Optimization Models for Empty Railcar Distribution Planning in Capacitated Networks

A Dissertation Presented

By

Ruhollah Heydari

to

The Department of Mechanical and Industrial Engineering

in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

in the field of

Industrial Engineering

Northeastern University
Boston, Massachusetts

January 2016
ABSTRACT

Because of spatial and temporal imbalances in the supply and demand of empty railcars/cars in freight transportation systems, empty equipment repositioning is inevitable. As the owners of rail network, trains and railcars, railway carriers charge their customers based on the days and the miles that a railcar is under load. Hence the cost of shipping empty cars to customers as well as the opportunity cost of extra inventory in customer facilities is on the railway’s shoulder. A reverse routing strategy suggesting to return the empty cars to the place they were loaded, imposes a 50% empty movement in a car cycle while a reload strategy suggesting to reload the cars at the point they become empty has 0% empty movement. The first strategy is good for frequent shippers only and still is too costly and the idealistic second strategy is not usually feasible because of the supply-demand imbalances. In this research we develop two formulations for the Empty Railcar Distribution problem, both aiming to minimize the total setup costs, total transportation costs, and total shortage penalties under supply limitation, demand satisfaction, customer preferences and priorities, and network capacity constraints.

We first formulate the problem as a path-based capacitated network flow model. Contrary to the traditional path-based formulations, the path connecting each supply-demand pair is given by an external application called Trip Planner which is defined on top of a time-space network. Another application called Pseudo Path Generator then generates alternative paths for each supply-order pair.

Then we formulate the problem as an arc-based capacitated multi-commodity network flow model where contrary to the path-based model, the car routing and car distribution decisions are integrated in a single model.

The path-based formulation is more practical for the United States railroads since in US railroad industry car routing and car distribution decisions are separated from each other and usually made by different departments while the integrated arc-based formulation is close to the Swedish Railway System. Both models are complex and because of the huge number of constraints and integer decision variables are hard to solve.

The models are implemented in Java and solved using Concert Technology of the IBM CPLEX solver. We also develop an Iterative Relaxation and Rounding Heuristic using Initial Basis for the path-based model. The comparison of path-based and arc-based formulations in both capacitated and uncapacitated modes confirmed the efficiency of the heuristic from both run time and solution quality perspectives.
ACKNOWLEDGEMENTS

I express my sincere gratitude and appreciation to my PhD advisor, Prof. Melachrinoudis of Northeastern University, for his encouragement, guidance, and continuous support over the years of my graduate studies, not only in research but also in other aspects of student life in the United States.

I thank Dr. Kubat and Dr. Turkcan from Northeastern University and Dr. Pranoto from Norfolk Southern Railway for being part of my PhD committee and providing me with their valuable comments on this research.

I would also like to thank Dr. Clark Cheng of Norfolk Southern for encouraging me to choose my Dissertation topic in the area of Car Distribution Problems; Fabio Colombo from Universita degli Studi di Milano, Italia (Team OR at UNIMI) for providing me with their group's 2011 RAS competition results; Nilay Roy from Research Computing Center of Northeastern University for his hours of technical support on distributed computing; Lisa O'Neill, Director of Graduate Student Services, for her endless support throughout my educational life at Northeastern University, and all other faculty, staff and friends who were always there when I needed their support.

I am indebted to my family for their relentless support and encouragement, during my graduate studies. And I want to express my greatest gratitude to them and specially to my mom whose patience, love, and praying has always been with me. Finally I will dedicate this work to my mom. Thank you maman!
## LIST OF TABLES

Table 2-1 A comparison among current models used by railway carriers and our model ........... 25  
Table 3-1 Parameters used to calculate the cost coefficients .............................................. 49  
Table 3-2 Demand node priorities based on customer priorities ........................................ 49  
Table 3-3 Illustration of customer profiles ........................................................................... 51  
Table 3-4 Model assumptions .............................................................................................. 55  
Table 4-1 Mathematical notation .......................................................................................... 58  
Table 5-1 Notation for the arc-based formulation ................................................................. 96  
Table 5-2 Permitted flows vs leak in the customer level ....................................................... 99  
Table 6-1 Networks used for verification and validation of the models and the solution procedures .................................................................................................................................. 113  
Table 6-2 Customers’ demand for a one-week time horizon ................................................. 118  
Table 6-3 Available supply of different pools in each yard .................................................. 119  
Table 6-4 Performance measures (optimal assigned cars; optimal objective value) for different models for two scenarios with different shortage penalties ................................................................. 128  
Table 6-5 Different models, variable setup and algorithms to solve them ............................. 129  
Table 6-6 Parameters used to generate case example for sensitivity analysis ...................... 135  
Table 6-7 Parameters used to generate case examples for computational complexity calculations .................................................................................................................................. 143
LIST OF FIGURES

Figure 3-1 An itinerary for a flight trip................................................................. 27
Figure 3-2 Underlying networks used in the railroad optimization models.......................... 33
Figure 3-3 The time-space network (a) and its associated multigraph (b)............................ 37
Figure 3-4 Time-space block-train network associated to Figure 3-2(f) ............................. 38
Figure 3-5 Time-space resource-constrained block network (colors for visualization)........ 41
Figure 3-6 Time-space resource-constrained block network (the main representation)........ 42
Figure 3-7 Customers on time-space resource-constrained block network ...................... 43
Figure 3-8 Bipartite supply-demand multigraph .................................................................. 44
Figure 3-9 Costs on the time space network...................................................................... 49
Figure 3-10 Classifying fleet for empty railcar distribution .............................................. 54
Figure 4-1 Shortage cost function.................................................................................... 62
Figure 4-2 Car distribution model in three different modes ............................................ 64
Figure 4-3 An arc-train network with only one path between any O-D pair (a-1) can be transformed to a resource-constrained bipartite graph (a-2), while the one with more than one path (b-1) is transformed to a bipartite multigraph (b-2). .................................................................................... 68
Figure 4-4 Lattice path of length four in in $\mathbb{Z}^2$ ......................................................... 70
Figure 4-5 Six lattice paths and their associated permutation words.................................. 71
Figure 4-6. A trip plan and its associated pseudo paths when a) no delay is permitted, and b) a maximum delay of two days is acceptable........................................................................... 75
Figure 5-1 Delay arcs in the customer level ...................................................................... 98
Figure 5-2 Permitted flow vs leak in customer level .......................................................... 99
Figure 5-3 Dealing with delay arc flow using a new variable ............................................ 101
Figure 6-1 Static networks associated with the Tiny and Supper Tiny Networks: a) physical rail network, b) train network, c) blocking plan and d) block to train assignment.......................... 113
Figure 6-2 Static networks associated with the Medium Network: a) physical rail network, b) train network and c) blocking plan .......................................................... 114
Figure 6-3 Static networks associated with the Large Network: a) Physical rail network, b) train network and c) blocking plan.............................................................................. 115
Figure 6-4 Detailed supply-order assignment suggested by PA with a) low shortage and b) high shortage penalties......................................................................................... 123
Figure 6-5 Detailed supply-order assignment suggested by AT with a) low shortage and b) high shortage penalties......................................................................................... 124
Figure 6-6 Detailed supply-order assignment suggested by NC with a) low shortage and b) high shortage penalties......................................................................................... 125
Figure 6-7 Detailed supply-order assignment suggested by NCActual with a) low shortage and b) high shortage penalties ........................................................................................................ 126
Figure 6-8 Optimal assignment from noncapacitated model and its optimistic behavior .... 127
Figure 6-9 Logistic function for creating daily demand ................................................................ 132
Figure 6-10 Path-Based formulation performance measures at the termination time when capacity increases: a) objective value, b) order fulfillment, and c) run time ................................................. 139
Figure 6-11 Path-Based formulation performance measures at the termination time when shortage penalties increase: a) objective value, b) order fulfillment, and c) run time ..................... 142
Figure 6-12 Average solution time of AT-IRRH2 with initial basis (solid green circles) along with its estimation from the regression model (empty blue circles) ........................................ 144
Figure 6-13 Average run time vs estimated run time from a power law model ......................... 145
Figure 6-14 The performance measures of IRRH2 per iteration when the problem size increases. a) EpGap, b) solution time, and c) their trade-off ................................................................. 147
# TABLE OF CONTENTS

1 Introduction .................................................................................................................. 1
   1.1 Problem Background ............................................................................................ 1
   1.2 Problem Statement ............................................................................................... 2
   1.3 Motivation and Objectives of the Dissertation .................................................. 3
   1.4 Dissertation Outline ............................................................................................ 6

2 Literature Review ......................................................................................................... 8
   2.1 Transportation Problem ...................................................................................... 11
   2.2 Transshipment and Network Flow Problem ..................................................... 13
   2.3 Multi Commodity Network Flow Problem ...................................................... 18
   2.4 Economies of Scale ............................................................................................ 20
   2.5 Survey Papers ..................................................................................................... 22
   2.6 Contribution of the Research ............................................................................ 22

3 Important Terminologies and Components of the Problem ....................................... 26
   3.1 Important Terminologies ..................................................................................... 26
   3.2 Network Representation ..................................................................................... 29
      3.2.1 Static Networks ............................................................................................. 29
      3.2.2 The Time-Space Network .......................................................................... 35
      3.2.3 The Bipartite Multigraph ........................................................................... 44
      3.2.4 Network Capacities .................................................................................... 45
      3.2.5 Cost Components ....................................................................................... 45
   3.3 Fleet ..................................................................................................................... 50
   3.4 Customer, Customer profile, Demand node and Demand .................................. 50
   3.5 Supply .................................................................................................................. 52
   3.6 General Assumptions on the Components of the Model .................................... 55

4 Path-Based Formulation for Car Distribution Problem ............................................... 56
4.1 A General Integer Programming Formulation .......................................................... 56
4.2 Pure Allocation Model on a Bipartite Resource Constrained Graph ...................... 65
4.3 Allocation-Timing Model on a Bipartite Resource Constrained Multigraph .......... 69
   4.3.1 An Algorithm for Generating Pseudo-paths of a Trip plan .............................. 69
   4.3.2 A Lagrangian Heuristic .................................................................................. 76
   4.3.3 An Iterative Relaxation and Rounding Heuristic ............................................. 83
4.4 Noncapacitated Model and its Optimistic Behavior on a Capacitated Network...... 86
4.5 Trip Planning Algorithm ......................................................................................... 91
   4.5.1 Trip Planning on the Time-Space Block-Train Network ................................ 92
   4.5.2 Trip Planning on the Time-Space Resource Constrained Blocking Network .... 93
5 Arc-Based Formulation for the Car Distribution Problem ....................................... 94
   5.1 Allocation-Routing Model .................................................................................... 94
   5.2 Formulating the Anti-Leak Rule using Extra Constraints .................................. 100
   5.3 Formulating the Anti-Leak Rule with Redefining the Decision Variables ............ 100
6 Numerical Results ........................................................................................................ 105
   6.1 Verification and Validation .................................................................................... 107
   6.2 Illustrative Example ............................................................................................. 116
      6.2.1 Input data ........................................................................................................ 116
      6.2.2 Model Results, Interpretation, and Remarks .................................................. 119
   6.3 Performance of Models and Solution Procedures in Larger Scale Problems ...... 128
      6.3.1 Generating case examples .............................................................................. 130
      6.3.2 Sensitivity Analysis on Network Capacities .................................................. 134
      6.3.3 Sensitivity Analysis on Shortage Penalties ..................................................... 140
      6.3.4 Average Time Complexity of the Proposed Heuristic .................................. 142
      6.3.5 Solution Quality of the Proposed Heuristic .................................................. 146
7 Summary, Results and Future Research ..................................................................... 148
   7.1 Summary ................................................................................................................ 148
   7.2 Results ..................................................................................................................... 151
7.3 Future Research .......................................................... 151

References ............................................................................. 153
Chapter 1

Introduction

1.1 Problem Background

 Freight railroads receive shipment requests from customers (internal or external) to transport cars. External customers, also referred to as customers, send their empty car requests to the railroad's car distribution unit. Once a customer asks the car distribution unit to send one empty railcar to its facility, the car distribution unit will allocate an empty railcar in the network that is suitable for the customer request and, as an internal customer, will send a shipment request to the railroad operation department. The operation department then generates a trip plan indicating the path the car will follow from its origin to its destination. Once it arrives at the customer facility, the car will be loaded by the customer. Next, the customer sends a shipment request to the railroad to pull the loaded car to the destination they determine. The operation department then generates another trip plan to take this loaded car to its destination. A shipment will have attributes such as origin, destination, car type, physical dimensions, freight type, etc., which will be used by the trip planning application to generate the trip plan for this shipment. Once it arrives at the destination, the car will be unloaded and the railroad will be notified that the car is empty. Once the empty car is released by the customer, one car cycle has finished and the car can be allocated again to other customers by the car distribution unit. In this research we mainly
focus on the part of the decision that is made by the car distribution unit of the railroad in allocating empty railcars, supply, to customer order, demand.

