ENERGY GLEANING TO INCREASE THE EFFICIENCY OF
2-AXIS TIME-POSITION TRACKING PHOTOVOLTAIC ARRAYS
UNDER VARIABLY CLOUDY SKIES

A Dissertation Presented
by
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to
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Doctor of Philosophy
in the field of
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This dissertation is dedicated to
my husband, Michael,
my son, Jerry, and
my daughter, Evelyn,
my mother and father, and
mother-engineers everywhere.
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List of Acronyms

AOI    Angle Of Incidence
ASTM  American Society for Testing and Materials
CWOP  Citizens Weather Observer Program
DE    Differential Evolution
DHI   Diffuse Horizontal Irradiance
DNI   Direct Normal Irradiance
DTS   Direct Toward the Sun
EDT   Eastern Daylight Time
EST   Eastern Standard Time
GHI   Global Horizontal Irradiance
H     Horizontal
HDKR  Hay, Davies, Klucher, and Reindl
MAE   Mean Absolute Error
MBE   Mean Bias Error
MFRSR MultiFilter Rotating Shadow Band Radiometer
MGSI  Maximum Global Solar Irradiance
NASA  National Aeronautics and Space Administration
NRMSE Normalized Root Mean Squared Error
POA   Plane of Array
PV    Photovoltaic
PVPDC PV Performance Modeling Collaborative
RMSE  Root Mean Squared Error
RSR   Rotating Shadowband Radiometer
SF    Soiling Factor
SSE   Surface Meteorology and Solar Energy
TA    Tracking Advantage
TMY3  Typical Meteorological Year
UTC   Universal Coordinated Time
Nomenclature

$A_i$ Anisotropy index
$E_0$ Reference irradiance, typically 1000 Wh/m$^2$
$g(k_T)$ Correlation that relates the ratio $I_d/I$ to clearness index
$I$ Irradiation on a horizontal surface (Wh/m$^2$)
$I_b$ Beam irradiation on a horizontal surface (Wh/m$^2$)
$I_{b,T}$ Beam irradiation on a tilted surface (Wh/m$^2$)
$I_c$ Critical hourly global solar radiation (Wh/m$^2$)
$I_d$ Diffuse irradiation on a horizontal surface (Wh/m$^2$)
$I_{d,T}$ Diffuse irradiation on a tilted surface (Wh/m$^2$)
$I_H$ Hourly global irradiance on horizontal surface (W/m$^2$)
$I_o$ Extraterrestrial irradiation on a horizontal surface for an hour period (Wh/m$^2$)
$I_{r,T}$ Reflected irradiation on a tilted surface (Wh/m$^2$)
$I_{sc}$ Short-circuit current (Amps)
$I_{sc,DT}$ Short-circuit current on a panel directly pointed at the sun (Amps)
$I_{sc,H}$ Short-circuit current on a horizontal panel (Amps)
$I_T$ Irradiation on a tilted surface, (Wh/m$^2$)
$I_{T,PV}$ Irradiation absorbed on cell surface of a tilted PV panel (Wh/m$^2$)
$k_T$ Hourly clearness index
$M$ Air mass
$R_b$ Ratio of beam radiation on a tilted plane to that on a horizontal plane
$R_b$ Ratio of average beam radiation on tilted surface to that on a horizontal surface
### Greek Letters

<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$\beta$</td>
<td>Slope or tilt angle of collector with respect to horizontal (degrees)</td>
</tr>
<tr>
<td>$\beta_{\text{optimal}}$</td>
<td>Optimal tilt angle of collector with respect to horizontal (degrees)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Surface azimuth angle (degrees)</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>Solar azimuth angle (degrees)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Earth declination angle (degrees)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of incidence (degrees)</td>
</tr>
<tr>
<td>$\theta_z$</td>
<td>Zenith angle (degrees)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Hour angle (degrees)</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Ground reflectance</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Latitude (degrees)</td>
</tr>
<tr>
<td>$(\tau\alpha)_b$</td>
<td>Transmittance-absorptance product of beam radiation stream</td>
</tr>
<tr>
<td>$(\tau\alpha)_d$</td>
<td>Transmittance-absorptance product of diffuse radiation stream</td>
</tr>
<tr>
<td>$(\tau\alpha)_r$</td>
<td>Transmittance-absorptance product of reflected radiation stream</td>
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Acknowledgments

“Faith is taking the first step even when you don’t see the whole staircase.” – Martin Luther King, Jr.

“Look for the helpers. You will always find people who are helping.” – Fred Rogers’ mother

I began this journey with a leap of faith, not knowing where it would lead. Although the path has been non-linear and uncertain at times, there has always been someone who was willing to help. I am most grateful for all of those helpers who supported and encouraged me along the way.

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Abstract of the Dissertation

ENERGY GLEANING TO INCREASE THE EFFICIENCY OF 2-AXIS TIME-POSITION TRACKING PHOTOVOLTAIC ARRAYS UNDER VARIABLY CLOUDY SKIES

by

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Dr. Michael Silevitch, Advisor

Positioning a photovoltaic (PV) array in the optimal orientation increases the collection of solar radiation and the production of electricity. Many methods for determining the optimal tilt angle of a fixed PV array have been reported in the literature; however, few methods have been proposed for finding the optimal tilt angle of a 2-axis time-position tracking PV array. This dissertation derives and validates a simple formula for directly calculating the optimal tilt angle of a 2-axis time-position tracking PV array under varying sky conditions. By modifying the conventional tilt angle as the sky conditions change, the tracking PV array can glean the additional small amounts of irradiation that are overlooked and unused on cloudy days. The validity of this formula was verified using 36 months of weather data from an installation in the northeastern United States where clear skies occur about 40% of the time. Simulations indicated that modifying the conventional tracking angles in response to changing cloud cover results in 2.26% increase in collected insolation and 2.33% increase in AC energy over a 36-month period when compared with conventional 2-axis time-position tracking. During hourly and sub-hourly intervals with cloud cover, the increase in energy collection can reach up to 50% more than for conventional 2-axis tracking. The ability to modify the conventional tracking angles in response to changing cloud cover allows the PV array to glean the previously uncollected energy, thereby capturing more of the total available irradiation and increasing electrical power production.
Chapter 1

Introduction

1.1 Overview of the problem

There are three types of photovoltaic (PV) installations — fixed, single-axis tracking, and 2-axis tracking. Commercial 2-axis tracking arrays are divided into two categories: trackers that use light sensors to find the brightest point in the sky and trackers that use an astronomical algorithm based on the sun’s apparent position. The latter type of tracker is sometimes called a time-position or chronological tracking array.

Although more costly and complex than fixed PV arrays, the advantage of 2-axis tracking arrays is the efficiency in which they capture the solar radiation, especially on clear, sunny days. There is an average of 34% more irradiance collected by a 2-axis tracking array when compared with a fixed array in the same location [1]. For locations in the continental United States the researchers in [2] calculated a 25 to 45% increase in annual irradiance collected by a 2-axis time-position tracking PV array in comparison with the irradiance collected by a fixed PV array that is positioned at optimal tilt and azimuth angles. The researchers also cited [3] and noted that an additional increase in irradiance may result by placing a 2-axis tracking array in a horizontal position when the amount of diffuse irradiance is great.

Although 2-axis trackers are typically installed in dry, sunny locations with consistently clear skies, the number of installations in cloudier locations has rapidly increased in recent years. This results in greater irradiance collection on clear, sunny days, when the total irradiation on the PV array is composed primarily of beam irradiation from the sun. However, it reveals a disadvantage of 2-axis time-position PV arrays in that they continue to track the apparent position of the sun even when it is obscured by clouds. On cloudy days the beam irradiation is very small and the total
CHAPTER 1. INTRODUCTION

available irradiation is composed primarily of diffuse irradiance that is scattered from the clouds and ground. A 2-axis tracker that tracks the sun on cloudy days may fail to collect the maximum amount of available diffuse irradiation.

Figure 1.1 shows the irradiance measured on a cloudy day by two sensors at a 2-axis time-position tracking installation located in Middlebury, Vermont, USA. A sensor measuring Plane of Array (POA) irradiance is mounted on one of the PV arrays tracking the sun. About 100 meters north of this POA irradiance sensor, a second sensor measuring global horizontal irradiance (GHI) is mounted at a tilt angle of zero degrees. As seen in the GHI measured by the horizontal sensor exceeds the POA irradiance measured by the sensor that is tracking the sun. This suggests that more irradiance could be captured by adjusting the conventional 2-axis tracking tilt angle to a more horizontal position. Each instance where the GHI exceeds the POA irradiance represents an opportunity to collect more of the total available irradiance and generate more electrical power.

![Irradiance Graph](image)

Figure 1.1: 15-minute measurements of Plane of Array (POA) irradiance and global horizontal irradiance (GHI) during an overcast day in Middlebury, Vermont (October 29, 2014). More irradiance is measured by the GHI sensor that is fixed horizontally (tilt angle of 0°) than by the POA sensor that is mounted on the 2-axis tracking PV array.

In this research, we ask the following questions:

1. How can we modify the conventional tilt angles for a 2-axis time-position tracking PV array so that the array collects the greatest amount of available solar radiation as the sky conditions vary?

2. What is the increase in solar radiation captured by a 2-axis time-position tracking PV array when the conventional tilt angle is modified as the sky conditions change?
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1.2 Energy gleaning — a new area of research

In 2009 Solar Energy published Nelson Kelly and Thomas Gibson’s paper “Improved photovoltaic energy output for cloudy conditions with a solar tracking system” [3]. Kelly and Gibson experimentally showed that a horizontal irradiance sensor collected more energy on overcast days than an irradiance sensor directly pointed at the sun. They proposed modifying the conventional tracking angles of 2-axis time-position tracking PV arrays on cloudy days by placing the array in a horizontal position.

During the past few years, a handful of researchers in Canada [4], Romania [5], Algeria [6] and the U.S. [7] have investigated this problem through simulations and experiments. Their results have confirmed that more irradiance could be collected on cloudy days if the 2-axis PV array did not track the sun, and their research has focused on finding the optimal orientation of the array on cloudy days.

We shall introduce the new term “energy gleaning” to describe what we, Kelly and Gibson, and other researchers are doing when adjusting the tilt angle of conventional 2-axis tracking arrays; that is, we are collecting additional small amounts of solar energy during cloudy intervals to improve the annual yield of 2-axis time-position tracking PV arrays. We borrow the term “gleaning” from the area of agriculture where it refers to the gathering of crops that are overlooked and left behind in the fields after the primary harvest. Energy gleaning collects the additional small amounts of power that are overlooked and otherwise unused on cloudy days by changing the orientation, in particular the tilt angle, of a 2-axis tracking PV array.

During hourly and sub-hourly intervals of varying cloud cover, the increase in collected irradiance could reach up to 50% more than for conventional 2-axis tracking [3]. By capturing these small amounts of additional irradiation throughout the year, the annual energy production of the PV installation could potentially increase by 0.5 to 3% when compared with conventional 2-axis tracking, depending on the installation’s location and climate [7].

1.3 Research contributions

1.3.1 Significance of the research

As mentioned in the previous section [1,2] over the past seven years a handful of researchers have proposed methods for determining the optimal tilt angle of 2-axis time-position arrays under cloudy skies. Their proposed methods (tracking advantage based on a correlation, optimization
techniques, or numerically finding the angle) may require knowledge of historical weather data, complicated equations, and/or computer programs.

In our dissertation, we propose a simple formula that directly calculates the optimal tilt angle under different sky conditions and at any time. Using the proposed S Formula, we analyze the potential increase in solar radiation incident on a hypothetical 2-axis tracking PV array for several U.S. cities located in different climates. We then focus on one specific 2-axis time-position tracking installation located in Middlebury, Vermont, USA, and use weather and power data to calculate the increase in collected solar radiation and AC power generation over three consecutive 12-month periods. Using a related formula for fixed PV arrays, we also examine how closely the formula estimates the optimal tilt angle for fixed arrays in different locations and climates in the U.S. and other countries.

The contributions of our research are as follows:

1. We derive a simple formula, called the “Parent S Formula”, that directly calculates the optimal tilt angle for a 2-axis PV array under varying sky conditions. This S Formula is mathematically-derived from the Isotropic Sky Model [8], and has the following features:
   - It takes into account all three components of irradiance (beam, diffuse, and ground-reflected) because it is mathematically derived from the equation for solar radiation absorbed on the cell surface of a photovoltaic panel.
   - It is a function of the conventional 2-axis tracking angle ($\theta_z$), which is multiplied by an expression containing the clearness index ($g(k_T) = I_d/I$), ground reflectance, and transmittance-absorptance products of the beam, diffuse and ground-reflected radiation streams.
   - It allows the tilt angle to be adjusted during any interval of time.
   - It allows the tilt angle to be adjusted in response to changing weather conditions.
   - It does not rely on historical weather data.
   - It explains mathematically the experimental results reported in the literature because it is a function of $\theta_z$, clearness index, and ground reflectance. The optimal angles calculated by the S Formula reduce to the conventional tracking angles $\beta = \theta_z$ under clear skies and the tilt angle of $\beta = 0^\circ$ under overcast skies as reported in [3,4]. The S Formula helps in understanding why another angle — other than $0^\circ$ or $\theta_z$ — may be the optimal angle for partially cloudy skies or for different values of ground reflectance as reported in [9,10].
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2. We present results for 21 U.S. cities showing the potential increase in yearly irradiance when a hypothetical 2-axis tracking PV array is allowed to adjust its tilt angle in response to varying sky conditions. We also show how well the optimal angles estimated by the Parent S Formula match those estimated using the numerical method.

3. We present results for a 2-axis time-position tracking array located in Middlebury, Vermont using weather data from the installation to calculate POA irradiance and AC power at the optimal angles predicted by the S Formula. We look at how closely the S Formula predicts the optimal tilt angles when compared with the numerical method. We compare the S Formula’s calculated yearly POA insolation with the values calculated numerically, when using conventional 2-axis tracking angles, and for each of the three methods mentioned in [11]. We then calculate and compare the yearly AC energy for the tilt angles calculated by the S Formula, the numerical method, and the conventional 2-axis tracking angles.

4. In related work, we derive from the Parent S Formula a family of formulas that can be used to calculate optimal tilt angles for 1-axis and fixed arrays. We present results showing how well the fixed S Formula estimates the optimal tilt angles for 21 U.S. cities, as well as the potential increase in yearly insolation at the optimal fixed tilt vs. latitude tilt.

5. We present the results showing how closely the S Formula for fixed PV arrays estimates the optimal tilt angles for 217 U.S. locations and 37 non-U.S. locations. We then compare the estimated S Formula optimal angles with those published at the NASA Surface Meteorology and Solar Energy website [12], as well as by other researchers who used numerical or optimization methods to find the optimal tilt angles at those same locations.

1.3.2 Contributions to knowledge and understanding

The journal Solar Energy has accepted for publication our manuscript “Energy gleaning for extracting additional energy and improving the efficiency of 2-axis time-position tracking photovoltaic arrays under variably cloudy skies”. In this paper, we present the S Formula and report the simulation results for 24 months of weather data. The term energy gleaning is also introduced [13].

In 2016, three journal articles written by researchers in Canada, Turkey, and Switzerland cited our earlier research on the S Formula that was published in conference proceedings in June 2013 [7].
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In the first article [14], researchers at the University of Ottawa examined the relationship of electricity tariffs on the optimal tilt angle of fixed PV arrays in three locations. The article’s introduction cites our research finding that the optimal tilt angles for fixed, south-facing PV arrays are $10^\circ$ to $12^\circ$ less than latitude for cloudier locations [7].

In the second article [15], researchers in Turkey proposed an algorithm for controlling the gel batteries in a hybrid wind/PV system. In discussing the specifications of the PV panels, they mentioned that the panels are fixed at a tilt angle that is optimal to the location (Soke, Turkey) and they cited [7] and another paper.

In the third article [16], researchers at ETH-Zurich have developed and tested new open-source software called the City Energy Analyst (CEA) that is an extension to the Geographic Information System ArcGIS v10.3 program. CEA allows architects and city planners to compare various scenarios for redeveloping urban areas by analyzing and optimizing multiple energy systems. The researchers are using the mathematically-derived S Formula for fixed PV arrays presented in [7] to calculate the optimal tilt angle of fixed, south-facing PV arrays located on flat roofs. This mathematically-derived formula has been revised in this dissertation.

1.4 Organization of the dissertation

This dissertation is organized as follows:

Chapter 2 begins with a broad overview of solar energy and tracking concepts in order to give the reader a general knowledge of the topic. We briefly discuss the following: solar angles, solar radiation streams, types of PV arrays, 2-axis tracking arrays, commercial trackers, and derivation of the conventional tracking angles. We then focus on the dissertation topic of improving the efficiency of 2-axis time-position tracking PV arrays. We present the research that has been published on this topic, and discuss the methods, results, and potential limitations of each method. Finally, we present the potential increase in yearly insolation when the tilt angles of a hypothetical 2-axis array are adjusted in response to changing sky conditions.

In Chapter 3 we start with the Liu and Jordan equation for solar radiation absorbed on the cell surface of a photovoltaic panel, and derive the proposed Parent S Formula for 2-axis tracking. We then derive a family of S Formulas that could be used for PV arrays in other configurations and orientations, such as 1-axis tracking and fixed PV arrays. Finally, we demonstrate the potential increase in yearly insolation using the proposed S formula for 2-axis tracking.
CHAPTER 1. INTRODUCTION

In Chapter 4 we look at the special case of the S Formula for a south-facing fixed array that was derived in the previous chapter. We begin the chapter by briefly reviewing the methods that have been proposed in the literature for finding the optimal angle of a fixed array. Although others have proposed taking the derivative of an irradiance model to solve for the optimal tilt angle of a fixed array, they do not compute the optimal tilt directly (as does the S Formula), but rather they use numerical or optimization techniques to solve for the optimal tilt angle. Furthermore, to the best of our knowledge, we do not believe that this approach of taking the derivative has been used to find the optimal tilt angle of a 2-axis tracking array.

The researchers in [16] are using the S Formula from [7] in their City Energy Analyst software to calculate the optimal tilt angle of fixed, south-facing PV arrays located on flat roofs. We shall show how well the formula estimates the yearly fixed optimal tilt angle by using the S Formula to calculate the optimal angles for 217 U.S. cities and 37 non-U.S. locations, and then comparing our results with those presented in the literature.

Next, we investigate the validity of the S Formula by using three years of measured weather data from a 2-axis tracking PV installation located in Middlebury, Vermont, U.S. where clear skies occur about 40% of the year. Chapter 5 discusses the installation parameters, instrumentation, and data quality. We also explain the criteria, process and MATLAB programs used to flag the data. We used the PVLIB Toolbox for MATLAB to calculate the POA irradiance and AC power for each 15-minute time interval. Inconsistent data and outliers were eliminated based on the flags. There are three screened data sets used in the calculations: November 1, 2013 to October 31, 2014, November 1, 2014 to October 31, 2015, and November 1, 2015 to October 31, 2016.

In Chapter 6 we compute yearly POA irradiance and AC power for five methods — the three methods proposed by Kelly and Gibson in [11], the numerical method, and the conventional 2-axis tracking angles— using the three 12-month screened data sets described in Chapter 5.

Chapter 7 explains the S Formula method for calculating the optimal tilt angles using measured diffuse and global horizontal irradiance data from the Middlebury College PV installation. POA irradiance and AC power were then calculated at the optimal angles using the PVLIB Toolbox for each of the three 12-month screened data sets.

In Chapter 8 we compare the results of the S Formula method for calculating the optimal tilt angles, yearly POA insolation, and yearly AC energy with the other methods (conventional 2-axis, numerical, and Kelly and Gibson’s three methods described in [11]). We also compare the results with the simulations and experiments reported in the literature. We examine the contribution of each of the irradiation streams to the overall increase in captured POA irradiation. We discuss the potential...
CHAPTER 1. INTRODUCTION

for additional energy gleaning under overcast skies due to the shift in the spectral components of diffuse irradiance. We propose several 2-axis tracking strategies for maximizing the capture of irradiation and for minimizing power consumption. The financial impact of the research is discussed, as well as areas of future work.

In Chapter 9, we discuss our research contributions, summarize our results, and present our conclusions.
Chapter 2

Background and literature review

2.1 Introduction

Chapter 2 begins with a broad overview of solar energy and tracking concepts in order to give the reader a general knowledge of the topic. We briefly discuss the following: solar angles, solar radiation streams, types of PV arrays, 2-axis tracking arrays, commercial trackers, and derivation of the conventional tracking angles. We then focus on the dissertation topic of improving the efficiency of 2-axis time-position tracking PV arrays. We present the research that has been published on this topic, and discuss the methods, results, and potential limitations of each method. Finally, we show the potential increase in yearly irradiance when the tilt angles of a hypothetical 2-axis array are adjusted in response to changing sky conditions.

2.2 Solar angles and solar radiation streams

Figure 2.1 shows the solar geometry and angles used in modeling irradiance. The north-south and east-west axes are perpendicular to each other and form a horizontal plane. The zenith line is normal to this horizontal plane. Two angles, the zenith angle $\theta_Z$ and the solar azimuth angle $\gamma_S$, describe the apparent position of the sun. The zenith angle is the angle between the zenith and the sun’s direct beam. The solar altitude angle $\alpha_S$ is the complement of the zenith angle. The solar azimuth angle $\gamma_S$ is the angle between the north-south axis and the sun’s projection onto the horizontal plane.

The position of a tilted surface is defined by two angles, the tilt angle $\beta$ and the surface azimuth angle $\gamma$. The tilt angle is the slope of the surface with respect to the horizontal plane. The
CHAPTER 2. BACKGROUND AND LITERATURE REVIEW

Surface azimuth angle is the angle between the north-south axis and the tilted surface. The angle of incidence $\theta$ is the angle between a line that is normal to the tilted surface and the incident light.

There are differences in the literature on the designation of the starting point on the north-south axis. In this dissertation, the convention in [8] is used where south is defined as $0^\circ$, north as $180^\circ$ or $-180^\circ$, east $-90^\circ$, and west $+90^\circ$. Other sources, such as PV_LIB Toolbox [17], designate north as $0^\circ$, east $+90^\circ$, south $+180^\circ$, and west $+270^\circ$.

Figure 2.1: $\alpha_s$ is solar altitude angle, $\beta$ is the tilt angle of the collector with respect to the horizontal, $\gamma$ is surface azimuth angle, $\gamma_s$ is solar azimuth angle, $\theta$ is angle of incidence, and $\theta_z$ is zenith angle. All angles are in degrees. Adapted from [8].

Figure 2.2 shows the sources of solar radiation that are incident upon a tilted PV panel. Beginning at the top of the atmosphere, the extraterrestrial radiation, $I_o$, is about 1367 W/m$^2$ [8]. This amount varies slightly throughout the year, depending on the day of the year. As the solar radiation enters the earth’s atmosphere, $I_o$ is attenuated by two processes—absorption and scattering. $I_o$ is absorbed by ozone, water vapor, and CO$_2$, and it is scattered by air vapor, water, and dust [8].

There are several radiation streams that reach the PV panel, as shown in Figure 2.2. The unscattered solar radiation that reaches the panel is called the “beam”, “direct”, or “direct beam” radiation in the literature. In this dissertation, we will primarily refer to it as beam radiation. However, we may use the terms direct or direct beam when discussing other researchers’ methods.

The solar radiation that is scattered throughout the sky dome creates three radiation streams.
CHAPTER 2. BACKGROUND AND LITERATURE REVIEW

Isotropic diffuse is scattered radiation throughout the sky dome that is uniformly received by the horizontal panel. Circumsolar diffuse results from forward scattering of the solar radiation, and it is concentrated in the part of the sky around the sun. Horizon brightening is mostly present in clear skies and is concentrated near the horizon. The sum of all three of these radiation streams is called “diffuse irradiation”.

The total irradiation reaching a horizontal panel on the earth, called “Global Horizontal Irradiance”, consists of the beam irradiation and the diffuse irradiation. It is sometimes denoted as GHI or I.

Tilting the panel at an angle of $\beta$ results in the collection of more beam irradiation but less diffuse irradiation. However, because the panel is now tilted, there is an additional radiation stream due to reflections from the ground, trees, and buildings. This radiation stream is called the reflected or ground-reflected irradiation.

The clearness index, $k_T$, indicates the amount of attenuation that the extraterrestrial irradiation undergoes as it travels through the atmosphere to the earth’s surface. It is the ratio of the irradiance on the horizontal surface of the earth to the solar radiation at the top of the atmosphere, or $\frac{I}{I_o}$. The clearness index can be used to indicate the sky conditions, or the level of cloudiness. It is a number between 0 and 1, with numbers close to 1 indicating clearer skies. On clear days, when the clearness index is close to 1, beam irradiation dominates, and GHI consists primarily of beam irradiation. On cloudy days, when the clearness index is close to 0, diffuse irradiation dominates, and GHI consists primarily of diffuse irradiation.

![Figure 2.2: Sources of radiation upon a PV panel that is tilted at an angle of $\beta$.](image)
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2.3 Types of PV arrays

In this section, we briefly review the types of PV arrays and their advantages and disadvantages, we look at various commercial 2-axis trackers, and we derive the conventional tracking angles for 2-axis time-position arrays.

2.3.1 Fixed, 1-axis tracking, and 2-axis tracking PV arrays

Figure 2.3 shows three types of photovoltaic (PV) installations — fixed, single-axis tracking, and 2-axis tracking. Several kinds of tracking arrays are reviewed in [18].

A fixed PV array is shown in Figure 2.3a. This type of array is installed at a fixed tilt angle and azimuth angle, and it does not rotate about an axis. Because it does not rotate about an axis and does not have any moving parts, it does not consume any energy. It is the least expensive of the three types; however, it captures the least amount of irradiance.

A 1-axis tracking PV array rotates about one axis. Three different configurations of this kind of array are shown in Figure 2.3b. In the top figure, the PV panel rotates about a fixed north-south axis. The middle figure shows a panel rotating about a fixed east-west axis. In the bottom figure, the panel is mounted on a vertical pole at a fixed tilt angle, and is then rotated about the vertical pole. A 1-axis array is more expensive and consumes energy due to the moving parts. However, the annual energy collection is 12 to 25% more than for a fixed array [19].

Figure 2.3c shows a 2-axis time-position tracking PV array. The panel is mounted on two rotating axis — a vertical axis and an east-west axis. Due to its moving parts, the 2-axis tracker consumes more energy, is more complex, and is more expensive than a 1-axis tracking PV array. However, its yearly annual energy collection is 30 to 45% more than for a fixed array [19].

In the past, it was considered cost-effective to install tracking PV arrays in consistently sunny locations, such as the southwestern United States. However, the number of 2-axis tracking PV installations has increased in northerly, humid climates where there is greater variability in sky conditions. As shown in Figure 2.4, AllEarth Renewables has installed over 1700 2-axis trackers, primarily in the northeastern U.S. [20].

Two-axis tracking PV arrays can be further divided into two categories: trackers that use light sensors to find the brightest point in the sky and trackers that use an astronomical algorithm based on the sun’s apparent position. The latter type of tracker is sometimes called a time-position or chronological tracking array.
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Figure 2.3: Three types of PV arrays. (a) Fixed array. (b) Three kinds of 1-axis arrays. (c) A 2-axis PV array.

Figure 2.4: Locations of AllEarth Renewables 2-axis tracking PV arrays in the U.S., as of 2014. Each marker represents a 2-axis PV installation. (Image downloaded from [20] in March 2014.)
Light-sensing 2-axis trackers use two or more light sensors to detect the available solar radiation. The data from the light sensors are compared and then used to orient the 2-axis tracker in the direction of the sunlight. Some of the disadvantages of using light-sensing 2-axis trackers are that they may not be able to track the sun when it is hidden by clouds, they may have difficulty finding the brightest point in the sky, or the light sensors may get dirty or fail.

Haosolar uses light sensors to find the brightest point in the sky during sunny sky conditions [21]. During cloudy skies the light sensors’ output is not used to control the array; instead, the tracking switches to an astronomical algorithm to allow the array to continue to track the sun. However, as shown in Figure 1.1, it is not optimal for the array to track the sun during cloudy conditions.

DEGER Energy uses three light sensors in its tracking method [22]. The light sensors are able to detect the intensity and angle of direction of the incoming light. By taking into account the beam, diffuse, and reflected light, the tracking angles are adjusted depending upon the amount of light that is present.

For 2-axis tracking arrays using an astronomical algorithm, the position of the sun is calculated based on well-known algorithms. For a time-position 2-axis tracking array, the tilt angle of the 2-axis tracker is the same as the solar zenith angle and the tracking array’s surface azimuth angle is the same as the solar azimuth angle. The disadvantage of this method is that the tracking array will continue to follow the sun when it is obscured by the clouds, and thus it may fail to capture the maximum available irradiance on cloudy days.

For both light-sensing and time-position 2-axis tracking arrays, the manufacturer may design the tracker so that its movements can be adjusted in response to shading or weather conditions. In installations where there are multiple tracking arrays, a backtracking algorithm is added to prevent the tracking array from being shaded by its neighboring trackers. During snowy conditions, the tracker can be positioned at an angle that allows the snow to slide off the panels.

In the case of strong winds, the International Electrotechnical Commission (IEC) Technical Specification IEC/TS 62727, “Photovoltaic systems Specifications for solar trackers” requires that a 2-axis tracking array be placed in a horizontal position when it is exposed to winds that exceed the specification’s maximum allowable wind speed [23]. An anemometer that measures wind speed and direction may be attached to an individual tracking array or located at a weather station at the installation.
2.3.2 Commercial 2-axis time-position tracking PV arrays

There are many ways to mechanically position a 2-axis time-position PV array to capture sunlight. In this section, we discuss four commercial 2-axis time-position PV arrays that use different mechanisms to adjust the tilt and azimuth angles.

Figure 2.5a shows the back of an AllEarth Renewables tracking array [24]. The tilt angle is adjusted by a hydraulic cylinder, and the azimuth angle is adjusted by a ring worm gear with a hydraulic motor. This tracker changes position approximately every ten minutes. Each tracker has an anemometer at the top of the array which can detect excessive winds and send a signal to place the array in the horizontal position.

The mechanism used by the TecnoSun Solar Systems 2-axis tracking array shown in Figure 2.5b uses only one drive to achieve 2-axis tracking [25]. Two PV modules are mounted on a support structure that rests on a pivoting vertical axis. A steel drive cable is looped around a traction ring, or drum, on one or more support structures. This steel cable is capable of moving up to 30 support structures from east to west.

To change the tilt angle, there is a uniquely-shaped ring that is connected to the drum on the support structure. This ring has been adjusted based on the tracker’s latitude. As the support structure rotates around the drum, a control lever connected to the PV modules adjusts the tilt angle throughout the day.

This tracker can be moved into a horizontal position during periods of high winds. During snowfall, the tracker can be positioned at an angle that allows the snow to slide off the modules. It also uses a back-tracking algorithm to prevent nearby modules from shading the tracker.

TecnoSun Solar’s 2-axis tracker modifies the astronomical tracking angles by using measurements from a sensor. From mid-February to October, the tracking array follows the sun. If no beam solar radiation is detected, the array is positioned at a tilt angle of 15° in order to capture the maximum available irradiation. From November to mid-February, the array does not track the sun; rather, it is placed at a fixed tilt angle of 15°. TechnoSun Solar has found that the tilt angle of 15° is optimal for Central Europe’s latitudes because it eliminates the shading that may occur due to the sun’s low elevation in the sky.

Figure 2.5c shows several QBotix trackers. Two robots periodically move around the track to adjust each tracker’s tilt and azimuth angles. One robot actuates the tracker’s slew drive to adjust the tilt angle, and a second robot actuates a passive linear actuator to adjust the azimuth angle [26].

The QBotix system uses a back-tracking algorithm to avoid shading from nearby modules,
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and it can move the trackers into a horizontal position under conditions of high winds. It should be noted that this company closed its business in September 2015 [27].

Figure 2.5d shows a large PV array made by Titan Tracker. This array uses electric motor gears and pinion-toothed wheels to adjust the tilt and azimuth angles. The array continuously moves around a circular track. The tilt is adjusted by a spindle attached to a tractor wheel that rolls along the track [28]. This tracker can be moved into a horizontal position during periods of high winds, and it can be tilted to shed snow.

Figure 2.5: Four commercial 2-axis time-position tracking PV arrays. (a) AllEarth Renewables. (Photographed by author on 11/18/2013.) (b) TecnoSun Solar Systems. (Downloaded from [25] in August 2015.) (c) QBotix. (Downloaded from [26] in August 2015.) (d) Titan Tracker. (Downloaded from [28] in August 2015.)

