Random Linear Packet Coding over Fading Channels with Long Delays

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To my brother, Razeen Ahmed, who will always be missed.
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We consider random linear packet coding over fading channels with long propagation delays, such as underwater acoustic channels, satellite communication and deep-space communication. We adapt random linear packet for such channels, particularly focusing on underwater acoustic channel. Underwater communication has garnered much interest in recent years with emerging applications in underwater sensor networks, warning systems, off-shore oil and gas platform operations, marine life monitoring, etc. Since electromagnetic waves do not propagate over long distances underwater, acoustic waves remain the preferred choice for a number of applications. The slow speed of sound in water, however, leads to long propagation delays which challenge the efficiency of traditional automatic repeat request (ARQ) techniques such as the stop-and-wait and its selective versions in acoustic communication systems.

We first propose a packet coding scheme where the number of coded packets to transmit is determined so as to achieve a pre-specified outage/reliability criterion, and investigate joint power and rate control with constrained resources. Using the channel state information which is obtained via feedback from the receiver, the transmitter adjusts its power and the number of coded packets such that the average energy per successfully transmitted bit
of information is minimized. Two optimization constraints are imposed: (a) the transmit power should not exceed a maximum level, and (b) the number of coded packets should not exceed a maximum value dictated by the desired throughput and delay.

We further extend the results to take into account the effect of inevitable channel estimation errors, and consider the case where the transmitter has only an estimate of the channel gain. We design adaptation policies for such a case based on minimum mean square error (MMSE) channel estimation, taking into account the effect of channel estimation errors in an optimal manner so as to satisfy the required outage/reliability criterion. Finally, we compare the proposed technique to standard automatic repeat request (ARQ) protocols for underwater communications in terms of the throughput efficiency. Analytical results show that substantial energy savings and improvements in throughput efficiency are available from adaptive power/rate control.

We finally investigate reliable data packet delivery employing random linear packet coding for half-duplex communication links with long delays. In order to achieve full reliability, we regard a group of coded packets as one super-packet, on which we apply an ARQ technique. Specifically, we group several super-packets and apply a selective stop-and-wait acknowledgment procedure to the so-obtained unit (a super-group). We explore adaptive power and rate control to improve the performance of the proposed grouped packet coding technique on fading channels. We compare the performance of the proposed technique to that of conventional stop-and-wait, as well as an ideal full-duplex benchmark. Results are presented for a point-to-point link, as well as for a broadcast network. For the broadcast network, we investigate two adaptation rules. According to the first rule, the transmitter adjusts its parameters in accordance with the average of the channel gains on each link (average link rule). Under the second rule, the transmitter adjusts its parameters in accordance with the lowest channel gain among the links (worst link rule). Using numerical analysis we show that the proposed grouped packet coding, applied to a half-duplex link, can achieve a throughput efficiency that is very close to that of a full-duplex link.
Chapter 1

Introduction

The problem on long propagation delays and high packet error rates form the central theme of this dissertation. There are several communication channels that experience long propagation delays such as the underwater acoustic communication, satellite communication, deep-space communication and terrestrial communication. Although we explore solutions for all such channels, we choose to focus on underwater acoustic communication in particular for this dissertation.

Underwater communication has garnered much interest in recent years with emerging applications in underwater sensor networks, warning systems, off-shore oil and gas platform operations, marine life monitoring, etc. There are three candidates for communication medium under water, i.e., electromagnetic waves, optical waves and acoustics. Since electromagnetic waves do not propagate over long distances underwater, and optical waves are prone to scattering, acoustic waves remain the preferred choice for a number of underwater applications. Acoustic waves are capable of traveling several hundred kilometers but bring a different set of challenges. With the tremendous advances in underwater technologies, there is an urgent need for reliable high-rate underwater communication systems.

As stated before, underwater acoustic channels bring with them, a number of challenges such as limited bandwidth, frequency dependent attenuation, slow propagation speed, and large Doppler shifts. These characteristics of the underwater channel implies that the traditional technologies that are developed for terrestrial wireless systems are not efficient
underwater, and hence there is a need for communication systems that are specifically designed to tackle these challenges.

Underwater acoustic links are severely bandwidth limited. At distances of a few kilometers, the available bandwidth reduces a few kilo Hertz. The limited bandwidth of the underwater acoustic channel is attributed the high absorption at higher frequencies as shown in Fig. 1.1.

![Figure 1.1: Absorption coefficient as a function of the frequency using Thorp’s formula [1].](image)

The nominal speed of sound in water is $\sim 1500 \text{ m/s}$ which is much lower in comparison to the speed of electromagnetic waves in air. The slow speed of sound directly leads to long propagation delays. As an example, the typical propagation delay in acoustic underwater links can be on the order of several seconds. In contrast, radio propagation delay is typically on the order of milliseconds. The slow speed of sound in water also leads to significant Doppler shifts which in turn leads to high packet error rates. These two factors reduce the
efficiency of traditional automatic repeat request (ARQ) protocols which rely on timely feedback for their correct operation.

In this dissertation we explore packet coding as an alternative to traditional ARQ techniques for channels that experience long delays. The concept of network coding, also known as packet coded networks was introduced in [2]. In contrast to a regular network where nodes transmit packets as they are generated, the nodes in a packet coded network take a block of packets and combine them before transmission to maximize information flow. Throughput analysis [2] showed that employing network coding saves bandwidth compared to traditional networks. There are three main types of packet codes: random linear packet codes, Luby transform codes, and Raptor codes [3]. In this dissertation, we concentrate on the random linear packet codes for simplicity.

Packet coding has the potential to reduce the overall waiting delay, and since it is applied at the packet level (as opposed to bit-level coding), it can be easily implemented on top of any existing physical layer. Packet coding is also particularly well suited for multicast and broadcast scenarios, where it additionally increases the information throughput. In traditional broadcast networks, the receiving nodes lose packets independently and request re-transmission of specific packets from the transmitter individually. A re-transmitted packet thus only benefits the node that requested it. In packet-coded broadcast networks, the re-transmission of coded packets benefits all the requesting nodes at once.

A system employing packet coding buffers a block of \( M \) information-bearing packets at the transmitter and encodes them into a larger set of \( N \geq M \) coded packets to be transmitted [3]. At the receiver, the original information-bearing packets can be recovered from a subset of any \( M \) or more of the received packets. The long propagation delay of the underwater acoustic channel causes performance degradation because of the time spent sending feedbacks and acknowledgements (ACK). In a full-duplex link, the transmitter can ideally generate and send infinite coded packets until the receiver acknowledges that decoding was successful. However, most commercially available acoustic modems are half-duplex in nature. This necessitates a link layer design for underwater channel that does not rely on timely feedback for its operation. We propose a random linear packet coding
based protocol for underwater channels that determines the number of coded packets to transmit based on a pre-defined probability of successful decoding that is required at the receiver.

Typical underwater deployments have limited resources in terms of power and energy. Underwater systems are designed to last for long time on limited power because deployments are expensive and time consuming. Energy conservation becomes a priority to extend the life of underwater systems. In the interest of energy conservation, we address the issue of joint power and rate control for an underwater acoustic channel employing random linear packet coding. We choose the average energy per bit as a measure of performance. We use a block fading channel model in which the channel gain is decomposed into two parts: the large-scale slowly-varying part which admits feedback, and the small-scale fast-varying part which does not admit feedback and determines the bit error rate performance. A feedback link is used to convey the large-scale channel gain from the receiver to the transmitter, and adjust the transmit power and the number of coded packets.

The technique proposed earlier only guarantees a pre-defined reliability at the receiver. In order to achieve full reliability, we propose to combine random linear packet coding with traditional stop-and-wait ARQ technique. We show that such a combination is simple to implement and yet achieves throughput efficiency that is comparable to a full-duplex link and an optimal half-duplex link. We provide numerical results showing throughput comparison for a point-to-point link as well as a broadcast network.

1.1 Organization of the Dissertation

The dissertation is organized as follows:

- In Chapter 2, we present the analysis of random linear packet coding for a time-invariant channel, where the number of coded packets to transmit is determined so as to maintain a pre-defined probability of successful decoding at the receiver. Using numerical analysis and experimentally recorded signals, we show that a small redundancy is sufficient to maintain the pre-defined reliability at the receiver.
• In Chapter 3, we address the issue of joint power and rate control for an underwater acoustic channel employing random linear packet coding. We show numerical and experimental results for the available energy savings using joint power and rate control. We further extend the results to a case where the channel is not accurately known and an estimate of the channel state information is used. We also compare the proposed packet coding technique with the traditional stop-and-wait ARQ techniques in terms of the throughput efficiency. We show that the proposed technique has a much better throughput efficiency when compared to other techniques for both a point-to-point link as well as a broadcast network.

• In Chapter 4, we present a grouped packet coding technique to achieve full reliability using random linear packet coding. We present the analysis to combine the grouped packet coding technique with joint power and rate control. We compare the grouped packet coding technique with a full-duplex link and an optimal half duplex link to show that the proposed technique has a throughput efficiency that is comparable to the other techniques at lower complexity. We provide numerical results for a point-to-point link as well as a broadcast network.

• We summarize our conclusions in Chapter 5.

1.2 Publications

Following is a list of publications that emerged as a part of this dissertation.

1.2.1 Journals


• R. Ahmed and M. Stojanovic, “Reliable communication using packet coding over fading channels with long delays”, to be submitted.

### 1.2.2 Conference Publications


### 1.2.3 Invited Presentations

Chapter 2

Random Linear Packet Coding without Feedback

2.1 Introduction

The concept of network coding, also known as packet coded networks was introduced in [2]. In contrast to a regular network where nodes transmit packets as they are generated, the nodes in a packet coded network take a block of packets and combine them before transmission to maximize information flow. Throughput analysis [2] showed that employing network coding saves bandwidth compared to traditional networks. The work in [4] provided an algebraic framework for studying the coded networks. Channel with no erasure was assumed, and hence feedback was not necessary. The work in [5] showed that network nodes that generate random linear mappings from input links to output links achieve capacity in a non-erasure channel.

For networks with channel erasures, an approach based on either rateless coding [6] or block transmission has been used [7, 8]. In the rateless coding approach, a transmitting node sends packets until the receiving node sends an acknowledgment that the packets have been successfully decoded. This approach relies on a feedback link to perform successful decoding. Both network coding and standard Automatic Repeat reQuest (ARQ) schemes achieve reliability by detecting errors in the received packets. However, random linear
packet coding provides performance advantages over the simple ARQ.

Random linear packet coding for time-division duplex systems where each end of the link can either transmit or receive, but not both at the same time, was addressed in [9] and [10]. It was shown that there exists an optimal number of coded packets that need to be sent back-to-back before waiting for an acknowledgment (ACK). The optimization criterion used in [9] was minimization of the mean time required to transmit a block of packets, while [10] focused on minimizing the mean energy consumption.

In this chapter, we focus on an acoustic scenario in which no feedback is available. We compute the number of coded packets $N_M$ that need to be transmitted in order for a block of $M$ original packets to be decoded with pre-specified reliability. This number is a function of the packet erasure rate $P_E$, the number of original packets $M$, and a target probability of successful decoding $P_s^*$. In determining the number of coded packets necessary to provide the $M$ degrees of freedom, i.e. linearly independent combinations of the original data packets, that the receiver requires for successful decoding, there is a natural trade-off in the choice of redundancy. On the one hand, if redundancy is too high, the system is sending more packets than necessary, thus causing unnecessary delays. On the other hand, if it is too small, there are insufficiently many coded packets at the receiver to provide the $M$ degrees of freedom needed for successful decoding. We assess this trade-off analytically, as well as experimentally.

For a given number of original packets $M$, we send a pre-specified number of coded packets, and use the recorded signals to measure the number of packets needed for successful detection. We compare the so-obtained experimental values with the pre-specified design value $N_M$ corresponding to a pre-defined reliability. We find that such design offers good efficiency, i.e. that $N_M$ is not much greater than the average number of packets needed for successful detection.

The lack of widely accepted statistical channel models for underwater acoustic communication leads to unreliable results produced by simulation, while actual underwater experiments are costly and time consuming. To bridge this gap between unreliable simulation and expensive real time experiments, we test the packet coding algorithm in an in-air
testbed. Such a testbed also provides proof of concept for actual underwater experiments.

Packet coding algorithm using in-air experiments show that a small redundancy suffices to establish a reliable link. Our results are further verified with signals transmitted during the Kauai Acomms MURI (KAM’11) experiment, which included transmissions at 3km over a 25 kHz-35 kHz acoustic bandwidth. This fact indicates that random packet coding is a promising tool for increasing the reliability of acoustic links where acknowledgment feedback is not an option, or where excessive propagation delays limit its usefulness.

The rest of the chapter is organized as follows. In Section 2.2 we describe the transmitter and receiver design. In Section 2.3 we present the analysis for the optimal number of packets that need to be transmitted in order to ensure successful decoding. In Section 2.4 we discuss the set-up of the in-air testbed, and describe the KAM’11 experiment. In Section 2.5 we present the results from the in-air and KAM’11 experiments. Conclusions are summarized in Section 2.6.

2.2 System Description

2.2.1 Random Linear Packet Coding

The transmitter buffers a block of $M$ packets and stores each packet in a new row of the matrix $P$. The symbols in each packet, and the random coding coefficients are elements of a finite field of size $2^q$. The coded packets are now generated as

$$\mathbf{cp}(i) = \mathbf{a}(i) \otimes P; \quad i = 1,...,L$$

(2.1)

where $\mathbf{cp}(i)$ is the $i^{th}$ coded packet and $\mathbf{a}(i)$ is a vector of size $1 \times M$ containing the random coefficients that are used to generate that coded packet. All operations are performed in a field of size $2^q$. The random coefficients $\mathbf{a}(i)$ are appended to the end of the coded packet. Each coding coefficient is represented by $q$ bits.

The coding process is shown in Fig. 2.1, while Fig. 2.2 shows the structure of the coded packet. Each coded packet is of length $h + q(N_b + M)$, where $N_b$ is the number of symbols
in each original packet and \( h \) is the number of header bits in each packet. In addition, channel coding is applied to enable forward error correction.

![Diagram of coding algorithm](image)

**Figure 2.1: Coding algorithm.**

![Diagram of packet structure](image)

**Figure 2.2: Packet structure.**

### 2.2.2 Receiver Design

The packet decoding algorithm now receives packets and checks each for errors. If errors are found, i.e. a packet does not pass the cyclic redundancy check, the packet is discarded. Otherwise, the packet is forwarded to the decoder. The decoder removes the random coding
coefficients from each packet and stores them in a matrix \( A \). The remaining portion of the packet is stored in a matrix \( Q \). For every newly stored packet, the decoder checks the rank of the matrix \( A \). When \( \text{rank}(A) = M \), i.e., there are \( M \) linearly independent combinations of the original packets, the decoder solves for the original packets as

\[
P = A^{-1} \otimes Q
\]  

(2.2)

where all operations are carried out in the field of size \( 2^q \). Fig. 2.3 shows the block diagram of the decoding algorithm.

