of thermal cycling and moisture contact. The third program was the well-known “damp heat” test from the IEC 61215 industry qualification standard [14]—which involves prolonged exposure in a dark chamber to 85°C at 85% relative humidity. In each exposure program, samples were removed from the exposure chamber at different time intervals, to yield samples with different time lengths of exposure (c.f. Table 2).

Figure 19. Accelerated-exposure (AE) coupons supplied by PREDICTS2 group, as viewed from glass side (left) and backsheet side (right).
Table 1. AE exposure programs, performed on the coupons.

<table>
<thead>
<tr>
<th>Program</th>
<th>Irradiance (W/m²)</th>
<th>Chamber Temperature</th>
<th>Black Panel Temperature</th>
<th>Relative humidity</th>
<th>Method</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xenon Arc #1</td>
<td>0.8</td>
<td>65°C</td>
<td>90°C</td>
<td>20%</td>
<td>ASTM D789</td>
<td>102 minutes same as Xenon #1.</td>
</tr>
<tr>
<td>Xenon Arc #2</td>
<td>0.8</td>
<td>65°C</td>
<td>90°C</td>
<td>20%</td>
<td>ASTM D789</td>
<td>18 minutes with water spray.</td>
</tr>
<tr>
<td>Damp Heat</td>
<td>-</td>
<td>85°C</td>
<td>-</td>
<td>85%</td>
<td>IEC 81215</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Sample retrieval intervals for the AE coupons.

<table>
<thead>
<tr>
<th></th>
<th>0 hrs</th>
<th>500 hrs</th>
<th>1000 hrs</th>
<th>1500 hrs</th>
<th>2000 hrs</th>
<th>2500 hrs</th>
<th>3000 hrs</th>
<th>4000 hrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xenon Arc #1</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Xenon Arc #2</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Damp Heat</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

David C. Miller at NREL, has supplied Northeastern with coupons consisting of PVF/PET/EVA backsheet (BS) laminated to a piece of glass with a sheet of EVA (Figure 20, below). The samples are between 1.5 and 2.25 inches in width and length, and approximately 0.5 inches in thickness. The backsheet was measured to be 175 μm in thickness. Prior to lamination of the BS-EVA film to the glass, the samples were pre-drilled with 1-cm diameter holes, using a water-jet cutter. The holes were then plugged with solid Teflon rod, before the film was laminated to the glass. Prior to testing, a sample’s Teflon plug can be removed, leaving a freestanding circle of the film for blister testing.
Figure 20. PVF/PET/EVA backsheet/EVA/glass samples, supplied by David C. Miller at NREL.

NIST has provided Northeastern with samples of fresh PPE backsheet, as both freestanding film (see Figure 21) as well as BS/EVA/glass coupons (similar to those provided by PREDICTS).

Figure 21. Freestanding PPE backsheet samples (approx. 2 in. x 3 in. in size) provided by NIST.

We have already performed promising work using all of the above methods and samples. In the following chapters, we highlight the outcomes of some of these applications.
2. THERMAL BLISTER TEST (TBT)

We prepared thermal blister test (TBT) samples using the coupons provided by David Miller at NREL (see Section 1.7.2). A glass slide was glued onto the hole-side of each sample, covering the hole and sealing air inside (Figure 22, below). The adhesive used was Devcon® five-minute epoxy (bisphenol A diglycidyl ether resin). The hole was sealed at atmospheric pressure at room temperature (~23°C).

![Figure 22. TBT sample created by gluing glass slide to a TPE backsheet-EVA-glass sample.](image)

2.1. Thermal Cycling Test (TCT)

2.1.1. Method

We performed a modified version of the TBT—which we call the “thermal cycling test” (TCT)—on five such samples (Figures 22 and 23), wherein we cycled them between a hot-water and a cold-water bath, periodically (Figure 24). The samples were manually interchanged between the cold-water and hot-water bath, once a day. The cold-water bath consisted of water with ice in it (~5°C), and the hot-water bath consisted of water heated to
55⁰C. The samples were submerged in the hot-water bath during the day (approximately 9:00-10:30am to 7:00-8:30pm), and in the cold-water bath at night (during the reciprocal times of day). The experiment was performed for approximately 20 days. At the end of each day—before moving the sample from the hot-water bath to the cold-water bath—pictures were taken of the blister, for measuring the blister radius.

![Figure 23](image_url) The four samples tested.
We measured the blister radius, $a$, from the pictures taken of the blister at the end of each day, using the popular image analysis software, ImageJ (see Figure 25). A plot of $a$—as well as sample temperature—as a function of time, for all four samples, is shown in Figure 26. All of blisters started with an initial radius of 5mm, equal to the hole size.
Figure 25. Measuring blister area from the images of each blister, using ImageJ.

Figure 26. Blister radius, $a$, (colored curves; right-hand vertical axis) and sample temperature, (blue curve; left-hand vertical axis) as functions of time in days (bottom axis).

2.1.2. Results & Discussion

The results from Figure 26 show that the onset of delamination was inconsistent across the samples. The three samples that exhibited delamination (the purple, red, and orange
curves) each started delaminating after a different number of days had elapsed. Furthermore, two samples never exhibited delamination at all, for the entire duration of the experiment. This may suggest that the onset of delamination is sensitive to small differences between the samples—such as, for example, inconsistent sample preparation during the lamination process.

In addition to inconsistent delamination onset, the samples that did delaminate also exhibited different rates of delamination. For example, the first sample that delaminated (purple curve in Figure 26) did so at a slower rate than the second (red) and third (orange). In general, it appears that the later the onset of delamination, the more quickly it occurred. This may suggest that the bath had an “aging” effect on the adhesive/cohesive strength of the materials. The longer the sample sat in the bath, the weaker the adhesive/cohesive strength became. Once the energy barrier to delamination onset was overcome, the delamination occurred at a more rapid rate. Further experiments would be needed to verify this hypothesis, however.

For those samples that did delaminate, delamination sometimes occurred between different interfaces. For example, in the samples corresponding to the purple and red curves in Figure 26, delamination initiated at the EVA-glass interface, at the periphery of the hole, then migrated to the EVA-BS interface, within the first few days (Figures 27 and 28, below). In the sample with the orange curve, delamination occurred entirely between the glass-EVA interface (Figure 29, below).
Figure 27. Dissection of the TCT sample corresponding to the red curve in Figure 26. At the periphery of the hole, there is a “crater” in the EVA. This is where the crack migrated from the EVA-glass interface to the EVA-BS interface. Outside of the “crater”, the delamination interface is between the EVA and BS.

Figure 28. Delamination interfaces of sample with purple curve in Figures 26 and 31. Delamination initiated at the EVA-glass interface, at the periphery of the hole, then migrated to the EVA-BS interface.
Figure 29. Delamination interface of sample with orange curve in Figures 26 and 31. Delamination initiated and continued at the glass-EVA interface, for the duration of the experiment.

Ultimately, we are interested in determining the interfacial adhesion energy, $G_C$, of the delaminating interfaces, from our measured blister radius values, $a$. To do this, we assume a (reasonable) geometry of the blister—here, a spherical cap—and combine this with known relations between the blister radius, $a$, internal gage pressure, $\Delta p$, blister height, $w$, and strain energy release rate (SERR), $G$. Some of the relevant system variables are shown in Figure 30.
Figure 30. Thermal blister test variables, with blister radius, \( a \), internal gage pressure, \( \Delta p \), blister height, \( w \), strain energy release rate (SERR), \( G \), film thickness, \( h \), and film plane-strain elastic modulus, \( E' = \frac{E}{1-\nu^2} \), where, \( E \) is the elastic modulus and \( \nu \) is the Poisson ratio.

Arjun and Wan [31] derived the SERR for the pressurized blister test from the von Karman equation for a circular plate with both bending and stretching stresses. They assumed small displacements and an average, equibiaxial membrane stress, and found the relationship with the blister height, \( w \), and the gage pressure, \( \Delta p \), to be

\[
G = n \cdot \Delta p \cdot w
\]

where \( n \) is a numerical pre-factor that varies from \( \frac{1}{2} \) for the limit of a plate dominated by bending stresses (a.k.a. the “bending limit”) to \( \frac{5}{8} \) for a membrane dominated by stretching stresses (the “stretching limit”). In addition, they found that the film assumes the bending limit when \( \frac{w}{h} < 1 \), approximately, the stretching limit when \( 10 < \frac{w}{h} \), and a combination of bending and stretching (i.e. a “transition region”) in between. During the course of the experiment, we found that the blister height varied between approximately 0.5 mm and 3.5 mm, which, when compared to the film thickness of 175 \( \mu \)m, gives a range of \( 2.85 < \frac{w}{h} < 20 \). In the early part of the experiment, we are, therefore within the transition region. By noting that we are, however, within the stretching limit for most of the experiment—and being faced with the lack of a simple model for the transition region—we make the
approximation of applying the stretching model to our analysis—bearing in mind the
inexactness of this approach. We, therefore, use the following relation for the SERR
\[ G = \frac{5}{8} \cdot \Delta p \cdot w \] (4)

The classical solution for the constitutive relationship between the gage pressure, \( \Delta p \),
blister height, \( w \), and blister radius, \( a \), for a membrane under pure stretching [18] is given by
\[ w^3 = \frac{a^4}{2E' h} \Delta p \] (5)
where \( h \) is the thickness of the film, \( E' \) the plane-strain elastic modulus \( E' = \frac{E}{1-\nu^2} \), \( E \) the
elastic modulus, \( \nu \) is the Poisson ratio. The gage pressure, \( \Delta p \), inside the blister can be
related to the blister volume, \( V \), and temperature, \( T \), by the ideal gas law
\[ (p_{atm} + \Delta p) V = m R T \] (6)
where \( p_{atm} \) is atmospheric pressure at room temperature, \( m \) is the mass of air inside the blister,
and \( R \) is the ideal gas constant. The mass of the air inside the blister, \( m \), can be calculated
from the known initial volume, \( V_0 \), of the blister when it was sealed with the glass backing
plate at atmospheric pressure, \( p_{atm} \), and room temperature, \( T_0 \).
\[ p_{atm} V_0 = m R T_0 \] (7)
Finally, the spherical cap geometry assumption gives
\[ V = \frac{\pi}{6} w (3a^2 + w^2) \approx \frac{\pi}{2} w a^2 \] (8)
If we combine (5), (6), (7), and (8), eliminating \( \Delta p \), \( V \), and \( m \), we end up with
\[ w^4 + A w - B = 0 \]
where
\[ A = \left( \frac{1}{2} E' h p_{atm} \right) a^4 \]
\[ B = \left( \frac{1}{\pi} \frac{T}{E' h T_0} V_0 p_{atm} \right) a^2 \] (9)
From this, for a given measured blister radius, $a$, we can solve (9) numerically for the corresponding blister height, $w$. Substitution of this pair of values into (5) yields the gage pressure, $\Delta p$. Finally, substituting $w$ and $\Delta p$ into (4), we can obtain the SERR, $G$.

The results of these calculations are shown in Figure 31. Of foremost note is the fact that the adhesion energy values, in general, are very low. Novoa, et al measured the adhesion energy at the EVA/glass interface for coupons held at 55°C and 38% RH to be in the range of 1000-1500 J/m$^2$ [26]. Our low values are due to the fact that the gage pressure inside our blister is very low. It can be shown from (6) and (7) that, for a system with fixed blister volume, $V_0$, the pressure increase of the heated blister (55°C) from the unheated blister at room temperature (23°C) is only 0.11 bar. This is the maximum possible pressure increase; the actual pressure increase is even small, due to the increase in volume from expansion of the blister. Despite the extremely low driving force, we did see delamination, which indicates that the influence of one or more of the factors—time, temperature, and moisture—is significant.

For the samples that did delaminate, we see a slight apparent increase in the SERR, for the initial growth of the blister, before it started to decrease. We believe that this is simply a geometric phenomenon—an artifact of the spherical cap geometry assumption. This assumption is not able to accurately represent the transition from a flat membrane to a spherical cap, for the early blister growth. Instead, it assumes a spherical cap geometry, throughout, including at the very beginning.

It is important to note, for the samples that debonded, that we cannot equate the SERR as a critical interfacial SERR, $G_C$, of the interface, over the period that debonding occurred.
This is because this is only valid for time-independent debonding. Our results are very clearly time-dependent, as well as potentially temperature- and moisture-dependent. Therefore, we need a model that accounts for these factors. We will explore such modeling in the future, as we continue to utilize this method.

Figure 31. Adhesion energy, $G$ (colored curves; right-hand axis), and bath temperature (blue curve; left-hand axis), from the TCT test of five samples, as a function of time, in days.

2.2. Constant-Temperature Thermal Blister Test

In order to determine whether the thermal cycling actually influenced the delamination mechanics—or whether, in contrast, the delamination was almost entirely due to the high temperature of the hot bath exposure—we performed the same experimental technique as above, but at constant temperature.
2.2.1. Method

We constructed three samples (see Figure 32) and immersed them in a hot water bath at 55\(^\circ\)C—the same as the hot water bath above—for several days. Each day, we measured the blister size in the morning (approx. 8:30am) and evening (approx. 4:30pm).

Figure 32. The three samples from the constant-temperature TBT.

2.2.2. Results & Discussion

The evolution of the blister radius over time is shown in Figure 33. The evolution of the SERR is shown in Figure 34.
Figure 33. Blister radius, \( a \), (colored curves; right-hand vertical axis) and sample temperature, (blue curve; left-hand vertical axis) as functions of time in days (bottom axis).

Figure 34. Strain energy release rate (SERR), \( G \) (green curve; right-hand axis), and bath temperature (blue curve; left-hand axis) as a function of time, in days.