As we can see from these four examples of commercial 2-axis time-position tracking arrays,
there are several different ways to mechanically adjust the tilt and azimuth angles of the tracker. The intervals at which the trackers change orientation can vary, too, with some trackers moving continuously while others move every few minutes or hourly. Many trackers have the capability to discontinue the conventional tracking algorithm and to change orientation during adverse weather conditions such as strong winds or snow.

The availability of information on the power consumption of 2-axis tracking arrays varies, with some companies giving a value in terms of a percentage of annual energy production while other companies provide a daily or yearly value. Other companies, such as TechnoSun Solar, do not give a value for power consumption \[25\]. The manufacturer of the Titan Tracker gives an approximate value of 0.5 kWh per day, with the caveat that the power consumption depends on several factors such as the location and wind loads \[28\]. MecaSolar manufactures a large 2-axis time-position tracking array that uses an electrically-driven mechanical jack to adjust the tilt and a gear motor with a cogged crown wheel to adjust the azimuth movement. It specifies the power consumption of the motors as 100 kWh per year \[29\]. The QBotix website stated that energy consumption of its robots was less than 0.1% of the energy production \[26\]. AllEarth Renewables specifies annual power consumption as less than 1% of system output for its tracking array \[24\].

Although the specific features may vary, what these commercial 2-axis time-position tracking PV arrays have in common is this: they all track the position of the sun under clear, partly cloudy, and overcast skies. However, as shown in Figure 1.1 more irradiance could be gleaned from the total available irradiation by not tracking the sun’s position. The focus of our research is to develop a simple formula that directly calculates the optimal tilt angle for a 2-axis PV array under varying sky conditions. This simple formula could potentially be used by any 2-axis time-position tracking PV array to point in the direction that maximizes the collection of total available radiation.

### 2.3.3 Tracking angles for a 2-axis time-position array during clear skies

The conventional tracking angles for a 2-axis time-position tracking array that is continuously tracking the sun under clear skies \[30, 8\] are:

\[
\beta = \theta_Z \tag{2.1}
\]

\[
\gamma = \gamma_S \tag{2.2}
\]

This section presents the derivation of these tracking angles. For a time-position 2-axis array, the goal is to maximize the irradiance collected by the receiver. For the maximum amount
of light to be transmitted through the collecting surface, the angle of incidence should equal zero degrees.

Recall that the equation for the angle of incidence is as follows [8]:

\[ \cos \theta = \cos \theta_z \cos \beta + \sin \theta_z \sin \beta \cos(\gamma_s - \gamma) \]  
(2.3)

If we assume that the only source of sunlight is the direct beam from the sun, then the goal is to minimize the angle of incidence \( \theta \). This happens when \( \theta = 0^\circ \), or when \( \cos \theta = 1 \).

\[ 1 = \cos \theta_z \cos \beta + \sin \theta_z \sin \beta \cos(\gamma_s - \gamma) \]  
(2.4)

To find the optimal tilt angle, we take the derivative of both sides of (2.4) and solve for \( \beta \).

\[ 0 = \frac{d}{d\beta}(\cos \theta_z \cos \beta + \sin \theta_z \sin \beta \cos(\gamma_s - \gamma)) \]  
(2.5)

\[ 0 = (- \cos \theta_z \sin \beta + \sin \theta_z \cos \beta \cos(\gamma_s - \gamma)) \]  
(2.6)

\[ \cos \theta_z \sin \beta = \sin \theta_z \cos \beta \cos(\gamma_s - \gamma) \]  
(2.7)

\[ \frac{\sin \beta}{\cos \beta} = \frac{\sin \theta_z}{\cos \theta_z \cos(\gamma_s - \gamma)} \]  
(2.8)

\[ \tan \beta = \tan \theta_z \cos \theta_z \cos(\gamma_s - \gamma) \]  
(2.9)

For a 2-axis time-position tracking array, the array tracks the apparent position of the sun from east to west. By setting the surface azimuth angle \( \gamma \) equal to the sun’s azimuth angle \( \gamma_s \), (2.9) reduces to:

\[ \tan \beta = \tan \theta_z \]  
(2.10)
Thus, in the case of a 2-axis time-position tracking array, the conventional tracking angles are:

\[
\begin{align*}
\beta &= \theta_z \quad (2.11a) \\
\gamma &= \gamma_s \quad (2.11b)
\end{align*}
\]

These angles assume that the only source of sunlight is the direct beam from the sun. However, research published in the past few years indicates that the conventional tilt angle of \(\beta = \theta_z\) is not necessarily optimal under partially cloudy or overcast skies.

### 2.4 Literature review: Methods for finding optimal tilt angles of 2-axis tracking PV arrays under cloudy skies

For fixed PV arrays, there have been many papers suggesting methods for calculating the yearly or monthly optimal tilt angle based on site parameters, weather conditions, cloudiness, or other parameters \([31, 2, 1, 7, 32]\). However, for 2-axis time-position tracking PV arrays, only a few papers have proposed methods for finding the optimal tilt angle under varying sky conditions. These methods are briefly summarized in the following subsections.

#### 2.4.1 Tracking advantage based on the ratio H/DTS

The authors in \([3]\) proposed modifying the conventional tracking angles of 2-axis time-position tracking PV arrays on cloudy days. They measured the irradiance on four overcast days using six different sensors on horizontal and tilted surfaces in Detroit, Michigan. The horizontal sensors measured 20 to 82% more irradiance throughout the day than the sensors that were directly pointed at the sun, with a mean increase of 47%. Their experimental results showed that a two-axis tracking module could collect 30 to 50% more energy on cloudy days if its tilt angle was changed to a horizontal position instead of being pointed directly at the sun. The researchers also noted that 50% of the days are cloudy in Detroit.

They proposed the concept of tracking advantage (TA), which compares the increase or decrease in solar radiation between the panel that is directly pointed at the sun (DTS) versus the horizontal panel (H), and they calculated it as

\[
TA = 1 - \frac{H}{\text{DTS}}
\]

\(2.12\)
CHAPTER 2. BACKGROUND AND LITERATURE REVIEW

In their patent \[11\], they proposed comparing the output of irradiance sensors in order to determine whether to position the 2-axis tracking array at the optimal tilt angle of 0°. In one embodiment of their invention, they use two irradiance sensors to determine when to track the sun. The first sensor (DTS) is directly pointed at the sun, while the second sensor (H) is placed horizontally. When the irradiance measured by the DTS sensor is greater than the irradiance measured by the H sensor by a predetermined value, the PV array tracks the sun \((\beta = \theta_z)\). Otherwise, the PV array is placed horizontally \((\beta = 0^\circ)\).

In a second embodiment, both irradiance sensors are placed horizontally. The first sensor is shaded from the sun and measures the diffuse horizontal irradiance (DHI). The second sensor (GHI) is exposed to the sun and measures the direct horizontal and diffuse horizontal irradiance. The array tracks the sun when the measured GHI exceeds the measured diffuse horizontal irradiance by a predetermined value. Otherwise, the PV array is placed at a tilt angle of 0°.

In another embodiment, a single irradiance sensor (DTS) is pointed at the sun. When the measured DTS irradiance is greater than a predetermined value, the PV array tracks the sun. Otherwise, the PV array is placed horizontally. The patent suggests a value of 250 W/m² as the boundary between tracking vs. non-tracking.

In a subsequent paper \[33\], the same researchers conducted additional experiments using four identical PV arrays that were fixed at four different tilt angles: 0°, latitude tilt, latitude−15°, and latitude+15° with a LI-COR irradiance sensor mounted on each array. They assumed that at solar noon the position of one of the arrays would be approximately the same as a 2-axis tracking array. For 30 minutes during solar noon, they recorded the irradiance measured on a horizontal sensor (H) and the irradiance measured on a sensor pointed directly toward the sun (DTS), and then calculated the ratio \(H/DTS\). They noted that the ratio of \(H/DTS = 1\) represents the boundary between tracking vs. non-tracking, and that the tracking advantage occurs when \(H/DTS < 1\). They determined that the value of irradiance that is the boundary between the conventional tracking angle and the modified angle of 0° is 340 W/m². Based on the experimental data, they reported that a PV array placed in a horizontal position (tilt angle of 0°) on a cloudy day would collect 50% more irradiance than an array pointed directly toward the sun. The annual increase in collected irradiance in Detroit was estimated to be about 1% when modifying the conventional 2-axis tracking tilt angle.

One limitation of this approach is that only two components of available irradiance are considered in the \(H/DTS\) ratio — the beam irradiance and the diffuse irradiance. The third component, ground-reflected irradiance, is not taken into account when calculating the tracking advantage (TA). A second limitation is that it gives only two options for positioning the 2-axis tracking array: place
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the array in a horizontal position ($0^\circ$) when the sky is overcast, or track the sun’s path ($\theta_z$). However, on partially cloudy days there may exist a more optimal tilt angle with a value between $0^\circ$ and $\theta_z$, where the maximum irradiance may be collected, as suggested in [9]. Another limitation of this method is that it requires one or more irradiance sensors, which may add to the complexity of the system.

In a related study by [6], the irradiance received by the PV array and the generated electrical power were simulated in MATLAB for five solar tracking systems under sunny, partially cloudy, and overcast skies for a location in Algeria. For overcast skies, the results confirmed those of [3, 33] that the optimal tilt angle is $0^\circ$ for a 2-axis tracking array.

2.4.2 Tracking advantage based on the critical hourly global solar radiation

In [10], the tracking advantage concept was extended by using the Liu and Jordan isotropic irradiance model [8] which considers all three components of solar irradiance (beam, diffuse, and ground-reflected) in calculating the total irradiance on a tilted surface $I_T$. They divided $I_T$ by the irradiance on a horizontal surface $I_H$ to obtain the ratio $I_T/I_H$, which resulted in the following equation:

$$\frac{I_T}{I_H} = \left(1 - \frac{I_{H,d}}{I_H}\right) R_b + \frac{I_{H,d}}{I_H} \left(\frac{1 + \cos \beta}{2}\right) + \rho_g \left(\frac{1 - \cos \beta}{2}\right)$$  \hspace{1cm} (2.13)

where

$\beta$ = tilt angle of the panel
$\rho_g$ = ground reflectance
$R_b$ = ratio of beam radiation on a tilted plane to that on a horizontal plane
$I_{H,d}$ = the hourly diffuse irradiation on a horizontal plane

They defined the critical hourly global solar radiation, $I_c$ as the value of $I_H$ for which $I_T/I_H = 1$. The tilted surface collects less solar radiation than a horizontal surface when the horizontal irradiance is less than the critical hourly global solar radiation, or $I_H < I_C$. By setting the condition that $I_C = I_T = I_H$, they derived a formula for the ratio of $I_{H,d}/I_H$ where $I_{H,d}$ represents the diffuse irradiance on a horizontal collector.

$$\frac{I_{H,d}}{I_C} = \frac{I_{H,d}}{I_H} = \frac{1 - \rho_g \left(\frac{1 - \cos \beta}{2}\right) - R_b}{\left(\frac{1 + \cos \beta}{2}\right) - R_b}$$  \hspace{1cm} (2.14)
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The decision to track the sun during each hour of the day is determined by estimating the value of \( I_C \).

\[
I_C = k_{tc} \cdot I_{H,0}
\]  
(2.15)

where

\( I_{H,0} \) = the extraterrestrial solar radiation on a horizontal surface

\( k_{tc} \) = the critical hourly clearness index for cloudy or partly sunny/cloudy skies.

The variable \( k_{tc} \) is calculated by substituting (2.14) into a rearranged Orgill and Hollands’ correlation \( k_{tc} \) for diffuse solar radiation.

\[
k_{tc} = \begin{cases} 
1 - \frac{I_{H,d}}{I_C} & \text{for } \frac{I_{H,d}}{I_C} > 0.91 \text{ for cloudy skies} \\
\frac{1.557 - \frac{I_{H,d}}{I_C}}{1.84} & \text{for } 0.177 \leq \frac{I_{H,d}}{I_C} \leq 0.91 \text{ for partly sunny/cloudy days}
\end{cases}
\]  
(2.16)

The researchers used commercial software to calculate the electricity produced by a hypothetical PV system connected to the grid. Under cloudy skies, the hourly increase in electricity production for the horizontal PV panel was calculated to be 18.8 to 46.9% greater than for the tracking panel on a day in winter, and 2.3% to 24.7% on a day in summer. An interesting result occurred for a cloudy winter’s day with high ground reflectance due to snow. In this case the simulation results indicated that, when compared with a panel that is directly pointed at the sun, there was a slight advantage (\( \approx 0.9\% \)) to placing the tracking PV panel in a fixed, south-facing position at a tilt angle of 60° throughout the day.

The simulations in [10] were validated by experimental results reported in [4]. To verify the observations reported in [3, 33] that 2-axis tracking panels in a horizontal position collect more solar radiation on overcast days, the Canadian researchers conducted a series of experiments. They used two identical solar panels — one mounted horizontally (tilt angle of 0°) and the other mounted on a 2-axis tracker. In the first experiment, the 2-axis tracking panel was placed in a fixed position facing south at a tilt angle of 60°. During several cloudy days in February and March 2014, pyranometers mounted on the two panels recorded the solar radiation collected at solar noon. It was found that the horizontal panel collected 3.63% to 23.8% more irradiance than the panel tilted at 60°. In the second experiment, the 2-axis tracking panel followed the sun’s path. The short circuit current of the 2-axis tracking panel and the horizontally-placed PV panel were measured over the course of several days. On one particular cloudy day (May 28, 2014), the short-circuit current of the PV panel in the
horizontal position was 4.1% to 32.3% greater during overcast periods than the short-circuit current for the 2-axis tracking PV panel. In the third experiment, 8-ohm resistive loads were installed on each panel and the energy production was measured. Over the course of one cloudy day (June 11, 2014), the total energy produced by the PV panel placed in the horizontal position was 27.37% more than that produced by the 2-axis tracking PV panel.

As a result of the simulation and experiments, the researchers proposed the following tracking algorithm in [10]: Under clear skies or partly sunny/partly cloudy skies ($I_H > I_C$), the 2-axis tracking panel follows the sun using the conventional tracking angles. The 2-axis tracking panel is placed in a horizontal position (tilt angle of $0^\circ$) when the sky is overcast and it is not snowing or there is no snow on the ground. However, when it is snowing or there is snow on the ground, and the sky is overcast, then the panel is placed in a fixed position, facing south, at the location’s optimal tilt angle. In their conclusions, the researchers mentioned plans to conduct experiments with the 2-axis PV tracker connected to the grid. They also planned to develop an algorithm that uses weather forecasts to determine the position that maximizes the electricity production of the 2-axis tracking array.

A limitation to this proposed method is as follows: when $I_T < I_H$ in (2.13), there is no tracking advantage and thus the researchers recommend placing the 2-axis tracking panel in a horizontal position (tilt angle $\beta = 0^\circ$). However, the tilt angle of $\beta = 0^\circ$ is not necessarily the optimal solution. Other researchers have shown in simulations that the optimal tilt angle is not limited to either $\theta_z$ or $0^\circ$, but may vary between those two angles depending upon the sky conditions [9]. In fact, the Canadian researchers own simulation results showed that a fixed tilt angle of $60^\circ$ was optimal when the sky was overcast and the ground reflectance was high due to snow cover.

### 2.4.3 Differential evolution (DE) optimization under clear skies

The authors in [34, 35] optimized 2-axis tracking arrays for maximum irradiance collection and minimization of power consumption under clear, sunny skies. They used optimization techniques to create daily trajectories for the tilt and azimuth angles. They calculated 30-minute averages of measured irradiance data collected over five years in Slovenia, and selected only the data for three sunny, cloudless days in spring, summer, and fall for simulation. They measured the power consumed in starting the motors and in changing the tilt and azimuth angles. They created an irradiance-to-electrical power model of a PV installation consisting of poly-crystalline PV panels, a DC/DC converter, and an inverter. In MATLAB, they first simulated a 2-axis PV system using only the
averaged irradiance data and the conventional 2-axis tracking angles ($\beta = \theta_z$ and $\gamma = \gamma_s$) to calculate the ideal electrical energy produced over a 24-hour period. They developed a differential evolution (DE) optimization problem that considered the energy consumed by the tracking mechanism as the tilt and azimuth angles were changed in different increments and then returned to a starting position at the end of the day, as well as the electrical energy produced by the array over the 24-hour period. They compared their DE optimization results with three other options – an array fixed at the daily optimal angles, an array changing angles every three hours, and a continuously tracking array. They found that the DE optimization method had a 10% to 50% increase in net energy produced than the other three options. Although they only considered sunny, cloud-free days, the authors noted that maximum energy gain could be achieved even on cloudy days by using methods to predict the movement of clouds.

In a subsequent paper [36], the authors proposed a new algorithm for moving a two-axis tracking array. They considered the electrical energy produced by the system, taking into account the efficiency of the module, the inverter, and the area of the panel and the irradiance. They developed their algorithm by taking the second derivative of the electrical energy function, and moving the tracking mechanism only when needed. Simulations were done using measured data for a sunny day for three cases — a continuously tracking array, an array that changed position every two hours, and an array that changed angles only at the times given by the proposed algorithm.

Although the researchers considered the energy produced and consumed by the complete tracking system, the study was limited to sunny days.

2.4.4 Artificial neural network (ANN) techniques combined with differential evolution (DE) optimization

In [9], an artificial neural network (ANN) method using the radial basis function (RBF), the unscented transformation (UT) method, and differential evolution (DE) optimization were used to generate daily trajectories for a 2-axis PV array with the goal of maximizing electricity production for clear, partially cloudy, and cloudy days. Weather data from Washington, DC was used to train and validate the RBF neural network. The bias and variance of the irradiance data were calculated in order to model uncertainty in a weather forecast. To develop the system model of the PV array, the authors used the UT method to calculate the Plane of Array (POA) irradiance, and they assumed a time-varying lumped thermal model to account for changes in the PV panels’ temperature. The weather data, the system model, and the energy consumed in positioning the array were all input
parameters to the DE algorithm. The DE algorithm solved for the optimal tilt and azimuth angles at 15-minute intervals for two sets of daily trajectories. The first set is a deterministic trajectory that assumes that the predicted weather will not change, while the second set is stochastic and takes into account the uncertainty in weather data and irradiance models. The daily trajectory was set once before sunrise based on a day-ahead weather forecast.

Simulation results showed that on a clear day, the conventional tracking angles ($\beta = \theta_z$ and $\gamma = \gamma_s$) were optimal for both types of trajectories. On an overcast day, the optimal positions were a tilt angle of 0° and an azimuth angle oriented due south for the deterministic trajectory. However, for the stochastic trajectory, the optimal tilt angles were found to be 20° to 30° less than the clear sky optimal angles ($\theta_z$) and the azimuth angles were $\gamma = \gamma_s$. On the partly cloudy day, the irradiance profile was primarily diffuse in the morning and beam irradiance in the afternoon. Throughout the day, the optimal azimuth angle was $\gamma_s$ for both types of trajectories. During the afternoon (clear sky), the conventional tracking angles of $\beta = \theta_z$ were optimal for both the deterministic and stochastic trajectories. However, in the morning when the sky was cloudy, the optimal tilt angle varied from about 15° to $\theta_z$ for the deterministic trajectory, and from 35° to $\theta_z$ for the stochastic trajectory. The electricity produced by a 2-axis tracking array using the DE trajectory was 0.3% greater than for an array using the conventional tracking angles, and 1.27% greater than for an array that continuously adjusted its tracking angles to maximize irradiance collection irrespective of the energy consumed by the tracker. These results indicate that the optimal tilt angles depend on the cloudiness of the sky and are not limited to the values of 0° and $\theta_z$.

These optimization methods have a couple of limitations. They require historical weather data in order to develop the daily trajectories. The trajectories are set once per day before sunset; thus, it may not be possible to alter the trajectory in response to sudden changes in sky conditions.

2.4.5 Mathematical model of maximum global solar irradiance (MGSI)

In [5] the performance of a 2-axis tracking PV module and a south-facing PV module fixed at the optimal tilt angle was compared for three days (sunny, cloudy, and partially cloudy) for a location in Romania. One year of data was collected on irradiance, electricity production, and power consumed in moving the tracking array. The researchers reported that placing the tracking array in a horizontal position on cloudy days resulted in increased irradiance collection and less power consumption in moving the actuators.

A mathematical model for finding the direction of the maximum global solar irradiance
CHAPTER 2. BACKGROUND AND LITERATURE REVIEW

(MGSI) was proposed in [37]. The optimal angle was derived from the equation for solar radiation incident on a surface,

\[ G_{\text{collected}} = \frac{B_h}{\sin \alpha} \cos(\alpha - \alpha_g) + \frac{D_h}{2} (1 + \sin \alpha_g) \]  

(2.17)

where the parameters were defined by the following notation:

- \( B_h \) = normal component of beam irradiance
- \( D_h \) = normal component of diffuse irradiance
- \( \alpha \) = elevation angle of the sun
- \( \alpha_g \) = elevation angle of the surface

After taking the derivative and solving for the tilt angle of the collector, the researchers proposed the following equations for describing the normal to the collector’s surface:

\[ \alpha_g = \arctan \left( \frac{1 + \frac{D_h}{2 B_h} \tan \alpha}{\tan \alpha} \right) \]  

(2.18)

\[ \Psi_g = \Psi \]  

(2.19)

where \( \Psi_g \) is the azimuth angle of the collector surface and \( \Psi \) is the sun’s azimuth angle.

Using measured weather data from three days in June 2012 (sunny, cloudy, and partially cloudy), the researchers simulated and compared the results of two tracking algorithms. The first algorithm tracks the sun’s position by changing the angles every hour. The second algorithm, referred to as the adaptive algorithm, adjusts the tracking position in unequal steps of time to the direction of the maximum solar irradiance, \( G_{\text{max}} \). Their simulations indicated that there is about an 8% increase in collected solar radiation on a cloudy day when the adaptive algorithm is used.

In a subsequent paper [38], they proposed an adaptive global irradiance tracking algorithm that positions the tracker in the direction of the maximum solar irradiance using one or more sensors that measure the global horizontal and diffuse irradiances. The clearness index, \( k = \frac{D_h}{G_h} \) was calculated over multiple time periods to determine whether the sky at a given time was clear, partially cloudy, or overcast. The decision to adjust a tracking angle depended on the measured clearness index and the calculated threshold clearness indices for clear and overcast skies. Measured weather data was used in simulating the adaptive global irradiance tracking algorithm and the sun-tracking algorithm. When compared with the sun-tracking algorithm, the adaptive algorithm collected about
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10% more solar irradiance on a cloudy day, and consumed less energy because it required fewer steps to adjust the tracking mechanism. On partially cloudy and sunny days, the amount of collected irradiance and steps for adjusting the angles were about the same for both tracking algorithms.

The proposed algorithms allow the possibility of optimal tilt angles between $0^\circ$ and $\theta_z$ depending upon the available direct irradiance and the diffuse irradiance. However, this approach does not take into account the ground-reflected irradiance, and it requires historical weather data to establish the threshold values for clearness indices. Another drawback is that the tracking mechanism depends upon one or more sensors for measuring the diffuse and global irradiances; these sensors could become soiled or even fail to operate.

2.4.6 Numerical methods using irradiance models and weather data

Although this method was used in [1] to compare the increase of solar radiation received by 2-axis time-position tracking arrays versus fixed and azimuth-tracking PV arrays, the conventional 2-axis tracking angles ($\beta = \theta_z$ and $\gamma = \gamma_S$) were used in calculating POA irradiance. Thus, the results did not take into account the possible increase in POA irradiance if the array could be placed in a more horizontal position on cloudy days.

In [7] the yearly POA irradiance was calculated for a 2-axis tracking PV array in 21 U.S. locations using Typical Meteorological Year 3 (TMY3) [39] data in MATLAB. The TMY3 database contains hourly weather data that has been averaged over a 15-year period for over 300 geographic locations. The data provides hourly values for beam irradiance, diffuse horizontal irradiance, global horizontal irradiance, and clearness index.

A MATLAB program was written to calculate the total solar radiation incident on the 2-axis tracking array for every hour of the year using the TMY3 solar radiation data and the Hay, Davies, Klucher, and Reindl (HDKR) anisotropic sky model [8] for $I_T$.

\[
I_T = I_b R_b + I_d \left[ (1 - A_i) \left( \frac{(1 + \cos \beta)}{2} \right) \left( 1 + f \sin^2 \left( \frac{\beta}{2} \right) \right) + A_i R_b \right] + I \rho_g \left( \frac{1 - \cos \beta}{2} \right) \tag{2.20}
\]

where

- $I_b$ = beam irradiance
- $I_d$ = horizontal diffuse irradiance
- $I$ = horizontal irradiance
- $A_i$ = anisotropy index
CHAPTER 2. BACKGROUND AND LITERATURE REVIEW

\[ f = \text{modulating index and is defined as } f = \sqrt{\left( \frac{I_b}{T} \right)} \]

\[ \beta = \text{tilt angle of panel} \]

\[ \rho_g = \text{ground reflectance} \]

\[ R_b = \text{ratio of beam radiation on a tilted plane to that on a horizontal plane, and is defined as} \]

\[ R_b = \frac{\cos \theta}{\cos \theta_z} \quad (2.21) \]

It was assumed that the tracking array could be placed at a non-conventional tilt angle; i.e, \( \beta \) did not necessarily have to equal \( \theta_z \). The hourly solar radiation on the tilted surface was calculated as the tilt angle was varied from 0° to 90° in fixed steps of 1° for each hour from sunrise to sunset on every day of the year. The tilt angle with the largest POA irradiance was declared the optimal tilt angle for the hour. The largest POA irradiance values for all hours were then summed to obtain the annual POA irradiance.

Depending on the local climate, the yearly POA irradiation collected by a 2-axis tracker positioned at optimal tilt angles was 0.56% to 3.4% greater than the yearly POA irradiation calculated using the conventional 2-axis tracking angles. In Figure 2.6, the potential increase in POA irradiance for nine U.S. locations is superimposed on a map of daily solar radiation. The largest increases in yearly collected irradiation occurred at locations at northerly latitudes that receive less solar radiation. For Detroit, the increase in yearly POA irradiation captured by a 2-axis tracking array at optimal tilt angles versus at the conventional 2-axis tilt angles was 1.88%, in comparison with 1% estimated in [33]. For Burlington, Vermont, the increase in yearly POA irradiation was 2.57%. Furthermore, the optimal tilt angles for collecting maximum irradiation ranged between 0° and \( \theta_z \) depending upon the clearness of the sky.

Figure 2.7 shows the relationship between the cloudiness of a location and the potential increase in annual POA irradiation when optimized 2-axis tracking is used. The NASA Annual Insolation Clearness Index for the location was used as an indicator of cloudiness [12]. As can be seen in the figure, locations that experience fewer clear days benefit the most from optimized 2-axis tracking. Locations with climates where sunny, clear skies are predominant throughout the year have little or no increase in POA irradiation when optimized 2-axis tracking is used.

The results for all 21 locations are listed in Table 2.1. These results indicate the potential increase in collected irradiation when the conventional 2-axis angle is adjusted to a more optimal angle in response to sky conditions.
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Figure 2.6: Potential increase in POA irradiance for optimizing 2-axis tracking using numerical method when compared with conventional 2-axis tracking. (Map of U.S. is from [40]).

Figure 2.7: Increase in annual POA irradiation for optimized 2-axis tracking (numerical method) when compared with conventional 2-axis tracking vs. Annual Insolation Clearness Index.
Table 2.1: Comparison of annual solar radiation absorbed on the cell surface for conventional 2-axis tracking vs. optimized 2-axis tracking using the numerical method to find optimal tilt angles. The PV panel’s orientation is changed once every daylight hour. Twenty-one U.S. locations are shown.

<table>
<thead>
<tr>
<th>Location</th>
<th>NASA Annual Insolation (deg N)</th>
<th>Latitude</th>
<th>Annual absorbed radiation using conventional 2-axis tracking (kWh/m²)</th>
<th>Annual absorbed radiation using numerical method to optimize tilt angles (kWh/m²)</th>
<th>Increase in annual absorbed radiation at optimized vs. conventional 2-axis tilt angles (kWh/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Olympia, WA</td>
<td>0.45</td>
<td>46.967</td>
<td>1660.11</td>
<td>1709.40</td>
<td>2.97%</td>
</tr>
<tr>
<td>Youngstown, OH</td>
<td>0.45</td>
<td>41.250</td>
<td>1778.94</td>
<td>1830.98</td>
<td>2.93%</td>
</tr>
<tr>
<td>Syracuse, NY</td>
<td>0.45</td>
<td>43.117</td>
<td>1929.70</td>
<td>1978.78</td>
<td>2.54%</td>
</tr>
<tr>
<td>Erie, PA</td>
<td>0.46</td>
<td>42.083</td>
<td>1827.34</td>
<td>1879.51</td>
<td>2.85%</td>
</tr>
<tr>
<td>Burlington, VT</td>
<td>0.46</td>
<td>44.467</td>
<td>1925.41</td>
<td>1974.93</td>
<td>2.57%</td>
</tr>
<tr>
<td>Zanesville, OH</td>
<td>0.46</td>
<td>39.950</td>
<td>1662.44</td>
<td>1705.02</td>
<td>2.56%</td>
</tr>
<tr>
<td>Detroit, MI</td>
<td>0.46</td>
<td>42.217</td>
<td>1953.82</td>
<td>1990.51</td>
<td>1.88%</td>
</tr>
<tr>
<td>Elkins, WV</td>
<td>0.46</td>
<td>38.883</td>
<td>1805.94</td>
<td>1856.35</td>
<td>2.79%</td>
</tr>
<tr>
<td>Bluefield, WV</td>
<td>0.46</td>
<td>37.270</td>
<td>1992.05</td>
<td>2033.30</td>
<td>2.07%</td>
</tr>
<tr>
<td>Traverse City, MI</td>
<td>0.47</td>
<td>44.733</td>
<td>1850.05</td>
<td>1901.37</td>
<td>2.77%</td>
</tr>
<tr>
<td>Eugene, OR</td>
<td>0.47</td>
<td>44.050</td>
<td>1979.07</td>
<td>2021.32</td>
<td>2.14%</td>
</tr>
<tr>
<td>Quillayute, WA</td>
<td>0.47</td>
<td>47.933</td>
<td>1553.73</td>
<td>1607.83</td>
<td>3.48%</td>
</tr>
<tr>
<td>Boston, MA</td>
<td>0.48</td>
<td>42.367</td>
<td>2059.91</td>
<td>2098.36</td>
<td>1.87%</td>
</tr>
<tr>
<td>Knoxville, TN</td>
<td>0.48</td>
<td>35.817</td>
<td>2194.10</td>
<td>2230.61</td>
<td>1.66%</td>
</tr>
<tr>
<td>Int’l Falls, MN²</td>
<td>0.49</td>
<td>48.567</td>
<td>1951.73</td>
<td>2005.51</td>
<td>2.76%</td>
</tr>
<tr>
<td>Asheville, NC</td>
<td>0.49</td>
<td>35.433</td>
<td>2274.87</td>
<td>2310.01</td>
<td>1.55%</td>
</tr>
<tr>
<td>Sioux City, IA</td>
<td>0.50</td>
<td>42.383</td>
<td>2322.68</td>
<td>2361.35</td>
<td>1.66%</td>
</tr>
<tr>
<td>Medford, OR</td>
<td>0.56</td>
<td>42.190</td>
<td>2419.20</td>
<td>2459.36</td>
<td>1.66%</td>
</tr>
<tr>
<td>Key West, FL</td>
<td>0.61</td>
<td>24.550</td>
<td>2591.54</td>
<td>2606.06</td>
<td>0.56%</td>
</tr>
<tr>
<td>Bakersfield, CA</td>
<td>0.62</td>
<td>35.433</td>
<td>2818.00</td>
<td>2837.89</td>
<td>0.71%</td>
</tr>
<tr>
<td>Las Vegas, NV</td>
<td>0.62</td>
<td>36.083</td>
<td>3255.49</td>
<td>3284.18</td>
<td>0.88%</td>
</tr>
</tbody>
</table>

² International Falls, MN
CHAPTER 2. BACKGROUND AND LITERATURE REVIEW

2.5 Summary

Although the specific features of commercial 2-axis time-position tracking PV arrays may vary, we found that they share a common feature—they all track the apparent position of the sun under clear, partly cloudy, and overcast skies. As shown in Figure 1.1, more irradiance could be gleaned from the total available irradiance by not tracking the sun’s position.

We summarized the published research on finding the optimal angles of 2-axis tracking arrays under varying sky conditions. For all proposed methods—tracking advantage, optimization, and numerical—the captured irradiation increased when the conventional 2-axis tracking angles were changed to a more optimal angle. However, a few of the limitations of the proposed methods are:

- The contributions of all three irradiance stream (beam, diffuse, and ground-reflected) were not considered.
- The optimal angle was limited to 0° or θz.
- Historical weather data was required to create daily trajectories.
- One or more sensors were required.
- Computational power was required.