1.2 Problem Statement

This research primarily deals with developing effective models for the distribution of empty railcars in order to fulfill customer demand over a multi-period horizon, under the constraints of supply and demand and network capacities. Furthermore, the research will address various issues including customer priorities, order expiration dates, and permitted substitutions. Also, since the car distribution problem is an operational decision, suitable real-time solution methods will be developed and validated in order to be useful to the railroad carriers.

Although in the literature the words distribution, allocation, repositioning, and assignment are used interchangeably to address this problem, even if it is combined with other optimization problems, e.g. with car routing problem, we will slightly differentiate between these terms in this research. We first use the term "empty car/railcar distribution" as a general term to refer to the repositioning of empty railcars from supply locations to demand locations, and then divide car distribution problems into two categories. The first category covers the pure allocation, or assignment, of supply to demand, where the route between supply and demand locations, known as a trip plan in railroad language, is given by an independent application and is out of the scope of the allocation model. The second category, however, simultaneously covers both the allocation of empty cars to the demand for cars and their associated routings in an integrated allocation-routing model. Clearly, the aggregated decision from the optimal car routing and car allocation problems might be sub-optimal compared to the one achieved from the integrated allocation-routing model.
However, the integration of allocation and routing decisions might not be applicable to all railroads as the decision makers might be different. Whether the first or the second modeling category is suitable for a certain railroad depends on the organization and the policies of that railroad. For example, U. S. Class 1 railroads, currently separate the car allocation and car routing decisions (Gorman et al., 2011), while Swedish National Railroad combines them (Holmberg et al., 1998).

1.3 Motivation and Objectives of the Dissertation

Because of spatial imbalances in the supply and demand of empty railcars in the freight transportation system, empty equipment repositioning is inevitable. Historically, railroads have used different repositioning methods, including dedicated pools (where a car is dedicated to a customer and will be returned to the same customer once it becomes empty), single-car allocation (ad hoc rules of assigning empty cars to customers; for example, supply covered coil demand of Pennsylvania from Conway Hub), offline optimization models (optimizing the assignment of projected supply and forecasted demand, as in old CSX Sentinel system), and most recently, real time optimization models, implemented by railway carriers CSX, BNSF, UP and CP (Gorman et al., 2010).

In 1997, CSX implemented their real-time car distribution system, Dynamic Car Planning (DCP), formulated as a single commodity network flow problem which was solved using a proprietary LP algorithm. By investing $5 million in DCP, CSX has claimed a reduction of 7% in empty railcar movement, translated into annual savings of $50 million (Gorman et al., 2010).

In 2000, BNSF launched their real-time car distribution system, Equipment Distribution Optimization (EDO), which is formulated as a transportation problem. BNSF
1.3 Motivation and Objectives of the Dissertation

reported an annual savings of $13 million after installing this $3 million system (Gorman et al., 2011).

Another North American Class 1 Railroad, UP has reported 35% ROI by reducing manpower needed for the demand fulfillment process and a significant reduction of costs by allowing the carrier to manage the cost of the miles moved while meeting the delivery date for empty cars. (Narisetty et al., 2008).

In Europe, the European Commission (EC) has funded a number of research works, among them the WagonLink (railcar reservation system), CroBIT (data exchange platform) and F-MAN (real-time track-and-trace and railcar management) projects in order to investigate the deficiencies of the railcar management system and propose solutions for its advancement (Ballis and Dimitriou, 2010). Also Swedish State Railway has developed the kernel of their empty railcar distribution system based on a multi-commodity network flow model presented by Holmberg et al. (1998).

Despite the vast studies performed in the railroad optimization modeling, there is still a need for another realistic modeling and optimization of the empty railcar distribution problem. In particular, there is no previous work that optimizes the empty railcar distribution decisions while simultaneously considering the resource (capacity) availabilities in the service network and the car type substitutability for the customer demands as well as separating the car distribution and car routing decision in accordance to the US railroad practices.

Current practices of the U. S. railroads do not consider network capacities in car allocation or in trip planning models. Ignoring capacity limits on the network will make the allocation and routing decisions independent of each other. This by itself justifies why
these decisions are currently separated from each other in the U.S. railroads. However, formulating the problem as a classical transportation model, or as an uncapacitated minimum cost flow formulation, might deliver solutions that are practically infeasible or feasible with higher costs in the real application under the presence of the network capacities. A capacitated multi-commodity network flow formulation, used for the Swedish National Railway, is also not fully applicable to the U.S. railroads mainly because it combines car routings and car distribution decisions.

The situation in US railroads is changing however. As presented in INFORMS (https://www.informs.org/content/download/239255/2274025/.../SC1.pdf), Norfolk Southern's Next Generation car routing system (ABC-NG) will consider network capacities (block capacities and train capacities) while creating the trip plans. Once network capacities are considered in the trip plan application, considering them in the allocation model seems inevitable. It is noteworthy that one of the principal causes of uncertainty in the travelling time arises from ignoring the network capacities in both the car allocation and the car routing decision making processes. By formulating the problem on a capacitated network, we can decrease the level of uncertainty in travel times, and as a result construct a deterministic model. Furthermore once capacities are considered during car routing and car distribution decision making, integrating empty car allocation and car routing decision could be the next logical step to lower the railroad operational costs. This is the reason we build our car distribution model, complying with current organization of US railroad which separates the car distribution and car routing decisions, on top of a capacitated network and compare its performance with both noncapacitated model and an integrated allocation-routing model. The objective of this research can be summarized as:
• to develop realistic optimization models for the distribution of empty railcars and
• to develop effective solution methods for solving those models and finally
• to validate the accuracy of the developed models and to verify the accuracy of computer programs developed for those models using verification and validation techniques

1.4 Dissertation Outline

The Dissertation is organized as follows. In Chapter 2 the literature of empty railcar distribution problem is studied, from both practical and theoretical perspectives. The previous research is classified in three categories based on the formulation: Transportation problem, Transshipment and network flow, and multi commodity network flows. At the end of Chapter 2, the gap between the literature and the reality is identified and the contribution of this research is described.

Important terminologies, vastly used in the future chapters, as well as the components of the input data set are described in Chapter 3 before presenting car distribution models in Chapter 4 and 5.

A path-based formulation for the capacitated car distribution problem is presented in Chapter 4 in two different modes: mode 1) The pure allocation, or simply the allocation model, where each feasible supply-demand pair is connected using only one path; and mode 2) the allocation-timing model, where more than one path is allowed between them. A third formulation where all feasible paths are considered can be transformed to an arc-based formulation and is discussed in Chapter 5.
The arc-based formulation presented in Chapter 5 is defined on a time-space network and integrates car routing and car distribution decisions in a single Allocation-Routing model.

The numerical techniques and results are presented in Chapter 6. First the verification and validation techniques are described then the implementation of the path-based and arc-based formulations of the car distribution problem is illustrated in a small size example on a medium size network and finally the average time complexity of the developed heuristics is calculated by generating random problem instances of various sizes.
Chapter 2

Literature Review

The empty car distribution problem has been studied for half a century from both a theoretical and a practical perspective. Dejax and Crainic (1987) describe several criteria that might be used to classify the numerous works that appeared in the literature of the empty flow and fleet sizing models. In this section we slightly redefine those criteria to be suitable for the empty car distribution problem.

Problem statement criteria:

Optimization focus or the level of integration: in a pure car allocation model the decision made is car-to-order assignments only (Gorman et al., 2010, 2011). Other optimization problems such as car routing and train and crew scheduling are optimized in different applications. In an integrated car distribution model however, car allocation might be integrated with other optimization problems. For example, it can be integrated with the car routing problem and be formulated as a transshipment problem (Holmberg et al., 1998). Or it can be integrated with the train routing problem (Haghani, 1989).

Fleet homogeneity: in a homogeneous fleet (case of the single commodity problem) all cars are eligible to be allocated to all orders. In a heterogeneous fleet (case of the multi-commodity problem), there are several types of cars that might be partially or totally
substituted for each other. It is noteworthy that if the heterogeneous fleet can be divided into multiple groups that cannot be substituted for each other like tank cars and coil cars and if cars in different groups do not share resources with each other like in uncapacitated networks, then the problem can be decomposed to multiple homogeneous fleet sub-problems (Holmberg et al., 1998).

**Transportation mode**: while container management and intermodal transportations has been studied vastly (Choong et al., 2002; Wang and Wang, 2007; Dong and Song, 2009; Dang et al., 2012; Song and Dong, 2013), in this research we will focus on rail cars only and will not study intermodal car distribution case.

**Type of flow**: while most of car distribution problems focus on the empty flow only, there are some problems that consider both empty and load movements (Bandeira et al., 2009; Mesa-Arango et al., 2013).

**Modeling assumptions criteria:**

**Time domain**: some researchers consider only static cases (Misra, 1972), while some others take time into consideration and formulate the problem as a dynamic model (White, 1972; Jordan and Turnquist, 1983; Powell, 1987; Godfrey and Powell, 2002).

**Level of certainty**: some researchers treat the problem as a deterministic problem and perform frequent runs to overcome the dynamics of data (Gorman et al., 2010, 2011), while others exclusively consider the stochasticity of supply, demand or network data as part of formulation (Jordan and Turnquist, 1983; Topaloglu and Powell, 2005; Powell and Topaloglu, 2006; Topaloglu and Powell, 2006; Lam et al., 2007; Topaloglu, 2007; Cao et al., 2008; Erera et al., 2009).
Side constraints: in addition to the supply and demand constraint, a car distribution problem might have some side constraint, for example network capacity constraints (Holmberg et al., 1998).

Problem formulation criteria:

Single objective or multi objective: while minimizing the total allocation cost is the most common objective in the literature, some researchers have considered other objectives, e.g. minimizing the total tardiness (Spieckermann and Voß, 1995).

Modeling technique: different researchers has used different techniques such as: general mathematical models (LP, IP, MLP, MIP, etc.), network models, simulation models or stochastic models to formulate the car distribution problem.

Solution criteria:

Solution precision: Exact algorithms (e.g. network algorithms or simplex algorithm) were used when the problem size was small (Misra, 1972) or the model was simplified, by for example relaxing the capacities, to make it practical for real word implementations (Gorman et al., 2011), while heuristics or metaheuristics were developed mainly for the integrated models when the state-of-the-art software were not able to solve the complex formulations (Holmberg and Yuan, 2003; Holmberg et al., 2008; Joborn et al., 2004).