In contrast to the TCT tests, the blisters for the constant-temperature TBT test took only three days to reach terminal blister growth. Furthermore, the onset of delamination occurred within the first day, for all three samples. It is difficult to ascertain whether or not this is the
result of the changed testing condition—or whether it is simply the result of variation in material properties between samples—because delamination onset within the first day is a roughly equivalent time period to the first hot-water bath exposure in a TCT test. More tests would need to be performed to determine this.

In addition to more rapid onset and rate of delamination, close observation of the samples also showed that the delamination occurred entirely at the glass-EVA interface, for the duration of the experiment, for all three samples. This is the same interface as the single, orange-curved sample in Figures 26 and 31 from the TCT test, but differs from the purple- and red-curved samples. It is possible that the mechanical or thermal cycling of the TCT test tends to induce a transition from the glass-EVA interface to the BS-EVA interface, but more experiments would need to be conducted to verify this.

Despite these speculative hypotheses, the accelerated rate of delamination of this constant-temperature test—in contrast with the slightly slower rate of the TCT tests—suggests that the constancy of exposure to the heat and moisture of the hot water bath contributes significantly to the rate of delamination in a TCT test. We plan to conduct more experiments to verify our observations, as well as to refine the method further.

3. SHAFT-LOADED BLISTER TEST (SLBT)

3.1.1. Method

In addition to our work on the TBT, we have also performed some significant work with the SLBT. We performed our first tests using the BS/EVA/glass coupons provided by David
Miller at NREL (c.f. Figure 20). We performed the tests using a TA-XT Plus Texture Analyzer by Texture Technologies (Hamilton, MA, USA). To conduct the tests, we placed a coupon, backsheet-side-down, on a platform, with cameras positioned below and on the side of the sample to capture overhead-view and side-view images of the blister, respectively (see Figure 35). Using a 5-mm diameter, flat-ended cylindrical probe, we performed the test at a displacement-controlled rate of 30 \( \mu m/s \).

![Figure 35](image-url) Setup of an SLBT on a BS/EVA/glass coupon, using a TA-XT Texture Analyzer by Texture Technologies (Hamilton, MA, USA).

3.1.2. Results & Challenges

The results of this test are shown in Figure 36. The force increased monotonically and almost linearly with displacement, until final rupture at approximately 5 mm displacement, 190 N force—whereat the punch punctured the film. Post-test sample inspection and image analysis revealed the film underwent extensive plastic stretching, during most of the test, but the blister radius only grew by approximately 1-2 mm (see Figure 37). From this, it became
apparent that (i) the adhesion in this material system is very large, compared to the mechanical properties of the film, and (ii) a plate, of some sort, was needed to provide a backing to the backsheet, to limit the plastic stretching. Furthermore, analysis of the camera footage revealed that the exact edge of the blister was difficult to ascertain from the video footage, due to the opacity of the white backsheet (Figure 37). Finally, from a review of the existing models relating the SERR to the measured variables for the SLBT, it became apparent that almost all of them analyzed the system as an ideal point-loaded shaft contacting the film. Since it was impractical for us to use such an idealized model for our experiments, it became apparent that a new model for the SLBT was needed—one which modeled the shaft-film contact as a finite-sized area.

![Image of force-displacement curve](image)

**Figure 36.** Force-displacement results of a SLBT performed on a BS/EVA/glass sample from David Miller at NREL. The force increased monotonically and almost linearly with displacement, until final rupture at approximately 5 mm displacement, 190 N force.
3.1.3. New Theoretical Model

To address the need for a new SLBT, we undertook an analysis of the SLBT, based upon an approximation of the film as a circular plate undergoing small deformations. We modeled the system where the punch is a flat-ended cylinder, and considered the two configurations of the “clamped” and “free” SLBTs (c.f. Figure 38). For each configuration, we investigated the two limits of a bending-dominated film response (i.e. the “bending limit”) and a stretching-dominated film response (i.e. the “stretching limit”).
Figure 38. Schematics of clamped and free SLBTs, with experimentally measured variables: $F$, the applied punch force, $a$, the blister radius, $w_0$, the blister height, and, $w_c$, the punch displacement. The known, fixed geometric parameters of the system are: $c$, the punch radius, and, $h$, the film thickness.
While the details of the full analysis are given in the appendix, a brief description is given here. For each of the four scenarios—clamped SLBT–bending limit, clamped SLBT–strecthing, free–bending, and free-stretching—we derived analytical relations between the experimentally measurable variables—applied force, $F$, punch displacement, $w_c$, and blister radius, $a$—and the SERR, $G$. Plots of normalized versions of these quantities for the clamped-bending scenario, as an example, are shown in Figure 39. Each plot approaches the asymptote of the point-load (PL) limit for sufficiently large blister-to-punch radius ratio, $\frac{a}{c}$, or sufficiently large punch displacement-to-film thickness ratio, $\frac{w_c}{h}$. We found this to be true, in all four scenarios. This is attractive, in that the relations for the PL limit are elegant and simple. The results for the other three scenarios are presented in the appendix. We plan on using these new models in future tests, once we have resolved the issue of measuring the blister radius (Section 3.1.5, below).
\[
\left( \frac{6c^2}{\pi E' h^4} \right) F
\]

\[
F \rightarrow \left[ \frac{8\pi^2 E' h^3}{3} \right]^{1/2}
\]

\[
\frac{w_c}{h} \rightarrow \left[ \frac{3c^2}{\pi^{1/2} E' h^4} \right] a^2
\]
Figure 39. Plots of the experimentally measurable variables—applied force, $F$, punch displacement, $w_c$, and blister radius, $a$—as well as the SERR, $G$, for the clamped-bending scenario. All of the relationships approach the point-load (PL) limit for sufficiently large blister-to-punch radius ratio, $\frac{a}{c}$, or large punch displacement-to-film thickness ratio, $\frac{w_c}{h}$.
3.1.4. Use of a Backing Plate

To address the need for a backing plate for the film, we explored the use of steel, PMMA (acrylic glass), and titanium (see Figure 40). The resulting higher loads of this system prompted us to move to a larger testing apparatus—an Instron 5582 Universal Testing Machine, capable of loads up to 100kN—and we were subsequently able to perform tests without excessive plastic stretching of the film. However, the addition of the backing plate only exacerbated the challenge of optically measuring the blister size, as there was no longer a blister profile with distinct slope transitions to mark the approximate edge of the blister.

3.1.5. Measuring the Blister Radius

Several attempts were made to address this issue, including attempting to extract the blister radius from the slope of an intermediate unloading curve on the force-displacement curve (Figure 41), as well as using Moire interferometry (Figure 42). It was during the process of exploring these alternatives that we happened upon the v-peel test. The method is outlined in Section 1.5.4, but, essentially, it is a one-dimensional variant of the SLBT. Attractively, it allows direct location of the crack locations, by use of a side-view camera. Due to the relative ease of this approach in comparison with the SLBT, we decided to temporarily forestall our work with the SLBT, in favor of this test, instead. In the future, we plan to resume our pursuit of the SLBT, and attempt to resolve the experimental challenges we encountered.
Figure 40. PV module SLBT sample, showing: (a) the glass side of the sample with the aluminum superstrate plate adhered for rigidity; (b) backsheet side of sample showing titanium backing plate; (c) side profile view of sample.

Figure 41. Illustration of the method of using the slope of the force-displacement curve to deduce the size of the blister. The slope is given by $m$, the blister radius by $a$, and the flexural rigidity of the film by $D = \frac{E' h^3}{12}$. 
4. **V-PEEL TEST**

We have performed v-peel tests on a number of different samples, including fresh and AE coupons provided by NIST and PREDICTS2, as well as actual FE modules provided by PREDICTS2. The method is outlined in Section 1.5.4. Here, we present the results of a test on an FE module sample, as an example of application of the method. Tabulation of the results from a number of tests performed on samples from the PREDICTS2 project are presented in Tables 3 and 5, at the end of this section.

4.1.1. **Method**

A picture of a representative FE module section is shown in Figure 18. We prepared testing samples by cutting out small strips of the module, 25mm in width, \( b \), by 75mm in total length. We then adhered a titanium plate—0.5mm in thickness, \( h \)—to the backsheet side of each strip (see Figure 43, parts a and b). Titanium was selected as the backing material.
because its high yield stress allows for more force to be applied in the linear elastic regime. Furthermore, for sufficiently low punch displacements, its high flexural rigidity forces the system to be in a pure-bending mode of deformation, which allows a bending-limit model for the v-peel test to be applied. After adhering the backing plate, a thick, rigid top plate of aluminum—5mm in thickness—was adhered to the top of the module glass. This made the tempered module glass—which was cracked during the module cutting process—rigid again, so as to form a rigid superstrate (c.f. Figure 43, parts c and d). Finally, using a diamond-coated glass-cutting drillbit, a slot was machined through the module glass and film stack, all the way down to the titanium backing plate. During this entire fabrication process, care was taken so as to not introduce premature delamination between any of the sample layers. The final sample was comprised of the module piece “sandwiched” between two metal plates (c.f. Figure 43, part e).

Figure 43. Samples prepared from field-weathered modules: (a) front-side view of module strip, (b) backside view of module strip, showing titanium plate adhered to the backsheet, (c) backside view of strip adhered to rigid Al metal superstrate, (d) frontside view of strip-superstrate assembly, showing slot through superstrate, (e) side-view of assembly.

We conducted our tests on these samples by quasi-statically loading each sample to a final, target displacement, then unloading, at the same rate, to the original, starting position. For each sample, we performed a succession of such loading/unloading cycles, each time to a
greater target displacement. We conducted our experiments on an Instron® 5582 Universal Testing Machine. The loading rate was selected to be $10 \mu \text{m/s}$.

### 4.1.2. Results & Discussion

A representative set of results from one of our experiments is shown in Figures 44 and 45. The applied force, $F$, and blade displacement, $w_0$, were measured by the Instron 5582 (Figure 44), and video footage of the propagation of the crack front was recorded by a side-view camera (Figure 45). Each loading/unloading curve in Figure 44 shows three stages: loading, during which the film deformed, but did not delaminate; delamination, during which the film delaminated at an interface and the crack front propagated; and unloading, during which the load was removed and no further delamination occurred. During unloading, the crack did not “heal”. Therefore, during the delamination portion of each loading/unloading cycle, the two opposing crack fronts propagated incrementally further in opposite directions. The camera footage was used to measure the distance between the two crack fronts (Figure 45).

In this particular sample, delamination occurred at the module-glass/top-encapsulant interface (Figure 47). The encapsulant material, in this case, was ethylene co-vinyl acetate (EVA). Other modules delaminated at other interfaces, such as the Ti-backing-plate/backsheet-outer-layer interface, or between two layers of backsheet (see Tables 3 and 5). In general, delamination occurred at the weakest interface.
Figure 44. Force-displacement, as well as adhesion energy, results from a V-peel test conducted on a PV module sample. The force-displacement cycling curves are associated with the left-hand axis, while the adhesion energy curve (curve with circled dots) is plotted against the right-hand axis.

Figure 45. Side-image from the V-peel test, showing the measured distance between crack fronts, $2I$. 
Figure 46. Post-test sample, separated along the delamination interface. Here, the delamination interface was at the module glass-top encapsulant interface.

Figure 47. Schematic of layers of module sample (not drawn to scale), as well as delamination interface.

We calculated the adhesion energy using a model derived by Wan [32] for the V-peel test under a pure-bending deformation mode. The adhesion energy, $G$, is related to the applied force, $F$, punch displacement, $w_0$, distance between cracks, $2l$, and sample width, $b$, according to

$$G = \frac{3}{2} \frac{Fw_0}{(2l)b}$$

(10)

This model is derived from plate theory and fracture mechanics, where the film stack and backing plate are modeled as a linearly elastic plate with clamped edges, and the fracture mode is assumed to be purely mode I. The plate is assumed to undergo pure-bending deformation and small bending angles. These assumptions are an idealization. In reality, both bending
and stretching-induced stresses are present in a V-peel system undergoing delamination (see Figure 48). Furthermore, because of this, the fracture mode is a combination of both mode I and mode II fracture. However, when the stretching stresses are small, relative to the bending stresses, this model can be applied. In our application of the model, we assume that the flexural rigidity of the backing plate dominates the mechanical response of the overall plate-film stack, and that the comparatively small flexural rigidity of the film stack can be ignored.

The values for the adhesion energy in Figure 44 show some inconsistency as the punch displacement increases. In particular, they gradually increase as punch displacement increases, from an initial value of 1653 J/m² to a final value of 2211 J/m². We believe that this is due to the gradual inclusion of increasingly higher stretching stresses in the backing plate (see Figure 48). This is illustrated by increasing deviation from linearity that the loading and unloading curves exhibit, as the punch displacement is increased. Since our model only accounts for a pure-bending situation, it overestimates the adhesion energy when tensile stresses begin to become comparable, in magnitude, to the bending stresses [32]. This could be mitigated by using a thicker backing plate, which would ensure that bending stresses are dominant during more of the delamination process. Alternatively, a more general analytical model—one that incorporates the inclusion of stretching behavior in the system—could be used to analyze our results.
In addition to the inconsistency exhibited by the adhesion energy values, there is also some hysteresis in the mechanical response of the materials. This is evident in Figure 44, where the loading curve from one cycle does not coincide with the unloading curve of the previous cycle. Our model assumes linearly elastic materials, and in such a system, these curves should coincide. The hysteresis exhibited by the curves clearly shows that there is some dependency of the response of the material on the loading and unloading rate. This is quite possibly due to the viscoelasticity of the polymers in the module, particularly EVA. We attempted to mitigate the rate-dependency, as much as possible, by choosing a sufficiently slow loading/unloading rate. In addition, we chose a rate of 10µm/s—the same as Novoa, et al [25], who conducted single cantilever beam (SCB) experiments on laboratory-made coupons of backsheet laminated to glass. While we currently do not have a model that can account for viscoelastic effects, we will investigate the possibility of incorporating it into future models.

