A COMPACT DRIVE SYSTEM FOR GEARED ROBOTIC JOINTS AND ACTUATION MECHANISMS

A Dissertation Presented

By

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Abstract

Robotic manipulators require compact Joint Drive Systems (JDS) that typically comprise an actuator, a transmission and a joint structure that can deliver high torques through stiff mechanical ports. Today’s conventional drive systems are made from off-the-shelf actuators and multi-stage planetary transmissions or harmonic drives. This current practice has certain benefits such as short development time due to the availability of mechanical components, however it lacks a system-level integration that accounts for the actuator structure and size, output force, transmission structure, gear ratio and strength, and the robot’s joint structure, and often leads to long and bulky assemblies with large number of parts. This research is aimed at developing a new robotic hardware that simplifies the complexity of the robot drive system into one component which is optimized for its size and maximum torque density. This is accomplished by designing the robotic joint around a special transmission which, when numerically optimized, can produce unlimited gear-ratios from only two stages. The design is computerized to obtain all the valid solutions that satisfy its kinematic and constitutive relationships. The theoretical results demonstrates the potential of an example device for which a proof-of-concept prototype was designed and fabricated that could deliver more than 200 Nm of torque in a package as small as a human elbow joint. Compared to conventional precision drive systems such as harmonic drives, the proposed actuator offer similar advantages such as high-torque in a compact assembly but with the potential of better stiffness characteristics and lower transmission friction and more predictable output speed response; and when compared to existing space-flight actuators, the proposed design leads to shorter assemblies with significantly lower number of parts for the same output torque. The behavior of one prototype was experimentally characterized where simple but accurate models for the transmission friction, stiffness and kinematic error are obtained. A dynamic model of sufficient complexity is proposed that captures the open-loop velocity dynamics with good accuracy. An analytical stiffness model was also developed for the transmission mechanism using the bending flexure of gear-teeth and transmission geometry, and finally the dynamic impact of using shorter drive systems in manipulation is studied. The proposed technology combines precision with size and torque and as such could have immediate technological implications in many robotic and motion control applications.

Dissertation supervisor:
Dr. Nader Jalili
Professor of Mechanical Engineering
In memory of Professor Constantinos Mavroidis,

“No man can reveal to you aught but that which already lies half asleep in the dawning of your knowledge. The teacher who walks in the shadow of the temple, among his followers, gives not of his wisdom but rather of his faith and his lovingness.
If he is indeed wise he does not bid you enter the house of wisdom, but rather leads you to the threshold of your own mind.
The astronomer may speak to you of his understanding of space, but he cannot give you his understanding.
The musician may sing to you of the rhythm which is in all space, but he cannot give you the ear which arrests the rhythm nor the voice that echoes it.
And he who is versed in the science of numbers can tell of the regions of weight and measure, but he cannot conduct you thither.
For the vision of one man lends not its wings to another man.
And even as each one of you stands alone in God's knowledge, so must each one of you be alone in his knowledge of God and in his understanding of the earth.”

- Khalil Gebran
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**Biographical Note**

The author received his BS in Mechanical Engineering from the University of Massachusetts in Lowell where he graduated among the top of his class. He entered the department of Mechanical and Industrial Engineering at Northeastern in June of 2011. Brassitos received a NASA Space Technology Fellowship for the development of “A Compact Drive System for Planetary Rovers and Space Manipulators,” and received the 2016 James W. Healy Fellowship Award for Excellence in Research and Innovations from the Mechanical and Industrial Engineering Department at Northeastern. The author has three patent applications, one book chapter, two journal paper submissions and three conference paper publications (see appendix D for curriculum vitae).
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Chapter 1: Introduction

1.1 Problem Description

Many robotic applications require compact Joint Drive Systems (JDS) that can apply high
torques at low speeds such as space robots. Conventional drive systems are developed by serially
coupling an actuator to a high gear-ratio transmission of some sort, such as electric motors with
harmonic drives or with planetary gear trains. Despite its popularity, this practice often leads to
long and bulky assemblies with many parts, which not only increase the size and complexity of
the robot but also reduces its workspace [1]. In addition, conventional compact high gear-ratio
transmissions such as harmonic drives have high friction and low stiffness, which hinder their
abilities to operate as pure torque amplifiers in the absence of elaborate torque sensing schemes
[2]. Harmonic drives are also limited to gear ratios below 1:320.

Alternatively, conventional planetary gear trains require multiple stages to achieve high gear
ratios. This leads to complex and bulky assemblies with excessive number of parts. Other types
of robotic drive systems are based on piezoelectric, shape memory alloys, magneto-rheological
and electroactive-polymer actuators. These have had limited success in developing fully
functional drive systems or are still in the experimental stages or have not reached conclusive
results. Therefore, the development of compact and efficient drive systems could improve the
performance of many robotic and motion control systems, particularly mobile applications with
challenging torque and size requirements. Such drive systems could further enable the birth of
new technologies that are not possible with existing drive systems.
1.2 Dissertation Significance

The joints of a robot play a significant role in determining its overall size, weight, performance and cost. They influence its dynamics and payload capacity just as they affect its manipulation dexterity and range of motion. To date, robots have been very successful in industrial factories yet are not meeting the needs of many mobile platforms with challenging requirements on the drive system’s weight and size. For example, a space robotic arm requires small and lightweight joints to reduce the cargo launch costs. Likewise, a prosthetic limb requires small and lightweight joints to not impede the patient’s gait. The same need for compact and lightweight drive systems also exists in many consumer products, manufacturing, and energy systems. Most of the advancements in robotic drive systems so far have been in control methods aimed at maximizing the performance of aging hardware technologies such as harmonic drives. As such, to improve the force capacity and reduce the size and mass of current robotic systems, hardware-level advancements in actuator/transmission concepts are necessary. The key requirements for such concepts are high torque density and low friction, high structural rigidity and stiffness, compact and versatile transmissions with a wide range of gear ratios and low number of parts. The development of such concepts could lead to significantly more effective robotic systems and potentially new products that are not possible with commercially available actuator/transmission devices [3].
1.3 Dissertation Contributions

This thesis presents original hardware advancements in the area of robotic drive systems. The main contributions are:

➢ The development of a novel concept for an integrated drive system involving the motor and transmission and joint structure in one compact assembly that is numerically optimized for its maximum torque density.

➢ The theoretical formulation and physical development of a compact super-reduction transmission whose gear ratio is independent of its size and number of parts.

➢ The use of structural symmetry to balance the internal loads of a planetary cluster and to eliminate its carrier and individual planet bearings.

➢ The employment of dual-functions components to systematically reduce the complexity and number of parts of an actuator/transmission system.

➢ This research presents the first successful demonstration of a working gear-bearing mechanism.

➢ The development of a novel drive system with the following characteristics:
  ○ Unlimited possibilities of gear-ratios from 1:1 to more than 1:10000 - all independent of the size and weight and number of parts of the drive system.
  ○ Low-friction, high-stiffness transmission allow for near-pure actuator torque amplification.
  ○ Fully integrated drive system allows the development of modular robots with interchangeable drive systems.
1.4 Dissertation At-A-Glance:

This research addresses the development of a compact integrated drive system for space robots. Chapter 2 introduces the state of the art in robotic drive systems for space and civilian applications while describing current challenges and opportunities. Chapter 3 describes the concept development of a novel drive system and the numerical optimization of its gearbox. Chapter 4 describes the development of a custom measurement setup through which the transmission stiffness, friction and kinematic error are studied. An open-loop model of sufficient complexity is proposed in chapter 4 that captures the dominant dynamics of the transmission; and finally chapter 5 describes the dynamic impact of using shorter actuators in manipulation and introduces other areas of research where the proposed technology may be useful. This document is written in a way to be understood by a senior engineering student who would like to further study and contribute to the art of gear bearing drive transmissions.
Chapter 2 : Background

2.1 Introduction

A lot of research has been done on robotic Joint Drive Systems (JDS) over the past four decades. Most of it has focused on developing and understanding actuators or transmissions independently of each other. Very limited research addressed the integration and physical placement of actuators and transmissions within the robot’s joint and virtually no research addressed the use of differential transmissions as viable alternatives to high-gear ratio harmonic drives. The objective of this research is to study the design colocation of actuators and transmissions within the robotic joint and develop the corresponding robotic hardware. This first chapter is aimed at presenting the state-of-the-art in robotic drive systems while describing their common properties and characteristics and is organized as follows: It begins by defining the JDS and explaining its significant aspects and influence on the robot’s performance, and then recounts the major space robots that have been developed to date in orbiting platforms and planetary exploration while discussing some of their main actuation characteristics and major shortcomings.

2.2 Definition of the Robot Joint Drive System (JDS)

A robot JDS joins and drives two links of the robot relative to each other (Figure 2-1). To perform its functions, the JDS must contain 1) an actuator to supply a force or torque, 2) a transmission to amplify the actuator force, and 3) a joint structure that limits the mobility of the links to one degree of freedom while bearing the loads in the remaining degrees of freedom.
Independent of its size and weight, the JDS dynamics are mainly driven by its actuator/transmission characteristics such as gear ratio (i.e. transmission gain), stiffness, inertia, friction, and backlash. These properties play a key role in the performance of the robot as well as in its control system development. For example, a low-stiffness transmission reduces the force bandwidth of the drive system and introduces instabilities under high gain feedback loops [4]. Also, transmission friction elevates the actuator starting torque requirements and increases its size and reduces its accuracy. In the case of transmission limitations such as nonlinear friction and/or stiffness, nonlinear controllers are used to improve the input/output torque relationship of the transmission [5].

2.3 Parameters Definition:

**Torque density**

The torque density (Nm/kg) represents the amount of torque that a drive system can deliver per unit weight. It is a function of the actuator torque and transmission’s gear ratio and material strength limitations. With the increasing interests in mobile robots for civilian and space
applications, the higher the torque-to-weight of a drive system, the less of a side effects it would have on the overall performance of the robot.

**Stiffness**

This property describes the transmission’s input/output torsional flexibility (Nm/deg) under static and dynamic loads. While most transmissions can act as pure displacement or speed reducers, they rarely act as pure torque amplifiers. This is because in order to act as a pure torque amplifier, the transmission would need to have infinite stiffness and zero backlash and hysteresis and also zero friction. The key aspects of transmission stiffness in relation to robotic drive systems are studied in more details next. However, the mere existence of stiffness leads to oscillations in the open loop position/force response of the drive system. These oscillations become an issue when the stiffness is relatively low such that the resonance frequencies are excited by the operating speeds. To suppress these oscillations, researchers have compensated for the torque ripple by measuring the torque travelling through the transmission and applying various control schemes.

A recent survey on the control of flexible-joint robots can be found in [6]. While all these methods are valid, they requires more sophisticated instrumentations than the case where the transmission is as stiff as the links of the robot. It would therefore make sense to consider a drive system with a high stiffness such that the frequency of oscillations is much higher than the operating frequencies. The high stiffness also improves the accuracy of the drive system by resulting in lower deflections under high loads. Another important aspect of transmission stiffness is its stiffening property. This occurs while the transmitted load removes the clearances and play between the various parts, which are subject to manufacturing errors and assembly fits. As the force propagates from the transmission input to its output, the load bearing surfaces come
in contact with each other and increase the number of load-sharing parts and thus result in a higher stiffness value. The transmission stiffness is normally measured by holding the transmission’s input while applying static loads on its output along various angular positions. A general representation of a transmission stiffness is presented in Figure 2-2 showing its various properties and characteristics.

![Graphical representation of various transmission stiffnesses curves](image)

**Figure 2-2**: Graphical representation of various transmission stiffnesses curves

It is noted that the stiffness measurements can be contaminated by the static friction in the transmission since a portion of the applied torque is needed to overcome the static friction in the gearing.

**Friction**

The frictional losses in a transmission can influence the drive system performance in multiple ways. Friction can be divided into static, kinetic and viscous losses. Static friction is a function of the breakaway torque of the transmission and the kinetic friction at constant speeds. The larger the static friction, the larger the required actuator torque and so the actuator size. Hence from a size perspective, a JDS with a large static friction is normally bigger than a same-performance JDS with lower static friction. Static friction also influences the resolution or minimum step that a drive system can consistently make between point-to-point movements. The lower the stiction,
the less of an effect it will have on the minimum step of the transmission, allowing for a smoother and more precise position control. The kinetic friction due to fluid lubrication is dynamically well understood and is proportional to the velocity, and the coulomb friction is known to be proportional to the normal forces (e.g. pre-load forces) at the points of contact.

**Rotary Inertia**

The rotary inertia in the actuator/transmission elements influences the responsiveness of the drive system. The distribution of inertia in the robot influences the kinetic energy and bandwidth of the entire robot. For a given robot architecture, it is important to minimize the drive system inertia and mass particularly when it is mounted at a high acceleration point such as a robot extremity or end effector. To relate the various drive system parameters, Table 2-1 shows a mapping between the system-level parameters, indicating what parameter(s) should be changed to manage a desired system-level performance.

**Table 2-1: Correlation between the JDS and the system-level parameters**

<table>
<thead>
<tr>
<th>Joint Drive System Parameter</th>
<th>Speed</th>
<th>Force</th>
<th>Effective Mass</th>
<th>Acceleration</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear Ratio</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Speed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Torque</td>
<td></td>
<td>x</td>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Stiffness</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Efficiency</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inertia</td>
<td></td>
<td>x</td>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Friction</td>
<td>x</td>
<td>x</td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>
It is evident from Table 2-1 that the transmission’s gear ratio has the largest influence on the drive system’s stiffness, efficiency, mass, inertia, and friction. The transmission’s gear ratio can amplify/reduce all other parameters of the robot, and as such it should be designed with the following characteristics:

- Produce a wide range of gear ratios
- Minimize friction
- Minimize number of parts
- Minimize backlash
- Maximize torque capacity
- Maximize stiffness
- Minimize inertia

2.4 Transmission Research (Harmonic Drives)

During the last four decades, a significant amount of research was performed to develop and understand the behavior of compact robotic transmissions such as harmonic drives [7]. Harmonic drives are primarily useful to develop compact, zero-backlash, high torque drive systems for manipulation robots. The harmonic drive operation is based on a unique mechanism that comprises three concentric components denoted by the Wave Generator, Flexspline, and Circular Spline as illustrated in Figure 2-3.

![Figure 2-3: A harmonic drive arrangement [8]](image)
The Wave Generator consists of a bearing that is press fit within an elliptically shaped steel disk and inserted within the Flexspline. The Flexpline is a compliant thin-walled steel cup that conforms to the shape of the wave generator and has teeth on its external diameter that mates with the Circular Spline. The Circular Splines consists of a rigid steel ring with teeth on its internal diameter. Harmonic drives are designed with the flexpline having two teeth less than the Circular Spline such that when the Wave Generator rotates one revolution, the Circular Spline is shifted by two teeth. This mechanism yields a large mechanical advantages between the Wave Generator and flexpline. Because the load is continuously shared among multiple teeth, the harmonic drive transmission tends to have a high torque capacity. The zero-backlash characteristic is due to the fact that the flexpline is radially pre-loaded against the circular spline at all times closing the radial and tangential clearances. The compact size of the harmonic drive combined with its ability to produce high gear ratios and zero backlash has made it very popular among robotic designers across virtually all application disciplines. A schematic figure of the harmonic drive cross sections is presented in Figure 2-4.

![Harmonic Drive Cross Sections](image)

**Figure 2-4:** Schematic of the harmonic drive cross-sections

Despite its popularity, the two major operational issues in harmonic drives are high friction and low stiffness. Fiction is traced to the fundamental operating principle of the harmonic drive which relies on the sliding teeth friction between the Flexspline and Circular Spline to drive the
output. Another source of friction in harmonic drives is the high radial pre-load of the Flexspline into the housing. A schematic of the harmonic drive teeth operation is shown in Figure 2-5. As illustrated, teeth operation in harmonic drives behave as wedges which, when radially pushed, lead to rotational movement between the Flexspline and Circular spline. For this reason, the teeth on harmonic drive components are often referred to as a “teeth system” since they walk over each other as opposed to traditional gearing mechanism which roll over each other. Controlling the angle of the wedges dictates the gear ratio of the transmission and influences its strength capacity. The maximum attainable gear-ratio of harmonic drive transmissions is 320:1 [9].

![Figure 2-5: Harmonic drive teeth operation model [9]](image)

Because of its uncommon operation, harmonic drives do not function as pure torque amplifiers due to their nonlinear stiffness and friction. Friction in harmonic drives has been extensively studied by many researchers such as in [10] and [11] and is well known to exhibit non-linear behaviors which were found to depend on the angular position of the wave generator and the amount of radial pre-load in the assembly and type and amount of lubrication.
Furthermore, the operation of the harmonic drive transmission is built upon the continuous
deformation of its core component, the flexpline. This flexibility creates a low-stiffness load path
that reduces the operating bandwidth of the robot, develops resonance, and produces a backlash-
like effect [10,12]. As a result, the open-loop velocity response of the harmonic drive is not only
contaminated by vibrations but also by unpredictable speed jumps following resonant regions.
This phenomena was first observed by Tuttle (1992) and is illustrated in Figure 2-6 where the
output velocity time response fails to scale correctly with linearly increasing motor current.

![Figure 2-6: Typical open-loop velocity response of two harmonic drive assemblies [9]](image)

According to Tuttle, this behavior is due to the amplification of friction around the resonant
zones which act as energy sinks for the transmission rotational energy. The system escape
resonance only when the lubricant reaches a certain viscosity state where the frictional forces are
overcome by the supplied motor torque. Because of these high nonlinearities harmonic drives
have posed serious control challenges for robotic designers and were subject to extensive research efforts during the past thirty years.

In general, although many researchers were successful at linearizing the input/output torque relationship of the harmonic drive to near ideal torque source, the price was elaborate controls and expensive sensing schemes which involved placing strain gauges on the flexpline and closing the torque loop using various control schemes. Most harmonic drive transmission research falls under two categories: 1) free output motion, and 2) restrained output motion. The first category aims to model the transmission behavior in which it drives an inertia that is free to move or rotate, whereas the second category aims to characterize the transmission when its output is pressing against a hard surface, such as in grinding, cutting or sanding applications. In order to develop accurate control schemes for both free and restrained motions, researchers had to first model the dynamics of the transmission such as friction, stiffness, hysteresis and kinematic error, and then propose corresponding controllers with varying degree of complexities depending on the level of desired accuracies.

Tuttle was among the first to extensively model the open loop behavior of the harmonic drive in free-motion proposing a non-linear oscillatory term for the static friction, an excitation model for the kinematic error, and a stiffness model based on the gear-teeth meshing mechanism. Tuttle noted the non-linear stiffening property of the harmonic drive transmission and noted the difficulties in accurately modeling its stiffness and hysteresis dissipation as well as its unpredictable speed-jumps during resonance.

Other researchers such as Kircanski and Goldenberg [13] modeled the harmonic drive transmission under constrained motion in details and proposed simple models for soft-windup, hysteresis and friction in which the Strubeck effect is most dominant. Despite the difficulties in
obtaining accurate theoretical models for stiffness and hysteresis, the experimental stiffness of harmonic drive can be accurately captured using cubic polynomials as in [10]. The harmonic drive company recommends using three linear piecewise functions that capture the stiffening property at low, medium and high torques [14].

One of the most difficult parameters to mathematically model is the hysteresis dissipation in the stiffness curve, which is caused by the structural damping of the flexpline. Fairly complex models for hysteresis were proposed by Seyffeth [15] and by Dhadouadi and Ghorbel [16] who represented hysteresis by a combination of a non-linear stiffness component and a damping component. Due to the complexity of implementing this model from a control perspective, the effect of hysteresis is often ignored. Most recently, a high-fidelity harmonic drive model was proposed by Preissner [17] in which the transmission model accounts for viscous friction, non-linear stiffness, looping behavior of hysteresis and kinematic error.

Other researchers such as Legnani [18] also attempted to model the stiffness, friction and position accuracy of harmonic drives and noted the inherent difficulties in finding an accurate model for the system.

On the control aspects of harmonic drives, Spong [19] was among the first to propose an adaptive control law based on rigid-body dynamic with a linear correction term that damps out the elastic oscillations of the output. Tomei (1990) proposed a non-linear observer for flexible joint robots in [20]. Readman (1992) developed a gain-scheduling velocity feedback law that stabilized the fast modes dynamics resulting from the joint flexibility in [21]. HD Taghridad (1997) developed a pioneering frequency-based controller that linearizes the input/output torque relationship of harmonic drives by sensing and filtering the torque within the flexpline using a robust-adaptive kalman filter [22]. A similar sensing approach was proposed by Lessard et al
(2014) but used the feed-forward control to attenuate the joint vibration [23]. Although successful, these methods required knowing the torque within the flexpline component which is instrumentally expensive to acquire.

To circumvent this, torque estimation was done based on estimating the output deflection of the flexspline as proposed by Zhang (2015) [24]. Up to this date, the use of harmonic drive transmission remains highly desirable in a wide variety of robotic applications. Depending on the nature of application and required positioning accuracy, feedback control can be used to improve the transmission performance in both free and constrained motion. The cost is rooted in the required control instrumentations which differs from one particular assembly to another as a result of varying manufacturing errors and pre-load forces.

Harmonic drives have been extensively used in space applications such as the Mars Explorer, International Space Station’s Remote Manipulator System and many Solar Array Drive Mechanisms (SADM’s) such as SARA 21 and the SEPTA series, in addition to planetary rovers such as MER and the Mars Science Laboratory.