Solution validation: some models are tested on the real world networks as case studies. For example Sherali and Suharko (1998) uses TTX data to validate their model and Spieckermann and VoB uses CARGOWAGGON GmbH of Germany for validation purposes. Some other works are published results of the real world implementation of the
models (Holmberg et al., 1998; Narisetty et al., 2008; Gorman et al., 2010, 2011; Engels, 2011). The majority of the works however, discuss the theory of the car distribution problem.

From a modeling perspective, the problem has been mostly formulated as a transportation problem, pure or with side constraints, network flow problem (for example transshipment problem, pure or with side constraints) and with multi commodity flows. In the next section, we review these models as well as the relevant literature of the empty railcar distribution.

### 2.1 Transportation Problem

The transportation problem was formulated first by Hitchcock in 1941 and then independently by Koopmans in 1951, and it is therefore also referred to as the Hitchcock-Koopmans problem.

The problem is defined over a bipartite graph of supply nodes on one side and demand nodes on the other side and seeks the optimal distribution plan of units of a single product from supply points \( i \in I \) to demand points \( j \in J \) while minimizing the total transportation cost.

Let \( s_i \) be the amount of supply at a supply node \( i \in I \) and \( d_j \) be the demand at a demand node \( j \in J \). Also, let \( c_{ij} \) and \( x_{ij} \) be the unit transportation cost between \( i \) and \( j \), and the amount transported between these nodes, respectively. The mathematical programming of the transportation problem then is given as:

\[
\begin{align*}
\text{Min} & \quad \sum_{j \in J} \sum_{i \in I} c_{ij} x_{ij} \\
\text{Subject to:} & \quad \sum_{i \in I} x_{ij} = d_j, \quad \forall j \in J \\
& \quad \sum_{j \in J} x_{ij} = s_i, \quad \forall i \in I \\
& \quad x_{ij} \geq 0, \quad \forall i \in I, j \in J
\end{align*}
\]

(2-1)
The objective function (2-1) minimizes the total transportation cost. The constraint set (2-2) stipulates that the total shipments from a node must be equal to its supply; similarly, the constraint set (2-3) states that the total shipments to a demand node must satisfy its demand. Finally constraint set (2-4) prevents negative flows in the network. In order to guarantee the feasibility of this model, total supply must be equal to total demand, i.e. $\sum_{i \in I} s_i = \sum_{j \in J} d_j$.

Several algorithms have been developed to solve this problem including but not limited to the one by Hitchcock himself, and those by Dantzig (1951), and Ford and Fulkerson (1956). In textbooks however, the problem is solved by a version of simplex method called transportation simplex.

One important feature of the transportation problem (and in general any network flow problem) is the unimodularity of the constraints' coefficient matrix. As a result, if the supply and demand quantities, i.e. right hand sides, are integers, then the optimal solution will also be integer valued, in other words in order to solve the integral transportation problem, it is sufficient to solve the LP relaxation of the problem. This characteristic along with the easy formulation of the problem has motivated many researchers and practitioners to use transportation model in the empty railcar distribution formulations.

Misra's (1972) article was the first published work that formulated the problem as a transportation model minimizing the total travel time over a static time period. He proposed transportation simplex as a solution method. In the same paper, he also covered
the capacitated model, where some arcs of the rail network, called bottlenecks, have capacity limitations. His capacitated model integrates the car allocation and routing together and is solved by the simplex method. Sherali and Suharko (1998) presented two formulations for the problem of repositioning TTX cars, one of which is a capacitated transportation model. A detailed discussion on the cost function formulation of their model is presented in Suharko (1997). Their second model considers economies of scale and will be discussed later. While transportation problem formulation seems too simple to be used by the railroads, it has actually been deployed in practice. In 2000, BNSF launched their real-time car distribution system, Equipment Distribution Optimization (EDO), which is formulated as a transportation problem. BNSF reported an annual savings of $13 million after installing this $3 million system (Gorman et al., 2011).

2.2 Transshipment and Network Flow Problem

The objective of a minimum cost network flow problem is to determine the least cost shipment of a commodity through a network in order to satisfy demands at certain nodes from available supplies at other nodes. Unlike the transportation problem, the network here is not necessarily a bipartite graph. In fact, the transportation problem is a special case of the minimum cost flow problem.

Let $G(V, E)$ be a finite, connected, directed and weighted network with node set $V$ of $n$ nodes and edge set $E$ of $m$ directed arcs. Associated with each arc $(i, j) \in E$ there is a cost $c_{ij}$ representing the cost per unit flow on that arc as well as two bounds, $l_{ij}$ and $u_{ij}$, respectively representing the minimum and maximum amount that can flow on the arc. Each node $i \in V$ has an integer weight $b_i$ representing its supply/demand. If $b_i > 0$, node $i$ is a supply node; if $b_i < 0$, node $i$ is a demand node with a demand of $-b_i$; and if
$b_i = 0$, node $i$ is a *transshipment node*. The decision variables in the minimum cost flow problem are arc flows represented by $x_{ij}$. The mathematical formulation of the problem is as follows (Ahuja et al., 1993):

\[
\begin{align*}
\text{Min} & \quad \sum_{(i,j) \in E} c_{ij}x_{ij} \quad (2-5) \\
\text{s.t.} & \\
\sum_{(j:i,j) \in E} x_{ij} - \sum_{(j,i,j) \in E} x_{ji} &= b_i, \quad \forall i \in V, \quad (2-6) \\
l_{ij} \leq x_{ij} \leq u_{ij}, \quad \forall (i,j) \in E, \quad (2-7)
\end{align*}
\]

where $\sum_{i=1}^n b_i = 0$. A matrix representation of the problem is as follows:

\[
\begin{align*}
\text{Min} & \quad cx \quad (2-8) \\
\text{s.t.} & \\
Ax &= b, \quad (2-9) \\
l \leq x \leq u, \quad (2-10)
\end{align*}
\]

where $A$ is an $n \times m$ matrix, called the *node-arc incidence matrix* of the minimum cost flow problem. It is proven that the node-arc incidence matrix of the network flow problem is totally unimodular and therefore unimodular and as a result, every basic feasible solution of the problem is an integer for any integer vector $b$.

The constraints in set (2-9) are known as *mass balance constraints*. The mass balance constraint states that the outflow (i.e., the flow emanating from the node) minus the inflow (i.e., the flow entering the node) must equal the supply/demand of the node. If the node is a supply node, its outflow exceeds its inflow; if the node is a demand node, its inflow exceeds its outflow; and if the node is a transshipment node, its outflow equals its
inflow. The flow must also satisfy the lower bound and capacity constraints (2-10), which are referred to as flow bound constraints. A minimum cost flow model without any constraint on arc flow bounds is called a transshipment model.

As stated in Ahuja et al. (1993), the minimum cost flow model is the most fundamental of all network flow problems and is widely used in different applications such as supply chain management; production lines; call routing through telephone systems, and empty railcar distribution systems.

The minimum cost flow problems used in empty railcar distributions can be divided into two categories based on the nature of the transshipment nodes: Those with temporal transshipment nodes and those with temporal or physical transshipment nodes. In the first group, cars flowing from supply nodes to demand nodes do not travel through any other physical node of the network, or at least we can say the model does not deal with the movements through the intermediate nodes. Transshipment nodes here are not physical nodes but temporal nodes added to the network to transform, for example, a static transportation problem (capacitated transportation problem) to a dynamic model formulated as transshipment problem (minimum cost flow problem). The decision variables of these models can be easily translated to supply-to-demand assignment decisions. In fact the operational department will know how many cars, at what time, and from what supply nodes, are planned to be sent to what demand nodes. The second group consists of the models in which the flow of cars, from the supply nodes to the demand nodes, is physically transferred through other nodes of the network. The solution of these models gives the flow of cars on each arc of the network, but does not explicitly offer any supply to demand (origin to destination) assignment. In the United State railways, the
origin and destination of each car movement have to be specified as the output of the car
distribution decision. Ahuja et al. (1993) describe an algorithm to decompose the arc flow
solution into path flows. The output of such an algorithm however, suggests that both the
car assignment and the car routing decisions are made at once which is not in alignment
with the US railroad processes and policies that separate these decisions (Gorman et al.,
2011).

The research by White and Bomberault (1969) was the first to formulate the car
distribution problem as a transshipment problem in a time-space network. It was the first
time that dynamics were added to the car distribution model. The arcs of this network
represent the scheduled trains and the nodes stand for the train events, i.e. train departure
and arrivals. The underlying time-space network was called the dynamic transshipment
network three years later in an extension of this paper to the empty container allocation
problem (White, 1972). Each node in the dynamic transshipment network represents a
location in a specific time. Nodes are connected to one another by movement arcs and
inventory arcs. A movement arc (referred to as a physical arc in their work) represents a
movement from one location to another and is associated with a travel cost, while an
inventory arc (referred to as temporal arcs in their work) connects two nodes corresponding
to one location in two consecutive days and is associated with a storage cost. The network
is acyclic and no backward-time arc is allowed. They used an adoption of the out-of-kilter
algorithm to solve their model. White and Bomberault (1969) and White (1972) both
emphasized the importance of car substitution and network capacities in the real world
railroad operations and suggested that these concepts should be studied in future research.
In his Master thesis, Ouimet (1972) studied the capacitated version of White and
Bomberault (1969), which by definition is considered a minimum cost network flow problem, and suggested an out-of-kilter algorithm to solve it. The fleet however was considered homogeneous.

Herren (1977) considered another minimum cost flow formulation for the car distribution problem at the Swiss Federal Railroad (SBB). As the model was built to be sent to operations, the fleet, this time, was considered heterogeneous with substitution possibilities and the resulted transshipment problem was solved by a modified out-of-kilter algorithm. Turnquist and Markowicz (1989) presented a new time-space schema for the problem. In their model each location in each day is represented by at least three nodes: one demand node plus one supply node and one collector node for each car type. For example if there are two car types, each location-time will be represented by five nodes (one demand node, two supply nodes and two collector nodes. In addition to the supply and demand nodes and movement and inventory arcs presented in White and Bomberault (1969), they introduced collector nodes and reverse arcs that enabled them to formulate the backorder in their model. Using this new network representation, they were able to formulate the problem with partial substitution as a single commodity network flow problem. The research was tested and implemented at CSX railroad from 1990 to 1996 (Newman et al., 2002). Then, in 1997, CSX implemented their real-time car distribution system, Dynamic Car Planning (DCP), formulated as another single commodity network flow problem which was solved using a proprietary LP algorithm (Gorman et al., 2010). CSX formulation's underlying network contains four layers of nodes: one source node, multiple supply nodes, multiple demand nodes, and one sink node, without the introduction of intermediate nodes between supply and demand nodes. While there is no capacity
restriction on the arcs, demand nodes have a restricted inflow equal to the customer demand. This restriction on the inflow of a node is also known as the node capacity. In the formulation of minimum cost flow problem presented at the beginning of this section, network arcs are capacitated but there is no restrictions on the node capacities. However they were still able to use minimum cost flow algorithms to solve the problem, as a node with a restricted inflow could be replaced by two unrestricted nodes connected by an arc of the relevant capacity. In their case, such a change is not necessary, as each demand node has only one single outflow arc and the demand nodes' capacities can be dropped after enforcing a capacity limit of the same amount to their associated single outflow arcs.

2.3 Multi Commodity Network Flow Problem

The multicommodity flow problem deals with the shipment of several commodities along the arcs of an underlying network. Depending on the application, the commodities are either distinguished by their physical characteristics or simply by their origin and destination nodes. In the empty car distribution problem, where different car types need to be assigned to different customer demands, the first definition of commodity is a good fit, while the second type of application can be used in the routing of the loaded cars in railroads when the origin and the destination of each load is known in advance.