We also showed the potential increase in yearly irradiation when the tilt angles of a hypothetical 2-axis array were adjusted in response to changing sky conditions. Results for the numerical method and the HDKR irradiance model indicated that, when using optimal tilt angles, the potential increase in yearly irradiation ranged from 0.56% to 3.48% when compared with conventional 2-axis tracking. Cloudier locations experienced higher increases in captured yearly irradiation.

The focus of our research is to develop a simple formula that directly calculates the optimal tilt angle for a 2-axis PV array under varying sky conditions. This simple formula could potentially be used by any 2-axis time-position tracking PV array to point in the direction that maximizes the collection of total available radiation. In Chapter 3 we shall derive a formula for directly calculating the optimal tilt angles of a 2-axis time-position tracking array.
Chapter 3

S Formula method: Derivation

3.1 Introduction

In Chapter 3, we start with the Liu and Jordan equation for solar radiation absorbed on the cell surface of a photovoltaic panel, and derive the proposed Parent S Formula for 2-axis tracking. We then derive a family of S Formulas that could be used for other types of PV arrays in other orientations, such as 1-axis tracking and fixed PV arrays. Finally, we demonstrate the potential increase in yearly irradiance using the proposed S Formula for 2-axis tracking.

3.2 Tracking angles for a 2-axis time-position array under varying sky conditions

In the Liu and Jordan isotropic irradiance model, the amount of solar radiation on a tilted surface $I_T$ is the sum of the beam radiation $I_{b,T}$, diffuse radiation $I_{d,T}$, and reflected radiation $I_{r,T}$, where the unit for all irradiation is Wh/m$^2$. However, this model can be modified to take into account incident radiation, air mass, and incidence angle to find the solar radiation $I_{T,PV}$ absorbed on the cell surface of a photovoltaic (PV) panel. Symbols are defined in the nomenclature section at the beginning of this dissertation.

$$I_T = I_{b,T} + I_{d,T} + I_{r,T}$$ (3.1)
CHAPTER 3. S FORMULA METHOD: DERIVATION

Using Equation 5.12.1 from [8], \( I_{T, PV} \) is found to be

\[
I_{T, PV} = M \{(\tau \alpha)_b I_{b,T} + (\tau \alpha)_d I_{d,T} + (\tau \alpha)_r I_{r,T}\}
= M \{(\tau \alpha)_b I_b R_b + (\tau \alpha)_d I_d \frac{(1 + \cos \beta)}{2} + (\tau \alpha)_r I_\rho \frac{(1 - \cos \beta)}{2}\}
\]

(3.2)

where

\(\beta\) = tilt angle of panel

\(\rho_g\) = ground reflectance

\(R_b\) = ratio of beam radiation on a tilted plane to that on a horizontal plane, and

\[
R_b = \frac{\cos \theta}{\cos \theta_z}
\]

(3.3)

\(I_d = I \cdot g(k_T)\) is the hourly diffuse irradiation where \(g(k_T)\) is defined by correlations proposed by Erbs, Orgill and Hollands, Lam and Li, and others [41 42 43 44 45 46 47]. These correlations are shown in Appendix B.

\(I\) = irradiation on a horizontal surface \(k_T = \) hourly clearness index and is defined as the ratio of the hourly irradiation on a horizontal surface to the hourly extraterrestrial radiation, and

\[
k_T = \frac{I}{I_o}
\]

(3.4)

\(I_b\) = hourly beam irradiation on a horizontal surface, which is calculated as

\[
I_b = I - I_d
\]

(3.5)

\((\tau \alpha)_b\) = transmittance-absorptance product of the beam radiation stream

\((\tau \alpha)_d\) = transmittance-absorptance product of the diffuse radiation stream

\((\tau \alpha)_r\) = transmittance-absorptance product of the reflected radiation stream

\(M\) = air mass modifier

\(\theta\) = angle of incidence of beam radiation on a surface, and

\[
\cos \theta = b_1 \cos \beta + b_2 \sin \beta + b_3 \cos \beta + b_4 \sin \beta + b_5 \sin \beta
\]

(3.6)
where the coefficients $b_1, b_2, b_3, b_4,$ and $b_5$ are defined as

\begin{align*}
    b_1 &= \sin \delta \sin \phi \\
    b_2 &= -\sin \delta \cos \phi \cos \gamma \\
    b_3 &= \cos \delta \cos \phi \cos \omega \\
    b_4 &= \cos \delta \sin \phi \cos \gamma \cos \omega \\
    b_5 &= \cos \delta \sin \gamma \sin \omega
\end{align*}

$\theta_z$ = zenith angle of the sun, and

$$
    \cos \theta_z = \cos \delta \cos \phi \cos \omega + \sin \delta \sin \phi = b_3 + b_1
$$

Substituting (3.3) through (3.8) into (3.2) results in the following expression:

\[
    I_{T, PV} = M \left\{ \tau_\alpha b I_o k_T (1 - g(k_T)) \left[ (b_1 + b_3) \cos \beta + (b_2 + b_4 + b_5) \sin \beta \right] + \right. \\
    & \left. \tau_\alpha d I_o k_T g(k_T) \left[ \frac{1 + \cos \beta}{2} \right] + \tau_\alpha r I_o k_T \rho g \left( \frac{1 - \cos \beta}{2} \right) \right\} \tag{3.9}
\]

We then take the derivative of (3.9) with respect to $\beta$ and set it equal to zero.

\[
    \frac{dI_{T, PV}}{d\beta} = M \left\{ \tau_\alpha b I_o k_T (1 - g(k_T)) \left[ \frac{(b_1 + b_3) \cos \beta}{\cos \theta_z} - (b_2 + b_4 + b_5) \sin \beta \right] + \right. \\
    & \left. \tau_\alpha d I_o k_T g(k_T) \left[ \frac{1 + \cos \beta}{2} \right] + \tau_\alpha r I_o k_T \rho g \frac{\sin \beta}{2} \right\} = 0 \tag{3.10}
\]

The $M$ variable drops out, resulting in

\[
    (\tau_\alpha b I_o k_T (1 - g(k_T)) \left[ \frac{(b_1 + b_3) \sin \beta}{\cos \theta_z} - (b_2 + b_4 + b_5) \cos \beta \right] \\
    = - (\tau_\alpha b I_o k_T (1 - g(k_T)) \left[ \frac{(b_2 + b_4 + b_5) \cos \beta}{\cos \theta_z} \right] \tag{3.11}
\]

Collecting the terms on the left-hand side of (3.11),

\[
    \sin \beta \left[ (\tau_\alpha b I_o k_T (1 - g(k_T)) \left[ \frac{(b_1 + b_3) \sin \beta}{\cos \theta_z} - (b_2 + b_4 + b_5) \cos \beta \right] \\
    = - \cos \beta \left[ (\tau_\alpha b I_o k_T (1 - g(k_T)) \left[ \frac{(b_2 + b_4 + b_5)}}{\cos \theta_z} \right] \right] \tag{3.12}
\]

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CHAPTER 3. S FORMULA METHOD: DERIVATION

Rearranging terms in (3.12) and using \( \tan \beta = \frac{\sin \beta}{\cos \beta} \) yields

\[
\tan \beta = - \frac{(\tau \alpha)_b I_o k_T (1 - g(k_T)) \left( \frac{b_2 + b_4 + b_5}{\cos \theta_z} \right)}{(\tau \alpha)_b I_o k_T (1 - g(k_T)) \left( \frac{b_1 + b_3}{\cos \theta_z} \right) - \frac{(\tau \alpha)_d I_o k_T g(k_T)}{2} - \frac{(\tau \alpha)_r I_o k_T \rho_g}{2}}
\]

(3.13)

Canceling out the terms \( I_o k_T \) results in

\[
\tan \beta = - \frac{(\tau \alpha)_b (1 - g(k_T)) \left( \frac{b_2 + b_4 + b_5}{\cos \theta_z} \right)}{(\tau \alpha)_b (1 - g(k_T)) \left( \frac{b_1 + b_3}{\cos \theta_z} \right) - \frac{(\tau \alpha)_d g(k_T)}{2} + \frac{(\tau \alpha)_r \rho_g}{2}}
\]

(3.14)

Multiplying the numerator and denominator by \( \frac{\cos \theta_z}{(1 - g(k_T))} \) and canceling out the minus signs yields

\[
\tan \beta = \frac{(\tau \alpha)_b (b_2 + b_4 + b_5)}{(\tau \alpha)_b (b_1 + b_3) + (b_1 + b_3) \left( \frac{(\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho_g}{2(1 - g(k_T))} \right)}
\]

(3.15)

Substituting (3.8) into (3.15) yields

\[
\tan \beta = \frac{(\tau \alpha)_b (b_2 + b_4 + b_5)}{(\tau \alpha)_b (b_1 + b_3) + (b_1 + b_3) \left( \frac{(\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho_g}{2(1 - g(k_T))} \right)}
\]

(3.16)

Substituting the constants shown in (3.7a) to (3.7e) into (3.16) results in

\[
\tan \beta = \left[ \frac{- \sin \delta \cos \phi \cos \gamma + \cos \delta \sin \phi \cos \gamma \cos \omega + \cos \delta \sin \gamma \sin \omega}{\sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega} \right] \times \left( \frac{(\tau \alpha)_b}{(\tau \alpha)_b + \frac{(\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho_g}{2(1 - g(k_T))}} \right)
\]

(3.17)

We then multiply the numerator and denominator of the second term by \( 2(1 - g(k_T)) \).

This is important because it will allow the second term to be 0 when \( k_T \) is 1, which occurs when the clearness index is nearly zero.

\[
\tan \beta = \left[ \frac{(- \sin \delta \cos \phi \cos \gamma + \cos \delta \sin \phi \cos \gamma \cos \omega + \cos \delta \sin \gamma \sin \omega)}{(\sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega)} \right] \times \left[ \frac{2(\tau \alpha)_b (1 - g(k_T))}{2(\tau \alpha)_b (1 - g(k_T)) + (\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho_g} \right]
\]

(3.18)

Observe that the denominator of the first term in (3.17) and (3.18) is \( \cos \theta_z \). Observe that the numerator is the equation for the angle of incidence of a vertical wall \( +90 \degree \). We shall now show that the numerator of the first term equals \( \sin \theta_z \cdot \cos(\gamma_s - \gamma) \).
CHAPTER 3. S FORMULA METHOD: DERIVATION

Recall that there are two equations for calculating the angle of incidence, \( \theta \). These are presented in [8] as Equations 1.6.2 and 1.6.3. One equation (3.6) is a function of the latitude, declination angle, hour angle, surface tilt angle, and surface azimuth angle. The other equation is a function of the zenith angle, solar azimuth angle, surface tilt angle, and surface azimuth angle.

\[
\cos \theta = \cos \theta_z \cos \beta + \sin \theta_z \sin \beta \cos (\gamma_s - \gamma) \tag{3.19}
\]

We set (3.6) and (3.19) equal to each other

\[
\sin \delta \sin \phi \cos \gamma + \cos \delta \cos \phi \cos \omega \sin \beta \\
+ \cos \delta \sin \phi \cos \omega \sin \beta + \cos \delta \sin \gamma \sin \omega \sin \beta \\
= \cos \theta_z \cos \beta + \sin \theta_z \sin \beta \cos (\gamma_s - \gamma) \tag{3.20}
\]

For \( \beta = 90^\circ \), \( \cos 90^\circ = 0 \) and \( \sin 90^\circ = 1 \). Substituting these values into (3.20) results in the following equation:

\[
- \sin \delta \cos \phi \cos \gamma + \cos \delta \sin \phi \cos \gamma \cos \omega + \cos \delta \sin \gamma \sin \omega = \sin \theta_z \cos (\gamma_s - \gamma) \tag{3.21}
\]

Notice that the left-hand side of (3.21) is the same as the numerator of (3.18). Substituting the right-hand side of (3.21) into the numerator of (3.18), as well as substituting \( \cos \theta_z \) into the denominator, yields a simpler and more meaningful equation.

\[
\tan \beta = \left[ \frac{(\sin \theta_z \cos (\gamma - \gamma_s))}{\cos \theta_z} \right] \left[ \frac{2(\tau \alpha)_{b}(1 - g(kT))}{2(\tau \alpha)_{b}(1 - g(kT)) + (\tau \alpha)_{d} g(kT) - (\tau \alpha)_{r} \rho g} \right] \tag{3.22}
\]

Recall that in (2.10) the conventional tilt angle of a time-position 2-axis solar tracker is \( \theta_z \). The first term on the right-hand side of (3.22) contains the variable \( \theta_z \); thus, the optimal tilt angle is related to the sun’s zenith angle. The second term on the right-hand side of (3.22) depends on the clearness index; thus, it modifies the conventional tilt angle as the cloudiness of the sky changes.

If we assume that the tracking surface’s azimuth angle will be the same as the sun’s azimuth angle, \( \gamma = \gamma_s \), then the equation for the optimal tilt angle is

\[
\tan \beta_{\text{optimal}} = [\tan \theta_z] \left[ \frac{2(\tau \alpha)_{b}(1 - g(kT))}{2(\tau \alpha)_{b}(1 - g(kT)) + (\tau \alpha)_{d} g(kT) - (\tau \alpha)_{r} \rho g} \right] \tag{3.23}
\]
CHAPTER 3. S FORMULA METHOD: DERIVATION

For overcast skies, the correlation \( g(k_T) \) approaches 1, so the numerator of the second term on the right-hand side of (3.23) will approach 0. Thus will result in \( \beta_{\text{opt}} \approx 0^\circ \), which is in agreement with the optimal tilt angle proposed in [3]. For clear, sunny skies, the correlation \( g(k_T) \) approaches 0, so the second term on the right-hand side of (3.23) will approach 1. This will result in \( \beta_{\text{opt}} \approx \theta_z \), which is in agreement with the conventional tilt angle shown in (2.10).

The optimal tilt angle calculated in this mathematically-derived formula (3.23) is a function of the zenith angle, the clearness index, the correlation \( g(k_T) \), the ground reflectance, and the transmittance-absorptance products of the beam, diffuse and reflected radiation streams. This formula can be used by a 2-axis time-position tracking array to adjust the PV panel’s tilt angle in response to changing sky conditions.

Recalling that \( g(k_T) = \frac{I_d}{I} \), the optimal tilt angle can be calculated by substituting measured or forecast data for diffuse irradiance and global horizontal irradiance into (3.18), (3.22) and (3.23).

\[
\tan \beta = \frac{\left( -\sin \delta \cos \phi \cos \gamma + \cos \delta \sin \phi \cos \gamma \cos \omega + \cos \delta \sin \gamma \sin \omega \right)}{\left( \sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega \right)} \times \frac{2(\tau \alpha)_b(1 - I_d)}{2(\tau \alpha)_b(1 - I_d) + (\tau \alpha)_d \frac{I_d}{I} - (\tau \alpha)_r \rho_g}
\]

\[\tan \beta = \tan \theta_z \cos(\gamma - \gamma_s) \left[ \frac{2(\tau \alpha)_b(1 - I_d)}{2(\tau \alpha)_b(1 - I_d) + (\tau \alpha)_d \frac{I_d}{I} - (\tau \alpha)_r \rho_g} \right]
\]

3.3 A family of S Formulas for calculating the optimal tilt angles of PV arrays

From the two versions of the Parent S Formulas (3.18) and (3.22), we can derive a family of formulas for calculating the optimal tilt angle for different configurations of PV panels. In the case of the 1-axis PV panel that continuously rotates about a fixed east-west axis and faces due south (\( \gamma = 0^\circ \)) (see Figure 2.3b), the formulas for the optimal tilt angle as a function of \( g(k_T) \) and \( \frac{I_d}{I} \) are

\[
\tan \beta = \tan \theta_z \cos \gamma_s \left[ \frac{2(\tau \alpha)_b(1 - g(k_T))}{2(\tau \alpha)_b(1 - g(k_T)) + (\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho_g} \right]
\]
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\[ \tan \beta = \tan \theta_z \cos \gamma_s \left[ \frac{2(\tau \alpha)_b(1 - \frac{I_d}{T})}{2(\tau \alpha)_b(1 - \frac{I_d}{T}) + (\tau \alpha)_d \frac{I_d}{T} - (\tau \alpha)_r \rho_g} \right] \]  (3.28)

In some locations, the tilt angle of a 1-axis PV array may be manually changed seasonally, monthly, or at other times of the year to collect the maximum amount of irradiance [1, 48, 49, 50, 51, 52]. Using the Parent S Formula (3.18) we can derive a formula for the optimal tilt angle of a 1-axis PV array that is installed at a fixed azimuth angle \( \gamma \) and is rotated on an east-west axis once a day, once a month, or once a season. At solar noon, the hour angle \( \omega \) equals 0°. If we assume that the tilt angle at solar noon represents an average optimal tilt angle for the day, then we can set the hour angle \( \omega \) equal to 0° in the Parent S Formula. The variable \( n \) is the number corresponding to the day of the year, as defined in [8]. This number, corresponding to the day that the array's tilt is manually adjusted, is used in (3.18) to obtain

\[ \tan \beta = \cos \gamma \left[ \frac{(- \sin \delta \cos \phi + \cos \delta \sin \phi)}{(\sin \delta \sin \phi + \cos \delta \cos \phi)} \right] \left[ \frac{2(\tau \alpha)_b(1 - g(k_T))}{2(\tau \alpha)_b(1 - g(k_T)) + (\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho_g} \right] \]  (3.29)

Substituting the trigonometric identities \( \cos(\phi - \delta) = \cos \phi \cos \delta + \sin \phi \sin \delta \) and \( \sin(\phi - \delta) = \sin \phi \cos \delta - \cos \phi \sin \delta \) into (3.29) results in

\[ \tan \beta = \cos \gamma \left[ \frac{\sin(\phi - \delta)}{\cos(\phi - \delta)} \right] \left[ \frac{2(\tau \alpha)_b(1 - g(k_T))}{2(\tau \alpha)_b(1 - g(k_T)) + (\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho_g} \right] \]  (3.30)

The first fraction in (3.30) is equal to \( \tan(\phi - \delta) \). Thus, the formulas for the optimal tilt angle of a 1-axis PV panel that is installed at a fixed azimuth angle and that rotates about an east-west axis as a function of \( g(k_T) \) and \( \frac{I_d}{T} \) are

\[ \tan \beta = \tan(\phi - \delta) \cos \gamma \left[ \frac{2(\tau \alpha)_b(1 - g(k_T))}{2(\tau \alpha)_b(1 - g(k_T)) + (\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho_g} \right] \]  (3.31)

\[ \tan \beta = \tan(\phi - \delta) \cos \gamma \left[ \frac{2(\tau \alpha)_b(1 - \frac{I_d}{T})}{2(\tau \alpha)_b(1 - \frac{I_d}{T}) + (\tau \alpha)_d \frac{I_d}{T} - (\tau \alpha)_r \rho_g} \right] \]  (3.32)

A formula for calculating the yearly optimal tilt angle of a PV panel installed at a fixed azimuth angle can be derived from (3.31) and (3.32). Although the earth’s declination angle \( \delta \) varies
throughout the year from -23.5° to +23.5°, it has an average yearly value of 0°. By setting \( \delta = 0^\circ \) in (3.31) and (3.32), the formulas for the yearly optimal tilt angle for a fixed, south-facing PV panel as a function of \( g(k_T) \) and of \( \frac{I_d}{I} \) are

\[
\tan \beta = \tan \phi \cos \gamma \left[ \frac{2(\tau \alpha) b(1 - g(k_T))}{2(\tau \alpha)_b(1 - g(k_T)) + (\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho_g} \right]
\] (3.33)

\[
\tan \beta = \tan \phi \cos \gamma \left[ \frac{2(\tau \alpha)_d(1 - \frac{I_d}{I})}{2(\tau \alpha)_b(1 - \frac{I_d}{I}) + (\tau \alpha)_d \frac{I_d}{I} - (\tau \alpha)_r \rho_g} \right]
\] (3.34)

It is very easy to extend (3.33) and (3.34) to the case of a fixed, south-facing PV array with a surface azimuth angle of \( \gamma = 0^\circ \). The resulting equations are

\[
\tan \beta = \tan \phi \left[ \frac{2(\tau \alpha) b(1 - g(k_T))}{2(\tau \alpha)_b(1 - g(k_T)) + (\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho_g} \right]
\] (3.35)

\[
\tan \beta = \tan \phi \left[ \frac{2(\tau \alpha)_d(1 - \frac{I_d}{I})}{2(\tau \alpha)_b(1 - \frac{I_d}{I}) + (\tau \alpha)_d \frac{I_d}{I} - (\tau \alpha)_r \rho_g} \right]
\] (3.36)

Tables 3.1 and 3.2 show the family of S Formulas. The formulas in Table 3.1 can be used if both the clearness index \( k_T \) and an appropriate correlation \( g(k_T) \) are known. Table 3.2 shows the formulas as functions of \( \frac{I_d}{I} \). If weather data or forecasts are available, then the measured or forecast values of \( I_d \) and \( I \) can be used in the formulas shown in Table 3.2.
Table 3.1: The S Formula family of equations for directly calculating the optimal tilt angles of PV arrays using the clearness index $k_T$ and the correlation $g(k_T)$

<table>
<thead>
<tr>
<th>Name</th>
<th>Formula</th>
<th>Eqn.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$\beta_{\text{optimal}}$</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parent S Formula Any day &amp; any time.</td>
<td>$\tan \beta = \left[ \frac{- \sin \delta \cos \phi \cos \gamma + \cos \delta \sin \phi \cos \gamma \cos \omega + \cos \delta \sin \gamma \sin \omega}{\sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega} \right] \times \left[ \frac{2(\tau \alpha)_b(1 - g(k_T))}{2(\tau \alpha)_b(1 - g(k_T)) + (\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho g} \right]$</td>
<td>(3.18)</td>
</tr>
<tr>
<td><strong>$\beta_{\text{optimal}, 2-\text{axis}}$</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alternating Parent S Formula Any orientation. Any day &amp; any time.</td>
<td>$\tan \beta = \tan \theta_z \cos(\gamma - \gamma_s) \left[ \frac{2(\tau \alpha)_b(1 - g(k_T))}{2(\tau \alpha)_b(1 - g(k_T)) + (\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho g} \right]$</td>
<td>(3.22)</td>
</tr>
<tr>
<td><strong>$\beta_{\text{optimal}, 1-\text{axis}}$</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>South-facing array that is rotated on a single East-West axis. Any day &amp; any time.</td>
<td>$\tan \beta = \tan \theta_z \cos \gamma_s \left[ \frac{2(\tau \alpha)_b(1 - g(k_T))}{2(\tau \alpha)_b(1 - g(k_T)) + (\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho g} \right]$</td>
<td>(3.27)</td>
</tr>
<tr>
<td><strong>$\beta_{\text{optimal}, \text{daily}}$</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Array is mounted on a single East-West axis and tilt is adjusted once per day or month. $\phi = \text{latitude}$ and $\gamma = \text{surface azimuth angle}$ (or, $0^\circ$ if south-facing).</td>
<td>$\tan \beta = \tan(\phi - \delta) \cos \gamma \left[ \frac{2(\tau \alpha)_b(1 - g(k_T))}{2(\tau \alpha)_b(1 - g(k_T)) + (\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho g} \right]$</td>
<td>(3.31)</td>
</tr>
<tr>
<td><strong>$\beta_{\text{optimal}, \text{yearly}}$</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed array with $\phi = \text{latitude}$ and $\gamma = \text{surface azimuth angle}$ (or, $0^\circ$ if south-facing).</td>
<td>$\tan \beta = \tan \phi \cos \gamma \left[ \frac{2(\tau \alpha)_b(1 - g(k_T))}{2(\tau \alpha)_b(1 - g(k_T)) + (\tau \alpha)_d g(k_T) - (\tau \alpha)_r \rho g} \right]$</td>
<td>(3.33)</td>
</tr>
</tbody>
</table>
Table 3.2: The S Formula family of equations for directly calculating the optimal tilt angles of PV arrays using $I_d$ and $I$

<table>
<thead>
<tr>
<th>Name</th>
<th>Formula</th>
<th>Eqn.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{\text{optimal}}$</td>
<td>$\tan \beta = \left[ \frac{(- \sin \delta \cos \phi \cos \gamma + \cos \delta \sin \phi \cos \gamma \cos \omega + \cos \delta \sin \gamma \sin \omega)}{(\sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega)} \right] \times \left[ \frac{2(\tau \alpha)_b(1 - \frac{L_f}{F})}{2(\tau \alpha)_b(1 - \frac{L_f}{F}) + (\tau \alpha)_d \frac{L_f}{F} - (\tau \alpha)_r \rho_g} \right]$</td>
<td>(3.24)</td>
</tr>
<tr>
<td><strong>Alternate Parent S Formula</strong></td>
<td>$\tan \beta = \tan \theta_z \cos(\gamma - \gamma_s) \left[ \frac{2(\tau \alpha)_b(1 - \frac{L_f}{F})}{2(\tau \alpha)_b(1 - \frac{L_f}{F}) + (\tau \alpha)_d \frac{L_f}{F} - (\tau \alpha)_r \rho_g} \right]$</td>
<td>(3.25)</td>
</tr>
<tr>
<td>$\beta_{\text{optimal,2-axis}}$</td>
<td>$\tan \beta = \tan \theta_z \cos \gamma_s \left[ \frac{2(\tau \alpha)_b(1 - \frac{L_f}{F})}{2(\tau \alpha)_b(1 - \frac{L_f}{F}) + (\tau \alpha)_d \frac{L_f}{F} - (\tau \alpha)_r \rho_g} \right]$</td>
<td>(3.26)</td>
</tr>
<tr>
<td>$\beta_{\text{optimal,1-axis}}$</td>
<td>$\tan \beta = \tan \theta_z \cos \gamma_s \left[ \frac{2(\tau \alpha)_b(1 - \frac{L_f}{F})}{2(\tau \alpha)_b(1 - \frac{L_f}{F}) + (\tau \alpha)_d \frac{L_f}{F} - (\tau \alpha)_r \rho_g} \right]$</td>
<td>(3.28)</td>
</tr>
<tr>
<td>$\beta_{\text{optimal,daily}}$</td>
<td>$\tan \beta = \tan(\phi - \delta) \cos \gamma \left[ \frac{2(\tau \alpha)_b(1 - \frac{L_f}{F})}{2(\tau \alpha)_b(1 - \frac{L_f}{F}) + (\tau \alpha)_d \frac{L_f}{F} - (\tau \alpha)_r \rho_g} \right]$</td>
<td>(3.32)</td>
</tr>
<tr>
<td>$\beta_{\text{optimal,yearly}}$</td>
<td>$\tan \beta = \tan \phi \cos \gamma \left[ \frac{2(\tau \alpha)_b(1 - \frac{L_f}{F})}{2(\tau \alpha)_b(1 - \frac{L_f}{F}) + (\tau \alpha)_d \frac{L_f}{F} - (\tau \alpha)_r \rho_g} \right]$</td>
<td>(3.34)</td>
</tr>
</tbody>
</table>

* $\phi$ = latitude and $\gamma = $ surface azimuth angle (or, $0^\circ$ if south-facing).
3.4 Optimized 2-axis tracking for 21 U.S. locations in different climates

In this section, we present the potential increase in yearly POA irradiance when the tilt angles of a hypothetical 2-axis tracking array are modified in response to the cloudiness of the sky. An earlier version of the Parent S Formula, a correlation for \( g(k_T) \), and TMY3 data were used to estimate the hourly optimal tilt angles for 21 different U.S. locations.

The optimal tilt angle for each one hour period was computed using the S Formula (3.17). The hourly clearness index, \( k_T \), was calculated using the TMY3 data for global horizontal irradiance (GHI or \( I \)) and extraterrestrial irradiance \( I_o \) in (3.4). We assumed a value of 0.2 for \( \rho_g \), and a value of 1 for the transmittance-absorptance products \( (\tau_\alpha)_b \), \( (\tau_\alpha)_d \), and \( (\tau_\alpha)_r \). The panel’s azimuth angles were set to \( \gamma_s \).

The following correlation for \( g(k_T) \) was substituted in the S Formula (3.17):

\[
g(k_T) = \begin{cases} 
0.977 & \text{for } k_t \leq 0.15 \\
1.237 - 1.361k_t & \text{for } 0.15 < k_t \leq 0.7 \\
0.273 & \text{for } k_t \geq 0.7
\end{cases}
\]  

A MATLAB program was written to calculate the total solar radiation incident on the 2-axis tracking array for every hour of the year at the optimal tilt angles. The Hay, Davies, Klucher, and Reindl (HDKR) anisotropic sky model (2.20) and TMY3 solar radiation data were used.

Table 3.3 presents the mean annual irradiation for two 2-axis tracking PV arrays — one using the conventional 2-axis tracking angles and the other using the optimal tilt angles predicted by (3.17). For a 2-axis tracking system, there was an increase in annual incident radiation for 20 out of 21 locations when the tilt angle was adjusted in response to sky conditions using the S Formula method. This increase ranged from 0.09% to 2.01%, with the greatest increase occurring at cloudier locations. For one location – Key West, Florida – there was a small decrease of 0.02% in yearly insolation when the optimal tilt angles predicted by the S Formula were used instead of the conventional 2-axis tracking angles.

Figure 3.1 shows the relationship between the cloudiness of a location and the potential increase in annual POA irradiation when optimized 2-axis tracking using the S Formula’s optimal angles are used. The NASA Annual Insolation Clearness Index for the location was used as an indicator of cloudiness [12]. As can be seen in the figure, locations that experience fewer clear days benefit the most from optimized 2-axis tracking. Locations with climates where sunny, clear skies
Table 3.3: Comparison of annual solar radiation absorbed on the cell surface for conventional 2-axis tracking vs. optimized 2-axis tracking using the S Formula to directly calculate optimal tilt angles. The PV panel’s orientation is changed once every daylight hour. Twenty-one U.S. locations are shown.

<table>
<thead>
<tr>
<th>Location</th>
<th>NASA Annual Insolation Clearness Index K</th>
<th>Annual absorbed radiation using 2-axis tracking (kWh/m²)</th>
<th>Annual absorbed radiation using S Formula method to optimize tilt angles (kWh/m²)</th>
<th>Increase in annual absorbed radiation at optimized vs. conventional 2-axis tilt angles (kWh/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Olympia, WA</td>
<td>0.45</td>
<td>1660.11</td>
<td>1688.97</td>
<td>1.74%</td>
</tr>
<tr>
<td>Youngstown, OH</td>
<td>0.45</td>
<td>1778.94</td>
<td>1809.02</td>
<td>1.69%</td>
</tr>
<tr>
<td>Syracuse, NY</td>
<td>0.45</td>
<td>1929.70</td>
<td>1953.89</td>
<td>1.25%</td>
</tr>
<tr>
<td>Erie, PA</td>
<td>0.46</td>
<td>1827.34</td>
<td>1855.78</td>
<td>1.56%</td>
</tr>
<tr>
<td>Burlington, VT</td>
<td>0.46</td>
<td>1925.41</td>
<td>1949.15</td>
<td>1.23%</td>
</tr>
<tr>
<td>Zanesville, OH</td>
<td>0.46</td>
<td>1662.44</td>
<td>1695.44</td>
<td>1.98%</td>
</tr>
<tr>
<td>Detroit, MI</td>
<td>0.46</td>
<td>1953.82</td>
<td>1974.94</td>
<td>1.08%</td>
</tr>
<tr>
<td>Elkins, WV</td>
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<td>1805.94</td>
<td>1835.66</td>
<td>1.65%</td>
</tr>
<tr>
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<td>1992.05</td>
<td>2020.28</td>
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</tr>
<tr>
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<td>1873.20</td>
<td>1.25%</td>
</tr>
<tr>
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<td>1979.07</td>
<td>2000.70</td>
<td>1.09%</td>
</tr>
<tr>
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<td>1553.73</td>
<td>1585.00</td>
<td>2.01%</td>
</tr>
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<td>2059.91</td>
<td>2081.48</td>
<td>1.05%</td>
</tr>
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<td>2194.10</td>
<td>2213.57</td>
<td>0.89%</td>
</tr>
<tr>
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<td>1951.73</td>
<td>1975.09</td>
<td>1.20%</td>
</tr>
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<td>2291.03</td>
<td>0.71%</td>
</tr>
<tr>
<td>Sioux City, IA</td>
<td>0.50</td>
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<td>0.66%</td>
</tr>
<tr>
<td>Medford, OR</td>
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<td>2439.36</td>
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</tr>
<tr>
<td>Key West, FL</td>
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<td>2590.94</td>
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<tr>
<td>Bakersfield, CA</td>
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</tr>
<tr>
<td>Las Vegas, NV</td>
<td>0.62</td>
<td>3255.49</td>
<td>3258.44</td>
<td>0.09%</td>
</tr>
</tbody>
</table>

* a International Falls, MN
CHAPTER 3. S FORMULA METHOD: DERIVATION

dominate throughout the year have little or no increase in POA irradiation when optimized 2-axis tracking is used.