2.5 Transmission Research (Planetary Transmissions)

Other commonly used high gear-ratio transmissions in space mechanisms are planetary gear trains or (PGT) such as in [25], [26] and [27]. The need for high gear-reductions is important across many space applications as they are operated in high torques and low speeds. For example, the European Robotic Arm uses a four-stage planetary gearbox with a gear-ratio of 450:1 on its joints [26]. To reduce the complexity and number of parts in their multi-stage transmission, the ring gears are shared between the first and last two stages. Similarly, the drive systems on the Mars Exploration Rover [27] employs transmissions that are made of 3-5 stages with gear reductions ranging from 1528:1 to more than 8000:1. Consequently, the
implementation of PGTs in space mechanisms often lead to cumbersome assemblies with large number of parts such as planet carriers, carrier bearings and individual planet bearings, which not only increase the complexity but also reduces the reliability of the drive system. There has been minimal investigations on the advantages of using various planetary transmission design in the context of developing robotic joints.

Recent research on robotic drive systems addressed the optimization problem of a motor/transmission joint for its largest torque-per-inertia under the assumption that high gear ratios adds mass, inertia and frictional losses [28]. Its results suggest that the largest motor and smallest transmission within the size envelope of the joint is optimal. However this approach does not consider applications requiring high torques under limited current capacity, such as space robots. Other types of compact transmissions in the literature of robotics and motion control are cycloidal reducers [29,30] or hybrid combinations of planetary and cycloidal gearing, known as RV reducers. The analyses of these mechanisms revealed that although they have higher efficiency compared to harmonic drives, they suffer from considerable backlash and large transmission errors [31]. As a result, they have not seen wide use in the robotics community. In addition, there seem to be a gap in the literature pertaining the maximum allowable gear-ratios of cycloidal drives relative to their maximum output torque.

2.6 Actuators Research

In the area of conventional compact high torque/force density actuators, hydraulic and pneumatic actuators are ranked with the highest force-to-weight and force-to-volume ratios. However, they require complementary sources of energy power, such as pressurized air source or pump, which adds weight, reduces efficiency and makes it difficult to function as powered joints for fully portable robotic systems, such as space robots. Alternatively, the DC motor technology is
considered the most mature source of actuation that permits the development of fully portable, compact, and efficient high-torque density robotic drive systems. The well-proven DC motor technology exhibits a number of desirable features such as linearity, high bandwidth, accuracy, efficiency, low friction and low-cost. However, DC motors suffer from limited torque output due to the low electromagnetic forces attainable at small scales. To overcome this problem, DC motors are coupled with high gear-ratio transmissions to amplify their torques. In this thesis, the term actuator refers to an electric motor, and the term drive system refers to a motor and transmission and a load-bearing structure.

2.7 Drive systems in space robots

Space robots can be separated into 1) orbital platforms such as the International Space Station and satellite systems and 2) planetary rovers such as the family of the Mars exploration rovers. Both of which employ drive systems throughout their positioning and manipulation mechanisms.

**DEXARM – European Space Agency**

DEXARM is a space robotic arm that was built by the company Selex Galileo on behalf of the European Space Agency for an orbiting robotic platform [32]. The arm was intended to match the size and dexterity of a human arm with a maximum payload capacity of 10 kg on earth and 500 Kg in space. The key technological contribution of the arm was its integrated joint drive system which was built around the harmonic drive transmission [33]. The performance parameters of the arm are shown below in Figure 2-7:
The joints are actuated by brushless motors and Harmonic Drive transmissions and equipped with brakes and two position sensors per joint that are mounted on its input/output as shown in Figure 2-8. The joint-level controllers could be driven in position, torque and impedance control modes by using position and torque sensors feedback mounted on the harmonic drive output.

Table 2-2: Performance parameters of the DEXARM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average positioning accuracy of 1 mm and repeatability at 1-g</td>
<td>0.065 mm</td>
</tr>
<tr>
<td>Maximum Cartesian velocity</td>
<td>15 cm/s</td>
</tr>
<tr>
<td>Maximum joint velocity</td>
<td>0.5 rad/s</td>
</tr>
<tr>
<td>Mass</td>
<td>25 Kg</td>
</tr>
<tr>
<td>Arm power consumption</td>
<td>100 W</td>
</tr>
<tr>
<td>Length</td>
<td>1.2 m</td>
</tr>
<tr>
<td>Back-drivable joints</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2-7: The DEXARM engineering model [31]

Figure 2-8: Joint Description of the DEXARM [32]
The performance characteristics of the joint drive system are shown in Table 2-3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>2.9 kg</td>
</tr>
<tr>
<td>Size (overall enclosure)</td>
<td>D = 112 mm, L = 175.2 mm</td>
</tr>
<tr>
<td>Central hole for cable passing</td>
<td>D = 16.5 mm</td>
</tr>
<tr>
<td>Maximum output torque</td>
<td>100 Nm</td>
</tr>
<tr>
<td>Worst case power consumption</td>
<td>20 W</td>
</tr>
<tr>
<td>Nominal power consumption</td>
<td>7.7 W</td>
</tr>
<tr>
<td>Minimum output shaft speed</td>
<td>6.3e-6 rad/s</td>
</tr>
<tr>
<td>Maximum output shaft speed (at no load)</td>
<td>0.5 rad/s</td>
</tr>
<tr>
<td>Positioning accuracy</td>
<td>0.01 deg (1.74e-4 rad)</td>
</tr>
<tr>
<td>Positioning repeatability</td>
<td>0.00016 deg (2.8e-6 rad)</td>
</tr>
<tr>
<td>Operative temperature</td>
<td>from -30 °C to +70 °C</td>
</tr>
<tr>
<td>Non-operative temperature</td>
<td>from -55 °C to +125 °C</td>
</tr>
</tbody>
</table>

The joint was modeled as a lumped two-mass system with linear springs and dampers. The author’s model covered the inertia of the motor and load, in addition to the torsional flexibility and friction of the harmonic drive which was taken to be a function of the wave generator angular position. The friction model accounted for both the viscous and Coulomb friction forces. The system was further refined with a motor torque ripple model describing the torque pulses based on the rotor position and current magnitude. The authors used the transient velocity response for a current step to identify the viscous and static friction, and reported 0.1129 Nm for the motor breakaway torque (i.e. static friction) and 0.001335 Nm.rad/s for viscous damping. The authors noted a high dependence between harmonic drive friction and external temperature. The authors determined that the low frequency oscillations were due to the friction in the harmonic drive (1 and 2 x motor speed) while the midrange frequency content (6 x motor speed) is due to the joint flexibility and resonance frequency in the Flexspline) and the high frequency contents (10-20 x motor speed) were due to the brushless motor torque ripple.

The authors concluded that the quality of motion is mostly downgraded because of the joint torsional flexibility which is traced to the harmonic drive flexibility. To perform torque control, the authors remodeled the joint and environment as a two-mass system with a spring/damper
environment port. The authors developed stable force controllers based on P and PID methods. The P-controller was subject to high disturbances near the resonance frequency, whereas the PID controller eliminated the resonance frequency however became unstable at sufficiently high environment stiffness. To summarize, the DEXARM employed one of the very first fully integrated drive systems in space robots. However due to the high torsional flexibility of the joint transmission, elaborate control and sensing schemes were employed to mitigate the effect of disturbances on the position, velocity and torque responses.

**German Aerospace Center (DLR)' Light Weight Robots LWR I, II, III**

The German Aerospace center developed three generations of lightweight robotic arms from 1990 to 2003 aimed at matching the human arm payload-to-weight capacity [34]. The first prototype (LWR I) was an 18 Kg manipulator whose payload-to-weight capacity was around 1:2. The robot’s drive system required a custom precision planetary gearhead with a 1:600 gear ratio. This was later determined to be too complex to be manufactured due to extreme part tolerances. The LWR II replaced the gearing of LWR I with a custom aluminum harmonic drive and became the highest payload-to-weight robot following its development in year 2000. The robot weighted 18 Kg and was able to lift a 7 Kg payload.

![Figure 2-9: Three generations of the DLR robots (Left to right LWR I, II, & III) [35]](image)
The third and most advanced DLR robot is the LWR III. Similar to LWR II, this robot used a lightweight harmonic drive transmission but with strain gages mounted on its output member (i.e. flexspline) allowing for torque and impedance control. The motors were upgraded to custom-made RoboDrive motors which exhibited small power loss and very low torque ripple. As a result of this drive system design, the robot’s payload-to-weight was about 1:1. It weighted 14 kg and lifted 15 kg at low speeds. The joint of the LWR III is shown in Figure 2-10.

![Figure 2-10: Description of the LWR III drive system [33]](image)

Although the performance of this manipulator was well ahead of its predecessor, it required a custom-made harmonic drive transmission with embedded stain gauges on its flexspline. The low and non-linear stiffness of the harmonic drives combined with the noisy strain gauges signals posed several challenges that had to be addressed using expensive instrumentation and control schemes.
Robotic Rovers in Space: Pathfinder

The first successful landing of a robotic rover in space was the Mars Sojourner rover in 1996. The wheels on this rover were independently actuated and geared with a 2000:1 transmission ratio yielding a vehicle speed of 40 cm/min. There have been no publications detailing the motor type and size. The rover had a mass of 11.5 Kg and is illustrated in Figure 2-11.

![Sojourner rover](publicdomain) - NASA copyright policy states that "NASA material is not protected by copyright unless noted"

Spirit and Opportunity

The second generation of planetary rovers were Spirit and Opportunity launched in June and July of 2003 as a part of the Mars Exploration Rover Mission (MER) [36]. The rovers hosted an Instrument Deployment Device (IDD) comprising a five degree-of-freedom robotic arm giving the rovers the ability to access the rocks and soil of the Martian environment. The design of the IDD drive systems was subject to the following criteria:

- Survive accelerations up to 42 g’s resulting from launch, flight maneuvers, entry, and descend.
- Operate within a temperature range of -70C to +45C.
- Be as low mass as possible and allow the IDD to stow in very confined launch volume.
The rover and its corresponding Azimuth actuator system are illustrated in Figure 2-12.

**Figure 2-12:** Spirit / Opportunity rovers and corresponding arm drive systems. (Public domain)

Despite the use of exotic materials such as titanium to minimize the weight of the joint, it was still long and bulky with a large number of parts, mainly due to its multi-stage transmission design. A list of the various IDD drive systems specifications are shown in Table 2-4 [27].

**Table 2-4:** Specifications of the MER arm drive systems

<table>
<thead>
<tr>
<th>Gear Reduction Type</th>
<th>Azimuth</th>
<th>Elevation</th>
<th>Elbow</th>
<th>Wrist</th>
<th>Turret</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear Ratio</td>
<td>813.7:1</td>
<td>813.7:1</td>
<td>813.7:1</td>
<td>1528:1</td>
<td>1528:1</td>
</tr>
<tr>
<td>Total Range of Motion [rad]</td>
<td>2.75</td>
<td>1.25</td>
<td>5.10</td>
<td>5.85</td>
<td>5.95</td>
</tr>
<tr>
<td>Static Torque Capability [Nm]</td>
<td>65</td>
<td>65</td>
<td>40</td>
<td>17</td>
<td>28</td>
</tr>
<tr>
<td>Operational Output Torque Capability [Nm]</td>
<td>45</td>
<td>45</td>
<td>20</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>No-load Speed +23°C, 28V [rad/s]</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>No-load Speed –70°C, 28V [rad/s]</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>No-load Current +23°C [A]</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>No-load Current –70°C [A]</td>
<td>0.16</td>
<td>0.16</td>
<td>0.12</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Torque/Current Slope +23°C [Nm/A]</td>
<td>180</td>
<td>180</td>
<td>180</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>Torque/Current Slope –70°C [Nm/A]</td>
<td>140</td>
<td>140</td>
<td>140</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>No-load Mechanical Accuracy [rad]</td>
<td>0.0015</td>
<td>0.0015</td>
<td>0.0020</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td>Min. Stop and Hold Torque [Nm]</td>
<td>28</td>
<td>28</td>
<td>20°</td>
<td>5.6</td>
<td>3</td>
</tr>
<tr>
<td>Motor Detent Torque Strength [mNm]</td>
<td>6°</td>
<td>6°</td>
<td>6°</td>
<td>6°</td>
<td>2</td>
</tr>
<tr>
<td>Mass [g]</td>
<td>590°</td>
<td>480°</td>
<td>405</td>
<td>380</td>
<td>350</td>
</tr>
</tbody>
</table>
A common characteristic between these various drive systems is their high gear-ratio transmissions that typically involve four to five stages of reduction. Some of the major tests that were performed on the MER drive systems were current vs. speed vs. torque characterization in addition to backdriving torque and backlash identification.

Interviews conducted with the design engineers at the Jet Propulsion Laboratory indicated that during the development of these joints they noted the challenges of creating parts that are strong yet lightweight enough which posed stringent limitations on the manufacturing, testing, and operation of the joints. In addition, the use of non-absolute encoders required the initialization of the arm using mechanical hard stops before operation. To perform this task, the motors would need to be run in open loop current control until contact is made. This led to potential gearbox damage because any small fluctuations in current led to very large contact torques, which should be addressed in future designs.

**Phoenix Lander**

The third successful rover in space was the Phoenix lander and rover [37]. The joint drive systems of the phoenix arm consisted of brushed DC motors with a multi-stage reduction system consisting of planetary gear set combined with a harmonic drive system. The brake in the arm was achieved using a magnetic detent that locks the motor rotor. Position sensing was accomplished using traditional quadrature encoders which are initialized using hard stops located at the maximum travel of the arm.

It is noted that the joints were equipped with a heater and temperature sensor that would ensure the motor is operated at

![Figure 2-13: Engineering model of the 2.4 m long phoenix robotic arm. (Public domain)](image-url)
sufficiently high temperature. The arm was designed to withstand temperatures as low as -90°C in a CO₂ filed atmosphere.

**Mars Science Laboratory**

The latest and most advanced robotic rover sent to Mars was the Mars Science Laboratory (MSL). Launched in 2008, the MSL mission was to scoop and examine the Martian soil for minerals supportive of microbial life. The rover is equipped with the following 5 degrees of freedom robotic arm illustrated in Figure 2-14 [38].

![Figure 2-14: The MSL robotic arm displayed in stow position for launch. (Public domain)](image)

The arm and turret together have a mass of approximately 100 Kg and a reach of over 2 meters. The applicable forces on the MSL arm were obtained from the launch trajectory acceleration, rocket launch vibrations data, rover induced loads from driving, and the contact forces resulting from pre-loading the end-effector (percussive drill) against Martian rocks. See [39] for more details. Due to the large span of the arm, compensation models for the joints stiffness and thermal distortion were implemented as in [40]. The MSL drive systems weighted 7.8 Kg for the Elevation, Azimuth, and Elbow joints and 4.2 Kg for the wrist and turret joints. An image of the elevation joint obtained from JPL is shown in Figure 2-15.
As a result of the large weight of the joints, the final mass of the arm to its turret interface was around 65 Kg \cite{source}. With the links being made from lightweight material, the actuators accounted for most of the arm mass. It is evident that current space robots lack the required mass and volume efficiencies of a seamless integration between the arm and its drive systems. The review of literature shows that current flight actuators are not only heavy but also highly sophisticated with many moving parts, which increases their manufacturing costs and reduces their reliability. In addition, the long length of these transmissions acts as mechanical stops that increases the stowing size of the arm during launch. This multifaceted design problem is addressed in the next chapter through the novel design and optimization of a robotic joint using gear bearings and brushless outrunner motor technology.
Chapter 3 : Mechanical Design of the Gear Bearing Drive (GBD)

3.1 Introduction

In the design concept presented here, the motor and transmission and joint are integrated with dual-function components aimed at increasing the torque density of the robot’s joint drive system. The proposed mechanism is inspired by the Gear Bearing concept which was first conceived by John Vranish [41] and later actuated by Brian Weinberg [42] using an integral brushless motor. The works of [41] and [42] demonstrated preliminary proof of concepts that however suffered from low efficiencies under high loads. Further research on the GBD’s transmission revealed that the planets are kinematical under constrained and does not maintain the alignment of gear mesh under large loads. With new design improvements by Brassitos and Mavroidis [43 - 46], the first successful demonstration of a working gear bearing drive mechanism was completed. Two plastic mockups and two metal prototypes were fabricated and tested on a custom-built experimental setup.

In this concept, the motor is integrated within a unique transmission that is capable of producing any gear ratio from 1:1 to more than 1:8000 using two stages and a fixed set of gears. The concept employs ground symmetry to balance the internal loads of its transmission and to rigidly secure its joint output interface. A numerical optimization analysis is conducted over a range of values for each transmission component while considering the applicable motor size, resultant transmission ratio and output torque causing GBD failure. The numerical analysis identified an optimal set of motor/transmission parameters (i.e., motor diameter and torque, transmission gearing-ratio, gear teeth values and size and strength) that would result in the highest GBD torque density. The novelty of the design combined with its numerical optimization leads to a very
compact robotic drive system assembly that is volumetrically smaller than a human elbow joint and capable of supplying more than 200 Nm of torque.

3.2 Concept Development

The GBD’s transmission is a two-stage differential planetary compound as schematically depicted in Figure 3-1. The input to this mechanism is the sun gear ($N_2$) and the output is the ring gear ($N_5$). The mechanism is fixed to ground using the first stage ring gear ($N_1$). The two planets ($N_4, N_6$) from both stages are rigidly attached and as such behave as one rigid body.

![Figure 3-1: Schematic representation of the differential planetary transmission](image)

The transmission is driven by an external-rotor motor embedded within its sun gear as shown in Figure 3-2. A set of cylindrical roller surfaces hold the radial position of the planetary cluster, hence eliminating the need for a carrier, planets, motor and carrier bearings. Ground-ground symmetry is applied to balance the internal yaw moments which would otherwise act on the planets due to the ground-output moment couple.

The key to this concept’s high torque advantage is described in the free body diagrams of the planet-planet coupling ($N_4, N_6$) shown in Figure 3-3. The gears are represented by their pitch
diameters for simplicity where $D_1, D_2, D_4, D_5, D_6$ denote the pitch diameters of the ground ring gear, first stage sun gear, first stage planet, second stage ring gear, and second stage planet respectively, and $T_{in}$ and $T_{out}$ are the input and output torques of the mechanism.

At equilibrium, the sum of moments acting about the instantaneous axis of rotation yields the torque advantage of the mechanism as:

$$T_{out} = \frac{2D_4D_5}{D_2(D_4 - D_6)} T_{in}$$

(1)
Equation 1 shows that the output torque is inversely proportional to the difference between the planets pitch diameters \((D_4-D_6)\) such that the gear ratio is mainly dictated by the planets gears and rather independent of the size of the transmission. This is due to the fact that the input motor force acts on a moment arm \(D_4\) while the output force acts on a much smaller moment arm equivalent to \((D_4-D_6)/2\). As a result of this relationship, one can adjust the planets pitch diameters to produce very high gear-ratios without the need for adding more stages. The numbers of gear teeth that corresponds to such high ratios are justified in more details in the subsequent section using numerical methods. Furthermore, it has been shown in [47] that the actuator’s gear-ratio significantly influence the magnitude and distribution of kinetic energy within robotic manipulators and as such can improve their spatial precisions by reducing the effects of their inertial forces. This further advocates the importance of developing robotic joints that are capable of producing large ranges of gear-ratios by design.

Another key characteristic of this concept is the use of structural symmetry to balance the internal loads inside the transmission, which would otherwise require additional load-bearing supports and components. The free body diagram shown in Figure 3-3 shows that the forces that act on the planets subassembly lie in two different planes. This produces a yaw moment that tends to distort the parallelism and perpendicularity of the planets with respect to their plane of rotation.

To counter this yaw moment, the output stage is placed between two symmetric ground stages such that the output planets rest under the equilibrium of the double-shear loading as shown in Figure 3-4.
Furthermore, this novel configuration allows to rigidly secure the output between two ground structures through duplex bearing arrangements (e.g., back-to-back, tandem, face-to-face) to produce a stiff GBD output across all types of loadings. The cross-axis ground reaction forces provide support against thrust and radial loads as shown in Figure 3-5.

The GBD is mated using dual-functions components to simplify its assembly and reduce the number of parts. The dual-function components consist of cylindrical roller surfaces adjacent to the gear components. These surfaces locate the planetary cluster radially, thereby eliminating the need for conventional planetary carriers and corresponding bearings, while also maintaining the
stator-to-rotor airgap (see Figure 3-2). The roller surfaces have a rolling diameter equal to the adjacent gear pitch diameter to synchronize the gear traction and rolling motion as shown in Figure 3-6.

In addition, the planetary cluster is axially retained through an abutment between the planar surface of the rollers and ring gear teeth crowns. This is because the planet roller diameter is radially greater than the minor diameter of the ring gear. In this configuration, the gear teeth action and bearing support functions are integrated with each other leading to a highly compact drive system structure. The motor consists of an external magnetic rotor and a hollow stator lamination. The rotor is embedded within the sun gear while the stator is fixed to the ground component. In the concept, the stator-to-rotor air gap is maintained by the same roller surfaces that radially locate and align the remaining planetary cluster.