The minimum cost multicommodity flow problem can be formulated as follows (Ahuja et al., 1993):

\[
\begin{align*}
\text{Min} & \quad \sum_{k \in K} c^k x^k \\
\text{s.t.} & \quad Ax^k = b^k \quad \forall k \in K,
\end{align*}
\]  

(2-11) (2-12)
2.3 Multi Commodity Network Flow Problem

\[
\sum_{k \in K} x_{ij}^k \leq u_{ij} \quad \forall (i,j) \in E, \quad (2-13)
\]
\[
0 \leq x_{ij}^k \leq u_{ij}^k \quad \forall (i,j) \in E, \forall k \in K, \quad (2-14)
\]

where \( A \) is the \textit{node-arc incidence matrix} of the network, the decision variable \( x_{ij}^k \) represents the flow of commodity \( k \) on arc \((i,j)\), and \( x^k, b^k \) and \( c^k \) represent the flow vector, the supply/demand amount vector, and per unit cost vector for commodity \( k \) respectively. Each arc \((i,j)\) has two sets of capacity constraints: an \textit{individual commodity capacity}, \( u_{ij}^k \), limiting the flow of commodity \( k \) on the arc and an \textit{arc bundle capacity}, \( u_{ij} \), limiting its total flow of all commodities.

A minimum cost multicommodity flow problem seeks the optimal allocation of the capacity of each arc to the individual commodities while minimizing the overall flow costs (2-11). Each commodity has its own mass balance constraints (2-12), but the arc capacities are shared among all commodities (2-13). In some applications, the flow of each commodity on each arc is also bounded (2-14).

Unlike the single commodity network flows, the integrality of the multicommodity network flow problems’ solution is not guaranteed. It is because the unimodularity of the constraints coefficient matrix is violated as a side effect of the arc bundle capacities.

The multicommodity flow problem has many practical applications, such as the transportation of passengers within a city, the transmission of messages in a communication network between different origin-destination (O-D) pairs and the distribution of empty (or loaded) cars sharing the same transportation network. The introduction of this model to the empty railcar distribution problem goes back to the end of the 20th century.
2.4 Economies of Scale

Joborn's (2001) Ph.D. dissertation and Holmberg's et al. (1998) research for the Swedish National Railway were the first published works that formulated the empty railcar distribution problem as a multi commodity network flow model. In their model, no substitution was allowed but different commodities (car types) shared same capacitated trains (arcs). One approximate solution was to list commodities in an order and to solve a series of inter-related single commodity problems, where the arc capacities needed to be updated after running for each commodity. However, in order to find the global optimal solution they used two methods. The first one was to use a Lagrangian based heuristic and the second one was to solve by CPLEX package. Joborn et al. (2004) expanded this non-substitutable multi commodity network flow model by charging fixed costs on origin-destination pairs to exploit economies of scale that enforce multiple empty cars to be grouped and sent in large clusters. The detail of their model is presented in Section 2.4.

2.4 Economies of Scale

Increasing the profit by means of lowering the marginal cost of transportation has been considered in many railroad-related researches. In fact the motivation behind the block design problem that arises in the tactical level of decision making in the railroad industries is to gain economies of scale by moving larger numbers of cars in groups rather than individual cars travelling from their origins to their destinations.

To the best of our knowledge, Turnquist's (1994) technical report was the first attempt to formulate economies of scale in the car distribution model. He used a time-space network similar to the one in Turnquist and Markowicz (1989) for his uncapacitated network design formulation. Economies of scale was gained by charging a fixed cost on the O-D pairs utilized in the distribution and the objective was to minimize both
transportation costs (variable costs) and the number of O-D pairs (fixed costs). His case study results showed a near 50% improvement in all criteria he listed for the car utilization, however, with an extremely high cost on execution time. For example the running time for a network with 4k nodes and 14k arcs, jumped from 1 minute to 14 hours, what he described as "the operation was successful, but the patient died!". As computer technology substantially improved from 1994 to 2004, Joborn et al. (2004) extended Turnquist's work, by adding the capacities on the trains that pull cars from their origin to their destinations while charging a fixed cost on the utilized paths. A path, comparing to the O-D pair considered in Turnquist (1994), also includes the routing of cars. As a result, there might be multiple paths, and consequently multiple fixed costs associated to a single O-D pair. They proposed Tabu Search metaheuristics to solve their model. Holmberg et al. (2008) developed a method to find feasible solutions and dual bounds for the same problem using a Lagrangian heuristic. Since the Lagrangian heuristic yields feasible integer solutions and, unlike the Tabu Search which yields an upper bound only, it presents both lower and upper bounds on the optimal value of the objective function; it can be used as a basis for a branch-and-bound algorithm to find the optimal solution. The branch and bound implementation however, is not presented in their paper.

In another effort to formulate the economies of scale, Suharko (1997) in his PhD Dissertation limited the number of super destinations that cars can be sent to from each origin. His model can be described as a capacitated transportation problem with an upper bound on the number of super demand nodes served by each supply node, where a super demand node is a set of demand nodes in a neighborhood. The results of his dissertation were published in Sherali and Suharko (1998).
2.5 Survey Papers

In 1987, Dejax and Crainic published the first survey paper on “empty flows and fleet management models in freight transportation”. The paper was not limited to empty railcar allocation literature only, and container allocation and truck allocations were also covered. From a decision level point of view, they covered the tactical decision of the fleet sizing in addition to the operational car allocation problem. In the same year, Haghani (1987) published another survey paper on railcar distribution and train routing. Crainic and Laporte (1997), Newman et al. (2002) and, in more details, Crainic (2003) investigated the literature of different levels of decision making processes in railroads from operational to tactical, to strategic planning.

2.6 Contribution of the Research

Despite the vast studies performed in the railroad optimization modelling and particularly in the car distribution problem, to the best of our knowledge, there is no previous work that optimizes the empty railcar distribution decisions while simultaneously considering the resource (capacity) availabilities in the service network and the car type substitutability for the customer demands while complying with the railroad decision making processes on separating car distribution and car routing decisions.

Currently, the U. S. Class 1 Railroads use a method named "first train available", practically noncapacitated shortest path, for the trip planning purposes. The abstract information delivered by this trip plan, such as the arrival time and the travel distance, is then used as part of the parameters of the car distribution model. However, in practice, the first train available might not have enough capacity to carry extra cars and as a result the
practical departure time and consequently the actual arrival time might be later than what has been considered in the model.

Cars are carried on blocks, and blocks are pulled by trains. Both blocks and trains have capacity limits. Blocks are created on the classification yards. Normally each block is created on one rail track in the classification yard. Tracks in classification yards have fixed length and as a result each block also has a length. Trains travel over the physical railroad network. Depending on the geography of the area, the track attributes, and the signaling requirements, certain segments of the network might have limitations on the maximum train length and weight. Assuming that train length constraints have been already satisfied at the tactical level of the block to train assignment decision making process, the train weight limits still need to be considered in the operational models such as trip planning and car distributions.

In fact, formulating the problem as a classical transportation model, or as an uncapacitated minimum cost flow formulation, might deliver solutions that are practically infeasible or feasible with higher costs in the real application under the presence of the network capacities.

Taking advantage of their simple formulations, railroads have been able to model the 1 to 1 car type substitution by means of a feasibility engine and charging a Big M cost on incompatible supply-order pairs. A 1 to 1 substitution means that one car of a certain type, can be substituted by one car of another certain type(s). This kind of substitution relies on the assumption that all cars eligible for a specific demand have the same size or volume. The only work that formulates the car substitutions among different car types with a substitution ratio not necessarily equal to one seems to be Engel's (2011) Ph. D.
2.6 Contribution of the Research

dissertation for DB Schenker Rail Deutschland AG of the European railroad network. However Engel's formulation also overlooks the network capacities. A capacitated multicommodity network flow formulation used in Holmberg's et al. (1998) research for the Swedish National Railway, is also not fully applicable here since it considers capacity limits on trains only and overlooks block capacities, does not allow substitutions between different car types, and assumes that all cars have similar length and weight, and regardless of their types, occupy one unit of train capacity.

Car distribution is an operational decision. A good car distribution plan should also consider the yard operation loads. Blocks are created in classification tracks of classification yards. Sometime the number of blocks that are planned, in tactical level, to be built in daily basis in a classification yard is more than the number of classification tracks available in that yard. While block design is a tactical decision and cannot be altered in the operational level, block creation is an operational task and if in a specific day there is no car assigned to a block, that block will be automatically canceled. In our model by charging a block setup cost, only on the blocks located in critical classification yards, i.e. yards with more number of blocks than their classification tracks, and only if there is no car already planned on them, we try to ease the job of classification yard car dispatchers as well.

In this research, we will build our car distribution model based on practical assumptions such as separation of car routing and car distribution decisions while considering network capacities including block capacity constraints as a limit on the total length of the block and train capacity as a limit on the total weight of the train, car type substitution with different substitution ratios, customer order time windows (order
2.6 Contribution of the Research

expiration date) and we will try to ease the dispatcher’s job in classification yards by trying to lower the number of blocks created in the critical yards. Table 2-1 shows a comparison between current applications and our main model. This Model by itself can be run in different modes that will be discussed in detail in Chapter 4 and 5 and their performance, along with a noncapacitated model performance, will be compared in Chapter 6.

Table 2-1 A comparison among current models used by railway carriers and our model

<table>
<thead>
<tr>
<th>Railway</th>
<th>Green Cargo - Sweden</th>
<th>UP</th>
<th>CSX</th>
<th>BNSF</th>
<th>Our Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>System name</td>
<td>-</td>
<td>TCM</td>
<td>DCP</td>
<td>EDO</td>
<td>Path-based capacitated network flow</td>
</tr>
<tr>
<td>Formulation</td>
<td>Multi-commodity network flow</td>
<td>Transp. problem</td>
<td>Cons. of flow</td>
<td>Transp. problem</td>
<td>-</td>
</tr>
<tr>
<td>Solution method</td>
<td>Tabu search</td>
<td>LP</td>
<td>LP</td>
<td>LP</td>
<td>IP</td>
</tr>
<tr>
<td>Substitution</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Setup cost on new critical blocks</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Capacitated (block, train)</td>
<td>Yes (train)</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes (both blocks and trains)</td>
</tr>
<tr>
<td>Combines car routing and car distribution?</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Total Car-Management (TCM); Dynamic Car Planning (DCP); Equipment Distribution Optimization (EDO)
In this section, we describe the important terminologies as well as the components of the input set of our models. First we explain the important terms that we will use throughout this research and then describe the underlying capacitated service network used in our formulation, followed by overviewing the fleet, supply and demand.

3.1 Important Terminologies

A railcar trip plan is similar to a passenger itinerary in the airline industry. In fact, despite the fact that planes fly in the sky and trains run on rail tracks, the underlying service network in railroads is similar to the one in the airline industry. Figure 3-1 shows a passenger itinerary for a customer who is planning to travel from Boston, MA to San Francisco, CA on Tue, Nov 11.
3.1 Important Terminologies

Figure 3-1 An itinerary for a flight trip

Based on this itinerary, this 2-leg trip has a connection in Minneapolis, MN. Minneapolis might or might not be the final destination of the individual passengers in flight 2027, however, all passengers on this flight will travel in the same plane from Boston to Minneapolis (aggregation). In Minneapolis, all passengers will leave the plane and our passenger will take flight 2209 toward San Francisco with other passengers who either start their trip from Minneapolis or have a flight connection there and San Francisco is either their final destination or their connection airport.

A railcar trip plan is akin to a passenger itinerary in the airline industry. In this context, passengers are railcars, flight legs are called blocks, and flights are the trains\(^1\). A trip plan determines the sequence of blocks the railcar will use, the trains that will carry these blocks, and the timing of these movements.