We are interested in finding the number of packets \( N_M \), that need to be transmitted in order for the \( M \) original packets to be detected with a desired reliability.

![Figure 2.3: Decoding algorithm.](image-url)
2.2.3 Example of Packet Coding

In this section we will demonstrate random linear packet coding with a small example. We will assume that there are not header bits, a block size of $M = 2$, and that each packet has $N_b = 9$ bits. We will assume that coding is done over a Galois field of size $2^3$.

The two packets are

$$p_1 = [111011110]$$

$$p_2 = [001011111]$$

We would like to perform encoding and decoding of the packets in $GF(2^3)$. Hence, we group three bits together and use the corresponding representation in $GF(2^3)$ as

$$p_1 = [7 \ 3 \ 6]$$

$$p_2 = [1 \ 3 \ 7]$$

Forming the matrix $P$ as

$$P = \begin{bmatrix} 7 & 3 & 6 \\ 1 & 3 & 7 \end{bmatrix}$$

We would like to note that all the operations that follow are performed in a Galois field of size 8. In order to perform the multiplication and addition of elements, we use the primitive polynomial $D^3 + D + 1$.

The first coded packet is generated by generating the random coefficients $a(1)$.

$$a(1) = [6 \ 7]$$
The first coded packet is generate as \( cp(1) = a(1) \otimes P \), i.e.,

\[
\begin{bmatrix}
3 & 3 & 1 & 6 & 7
\end{bmatrix}
\]

Note that the coefficients \( a(1) \) are appended to the end of the packet. The second coded packet is generated similarly using \( a(2) = [5 \ 0] \) as

\[
\begin{bmatrix}
6 & 4 & 3 & 5 & 0
\end{bmatrix}
\]

At the receiver, assuming both packets are correctly received, the received bits are stored in \( Q \) as

\[
Q = \begin{bmatrix}
3 & 3 & 1 \\
6 & 4 & 3
\end{bmatrix}
\]

The coefficients are stored in \( A \) as

\[
A = \begin{bmatrix}
6 & 7 \\
5 & 0
\end{bmatrix}
\]

We see that the rank(\( A \)) = 2 and hence we perform the operation \( P = A^{-1} \otimes Q \) to recover the original packets, i.e.,

\[
\begin{bmatrix}
6 & 7 \\
5 & 0
\end{bmatrix}^{-1} \otimes \begin{bmatrix}
3 & 3 & 1 \\
6 & 4 & 3
\end{bmatrix} = \begin{bmatrix}
7 & 3 & 6 \\
1 & 3 & 7
\end{bmatrix}
\]

### 2.3 Optimal Number of Coded Packets

In a system with systematic coding, the transmitter first sends the original \( M \) packets, and then follows with the coded packets. The receiver, upon observing the first \( M \) packets, tries to decode them. It will succeed in doing so if all the \( M \) packets were received correctly. The probability of successful decoding on this first attempt is \((1 - P_E)^M\) where \( P_E \) denotes the packet erasure (error) rate. If \( i \) packets are received in error, the receiver then looks at
the next batch of $i$ packets to complete the $i$ missing degrees of freedom. This process can be described by a Markov chain, where state $i$ denotes the state in which the receiver needs $i$ more degrees of freedom ($i$ more correct packets assuming that each brings an additional degree of freedom). Upon receiving the $i$ packets, the receiver will transition into a next state which can be any of the states $i-1, i-2, \ldots, 0$, depending on how many of the $i$ packets are received without error. The receiver completes detection upon reaching state 0. The probability of going from state $i$ to state $i-j$ is the probability of receiving $j$ packets correctly out of the $i$ packets, which is given by

$$P_{i \rightarrow i-j} = \binom{i}{j} (1-P_E)^j P_E^{i-j} \quad (2.3)$$

Let $L_i$ denote the number of packets that still need to be transmitted to complete detection when in state $i$. The average number of packets needed for completion will then be given by

$$\overline{L}_i = \sum_{j=0}^{i} P_{i \rightarrow i-j} (i + \overline{L}_{i-j}) \quad (2.4)$$

From this recursion, we can compute $\overline{L}_M$, the average number of packets that need to be transmitted in order to successfully decode the $M$ original packets.

For a system that has no feedback, the number of transmitted packets needs to be set a-priori. We determine this number so as to achieve pre-specified level of reliability, i.e. we require that probability of successful decoding of the $M$ original packets to be at least $P_s^*$. For a given $P_E$, if $N$ packets are transmitted, the probability that $m$ packets are received correctly is given by

$$P_m = \binom{N}{m} (1-P_E)^m P_E^{N-m} \quad (2.5)$$

The probability of $M$ packets being received correctly when $N$ packets are sent is given by

$$P_s(N) = \sum_{m=M}^{N} P_m \quad (2.6)$$
To ensure a pre-determined level of reliability $P_s^*$, we need

$$P_s(N) \geq P_s^*$$

The smallest $N$ for which this inequality holds is the minimum number of packets that need to be transmitted in order for the receiver to detect the $M$ original packets with probability $P_s^*$. We denote this number by $N_M$, and note that it depends on the packet error rate $P_E$, desired probability of success $P_s^*$, and the number of the original packets $M$. Fig. 2.5 shows the values of $N_M$ and the corresponding redundancy, computed as $R_M = (N_M - M)/M$. Note that at a reasonable $P_E$, say $10^{-2}$ or less, $N_M$ is not far from $L_M$ which in turn is very close to $M$. In other words, the redundancy is low when $P_E$ is not too high.

Figure 2.4: $N_M$ and $L_M$ as a function of $M$. 
2.3. Optimal Number of Coded Packets

Figure 2.5: Redundancy corresponding to $N_M$ as a function of $M$.

Fig. 2.6 shows the dependence of $N_M$ on the packet erasure rate $P_E$.

Figure 2.6: $N_M$ and $\bar{L}_M$ vs $P_E$.

To assess the actual distribution of $L_M$, computer simulation was performed. Simula-
tion consists of a finite number of trials $N_e$ in each of which the transmitter sends packets to the receiver until the receiver can successfully decode the $M$ original packets. Each trial produces a realization of the random variable $L_M$ which are collected to form a histogram, i.e. an estimate of the probability $P\{L_M = l_m\}$. Fig. 2.7 shows the results of simulation for different values of $M$ at $P_E = 10^{-2}$. We note that the minimum observed value of $L_M$ and the ensemble average are very close to $\bar{L}_M$. Also, the maximum observed value of $L_M$ is relatively close to $\bar{L}_M$. Hence, we conjecture that the deviation of $L_M$ around $\bar{L}_M$ is not large at this value of $P_E$. This fact in turn implies that our design parameter $N_M$ could indeed be a good measure of the number of packets that need to be transmitted in a system with no feedback.

![Figure 2.7: Simulation results.](image-url)


2.4 Experimental Set-up

2.4.1 In-air Experiments

As stated earlier, the lack of widely accepted statistical channel models for underwater acoustic communication makes simulation results unreliable. An intermediate solution is proposed by developing an in-air acoustic testbed to perform proof-of-concept of various underwater communication technologies.

The in-air testbed was developed using the Edirol FA-101 board. Edirol FA-101, a feature-packed 10×10 audio interface which can handle 10 inputs/outputs at 24-bit/96kHz, is used as the data acquisition interface. The Edirol FA-101 sound card connects to the computer using a high speed IEEE 1394 port. Once correctly set, this device can be made to act as the primary sound card for a computer which makes it easier to use MATLAB for signal generation and processing. The transmitter is a multimedia speaker connected to the Edirol FA-101 board using a stereo to mono 1/8th to 1/4th inch connector. The

Figure 2.8: Histograms. Each histogram corresponds to one value of M.
receiver is a capacitive microphone connected to the XLR port of the Edirol FA-101 sound card. Edirol FA-101 powers the microphone with a 48V phantom power supply through the XLR port. The transmitter and receiver block diagrams of the in-air testbed are shown in Fig. 2.9.

![Transmitter block diagram.](image1)

(a) Transmitter block diagram.

![Receiver block diagram.](image2)

(b) Receiver block diagram.

Figure 2.9: In-air testbed.

### 2.4.2 KAM’11 Experiment

The Kauai Acomms MURI (KAM’11) experiment was conducted in June 2011 off the coast of Kauai, HI in the Pacific Missile Range Facility (PMRF). The transmitter was an 8-element vertical source consisting of ITC-1001 transducers with a 7.5m inter-element separation. The receiver was an 16-element vertical array with a 3.75m inter-element spacing. The operational area of KAM’11 is shown in Fig. 2.10. The transmitter was located at Station 01 denoted by MPL-SRA1 in the legend and the receiver was located at Station 07 denoted by MPL-VLA1 in the legend. The distance between the transmitter and the receiver was 3km. The exact location of the transmitter and receiver used are given in Table 2.1.
2.4. Experimental Set-up

Figure 2.10: Operational area of KAM’11.

Table 2.1: Experimental details of the KAM’11 experiment.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>Kauai, HI</td>
</tr>
<tr>
<td>Tx Location</td>
<td>22 7.583° N 159 48.532° W</td>
</tr>
<tr>
<td>Tx Depth</td>
<td>100m</td>
</tr>
<tr>
<td>Rx Location</td>
<td>22 9.069° N 159 47.940° W</td>
</tr>
<tr>
<td>Rx Depth</td>
<td>106m</td>
</tr>
<tr>
<td>Day</td>
<td>29 June 2011</td>
</tr>
</tbody>
</table>

2.4.3 System Parameters

Two designs were used for random linear packet coding, one with $M = 7$ and another with $M = 20$ original packets in the KAM’11 experiment. The parameters used for the two designs are detailed in Table 2.2. For each design, the number of coded packets was set a-priori for the experiments.
Table 2.2: Parameters used in the two designs of packet coding.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Design1</th>
<th>Design2</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of symbols in a packet</td>
<td>110</td>
<td>113</td>
</tr>
<tr>
<td>No of original packet $M$</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>No of symbols in coded packet</td>
<td>130</td>
<td>120</td>
</tr>
<tr>
<td>No of coded packets</td>
<td>40</td>
<td>42</td>
</tr>
</tbody>
</table>

2.5 Results

In real-time experiments, the transmitter cannot send coded packets continuously till the receiver can successfully decode the $M$ original packets as there is no instantaneous feedback. To estimate the distribution of $L_M$ in such a scenario, the transmitter sends a certain pre-determined number of coded packets $N_p$. In such an experiment, we call the number of packets needed for successful decoding $K_M$, to acknowledge the fact that we do not transmit continuously until the receiver can successfully decode $M$ original packets. When $N_p$ is large enough that there are no failures to decode, we may think of $K_M$ as a good approximation for $L_M$. We denote the ensemble average of $K_M$ by $\bar{K}_M$. We are interested in comparing this number with the design value $N_M$ computed according to (2.6) with $P_s^* = 0.999$. Reducing $N_p$ to $N_M$ will yield in an actual implementation of a practical system with no feedback.

2.5.1 In-air Testing

Using the in-air testbed gives the opportunity to test the packet coding algorithm for different probability of packet erasures. An experiment was conducted where the transmission power and distance between the transmitter and receiver were adjusted to produce different packet erasure rates (which were measured).

Fig. 2.11 summarizes the experimental results. The plots show the values of $\bar{K}_M$ measured in $N_e = 10,000$ trials. Shown also are the analytical values $\bar{L}_M$ and $N_M$ obtained from (2.4) and (2.6). We note that the ensemble average closely follows $\bar{L}_M$, but more importantly, that the design value $N_M$ calculated for $P_s^* = 0.999$ is not much greater than the recorded values, and that the design value $N_M$ calculated for $P_s^* = 0.9999$ is completely
satisfactory. The implication for a practical system in which there is no feedback is that it can be safely designed based on the concept of pre-specified reliability, i.e. that doing so will not incur an undue penalty in redundancy.

Figure 2.11: Measured average number of packets $\overline{K}_M$, analytical value of $N_M$ for $P^*_s = 0.99$, and analytical average value $\overline{L}_M$ are plotted for different $P_E$ corresponding to $M = 20$.

Fig. 2.12 shows the histogram of $K_M$ corresponding to $M = 20$ ($N_p = 70$). Simulation results are shown alongside for comparison.
2.5. Results

In-Air Experiment.

Simulation experiment.

Estimate of $P(K_M = M + \Delta M)$

Fig. 2.12: Histogram of $K_M$ for $M = 20$ and two different values of $P_E$.

Fig. 2.13 shows the histogram corresponding to $M = 30$ at $P_E = 0.03$. In both cases, experimentally obtained $K_M$ follows the same trend as $L_M$ generated by simulation.
2.5. Results

Figure 2.13: Histogram of $K_M$ for $M = 30$.

2.5.2 KAM’11

In the KAM’11 experiment, pre-packaged signals were transmitted. After demodulation and detection, the total number of packets $K_M$ that provided the $M$ degrees of freedom was measured. Table 2.3 shows the results for design 1. The packet erasure probability was estimated to be $5 \times 10^{-2}$. The corresponding value of $N_M$ for a $P_* = 0.9999$ and $M = 20$ is found to be 28, while the corresponding $\bar{L}_M$ is 22.

Table 2.3: Experimental results from the KAM’11 experiment employing Design 1.

<table>
<thead>
<tr>
<th>Day</th>
<th>180</th>
<th>180</th>
<th>180</th>
<th>180</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time(Hrs)</td>
<td>06:18</td>
<td>08:18</td>
<td>10:18</td>
<td>12:18</td>
<td>14:18</td>
</tr>
<tr>
<td>$K_M$</td>
<td>22</td>
<td>20</td>
<td>22</td>
<td>24</td>
<td>21</td>
</tr>
</tbody>
</table>

Similarly for design 2 with $M = 7$, for which the results are shown in Table 2.4, the
average packet erasure probability was estimated to be $P_E = 1 \times 10^{-2}$. The corresponding value of $N_M$ is found to be 12.

Table 2.4: Experimental results from the KAM’11 experiment employing Design 1.

<table>
<thead>
<tr>
<th>Day</th>
<th>180</th>
<th>180</th>
<th>180</th>
<th>180</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time(Hrs)</td>
<td>06 : 19</td>
<td>08 : 19</td>
<td>10 : 19</td>
<td>12 : 19</td>
<td>14 : 19</td>
</tr>
<tr>
<td>$K_M$</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>

2.6 Conclusions

This work provides a preliminary experimental analysis of random linear packet coding for an underwater channel. Because of the limited availability of feedback, a packet coding algorithm that generates packets according to a predefined redundancy was analyzed. The goal was to theoretically calculate, and verify experimentally, the number of packets that need to be transmitted in order for the receiver to detect a block of $M$ original data packets with pre-specified probability. This number depends on the number of original packets that are being sent, as well as on the packet erasure rate of the channel.