As a final note, Novoa, et al [26] conducted SCB experiments on laboratory-made coupons of EVA encapsulant laminated to glass. They measured an adhesion energy of 2,150 J/m² at the glass-EVA interface. While there are caveats with directly comparing our results—
conducted on a module with 3 ½ years of field exposure—with their results—conducted on unexposed, laboratory-made coupons—the comparison indicates, at least, that we are both within the same order of magnitude.

4.1.3. Application to PREDICTS2 Project

As part of our PREDICTS2 work, we performed v-peel tests on a number of samples—both FE modules and AE coupons—the results of which are shown in Tables 3 and 5, respectively. The FE modules covered a number of different makes and models; deployment locations, climate zones, and time-periods; and backsheet types (c.f. Section 1.7.1). For the FE modules results, each row in Table 3 corresponds to a different module. Climate zone descriptions are provided in Table 4. The AE coupons encompassed three different backsheet types, pulled at three different points in time during a single UV exposure program (c.f. Section 1.7.2). Regardless of sample type, for each test we noted the interface at which delamination occurred, and calculated the adhesion energy of the interface. The adhesion energy numbers are given as a range of values, because the calculation results do not yield a single value (c.f. Figure 44). Typically, we used the latter three to five values from the adhesion energy curve, because, by this point, we assumed that the behavior of the system had reached a steady-state.

Among the FE module results, it is difficult to see obvious trends between the relevant variables—climate zone, deployment duration, backsheet type, and delamination interface—and the adhesion energy. However, there may be a correlation between deployment duration and adhesion energy—namely, that longer deployment durations are correlated with lower adhesion energies. Figure 49 shows a plot of the results listed in Table 3. While there is no
clear trend among the data for the modules between 2 – 6 years, there is a clear decrease in adhesion energy between that group and the 27-28 year group, whose adhesion energies were so low that they fell apart during sample prep.

Among the AE coupon results in Table 5, trends are easier to discern. The data for the two backsheets that yielded measurements—PA/PA/PA and PVF/PET/EVA—are plotted in Figure 50. The trend between exposure time and adhesion energy is more definite than for the FE module results. For the two backsheets that yielded measurable adhesion energies, there is a clear decrease in adhesion energy with exposure time. Furthermore, the PA/PA/PA backsheet appears to, on average, have slightly lower adhesion energy than the PVF/PET/EVA backsheet.

Table 3. Results of v-peel tests on PREDICTS2 FE module samples. Climate zone types are listed in Table 4.
Table 4. Climate zone descriptions for the climate zones listed in Table 3.

<table>
<thead>
<tr>
<th>Köppen-Geiger Climate Zone</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cfa</td>
<td>Temperate or subtropical hot summer climate</td>
</tr>
<tr>
<td>Csa</td>
<td>Dry summer or Mediterranean climate</td>
</tr>
<tr>
<td>Csb</td>
<td>Temperate, dry and warm summer</td>
</tr>
<tr>
<td>Cwb</td>
<td>Oceanic and subtropical highland climate</td>
</tr>
<tr>
<td>Dfa</td>
<td>Hot summer continental climate</td>
</tr>
<tr>
<td>Dfb</td>
<td>Warm summer continental climate</td>
</tr>
</tbody>
</table>

Figure 49. Average adhesion energy vs. deployment duration for the field-exposed modules in Table 3. Here, there is a clear decrease in adhesion energy with exposure time.

Table 5. Results of v-peel tests on PREDICTS2 AE coupon samples.

<table>
<thead>
<tr>
<th>Backsheet Type</th>
<th>Exposure Treatment</th>
<th>Exposure Time</th>
<th>Delamination Interface</th>
<th>Adhesion Energy (J/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA/PA/PA</td>
<td>Xenon Arc #1</td>
<td>0 hrs</td>
<td>Inner-PA/Core-PA</td>
<td>1850-1950</td>
</tr>
<tr>
<td>PA/PA/PA</td>
<td>Xenon Arc #1</td>
<td>2000 hrs</td>
<td>Inner-PA/Core-PA</td>
<td>750-800</td>
</tr>
<tr>
<td>PA/PA/PA</td>
<td>Xenon Arc #1</td>
<td>4000 hrs</td>
<td>Outer-PA/Core-PA</td>
<td>200-550</td>
</tr>
<tr>
<td>PET/PET/EVA</td>
<td>Xenon Arc #1</td>
<td>0 hrs</td>
<td>- Testing epoxy failure.</td>
<td>-</td>
</tr>
<tr>
<td>PET/PET/EVA</td>
<td>Xenon Arc #1</td>
<td>2000 hrs</td>
<td>- Testing epoxy failure.</td>
<td>-</td>
</tr>
<tr>
<td>PET/PET/EVA</td>
<td>Xenon Arc #1</td>
<td>4000 hrs</td>
<td>- Testing epoxy failure.</td>
<td>-</td>
</tr>
<tr>
<td>PVF/PET/EVA</td>
<td>Xenon Arc #1</td>
<td>0 hrs</td>
<td>Outer-PVF/Core-PET</td>
<td>2250-3200</td>
</tr>
<tr>
<td>PVF/PET/EVA</td>
<td>Xenon Arc #1</td>
<td>2000 hrs</td>
<td>Outer-PVF/Core-PET</td>
<td>2150-2250</td>
</tr>
<tr>
<td>PVF/PET/EVA</td>
<td>Xenon Arc #1</td>
<td>4000 hrs</td>
<td>Within Outer-PVF</td>
<td>950-1550</td>
</tr>
</tbody>
</table>
Figure 50. Average adhesion energy vs. exposure time for the PA/PA/PA and PVF/PET/EVA backsheets from Table 5. Here, there is a clear decrease in adhesion energy with exposure time.

More tests are needed to verify the reliability of our data, as well as establish better trends between the relevant variables. In particular, it would be illuminating to test FE modules with deployment durations that are intermediate to the 2-6-year and 27-28-year periods for which we have data. This would better establish a trend between deployment duration and adhesion energy. Over the remaining 1 ½ years of this project, we will continue to expand our collection of modules.

Ultimately, if we are able to establish clear trends among the variables within the FE modules—as well as within AE coupons—we will then work to develop a model that links the two. That is, for example, we hope to link the correlation between adhesion energy and deployment duration in FE modules, to the correlation between adhesion energy and exposure
time in AE coupons. Such a model would then allow prediction of backsheet lifetime from AE tests—a tool that would be of great value to the industry.

5. PRESSURIZED BLISTER TEST (PBT)

5.1.1. Method

In addition to the above tests, we have performed some promising work with the pressurized blister test. The general method is outlined in Section 1.5.2. We have performed test on some of the BS/EVA/glass samples supplied by David Miller at NREL (see Section 1.7.2). From these coupons, we produced PBT samples by first epoxying a double-wide (2 in. x 3 in. x 1 mm) glass slide—with a 5-mm hold drilled through the center—to the back of the sample, covering the blister hole, but leaving access through the hole in the slideglass (Figure 51b). In conjunction with this, we constructed a 2-in. x 3-in. plate of PMMA clear acrylic, with a short length of PE-90 polyethylene tubing (OD: approx. 0.05 in.) epoxied through the hole (Figure 51a). Finally, we epoxied the plate onto the slideglass, forming a final PBT blister sample (Figure 51c).
Figure 51. Construction of a PBT sample.

We performed tests using the setup shown in Figure 52. Due to the strong adhesion observed in these samples at room temperature, during the SLBTs (see Section 3), we decided to immerse the test sample in 55°C water during testing. This is just 5-10°C below the melting temperature range of 60-65°C of EVA, so we expected a drop in adhesion between the EVA and adjacent layers. Furthermore, studies on EVA [33] have shown that moisture causes a significant, but recoverable, in situ drop in adhesion at these interfaces.
The sample was immersed in the water for 5-10 minutes, prior to testing, and left immersed during testing. Testing was conducted by injecting 0.75 bar of air pressure into the blister using the syringe, holding for a fixed period of time, releasing for the same amount of time, then repeating. The pressure was cycled in this manner until either: (a) the blister growth approached the edge of the sample, or (b) the duration of the test—which was performed manually—approached two hours. Four experiments were run, each with a different hold (and, equivalently, release) time: 60s, 45s, 30s, and 20s. The blister area was measured, periodically, from video footage taken during the test (see Figure 53). The popular image analysis program, ImageJ, was used to measure the area of the blister, \(A\), in mm\(^2\). From this a mean blister radius, \(\bar{a}\)—which represents the radius of a circular blister of equivalent area—was calculated using \(\pi \bar{a}^2 = A\).
5.1.2. Results & Discussion

The results are plotted in Figure 53, where the mean blister radius is plotted versus number of cycles, $N$. For all four tests, delamination occurred at the same interface throughout the test: the glass/EVA interface. One exception was encountered in the 30-second sample, which exhibited a transition to the BS/EVA interface after approximately 100 cycles (see Figure 53). Collectively, the data in Figure 53 shows that, for longer hold/release times (i.e. slower cycling frequency), delamination occurred more quickly, while for shorter hold/release times (i.e. faster frequency), it occurred more slowly. Summarily, these results suggest that the crack growth rate in these materials and test conditions exhibits a cycling frequency dependency.
Figure 53. Mean blister radius, $\bar{a}$, vs. no. of cycles, $N$, for cyclic blister tests with holding (and, equivalently, release) times of 60s, 45s, 30s, and 20s. Pictures of the blister corresponding to points A, B, C, and D, on the 30-s curve, are shown above the plot.

It is plausible that this apparent frequency dependency would vanish if the blister size were plotted against only the time during which the blister presumably grew—i.e. the elapsed hold time. Such a plot is shown in Figure 54. The plot reveals that the rate dependency still stands.
Figure 54. Mean blister radius, $\bar{a}$, vs. elapsed hold time. The rate dependency of the blister growth rate on frequency, shown in Figure 53, is also shown here.

The frequency dependency of the crack growth rate that we are witnessing contrasts with the conventional mode of fatigue that traditionally characterizes fatigue crack growth in materials. Conventionally—in non-viscoelastic materials such as metals and ceramics—fatigue crack growth rate is a function purely of the number of loading cycles and the applied stress. This is depicted in Figure 55. Here, a fatigue specimen with a crack of initial length, $a_0$, has been subjected to a sawtooth cyclic applied stress, that varies between 0 and $S$. As a result, the crack length, $a$, gradually increases with the number of cycles, $N$. The resulting “$a$-N curve” is depicted for three different values of $S$, from a large maximum applied stress, $S_1$, to a small maximum applied stress, $S_3$. The curves show that the rate of crack growth, $\frac{da}{dN}$, is greater when $S$ is greater. It is important to note that these results are conventionally
independent of the cycling frequency. That is, the crack growth rate, $\frac{da}{dN}$, is only a function of the applied stress range, $\Delta S = S_{\text{max}} - S_{\text{min}}$, and the number of cycles, $N$,

$$\frac{da}{dN} = f(S, N)$$  \hspace{1cm} (11)

(Note, here, that $\Delta S = S - 0 = S$.) This can alternatively be expressed as,

$$\frac{da}{dN} = g[\Delta K(\Delta S, a)]$$  \hspace{1cm} (12)

where $\Delta K = K_{\text{max}} - K_{\text{min}}$, is the range of stress intensity factors during cycling, which is a function of the range of applied stress, $\Delta S$, and the crack length, $a$. This is typically expressed as Paris’s law [23]

$$\frac{da}{dN} = C(\Delta K)^m$$  \hspace{1cm} (13)

where $C$ and $m$ are material fatigue constants. The relation, in other words, states that crack growth rate is purely a function of the number of cycles, $N$, and the intensity of the loading, $\Delta K$. It has no dependency upon loading frequency. A sample loaded at a fast rate has the same crack growth rate as a sample loaded at a slow rate.
Figure 55. Typical “a-N curve” from cyclic fatigue loading tests. A fatigue specimen (top-left) with initial crack length, $a_0$, is subjected to a sawtooth cyclic loading (bottom-right), varied from a minimum applied stress, $\theta$, to a maximum applied stress, $S$. Curves are shown for three different maximum applied stresses, $S$, with $S_1 > S_2 > S_3$. The curves show that crack growth rate, $\frac{da}{dN}$, depends only upon $S$ and number of cycles, $N$, and that $\frac{da}{dN}$ increases with increasing $S$.

Our findings contrast with this behavioral law. We witness a fatigue behavior that is more represented by Figure 56. The maximum applied stress, $S$, is constant—because our maximum applied pressure is constant—which the frequency, $\omega_1$, is varied. High frequencies correspond to slower delamination rates, and vice-versa, such that $\omega_1 < \omega_2 < \omega_3$. To account for this frequency-dependency, one hypothetical model we put forth is that one or both of the fatigue constants, $C$ or $m$, in (13) is frequency-dependent. Perhaps, hypothetically, Paris’s Law might be, therefore, rewritten as

$$\frac{da}{dN} = C(\omega)\Delta K^m(\omega)$$

While this is purely hypothetical, we are in the process of searching for a satisfactory model to explain our results.
Figure 56. Schematic of the $a$-$N$ curves obtained from our PBT results (c.f. Figure 53). The maximum applied stress, $S$, is constant, because the maximum applied pressure, during the experiment, was constant. Meanwhile, the cycling frequency, $\omega$, is varied between three different values, with $\omega_1 < \omega_2 < \omega_3$. The curves show that crack growth rate, $\frac{da}{dN}$, depends not only upon $S$ and $N$, but also upon $\omega$, with $\frac{da}{dN}$ increasing with decreasing $\omega$.