---
\(^1\) However in the railroad industry, contrary to the airline industry, a train might have intermediate stops to pick up or set off blocks and as a result it might carry more than one block at the same time.
3.1 Important Terminologies

Similar to airline passengers, a set of cars can be grouped at a yard, then separated into individual cars at a subsequent yard and again aggregated with different cars into a different group. A set of railcars that are moved as a single group from one rail yard (block's origin) to another (block's destination) is called a block and the process of aggregation and separation of cars in the yards is referred to as Blocking or Classification. Block’s origin and destination are predesigned in the tactical level. In fact block is a predesigned origin-destination pair where all cars will be treated as a group as long as they are in the same block. It is important to differentiate between blocks and shipments. 100 cars moving from Boston to San Francisco are not necessarily considered a block the same way that a group of 100 students flying from Boston to San Francisco doesn’t necessarily mean there is a direct flight between these two cities. The individual cars within the block might have different origins and destinations but will be treated as a unit in the part of the trip which they share as a block.

Trip plans are generated based on the operating plan which consists of three main components: a blocking plan, train routings, and a train makeup plan. Each of these components by itself is an optimization problem that arises at tactical level. A blocking plan can be derived from a block design problem that determines which blocks need to be made at each rail yard, and what type of railcars are to be placed into each block. The train routing problem is to identify the origin, destination, routes, frequencies, and timetables of all trains to minimize the cost of carrying cars from their origins to their destinations. Once the blocking plan and the train routings are ready, the next step is to determine which trains should carry which blocks (train makeup or block to train assignment problem). Train timetables and routes should be designed in such a way that they are consistent with the
rail network and the blocks to be transported. That is why the train routing and the train makeup problem are often integrated into a single problem, called the train design problem.

Operating plan development is a tactical decision and is considered a service network design problem (SNDP). In the operational level of decision making such as the empty railcar allocations or car routing (trip planning) we realistically assume that the operating plan is already developed and the operational decisions should be consistent with this plan.

### 3.2 Network Representation

The network referred to in most part of this research, is the service network also known as the railroad operating plan. The operating plan is designed on top of the physical (geographical) railroad network where the track segment’s maximum tonnage enforces the train capacities and the classification track lengths determine the block capacities. In this section, we first present the static networks, i.e. without time dimension, including the physical network and the static operating plan. This will give us the opportunity to explain in detail the operating plan and the steps that are taken at the tactical level to design this plan. Then, we add the time dimension to the network and create a time-space network which is suitable for our dynamic car distribution model.

#### 3.2.1 Static Networks

Figure 3-2(a) shows a geographical/physical railroad network of seven nodes and nine arcs, where nodes correspond to the rail yards, and arcs correspond to the rail tracks, also known as rail segments, connecting these yards. Designing/redesigning the geographical railroad network is a strategic decision made by the highest management, and in this research, we realistically assume that the physical network is given. Three trains $A$, $B$, and $C$...
3.2 Network Representation

$B$, and $C$ traveling in this network have been designed at the train routing level and are shown in Figure 3-2(b). All trains start at node 1 and terminate at node 7, excepted train $A$ which terminates at node 6. Train $A$ uses path 1-2-6 while train $B$ and $C$ use path 1-4-5-6-7 and 1-2-3-7 respectively. The blocking plan for this railroad consists of six blocks $b_1, \ldots, b_6$. Figure 3-2(c) shows these blocks and regardless of the path, each block is travelling in the railroad network. The block-to-train assignments are planned at the train makeup level and the result is illustrated in Figure 3-2(d). Each block might use more than one train segment or even more than one train from its origin to its destination; however, it cannot be assigned to more than one train over one single railroad arc. For example block $b_1$ takes only train $A$ over only one train segment 2-6, while block $b_6$ takes the train $B$ over the segments 5-6 and 6-7. Thus, as a train travels from its origin to its destination, it visits various yards to pick up blocks (e.g. train $C$ at yard 1 picks up block $b_3$), to drop off blocks (e.g. train $C$ at yard 7 drops off block $b_5$) or to perform both (e.g. train $C$ at yard 2 drops off block $b_3$ and picks up block $b_5$). Typically, several blocks travel on a train at any segment (e.g. blocks $b_1$ and $b_4$ travel on train $A$ on segment 2-6). On its way from the origin to the destination, a block may also travel on several trains. Transferring a block between trains without re-classification of individual cars at yards is called a block swap. For example, block $b_4$ requires a block swap at yard 6 where it is set off by train $A$ and is picked up by train $B$. During a block swap, the entire block of cars remains coupled together while moving to another train, requiring less classification work comparing to the reclassification of all individual cars in the block. In order to easily refer to different segments of the trains moving on the network, in Figure 3-2(e) we have named each train segment separately. For example arcs $C_1$, $C_2$ and $C_3$ in this extended train network represent...
train $C$ on segments 1-2, 2-3, and 3-7, respectively. From now on, unless specified otherwise, with train we mean the train segment and with train network we mean the extended train network. Figure 3-2(f) shows the block-train network. Each arc of this network uniquely represents a block moving on a train. For example block $b_4$ is represented by three arcs $b_4: A_1$, $b_4: A_2$, and $b_4: B_4$. This network can be used to find the trip plan for a given shipment. There is only thing we need to be careful about: Although a block might be represented by more than one arc, a shipment can be assigned to a block only at the block's origin (solid circle tail) and, if necessary, re-assigned to the next block only in the current block's destination (triangle shape head) and not in the intermediate stops (open arrow head) of the train. For instance, for a shipment from node 2 to node 7, paths $b_5: C_2$-$b_5: C_3$ and $b_1: A_2$-$b_2: B_4$ are feasible, while paths $b_4: A_2$-$b_2: B_4$ and $b_1: A_2$-$b_6: B_4$ are infeasible. Also for a shipment from node 1 to 7, a path that starts with arc $b_4: A_1$ is feasible only if it is followed with arcs $b_4: A_2$ and $b_4: B_4$. Another way of illustrating the block-train network is to take the blocking network from the blocking plan level with additional attributes on each arc, representing the trains carrying the block from its origin to its destination. This representation, will allow us to formulate the capacities of blocks and trains in a network flow model. In this formulation, the block capacities can be simply considered as arc capacities, while the trains can be considered as shared resources. Each car will use one unit of resource from each one of the trains appearing in the arc's attributes. This new representation is illustrated in Figure 3-2(g) and will be called resource-constrained blocking network.

Both the block-train network of Figure 3-2(f) and the resource-constrained blocking network of Figure 3-2(g) are defined in a static environment. In the railroad network,
3.2 Network Representation

blocks are statically defined as pairs of origin-destination and some other attributes such as car type, freight type, etc. However, train schedules are dynamic. Some trains might run every day or even more than once a day, while some others might run less than seven times a week. A time-space network will add this dynamicity to the network representation.
Figure 3-2 Underlying networks used in the railroad optimization models
3.2 Network Representation

Figure 3-2 continued. Underlying networks used in the railroad optimization models
3.2 Network Representation

3.2.2 The Time-Space Network

In a time-space block-train network, each node represents a specified yard at a particular time associated with an event. An event might be a train event or a block event and is associated to a departure or arrival at a yard. Train events are train departures or arrivals in a yard and might or might not be coupled with a classification event, i.e. picking up or setting off blocks. The timing of the train events comes from the train timetables. Block events are block departures and arrivals where the departure time of a block is the departure time of the first train carrying the block and the block arrival time is the arrival time of the final train that takes the block to its destination.

The arcs of the time-space network can be trains or blocks. Depending on the type of the arcs we define two time-space networks. But before that let us consider a special case where each train is carrying only one block and each block is carried by only one train. Figure 3-3(a) shows such a time-space network. There are three types of arcs in this network: Moving arcs (shown by solid straight arrows in Figure 3-3(a)), are predefined blocks of the network assigned to prescheduled trains. Inventory arcs (shown by dashed straight arrows in Figure 3-3(a)) and delay arcs (shown by curved arrows in Figure 3-3(a)) are temporal arcs, respectively representing holding empty cars at a location from one time period to another, and filling backordered demand from an earlier time period before their expiration dates.

Going back to the block-train network in Figure 3-2(f)-(g), let us consider a shipment from node 2 to node 7. In a static network without any capacity limits, the paths $b_5: C_2-b_5: C_3$ and $b_1: A_2-b_2: B_4$ are both feasible for this shipment. In practice, however, trains' trips are repeated during a week.
3.2 Network Representation

Figure 3-3 shows three paths, all associated to the static path $b_1: A_2-b_2: B_4$, but this time on a time-space network of blocks and trains. The sub indices T,W,R,F for train (block) represent the day in which the train leaves the yard (block is created). For example, arc $b_{1T}: A_{2T}$ refers to the block $b_1$ which is created on Tuesday and is assigned to train $A_2$ on Tuesday. The trip from $s$ to $d$ is not a direct trip and has a connection on the classification yard 6. The green path starts from yard 2 on Tuesday when Train $A_{2T}$ pulls block $b_{1T}$, which is carrying the car, to the block destination at yard 6. In the classification yard 6, the car will be reclassified to the block $b_{2R}$ which is planned to be pulled by train $B_{4R}$ on Thursday. Finally, the car will arrive at the customer yard 7 (end of the physical trip) on Sunday, which is one day later than the order date of customer $d$. This explains the reason why a temporal delay arc connects this car to its corresponding demand day.

The blue path is generated by a similar trip plan, but the trip starts and ends one day later. Another possible path from $s$ to $d$ is to start with train $A_{2T}$, wait one day in classification yard 6, and finally take the train $B_{4F}$. Practically, if the number of cars assigned to a block is more than the block size, the remaining cars will be classified in the next day. While the mileage cost of both Green and Blue paths are similar, the total cost associated to the blue path is likely to be higher as a result of the inventory costs at the beginning of the trip, and an extra day of delay at the destination.
3.2 Network Representation

3.2.2.1 The Time-Space Block-Train Network

The time-space network of Figure 3-3(a) is not realistic since in practice trains might carry multiple blocks at a time and a block might be carried on multiple trains (train segments) throughout its destination. Therefore a single arc cannot represent a single train and a single block at the same time. The time-space network presented in this section is a block-train network, where events are train events. This network is a multigraph and there exist one arc for each train and each block carried on that train.

Figure 3-4 shows the entire time-space block-train network associated with the static block-train network of Figure 3-2(f) in a 6-day time horizon. Similar to the static block-train network arrows have two different types of tail and heads, where only a solid circle represents a block origin and a triangle shape head represents a block destination. Unlike Figure 3-3(a), in this figure we do not show the delay arcs, since delays are associated to the shipments and not to the operating plan of the railroad.
3.2 Network Representation

Figure 3-4 Time-space block-train network associated to Figure 3-2(f)
3.2.2.2 The Time-Space Resource Constrained Blocking Network

The time space network presented in train-block network is defined on top of the train networks. In US railways, shipments are assigned to blocks and blocks are carried by trains. This is why a service network defined on top of the blocking network, rather than the train network, is more applicable to the US railways. Figure 3-5 shows the resource constrained blocking network associated to the time space block-train network of Figure 3-4. Nodes in this network are associated to block events in classification yards, where a block event means a block departure or arrival at a classification or serving yard. Timing of block events is known beforehand from block to train assignment plan in tactical level. Arcs are the blocks and the inventory arcs connecting the nodes associated to the same yard and trains carrying those blocks are listed as the resources used by the blocks. Figure 3-6 is basically the same as Figure 3-5; with less visualization on the train changes. In this research, unless specified otherwise, whenever talking about time-space network, we refer to Figure 3-6. Customers are located in the nodes of the time-space network. Figure 3-7 shows three customer profiles on the time-space blocking network, two of them, C1 and C2, are located in yard 2 and the third one is located in yard 7. Demand nodes are shown by rectangular and those associated with the same customer profile in different days are connected using delay arcs. It is noteworthy that this network can be considered as a two-layer network: a) the time-space network layer which is constructed by inventory arcs and moving arcs (blocks) and b) the customer layer which is constructed by the delay arcs. The two layers are connected using intraarcs that transfer the flow of cars from the time-space network to the customers.