3.3 Numerical Optimization and Strength Analysis

Given the significance of torque density (torque per unit weight) in the robot’s joint drive system, a numerical optimization study was conducted on the analytical model to evaluate its torque density over a range of values of gear parameters. Traditionally, gear design is an iterative process, however in this research project we computerize the design by solving all the solutions of this
arrangement that are within a given output diameter. To limit the scope of the optimization, the following design assumptions were made:

1. The motor rotor diameter is nearly equal or less than the sun gear bore diameter.
2. The minimum number of teeth on the planet pinions is 10 (or greater) to avoid gear undercut.
3. The amplified motor torque by gear ratio is lower than the failure torque of the transmission.
4. Standard diametral pitches vary from 10 to 96 teeth per inch.

Knowing that the planets must orbit at the same radial distance from the center axis, denoted by $K$ in Figure 3-1, the following relationship can be written:

$$\frac{D_2}{2} + \frac{D_4}{2} = \frac{D_5}{2} - \frac{D_6}{2} = K$$

(2)

By recognizing the fact that the pitch diameter is equal to the number of teeth over the diametral pitch, or $D = \frac{N}{P}$, Eq. 2 can be re-written in terms of the number of teeth and the diametral pitches as in Eq. 3.

$$\frac{N_2}{P_1} + \frac{N_4}{P_1} = \frac{N_5}{P_2} - \frac{N_6}{P_2} = 2K$$

(3)

Where $P_1$ and $P_2$ are the diametral pitches of the first and second stages respectively, and $N_4$ and $N_6$ are the number of teeth on the planet gears, with their values assumed to range from 10 teeth per gear to an intermediate arbitrary value (e.g., 30) in increment of 1 tooth. Furthermore, $P_1$ and $P_2$ must have specific pitch values to use standard stock gear cutters. Knowing the approximate range of values for $N_4, N_6, P_1$ and $P_2$, Eq. 3 can be used to solve for the sun and output ring gears number of teeth, denoted by $N_2$ and $N_5$, as:

$$N_2 = 2KP_1 - N_4$$

(4)

$$N_5 = 2KP_2 + N_6$$

(5)

$$N_1 = N_2 + 2N_4$$

(6)
The parameters shown in Table 3-1 were used to populate the transmission variables over their possible combinations for planets orbit radius increasing from 1 to 2 inches by increment of 0.1 inches. By letting the arm radius vary from 1 to 2 inches, the transmission parameters are not only populated for different number of teeth but also for the physical diameter of the transmission.

**Table 3-1: Boundaries of optimization parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>1:0.1:2 [in]</td>
</tr>
<tr>
<td>N₄</td>
<td>10:1:30</td>
</tr>
<tr>
<td>N₆</td>
<td>10:1:30</td>
</tr>
<tr>
<td>P₁</td>
<td>5:1:96 [teeth / in]</td>
</tr>
<tr>
<td>P₂</td>
<td>5:1:96 [teeth / in]</td>
</tr>
</tbody>
</table>

Using an exhaustive computation search in which the transmission configurations that has non-integer values for N₂ and N₅ are eliminated, we obtained approximately 2.5 million solutions with ratios varying from -5000:1 to +5000:1. For each of these configurations, the gear ratio is solved using Eq. 7 (see Appendix A for derivation):

\[
\frac{T_{out}}{T_{in}} = \frac{1 + \frac{N₁}{N₂}}{1 - \frac{N₁N₆}{N₄N₅}}
\]  

(7)

Because the teeth geometry can be extracted from the diametral pitches and number of teeth, it is possible to perform strength analysis on every transmission configuration during the computational loop. A static force analysis shows that the planets are the weakest components in the transmission and therefore limit the maximum output torque of the GBD. Specifically, the mechanical power is transferred along three mesh points (A, B, C) as shown in Figure 3-7.
Where the pitch line velocities (tooth-passing speed) and transmission forces at the mesh points are given by (see appendix A for derivation):

\[ V_{\text{in/\text{arm}}} = \frac{N_1}{N_1 + N_2} \frac{D_2}{2} w_{\text{input}} \]
\[ V_{\text{ground/\text{arm}}} = \frac{N_2}{N_2 + N_1} \frac{D_1}{2} w_{\text{input}} \]
\[ V_{\text{out/\text{arm}}} = \frac{N_6N_1}{N_6N_4} \left( \frac{N_2}{N_2 + N_1} \right) \frac{D_5}{2} w_{\text{input}} \]

\[ F_{\text{in/\text{arm}}} = \frac{(D_4 - D_6)}{D_4D_5} T_{\text{out}} \]
\[ F_{\text{ground/\text{arm}}} = \frac{(D_4 + D_6)}{D_4D_5} T_{\text{out}} \]
\[ F_{\text{out/\text{arm}}} = \frac{2T_{\text{out}}}{D_5} \]

Using the pitch-line velocities and forces obtained above, it is possible to obtain the dynamic factors and corresponding failure forces for all the possible transmission configurations. For this, the planet stresses are computed using the AGMA gear rating criteria involving the geometric, material, mounting and reliability factors associated with each gear component. The output forces that would cause failure on the three mesh points are calculated using the contact and bending stresses as per [48]. The failure force that would cause teeth failure due to contact stress is given by Eq. 8: (see Shigley Design)

\[ F_c = \frac{bdI}{K_oK_vK_sK_mC_f} \left( \frac{S_cC_LC_H}{C_pK_TK_R} \right)^2 \]

\( (8) \)
The failure force that would cause teeth failure due to bending stress is given by Eq. 9:

\[ F_b = \frac{S_t b J K_l}{K_T K_R K_O K_v P K_s K_M} \]  

(9)

The dynamic factor is given by Equation 10 where \( V_i \) is the teeth passing speed along the three mesh points.

\[ K_{v,i} = \frac{50 + \sqrt{V_i \text{ ft}}} {50 \text{ s}} \]  

(10)

The other remaining factors such as overload and surface hardness are selected according to the manufacturing / assembly process. The lifetime is assumed \( 10^7 \) load cycles on all components.

Knowing the rating factors and individual failure forces on all the gear components, it is possible to obtain the maximum output torque for every transmission configuration as shown below.

**Point A (Bending stress failure)**

By substituting the pitch force into the stress equations, we can obtain the output torque that would cause bending failure on point A at the input planet as:

\[ T_{out,b,A} = \frac{n D_4 D_5} {D_4 - D_6} \frac{S_t b J K_l} {K_T K_R K_O K_v P_1 K_s K_M} \]  

(11)

Where the variable stress factors are given by:

\[ J = 0.07 \log(N_s) + 0.03 (\text{for 20 degree pressure angle}) \]  

(12)

\[ K_{v, A} = \frac{50 + \sqrt{V_{in/arm}}}{50} \]  

(13)

And the pitch line velocity is given by:

\[ V_{in/arm} = \frac{N_1}{N_1 + N_2} \frac{D_2}{2} W_{\text{input}} \left( \frac{1}{12} \right) \text{ ft/s} \]  

(14)
The constant stress factors are given by:

*Overload factor* $K_O = 1.25$ (moderate shock)

*Size factor* $K_s = 1$ (*uniform material*)

*Mounting factor* $K_m = 1.6$ (*gear width < 2, flexible mounting*)

*Temperature factor* $K_t = 1$ (*Stable oil lubrication temperature < 160F*)

*Life factor* $K_L = 1$ for $10^7$ cycles

*Reliability factor* $K_R = 1.25$

*Hardness* = 55 RC

*And allowable yield strength, $S_t = 60$ KSI*

And $P_1, N_4, D_4, D_5, D_6$ are available from the loop information where $n$ is the number of planets.

**Point A (Contact stress failure)**

Similarly, the output torque that would cause failure due to contact stress at point A of the input planet is given by:

$$T_{out,c,A} = \frac{D_4 D_5}{D_4 - D_6} \frac{bdl}{K_o K_v K_s K_m C_f} \left( \frac{S_c C_L C_H}{C_p K_T K_R} \right)^2$$

(15)

Where,

*Geometry factor* $l = \frac{m_G}{2(m_G + 1)} \sin(\theta) \cos(\theta)$, where $m_G = N_2/N_4$ (16)

*Elastic coefficient* $C_p = 0.564 \left[ \frac{1}{E_p} + \frac{1}{E_g} \right]^2$ (17)

And the constant stress factors are given by:

*Surface condition factor* $C_f = 1$ for 65 RA surface finish

$S_c = 180$ KSI for surface hardness 55 RC
Hardness Ratio Factor \( C_H = 1.0 + A \left( \frac{N_2}{N_4} - 1 \right) \) for \( H_{Bp} < 1.7H_{Bg} \) (18)

Where, \( A = 8.89(10^{-3}) \left( \frac{H_{Bp}}{H_{Bg}} \right) - 8.29(10^{-3}) \)

Life Factor \( C_L = 1 \) for 107 cycles

Steel modulus of elasticity \( E_g = E_p = 30,000 \) (10^3) psi for gear and pinion

Poisson’s ratio \( v_g = v_p = 0.33 \)

**Point B (Bending stress failure)**

Similarly to point A, we can obtain the output torque that would cause bending failure on point B at the input planet as:

\[
T_{out,b,B} = \frac{n D_4 D_5 S_1 b J K_L}{D_4 + D_6 K_T K_R K_G K_v K_p K_1 K_S K_M} \]

(19)

Where,

\[
J = 0.07 \log(N_4) + 0.03 \text{ (for } 20 \text{ degree pressure angle)} \]

(20)

\[
K_{v,B} = 50 + \frac{\sqrt{V_{\text{Ground/arm}}}}{50} \]

(21)

\[
V_{\text{Ground/arm}} = \frac{N_2}{N_2 + N_1} \frac{D_1}{2} w_{\text{input}} \left( \frac{1}{12} \right) \text{ ft/s} \]

(22)

**Point B (Contact stress failure)**

The output torque that would cause contact stress failure on point B at the input planet is given by:

\[
T_{out,c,B} = \frac{n D_4 D_5}{D_4 + D_6} \frac{b d l}{K_T K_R K_p K_S K_m C_f} \left( \frac{S_c C_L C_H}{C_p K_T K_R} \right)^2 \]

(23)
Where,

\[ \text{Geometry factor } I = \frac{m_G}{2(m_G + 1)} \sin(\emptyset) \cos(\emptyset), \text{where } m_G = N_1/N_4 \] (24)

\[ \text{Hardness Ratio Factor } C_H = 1.0 + A \left( \frac{N_2}{N_4} - 1 \right) \text{ for } H_{BP} < 1.7H_{BG} \] (25)

**Point C (Bending stress failure)**

Similarly, we can obtain the output torque that would cause bending failure on point C at the output planet as:

\[ T_{out\_b\_c} = \frac{nD_5}{2} \frac{S_l b J K_L}{K_T K_R K_0 K_v P_1 K_s K_M} \] (26)

Where,

\[ J = 0.07 \log(N_4) + 0.03 \text{ (for 20 degree pressure angle)} \] (27)

\[ K_{v\_c} = \frac{50 + \sqrt{V_{out\_arm}}}{50} \] (28)

\[ V_{out\_arm} = \frac{N_6 N_1}{N_5 N_4} \left( \frac{N_2}{N_2 + N_1} \right) \frac{D_5}{2} \frac{w_{input}}{12} \text{ ft/s} \] (29)

**Point C (Contact stress failure)**

The output torque that would cause contact stress failure on point C at the output planet is given by:

\[ T_{out\_c\_c} = \frac{nD_5}{2} \frac{bd I}{K_0 K_s K_m C_f} \left( \frac{S_c C_L C_H}{C_p K_T K_R} \right)^2 \] (30)

Where,

\[ \text{Geometry factor } I = \frac{m_G}{2(m_G + 1)} \sin(\emptyset) \cos(\emptyset), \text{where } m_G = N_5/N_6 \] (31)

\[ \text{Hardness Ratio Factor } C_H = 1.0 + A \left( \frac{N_5}{N_6} - 1 \right) \text{ for } H_{BP} < 1.7H_{BG} \] (32)
Because the teeth geometry is available during the computational loops, it is also possible to estimate the weight of the gears from their surface area and face width and mass density, and hence obtain a value for the torque density for each transmission configuration. The surface area of the external gear face is reconstructed using three geometric sections $A_1$, $A_2$, and $A_3$ corresponding to the gear rim surface (i.e. surface below the gear root diameter), the surface between the base circle diameter and pitch diameter and finally the area above the pitch diameter as illustrated in Figure 3-8.

![Figure 3-8: Estimation of the surface area of the gears](image)

It is known from AGMA (2001-DO4) that the base circle diameter is given by:

$$D_{base} = D_i - \frac{0.157}{P_{d,i}}$$  \hspace{1cm} (33)

Where $D_i$ is the pitch diameter of the corresponding gear, and $P_{d,i}$ is its diametral pitch. The whole depth (i.e. distance between the top of the tooth to the base circle) is given by:
\[ WD = \frac{2.157}{P_{d,i}} \]  

(34)

The minimum critical rim thickness \((RT)\) that produces a unity stress factor on the bending stress is 1.2 times the whole depth. Therefore, we can solve for the minimum rim thickness distance as:

\[ RT = \frac{2.588}{P_{d,i}} \]  

(35)

The addendum distance \((a)\) is given by:

\[ a = \frac{1}{P_{d,i}} \]  

(36)

The dedendum distance \((d)\) is given by:

\[ d = WD - \frac{1}{P_{d,i}} = \frac{1.157}{P_{d,i}} \]  

(37)

The tooth thickness \((t)\) distance at pitch diameter is given by:

\[ t = \frac{1.5708}{P_{d,i}} \]  

(38)

Using the relationships above and geometric relations of Figure 3-8, the areas of surface \(A_1, A_2\) and \(A_3\) can be written as:

\[ A_1 = \frac{\pi}{4} \left( D_{base}^2 - (D_{base} - 2RT)^2 \right) \]  

(39)

Note that the area \(A_1\) is a function of the gear pitch diameter and diametral pitches only, which are both available in the computational loop. The area of surface \(A_3\) can be shown to be:

\[ A_3 = at - a^2 \tan(\phi) \]  

(40)

Where \(\phi\) is the pressure angle of the teeth involute. The surface area \(A_2\) can be written as:

\[ A_2 = dt + d^2 \tan(\beta) \]  

(41)
Where the angle $\beta$ is a geometric variable as illustrated in Figure 3-8. By comparing this variable to the CAD model, it was determined that $\beta = 17.37$ degrees produces results that are within 3% of the simulation model. Therefore it is used to estimate the torque density of all the transmission configurations. Finally, the total surface area of the spur gear face is given by:

$$A_{total} = A_1 + N(A_2 + A_3)$$

(42)

Where $N$ is the number of teeth on the corresponding gear. The total weight of the gear is given by:

$$W_{gear} = \rho b A_{total}$$

(43)

Where $\rho$ is the mass density of the gear and $b$ is its face width.

The face surface of internal gears can be approximated by two subareas corresponding to the gear teeth and gear rim thickness. The gear face area is obtained using a similar procedures to external gears and is illustrated in Figure 3-9.

**Figure 3-9**: Estimation of the surface area of internal gears
Using a trapezoidal surface, it can be shown that the internal gear teeth area can be approximated as:

\[ B_2 = 2a\left(\frac{t}{2} - a\tan(\varphi)\right) + a^2\tan(\varphi) + dt + d^2\tan(\varphi) \]  \hspace{1cm} (44)

Where \(a, d,\) and \(t\) are the addendum, dedendum, and gear teeth thickness respectively. We note that this approximation produces a maximum of 3% error between the analytical estimation and the CAD model as available from the part Mass Properties of the CAD software.

![Figure 3-10: Gear weight estimation using](image)

Now that a method for obtaining the torque density for any transmission configuration is available, the design can be computerized using the flow-chart diagram illustrated in Figure 3-11 in order to obtain all the possible solutions of this design arrangement. The computational
algorithm returns a matrix where each row corresponds to one design configuration while the columns are its respective parameters.

![Flow chart of the computational algorithm](image)

**Figure 3-11:** Flow chart of the computational algorithm

A representative sample of solutions is presented in Table 3-2.

**Table 3-2:** Solutions format to the GBD configurations

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>-4709.78</td>
<td>1.14</td>
<td>0.66</td>
<td>0.66</td>
<td>2.66</td>
<td>2.66</td>
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<td>27</td>
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<td>486.52</td>
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</tr>
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<td>-4681.72</td>
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<td>0.68</td>
<td>2.67</td>
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<td>115</td>
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<td>107</td>
<td>501.66</td>
<td>2.16</td>
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</tbody>
</table>

The GBD torque density is calculated from the failure torque and estimated weight of the gears and motor. Figure 3-12 shows the relationship between the gear ratio and the GBD torque density. Each point represents one drive system configuration. The negative ratios indicate a reverse of direction between the input and output motions of the transmission.
Because the relationship between gear ratio and torque density is limited by the motor ability to supply enough torque under a particular gear ratio, the GBD solutions were then filtered using the motor diameter (taken from the sun gear bore diameter) over required motor torque (GBD failure torque over gear ratio). This process eliminates the GBD configurations with over and under sized motors. These configurations are illustrated in Figure 3-13. The motor sizing classification was estimated according to data supplied by BEI KIMCO Magnetics Inc. (www.beikimco.com).
Following the analysis of the data, one actuator combination having the highest possible torque density with a GBD diameter of 4.5” was selected while considering system manufacturability and use of standard bearing components. These specifications are listed in Table 3-3. It is apparent from Table 3-3 that only small differences in the planets pitch diameters (17.65 mm, 17.57 mm) are enough to produce a high gear ratio of 900:1.

<table>
<thead>
<tr>
<th>Table 3-3: Prototype specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear Ratio</td>
</tr>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Diameter</td>
</tr>
<tr>
<td>Weight</td>
</tr>
<tr>
<td>Ground Planet Number of Teeth</td>
</tr>
<tr>
<td>Output Planet Number of Teeth</td>
</tr>
<tr>
<td>Sun Gear Number of Teeth</td>
</tr>
<tr>
<td>Ground Ring Gear Number of Teeth</td>
</tr>
<tr>
<td>Output Ring Gear Number of Teeth</td>
</tr>
<tr>
<td>Stage 1 Diametral Pitch</td>
</tr>
</tbody>
</table>
### Stage 2 Diametral Pitch

<table>
<thead>
<tr>
<th>Component</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun Gear Pitch Diameter</td>
<td>55.95 mm [2.202 in]</td>
</tr>
<tr>
<td>Ground Planet Pitch Diameter</td>
<td>17.65 mm [0.694 in]</td>
</tr>
<tr>
<td>Output Planet Pitch Diameter</td>
<td>17.57 mm [0.691 in]</td>
</tr>
<tr>
<td>Ground Ring Pitch Diameter</td>
<td>91.31 mm [3.594 in]</td>
</tr>
<tr>
<td>Output Ring Pitch Diameter</td>
<td>91.25 mm [3.592 in]</td>
</tr>
<tr>
<td>Rated Torque</td>
<td>271 Nm [2998.5 lb.in]</td>
</tr>
</tbody>
</table>

**Concept A:**

Several design concepts were brainstormed to build a physical model corresponding to the specifications shown in Table 3-3. A significant design effort was placed on using standard mechanical components when applicable to reduce the cost of production. In a preliminary concept shown in Figure 3-14, a gear with smaller face width is placed at a longer moment arm to balance the yaw moment that was previously described in Figure 3-4. This concept uses an arrangement of a tapper needle bearing with a thrust bearing to pre-load the output ring using a locknut.

![Figure 3-14: Concept A](image)

**Figure 3-14:** Concept A
**CONCEPT B:**

A similarly-related concept is presented in Figure 3-15 which places a thrust bearing on the interior of the output ring gear such that it is available to drive a coaxial shaft.

![Figure 3-15: Concept B](image)

**CONCEPT C:**

A third concept uses a back-to-back taper needle bearing arrangement which is known for its axial and bending rigidity to retain the output member. It was determined that this would be the most complex and expensive approach since taper bearings are made from three components and tends to be bulky and difficult to smoothly integrate within the design.

![Figure 3-16: Concept C](image)
**CONCEPT D:**

In the last concept, a dual-row thrust bearing is used to retain the verticality of the output ring gear without over-constraining the mechanism, however, after consulting several bearing vendors it was determined that this concept is prohibitively expensive.

