DYNAMIC PROGRAMMING FOR REPLACEMENT THRESHOLD AND INSPECTION SCHEME OPTIMIZATION IN CONDITION-BASED MAINTENANCE

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PREFACE

This thesis is submitted for the degree of Master of Science at Northeastern University. The research described herein was conducted under the supervision of professor Sagar Kamarthi and professor Md. Noor-E-Alam in the Department of Industrial Engineering, Northeastern University, from June 2015 to November 2016.

This work is to the best of knowledge original, except acknowledgements and references are made to previous work. Neither this, nor any substantially similar thesis has been or is being submitted for any other degree, diploma or other qualifications at any other institution.

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ABSTRACT

In condition-based maintenance, preventative replacement thresholds and inspection schemes play important roles in maintenance performance. Previous research considers cost as the main objective for measuring maintenance performance. Consequently, the average cost per unit time is the only objective in a single-unit system. The intention of this study was to investigate how the average cost per unit time varies through changing replacement thresholds and inspection schemes. The goal is to simultaneously determine an optimal replacement threshold and inspection scheme using dynamic programming. The heterogeneity of hazard rates over stages in the life of equipment entails less frequent inspections when the equipment is in good condition and more frequent inspections as the equipment progressively deteriorates. Therefore, the proposed condition-based inspection scheme presented in this thesis is non-periodic and condition-based. A method based on dynamic programming is developed to implement a condition-based inspection scheme. Furthermore, the threshold for preventative replacement and the corresponding inspection scheme are simultaneously determined. Finally, the optimal average maintenance cost is obtained.

Keywords: condition-based maintenance; control limit; proportional hazards model; dynamic programming.
1 INTRODUCTION
1.1 Overview

Condition-based maintenance (CBM) is a maintenance strategy that utilizes information analysis for preventing equipment breakdown. In recent years, CBM received much attention in the literature due to the achievement of maintenance improvement. Jardine et al. (2006) summarized hundreds of papers, which are organized into the categories of diagnostics and prognostics. The authors note that CBM consists of three main steps: data acquisition, data processing and maintenance decision-making. Three data processing techniques are presented: waveform data analysis, value type data analysis, data analysis combining event data and condition monitoring data. Remaining useful life (RUL) has attracted researcher’s attentions in recent years as a widely used prognostic approach. A pattern-based prognostic model for remaining lifetime classification and prediction was demonstrated by Ragab (2015). Banjevic and Jardine (2006) developed extensive methods for analyzing remaining useful life based on a control limit policy. Pham et al. (2012) developed a three-stage method for assessing the machine health degradation and forecasting the RUL. Pham first uses an identification model for recognizing the dynamic equipment behavior. The authors then estimate the survival function of the equipment, using Cox’s proportional hazard model. Finally, support vector machines, are used to predict the RUL. Goode et al. (2000) used a statistical control method to discretize machine lifetime into a stable zone (I-P interval) and a failure zone (P-F interval). The Weibull distribution is used to separately fit those two intervals and derive the machine prognosis. Wang (2007) built up a probabilistic two-stage prognostic model to predict remaining component lifetime. This method differs from most of the prognostic approaches found in the literature, which adopt a certain number of
condition indicators or setting a threshold in predicting remaining useful life. Ragab et al. (2014) proposed a reliability-based prognostic methodology through combining the Kaplan-Meier estimation technique with the Logical Analysis of Data (LAD). Artificial neural networks (ANN) are used as prognosis tools to directly quantify equipment lifetime and remaining useful life or indirectly quantify uncertainty in prognostic analysis. A model using Bayesian evidence to train a series of multi-layer perceptron networks was proposed by Pierce (2008). An interval set techniques for the quantification of uncertainty in a neural network regression model has been considered. Tian et al. (2010) developed an ANN approach utilizing both failure and suspension condition monitoring histories.

Preventative maintenance shows some similarities to condition-based maintenance in their consideration of equipment deterioration and reliability analysis. Liu et al. (2014) developed a dynamic value-based preventative maintenance policy for multiple-component system. The proposed policy is aimed at determining which component to maintain and to what degree the component ought to be maintained. Leger (1999) developed a supervisory control model for predicting mean time to failure. Bairong (2013) used neural networks to find the optimal failure probability threshold value. Due to considerable uncertainty in the deterioration process, improved analysis accuracy is the goal of several researchers. Zhigang (2013) developed a method to quantify the remaining life prediction uncertainty. Gebraeel (2005) used a Baysian updating approach by continuously updating the stochastic parameters of the deterioration model. Fitouchi and Nourelfath (2012) integrate noncyclical preventative maintenance scheduling and production planning. The proposed model is used to simultaneously determine the optimal production plan and preventative maintenance scheduling in a multi-objective
measurement, which include preventative and corrective maintenance cost, setup costs, holding costs, backorder costs and production costs.

Inspection Scheduling is also a critical area of research. Golmakani and Moakedi (2012) proposed a model to find the optimal periodic inspection interval in a two-component repairable system with failure interaction. Golmakani and Moakedi extended this result to a multi-component system (Golmakani and Moakedi 2012). Periodic inspection has been extensively investigated in literature. Nakagawa (1984), Taghipour et al. (2010), Taghipour and Banjevic (2011) and Kallen and Noortwijk (2006) present various approaches in inspection scheduling. Although periodic inspection is commonly implemented, its optimality is suspect due to an increasing deterioration rate over time. Nakagawa (2009) exhibited his research on multiple inspection scheduling strategies, including periodic inspection, random inspection and non-periodic sequential inspections. Goode et al. (2000) developed a model to schedule inter-inspection condition monitoring interval given a risk level. A simple model derived from Christer and Wang (1995) quantifies the optimal time for subsequent inspections based on the wear condition. Okumura (1997) utilized a delay-time model to optimize the sequential inspection interval. The measurement is to minimize the average cost per unit time. Non-periodic inspection received substantial research attention as well. Barker et al. (2009), Castanier et al. (2003), Castanier et al. (2005), Deodatis et al. (1992), Deodatis et al. (1996), Golmakani and Moakedi (2012), Lin et al. (2015), Lin et al. (2010) and Pereira et al. (2010) investigated optimization of inspection scheduling.

Optimization of the preventative replacement threshold and inspection scheduling play a significant role in evaluating system performance from the economic and
reliability perspectives. A model based on random coefficient growth (Lu and Meeker 1993), was developed by Wang (2000). In this model, three objective functions are outlined: expected cost, expected downtime and the reliability measure. All three are considered independently to determine the optimal critical level for preventative replacement, as well as the optimal constant interval. Banjevic et al. (2001) proposed an approach known as a control limit policy. An iterative algorithm and a recursive procedure were developed in this work in order to obtain the optimal threshold for preventative replacement. Golmakani (2011) extended the research of Makis and Jardine (1992). His work aimed at finding the optimal replacement policy and periodic inspection interval. By stratifying a control limit into several stages, a multi-level control-limit rule was implemented by Grall (2002) to determine the optimal critical maintenance threshold and inspection choices. Huynh et al. (2010) provided a model to assess the value of condition monitoring information for the maintenance decision-making. And the developed condition-based periodic inspection/ replacement policy is compared with benchmark time-based block replacement policy. Nakagawa and Yasui (1991) suggest that five replacement policies, which are inspection, periodic replacement, block replacement, parallel systems and cumulative damage, can be transformed into replacement models with threshold levels.