An in-air testbed was used to emulate a shallow underwater channel and test the performance of the packet coding algorithm. The in-air testbed provided a flexible platform for adjusting the transmitted power, thus allowing us to experiment with different values of the packet erasure rate. Packet coding was also tested in real underwater scenarios during the KAM’11 experiment. The experimentally recorded number of packets needed for successful decoding was found to closely follow the design value, thus supporting the proposed method for determining a-priori the number of coded packets that need to be transmitted in a system with no feedback. The attendant redundancy is very low (for example, 10% for 20 original packets at a packet erasure rate of 0.01 at $P_S^* = 0.999$). Future work will concentrate on using adaptive power and rate control to enable optimal performance on channels with time-varying packet erasure rate.
Chapter 3

Joint Power and Rate Control

3.1 Introduction

In this chapter, we explore joint power and rate control for an underwater acoustic link employing random linear packet coding. A system employing packet coding buffers a block of \( M \) information-bearing packets at the transmitter and encodes them into a larger set of \( N \geq M \) coded packets to be transmitted [3]. At the receiver, the original information-bearing packets can be recovered from a subset of any \( M \) or more of the received packets. The concept of packet coded networks was first introduced in [2] where it was shown that employing packet coding improves the overall throughput efficiency as compared to traditional networks.

Random linear packet coding for underwater acoustic communication has been studied in [31–35]. Rateless coding for reliable file transfer in underwater acoustic networks was considered in [31] where a feedback link was used to inform the transmitter when to stop sending coded packets. Since the feedback was used less frequently as compared to traditional ARQ techniques, the overall system performance was shown to improve. Optimal broadcasting policy for underwater acoustic networks based on random linear packet coding was investigated in [32], showing performance improvements over traditional ARQ techniques. Optimal schedules for random linear packet coding in half duplex links were investigated in [33], which provided the optimal number of coded packets to minimize the
average time (or energy) needed to complete the transmission of a block of information-bearing packets. Random linear packet coding in the absence of a feedback link was introduced in [34] where the number of coded packets to transmit was determined such that the receiver can decode the original packets with a pre-specified reliability.

The work referenced so far addressed a time-invariant channel with a fixed packet erasure rate. Random linear packet coding for a fading channel with time-varying link conditions was considered in [35], where adaptive power/rate control was explored to overcome the effects of channel fading. In [36], standard optimization techniques were used to determine the trade-off between allocation of redundancy to packet-level erasure coding and physical layer channel coding. In [37], an analysis based on differential equations was utilized to analyze the throughput achievable with packet coding, and to design a dynamic power control algorithm that achieved higher multicast throughput. The trade-off between channel coding and ARQ in a Rayleigh block-fading channel was considered in [38]. Performance comparison was made between a heavily coded system which uses fewer re-transmissions and a lightly coded system with more re-transmissions, to show that a lightly coded system increases the system throughput. In [39], the authors employed network coding and adaptive power control to improve network performance in a broadcast cellular network. Cross-layer optimization of physical layer modulation and coding to maximize the system throughput for wireless fading channels was considered in [40]. For channels that experience severe fading, [41] considered the optimization of packet-level and bit-level coding, and concluded that performance is improved by adding more redundancy to erasure correction coding across packets. In [42], the problem of joint optimization of the mean throughput and packet loss rate in network-coded systems was considered. It was found that a feedback-free packet coding approach provided better performance in terms of the mean throughput and packet loss rate. In [43], the problem of joint power and rate control for a block-fading channel was addressed. The authors defined utility and cost function to specify an optimization framework that aims at maximizing the transmission rate while minimizing the power consumption.

In this chapter we address the issue of joint power and rate control for an under-
water acoustic channel employing random linear packet coding. In [35], we described a framework that combines adaptive power control with random linear packet coding. The average energy per bit was chosen as a measure of performance, since underwater systems have limited resources, and conserving energy aims at increasing the system lifetime. We have shown that when employing adaptive power control, there exists an optimal number of coded packets that minimizes the average energy per bit. Similarly, when employing adaptive rate control, there exists an optimal transmit power that minimizes the average energy per bit. In the present work, we aim to perform joint power and rate control with constrained resources. We use a block fading channel model in which the channel gain is decomposed into two parts: the large-scale slowly-varying part which admits feedback, and the small-scale fast-varying part which does not admit feedback and determines the bit error rate performance. A feedback link is used to convey the large-scale channel gain from the receiver to the transmitter, and adjust the transmit power and the number of coded packets. We define two constraints on the available resources: (a) the transmit power cannot exceed a maximum level, and (b) the number of coded packets cannot exceed a maximum value. Under these two constraints, we provide a framework to perform joint power and rate control which aims to minimize the average energy per successfully transmitted bit of information.

In order to implement the adaptation policy, the channel gain needs to be known at the transmitter. However, in practical systems, the channel gain is not known accurately and only its estimate is available at the transmitter. Using an estimate in place of the true value, we introduce a safety margin to develop adaptation policies that ensure a desired outage/reliability. Following [44], we model the large-scale channel as a log-normally distributed process whose dynamics obey a first-order auto regressive (AR-1) process. Assuming a minimum mean squared error (MMSE) estimate of the channel gain, we provide analytical expressions for system performance under channel uncertainty. Finally, we compare the proposed packet coding technique with traditional ARQ techniques and show performance improvements in terms of throughput efficiency. We also provide experimental results from the Surface Process Acoustic Communication Experi-
3.2 System Model

On a channel with large-scale gain $G$, the average signal power at the receiver is given by $P_R = GP_T$, where $P_T$ is the transmit power. We assume a block fading model where the channel gain remains constant over a block of packets, but may vary from one block to another. The duration over which the channel gain remains constant is referred to as the coherence time of the channel, and is represented by $T_c$. The large-scale channel gain is varying slowly, and hence its value can be sent via feedback to the transmitter. In each block, the transmitter buffers a block of $M$ packets and encodes them into $N \geq M$ packets for transmission over the channel. The transmitter employs random linear packet coding to generate the coded data packets. Each original information packet contains $N_b$ bits. A coded packet is generated as a linear combination of the $M$ original data packets. The random coding coefficients that are used to generate the coded packets are appended to the end of each coded packet. Each coding coefficient is represented by $q$ bits when the field over which encoding takes place is GF($2^q$) [3]. Thus, the total number of bits in each packet is $C_b = N_b + qM + h$, where $h$ represents any additional overhead, including the header and the bits for cyclic redundancy check (CRC). The structure of a coded packets is shown in Fig. 2.2.

Each transmitted packet contains $K_b$ bits, $C_b$ of which are from the coded packet and
$K_b - C_b$ are channel coding bits. The duration of each packet is $T_p = K_b / R_b$, where $R_b$ is the bit rate in the channel, and the effective information rate is $N_b / T_p$. After every block of packets, the transmitter waits for a feedback from the receiver which contains the channel gain information.

The signal-to-noise ratio (SNR) at the receiver is given by $\gamma = P_R / P_N = G P_T / P_N$, where $P_N$ is the noise power. The bit error rate (BER) is a function of the SNR and is represented by $P_e(\gamma)$. The corresponding packet error rate is determined by $P_e(\gamma)$, e.g. as $P_E(\gamma) = 1 - (1 - P_e(\gamma))^{C_b}$. Since the channel gain $G$ is randomly varying, so is the packet error rate $P_E(\gamma)$. We note that packet coding does not replace channel coding, but works along with it to improve the throughput efficiency.

The probability of successful decoding, defined as the probability that at least $M$ out of the $N$ coded packets are received correctly is given by

$$P_s(\gamma) = \sum_{m=M}^{N} \binom{N}{m}(1 - P_E(\gamma))^m P_E^{N-m}(\gamma)$$  

(3.1)

We wish to maintain a pre-defined success rate $P_s^*$ at the receiver. We can now define the outage probability as the probability that $P_s$ falls below the pre-defined value $P_s^*$, i.e.,

$$P_{out} = \mathbb{P}\{P_s(\gamma) < P_s^*\}$$  

(3.2)

We chose the average energy per bit as the figure or merit to determine the optimal transmit power and the number of coded packets. In many underwater applications, power is limited, and hence minimizing the average energy per bit aims at increasing the system lifetime. The average energy per successfully transmitted bit is given by

$$\bar{E}_b = \frac{1}{R_b} \frac{\mathbb{E}\{NP_T\}}{P_s^* M} \frac{C_b}{N_b}$$  

(3.3)

The average energy per bit is influenced by both the number of coded packets and the
transmit power. On the one hand, increasing the transmit power leads to a higher SNR, and hence fewer coded packets would be necessary, but on the other hand, increasing the transmit power also directly increases the average energy per bit. It is this trade-off that we wish to exploit by finding the optimal values for the transmit power and the number of coded packets.

In a practical system, resources are limited. Keeping this fact in mind, we impose two constraints:

1. The transmit power cannot exceed a maximal level, $P_{T,max}$. This level is dictated by the hardware system constraints or by the total budget.

2. The number of coded packets cannot exceed a maximal value, $N_{max}$. This value is determined so as to satisfy the following requirements: (i) a block must not last longer than a value dictated by the coherence of the channel gain $T_c$; (ii) the decoding delay $T_d$ must not exceed a maximum tolerable value $T_{d,max}$; and (iii) the average bit rate must not fall below a tolerable minimum $R_{b,min}$. $N_{max}$ is thus given as

$$N_{max} = \min\left\{ \frac{T_c}{T_p}, \frac{T_{d,max}}{T_p}, \frac{N_b P_s^* M}{T_p R_{b,min}} \right\}$$

(3.4)

3.3 Optimization procedure

In this section, we define the optimization procedure to determine the transmit power and the number of coded packets so as to maintain a pre-defined reliability while minimizing the average energy per successfully transmitted bit of information. For different values of $N$ starting from $M$ and increasing to $M+1, M+2 \ldots$, Fig. 3.1 shows the relationship between $P_s$ and $P_E$. As seen in the figure, for a desired $P_s^*$, and for every candidate value $N$, there is a corresponding value $P_E^*$. Given a specific small-scale fading type and a modulation/coding/diversity scheme, the value $P_E^*$ corresponds to a particular value of the SNR, which we denote as $\gamma^*(N)$. 
Figure 3.1: Probability of successful decoding vs. probability of packet error. For a desired $P_s^*$ and a chosen value of $N$, the figure points to a value of $P_E^*$ which in turn points to a necessary SNR $\gamma^*(N)$.

The relationship between the number of coded packets $N$ and the corresponding $\gamma^*(N)$ is summarized in Fig. 3.2. Also plotted in Fig. 3.2 is the product $N\gamma^*(N)$ which is meaningful for minimizing the average energy per bit. For purposes of illustration, we assume Rician fading, and differentially coherent detection with no coding or diversity. Note that our analysis does not change with the change in any of these assumptions; only numerical results do.
3.3. Optimization procedure

Figure 3.2: The SNR $\gamma^*(N)$ and the product $N\gamma^*(N)$ as functions of $N$. The product $N\gamma^*(N)$ is relevant for minimizing the average energy per bit.

For a given channel gain $G$, the transmit power needed to achieve $\gamma^*(N)$ is

$$P_T^* = \gamma^*(N)P_N/G$$

(3.5)

where $P_N$ is the noise power. Fig. 3.3 shows the product $NP_T^*$ as a function of $N$ for various values of the gain $G$. It is important to note that the value of $N$ which minimizes $NP_T^*$ is the same as that which minimizes $N\gamma^*(N)$. Hence, this value does not depend on the channel gain $G$. We denote this value by $N_{opt}$ as it is the value that minimizes the average energy per bit $\bar{E}_b$ in the absence of any constraints.

In order to accommodate the constraints stated earlier, the optimization procedure is
3.3. Optimization procedure

Figure 3.3: To minimize the average energy per bit, the number of coded packets and the transmit power should be chosen such that the product $NP_T^*$ is minimized (subject to system constraints $P_{T,\text{max}}, N_{\text{max}}$).

Conducted as follows:

1. If the optimal number of coded packets is less than the maximum number of coded packets allowed, i.e., $N_{\text{opt}} \leq N_{\text{max}}$, we choose to transmit $N = N_{\text{opt}}$ coded packets.
   
   If $N_{\text{opt}} > N_{\text{max}}$ we choose $N = N_{\text{max}}$.
2. For a given $G$, we calculate the threshold $\tilde{P}_T = \gamma^*(N)P_N/G$ using the value of $N$ chosen above. If this threshold is below the maximum system power $P_{T,\text{max}}$, we set the transmit power to $P_T = \tilde{P}_T$. Otherwise (i.e. if $\tilde{P}_T > P_{T,\text{max}}$), we have to make a choice based on $N$ that was used to calculate $\tilde{P}_T$, i.e., $N = N_{\text{max}}$ or $N = N_{\text{opt}}$. If $N = N_{\text{max}}$, we have exhausted the system resources, i.e., there is no combination of $N$ and $P_T$ that will satisfy $P_s \geq P_s^*$, and hence we shut the transmission off. If $N = N_{\text{opt}}$, we may increase $N$ above $N_{\text{opt}}$ (but not above $N_{\text{max}}$) and see if there exists a value of $N$ for which $\tilde{P}_T$ drops to $P_{T,\text{max}}$. If such a point exists, we choose that combination; otherwise, we shut off.

From the above discussion, it is clear that two cases exist for the system operation. The first case corresponds to the maximum number of coded packets being less than or equal to the optimal number of coded packets, $N_{\text{max}} \leq N_{\text{opt}}$. The second case occurs when $N_{\text{max}} > N_{\text{opt}}$. Below, we analyze each of these cases in detail.

### 3.3.1 Case 1: $N_{\text{max}} \leq N_{\text{opt}}$

When $N_{\text{max}} \leq N_{\text{opt}}$, the optimization procedure is straightforward. The number of packets is kept fixed at $N = N_{\text{max}}$, and the power is varied as $P_T = \gamma^*(N_{\text{max}})P_N/G$ if this value is below $P_{T,\text{max}}$; otherwise, transmission is turned off, i.e.,

$$P_T = \begin{cases} 
\gamma^*(N_{\text{max}})P_N/G, & \text{if } \gamma^*(N_{\text{max}})P_N/G \leq P_{T,\text{max}} \\
0, & \text{otherwise}
\end{cases} \quad (3.6)$$

Outage occurs when $P_T G/P_N < \gamma^*(N)$, i.e. when $G < \gamma^*(N_{\text{max}})P_N/P_{T,\text{max}}$ in this case. Under the assumption that the large-scale channel gain can be modeled as log-normally distributed, i.e., $10 \log_{10} G \sim \mathcal{N}(\bar{g}, \sigma_g^2)$, the probability of outage is

$$P_{\text{out}} = \mathbb{P}\{G < \gamma^*(N_{\text{max}})P_N/P_{T,\text{max}}\} = Q\left(\frac{\bar{g} - 10 \log_{10} G_{\text{out}}}{\sigma_g}\right) \quad (3.7)$$

where $Q$ denotes the Q-function, $Q(x) = 1/2\operatorname{erfc}(x/\sqrt{2})$. There are two ways in which the
system design can be specified. One can choose to specify the maximum transmit power $P_{T,\text{max}}$, in which case the outage probability follows from the above expression, or by a desired $P_{\text{out}}$, in which case the corresponding value of $P_{T,\text{max}}$ can be obtained from the above expression.