![Figure 3-17: Concept D](image)

### 3.4 Mechanical Design

A design matrix was established between the various concepts while considering structural strength and heat dissipation, manufacturing tolerances and planetary gear cluster alignment. The final CAD model is presented in Figure 3-18. Finite element strength and thermal analysis were performed on the mechanism to assure performance according to specifications (Table 3-3). Some of the major design challenges facing this concept were the ground-ground alignment accuracy and motor heat dissipation. The results of the finite element analysis supported the analytical strength and thermal models, and validated the torque capacity of the transmission and heat dissipation of the motor. The exploded assembly is illustrated in Figure 3-20.
Figure 3-18: A cross section of the final drive system
Figure 3-19: Rendered 3D figure of the design

Figure 3-20: Exploded assembly
3.5 Thermal Analysis

In order to understand how heat flows within the drive system, a thermal circuit was developed as shown in Figure 3-21. The motor was approximated as an ideal source of constant power (100W) applied at the cylindrical and planar faces of the ground member. It was assumed that heat will be dissipated via conduction then convection from the exterior surfaces using natural convection.

\[ Q_1: \text{Heat conduit through ground plate} \]

\[ Q_2: \text{Heat conduit through central shaft} \]

\[ Q_3: \text{Heat conduit through central shaft} \]

\[ T_m: \text{Motor Coils Temperature} \]

\[ T_A: \text{Ambient Temperature} \]

\[ T_m: \text{Motor Coils Temperature} \]

\[ T_{ambient}: \text{Ambient Temperature} \]

\[ Q_1, Q_2, Q_3: \text{Heat conduits through the system} \]

\[ h_{conv}: \text{Convection coefficient} \]

\[ T_{ambient}: \text{Ambient Temperature} \]

**Figure 3-21**: Steady-state thermal circuit of the Gear Bearing Drive actuator

An analytical model was developed based on the total resistance and temperature drop across the system terminals (motor and ambient temperatures). It was determined that roughly 90 inches square of heat sink are needed for heat to be fully convected off the exterior surfaces of the grounds in natural convection. The thermal calculations were further supported with finite element analysis as shown in Figure 3-22.
Since heat favors the least-resistance path, a significant portion of the motor heat will flow directly through the ground. The goal of this model was to develop a low thermal resistance to the ambient air that would keep the motor temperature below 120°C when it is running at peak power of 100 Watt. The model shows that the motor coil temperatures is 116°C at peak power at ambient temperature of 24°C. It is expected that the nominal motor temperature would be less at the rated operating conditions, especially in space applications where the ambient temperature is extremely low.

### 3.6 Strength Analysis

The FE strength analysis computed the actuator stresses under peak output torque of 300 Nm as shown in Figure 3-23. Due to the double-shear design, the output torque is equally shared between the ground stages leading to low stresses throughout the retaining structure. The load line is shown to travel through the stiffer medium (pseudo-ground) into the central shaft and into the mounting flange without developing high stresses in the Aluminum component.
Figure 3-23: FE analysis performed on the drive system ground structure

A finite element analysis model of the planets was developed to characterize the von Mises stress distribution at the gear teeth, particularly the stress localization near the dowel hole. As expected, the highest stress is recorded at the teeth root where the bending moment is largest.

Figure 3-24: Finite element analysis on the output planets pinion

The corresponding theoretical bending stress is determined using the AGMA equation accounting for the geometric factor ($Y_j = 0.26$ for $N=11$):
\[ \sigma_b = \frac{W_t P_d}{F Y_j} = \frac{(367.3 \text{ lbs})(16 \text{ teeth/inch})}{(0.350 \text{ in})(0.26 \text{ teeth/inch})} = 64580 \text{ PSI (445.2 MPA)} \]  

(45)

Where \( W_t \) is the observed transmission force on the planet teeth under an output torque of 300 Nm divided among four planets. It is given by:

\[ W_t = \frac{T_{\text{out}}}{2D_5} = \frac{300 \text{Nm}}{2(91.25)10^{-3} \text{m}} = 1634 N(367.3 lb) \]  

(46)

Where the diametral pitch, \( P_d \), and pitch diameter \( D_5 \) are given in Table 3-3 as 16 teeth-per-inch and 91.25 mm respectively. The analytical and simulation results are within 6.26% of each other and are therefore assumed satisfactory. Although the gear rating factors such as overload, reliability, dynamic, and alignment are considered in the analytical models, they are assumed unity throughout the finite element simulation as they would require a comprehensive study that involves the flexibility of bearings and retaining components which are beyond the scope of this study.

Because the mechanism uses two parallel ground structures, it is important to ensure that the clocking accuracy between the ground-ring gears is maintained at the maximum load. The deflection results obtained from finite element simulation (see Fig. 3-25) show marginal material movement at the order of 0.03 mm which is equivalent to 1 thousands of an inch and is within the typical precision requirements of planetary gear sets. As such, the design is adopted for the manufacturing of the drive system, which is further detailed in the future sections.
3.7 Plastic Mockup I

To better understand and visualize this concept, a preliminary plastic mockup was fabricated using additive manufacturing as shown in Figure 3-26. The mockup successfully validated the theoretical gear-ratio and assembly process and smooth gearing of the design.

Figure 3-26: Plastic mockup of the proposed GBD (without cover/motor)

To ensure that the parts fit together with just enough clearance to operate, a worst-case tolerance analysis was conducted on the precision parts (i.e. rollers). It was determined that the concentricity error on the sun gear must not be too loose since it would result in a large stator-to-
rotor airgap and neither too tight that it would result in rollers assembly interference. This is graphically shown in Figure 3-27.

![Figure 3-27: Tolerance stack-up analysis of the rollers assembly](image)

After 3D printing the parts, the critical dimensions were measured and compared to the nominal drawings as shown in Table 3-4 to assess the required fit. Several printing iterations were enough to relax the tolerances on the parts while maintaining a functional prototype.

![Figure 3-28: Assembly tolerance diagram](image)
Table 3-4: Tolerance analysis of the GBD2 assembly

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground Base</td>
<td>1</td>
<td>Stator mount shaft</td>
<td>A</td>
<td>0.332</td>
<td>0.333</td>
</tr>
<tr>
<td>Ground Base</td>
<td>1</td>
<td>Ground ring roller diameter</td>
<td>B</td>
<td>2.389</td>
<td>2.393</td>
</tr>
<tr>
<td>Ground Base</td>
<td>1</td>
<td>Ground ring mount</td>
<td>C</td>
<td>2.710</td>
<td>2.702</td>
</tr>
<tr>
<td>Ground Base</td>
<td>1</td>
<td>Inner bearing race mount</td>
<td>D</td>
<td>2.998</td>
<td>2.998</td>
</tr>
<tr>
<td>Ring Gear</td>
<td>2</td>
<td>Outer rim diameter</td>
<td>O</td>
<td>2.700</td>
<td>2.702</td>
</tr>
<tr>
<td>Ball Bearing</td>
<td>3</td>
<td>ID</td>
<td>F</td>
<td>3.000</td>
<td>3.001</td>
</tr>
<tr>
<td>Ball Bearing</td>
<td>3</td>
<td>OD</td>
<td>E</td>
<td>3.500</td>
<td>3.501</td>
</tr>
<tr>
<td>Sun Roller</td>
<td>4</td>
<td>ID</td>
<td>H</td>
<td>1.253</td>
<td>1.250</td>
</tr>
<tr>
<td>Sun Roller</td>
<td>4</td>
<td>OD</td>
<td>G</td>
<td>1.512</td>
<td>1.517</td>
</tr>
<tr>
<td>Planet Roller</td>
<td>5</td>
<td>ID</td>
<td>I</td>
<td>0.402</td>
<td>0.409</td>
</tr>
<tr>
<td>Planet Roller</td>
<td>5</td>
<td>OD</td>
<td>J</td>
<td>0.476</td>
<td>0.474</td>
</tr>
<tr>
<td>Output</td>
<td>6</td>
<td>Bearing mount</td>
<td>K</td>
<td>3.501</td>
<td>3.504</td>
</tr>
<tr>
<td>Sun Gear</td>
<td>7</td>
<td>ID</td>
<td>M</td>
<td>1.099</td>
<td>1.099</td>
</tr>
<tr>
<td>Sun Gear</td>
<td>7</td>
<td>Sun roller mount</td>
<td>L</td>
<td>1.250</td>
<td>1.249</td>
</tr>
<tr>
<td>Plane s/a</td>
<td>8</td>
<td>Planet roller mount</td>
<td>N</td>
<td>0.400</td>
<td>0.405</td>
</tr>
<tr>
<td>Sun gear aux</td>
<td>10</td>
<td>ID</td>
<td>Q</td>
<td>1.228</td>
<td>1.232</td>
</tr>
</tbody>
</table>

Following the development of the mockup, an off-the-shelf brushless outrunner motor was modified to be integrated within the mechanism. The motor stator and rotor were press fit within the sun gear and on the central shaft respectively as shown in Figure 3-29.

Figure 3-29: Integral motor assembly of the GBD actuator

The two thin sections ball bearing were press-fit between the ground plates and actuator output. A hex feature was made on the central shaft and pseudo-ground plate to lock the system axially while maintaining the gear clocking between the ground stages as shown in Figure 3-30.
A jig fixture was also fabricated to control the radial position and clocking of the planets during the assembly process. The assembly parts are presented in Figure 3-31.

**Figure 3-30:** Ground plates / back-to-back angular bearing assembly process

**Figure 3-31:** 3D-printed actuator parts
The jig assembly is shown in Figure 3-32. This particular design used an off-the-shelf brushless motor that is typically used in the RC industry which required a major operation to be fitted on the actuator, and presented several challenges to maintain a constant airgap. As such, a more elaborate design was developed based on a custom-built brushless motor developed by the company BEI / Kimco magnetics.

![Assembly jig of the mockup prototype](image)

**Figure 3-32: Assembly jig of the mockup prototype**

### 3.8 Plastic Mockup II

The second design accounted for the bearing pre-load using a custom-made wave spring (see Figure 3-18) and incorporated Omni seals within the retaining structure to enable the use of oil lubrication. The second plastic mockup was developed using high-precision additive manufacturing technology through *3D Systems Projet 3500 HDmax* and is shown in **Figure 3-33**.

![Plastic mockup of the proposed GBD (without cover/motor)](image)

**Figure 3-33: Plastic mockup of the proposed GBD (without cover/motor)**
3.9 High Precision Metal Prototype

The high precision metal prototype was built following the design lessons and insights learned from the plastic prototypes. The design of gear teeth involutes was made using the software *Integrated Gear Software (IGS)* by University Technical Systems Inc. See appendix B for detailed gear and parts drawings. The engineering model is presented in Figure 3-34.

![Figure 3-34: High precision metal prototype of the Gear Bearing Drive](image)

In this design, the transmission uses standard diametral pitches such that Stage 1 is 18 pitch and uses a sun gear with 38 teeth, a planet with 12 teeth and a ring gear with 62 teeth. The second stage is 16 pitch and uses a planet with 11 teeth and ring with 57 teeth. The center distance for both stages is 1.450 +/- 0.001 inches. The resultant gear ratio is 900 to 1. A schematic of the planets subassembly is shown in Figure 3-35.
According to the motor datasheet, its maximum torque is 50 oz.in or 3.125 in.lb. This produces a maximum output torque of $3.125 \times 900 = 2812.5 \text{ in.lb} = 234.4 \text{ ft.lb}$ or 317 Nm. Assuming the load is divided equally among the 4 output planets, the torque on one output planet is $2812.5 \times \frac{11}{57} / 4 = 135.69 \text{ in.lb}$. The strength analysis is done by running the 11 tooth output planet with a 1,000 tooth external gear using the IGS software. This analysis includes the effects of lead mismatch. The resultant stresses on the output planet are:

- Bending stress = 80,786 lb/in²
- Contact stress = 313,846 lb/in²

A separate analysis was done by taking into account the fact that the internal gear tooth is concave. The contact with the planet convex surface and without lead mismatch is much less. The contact stress without lead mismatch was found to be 200,177 lb/in². The material used for the output planets is 4150 steel quenched and tempered to RC 57-61. The load on the output planet is divided between the two input planets due to the drive symmetry. The torque on one input planet is therefore 67.845 lb.in. The face width of the input planet is 0.242 inches. The

**Figure 3-35**: Schematic of the planets subassembly

Notes:
1. Bolt grade 8 socket head cap screw, Length 1.75, (1/4 28 UNF 3A)
2. Nut (1/4 28 UNF 3B)
3. Roller planet output OD .6960-.0004”
4. Planet input
5. Output planet
6. Output ring
7. Ring input
8. Pin
9. Version of roller with internal key
strength analysis on the input planet is done by running the pinion with a 1,000 tooth external gear. The resultant stresses on the ground planet are:

- Bending stress = 74,982 lb/in$^2$
- Contact stress = 278,138 lb/in$^2$

The contact stress on the ground planet without lead mismatch was found to be 157,762 lb/in$^2$.

The material for the input planet is chosen as 4150 steel quenched and tempered to RC 57-61.

The stresses on the sun gear due to a torque of 1.562 in.lb and face width of 0.242 in are:

- Bending stress = 6,632 lb/in$^2$
- Contact stress = 44,073 lb/in$^2$

This low stress level means that the sun gear can be 300 Brinell hardness or RC 32-36. The sun gear material is chosen as hiperco 50A due to its high magnetic saturation property which would act as a backiron for the magnetic flux of the rotor. The planet gears and rollers are mounted on a common shaft and secured via a bolt and nut assembly. The alignment of the planet teeth is achieved by the friction caused by the tension of the bolt. The planets assembly is shown in Figure 3-36.

![Figure 3-36: Engineering model of the planets assembly](image)

The phase between the planets teeth is held using an axial force from a through bolt that carries the planets torque (67.845 in.lb) by friction. Using a coefficient of friction of 0.12 between the
steel parts, the torque of the bolt/nut assembly is determined as 2,661 in.lb. A grade 8 bolt is used to produce a tension force of 2,661 lb when torqued at 147.2 in.lb. The procedure for assembly is as follows:

1- Assemble each planet subassembly with the bolt hand tight.
2- Carefully increase the torque on each of the planets assemblies.
3- Roll the gears to make sure the planets are properly aligned.
4- Increase the torque in small steps and keep rolling the assembly to ensure the planets remain lined up properly. If the planets become misaligned during any of the steps, this procedure is repeated.

The radial force on the rollers and moment on the output planet hollow shaft are also calculated to ensure there is enough face width on the roller and enough thickness in the hollow shaft to carry the bending moment. The radial force from the output planet on the rollers is the torque on the output planet divided by its base radius. As such:

\[ F_{out} = \frac{135.69 \text{ in.lb.}}{0.6460 \text{ in}} / 2 = 420.1 \text{ lb} \]

Similarly, the force from the input planet on the rollers is the torque on the input planet divided by the base radius.

\[ F_{in} = \frac{67.845 \text{ in.lb.}}{0.6265 \text{ in}} / 2 = 216.58 \text{ lb} \]

These forces are added and projected over the corresponding pressure angles between the planets and ring gears. The net forces from the planets on the rollers are obtained by multiplying these forces by the sine of the pressure angle.

\[ F_{out-net} = 420.1 \sin (20) = 153.4 \text{ lb} \]
\[ F_{in-net} = 216.58 \sin (20) = 94.65 \text{ lb} \]

The total radial force on the rollers is therefore 153.4 + 2\times94.65 = 342.7 lb. This force is divided between two rollers such that the force on each roller is 171.35 lb. Knowing the radial force on
the rollers, the moment on the output planet hollow shaft is determined as $171.35 \text{ lb} \times 0.4463 \text{ in} = 76.47 \text{ in.lb}$. With $\text{OD} = 0.2996 \text{ in}$ and $\text{ID} = 0.256 \text{ in}$, the bending stress under maximum load is determined as $62,033 \text{ lb/in}^2$, which is well below the yield stress of the 4150 steel (quenched and tempered to RC 57-61) and is considered satisfactory. The gears were manufactured by the company Butler Gear Corporation thanks to a funding supplement from the Jet Propulsion Laboratory. The Gear Bearing Drive is actuated by a brushless motor that is integrated within its sun gear. The motor components were custom developed by the company BEI / KIMCO magnetics on behalf of Northeastern University. The motor parameters are shown in Table 3-5.

<table>
<thead>
<tr>
<th>MOTOR PARAMETERS*</th>
<th>UNITS</th>
<th>SYM</th>
<th>NOM. VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PEAK TORQUE</td>
<td>OZ-IN</td>
<td>$T_p$</td>
<td>50.0</td>
</tr>
<tr>
<td>MOTOR CONSTANT</td>
<td>OZ-IN/$\sqrt{\text{Watt}}$</td>
<td>$K_M$</td>
<td>8.70</td>
</tr>
<tr>
<td>ELECTRICAL TIME CONSTANT</td>
<td>MILLISECOND</td>
<td>$\tau_F$</td>
<td>TBD</td>
</tr>
<tr>
<td>MECHANICAL TIME CONSTANT</td>
<td>MILLISECOND</td>
<td>$\tau_M$</td>
<td>12.8</td>
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<tr>
<td>DAMPING FACTOR (ZERO IMPEDANCE)</td>
<td>OZ-IN/(RAD/SEC)</td>
<td>$f_0$</td>
<td>0.53</td>
</tr>
<tr>
<td>ROTOR INERTIA</td>
<td>OZ-IN–SEC$^2$</td>
<td>$J_M$</td>
<td>6.8x10$^{-3}$</td>
</tr>
<tr>
<td>THEO. NO–LOAD SPEED @ 48 VDC</td>
<td>RPM</td>
<td>$S_0$</td>
<td>3668</td>
</tr>
<tr>
<td>SPEED @ 21.3 OZ–IN &amp; 48 VDC</td>
<td>RPM</td>
<td>$S_1$</td>
<td>3000</td>
</tr>
<tr>
<td>THEO. ACCELERATION @ $T_p$</td>
<td>RAD/SEC$^2$</td>
<td>$\alpha_T$</td>
<td>7.35x10$^3$</td>
</tr>
<tr>
<td>MAX ALLOWABLE WINDING TEMP</td>
<td>°C</td>
<td>TEMP</td>
<td>155</td>
</tr>
<tr>
<td>NUMBER OF PHASES/WINDING TYPE</td>
<td></td>
<td></td>
<td>3/Y</td>
</tr>
<tr>
<td>NUMBER OF POLES</td>
<td></td>
<td></td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WINDING CONSTANTS*</th>
<th>UNITS</th>
<th>TOL</th>
<th>SYM</th>
<th>NOM. VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC RESISTANCE</td>
<td>OHMS</td>
<td>±12.5%</td>
<td>R</td>
<td>4.14</td>
</tr>
<tr>
<td>VOLTAGE @ $T_p$</td>
<td>VOLTS</td>
<td>NOMINAL</td>
<td>$V_p$</td>
<td>11.7</td>
</tr>
<tr>
<td>CURRENT @ $T_p$</td>
<td>AMPERES</td>
<td>NOMINAL</td>
<td>$I_p$</td>
<td>2.82</td>
</tr>
<tr>
<td>TORQUE SENSITIVITY</td>
<td>OZ–IN/AMP</td>
<td>±10%</td>
<td>$K_T$</td>
<td>17.7</td>
</tr>
<tr>
<td>BACK EMF CONSTANT</td>
<td>VOLTS/(RAD/SEC)</td>
<td>±10%</td>
<td>$K_B$</td>
<td>0.125</td>
</tr>
<tr>
<td>INDUCTANCE @ 1 KHZ</td>
<td>MILLIHENRY</td>
<td>±30%</td>
<td>L</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Before installing the motor on the drive system, it was tested and characterized on a standalone testbed to isolate the motor and transmission dynamics. A CAD model of the motor test assembly is shown in Figure 3-37. The assembly consists of an aluminum housing that uses a back iron sleeve to retain the magnetic flux and two bearings to maintain the motor airgap.
The engineering model is shown in in Figure 3-38.

The motor assembly was then connected to a dynamic torque sensor and hysteresis brake, which allows the testing of the motor under incremental friction loads. The motor tesbed is presented in Figure 3-39. The experimental results are presented in Chapter 4.
3.10 Example Application

The adaptation of the actuator into a planar robotic arm is presented in Figure 3-40. In addition to its compact size and large torque output, the actuator structure supports both single and double shear links on the output and ground members respectively. When mounted on a double shear link, the torsional stiffness of the joint is greatly increased as it acts in parallel to the central shaft. Furthermore, all feedback and communication electronics are integrated and housed within the drive system assembly.

![3D rendering of the proposed arm concept](image)

**Figure 3-40**: 3D rendering of the proposed arm concept

The following robotic arm design possess both high payload-to-weight and a very compact profile, allowing the arm to perform exceptionally well in mobile deployment applications. The key enabling technology for the arm is its compact actuation system, which is able to supply high
torques and provide a rigid joint structure, allowing the arm to manipulate heavy payloads with
dexterity and precisions.

Figure 3-41: Serial robotic arm concept shown in a folded and extended position.

The all-in-one nature of this design facilitates the development of modular, high-payload systems
that are reconfigurable and adaptable to the task at hand. These joints can all have a similar
compact standard size but each, according to its gear ratio, can deliver a different performance.
This approach could improve modern manipulation by introducing various modular joints that
can be substituted for different tasks, such as manipulating heavy objects slowly and precisely, or
moving lighter objects with speed and agility.

3.11 Case Study I: GBD Comparison Against Conventional Space Robots Actuators

To assess the viability of this technology in space applications, a comparative analysis is performed
relative to a flight actuator supplied by NASA’s Jet Propulsion Laboratory. The standard model of
a geared space actuator is presented in Figure 58. It comprises of a motor assembly, multi-stage
planetary gear train, and output mechanical port driving the robot’s joint. In this design, the mechanical power travels between the stages through a carrier arm as shown in Figure 3-42.

Because of this arrangement, the gear ratio (per stage) is proportional to the carrier arm radius over the sun gear pitch radius. This imposes constraints on the sun gear pitch radius and leads to sun gear undercut when the ratio-per-stage exceeds 8:1. As a result of this limitation, this arrangement requires multiple stages to achieve high ratios, which leads to long, bulky and complex assemblies with large number of parts and more weight. The detailed comparison is shown in Table 3-6.