The condition-based maintenance literature referencing prognostic and diagnostic methods present various approaches to condition information collection, equipment stochastic deterioration process modeling and objective function build-up. Condition-based maintenance shows some similarities with preventative maintenance. They both work to find a maintenance strategy to prevent equipment failures. In preventative
maintenance, a threshold for the equipment is usually aged-based or provided by the manufacturer. In contrast, a threshold in condition-based maintenance is decided by equipment condition and captured from equipment condition information analysis. Most proposed inspection schemes are periodic. Non-periodic inspection, such as sequential inspection and random inspection, has proven superior to periodic inspection in several scenarios. Prognostic approaches in predicting equipment remaining useful life inevitably contain uncertainties, which are difficult to remedy and negatively affect prediction accuracy.

1.2 Statement of Problem

Periodic inspection and alarm thresholds for replacement make contributions in maintaining equipment reliability and reducing maintenance-related cost. Figure 1 visually displays the general deterioration process. The rate of deterioration goes up as equipment ages without maintenance intervention. Due to the heterogeneity of the hazard rate over the useful life, equipment needs fewer inspections at an early age and an increasing number of inspections later on. This is attributed to decreasing equipment reliability and increasing failure risk over the life of the equipment. Non-periodic inspection (Cui et al. 2004, Pedone et al. 2009, Chun 2008, Chang et al. 2009, Dieulle et al. 2003, Goldberg et al. 2008, Boros et al. 2011, Jing 2009 and Vaughan 2001) has proven advantageous in rationalizing inspection scheduling. Therefore, condition-based inspection has been proposed for the scheduling of inspection activities based on equipment condition. Although the non-periodic inspection schedule has been widely used as a factor to be optimized, equipment condition information is rarely used to augment the schedule. Some papers have demonstrated the utilization of equipment
condition information in choosing the inter-inspection interval. These papers assume that the equipment condition information and inter-inspection interval follows a linear/nonlinear function, and the obtained inspection scheme is usually sub-optimal. Some heuristic search algorithms can decrease the computational difficulties and complexities, but the finally calculated relation is suboptimal as well.

Very few papers address the optimization of the preventative replacement threshold and the non-periodic inspection scheme simultaneously. Jiang (2010) presented a flexible degradation model to optimize the alarm threshold and sequential inspection scheme under the given two cost models. By utilizing equipment condition information, a condition-based inspection scheduling function was presented by Dieulle et al. (2003) to optimize the given objective, where the relationship between inter-inspection interval and equipment condition follows an assumed linear inspection scheduling function.

Figure 1: Deterioration process
Golmakani (2011) developed a two-phase, age-based inspection for the determination of an optimal replacement threshold and an optimal condition-based inspection scheme. However, relaxing the inspection cost assumption is impractical and failure in simultaneous optimization of replacement threshold and inspection scheme becomes a critical drawback. All the proposed approaches lack simultaneity in identifying the optimal preventative replacement threshold and the condition-based inspection scheme.

In order to make inspection scheduling more practical, equipment condition information needs to be captured and utilized. Moreover, a correlation exists between the preventative replacement threshold and the inspection schedule. Equipment with a high replacement threshold has a long lifetime and requires more inspections within its lifetime. Setting a low replacement threshold shortens the lifetime for equipment and reduces the number of inspections accordingly. A high threshold incurs frequent replacement and a low threshold incurs less frequent replacement. Due to the equal importance of the preventative maintenance threshold and inspection scheduling, tradeoffs need to be maintained under the given criteria.

1.3 Objective

Our objective is to simultaneously optimize the preventative replacement threshold and condition-based inspection scheme utilizing equipment condition information. The proposed approach in this present research is based on dynamic programming. It aims to minimize the average cost per unit time while simultaneously identifying the optimal preventative threshold and inspection scheme.

In this work, the proposed dynamic programming approach is used to simplify the decision of inspection distribution by breaking it down into a sequence of decisions. The
control limit policy is adopted from Makis and Jardine (1992), and identifies the optimal control limit for preventative replacement. The proposed approach succeeds in simultaneously optimizing the preventative replacement threshold and non-periodic inspection scheme.

The remainder of this paper is organized as follows: Section 2 includes brief description for deterioration modeling and maintenance decision making. Explicit description is presented in presenting the proposed dynamic optimization approach. Section 3 provides a case study comparing periodic inspection and condition-based inspection. Section 4 provides a brief discussion and conclusions. Finally, section 5 details directions for future research.

2 PROPOSED METHODOLOGY

2.1 Condition Information Utilization

2.1.1 Deterioration Process Modeling

Multiple statistical models have been used to describe and model a stochastic process, i.e. a Gamma distribution (Dieulle et al. 2003, Park 1988, Park 1988 Chelbi and Daoud 1999), proportional hazard model (PHM) (Banjevic and Jardine 2001, Glomakani and Hamid 2012, Tian 2010), random coefficient approach (Wang 2000, Egger and Jan 2016, Westerlund and Paresh 2015), Bayesian approach (Gebraeel et al. 2005, Kumar and Pravin 2015, C Genovese 2004). PHM has been chosen in this work, since it can accommodate equipment condition information and age in the model. PHM can also indicate equipment failure rate under the given age of the equipment and monitored condition.

Cox and Oake’s proportional hazard model (PHM) (Cox and Oakes 1984) utilizes a baseline Weibull hazard function and time dependent stochastic covariates to describe the
equipment deterioration process. This model accommodates equipment age and time-dependent condition data and show equipment failure behavior at various states. In this work, we only consider single critical covariate in a single-unit system. The hazard function is provided below to describe equipment failure rate:

\[ h(t, Z(t)) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} \exp\{\gamma Z(t)\} \]  

(1)

\( \beta \) and \( \eta \) are the shape and scale parameters of the Weibull distribution; \( h_0(s) = \frac{\beta}{\eta} (s/\eta)^{\beta-1} \) is the baseline hazard rate by taking into account age of the equipment; \( Z(t) \) is the covariate; \( \exp\{\gamma Z(t)\} \) takes into account equipment health condition.

Because dependencies (Zhang and Yang 2015) between components within a multi-component system are very complex, the deterioration process for each component requires a more complex function than shown in (1). This work considers single-component system only.

### 2.1.2 Markov Process

Prior to equipment condition information analysis, a data processing procedure needs to be carried out as to make the collected data useful and the analysis procedure easier. Let \( Z(t) \) denote the equipment condition at time \( t \). \( Z(t) \) is the data collected from the critical component, which is indicative of the equipment condition. \( Z(t) \) can be represented as a value, a waveform or multidimensional data structure. Through data acquisition and data processing, \( Z(t) \) is discretized into multiple states \{0, 1, 2, ..., \( m \}\}. Practically, this discretization is based on analysis of equipment’s physical behavior. Boundaries used for discretizing continuous data type into stratified zones are significant events indicating equipment condition change. Markov processes are used to model stochastic transition
probabilities from one stage to another. Let $P_{ij}(k)$ denote the conditional transition probability from the transition probability matrix $P(k)$ for the covariate $Z(k\Delta)$ given equipment condition $i$ at inspection time $k\Delta$ and state changes to $j$ at time $(k+1)\Delta$, here $\Delta$ is the basic inspection interval. The transition probability matrix is based on the basic inspection interval $\Delta$ and positive integer $k$. Given equipment in state $i$ at time $k\Delta$ the probability there is no failure during time $[k\Delta, (k + 1)\Delta]$ and state $j$ is detected at inspection time $(k + 1)\Delta$. State transition probabilities can be estimated with the adoption of maximum likelihood method. It is assumed that the component is not self-repairable.