The average energy per bit is

$$\bar{E}_b = \frac{1}{R_b} \frac{N_{\text{max}} P_T C_b}{P_s^* M N_b}$$

(3.8)

The average power $\bar{P}_T$ can be computed as

$$\bar{P}_T = \gamma^*(N_{\text{max}}) P_N \int_{G_{\text{out}}}^{+\infty} \frac{1}{x} p_G(x) dx$$

(3.9)

where $p_G(\cdot)$ represents the probability density function (p.d.f.) of the channel gain $G$. The factor $F_G(G_{\text{out}})$ can be computed in closed form (see Appendix A) as

$$F_G(G_{\text{out}}) = 10^{-\tilde{g}/10} e^{\left(\frac{\ln 10}{10}\right)^2 \sigma_g^2/2} Q\left(\frac{\ln 10}{10} \sigma_g - Q_{\text{inv}}(P_{\text{out}})\right)$$

(3.10)

In order to test the performance of the adaptation scheme, we compare it with a system that operates at a fixed power and rate. If there is no adaptive control, transmit power is kept fixed at some $P_{T,\text{fix}}$, and the number of packets is also fixed at $N_{\text{fix}}$. The SNR, $\gamma = P_{T,\text{fix}} G / P_N$, changes with the gain, and so do the probabilities $P_E$ and $P_s$. Outage occurs when $\gamma < \gamma^*(N_{\text{fix}})$, and the probability of outage for a log-normally distributed $G$ is

$$P_{\text{out}} = \mathbb{P}\{\gamma < \gamma^*(N_{\text{fix}})\} = \mathbb{P}\{G < \gamma^*(N_{\text{fix}}) P_N / P_{T,\text{fix}}\} = Q\left(\tilde{g} - 10 \log_{10} G_{\text{fix}} / \sigma_g\right)$$

(3.11)

The power needed to keep the outage at a pre-specified level $P_{\text{out}}$ is thus $P_{T,\text{fix}} = \gamma^*(N_{\text{fix}}) P_N / G_{\text{fix}}$, where $10 \log_{10} G_{\text{fix}} = \tilde{g} - \sigma_g Q^{-1}(P_{\text{out}})$. Compared to a non-fading case ($\sigma_g = 0$), the trans-
mit power is increased by a fixed margin $\sigma_g Q^{-1}(P_{out})$. For example, if $\sigma_g=10$ dB and $P_{out} = 10\%$, the margin is about 13 dB (for 1% outage, it is 23 dB).

![Figure 3.4: Average energy per bit as a function of the block size $M$ for Case 1 ($N_{\text{max}} \leq N_{\text{opt}}$). The slight increase in the average energy per bit at larger block sizes is due to the packet coding overhead. It should be noted that the block duration must be kept below the coherence time of the channel.](image-url)
3.3. Optimization procedure

Figure 3.5: Average energy per bit as a function of the number of information bits per packet $N_b$ for Case 1 ($N_{\text{max}} \leq N_{\text{opt}}$). At very small packet sizes, energy consumption is dominated by the packet coding overhead. Larger packet sizes are favorable as they lead to higher average throughput.
3.3. Optimization procedure

Using adaptive power and rate control
Using fixed power and rate

Figure 3.6: Average energy per bit as a function of the standard deviation $\sigma_g$ for Case 1 ($N_{\text{max}} \leq N_{\text{opt}}$).

For Case 1, the number of coded packets for the fixed scheme is $N_{\text{fix}} = N_{\text{max}}$. Fig. 3.4 shows the average energy per bit as a function of the block size $M$. As seen from the figure, larger block size leads to a higher average energy per bit consumption because of the additional packet coding overhead. One has to keep in mind that the block size cannot become so large that it violates the block fading model. The value of the block size $M$ is thus a design choice made based on the coherence time of the channel. We can see that savings of approximately 6 dB-8 dB are available by employing adaptive power and rate control. Fig. 3.5 shows the average energy per bit as a function of the packet size (number of information bits) $N_b$. The packet coding overhead at smaller packet sizes is high, leading to a higher energy per bit consumption. Larger packet sizes are favorable as they lead to a higher average throughput as well as lower average energy per bit. Fig. 3.6 shows the
average energy per bit as a function of the standard deviation $\sigma_g$. The savings available from adaptive power and rate control increase with $\sigma_g$. Fig. 3.7 shows the average transmit power as a function of $P_s^*$ and Fig. 3.8 shows the average transmit power as a function of the outage probability $P_{out}$. We also note that the value of $\bar{g}$ does not change the results but merely shifts the curves as the change in $\bar{g}$ only shifts $P_T$ by the same amount.

Figure 3.7: Transmit power as a function of the reliability $P_s^*$ for Case 1 ($N_{\text{max}} \leq N_{\text{opt}}$).
42 3.3. Optimization procedure

We also would like to evaluate the effect of channel coding on the average energy per bit. In addition to the packet coding across packets, we add a Hamming code for error correction in each packet. Fig. 3.9 shows the average energy per bit as a function of the code rate of the Hamming code. We can see that the average energy per bit increases with increasing bit level coding, indicating that it is beneficial to add more redundancy at the packet level as against bit level.
3.3. Optimization procedure

Figure 3.9: Energy per bit as a function of the code rate of channel coding for Case 1 ($N_{\text{max}} \leq N_{\text{opt}}$).

3.3.2 Case 2: $N_{\text{max}} > N_{\text{opt}}$

When $N_{\text{max}} > N_{\text{opt}}$, the adaptation policy is somewhat more involved. We now have three regions of operation, two in which the system is on, and one in which the transmission is shut off. When the system is on, there are further two modes of operation, one in which the number of packets is kept fixed at $N = N_{\text{opt}}$ and the power is varied, and another in which the number of coded packets $N$ is varied between $N_{\text{opt}}$ and $N_{\text{max}}$ the power is kept fixed at $P_T = P_{T,\text{max}}$.

If the gain is high enough such that $\gamma^*(N_{\text{opt}})P_N/G \leq P_{T,\text{max}}$, the number of packets is set to $N = N_{\text{opt}}$, and the power is varied as $P_T = \gamma^*(N_{\text{opt}})P_N/G$. As the gain diminishes,
the break point occurs at

$$G_{\text{break}} = \gamma^*(N_{\text{opt}})P_N/P_{T,\text{max}}$$  \hspace{1cm} (3.12)$$

If the gain drops below this value, but there exists a value \(N \leq N_{\text{max}}\) such that \(\gamma^*(N)P_N/G = P_{T,\text{max}}\), that value is chosen, and the power is kept at \(P_T = P_{T,\text{max}}\). Rate adaptation is thus performed as \(N = \gamma_{\text{inv}}^*(P_{T,\text{max}}G/P_N)\), where the \(\gamma_{\text{inv}}^*(\cdot)\) denotes the inverse function of \(\gamma^*(\cdot)\). This function follows from the relationship between \(N\) and \(\gamma^*\) shown in Fig. 3.2.

The outage point occurs at

$$G_{\text{out}} = \gamma^*(N_{\text{max}})P_N/P_{T,\text{max}}$$  \hspace{1cm} (3.13)$$

If the gain drops below this value, there is no solution for \((N, P_T)\) that satisfies the required \(P_s^*\), and transmission is shut off.

The probability of outage is again given by the expression (3.7), but now with \(N_{\text{max}} > N_{\text{opt}}\). If the system is designed based on a given \(P_{T,\text{max}}\), then this outage probability represents the best that can be achieved. Alternatively, if rate adjustment is not an option, one can settle for operating only with \(N_{\text{opt}}\), in which case the outage will be imposed whenever the gain drops below \(G_{\text{break}}\). In either case, the design can also be carried “backwards,” i.e. starting with a desired \(P_{\text{out}}\) and determining the necessary \(P_{T,\text{max}}\) from it.

The adaptation policy can be visualized in Fig. 3.10, by following the solid straight lines in the direction of increasing (or decreasing) gain. For instance, starting with a very high gain and going in the direction of decreasing gain, the system follows the vertical line labeled \(N_{\text{opt}}\), moving upwards until the point labeled \(G_{\text{break}}\) is reached. There, a 90° turn is taken to the right, and the system continues to follow the horizontal line labeled \(P_{T,\text{max}}\), until the point labeled \(G_{\text{out}}\) is reached. Thereafter, the system is shut off.
Figure 3.10: For a given channel gain $G$, each curve represents the points $(N, P_T)$ that satisfy the outage requirement. For $G < G_{\text{out}}$, there are no such points, and transmission is shut off. In the region between $G_{\text{out}}$ and $G_{\text{break}}$, the power is kept fixed at $P_{T,\text{max}}$ and the rate is chosen at the crossing point between the line $P_{T,\text{max}}$ and the given $G$. In the region $G > G_{\text{break}}$, the number of packets is kept fixed at $N_{\text{opt}}$, and the power is chosen at the crossing point between the line $N_{\text{opt}}$ and the given $G$.

The adaptation policy for the case $N_{\text{max}} > N_{\text{opt}}$ is thus summarized as follows:

$$N = \begin{cases} 
0 & \text{if } G < G_{\text{out}} \\
\gamma^*_\text{inv} \left( \frac{P_{T,\text{max}} G}{P_N} \right), & \text{if } G \in [G_{\text{out}}, G_{\text{break}}] \\
N_{\text{opt}}, & \text{if } G > G_{\text{break}} 
\end{cases} \quad (3.14)$$
3.3. Optimization procedure

\[ P_T = \begin{cases} 
0, & \text{if } G < G_{\text{out}} \\
 P_{\text{T, max}}, & \text{if } G \in [G_{\text{out}}, G_{\text{break}}] \\
\gamma^*(N_{\text{opt}})P_N \frac{G}{G_{\text{break}}}, & \text{if } G > G_{\text{break}} 
\end{cases} \tag{3.15} \]

The values \( G_{\text{break}} \) and \( G_{\text{out}} \) are defined by the expressions (3.12) and (3.13).

Fig. 3.11 summarizes an adaptation policy of this type.

![Figure 3.11: Rate (top) and power (bottom) control policy for the case when \( N_{\text{opt}} < N_{\text{max}} \).](image)

The average energy per bit is now given by

\[
E_b = \frac{1}{R_b} \frac{C_b}{P_x M N_b} \left[ P_{\text{T, max}} \int_{G_{\text{out}}}^{G_{\text{break}}} \gamma^* \left( \frac{x P_{\text{T, max}}}{P_N} \right) p_G(x) dx + N_{\text{opt}} \gamma^*(N_{\text{opt}}) P_N \int_{G_{\text{break}}}^{+\infty} \frac{1}{F_G(G_{\text{break}})} - p_G(x) dx \right] \tag{3.16}
\]
Figure 3.12: Average energy per bit as a function of the block size $M$ Case 2 ($N_{\text{max}} > N_{\text{opt}}$). As in Case 1, there is a slight increase in the average energy per bit at larger block sizes, which is caused by the packet coding overhead.

- $N_b=1000$ bits, $q=8$
- $h=16$ bits
- $R_b=10$ kbps
- $P^*_s=0.99$
- $P_{\text{out}}=0.1$
- $\bar{g}=-10$ dB, $\sigma_g = 5$ dB
- $P_N=25$ dB
- $T_c=15$ s
- Rice $K=5$ dB
Figure 3.13: Average energy per bit as a function of the number of information bits per packet $N_b$ Case 2 ($N_{\text{max}} > N_{\text{opt}}$).
Figure 3.14: Average energy per bit as a function of the standard deviation $\sigma_g$ Case 2 ($N_{\text{max}} > N_{\text{opt}}$). As seen in Case 1, higher energy savings are available from packet coding when $\sigma_g$ increases.

The system performance for $N_{\text{max}} > N_{\text{opt}}$ is shown in Figs. 3.12, 3.13 and 3.14. For the benchmark case with fixed power and rate, we choose $N_{\text{fix}} = N_{\text{opt}}$. As before, we plot the energy per bit as a function of the block size (Fig. 3.12), the packet size (Fig. 3.13), and the standard deviation $\sigma_g$ of the channel gain (Fig. 3.14). We observe a similar trend as in Case 1, although a slightly higher energy savings are available in this case since the transmitter has an additional degree of freedom (number of coded packets).
3.4 Adaptive power and rate control in the presence of channel estimation errors

Our analysis so far was based on the assumption that perfect channel state information is available to the transmitter. However, in practice, due to the noisy estimation carried out at the receiver, and the feedback delay, the transmitter only has an estimate of the channel gain. The discrepancy between the estimate $\hat{G}$ and the true channel gain $G$ has to be taken into account to meet a desired outage criterion. Specifically, this discrepancy will lead to additional outage events, which, together with the outage due to shut-off, now contribute to the overall outage rate, and the system has to be designed such that the overall outage remains at the design value. In order to keep the overall outage at the design value, we introduce an additional margin $C$ to control the outage due to estimation errors. With the inclusion of the margin, we formulate the adaptive power and rate control policy for each of the cases discussed earlier, now utilizing the estimated channel gain.

3.4.1 Case 1: $N_{\text{max}} \leq N_{\text{opt}}$, with imperfect channel knowledge

For this case, we fix the number of coded packets at $N_{\text{max}}$ and adapt the power as

$$P_T = \begin{cases} C\gamma^*(N_{\text{max}})P_N/\hat{G}, & \text{if } C\gamma^*(N_{\text{max}})P_N/\hat{G} \leq P_{T,\text{max}} \\ 0, & \text{otherwise} \end{cases} \quad (3.17)$$

This is the same policy as before, except that the gain is replaced by its estimate, and the margin $C$ is introduced. The margin provides an additional degree of freedom needed to meet the outage requirement.

The probability of outage is now dictated by two factors: outage when the system is turned off, and outage that occurs when the system is turned on but we have an imperfect channel information. In other words,

$$P_{\text{out}} = P_{\text{off}} + P_{\text{on}}P_{\text{out|on}} \quad (3.18)$$
where $P_{\text{off}} = 1 - P_{\text{on}}$ is the probability that transmission is shut off,

$$P_{\text{off}} = \mathbb{P}\{ C \gamma^* (N_{\text{max}}) P_N / \hat{G} > P_{T, \text{max}} \} = \mathbb{P}\{ \hat{G} < C \gamma^* (N_{\text{max}}) P_N / P_{T, \text{max}} \}_{G_{\text{out}}}$$

and

$$P_{\text{out}|\text{on}} = \mathbb{P}\{ CG / \hat{G} < 1 \mid \hat{G} \geq CG_{\text{out}} \}$$

is the probability of outage when transmission is active.