One possible explanation for the underlying cause of frequency-dependence of the growth rate is that the crack velocity is transient due to the viscoelasticity of the materials, and, therefore, must accelerate from a zero velocity, each time the crack is loaded. Such a hypothetical scenario is depicted in Figure 57.
Figure 57. Hypothetical time dependency of crack growth during cycling, showing a transient crack propagation velocity. For sufficiently short cycle times (i.e. sufficiently high frequency), crack propagation would be quite slow. This would explain the frequency-dependence of our results.

The theory of fracture mechanics of viscoelastic materials might shed some light into this hypothesis. From [23], the crack tip opening displacement (CTOD), \( \delta_c \), for a Dugdale crack tip is given by

\[
\delta_c = \frac{K_{lc}^2}{\sigma_{cr}(t)E(t)}
\]  

(15)

where \( K_{lc} \) is the critical stress intensity factor required to propagate the crack; \( \sigma_{cr}(t) \) is the stress required to cause crazing (crack propagation) at the crack tip; and \( E(t) \) is the time-dependent elastic modulus of the viscoelastic material. The latter can be represented, for many polymers, by a simple power relationship

\[
E(t) = E_1 t^{-n}
\]  

(16)

where \( E_1 \) and \( n \) are material constants. Furthermore, the crazing stress, \( \sigma_{cr}(t) \), is time-dependent, but is related to the time-independent crazing strain, \( \epsilon_{cr} \), by

\[
\sigma_{cr}(t) = E(t)\epsilon_{cr}
\]  

(17)

Substituting (16) and (17) into (15), we obtain
\[ \delta_c = \frac{K_{fc}^2}{\epsilon_c E_1^2} t^{2n} \]  

(18)

Generalizing, we write

\[ \delta_c \propto \frac{K_f^2}{\epsilon_c E_1^2} t^{2n} \]  

(19)

from which we see that, for a given specimen geometry and fixed loading, \( K_f \)—which, in our constant-pressure experiments, is the case—as well as a given set of material properties, \( \epsilon_c \) and \( E_1 \)—which are constant—the crack tip opening, \( \delta_c \), increases with time. If crack propagation does not occur until a critical crack tip opening is reached, then there is a time delay between initial loading and when crack propagation occurs. While more development would be needed to ultimately model the behavior we see, viscoelastic fracture mechanics does, at least, appear to qualitatively corroborate it.

We are currently in the process of repeating these experiments to verify our findings. To this end, we have designed and constructed an automated pressure cycling apparatus for performing the experiments. We will present the results of these tests in future work.

6. CONCLUSIONS

In summary, we have conducted adhesion tests on PV material systems using four variants of the blister test: the thermal blister test (TBT), shaft-loaded blister test (SLBT), v-peel test, and pressurized blister test (PBT). For three out of the four methods—the TBT, v-peel test, and PBT—we have succeeded in demonstrating the applicability of the method to PV systems. For the TBT, we have been able to generate delamination in a manner that likely closely mimics the process in PV modules. We have been able to calculate the strain
energy release rate of the system, based on the measured blister radii. Furthermore, we witnessed a possible relationship between delamination rate and cycling frequency. Additional work still needs to be done to verify this apparent trend, as well as to incorporate viscoelasticity into the fracture mechanics.

For the shaft-loaded blister test, we succeeded in deriving a new set of analytical models that explore the range of applicability of the existing point-load models in the literature, as well as offer analytical solutions to the more real-world scenarios of finite-sized punches. Additional work is planned for the near future to experimentally verify the models. Should they prove to be valid, we plan on publishing our results in a set of full-length journal papers, as well as on applying them to SLBT tests on PV materials, once the practical obstacles are overcome.

For the v-peel test, we have been able to apply the test to over twenty PV material samples, and to derive the interfacial adhesion energy of the weakest interface within each system. The data from these measurements will soon be used in the statistical engine of the PREDICTS2 project, to correlate adhesive strength with other degradation phenomena in PV systems. We plan on applying the test to additional PV modules and coupons, as well as refining the model to account for viscoelasticity and stretching behavior in the fracture mechanics.

Finally, for the pressurized blister test, we have succeeded in conducting cyclic tests on fresh coupon samples, and we have witnessed a possible correlation between cycling rate and frequency—similar to that seen in the thermal blister tests. While a preliminary survey of viscoelastic fracture mechanics offers some corroboration of this correlation, verification of
our results is needed, and we are in the process of conducting additional tests. Should they be confirmed, we will pursue a more rigorous development of the fracture mechanics into a more complete model.

Based on our findings thus far, we believe that, with further refinement of these methods, we will have a set of techniques that can be applied to understanding delamination mechanics in PV systems—a set that can be of benefit to both the PV industry and the academics community, at large. With the need to curb greenhouse gas emissions and develop alternative energy sources more critical than ever, having a body of measurement science to advance the performance of these energy sources and drive up their economic competitiveness is essential. We hope that our work in this area can make a contribution to this critical effort.
7. REFERENCES


8. APPENDIX
FRACTURE MECHANICS OF THE FINITE-PUNCH SHAFT-LOADED BLISTER TEST AT THE BENDING AND STRETCHING LIMITS

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ABSTRACT

We derive an analytical model for the axisymmetric shaft-loaded blister test (SLBT), and study the theoretical behavior of the system when the punch size is varied. We show that, within certain limitations, a point-load model can be applied to the system, to give simple relations between the film-substrate energy of adhesion and the measured variables of applied punch force, blister height, and blister radius.
1. INTRODUCTION

A number of testing methods exist for evaluating thin-film adhesion, including the peel test, double-cantilever beam (DCB) test, single-cantilever beam (SCB) test, and blister test, among others [1]. The peel test, while simple and intuitive, has the disadvantages of large deformation of the unbonded film and a plane stress-to-plane strain transition from the center of the peel to the outer edge of the peel, at the debonding front. The DCB and SCB tests do not suffer from these shortcomings, but—in the case of a thin film only bonded to one surface—require the lamination of an additional stiff backing material in order to form the beam geometry. The blister test—with its axisymmetric debond geometry, low debonding angles, and lack of a need for a backing material—avoids all of these shortcomings.

Two common variations of the blister test exist: the pressurized blister test (PBT)—wherein a working gas is injected into the blister under pressure, driving delamination between the film and substrate—and the shaft-loaded blister test (SLBT)—wherein a shaft is inserted into the hole in the substrate, and driven against the film [2]. The PBT suffers from the limitations of unstable delamination during pressure-controlled experiments, while the more stable volume-controlled approach requires an incompressible fluid and cumbersome equipment setup. The SLBT, on the other hand, is stable for the displacement-controlled test, and is easy to perform using readily-available displacement-controlled compression testing apparatuses.

Most models for the SL BT in the literature represent the contact between the punch and film as a point load. When combined with simplifying assumptions that make the governing equations tractable, this approach yields analytical solutions that are elegant and simple. Malyshev and Salganik [3], for example, first proposed the use of the shaft-loaded blister test,
but their configuration was for the point-load case. Jensen [4] solved numerically the general von Karman equations for the point-loaded shaft-loaded blister test, for both small and large displacements. Many investigators have derived analytical solutions to the von Karman equations—or their simplified derivatives—by restricting the film to small out-of-plane displacement. Wan and Mai [5, 6], Williams [7], and Afferrante, et al [8] derived analytical models for the point-load case under the stretching-limit. Wan [9], and Guo, et al [2] studied the point-load bending-to-stretching transition. Wan and Liao [10] analyzed the case for a spherically-tipped shaft of finite size, but without delamination. Wan, et al. [11] extended this analysis for to several other punch geometries—including the point-load and finite-sized cylindrical punch. However, no one has yet derived useful relations for determining the fracture toughness from the experimentally measured variables for a finite-sized punch—a configuration which is common in experimental work. We have developed a new theoretical model for shaft-loaded blister test, using this configuration, wherein the influence of the finite size of the punch is taken into account.
2. **OVERVIEW**

Figure 1 shows a depiction of our system. A thin film, of thickness, \( h \), is adhered to a rigid substrate with a circular hole in it. The film over the hole is thus free-standing. A circular punch, of radius, \( c \), is brought into contact with the film, then driven with an applied force, \( F \), causing the film to delaminate from the substrate. The shape of the delaminating film is that of a circular blister, with gradually increasing radius, \( a \).

We study two cases: that of a “clamped” SLBT—in which the film is clamped or glued to the top of the punch, and consequently constrained from in-plane or out-of-plane deformation (c.f. Fig. 1a)—and that of a “free” SLBT—in which the film above the punch is unconstrained, and free to undergo in-plane or out-of-plane deformation (c.f. Fig. 1b).
Figure 1. Schematics of clamped and free SLBTs.

(a) Clamped SLBT

(b) Free SLBT
3. THEORY

In our analysis, we assume that the film is linearly elastic, with elastic modulus, $E$, Poisson’s ratio, $\nu$, plane-strain elastic modulus, $E' = \frac{E}{1-\nu^2}$, and flexural rigidity, $D = \frac{E' h^3}{12}$. The equations relating the film profile, $w(r)$, to the material properties, internal membrane stresses, $N_r$ (radial) and $N_t$ (tangential), and internal shear force, $q(r)$, are given by [2]:

$$
-D \left( \frac{d^3 w}{dr^3} + \frac{1}{r} \frac{d^2 w}{dr^2} - \frac{1}{r^2} \frac{dw}{dr} \right) + N_r \frac{dw}{dr} = -q(r)
$$

$$
\frac{d}{dr} (N_r + N_t) + \frac{E h}{2r} \left( \frac{dw}{dr} \right)^2 = 0
$$

(1)

$$
N_t = \frac{d}{dr} (r N_r)
$$

The internal shear force is related to the loading function, $\Psi(r)$, by

$$
\Psi(r) = \frac{1}{r} \frac{d}{dr} [r q(r)]
$$

(2)

Wan, Guo, and Dillard [11] showed that, for both the clamped and free configurations, the film’s elastic response is dominated by bending stresses at small punch displacements, and by stretching stresses at large punch displacements. Henceforth, we define these two limits as the “bending limit” and “stretching limit” of the film behavior. At these two limits, the relationships between the measured parameters, $F$, $w_c$, and $a$, as well as the strain energy release rate (SERR), $G$, take on certain, fairly simple analytical forms. Between the two limits, there is a “bending-to-stretching” transition, wherein these relationships are no longer analytical, or intractably complicated. In this paper, in order to avoid the complexity of modeling the transition, we model only the two limits: bending and stretching.
3.1. Assumptions

Equations (1) are nonlinear and analytically intractable. We employ the following assumptions, to make them analytically workable:

i. Out-of-plane displacements of the film are small \( \left( \frac{dw}{dr} \ll 1 \right) \).

ii. The membrane stresses are equibiaxial \( (N_r = N_t \equiv N) \).

iii. The induced membrane stress, \( N_m \), is uniform across the membrane, and takes on an average value given by

\[
N_m = \frac{Eh}{2a^2(1-\nu^2)} \int_0^a \left( \frac{dw}{dr} \right)^2 r dr
\]

iv. Initial membrane pre-stresses, \( N_0 \), are negligible. Therefore, the total membrane stress, \( N \), is only comprised of the induced membrane stress, \( N_m \):

\[ N = N_m \]

v. Fracture occurs in a purely Mode I (opening mode) manner.

3.2. Bending Limit

We now simplify our governing equations, for the case of the bending limit. In the bending limit, any stretching stresses are negligible (i.e. \( N_r = N_t \approx 0 \)). Applying this, as well as the above assumptions, to our governing equations it can be shown that they reduce to:

\[
-D \left( \frac{d^3w}{dr^3} + \frac{1}{r} \frac{d^2w}{dr^2} - \frac{1}{r^2} \frac{dw}{dr} \right) = -q(r)
\]  \hspace{1cm} (3)

3.3. Stretching Limit
In the stretching limit, membrane stresses \((N_r \text{ and } N_t)\) become dominant. Applying this, as well as the above assumptions, to our governing equations it can be shown that they reduce to:

\[
N \frac{dw}{dr} = -q(r)
\]

(4)

\[
N_t = N_r
\]

3.4. Non-Dimensionalization

It is convenient to express our system in terms of non-dimensional variables. In our analysis, we will use two non-dimensionalizations. We will first non-dimensionalize our system in accordance with the analysis presented in [2], in order to show that our analysis matches the limiting case of the point-loaded blister presented in that work. We will then perform a non-dimensionalization that is more physically relevant to our system, in order to provide results that are more meaningful in the case in which the punch is of finite—rather than infinitesimal—size. For reasons which will become apparent later, we will henceforth refer to these as the “thickness-based” and “punch radius-based” non-dimensionalizations, respectively. In both cases, as we will see, the form of the governing equations is the same, with only some of the boundary conditions changing.