### 2.2 Optimization Criteria

Various optimization criteria are used to solve maintenance optimization problems, though cost is most frequently encountered. In the manufacturing industry, availability and reliability also play very important roles as well in evaluating equipment performance. Tsai et al. (2003) studied preventative maintenance in a multi-component system, where the optimization process was based on availability. Mohanta et al. (2005) presented a set of results for the optimization of captive power plant maintenance scheduling. The authors used reliability maximization to formulate the optimization problem. Tian et al. (2009) used physical programming to optimize the preventative threshold in a multi-objective setting. Two objectives, reliability and cost, are considered when identifying the optimal threshold.

Cost is considered as the only objective in this work. Based on the replacement policy, a preventative replacement will be performed if the failure rate exceeds a control limit or replacement performs during a failure event. If neither a failure event nor preventative maintenance occur, the equipment continues working. Equipment is assumed to be
restored as good as new after the replacement. Let $d$ denote the control limit, which is non-negative; and $T_d$ denotes the time when the failure rate of the equipment exceeds $d$. A cost calculation model adopted from Benjevic et al. (2001) is used to calculate the expected average cost per unit time incurred from replacement:

$$C_{\text{wins}} = \frac{C(1 - P(T \leq T_d)) + (C + K)P(T \leq T_d)}{E(\min\{T, T_d\})} = \frac{C(1 - Q(d)) + (C + K)Q(d)}{W(d)}$$  \hspace{1cm} (2)$$

where $C$ is the preventative replacement cost per replacement; $C + K$ is the failure replacement cost per replacement; $P(T \leq T_d) = Q(d)$, and denote the probability that failure replacement takes place before equipment failure rate reaches control limit $d$. $(1 - Q(d))$ is the probability that preventative replacement occurs. $E(\min\{T, T_d\}) = W(d)$ is the expected time until replacement, regardless of preventative or failure replacement. In order to compute $Q(d)$ and $W(d)$, a form of recursive computation was proposed by Makis and Jardine (1992). It is given as follows: $t_i = \inf\{t \geq 0| K \times h(t, i) \geq d_i\}$, here $i \in S$ is the state of covariate, $S = \{0, 1, 2, \ldots, m\}$, and $(k_i - 1)\Delta \leq t_i < k_i\Delta$. Assuming the current age of the equipment is $k\Delta$ and corresponding state is $i$; $Q(k, i)$ is the probability of failure replacement and $W(k, i)$ is the expected time until replacement. A backward recursion procedure is used to calculate $W(0, 0)$ and $Q(0, 0)$. An iterative procedure $d^{(r)} = C_{\text{wins}}(d^{(r-1)})$, $r = 1, 2, 3, \ldots, d^{(r)} > 0$, which was proposed by Makis and Jardine (2006), is used to determine the optimal control limit $d^*$. Makis and Jardine proved that $d$ is non-increasing given $r \geq 2$ and is optimal when the iteratively assigned value of $d$ satisfies $C_{\text{wins}}(d^*) = d^*$. According to Mavkovian property and the estimation algorithm for conditional reliability, the proposed control limit policy was adopted and extended by Hamid [6] to find the optimal control limit and the optimal constant inspection interval.
Because the proposed optimization approach is dynamic, inspection options change from one iteration to the next. The cost function for inspection will be given in the following sections.

2.3 Proposed Dynamic Programming

Dynamic programming is a recursive optimization algorithm used for solving complex decision-making problems. Its key trait is the memorability in extending the optimization procedure from solving and combining sub-problems to optimizing systematic decision-making. Wei et al. (2015) developed a method called distributed iterative adaptive dynamic programming (ADP) and utilized it to optimize multi-battery coordination control for household energy management systems. Zhu et al. (2015) present a model based on dynamic programing for production energy optimization. There are two objective functions, one which aims at cost savings and the other which aims to maximize energy efficiency. This creates a multi-objective scenario in which the authors successfully apply a dynamic programming solution.

We assume that the basic inspection interval is \( \Delta \) and inter-inspections are multiples of \( \Delta \). The proposed dynamic programming model enables all alternative inspection schemes to be considered before reaching the optimal one.
Figure 2 illustrates how the proposed dynamic programming works. To design the number of stages, we use the function
\[ t_i = \inf \{ t \geq 0 \mid K \times h(t, i) \geq d \} \].
Assuming there is no failure prior to reaching the control limit \( d \), \( k_0 - 1 \) gives the longest lifetime before disposing the component. As Figure 2 shows, \( k_0 - 1 \) is assigned to be the number of stages given the initial control limit \( d^{(1)} \). Let \( S_{ni} \) be the amount of inspections available for allocation at stage \( n \) given equipment state \( i \) before exceeding time \( (k_i - 1)\Delta \), \( n \) is a positive integer less than \( k_0 - 1 \). \( x_n \) is the inter-inspection interval from stage \( n \) to stage \( n + 1 \). \( S_{nj} - (x_1 + x_2 + \ldots + x_{n-1}) \) is the remaining number of inspections available for allocation and state changes from \( i \) to \( j \) after the inter-inspection interval \( x_n \Delta \). Inspection scheduling begins at the start node \( S_{10} \) and terminates when \( S_{ni} = 0 \). Because the deterioration rate increases over time and scheduling one inspection per cycle is impractical, it is assumed that \( x_{n-1} \geq x_n \) and \( n \geq 2 \). At stage \( n = 1 \), the initial equipment state is 0, hence \( S_{10} = k_0 - 1 \).

The first inspection time is \( x_1 \Delta \), where \( x_1 \) can be any positive integer which is less than or equal to \( k_0 - 2 \). Therefore, the number of inspections available for allocation to stage 3 at the second stage is \( S_{2i} - x_1 \), where \( S_{2i} \leq k_i - 1 \). When equipment is at stage \( n \), number of inspections still available for allocation to next stage is \( S_{nj} - (x_1 + x_2 + \ldots + x_{n-1}) \), \( x_1 + x_2 + \ldots + x_{n-1} \) is the total length of inter-inspection starting from stage 1 \( (t = 0) \) to stage \( n \), here let \( X_n = x_1 + x_2 + \ldots + x_{n-1} \).
A critical trait of our proposed dynamic programming approach is that the number of stages will dynamically change because the iteration procedure constantly reassigns new value to control limit $d^{(r)}$ before obtaining the optimal control limit $d^*$. As illustrated in figure 3, given an initial value of $d^{(1)}$, the number of stages at the first iteration is $k_0 - 1$. This is computed from $k_0 = \inf\{t \geq 0 \mid K \times h(t, 0) \geq d^{(1)}\}$.