Table 3-6: GBD comparison against a Mars Science Laboratory actuator

<table>
<thead>
<tr>
<th>Comparison Metric</th>
<th>Proposed GBD</th>
<th>Standard Flight Actuator A338 Actuator (MSL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>120 mm</td>
<td>95 mm</td>
</tr>
<tr>
<td>Length</td>
<td>57 mm</td>
<td>152 mm</td>
</tr>
<tr>
<td>Total number of parts</td>
<td>14</td>
<td>70</td>
</tr>
<tr>
<td>Number of moving parts</td>
<td>6</td>
<td>32</td>
</tr>
<tr>
<td>Nominal efficiency</td>
<td>90-95%*</td>
<td>95-99%</td>
</tr>
<tr>
<td>Rated Torque</td>
<td>271 Nm*</td>
<td>165 Nm</td>
</tr>
<tr>
<td>Nominal Speed</td>
<td>2.5 rpm*</td>
<td>1.1 rpm</td>
</tr>
</tbody>
</table>

*Estimated from analytical model
It is evident that the proposed GBD concept drastically reduces the number of parts of a conventional robot’s joint drive system, leading to compact and more reliable space systems. Reducing the number of parts is strongly correlated with improved reliability and lower risk of failure which is of paramount importance to space missions.

![Diagram of transmission model comparison between a conventional flight actuator vs the proposed drive system](image)

**Figure 3-43:** Transmission model comparison between a conventional flight actuator vs the proposed drive system

**3.12 Case Study II: GBD Comparison with a Conventional Harmonic Drive Transmission**

A comparative analysis between the proposed drive system and harmonic drives is subject to application requirements since harmonic drives are sole transmissions with highly non-linear stiffness and friction. However, a key distinction between the GBD transmission and harmonic drive is the low efficiency of harmonic drives which is traced to the sliding teeth friction occurring
in its transmission. To illustrate these differences, one GBD configuration with a gear ratio of 1:2116 is compared against size 25 harmonic transmission in Table 3-7. In the case of a space application where speed is secondary to torque, the integrated GBD concept could lead to a smaller and more efficient drive system compared to a space robot drive system that uses a harmonic drive transmission. This is due to the fact that the high gear ratio of the GBD transmission reduces the maximum torque requirement on the motor, thereby allowing the use of a smaller motor which occupies less space, consumes less current, and generates less heat compared to a motor driving a harmonic drive transmission. Also, because the GBD has a high gear-ratio, it is not backdrivable, and as such it does not require a brake on the motor shaft, which would consume volume and electric energy. A study of energy efficiency vs gear ratio is presented in Chapter 5.

In addition, the GBD stiffness is likely to be higher than the harmonic drive because of its rigid gearing components, as opposed to the harmonic drive flexibility. Increasing the GBD stiffness and torque density can be achieved by adding more planets into the GBD assembly. This is made possible due to the GBD carrier-less design, which relaxes the kinematic constraints on the mechanism to accept more planets as opposed to a carrier-based design.

3.13 Chapter Conclusions

A design concept and a mockup prototype for a compact joint drive system were developed in this chapter. With the motor and transmission and joint structure integrated and optimized together, higher torque densities could be achieved relevant to existing drive systems involved in space robotic applications. The proposed technology overcomes gearing packaging, efficiency and reliability problems of current space drive systems, allowing the development of high payload-to-weight robots that are not possible with conventional drives. An example of a high payload-to-
weight robotic arm was proposed based on the GBD concept with the ability to deploy and operate in highly confined spaces.

**Table 3-7:** GBD comparison against a drive system that uses a harmonic drive transmission

<table>
<thead>
<tr>
<th>Comparison Metric</th>
<th>Proposed GBD</th>
<th>Harmonic Drive Transmission Size 25 (CSF, SHF, SHD series)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>120 mm</td>
<td>107 mm</td>
</tr>
<tr>
<td>Length</td>
<td>57 mm</td>
<td>52 mm</td>
</tr>
<tr>
<td>Total number of parts</td>
<td>14</td>
<td>5</td>
</tr>
<tr>
<td>Number of moving parts</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Nominal efficiency</td>
<td>90-95%*</td>
<td>67-70%</td>
</tr>
<tr>
<td>Nominal Torque</td>
<td>177 Nm*</td>
<td>178 Nm</td>
</tr>
<tr>
<td>Nominal Speed</td>
<td>2 rpm*</td>
<td>12 rpm</td>
</tr>
<tr>
<td>Gear Ratio</td>
<td>1:2116</td>
<td>1:160</td>
</tr>
<tr>
<td>Required Motor Torque (assumed no friction losses)</td>
<td>0.08 Nm</td>
<td>1.11 Nm</td>
</tr>
</tbody>
</table>
Chapter 4 : Experimental Results and Dynamic Model Development

4.1 Introduction

An experimental setup was developed to characterize the behavior of the transmission by collecting its input and output positions, velocities and torques. This was done by placing an encoder and a torque sensor at the input and output ports of the motor and transmission. The output torque sensor measures the load torque of the transmission while the input torque sensor measures the supplied motor torque. Both sensors were calibrated using precision weights. A large inertia with a built in magnetic brake is connected to the output port of the transmission. A non-contact magnetic brake is employed to enable precise friction torque control. All sensors and command signals are interfaced with dSPACE and manipulated using Simulink. A CAD model of the experimental setup is shown in Figure 4-1.

Figure 4-1: CAD model of the experimental setup
### Table 4-1: Description of experimental data acquisition

<table>
<thead>
<tr>
<th>Drive System Parameter</th>
<th>Acquisition Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Torque</td>
<td>The input torque is measured using a non-contact shaft-to-shaft rotary torque sensor. The torque signal is collected using DSPACE and filtered using an RC filter with a 100 HZ cutoff frequency. The torque sensor was calibrated in both clockwise and counterclockwise directions using precision weights. It is validated using the drive motor current and torque constant.</td>
</tr>
<tr>
<td>Output Torque</td>
<td>The output torque is measured using a dynamic torque sensor mounted at the output port of the transmission.</td>
</tr>
<tr>
<td>Motor Speed and Output Shaft Speed</td>
<td>The input and output speeds are acquired through high resolution Tonic RESM rotary encoder systems mounted on the motor rotor input shaft, and torque sensor shaft. The total encoder resolution is around 5-7 arcsec.</td>
</tr>
<tr>
<td>Motor Current</td>
<td>For 3-phases AC motors, the current drawn by the motor is measured using three current shunt sensors enclosing each wire cross section. The current is correlated with the controller output current for accuracy verification.</td>
</tr>
<tr>
<td>Motor Voltage</td>
<td>The motor voltage is collected using an oscilloscope for AC motors, and standard multimeter for DC motors.</td>
</tr>
<tr>
<td>Input Electrical Power</td>
<td>The input power is determined from the product of motor current and motor voltage. The power to the controller is measured using an AC powermeter.</td>
</tr>
<tr>
<td>Output Mechanical Power</td>
<td>The mechanical power is determined from the product of output speed from the output encoder and output torque sensor.</td>
</tr>
<tr>
<td>Backlash</td>
<td>The backlash is collected from the relative position difference between the two high precision encoders. A dedicated experimental station was developed for measuring backlash and hysteresis (See Fig. 4-6)</td>
</tr>
<tr>
<td>Mechanism Rotary Stiffness</td>
<td>The gear stiffness is calculated from the relative twist in the mechanism after the backlash has been cleared. In this test, the motor rotor is locked while an output torque is applied at its output.</td>
</tr>
<tr>
<td>Peak and Continuous Torques</td>
<td>The peak and continuous torques are determined from the torque-speed curve. The peak torque is the maximum torque delivered by the drive system at the rated motor voltage, the continuous torque is the torque at which the actuator delivers the most power.</td>
</tr>
<tr>
<td>Backdriving Torque</td>
<td>The system is backdriven through the brake using a wrench with an elongated moment arm. The backdriving torque is recorded by the torque sensor when the motor is on the verge of backspinning.</td>
</tr>
</tbody>
</table>
The final experimental setup included a dynamic torque sensor that measures the incoming motor torque and speed into the transmission as presented in Figure 4-2.

**Figure 4-2:** Engineering model of the experimental setup
4.2 Kinematic Error Characterization

For applications that require precise position control of the transmission output, a good understanding of the transmission kinematic error and backlash is necessary. Although the transmission’s gear-ratio per output revolution is constant as dictated by the number of teeth on the gears, the instantaneous gear-ratio changes as it depends on the manufacturing variations along the effective pitch diameter of the gears. As such, the transmission error, defined as the difference between the expected output displacement and the actual output displacement, varies along the angular position of the input shaft. However, the aggregate error averages to zero after one full output rotation to maintain the discrete mechanical advantage of the transmission. As a result of its cyclic nature, this error acts as an exciter to the transmission elements and causes noise and vibrations corresponding to its frequency contents. The kinematic error equation is presented in Equation 47, where $N$ is the gear-ratio of the transmission. The transmission parameters are listed in Table 4-2.

$$\phi_{error} = \phi_{out} - \frac{\phi_{in}}{N}$$ (47)

<table>
<thead>
<tr>
<th>Component</th>
<th>Pitch Diameter [mm]</th>
<th>Number of Teeth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>44.01</td>
<td>52</td>
</tr>
<tr>
<td>Input Planet</td>
<td>21.99</td>
<td>26</td>
</tr>
<tr>
<td>Ground Ring</td>
<td>88.05</td>
<td>104</td>
</tr>
<tr>
<td>Output Planet</td>
<td>19.89</td>
<td>25</td>
</tr>
<tr>
<td>Output Ring</td>
<td>85.93</td>
<td>108</td>
</tr>
<tr>
<td>Gear-Ratio</td>
<td>-39.5:1</td>
<td></td>
</tr>
</tbody>
</table>

To capture the kinematic error of the gear bearing drive transmission over one full output rotation, the motor was commanded in position mode under a low but non-zero velocity using a PID controller. The low motor velocity was used to avoid contaminating the kinematic error.
signature by possible high-speed effects such as resonance. The error is plotted as a function of the number of input revolutions as shown in Figure 4-3.

Figure 4-3: Transmission error displayed by the gear bearing drive transmission

The kinematic error is shown to vary from 0.0982 to -0.1576 degrees over the course of one output revolution. To identify the sources of this error, a spectral analysis of the error waveform was performed relative to the number of input revolutions as illustrated in Figure 4-4. The FFT spectrum clearly shows three major frequency component located at 0.02532, 1.004 and 1.367 cycles-per-input-revolutions. The largest error component, located at 0.02532 cycles-per-input-revolution, is primarily due to the inaccuracies in the assembly of the output ring gear, which cycles once every 39.5 input revolutions and matches with the frequency spectrum where 1/0.02532 = 39.5. The second error component, located at 1.004 cycles-per-input-revolution, is due to the placement of the sun gear and is contributing around 0.01 degrees in the kinematic error.
Figure 4-4: Frequency analysis of the kinematic error

Significant error losses are observed at 1.367 cycles-per-input-revolution. This error component is due to the teeth-passing frequency of the output planets and output ring gear and is attributed to the placement of the output planets and output ring gear. Specifically, the planets complete
one full revolution with respect to the ground ring gear when the input (sun gear) is displaced by 26 teeth relative to the input planet, equivalent to 26/52 input revolutions. At the same time, the output planets would have traveled a distance of 25/108 input revolutions along the output ring gear pitch diameter. Therefore, the planets complete one full revolution every 26/52 + 25/108 = 0.73148 input revolution. This error cycles once every 1/0.731 = 1.367 input revolutions and matches exactly with the frequency spectra in Figure 4-4. Interestingly, no errors are observed at the tooth passing frequency of the input planets and ground ring gear, which would have appeared at 1/ (26/52 + 26/104) or 1.5 cycles-per-input-revolutions. These results are due to the fact that the input ring gear was floating while the output ring gear was constrained in our experimental setup. As a result, the floating ground ring had the freedom to slightly move radially and absorb the placement error of the input planets. Having measured the magnitude and phase of the individual error contributions, it is possible to mathematically reconstruct this error behavior using a harmonic series. The aggregate effect is compiled into Equation 18:

$$\phi_{error} = A_r \sin(w_r \phi_m + \beta_r) + A_{p-r} \sin(w_{p-r} \phi_m + \beta_{p-r}) + A_s \sin(w_s \phi_m + \beta_s)$$  \hspace{1cm} (48)$$

Where $A_r$, $A_{p-r}$ and $A_s$ are the error amplitudes of the ring, planet and sun gear respectively, and $w_r$, $w_{p-r}$ and $w_s$ are their frequency of occurrence in terms of input revolutions. These parameters are identified from the spectral analysis of Figure 4-4 and are listed in Table 4-3.

**Table 4-3:** Error contributions of gear bearing drive transmission’s components

<table>
<thead>
<tr>
<th>Component</th>
<th>Frequency [Cycles-per-input-revolutions]</th>
<th>Error Amplitude [Degrees]</th>
<th>Phase [Radians]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Ring</td>
<td>0.02532</td>
<td>0.0530</td>
<td>1.8</td>
</tr>
<tr>
<td>Planets/Rings</td>
<td>1.367</td>
<td>0.01745</td>
<td>6.095</td>
</tr>
<tr>
<td>Sun Gear</td>
<td>1.004</td>
<td>0.01154</td>
<td>6.658</td>
</tr>
</tbody>
</table>

Based on these results, the major components of the gear-bearing-drive transmission kinematic error are due to the assembly placement between the ring and output planets. The high frequency
excitations are located at $F_{\text{high}} = 1 / (N_{\text{planet}}/N_{\text{sun}} + N_{\text{planet}}/N_{\text{ring}})$ and 1 times the motor speed. These frequency components should be considered when designing the gear bearing drive for high-speed applications to reduce vibrations and to also avoid exciting resonant frequencies. Since compliance and speed can also influence the kinematic error, it is recommended that future experiments be performed to characterize the kinematic error under truly dynamic circumstances.

![Figure 4-5: More detailed segment of the kinematic error and its simulation results](image)

### 4.3 Stiffness and Hysteresis Loss

The stiffness of the gear bearing drive transmission was measured at both its input and output shafts. The output stiffness was obtained by rigidly locking the input shaft while applying incremental loads on its output shaft. To capture both backlash and hysteresis, a bidirectional arm with a sliding weight as shown in Figure 4-6 was fabricated to produce a continuous torque profile without unloading the transmission output as presented in Figure 4-7.

![Figure 4-6: Measurement setup for the output stiffness](image)
Similarly, the input stiffness was recorded by locking the transmission output shaft and measuring the deflection and torque at the input shaft. We note the placement of the encoder before the torque sensor along the load line, such that the encoder is blind to the deflections occurring inside the torque sensor. The stiffness data was acquired multiple times and the results are within 10 to 15% of the mean values.

**Figure 4-7:** Stiffness curve of the transmission output

A data regression of the input and output stiffness measurement are illustrated in Figure 4-8 and Figure 4-9. The hysteresis dissipation in the output stiffness varies between approximately 2-3 Nm and the backlash is determined to be 0.2 degrees. It is noted that the backlash in planetary gearsets depends on many variables such as manufacturing errors, mounting tolerances and bearing play.
Both stiffnesses exhibit a stiffening behavior at low torques that become near linear at higher torques. This behavior is likely due to clearances and misalignments in the transmission assembly that causes non-uniform load sharing in the assembly. A linear regression analysis
shows that the input and output stiffnesses curves can be accurately captured by cubic polynomials, which are free of torsional wind up or abrupt changes in stiffness associated with highly non-linear behaviors.

A preliminary stiffness model is proposed for the input and output stiffness based on the geometry of the transmission and the gear teeth placement. A simplified schematic is shown in Figure 4-10 where the gear teeth are modeled as linear springs. In this model, $K_2$ is the equivalent stiffness of the ring and planet teeth pair, $K_1$ is the equivalent stiffness for the sun gear and input planet teeth pair, and $K_3$ corresponds to the equivalent stiffness of the output ring gear and output planet teeth pair.

The effective stiffness observed at the transmission output when the motor shaft is locked can be written as:

$$K_{\text{eff, out}} = \frac{F_{\text{out}}}{X_3}$$

(49)
The planet subassembly behaves as one rigid body that rotates and translates with respect to an instantaneous center of rotation, located at the ground ring and input planet mesh point. This is graphically illustrated in Figure 4-11.

**Figure 4-11:** Force balance on the planet subassembly

The equilibrium of static forces and moments on the planet subassembly lead to the following relationships:

\[
F_{\text{out}}(b - d) = 2F_{\text{in}}d
\]

\[
F_{\text{in}} + F_{\text{out}} = F_g
\]

The forces within the individual springs can be written as:

\[
F_g = K_2X_0
\]

\[
F_{\text{out}} = K_3(X_3 - X_4)
\]

\[
F_{\text{in}} = -K_1X_2
\]

Using the above relationships, the deflection of the ground ring gear can be solved for as:

\[
X_0 = \frac{X_4(b + d)}{2b} + \frac{X_2(b - d)}{2b}
\]

In order to obtain the effective stiffness with respect to the output force, the expressions of the ground and input forces are solved in terms of the output force \(F_{\text{out}}\) such that:

\[
F_{\text{in}} = F_{\text{out}} \frac{(b - d)}{2d} \quad \text{and} \quad F_g = F_{\text{out}} \frac{(b + d)}{2d}
\]
The effective deflection due to the output force can be shown to be:

\[ X_3 = \frac{F_{out}}{K_3} + X_4 = \frac{F_{out}}{K_3} + \frac{2b}{b + d} \frac{F_{out}(d + b)}{2K_2d} + \frac{(d - b)(d - b)F_{out}}{2bK_1b} \]  \hspace{1cm} (57)

The total effective stiffness of the transmission as seen at the output port is the ratio of output force over the inertial deflection at the output is written in Eq. 58:

\[ K_{eff, out} = \frac{F_{out}}{X_3} = \frac{1}{\frac{1}{K_3} + \frac{b}{dK_2} + \frac{(d - b)^2}{4b^2K_1}} \]  \hspace{1cm} (58)

In the case where the transmission has a high mechanical advantage, i.e. \( b \approx d \), the effective transmission stiffness reduces to a two-spring-in-series such as:

\[ K_{eff, out} = \frac{d}{\frac{b}{K_3} + \frac{b}{K_2}} \]  \hspace{1cm} (59)

Using a similar approach, the effective input stiffness can be formulated as:

\[ K_{eff, in} = \frac{1}{\frac{1}{K_1} + \frac{2b(b + d)}{(b - d)^2K_2} + \frac{2d(b + d)}{(b - d)^2K_3}} \]  \hspace{1cm} (60)

Due to the relatively low torque at the input shaft compared to the output shaft, the deflection at the sun gear can be assumed negilible where \( K_1 \) is infinitly high. As a result, a new expression for the input stiffness would be:

\[ K_{eff, in} = \frac{1}{\frac{2b(b + d)}{(b - d)^2K_2} + \frac{2d(b + d)}{(b - d)^2K_3}} \]  \hspace{1cm} (61)

Since \( K_2 \) and \( K_3 \) are the stiffnesses of the gear pairs of the ground_ring/input_planet and output_ring/output_planet respectively, their values can be approximated to be equal since the two stages would have near-identical diamteral pitches. We then let \( K_2 = K_3 = K_{avg} \) and substitute these in both the input and output stiffnesses and obtain:
\[ K_{\text{eff.in}} = \frac{K_{\text{avg}}}{2b(b + d)} \frac{2d(b + d)}{(b - d)^2} \quad \text{and} \quad K_{\text{eff.out}} = \frac{dK_{\text{avg}}}{d + b} \] (62)

Using the updated values for the input and output stiffness, their ratio is obtained as:

\[ \frac{K_{\text{eff.out}}}{K_{\text{eff.in}}} = \frac{2d(b + d)}{(b - d)^2} \] (63)

Equation 63 presents an important dimensionless equation that relates the input and output stiffnesses of the gear bearing drive transmission, which suggests that the transmission stiffness is mainly characterized by the geometric parameters of the planets gears.

### 4.4 Dynamic Friction Measurements

The gear bearing drive transmission exhibits finite power losses due to its gear-mesh friction and velocity-dependent damping. The static friction is determined from the minimum breakaway torque that triggers a steady output motion and is measured along four equally spaced planets locations. The average starting torque of the GBD transmission is determined to be 0.0165 Nm.

<table>
<thead>
<tr>
<th>Planets Location [degrees]</th>
<th>0</th>
<th>90</th>
<th>180</th>
<th>270</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting torque [Nm]</td>
<td>0.016</td>
<td>0.017</td>
<td>0.016</td>
<td>0.017</td>
</tr>
</tbody>
</table>

Table 4-4: Starting torque of the Gear Bearing Drive transmission

These relatively low friction values are due to the fact that there are no preload forces in the assembly of the GBD transmission. To study the velocity-dependent frictional losses, the input and output transmission velocities were recorded for input current steps that varies from 0.5 to 1.1 Amp by a 0.1 Amp increment. The current loop was tuned using a Copley Controller (Junus, Model: 800-1468) to reach the commanded motor current in less than 2.5 milliseconds. The input and output unfiltered velocities are presented in Figure 4-12 and Figure 4-13. The output velocity is negative because the transmission output move in the opposite direction of the input. From the two plots shown below, several observations about the dynamic friction losses in the gear bearing drive transmission can be made. First, the input velocity response displays a strong
first order behavior in the transient phases. There are some velocity fluctuations at low currents which are attributed to a low cyclic frictional torque that becomes more dominant at low motor torques and also due to the kinematic error as will be shown later. Second, the steady state input velocity does not scale up linearly with the input current at higher speeds, suggesting the presence of non-linear friction torque. These dynamic losses are influenced by many factors such as type and amount of lubrication and gear operating and mass imbalances resulting from manufacturing tolerances.