It is worth mentioning that the car routing (trip planning) between supply and demand nodes is not an output of the distribution model, but an input to it. The selection
3.2 Network Representation

of either the green or the blue path in Figure 3-3 will result in the same assignment of $s$ to $d$, from the same physical route but in different days. In other words, the output of the assignment model is the decision about when and where to send the car, while the corresponding route for the trip is decided by the trip plan application.
Figure 3-5 Time-space resource-constrained block network (colors for visualization)
3.2 Network Representation

Figure 3-6 Time-space resource-constrained block network (the main representation)
Figure 3-7 Customers on time-space resource-constrained block network
3.2 Network Representation

3.2.3 The Bipartite Multigraph

Similar to the transportation problem formulation, network used for car distribution in noncapacitated networks is a bipartite supply-demand graph. Similar graph can be used for the path-based formulation of the capacitated model. However if more than one path exists between some supply-demand pairs, the network will be a bipartite multigraph.

Nodes of this graph as is shown in Figure 3-8 are from two layers of supply nodes and demand nodes. In this graph there is one supply node for each yard, for each time period and for each pool available at that yard and there is one demand node for each customer profile, for each day. One super supply node and one super demand node will be added to maintain the supply-demand balance. Eligible and feasible supply and demand nodes are connected using at least one arc where, as will be discussed in Section 3.5, eligibility is defined based on attribute matching between supply-order pair and feasibility is defined based on order fulfilment before its expiration date.

Figure 3-8 Bipartite supply-demand multigraph
3.2.4 Network Capacities

Cars are carried on blocks, and blocks are pulled by trains. Both blocks and trains have capacity limits. Blocks are created on the classification yards. Normally each block is created on one rail track in the classification yard. Tracks in classification yards have fixed length and as a result each block also has a length. Furthermore, the number of blocks created in a classification yard sometimes exceeds the number of classification tracks in the yard; hence reducing the number of blocks to be built, by charging a setup cost on blocks, might improve the yard operations. However, before running the empty car distribution model, many of these blocks are already planned as a result of currently planned loaded or empty car movements. This is the reason that the setup cost should be charged only on the blocks that will be created in bottleneck yards and only when they are created as a result of empty car distribution model.

Trains travel over the physical railroad network. Depending on the geography of the area and the rail track attributes, certain segments of the network might have limitations on the maximum train length and weight. Assuming that train length constraints have been already satisfied at the tactical level of the block to train assignment decision making process, the train weight limits still need to be considered in the operational models such as trip planning and car distributions.

3.2.5 Cost Components

While the complexity of linear programs mainly depends on the number and type of constraints and decision variables, the usefulness of the optimal solution also depends on the cost coefficients. This means that formulating the problem as a complex mathematical model that considers all business rules and limitations is not sufficient for
3.2 Network Representation

having a desirable output unless the coefficients of the objective function are realistically chosen and calibrated. In this research, instead of randomly generating the cost coefficients, we develop a systematic approach on calculating the cost coefficients.

Figure 3-9 shows part of the time-space network. Let’s consider a shipment of a railcar of pool k from supply node m to the demand node n on the bold dark blue path. The costs associated with this shipment are $c_i^k$ on inventory arc i, $c_b^k$ on block b, $c_{n+1}^k$ the fulfillment cost/credit, and $c_d^{delay}$ the delay cost. In mathematical language if we note this path by l the total cost associated to it is calculated as $c_l^k = c_i^k + c_b^k + c_{n+1}^k + c_d^{delay}$.

Now we explain each term in this function. $c_i^k$ is the unit inventory cost and as stated in (3-1) depends on the number of days in the inventory arc and the car hire rate per day. Car hire is the rate that the railroad is paying for leasing a car or the rate that a railroad can lease out its car to other railroad if not used in its own network. The “$” sign in cost parameter definitions (3-1) to (3-7) represents a constant rate and the suggested rates are given in Table 3-1. $c_b^k$ is the variable cost of block b and as in (3-2) has three components: a constant part as classification cost for each car at the block origin classification yard and two other parts that depend on how many days and miles it takes from block’s origin to its destination. $c_n^k$ is the cost/credit of fulfilling one unit of demand of customer n by a car of pool k and based on (3-3) depends on how good fit the car is to the order and how important the customer, more specifically the demand node, is to the railroad. The best way to measure the customer importance is to see how much profit will be collected by fulfilling the customer’s demand. However in the railroad industry, the loaded destination of an empty car requested by a customer is not known unless it is loaded, which clearly is not the case in the car distribution planning time. However, by analyzing the historical data,
railroad carrier can prioritize some customers and give a virtual credit for fulfilling demand of high priority customers. It is also important to be clear about what a higher priority customer implies: An absolute customer priority over another customer means that if two customers are at the same location, the high priority customer’s demand, no matter for what day, is of more importance than the other customer’s. A proportional priority applies the priorities only for the same days. A middle approach is presented in Table 3-2 where if three customers 1, 2, and 3 of priority 1, 2, and 3 are located in the same yard, the demand of customer 3 in days 1 and 2 has higher priority over customer’s 2 demand for day 1. However this customer’s demand for day 3 will be satisfied after customer 2 demand for day 1 is satisfied. Finally \( c_d^{delay} \) in (3-4) is the adjusted delay cost and depends on how late the shipment will arrive to the customer. As mentioned before the fulfillment cost/credit considered in \( c_t^k \) is \( c_{n+1}^k \) and not \( c_n^k \). This is why the delay cost \( c_d^{delay} \) is adjusted by \( c_n^k - c_{n+1}^k \) to reflect the correct cost coefficients.

The above mentioned cost components are associated with the variable costs of assignment. Two other cost components are the fixed cost associated with block setups (3-5) and the shortage penalties.

In real world applications the supply and demand are not necessarily balanced. In fact in our application, since supply is usually on hand while demand is for a two week time horizon, supply is always less than demand. Hence shortage is inevitable. Each unit of unmet demand is penalized by \( c_n \). Designing the shortage penalty is extremely important. Assigning a very large number, a Big M, to \( c_n \) will transform the problem from an assignment cost minimization problem to an order fulfilment maximization problem regardless of customer priorities and their pool preferences. This is why a trade-off between
3.2 Network Representation

Assignment costs and shortage penalties must be considered while assigning the shortage penalties. The shortage penalty $c_n$ depends on how much money the railroad is willing to spend on pulling an empty railcar in general to satisfy an order and specifically this order. Equation (3-7) shows how to calculate a minimum penalty cost based on how many miles and days and classifications, in general, a railroad is willing to spend to fulfill a general customer demand. This value is used in (3-6) to calculate the unmet demand penalty for a specific demand node where a demand node with higher priority is penalized more compared to the ones with lower priorities.

$$c^k_i = \text{numWaitingDays} \times \text{Car Hire Per Day}$$  \hspace{1cm} (3-1)

$$c^k_b = \text{numTravelDays} \times \text{Car Hire Per Day} + \text{numTravelMiles} \times \text{Per Mile} + 1 \times \text{Classification Price}$$  \hspace{1cm} (3-2)

$$c^k_n = (\text{pref} (k) - 1) \times \text{Pref Value} - \text{priority} (n) \times \text{Priority Value}$$  \hspace{1cm} (3-3)

$$c^d_{\text{delay}} = \text{delay} \times \text{Late Penalty} + c^k_{\text{nd}} - c^k_{\text{no}}, \text{ where no and nd are the origin and destination of delay arc } d.$$  \hspace{1cm} (3-4)

$$f_b = \text{Block Setup Cost (b)}$$  \hspace{1cm} (3-5)

$$c_n = [\text{minPenCost} + \text{acceptedDelay} (n) \times \text{Late Penalty} +$$

$$(\text{worseAcceptedPoolPref} (n) - 1) \times \text{Pref Value}] \times (1 + \text{priority} (n))$$  \hspace{1cm} (3-6)

$$\text{minPenCost} = \max\text{AcceptedMile4EmptyTrip} \times \text{Car Hire Per Mile} +$$

$$(\max\text{AcceptedDays4EmptyTrip} \times \text{Car Hire Per Day} +$$

$$(\max\text{AcptdNumOfClassfication4EmptyTrip} \times \text{Classification Price})$$  \hspace{1cm} (3-7)
### Table 3-1 Parameters used to calculate the cost coefficients

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Suggested Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$carHirePerDay</td>
<td>24</td>
</tr>
<tr>
<td>$carHirePerMile</td>
<td>0.83</td>
</tr>
<tr>
<td>$classificationPrice</td>
<td>50</td>
</tr>
<tr>
<td>$latePenalty</td>
<td>100</td>
</tr>
<tr>
<td>$prefValue</td>
<td>50</td>
</tr>
<tr>
<td>$priorityValue</td>
<td>100</td>
</tr>
<tr>
<td>acceptedDelay(n)</td>
<td>0,1,2,3,4</td>
</tr>
<tr>
<td>maxAcceptedMile4EmptyTrip</td>
<td>400</td>
</tr>
<tr>
<td>maxAcceptedDays4EmptyTrip</td>
<td>6</td>
</tr>
<tr>
<td>maxAcptdNumOfClassfication4EmptyTrip</td>
<td>4</td>
</tr>
<tr>
<td>pref (k)</td>
<td>1,2,3 where 1 is the best match</td>
</tr>
<tr>
<td>priority(n)</td>
<td>[0-3] where 3 has highest priority</td>
</tr>
</tbody>
</table>

\[
f_b = (\text{uni} \ (0, 1) < \text{ratioOfBlocksWithSetupCost}) \ast 
    c_{\text{maxBlockFixedCost}} \ast \text{uni} \ (0, 1)
\]

\[
c_{\text{maxBlockFixedCost}} = 3000
\]

\[
\text{ratioOfBlocksWithSetupCost} = 0.1
\]

### Table 3-2 Demand node priorities based on customer priorities

<table>
<thead>
<tr>
<th>Customer Priority</th>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1.0</td>
<td>0.7</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2.1</td>
<td>1.4</td>
<td>1.0</td>
<td>0.7</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>3.2</td>
<td>2.1</td>
<td>1.5</td>
<td>1.0</td>
<td>0.7</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

### Figure 3-9 Costs on the time space network
3.3 Fleet

Freight cars are categorized in different car types mainly based on the nature of goods which are going to be shipped by the car. For example, box cars are fully enclosed cars with side or end doors and are good for transporting moisture-susceptible goods; coil cars are good for carrying heavy sheet metal rolls; and flatcars are mainly used for shipping automobiles. Generally speaking, substitution across car types is typically not possible and traditionally, different car types are managed separately. However, all the cars of the same type are not exactly the same. For example, boxcars might have one door or two. Furthermore, the doors might be plug doors which open with a hand crank, or sliding doors, which may require opening with a forklift. Some customers might have a preference between cars from the same type. This is one of the reasons why railroads further classify cars within a car type into smaller groups named pools. From order fulfillment perspective, cars within a pool can be considered exactly the same. The number of pools in a single car type typically ranges between 5 and 20. As a general rule, a customer will require a number of cars from a particular set of pools along with their preferences or will specify a car type as well as some extra requirements which can be translated into priorities among pools within that car type. Substitutability is an attribute of demand and not car type; in other words substitutability is demand specific. This means that pool 1, in car type 1, might be substitutable by pool 2, also in car type 1, for customer A, while customer B only accepts pool 1.