At the second iteration, this procedure re-updates control limit by assigning a new value to $d^{(2)}$. The procedure terminates when $d^{(r)} = C_{wins}(d^{(r-1)})$. A relatively large initial value is assigned to $d^{(1)}$ in order to produce a network with an initial number of stages larger than the optimal number of stages. This exhaustively searches all potential inspection to guarantee the finally acquired threshold and inspection scheme are optimal.
Additionally, a small $\delta^{(1)}$ tends to expand the number of stages. This would increase the number of stages from the first iteration to the second iteration. Scheduling through the inspection network also needs to be redesigned, though this will increase the complexity and computational difficulty.

Markov processes were proposed to model the stochastic transition probabilities, since the inter-inspection intervals $x_n\Delta$ is not identical over stages. The transition probability matrix for the time interval is computed via the Markovian property. We should note that all time intervals are multiples of the basic inspection interval. We first estimate the basic transition probability matrixes from the equipment’s historical failure and suspension data, taking advantage of the Markovian property of the covariate process. The estimated basic transition probability matrix, acquired under the basic time interval, will be used as the one-step transition baseline to estimate the multiple-step transition probability matrix. Let $P(k)$ be the initial one-step transition probability matrix. The transition probability matrix for any alternative inspection interval $x\Delta$ and corresponding $x$-step transition matrix can also be computed. We assume that the transition rates are equal within any discretized time interval and equal for each time interval. The estimation of $x$-step transition probability matrix is given $P^x(k) = [P(k)]^x$. We assume that the inspection intervals are multiples of $\Delta$ and state transition is a non-decreasing stochastic process. Therefore, state transitions can happen at any time within an inter-inspection time larger than or equal to $2\Delta$. In order to calculate the average replacement cost (2) from our proposed dynamic programming approach, the recursive computation procedure provided by Makis and Jardine (1992) is used. Three cases are generated: $X_n \geq k_i - 1$, $X_n = k_i - 1$ and $X_n < k_i - 1$. When $X_{n+1} < k_i - 1$, $X_{n+1} = X_n + x_n$, 
there is no replacement and the subsequent inspection is scheduled at time interval \(x_n\). This procedure ends when the total inspection length equal to \((k_i - 1)\Delta\). When \(X_n = k_i - 1\) replacement occurs at time \(t_i\). We assume that state change can only be detected at the inspection times and the serviced equipment is as good as new at the end of each cycle.

Given the control limit \(d\) and chosen inspection scheme \((x_1, x_2, \ldots, x_{n-1})\), the recursive computational procedures are presented below to compute \(Q(0,0)\) and \(W(0,0)\):

\[
Q(X_n, i) =
\begin{cases}
0 & X_n > k_i - 1 \\
1 - R(X_n, i, t_i - X_n\Delta) & X_n \leq k_i - 1 & \& X_{n+1} > k_i - 1 \\
1 - R(X_n, i, x_n) + \sum_{r=i}^{m} Q(X_{n+1}, r) \left[ \prod_{k=X_n}^{X_n+x_n-1} P(k) \right] R(X_n, i, x_n) & X_n < k_i - 1
\end{cases}
\]

\[
W(X_n, i) =
\begin{cases}
0 & X_n > k_i - 1 \\
\int_{0}^{t_i-X_n\Delta} R(X_n, i, s) \, ds & X_n \leq k_i - 1 & \& X_{n+1} > k_i - 1 \\
\int_{0}^{x_n} R(X_n, i, s) \, ds + \sum_{r=i}^{m} W(X_{n+1}, r) \left[ \prod_{k=X_n}^{X_n+x_n-1} P(k) \right] R(X_n, i, x_n) & X_n < k_i - 1
\end{cases}
\]

where \(X_n\Delta\) denotes the current inspection time, and the conditional probability function is given as follows:

\[
R(X_n, i, t) = \exp \left\{ -\exp\{yi\} \int_{X_n\Delta}^{X_n\Delta+t} h_0(s) \, ds \right\}
\]

\(h_0(s)\) is the baseline hazard rate which takes into account only the age of the equipment.

Given the control limit \(d\), \(Q(X_n, i)\) is the probability of failure replacement and \(W(X_n, i)\) is the expected time until replacement when equipment age is \(X_n\Delta\) and the corresponding state is \(i\). \(R(X_n, i, x_n)\) is given below:
\[ R(X_n, i, x_n) = \exp \left\{ -\exp \{\gamma i\} \left( \frac{1}{\eta} \right)^\beta \left( (X_n + \Delta)^\beta - (X_n^0)^\beta \right) \right\} \]

\[ - \sum_{j=1}^{x_n-1} \sum_{l=1}^{m} \left\{ \frac{X_n + f - 1}{P(k)} \right\} \exp\{\gamma l\} \left( \frac{1}{\eta} \right)^\beta \times \left( (X_n + (f + 1)\Delta)^\beta - (X_n + f\Delta)^\beta \right) \]

where equipment state at time \(X_n\Delta\) is \(Z(X_n\Delta), Z(X_n\Delta) = i\). Given the difference \(t_i - X_n\Delta\), because the adopted conditional reliability calculation algorithm is captured based on the basic constant inspection, two cases are considered, the estimation of \(R(k_i - 1, i, t_i - X_n\Delta)\) is given when \(X_n = k_i - 1\)

\[ R(X_n, i, t_i - X_n\Delta) = \exp \left\{ -\exp \{\gamma i\} \int_{X_n\Delta}^{t_i} h_0(s)ds \right\} \]

when \(X_n < k_i - 1\) and \(X_n + 1 > k_i - 1\)

\[ R(X_n, i, t_i - X_n\Delta) = \exp \left\{ -\exp \{\gamma i\} \left( \frac{1}{\eta} \right)^\beta \left( (X_n + \Delta)^\beta - (X_n^0)^\beta \right) \right\} \]

\[ - \sum_{j=1}^{[\frac{t_i - X_n\Delta}{\Delta}] - 1} \sum_{l=1}^{m} \left\{ \frac{X_n + f - 1}{P(k)} \right\} \exp\{\gamma l\} \left( \frac{1}{\eta} \right)^\beta \times \left( (X_n + (f + 1)\Delta)^\beta - (X_n + f\Delta)^\beta \right) \]