3.4.1. Thickness-Based Non-Dimensionalization

We use the below variable definitions. Figure 2 shows our system represented in these non-dimensional variables.
\[ \xi = \frac{r}{h} \quad W = \frac{w}{h} \quad \Gamma = \left( \frac{h^2}{2\pi D} \right) \quad G = \left( \frac{6}{\pi E'h} \right) G \]

\[ \eta = \frac{c}{h} \quad \phi = \left( \frac{h}{2\pi D} \right) F = \left( \frac{6}{\pi E'h^2} \right) F \quad Q(\xi) = \left( \frac{h^2}{D} \right) q(r) = \left( \frac{12}{E'h} \right) q(r) \]

\[ \zeta = \frac{a}{h} \quad \Upsilon = \left( \frac{1}{2\pi D} \right) U = \left( \frac{6}{\pi E'h^3} \right) U \quad \beta^2 = \left( \frac{12}{E'h} \right) N \]

Table 1. Non-dimensional variables for the “thickness-based” non-dimensionalization (ND). Observe that we non-dimensionalize the radial dimensions, \( r, c, \) and \( a \), with respect to the thickness, \( h \), of the film. In the “punch radius-based” non-dimensionalization later, we will non-dimensionalize these dimensions with respect to the punch radius, \( c \)—a more physically meaningful approach.
Figure 2. Clamped and free SLBT configurations in terms of the “thickness-based” non-dimensional variables.
3.5. Bending Limit

After performing this non-dimensionalization, our governing equation for the bending limit becomes

\[
\frac{d^3 W}{d\xi^3} + \frac{1}{\xi} \frac{d^2 W}{d\xi^2} - \frac{1}{\xi^2} \frac{dW}{d\xi} = Q(\xi)
\]  

(5)

As will become apparent later, the solution of this equation is facilitated by expressing it in terms of the slope, \( \Theta = \frac{dW}{d\xi} \), and multiplying by \( \xi^2 \):

\[
\xi^2 \frac{d^2 \Theta}{d\xi^2} + \xi \frac{d\Theta}{d\xi} - \Theta = \xi^2 Q(\xi)
\]

3.6. Stretching Limit

After non-dimensionalization, our stretching-limit equations become

\[
\beta^2 \frac{dW}{d\xi} = Q(\xi)
\]

(6)

\[
\beta^2 = \frac{6}{\xi^2} \int_{0}^{\xi} \left( \frac{dW}{d\xi} \right)^2 \xi \, d\xi
\]

3.7. Strain Energy Release Rate

The strain energy release rate (SERR), \( G \), is generally defined as

\[
G = \frac{d}{dA} (W - U_e)
\]

where \( dA \) is an infinitesimal increase in area of the crack interfaces, \( W \) is the external work put into the system, and \( U_e \) is the elastic energy stored in the film. For a fixed-load configuration, this becomes (non-dimensionalized)
\[
\Gamma = -\frac{d}{d(\pi \xi^2)} \left[ \left( \int_0^{W_0} \phi \, dW_\eta - \phi \, W_\eta \right)_{fixed \, \xi \, fixed \, \phi} \right]
\] (7)

while for a fixed-displacement configuration, it becomes

\[
\Gamma = -\frac{d}{d(\pi \xi^2)} \left[ \left( \int_0^{W_0} \phi \, dW_\eta \right)_{fixed \, \xi \, fixed \, \phi} \right] \]

(8)

For the clamped SLBT, it is more meaningful to express the blister area as \( A = \pi (a^2 - c^2) \) than as \( A = \pi a^2 \), the latter of which is more meaningful for the free SLBT. It can be shown that this alternative area definition does not alter the forms of (7) and (8), namely,

\[
\frac{d}{d(\pi a^2)} = \frac{d}{d(\pi (a^2 - c^2))} = \frac{d}{d(\pi (\xi^2 - \eta^2))} = \frac{d}{d(\pi \xi^2)}
\]

3.8. Debonding Criterion

Debonding between the film and substrate occurs when the SERR, \( \Gamma \), exceeds the critical SERR, \( \Gamma_C \), of the interface (i.e. \( \Gamma \geq \Gamma_C \)). At equilibrium

\[
\Gamma = \Gamma_C
\]

(9)

and the punch applied force, \( \phi \), punch displacement, \( W_\eta \), and blister radius, \( \zeta \), exist at defined equilibrium values.
4. ANALYSIS – THICKNESS-BASED NON-DIMENSIONALIZATION

Having derived our general relations, we now analyze our two testing configurations.

4.1. Clamped SLBT

For the clamped SLBT, the film has a loading function defined by a line force at \( r = c \):

\[
\psi(r) = \frac{F}{2\pi c} \delta(r - c), \quad (c \leq r \leq a)
\]

where \( \delta(r) \) is the Dirac delta function. Substituting this into (2), and integrating to solve for the shear force function, \( q(r) \), we obtain:

\[
q(r) = \frac{F}{2\pi r} \quad (c \leq r \leq a)
\]  

(10)

4.1.1. Bending Limit

Substituting (10) into (3), we obtain for our governing equation:

\[
-D \left( \frac{d^3 w}{dr^3} + \frac{1}{r} \frac{d^2 w}{dr^2} - \frac{1}{r^2} \frac{dw}{dr} \right) = -\frac{F}{2\pi r} \quad (c \leq r \leq a)
\]

or, in non-dimensional form (after substitution of \( \Theta = \frac{dw}{d\xi} \))

\[
\frac{d^3 W}{d\xi^3} + \frac{1}{\xi} \frac{d^2 W}{d\xi^2} - \frac{1}{\xi^2} \frac{dW}{d\xi} = \frac{\phi}{\xi} \quad (\eta \leq \xi \leq \zeta)
\]  

(11)

By definition of our coordinate axes, one of our boundary conditions is \( W(\xi)|_{\xi=\zeta} = 0 \).

Furthermore, in order for the bending stresses in the film to be non-singular, the slope must be continuous, over the entire film, giving the remaining two boundary conditions: \( \frac{dW}{d\xi} \bigg|_{\xi=\eta} = 0 \) and \( \frac{dW}{d\xi} \bigg|_{\xi=\zeta} = 0 \). Expressing (11) in terms of slope, we obtain the following
\[
\frac{\xi^2 d^2 \Theta}{d\xi^2} + \xi \frac{d\Theta}{d\xi} - \Theta = \phi \xi
\]  

(12)

the homogeneous form of which is the Cauchy-Euler equation. By solving (12), differentiating according to \( \Theta = \frac{dW}{d\xi} \), then imposing the above boundary conditions, we obtain the final profile relation

\[
W(\xi) = \frac{\phi}{8} [A(\eta, \zeta) \xi^2 + B(\eta, \zeta) \ln \xi + 2\xi^2 \ln \xi + X(\eta, \zeta)]
\]

where

\[
A = -\frac{(\zeta^2 - \eta^2) + 2(\zeta^2 \ln \zeta - \eta^2 \ln \eta)}{\zeta^2 - \eta^2}
\]

(13)

\[
B = \frac{4\xi^2 \eta^2 \ln \left(\frac{\xi}{\eta}\right)}{\zeta^2 - \eta^2}
\]

\[
X = \frac{\zeta^4 + \zeta^2 \eta^2 \left[2 \ln \left(\frac{\xi}{\eta}\right) - 4 \ln \xi \ln \left(\frac{\xi}{\eta}\right) - 1\right]}{\zeta^2 - \eta^2}
\]

We define the non-dimensional punch displacement as \( W_\eta \equiv W(\xi = \eta) \) and the non-dimensional central displacement of the film as \( W_0 \equiv W(\xi = 0) \). In the clamped SLBT, the two are equal. Solving the resulting equation for \( \phi \), we obtain for our constitutive relation

\[
\phi = \left\{ \frac{8(\zeta^2 - \eta^2)}{\left((\zeta^2 - \eta^2)^2 - 4\xi^2 \eta^2 [(\ln \xi)^2 + 2 \ln \xi \ln \eta - (\ln \eta)^2]\right)} \right\} W_\eta
\]

(14)

Here, we see a linear relationship between \( \phi \) and \( W_\eta \), which matches the findings from [2].
\( \phi \) vs. \( \zeta \)

Substituting (14) into (7) and (9), then solving for \( \phi \), we obtain the relationship between the applied force, \( \phi \), the adhesion energy at the film-substrate interface, \( \Gamma_C \), and the punch and blister radii, \( \eta \) and \( \zeta \)—during delamination of the film from the substrate

\[
\phi = 4 \pi^{1/2} \left\{ \frac{(\zeta^2 - \eta^2)^2}{[f(\zeta, \eta)]^2 - 4\eta^2 \ln \eta [f(\zeta, \eta)] - 4\eta^4 (\ln \eta)^2} \right\}^{1/2} \Gamma_C^{1/2}
\]

(15)

where

\[
f(\zeta, \eta) = (\zeta^2 - \eta^2) - 2\eta^2 \ln \zeta
\]

For a point load (\( \eta \rightarrow 0 \)), the limit of this expression is

\[
\phi = 4 \pi^{1/2} \Gamma_C^{1/2}
\]

(16)

The authors in [2] also found this constant-force relationship for the point-loaded case. What it implies is that, once the load reaches the critical constant value, delamination will run away, catastrophically.

\( W_\eta \) vs. \( \zeta \)

For a displacement-controlled setup, substituting (14) into (8) and (9), and solving for \( W_\eta \), we obtain the relationship between the punch displacement, \( W_\eta \), the interfacial adhesion energy, \( \Gamma_C \), and the punch and blister radii, \( \eta \) and \( \zeta \)—during delamination of the film

\[
W_\eta = \frac{\pi^{1/2}}{2} \left\{ \frac{[f(\zeta, \eta)]^2 - 4\eta^2 \ln \eta (\zeta^2 - 2\eta^2 \ln \zeta - \eta^2) - 4\eta^4 (\ln \eta)^2}{\{(\zeta^2 - \eta^2)^2 - 4\zeta^2 \eta^2 [(\ln \zeta)^2 + 2\ln \zeta \ln \eta - (\ln \eta)^2]\}^{1/2}} \right\}^{1/2} \Gamma_C^{1/2}
\]

(17)

where \( f(\zeta, \eta) \) is given in (16).
It can be shown that this displacement-controlled result is equivalent to the load-controlled result by simple substitution of (14) into (17). For the point-load limit (\(\eta \to 0\)), this expression reduces to

\[
W_\eta = \frac{\pi^{1/2}}{2} \zeta^2 \Gamma_C^{1/2}
\]  

(18)

This quadratic relationship also matches the point-load bending limit found in [2].

\(\phi\) vs. \(W_\eta\)

It is well-known that the fixed-load and fixed-displacement results are equivalent, for quasi-static delamination. Therefore, for a given, fixed pair of values for punch size, \(\eta\), and interfacial adhesion energy, \(\Gamma_C\), (15) and (17) give the full description of the point-load behavior of the system, when treated as parametric functions of \(\zeta\). Since \(\phi\) is a constant for a given interfacial adhesion energy, \(\Gamma_C\), a non-dimensionalized force-displacement curve, obtained from an experiment, would theoretically exhibit a constant force, as shown in Figure 3.
Figure 3. Applied force, $\phi$, vs. punch displacement, $W_0$, for interfacial adhesion energy $\Gamma_C = 0.01$, showing the theoretically-predicted constant-force behavior. The force is dependent only upon $\Gamma_C$, and the relationship is given by (16).

$$\Gamma \frac{\phi W_0}{\pi (\zeta^2 - \eta^2)}$$

A normalized SERR can be expressed as, $\frac{g}{F_w e/A}$, where $A$ is the total area of the blister, here given by $\pi (a^2 - c^2)$. The non-dimensional result is obtained by combination of (15), and (17):
\[
\frac{\Gamma_c}{\phi W_\eta / \pi (\zeta^2 - \eta^2)} = \frac{1}{2} \frac{\left[f(\zeta, \eta)\right]^2 - 4 \eta^2 \ln \eta \left[f(\zeta, \eta)\right] - 4 \eta^2 \left[\ln \eta\right]^2}{(\zeta^2 - \eta^2)^2 - 4 \zeta^2 \eta^2 \left[(\ln \zeta)^2 + 2 \ln \zeta \ln \eta - (\ln \eta)^2\right]}
\]

where \(f(\zeta, \eta)\) is given in (16).

The point-load limit \((\eta \to 0)\) of this expression can be shown to approach the value of \(1/2\), which matches the results in [2]. This gives the elegant and simple relationship between the interfacial adhesion energy and the measured variables of

\[
G = \frac{1}{2 \pi (a^2 - c^2)} \frac{F w_c}{\phi W_\eta} 
\]

for the point-load case.

4.1.2. Stretching Limit

Substituting (10) into our governing equations for the stretching limit, (4), we obtain, after non-dimensionalization:

\[
\beta^2 \frac{dW}{d\xi} = -\frac{\phi}{\zeta} \quad (\eta \leq \xi \leq \zeta) 
\]

(20)

\[
\beta^2 = \frac{6}{\zeta^2 - \eta^2} \int_\eta^\zeta \left(\frac{dW}{d\xi}\right)^2 \xi d\xi 
\]

(21)

For the stretching limit, by noting that the membrane stress, \(\beta\), and applied force, \(\phi\), are independent of \(\xi\), we can directly integrate (20) to yield the membrane profile. Furthermore, we can solve (20) for \(\frac{dW}{d\xi}\) and substitute into (21) to obtain the average membrane stress over the
film. Substituting the result of the latter into the result of the former, and applying our only boundary condition, \( W|_{\xi=\zeta} = 0 \), we obtain for the final profile relation

\[
W(\xi) = \left[ \frac{(\zeta^2 - \eta^2)\phi}{6 \ln \left(\frac{\zeta}{\eta}\right)} \right]^{1/3} \ln \left(\frac{\zeta}{\xi}\right) \quad (\eta \leq \xi \leq \zeta) \tag{22}
\]

The punch displacement, \( W_\eta \), is given by \( W_\eta \equiv W(\xi = \eta) = W_0 \), which can be solved for \( \phi \) to yield the constitutive relation

\[
\phi = \frac{6}{(\zeta^2 - \eta^2) \left[ \ln \left(\frac{\zeta}{\eta}\right) \right]^2} W_\eta^3 \tag{23}
\]

This cubic relationship between \( \phi \) and \( W_\eta \) is the expected relationship for the stretching limit, based on [2] and others.

\( \phi \) vs. \( \zeta \)

Substituting (23) into (7) and (9), and solving for \( \phi \), we obtain the relationship between \( \phi \), \( \Gamma_c \), \( \eta \), and \( \zeta \), during debonding:

\[
\phi = 4^{3/4} \times 6^{1/4} \pi^{3/4} \left\{ \frac{\zeta^6 (\zeta^2 - \eta^2)^2 \ln \left(\frac{\zeta}{\eta}\right)}{\left[(\zeta^2 - \eta^2) + \zeta^2 \ln \left(\frac{\zeta}{\eta}\right)\right]^3} \right\}^{1/4} \Gamma_c^{3/4} \tag{24}
\]

The limit of this expression for a point load (\( \eta \to 0 \)) is \( \phi \to 0 \). However, a log-log plot of \( \phi \) vs. \( \zeta \) for small \( \eta \) reveals that the relationship between \( \phi \) and \( \zeta \) is always linear (c.f. Fig. 4). This matches the result found in [2].
Figure 4. Log-log plot of $\phi$ vs. $\zeta$ for $\eta=10^{-6}$ and $\Gamma_{C}=0.01$.