Figure 3.1: Increase in annual POA irradiation for optimized 2-axis tracking (S Formula method) when compared with conventional 2-axis tracking versus Annual Insolation Clearness Index. Locations with lower annual clearness indices show greater increases in annual irradiation.

3.5 Summary

In this chapter, we derived a simple formula, the Parent S Formula for estimating the optimal tilt angle of a 2-axis time-position tracking array under varying sky conditions. For overcast skies, the S Formula estimates the optimal tilt angle to be $\beta_{\text{optimal}} \approx 0^\circ$, which is in agreement with the optimal tilt angle proposed in [3]. For clear, sunny skies, the S Formula estimates the optimal tilt angle to be $\beta_{\text{optimal}} \approx \theta_z$, which is in agreement with the conventional tilt angle shown in (2.10).

We then presented results showing the potential increase in yearly POA irradiance if the tilt angles of a hypothetical 2-axis tracking array could be modified in response to the cloudiness of the sky. The S Formula, a correlation for $g(k_T)$, and TMY3 data for $k_T$ were used to estimate the hourly optimal tilt angles for 21 different U.S. locations. Results indicated that the yearly POA irradiation could increase up to 2% for cloudier locations when using the optimal tilt angles estimated by the S Formula instead of the conventional 2-axis tracking tilt angles. For Burlington, Vermont, the potential increase in yearly POA irradiation was 1.23% when using the S Formula method’s tilt angles versus the conventional 2-axis tracking tilt angles.
We also presented a table of formulas based on the Parent S Formula that may be used for finding the optimal tilt angles of other types of PV arrays, such as 1-axis tracking and fixed, south-facing arrays. In Chapter 4 we shall use one of these formulas to estimate the yearly optimal tilt angle of fixed, south-facing PV arrays.
Chapter 4

S Formula method for fixed, south-facing PV arrays

4.1 Introduction

In Chapter 4 we look at the special case of the S Formula for a south-facing fixed array that was derived in the previous chapter. We first briefly review the methods that have been proposed in the literature for finding the optimal tilt angle of a fixed array. We discuss other researchers’ methods of taking the derivative of an irradiance model and then solving for the optimal tilt angle using numerical or optimization techniques. We show how closely the formula estimates the yearly fixed optimal tilt angle by using the S Formula to calculate the optimal angles for 217 U.S. cities and 37 non-U.S. locations, and then comparing our results with those presented in the literature.

4.2 Literature review: Methods for finding yearly optimal tilt angles of fixed PV arrays

For fixed arrays, there have been many papers published on the optimal tilt angles that will maximize the yearly, monthly or seasonal collection of irradiation. In [32], the author presented several methods for finding the optimal orientation of fixed photovoltaic (PV) arrays. Here we briefly summarize each method described in [32], and also present a method that was not covered in [32].

Rules of thumb: In [32], these are called “tilt angle-latitude relations” because the tilt angle is calculated by adding or subtracting a constant value (in degrees) to the location’s latitude.
These equations are approximate and do not take into account the variability of the cloud conditions at the location.

**Correlations:** In [32], the author presents several correlations that other researchers have proposed for different locations. These correlations relate the optimal tilt angle to one or more parameters such as clearness index, latitude, day of the year, and declination angle.

**Numerical methods based on weather data and irradiance models:** Measured weather data is used in solar radiation equations to estimate the monthly, seasonal or yearly tilt angles. The tilt angle is varied from 0° to 90° in fixed steps, and the hourly solar radiation on the tilted surface (POA irradiance) is calculated for every hour of the year at each tilt angle. The hourly POA irradiance values for a particular tilt angle are then summed to find the total yearly POA irradiance at that tilt angle. The set of total yearly POA irradiance values are compared to find the maximum total yearly POA irradiance value. The tilt angle that corresponds to this maximum total yearly POA irradiance value is considered the optimal tilt angle for the fixed array. In [1], the optimum tilt angles and the mean annual irradiance for 217 U.S. locations were calculated for four configurations of fixed arrays: tilt angle is changed monthly, tilt angle is changed six times per year, tilt angle is changed twice per year, and tilt angle is not changed. This method was used in [7] to calculate the yearly optimum tilt angles and annual irradiance for 21 locations in the U.S. Both [1] and [7] varied the tilt angle from 0° to 90° in fixed steps of 1°.

**Optimization algorithms:** In [32] the authors review the optimization algorithms used by other researchers to determine the optimal tilt angle. These algorithms are the genetic algorithm (GA), simulated annealing (SA), and particle-swarm optimization (PSO). In [2] the authors calculated the optimal tilt angles and optimal azimuth angles for the continental United States using unconstrained linear optimization in MATLAB.

**Artificial Neural Network (ANN) techniques:** In [32], the authors briefly summarized the ANN techniques that have been used in finding the optimal tilt angle. These techniques are sequential neural-network approximation and orthogonal arrays (SNAOA), radial basis function neural network (RBFNN), and generalized regression neural network (GRNN).

**Mathematically-derived:** This category was not described in [32], but has been used in [7, 53, 54, 55, 31]. The optimal tilt angle of a fixed array is calculated by taking the derivative of a solar radiation equation with respect to the tilt angle $\beta$, setting it equal to 0, and then finding $\beta$ as a function of the remaining variables.

In [53], researchers calculated the annual, semi-annual, and monthly optimum tilt angles and the collected irradiance for three locations. In the Liu and Jordan isotropic model for irradiance
on a tilted surface, they substituted the monthly mean values of the diffuse radiation, the daily global solar radiation on a horizontal surface, and the ratio of the average beam radiation on the tilted surface to that on a horizontal surface, $R_b$. Taking the derivative of $R_b$, they proposed the following equation for the optimal tilt angle of the fixed surface, $\beta_{\text{optimal}}$:

$$
\beta_{\text{optimal}} = \tan^{-1} \left( \frac{(1 - \lambda) \cdot \alpha}{(1 - \frac{\lambda \cdot \rho_g}{2})} \right)
$$

(4.1)

where $\alpha$ is defined by the following expression:

$$
\alpha = \tan \phi \sin \omega' - \omega' \tan \delta \sin \omega + \omega \tan \phi \tan \delta
$$

(4.2)

The remaining symbols are defined as follows:

$\phi$ = latitude
$\omega$ = sunset hour angle
$\omega'$ = sunset hour angle for the tilted surface
$\delta$ = declination hour
$\rho_g$ = ground reflectance

where

$D_m$ = monthly mean of the diffuse radiation on a horizontal surface
$G_m$ = monthly mean of the daily global radiation on a horizontal surface for a given month

It should be noted that the paper does not show the steps between the starting equation for irradiance on a tilted surface and the final result shown in (4.1).

In [54] the optimal tilt angle of a collector is found by taking the derivative of the angle of incidence (AOI) with respect to the tilt angle. The proposed formula is:

$$
\tan \beta_{\text{optimal}} = \frac{-\sin \delta \cos \phi \cos \gamma + \cos \delta \sin \phi \cos \gamma \cos \omega + \cos \delta \sin \gamma \sin \omega}{\sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega}
$$

(4.3)

where

$\phi$ = latitude
$\omega$ = hour angle [note: this is defined differently than in [4.2]
$\delta$ = declination hour
$\gamma$ = surface azimuth angle
CHAPTER 4. S FORMULA METHOD FOR FIXED, SOUTH-FACING PV ARRAYS

The researchers took the second derivative of (4.3) to verify that it is less than zero (i.e., maximum value). Although (4.3) is valid for any time of the day and surface azimuth angle, it only maximizes the beam component of solar radiation, and does not include the irradiance contributed by the diffuse and reflected components. Thus, it is only valid for clear, sunny days.

In [55] the researchers investigated the sensitivity of the optimal tilt angle to various values of ground reflectance and at different latitudes. They started with the equation for the Liu and Jordan isotropic irradiance model for calculating irradiance on a tilted surface, which is the sum of the beam, diffuse, and reflected irradiance components. They took the integral of this equation over the year, and then took the partial derivative with respect to the tilt angle. After rearranging terms, the optimal tilt angle was found to be a function with integrals in the numerator and the denominator. The researchers then used numerical methods to solve for the optimal tilt angle. Although this proposed equation takes into account the three components of solar radiation on the tilted surface, it does not directly solve for the optimal tilt angle.

In [31] the monthly optimum tilt angles and monthly mean daily global solar radiation were calculated for several European locations. The author considered both the isotropic and anisotropic models of diffuse irradiance. The author derived three partial derivative equations by taking the derivative of the solar radiation on a tilted plane with respect to tilt angle, surface azimuth angle, and hour angle. Each equation was set to 0, and then the optimal tilt angle, optimal surface azimuth angle and optimal hour angle were found using iterative methods. The author found that the maximum solar irradiance occurred when the surface azimuth angle was set at due south and the hour angle was at solar noon.

4.3 S Formula method for calculating the yearly optimal tilt angles of fixed, south-facing PV arrays

A general formula that was dependent on the location’s clearness index was proposed by [7] for calculating the optimal tilt angle. This formula was different than the ones proposed by [53, 54, 55, 31] because it considered all three sources of irradiance, and it directly calculated the optimal tilt angle. This first published version of the S Formula for fixed, south-facing PV arrays is shown here:
CHAPTER 4. S FORMULA METHOD FOR FIXED, SOUTH-FACING PV ARRAYS

\[
\tan \beta_{\text{optimal}} = \tan \phi \cos \gamma \left[ \frac{1}{1 + \frac{[(\tau \alpha)_d g(k_T)-(\tau \alpha)_r \rho_g]}{2(1-g(k_T))}} \right]
\] (4.4)

For a south-facing fixed array, \( \gamma = 0^\circ \). It should be pointed out that the number “1” in the numerator and denominator should be \((\tau \alpha)_b\). The parameter \((\tau \alpha)_b\) was inadvertently omitted in the numerator and denominator in \([7]\). However, if we assume that the transmittance-absorptance product of beam radiation stream is ideal \((\tau \alpha)_b = 1\), then the results reported in \([7]\) do not change.

To calculate the yearly optimal tilt angle of a fixed, south-facing array, the only parameters that are required are the location’s latitude, the ground reflectance, the transmittance-absorptance products of the beam, diffuse, and reflected radiation streams, the average clearness index, and the correlation \(g(k_T)\). There are many correlations relating the diffuse horizontal irradiance to the global horizontal irradiance \(g(k_T)\). In this study, we used the following correlation for \(g(k_T)\) \([43]\):

\[
g(k_T) = \begin{cases} 
0.977 & \text{for } k_t \leq 0.15 \\
1.237 - 1.361k_t & \text{for } 0.15 < k_t \leq 0.7 \\
0.273 & \text{for } k_t \geq 0.7 
\end{cases}
\] (4.5)

The average annual insolation clearness index for \(k_T\) was used for each location, and the ground reflectance \(\rho_g\) was set to a value of 0.2. Values of 1 were assumed for the transmittance-absorptance products \((\tau \alpha)_d\) and \((\tau \alpha)_r\).

The proposed S Formula in \([7]\) and shown here as \((4.4)\) was used to estimate the yearly optimal tilt angles for a hypothetical fixed, south-facing array in 21 U.S. locations. As Table 4.1 shows, the formula’s estimated tilt angles were found to be within 1° to 5° of the values calculated using the numerical method. Although a rule of thumb is to install a fixed PV array at a tilt angle equal to its latitude \([32]\), the results show that a tilt angle that is 12° to 16° less than latitude is more optimal for cloudier locations.

To obtain the yearly POA irradiation, the Hay, Davies, Klucher, and Reindl (HDKR) anisotropic sky model \((2.20)\) and TMY3 weather data for each location were used to calculate the total radiation incident on the panel for each hour of the day and for each day of the year. Table 4.2 presents the mean annual irradiation for a south-facing PV panel for three cases: latitude tilt, optimal tilt angle found numerically, and estimated tilt angle predicted by \((4.4)\). For a fixed, south-facing PV array, there was an increase in annual incident radiation for 19 out of 21 locations when the tilt angle
was fixed at an angle that was optimal for the location’s climate. When compared with the yearly insolation calculated for the array fixed at a tilt angle equal to the location’s latitude, this increase ranged from 0.05% to 2.52% using the optimal angles calculated by the S Formula, and 0.03% to 2.65% using the numerical method’s optimal angles. The greatest increase in POA irradiance occurred at the cloudier locations such as Youngstown, Ohio and Olympia, Washington. For these locations, the rule-of-thumb of installing a fixed PV array at a tilt angle equal to the location’s latitude tilt may result in a 2% or more loss in captured insolation over the course of a year.

Table 4.1: Yearly optimal tilt angles for fixed, south-facing PV arrays for 21 U.S. locations. Optimal angles are calculated using the numerical method and directly using the S Formula.

<table>
<thead>
<tr>
<th>Location</th>
<th>NASA Annual Insolation Clearness Index K</th>
<th>Fixed at latitude tilt, (deg)</th>
<th>Fixed at optimal tilt angle, numerical method, (deg)</th>
<th>Fixed at optimal tilt angle, S Formula method, (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Olympia, WA</td>
<td>0.45</td>
<td>46.97</td>
<td>31</td>
<td>34.38</td>
</tr>
<tr>
<td>Youngstown, OH</td>
<td>0.45</td>
<td>41.25</td>
<td>27</td>
<td>29.26</td>
</tr>
<tr>
<td>Syracuse, NY</td>
<td>0.45</td>
<td>43.12</td>
<td>31</td>
<td>30.89</td>
</tr>
<tr>
<td>Erie, PA</td>
<td>0.46</td>
<td>42.08</td>
<td>27</td>
<td>30.58</td>
</tr>
<tr>
<td>Burlington, VT</td>
<td>0.46</td>
<td>44.47</td>
<td>33</td>
<td>32.71</td>
</tr>
<tr>
<td>Zanesville, OH</td>
<td>0.46</td>
<td>39.95</td>
<td>29</td>
<td>28.73</td>
</tr>
<tr>
<td>Detroit, MI</td>
<td>0.46</td>
<td>42.22</td>
<td>31</td>
<td>30.7</td>
</tr>
<tr>
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<td>0.46</td>
<td>38.88</td>
<td>29</td>
<td>27.82</td>
</tr>
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<td>0.46</td>
<td>37.27</td>
<td>32</td>
<td>26.47</td>
</tr>
<tr>
<td>Traverse City, MI</td>
<td>0.47</td>
<td>44.73</td>
<td>30</td>
<td>33.56</td>
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<td>44.05</td>
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<td>32.93</td>
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<td>47.93</td>
<td>35</td>
<td>36.57</td>
</tr>
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<td>42.37</td>
<td>36</td>
<td>31.98</td>
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<td>Knoxville, TN</td>
<td>0.48</td>
<td>35.82</td>
<td>29</td>
<td>24.75</td>
</tr>
<tr>
<td>International Falls, MN</td>
<td>0.49</td>
<td>48.57</td>
<td>38</td>
<td>38.38</td>
</tr>
<tr>
<td>Asheville, NC</td>
<td>0.49</td>
<td>35.43</td>
<td>32</td>
<td>26.45</td>
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<td>42.38</td>
<td>35</td>
<td>33.06</td>
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<td>42.19</td>
<td>31</td>
<td>35.69</td>
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<td>0.61</td>
<td>24.55</td>
<td>23</td>
<td>21.26</td>
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<td>Bakersfield, CA</td>
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<td>35.43</td>
<td>28</td>
<td>31.54</td>
</tr>
<tr>
<td>Las Vegas, NV</td>
<td>0.62</td>
<td>36.08</td>
<td>33</td>
<td>32.16</td>
</tr>
<tr>
<td>Location</td>
<td>Latitude (deg N)</td>
<td>Annual absorbed radiation (kWh/m²)</td>
<td>Annual absorbed radiation (kWh/m²)</td>
<td>Increase in annual absorbed radiation at Numerical optimal tilt vs. latitude tilt</td>
</tr>
<tr>
<td>-------------------</td>
<td>------------------</td>
<td>-----------------------------------</td>
<td>------------------------------------</td>
<td>---------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Olympia, WA</td>
<td>46.97</td>
<td>1279.27</td>
<td>1313.13</td>
<td>2.65%</td>
</tr>
<tr>
<td>Youngstown, OH</td>
<td>41.25</td>
<td>1406.39</td>
<td>1434.61</td>
<td>2.01%</td>
</tr>
<tr>
<td>Syracuse, NY</td>
<td>43.12</td>
<td>1485.57</td>
<td>1509.96</td>
<td>1.64%</td>
</tr>
<tr>
<td>Erie, PA</td>
<td>42.08</td>
<td>1431.15</td>
<td>1466.55</td>
<td>2.47%</td>
</tr>
<tr>
<td>Burlington, VT</td>
<td>44.47</td>
<td>1483.86</td>
<td>1504.82</td>
<td>1.41%</td>
</tr>
<tr>
<td>Zanesville, OH</td>
<td>39.95</td>
<td>1328.85</td>
<td>1346.15</td>
<td>1.30%</td>
</tr>
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<td>42.22</td>
<td>1522.66</td>
<td>1541.58</td>
<td>1.24%</td>
</tr>
<tr>
<td>Elkins, WV</td>
<td>38.88</td>
<td>1471.17</td>
<td>1484.61</td>
<td>0.91%</td>
</tr>
<tr>
<td>Bluefield, WV</td>
<td>37.27</td>
<td>1539.25</td>
<td>1544.27</td>
<td>0.33%</td>
</tr>
<tr>
<td>Traverse City, MI</td>
<td>44.73</td>
<td>1423.71</td>
<td>1457.55</td>
<td>2.38%</td>
</tr>
<tr>
<td>Eugene, OR</td>
<td>44.05</td>
<td>1479.74</td>
<td>1507.53</td>
<td>1.88%</td>
</tr>
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<td>47.93</td>
<td>1227.21</td>
<td>1249.33</td>
<td>1.80%</td>
</tr>
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<td>42.37</td>
<td>1617.46</td>
<td>1625.55</td>
<td>0.50%</td>
</tr>
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<td>Knoxville, TN</td>
<td>35.82</td>
<td>1710.45</td>
<td>1718.65</td>
<td>0.48%</td>
</tr>
<tr>
<td>International Falls, MN</td>
<td>48.57</td>
<td>1466.89</td>
<td>1484.73</td>
<td>1.22%</td>
</tr>
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<td>Asheville, NC</td>
<td>35.43</td>
<td>1775.38</td>
<td>1777.87</td>
<td>0.14%</td>
</tr>
<tr>
<td>Sioux City, IA</td>
<td>42.38</td>
<td>1740.79</td>
<td>1752.15</td>
<td>0.65%</td>
</tr>
<tr>
<td>Medford, OR</td>
<td>42.19</td>
<td>1763.54</td>
<td>1788.50</td>
<td>1.42%</td>
</tr>
<tr>
<td>Key West, FL</td>
<td>24.55</td>
<td>2007.72</td>
<td>2008.29</td>
<td>0.03%</td>
</tr>
<tr>
<td>Bakersfield, CA</td>
<td>35.43</td>
<td>2062.39</td>
<td>2077.90</td>
<td>0.75%</td>
</tr>
<tr>
<td>Las Vegas, NV</td>
<td>36.08</td>
<td>2323.74</td>
<td>2327.06</td>
<td>0.14%</td>
</tr>
</tbody>
</table>
CHAPTER 4. S FORMULA METHOD FOR FIXED, SOUTH-FACING PV ARRAYS

In [16], researchers at ETH-Zurich have developed and tested new open-source software called the City Energy Analyst (CEA) that is an extension to the Geographic Information System ArcGIS v10.3 program. CEA allows architects and city planners to compare various scenarios for redeveloping urban areas by analyzing and optimizing multiple energy systems. The researchers are using the mathematically-derived formula (4.4) and correlation (4.5), both presented in [7], to calculate the optimal tilt angle of fixed, south-facing PV arrays located on flat roofs.

4.4 Optimal tilt angles of fixed, south-facing arrays: Comparing the S Formula’s results with other researchers’ results

In October 2012, the author of this dissertation proposed a related and similar formula for calculating the optimal yearly tilt angle for a south-facing PV array:

\[
\tan \beta_{optimal} = \tan \phi \left[ \frac{1}{1 + \frac{g(k_T) - \rho_g}{2(1-g(k_T))}} \right]
\]  
(4.6)

where \( \phi \) is the latitude of the location,
\( \rho_g \) is the ground reflectance, and
\( g(k_T) \) is a correlation that relates the ratio of the diffuse and global horizontal irradiances to the clearness index, \( k_T \).

This equation does not consider the transmittance-absorptance products of the beam, diffuse and reflected radiation streams.

There are many correlations relating the diffuse horizontal irradiance to the global horizontal irradiance \( g(k_T) \). Some correlations depend only on the clearness index \( k_T \) of the location, while others are functions of one or more parameters such as the clearness index, solar altitude, sunshine hours, ambient temperature, and relative humidity. If we wish to use the S Formula to find the optimal tilt angle for any location on earth, then we need a set of publicly available \( k_T \) values and the \( g(k_T) \) correlation derived using those \( k_T \) values.

The NASA Surface meteorology and Solar Energy (SSE) website allows the user to find meteorological parameters for a given latitude and longitude. It also lists the correlations used to calculate the fraction of the diffuse horizontal irradiation to the global horizontal irradiation [12]. The correlations are listed in Appendix B.
CHAPTER 4. S FORMULA METHOD FOR FIXED, SOUTH-FACING PV ARRAYS

Figure 4.1 outlines a method for finding the yearly optimum tilt angle for a fixed solar collector at any location using the S Formula (4.6) presented in [7], and by using the NASA SSE clearness index and the NASA SSE correlations for $g(k_T)$. The accuracy of the angles predicted by the S Formula can then be validated by comparing it with the yearly optimum tilt angles published on the NASA SSE website.

We used the process shown in Figure 4.1 to analyze the accuracy of the S Formula’s optimal angles for 217 U.S. locations and 37 non-U.S. locations. Of the non-U.S. locations, nine were located in Turkey [48,56], one in Saudi Arabia [49], one in Brunei [57], 16 in Iran [58], two in India [50,51], and 8 in China [52]. The NASA SSE correlations and the NASA SSE clearness index values for each location were substituted into (4.6) and the yearly optimal tilt angles were then calculated.

For each location, we compared the S Formula’s calculated optimal angles with the annual optimal tilt angles listed on the NASA SSE website. For all 217 U.S. locations, the S Formula’s calculated optimal tilt angles were within $2.1^\circ$ of the optimal angles published on the NASA SSE website. For 36 out of 37 non-U.S. locations, the S Formula’s optimal tilt angles were within $2^\circ$ of the NASA SSE optimal angles.
## Table 4.3: Comparison of yearly optimal tilt angles listed on NASA SSE website, calculated by the S Formula, and reported by other researchers for 37 locations.

<table>
<thead>
<tr>
<th>Location</th>
<th>Latitude (°N)</th>
<th>Longitude (°E)</th>
<th>SSE Annual Insolation Clearness Index</th>
<th>Yearly Optimal Tilt Angle (°)</th>
<th>Reported in literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beijing, China</td>
<td>39.9</td>
<td>116.41</td>
<td>0.55</td>
<td>36.9</td>
<td>37.2</td>
</tr>
<tr>
<td>Chengdu, China</td>
<td>30.66</td>
<td>104.06</td>
<td>0.36</td>
<td>23.1</td>
<td>22.5</td>
</tr>
<tr>
<td>Guangzhou, China</td>
<td>23.13</td>
<td>113.26</td>
<td>0.4</td>
<td>19.2</td>
<td>17.5</td>
</tr>
<tr>
<td>Kunming, China</td>
<td>25.04</td>
<td>102.72</td>
<td>0.53</td>
<td>23.4</td>
<td>22.3</td>
</tr>
<tr>
<td>Shanghai, China</td>
<td>31.23</td>
<td>121.47</td>
<td>0.44</td>
<td>25.9</td>
<td>25.7</td>
</tr>
<tr>
<td>Shenyang, China</td>
<td>41.81</td>
<td>123.43</td>
<td>0.55</td>
<td>38.9</td>
<td>39.1</td>
</tr>
<tr>
<td>Xi’an, China</td>
<td>34.26</td>
<td>108.94</td>
<td>0.47</td>
<td>29.2</td>
<td>29.5</td>
</tr>
<tr>
<td>Yinchuan, China</td>
<td>38.49</td>
<td>106.23</td>
<td>0.57</td>
<td>35.4</td>
<td>36.3</td>
</tr>
<tr>
<td>Brunei, Darussalam</td>
<td>4.9</td>
<td>115</td>
<td>0.5</td>
<td>15.6</td>
<td>4.1</td>
</tr>
<tr>
<td>Khatkar Kalan, India</td>
<td>31.16</td>
<td>76.02</td>
<td>0.61</td>
<td>29.2</td>
<td>29.8</td>
</tr>
<tr>
<td>New Delhi, India</td>
<td>28.64</td>
<td>77.22</td>
<td>0.57</td>
<td>26.6</td>
<td>26.5</td>
</tr>
<tr>
<td>Abadan, Iran</td>
<td>30.2</td>
<td>48.3</td>
<td>0.58</td>
<td>27.2</td>
<td>28.2</td>
</tr>
<tr>
<td>Bandar Abbas, Iran</td>
<td>27.1</td>
<td>56.27</td>
<td>0.59</td>
<td>25.1</td>
<td>25.4</td>
</tr>
<tr>
<td>Booshehr, Iran</td>
<td>28.6</td>
<td>51.52</td>
<td>0.58</td>
<td>26.2</td>
<td>26.7</td>
</tr>
<tr>
<td>Esfahan, Iran</td>
<td>32.4</td>
<td>51.68</td>
<td>0.61</td>
<td>29.8</td>
<td>31.0</td>
</tr>
<tr>
<td>Ghaemshahr, Iran</td>
<td>36.3</td>
<td>52.86</td>
<td>0.46</td>
<td>30.6</td>
<td>31.1</td>
</tr>
<tr>
<td>Kerman, Iran</td>
<td>30.15</td>
<td>57.07</td>
<td>0.6</td>
<td>27.8</td>
<td>28.6</td>
</tr>
<tr>
<td>Kermanshah, Iran</td>
<td>34.2</td>
<td>47.07</td>
<td>0.59</td>
<td>30.8</td>
<td>32.4</td>
</tr>
<tr>
<td>Mashhad, Iran</td>
<td>36.5</td>
<td>59.6</td>
<td>0.55</td>
<td>32.6</td>
<td>33.8</td>
</tr>
<tr>
<td>Orumie, Iran</td>
<td>37.3</td>
<td>45.07</td>
<td>0.58</td>
<td>33.7</td>
<td>35.3</td>
</tr>
<tr>
<td>Rasht, Iran</td>
<td>37.1</td>
<td>49.58</td>
<td>0.46</td>
<td>30.8</td>
<td>31.9</td>
</tr>
<tr>
<td>Shiraz, Iran</td>
<td>29.4</td>
<td>52.53</td>
<td>0.61</td>
<td>27.3</td>
<td>28.0</td>
</tr>
<tr>
<td>Tabriz, Iran</td>
<td>38.5</td>
<td>46.29</td>
<td>0.55</td>
<td>34.5</td>
<td>35.8</td>
</tr>
<tr>
<td>Tehran, Iran</td>
<td>35.4</td>
<td>51.42</td>
<td>0.57</td>
<td>31.7</td>
<td>33.2</td>
</tr>
<tr>
<td>Yazd, Iran</td>
<td>31.5</td>
<td>54.36</td>
<td>0.62</td>
<td>29.3</td>
<td>30.3</td>
</tr>
<tr>
<td>Zahedan, Iran</td>
<td>29.3</td>
<td>60.86</td>
<td>0.61</td>
<td>27.3</td>
<td>27.9</td>
</tr>
<tr>
<td>Zanjan, Iran</td>
<td>36.4</td>
<td>48.49</td>
<td>0.56</td>
<td>32.7</td>
<td>33.9</td>
</tr>
<tr>
<td>Madinah, Saudi Arabia</td>
<td>24.5</td>
<td>9.6</td>
<td>0.65</td>
<td>24.2</td>
<td>23.9</td>
</tr>
<tr>
<td>Adana, Turkey</td>
<td>36.59</td>
<td>35.18</td>
<td>0.59</td>
<td>33.1</td>
<td>34.8</td>
</tr>
<tr>
<td>Ankara, Turkey</td>
<td>39.57</td>
<td>32.53</td>
<td>0.53</td>
<td>35.2</td>
<td>36.3</td>
</tr>
<tr>
<td>Diyarbakir, Turkey</td>
<td>37.55</td>
<td>40.12</td>
<td>0.56</td>
<td>33.5</td>
<td>35.1</td>
</tr>
<tr>
<td>Erzurum, Turkey</td>
<td>39.55</td>
<td>41.16</td>
<td>0.55</td>
<td>35.6</td>
<td>36.8</td>
</tr>
<tr>
<td>Istanbul, Turkey</td>
<td>40.58</td>
<td>29.05</td>
<td>0.48</td>
<td>35.6</td>
<td>35.9</td>
</tr>
<tr>
<td>Izmir, Turkey</td>
<td>38.24</td>
<td>27.1</td>
<td>0.55</td>
<td>34</td>
<td>35.5</td>
</tr>
<tr>
<td>Izmir, Turkey</td>
<td>38.24</td>
<td>27.1</td>
<td>0.55</td>
<td>34</td>
<td>35.5</td>
</tr>
<tr>
<td>Samsun Turkey</td>
<td>41.17</td>
<td>36.2</td>
<td>0.47</td>
<td>36.5</td>
<td>36.2</td>
</tr>
<tr>
<td>Trabzon, Turkey</td>
<td>41</td>
<td>39.43</td>
<td>0.5</td>
<td>36.6</td>
<td>36.9</td>
</tr>
</tbody>
</table>
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We then compared the S Formula’s calculated optimal angles with the values that were calculated by other researchers using numerical or optimization methods. When compared with the optimal tilt angles reported in [1], the S Formula’s optimal angles were within 3° for 76% of the U.S. locations. For 75% of non-U.S. locations, the S Formula’s optimal angles were within 3° of the values calculated by other researchers using numerical or optimization methods. Table 4.3 compares yearly optimal tilt angles listed on NASA SSE website, calculated by the S Formula, and reported by others in the literature for 37 locations around the world. It should be noted that a solar collector positioned within 3° of the optimal tilt angle will receive 99.9% of the available solar radiation.

4.5 Summary

In this chapter we presented a literature survey of the different methods for finding the yearly optimal tilt angle of fixed, south-facing PV arrays. Other researchers have proposed taking the derivative of an irradiance model to find the optimal tilt angle of a fixed array; however, they use numerical or optimization techniques to solve for the optimal tilt angle instead of directly calculating the angle.

The S Formula is a simple method for directly calculating the yearly optimal tilt angle of a fixed, south-facing array. The only parameters that are required are the location’s latitude, the ground reflectance, the transmittance-absorptance products of the beam, diffuse, and reflected radiation streams, the average clearness index, and the correlation \( g(kT) \).

To analyze the validity of the S Formula for fixed PV arrays, we calculated the yearly optimal tilt angles for 217 U.S. locations and 37 non-U.S. locations and compared the results with published values. For U.S. locations, the S Formula’s calculated optimal tilt angles were within 2.1° of the optimal angles published on the NASA SSE website. When compared with the optimal tilt angles reported in [1], the S Formula’s optimal angles were within 3° for 76% of the locations.

For 36 out of 37 non-U.S. locations, the S Formula’s yearly optimal tilt angles were within 2° of the optimal angles published on the NASA SSE website. For 75% of the non-U.S. location, the S Formula’s optimal angles were within 3° of the values calculated by other researchers using numerical and other methods. These results suggest that the S Formula can closely estimate the optimal tilt angles of fixed PV arrays located in different climates.

A PV array positioned within 3° of the optimal tilt angle will receive 99.9% of the available solar radiation. However, an array that is installed at a tilt angle that is 12° greater than the optimal tilt angle will fail to capture 2% or more of the available irradiance. Thus, for cloudier locations the
rule-of-thumb convention of installing a fixed PV array at a tilt angle equal to the location’s latitude tilt may result in a 1 to 3% loss in captured insolation over the course of a year.