The estimations of \(\int_0^{x_n} R(X_n, i, s)ds\) is given by:

\[ \int_0^{x_n} R(X_n, i, s)ds = \int_0^{x_n} \exp \left\{ -\exp \{\gamma i\} \left( \frac{1}{\eta} \right)^\beta \left( (X_n + s)^\beta - (X_n^0)^\beta \right) \right\} ds \]
\[ + \sum_{n=1}^{x_n-1} \int_{n\Delta}^{(n+1)\Delta} \exp \left\{ -\exp \left\{ \gamma i \left( \frac{1}{\eta} \right) \right\} \left( (X_n \Delta + \Delta)^\beta - (X_n \Delta)^\beta \right) - \sum_{f=1}^{n-1} \sum_{l=i}^{m} \left( \prod_{k=X_n}^{X_n+f-1} p^{(\Delta)}(k) \exp \left\{ \gamma l \left( \frac{1}{\eta} \right) \right\} \right) \times (X_n + (f+1)\Delta)^\beta - (X_n + (f\Delta))^\beta \right\} \left( X_n + s \right)^\beta - (X_n + n\Delta)^\beta \right\} ds 
\]

if \( X_n = k_i - 1 \)

\[ \int_0^{t_i - X_n\Delta} R(X_n, i, s) \, ds = \]

\[ \int_0^{t_i - (k_i-1)\Delta} \exp \left\{ -\exp \left\{ \gamma i \left( \frac{1}{\eta} \right) \right\} \left( (k_i-1)\Delta + s \right)^\beta - (k_i-1)\Delta)^\beta \right\} ds \]

when \( X_n < k_i - 1 \) and \( X_{n+1} > k_i - 1 \)

\[ \int_0^{t_i - X_n\Delta} R(X_n, i, s) \, ds = \int_0^\Delta \exp \left\{ \exp \left\{ \gamma i \left( \frac{1}{\eta} \right) \right\} \times ((X_n \Delta + s)^\beta - (X_n \Delta)^\beta) \right\} ds \]

\[ + \sum_{f=1}^{ \left\lceil \frac{t_i - X_n}{\Delta} \right\rceil - 1} \int_{n\Delta}^{(n+1)\Delta} \exp \left\{ -\exp \left\{ \gamma i \left( \frac{1}{\eta} \right) \right\} \left( (X_n \Delta + \Delta)^\beta - (X_n \Delta)^\beta \right) - \sum_{f=1}^{n-1} \sum_{l=i}^{m} \left( \prod_{k=X_n}^{X_n+f-1} p^{(\Delta)}(k) \exp \left\{ \gamma l \left( \frac{1}{\eta} \right) \right\} \right) \times (X_n \Delta + (f+1)\Delta)^\beta - (X_n \Delta + f\Delta)^\beta \right\} \left( X_n \Delta + s \right)^\beta - (X_n \Delta + n\Delta)^\beta \right\} ds \]
As presented before, the iteration procedure \( d^{(r)} = C_{\text{wins}}(d^{(r-1)}) \), \( r = 1, 2, 3, \ldots \), \( d^{(r)} \) is positive and non-increasing given an initial large \( d^{(1)} \). Prior to iteratively assigning the value of \( C_{\text{wins}}(d^{(r-1)}) \) to \( d^{(r)} \), an inspection scheme should be chosen. Because dynamic programming allows each node having multiple alternatives for the next inspection, there are multiple ways forwarding from the starting node to the end node. Because the number of stages comes from \( t_i = \inf \{ t \geq 0 | K \times h(t, i) \geq d^{(1)} \} \), the iteration procedure \( d^{(r)} = C_{\text{wins}}(d^{(r-1)}) \) iteratively updates the value of \( d \). The number of stages is changed accordingly, since \( d^{(r)} \) is non-increasing for \( r \geq 2 \) (Makis and Jardine 1992). In our work, a relatively large value is initially assigned to \( d^{(1)} \) after a set of pre-analytic tests, the intention is to guarantee \( d^{(r)} \) is non-increasing and all possible inspection schemes can be completely tested.

Figure 3 demonstrates that the number of stages updating procedure from one iteration to the next iteration. With an initial large control limit \( d^{(1)} \), the number of alternative inspection schemes is large. As the control limit \( d \) decreases, inspection network shrink with the decreased control limit \( d \). Some inspection schemes, e.g. \((x_1, x_2, \ldots, x_{n-1})_1\) and \((x_1, \ldots,
\( x_2, \ldots, x_{n-1}) \) have different inspection routes from the start node to the end node. The newly updated inspection network may generate two different inspection schemes from those previously selected.

Figure 4: Inspection rescheduling algorithm

Figure 4 is used to illustrate the inspection rescheduling algorithm. Lines with marked with blue denotes the inspection scheme chosen from all the alternative inspection schemes, given the initial inspection scheme \( d^{(1)} \). Here number of stages is \( k_0 - 1 \). Use Equations (3) and (4) are used to compute the corresponding preventative replacement threshold cost (2), which is assigned to \( d^{(2)} \). At the second iteration, \( d^{(2)} \) is used, where the number of stages of the newly updated inspection network is less than \( k_0 - 1 \). This can be obtained from the control limit policy \( t_i = \inf \{ t \geq 0 \mid K \times h(t, i) \geq d^{(2)} \} \). The vertical dashed line on figure 4 separates multiple iteration procedure and indicates that the preventative
replacement time is decreasing. The total length of the early chosen inspections \((x_1, x_2, \ldots, x_{n-1})\) is decreasing as well. Any inspection scheme which differs from earlier inspection schemes reduces to a state which has the same number of stages and the same inspection route. One inspection schemes is retained while the rest of those inspection schemes are deleted from the list of alternatives. Deleted inspection schemes are not considered during future iterations.

In section 2.2, the cost function for the preventative replacement threshold is given. The cost function below shows the computation of inspection scheme \((x_1, x_2, \ldots, x_{n-1})\):

\[
C_{\text{ins}}(d) = \frac{C_{\text{in}}N_{\text{in}}}{x_1 + x_2 + \cdots + x_{k_0-1}}
\]

where \(C_{\text{ins}}\) is the average inspection cost per unit time; \(C_{\text{in}}\) is the cost incurred from one inspection; \(N_{\text{in}}\) is number of inspections within time \((k_0 - 1)\Delta\) given the control limit \(d\). Because \(x_{n-1} \geq x_n\) and the initial \(d^{(1)}\) is relatively large, \(d^{(r)}\) is non-increasing and number of stages decreases from one iteration to the next. By the same reasoning, \(C_{\text{ins}}\) is also non-increasing.

By aggregating the replacement and inspection cost, the overall average cost per unit time is given below:

\[
C_{\text{total}} = C_{\text{win}}(d) + C_{\text{ins}}(d)
\]  

Because the recursive calculation algorithm works backwards, the final cost function associated with maintenance and inspections computed when the algorithm reaches stage one of the inspection network.

Let \(f_n(S_n, x_1, x_2, \ldots, x_{n-1})\) denote the expected total cost at stage \(n\) given a specified control limit \(d\) and the chosen inspection scheme \((x_1, x_2, \ldots, x_{n-1})\), where \(S_1\) is still the number of inspections available for allocation, and \(f_1(S_1, x_1, x_2, \ldots, x_{n-1}) = C_{\text{total}}\). Unlike
dynamic programming applied in solving general problems where a set of optimal values is obtained backward and there exists an optimal value at each node, the optimal maintenance cost is only acquired at node $S_{10}$. Due to the constantly changing control limit $d$, the solution gradually approaches optimality under every alternative inspection route ($x_1, x_2, \ldots, x_{n-1}$). The optimal total average cost $f^*_i(S_1, x_1, x_2, \ldots, x_{n-1}; d^*)$ for each chosen inspection scheme ($x_1, x_2, \ldots, x_{n-1}$) is obtained after iteratively rescheduling inspection scheme and deleting any overlapping inspections. Through comparisons of a set of costs obtained from each inspection scheme, the minimum total average cost, $f^*_i(S_1, x_1^*, x_2^*, \ldots, x_{n-1}^*; d^*)$ are ultimately obtained. This solution includes the optimal inspection scheme and control limit.