Figure 4-12: Input velocity step-responses
The friction in the gear bearing drive transmission is modeled to account for the velocity-independent friction and the dynamic friction losses. The prototype used a dense lithium grease which resulted in more friction than using transmission oil. From the results illustrated in Figure 4-14 the velocity-independent friction can be approximated by a constant torque offset which remain unchanged until about 1400 rpm, while the velocity-dependent damping can be accurately captured by a quadratic function.
Furthermore, the DC motor torque constant is captured dynamically using linear regression of the input torque and input current data and is within 1% of the motor datasheet specification.
The efficiency curves for various constant output torques are presented in Figure 4-16. The efficiency improves with higher torques and reaches a maximum of 83%. The drop in efficiency at higher speeds is due to the amplification of frictional losses. The efficiency values of this first prototype are deemed acceptable given that it uses grease lubrication. It is expected that the efficiency of a more elaborate design that uses oil lubrication with proper sealing would be higher.

![Efficiency measurements of the gear bearing drive transmission](image)

**Figure 4-16:** Efficiency measurements of the gear bearing drive transmission

### 4.5 Modeling Considerations

Following the analysis of the experimental data, a simple lumped parameter model for the drive system is proposed that accounts for the effects of the input and output stiffnesses, kinematic error, friction and input and output inertias. The kinematic error is placed after the transmission.
gain such that to excite the output stiffness. The transmission gain is assumed to be a pure torque amplifier between the input stiffness and kinematic error.

**Figure 4-17**: Schematic of the transmission dynamic model

The model is segmented into individual elements that describe the constitutive relationships along the dynamic system as shown in Figure 4-18.

**Figure 4-18**: Constitutive relationships of the dynamic model

The values of friction, kinematic error, and stiffness are computed from the experimental data that was presented in the previous section, where:
\[ f_{friction}(\dot{\phi}_m) = \begin{cases} 0.0125 \text{Nm}, & 0 < \dot{\phi}_m < 140 \text{ rad/s} \\ -6.5437 \times 10^{-8} \dot{\phi}_m^2 + 0.00010073 \dot{\phi}_m - 0.000228 \text{Nm}, & \dot{\phi}_m \geq 140 \text{ rad/s} \end{cases} \]

The input and output stiffnesses are found from the linear regression of the torque profiles which were presented in the previous section as:

\[ K_{in} = \frac{T_{in}}{\phi_m - \phi_{k,in}} \]  \hspace{1cm} (64)

Where the input and output stiffnesses are written as:

\[ K_{in}(\phi_m, \phi_{k,in}) = (K_{in,1})(\phi_m - \phi_{k,in}) + K_{in,0} \]  \hspace{1cm} (65)

\[ K_{out}(\phi_m, \phi_{e2}) = (K_{out,1})(\phi_{e2} - \phi_{out}) + K_{out,0} \]  \hspace{1cm} (66)

The stiffness coefficients \( K_{in,1}, K_{in,0}, K_{out,1}, K_{out,0} \) are equal to 0.545, 0.647, 68615, 968.76 respectively as determined from the quadratic regressions shown in Figure 4-8 and Figure 4-9.

The third terms of the regression coefficients are due to the residual torques in the transmission and are therefore removed from the effective stiffness values. By assuming that the mechanical energy is conserved across the springs and kinematic error elements, the absolute deflections in the input and output springs can be solved in terms of the motor and load deflections and kinematic error as in Equations 67-68:

\[ \phi_{k,in}(\phi_m, \phi_{out}, \phi_{error}) = \frac{N^2 K_{in} \phi_m + N K_{out} \phi_{out} - N K_{out} \phi_{error}}{K_{out} + N^2 K_{in}} \]  \hspace{1cm} (67)

The kinematic error introduces a position error in the transmission output where:

\[ \phi_{e2}(\phi_{k,in}, \phi_{error}) = \phi_{e1} + \phi_{error} = \frac{\phi_{k,in}}{N} + \phi_{error} \]  \hspace{1cm} (68)

The error is constructed using the parameters identified in Table 4-3 such that:

\[ \phi_{error}(\phi_m) = 0.0530 \frac{\pi}{180} \sin\left(\frac{0.02532\phi_m}{2\pi} + 1.8\right) \]

\[ + 0.01745 \frac{\pi}{180} \sin\left(\frac{1.367\phi_m}{2\pi} + 6.09\right) \]  \hspace{1cm} (69)
The equations of motion for the motor and load were derived using both energy and direct methods as shown in Equations 70 and 70 respectively.

\[ J_m \ddot{\phi}_m + K_{in,0}(\phi_m - \phi_{k,in}) + K_{in,1}(\phi_m - \phi_{k,in})^2 + f_{friction} = K_T i \]  

(70)

And,

\[ J_L \ddot{\phi}_{out} + K_{out,0}\left(\frac{\phi_{k,in}}{N} + \phi_{error} - \phi_{out}\right) + K_{out,1}\left(\frac{\phi_{k,in}}{N} + \phi_{error} - \phi_{out}\right)^2 + F_L = 0 \]  

(71)

By substituting Equations 67-69 into 70 and 71, the equations of motion were recast as functions of the input and output states as in Equations 72 and 73. The kinematic error appears as a forcing function on both the input and output stiffnesses where:

\[ J_m \ddot{\phi}_m + \phi_m \left( K_{in} - \frac{N^2K_{out}^2}{K_{out} + N^2K_{in}} \right) + f_{friction} = \frac{K_{in}K_{out}N}{K_{out} + N^2K_{in}} \phi_{out} + \frac{K_{in}K_{out}N}{K_{out} + N^2K_{in}} \phi_{error} + K_T i \]  

(72)

And,

\[ J_L \ddot{\phi}_{out} + \phi_{out} \left( K_{out} - \frac{K_{out}^2}{K_{out} + N^2K_{in}} \right) + F_L = \frac{K_{in}K_{out}N}{K_{out} + N^2K_{in}} \phi_m + \left( \frac{k_{out}^2}{K_{out} + N^2K_{in}} - K_{out} \right) \phi_{error} \]  

(73)

Finally, the system of equations was placed in state-space and solved using the Runge-Kutta algorithm of MATLAB where the stiffness, friction, and kinematic error are updated following every loop iteration (See appendix C). Only minor modifications to the damping values were made before good model predictions are achieved in the open loop response. The modifications were within 6% of the original damping values. Comparison between the experimental data and
simulation results are presented in Figure 4-19 to Figure 4-21 where the system is subject to input current step functions increasing from 0.6 to 1 Amp.

**Figure 4-19:** Comparison between the measured and simulated input velocity response

**Figure 4-20:** Comparison between the measured and simulated output velocity response
The contribution of the kinematic error due to the output ring gear is shown to approximate the vibrations of the output velocity and therefore a more accurate manufacturing would reduce the amplitudes of these oscillations.

**Figure 4-21**: Simulation of open loop output velocity step-responses with kinematic error

The comparison between the measured and simulated velocity response under free sinusoidal motion is illustrated in Figure 4-22. It is evident that the fidelity of simulation to actual measurements is significant and valid over the entire speed and dynamic range of the drive system.
The analysis of the open-loop time response shows that the GBD behavior is dominated by linear dynamics with finite frictional losses and negligible stiffness effects. It is determined that the open-loop behavior is highly sensitive to friction and geometric errors as opposed to stiffness. A more in-depth analysis of the transmission stiffness will be performed in future work where the transmission will be operated in constrained-motion in which a suitable force controller will be developed.

4.6 Chapter Conclusions

In this chapter, the open-loop behavior of the gear bearing drive transmission is tested and characterized. Simple but accurate models for the transmission’s friction, stiffness and kinematic error are obtained. A preliminary dynamic model is proposed that captures the open-loop response of the system with good accuracy, including contribution of the kinematic error.
Analysis of the dynamic model suggests that the transmission behavior is dominated by linear dynamics that are characterized by finite frictional losses and geometric errors in its gear mesh. In addition, the transmission shows no unpredictable open-loop dynamics or transmission resonance within its speed range, making its closed-loop control problem a simple task. These desirable characteristics combined with its compact size and light weight make the gear bearing drive a useful actuator in a variety of applications, including robotic arms, prosthetics, powered winches and bionics.
Chapter 5 : Impact on Manipulation

In the following chapter, we study the impact of using the Gear Bearing Drive concept versus a more conventional design in the manipulator arm shown in Figure 5-1. The kinematic workspace and energy consumption of the arm are compared for both designs during various tasks.

The two drive systems have a near-similar torque output but different weight, size and gear-ratio as listed in Table 5-1. It is assumed that the transmission stiffness is linear and that there are no backlash or play in the mechanism. The arm consists of two joints with relative compliance and structural damping between the motors and transmissions. Each joint includes motor inertia ($J_m$) and viscous damping between the rotor and housing ($b_1$), transmission stiffness ($K_t$) and damping ($b_t$), viscous damping between the transmission and housing ($b_2$) and transmission inertia and mass, $J_t$ and $m_a$ respectively. The length of the drive system is ($L_a$) and its gear ratio is $N$. The payload of the arm is assumed to be a point mass at the extremity of the link.

The states ($\phi_1$) and ($\theta_1$) represent the inertial angular displacements of the motor and transmission respectively for Joint 1, and ($\phi_2$) and ($\theta_2$) defines the same states for Joint 2.
Figure 5-1: Two degrees of freedom robotic arm model
The parameters of a conventional design supplied by NASA JPL and the proposed actuator concept are listed in Table 5-1.

**Table 5-1: Parameters of a conventional design vs proposed concept**

<table>
<thead>
<tr>
<th>Comparison Metric</th>
<th>Standard Flight Actuator A338 (MSL)</th>
<th>Proposed actuator concept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>95 mm</td>
<td>120 mm</td>
</tr>
<tr>
<td>Length ($L_a$)</td>
<td>152 mm</td>
<td>58 mm</td>
</tr>
<tr>
<td>Nominal Torque</td>
<td>165 Nm</td>
<td>180 Nm</td>
</tr>
<tr>
<td>Nominal Speed</td>
<td>1.1 rpm</td>
<td>2.5 rpm</td>
</tr>
<tr>
<td>Mass</td>
<td>2.6 Kg</td>
<td>1.1 Kg</td>
</tr>
<tr>
<td>Gear-Ratio</td>
<td>160</td>
<td>900</td>
</tr>
<tr>
<td>Total number of parts</td>
<td>70</td>
<td>14</td>
</tr>
<tr>
<td>Number of moving parts</td>
<td>32</td>
<td>6</td>
</tr>
</tbody>
</table>

In order to perform a comparative analysis between the two designs and their effect on the manipulator performance, the dynamic and kinematic relationships between the manipulator and the drive system are derived. A generic drive system is modeled as a two-inertia, fourth-order system with compliance and structural damping between the motor and transmission. The transmission element is assumed to act as a linear torque amplifier with torque efficiency $n$. A free body diagram of the drive system is presented in Figure 5-2.

**Figure 5-2: Schematic and free body diagram of a generic drive system model**
The equation of motion for joint 1’s motor is shown in Eq. 74. All equations of motion were derived using the direct method and verified using the Lagrangian formulation.

\[ J_m \left( \frac{d^2}{dt^2} \Phi_1(t) \right) + (bI + bt) \left( \frac{d}{dt} \Phi_1(t) \right) - Tm_1 + Kt \Phi_1(t) - Kt N \theta_1(t) - bt N \left( \frac{d}{dt} \theta_1(t) \right) \]  

(74)

The output torque of joint 1’s transmission, \( T_L \), is equal to the rate of change of the manipulator’s angular momentum with respect to time in the \( z \)-direction.

\[ T_L = \frac{dL}{dt}. \hat{z} \]  

(75)

Substituting the angular momentum equation into Eq. 74, the equation of motion for joint 1’s transmission is obtained as:

\[ (mL^2 + mL^2 \cos(\theta_2(t))^2 + maL^2 + Ja) \left( \frac{d^2}{dt^2} \theta_1(t) \right) - mL \left( \frac{d^2}{dt^2} \theta_2(t) \right) L_1 \sin(\theta_2(t)) + \left( -2 mL^2 \cos(\theta_2(t)) \sin(\theta_2(t)) \right) \left( \frac{d}{dt} \theta_2(t) \right) + N^2 bt n + b_2 \left( \frac{d}{dt} \theta_1(t) \right) - mL \left( \frac{d}{dt} \theta_2(t) \right)^2 L_1 \cos(\theta_2(t)) + n N \left( -bt \left( \frac{d}{dt} \Phi_1(t) \right) + Kt \left( -\Phi_1(t) + N \theta_1(t) \right) \right) \]  

(76)

Equation 76 reveals a coupling term between the joints acceleration due to the drive system length, \( L_a \). This coupling term (highlighted in underline) introduces an inertial torque in transmission 1 that is related to the angular acceleration of joint 2. This is explored in more depth in the following section. The equation of motion for motor 2 is obtained as:

\[ J_m \left( \frac{d^2}{dt^2} \Phi_2(t) \right) + (bI + bt) \left( \frac{d}{dt} \Phi_2(t) \right) - Kt \Phi_2(t) - Kt N \theta_2(t) - Tm_2 - bt N \left( \frac{d}{dt} \theta_2(t) \right) \]  

(77)

The end effector equation of motion is obtained as:

\[ (mL^2 + Ja) \left( \frac{d^2}{dt^2} \theta_2(t) \right) - m \left( \frac{d^2}{dt^2} \theta_1(t) \right) L_1 \sin(\theta_2(t)) + \left( N^2 bt n + b_2 \right) \left( \frac{d}{dt} \theta_2(t) \right)^2 L_1 \cos(\theta_2(t)) \sin(\theta_2(t)) + n N \left( K \Phi_2(t) + N \theta_2(t) \right) \]  

(78)

Finally, the system of equations is numerically solved using SIMULINK as illustrated in Figure 5-3, where \( p_i \) and \( q_i \) denotes \( \Phi_i \) and \( \theta_i \) respectively. This model describes the full dynamic
interaction between the links of the robot and input torques under a given payload $W$ and a gravity value.

**Figure 5-3:** SIMULINK model of the 2DOF manipulator

The dynamic inputs to this model are the motor torques and the outputs are the states of the joints. The graphical model of transmission 1 is shown in Figure 5-4 where an integration scheme is used to acquire the velocity and position data from acceleration.
Now that a simulation model is available, one can study the effect of various drive system parameters on the manipulator end-effector response. The kinematic data such as workspace size is extracted from the dynamic model at steady-state conditions.

**Figure 5-4:** Transmission 1 model
5.1 Impact of Drive System Length on Workspace Size

As seen in the equations of motion, the dynamically dominant size parameter of the drive system is its length along its axis of rotation. In this particular example, the conventional design is longer by 9.4 cm than the proposed concept for near-similar torque output. The volumetric workspace of the end effector is shown to increase by 120,000 cm$^3$ using the proposed concept.

![Figure 5-5: Workspace size comparison](image)

A more general analysis is performed for drive systems with length $La = 10, 20$ and $30$ cm in Figure 5-6, Figure 5-7 and Figure 5-8 respectively. The workspace is shown to increase by as much as 14% when the drive system length is reduced by 20 cm.
Figure 5-6: Manipulator workspace reach with La = 30 cm

Figure 5-7: Manipulator workspace reach with La = 20 cm
Clearly, the length of the drive system affect the kinematic workspace and stow size of the manipulator. In space applications, for instance, it is desired to stow the manipulator in the least possible volume during launch. This allows more room for scientific instruments or supplies.

### 5.2 Impact of Drive System Length on the Manipulator Joint-to-Joint Inertial Coupling

In this simulation, the joint-to-joint inertial torques of the manipulator are evaluated for various drive system lengths. These inertial coupling are reactive torques that are developed when either of the joint accelerates. These torques are proportional to the drive system length as was shown earlier in Eq. 76. In this example, the payload is simulated during a free-fall motion from the horizontal position ($\theta_2=0$) for different drive system lengths ($L_a = 10, 20$ and $30$ cm). The angular position response clearly shows an inertial disturbance torque on Joint 1 caused by the couple resulting from the drive system length as illustrated in Figure 5-9.
Figure 5-9: Joints response under payload free-fall with $L_a = 10$ cm

The joints position responses for $L_a = 20$ cm and $30$ cm are presented in Figure 5-10 and Figure 5-11 respectively. The results clearly shows increasing oscillations amplitudes on Joint 1 with an increasing $L_a$.

Figure 5-10: Joints response under payload free-fall with $L_a = 20$ cm
These results suggest that shortening the drive system length reduces the inertial coupling between the joints of the robot. This simplifies the robot control problem in which compensating for non-linear terms require additional instrumentation and sophisticated control schemes.

5.3 Impact of the Drive System Gear-Ratio on the Manipulator Energy Efficiency

In the GBD concept, the gear-ratio solely depends on the difference between the planets pitch diameters, and as such it is independent from the number of parts or stages in the transmission. This unique property allows the development of drive systems with high gear-ratio transmissions without compromising the efficiency or size or mass of the drive system. In the following simulation, we study the effect of the drive system gear-ratio on the energy consumption of the manipulator and examine the potential of using high gear-ratio transmissions to improve the energy efficiency of the robot. We will first show analytically how the gear-ratio can reduce the apparent load friction at the motor, and then deduce a relationship between the energy consumption and gear-ratio for a given friction load.
To simplify the analysis, the model presented in Figure 5-2 is adjusted to make the transmission stiffness fully rigid. If the transmission stiffness is sufficiently high, the motions of the load and motor will be coupled by the gear-ratio as shown in Figure 5-12.

The equation of motion at the motor can be shown to be:

\[ T_m = \left( J_m + \frac{J_L}{N^2} \right) \dot{\theta}_1 + \left( b_1 + \frac{b_2}{N^2} \right) \dot{\theta}_1 + \frac{T_f + T_l}{N} \]  

(79)

The gear ratio reduces the transmission viscous damping by \( 1/N^2 \) and cumulative friction (coulomb/stiction/kinetic) by \( 1/N \). The total energy efficiency of the joint can be written as:

\[ \text{Energy Efficiency} = \frac{\text{Work}_{\text{out}}}{\text{Work}_{\text{in}}} \]  

(80)

During steady state, the terminal velocity is found from Equation 79 by setting \( \ddot{\theta}_1 = 0 \). The terminal velocity is a function of the input torque and friction at given damping values. The input and output energies are written as \( \text{Work}_{\text{in}} = T_m \theta_1 \) and \( \text{Work}_{\text{out}} = T_L \theta_2 \). Substituting the terminal velocity into Eq. 80, the total energy efficiency can be solved as:

\[ \text{Energy Efficiency} = 1 - \frac{T_f}{NT_m} - \frac{b_2 \dot{\theta}_0}{T_m N^2} \]  

(81)

Equation 81 shows that increasing the gear-ratio of the transmission improves the total energy efficiency of the drive system under a given friction load. This is because the gear ratio reduces the output friction reflected at the motor through the mechanical advantage. The energy efficiency is improved at the cost of longer settling times. To illustrate these observations, the energy efficiency of the arm is computed while lifting a 25 Kg payload from the horizontal.
position to a vertical position using both a conventional drive system and the GBD concept (Figure 5-13).

![Figure 5-13: Energy efficiency comparison](image)

In this example, both designs deliver approximately the same output torque however using different gear-ratios. The conventional design uses a harmonic drive with a gear-ratio of 1:160 and a motor torque of 1.8 Nm, and the GBD concept uses a transmission with 1:900 gear ratio and motor torque of 0.33 Nm. Initially, the instantaneous energy efficiency is ideal because the initial work done by the motor is converted into output work. The energy efficiency begin to drop as a result of the frictional losses until it settles to 98% for the proposed concept and 89% for the conventional design. Although these results are subject to several assumptions, they clearly show that the proposed actuator concept can improve the energy efficiency of the
manipulator by increasing the gear-ratio and reducing the motor torque. For a 25 Kg payload, the total energy reduction would be 24.5 J.

The cost of reducing energy consumption is longer settling time, which is approximately 5.6 times the conventional design settling time. The proposed drive system would be at an advantage when energy consumption is more important than settling time, such as space applications. The electric losses in the drive system are also shown to decrease with increased gear-ratio as in Eq. 82, where $n$ is the mechanical efficiency of the transmission, $T_f$ is the frictional torque and $R$ is the motor coil resistance.

$$P_{loss} = \frac{RT_f^2}{(1-n)^2 K_t^2 N^2} \frac{1}{N^2}$$

(82)

Increasing the gear-ratio reduces the required motor torque and consequently the motor current. Reducing the motor current diminishes the power losses in the motor coil due to heat generation and improve the overall efficiency of the drive system.