In the remaining chapters of this dissertation, we shall investigate the use of the Parent S Formula to calculate the optimal tilt angles of a 2-axis tracking PV installation in Middlebury, Vermont. Background information on the installation, instrumentation, and the data set shall be presented in Chapter 5.
Chapter 5

Methodology for validating the S Formula

5.1 Introduction

In Chapter 5, we discuss the methodology for testing the validity of the S Formula for 2-axis tracking PV arrays. We discuss the installation parameters and the measurement instrumentation at this location. We review the programs and data that were used in simulating the 2-axis tracking array.

The simulations used 36 months of measured weather data from a 2-axis tracking PV installation located in Middlebury, Vermont, USA. The PV_LIB Toolbox for MATLAB calculated the POA irradiance and AC power for each 15-minute interval. To create a valid set of data, additional MATLAB programs flagged inconsistent data according to specific criteria. Since this location experiences clear skies about 40% of the year, we would expect this location to benefit from the energy gleaning potential of the S Formula method.

5.2 PV installation and instrumentation at Middlebury College

5.2.1 Location and installation

In this research, weather and power data from a 2-axis tracking PV installation at Middlebury College in Middlebury, Vermont were used. The latitude and longitude of the installation is 44° North and 73.2° West. As shown in Figure 5.1, this 143 kW installation consists of 34
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AllEarth Renewables 2-axis tracking PV arrays. Twenty Evergreen ES-A 210 multicrystalline silicon photovoltaic modules with a total DC capacity of 4200 Watts are mounted on each tracker, and each tracker has an SMA America SB4000US 240V inverter. Since the surrounding ground cover is grass, an albedo or ground reflectance of 0.2 was assumed for the simulations [59].

Figure 5.1: Middlebury College’s 143 kW 2-axis tracking PV installation located in Middlebury, Vermont. (Photographed by author on February 2, 2016.)

5.2.2 Instrumentation and weather stations

Figure 5.2 shows the locations of the sensors in Middlebury. Two sets of instrumentation are located at the PV installation. One tracking array, marked as PV-17, has a Kipp & Zonen CMP11 pyranometer mounted at the top of the array that measures POA irradiance, and a Draker KL206 sensor measuring the back-of-module temperature, as shown in Figure 5.3. The CMP11 is a thermopile detector with a spectral range of 285 to 2800 nm at its 50% points, as illustrated in Figure 5.4. The CMP11’s change in stability is less than ±5% per year. The DC current and DC voltage of PV-17 tracking array are also recorded.

A second set of instrumentation is located about 109 meters northwest of the installation. Figure 5.5 shows the Irradiance Inc. RSR2 rotating shadowband radiometer (RSR) with a LI-COR LI-200SA irradiance sensor. The rotating shadowband radiometer has a shading ring and measures global horizontal irradiance (GHI), diffuse horizontal irradiance (DHI), and direct normal irradiance (DNI). The LI-200SA is a silicon photovoltaic detector with a spectral range of 380 to 1155 nm.
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Figure 5.2: Location of installation and sensors at Middlebury College, Vermont. (Downloaded from [60] in April 2016)

Figure 5.3: Location of POA irradiance sensor and back of module temperature sensor that are mounted on the PV-17 2-axis tracking array. (Photographed by author on February 2, 2016.)
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Figure 5.4: Spectral response of the Kipp & Zonen CMP11 pyranometer. For wavelengths above approximately 400 nm, CMP3’s line overlaps the CMP11’s due to their similar spectral responses. (Image downloaded from [61] in February 2017.)

(Figure 5.6). According to the manufacturer’s data sheet, the LI-200SA has an error of ±5% under daylight conditions and change in stability of less than ±2% over a one-year period [62]. There is also a Campbell Scientific CS215 ambient temperature and humidity sensor, and a Met One 034B anemometer and wind vane.

In addition, Middlebury College has a Citizens Weather Observer Program (CWOP) weather station, Station ID D2416, located at the college’s athletic field approximately 1.27 km southeast from the rotating shadowband radiometer. Figure 5.7 shows the Davis Instruments Vantage Pro2 Station which measures solar radiation, temperature, relative humidity, wind speed, and wind direction. Data is recorded about every 10 minutes.

A weather station at the Middlebury State Airport, Station ID K6B0, records weather data and conditions approximately every 20 minutes. It is located about 7.5 km southeast from Middlebury College’s PV installation. The reported weather conditions are clear, partly cloudy, mostly cloudy, overcast, haze, fog, ice fog, thunder, light drizzle, moderate drizzle, heavy drizzle, light rain, moderate rain, heavy rain, light snow, moderate snow, heavy snow, light rain thunder shower, heavy rain thunder shower, moderate thunder shower, and unknown precipitation.
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Figure 5.5: Middlebury College’s Rotating Shadowband Radiometer. (Photographed by author on February 2, 2016.)

Figure 5.6: Spectral response of the LI-COR LI-200SA pyranometer. (Image downloaded from [62] in February 2017.)
Figure 5.7: Middlebury College’s weather station D2416 in Middlebury, Vermont. (Photographed by author on February 2, 2016.)
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5.3 Programs and data for simulations

The flowchart shown in Figure 5.8 presents the MATLAB programs that were used to create the input data files, to process the data, and to screen the data in order to generate a valid data set. The MATLAB programs can be divided into two categories: programs that were used to calculate POA irradiance and AC power, and programs that were used to re-format and flag data. The input data sets can also be divided into two categories: data that was used to calculate POA irradiance and AC power, and data that was used for flagging and screening.

5.3.1 Process for calculating POA irradiance and AC power

Referring to Figure 5.8, the data from Middlebury College’s PV array was imported into the computer program “PVLIB_rev4.m”. The Draker Monitoring data from Middlebury College’s PV installation consisted of 81 columns of power and weather data that were averaged and recorded at 15-minute intervals. The following data was used in the simulations: local timestamp, GHI, DNI, DHI, and ambient temperature. Three 12-month periods of data were separately analyzed — November 1, 2013 to October 31, 2014, November 1, 2014 to October 31, 2015, and November 1, 2015 to October 31, 2016. Fifteen days of data were missing from 2015, and approximately three days of data were missing from 2016.

The program “PVLIB_rev4.m” was written in MATLAB and called PV_LIB Toolbox functions. Descriptions of the PV_LIB Toolbox’s functions as well as the installation parameters required by PV_LIB Toolbox can be found in Appendix A. PV_LIB Toolbox’s has six diffuse irradiance models. For this research, the Isotropic Sky model (“pvl_isotropic” A.8.7.2) was used in the simulations. The purpose of “PVLIB_rev4.m” was to import the 15-minute weather data from a 2-axis PV tracking array (GHI, DNI, DHI, and ambient temperature) and then use this data in the PV_LIB functions to calculate POA irradiance and AC power. The flowchart shown in Figure A.3 in Appendix A illustrates this process. “PVLIB_rev4.m” calculated POA irradiance and AC power at the conventional 2-axis tracking angles.

In addition, this program used the numerical method to find the optimal tilt angle. At each timestamp, it varied the tilt angle from 0° to 90° in one-degree increments, calculated the POA irradiance at each 1° angle, and then saved the angle where the maximum POA irradiance occurred. It then calculated the AC power at the optimal tilt angle.

Finally, “PVLIB_rev4.m” exported the weather data and sun angles to another MATLAB program, “SFormula_rev3.m”. This program used the measured DHI and GHI data to directly
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Figure 5.8: Flowchart of programs for processing data.
compute the optimal tilt angle using the S Formula, Eqn. (3.26). Using this optimal tilt angle, the program then calculated the POA irradiance and the AC power.

As a result of these two programs, “PVLIB_rev4.m” and “SFormula_rev3.m”, three sets of prescreened data for each 15-minute interval were exported:

1. Conventional 2-axis tracking tilt angles, POA irradiance, and AC power.
2. Numerical method’s optimal tilt angles, POA irradiance, and AC power.
3. S Formula method’s optimal tilt angles, POA irradiance, and AC power.

In the next section, we shall discuss the programs and input data that were used to flag and screen the prescreened data sets to create the final valid data sets.

5.3.2 Process for re-formatting and flagging data

In the previous section, the tilt angles, POA irradiance, and AC power were calculated at each 15-minute interval. However, there were some intervals in which the data was not valid. To help in identifying invalid data sets, data from the MesoWest stations D2416 and K6B0 were used to identify and flag invalid prescreened data. Because station D2416’s data is recorded every 10 minutes and station K6B0’s is recorded every 20 minutes, the MATLAB program “MC_data_flags_midnight.m” was written for aligning both stations’ data to the Draker Monitoring data’s 15-minute timestamps. We assumed persistence in the data; that is, if the weather observation at K6B0 was overcast at at 11:20 am, then it would continue to be overcast at 11:30 am.

Referring to Figure 5.8, a MATLAB program called “Wunderground2Meso_D2416.m” was written for converting the Weather Underground data format to the MesoWest format for station D2416. From November 1, 2015 through January 5, 2016 and from January 20, 2016 to May 2, 2016, station D2416 was not transmitting data to the MesoWest site. However, it was transmitting data to the Weather Underground website KVTMIDDLL2 [63]. When compared with the MesoWest data format, the Weather Underground data was formatted differently. The timestamp format was different, the columns were arranged in a different order, and the Weather Underground site used English units. Once the Weather Underground data was re-formatted, it was imported into the program “MC_data_flags_midnight.m” to be aligned with the Draker Monitoring data’s 15-minute timestamps.
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The MATLAB program “ClearSky.m” and function “clearsky_masters.m” calculated Masters’ clear sky model for solar insolation, and flagged data with the goal of identifying outliers [64].

The MATLAB program “MC_model_MonthYear.xlsx” eliminated data sets based on a single flag, such as a weather condition, and recalculated error statistics for calculated POA irradiance and measured POA irradiance, and calculated AC Power and measured AC Power.

5.3.3 Data flagging and screening to create valid data sets

All data points were examined against established criteria and flagged. Nighttime intervals were flagged first and eliminated from the data set. About 25% of the daytime data points were removed based on criteria suggested by [65, 66] or for other reasons, as listed below:

- Data occurred at sunrise and sunset, or when the zenith angle was greater than 87°.
- GHI was greater than the corresponding extraterrestrial irradiance value.
- DNI was greater than the corresponding extraterrestrial irradiance value.
- POA irradiance was less than or equal to 0 W/m². (Negative values were occasionally recorded.)
- GHI was 25% greater than the ideal Clear Sky GHI model as calculated in [64].
- DNI was 25% greater than the ideal Clear Sky DNI model as calculated in [64].
- Data occurred during periods of precipitation as indicated by the weather conditions recorded at Middlebury State Airport.
- AC power, as calculated in PV_LIB, was less than 0 W.

The weather observations recorded at Middlebury State Airport were used to flag and eliminate periods of precipitation from the original Draker data set. About 38.07% of the daytime values were classified as clear during the period of November 1, 2013 to October 31, 2014, about 39.82% during the period of November 1, 2014 to October 31, 2015, and about 43.25% during the period of November 1, 2015 to October 31, 2016.

Table 5.1 shows the weather conditions for the three 12-month data sets used in the simulations. Each data set represents a 15-minute interval. Over the three-year period, clear skies
occurred during 45% of the intervals and overcast days about 31% of the time. About 22% of the intervals were classified as either partly cloudy or mostly cloudy. These intervals of varying cloud cover represent opportunities for energy gleaning by modifying the conventional 2-axis tracking tilt angle.

Table 5.1: Description of the weather conditions for each data set used in simulations. Each data point represents a 15-minute interval.

<table>
<thead>
<tr>
<th>Weather observation</th>
<th>YEAR 1</th>
<th>YEAR 2</th>
<th>YEAR 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear</td>
<td>42.53%</td>
<td>44.35%</td>
<td>47.69%</td>
</tr>
<tr>
<td>Partly cloudy</td>
<td>12.61%</td>
<td>12.34%</td>
<td>12.03%</td>
</tr>
<tr>
<td>Mostly cloudy</td>
<td>9.98%</td>
<td>10.66%</td>
<td>9.47%</td>
</tr>
<tr>
<td>Overcast</td>
<td>32.54%</td>
<td>30.65%</td>
<td>29.77%</td>
</tr>
<tr>
<td>Fog</td>
<td>0.99%</td>
<td>1.24%</td>
<td>0.64%</td>
</tr>
<tr>
<td>Haze</td>
<td>0.32%</td>
<td>0.43%</td>
<td>0.23%</td>
</tr>
<tr>
<td>Thunder</td>
<td>0.20%</td>
<td>0.32%</td>
<td>0.13%</td>
</tr>
<tr>
<td>Thunder; fog</td>
<td>0.00%</td>
<td>0.01%</td>
<td>0.00%</td>
</tr>
<tr>
<td>No reported observation</td>
<td>0.82%</td>
<td>0.00%</td>
<td>0.04%</td>
</tr>
</tbody>
</table>

5.3.4 Screened data sets

Graphs of PV_LIB’s calculated POA irradiance versus the measured POA irradiance for each of the three 12-month screened data sets are shown in Figures 5.9 and 5.10. Figure 5.9 shows all the 15-minute intervals for the weather conditions listed in Table 5.1, while Figure 5.10 plots a subset of the data containing only clear, partly cloudy, and mostly cloudy intervals. The dashed line indicates the ideal relationship of calculated POA irradiance equal to measured POA irradiance. Although the calculated and measured POA irradiance curves exhibit some linearity, there are many outliers. This may be because of variation in the sensors. The measured POA irradiance data is recorded by a LI-COR sensor mounted at the top of the PV-17 tracking array. However, the POA irradiance calculated in PV_LIB uses the measured irradiance data from the rotating shadowband radiometer located 109 meters north of the installation.
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(a) Year 1, Nov. 1, 2013 to Oct. 31, 2014

(b) Year 2, Nov. 1, 2014 to Oct. 31, 2015

(c) Year 3, Nov. 1, 2015 to Oct. 31, 2016

Figure 5.9: POA irradiance calculated in PV_LIB vs. POA irradiance measured by PV-17’s sensor for all 15-minute intervals in the valid data set.
Figure 5.10: POA irradiance calculated in PV_LIB vs. POA irradiance measured by PV-17’s sensor for clear, partly cloudy, and mostly cloudy intervals in the valid data set. Each point represents a 15-minute interval in the valid data set.
5.4 Summary

In this chapter we discussed the methodology for testing the validity of the S Formula for 2-axis tracking PV arrays. Three 12-month periods of measured weather data from a 2-axis tracking PV installation located in Middlebury, Vermont, were used in the simulations. The PV_LIB Toolbox for MATLAB calculated the POA irradiance and AC power for three cases: conventional 2-axis tracking, the numerical method, and the S Formula method. Inconsistent data were flagged and eliminated to create a valid set of data.

Over the three-year period, clear skies occurred during 45% of the intervals and overcast days about 31% of the time. About 22% of the intervals were classified as either partly cloudy or mostly cloudy. These intervals of varying cloud cover represent opportunities for energy gleaning by modifying the conventional 2-axis tracking tilt angle.

In Chapter 6 we shall use the flagged and screened data sets to evaluate other methods for positioning a 2-axis tracking array on cloudy days, and in Chapter 7 we shall compare and discuss the results.
Chapter 6

Energy gleaning using Kelly-Gibson and numerical methods

6.1 Introduction

In this chapter, we use the data from the irradiation sensors at Middlebury College to evaluate the three methods proposed by Nelson Kelly and Thomas Gibson for increasing the POA irradiation collection on overcast days [11]. We then calculate and compare the yearly POA irradiation and AC energy at the optimal tilt angles calculated by the numerical method and at the conventional 2-axis tracking angles.

6.2 Three methods proposed by Kelly and Gibson

In their patent [11], Kelly and Gibson proposed comparing the output of one or more irradiance sensors in order to determine whether to position a 2-axis tracking array at the tilt angle of \(0^\circ\) on cloudy days. In this section, we use the screened data to calculate the yearly collected irradiation for each of three methods mentioned in the patent. Since we do not have the capability to alter the position of the tracking array, we will use the measured POA data to represent a tracker pointed at the sun, and the measured GHI data to represent a tracker that has been repositioned to a horizontal position (\(\beta = 0^\circ\)).
CHAPTER 6. ENERGY GLEANING USING KELLY-GIBSON AND NUMERICAL METHODS

6.2.1 Method 1: DTS vs GHI

In the first method, two irradiance sensors are used to determine when to track the sun. The first sensor (DTS) is directly pointed at the sun, while the second sensor (H) is placed horizontally. When the irradiance measured by the DTS sensor is greater than the irradiance measured by the H sensor by a predetermined value, the PV array tracks the sun \( (\beta = \theta_z) \). Otherwise, the PV array is placed horizontally \( (\beta = 0^\circ) \).

In our analysis, we used the measured POA irradiance data from the sensor located at the top of one of the 2-axis trackers at Middlebury. For the second sensor, we used the measured GHI data from the Rotating Shadowband Radiometer (RSR) sensor that is placed horizontally. The algorithm is this:

For \( \text{POA} \geq \text{GHI} \) ⇒ Panel tracks the sun ⇒ Use POA data
For \( \text{POA} < \text{GHI} \) ⇒ Place panel horizontally ⇒ Use GHI data

For each 15-minute interval, the POA and GHI measured data is compared to determine which data should be counted for that interval. All the end of the 12-month period, the irradiance data is then summed to get the total collected irradiance for the year.

6.2.2 Method 2: GHI vs DHI

In the second method, both irradiance sensors are placed horizontally. The first sensor is shaded from the sun and measures the diffuse horizontal irradiance (DHI). The second sensor (GHI) is exposed to the sun and measures the direct horizontal and diffuse horizontal irradiance. The array tracks the sun when the measured GHI exceeds the measured diffuse horizontal irradiance by a predetermined value. Otherwise, the PV array is placed at a tilt angle of \( 0^\circ \).

In our analysis, we compare the measured diffuse horizontal irradiance and the measured GHI data from the RSR sensor, and we set the predetermined value to be 1 W/m\(^2\). Since measured \( \text{GHI} - \text{measured DHI} = \text{measured direct horizontal irradiance} \), we check to see whether the measured direct horizontal irradiance is greater than 1 W/m\(^2\). The algorithm is this:

For Direct Horizontal \( \geq 1 \text{ W/m}^2 \) ⇒ Panel tracks the sun ⇒ Use POA data
For Direct Horizontal \(< 1 \text{ W/m}^2 \) ⇒ Place panel horizontally ⇒ Use GHI data

For each 15-minute interval, the value of direct horizontal irradiance is checked to see whether POA and GHI measured data should be counted for that interval. All the end of the 12-month period, the irradiance data is then summed to get the total collected irradiance for the year.
CHAPTER 6. ENERGY GLEANING USING KELLY-GIBSON AND NUMERICAL METHODS

6.2.3 Method 3: DTS vs Predetermined threshold

In the third method, a single irradiance sensor (DTS) is pointed directly at the sun. When the measured DTS irradiance is greater than a predetermined value, the PV array tracks the sun. Otherwise, the PV array is placed horizontally. The patent suggests a value of 250 W/m$^2$ as the boundary between tracking sun or placing the array horizontally. However, when we found that more irradiance could be collected throughout the 12-month period when the boundary was set at a lower value of 150 W/m$^2$. The algorithm is this:

- For DTS $\geq$ 150 W/m$^2$ ⇒ Panel tracks the sun ⇒ Use POA data
- For DTS $< 150$ W/m$^2$ ⇒ Place panel horizontally ⇒ Use GHI data

For each 15-minute interval, the value of direct horizontal irradiance is checked to see whether POA and GHI measured data should be counted for that interval. All the end of the 12-month period, the irradiance data is then summed to get the total collected irradiance for the year.

6.2.4 Results of Kelly and Gibson’s three methods

Table 6.1 shows the increase in POA vs the conventional tracking angles for three of the methods proposed by the patent [11]. For comparison, we have included the measured POA irradiance from the sensor that is mounted on the 2-axis tracker. Since it follows the conventional 2-axis tracking angles, this measured POA irradiance will serve as a baseline value for the conventional tracking method.

All three methods showed an increase in yearly insolation versus the conventional 2-axis tracking method, with Method 1 (DTS vs H) having the biggest gains with an increase of 1.05% for the first year, 0.72% for the second year, and 0.81% for the third year. Method 2 (GHI vs Diffuse) and Method 3 (DTS vs Limit) had similar increases of 0.23 to 0.53%.

The values for Method 1 of 1.05%, 0.72%, and 0.81% are about the same as the 1% annual increase in collected irradiance estimated for Detroit in 1% [33]. This seems reasonable since Middlebury, Vermont and Detroit, Michigan have similar latitudes and climates. The limitation of all three methods is that there are only two choices for the tracking tilt angle: $\beta = \theta_z$ or $\beta = 0^\circ$. As we have discussed earlier, there are other optimal angles during partly cloudy skies.
Table 6.1: Yearly POA Insolation calculated for four tracking algorithms. Conventional 2-axis tracking vs. three methods proposed by Kelly and Gibson in [11].

<table>
<thead>
<tr>
<th>Method</th>
<th>YEAR 1</th>
<th>YEAR 2</th>
<th>YEAR 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(kWh/m²) Increase over Conventional 2-axis tracking %</td>
<td>(kWh/m²) Increase over Conventional 2-axis tracking %</td>
<td>(kWh/m²) Increase over Conventional 2-axis tracking %</td>
</tr>
<tr>
<td>Conventional 2-axis tracking</td>
<td>1844.21 —</td>
<td>1811.02 —</td>
<td>1950.98 —</td>
</tr>
<tr>
<td>Method 1: DTS vs H</td>
<td>1863.62 1.05%</td>
<td>1824.10 0.72%</td>
<td>1966.70 0.81%</td>
</tr>
<tr>
<td>Method 2: GHI vs Diffuse</td>
<td>1854.02 0.53%</td>
<td>1817.98 0.38%</td>
<td>1960.72 0.50%</td>
</tr>
<tr>
<td>Method 3: DTS vs Limit</td>
<td>1850.13 0.32%</td>
<td>1815.19 0.23%</td>
<td>1958.25 0.37%</td>
</tr>
</tbody>
</table>
CHAPTER 6. ENERGY GLEANING USING KELLY-GIBSON AND NUMERICAL METHODS

6.3 A numerical method

6.3.1 Overview of the numerical method

As mentioned in Section 2.4.6, one method for finding the optimal tilt angle of a PV array is to use weather data and irradiance models to compute the POA irradiance received by the collecting surface as the tilt angle is varied from 0° to 90° \(^{12}\). Figure 6.1 gives an overview of the process used in finding the optimal tilt angle. A program written in MATLAB and functions from the PV_LIB Toolbox were used in the simulations. The PV_LIB Toolbox was developed for use in MATLAB by Sandia National Laboratories \(^{17}\). The PVPMC website describes in detail the models that are the basis of each function.

The following parameters for the installation, previously given in Section 5.2.1 were provided to the program: latitude, longitude, ground reflectance, PV module part number, inverter part number, and the number of panels and strings. A soiling factor of 0.02 was assumed in calculating the AC power. The local timestamp was converted to solar time in the program. The 15-minute weather data (POA, GHI, DHI, DNI, and ambient temperature) that had been screened according to the criteria in Section 2.1.3 were also inputs to the program. There were 13272 data points, each representing a 15-minute interval, for the period of November 1, 2013 to October 31, 2014, 12945 data points for the period of November 1, 2014 to October 31, 2015, and 13637 data points for the period of November 1, 2015 to October 31, 2016. Each 12-month period was run separately.

It was assumed that the surface continues to track the sun from east to west (\(\gamma = \gamma_s\)). During each time period, the tilt angle was varied from 0° to 90° in one-degree increments and the POA irradiance was calculated at each value of \(\beta\) using the isotropic diffuse sky model \(^{8}\) \(^{67}\). The optimal tilt angle \(\beta_{\text{optimal}}\) was the angle at which the maximum POA irradiance occurs during that particular time period. The value of maximum POA was then used to calculate the AC power at that interval.

POA irradiance and AC power at the conventional tracking angles (\(\theta_z\) and \(\gamma = \gamma_s\)) were also calculated in MATLAB and the PV_LIB Toolbox with the same installation parameters and weather data. This is not shown in Figure 6.1

6.3.2 Results of the numerical method

Figure 6.2 displays the results of the numerical method (Section 6.3 and Figure 6.1) for three days in 2014. The conventional 2-axis tracking angles are shown for all daytime intervals,
CHAPTER 6. ENERGY GLEANING USING KELLY-GIBSON AND NUMERICAL METHODS

Figure 6.1: Finding $\beta_{\text{optimal}}$ by calculating POA irradiance as tilt angle is varied from 0° to 90°. $POA_{\text{max}}$ is the POA irradiance occurring at the optimal tilt angle, $\beta_{\text{optimal}}$, and is used in calculating the AC Power in PV_LIB Toolbox.
including those intervals at sunrise and sunset when $\theta_z$ may be greater than $87^\circ$. On the overcast day (Figure 6.2a), the optimal tilt angle is about $0^\circ$. On the clear day (Figure 6.2b), the optimal tilt angle is near the conventional angle of $\theta_z$, except at around 06:30 when the angle is near $0^\circ$ during fog conditions. In Figure 6.2c, the optimal tilt angle changes throughout the day, starting at about $0^\circ$ before 08:00, varying between $25^\circ$ and $\theta_z$ through 15:30, approaching $0^\circ$, and finally ending at about $\theta_z$.

Tables 6.2 and 6.3 show the increase in POA insolation and AC energy when using the numerical method.
Chapter 6. Energy Gleaning Using Kelly-Gibson and Numerical Methods

Figure 6.2: Results of MATLAB simulations showing the optimal tilt angles for three days in 2014 (a) an overcast day (May 28th), (b) a clear day (August 19th), and (c) a day with a mix of clouds and clear skies (August 27th). The red dotted trace represents the optimal tilt angles calculated by the numerical method where POA irradiance is calculated as the tilt angle is varied from $0^\circ$ to $90^\circ$. Shown for comparison are the conventional tilt angles of $\beta = \theta_z$ (black solid line). Each step represents a 15-minute interval.
Table 6.2: Yearly POA Insolation calculated for two sets of tilt angles - conventional 2-axis tracking and optimal angles found using the numerical method.

| Method                      | YEAR 1  
|                            | Nov. 1, 2013 to Oct. 31, 2014 | YEAR 2  
|                            | Nov. 1, 2014 to Oct. 31, 2015 | YEAR 3  
|                            | Nov. 1, 2015 to Oct. 31, 2016 |
| Conventional 2-axis tracking | Insolation (kWh/m²) | Increase over Conventional 2-axis tracking % | Insolation (kWh/m²) | Increase over Conventional 2-axis tracking % | Insolation (kWh/m²) | Increase over Conventional 2-axis tracking % |
| Conventional 2-axis tracking | 1616.82 | — | 1570.50 | — | 1712.11 | — |
| Numerical method            | 1654.83 | 2.35% | 1607.02 | 2.33% | 1747.97 | 2.09% |
Table 6.3: Yearly AC energy calculated for two sets of tilt angles - conventional 2-axis tracking and optimal angles found using the numerical method.

<table>
<thead>
<tr>
<th>Method</th>
<th>YEAR 1</th>
<th></th>
<th>YEAR 2</th>
<th></th>
<th>YEAR 3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AC energy (kWh/m²)</td>
<td>Increase over Conventional 2-axis tracking %</td>
<td>AC energy (kWh/m²)</td>
<td>Increase over Conventional 2-axis tracking %</td>
<td>AC energy (kWh/m²)</td>
<td>Increase over Conventional 2-axis tracking %</td>
</tr>
<tr>
<td>Conventional 2-axis tracking</td>
<td>6244.70</td>
<td>—</td>
<td>6066.26</td>
<td>—</td>
<td>6606.20</td>
<td>—</td>
</tr>
<tr>
<td>Numerical method</td>
<td>6395.83</td>
<td>2.42%</td>
<td>6211.56</td>
<td>2.40%</td>
<td>6748.54</td>
<td>2.15%</td>
</tr>
</tbody>
</table>
6.4 Summary

Data from the irradiance sensors at Middlebury College was used to evaluate the three methods proposed by Nelson Kelly and Thomas Gibson for increasing the POA irradiation collection on overcast days. All three methods showed an increase in yearly insolation versus the conventional 2-axis tracking method; however, Method 1 (DTS vs H) showed the largest gains with increases of 1.05% for the first year, 0.72% for the second year, and 0.81% for the third year. These values were similar to the 1% annual increase in collected irradiation estimated for Detroit when the panel was placed horizontally instead of pointing directly at the sun. The limitation of all three methods is that there are only two choices for the tracking tilt angle: $\beta = \theta_z$ or $\beta = 0^\circ$.

For the numerical method for finding the optimal tilt angles, the yearly increases in insolation and AC energy were 2.35% and 2.42% for the first year, 2.33% and 2.40% for the second year, and 2.09% and 2.15% for the third year. In Chapter 7, we shall compute the POA irradiation and AC energy at the optimal tilt angles calculated by the Parent S Formula.
Chapter 7

S Formula method’s optimal angles for 2-axis tracking

7.1 Introduction

In Chapter 7 we explain the S Formula method for finding the optimal tilt angles of 2-axis time-position tracking PV arrays under different sky conditions. Optimal tilt angles are calculated for three 12-month periods using screened data sets. We substitute measured diffuse irradiance and global horizontal irradiance data from the installation into the S Formula to directly calculate the optimal tilt angle during each 15-minute interval. POA irradiation and AC energy are then computed at the optimal angles using the PV_LIB Toolbox.

We also present the results of the simulations for three days — an overcast day, a sunny day, and a day with mixed sky conditions. We illustrate the potential increase in POA irradiance during each 15-minute interval. We show the yearly change in POA irradiation, as well as the yearly change in the beam, diffuse and reflected irradiation streams, for both the S Formula method and conventional 2-axis tracking.

7.2 Validating the S Formula method with weather data and the PV_LIB Toolbox

The flowchart in Figure 7.1 presents an overview of the process. The input parameters and screened data sets were the same as the ones used for the numerical method (see Section 6.3). We
CHAPTER 7. S FORMULA METHOD’S OPTIMAL ANGLES FOR 2-AXIS TRACKING

assumed that the 2-axis tracking PV array tracked the sun from east to west using the conventional 2-axis azimuth tracking angles \( (\gamma = \gamma_s) \). A new MATLAB program was written to directly calculate \( \beta_{\text{optimal}} \) using (3.23). However, the same PV_LIB functions used for the numerical method were again used in calculating POA irradiance and AC power at the optimal tilt angles calculated by the S Formula (3.23) in Section 3.2. It was assumed that the transmittance-absorptance products were ideal, so \( (\tau \alpha)_b, (\tau \alpha)_d, \) and \( (\tau \alpha)_r \) were set to 1. Since \( g(k_T) = \frac{I_d}{I} \), the measured values for \( I_d \) and \( I \) were substituted in (3.23) to calculate the optimal tilt angle.

Figure 7.1: The S Formula calculates \( \beta_{\text{optimal}} \) during each 15-minute interval. \( \beta_{\text{optimal}} \) is then used to calculate POA irradiance and AC Power.

The following equations for Mean Bias Error, Mean Absolute Error, and Root Mean Square Error from [68] were used to analyze the errors between the optimal tilt angles found through the numerical method (Figure 6.1) and the optimal tilt angles calculated using the S Formula (Figure 7.1):

```
\[
\text{INPUT DATA}
\]
```

```
\[
\text{Tracking Angles}
\]
```

```
\[
\text{Day & Time} \quad \text{Measured
irradiance} \quad \text{Site parameters} \quad \text{Measured
Ambient temperature} \quad \text{Installation
parameters} \quad \text{Soiling factor} \quad \text{PV
modules} \quad \text{Inverters}
\]
```

```
\[
\text{Heat} \quad \text{DNI}
\]
```

```
\[
\text{INPUT DATA}
\]
```

```
\[
\text{Measured
irradiance} \quad \text{Site parameters} \quad \text{Measured
Ambient
temperature} \quad \text{Installation
parameters} \quad \text{Soiling factor} \quad \text{PV
modules} \quad \text{Inverters}
\]
```

```
\[
\text{INPUT DATA}
\]
```

```
\[
\text{Tracking Angles}
\]
```

```
\[
\text{Day & Time} \quad \text{Measured
irradiance} \quad \text{Site parameters} \quad \text{Measured
Ambient temperature} \quad \text{Installation
parameters} \quad \text{Soiling factor} \quad \text{PV
modules} \quad \text{Inverters}
\]
```

```
\[
\text{Heat} \quad \text{DNI}
\]
```

```
\[
\text{INPUT DATA}
\]
```

```
\[
\text{Measured
irradiance} \quad \text{Site parameters} \quad \text{Measured
Ambient
temperature} \quad \text{Installation
parameters} \quad \text{Soiling factor} \quad \text{PV
modules} \quad \text{Inverters}
\]
```

```
\[
\text{INPUT DATA}
\]
```

```
\[
\text{Tracking Angles}
\]
```

```
\[
\text{Day & Time} \quad \text{Measured
irradiance} \quad \text{Site parameters} \quad \text{Measured
Ambient temperature} \quad \text{Installation
parameters} \quad \text{Soiling factor} \quad \text{PV
modules} \quad \text{Inverters}
\]
```

```
\[
\text{Heat} \quad \text{DNI}
\]
```
CHAPTER 7. S FORMULA METHOD’S OPTIMAL ANGLES FOR 2-AXIS TRACKING

\[
MBE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - x_i)}
\]  \hspace{1cm} (7.1)

\[
MAE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} |y_i - x_i|}
\]  \hspace{1cm} (7.2)

\[
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - x_i)^2}
\]  \hspace{1cm} (7.3)

where \( y_i = i^{th} \beta_{optimal} \) found using numerical method, \( x_i = i^{th} \beta_{optimal} \) calculated using the S Formula, and \( N \) = number of data points.