3. CASE STUDY

We assume that the estimated values for parameters in the hazard function $h_0(s) = \frac{\beta}{\eta(t/\eta)^{\beta-1}} \exp\{\gamma Z(t)\}$, where $\beta = 2$, $\eta = 14$, $\gamma = 0.8$. There are two states $S=\{0, 1\}$ and $Z(t)$ belongs to $S$ for $t > 0$. The basic inspection interval $\Delta$ is one month. One preventative replacement costs $10,000$, while a failure replacement costs $100,000$ (cost incurred from failure replacement contains equipment breakdown cost, replacement cost, setup up cost, labor cost). Each inspection incurs a cost of $1,000$, where $C = 10$, $K = 90$ and $C_{in} = 1$. The estimated one-step transition probability matrix from historical data and suspension data is:

$$P = \begin{pmatrix} 0.7 & 0.3 \\ 0 & 1.0 \end{pmatrix}$$
In order to make the initial value assigned to $d^{(1)}$ rational and reduce the calculation complexity, a set of pre-analytic tests on $d^{(1)}$ is carried out. The initially assigned a value $d^{(1)}$ that is too large would make computation complexity very difficult at the first iteration. Alternatively a value that is too small may generate incomplete inspection scheme alternatives and augment the calculation workload at the following iteration process. Given the assigned value $d^{(1)}$, the recursive algorithm $t_i = \inf\{t \geq 0| K \times h(t, i) \geq d\}$ is used to calculate $t_i$ and $k_i$. Here the maximum number of stages is captured, which is $k_0 - 1$. $k_0 - 1$ is also used as the initial number of stages. In this work, both the periodic inspection and the proposed condition-based inspection will be considered. Calculation results for both are presented separately. Figure 5 displays the relation between inspection interval and average inspection cost (both periodic inspection and non-periodic inspection). Average inspection cost per unit time has a negative relation with inter-inspection interval, which is that increasing inter-inspection interval will reduce the average cost per unit time.
incurred from inspection action.

![Graph](Image)

Figure 6: Preventative replacement cost VS control limit.

### 3.1 Periodic Inspection

Table 1. Calculation to determine optimal threshold for inspection interval \(x = 1\).

<table>
<thead>
<tr>
<th>(d^{(r)})</th>
<th>(t_0)</th>
<th>(t_1)</th>
<th>(k_{0-1})</th>
<th>(k_{1-1})</th>
<th>(W(0,0))</th>
<th>(Q(0,0))</th>
<th>(C_{\text{wins}})</th>
</tr>
</thead>
<tbody>
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<td>2.691</td>
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<td>2</td>
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<tr>
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<td>2.4853</td>
<td>5</td>
<td>2</td>
<td>3.3115</td>
<td>0.07541</td>
<td>5.06922</td>
</tr>
<tr>
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<td>2.4802</td>
<td>5</td>
<td>2</td>
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<td>0.07518</td>
<td>5.06921</td>
</tr>
<tr>
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<td>5.5198</td>
<td>2.4802</td>
<td>5</td>
<td>2</td>
<td>3.30744</td>
<td>0.07518</td>
<td><strong>5.06921</strong></td>
</tr>
</tbody>
</table>

Table 2. Calculation to determine optimal threshold for inspection interval \(x = 2\).

<table>
<thead>
<tr>
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<th>(t_0)</th>
<th>(t_1)</th>
<th>(k_{0-1})</th>
<th>(k_{1-1})</th>
<th>(W(0,0))</th>
<th>(Q(0,0))</th>
<th>(C_{\text{wins}})</th>
</tr>
</thead>
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<tr>
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<td>5.9385</td>
<td>2.6684</td>
<td>5</td>
<td>2</td>
<td>3.54381</td>
<td>0.09897</td>
<td>5.3353</td>
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<tr>
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<td>3.53331</td>
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Table 3. Calculation to determine optimal threshold for inspection interval $x = 3$.

<table>
<thead>
<tr>
<th>$d^{(k)}$</th>
<th>$t_0$</th>
<th>$t_1$</th>
<th>$k_0-1$</th>
<th>$k_1-1$</th>
<th>$W(0,0)$</th>
<th>$Q(0,0)$</th>
<th>$C_{wins}$</th>
</tr>
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<td>5.5</td>
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<tr>
<td>6.20902</td>
<td>6.7609</td>
<td>3.0379</td>
<td>6</td>
<td>3</td>
<td>3.83993</td>
<td>0.1333</td>
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<td>2</td>
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</tr>
<tr>
<td>5.67042</td>
<td>6.1715</td>
<td>2.7744</td>
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<td>2</td>
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Table 4. Optimal cost for constant inspection.

<table>
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<tr>
<th>$x_n$</th>
<th>$t_0$</th>
<th>$t_1$</th>
<th>$d$</th>
<th>$C_{wins}(d^*)$</th>
<th>$C_{ins}$</th>
<th>$C$</th>
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<tr>
<td>1</td>
<td>5.5198</td>
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<td>5.06921</td>
<td>5.06921</td>
<td>1</td>
<td>6.06921</td>
</tr>
<tr>
<td>2</td>
<td>5.7722</td>
<td>2.5936</td>
<td>5.30099</td>
<td>5.30099</td>
<td>0.5</td>
<td>5.80099</td>
</tr>
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<td>3</td>
<td>6.1745</td>
<td>2.7744</td>
<td>5.67042</td>
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<td>0.33</td>
<td>6.00042</td>
</tr>
<tr>
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<td>0.25</td>
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<td>3.0547</td>
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<td>6.24329</td>
<td>0.2</td>
<td>6.44329</td>
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</table>

Table 4 shows the minimum total cost and optimal control limit for the constant inspection interval $x_n = 1, 2, 3, 4, 5$, where inspections are performed every $x_n \Delta$. The optimal constant inspection interval is $x_n = 2$, and the corresponding minimum total cost $C_{total} = 5.93725$ and optimal control limit $d^* = 5.43725$. The relationship between the control limit (threshold) for preventative replacement threshold and the average preventative replacement cost per unit time is show in figure 5. Some obvious observations are identified: A small control limit generally incurs a high cost. A high threshold also incurs high cost. Given the inspection interval, the preventative replacement cost is minimized with a proper control limit, and the minimized cost varies for different inspection intervals.

### 3.2 Condition-based Inspection Scheme

After pre-analysis, $d^{(1)} = 5.5$ is used as the initial control limit to design number of stages. Because $d^{(1)} = 5.5$ is a relatively large initial value, the optimal control limit does
not fluctuate too far from for all the alternative inspection schemes. Additionally, after the first iteration, \( d^{(r)} \) is not less than and very close to \( d^{(1)} \). In most cases, no inspection rescheduling is needed.

Table 5. Calculation to determine optimal threshold for inspection scheme (2,1,1,1,0).