For the log-normally distributed gain, we assume that MMSE estimation is employed on the dB scale, i.e. that the gain $g = \bar{g} + \Delta g$ is estimated as $\hat{g} = \bar{g} + \Delta \hat{g}$, such that the estimation error $e = \Delta g - \Delta \hat{g}$ is orthogonal to the estimate $\Delta \hat{g}$. In that case, the outage probability reduces to

$$P_{\text{out}} = \mathbb{P}\{ \Delta \hat{g} < c + g_{\text{out}} - \bar{g} \} + (1 - \mathbb{P}\{ \Delta \hat{g} < c + g_{\text{out}} - \bar{g} \}) \frac{\mathbb{P}\{ e_g < -c \mid \Delta \hat{g} > c + g_{\text{out}} - \bar{g} \}}{\mathbb{P}\{ e_g > c \}}$$

where $c = 10 \log_{10} C$, $g_{\text{out}} = 10 \log_{10} G_{\text{out}}$, and we use the fact that the zero-mean Gaussian error $e_g$ is orthogonal to the MMSE estimate $\Delta \hat{g}$. Expressing the constituent probabilities in terms of the $Q$-function, we obtain

$$P_{\text{out}} = Q\left( \frac{\bar{g} - g_{\text{out}} - c}{\sigma_{\bar{g}}} \right) + \left[ 1 - Q\left( \frac{\bar{g} - g_{\text{out}} - c}{\sigma_{\bar{g}}} \right) \right] Q\left( \frac{c}{\sigma_e} \right)$$

where $\sigma_{\bar{g}}^2 = E\{ \Delta \hat{g}^2 \}$ and $\sigma_e^2 = E\{ e_g^2 \}$.

Based on [44], we further assume that $\Delta g$ obeys an auto-regressive Gauss-Markov process of order 1, described by the one-step correlation coefficient $a$. This model applies to a class of shallow water channels for which it was validated experimentally. The details of the model can be found in [44]. Following this,

$$\Delta g[n] = a \Delta g[n-1] + w_g[n]$$

$$g[n] = \bar{g} + \Delta g[n]$$
where the clock $n$ ticks with each block, i.e., one feedback cycle of duration $T_s$, and $w_g[n]$ represents the process noise which is Gaussian, zero-mean and independent of $\Delta g[n - 1]$. Assuming a stationary process with known statistics, the estimate of the gain is given by

\begin{align*}
\Delta \hat{g}[n] &= a \Delta g[n - 1] \\
\hat{g}[n] &= \bar{g} + \Delta \hat{g}[n]
\end{align*}

In this case, we have that $\sigma_{\hat{g}}^2 = a^2 \sigma_g^2$, and $\sigma_e^2 = (1 - a^2) \sigma_g^2$. Both variances are thus specified through the parameter $a$, which is in turn related to the Doppler bandwidth $B_g$ of the process $g[n]$ via $a = e^{-\pi B_g T_s}$. Substituting for the variance, the outage probability can now be expressed as

$$P_{out} = Q \left( \frac{\bar{g} - g_{out} - c}{a \sigma_g} \right) + \left[ 1 - Q \left( \frac{\bar{g} - g_{out} - c}{a \sigma_g} \right) \right] Q \left( \frac{c}{\sqrt{1 - a^2 \sigma_g}} \right)$$

(3.27)

Given the statistics of the log-normal slow fading (the mean and variance $\bar{g}, \sigma_g^2$, and the correlation factor $a$) the above expression can be used to determine the margin that will meet a desired outage requirement. Fig. 3.15 shows the probability $P_{out}$ as a function of the margin $C$. As the correlation $a$ increases, the channel becomes more predictable, and hence a lower $C$ suffices to reach a desired $P_{out}$. 
3.4. Adaptive power and rate control in the presence of channel estimation errors

![Graph showing $P_{out}$ as a function of $C$.](image)

Figure 3.15: $P_{out}$ as a function of the margin $C$. A design value of $P_{out}$ implies the necessary margin $C$ for a given set of statistical parameters $\bar{g}$, $\sigma_g$ and $a$.

The average energy per bit is given by

$$
\bar{E}_b = \frac{1}{R_b P_s^* M N_b} C \gamma^* (N_{max}) P_N \int_{CG_{out}}^{+\infty} \frac{1}{F_G(x)} p_{\hat{G}}(x) dx
$$

(3.28)

where $p_{\hat{G}}(\cdot)$ denotes the p.d.f. of the estimate $\hat{G}$.

Figs. 3.16, 3.17 and 3.18 show the performance for Case 1 with imperfect channel knowledge. Different performance curves correspond to different values of the correlation factor $a$. It should be noted that $a = 1$ corresponds to the ideal case of perfect channel knowledge.
3.4. Adaptive power and rate control in the presence of channel estimation errors

![Figure 3.16: Average energy per bit as a function of the block size $M$ Case 1 with imperfect channel knowledge.](image)

Figure 3.16: Average energy per bit as a function of the block size $M$ Case 1 with imperfect channel knowledge.
3.4. Adaptive power and rate control in the presence of channel estimation errors

Figure 3.17: Average energy per bit as a function of the number of information bits per packet $N_b$ Case 1 with imperfect channel knowledge.
Figure 3.18: Average energy per bit as a function of the standard deviation $\sigma_g$ Case 1 with imperfect channel knowledge.
3.4.2 Case 2: $N_{\text{max}} > N_{\text{opt}}$ with imperfect channel knowledge

In this case, since we adapt both power and rate in accordance with the channel gain, we define the adaptation policy as

$$N = \begin{cases} 
0 & \text{if } \hat{G} < CG_{\text{out}} \\
\gamma_{\text{inv}}^{*} \left( \frac{P_{T,\text{max}} \hat{G}}{CP_{T}} \right), & \text{if } \hat{G} \in [CG_{\text{out}}, CG_{\text{break}}] \\
N_{\text{opt}}, & \text{if } \hat{G} > CG_{\text{break}}
\end{cases}$$

(3.29)

$$P_{T} = \begin{cases} 
0, & \text{if } \hat{G} < CG_{\text{out}} \\
P_{T,\text{max}}, & \text{if } \hat{G} \in [CG_{\text{out}}, CG_{\text{break}}] \\
\frac{C \gamma^{*}(N_{\text{opt}})P_{N}}{\hat{G}}, & \text{if } \hat{G} > CG_{\text{break}}
\end{cases}$$

(3.30)

where $G_{\text{out}} = \gamma^{*}(N_{\text{max}})P_{N}/P_{T,\text{max}}$ and $G_{\text{break}} = \gamma^{*}(N_{\text{opt}})P_{N}/P_{T,\text{max}}$ as before.

There are again three regions of operation, one in which the system is shut off, the other two in which the system is on. We define $r_{1}$ as the event where $\hat{G} \in [G_{\text{out}}, G_{\text{break}}]$ and $r_{2}$ as the event where $\hat{G} > G_{\text{break}}$. When the system is on, the system can operate in conditions of either $r_{1}$ or $r_{2}$. The outage probability can thus be written as

$$P_{\text{out}} = P_{\text{off}} + P_{r_{1}}P_{\text{out}|r_{1}} + P_{r_{2}}P_{\text{out}|r_{2}}$$

(3.31)

where

$$P_{\text{off}} = \mathbb{P}\{C \gamma^{*}(N_{\text{max}})P_{N}/\hat{G} > P_{T,\text{max}}\} = \mathbb{P}\{\hat{G} < CG_{\text{out}}\}$$

(3.32)

The probabilities of outage conditioned on $r_{1}$ and $r_{2}$ are

$$P_{\text{out}|r_{1}} = \mathbb{P}\{CG/\hat{G} < 1 \mid \hat{G} \in [G_{\text{out}}, G_{\text{break}}]\}$$

(3.33)
which reduces to

\[ P_{\text{out}|r_1} = \mathbb{P}\{e_g < -c \mid \hat{\Delta} \in [c + g_{\text{break}} - \bar{g}, c + g_{\text{out}} - \bar{g}]\} = \mathbb{P}\{e_g > c\} \quad (3.34) \]

and,

\[ P_{\text{out}|r_2} = \mathbb{P}\{CG/\hat{G} < 1 \mid \hat{G} \geq CG_{\text{break}}\} = \mathbb{P}\{e_g < -c \mid \hat{\Delta} > c + g_{\text{break}} - \bar{g}\} \quad (3.35) \]

Expressing the constituent probabilities in terms of the \( Q \)-function for the case of log-normal variation, and assuming that \( \Delta g \) obeys an auto-regressive Gauss-Markov process of order 1, we obtain

\[ P_{\text{out}} = Q\left(\frac{\bar{g} - g_{\text{out}} - c}{a\sigma_g}\right) + \left[1 - Q\left(\frac{\bar{g} - g_{\text{out}} - c}{a\sigma_g}\right)\right] Q\left(\frac{c}{\sqrt{1 - a^2\sigma_g}}\right) \quad (3.36) \]
3.4. Adaptive power and rate control in the presence of channel estimation errors

![Graph showing average energy per bit as a function of the block size M. Case 2 with imperfect channel knowledge.]

Figure 3.19: Average energy per bit as a function of the block size $M$ Case 2 with imperfect channel knowledge.
3.4. Adaptive power and rate control in the presence of channel estimation errors

Figure 3.20: Average energy per bit as a function of the number of information bits per packet $N_b$ Case 2 with imperfect channel knowledge.
3.4. Adaptive power and rate control in the presence of channel estimation errors

The average energy per bit is now given as

\[
E_b = \frac{1}{R_b P_s^* M N_b} C_b \left[ P_{T,\text{max}} \int_{C_G_{\text{out}}}^{C_{G_{\text{break}}}} \frac{x P_{T,\text{max}}}{C P_N} p_G(x) dx + N_{\text{opt}} C \gamma^* (N_{\text{opt}}) P_N \int_{C_{G_{\text{break}}}}^{+\infty} \frac{1}{2} p_G(x) dx \right] (3.37)
\]

Figs. 3.19, 3.20 and 3.21 show the performance for Case 2 with imperfect channel information. The results show a trend similar to Case 1, although slightly higher energy savings is available since the transmitter can now adapt the rate in addition to adapting the power.
3.5 Comparison with other ARQ techniques

In this section, we compare the proposed packet coding technique with traditional ARQ techniques. Given the half-duplex operation of acoustic modems, we are limited to the Stop and Wait (S&W) family of ARQ protocols. We consider the basic S&W protocol, as well as its modifications proposed in [30] to optimize the throughput over long-delay acoustic channels.

The simplest ARQ protocol is the basic S&W which we call S&W-1, where the transmitter sends a packet and waits for the acknowledgment (ACK). If the ACK is not received within a pre-specified amount of time (time-out), or a negative ACK is received, the packet is re-transmitted. In the modified version S&W-2 [30], the transmitter sends a group of packets, say $M$, and waits for the acknowledgment. The receiver checks individual received packets and sends acknowledgments at the end of $M$ packets. The packets that are negatively acknowledged are grouped together with new packets to form the next group of $M$ packets. In the modified version S&W-3, the transmitter sends out a group of $M$ packets and waits for the acknowledgment, but only those packets that are negatively acknowledged are transmitted in the next cycle. The transmitter keeps attempting re-transmission until all the $M$ packets are correctly acknowledged. We only consider S&W-2 in this comparison as it outperforms S&W-3 in terms of throughput efficiency.

The propagation delay introduced by the communication channel is $T_{\text{prop}} = d/c$, where $d$ is the distance between the transmitter and receiver, and $c$ is the speed of sound in water (nominally 1500 m/s). The duration of the ACK packet is $T_{\text{ack}} = N_{\text{ack}}/R_b$, where $N_{\text{ack}}$ is the number of bits in the ACK packet. Although each of the techniques may have a slightly different ACK duration, we assume the same ACK duration, since it is negligible in comparison to the packet duration. We define the round-trip waiting time as $T_w = 2T_{\text{prop}} + T_{\text{oh}}$, where $T_{\text{oh}}$ is any overhead time that includes the time needed for synchronization.
3.5.1 Throughput efficiency of packet coding

A system employing packet coding buffers a block of $M$ packets and encodes them into $N \geq M$ packets. The time taken to transmit $N$ packets is

$$T_{PC}(N) = NT_p + T_{ack} + Tw$$

(3.38)

The efficiency is given by the ratio of useful time to the total time invested in transmission,

$$\eta_{PC} = \frac{P^*_s M(N_b/R_b)}{T_{PC}(N)}$$

(3.39)

This efficiency corresponds to a certain reliability/outage specification, i.e., to a certain $P^*_s$ and $P_{out}$. When joint power and rate control is employed, we have two cases as described earlier. For Case 1, i.e., $N_{max} \leq N_{opt}$, the number of packets is fixed at $N_{max}$, and hence the time taken to transmit is $T_{PC}(N_{max})$. The corresponding efficiency is

$$\eta_{PC,1} = \frac{P^*_s M(N_b/R_b)}{T_{PC}(N_{max})}$$

(3.40)

For Case 2, i.e., $N_{max} > N_{opt}$, under the assumption that perfect channel knowledge is available, the average number of coded packets is given by

$$\bar{N} = \int_{G_{out}}^{G_{break}} \gamma_{inv}^*(xP_{T,max}/P_N)p_G(x)dx + N_{opt} \int_{G_{break}}^{+\infty} p_G(x)dx$$

(3.41)

The corresponding efficiency is given by

$$\eta_{PC,2} = \frac{P^*_s M(N_b/R_b)}{T_{PC}(N)}$$

(3.42)

If perfect channel knowledge is not available, an estimate of the channel gain will be used along with the additional margin as discussed in Sec. 3.4. For simplicity, we consider only the case of perfect channel knowledge for the present comparison as it suffices to illustrate
3.5. Comparison with other ARQ techniques

the point, and yields results similar to those that take imperfect channel knowledge into account.

3.5.2 Throughput efficiency of S&W-1

Theoretically, S&W techniques are designed to provide reliable transmission, i.e. a success rate of $P_s = 1$. Since this would entail a possibility of infinitely many re-transmissions, a time-out mechanism is used in practice to limit the number of re-transmissions to some maximum value $L^*$. As a result, a re-transmission may fail, effectively leading to outage.

In order to make a fair comparison between an ARQ technique and the packet coding technique that has a reliability of $P_{s}^*$, we choose $L^*$ such that the S&W-1 technique has the same reliability.

We assume that the S&W-1 transmitter has the same $P_{T,\text{max}}$, and hence the best achievable SNR when the gain is $G_{\text{out}}$, is $\gamma_{\text{ARQ}}^* = P_{T,\text{max}} G_{\text{out}} / P_N$. Power control for the ARQ system is implemented as

$$P_T = \begin{cases} \gamma_{\text{ARQ}}^* P_N / G, & \gamma_{\text{ARQ}}^* P_N / G \leq P_{T,\text{max}} \\ 0, & \text{otherwise} \end{cases}$$

(3.43)

The packet error rate is now given by $P_{E}(\gamma_{\text{ARQ}}^*)$ and the reliability of the S&W-1 protocol can be defined as the probability that at least one transmission is successful out of the $L$ re-transmission attempts, i.e.,

$$P_{s,SW1} = \sum_{\ell=1}^{L} \binom{L}{\ell} (1 - P_{E}(\gamma_{\text{ARQ}}^*))^\ell P_{E}^{L-\ell}(\gamma_{\text{ARQ}}^*)$$

(3.44)

The maximal number of re-transmissions $L^*$ is now obtained as the smallest $L$ for which $P_{s,SW1} \geq P_{s}^*$.