7.3 Results using the S Formula method with weather data and the PV_LIB Toolbox

It should be clarified that the figures and tables presented in this section are simulation results. As explained in Section 5, measured weather data and the installation parameters of an actual 2-axis tracking PV array were provided as input to the programs shown in Figures 6.1 and 7.1. However, no experiments have been performed using the proposed S Formula.

Figure 7.2 shows the optimal tilt angles that were calculated using the S Formula (Section 3.2 and Figure 7.1) on the same three days. The plots are similar to the ones shown in Figure 6.2, with the calculated optimal tilt angle of about 0° on the overcast day (Figure 7.2a), near the conventional 2-axis tracking angle of \( \theta_z \) on the clear day (Figure 7.2b), and varying between 0° and \( \theta_z \) on the day with variable weather (Figure 7.2c).

The plots shown in Figures 6.2 and 7.2 appear to be the same for both the numerical and S Formula methods. However, there were sometimes small differences between the angles that are not visible in Figures 6.2 and 7.2. To evaluate the errors between the two methods, the MBE, MAE, and RMSE were calculated (see Table 7.1).

Figure 7.3 shows the calculated POA irradiance values for a 2-axis tracking array whose tilt is being adjusted using the S Formula’s optimal tilt angles. During most of the overcast day (Figure 7.3a), more POA irradiance was collected using the S Formula’s optimal tilt. On the clear day
CHAPTER 7.  S FORMULA METHOD’S OPTIMAL ANGLES FOR 2-AXIS TRACKING

Figure 7.2: Results of MATLAB simulations showing the optimal tilt angles for three days in 2014 - (a) an overcast day (May 28th), (b) a clear day (August 19th), and (c) a day with a mix of clouds and clear skies (August 27th). The blue dotted trace represents the optimal tilt angles calculated by the proposed S Formula. Shown for comparison are the conventional tilt angles of $\beta = \theta_z$ (black solid line). Each step represents a 15-minute interval.
Table 7.1: MBE, MAE, and RMSE between the optimal tilt angles found using the S Formula’s method of directly calculating the optimal tilt angle, and the numerical method of varying the tilt angle to find the maximum POA irradiance.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>YEAR 1</th>
<th>YEAR 2</th>
<th>YEAR 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>MBE, Mean Bias Error</td>
<td>0.0268</td>
<td>0.0311</td>
<td>0.0322</td>
</tr>
<tr>
<td>MAE, Mean Absolute Error</td>
<td>0.2459</td>
<td>0.2464</td>
<td>0.2361</td>
</tr>
<tr>
<td>RMSE, Root Mean Square Error</td>
<td>0.4579</td>
<td>0.4413</td>
<td>0.4206</td>
</tr>
</tbody>
</table>

(Figure 7.3b), the calculated POA irradiances for both the S Formula and conventional methods are about the same. On the day with variable weather (Figure 7.3c), the S Formula’s tilt angles captured more of the POA irradiance during the periods of overcast skies.

In reviewing the literature (Section 2), some researchers gave a range of values for the percentage increase in POA irradiance when the PV panel is placed horizontally during cloudy periods. The histogram displayed in Figure 7.4a shows the percentage increase in POA irradiance when using the S formula’s optimal tilt angles versus the conventional tilt angles. Each column label indicates a range of values. For example, the column labeled “15%” means “10% < x% ≤ 15%” where x% is the percentage increase in POA irradiance. For all three years, more than half of the overcast intervals showed an increase of 15% or more in POA irradiance using the S Formula’s optimal tilt angles instead of the conventional 2-axis tracking tilt angles. In particular, 58.7% of overcast intervals in Year 1 had an increase of 15% or more in POA irradiance, 55.5% of overcast intervals in Year 2, and 64.0% of overcast intervals in Year 3. Figure 7.4b shows the net increase in POA irradiance in W/m² when the S Formula’s optimal angles are used versus the conventional 2-axis tracking tilt angles. The percentage of overcast intervals with an increase in POA irradiance of 20 W/m² or more was 46.2% for Year 1, 45.3% for Year 2, and 46.2% for Year 3.

Figure 7.5 shows the distribution of the increase in POA irradiance for four different sky conditions: clear, partly cloudy, mostly cloudy, and overcast. Note that the large value for clear skies is limited to 1200 15-minute intervals in order to allow the distribution of the smaller values to be seen. There is a similar pattern for clear, partly cloudy, and mostly cloudy conditions. The
CHAPTER 7. S FORMULA METHOD’S OPTIMAL ANGLES FOR 2-AXIS TRACKING

Figure 7.3: Results of MATLAB simulations showing the POA irradiance collected for three days in 2014 - (a) an overcast day (May 28th), (b) a clear day (August 19th), and (c) a day with a mix of clouds and clear skies (August 27th). The blue dotted trace represents the POA calculated by a hypothetical 2-axis tracking PV array using the proposed S Formula to adjust the tilt angle. Shown for comparison is the POA collected by the hypothetical array using conventional tilt angles of $\beta = \theta_z$ (black solid line). Each step represents a 15-minute interval.
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Figure 7.4: Histograms showing the (a) percentage increase and (b) increase in W/m² of POA irradiance during overcast skies when optimal angles are calculated using the S Formula versus the conventional 2-axis tracking tilt angles. Intervals of overcast sky conditions for three 12-month periods – Nov. 1, 2013 to Oct. 31, 2014 (Year 1), Nov. 1, 2014 to Oct. 31, 2015 (Year 2), and Nov. 1, 2015 to Oct. 31, 2016 (Year 3).
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The highest number of intervals has a minimal change in POA irradiance (5 W/m²) or less, with fewer intervals showing increases in POA irradiance as high as 60 W/m². However, there is a different distribution for overcast skies for each 12-month period. The highest number of overcast intervals occur in the range of 10 to 25 W/m² and then gradually reduce in number as the change in POA irradiance increases.

The impact of capturing these small additional amounts of irradiance over the course of a year is shown in Figure 7.6. Each subfigure displays the yearly contribution of the three irradiation streams (beam, diffuse, and reflected) for conventional 2-axis tracking and for the S Formula method. For all three years, there is a slight loss in captured beam irradiation (about 0.5%). The reflected irradiation also decreases about 25% to 29%; however, this is a small loss when we consider that the contribution of reflected irradiation to the POA irradiation is very small. Each year shows a gain of approximately 13% in diffuse irradiation, which is a significant contribution to the yearly increase in POA irradiation.

This gain in yearly diffuse irradiation that is captured using the S Formula method results in an overall increase in POA irradiation and AC power, as shown in Tables 7.2 and 7.3. Table 7.2 presents the yearly POA insolation and Table 7.3 lists the yearly AC energy, as calculated in PV_LIB for three sets of tilt angles: conventional 2-axis tracking, optimal tilt angles found by the numerical method, and optimal tilt angles calculated directly using the S Formula. For the first year (Nov. 1, 2013 to Oct. 31, 2014), yearly POA insolation increased 2.35% and yearly AC energy increased 2.42% when using the S Formula’s optimal angles. For the second year (Nov. 1, 2014 to Oct. 31, 2015), yearly POA insolation increased 2.33% and AC energy increased 2.40% when using the S Formula’s optimal angles. For the third year (Nov. 1, 2015 to Oct. 31, 2016), yearly POA insolation increased 2.10% and AC energy increased 2.16% when using the S Formula’s optimal angles.
CHAPTER 7. S FORMULA METHOD’S OPTIMAL ANGLES FOR 2-AXIS TRACKING

Figure 7.5: Increase in POA irradiance during clear, partly cloudy, mostly cloudy, and overcast skies when optimal angles are calculated using the S Formula versus the conventional 2-axis tracking tilt angles.
CHAPTER 7. S FORMULA METHOD’S OPTIMAL ANGLES FOR 2-AXIS TRACKING

Figure 7.6: Comparison of beam, diffuse, and reflected irradiation collected by conventional 2-axis tracking versus using the S Formula method.
Table 7.2: Yearly POA Insolation calculated for three sets of tilt angles - conventional 2-axis tracking, optimal angles found using the numerical method, and S Formula’s optimal angles.

<table>
<thead>
<tr>
<th>Method</th>
<th>YEAR 1</th>
<th></th>
<th>YEAR 2</th>
<th></th>
<th>YEAR 3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yearly</td>
<td>Increase over</td>
<td>Yearly</td>
<td>Increase over</td>
<td>Yearly</td>
<td>Increase over</td>
</tr>
<tr>
<td></td>
<td>Insolation</td>
<td>Conventional</td>
<td>Insolation</td>
<td>Conventional</td>
<td>Insolation</td>
<td>Conventional</td>
</tr>
<tr>
<td></td>
<td>(kWh/m²)</td>
<td>2-axis tracking</td>
<td>(kWh/m²)</td>
<td>2-axis tracking</td>
<td>(kWh/m²)</td>
<td>2-axis tracking</td>
</tr>
<tr>
<td>Conventional 2-axis tracking</td>
<td>1616.82</td>
<td>—</td>
<td>1570.50</td>
<td>—</td>
<td>1712.11</td>
<td>—</td>
</tr>
<tr>
<td>Numerical method</td>
<td>1654.83</td>
<td>2.35%</td>
<td>1607.02</td>
<td>2.33%</td>
<td>1747.97</td>
<td>2.09%</td>
</tr>
<tr>
<td>S Formula method using measured DHI and GHI</td>
<td>1654.83</td>
<td>2.35%</td>
<td>1607.03</td>
<td>2.33%</td>
<td>1747.98</td>
<td>2.10%</td>
</tr>
</tbody>
</table>
Table 7.3: Yearly AC energy calculated for a 2-axis tracking PV array. Results are listed for three sets of tilt angles—conventional 2-axis tracking, optimal angles found using the numerical method, and S Formula’s optimal angles.

<table>
<thead>
<tr>
<th>Method</th>
<th>YEAR 1</th>
<th>YEAR 2</th>
<th>YEAR 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yearly AC Energy (kWh)</td>
<td>Increase over Conventional 2-axis tracking %</td>
<td>Yearly AC Energy (kWh)</td>
</tr>
<tr>
<td>Conventional 2-axis tracking</td>
<td>6244.70</td>
<td>—</td>
<td>6066.26</td>
</tr>
<tr>
<td>Numerical method</td>
<td>6395.83</td>
<td>2.42%</td>
<td>6211.56</td>
</tr>
<tr>
<td>S Formula method using measured DHI and GHI</td>
<td>6395.87</td>
<td>2.42%</td>
<td>6211.60</td>
</tr>
</tbody>
</table>
CHAPTER 7. S FORMULA METHOD’S OPTIMAL ANGLES FOR 2-AXIS TRACKING

7.4 Summary

In this chapter we discussed the S Formula method for finding the optimal tilt angles under different sky conditions. We then presented our results from simulating the S Formula method in MATLAB and the PV_LIB Toolbox using measured weather data.

Adjusting the 2-axis tracking arrays tilt angle in response to varying sky conditions allows the 2-axis tracking array to capture more of the available irradiation during 15-minute intervals. For 45% of the intervals with overcast skies, the minimum increase in POA irradiance was 20 W/m² using the S Formula’s optimal angles versus the conventional tilt angles.

When comparing the yearly contribution of the three irradiation streams (beam, diffuse, and reflected) for both conventional 2-axis tracking and for the S Formula method, we observed that there is a significant gain of about 13% in diffuse irradiation for each year when the S Formula method is used. This gain in yearly captured diffuse irradiation compensates for the slight loss in captured beam irradiation (about 0.5%) and reflected irradiation, thereby resulting in an overall increase in POA irradiation and AC energy. The increases in annual insolation and AC energy were 2.35% and 2.42% for the first year, 2.33% and 2.40% for the second year, and 2.10% and 2.16% for the third year.

In Chapter 8 we shall compare the results of the S Formula method for calculating the optimal tilt angles, yearly POA irradiation, and yearly AC energy with five methods—conventional 2-axis tracking, numerical, and Kelly and Gibson’s three methods [11]. We shall also compare the S Formula method’s results with the simulations and experiments reported in the literature.
8.1 Introduction

In Chapter 8, we compare the results of the S Formula method for calculating the optimal tilt angles, yearly POA irradiation, and yearly AC energy with five other methods: conventional 2-axis, numerical, and Kelly and Gibson’s three methods. We compare the S Formula method’s results with the simulations and experiments reported in the literature. We examine the contribution of each of the irradiation streams to the overall increase in captured POA irradiation, and we discuss the potential for additional energy gleaning under overcast skies due to the shift in spectral components. We propose several 2-axis tracking strategies for maximizing the capture of irradiation and for minimizing power consumption. We examine the financial impact of energy gleaning. Finally, we propose areas of future study on this research topic.

8.2 Comparing results: S Formula, Kelly and Gibson, and numerical methods

As shown in (3.23) in Section 3.2, the S Formula relates the tangent of the optimal tilt angle to the tangent of the conventional 2-axis tilt angle ($\theta_z$), multiplied by a function of $g(k_T)$, $\rho_g$, and $(\tau\alpha)_b$, $(\tau\alpha)_d$, and $(\tau\alpha)_r$. Since the clearness index, $k_T$, is an indicator of sky conditions, this function modifies the conventional 2-axis tracking tilt angle as the sky conditions change. For
clear, sunny skies, the correlation \( g(k_T) \) approaches 0, thereby causing the modifying function (that is, the second term on the right-hand side of (3.23)) to approach 1. This reduces to \( \beta_{optimal} \approx \theta_z \), which is in agreement with the conventional 2-axis tilt angle shown in (2.11a). In Figure 7.2b, the S Formula’s optimal tilt angles approximately equal the conventional tilt angle (\( \beta_{optimal} \approx \theta_z \)). The weather conditions for this day (August 19, 2014) were clear skies, except for a short period of fog occurring at around 6:30 a.m.

For overcast skies the correlation \( g(k_T) \) approaches 1, which causes the numerator of the modifying function in (3.23) to approach 0. Thus, the S Formula reduces to \( \beta_{optimal} \approx 0^\circ \).

For overcast skies the correlation \( g(k_T) \) approaches 1, which causes the numerator of the modifying function in (3.23) to approach 0. Thus, the S Formula reduces to \( \beta_{optimal} \approx 0^\circ \). Figure 7.2a shows that the S Formula’s optimal tilt angles are about \( 0^\circ \). According to the weather observations at Middlebury State Airport, on this day (May 28, 2014) there were overcast skies starting at 6:30 am and continuing through the rest of the day. The tilt angles calculated by the S Formula agree with the recommendations of several researcher to place the PV module in a horizontal position (\( \beta_{optimal} = 0^\circ \)) on overcast days [5, 4, 3, 33, 6, 10, 7]. In [9] the PV modules optimal tilt angle was found to be \( 0^\circ \) for the deterministic trajectory; however, for the stochastic trajectory, which takes into account the uncertainty in weather data and irradiance models, the optimal tilt angle was \( 20^\circ \) to \( 30^\circ \) less than the zenith angle.

On the overcast day (May 28, 2014) the S Formula’s POA irradiance exceeded the conventional 2-axis tracking’s POA irradiance during each 15-minute interval, as illustrated in Figure 7.3a. The increase in POA irradiance when using the S Formula’s optimal tilt angles ranged from 3.14% at midday to 52.49% before sunset in comparison with the POA irradiance calculated for the conventional 2-axis tracking tilt angles. In Section 8.3 we shall discuss how these simulated results are similar to the results obtained experimentally by researchers in Montreal, Canada on the same day.

In Figure 7.2c, the S Formula’s optimal tilt angles vary between \( 0^\circ \) and \( \theta_z \). According to weather observations from the Middlebury State Airport, the sky conditions changed throughout the day with periods of mostly or partly cloudy, overcast, and clear skies. The trace in Figure 7.2c is similar to the traces in Figures 7.2a and 7.2b, that is, the optimal tilt angle is \( 0^\circ \) during overcast skies and the optimal angle is \( \theta_z \) during periods of clear skies. However, as Figure 7.2c shows, there are also other angles — neither \( 0^\circ \) nor \( \theta_z \) — that are optimal during periods of varying cloud cover. This agrees with the simulations of [9] which showed that the optimal tilt angle varied from about \( 15^\circ \) to \( \theta_z \) for the deterministic trajectory, and from \( 35^\circ \) to \( \theta_z \) for the stochastic trajectory.

The optimal tilt angles found using the numerical method (Section 6.3) were compared with the S Formula’s angles. The optimal tilt angles shown in Figure 6.2 (numerical method) and
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Figure 7.2 (S Formula) are similar. The low values of MBE, MAE, and RMSE listed in Table 7.1 indicate that the S Formula is able to estimate the numerical method’s optimal tilt angles. In this study, measured diffuse horizontal irradiance and global horizontal irradiance data from the instrumentation sensors were substituted into the S Formula; however, the statistical errors may be larger if using a correlation for $g(kT)$. Identifying an appropriate correlation for $g(kT)$ to use in the S Formula is an area for future study.

Figure 7.4a shows the percentage increase in POA irradiance when using the S Formula’s optimal angles instead of the conventional tracking angles. All 15-minute intervals that occurred during overcast skies throughout the year were distributed into bins of 5%. The largest number of overcast intervals had an increase in POA irradiance of 5% or less. The remaining overcast intervals are somewhat evenly distributed with POA increases ranging from 10% to 50%. This range is similar to that reported in [3] where the horizontal sensors measured 20% to 82% more irradiance throughout an overcast day than the sensors pointed directly toward the sun.

Capturing these small amounts of increased POA irradiance throughout the year potentially leads to an increase in the yearly insolation and AC energy, as shown in Tables 7.2 and 7.3. The increase in POA insolation was 2.35% for the first year, 2.33% for the second year, and 2.10% for the third year. These results are similar to the 2.57% increase in yearly insolation that was calculated in [7] for Burlington, Vermont, using the numerical method, the Hay, Davies, Klucher, and Reindl (HDKR) anisotropic sky model and TMY3 weather data.

Table 8.1 shows the increase in yearly POA insolation for all five methods considered in this research. As can be seen from the table, the S Formula method for finding the optimal tilt angles of 2-axis tracking PV arrays does as well as the numerical method in increasing the capture of available solar radiation over a one-year period.

8.3 Comparing experimental results in Montreal with S Formula’s PV_LIB results for Middlebury

As discussed in Section 8.2 on the overcast day (May 28, 2014) the S Formula’s POA irradiance exceeded the conventional 2-axis tracking array’s POA irradiance during each 15-minute interval. The increase in POA irradiance ranged from 3.14% at midday to 52.49% before sunset. On the same day (May 28, 2014) and 160 km north of Middlebury, Vermont, the short circuit currents of two identical solar panels were recorded under overcast skies in Montreal, Canada [4]. One
Table 8.1: Percentage increase in yearly POA Insolation for Kelly and Gibson’s three methods \[11\], numerical method, and the S Formula method, when compared with the conventional 2-axis tracking.

<table>
<thead>
<tr>
<th>Method</th>
<th>Increase in POA insolation over conventional 2-axis tracking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>YEAR 1</td>
</tr>
<tr>
<td>Conventional 2-axis tracking</td>
<td>—</td>
</tr>
<tr>
<td>Method 1: DTS vs H</td>
<td>1.05%</td>
</tr>
<tr>
<td>Method 2: GHI vs Diffuse</td>
<td>0.53%</td>
</tr>
<tr>
<td>Method 3: DTS vs Limit</td>
<td>0.32%</td>
</tr>
<tr>
<td>Numerical method</td>
<td>2.35%</td>
</tr>
<tr>
<td>S Formula method using measured DHI and GHI</td>
<td>2.35%</td>
</tr>
</tbody>
</table>
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panel was mounted at a tilt angle of $0^\circ$ and the other mounted on a 2-axis tracking array. Both panel’s short circuit currents were measured every hour for 12 hours. During overcast periods, the fixed horizontal panel’s short-circuit current, $I_{sc,H}$, was 4.1% to 32.3% greater than the tracking PV panel’s short-circuit current, $I_{sc,DTS}$.

In another experiment by the same researchers [10,4], 8-ohm resistive loads were installed on each panel and the energy production was measured. Over the course of one cloudy day (June 11, 2014), the total energy produced by the PV panel placed in the horizontal position was 27.37% more than that produced by the 2-axis tracking PV panel.

In Figures 8.1 and 8.2, we compare the experimental results reported in [10,4] with the measured irradiance data and simulation results from Middlebury, Vermont. Since the experimental results from Montreal were reported in solar time, we first converted the timestamp from solar time to local Eastern Daylight Time (EDT), and then aligned the data to correspond with the nearest EDT hourly timestamp of the Middlebury, Vermont data. In [4], the location of the 2-axis tracking array is given as latitude of $45^\circ29'$ North and longitude of $73^\circ33'$ West, which converts to decimal as $45.48^\circ$ and $73.55^\circ$. The Middlebury College installation’s longitude is $73.185854^\circ$ West, which is approximately the same longitude as Montreal, Canada. The difference in solar time between the Montreal, Canada and Middlebury, Vermont is approximately two minutes.

To get an understanding of the weather conditions that occurred throughout the day, we downloaded weather observations from the Montreal-Pierre Elliott Trudeau International Airport [69]. Table 8.2 compares the weather conditions for both locations on May 28, 2014 and June 11, 2014. It should be noted that the Montreal airport reported two weather observations at timestamps 14:00 on May 28 and 08:00 on June 11.

First, we compare the results for May 28, 2014. We see from Table 8.2 that Middlebury State Airport reported overcast skies throughout the day. Montreal experienced rainy or overcast conditions for most of the day, except for an observation of partly cloudy skies at 19:00. In Figure 8.1 there are three graphs of data for May 28, 2014 that exhibit similar curves. In Figure 8.1a the experimental results of [4] are plotted. Throughout most of the day, the short-circuit current on the horizontal panel, $I_{sc,H}$, slightly exceeds the short-circuit current of the panel that is pointed directly at the sun, $I_{sc,DTS}$, with the exception occurring at the timestamp of 18:00. In Figure 8.1b, the measured GHI data slightly exceeds the measured POA irradiance throughout the overcast day at Middlebury. Results of the simulations in PV_LIB are shown in Figure 8.1c. We see that the simulated POA irradiance collected at the optimal tilt angles predicted by the S Formula exceeds the POA irradiance collected by a conventional 2-axis tracking array.
(a) $I_{SC,H}$ and $I_{SC,DTS}$, Montreal, Canada. Rated short-circuit current is 7.9 amps.

(b) GHI and POA irradiance, Middlebury, Vermont

(c) Simulated POA irradiance for conventional 2-axis tracking and S Formula, Middlebury, Vermont

Figure 8.1: Results for two locations on May 28, 2014. (a) Experimental results for Montreal, Canada [10, 4], (b) Measured irradiance data for Middlebury, Vermont, and (c) Simulated results for Middlebury, Vermont
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Figure 8.2: Results for two locations on June 11, 2014. (a) Experimental results for Montreal, Canada [10][4], (b) Measured irradiance data for Middlebury, Vermont, and (c) Simulated results for Middlebury, Vermont.
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Table 8.2: Weather observations for Middlebury, Vermont and Montreal, Canada on May 28 and June 11, 2014.

<table>
<thead>
<tr>
<th>Local Time (EDT)</th>
<th>May 28, 2014</th>
<th>June 11, 2014</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Middlebury, Vermont</td>
<td>Middlebury, Vermont</td>
</tr>
<tr>
<td></td>
<td>Montreal, Canada</td>
<td>Montreal, Canada</td>
</tr>
<tr>
<td>06:00</td>
<td>overcast</td>
<td>clear</td>
</tr>
<tr>
<td></td>
<td>light drizzle</td>
<td>scattered clouds</td>
</tr>
<tr>
<td>07:00</td>
<td>overcast</td>
<td>clear</td>
</tr>
<tr>
<td></td>
<td>light rain</td>
<td>scattered clouds</td>
</tr>
<tr>
<td>08:00</td>
<td>overcast</td>
<td>clear</td>
</tr>
<tr>
<td></td>
<td>light drizzle</td>
<td>clear; mostly cloudy</td>
</tr>
<tr>
<td>09:00</td>
<td>overcast</td>
<td>clear</td>
</tr>
<tr>
<td></td>
<td>light drizzle</td>
<td>mostly cloudy</td>
</tr>
<tr>
<td>10:00</td>
<td>overcast</td>
<td>clear</td>
</tr>
<tr>
<td></td>
<td>light drizzle</td>
<td>mostly cloudy</td>
</tr>
<tr>
<td>11:00</td>
<td>overcast</td>
<td>overcast</td>
</tr>
<tr>
<td>12:00</td>
<td>overcast</td>
<td>light rain</td>
</tr>
<tr>
<td></td>
<td>overcast</td>
<td>overcast</td>
</tr>
<tr>
<td>13:00</td>
<td>overcast</td>
<td>light rain</td>
</tr>
<tr>
<td></td>
<td>overcast</td>
<td>overcast</td>
</tr>
<tr>
<td>14:00</td>
<td>overcast</td>
<td>light rain</td>
</tr>
<tr>
<td></td>
<td>partly cloudy; overcast</td>
<td>light rain</td>
</tr>
<tr>
<td>15:00</td>
<td>overcast</td>
<td>light rain</td>
</tr>
<tr>
<td></td>
<td>overcast</td>
<td>light rain</td>
</tr>
<tr>
<td>16:00</td>
<td>overcast</td>
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</tr>
<tr>
<td></td>
<td>overcast</td>
<td>moderate rain</td>
</tr>
<tr>
<td>17:00</td>
<td>overcast</td>
<td>light rain showers</td>
</tr>
<tr>
<td>18:00</td>
<td>overcast</td>
<td>moderate drizzle</td>
</tr>
<tr>
<td></td>
<td>overcast</td>
<td>light rain showers</td>
</tr>
<tr>
<td>19:00</td>
<td>overcast</td>
<td>light rain</td>
</tr>
</tbody>
</table>

Next, we compare the results for June 11, 2014. From Table 8.2, we see that Middlebury State Airport experienced clear skies from sunrise to 11:00, while there was a mix of weather observations (scattered clouds, clear skies, and mostly cloudy) reported by the Montreal airport during that same period. After 11:00 the weather was rainy or overcast for both locations. Figure 8.2a compares the DC energy production of the experimental setup in Montreal. The horizontal panel produced more DC energy than the panel that was pointed directly at the sun for most of the day; however, at 14:00 the horizontal panel produced less DC energy. In Figure 8.2b there was more POA irradiance captured than GHI during the morning hours when skies were clear. Based on the PV_LIB simulation results, we see from Figure 8.2c that the simulated AC power using the S Formula method was similar to that of a conventional 2-axis array that tracks the sun during clear skies. However, after 11:00 when the skies are overcast or there is light rain, the AC power captured by the S Formula slightly exceeds the AC power produced using the conventional tracking angles.
8.4 A closer look at the S Formula method’s three irradiance streams

In this section we look at the beam, diffuse, and ground-reflected components of the POA irradiance for May 28 and June 11, 2014. These are the two days discussed in Section 8.3.

Figure 8.3 shows the POA irradiance calculated using the conventional 2-axis tracking tilt angles and the optimal tilt angles predicted by the S Formula. As was shown in Table 8.2 in the previous section, weather observations at the Middlebury State Airport indicated that the skies were overcast throughout the day on May 28. On June 11 the skies were clear until 11:00, and then became overcast or rainy during the remainder of the day. Throughout both days, the POA irradiance captured using the S Formula’s optimal tilt angles is about the same or exceeds the POA irradiance collected by the conventional 2-axis tracking angles.

Figure 8.3: POA irradiation collected by conventional 2-axis tracking versus the S formula method on (a) May 28, 2014 and (b) June 11, 2014.
The POA irradiance plotted in Figure 8.3 is the sum of three irradiance streams – beam, diffuse, and ground-reflected. Each irradiance stream is separately plotted in Figures 8.4 and 8.5 for May 28 and June 11, respectively. Each graph compares the irradiance calculated using the conventional 2-axis tracking tilt angles and the irradiance calculated using the S Formula’s optimal tilt angles. It should be noted that the range of the y-axis varies for each graph.

As seen in Figure 8.4 for the overcast day of May 28th, the amount of ground-reflected irradiance captured using the S Formula’s method is about 0 W/m². This is because the optimal tilt angle under overcast skies is $0^\circ$. Although the conventional 2-axis tracking tilt angles capture more beam and ground-reflected irradiance, the contribution of the beam and ground-reflected irradiance streams to the POA irradiance collected by an array tracking the sun during overcast skies is minimal. Figure 8.4b shows that most of the contribution to POA irradiance comes from diffuse irradiance. Positioning the 2-axis tracker at the optimal angles calculated by the S Formula captures more of the diffuse irradiance during each interval, thereby resulting in more POA irradiance.

Figure 8.5 shows the irradiance streams for June 11, 2014. During the morning hours when clear skies were observed, the beam irradiance stream is the principal component of POA irradiance. The S Formula method’s optimal angles and the conventional 2-axis tracking angles are able to capture approximately the same amount of beam irradiance. However, as the morning progresses and the sky conditions change, the amount of beam irradiance decreases and the diffuse irradiance increases. During the afternoon hours when the skies are overcast or rainy, the diffuse irradiance stream makes up a greater portion of POA irradiance. Similar to what is observed for May 28, the conventional 2-axis tracking angles collect more of the ground-reflected irradiance stream under cloudy skies. However, the contribution of the ground-reflected stream to the POA irradiance is minimal. Diffuse irradiance is the dominant contributor to POA irradiance during cloudy periods, and thus the 2-axis tracking array that uses the S Formula’s optimal angles will capture more POA irradiance.

In summary, under clear skies the irradiance captured by positioning the tracking array at the S Formula’s optimal tilt angles is about the same as the irradiance captured using the conventional 2-axis tracking angles. This is because the S Formula’s optimal tilt angles is $\beta = \theta_z$ under clear skies. Under overcast skies, a tracking array positioned at the S Formula’s angles will collect slightly less beam and ground-reflected irradiance than the conventional 2-axis tracking array. However, the contribution of the beam and ground-reflected irradiance streams is very small during overcast periods, and the diffuse irradiance stream is the primary contributor to POA irradiance. As seen in Figures 8.4b and 8.5b, placing the array at the S Formula’s optimal angles results in a significant
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(a) Beam irradiance

(b) Diffuse irradiance

(c) Ground-reflected irradiance

Figure 8.4: Comparison of beam, diffuse, and reflected irradiation collected by conventional 2-axis tracking versus using the S formula method on May 28, 2014. Note that the range of the y-axis is different for each graph.
Figure 8.5: Comparison of beam, diffuse, and reflected irradiation collected by conventional 2-axis tracking versus using the S formula method on June 11, 2014. Note that the range of the y-axis is different for each graph.
increase in the amount of diffuse irradiance that is collected, and this in turn leads to an increase in the POA irradiance.

8.5 Spectral components of diffuse irradiance under cloudy skies: Potential for increased energy gleaning

In Section 2.2 we presented an overview of the solar resource and the three irradiation streams. We mentioned that the extraterrestrial radiation at the top of the atmosphere is about 1367 W/m², and that this amount varies slightly throughout the year depending on the day of the year [8]. However, the amount of extraterrestrial radiation is distributed among a range of wavelengths from about 280 nm to 2500 nm. As the solar radiation enters the earth’s atmosphere, the extraterrestrial radiation is attenuated by two processes—absorption and scattering. It is absorbed by ozone, water vapor, and CO₂, and it is scattered by air vapor, water, and dust [8]. Furthermore, as a result of these processes, the global horizontal irradiance (GHI) and its three component irradiation streams have spectral distributions that can vary depending on the climate.

Figure 8.6 shows the spectral irradiance distribution for the Extraterrestrial Radiation at the top of the atmosphere, the GHI on a surface that is tilted at an angle of 37°, the sum of direct normal irradiance (DNI) and circumsolar irradiance, and diffuse horizontal irradiance (DHI). This graph of the ASTM G-173-03 Reference Solar Spectral Irradiance at an air mass (AM) of 1.5 was created from a table of data available at [70]. Under clear skies, the spectral distribution of GHI and the sum of DNI and circumsolar irradiance are nearly the same, with the maximum amount of irradiance occurring at the same wavelength. However, the spectral distribution of diffuse irradiance has shifted towards the shorter wavelengths.