<table>
<thead>
<tr>
<th>(d^{(r)})</th>
<th>(t_0)</th>
<th>(t_1)</th>
<th>(k_{0-1})</th>
<th>(k_{1-1})</th>
<th>(W(d))</th>
<th>(Q(d))</th>
<th>(C_{wins})</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
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<td>2.5009</td>
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<td>2</td>
<td>3.29465</td>
<td>0.076</td>
<td>5.11154</td>
</tr>
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<td>5.11154</td>
<td>5.5659</td>
<td>2.5009</td>
<td>5</td>
<td>2</td>
<td>3.29465</td>
<td>0.07601</td>
<td>5.11154</td>
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</table>

Table 6. Calculation to determine optimal threshold for inspection scheme (2,2,1,0,0).

<table>
<thead>
<tr>
<th>(d^{(r)})</th>
<th>(t_0)</th>
<th>(t_1)</th>
<th>(k_{0-1})</th>
<th>(k_{1-1})</th>
<th>(W(0,0))</th>
<th>(Q(0,0))</th>
<th>(C_{wins})</th>
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<td>2</td>
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<td>2</td>
<td>3.45271</td>
<td>0.088798</td>
<td>5.21094</td>
</tr>
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</table>

Table 7. Calculation to determine optimal threshold for inspection scheme (3,1,1,0,0).

<table>
<thead>
<tr>
<th>(d^{(r)})</th>
<th>(t_0)</th>
<th>(t_1)</th>
<th>(k_{0-1})</th>
<th>(k_{1-1})</th>
<th>(W(0,0))</th>
<th>(Q(0,0))</th>
<th>(C_{wins})</th>
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<td>2</td>
<td>3.51178</td>
<td>0.09123</td>
<td>5.18552</td>
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</table>

Table 8. Calculation to determine optimal threshold for inspection scheme (3,2,0,0,0).

<table>
<thead>
<tr>
<th>(d^{(r)})</th>
<th>(t_0)</th>
<th>(t_1)</th>
<th>(k_{0-1})</th>
<th>(k_{1-1})</th>
<th>(W(0,0))</th>
<th>(Q(0,0))</th>
<th>(C_{wins})</th>
</tr>
</thead>
<tbody>
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<td>5</td>
<td>2</td>
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<td>5.29811</td>
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<tr>
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<td>5.7691</td>
<td>2.5922</td>
<td>5</td>
<td>2</td>
<td>3.60713</td>
<td>0.10123</td>
<td>5.29811</td>
</tr>
</tbody>
</table>

Table 9 shows the minimum total cost and optimal control limit for our condition-based inspection. The optimal inspection scheme is \( x_1 = 3 \) and \( x_2 = 2 \), here there are two inspections for each cycle, the first inspection performs at time \( 3\Delta \) and the second
inspection performs at time $5\Delta$; the corresponding minimum total cost $C_{\text{total}} = 5.69811$ and the optimal control limit $d^* = 5.29811$.

Table 9. Optimal cost for condition-based inspection.

<table>
<thead>
<tr>
<th>$x_n$</th>
<th>$t_0$</th>
<th>$t_1$</th>
<th>$d^*$</th>
<th>$C_{\text{win}}(d^*)$</th>
<th>$C_{\text{ins}}$</th>
<th>$C$</th>
</tr>
</thead>
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<tr>
<td>(2,1,1,1)</td>
<td>5.5659</td>
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<td>5.11154</td>
<td>5.11154</td>
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<tr>
<td>(2,2,1)</td>
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<td>5.21094</td>
<td>5.21094</td>
<td>0.6</td>
<td>5.81094</td>
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<tr>
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<td>5.18552</td>
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<tr>
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<td>5.29811</td>
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<td>5.48843</td>
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</table>

Because the inspection rescheduling progresses dynamically, the inspection scheme varies from one iteration to the next. Like periodic inspection, a proper control limit would minimize the preventative replacement cost. Either a small or a large control limit would augment the preventative replacement maintenance cost.

4. CONCLUSION

As the deterioration rate of equipment increases over time, failure risk incurred from equipment degradation increases as well. Inspection is used to detect equipment health condition so as to maintain equipment functionality and reliability. Because inspection incurs a cost, frequent inspection can maintain high reliability but also generate high costs. From the economic and practical perspective, this necessitates an intelligent inspection schedule. Additionally, an appropriate preventative replacement threshold is needed to reduce failure risk and the losses caused by those failures. A small preventative replacement threshold maintains reliability and functionality, it also requires frequent replacement of the equipment. Frequent replacement increases the cost associated with maintenance. Moreover, equipment may be disposed even though it is still in very good condition. This produces unnecessary waste and increases manufacturing cost. Conversely,
a large preventative threshold entails less frequent preventative replacement. While this may elongate the life of the equipment, it does so at a high risk of failure. This can be dangerous for personnel using the equipment.

Our proposed approach shows its efficiency in simultaneously optimizing the inspection and preventative replacement. Condition-based inspection is shown to be more cost-effective than periodic inspection in many cases. The algorithm enables all possible inspection schemes to be searched, which guarantees a globally optimal solution.

By assigning a large initial control limit, the number of stages decrease as the iteration procedure keeps working from one iteration to the next. This has the disadvantage of generating many alternative inspection schemes when the optimal threshold is large. Because dynamic programming is an exhaustive search algorithm, testing all the generated inspection options will inevitably increase the computational difficulties.

5. FUTURE WORK

The proposed model simultaneously optimizes the replacement threshold and inspection scheme through intensive iterative calculation. As an exhaustive search algorithm, the proposed dynamic programming model has been proven its feasibility and superiority in simultaneously optimizing the preventative replacement threshold and inspection scheme. From the perspective of optimization, the exhaustiveness circumvents the local optimum. However, from the perspective of computational complexity, as the number of alternative inspection schemes increases, computational difficulty and complexity also increase. We therefore recommend further investigation into heuristics for this problem. Because heuristic solutions may be sub-optimal, care should be exercised during their use. Advanced improvements have been underway as to make the proposed
model less calculation-intense. Mathematical optimization problems commonly involve more than one objective, which are usually conflicting with each other. In the manufacturing industry, equipment reliability, manufacturing cost, energy efficiency are usually important and conflicting with each other. But those criteria need to be taken into account at the same time. Multi-objective optimization is already applied in some research fields. Tradeoffs between multiple objectives, which are usually conflicting with each other, need to be taken into account to find out the optimal decisions.