The average time taken to transmit a packet is

$$T_{SW1} = \sum_{\ell=1}^{L^*} P_{E}^{\ell-1}(\gamma_{\text{ARQ}}^*)(1 - P_{E}(\gamma_{\text{ARQ}}^*))\ell T_{ARQ}(1)$$

(3.45)
where $T_{ARQ}(1) = \left(\frac{K_b}{R_b}\right) + T_{ack} + T_w$ and $K_b$ is the total number of bits in a packet (now without the packet coding overhead). The above reduces to

$$T_{SW1} = \left(1 - P_E^*(\gamma_{ARQ}^*)\right) - L^* P_E^*(\gamma_{ARQ}^*)$$

$$T_{ARQ}(1)$$

(3.46)

The corresponding throughput efficiency is

$$\eta_{SW1} = \frac{P^* N_b}{T_{SW1}}$$

(3.47)

### 3.5.3 Throughput efficiency of S&W-2

The S&W-2 protocol can be regarded as $M$ S&W-1 protocols operating in parallel with the average time taken to transmit a packet on one of the $M$ links given by

$$T_{SW2} = \left(1 - P_E^*(\gamma_{ARQ}^*)\right) - L^* P_E^*(\gamma_{ARQ}^*)$$

$$T_{ARQ}(M)$$

(3.48)

where $T_{ARQ}(M) = M((K_b/R_b) + T_{ack}) + T_w$. The corresponding efficiency is given as

$$\eta_{SW2} = \frac{P^* M N_b}{T_{SW2}}$$

(3.49)

The throughput efficiency of various techniques is shown as a function of the block size $M$ (Fig. 3.22) and the packet size $N_b$ (Fig. 3.23). These results show that the packet coding technique outperforms the S&W techniques while maintaining the same reliability. Particularly, for larger packets, the efficiency of the S&W techniques reduces, while packet coding combined with adaptive power and rate control continues to operate with acceptable throughput efficiency.
3.5. Comparison with other ARQ techniques

Figure 3.22: Throughput efficiency as a function of the block size $M$. The efficiency of S&W-1 is included as a benchmark (it does not depend on $M$). Although longer blocks are favorable for increased efficiency, it should be noted that they require larger buffers to store the $M$ packets until they are positively acknowledged.
Figure 3.23: Throughput efficiency as a function of the number of information bits in a packet $N_b$.

Fig. 3.24 shows the throughput efficiency for a system with negligible waiting time. As we can see S&W-2 outperforms the packet coding at small packet sizes because of the large coding overhead associated with packet coding. In a system with no delay, packet coding is only beneficial at larger packet sizes.
3.5. Comparison with other ARQ techniques

Fig. 3.24 shows the average energy per bit as a function of the packet size. When we have smaller packets, the coding overhead leads to additional energy per bit. When we have larger packets, this additional redundancy becomes negligible and hence the proposed packet coding technique uses lesser average energy per bit when compared to the S&W technique. It should be noted that the average energy per bit consumed by both S&W techniques are same since S&W-2 is essentially the equivalent of $K$ S&W-1 links operating in parallel.
3.6 Experimental results for the point-to-point Link

In order to quantify the performance of adaptive power and rate control on an acoustic channel, we use the experimentally recorded values of the channel gain from the Surface Processes Acoustic Communication Experiment (SPACE) conducted off the coast of Martha’s Vineyard in the fall of 2008. In this experiment, a pseudo-random channel probing sequence of length 4095 was transmitted repeatedly, modulated using binary phase shift keying (BPSK) onto a carrier of frequency of 12.5 kHz. The transmitter and receiver were separated by a distance of 1 km. The ocean depth in the region was 10 m and the transmitter and receiver were fixed at a height of 2 m and 4 m from the ocean floor, respectively.

Figure 3.25: Average energy per bit as a function of the number of information bits in a packet $N_b$. 

- $M=25$
- $R_b=10$kbps
- $P_S^*=0.99$
- $P_{\text{out}}=0.1$
- $g=-10$; $\sigma_g=10$
- $P_N=25$dB
- $T_c=15$s
- Rice $K=5$dB
- $T_w=0.1$s
Figure 3.26: Channel gain recorded during the SPACE-08 experiment.
3.6. Experimental results for the point-to-point Link

![Histogram of the locally averaged gain (dB scale). The averaging window is 10 s.](image1)

![Auto-correlation of the channel gain (dB scale). The channel remains stable over several seconds.](image2)

The average channel gain measured from the data is $\bar{g} = -40$ dB and the channel gain standard deviation is $\sigma_g = 3.9$ dB. The 3 dB Doppler bandwidth of the fitted AR-1 model is estimated to be $B_g = 0.02$ Hz, corresponding to the channel correlation coefficient of $a = $
0.77 for $T_s = 4.5$ s. The experimentally recorded gain values are shown in Fig. 3.26 along with its histogram (Fig. 3.27) and auto-correlation function (Fig. 3.28). We note that these values are characteristic of the particular SPACE’08 experiment, while other environments may exhibit different statistics. For instance, [44] reports on coherence times $T_c \sim 1/B_g$ of 180 s and 250 s observed in locations near Rhode Island and Hawaii, respectively. These coherence times are considerably longer than in the SPACE’08 experiment, thus allowing for more accurate estimation of the channel gain.

The measured channel statistics were used to assess the performance of power/rate control. Figs. 3.29 and 3.30 show the results obtained for Case 1, ($N_{\text{max}} \leq N_{\text{opt}}$). We can see the average energy savings of about 6 dB when the channel gain is perfectly known and about 4 dB with the estimated channel.
Figure 3.29: Experimental results of the average energy per bit as a function of the block size $M$ for Case 1.
3.6. Experimental results for the point-to-point Link

Figure 3.30: Experimental results of the average energy per bit as a function of the number of information bits per packet \( N_b \) for Case 1.

Figs. 3.31 and 3.32 show the experimental results for Case 2, \( (N_{\text{max}} > N_{\text{opt}}) \). As with the analytical results, compared with Case 1, slightly higher average energy savings are available by using adaptive power and rate control (about 9 dB with perfect channel knowledge, and about 6 dB with estimated channel).
3.6. Experimental results for the point-to-point Link

Figure 3.31: Experimental results of the average energy per bit as a function of the number of information bits per packet $M$ for Case 2.
3.7 Results for the Broadcast Network

In this section we consider an underwater network with a leader node that wishes to broadcast information to $D$ receiver nodes. In each broadcast cycle, the leader buffers a block of $M$ packets and encodes them into $N \geq M$ packets for transmission. The leader transmits at a power $P_T$ over a channel with large-scale channel gain $G_i$. The signal power at the $i^{th}$ receiver is $P_{R,i} = G_i P_T$. The signal-to-noise ratio at the $i^{th}$ receiver is given by $\gamma_i = P_{R,i}/P_N = G_i P_T/P_N$, where $P_N$ is the noise power. The probability of bit error (BER) is a function of the signal-to-noise ratio and is represented by $P_e(\gamma)$. The corresponding packet error rate for each link is given as $P_E(\gamma_i) = 1 - (1 - P_e(\gamma_i))^{C_b}$.

Figure 3.32: Experimental results of the average energy per bit as a function of the number of information bits per packet $N_b$ for Case 2.
3.7.1 Adaptation policy for broadcast network when $N_{\text{max}} \leq N_{\text{opt}}$

In this case, the number of coded packets has to be fixed at $N = N_{\text{max}}$ since $N_{\text{max}}$ is less than $N_{\text{opt}}$. The transmit power can be adapted according to two rules: (a) worst link rule, and (b) average link rule.

When employing the worst link rule, the leader adapts its transmit power $P_T$ in accordance with the link that has the worst (least) channel gain. We define the set $\mathcal{I} = \{i | G_i > G_{\text{out}}\}$. The worst channel gain is computed as

$$G_{\text{min}} = \min_{i \in \mathcal{I}} G_i$$ \hspace{1cm} (3.50)

The adaptation policy can be expressed as

$$P_T = \begin{cases} 
0, & |\mathcal{I}| = 0 \\
\gamma^*(N_{\text{max}})P_N/G_{\text{min}}, & G_{\text{min}} > G_{\text{out}}
\end{cases}$$ \hspace{1cm} (3.51)

When employing the average link rule, the leader adapts its transmit power $P_T$ in accordance to the average of the channel gains on each link. The average channel gain is given as

$$G_{\text{av}} = \frac{1}{|\mathcal{I}|} \sum_{i \in \mathcal{I}} G_i$$ \hspace{1cm} (3.52)

The adaptation policy can now be expressed as

$$P_T = \begin{cases} 
0, & |\mathcal{I}| = 0 \\
\gamma^*(N_{\text{max}})P_N/G_{\text{av}}, & \text{otherwise}
\end{cases}$$ \hspace{1cm} (3.53)
3.7.2 Adaptation policy for broadcast network when $N_{\text{max}} > N_{\text{opt}}$

The adaptation policy for the case $N_{\text{max}} > N_{\text{opt}}$ using the worst link rule is

$$
N = \begin{cases}
0 & \text{if } G_{\text{min}} < G_{\text{out}} \\
\gamma_{\text{inv}}^{*} \left( \frac{P_{T,\text{max}} G}{P_N} \right), & \text{if } G_{\text{min}} \in [G_{\text{out}}, G_{\text{break}}] \\
N_{\text{opt}}, & \text{if } G_{\text{min}} > G_{\text{break}}
\end{cases}
$$

(3.54)

The adaptation policy for the case $N_{\text{max}} > N_{\text{opt}}$ using the average link rule is

$$
N = \begin{cases}
0 & \text{if } G_{\text{av}} < G_{\text{out}} \\
\gamma_{\text{inv}}^{*} \left( \frac{P_{T,\text{max}} G}{P_N} \right), & \text{if } G_{\text{av}} \in [G_{\text{out}}, G_{\text{break}}] \\
N_{\text{opt}}, & \text{if } G_{\text{av}} > G_{\text{break}}
\end{cases}
$$

(3.55)

3.8 Simulation Results

In this section we evaluate the performance of each technique using simulation. We consider a network of $D = 5$ nodes that are equidistant from each other. We further assume that the all links have a channel gain that is log-normal distributed with mean gain $\bar{g} = -10\text{dB}$. 
We can see an average savings of about $6 - 12$ dB by employing the average link rule and $2 - 6$ dB using the worst link rule. The average link rule does lead to a higher outage ($\sim 26\%$) than the system is designed for (10%) because the leader is being conservative and adapting to the average of the links, thus starving some of the nodes. While employing the worst link rule, the measured outage was $\sim 12\%$ because the leader also transmits to those nodes which have a very poor link ($G_i < G_{\text{out}}$) and hence were not considered during the calculation of $G_{\text{min}}$.

Fig. 3.33 shows the average energy per bit as a function of the block size $M$. Although larger block sizes leads to lower average energy per bit, one has to keep in mind that the block sizes cannot become so large that they violate the block fading model. The value of the block size $M$ is thus a design choice made based on the coherence time of the channel. Fig. 3.34 shows the average energy per bit as a function of the packet size $N_b$. Despite lowering the average energy per bit, smaller packet sizes will not be favored in the interest of higher throughput. Fig. 3.35 shows the average energy per bit as a function of the fading parameter $\sigma_g$. The savings available by employing adaptive power and rate control increases with an increase in the channel fading.
3.8. Simulation Results

Figure 3.33: Average energy per bit as a function of the block size $M$ for Case 1. Although larger blocks are favorable as they lead to lower average energy per bit, it is important to keep in mind that the block fading model should not be violated.
Figure 3.34: Average energy per bit as a function of the number of bits in each packet for Case 1. Although smaller packets use less energy per bit, the average throughput will suffer with smaller packet sizes.
3.8. Simulation Results

The performance of the system for $N_{\text{max}} > N_{\text{opt}}$ is shown below. As in the previous case, we plot the energy per bit as a function of block size (Fig. 3.36), packet size (Fig. 3.37), and channel fading parameter $\sigma_g$ (Fig. 3.38). Higher energy savings are available in Case 2 because the leader now has an additional degree of freedom for its optimization.
Figure 3.36: Average energy per bit as a function of the block size $M$ for Case 2. As in Case 1, larger block sizes are favorable as long as they do not violate the block fading assumption.
Figure 3.37: Average energy per bit as a function of the packet size $N_b$ for Case 2. Although smaller packets lead to lower average energy per bit, design choice for should also consider the average throughput.
3.9 Conclusion

We considered random linear packet coding as an alternative to traditional ARQ techniques whose efficiency is compromised by the long round-trip delay of acoustic channels. We proposed a system design in which the number of coded packets to transmit is determined based on pre-specified reliability, hence reducing the need for frequent feedbacks.

Most underwater deployments are offshore and hence have limited power supply for their operation. In such scenarios it becomes essential to prolong the system’s lifetime by saving the available energy. To that end, our system employs power and rate control which are optimized so as to minimize the average energy invested per successfully transmitted bit of information. It does so within the constraints on maximal transmit power and maximal coding length (the latter follows from the acceptable decoding delay and bit rate, as well
as channel dynamics). The existence of optimal coding length was established, giving rise to two control policies, depending on whether the optimal packet coding length is below or above the allowable maximum. The control policies were further extended to accommodate practical situations in which perfect knowledge of the channel conditions is not available at the transmitter, and an estimate has to be used instead. We analyzed such a case based on MMSE channel estimation, taking into account the effect of channel estimation errors by introducing an optimally designed margin so as to satisfy the required outage probability. We showed both analytically and using experimentally measured channel statistics that savings of $5 - 9$ dB are achievable using the adaptive power/rate control. Finally, the proposed technique was compared to traditional ARQ techniques to show that employing packet coding with joint power and rate control increases the throughput efficiency.

We analyzed joint power and rate control for an underwater broadcast network employing random linear packet coding. We proposed a system design based on reliability to improve the overall network efficiency which is otherwise hampered by the long propagation delays of underwater acoustics.

We proposed two adaptation policies: (a) worst link rule where the transmitter adapts in accordance with the link that has the least channel gain, and (b) average link rule where the transmitter adapts in accordance to the average of the channel gains on each link. We showed using simulations that an average energy savings of $6 - 12$ dB, and $2 - 6$ dB is available by employing the worst link rule and average link rule, respectively.
Chapter 4

Reliable Packet Delivery
Employing Packet Coding

4.1 Introduction

In this section we investigate reliable packet delivery using random linear packet coding for channels that experience long delays and time-varying propagation conditions that contribute to the high bit (packet) error rates.

Traditionally, automatic repeat request (ARQ) protocols such as stop-and-wait (S&W), go-back-N, and selective-repeat are used to make a link reliable. The half-duplex constraint further limits the choice of a data link layer protocol to the S&W type. Coupled with long propagation delays such as those of acoustic links, these techniques become inefficient as they rely on waiting for feedback from the receiver.