In this section we will present other researchers’ findings on how the spectral distribution of diffuse irradiation shifts under cloudy skies. We will see that different types of irradiance sensors and PV modules may have different spectral responses depending upon the material. We will suggest when one type of sensor or module would be a better fit for a given application. We will review the specifications of the sensors and PV modules that are installed at Middlebury College, and which were used in the PV_LIB simulations. Finally, we will review our results to see if additional irradiation collection is possible under cloudy conditions.

The researchers in [71] used a MultiFilter Rotating Shadow Band Radiometer (MFRSR) to record global irradiance and diffuse irradiance across a range of wavelengths. The purpose of
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Figure 8.6: Graph of ASTM AM 1.5 Reference Solar Spectral Irradiance showing the spectral components of Extraterrestrial radiation, the sum of direct normal and circumsolar irradiance, and diffuse horizontal irradiance. (Graph generated from data available at [70]).

their research was to determine whether the ratio of diffuse to global could be used as a method for screening occurrences of clouds for radiometers. They plotted curves of the spectral irradiance in W/m²·nm versus time for three wavelengths – 413 nm, 613 nm, and 867 nm. For the case of global irradiance, the spectral irradiance for the 613 nm curve was the highest throughout the day, followed by 413 nm and then 867 nm. This ordering of the curves from highest (613 nm) to lowest (867 nm) was true for the day with clear skies, the day with clouds forming during the afternoon, and the day when there were clouds during the midday hours. However, for diffuse irradiance, the order of the curves from highest to lowest spectral irradiance was 413 nm, 613 nm, and 867 nm. Furthermore, on the day with the midday periods of cloudiness, there were sharp increases in the spectral irradiance curves for 613 nm and 867 nm. The spikes in the 613 nm curve appear to be almost equal to the 413 nm spectral irradiance values. This appears to indicate that the spectral distribution of diffuse irradiance shifts towards longer wavelengths during cloudy sky conditions.

Another group of researchers used 15 different commercial and experimental shaded pyranometers to measure the diffuse horizontal irradiance on clear and cloudy days [72]. Their goal was to develop a standard method for measuring diffuse horizontal irradiance. Figure 8.7 shows a plot of the normalized diffuse irradiance as a function of wavelength for three days—a “clean” day
with clear skies and no haze, a clear day with a hazy sky, and a cloudy day. On the cloudy day, the spectral distribution of the diffuse irradiance has shifted to the longer wavelengths with the maximum occurring at about 500 nm. Between the wavelengths of 500 nm and 1000 nm, the shape of the spectral distribution has become wider and higher than for the case of the clear sky. For example, at about 750 nm the normalized diffuse irradiance has increased from about 0.2 for clear, clean skies to 0.7 for cloudy skies.

Figure 8.7: Spectral distribution of diffuse irradiance under clear, hazy, and cloudy skies. Under cloudy skies, the distribution of diffuse irradiance shifts to longer wavelengths. Reprinted from [72] with permission of John Wiley & Sons, Inc.

In Section 5.2.2 we briefly discussed the two sensors that measure irradiance at Middlebury College’s PV installation. The CMP11 and LI-200SA are composed of different materials, and thus they have different parameters and spectral responses.

The CMP11 is a thermopile detector with broad spectral response. Its spectral range is specified as 285 to 2800 nm at its 50% points. Spectral selectivity is less than 3% from 350 to 1500 nm [61]. At the Middlebury College installation, the CMP11 is mounted at the top of the PV-17 tracking array and measures POA irradiance.

The LI-200SA has a silicon photodetector, and this results in a narrower, non-constant
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spectral response from 380 to 1155 nm [62]. This sensor is mounted on the Rotating Shadowband Radiometer (RSR2), and it is used to measure the GHI and DHI irradiance. Once every three seconds, the RSR2 measures GHI. One-minute averages of the GHI data points are calculated. The RSR2’s shadowband is rotated over the LI-200SA at least once every 30 seconds. However, the shadowband will rotate more frequently under cloudy skies. The direct normal irradiance is then calculated from the GHI and DHI data.

The spectral responses of the LI-200SA and the Kipp & Zonen CMP11 were previously presented in Figure 5.6 and Figure 5.4 in Section 5.2.2. For ease of comparison, Figure 8.8 displays the spectral responses of the LI-200 and the CMP21 (a cousin of the CMP11) [73]. This graph also shows the spectral distributions of global horizontal irradiance (GHI) and diffuse horizontal irradiance (DHI). The authors calculated the GHI and DHI values in the SMARTS program [74] using the conditions found in the American Society for Testing and Materials ASTM G-173-03 [75]. This graph shows the near-constant response of the CMP21 and CMP11 over a wide range of wavelengths from approximately 300 to 2500 nm. From the graph, we can see that the LI-200SA’s spectral response is not flat; rather, it has an almost linearly increasing response between the wavelengths of 380 nm to 1000 nm, and then it rapidly decreases and drops off at 1155 nm.

Figure 8.8: Comparing the spectral responses of LI-200 and CMP21 pyranometer with the spectral components of GHI and DHI. The CMP21 has the same spectral response as the CMP11. Reprinted from [73]; used in accordance with the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

As was mentioned in Section 5.2.1, each 2-axis tracking PV array at Middlebury College’s
installation holds twenty Evergreen ES-A 210 multicrystalline silicon photovoltaic modules. Figure 8.9 shows the normalized spectral response for five photovoltaic technologies: crystalline silicon (c-Si), amorphous silicon (a-Si), cadmium telluride (CdTe), high-efficiency crystalline silicon (high-eff c-Si), and small-band-gap copper indium gallium selenide (CIGS) \cite{76}. As this figure shows, there is a wide variation between the spectral response of different PV technologies. One would expect the spectral responses of the c-Si and the high-efficiency c-Si to be about the same because they are made from similar materials. However, it has been reported in the literature that there can be variations in the spectral response of PV modules from the same technology \cite{77}.

As discussed in \cite{78}, \cite{79}, the decision to use a broadband thermopile pyranometer such as the CMP11 or a silicon photodetector pyranometer such as the LI-200SA with a narrower spectral response depends on the application. Typically, the thermopile pyranometers are used for meteorology in order to get accurate measurements of the solar resource. Thermopile pyranometers such as the CMP11 are able to measure the irradiance over a wide band of wavelengths; thus, they would be able to measure the diffuse irradiance that is present at the shorter wavelengths on clear days. However, as we can see from Figure 8.9 much of that irradiance cannot be converted into electricity because it is outside the PV module’s spectral response. In the case of a multicrystalline silicon module,
a narrowband silicon photodetector that matches the module’s spectral response may be a better indicator of how much of the available solar radiation can be converted to electricity.

In this research, the results reported in Sections 6.3.2 and 7 were calculated in PV_LIB using the measured data from the LI-200SA silicon irradiance sensor located on the RSR2. When the S Formula’s optimal tilt angles are used to adjust a 2-axis tracking PV array, our results indicated a 2.10% to 2.35% annual increase in POA irradiation versus using the conventional 2-axis tracking angles.

Now that we know that the spectral response of diffuse irradiance shifts to longer wavelengths under cloudy skies, is there the potential for additional energy gleaning?

First, it should be noted that PV_LIB does not model the spectral distribution of the measured data or the spectral response of the PV module. Thus, these results of a 2.10% to 2.35% annual increase in POA irradiance do not take into account the potential increase in irradiance that results when the spectral distribution of the diffuse irradiance shifts to longer wavelengths.

Second, the potential increase could be estimated by looking at Figure 8.10. In this figure, we show the spectral distribution of diffuse irradiance under clear, hazy, and cloudy skies that was reported in [72], and superimpose the spectral response of crystalline silicon PV module. Under cloudy skies, the distribution of diffuse irradiance shifts to longer wavelengths within the response curve of the PV module.

### 8.6 Tracking strategies to maximize irradiance capture and minimize power consumption

In this section we will briefly discuss the power consumed by 2-axis tracking arrays, and in particular, the AllEarth Renewables tracking array. We will look at data from six days with various sky conditions, and we will give recommendations on when to adjust the tilt and the azimuth angles in order to minimize the power consumed when gleaning the additional available irradiation. Finally, we will present the total net increase in AC energy due to the additional irradiation that is captured and due to the decrease in power consumption that results by not adjusting the tilt and/or azimuth angles.

In Section 2.3.2 we discussed commercial 2-axis time-position tracking arrays, the different mechanisms employed to adjust the tilt and azimuth angles, and information on the power consumed. As mentioned earlier, the availability of information on the power consumption of 2-axis tracking
Figure 8.10: Spectral distribution of diffuse irradiance under clear, hazy, and cloudy skies shown with spectral response of crystalline silicon PV module. Under cloudy skies, the distribution of diffuse irradiance shifts to longer wavelengths within the response curve of c-Si. Adapted from [72], which is reprinted with permission from John Wiley & Sons, Inc.
arrays varies, with some companies giving a value in terms of a percentage of annual energy production while other companies provide a daily or yearly value. The manufacturer of the Titan Tracker gives an approximate value of 0.5 kWh per day\(^{28}\). MecaSolar specifies the power consumption of the motors as 100 kWh per year\(^{29}\). By dividing this yearly amount by the number of days in the year, we can estimate an average daily power consumption of 0.274 kWh.

AllEarth Renewables specifies annual power consumption as less than 1% of system output for its tracking array\(^{24}\). Its hydraulic power unit (HPU) is a 0.18-kW single-phase AC motor manufactured by Hydro-Tek\(^{80}\). A hydraulic motor drives the ring-worm gear that rotates the array about its vertical supporting pole. A hydraulic cylinder adjusts the array’s tilt angle. In addition to the HPU, there are several components that require electrical power\(^{81}\). There are two electronic circuit boards inside the control box\(^{82}\). One of the boards is populated with a wireless communications radio, a Global Positioning System (GPS) unit, and LED indicator lights. The anemometer that is used to measure wind speed and direction requires power to operate.

In the research discussed in Section\(^{2.4.3}\) the authors measured the power consumption of an industrial 2-axis tracking array\(^{35}\). These results were then used in creating daily trajectories for the tilt and azimuth angles of 2-axis tracking arrays under clear, sunny skies. The tracking array held seven 105-Watt PV panels, and its angles were adjusted using permanent magnet dc motors (PMDC) that were powered by 24V batteries. The researchers measured the power needed to start the motors and to change tilt and azimuth angles. The results were presented as two linear plots – one representing the energy consumption per change in tilt angle, and the other as the energy consumption per change in azimuth angle. A 70° change in tilt angle consumed 2 Wh of energy, while a 150° change in surface azimuth angle consumed 8 Wh of energy. These results were later used by the researchers in\(^{9}\) to create daily trajectories for a 2-axis tracking array under clear, partly cloudy, and cloudy skies.

We will base our analysis on the information listed in AllEarth Renewables’ data sheet; that is, the annual power consumption of the 2-axis tracking array is less than 1% of system output. The actual AC energy produced by the PV-17 tracker from November 1, 2013 to October 31, 2014 was 6965.4 kWh. Based on the data sheet’s stated value of less than 1% of system output, the average energy consumption is 69,654 kWh per year or about 190 Wh per day. Furthermore, if we consider that there were 14,533 15-minute intervals of daylight during the 12-month period, we can estimate the energy consumption as 4.8 Wh per 15-minute period. This value of 4.8 Wh could be used to determine a threshold for deciding whether or not to adjust the tilt angle.

We will now present six different scenarios that describe how the 2-axis tracker could be
oriented throughout the day using the S Formula’s optimal angles to increase the capture of POA irradiation and also minimize power consumption. It should be noted that at sunset, the tracker is facing nominally westward with the panels placed horizontally. The tracker is stowed in a horizontal position during the night, and rotates to its starting position for the next day. In the morning at sunrise, the tracker is facing nominally eastward. It should be noted that Figures 8.11c through 8.16c observe PVLIB Toolbox’s convention of due south being equal to 180°.

**Scenario 1: May 28, 2014, An overcast day in late spring:** Figure 8.11 shows the tracking strategy for an overcast day on May 28, 2014. Since the S Formula predicts the optimal tilt angle to be 0°, the array is placed in a horizontal position throughout the day. The additional capture of POA irradiance using the S Formula’s optimal tilt angles results in a net increase in AC energy of 6 Wh to 42 Wh when compared with the conventional 2-axis tracking angles. Instead of tracking the sun using the solar azimuth angles, the array’s azimuth position is held at the starting angle of 75.8° throughout the day, thereby reducing the amount of energy consumed by the tracker.

**Scenario 2: December 28, 2013, An overcast day in winter:** Figure 8.12 shows the tracking strategy for an overcast day in winter (December 28, 2013). Since the S Formula predicts the optimal tilt angle to be 0°, the array is placed in a horizontal position throughout the day. The additional capture of POA irradiance using the S Formula’s optimal tilt angles results in a net increase in AC energy of 8 Wh to 22 Wh when compared with the conventional 2-axis tracking angles. Instead of tracking the sun using the solar azimuth angles, the array’s azimuth position is held at the starting angle of 133.8° throughout the day, thereby reducing the amount of energy consumed by the tracker.

This scenario is similar to the previous one. However, it should be noted that the conventional 2-axis tilt angles are greater in winter than in the summer because the sun is lower in the sky during the winter. Furthermore, the range of tilt angles during a winter’s day is narrower than on a summer’s day. The conventional 2-axis azimuth angles also differ significantly between the two days. During the summer, the surface azimuth angles have a greater rate of change around the noontime hours, whereas the change is more linear during the winter.

**Scenario 3: February 20, 2014, Clear skies in the morning and overcast skies in the afternoon:** Figure 8.13 shows the tracking strategy for a day that begins with clear skies in the morning, and then changes over to overcast skies in the afternoon. The S Formula predicts the optimal tilt angles to be the conventional 2-axis tilt angles, θz, under clear skies. After 12 noon, the tracking array’s optimal tilt angles would decrease to 0°. By placing the array in the horizontal position during the afternoon, there is a net increase in AC energy of 5 Wh to 64 Wh when compared with the conventional 2-axis tracking angles. During the morning hours, the array would track the
Figure 8.11: An overcast day (May 28, 2014). Net increase in AC energy due to greater collection of POA irradiation by the S Formula’s optimal tilt angles instead of the conventional 2-axis tracking angles. (a) Net increase in AC energy. (b) S Formula’s optimal tilt angles vs. conventional 2-axis tilt angles. (c) Surface azimuth angle is kept constant and does not track the sun. Note: Due south = 180°.
Figure 8.12: An overcast day (Dec. 28, 2013). Net increase in AC energy due to greater collection of POA irradiation by the S Formula’s optimal tilt angles instead of the conventional 2-axis tracking angles. (a) Net increase in AC energy. (b) S Formula’s optimal tilt angles vs. conventional 2-axis tilt angles. (c) Surface azimuth angle is kept constant and does not track the sun. Note: Due south = 180°.
sun using the solar azimuth angles. However, as the cloudy skies begin to appear about noontime, the array’s azimuth position would be maintained at an angle of about 180° throughout the remainder of the day. After sunset, the array would be rotated about 65° (instead of 130°) to the next day’s starting position, facing nominally eastward. Thus, the potential savings in energy consumption is the result of three sources: not adjusting the tilt angle, not adjusting the surface azimuth angle, and rotating the array only half the conventional azimuth angular distance to the next day’s starting position.

**Scenario 4: December 4, 2013, Overcast skies in the morning and clear skies in the afternoon:** Figure 8.14 shows the tracking strategy for a day that begins with overcast skies in the morning, and then changes to clear skies in the afternoon. The S Formula predicts the optimal tilt angles to be 0° during the morning hours, and then increasing to the conventional 2-axis tilt angles, \( \theta_z \), under clear skies in the afternoon. By placing the array in the horizontal position during the morning, there is a net increase in AC energy of 11 Wh to 49 Wh when compared with the conventional 2-axis tracking angles. The array would track the sun using the solar azimuth angles throughout the day.

**Scenario 5: June 9, 2014, Overcast skies in the morning, clear skies midday, and cloudy skies in the afternoon:** Figure 8.15 shows the tracking strategy for a day that begins with overcast skies in the morning, changes to clear skies during the midday hours, and then ends with cloudy skies in the afternoon. The S Formula predicts the optimal tilt angles to be 0° during the morning hours, and then increasing to the conventional 2-axis tilt angles \( \theta_z \) for the midday hours, and then varying between 0° and near \( \theta_z \) in the afternoon. By placing the array in the horizontal position during the morning, there is a net increase in AC energy of 5 Wh to 54 Wh when compared with the conventional 2-axis tracking angles. For the afternoon hours, there is a net increase in AC energy of 7 Wh to 58 Wh. The array would track the sun using the solar azimuth angles throughout the day.

**Scenario 6: March 16, 2014, A day with varying sky conditions throughout the day:** Figure 8.16 shows the tracking strategy for a day with variably cloudy skies throughout the day. The S Formula predicts the optimal tilt angles to be 0° to almost \( \theta_z \). There is a net increase in AC energy of 0.4 Wh to 70 Wh when compared with the conventional 2-axis tracking angles. A possible strategy would be to set the optimal tilt angles to \( \theta_z - 15° \). The array would track the sun using the solar azimuth angles throughout the day.

Table 8.3 summarizes the tracking strategy for each daily scenario and the potential net increase in total AC energy for each day. The total AC energy is the sum of the daily increase in AC energy using the S Formula and the estimated energy savings when the array is not moving.

Based on our earlier calculation of 190 Wh for the average daily energy consumption, we...
Figure 8.13: A day with clear skies in the morning and overcast skies in the afternoon (Feb. 20, 2014). Net increase in AC energy due to greater collection of POA irradiation by the S Formula’s optimal tilt angles instead of the conventional 2-axis tracking angles. (a) Net increase in AC energy. (b) S Formula’s optimal tilt angles vs. conventional 2-axis tilt angles. (c) Surface azimuth angles track the sun in the morning, but cease tracking in the afternoon. Note: Due south = 180°.
Figure 8.14: A day with overcast skies in the morning and clear skies in the afternoon (Dec. 4, 2013). Net increase in AC energy due to greater collection of POA irradiation by the S Formula’s optimal tilt angles instead of the conventional 2-axis tracking angles. (a) Net increase in AC energy. (b) S Formula’s optimal tilt angles vs. conventional 2-axis tilt angles. (c) Surface azimuth angles track the sun all day. Note: Due south = 180°.
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(a) Net increase in AC energy

(b) Tilt angles - proposed vs. conventional

(c) Surface azimuth angles - proposed vs. conventional

Figure 8.15: A day with overcast skies in the morning, clear skies midday, and overcast skies in the afternoon (June 9, 2014). Net increase in AC energy due to greater collection of POA irradiation by the S Formula’s optimal tilt angles instead of the conventional 2-axis tracking angles. (a) Net increase in AC energy. (b) S Formula’s optimal tilt angles vs. conventional 2-axis tilt angles. (c) Surface azimuth angles track the sun all day. Note: Due south = 180°.
Figure 8.16: A day with varying sky conditions throughout the day (March 16, 2014). Net increase in AC energy due to greater collection of POA irradiation by the S Formula’s optimal tilt angles instead of the conventional 2-axis tracking angles. (a) Net increase in AC energy. (b) S Formula’s optimal tilt angles vs. conventional 2-axis tilt angles. (c) Surface azimuth angles track the sun all day. Note: Due south = 180°.
made the following assumptions:

1. The average daily value is split evenly between the energy required to adjust the tilt angles and the energy required to adjust the azimuth angles. In other words:
   - Average daily energy consumption to adjust the tilt is 95 Wh.
   - Average daily energy consumption to adjust the surface azimuth angles is 95 Wh.

2. When the tilt angles are being adjusted for only half the day, then the average daily value for the energy required to adjust the tilt angles is divided by 2. A similar adjustment is calculated for the surface azimuth angles. In other words:
   - Adjusting tilt angles during half the day (morning only or afternoon only) is 47.5 Wh.
   - Adjusting surface azimuth angles during half the day (morning only or afternoon only) is 47.5 Wh.

As mentioned earlier in this section, there are several components that require electrical power such as the wireless communications radio, a GPS unit, LED indicator lights, the anemometer, and other control circuitry. These components will still consume energy even when the 2-axis tracker is not tracking the sun. Thus, the numbers that are shown in Table 8.3 for the potential daily gain in AC energy when the 2-axis tracking array is not tracking the sun should only be seen as rough estimates.
Table 8.3: Tracking strategy and increase in AC energy for 6 days of different sky conditions in Middlebury, Vermont.

<table>
<thead>
<tr>
<th>Date</th>
<th>Sky conditions</th>
<th>Tilt angle</th>
<th>Surface azimuth angle</th>
<th>Daily gain in AC energy due to increased capture of irradiance when using proposed tracking strategy vs conventional 2-axis tracking (Wh)</th>
<th>Daily gain in AC energy due to not moving tracking mechanism during cloudy skies (Wh)</th>
<th>Total daily gain in AC energy when using proposed tracking strategy vs conventional 2-axis tracking (Wh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 28, 2014</td>
<td>Overcast all day</td>
<td>$\beta = 0^\circ$</td>
<td>Do not move. Hold $\gamma$ at $\gamma_{sunrise}$</td>
<td>1014</td>
<td>190</td>
<td>1204</td>
</tr>
<tr>
<td>Dec. 28, 2013</td>
<td>Overcast all day</td>
<td>$\beta = 0^\circ$</td>
<td>Do not move. Hold $\gamma$ at $\gamma_{sunrise}$</td>
<td>435</td>
<td>190</td>
<td>625</td>
</tr>
<tr>
<td>Feb. 20, 2014</td>
<td>Clear morning, overcast afternoon</td>
<td>$\beta = \theta_z$, then decrease to $\beta = 0^\circ$</td>
<td>$\gamma = \gamma_s$ then hold at $\gamma_{noon}$</td>
<td>647</td>
<td>95</td>
<td>742</td>
</tr>
<tr>
<td>Dec. 4, 2013</td>
<td>Overcast morning, clear afternoon</td>
<td>$\beta = 0^\circ$, then increase to $\beta = \theta_z$</td>
<td>Track sun all day. $\gamma = \gamma_s$</td>
<td>420</td>
<td>47.5</td>
<td>467.5</td>
</tr>
<tr>
<td>June 9, 2014</td>
<td>Overcast morning, clear midday, cloudy afternoon</td>
<td>$\beta = 0^\circ$, increase to $\beta = \theta_z$, decrease to $\beta = 0^\circ$</td>
<td>Track sun all day. $\gamma = \gamma_s$</td>
<td>1101</td>
<td>47.5</td>
<td>1148.5</td>
</tr>
<tr>
<td>March 16, 2014</td>
<td>Mixed skies all day</td>
<td>For future research. A solution may be $\beta = \theta_z - 15^\circ$</td>
<td>Track sun all day. $\gamma = \gamma_s$</td>
<td>793</td>
<td>0</td>
<td>793</td>
</tr>
</tbody>
</table>
8.7 Financial impact of energy gleaning

Table 8.4 shows the potential cost savings that could result if the optimal tilt angles of a 2-axis time-position tracking PV array could be modified in response to varying cloud cover. We assumed that cost of electricity in Vermont for residential customers is $0.1778 per kWh\[83\]. For a single 4.2 kW 2-axis tracking array, similar to those installed at Middlebury College, the yearly cost savings is modest value of about $26. If we scale this cost savings to a larger installation of 34 2-axis tracking arrays, then the savings ranges from about $860 to $913 per year.

Although these yearly cost savings are small, we should consider that overcast skies may cover a very large region, such as the 160 km distance between Middlebury, Vermont and Montreal, Canada. This very large region may experience a considerable short-term increase in insolation throughout the day during intervals of cloudy or overcast days, such as the increase of 27.37% in daily DC energy that occurred in the Montreal experiments.

Table 8.4: Potential yearly cost savings for Middlebury, Vermont when using S Formula for 2-axis tracking. Average price of electricity in Vermont is $0.1778 per kWh as of October 2016\[83\]

<table>
<thead>
<tr>
<th>Year</th>
<th>AC Energy (kWh)</th>
<th>Number of 2-axis tracking PV arrays</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Year 1</td>
<td>151.16</td>
<td>$26.88</td>
</tr>
<tr>
<td>Year 2</td>
<td>145.34</td>
<td>$25.84</td>
</tr>
<tr>
<td>Year 3</td>
<td>142.39</td>
<td>$25.32</td>
</tr>
</tbody>
</table>

8.8 Future work

During this research, we have identified the following areas for future study:

- In this research, we assumed that the ground reflectance was 0.2 throughout the year. An area for future study is how closely the S Formula estimates the optimal tilt angles at larger values of ground reflectance, such as when the ground is covered with snow. In [4], it was suggested that when the sky is overcast and snow is present, either falling or on the ground, that the 2-axis tracking PV array be fixed in a south-facing position at the location’s optimal tilt angle, rather than placed horizontally. As mentioned earlier in this section, the S Formula is the tangent of the conventional 2-axis tilt angle multiplied by a function of $g(k_T), \rho_g$, and $(\tau \alpha)_b$.\[126\]
(τα)_d, and (τα)_r. This multiplying function modifies the conventional 2-axis tilt angle. A quick calculation using the S Formula may indicate why the horizontal position is not optimal. Assuming that \( g(k_T) = \frac{I_d}{I} = 0.1 \) for an overcast sky, the multiplying term is 0.222 for \( \rho_g = 0.2 \); however, the multiplying term increases to 0.677 for \( \rho_g = 0.8 \) for snow.

- We propose examining the potential for additional energy gleaning by taking into account the spectral response of diffuse irradiance as it shifts to longer wavelengths under cloudy skies.

- In this research, we used measured diffuse and global horizontal irradiance values in the S Formula. In a real-life implementation, one or two irradiance sensors would be required. It would be better to find a method that depends only on the clearness index \( k_T \); however, this would involve identifying an appropriate correlation for \( g(k_T) \) to use in the S Formula.

- To validate our simulation results, we propose experiments using a 2-axis tracking PV array that can be adjusted using the proposed S Formula. These experiments would also examine the power consumption of the tracking mechanism.

8.9 Summary

In this chapter we compared the results of the S Formula method for calculating the optimal tilt angles, yearly POA irradiation, and yearly AC energy with five other methods: conventional 2-axis tracking, numerical, and Kelly and Gibson’s three methods [11]. The yearly increase in POA irradiation was 2.1% to 2.35% for both the numerical and S Formula methods. The optimal tilt angles calculated by the S Formula closely agreed with the optimal angles that were found numerically. On clear, sunny days, the S Formula’s optimal tilt angles are nearly the same as the conventional 2-axis tracking angles. On overcast days, the S Formula’s optimal tilt angle was approximately 0°. On variably cloudy days, the S Formula predicted optimal tilt angles between 0° and the conventional 2-axis tracking tilt angle of \( \theta_z \).

During periods of overcast skies, the S Formula’s optimal tilt angles agree with the recommendations of several researchers to place the PV module in a horizontal position. On two particular days, the S Formula’s simulation results were in agreement with published experimental results at a location 160 km away.

We took a closer look at the contribution of each of the irradiance streams to the overall increase in captured POA irradiance. When placing the panel in a horizontal position during overcast
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periods, the amount of beam irradiance and ground-reflected irradiance decreased, and the amount of
diffuse irradiance increased. However, the contribution of the beam and ground-reflected irradiance
streams was very small during overcast periods; thus, the diffuse irradiance stream was the primary
contributor to the increase in POA irradiance.

We considered the potential for additional energy gleaning during cloudy intervals due to
the spectral shift of diffuse irradiance to longer wavelengths under overcast skies.

We recommended tracking strategies for adjusting the tilt and the azimuth angles in order
to maximize irradiation and minimize power consumption. We estimated a daily gain in AC energy
of 467 Wh to 1200 Wh, which is dependent on the sky conditions throughout the day.

We examined the financial impact of energy gleaning. Although the yearly electricity cost
savings is about $26 for one tracking array, the cost savings for a larger installation of 34 2-axis
tracking arrays is estimated at approximately $860 to $913 per year.

Finally, we identified the following areas for future research on the S Formula and energy
gleaning: estimating the optimal tilt angle when the ground is highly reflective due to snow, iden-
tifying an appropriate correlation for $g(k_T)$ to use in the S Formula, examining the potential for
additional energy gleaning due to the spectral shift of diffuse irradiance under overcast skies, and
experimentally validating the S Formula method with a 2-axis tracking PV array.
Chapter 9

Conclusions

In this dissertation, we proposed a simple formula called the “S Formula” that directly calculates the optimal tilt angle for different array orientations, at any time, and under different sky conditions. We used the proposed S Formula to analyze the potential increase in solar radiation incident on a hypothetical 2-axis tracking PV array for several U.S. cities located in different climates. We then focused on one specific 2-axis time-position tracking installation located in Middlebury, Vermont, USA. We calculated the increase in captured irradiation and AC power generation over three consecutive 12-month periods using weather data from the PV installation. We used a relative of the S Formula for fixed PV arrays to examine how closely the formula estimates the optimal tilt angle for fixed arrays in different locations and climates in the U.S. and other countries.

The following are the contributions of our research to knowledge and understanding:

1. We derived a simple formula, called the “Parent S Formula”, that directly calculated the optimal tilt angle for a 2-axis PV array in varying sky conditions. This S Formula is mathematically-derived from the Isotropic Sky Model [8], and has the following features:

   - It takes into account all three components of irradiance (beam, diffuse, and ground-reflected) because it is mathematically derived from the equation for solar radiation absorbed on the cell surface of a photovoltaic panel.
   - It is a function of the conventional 2-axis tracking angle ($\theta_z$), which is multiplied by an expression containing the clearness index ($g(k_T) = I_d/I$), ground reflectance, and transmittance-absorptance products of the beam, diffuse and ground-reflected radiation streams.
   - It allows the tilt angle to be adjusted during any interval of time.
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- It allows the tilt angle to be adjusted in response to changing weather conditions.
- It does not rely on historical weather data.
- It explains mathematically the experimental results reported in the literature because it is a function of $\theta_z$, clearness index $k_T$, and ground reflectance $\rho_g$. The optimal angles calculated by the S Formula reduce to the conventional tracking angles $\beta = \theta_z$ under clear skies and the tilt angle of $\beta = 0^\circ$ under overcast skies as reported in [3, 4]. The S Formula helps us to understand why another angle – other than $0^\circ$ or $\theta_z$ – may be the optimal angle for partially cloudy skies or for different values of ground reflectance as reported in [9, 10].

2. We presented results for 21 U.S. cities showing the potential increase in yearly POA irradiance when a hypothetical 2-axis tracking PV array is allowed to adjust its tilt angle in response to varying sky conditions. When the optimal angles were calculated numerically with the HDKR irradiance model, the annual insolation increased from 0.56% to 3.48% when compared with conventional 2-axis tracking, with the greatest increase occurring in cloudier locations. When the optimal angles were calculated using a correlation and the S Formula, 20 out of 21 locations showed an increase in yearly insolation that ranged from 0.09% to 2.01% when compared with the conventional 2-axis tracking. The greatest increases in yearly insolation occurred at cloudier locations.

3. We presented results for a 2-axis time-position tracking array located in Middlebury, Vermont using weather data from the installation to calculate POA irradiance and AC power at the optimal angles predicted by the S Formula. We looked at how well the S Formula predicted the optimal tilt angles when compared with the numerical method. We compared the S Formula’s calculated POA insolation with the values calculated numerically, when using conventional 2-axis tracking angles, and for each of the three methods proposed by Kelly and Gibson in [11]. We then calculated and compared the yearly AC energy for the tilt angles calculated by the S Formula, the numerical method, and the conventional 2-axis tracking angles.

The optimal tilt angles that were calculated using measured diffuse horizontal irradiance (DHI) and global horizontal irradiance (GHI) data in the Parent S Formula are similar to those found using the numerical method. This indicates that the S Formula is able to estimate the numerical method’s optimal tilt angles.
CHAPTER 9. CONCLUSIONS

Results indicated that adjusting the 2-axis tracking array’s tilt angle in response to varying sky conditions allowed the array to capture more of the available irradiance during 15-minute intervals. For 45% of the intervals with overcast skies, the minimum increase in POA irradiance was 20 W/m² using the S Formula’s optimal angles versus the conventional tilt angles. The increases in annual insolation and AC energy were 2.35% and 2.42% for the first year, 2.33% and 2.40% for the second year, and 2.10% and 2.16% for the third year.