5.4 Applications Area I: (Robotic Manipulation Systems)

The following robotic arm leverages from the unique capabilities of the Gear Bearing Drive system which can operate as an actuator providing very high torques and as a joint providing cross-axis support. As such, the arm possess both high strength and exceptional precision in a lightweight and compact package. The arm is designed to be modular, composed of a series of GBD modules and an end-effector module. This modularly allows the fast design and prototyping of these arms at any size and with any number of required degrees of freedom at the end effector. The proposed actuator’s all in-one-nature facilitates this modular design, making the robot reconfigurable and highly adaptable to the task at hand.
5.5 Applications Area II: (Medical Devices)

Stroke affects more than 750,000 people each year and claims more than 160,000 lives in the US annually [49]. Roughly eighty percent of stroke survivors display some form of motor deficit that results in their loss of mobility due to muscle weakness and spasticity, consequently impaired gait is the primary contributor to post stroke disability. In the area of lower limb rehabilitation, powered ankle-foot orthoses (AFOs) play an effective role in restoring the plantarflexion and dorsiflexion neuromuscular abilities of the ankle such as providing propulsion during push off, mitigating drop foot, and stabilizing the ankle. Most of the AFOs developed to date are relatively complex and bulky systems limited to specialized research hospitals and clinics, and therefore inaccessible to a large portion of the impaired population [50]. The development of fully-portable lightweight AFOs can increase the intensity of rehabilitation beyond the hospital through active home-based rehabilitation, which reduces recovery periods, and improve walking comfort during simple home or work activities [51].

One of the most promising portable powered AFOs is the recently developed University of Illinois PPAFO [52]. The device uses a bi-directional pneumatic air actuator with a pressure regulator to control the assistive ankle torque. Experimental results have shown that it was capable of providing appropriately untethered powered assistance. However, the major drawback to this approach is the
unconventional source of power, which requires the user to carry an external compressed-air bottle during walking. Another promising device intended for daily wear is the MIT Variable-Impedance AFO [53]. The device is based on a DC motor coupled to a ball-screw transmission and mounted in series with a torsion spring- also known as Series Elastic Actuator. The resultant actuator was too heavy and power intensive to be practical in commercially available AFOs. Other fully active AFOs have been developed such as the University of Michigan AFO and Arizona State University Robotic Gait Trainer [54, 55]. Both are actuated via pneumatic muscle actuators and consequently tethered and bulky systems. Semi-active AFOs were also developed by other groups such as Osaka University in Japan and Halmstad University in Sweden utilizing magneto rheological dampers to provide resistive plantarflexion torques. These devices rely on purely passive mechanical components such as electrically-modulated dampers and/or clutch-activated springs which can only dissipate, store or release energy and lack the ability to provide net power to the ankle during propulsion.

Up to this date, portable powered systems for ankle retraining have not been commercialized or used outside specialized hospitals due to the lack of adequate off-the-shelf actuator technology [56]. In order to facilitate the development of these devices into user-friendly portable wearable systems for both home and clinical rehabilitation, new forms of actuation must be developed with key abilities such as very high torque/force output, lightweight, unobtrusive and energy efficient. The GBD actuator can potentially fill this gap by providing an efficient source of high-torque, battery-powered actuation as illustrated in the conceptual AFO device shown in Figure 5-15.
Such a robotic-retainer will benefit the impaired population by improving their and consequently their quality of life and present a viable area of research to be addressed. In similar way, the GBD actuator could be useful in the development of multi-degrees of freedom prosthetic devices such as the upper limb elbow / arm prosthesis shown in Figure 5-16.

Figure 5-16: Compact elbow / arm prosthesis concept
5.6 Design Embodiment II

The following concept is an inversion of the Gear Bearing Drive transmission which eliminates the need for the ring gears by using an arm carrier. The summation of forces clearly show a differential term \((b-d)\) which could be optimized for high-ratio application in a similar way as discussed in Chapter 3.

**Figure 5-17:** Alternative high gear-ratio embodiment of the inverted transmission

The corresponding mechanism is illustrated in Figure 5-18.

**Figure 5-18:** CAD model of an inverted GBD transmission
Although this particular design eliminates some parts when compared to the original Gear Bearing Drive, it has a slower response time due to the fact that it is driven through its carrier, which has a much larger inertia than that of the sun gear as in the case of the original GBD transmission.

5.7 Final Conclusion Remarks

A new robotic hardware that simplifies the complexity of the robot drive system into one component is developed. The design is numerically computerized to assure that the final actuator system is optimized for its maximum torque density within a given solution envelope. Several preliminary plastic mockups were developed followed by a high-precision metal system that is later characterized on a custom-built experimental setup. It was shown that simple but accurate models for the transmission’s friction, stiffness and kinematic error can be obtained. A preliminary dynamic model of sufficient complexity is proposed that captures the open-loop response of the system with good accuracy, including the contribution of kinematic error. Analysis of the system dynamics suggests that the Gear Bearing Drive behavior is dominated by linear dynamics that are characterized by finite friction dissipation and geometric errors in its gear mesh resulting from imperfect manufacturing. In addition, the transmission shows no unpredictable open-loop dynamics or transmission resonance within its speed range, making its closed-loop control problem a simple task. These desirable characteristics combined with its compact size and light weight makes this class of actuators useful in a variety of applications, including robotic arms, prosthetics, manufacturing and virtually any other motion-control application that demands compact, lightweight and powerful actuators.
5.8 Future Work

Although the drive system performs exceptionally well in unconstrained motion, it lacks the ability to perform stable force-control when in contact with a stiff environment. High quality force control is needed in a variety of commercial applications that involve interaction forces with the environment such as welding, sanding, cutting, etc. Some of the immediate next steps on this project would be:

- Research and develop a force sensor (e.g. strain gage, piezoelectric) to be integrated within transmission structure such that the stiffness is not reduced by a modular torque sensor unit.

- Develop a dynamic model for the transmission system in hard-contact applications and a corresponding controller.

- Perform testing and demonstration experiments showing the drive performing tasks that require both force and position control such as grinding, cutting or welding.
Appendix A

In this appendix, we obtain the teeth-passing velocities and forces and gear-ratio of the transmission diagram below (Figure A-1).

The teeth-passing velocity at the mesh points is the relative velocity between the corresponding gear and its virtual arm carrier. As such, the mesh velocities at points A, B and C are \( w_{2/arm} \), \( w_{1/arm} \) and \( w_{5/arm} \). The objective is to obtain the relationships between the pitch-line velocities as functions of the input shaft speed \( w_2 \).

The input velocity \( w_2 \) can be separated in two terms, 1) the input velocity relative to the arm and 2) the velocity of the arm itself, such that:

\[
    w_2 = w_{2/arm} + w_{arm} \quad A - 1
\]

The velocity of the input planet with respect to the arm can be written as:

\[
    w_{4/am} = -\frac{N_2}{N_4}w_{2/arm} \quad A - 2
\]

The negative sign is used to illustrate the reverse of motion in externally meshing gears whereas a positive sign is used for internally meshing gears. The relative velocity of the ground ring gear with respect to the arm is written as:

\[
    w_{1/am} = \frac{N_4}{N_1}w_{4/am} \quad A - 3
\]

The absolute velocity of the ground ring gear is zero, therefore:

\[
    w_1 = w_{1/am} + w_{am} = 0 \quad A - 4
\]

By substituting Equations A-3 and A-2 into A-4, we obtain:

\[
    \frac{N_4}{N_1} \left( -\frac{N_2}{N_4} \right) w_{2/arm} + w_{arm} = 0 \quad A - 5
\]

Equation A-1 can be rearranged to solve for \( w_{2/arm} \):

\[
    w_{2/arm} = w_2 - w_{arm} \quad A - 6
\]
By substituting Equation A-6 into A-5,

\[-\frac{N_2}{N_1}(w_2 - w_{arm}) + w_{arm} = 0\]  \hspace{1cm} A - 7

Finally, the arm velocity is solved a function of the absolute input velocity $w_2$ as given by:

\[w_{arm} = \left(\frac{N_2}{N_2 + N_1}\right)w_2\]  \hspace{1cm} A - 8

The angular velocity of the sun gear with respect to the arm can then be obtained as:

\[w_{2/\text{arm}} = \left(\frac{N_1}{N_2 + N_1}\right)w_2\]  \hspace{1cm} A - 9

Because the input and output planets are rigidly coupled together, they have the same angular velocity such that:

\[w_{6/\text{arm}} = w_{4/\text{arm}}\]  \hspace{1cm} A - 10

The angular velocity of the output ring gear with respect to the arm is given by:

\[w_{5/\text{arm}}N_5 = w_{6/\text{arm}}N_6\]  \hspace{1cm} A - 11

The angular velocity of the output ring gear as a function of the input speed is given by:

\[w_{5/\text{arm}} = -\frac{N_6N_1}{N_5N_4}\left(\frac{N_2}{N_2 + N_1}\right)w_2\]  \hspace{1cm} A - 12

And,

\[w_{1/\text{arm}} = -w_{arm} = -\left(\frac{N_2}{N_2 + N_1}\right)w_2\]  \hspace{1cm} A - 13

Finally, the pitch line velocities at the input sun gear and output ring gear are obtained from the angular velocities with respect to the arm as:

\[V_A = w_{2/\text{arm}} \frac{D_2}{2} = \left(\frac{N_1}{N_1 + N_2}\right)\frac{D_2}{2}w_2\]  \hspace{1cm} A - 14

\[V_B = w_{1/\text{arm}} \frac{D_1}{2} = -\left(\frac{N_2}{N_1 + N_2}\right)\frac{D_1}{2}w_2\]  \hspace{1cm} A - 15

\[V_C = \frac{N_6N_1}{N_5N_4}\left(\frac{N_2}{N_1 + N_2}\right)\frac{D_5}{2}w_2\]  \hspace{1cm} A - 16
The transmission gear ratio is equal to the input speed over the output speed. It is given by:

\[
N = \frac{w_2}{w_5} = \frac{w_2}{w_{5/arm} + w_{arm}} = \frac{w_2}{\frac{N_6 N_1}{N_5 N_4} \left( \frac{N_2}{N_2 + N_1} \right) w_2 + \left( \frac{N_2}{N_2 + N_1} \right) w_2} = \frac{N_1}{N_2} + 1
\]

Now, let \( F_A \) and \( F_B \) represent the input and output forces applied at the sun gear and output ring gear mesh points respectively, that is on points A and C. Let \( F_B \) be the ground reaction force at point B. Assuming the transmission is operating at a steady speed, the equilibrium of forces on the planets subassembly leads to equations A-17 and A-18:

\[
F_C (D_4 - D_6) = F_A (2D_4)
\]

\[A - 17\]

\[
F_B + F_A = F_C
\]

\[A - 18\]

Where Eq. A-17 is obtained by summing the moments about point C. From equations A-17 and A-18, we can obtain the mesh forces as a function of the output force, such that:

\[
F_B = F_C \left( \frac{D_4 + D_6}{2D_4} \right)
\]

\[A - 19\]

And,

\[
F_A = F_C \left( \frac{D_4 - D_6}{2D_4} \right)
\]

\[A - 20\]

Now, knowing the mesh forces and velocities, it is possible to obtain the total power transferred by the transmission under steady state conditions, where:

\[
P_{total} = F_A V_A + F_B V_B + F_C V_C
\]

\[A - 21\]

Assuming that the power loss percentage along each mesh point is \( n \), the total power loss is given by:

\[
P_{loss} = n(F_A V_A + F_B V_B + F_C V_C)
\]

\[A - 22\]

The efficiency of the transmission is be finally written as:

\[
Efficiency = \frac{(P_{in} - P_{loss})}{P_{in}} = 1 - \frac{P_{loss}}{F_A V_A} = 1 - n - \frac{n(F_B V_B + F_C V_C)}{F_A V_A}
\]

\[A - 23\]
### Appendix B

#### Figure B-1: Engineering drawing of the input planet gear

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#### Hob Rack Form Data

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</tr>
<tr>
<td>tip to chamfer</td>
<td>none</td>
</tr>
<tr>
<td>chamfer angle from vertical</td>
<td>NA</td>
</tr>
<tr>
<td>radius in boss root</td>
<td>0.0011 max</td>
</tr>
<tr>
<td>AGMA quality class</td>
<td>A</td>
</tr>
</tbody>
</table>

#### UTS Involute

<table>
<thead>
<tr>
<th>Chamfer Both Sides</th>
<th>0.007 IN inside diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notes</td>
<td>1. Material 4150 steel</td>
</tr>
<tr>
<td>2. Quench and temper R/C 57-61</td>
<td></td>
</tr>
<tr>
<td>3. Do not break edges of gear teeth.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PLANET INPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>QT = 10</td>
</tr>
<tr>
<td>SEE NOTES</td>
</tr>
</tbody>
</table>
Figure B.2: Engineering drawing of the output planet gear

OUTER PLANE

Figure B.2: Output planet gear engineering drawing.
Figure B-3: Engineering drawing of the output ring gear

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OUTPUT RING
QT = 1
SEE NOTES
Figure B.4: Engineering drawing of the ground ring gear.

---

RING INPUT

QT = 2

SEE NOTES

---

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Figure B.5: Engineering drawing of the sun gear.

- **Spur Gear Data**
  - **Part Name**: SUN GEAR
  - **Date**: A
  - **Number of Teeth**: 13
  - **Normal Diameter**: 1.8 to 1.807
  - **Pressure Angle**: 20646 to 6.000
  - **Clearance**: 0.0000
  - **Angular Deviation**: 0.0000
  - **Total Deviation**: 0.0000

- **Master Gear**
  - **Master Gear Part Number**: A
  - **Number of Teeth**: 13
  - **Pitch Diameter**: 2.875
  - **Outside Diameter**: 2.3238
  - **Major Diameter**: 1.502
  - **Measurement Over Two Teeth**: 2.3236 to 2.3238
  - **Clearance**: 0.0000 to 0.007

- **Hi-Lo Rake Form Data**
  - **Form Involute**: NEW
  - **Tooth Number**: NEW
  - **Teeth**: 13
  - **Angle of HPA**: 20
  - **Angle of TPA**: 20
  - **Angle of ECA**: 20
  - **Angle of EDA**: 20

- **Notes**
  - Material: Alumina (تايم)
  - Quench and Temper RQ3-26
  - Parties: sun gear
  - Material will be supplied by JPL

---

**SUN GEAR**

- **QT = 2**
- **SEE NOTES**
Figure B-6: Engineering drawing of the planet roller

Scale 10:1 internal detail produced by wire EDM

Notes:
1. Material 4150 steel
2. Quench and temper RC57-61

Integral key to fit slot in output planet .078125 ref centered on .3008 diameter. No angular looseness allowed.
Figure B-7: Engineering drawing of the ground plate – Part 1

Figure B-8: Engineering drawing of the ground plate – Part 2
Figure B-9: Engineering drawing of the ground plate – Part 3

Figure B-10: Engineering drawing of the ground plate – Part 4
Figure B-11: Engineering drawing of the output member

Figure B-12: Engineering drawing of the pseudo-ground plate
Figure B-13: Engineering drawing of the ring sleeve part

Figure B-14: Engineering drawing of the sun gear roller (encoder side)
Figure B-15: Engineering drawing of the sun gear roller (pseud-ground side)
Figure B.16: Ring gear seal drawing
This script computes the stresses on the Gear Bearing Drive transmission and determines its torque density for a wide range of input values. The American unit system is used throughout this article as gear strength rating and manufacturing are performed according to the AGMA (American Gear Manufacturing Association) recommendations, specifically AGMA 10.

clc
clear all

% We first initialize the parameters vectors
EE=[];
RR=[];
TT=[];
TD=[];
NN2=[];
NN4=[];
NN1=[];
NN5=[];
NN6=[];
DD2=[];
DD4=[];
DD1=[];
DD5=[];
DD6=[];
PP1=[];
PP2=[];
TW=[];
TTm=[];

% Parameters of the Aluminum prototype:
k=1.3
% p1=30
% p2=32
% N4= 26
% N6= 25

% Parameters of the Steel prototype:
k=1.450
% p1=18
% p2= 16
% N2=38
% N4=12
% N1=62
% N6=11
% N5= 57

b= 0.5; % Face width of all gears
n = 4; % Number of planets
Steel_density = 0.28359; % Measured in lb / in^3

for k=1:0.1:3; % Let the arm radius vary from 1 to 3 inches
    for p1=5:96; % Stage 1 diametral pitch can vary from 5 to 96 teeth/inch
        for p2=5:96; % Similarly, the stage 2 pitch vary from 5 to 96 teeth/inches
            % Code for computing stresses and torque density
        end
    end
end
for N4= 10:30; %The ground planet number of teeth vary from 10 to 30
for N6= 10:30; %Similarly, the stage 2 planets teeth vary from 10 to 30

N2 = 2*k*p1-N4; %Solve number of teeth on Sun Gear
N5 = 2*k*p2+N6; %Solve number of teeth on Output Ring
N1 = N2+2*N4; %Solve number of teeth on Ground Ring
Ratio = ((1+N1/N2)/(1-(N1*N6/(N4*N5))));

% Solve for the pitch diameters on individual gears:
D1 = N1/p1;
D2 = N2/p1;
D4 = N4/p1;
D5 = N5/p2;
D6 = N6/p2;

if mod(N2,1)==0 % Filter out gear ratios that non-integer values for N2 and N5
else
    continue
end
if mod(N5,1)==0
else
    continue
end
if abs (Ratio) < 50 % Eliminate configurations with gear-ratios that are smaller than 50:1 and larger than 8000:1
    continue
else
    end
if abs (Ratio) > 8000
    continue
else
    end
if D2 < 1 % This statement clears out the solutions that has a sun gear diameter smaller than 1 inch, which would be too small for embedding a brushless outrunner motor
    continue
else
    end
if D2 > 2
    continue
else
    end
if D4 < 0.3 %We also filter out the configurations that have a very small planet diameter (i.e. smaller than 0.3 inch)
    continue
else
    end
if D5 > 4 %We are also not interested in excessively large configurations for which the ring gear pitch diameter is larger than 4 inches
    continue
else
    end
% Below are the AGMA factors for bending stress computation:
Ko = 1.25; %The overload factor is 1.25 for limited to moderate shocks
Ks = 1; %Size factor is 1 for uniform material (small scale gears)
Km = 1.6; %Mounting factor is 1.6 for face width less than 2 inches
Kt = 1; %The temperature factor is 1 if the oil temperature does not rise above 160 F as in the case of this prototype
Kr = 1.25; %The reliability factor is 1.25 for 99.99% acceptance. Note that increasing reliability reduces maximum allowable stress
Kl = 1; % Life factor is 1 for 10 million cycles. Note that by reducing the operation lifetime we can increase the allowable stress
St = 60e3; %Tensile strength for 55RC steel

%Common factors for contact stress computation
Cf = 1; %For surface finish of 65RA
Sc = 190e3; %For surface hardness 55RC
E = 30000e3; %PSI for steel
u = 0.33; %Poisson's ratio
Theta = 20*pi/180; %Assume standard pressure angle of 20 degrees
Cl = 1; %Life factor for 10 million cycles. Same rules as for bending strength calculation
Ch = 1; %Hardness factor is assumed unity

% Input speed
Win = 2000*2*pi/60; % rad/s
% Face width (in)
b = 0.4;

%FAILURE OF GROUND PLANETS INTERFACE WITH THE SUN GEAR
% Bending strength computation:
Vp2 = N1/(N1+N2)*(D2/2)*Win/12; %Pitch line velocity
Kv2 = (50 + sqrt(Vp2))/50; %Dynamic factor.
J = (0.311*log(N4)+0.15)/4.5; %Input planet geometry factor
Tout_b_sun = n*D4*D5/(abs(D4-D6))*(b*J*Kl*St)/(Kt*Kr*Ko*Kv2*p1*Ks*Km)*0.1129;
% The 0.1129 converts lb.in to Nm. This is the failure torque due to excessive teeth bending stress
mG = N2/N4; %Gear ratio between sun gear and planet pinion
I = 0.5*sin(Theta)*cos(Theta)*(mG/(mG+1)); %Geometric factor of the input planet
Cp = 0.564*sqrt(E/(2*(1-u*u))); %AGMA elastic coefficient
Tout_c_sun = n*D4*D5/(abs(D4-D6))*b*D2*I/(Ko*Kv2*Ks*Km*Cf)*(Sc*Cl*Ch/(Kt*Kr*Cp))^2*0.1129; %This is the failure torque due to excessive contact stresses on the input planet teeth

%FAILURE OF GROUND PLANETS PINION INTERFACE WITH THE GROUND RING GEAR
% Bending strength analysis:
Vp1 = N2/(N1+N2)*(D1/2)*Win/12; %Pitch line velocity
Kv1 = (50 + sqrt(Vp1))/50; %Dynamic factor.
J = (0.311*log(N4)+0.15)/4.5; %Geometric factor
Tout_b_ring1 = n*D4*D5/(D4+D6)*(b*J*Kl*St)/(Kt*Kr*Ko*Kv1*p1*Ks*Km)*0.1129;
% Contact strength analysis:
mG = -N1/N4; %Gear ratio between ring gear and planet pinion
I = 0.5*sin(Theta)*cos(Theta)*(mG/(mG+1)); %Geometric factor
Cp = 0.564*sqrt(E/(2*(1-u*u))); %AGMA elastic coefficient
Tout_c_ring1 = n*D4*D5/(D4+D6)*b*D4*I/(Ko*Kv1*Ks*Km*Cf)*(Sc*Cl*Ch/(Kt*Kr*Cp))^2*0.1129;
% FAILURE OF OUTPUT PLANETS PINION INTERFACE WITH OUTPUT RING GEAR
% Bending strength analysis:
Vp5 = N6*N1*N2*D5*Win/(12*2*N5*N4*(N2+N1)); %Pitch line velocity
Kv5= (50 + sqrt(Vp5))/50;
J = (0.311*log(N6)+0.15)/4.5; %Dynamic factor
Tout_b_ring2 = n*D5/(2)*(b*J*K1*St)/(Kt*Kr*Ko*Kv5*p2*Ks*Km)*0.1129;