3.4 Customer, Customer profile, Demand node and Demand

A customer is a client located on the railroad network who orders empty railcar from railroad. Each customer might have one or more profiles. A profile consists of a set
of pools and the *preferences* among them. Preferences are integer values of 1, 2, or 3 and will be used while calculating the assignment costs. While the optimization model will try to fulfill the customers demand from higher preference pools, in the presence of supply limitation it is not guaranteed. In fact any assignment from the set of pools listed in the customer’s profile is acceptable to the customer, even if they all come from the customer’s lowest preference. Any criteria on the minimum or maximum number of cars from one specific pool among the accepted pools (as in case 3 in Table 3-3) should be handled explicitly, for example by introducing a new customer profile (case 3-1 and 3-2).

A *demand node* represents a customer profile on a specific day. For example, in a two-week ahead planning strategy, each customer profile will be associated with a maximum of 14 demand nodes and less than that if they do not order all days. *Demand* in this research means the number of cars that a customer has ordered from a specific profile and for a specific day, and basically is associated to a demand node. Furthermore, if part of customer’s demand is already fulfilled, the demand should be modified by excluding the fulfilled part. In this research unless specified otherwise, the demand refers to the modified demand and not to the original customer’s demand.

### Table 3-3 Illustration of customer profiles

<table>
<thead>
<tr>
<th>Case</th>
<th>Customer Location (serving yard)</th>
<th>Custome Profile</th>
<th>Accepted pools</th>
<th>Preference(s)</th>
<th>Two-week Demand</th>
<th>Extra condition</th>
<th>OK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X</td>
<td>123</td>
<td>X1</td>
<td>{1,2,3}</td>
<td>100</td>
<td>NA</td>
<td>OK</td>
</tr>
<tr>
<td>2</td>
<td>X</td>
<td>123</td>
<td>X2</td>
<td>{4,5}</td>
<td>50</td>
<td>NA</td>
<td>OK</td>
</tr>
<tr>
<td>3</td>
<td>Y</td>
<td>451</td>
<td>Y1</td>
<td>{1,2,3}</td>
<td>50</td>
<td>At least 10 from pool 1</td>
<td>NO</td>
</tr>
<tr>
<td>3-1</td>
<td>Y</td>
<td>451</td>
<td>Y1-1</td>
<td>{1}</td>
<td>10</td>
<td>NA</td>
<td>OK</td>
</tr>
<tr>
<td>3-2</td>
<td>Y</td>
<td>451</td>
<td>Y1-2</td>
<td>{1,2,3}</td>
<td>40</td>
<td></td>
<td>OK</td>
</tr>
</tbody>
</table>
3.5 Supply

Supply is the fleet that is available for assignment and is a subset of fleet that can be assigned to the customer orders. For a railcar to be considered as supply it should be suitable, serviceable, distributable, and optimizeable and to be considered eligible for a given customer profile it should match the order’s attributes (pool matching) and to be considered feasible for a given demand node (i.e. in a specific day) it should arrive before the order expiration date. Below we explain all these keywords.

Suitable: a car is considered suitable for the customer orders if its pool number matches at least one of the customer profiles in the planning horizon. If a car is not suitable it means it is out of the scope of the supply-order matching.

Serviceable: a railcar is serviceable if it is not in bad order or rejection status. A serviceable car is either currently serving a customer or on the way to a customer or available to serve a customer.

Distributable: a railcar is distributable if the car distribution department has the authority to distribute this car. For example, foreign cars that belong to other railway carriers, private cars that are property of a customer and only hauled by the railway, and dedicated pool cars that are usually assigned to major customers, unless there is an agreement, are not distributable.

Optimizeable: a railcar is considered optimizeable in operating station A and time T if it is guaranteed that it will be empty in that time and location, is not assigned to a customer or can be reassigned if it is already assigned to one. As a special case a suitable, serviceable and distributable car that is already empty (time zero) at station A and is not assigned to any customer is optimizeable now at station A and can be considered as supply.
**Eligible:** a car is eligible to be assigned to a customer profile if its pool number matches that profile.

**Feasible:** assignment of a car to a demand node is considered feasible if it is eligible and can arrive to customer facility before the order expiration date. Optimization model will be run on the feasible assignments to find the optimal assignments. The output of the model after CDU confirmation/modification is considered **Planned** and once is billed to a customer is considered **Assigned**.

Note: a situation of having cars not planned is most likely to happen when we are in a global or local surplus situation. A local surplus situation means we have excess of cars in an area that their assignment to the current demand is either not feasible or not economic. In order to avoid sleep of fleet, railroad should consider some storage facilities in different parts of the railroad network, as low priority customers; otherwise, the optimizer will leave them as not planned cars or leftover supply.
Figure 3-10 Classifying fleet for empty railcar distribution
3.6 General Assumptions on the Components of the Model

After describing the important railroad terminologies and introducing different components of the input set, we present in Table 3-4 a list of assumptions based on which our models are built in the next chapters.

<table>
<thead>
<tr>
<th>Table 3-4 Model assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Fleet is heterogeneous, but can be divided into different groups (car type: Box car, flatcar, coil car, etc.)</td>
</tr>
<tr>
<td>• No substitution is allowed across these groups (car types)</td>
</tr>
<tr>
<td>• Cars within a car type can be divided into smaller groups named pools. Cars inside a pool can be considered exactly the same</td>
</tr>
<tr>
<td>• Substitution across pools in the same car type is allowed</td>
</tr>
<tr>
<td>• Substitutability is demand specific</td>
</tr>
<tr>
<td>• Car supply and customer demand, both original and modified, for the planning horizon (14 days) are on hand</td>
</tr>
<tr>
<td>• Backorder is allowed but within a time window (order expiration date) for each order day. Demand not satisfied before its expiration date is considered unmet (lost).</td>
</tr>
<tr>
<td>• Network is capacitated and original and residual capacities of network, for all blocks and trains in the operating plan, are available to the model</td>
</tr>
<tr>
<td>• The car routing task is out of the scope of the car distribution model and is done by another application called “Trip Planner”.</td>
</tr>
</tbody>
</table>
4.1 A General Integer Programming Formulation

Let $\Psi$ represent the set of all pools of cars that appear in customers’ demand. Set $\Psi$ can be partitioned into a collection of some number of nonempty subsets of pools that are mutually exclusive and collectively exhaustive, representing different car types. Supply is a tuple $(K \subseteq \Psi, k \in K, m \in M, s_m^k)$ where $K \subseteq \Psi, k \in K$ represents a car type and car pool, respectively; $m \in M$ is a node in the time-space network, and $s_m^k$ is the number of cars of pool $k$ available at node $m$. Demand is also a tuple $(K \subseteq \Psi, n \in N_K, pri(n), K_n \subseteq K, pref(k), d_n, y_n^{\text{max}}, exp(n))$ where $K \subseteq \Psi$ is the car type that the demand belongs to; $n \in N_K$ represents the customer index in the time space network; $pri(n) \to R^+$ is function that returns the importance to fulfill this demand compared to other demands; $K_n \subseteq K$ is the set of pools which are eligible to fulfill customer needs, and $pref(K_n) \to R^{|K_n|}$ is a function representing the preference of customer/railroad within these eligible pools; $d_n$ is the modified demand of the customer and is calculated from subtracting part of the demand that has been already fulfilled, $a_n$, from the original demand, $d'_n$; and $exp(n)$ is order
expiration date after which date the demand is lost; meaning that if the car arrives after that date, the customer will not be able to use it against this demand.

Table 4-1 contains the indices, sets, parameters and decision variables that are used in the mathematical model. Some elements of the demand tuple such as $pri(n), pref(K_n), exp(n)$ are not listed in this table. This is not because these elements (parameters) are not important, but because they do not appear in the mathematical model directly. However, we use these parameters to calculate the cost coefficients associated with the objective function. Please see Section 3.2.5 for details on how to calculate the cost parameters. After introducing the model parameters, we start formulating the car distribution optimization model.

The objective function (4-1) minimizes the total cost of empty car repositioning plan which includes the setup cost of new blocks, the transportation costs, and the unmet demand penalties. Since in the scheduled railroads, general trains are unlikely to be canceled at the operational planning level, train setup cost is not considered in this model.

$$Min \sum_{b \in B} f_b z_b + \sum_{l \in L} \sum_{k \in K \subseteq \Psi} c^l_k x_{l,k} + \sum_{n \in N \subseteq K \subseteq \Psi} c_n y_n$$  \hspace{1cm} (4-1)

Constraint (4-2) assures that the number of cars of each pool moved from node $m$ does not exceed its available supply. Constraint (4-3) defines variable $y_n$ (unmet demand) as the demand of customer at node $n$ minus the number of cars that customer receives.
### 4.1 A General Integer Programming Formulation

#### Table 4-1 Mathematical notation

<table>
<thead>
<tr>
<th>Indices and Sets</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k \in K$</td>
<td>Index for pools that belong to a specific car type</td>
</tr>
<tr>
<td>$m \in M$</td>
<td>Index for supply nodes in a time-space network</td>
</tr>
<tr>
<td>$n \in N_k$</td>
<td>Index for demand nodes in a time-space network</td>
</tr>
<tr>
<td>$b \in B$</td>
<td>Index for blocks</td>
</tr>
<tr>
<td>$g \in G$</td>
<td>Index for trains</td>
</tr>
<tr>
<td>$p = {(b_1, g_1), (b_2, g_2), \ldots} \in P$</td>
<td>Index for trip plans. Each trip plan is an ordered set of blocks carrying the cars and the trains carrying the blocks, $b_1, b_2, \ldots \in B$ and $g_1, g_2, \ldots \in G$.</td>
</tr>
<tr>
<td>$l = (m, n, p) \in L$</td>
<td>Index for supply-demand paths, or simply saying paths, in a time-space network. A path $l = (m, n, p)$ connects supply node $m$ and demand node $n$ through trip plan $p$.</td>
</tr>
<tr>
<td>$L_m^o \subseteq L$</td>
<td>Set of all paths that originate from node $m$</td>
</tr>
<tr>
<td>$L_n^d \subseteq L$</td>
<td>Set of all paths that end at node $n$</td>
</tr>
<tr>
<td>$L_b \subseteq L$</td>
<td>Set of all paths that use block $b$</td>
</tr>
<tr>
<td>$L_g \subseteq L$</td>
<td>Set of all paths that use train $g$</td>
</tr>
<tr>
<td>$K \subseteq \Psi$</td>
<td>Set of pools representing a specific car type</td>
</tr>
<tr>
<td>$K_n \subseteq K$</td>
<td>Set of all pools eligible for demand node $n$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_b$</td>
<td>Setup cost for block $b$</td>
</tr>
<tr>
<td>$c_{lk}^k$</td>
<td>Unit cost for sending a car of pool $k$ on path $l$</td>
</tr>
<tr>
<td>$c_n$</td>
<td>Unit cost of unmet demand (penalty) for demand node $n$</td>
</tr>
<tr>
<td>$c_n'$</td>
<td>Unit cost of unmet demand (penalty) beyond for demand node $n$</td>
</tr>
<tr>
<td>$u_g$</td>
<td>Capacity of train $g$ (train tonnage)</td>
</tr>
<tr>
<td>$v_b$</td>
<td>Capacity of block $b$ (block length)</td>
</tr>
<tr>
<td>$\mu_{lk}^k$</td>
<td>Substitution ratio of a car of pool $k$ in path $l$. For example, $\mu_{lk}^k = 2$ means that a car from pool $k$ originating from node $m$, where $l \in L_m^o$, having destination at demand node $n$, where $l \in L_n^d$, will fulfill 2 units of demand of node $n$.</td>
</tr>
<tr>
<td>$\omega^k$</td>
<td>Weight of an empty car of pool $k$</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>Length of an empty car of pool $k$</td>
</tr>
<tr>
<td>$s_m^k$</td>
<td>Supply of cars of pool $k$ in supply node $m$</td>
</tr>
<tr>
<td>$d_n$</td>
<td>Demand of cars at demand node $n$ calculated as original demand, $d_n'$.  minus fulfilled demand so far, $a_n$.</td>
</tr>
<tr>
<td>$y_{n,\text{max}}$</td>
<td>Soft bound on the maximum unmet demand of demand node $n$ (Service level of demand node $n$).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{lk}$</td>
<td>number of cars of pool $k$ to be sent on path $l$ to fulfill the order of the associated customer</td>
</tr>
<tr>
<td>$z_b$</td>
<td>Binary variable, 1 if block $b$ is created, and 0 otherwise</td>
</tr>
<tr>
<td>$y_n$</td>
<td>Unmet demand of demand node $n$</td>
</tr>
<tr>
<td>$y_n'$</td>
<td>Unmet demand of demand node $n$, beyond its soft bound, $y_{n,\text{max}}$</td>
</tr>
</tbody>
</table>
4.1 A General Integer Programming Formulation

\[
\begin{align*}
\sum_{l \in L_m} x_{l}^{k} & \leq s_{m}^{k} \quad \forall \ m \in M, \forall \ k \in K \quad (4.2) \\
\sum_{l \in L_n} \sum_{k \in K} \mu_{l}^{k} x_{l}^{k} + y_{n} &= d_{n} \quad \forall \ n \in N_K \quad (4.3)
\end{align*}
\]