We found that the three methods proposed by Kelly and Gibson showed an increase in yearly insolation versus the conventional 2-axis tracking method, with Method 1 (DTS vs H) having the biggest gains with an increase of 1.05% for the first year, 0.72% for the second year, and 0.81% for the third year. Method 2 (GHI vs Diffuse) and Method 3 (DTS vs Limit) had similar increases of 0.23 to 0.53%. The limitation of all three methods, however, is that there are only two choices for the tracking tilt angle: \( \beta = \theta_z \) or \( \beta = 0^\circ \).

4. In related work, we derived from the Parent S Formula a family of formulas that can be used to calculate optimal tilt angles for 1-axis and fixed arrays. We presented results showing how closely the fixed S Formula estimates the optimal tilt angles for 21 U.S. cities, as well as the potential increase in yearly irradiance at the optimal fixed tilt versus latitude tilt. The S Formula’s estimated tilt angles were found to be within 1° to 5° of the values calculated using the numerical method. Although a rule of thumb is to install a fixed PV array at a tilt angle equal to its latitude \([32]\), the results show that a tilt angle that is 12° to 16° less than latitude is more optimal for cloudier locations. There was an increase in annual incident radiation for 19 out of 21 locations when the tilt angle was fixed at an angle that was optimal for the location’s climate. When compared with the yearly insolation calculated for the array fixed at a tilt angle equal to the location’s latitude, this increase ranged from 0.05% to 2.52% using the optimal angles calculated by the S Formula, and 0.03% to 2.65% using the numerical method’s optimal angles. The greatest increase in yearly insolation occurred at the cloudier locations. For these locations, the rule-of-thumb of installing a fixed PV array at a tilt angle equal to the location’s latitude tilt may result in a 2% or more loss in captured insolation over the course of a year.

5. We presented the results showing how well the S Formula for fixed PV arrays estimates the optimal tilt angles for 217 U.S. locations and 37 non-U.S. locations. We then compared the estimated S Formula optimal angles with those published at the NASA Surface Meteorology and Solar Energy website \([12]\), as well as by other researchers who used numerical or optimization methods to find the optimal tilt angles at those same locations. For U.S. locations,
CHAPTER 9. CONCLUSIONS

The S Formula’s calculated optimal tilt angles were within 2.1° of the optimal angles published on the NASA SSE website. When compared with the optimal tilt angles reported in [11], the S Formula’s optimal angles were within 3° for 76% of the locations. For 36 out of 37 non-U.S. locations, the S Formula’s yearly optimal tilt angles were within 2° of the optimal angles published on the NASA SSE website. For 75% of the non-U.S. location, the S Formula’s optimal angles were within 3° of the values calculated by other researchers using numerical and optimization methods. These results suggest that the S Formula can closely estimate the optimal tilt angles of fixed PV arrays located in different climates.

Finally, we have introduced a new term “energy gleaning” to describe what we, Kelly and Gibson, and other researchers are doing when adjusting the tilt angle of conventional 2-axis tracking arrays; that is, we are collecting additional small amounts of solar energy during cloudy intervals to improve the annual yield of 2-axis time-position tracking PV arrays. The term “gleaning” is borrowed from the field of agriculture where it refers to gathering the crops that are overlooked and left behind in the fields after the primary harvest. Energy gleaning collects the additional small amounts of power that are overlooked and otherwise unused on cloudy days by changing the orientation, in particular the tilt angle, of a 2-axis tracking PV array. This research indicates that the efficiency of 2-axis time-position tracking photovoltaic arrays can be improved under variably cloudy skies by using energy gleaning throughout the year.
Appendix A

PV_LIB Toolbox in MATLAB

A.1 Introduction

This appendix explains the parameters and models used in calculating POA irradiance and AC power for Middlebury College’s photovoltaic (PV) installation located in Middlebury, Vermont. This is a 143-kW grid-connected 2-axis tracking PV installation.

The inputs to the PV_LIB Toolbox for MATLAB are the parameters of the installation and the weather data provided by Middlebury College’s Draker Monitoring site. The functions in the PV_LIB Toolbox were developed by Sandia National Laboratories. The Middlebury College installation consists of 34 2-axis tracking arrays, each with its own inverter. The PV_LIB program calculates the POA irradiance and the AC power generated by one of the 2-axis arrays. The accuracy of the predicted AC power and the installation’s measured AC power can then be compared by calculating the errors.

A.2 User-provided parameters

A.3 Middlebury College’s PV installation parameters

Middlebury College has a 143 kW photovoltaic installation consisting of 34 AllEarth Renewables 2-axis tracking arrays. Each tracking array holds twenty Evergreen ES-A 210 multicrystalline photovoltaic modules for a DC capacity of 4200 Watts, as well as an SMA America SB4000US 240V inverter. Table A.1 lists the installation parameters that are required by the PV_LIB Toolbox.
APPENDIX A. PV_LIB TOOLBOX IN MATLAB

Table A.1: Installation parameters required by PV_LIB Toolbox.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
<td>44.010014°</td>
<td><a href="http://www.itouchmap.com">www.itouchmap.com</a> [84]</td>
</tr>
<tr>
<td>Longitude</td>
<td>-73.185854°</td>
<td><a href="http://www.itouchmap.com">www.itouchmap.com</a> [84]</td>
</tr>
<tr>
<td>Standard meridian</td>
<td>-75°</td>
<td><a href="http://www.itouchmap.com">www.itouchmap.com</a> [84]</td>
</tr>
<tr>
<td>Ground reflectance</td>
<td>0.2</td>
<td>PV Performance Modeling</td>
</tr>
<tr>
<td>Number of strings per tracker</td>
<td>1</td>
<td>AllEarth Renewables [24]</td>
</tr>
<tr>
<td>Number of PV modules per string</td>
<td>20</td>
<td>AllEarth Renewables [24]</td>
</tr>
<tr>
<td>PV module</td>
<td>Evergreen ES-A 210</td>
<td>AllEarth Renewables [24]</td>
</tr>
<tr>
<td>Inverter</td>
<td>SMA America SB4000US 240V</td>
<td>AllEarth Renewables [24]</td>
</tr>
<tr>
<td>Tilt angle</td>
<td>2-axis tracking</td>
<td></td>
</tr>
<tr>
<td>Surface azimuth angle</td>
<td>2-axis tracking</td>
<td></td>
</tr>
</tbody>
</table>

A.4 PV_LIB’s definitions of latitude, longitude, tilt angle, and surface azimuth angle

The PV_LIB Toolbox defines the direction of latitude, longitude, tilt angle, and surface azimuth angle as follows:

- Latitudes north of the equator are positive, and latitudes south of the equator are negative.

- Longitudes east of the prime meridian are positive, and longitudes west of the prime meridian are negative.

- Tilt angle is defined with respect to the horizontal. A PV panel that is placed horizontal has a tilt angle of zero degrees (0°). A PV panel that is placed vertically has a tilt angle of +90°.

- Surface azimuth angle is defined as follows: North = 0°, East = +90°, South = +180°, and West = +270°.

A.5 Performance coefficients for modules and inverters

The PV_LIB Toolbox contains two databases of performance parameters for PV modules and inverters. The Sandia PV Array Performance Model (SAPM) coefficients for the Evergreen ES-A 210 fa2 modules are listed in the database named “CECModuleDatabaseSAM2012.11.30.mat”. The performance parameter coefficients for the SMA America SB4000 inverter are listed in the Sandia
APPENDIX A. PV_LIB TOOLBOX IN MATLAB

Performance Model for Grid-Connected Photovoltaic Inverters database “SandiaInverterDatabaseSAMP2012.11.30.mat”.

These two databases are the sources of the module and inverter parameters used in calculating the AC power of the Middlebury College PV installation. The PV_LIB parameters for the Evergreen PV modules are shown in Figure A.1 and the PV_LIB parameters for the SMA inverters are shown in Figure A.2.

Figure A.1: Parameters for Evergreen ES-A-210-fa2 modules (from PV_LIB’s CEC Module database [17]).
Figure A.2: Parameters for SMA America SB4000US 240V (CEC2007) inverter (from PV_LIB’s SNL Inverter database [17]).
A.6 Draker Monitoring data format

The Draker Monitoring data for Middlebury College’s PV installation consists of 81 columns of power and weather data at 15-minute intervals. The following data was used in the simulations:

- Local timestamp
- Horizontal Irradiance (known as global horizontal irradiance)
- Horizontal Direct Irradiance (known as direct normal irradiance)
- Horizontal Diffuse Irradiance (known as diffuse horizontal irradiance)
- Ambient temperature

A.7 Other parameters

Additional values that are required to run the PV_LIB functions are:

- Soiling factor (SF) represents the amount of dust, snow, or other particles on the PV modules that reduce the received solar irradiation. SF is a value between 0 and 1, with 1 indicating a clean module. SF is set to 0.98 for the Middlebury College installation.
- The time zone is indicated by the standard meridian which is used to calculate the UTC offset for local standard time.
- Daylight Savings Time start date and end date are required for adjusting the UTC offset in order to correctly calculate the solar zenith and solar azimuth angles.
- Reference irradiance ($E_0$) is set to 1000 Wh/m$^2$.

A.8 PV_LIB Models

The functions in the PV_LIB Toolbox were developed for use in MATLAB by Sandia National Laboratories. The functions are publicly available at the PV Performance Modeling Collaborative (PVPMC) website [17]. The descriptions of the functions in the following sections are taken from the descriptions of the functions presented on the PVPMC website.
Figure A.3 shows a flowchart of the PV_LIB functions used in calculating the AC power for the Middlebury College PV installation. Each function is briefly described in the following sections.

A.8.1 pvl_sapmmoduledb

This function retrieves the Sandia PV Array Performance Model (SAPM) coefficients for the Evergreen ES-A-210-fa2 modules from the Excel version of Sandia PV Module database.

A.8.2 pvl_snlinverterdb

This function retrieves the SMA SB4000US inverter’s performance parameter coefficients from the database “SandiaInverterDatabaseSAM2012.11.30.mat”.

A.8.3 pvl_maketimestruct

This function creates a time structure from the MATLAB datenum function and UTC offset code. In addition to this function, code has been written to change the UTC offset during days that occur during Daylight Savings Time.

A.8.4 pvl_makelocationstruct

This function creates a location structure from the latitude and longitude coordinates of the location.

A.8.5 pvl_ephemeris

This function calculates the position of the sun and solar time. Input parameters are forecast time and location. The pressure is assumed to 101325 PA and the temperature is assumed to be 12° C.

A.8.6 pvl_getaoi

This function calculates the angle of incidence (AOI) between the sun’s beam and the surface of the PV module. The tilt angle, surface azimuth angle, sun’s zenith angle, and the solar azimuth angle are the inputs to this function.
Figure A.3: Flowchart for calculating Plane of Array Irradiance and AC Power for conventional 2-axis tracking and modified 2-axis tracking (optimal tilt angle is found by varying $\beta$ from 0 to 90°).
APPENDIX A. PV_LIB TOOLBOX IN MATLAB

A.8.7 Diffuse irradiance models

Six different functions are used to calculate the diffuse irradiance $E_d$ on a tilted surface.

A.8.7.1 pvl_haydavies1980

This function calculates the diffuse irradiance $E_d$ on a tilted surface using the Hay and Davies model for diffuse irradiance. This function assumes that the diffuse component due to horizontal brightening is zero. Inputs to this function are tilt angle ($\beta$), surface azimuth angle ($\gamma$), diffuse horizontal irradiance (DHI), direct normal irradiance (DNI), extraterrestrial normal irradiance (Hextra), the sun’s zenith angle ($\theta_z$), the sun’s azimuth angle ($\gamma_s$), the anisotropy index (AI), and the ratio $R_b$. The anisotropy index is defined as the ratio of the direct normal irradiation to the extraterrestrial normal irradiation. It is a measure of the transmittance of the direct radiation through the atmosphere, where AI is the portion of diffuse radiation due to circumsolar radiation and $(1 - AI)$ is the portion due to isotropic sky radiation. $R_b$ is defined as the ratio of the beam irradiance on a tilted surface to the beam irradiance on a horizontal surface. To prevent $R_b$ from being unreasonably large at sunrise and sunset, PV_LIB limits the denominator to a maximum value of 0.01745 which corresponds to a zenith angle of 89°.

$$E_d = DHI \cdot \left[ AI \cdot R_b + (1 - AI) \cdot \left( \frac{1 + \cos \beta}{2} \right) \right]$$ (A.1)

A.8.7.2 pvl_isotropic

This function calculates the diffuse irradiance $E_d$ on a tilted surface using the Isotropic Sky model for diffuse irradiance. This function assumes that the diffuse components due to horizontal brightening and circumsolar irradiance are both zero. Inputs to this function are tilt angle $\beta$ and diffuse horizontal irradiance (DHI).

$$E_d = DHI \cdot \left( \frac{1 + \cos \beta}{2} \right)$$ (A.2)

A.8.7.3 pvl_kingdiffuse

This function calculates the diffuse irradiance $E_d$ on a tilted surface using the King model for diffuse irradiance. Inputs to this function are tilt angle ($\beta$), diffuse horizontal irradiance (DHI),
APPENDIX A. PV_LIB TOOLBOX IN MATLAB

global horizontal irradiance (GHI), and the sun’s zenith angle ($\theta_z$). The first term is the Isotropic Sky model, and the second term is an empirical equation which takes into account the diffuse components due to horizontal brightening and circumsolar radiation.

$$E_d = DHI \cdot \left(\frac{1 + \cos^2 \beta}{2}\right) + GHI \cdot \left(\frac{(0.12 \cdot \theta_z - 0.04)(1 - \cos \beta)}{2}\right)$$  \hspace{1cm} (A.3)

A.8.7.4 pvl_klucher1979

This function calculates the diffuse irradiance $E_d$ on a tilted surface using the Klucher model for diffuse irradiance. This function takes into account the diffuse components due to horizontal brightening and circumsolar radiation. Inputs to this function are tilt angle ($\beta$), surface azimuth angle ($\gamma$), diffuse horizontal irradiance (DHI), global horizontal irradiance (GHI), the sun’s zenith angle ($\theta_z$), the sun’s azimuth angle ($\gamma_s$), and a clearness index $F$. The clearness index is defined as $(1 - \frac{DHI}{GHI})^2$. The angle of incidence (AOI) is calculated using the tilt angle, surface azimuth angle, sun’s zenith angle, and the sun’s azimuth angle. The second term in brackets takes into account the horizontal brightening effects, and the last term in brackets takes into account the circumsolar radiation.

$$E_d = DHI \cdot \left(\frac{1 + \cos^2 \beta}{2}\right) \cdot \left[1 + F \cdot \left(\frac{\sin \beta}{2}\right)^3\right] \cdot \left[1 + F \cdot (\cos AOI)^2 \cdot (\sin \theta_z)^3\right]$$  \hspace{1cm} (A.4)

A.8.7.5 pvl_perez

This function calculates the diffuse irradiance $E_d$ on a tilted surface using the Perez model for diffuse irradiance. This function takes into account the diffuse components due to horizontal brightening and circumsolar radiation. Inputs to this function are tilt angle ($\beta$), surface azimuth angle ($\gamma$), diffuse horizontal irradiance (DHI), direct normal irradiance (DNI), extraterrestrial normal irradiance (Hextra), the sun’s zenith angle ($\theta_z$), the sun’s azimuth angle ($\gamma_s$), relative air mass (AM), and a matrix of brightness coefficients. $F_1$ represents the circumsolar brightening coefficient, and $F_2$ represents the horizon brightening coefficient. The matrix of brightness coefficients consists of rows representing 8 ranges of the clearness parameter, $\epsilon$, from 0 to infinity, and six columns for the parameters $f_{11}, f_{12}, f_{13}, f_{21}, f_{22},$ and $f_{23}$. 

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APPENDIX A. PV_LIB TOOLBOX IN MATLAB

\[ E_d = DHI \cdot \left[ (1 - F_1) \left( \frac{1 + \cos \beta}{2} \right) + F_1 \cdot \frac{a}{b} + F_2 \cdot \sin \beta \right] \]  
(A.5)

\[ a = \max(0, \cos \theta) \]  
(A.6)

\[ b = \max(\cos 85, \cos \theta_z) \]  
(A.7)

\[ F_1 = \max \left[ 0, (f_{11} + f_{12} \cdot \Delta + \left( \frac{\pi \cdot \theta_z}{180} \right) \cdot f_{13}) \right] \]  
(A.8)

\[ F_2 = (f_{21} + f_{22} \cdot \Delta + \left( \frac{\pi \cdot \theta_z}{180} \right) \cdot f_{23}) \]  
(A.9)

\[ \Delta = AM \cdot \frac{DHI}{H_{extra}} \]  
(A.10)

\[ \epsilon = \frac{GHI + DNI}{GHI} + 1.041 \cdot \left( \frac{\pi \cdot \theta_z}{180} \right)^3 \]  
(A.11)

A.8.7.6 pvl_reindl1990

This function calculates the diffuse irradiance \( E_d \) on a tilted surface using the Reindl model for diffuse irradiance. This function takes into account the diffuse components due to horizontal brightening and circumsolar radiation. Inputs to this function are tilt angle (\( \beta \)), surface azimuth angle (\( \gamma \)), diffuse horizontal irradiance (DHI), direct normal irradiance (DNI), global horizontal irradiance (GHI), extraterrestrial normal irradiance (Hextra), the sun’s zenith angle (\( \theta_z \)), the sun’s azimuth angle (\( \gamma_s \)), the anisotropy index (AI), and the ratio \( R_b \). The anisotropy index is the defined as the ratio of the direct normal irradiation to the extraterrestrial normal irradiation. It is a measure of the transmittance of the direct radiation through the atmosphere, where AI is the portion of diffuse radiation due to circumsolar radiation and \( 1 - AI \) is the portion due to isotropic sky radiation. \( R_b \) is
defined as the ratio of the beam irradiance on a tilted surface to the beam irradiance on a horizontal surface. To prevent \( R_b \) from being unreasonably large at sunrise and sunset, PV_LIB limits the denominator to a maximum value of 0.01745 which corresponds to a zenith angle of 89\(^\circ\). The angle of incidence (AOI) is calculated using the tilt angle, surface azimuth angle, sun’s zenith angle, and the sun’s azimuth angle. The modulating index \( f \) is defined as the square root of the ratio of the direct horizontal irradiance to global horizontal irradiance.

\[
E_d = DHI \cdot \left[ AI \cdot R_b + (1 - AI) \cdot \left( \frac{1 + \cos \beta}{2} \right) \left( 1 + f \cdot \sin \left( \frac{\beta}{2} \right)^3 \right) \right]
\] (A.12)

### A.8.8 \texttt{pvl\_grounddiffuse}

This function estimates the ground-reflected irradiance \( E_g \) on the tilted surface given the tilt angle (\( \beta \)), global horizontal irradiance (GHI), and albedo.

\[
E_g = GHI \cdot \text{albedo} \cdot \left( 1 - \cos \frac{\beta}{2} \right)
\] (A.13)

### A.8.9 Plane of Array (POA) irradiance and Effective irradiance (\( E_e \))

Separate lines of code were written to calculate the Plane Of Array (POA) irradiance and the Effective irradiance \( E_e \). POA is the sum of all three irradiance streams (beam, diffuse, and ground-reflected diffuse) and is calculated as:

\[
POA = DNI \cdot \cos(AOI) + E_d + E_g
\] (A.14)

The effective irradiance is an adjustment to the plane of array (POA) irradiance to account for losses due to angle of incidence losses, soiling, and spectral mismatch. A simplified equation is used in calculating effective irradiance, where SF is the soiling factor and \( E_0 \) is the reference irradiance, which is assumed to be 1000 Wh/m\(^2\)

\[
E_e = \frac{POA}{E_0} \cdot SF
\] (A.15)
A.8.10  pvl_sapmcelltemp

This function uses the Sandia Module Temperature Model and the Sandia Cell Temperature Model to calculate module and cell temperatures. The input parameters to this function are POA irradiance, reference irradiance \( E_0 \), ambient temperature \( T_a \), wind speed (WS), and the Sandia PV Array Performance Model (SAPM) coefficients \((a, b, \Delta T)\) for PV modules found in the Sandia PV Module database.

The equation for module temperature is:

\[
T_m = \frac{POA}{E_0} \left( e^{a+bWS} + T_a \right) \tag{A.16}
\]

The equation for cell temperature is:

\[
T_c = T_m + \frac{POA}{E_0} \Delta T \tag{A.17}
\]

A.8.11  pvl_sapm

This function uses the Sandia PV Array Performance Model to calculate the 5 points on a PV module’s I-V curve \((V_{oc}, I_{sc}, I_x, I_{xx}, V_{mp}/I_{mp})\). The input parameters are effective irradiance \( E_e \), cell temperature, and the Sandia PV Array Performance Model (SAPM) coefficients for PV modules. The reference cell temperature is assumed to be 25° C.

A.8.12  pvl_snlinverter

This function calculates the AC power using Sandia’s Grid-Connected PV Inverter model. Inputs are the DC voltage and DC power calculated by the pvl_samp function, and the inverter’s performance parameter coefficients listed in the Sandia Inverter database.

A.9  PV_LIB’s modeled data versus the installation’s measured data

In order to understand how well PV_LIB models the array, we ran the MATLAB program “PVLIB_rev4.m” for each of the six diffuse models discussed in Section A.8.7. All daytime values were used.

Table A.2 shows the error statistics for POA irradiance and AC power. Normalized RMSE (NRMSE) is calculated by dividing POA irradiance’s RMSE by the value for 1 sun (1000 W/m²) and
Table A.2: Error statistics of measured data vs modeled data for Year 1, Nov. 1, 2013 to Oct. 31, 2014.

<table>
<thead>
<tr>
<th>Name of Diffuse model</th>
<th>POA irradiance (W/m²) (18920 data points)</th>
<th>AC power (W) (17221 datapoints)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MBE</td>
<td>MAE</td>
</tr>
<tr>
<td>haydavies1980</td>
<td>-24.05</td>
<td>56.26</td>
</tr>
<tr>
<td>isotropicsky</td>
<td>-49.45</td>
<td>70.77</td>
</tr>
<tr>
<td>kingdiffuse</td>
<td>-16.65</td>
<td>50.77</td>
</tr>
<tr>
<td>klucher1979</td>
<td>-27.14</td>
<td>52.86</td>
</tr>
<tr>
<td>perez</td>
<td>-13.27</td>
<td>46.15</td>
</tr>
<tr>
<td>reindl1990</td>
<td>-21.87</td>
<td>54.91</td>
</tr>
</tbody>
</table>

* NRMSE normalized for 1 sun = 1000 W/m²
* NRMSE normalized for AC nominal power of 4000 W

AC power’s RMSE by the array’s AC capacity (4000 W). The NRMSE for POA irradiance varied between the lowest value of 9.31% for the Perez diffuse model and the highest value of 10.96% for the isotropic diffuse model. For AC power, the NRMSE ranged from the lowest value of 11.78% for the Klucher1979 diffuse model to 12.57% for the reindl1990 diffuse model.

Figure A.4 presents two plots – the modeled AC power versus POA irradiance that was calculated in PV_LIB, and the measured AC power versus measured POA irradiance. All daytime data values are plotted, including those values that occur during intervals of precipitation. The relationship between modeled AC power versus POA irradiance is fairly linear. However, there are many outliers that occur in the plot of measured data, which may be due to the variability in measured irradiance data.

The SMA America SB4000US inverter has a maximum AC power rating of 4000 W. We can see in Figure A.4b that the measured AC power is clipped at 4000 W for values of measured POA irradiance greater than 1000 W/m². The AC power modeled in PV_LIB also exhibits clipping at 4000 W when the modeled POA irradiance exceeds 1000 W/m², as shown in Figure A.4a.

### A.10 Summary

In this appendix we explained the parameters, models, and functions used in the PV_LIB Toolbox for MATLAB to calculate the POA irradiance and AC power for Middlebury College’s photovoltaic (PV) installation.
APPENDIX A. PV_LIB TOOLBOX IN MATLAB

Figure A.4: Comparing modeled AC power vs. modeled POA irradiance and measured AC power to measured POA irradiance. Year 1 data. Each marker represents a 15-minute interval.
Appendix B

Correlations for ratio of DHI to GHI

B.1 Introduction

Many correlations have been proposed for relating the fraction of diffuse horizontal irradiance to global horizontal irradiance. Some correlations depend only on the clearness index of the location, while others are functions of one or more parameters such as the clearness index, solar altitude, sunshine hours, ambient temperature, and relative humidity. All of these correlations were derived using the process shown in Figure B.1. The symbol for the fraction of diffuse horizontal irradiance to the global horizontal irradiance may be represented as $I_{d}/I_{G}$, $H_{d}/H_{G}$, $f_{d}$, $f_{h}$, $k_{D}$ or $K$ in the correlations.

Figure B.1: Process used by researchers to develop correlations for the ratio of diffuse to global horizontal irradiance.
APPENDIX B. CORRELATIONS FOR RATIO OF DHI TO GHI

B.2 de Miguel, Bilbao, Aguiar, Kambezidis, and Negro hourly correlation

In [47] hourly and daily correlations were derived for 11 locations in southern France, Greece, Italy, Portugal, and Spain using hourly weather data for 4 to 13 years, depending on the location. The De Miguel, Bilbao, Aguiar, Kambezidis, and Negro hourly correlation is:

\[ f_h = \begin{cases} 
0.995 - 0.081k & \text{for } k \leq 0.21 \\
0.724 + 2.738k - 8.32k^2 + 4.967k^3 & \text{for } 0.21 < k \leq 0.76 \\
0.180 & \text{for } k > 0.76 
\end{cases} \quad (B.1) \]

B.3 Erbs, Klein, and Duffie correlation

The Erbs, Klein, and Duffie correlation was derived from one to two years of hourly weather data for four U.S. locations [41].

\[ \frac{I_d}{I} = \begin{cases} 
1.0 - 0.09k_T & \text{for } k_T \leq 0.22 \\
0.9511 - 0.1604k_T + 4.388k_T^2 - 16.638k_T^3 + 12.336k_T^4 & \text{for } 0.22 < k_T \leq 0.80 \\
0.165 & \text{for } k_T > 0.8 
\end{cases} \quad (B.2) \]

B.4 Lam and Li correlation

The Lam and Li correlation was derived from four years of hourly weather data from one location in Hong Kong [43].

\[ K = \begin{cases} 
0.977 & \text{for } k_t \leq 0.15 \\
1.237 - 1.361k_t & \text{for } 0.15 < k_t \leq 0.7 \\
0.273 & \text{for } k_t \geq 0.7 
\end{cases} \quad (B.3) \]
APPENDIX B. CORRELATIONS FOR RATIO OF DHI TO GHI

B.5 Mondol, Yohanis and Norton correlation

The Mondol, Yohanis and Norton correlation was derived from nine years of hourly weather data from one location in Northern Ireland [44].

\[
k_D = \begin{cases} 
0.98 & \text{for } k_T \leq 0.20 \\
0.61092 + 3.6259k_T - 10.171k_T^2 + 6.338k_T^3 & \text{for } 0.2 < k_T \leq 0.70 \\
0.672 - 0.474 & \text{for } k_T > 0.7 
\end{cases}
\]  

(B.4)

B.6 Orgill and Hollands correlation

The Orgill and Hollands correlation was derived from four years of hourly weather data from Toronto, Ontario [42].

\[
\frac{H_d}{H_T} = \begin{cases} 
1.0 - 0.249k_T & \text{for } 0 \leq k_T < 0.35 \\
1.557 - 1.84k_T & \text{for } 0.35 \leq k_T \leq 0.75 \\
0.177 & \text{for } k_T > 0.75 
\end{cases}
\]  

(B.5)

B.7 Reindl, Beckman, and Duffie 2nd correlation

In [46] three correlations were derived from at least one year of hourly weather data from five European and U.S. locations. The first correlation is a function of clearness index, solar altitude, relative humidity, and ambient temperature. This correlation is not used in this research. However, the other two correlations were used in this research. One of the correlations depends on the clearness index and solar altitude. It will be referred to as the Reindl, Beckman, and Duffie 2nd correlation in this dissertation.

\[
\frac{I_d}{I_T} = \begin{cases} 
1.020 - 0.254k_T + 0.0123 \sin \alpha & \text{for } 0 \leq k_T \leq 0.3 \text{ and constraint } \frac{I_d}{I_T} \leq 1.0 \\
1.400 - 1.749k_T + 0.177 \sin \alpha & \text{for } 0.3 < k_T < 0.78 \text{ and constraints } \frac{I_d}{I_T} \leq 0.97 \\
0.486k_T - 0.182 \sin \alpha & \text{for } k_T \geq 0.78 \text{ and constraint } \frac{I_d}{I_T} \geq 0.1 
\end{cases}
\]  

(B.6)
B.8 Reindl, Beckman, and Duffie 3rd correlation

The third correlation presented in \[46\] depends only on the clearness index, and we will refer to it as the Reindl, Beckman, and Duffie 3rd correlation.

\[
\frac{I_d}{I} = \begin{cases} 
1.020 - 0.248k_T & \text{for } 0 \leq k_T \leq 0.3 \text{ and constraint } \frac{I_d}{I} \leq 1.0 \\
1.45 - 1.67k_T & \text{for } 0.3 < k_T < 0.78 \\
0.147 & \text{for } k_T \geq 0.78
\end{cases}
\] (B.7)

B.9 NASA Surface meteorology and Solar Energy (SSE) correlation

The NASA Surface meteorology and Solar Energy website http://power.larc.nasa.gov/cgi-bin/cgiwrap/solar/grid.cgi?email=grid@larc.nasa.gov allows the user to find meteorological parameters for a given latitude and longitude. The document “Surface meteorology and Solar Energy (SSE) Release 6.0 Methodology, Version 3.1, March 1, 2012” \[12\] lists the correlations used to calculate the fraction of the diffuse irradiation to the global horizontal irradiation.

For latitudes between 45° South and 45° North:

\[
\frac{(H^\text{All})_\text{Diff}}{H^\text{All}} = 0.96268 - (1.45200 \cdot K_T) + (0.27365 \cdot K_T^2)
+ (0.04279 \cdot K_T^3) + (0.000246 \cdot SSHA) + (0.001189 \cdot NHSA)
\] (B.8)

For latitudes between 90° and 45° South, and 45° and 90° North:

If 0° ≤ SSHA ≤ 81.4°:

\[
\frac{(H^\text{All})_\text{Diff}}{H^\text{All}} = 1.441 - (3.6839 \cdot K_T) + (6.4927 \cdot K_T^2)
- (4.147 \cdot K_T^3) + (0.0008 \cdot SSHA) - (0.008175 \cdot NHSA)
\] (B.9)

If 81.4° < SSHA ≤ 100°:

\[
\frac{(H^\text{All})_\text{Diff}}{H^\text{All}} = 1.6821 - (2.5866 \cdot K_T) + (2.373 \cdot K_T^2)
- (0.5294 \cdot K_T^3) - (0.00277 \cdot SSHA) - (0.004233 \cdot NHSA)
\] (B.10)

If 100° < SSHA ≤ 125°:

\[
\frac{(H^\text{All})_\text{Diff}}{H^\text{All}} = 0.3498 + (3.8035 \cdot K_T) - (11.765 \cdot K_T^2)
+ (9.1748 \cdot K_T^3) + (0.001575 \cdot SSHA) - (0.002837 \cdot NHSA)
\] (B.11)
APPENDIX B. CORRELATIONS FOR RATIO OF DHI TO GHI

If \(125^\circ < \text{SSHA} \leq 150^\circ\):

\[
\frac{(H^{\text{All}})_{\text{Diff}}}{H^{\text{All}}} = 1.6586 - (4.412 \cdot \text{KT}) + (5.8 \cdot \text{KT}^2) - (3.1223 \cdot \text{KT}^3) + (0.000144 \cdot \text{SSHA}) - (0.000829 \cdot \text{NHSA})
\]

(B.12)

If \(150^\circ < \text{SSHA} \leq 180^\circ\):

\[
\frac{(H^{\text{All}})_{\text{Diff}}}{H^{\text{All}}} = 0.6563 - (2.893 \cdot \text{KT}) + (4.594 \cdot \text{KT}^2) - (3.23 \cdot \text{KT}^3) + (0.004 \cdot \text{SSHA}) - (0.0023 \cdot \text{NHSA})
\]

(B.13)

where:

- \(H^{\text{All}}\) = All sky total global solar radiation
- \((H^{\text{All}})_{\text{Diff}}\) = All sky diffuse solar radiation
- \(H^{\text{TOA}}\) = Solar radiation at the top of the atmosphere
- \(\text{KT} = \frac{H^{\text{All}}}{H^{\text{TOA}}}\)
- \(\text{SSHA}\) = sunset hour angle in degrees
- \(\text{NHSA}\) = noon solar angle from the horizon in degrees
Bibliography


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