The experimental meaning of this linear relationship is that, for a load-controlled experiment where the membrane response is stretching-dominated, the delamination will not proceed catastrophically after a critical force is reached, as in the bending-dominated case. For a finite-sized punch (i.e. $\eta > 0$), the limit of (24) as $\eta \rightarrow \zeta$ is

$$\phi = \frac{4 \times 2^{1/4} \pi^{3/4}}{3^{1/2}} \eta \Gamma_{C}^{3/4}$$

Thus, for a finite-sized punch, there is an initial, non-zero force required to initiate delamination, which depends upon the punch size and the interfacial adhesion energy.
\( \theta \) vs. \( \zeta \)

Since continuity of slope is not a requirement for the stretching limit, it is informative to investigate the slope of the film at the edge of the punch and at the outer edge of the blister, as they evolve with the system variables. From differentiation (22), the slope of the film is given by

\[
\frac{dW}{d\xi} = -\left[ \frac{\left(\zeta^2 - \eta^2\right)\phi}{6 \ln\left(\frac{\zeta}{\eta}\right)} \right]^{1/3} \frac{1}{\xi} \quad (\eta \leq \xi \leq \zeta)
\]

By noting that contact angle, \( \theta \), is related to slope by \( \theta(\xi) = \tan^{-1}\left(\frac{dW}{d\xi}\right) \), we define the contact angle at the edge of the punch (\( \xi = \eta \)) as \( \theta_\eta \)

\[
\theta_\eta \equiv \theta(\xi = \eta) = \tan^{-1}\left( -\left[ \frac{\left(\zeta^2 - \eta^2\right)\phi}{6 \ln\left(\frac{\zeta}{\eta}\right)} \right]^{1/3} \frac{1}{\eta} \right)
\]

and the contact angle at the outer edge of the blister (\( \xi = \zeta \)) as \( \theta_\zeta \)

\[
\theta_\zeta \equiv \theta(\xi = \zeta) = \tan^{-1}\left( -\left[ \frac{\left(\zeta^2 - \eta^2\right)\phi}{6 \ln\left(\frac{\zeta}{\eta}\right)} \right]^{1/3} \frac{1}{\zeta} \right)
\]

Figure 5 shows a plot of contact angle, \( \theta \), vs. punch radius, \( \eta \), for the contact angles at the punch edge (\( \xi = \eta \), blue) and at the blister outer edge (\( \xi = \zeta \), red). Here, we fix \( \zeta \) at a finite blister radius of 1, while we vary \( \eta \) between \( \zeta \) and 0 (the latter being the point-load limit).

When \( \eta = \zeta \), the film annulus is infinitesimally wide, and the angles consequently converge to
the same value. Interestingly, rather than converging to one or the other of the two intuitive limits, $0$ or $-\frac{\pi}{2}$, they converge to an intermediate value,

$$\theta(\eta \to \zeta) = -\tan^{-1}\left(\frac{2^{3/4}\pi^{1/4}}{3^{1/2}}\Gamma_{C}^{1/4}\right)$$

(28)

which is obtained by simply taking the limit of (26) or (27) as $\eta \to \zeta$ (or vice versa). The intermediacy of this limit is directly related to the fact that, for a finite-size punch (i.e. $\eta > 0$), the force is non-zero at the onset of delamination by (25).

In contrast to the mutually, intermediate value for $\eta \to \zeta$, when $\eta \to 0$ (the point-load limit), the contact angle at the punch edge and blister outer edge diverge to the separate limits of $-\frac{\pi}{2}$ and $0$, respectively. Thus, for the point-load limit, the contact angle is completely vertical at the location of the point load, and completely horizontal at the blister outer edge. Of course, these are theoretical limits; in an actual loading situation, the load will never be a point-load, so the contact angles will be intermediates of these two values.
Figure 5. Plot of $\theta$ vs. $\eta$ for $\Gamma_c = 0.01$ and $\zeta = 1$, of the film at the edge of the punch (blue) and at the outer edge of the blister (red). (Note that $0 \leq \eta \leq \zeta$, necessarily.)

$W_\eta$ vs. $\zeta$

Substituting (23) into (8) and (9), and solving for $W_\eta$, we obtain the relationship between $W_\eta$, $\Gamma_c$, $\eta$, and $\zeta$, during debonding:

$$W_\eta = \frac{2^{1/4} \pi^{1/4}}{3^{1/4}} \left( \frac{\zeta^2 (\zeta^2 - \eta^2)^2 \ln \left( \frac{\zeta}{\eta} \right)}{(\zeta^2 - \eta^2) + \zeta^2 \ln \left( \frac{\zeta}{\eta} \right)} \right)^{1/4} \Gamma_c^{1/4}$$

(29)

The limit of this as ($\eta \to 0$) is $W_\eta = \infty$. However, plotting log-log for small $\eta$ shows that the relationship between $W_\eta$ and $\zeta$ is linear, which matches the findings in [2] (c.f Fig. 6).
Figure 6. Log-log plot of $W_0$ vs. $\zeta$ for $\Gamma_C=0.01$ and $\eta=10^{-6}$.

$\phi$ vs. $W_\eta$

The relationship between $\phi$ and $W_\eta$ for the point-load case can be determined by plotting (24) and (29) parametrically as functions of $\zeta$, for a given pair of values for $\eta$ and $\Gamma_C$. Figure 7 shows such a plot, in log-log scale, for a small value of $\eta$. The relationship is linear, which matches the results of [2].
Finally, combination of (24) and (29) yields the normalized SERR,

\[
\frac{\Gamma_c}{\phi W_0/\pi (\zeta^2 - \eta^2)} = \frac{1}{4} \left[ 1 + \frac{\zeta^2 - \eta^2}{\zeta^2 \ln \left( \frac{\zeta}{\eta} \right)} \right]
\]  

(30)
For the point-load limit \((\eta \to 0)\), this approaches \(\frac{1}{4}\), which is the same limit as found in [2].

### 4.2. Free SLBT

In this system, the annulus outside of the punch (the “outer annulus”; \(c \leq r \leq a\)) has a loading function defined by a line force at \(r=c\), while the “inner circle” of film atop the punch \((0 \leq r \leq c)\) has a loading function of null. These give the piecewise-defined, non-dimensional shear function:

\[
Q(\xi) = \begin{cases} 
0 & (\xi < \eta) \\
\phi \frac{\xi}{\zeta} & (\eta \leq \xi \leq \zeta) 
\end{cases}
\]  

(31)

#### 4.2.1. Bending Limit

Substituting (31) into (3), and expressing in terms of the membrane slope, \(\Theta = \frac{dW}{d\xi}\), we obtain the piecewise-defined governing equations

\[
\begin{cases}
\xi^2 \frac{d^2\Theta}{d\xi^2} + \xi \frac{d\Theta}{d\xi} - \Theta = 0 & (\xi < \eta) \\
\xi^2 \frac{d^2\Theta}{d\xi^2} + \xi \frac{d\Theta}{d\xi} - \Theta = \phi \xi & (\eta \leq \xi \leq \zeta)
\end{cases}
\]

(32)

This is just a pair of Cauchy-Euler-type expressions as in the clamped SLBT bending case.

Solving them yields separate expressions for the “inner” portion of film atop the punch \((\xi < \eta)\)—which we will denote as \(\Theta_1(\xi)\) and \(W_I(\xi)\)—and the “outer” portion of film—the film annulus \((\eta \leq \xi \leq \zeta)\)—which we will denote as \(\Theta_0(\xi)\) and \(W_O(\xi)\).
By definition of our coordinate axes, \( W_0(\xi) \mid_{\xi=\zeta} = 0 \). Furthermore, in order for the bending stresses in the film to be non-singular, the slope must be continuous, over the entire film. Therefore, we have the addition boundary conditions:

\[
\frac{dW_I}{d\xi} \bigg|_{\xi=0} = 0, \quad \frac{dW_O}{d\xi} \bigg|_{\xi=\zeta} = 0.
\]

\[
\frac{dW_I}{d\xi} \bigg|_{\xi=\eta} = \frac{dW_O}{d\xi} \bigg|_{\xi=\eta}, \text{ and } \frac{d^2W_I}{d\xi^2} \bigg|_{\xi=\eta} = \frac{d^2W_O}{d\xi^2} \bigg|_{\xi=\eta}.
\]

Finally, where the two profile functions meet,

\[
W_I \mid_{\xi=\eta} = W_O \mid_{\xi=\eta}.
\]

Solving (32) for \( \Theta \), differentiating to yield the membrane profile, \( W \), and imposing our boundary conditions, we obtain the final set of profile relations

\[
\begin{align*}
W_I(\xi) &= \phi \left[ \Delta(\eta, \zeta) \xi^2 + E(\eta, \zeta) \right] \quad (\xi < \eta) \\
W_O(\xi) &= \phi \left[ \Phi(\eta, \zeta) \xi^2 + 2\eta^2 \ln \xi + 2\xi^2 \ln \xi + H(\eta, \zeta) \right] \quad (\eta \leq \xi \leq \zeta)
\end{align*}
\]

where

\[
\Delta = \left( \frac{(\xi^2 - \eta^2)}{\xi^2} \right) - 2 \xi^2 \ln \left( \frac{\xi}{\eta} \right)
\]

\[
E = \left( \frac{(\xi^2 - \eta^2)}{\xi^2} \right) - 2 \eta^2 \ln \left( \frac{\xi}{\eta} \right)
\]

\[
\Phi = -\left( \frac{(\xi^2 + \eta^2)}{\xi^2} \right) + 2 \xi^2 \ln \xi
\]

\[
H = \left( \frac{(\xi^2 + \eta^2)}{\xi^2} \right) - 2 \eta^2 \ln \zeta
\]

For the free SLBT, the punch displacement, \( W_\eta \), and central displacement, \( W_0 \), are, in general, not equal. The constitutive relation in terms of the punch displacement, \( W_\eta \), is given by,
\[ \phi = \left\{ \frac{8 \zeta^2}{\left( \zeta^4 - \eta^4 \right) - 4 \zeta^2 \eta^2 \ln \left( \frac{\zeta}{\eta} \right)} \right\} W_\eta \]  

while the constitutive relation in terms of the central displacement, \( W_0 \), is given by

\[ \phi = \left\{ \frac{8}{\left( \zeta^2 - \eta^2 \right) - 2 \eta^2 \ln \left( \frac{\zeta}{\eta} \right)} \right\} W_0 \]  

This is the expected linear relationship between \( \phi \) and \( W_0 \) found in [2] and others.

\( \phi \) vs. \( \zeta \)

Substituting (34) into (7) and (9), and solving for \( \phi \), we obtain the relationship between \( \phi \), \( \Gamma_c \), \( \eta \), and \( \zeta \), for debonding:

\[ \phi = 4 \pi^{1/2} \frac{\zeta^2}{\zeta^2 - \eta^2} \Gamma_c^{1/2} \]  

The limit of this expression for a point load (\( \eta \to 0 \)) is (16). This identicality is to be expected, as there is no difference in geometry of the film, once the punch reduces to a point. We again see that \( \phi = \text{const} \) for the point-load case.

\( W_\eta \) vs. \( \zeta \)

Substituting (34) into (8) and (9), and solving for \( W_\eta \), we obtain the relationship between \( W_\eta \), \( \Gamma_c \), \( \eta \), and \( \zeta \), for debonding:

\[ W_\eta = \frac{\pi^{1/2} \left( \zeta^4 - \eta^4 \right) - 4 \zeta^2 \eta^2 \ln \left( \frac{\zeta}{\eta} \right)}{2 \zeta^2 - \eta^2} \Gamma_c^{1/2} \]  

(37)
The limit of this expression for a point load ($\eta \to 0$) is

$$W_\eta = \frac{\pi^{1/2}}{2} \xi^2 \Gamma_C^{1/2}$$

(38)

This result matches the quadratic relationship found in [2].

$\phi$ vs. $W_\eta$

Since $\phi$ is a constant for a given interfacial adhesion energy, $\Gamma_C$, a non-dimensionalized force-displacement curve would also exhibit a constant force.

$$\Gamma \frac{\phi W_0 / \pi (\xi^2 - \eta^2)}{\phi W_\eta / \pi \xi^2}$$

Finally, combination of (36) and (37) yields the normalized SERR,

$$\Gamma_C \frac{\phi W_\eta / \pi \xi^2}{\phi W_0 / \pi (\xi^2 - \eta^2)} = 1 \quad \frac{(\xi^2 - \eta^2)^2}{\frac{1}{2} \left( \xi^4 - \eta^4 \right) - 4 \xi^2 \eta^2 \ln \left( \frac{\xi}{\eta} \right)}$$

For the point-load limit ($\eta \to 0$), this approaches $\frac{1}{2}$, which is the same limit as found in [2].

4.2.2. Stretching Limit

Substituting (31) into (4), we obtain the piecewise-defined governing equations

$$\begin{cases} 
\beta^2 \frac{dW}{d\xi} = 0 & \text{if } (\xi < \eta) \\
\beta^2 \frac{dW}{d\xi} = -\frac{\phi}{\xi} & \text{if } (\eta \leq \xi \leq \zeta) 
\end{cases}$$

(39)

and
\[
\beta^2 = \frac{6}{\zeta^2} \int_0^\zeta \left( \frac{dW}{d\xi} \right)^2 \xi d\xi \quad (0 \leq \xi \leq \zeta)
\]  

(40)

As in the bending limit, the solution of these equations yields a pair of separate expressions, \(W_I(\xi)\) and \(W_O(\xi)\), for the “inner” and “outer” portions of the film, respectively.