Optimization of ARQ techniques for underwater acoustic channels was discussed in [15]. Two modifications of the basic S&W techniques that draw on the concepts used in satellite communications were studied there, and their performances were compared in terms of the throughput efficiency. It was shown that grouping of packets and the use of selective acknowledgments (ACK) improves the throughput efficiency. Reliable data transfer from one half-duplex node to another was also addressed in [16], where the long propagation delay was exploited by allowing the two nodes to transmit simultaneously in
a juggling fashion. A comparative performance analysis of ARQ protocols for underwater acoustic networks was presented in [17]. A variation of the selective-repeat ARQ and two hybrid ARQ techniques were studied there for the multi-user underwater network, and their performance were compared in terms of the throughput efficiency and the average packet delay. Another variation of the selective-repeat ARQ was proposed in [18], where the authors used the long propagation delay of underwater channels to set up an interleaved time-division duplexed link. A transmission scheme for a continuous ARQ protocol over underwater acoustic channels was proposed in [19], where the authors used an idle period after the transmission of every packet to time the reception of acknowledgements for continuous transmission, similar to juggling.

In this paper, we explore random linear packet coding as an attractive addition for achieving link reliability. In a packet coded system, a group of $M$ information-bearing packets are encoded into $N \geq M$ coded packets for transmission [20]. The receiver can decode the original information bearing packets from a subset of any $M$ out of the $N$ coded packets. It should be noted that the packet coding does not replace channel coding, but can be used in addition to it. Since packet coding is performed at the packet level (as opposed to bit level), it is readily applicable to any existing physical layer technique.

Packet coding for acoustic channels was studied in [24, 25, 31–33] and for satellite networks in [26]. In [31], rateless codes were considered for reliable data transfer in underwater acoustic networks. It was shown there that the throughput efficiency improved since the feedback was used less often. In [33], the authors investigated optimal schedules for packet coding in a half-duplex link and showed that an optimal number of coded packets exists, which minimizes the time (or energy) required to complete the transmission of a group of packets. Optimal strategies for broadcasting information using random linear packet coding were addressed in [32], showing performance improvements over traditional ARQ techniques. Multi-hop reliable data transfer for an underwater acoustic network using fountain codes was proposed in [24], where, under the assumption of a half-duplex operation, the block size of each hop was adapted so as to optimize the end-to-end delay. Joint power and rate control for an acoustic link employing random linear packet coding was consid-
ered in [25]. It was shown there that a small additional redundancy suffices to maintain a pre-specified reliability at a relatively high level at the receiver.

In this paper, we extend the work of [25] to design a fully reliable link using packet coding in conjunction with an ARQ mechanism. In particular, we regard a group of \( N \) coded packets as one unit called a super-packet. We group \( L \) such super-packets together, forming a super-group which is then transmitted. The receiver sends a selective acknowledgment for the super-group. If a certain super-packet is negatively acknowledged, it is re-transmitted in the next round, along with any new units. Packet grouping hierarchy is illustrated in Fig. 4.1.

Figure 4.1: Grouping hierarchy of the proposed packet coding technique. \( M \) information-bearing packets are encoded into \( N \geq M \) packets for transmission. \( N \) coded packets form one super-packet. The super-packets are grouped into groups of \( L \), to form super-groups. A super-group is subject to a selective acknowledgment procedure.

We extend the adaptation policies defined in [25] to a link providing full reliability by employing the proposed grouped packet coding technique. We develop adaptation policies
so as to minimize the average energy per bit used by packet coding. Two constraints are imposed in the optimization: (a) the transmit power cannot exceed a maximum available level, dictated by the system budget and hardware restrictions, and (b) the number of coded packets cannot exceed a maximum value, dictated by the coherence time of the channel, the maximum permissible decoding delay and the minimum acceptable bit rate.

We compare the proposed technique with a full-duplex link (as a benchmark) and other techniques employing packet coding, in terms of the throughput efficiency. We show numerical results for a point-to-point link and a broadcast network. Random linear packet coding for a broadcast network has been explored in [28], where two adaptation rules based on achieving a pre-specified reliability were proposed for adaptive power and rate control. According to the first rule, the transmitter adjusts its parameters in accordance with the average of the channel gains on each link (average link rule). Under the second rule, the transmitter adjusts its parameters in accordance with the lowest channel gain among the links (worst link rule). In this paper, we combine the two techniques with the ARQ procedure to provide a network with full reliability. We show that the proposed grouped packet coding achieves throughput efficiency which is very close to that of an ideal full-duplex link.

The rest of the paper is organized as follows. We present the system model in Sec. 4.2. Sec. 4.3 is devoted to joint power and rate control. The results for a point-to-point link and broadcast network are presented in Sec. 4.4, and the conclusions are summarized in Sec. 4.5.

4.2 System Model

We review some of the system model and notations from Chapter 3 here before proceeding to our analysis.
4.2. System Model

4.2.1 Packet Coding

We consider a packet coded system in which the transmitter buffers a block of $M$ information-bearing packets, and encodes them into $N \geq M$ coded packets by employing random linear packet coding [20]. Each packet contains $N_b$ information bits. A coded packet is generated by linearly combining $M$ information packets. The random coding coefficients that are used to generate the coded packet are appended at its end. In addition, $h$ overhead bits are also added to each coded packet, to include the header and any bits for cyclic redundancy check (CRC). Thus, the total number of bits in each coded packet is

\[ C_b = N_b + qM + h \]  

(4.1)

where $q$ denotes the number of bits required to represent each coding coefficient when encoding is performed over the Galois field GF($2^q$). The structure of a coded packets is shown in Fig. 2.2.

Each transmitted packet contains $K_b$ bits, $C_b$ of which are from the coded packet and $K_b - C_b$ are the channel coding bits. Note that the packet coding does not replace channel coding, but works to improve performance in addition to channel coding. The packet duration of each transmitted packet is given by $T_p = K_b/R_b$, where $R_b$ is the bit rate in the channel.

The $N$ coded packets are transmitted over a channel which is characterized by the packet error rate $P_E$. The packet error rate is related to the bit error rate $P_e$ as $P_E = 1 - (1 - P_e)^{C_b}$. The receiver can decode the original information-bearing packets as soon as it receives any $M$ coded packets. A packet is deemed successfully received if it passes a standard procedure such as cyclic redundancy check.

We define the success rate or the reliability $P_s$ as the probability that at least $M$ out of the $N$ coded packets are received correctly, i.e.,

\[ P_s = \sum_{m=M}^{N} \binom{N}{m} (1 - P_E)^m P_E^{N-m} \]  

(4.2)
The minimum number of coded packets $N_M$ to transmit in each group is chosen such that a pre-defined probability of success $P_s^*$ is maintained at the receiver. In other words, $N_M$ is the smallest $N$ for which $P_s \geq P_s^*$.

### 4.2.2 ARQ

In [25], we analyzed packet coding that is designed to achieve a pre-specified success rate. When used without power / rate control, this design yields a system that needs no feedback, and as such is appealing for half-duplex systems with long delay. If power / rate control is used, feedback is needed to inform the transmitter of the channel state. That feedback can be used sparingly if the large-scale channel variation is slow. Namely, if the channel is deemed to stay more or less constant over several minutes, feedback is needed only as often.

The feedback that we considered in [25] was used only for conveying the large-scale, slowly changing channel gain, and not for ARQ. As a result, the scheme guaranteed only a pre-specified reliability, which is less than 100%. To achieve full reliability, an ARQ procedure must be used.

Traditional ARQ methods for the underwater channel are based on the variants of the Stop-and-Wait (S&W) protocol [15]. The first of these variants, S&W-1 protocol, transmits packets one by one, waiting for an acknowledgment (ACK) between each packet. We define throughput efficiency as the ratio of the useful packet time to the time taken to transmit a packet. Throughput efficiency of the S&W-1 protocol is given by

$$
\eta_{SW1} = (1 - P_E) \frac{N_b/R_b}{T_p + T_{ack} + T_w}
$$

where $T_{ack} = N_{ack}/R_b$ is the duration of the acknowledgment, and $T_w$ is the waiting time, which must be at least equal to the round-trip propagation delay plus any synchronization overhead. We note that the packet duration $T_p$ for S&W techniques contains only the $N_b$ information bits and $h$ header bits, i.e., no coding coefficients.

To overcome the throughput limitation of the conventional S&W-1 protocol, packet
grouping can be used with selective acknowledgments. The resulting S&W-2 and S&W-3 protocols have greater throughput efficiency, particularly on channels with long delay. Specifically, S&W-2 transmits groups of $K$ packets in each round before waiting for an acknowledgment. If a packet needs to be re-transmitted, it is included into the next group of $K$ packets that now contain both the new and the old packets. In contrast, S&W-3 transmits each group of $K$ packets until it is successfully received, i.e., no new packets are added when re-transmission is made. Since S&W-2 has better throughput performance than S&W-3 [15], we only consider S&W-2 in the present analysis. Throughput efficiency of the S&W-2 protocol is given by

$$\eta_{SW2} = (1 - P_E) \frac{K N_b/R_b}{K T_p + T_{ack} + T_w} \quad (4.4)$$

To extend the S&W-2 principle to a packet-coded system, we regard the $N_M$ coded packets (obtained from $M$ information packets) as one unit to which we refer as a super-packet. We group $L$ super-packets together to form a super-group. The receiver sends a selective acknowledgment for the super-group (see Fig. 4.1). If a certain super-packet is negatively acknowledged, it is re-transmitted in the next round (next super-group), along with any new super-packets.

The approach described above is somewhat wasteful, in the sense that if a super-packet is negatively acknowledged, one does not have to re-transmit all of its $N_M$ packets. It suffices to re-transmit only so many packets as there are missing degrees of freedom (extra coded packets needed to complete the decoding of the original $M$). However, the chances of a re-transmission being needed are contingent upon the pre-defined success rate $P_s^*$, which is a design parameter that can be controlled. If the success rate is set to a high value, e.g., $P_s^* = 0.99$, only one in every 100 super-packets will need to be re-transmitted on average. The resulting re-transmissions are not overly wasteful, especially in light of the procedure’s complexity. The grouped packet coding procedure is rather simple, as it does not require the missing degrees of freedom to be spelled out explicitly.

Although analogous to S&W-2, the grouped packet coding procedure differs in the fact that each unit is not one information packet, but one super-packet. Hence, the role played
in the original S&W-2 by the packet error rate $P_E$ is now played by $1 - P_s^*$. The success rate $P_s^*$ depends on $P_E$, but packet coding ensures that $1 - P_s^* \ll P_E$. In other words, the fact that $P_s^*$ can be controlled is used to improve the throughput efficiency. Our procedure also differs from the one considered in [33], where a single unit of $N \geq M$ coded packets is transmitted until all of its $M$ information-bearing packets have been successfully received. Such an operation is accomplished by providing feedback on the missing degrees of freedom (which diminish with repeated feedback).

The throughput efficiency of the grouped packet coding technique (GPC) is given by

$$\eta_{GPC} = \frac{P_s^*LMN_b/R_b}{LN_M T_p + T_{ack} + T_w} \quad (4.5)$$

It is evident that the throughput efficiency can be controlled using two parameters, $L$ and $P_s^*$. By doing so, grouped packet coding aims to overcome both the poor channel quality (high packet error rate), and the latency (long waiting time as compared to the packet duration).

Fig. 4.2 shows the throughput efficiency as a function of the super-group size $L$. Clearly, increasing $L$ leads to an increased throughput efficiency. However, it should be noted that a greater $L$ increases the decoding delay at the receiver, and also requires larger buffers to store all the packets at both ends of the link. A practical choice of $L$ might thus be limited by the storage space available at the transmitter and receiver, and the maximum permissible decoding delay.

Fig. 4.3 shows the throughput efficiency as a function of the group size $M$. Again, larger $M$ leads to higher throughput efficiency, but also requires a larger buffer to store the packets at the transmitter and receiver. As a result, the choice of $M$ is also dictated by the buffer size at the transmitter and receiver.
Finally, Fig. 4.4 shows the throughput efficiency as a function of the success rate $P^*_s$ for different values of the super-group size $L$. As expected, the throughput efficiency increases
with higher $P^*_s$.

![Graph showing throughput efficiency $\eta_{GPC}$ as a function of the success rate $P^*_s$.](image)

Figure 4.4: Throughput efficiency $\eta_{GPC}$ as a function of the success rate $P^*_s$.

### 4.3 Adaptive Power and Rate Control

Our analysis so far focused on a time-invariant channel with a fixed packet error rate $P_E$. On a time-varying channel, the bit error rate $P_e$ fluctuates with the signal-to-noise ratio (SNR), which is dictated by the channel gain $G$. The SNR is related to the transmit power $P_T$ and the noise power $P_N$ as $\gamma = G P_T / P_N$. The probability of packet error now varies with $\gamma$, and so does the probability of successful decoding,

$$P_s(\gamma) = \sum_{m=M}^{N} \binom{N}{m} (1 - P_E(\gamma))^m P_E^{N-m}(\gamma)$$  \hspace{1cm} (4.6)$$

The transmitter can adapt its power $P_T$, and the size of the super-packet $N$ in accordance with the large-scale channel gain $G$. The channel gain information is sent along with an ACK every $L$ super-packets. The choice of $L$ is now also limited by the coherence time.
of the channel along with the maximum permissible decoding delay and the buffer size at the transmitter and receiver. The time taken to send the \( L \) super-packets must not exceed the coherence time of the channel because a feedback is necessary to update the channel gain.

Adaptive power and rate control for an acoustic link employing packet coding were analyzed in Chapter 3 for the case with no ARQ. We choose the average energy per successfully transmitted bit of information as our figure of merit to determine the optimal transmit power and number of coded packets to transmit in each super-packet. We define the average energy per bit as

\[
\bar{E}_b = \frac{1}{R_b} \frac{\mathbb{E}\{NP_T\}}{P_s^* M N_b}
\]  

(4.7)

This is the average energy needed to achieve the pre-specified success rate \( P_s^* \) using packet coding. We wish to determine \( P_T \) and \( N \) that minimize the average energy per bit. It is evident from (4.7) that increasing the transmit power directly increases the average energy per bit. However, at the same time it reduces the packet error rate, which in turn allows for a smaller super-packet size \( N \) to be used to achieve a desired success rate \( P_s^* \). This trade-off between the transmit power and the super-packet size yields an optimization framework for minimizing the average energy per bit. In addition, we impose two optimization constraints

1. Transmit power is restricted to a maximum value \( P_{T,\text{max}} \) dictated by the hardware system constraints or by the total budget.

2. The total number of coded packets in each super-packet is restricted to \( N_{\text{max}} \), dictated by the coherence time of the channel \( T_c \), the maximum permissible decoding delay \( T_{d,\text{max}} \), and a necessary minimum average bit rate \( R_{b,\text{min}} \).

In this section, we analyze the joint power and rate control for Case 1 as described in Chapter 3. The second case would be a trivial extension and is addressed in detail in Chapter 3.