In this work, inspection is assumed to be perfect, which means any equipment default can be monitored and detected at the inspection time. Because of the uncertain operational conditions and material properties, equipment deterioration contains inevitable uncertainties. It is not possible to collect perfect information regarding the condition of equipment. Imperfect inspection (Kallen and Noortwijk 2005, Berrade et al. 2012, Kaio and Osaki 1986) has been applied in modeling uncertainties in the equipment deterioration process. Most research work in equipment maintenance assumes that the maintained equipment is restored ‘as good as new’. Practically, only critical component/components are monitored, and maintenance is performed mainly on faulty critical components. It is impractical to assume that the replaced failure component or maintenance equipment is restored to “good as new” condition. Imperfect maintenance (Do et al. 2015, Le and Tan 2013), must be considered in future iterations of this work. As manufacturing techniques advance, manufacturing systems becomes more complex. Some manufacturing lines may contain multiple critical components, which need to be monitored and maintained constantly. There exist economic and stochastic dependences in a multi-component system. Some dependence is very hard to be quantified. Even through the dependence can be
defined and modeled, the modeled dependences would increase the computation difficulties and complexities. Future research will be carried out in consideration of multi-component systems.
REFERENCES


Mohanta, Dusmanta Kumar, Pradip Kumar Sadhu, and Rupendranath Chakrabarti. "Deterministic and stochastic approach for safety and reliability optimization of captive power plant maintenance scheduling using GA/SA-based hybrid


Pham, Hong Thom, Bo-Suk Yang, and Tan Tien Nguyen. "Machine performance degradation assessment and remaining useful life prediction using proportional


APPENDIX

\[ R(X_n, i, x_n) = \exp \left\{ - \int_{X_n \Delta}^{X_n \Delta + x_n \Delta} \exp \{ \gamma Z(s) \} h_0(s) ds \right\} \]

\[ = \exp \left\{ - \sum_{f=0}^{x_n - 1} \int_{X_n \Delta + f \Delta}^{X_n \Delta + (f+1) \Delta} \exp \{ \gamma Z(s) \} h_0(s) ds \right\} \]

\[ = \exp \left\{ - \sum_{f=0}^{x_n - 1} \exp \{ \gamma Z(X_n \Delta + f \Delta) \} \int_{X_n \Delta + f \Delta}^{X_n \Delta + (f+1) \Delta} h_0(s) ds \right\} \]

\[ = \exp \left\{ - \exp \{ \gamma i \} \int_{X_n \Delta}^{X_n \Delta + \Delta} \frac{s^\beta}{\eta^\beta} \eta^{-1} ds \right\} \]

\[ = \exp \left\{ - \sum_{f=0}^{x_n - 1} \exp \{ \gamma Z(X_n \Delta + f \Delta) \} \left( (X_n \Delta + (f+1) \Delta)^\beta - (X_n \Delta)^\beta \right) \right\} \]

\[ = \exp \left\{ - \sum_{f=0}^{x_n - 1} \sum_{m=1}^{\infty} \left( \frac{1}{\eta} \right)^\beta \left( \prod_{k=X_n}^{X_n + f - 1} \right) \exp \{ \gamma l \} \left( \frac{1}{\eta} \right)^\beta \times \left( (X_n \Delta + (f+1) \Delta)^\beta - (X_n \Delta + f \Delta)^\beta \right) \right\} \]

\[ \int_0^{x_n} R(X_n, i, s) ds = \sum_{f=0}^{x_n - 1} \int_{f \Delta}^{(f+1) \Delta} R(j, i, s) ds \]
\[ \begin{align*}
&= \int_0^\Delta \exp \left\{ - \int_{X_{n\Delta}}^{X_{n\Delta} + s} \exp(\gamma Z(t)h_0(t)) \right\} \, ds + \int_\Delta^{2\Delta} \exp \left\{ - \int_{X_{n\Delta}}^{X_{n\Delta} + s} \exp(\gamma Z(t)h_0(t)) \right\} \, ds \\
&\quad + \int_{2\Delta}^{3\Delta} \exp \left\{ - \int_{X_{n\Delta}}^{X_{n\Delta} + s} \exp(\gamma Z(t)h_0(t)) \right\} \, ds \\
&\quad + \cdots + \int_{(n-1)\Delta}^{n\Delta} \exp \left\{ - \int_{X_{n\Delta}}^{X_{n\Delta} + s} \exp(\gamma Z(t)h_0(t)) \right\} \, ds \\
&= \int_0^\Delta \exp \left\{ - \exp(\gamma Z(X_{n\Delta})) \right\} \int_{X_{n\Delta}}^{X_{n\Delta} + s} h_0(t) \, dt \, ds \\
&\quad + \int_\Delta^{2\Delta} \exp \left\{ - \int_{X_{n\Delta}}^{X_{n\Delta} + \Delta} \exp(\gamma Z(X_{n\Delta})h_0(t)) \right\} \, ds - \int_{X_{n\Delta} + \Delta}^{X_{n\Delta} + 2\Delta} \exp(\gamma Z(X_{n\Delta} + \Delta)h_0(t)) \right\} \right\} \, ds \\
&\quad + \cdots + \int_{(n-1)\Delta}^{n\Delta} \exp \left\{ - \int_{X_{n\Delta}}^{X_{n\Delta} + \Delta} \exp(\gamma Z(X_{n\Delta} + \Delta)h_0(t)) \right\} \, ds \\
&= \int_0^\Delta \exp \left\{ - \exp \left( \frac{1}{\eta} \right)^\beta \left( (X_{n\Delta} + s)^\beta - (X_{n\Delta})^\beta \right) \right\} \\
&\quad + \sum_{n=1}^{\frac{x_n-1}{\Delta}} \int_{n\Delta}^{(n+1)\Delta} \exp \left\{ - \exp(\gamma t) \left( \frac{1}{\eta} \right)^\beta \left( (X_{n\Delta} + \Delta)^\beta - (X_{n\Delta})^\beta \right) \right\} \\
&\quad \quad + \sum_{n=1}^{\frac{n-1}{\Delta}} \sum_{j=1}^{n-1} \exp(\gamma Z(X_{n\Delta} + f\Delta)) \left( \frac{1}{\eta} \right)^\beta \left( (X_{n\Delta} + (f+1)\Delta)^\beta - (X_{n\Delta} + f\Delta)^\beta \right) \right\} \, ds \\
&\quad \quad - \exp(\gamma Z(X_{n\Delta} + n\Delta)) \left( \frac{1}{\eta} \right)^\beta \left( (X_{n\Delta} + s)^\beta - (X_{n\Delta} + n\Delta)^\beta \right) \right\} }
\[
\begin{align*}
&= \int_0^\Delta \exp \left\{ - \exp \left\{ \gamma i \right\} \left( \frac{1}{\eta} \right)^\beta \left( X_n \Delta + s \right)^\beta - (X_n \Delta)^\beta \right\} ds \\
&+ \sum_{n=1}^{x_n-1} \int_{n \Delta}^{(n+1) \Delta} \exp \left\{ - \exp \left\{ \gamma i \right\} \left( \frac{1}{\eta} \right)^\beta \left( X_n \Delta + \Delta \right)^\beta - (X_n \Delta)^\beta \right\} \\
&\quad \times \left( \prod_{k=X_{n+1}}^{X_{n+f-1}} p(\Delta) (k) \exp \left\{ \gamma i \right\} \left( \frac{1}{\eta} \right)^\beta \right) \\
&\quad \times \left( (X_n \Delta + (f+1) \Delta)^\beta - (X_n \Delta + (f \Delta))^\beta \right) \\
&\quad - \sum_{l=1}^{m} \left\{ \prod_{k=X_{n+1}}^{X_{n+f-1}} p(\Delta) (k) \exp \left\{ \gamma i \right\} \left( \frac{1}{\eta} \right)^\beta \right\} \\
&\quad \times \left( (X_n \Delta + s)^\beta - (X_n \Delta + n \Delta)^\beta \right)
\end{align*}
\]