For our boundary conditions, in addition to \(W_O|_{\xi=\zeta} = 0\), we have \(W_I|_{\xi=\eta} = W_O|_{\xi=\eta}\). Solving (39) and (40) in the same manner as (20) and (21) for the clamped SLBT, we obtain, for our final profile relations,

\[
\begin{aligned}
W_I &= \left\{ \frac{\zeta^2 \phi \left[ \ln \left( \frac{\zeta}{\eta} \right) \right]^2}{6} \right\}^{1/3} \quad (\xi < \eta) \\
W_O(\xi) &= \left[ \frac{\zeta^2 \phi}{6 \ln \left( \frac{\zeta}{\eta} \right)} \right]^{1/3} \ln \left( \frac{\zeta}{\xi} \right) \quad (\eta \leq \xi \leq \zeta)
\end{aligned}
\]  

(41)

Note that \(W_I\) does not depend upon \(\xi\), but, instead, is just a constant for a given \(\zeta, \eta,\) and \(\phi\). As such, it simply represents the film lying flat atop the punch, which is the expected profile of a film with no bending rigidity under the influence of no out-of-plane forces.

The punch displacement, \(W_\eta\), is given by \(W_\eta \equiv W_O(\eta) = W_I = W_0\), which can be solved for \(\phi\) to yield the constitutive relation

\[
\phi = \left\{ \frac{6}{\zeta^2 \left[ \ln \left( \frac{\zeta}{\eta} \right) \right]^2} \right\} W_\eta^3
\]  

(42)

This cubic relationship between \(\phi\) and \(W_\eta\) is the expected relationship for the stretching limit, based on [2] and others.
\( \phi \) vs. \( \zeta \)

Substituting (42) into (7) and (9), and solving for \( \phi \), we obtain the relationship between \( \phi \), \( \Gamma_c \), \( \eta \), and \( \zeta \), during debonding:

\[
\phi = 4^{3/4} \times 6^{1/4} \pi^{3/4} \left( \frac{\zeta^4 \ln \left( \frac{\zeta}{\eta} \right)}{\left[ 1 + \ln \left( \frac{\zeta}{\eta} \right) \right]^3} \right)^{1/4} \Gamma_c^{3/4}
\]  

(43)

The limit of this as \( \eta \to 0 \) is \( \phi \to 0 \), but we can observe a relationship if we plot log-log for small \( \eta \) (see Figure 8). Here, we see that \( \phi \) is linearly proportional to \( \zeta \), which is the result found in [2] for the point-load limit.

Figure 8. Log-log plot of \( \phi \) vs. \( \zeta \) for \( \eta = 10^{-6} \) and \( \Gamma_c = 0.01 \).
\[ \theta \text{ vs. } \zeta \]

Since we are only interested in the contact angles at the edge of the punch ($\xi = \eta$) and the outer edge of the blister ($\xi = \zeta$), we can take the slope of just the “outer” portion of the film, $W_o(\xi)$. This is given by

\[
\frac{dW_o}{d\xi} = -\left[ \frac{\xi^2 \phi}{6 \ln \left( \frac{\xi}{\eta} \right)} \right]^{1/3} \frac{1}{\xi} \quad (\eta \leq \xi \leq \zeta)
\]

From this, the contact angle at the edge of the punch, $\theta_\eta$, is

\[
\theta_\eta \equiv \theta(\xi = \eta) = \tan^{-1} \left( -\left[ \frac{\xi^2 \phi}{6 \ln \left( \frac{\xi}{\eta} \right)} \right]^{1/3} \frac{1}{\eta} \right)
\]

and the contact angle at the outer edge of the blister, $\theta_\zeta$, is

\[
\theta_\zeta \equiv \theta(\xi = \zeta) = \tan^{-1} \left( -\left[ \frac{\xi^2 \phi}{6 \ln \left( \frac{\xi}{\eta} \right)} \right]^{1/3} \frac{1}{\zeta} \right)
\]

Figure 9 shows a plot of contact angle, $\theta$, vs. punch radius, $\eta$, for the contact angles at punch edge ($\xi = \eta$, blue) and at the blister outer edge ($\xi = \zeta$, red). Again, here, we fix $\zeta$ at a finite blister radius of 1, while we vary the $\eta$ from $\zeta$ to 0 (the latter being the point-load limit). When $\eta = \zeta$, the film annulus is infinitesimally wide, and the angles consequently converge to the same value of $-\frac{\pi}{2}$. Thus, for a finite-sized punch, the film annulus is vertical when delamination initiates. We believe that this is due to the fact that, in our model of the free
SLBT, the film atop the punch is allowed to stretch. This appears also to be in keeping with the fact that the applied force is 0 when delamination initiates (c.f. Fig. 8).

For the case of the point-load limit (i.e. when \( \eta \rightarrow 0 \)), the contact angle at the punch edge and blister outer edge diverge to the separate limits of \(-\frac{\pi}{2}\) and 0, respectively—just as in the case of the clamped SLBT.

![Diagram](image)

Figure 9. Plot of \( \theta \) vs. \( \eta \) for \( \Gamma_C=0.01 \) and \( \zeta=1 \), of the film at the edge of the punch (blue) and at the outer edge of the blister (red).

\[ W_{\eta} \text{ vs. } \zeta \]

Substituting (42) into (8) and (9), and solving for \( W_{\eta} \), we obtain the relationship between \( W_{\eta} \), \( \Gamma_C \), \( \eta \), and \( \zeta \), during debonding:
\[ W_{\eta} = \frac{2^{1/4} \pi^{1/4}}{3^{1/4}} \left( \frac{\zeta^4 \left[ \ln \left( \frac{\zeta}{\eta} \right) \right]^3}{1 + \ln \left( \frac{\zeta}{\eta} \right)} \right)^{1/4} \Gamma_{C}^{1/4} \]  

(46)

The limit of this as \( \eta \to 0 \) is \( \infty \), but we can observe a relationship if we plot log-log for small \( \eta \) (see Figure 10). Here, we see that \( W_{\eta} \) is linearly proportional to \( \zeta \), which is the result found in [2] for the point-load limit.

![Log-log plot of \( W_0 \) vs. \( \zeta \) for \( \eta=10^{-6} \) and \( \Gamma_C=0.01 \).](image)

Figure 10. Log-log plot of \( W_0 \) vs. \( \zeta \) for \( \eta=10^{-6} \) and \( \Gamma_C=0.01 \).

**\( \phi \) vs. \( W_{\eta} \)**

The relationship between \( \phi \) and \( W_{\eta} \) can be obtained by plotting (43) and (46) parametrically as functions of \( \zeta \), for \( \Gamma_C = 0.01 \) and a near-zero value of \( \eta \) (see Fig. 11). Like
the case for the clamped SLBT (c.f. Fig. 7), the relationship is linear, which matches the results of [2].

![Graph](image)

Figure 11. Plot of $\phi$ vs. $W_0$ for $\Gamma_c=0.01$ and $\eta=10^{-6}$.

$$\Gamma \frac{\phi W_0}{\pi (\zeta^2 - \eta^2)}$$

Finally, combination of (43) and (46) yields the normalized SERR,
\[
\frac{\Gamma_C}{\phi W_\eta / \pi \zeta^2} = \frac{1}{4} \left[ 1 + \frac{1}{\ln \left( \frac{\zeta}{\eta} \right)} \right]
\]

(47)

which, for the point-load limit \((\eta \to 0)\), approaches \(\frac{1}{4}\)—the same limit as found in [2].

**4.3. Summary**

We have, thus far, shown that our current model matches that of [2] when reduced to a point load. We now proceed to non-dimensionalizing our system in a more physically meaningful manner.
5. ANALYSIS – PUNCH RADIUS-BASED NON-DIMENSIONALIZATION

In the present system, it is more physically meaningful to express the non-dimensionalized radial variables, $\xi$, $\eta$, and $\zeta$, in terms of the punch radius, $c$, rather than the film thickness, $h$. Having defined $\zeta$ in this way, we then formulate our other non-dimensional variables such that the form of our governing equations is the same as before. Doing this, we end up with the following non-dimensional variable definitions:

<table>
<thead>
<tr>
<th>$\xi = \frac{r}{c}$</th>
<th>$W = \frac{w}{h}$</th>
<th>$\Gamma = \left(\frac{c^4}{2\pi Dh^2}\right) \frac{G}{\left(\frac{6c^4}{\pi E'h^5}\right)G}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta = \frac{c}{c} = 1$</td>
<td>$\phi = \left(\frac{c^2}{2\pi Dh}\right) F = \left(\frac{6c^2}{\pi E'h^4}\right) F$</td>
<td>$Q(\xi) = \left(\frac{c^3}{Dh}\right) q(r) = \left(\frac{12c^3}{E'h^4}\right) q(r)$</td>
</tr>
<tr>
<td>$\zeta = \frac{a}{c}$</td>
<td>$Y = \left(\frac{c^2}{2\pi Dh^2}\right) U = \left(\frac{6c^2}{\pi E'h^5}\right) U$</td>
<td>$\beta^2 = \left(\frac{c^2}{D}\right) N = \left(\frac{12c^2}{E'h^3}\right) N$</td>
</tr>
</tbody>
</table>

Table 2. Non-dimensional variables for the “punch radius-based” non-dimensionalization.

With our governing equations the same, the only things that change are our boundary conditions. In fact, it can be shown that the only change, in all of our derived non-dimensionalized relations, is that $\eta = 1$ in our thickness-based relations. Therefore, we can simply replace $\eta$ with 1 in all of the relationships we derived earlier, to yield the relationships governing this new set of non-dimensional variables.

5.1. Clamped SLBT

5.1.1. Bending Limit

Figures 12, 13, 14, and 15 show the relationships between the relevant measured variables.
Figure 12 shows a linear-log plot of force, $\phi$, vs. blister radius, $\zeta$, for $\Gamma_C=0.01$, obtained by setting $\eta = 1$ in (15). We see that, when delamination is first initiating, and the blister radius has not grown (i.e. $\zeta = 1$), the force is infinite. As in the thickness-based non-dimensionalization, this is due to the zero-slope requirement at the inside and outside edges of the infinitesimally-wide blister annulus. As $\zeta$ increases, the force drops to finite values. For $\zeta \to \infty$, the limiting case of (16) is obtained, which is the point-load limit. Thus, for a blister radius that is sufficiently larger than the punch radius, we can treat the system as a point load.

When $\zeta = 10$, $\phi = 4.195 \sqrt{\pi} \sqrt{\Gamma_C}$, which is 104.9% of $4 \sqrt{\pi} \sqrt{\Gamma_C}$. Thus, by $\zeta = 10$, the blister radius has grown sufficiently such that the force behavior is within 5% of the point-load limit. However, according to [11], the bending-to-stretching transition occurs for approximately $15 < \zeta < 50$. Therefore, since the point-load “threshold” is close to the start of the transition, the point-load limit is of limited use. Nonetheless, we may still be able to use the portion of the curve for $\zeta < 15$, to characterize the adhesion energy. Theoretically, the critical SERR can be found solving (15) for $\Gamma_C$, then substituting the measured applied force, $\phi$, and blister radius, $\zeta$, (along with the known punch radius, $\eta$). Experimental verification of this, however, is needed. Such experimentation is in progress, and the results are pending.
Figure 12. Linear-log plot of force, $\phi$, vs. blister radius, $\zeta$, for $\Gamma_C=0.01$, obtained by setting $\eta = 1$ in (15). For $\zeta \to \infty$, the point-load limit of (16) is reached. For approximately $\zeta > 10$, the force is within 5% of this limit.

Figure 13 shows a log-log plot of punch displacement, $W_\eta$, vs. blister radius, $\zeta$, for $\Gamma_C=0.01$, obtained by setting $\eta = 1$ in (17). We see that $W_\eta \propto \zeta^n$, where $n$ is an exponent ranging from 2 to $\infty$. From the figure, it is apparent that $n \to 2$ for large $\zeta$. This is the point-load limit, reminiscent of (18). It can be shown that $n$ is within 5% of 2 when $\zeta > 15$ or $W_\eta > 15$, approximately. Therefore, we could treat the system as a point-load system by approximately this threshold. This is, again, however, the bending-to-stretching transition regime, which renders the point-load treatment invalid. Nonetheless, for $\zeta < 15$, one could
theoretically obtain the fracture toughness of the interface by solving (17) for $\Gamma_C$, then substituting the measured values for $W_\eta$ and $\zeta$, (along with $\eta = 1$).

Figure 13. Log-log plot of punch displacement, $W_\eta$, vs. blister radius, $\zeta$, for $\Gamma_C=0.01$, obtained by setting $\eta = 1$ in (17). For $\zeta \to \infty$, the point-load limit, where $W_\eta \propto \zeta^2$, is reached. For approximately $\zeta > 15$, the exponent, $n$, of $W_\eta \propto \zeta^n$ is within 5% of this limit.