% Contact strength
mG = -N5/N6; %Gear ratio between ring gear and planet pinion
I = 0.5*sin(Theta)*cos(Theta)*mG/(mG+1); %Geometric factor
Cp = 0.564*sqrt(E/(2*(1-u*u))); %AGMA elastic coefficient
Tout_c_ring2 = n*D5/2*b*D6*I/(Ko*Kv1*Ks*Km*Cf)*((Sc*Cl*Ch)/(Kt*Kr*Cp))^2*0.1129;
Contact_stress_planet2 = Cp*((900*Tm/D5)*Cf/(b*D6*I))^2/n;
T = min([Tout_b_sun Tout_c_sun Tout_b_ring1 Tout_c_ring1 Tout_b_ring2 Tout_c_ring2]);

% BEGIN ESTIMATION OF TORQUE DENSITY
WD1 = 2.157/p1; %Whole depth of stage 1
WD2 = 2.157/p2; %Whole depth of stage 2
tr1 = 1.2*WD1; %Rim thickness of stage 1
tr2 = 1.2*WD2; %Rim thickness of stage 2

%Estimating the mass of the sun gear:
Sun_root_circle_diameter = (N2-2)/p1;
Sun_bore_diameter = Sun_root_circle_diameter - 2*tr1;

%Computing the sun teeth surface using the square-method approximation:
Sun_tooth_thickness = 1.57/p1;
Sun_tooth_area1 = Sun_tooth_thickness*WD1;
Sun_teeth_area1 = N2*Sun_tooth_area1;

%Computing the sun teeth surface using trapezoidal approximation:
Sun_addendum = 1/p1;
Sun_dedendum = 1.157/p1;
t_sun = 1.5708/p1;
Sun_tooth_area2 = Sun_addendum*t_sun-Sun_addendum^2*tan(20*pi/180) + Sun_dedendum*t_sun+Sun_dedendum^2*tan(17.37*pi/180);
Sun_teeth_area2 = N2*Sun_tooth_area2;

%The sun gear mass is finally:
Sun_gear_area = Sun_teeth_area2 + pi/4*(Sun_root_circle_diameter^2 - Sun_bore_diameter^2);
Sun_gear_mass = Sun_gear_area*b*Steel_density;

% Estimating the mass of the input planet gear:
Planet1_root_circle_diameter = (N4-2)/p1;
Planet1_bore_diameter = Planet1_root_circle_diameter - 2*tr1;
%Computing the teeth surface using the square approximation:
Planet1_tooth_thickness = 1.57/p1;
Planet1_tooth_area1 = Planet1_tooth_thickness*WD1;
Planet1_teeth_area1 = Planet1_tooth_area1*N4;
Planet1_addendum = 1/p1;
t_Planet1 = 1.5708/p1;
Planet1_tooth_area2 = Planet1_addendum*t_Planet1 -
Planet1_addendum^2*tan(20*pi/180) + Planet1_dedendum*t_sun +
Planet1_dedendum^2*tan(17.37*pi/180);
Planet1_teeth_area2 = N4*Planet1_tooth_area2;
Planet1_gear_area = Planet1_teeth_area2 +
pi/4*(Planet1_root_circle_diameter^2 - Planet1_bore_diameter^2);
Planet1_gear_mass = Planet1_gear_area*b*Steel_density;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Estimating the mass of the ground sun gear:
Ring1_root_circle_diameter = (N1-2)/p1;
Ring1_bore_diameter = Ring1_root_circle_diameter + 2*tr1;
t_Ring1 = 1.5708/p1;
Ring1_tooth_area1 = t_Ring1*WD1;
Ring1_teeth_area1 = Ring1_tooth_area1*N1;
Ring1_addendum = 1/p1;
Ring1_dedendum = 1.157/p1;
Ring1_tooth_area2 = 2*Ring1_addendum*(t_Ring1/2 -
Ring1_addendum*tan(20*pi/180)) + Ring1_addendum^2*tan(20*pi/180) +
Ring1_dedendum*t_Ring1 + Ring1_dedendum^2*tan(20*pi/180);
Ring1_teeth_area2 = N1*Ring1_tooth_area2;
Ring1_gear_area = Ring1_teeth_area2 +
pi/4*(Ring1_bore_diameter^2 -
Ring1_root_circle_diameter^2);
Ring1_gear_mass = Ring1_gear_area*b*Steel_density;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Similarly, the mass of the output planet is calculated as:
Planet2_root_circle_diameter = (N6-2)/p2;
Planet2_bore_diameter = Planet2_root_circle_diameter - 2*tr2;
Planet2_tooth_thickness = 1.57/p2;
Planet2_tooth_area1 = Planet2_tooth_thickness*WD2;
Planet2_teeth_area1 = Planet2_tooth_area1*N6;
Planet2_addendum = 1/p2;
t_Planet2 = 1.5708/p2;
Planet2_tooth_area2 = Planet2_addendum*t_Planet2 -
Planet2_addendum^2*tan(20*pi/180) + Planet2_dedendum*t_Planet2 +
Planet2_dedendum^2*tan(17.37*pi/180);
Planet2_teeth_area2 = N4*Planet2_tooth_area2;
Planet2_gear_area = Planet2_teeth_area2 +
pi/4*(Planet2_root_circle_diameter^2 - Planet2_bore_diameter^2);
Planet2_gear_mass = Planet2_gear_area*b*Steel_density;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% The mass of the output ring gear is calculated as:
Ring2_root_circle_diameter = (N5-2)/p2;
\[ \text{Ring2 bore diameter} = \text{Ring2 root circle diameter} + 2 \times \text{tr2}; \]
\[ t_{\text{Ring2}} = \frac{1.57}{p2}; \]
\[ \text{Ring2 tooth area1} = t_{\text{Ring2}} \times \text{WD2}; \]
\[ \text{Ring2 teeth area1} = \text{Ring2 tooth area1} \times \text{N5}; \]
\[ \text{Ring2 addendum} = \frac{1}{p2}; \]
\[ \text{Ring2 dedendum} = \frac{1.157}{p2}; \]
\[ \text{Ring2 tooth area2} = 2 \times \text{Ring2 addendum} \times \left( \frac{t_{\text{Ring2}}}{2} - \text{Ring2 addendum} \times \tan\left(\frac{20\pi}{180}\right)\right) + \text{Ring2 addendum}^2 \times \tan\left(\frac{20\pi}{180}\right) + \text{Ring2 dedendum} \times t_{\text{Ring2}} + \text{Ring2 dedendum}^2 \times \tan\left(\frac{20\pi}{180}\right); \]
\[ \text{Ring2 teeth area2} = \text{N5} \times \text{Ring2 tooth area2}; \]
\[ \text{Ring2 gear area} = \text{Ring2 teeth area2} + \pi/4 \times (\text{Ring2 bore diameter}^2 - \text{Ring2 root circle diameter}^2); \]
\[ \text{Ring2 gear mass} = \text{Ring2 gear area} \times b \times \text{Steel density}; \]

\[
\text{Transmission weight} = (\text{Sun gear mass} + n \times (\text{Planet1 gear mass} + \text{Planet2 gear mass}) + \text{Ring2 gear mass} + \text{Ring1 gear mass}) \times 0.45392; \quad \text{in Kg}\]
\[
\text{Torque density} = \frac{T}{\text{Transmission weight}}; \]
\[
T_m = \text{abs}(T/Ratio); \]

\[\text{if } T_m/D2 > 0.25 \quad \text{Filter out configurations whose required motor torque is not sustainable within motor cavity size.} \]
\[\text{continue} \]
\[\text{else} \]
\[\text{end} \]
\[\text{if } T_m/D2 < 0.1 \quad \text{Filter out configurations whose required motor torque is not sustainable within motor cavity size.} \]
\[\text{continue} \]
\[\text{else} \]
\[\text{end} \]

%Begin Efficiency Calculation:
\[n2=0.005; \quad \text{Estimated mesh power loss between sun and ground planet} \]
\[n1=0.005; \quad \text{Estimated mesh power loss between ground ring and ground planet} \]
\[n5=0.005; \quad \text{Estimated mesh power loss between output ring and output planet} \]
\[Tout=300; \quad \text{Estimated output torque} \]

%The mesh loss at the interface between the sun gear and ground planet is:
\[P_{\text{sun}} = \text{abs}(n2 \times (N1/(N1+N2)) \times D2 \times (\text{win}) \times ((D4-D6)/(2 \times D4)) \times Tout/D5); \]
%The mesh loss between the ground ring and ground planet is:
\[P_{\text{ground ring}} = n1 \times (N2/(N1+N2)) \times D1 \times (\text{win}) \times (D4+D6)/(2 \times D4) \times Tout/D5; \]
%The mesh loss between the output ring and output planet is:
\[P_{\text{output ring}} = n5 \times N6 \times N1 \times N2/((N5 \times N4) \times (N1+N2)) \times \text{win} \times Tout; \]
%The total power loss is:
\[P_{\text{loss}} = P_{\text{sun}} + P_{\text{ground ring}} + P_{\text{output ring}}; \]
%The total input power can be written as:
\[P_{\text{in}} = Tout \times \text{win} \times Ratio + P_{\text{loss}}; \]
%The transmission efficiency can be written as:
\[E = (1 - P_{\text{loss}}/P_{\text{in}}) \times 100; \]

NN2=[NN2,N2]; \quad \text{%Collect values for N2} \]
\[\text{NN4}=[\text{NN4},N4]; \quad \text{%Collect values for N4} \]
\[\text{NN1}=[\text{NN1},N1]; \quad \%... \]
\[\text{NN5}=[\text{NN5},N5]; \]
\[\text{NN6}=[\text{NN6},N6]; \]
DD2=[DD2,D2];
DD4=[DD4,D4];
DD1=[DD1,D1];
DD5=[DD5,D5];
DD6=[DD6,D6];
RR = [RR,Ratio];
TT = [TT,T];
PP1=[PP1,p1];
PP2=[PP2,p2];
TW=[TW,Transmission_weight];
TTm=[TTm,Tm];
TD = [TD,Torque_density];
EE = [EE,E];
end
end
end
end

%The final solution is organized into the GBD matrix, defined as:

GBD = [PP1' PP2' NN2' NN4' NN1' NN5' NN6' DD2' DD4' DD1' DD5' DD6' RR' TD' TW' TTm']; % Collect the system variables for all loop values

plot (DD2*25.4, TTm,'+')
ylim ([0 0.45])

Kinematic Error Code:
clc
clear all
load data1.txt;
A=data1;
In_rot= A(:,2);        %Collect the number of input revolutions
err=A(:,1)-0.1507;     %Removed the DC bias from the measurements
k=39.5;
i = find(In_rot < k);
In_rot1=In_rot(1:size(i),1); %Cut the data to one output revolution, obtained after 39.5 input revolutions
err1 = err(1:size(i),1); %Write the kinematic error signal
%Length of signal
L= length(In_rot1);    %Identify number of data points for FFT process
T = max(In_rot1)/L;    %Obtain angle increment
Fs = 1/T;              %Obtain spatial sampling frequency
t=[0:T:max(In_rot1)]'; %Write spatial x-vector
X=err1;
Y = fft(X);  %Obtain the frequency and magnitude content of the error signal
% Divide the signal in two folds
P2 = abs(Y/L);
P1 = P2(1:L/2+1);
P1(2:end-1) = 2*P1(2:end-1);

%Obtain power spectral density.
xdft = fft(X);
xdft = xdft(1:L/2+1);
psdx = (1/(Fs*L)) * abs(xdft).^2;
psdx(2:end-1) = 2*psdx(2:end-1);
freq = 0:Fs/L:Fs/2; % Define frequency band
%Divide phase in two folds
PhaseY=unwrap(angle(Y));
M2 = PhaseY;
M1 = M2(1:L/2+1);
M1(2:end-1) = 2*M1(2:end-1);
%Write frequency vector
f = Fs*(0:(L/2))/L;
plot (In_rot1, err1),grid % Plot kinematic error vs input revolutions
ylabel ('Kinematic Error [deg]')
xlabel('Number of Input Revolutions')

%The for/end statement below reconstruct the kinematic error using the
%corresponding frequencies and magnitudes obtained
for i=1:1:1000
    t= (k/1000)*i;
    X_corr(i,1) =
        2*0.0265*sin(0.02532*2*pi*t+1.8)+2*0.01154*sin(1.004*2*pi*t+6.658)+2*0.01745*
        sin(1.367*2*pi*t+6.095+0.4)+ 2*0.01288*sin(1.688*2*pi*t-0.286);
    Pos(i,1)=t;
end

subplot(4,1,1)
plot(f, P1),grid
ylabel('Kinematic Error')
xlabel('Cycles per Input Revolutions')
xlim([0 5])
subplot(4,1,2)
plot(freq,psdx), grid
ylabel('Watts / Input Revolutions')
xlim([0 5])
subplot(4,1,3)
plot(f, M1),grid
xlabel('Number of Input Revolutions')
ylabel('Radians')
xlim([0 5])
subplot(4,1,4)
plot(Pos,X_corr, In_rot1, err1),grid
xlabel('Number of Input Revolutions')
ylabel('Kinematic Error')
legend ('simulation', 'experimental data')
xlim([0 79])
Simulation Model using MATLAB ODE45:
%The following script simulates a non-linear two degrees of freedom system
with parabolic friction and stiffness functions. The global variables are
transferred between the main script file (Main_model4.m) and the function
file (Model4.m).
clc
clear all
global J1 J2 b0 b1 b2 b3 N Load f0 A1 A2 A3 w1 w2 w3 phase1 phase2 phase3 p1
p2 q1 q2 t

% SYSTEM VALUES
% J1 = 0.00011     kg.m^2
% J2 = 0.0125     kg.m^2
% b0 = 0.0125;     Nm (starting torque)
% b1= -6.5437e-8;  Nm/(rad/s)^2
% b2= 0.00010673; Nm/(rad/s)
% b3= -0.00022825; Nm
% N=39.5; %Gear Ratio
% Update stiffness parameter from function file
%
N=39.5;
Load= 0; %Value of external output load (Nm)
%Input current (i) vary between 0.6 to 1 Amp
i= 0.8; %Simulate output velocity time-response under an input step current
% of 0.8 Amp.
b0=0.0125;
b1= -6.5437e-8;
b2=0.00010673; %Nm/(rad/s)
b3=0.00022825;
J1=0.000125; %Input Inertia
J2=0.0125; %Output Inertia

%Write magnitudes of kinematic error components
A1=2*(pi/180)*0.00530;
A2=2*(pi/180)*0.01745;
A3=2*(pi/180)*0.01154;

%Write frequency contents of kinematic error components
w1= 0.02532; %Cycles per input revolutions
w2= 1.367;
w3= 1.0040;

%Establish phase of kinematic error contents
phase1 = 1.8*pi/180;
phase2 =6.09*pi/180;
phase3=6.65*pi/180;
t0=0; tf=15; t=linspace(t0,tf,100); %Define simulation time vector
f0= (0.038909*i-0.0080545);%Write corresponding torque function from input
current. Refer to plot of torque vs. current to obtain the function’

coefficients

[t,x] =ode45(@model4,t,[0 0 0 0]'); %Solve nonlinear system found in function
file (moel4.m) using ODE45.

%Verify error forcing function
err=
A1.*sin(w1.*x(1)/(2*pi)+phase1)+A2*sin(w2.*x(1)/(2*pi)+phase2)+A3.*sin(w3.*x(1)/(2*pi)+phase3);
A = N*p1-p2/(N*N);
B = -2*N*p1.*x(:,1)-q1*N+2*p2.*x(:,3)/N-q2/N -2*p2*err/N;
C = q1*N.*x(:,1)+N*p1.*x(:,1).*x(:,1)-p2.*err.*err+2.*p2.*err.*x(:,3)-q2.*err+q2.*x(:,3);
x_k = (-B-sqrt(B.*B-4.*A.*C))/(2*A); %Assumes non-linear spring

%x_k = (-B-sqrt(B.*B-4.*A.*C))/(2*A); %Assumes non-linear spring

%Load experimental measurements obtained for input current 0.5 to 1.1 Amps.
load '5ampB.txt';
load '6ampB.txt';
load '7ampB.txt';
load '8ampB.txt';
load '9ampB.txt';
load '10ampB.txt';
load '11ampB.txt';
load 'dyna.txt';
load 'dyna1.txt';
load 'dyna2.txt';

%Load experimental measurements obtained for input current 0.5 to 1.1 Amps.
B5= X5ampB;
B6= X6ampB; %Note there was .192 seconds delay on acquisition trigger.
B7= X7ampB; %Note there was .056 seconds delay on acquisition trigger.
B8= X8ampB;
B9= X9ampB;
B10= X10ampB;
B11= X11ampB;
E = dyna;
E1= dyna1;
E2= dyna2;

plot(B6(:,1)-0.192,B6(:,4)*(2*pi/60), B7(:,1)-0.056,B7(:,4)*(2*pi/60),
B8(:,1)-0.212,B8(:,4)*(2*pi/60), B9(:,1)-0.256,B9(:,4)*(2*pi/60), B10(:,1)-
0.262,B10(:,4)*(2*pi/60))
plot (t, x(:,2), E1(:,1)-10.263, E1(:,3)*2*pi/60)
xlim([0 15])

%The following function file writes the non-linear system into a set of first
order differential equations.

function xp= model4 (t,x);
%Define global variable between main script and function file
global J1 J2 b0 b1 b2 b3 N err Load f0 A1 A2 A3 w1 w2 w3 phase1 phase2 phase3
p1 p2 q1 q2
xp = zeros(length(x),1); %Set initial conditions as zeros
err=
A1*sin(w1*x(1)/(2*pi)+phase1)+A2*sin(w2*x(1)/(2*pi)+phase2)+A3*sin(w3*x(1)/(2
*pi)+phase3);
p1 = 0.54;    %Coefficient of quadratic term of first spring
q1= 0.64;     %Coefficient of linear term of first spring
p2=68615;     %Coefficient of quadratic term of second spring
q2=988;       %Coefficient of linear term of second spring
x_k = (N*N*q1*x(1)+q2*N*x(3)-N*q2*err)/ (q2+N*N*q1);
F = b0*heaviside(140-x(2))+heaviside(x(2)-140)*(b1*x(2)*x(2)+b2*x(2)+b3)-
b0*heaviside(-x(2)); %Define nonlinear friction function. The term 140 rad/s
is due to independence of friction and speed below input speed of 140rad/s.
See Input Speed vs. Friction torque plot.
%xp(1)= x(2);
xp(2)= (1/J1)*(-F+(f0 +0.0078*sin(0.5*t)+0.0078*sin(2*t)))+q1*(x(1)-x_k)+p1*(x(1)-x_k)^2);
xp(3)= x(4);
xp(4)= (1/J2)*(-Load + q2*(x_k/N + err - x(3))+p2*(x_k/N+err-x(3))^2);
t=t
end

The data files are available on:

https://drive.google.com/file/d/0BzchhkqBCEdZRNlSNEpJQ1pVTFE/view?usp=sharing

END OF CODE END OF CODE END OF CODE END OF CODE
Appendix D: Curriculum Vitae and Biographical Information

Personal Data:
DOB/Place: February 26, 1987 / Beirut, Lebanon
Citizenships: Lebanon United States of America

Education:
Ph.D. Northeastern University, Boston, MA, December 2016, Mechanical Engineering
Dissertation title: A compact drive system for geared robotic joints and actuation mechanisms.
M.Sc. Northeastern University, Boston, MA, August 2013, Mechanical Engineering
(Major in Mechatronic Systems)
B.Sc. University of Massachusetts, May 2011, Mechanical Engineering
(1st Class honors)

Honors, Awards and Synergistic Activities:
- Recipient of a NASA Space Technology Fellowship award ($86,000/year for 2013, 2014, 2015, 2016)
- Recipient of the James Healy Fellowship award from Northeastern University for excellence in research and innovation ($8,000, 2016)
- Recipient of Outstanding Academic Achievement award from the University of Massachusetts – Lowell (2011)
- Invited speaker at Northeastern University’s annual Board of Trustees’ meeting
- Speaker at the Small Business Research and Innovation Conference (SBIR) in Atlanta, GA (2015)
- Awarded a $35,000 grant from the Jet Propulsion Laboratory’s Strategic Missions & Systems to develop the end-effector actuator of JPL’s RoboSimian robot.

Professional Experience and Appointments:
2013-present NASA Space Technology Fellow
Jet Propulsion Laboratory, Pasadena, CA
2011-present Doctoral Student: Department of Mechanical Engineering
Northeastern University, Boston, MA
2010-2011 Design Engineer: Autoliv Electronics – Radar Systems
1011 Pawtucket Blvd, Lowell, MA

Research Interests and Expertise:
Robotics, design and development of robotic drive systems and mechanisms, dynamic modeling and control of electro-mechanical systems.
Journal Papers / Conference Publications:


- Brassitos, E., Dubowsky, S., “Compact Drive System for Planetary Rovers and Space Manipulators”, proceedings of the 2015 ASME/IEEE International Conference on Advanced Intelligent Mechatronics.


Patents:

- Brassitos, E., “Drive system for Actuating Robotic Joints” INV-16004

Professional Society Activities:

- Technical Session Chair for IEEE/AME Advanced Intelligent Mechatronics, Busan, Korea (2015)
- Textbook reviewer for Springer (2012)

References


[14] Harmonicdrive.net, 2016 catalog, “Cup Type Component Sets & Housed Units.”