When \( N_{\text{max}} \leq N_{\text{opt}} \), we fix the super-packet size at \( N = N_{\text{max}} \) and vary the transmit
power as
\[ P_T = \begin{cases} \gamma^*(N_{\text{max}})P_N/G, & \text{if } \gamma^*(N_{\text{max}})P_N/G \leq P_{T,\text{max}} \\ 0, & \text{otherwise} \end{cases} \] (4.8)

We define the outage probability as the probability that \( P_s \) falls below the pre-defined value \( P^*_s \), i.e.,
\[ P_{\text{out}} = \mathbb{P}\{P_s(\gamma) < P^*_s\} \] (4.9)

For a log-normally distributed large-scale channel gain \( G \), i.e., \( 10 \log_{10} G \sim \mathcal{N}(\bar{g}, \sigma^2_g) \), the probability of outage is
\[ P_{\text{out}} = \mathbb{P}\{G < \gamma^*(N_{\text{max}})P_N/P_{T,\text{max}}\} = Q\left(\frac{\bar{g} - 10 \log_{10} G_{\text{out}}}{\sigma_g}\right) \] (4.10)

where \( Q \) denotes the Q-function, \( Q(x) = 1/2 \text{erfc}(x/\sqrt{2}) \).

The throughput efficiency of the grouped packet coding protocol when \( N_{\text{max}} \leq N_{\text{opt}} \) is given by
\[ \eta_{\text{GPC}} = P^*_s \frac{LMN_b/R_b}{LN_{\text{max}}T_p + T_{\text{ack}} + T_w} \] (4.11)

### 4.4 Numerical Results

In this section we present numerical results for the throughput efficiency of the proposed technique for a point-to-point link and a broadcast network. For comparison, we use the full-duplex (FD) link as the benchmark technique (a hypothetical, ideal case). The transmitter in this case sends packets back-to-back until an ACK is received from the receiver indicating that decoding was successful. The average time taken to complete the transmission of a group of \( M \) information packets on a full-duplex link is given by
\[ T_{\text{FD}} = M\frac{T_p}{1 - P_E} + T_w \] (4.12)
and the corresponding throughput efficiency is given by

\[ \eta_{FD} = \frac{MT_p}{T_{FD}} \]  \hfill (4.13)

We also compare our proposed technique to the optimal technique for a half-duplex link (HD,opt) proposed in [33], where the authors showed that an optimal number of coded packets can be obtained to minimize the average time needed to transmit a group of \(M\) packets. The throughput efficiency for optimal half-duplex technique is given by [33]

\[ \eta_{HD,\text{opt}} = \frac{MT_p}{T_M} \]  \hfill (4.14)

where \(T_M\) is the average time required to transmit a group of \(M\) packets. The closed form solution for \(T_M\) is provided in [33].

We assume that both benchmark techniques, use the same maximum transmit power \(P_{T,\text{max}}\) as the grouped packet coding. For a given channel gain \(G\), the transmitter (employing either FD or optimal HD) adapts its transmit power as

\[
P_T = \begin{cases} 
P_{T,\text{max}} G_{\text{out}} P_N / G, & \gamma^* P_N / G \leq P_{T,\text{max}} \\ 0, & \text{otherwise} \end{cases}
\]  \hfill (4.15)

### 4.4.1 Point-to-point Link

Fig. 4.5 show the throughput efficiency of the different methods as a function of the packet size \(N_b\) for \(L = 5\) (chosen so as to conform to the coherence time of the channel \(T_c = 15\) s). For numerical evaluation, we assume that the average bit error rate \(P_e(\gamma)\) is governed by Rician fading and differentially coherent detection with no coding or diversity, while the channel gain \(G\) is taken to be log-normally distributed, i.e., \(10 \log_{10} G \sim \mathcal{N}(\bar{g}, \sigma_g^2)\) [44]. The results of Fig. 4.5 show that grouped packet coding technique can achieve efficiency that is comparable to that of the optimal half-duplex (HD, opt) technique, but with much lower complexity.
Fig. 4.5 shows the throughput efficiency as a function of the packet size $N_b$. Larger group sizes are seen to be favored, as long as the group size $M$ and the super-group size $L$ together do not violate the coherence time of the channel. The choice of $M$ and $L$ also depends on the acceptable coding delay at the receiver.
4.4.2 Broadcast Network

In this section we present the simulation results for a broadcast network employing the grouped packet coding technique. We assume a broadcast network with a leader node that wishes to broadcast information to \( D \) receiver nodes. Each receiver nodes acknowledge the receipt of the super-packets. For simplicity, we assume that all the links have the same channel gain parameters \( \bar{g} \) and \( \sigma_g^2 \). We note that this is not a necessary condition, as the technique can be applied to a situation where the channel parameters are different on each link.

Letting the channel gain on the \( i^{th} \) link be denoted by \( G_i \), the received power at the \( i^{th} \) link is given by:

\[
P_i = P_s G_i \left( \frac{N_b}{L} \right)^{1/2} \left( 1 + \frac{P_N}{P_s} \right)^{-1}
\]

where:
- \( P_s \) is the transmitted power,
- \( N_b \) is the number of bits in a super-packet,
- \( L \) is the number of sub-packets in a super-packet,
- \( P_N \) is the noise power,
- \( T_c \) is the total time for the broadcast,
- \( T_w \) is the time window for the acknowledgment.

Figure 4.6: Throughput efficiency \( \eta \) as a function of the group size \( M \).
node is $P_{R,i} = G_i P_T / P_N$. After each super-group the receiving nodes send the channel gain $G_i$ back to the broadcasting node along with the ACK. After reading the channel gains on each link, the broadcasting node adapts the transmit power and rate. We consider two rules for the adaptation: (1) worst link rule, and (2) average link rule. The two rules are detailed below.

4.4.2.1 Worst Link Rule

When employing the worst link rule, the leader adapts its power and rate in accordance with the link that has the worst channel gain. For the case when $N_{\text{max}} \leq N_{\text{opt}}$, the size of the super-packet is fixed at $N_{\text{max}}$ and the transmit power is varied. We define the set $I = \{i|G_i > G_{\text{out}}\}$. The worst channel gain is computed as

$$G_{\text{min}} = \min_{i \in I} G_i$$ (4.16)

The adaptation policy is given by

$$P_T = \begin{cases} 
0, & |I| = 0 \\
\gamma^* (N_{\text{max}}) P_N / G_{\text{min}}, & \text{otherwise}
\end{cases}$$ (4.17)

Figs. 4.7 and 4.8 show the throughput efficiency as a function of the number of information bits per packet $N_b$ and the group size $M$. It should be noted that the other techniques that are used for comparison also use the worst link rule for their adaptation in this simulation. We can see that grouped packet coding has a throughput efficiency that is comparable to the optimal HD (or optimal half-duplex) technique but with much lower complexity.
4.4. Numerical Results

Figure 4.7: Throughput efficiency $\eta$ as a function of the packet size $N_b$ for a broadcast network employing the worst link rule.

- $M = 25, q = 8$
- $L = 5, h = 16$ bits
- $P_r^* = 0.99, D = 5$
- $P_{\text{out}} = 0.1$
- $\bar{g} = -10$ dB;
- $\sigma_g = 5$ dB
- $P_N = 25$ dB
- $T_c = 15$ s
- $T_w = 3$ s
- $R_b = 10$ kbps
4.4. Numerical Results

Figure 4.8: Throughput efficiency $\eta$ as a function of the group size $M$ for a broadcast network employing the worst link rule.

4.4.2.2 Average Link Rule

When employing the average link rule, the broadcasting node adapts its power and rate in accordance with the average of the channel gains on each link. We define the set $\mathcal{I} = \{i | G_i > G_{out}\}$. The average of the channel gain is given as

$$G_{av} = \frac{1}{|\mathcal{I}|} \sum_{i \in \mathcal{I}} G_i$$

(4.18)

The adaptation policy can now be expressed as
4.4. Numerical Results

\[ P_T = \begin{cases} 
0, & |\mathcal{I}| = 0 \\
g^\star (N_{\text{max}}) P_N / G_{\text{av}}, & \text{otherwise} 
\end{cases} \quad (4.19) \]

Figs. 4.9 and 4.10 show the throughput efficiency as a function of the number of information bits \( N_b \) and the group size \( M \), when employing the average link rule. We can see that employing the average link rule leads to a reduction in the throughput efficiency as compared to the worst link rule. The broadcasting node is now being conservative by adapting to the average channel gain and hence starving some of the nodes that are experiencing a worse link.

![Figure 4.9: Throughput efficiency \( \eta \) as a function of the packet size \( N_b \) for a broadcast network employing the average link rule.](image-url)
Figure 4.10: Throughput efficiency $\eta$ as a function of the group size $M$ for a broadcast network employing the average link rule.

Fig. 4.11 shows the throughput efficiency comparison of the two rules. The choice between the average link rule and the worst link rule can be made based on the system resources available and the throughput requirements. Fig. 4.12 shows the average energy per bit for a super-packet as a function of the group size $M$. The worst link rule uses about 2.5 dB more energy per bit than the average link rule. It can be seen from these two figures that the advantage in throughput efficiency using the worst link rule is achieved at the cost of more average energy per bit. In systems that are not resource constrained, worst link rule is advantageous, whereas average link rule is advantageous in systems that aim to conserve energy.
4.4. Numerical Results

Figure 4.11: Comparison of throughput efficiency $\eta$ as a function of the group size $M$ for the two rules.

- $N_b = 1000$ bits, $q = 8$
- $L = 5$, $h = 16$ bits
- $P_s^* = 0.99$, $D = 5$
- $P_{out} = 0.1$
- $\bar{g} = -10$ dB;
- $\sigma_g = 5$ dB
- $P_N = 25$ dB
- $T_c = 15$ s
- $T_w = 3$ s
- $R_b = 10$ kbps
We considered random linear packet coding for reliable data transfer on half-duplex communication links characterized by long delays and high packet error rates. We proposed a technique to combine random linear packet coding with an ARQ mechanism to achieve full reliability. To that end, regarded a group of coded packets as a super-packet and grouped several such super-packets to form a super-group. The number of coded packets in each super-packet was determined so that a pre-defined probability of successful decoding within a super-packet is maintained at the receiver. A stop-and-wait ARQ technique
was employed at the super-group level to achieve full reliability.

In order to address the effects of fading, we employed joint power and rate control. We analyzed the optimization of transmit power and rate in accordance with the large-scale channel gain such that an average energy per bit metric is minimized. The throughput efficiency of the proposed technique was compared to that of an ideal full-duplex link, optimal half-duplex technique, and S&W ARQ techniques. We presented results for a point-to-point link, as well as a broadcast network. For the point-to-point link, it was shown that our grouped packet coding technique can achieve throughput efficiency that was significantly better than that of the S&W ARQ techniques, while also comparable to the throughput of the full-duplex and optimal half-duplex link. Its advantage over the the optimal half-duplex system is the lower complexity of implementation.

For the broadcast network, we presented two rules for the optimization of the transmit power, the worst link rule, and average link rule. We showed that adapting to the worst link’s channel gain boosts the throughput, but requires more energy per bit.
Chapter 5

Conclusion

We investigated random linear packet coding as an attractive alternative to traditional ARQ techniques for channels that experience long propagation delays and high packet error rates. We focused our attention on the underwater acoustic channel and proposed techniques to over the high packet error rate and reduce delays.

In Chapter 2, we developed a packet coding technique in which the number of coded packets in each block was determined so as to maintain a pre-specified probability of successful decoding at the receiver. We showed theoretically, and verify experimentally, that the additional redundancy that need to be transmitted in order for the receiver to detect a block of original data packets with pre-specified probability was low. We showed that such a technique was feasible and easy to implement on top of any existing physical layer.

In Chapter 3, we extended the packet coding technique developed in Chapter 2 to employ joint power and rate control. Most underwater deployments are offshore and hence have limited power supply for their operation. In such scenarios it becomes essential to prolong the system’s lifetime by saving the available energy. We employed power and rate control which are optimized so as to minimize the average energy invested per successfully transmitted bit of information. We optimized transmit power and rate within the constraints of maximum transmit power and coding delays. We developed adaptation policies, and showed numerically, as well as experimentally, that energy savings on the order of
several dB were achievable using the proposed technique. We also provided adaptation policies for a broadcast network based on the average link rule and worst link rule.

Finally, in Chapter 4, we proposed the grouped packet coding technique, a combination of random linear packet coding and stop-and-wait ARQ to achieve full reliability. We regarded a group of coded packets as a super-packet and grouped several such super-packets to form a super-group. The number of coded packets in each super-packet was determined so that a pre-defined probability of successful decoding within a super-packet is maintained at the receiver. A stop-and-wait ARQ technique was employed at the super-group level to achieve full reliability. We also developed joint power and rate control along with grouped packet coding so as to minimize the average energy consumed per bit of successfully transmitted bit of information. We showed that the grouped packet coding technique had a throughput efficiency that was significantly better than the stop-and-wait techniques while also comparable to a full-duplex link employing packet coding.

We have proposed two alternatives to the traditional ARQ techniques for channels that experience long delays. We wish to emphasize that these two techniques are new, novel, easy to implement on top on any existing physical layer, and have low complexity. These features make the proposed techniques an attractive practical alternative to other techniques such as stop-and-wait and the optimal half-duplex technique.
Bibliography


[29] P. Qarabaqi and M. Stojanovic, “Statistical characterization and computationally ef-


based on fountain codes in underwater networks,” in *Proc. Wireless on Demand Net-

2012.


erasure coding and physical-layer channel coding in fading channels,” *IEEE Trans. 

networks,” in *Proc. 46th Annual Conference on Information Sciences and Systems 
(CISS)*, Mar. 2012.

[38] P. Wu and N. Jindal, “Coding versus ARQ in fading channels: How reliable should 


Appendix

5.1 Appendix A: Log-normal distribution

Given a log-normally distributed random variable \( G \), let \( G = e^A = 10^{B/10} \). Thus,

\[
A = \ln G, \; B = 10 \log_{10} G, \; A = \frac{\ln 10}{10} B
\]

and the corresponding mean values and variances satisfy

\[
m_A = \frac{\ln 10}{10} m_B, \; \sigma_A^2 = \left( \frac{\ln 10}{10} \right)^2 \sigma_B^2
\]

We now have

\[
F_G(G_{\text{out}}) = \int_{G_{\text{out}}}^{+\infty} \frac{1}{x} p_G(x) dx = \int_{A_{\text{out}}}^{+\infty} e^{-a} p_A(a) da = \int_{e^{A_{\text{out}}}}^{+\infty} e^{-a} \frac{1}{\sqrt{2\pi}\sigma_A} e^{-\frac{(a-m_A)^2}{2\sigma_A^2}} da
\]

\[
= e^{\frac{\sigma_A^2}{2} - m_A} Q\left( \sigma_A - \left( \frac{m_A - e^{A_{\text{out}}}}{\sigma_A} \right) \right)
\]

\[
= 10^{-m_B/10} e^{\left( \frac{\ln 10}{10} \right)^2 \sigma_B^2} Q\left( \frac{\ln 10}{10} \sigma_B - Q_{\text{inv}}(P_{\text{out}}) \right)
\]