$\phi$ vs. $W_\eta$

Figure 14 shows a plot of $\phi$ vs. $W_\eta$ for $\Gamma_C = 0.01$, obtained by setting $\eta = 1$ in (15) and (17), and plotting parametrically with respect to $\zeta$. We see that, as $W_\eta$ is driven, further, the force approaches that of the point load limit in (16) (here, $\phi=0.7090$). By $W_\eta=10$, $\phi=0.7361$, which is within 5% of the limit.
Figure 14. Log-log plot of $\phi$ vs. $W_\eta$ for $\Gamma_C=0.01$, obtained by setting $\eta = 1$ in (15) and (17), and plotting parametrically with respect to $\zeta$. For $W_\eta \to \infty$, the point-load limit of (16) is reached. For approximately $W_\eta > 10$, the force is within 5% of this limit.

$$\Gamma \frac{\phi W_0}{\pi (\zeta^2 - \eta^2)}$$

The normalized SERR is given by substituting $\eta = 1$ into (19). It can be shown that this approaches the limit $\frac{1}{2}$ as $\zeta \to \infty$, which is the point-load limit found in [2]. Figure 15 shows this evolution. It is a slower evolution toward the point-load limit than either the force, $\phi$, or the
punch displacement, \(W_\eta\), exhibited. By \(\zeta = 30\), \(\frac{G_A}{F_{w_0}}\) is 0.5198, which is within 5% of 0.5. This is, however, in the bending-to-stretching transition regime. Theoretically, however, we can use the portion of the curve for \(\zeta < 30\) to determine the fracture toughness by simply solving (19) for \(\Gamma_C\), the substituting the known values of \(\phi, W_\eta,\) and \(\zeta\) (along with \(\eta = 1\)). Experimental verification of this is pending.

Figure 15. Linear-log plot of normalized SERR, \(\frac{G_A}{F_{w_c}}\), vs. \(\zeta\), obtained by setting \(\eta = 1\) in (19). For \(\zeta \rightarrow \infty\), the point-load limit of \(1/2\) is reached. For approximately \(\zeta > 30\), the SERR is within 5% of this.
5.1.2. Stretching Limit

ϕ vs. ζ

The relationship between ϕ and ζ for the clamped SLBT in the stretching limit can be obtained by setting η = 1 in (24). Figure 16 shows this relationship for Γ_c=0.01. We see that the two are linearly related during the entirety of the system response, and there is no transition to a point-load equivalent behavior, at any point. Furthermore, we see that there is a finite force at which delamination initiates (i.e. ζ > 1). Taking the limit as ζ → 1 of (24), with η = 1, yields

\[ \phi = \frac{2^{9/4} \pi^{3/4}}{3^{1/2}} \Gamma_c^{3/4} \]  

This shows that the force required to initiate delamination is dependent only upon the adhesion energy, Γ_c, of the system. However, it contrasts with the point-load limit, wherein ϕ → 0 as η → 0. One implication of this non-zero initial-force is that we can theoretically measure Γ_c, experimentally, from measurement of the force required to initiate delamination.
Figure 16. Log-log plot of $\phi$ vs. $\zeta$ for $\Gamma_C=0.01$, obtained by setting $\eta = 1$ in (24). Note the linear relationship, and the absence of finite-punch to point-load transition. Furthermore, note the initial force of $\phi = \frac{2^{3/4}\pi^{3/4}}{3^{1/2}} \Gamma_C^{3/4}$ required to initiate delamination.

$\theta$ vs. $\zeta$

Figure 17 shows a plot of contact angle, $\theta$, vs. blister radius, $\zeta$, for the contact angles at punch edge ($\zeta = 1$, blue) and at the blister outer edge ($\zeta = \zeta$, red), obtained from setting $\eta = 1$ in (26) and (27), respectively. Here, we vary $\zeta$ from 1 (the punch radius) to $\infty$ (the case when the punch is infinitesimally small compared to the blister, and can therefore be treated as a point-load). When $\zeta \to 1$, the film annulus has infinitesimal width, and the angles consequently converge to the same value. This situation is equivalent to the case of the clamped SLBT under the thickness-based non-dimensionalization, when the punch radius, $\eta$, was taken to approach $\zeta$: 
the contact angles converged to (28). Indeed, this is verified by taking the limits of (26) and (27), with $\eta$ set to 1, as $\zeta \to 1$. Again, this intermediate limit implies that, for a finite-sized punch, the film annulus has a theoretically finite slope when delamination initiates, even though the annulus has infinitesimal width.

When the point-load limit is taken (i.e. when $\zeta \to \infty$), the contact angle at the punch edge and blister outer edge diverge to their separate limits of $-\frac{\pi}{2}$ and 0, respectively—just as in the case of the clamped SLBT under the thickness-based non-dimensionalization (c.f. Fig. 5). Thus, the case of $\zeta \to \infty$ for the punch radius-based ND is identical to the case of $\eta \to 0$ for the thickness-based ND.

Figure 17. Plot of $\theta$ vs. $\eta$ for $\Gamma_c=0.01$ and $\zeta=1$, of the film at the edge of the punch (blue) and at the outer edge of the blister (red).
$W_\eta$ vs. $\zeta$

Figure 18 shows the relationship between $W_\eta$ and $\zeta$—obtained by setting $\eta = 1$ in (29)—for $\Gamma_c=0.01$. We see that the two are linearly related for almost the entirety of the delamination path. There is some slight asymptotic behavior in the beginning, but it is negligible by the time the blister radius is 10x the punch radius. Furthermore, we see that, after 10x the blister radius, the slope is 1, which matches that of the point-load result.

![Log-log plot of $W_\eta$ vs. $\zeta$ for $\Gamma_c=0.01$, obtained by setting $\eta = 1$ in (29). For $\zeta \to \infty$, the point-load limit of $W_\eta \propto \zeta$, is reached.](image)

$\phi$ vs. $W_\eta$

Figure 19 shows plots of $\phi$ vs. $W_\eta$ for $\Gamma_c=0.01$, obtained by setting $\eta = 1$ in (24) and (29), and plotting parametrically with respect to $\zeta$. For infinitesimally small $W_\eta$, the relationship is
initially constant, but by between $10 < W_\eta < 100$ transitions to the linear point-load limit. This is mostly within the bending-to-stretching transition.

\[
\Gamma \frac{\phi W_0}{\pi (\zeta^2 - \eta^2)}
\]

Figure 19. Log-log plot of $\phi$ vs. $W_\eta$ for $\Gamma_c = 0.01$. For $10 < W_\eta < 100$, approximately, the point-load limit is reached.

\[
\Gamma \frac{\phi W_0}{\pi (\zeta^2 - \eta^2)}
\]

Figure 20 shows plots of $\frac{r}{\phi W_0/\pi (\zeta^2 - \eta^2)}$ vs. $\zeta$—obtained by setting $\eta = 1$ in (30)—for $\Gamma_c = 0.01$. The limit, as $\zeta \to \infty$ is $\frac{1}{4}$, which is the point-load limit. However, this limit is approached extremely slowly. For example, in order to be within 5% of the point-load limit (i.e. $\frac{r}{\phi W_0/\pi (\zeta^2 - \eta^2)} \leq 1.05 \times \frac{1}{4}$), $\zeta \geq 4.85 \times 10^8$, which is unrealistically large. So, in practice, we cannot use the point-load limit for the clamped SLBT under pure stretching. Despite this, we
may still be able to use (24) or (29) (or, alternatively, (48)) to compute $\Gamma_C$ from the measured $\zeta$, and $\phi$ or $W_\eta$. We are in the process of performing experimental verification of this.

Figure 20. Log-linear plot of $\frac{\pi \zeta^2}{\phi W_0/\pi(\zeta^2-\eta^2)}$ vs. $\zeta$ for $\Gamma_C=0.01$, obtained by setting $\eta = 1$ in (30). The relation approaches the point-load limit as $\zeta \to \infty$, but prohibitively slowly.
5.2. Free SLBT

5.2.1. Bending Limit

ϕ vs. ζ

Figures 21 and 22 show plots of ϕ and $W_\eta$ vs. ζ, respectively, for $\Gamma_C = 0.01$, obtained by setting $\eta = 1$ in (36). We see the transition to the point-load limit by approximately $\zeta = 10$. However, this happens to be the approximate upper bound of the pure-bending limit predicted in [2], wherein the bending-to-stretching transition occurred for approximately $15 < \zeta < 50$. Therefore, the point-load limit cannot be used to analyze a pure-bending free SLBT. Experimental evaluation will be performed to determine whether (36) can be used to compute $\Gamma_C$ from the measured $\zeta$ and $\phi$. 
Figure 21. Log-log plot of $\phi$ vs. $\zeta$ for $\Gamma_c=0.01$, obtained by setting $\eta = 1$ in (36). The point-load limit is attained by approximately $\zeta = 10$, but this is nearly within the bending-to-stretching transition regime of $15 < \zeta < 50$, approximately.

$W_\eta$ vs. $\zeta$

Figure 22 shows the relationship between $W_\eta$ and $\zeta$ for, obtained by setting $\eta = 1$ in (37). We see that, for approximately $\zeta > 5$, the behavior of the system is that of the point-load limit, which is characterized by the quadratic relationship given by (38).
Figure 22. Log-log plot of $W_\eta$ vs. $\zeta$ for $\Gamma_C=0.01$, obtained by setting $\eta = 1$ in (37). The point-load limit is reached by approximately $\zeta > 5$.

$\phi$ vs. $W_\eta$

Figure 23 shows the relationship between $\phi$ vs. $W_\eta$ for $\Gamma_C=0.01$, obtained by setting $\eta = 1$ in (36) and (37), and plotting parametrically with respect to $\zeta$. We see the transition to the point-load limit occur by approximately $\zeta = 5$. 
Figure 23. Plot of $\phi$ vs. $W_\eta$ for $\Gamma_0=0.01$. The force approaches the limiting value of $\frac{1}{2}$, which is the point-load limit in [2].
\[
\Gamma = \frac{\phi W_\eta}{\pi \zeta^2}
\]

Figure 24. Log-linear plot of \(\frac{G A}{F W_c} vs. \zeta\). The ratio approaches the limiting value of \(\frac{1}{2}\), which is the point-load limit found in [2].

5.2.2. Stretching Limit

Figures 25, 27, and 28 show the relationships between \(\phi\), \(W_\eta\), and \(\zeta\), for the stretching limit. In all three, the transition to the point-load limit occurs by approximately \(\zeta = 10\), at the greatest. However, in Figure 29, we see a much slower transition to the limit for the normalized
SERR, \( \frac{r_c}{\phi \eta / \pi \zeta^2} \). In fact, the value of the function does not even approach to within 5\% of the PL limit of \( \frac{1}{4} \) until \( \zeta \) is approximately \( 5 \times 10^8 \). For the more reasonable blister radii of 10 or 100, the error is 43\% and 22\%, respectively. Therefore, we can only apply the PL limit to the normalized SERR when a very rough measure is sought. Nonetheless, experimentation is in progress to verify whether (43) or (46) can be used to compute \( \Gamma_c \) from \( \phi, \eta, \) and \( \zeta \).

\( \phi \) vs. \( \zeta \)

Figure 25. Plot of \( \phi \) vs. \( \zeta \) for \( \Gamma_c=0.01 \), obtained by setting \( \eta = 1 \) in (43). The limiting relationship is linear, which is the point-load limit in [2], and it is reached very quickly after delamination initiates.
\[ \theta \text{ vs. } \zeta \]

Figure 26 shows a plot of contact angle, \( \theta \), vs. blister radius, \( \zeta \), for the contact angles at the punch edge \((\xi = 1, \text{ blue})\) and at the blister outer edge \((\xi = \zeta, \text{ red})\)—obtained by setting \( \eta = 1 \) in (44) and (45). Again, we vary \( \zeta \) from 1 (the punch radius) to \( \infty \) (the case when the punch can be treated as a point-load). When \( \zeta \to 1 \), the film annulus is infinitesimally wide, and the angles consequently converge to the same value of \(-\frac{\pi}{2}\). This situation is equivalent to the case of the free SLBT under the thickness-based non-dimensionalization, when the punch radius, \( \eta \), was taken to approach \( \zeta \).

When the point-load limit is taken (i.e. when \( \zeta \to \infty \)), the contact angle at the punch edge and blister outer edge diverge to their separate limits of \(-\frac{\pi}{2}\) and 0, respectively—just as in the case of the free SLBT under the thickness-based ND. Thus, again, the case of \( \zeta \to \infty \) for the punch radius-based ND is identical to the case of \( \eta \to 0 \) for the thickness-based ND.
Figure 26. Plot of $\theta$ vs. $\eta$ for $\Gamma_C=0.01$ and $\zeta=1$, of the film at the edge of the punch (blue) and at the outer edge of the blister (red).
$W_\eta$ vs. $\zeta$

Figure 27. Log-log plot of $W_0$ vs. $\zeta$ for $\Gamma_c=0.01$, obtained by setting $\eta = 1$ in (46). The limiting relationship is linear, which is the point-load limit in [2], and it is reached very quickly after the initial onset of delamination.
Figure 28. Log-log plot of $\phi$ vs. $W_0$ for $\Gamma_c=0.01$, obtained by setting $\eta = 1$ in (43) and (46), and plotting parametrically with respect to $\zeta$. The limiting relationship is linear, which is the point-load limit in [2]. It is reached by approximately $10 < \zeta < 100$. 

$\phi$ vs. $W_0$
\[
\frac{\Gamma}{\phi W_0 / \pi \zeta^2}
\]

![Log-linear plot of \( \frac{G_A}{FW_0} \) vs. \( \zeta \), obtained by setting \( \eta = 1 \) in (47). The point-load limit of \( \frac{1}{4} \) is approached very slowly.]

**Figure 29.**

6. **CONCLUSION**

Summarily, we have derived a new set of models for the shaft-loaded blister test, wherein the punch is of finite, rather than point-load size. We have shown that, for sufficiently high blister-to-punch radius ratio, the analysis reduces, theoretically, to a point-load solution. However, for the bending limit, the point-load limit lies within, or near to, the bending-to-
stretching transition regime. Therefore, it is of very limited usefulness. On the other hand, for the stretching limit, the point-load limit lies at an inaccessibly large blister-to-punch radius ratio. Therefore, we are confined to using the more general—but slightly less simple and elegant—analytical relationships, derived herein. Experimental work, however, is needed to verify these relationships. We are currently in the process of performing such work, and the results are forthcoming.

7. REFERENCES