Constraints (4.4) and (4.5) apply capacity limits on the trains and blocks, respectively.

\[
\begin{align*}
\sum_{l \in L_g} \sum_{k \in K, K \subseteq \Psi} \omega^{k} x_{l}^{k} & \leq u_{g} \quad \forall \ g \in G \quad (4.4) \\
\sum_{l \in L_b} \sum_{k \in K, K \subseteq \Psi} \tau^{k} x_{l}^{k} & \leq v_{b} z_{b} \quad \forall \ b \in B \quad (4.5)
\end{align*}
\]

Finally customer service levels are formulated as bounds on the unmet demand in constraint set (4.6), assuring that unmet demand of each demand node is less than a premised value.

\[
y_{n} \leq y_{n}^{\max} \quad \forall \ n \in N_K, \forall \ K \subseteq \Psi \quad (4.6)
\]

Furthermore, all variables are nonnegative, flow variables \(x_{l}^{k}\) are general integers and block creation variables \(z_{b}\) are binary. The shortage variables \(y_{n}\) are not necessarily integer if fractional numbers are permitted for \(\mu_{l}^{k}\), and their integrality is guaranteed if otherwise.

Assigning a bound on customer’s maximum accepted unmet demand in (4.6) is an effort to increase the equity and fairness among the customers while optimizing the efficiency of the system as a whole. Such bound might be enforced by the customer or might be assigned by the car distribution department of the railroad. However it is
important to keep in mind that by the time we are running the model some cars might have been already assigned to the customer. Such a situation is likely to happen when the model is run frequently, for example every one or two hours, if new supply of cars is detected or for re-optimizing the previously made decisions. Simply saying, the demand $d_n$ in this formulation might not necessarily be equal to what customer has asked in the first place (i.e. the original demand) but part of the demand that is not yet satisfied before this run (i.e. the modified demand). Equation (4-7) shows how the modified demand of demand node $n$, $d_n$, is calculated based on its original demand, $d'_n$, minus the part of demand that is fulfilled/planned so far, $a_n$.

$$d_n = d'_n - a_n \quad \forall n \in N_K, \forall K \subseteq \Psi$$  \hspace{1cm} (4-7)

However with regard to the fairness among customers, the service level should be applied against the original demand and not the modified demand. Hence, if the service level of node $n$, is noted by $0 \leq \rho_n \leq 1$, then the maximum acceptable unmet demand can be calculated as:

$$y_n^{max} = (1 - \rho_n) d'_n \quad \forall n \in N_K, \forall K \subseteq \Psi$$  \hspace{1cm} (4-8)

During the validation process using randomly generated problems, we noticed that in many cases the problem becomes infeasible. From a theoretical perspective, infeasibility can happen in many mathematical programs, but when it goes to real world applications, such a situation is not acceptable. After investigating the issue, the hard upper bounds on
unmet demand in constraint (4-8) were detected as the source of the infeasibility. In order to overcome this problem, the unmet demand above the upper bound was permitted but with a higher penalty cost. This change was performed by replacing the hard constraints in (4-8) by softer constraints as:

\[ y'_n \geq y_n - y_n^{max} \quad \forall \ n \in N_K, \forall \ K \subseteq \Psi \]  \hspace{1cm} (4-9)

and adding another term, \( \sum_{n \in N_K, K \subseteq \Psi} c'_n y'_n \), to the objective function. The new objective function will be:

\[
\text{Min} \sum_{b \in B} f_b z_b + \sum_{l \in L} \sum_{k \in K, K \subseteq \Psi} c^k_l \chi^k_l + \sum_{n \in N_K, K \subseteq \Psi} c_n y_n + \sum_{n \in N_K, K \subseteq \Psi} c'_n y'_n
\]  \hspace{1cm} (4-10)

Constraint (4-9), will resolve the potential infeasibility caused by the enforced service levels. The use of sign “\( \geq \)” in this constraint, instead of a “\( = \)” sign, prevents the model from another infeasibility situation in case \( y_n < y_n^{max} \). It should be noted that the corresponding term in the minimizing objective function (4-10), \( \sum_{n \in N_K, K \subseteq \Psi} c'_n y'_n \), will guarantee that in the optimal solution constraint (4-9) is satisfied as an equality ( \( y'_n = y_n - y_n^{max} \)) whenever \( y_n \geq y_n^{max} \) and \( y'_n = 0 \) whenever \( y_n < y_n^{max} \). The effective unmet demand cost then can be formulated as a piecewise linear function as in (4-11) and is illustrated in Figure 4-1.
4.1 A General Integer Programming Formulation

\[
\text{unitUnmetCost} = \begin{cases} 
    c_n, & \forall y_n \leq y_n^{\text{max}} \\
    c_n + c'_n, & \forall y_n > y_n^{\text{max}} 
\end{cases}
\]  \hspace{1cm} (4-11)

The path-based formulation of the car distribution problem is presented as follows:

**PathBased:**

\[
\begin{align*}
\text{Min} & \sum_{b \in B} f_b z_b + \sum_{l \in L} \sum_{k \in K \subseteq \Psi} c^k_l x^k_l + \sum_{n \in N_K \subseteq \Psi} c_n y_n + \\
& \sum_{n \in N_K \subseteq \Psi} c'_n y'_n \\
& \sum_{l \in L^m_n} x^k_l \leq s^k_m & \quad \forall m \in M, \forall k \in K \subseteq \Psi \\
& \sum_{l \in L^n_k} \mu^k_l x^k_l + y_n = d_n & \quad \forall n \in N_K, K \subseteq \Psi \\
& \sum_{l \in L_g} \sum_{k \in K \subseteq \Psi} \omega^k_l x^k_l \leq u_g & \quad \forall g \in G \\
& \sum_{l \in L_b} \sum_{k \in K \subseteq \Psi} \tau^k_l x^k_l \leq v_b z_b & \quad \forall b \in B \\
& y'_n \geq y_n - y_n^{\text{max}} & \quad \forall n \in N_K, \forall K \subseteq \Psi \\
x^k_l \geq 0 \text{ and integer} & \quad \forall l \in L, \forall k \in K \subseteq \Psi 
\end{align*}
\]  \hspace{1cm} (4-12 to 4-18)

Figure 4-1 Shortage cost function
Problem PathBased is a general (mixed) integer programming problem and because of the fixed costs on blocks, and the integrality requirements of decision variables is hard to solve. The complexity of the problem and whether we can solve it in a reasonable amount of time using state of the art software or combinatorial algorithms, highly depend on the number of decision variables. If we assume that the underlying network and the number of commodities are fixed, this will solely depend on the number of paths, i.e. \(|L|\). The number of allowed paths between any feasible supply-demand pair in a network might vary from one path to all possible paths. The former is the case when a trip planner enforces the car routing between each pair and is discussed in Section 4.2 and the latter is the case of an integrated allocation-routing model and is discussed in Chapter 5. As we will discuss in Section 4.3, there is a third case where the trip planner suggests (and not enforces) only one trip plan, but we try to build some extra feasible paths on top of the trip plan. The number of paths between supply-demand pairs in this case is something in between and considerably closer to one than to the number of all possible paths.

Trip plan based formulations for the empty car distribution problem, considered in this research, can be divided into two sub categories: 1) The pure allocation, or simply the allocation model, where each feasible supply-demand pair is connected using only one path; and 2) the allocation-timing model, where more than one path is allowed between them. A third category where all feasible paths are considered can be transformed to an arc-based formulation and will be discussed in Chapter 5. Figure 4-2 illustrates the main
components of the car distribution model in three different modes. Both Allocation and Allocation-Timing models assign empty cars to the customer orders. However in the former, the entire trip plan from the supply point to the customer location is given by the trip planner application, while in the latter, only the physical path is generated by the trip planner and the timing of the movements are an output of the distribution model. Next we describe the allocation and allocation-timing formulations.

Figure 4-2 Car distribution model in three different modes
4.2 Pure Allocation Model on a Bipartite Resource Constrained Graph

In railway industry, car routing and car distribution decisions are usually made separately. Hence let us assume that an external application, say the trip planner, will determine the optimal trip plan between every requested origin-destination pair. For example, the trip planner suggests the green path $b_{1T}: g_{1T}^R-b_{2R}^R: g_{2R}$ for $s_2-d$ pair in Figure 4-3(a-1). The algorithm that delivers this path is an internal of the trip planner and is out of the scope of the allocation model. Since there is at most one path between each origin-destination pair, we can consider this path, in the time-space network, as an arc, in a bipartite supply-demand graph, use blocks and trains involved in the path, as resources (Figure 4-3(a-2)) and transform the problem to a resource-constrained generalized transportation model. The transformation can be implemented as follows: replace each supply node $m$ with $|K|$ nodes, corresponding to the $|K|$ commodities, and connect these nodes to all demand nodes $n$, that were feasible to $m$, using arcs of length $c_{ik}^j$.

Enforcing cars to flow on a single capacitated path from $m$ to $n$ or to have to choose another destination might increase the total unmet demands or the overall assignment costs.

**Example 4-1:** Let us consider a network with only two supply nodes, one demand node and one intermediate classification yard in Figure 4-3(a-1). The supply nodes $s_1$ and $s_2$ each has a surplus of two cars of the same pool and the demand node $d$ requires two cars of a similar pool with a maximum delay of three days. Also assume that the path $b_{3T}: g_{3T}$ and path $b_{1T}: g_{1T}^R-b_{2R}^R: g_{2R}$ are suggested by the trip planner as the optimal paths for the $s_1-d$ and $s_2-d$ pairs respectively, where the assignment cost of the first path is considerably higher than the second one and the unmet demand penalty is extremely high.
compared to the assignment cost. Let’s consider three scenarios and investigate the behavior of our model.

Scenario 1: If there is no capacity limit (or a limit of more than two cars which will make the capacity constraint to be redundant) on the arcs $b_{1T}: g_{1T}$ and $b_{2R}: g_{2R}$, then both cars in $s_2$ would be planned to $d$.

Scenario 2: But if arc $b_{2R}: g_{2R}$ has a capacity limit of one car then the previous assignment will not be feasible anymore. The optimal solution will then be to send one car from $s_1$ and one car from $s_2$, which apparently has a higher assignment cost than the first scenario.

Scenario 3: Now let’s assume that arc $b_{2R}: g_{2R}$ has a capacity limit of one car and arc $b_{3T}: g_{3T}$ is full. Then the optimal (and the only feasible) solution is to provide only one car from $s_2$, resulting in one unit of unmet demand.

Now let see what an experienced car dispatcher would traditionally, without using a mathematical model, do in scenario 3. A car dispatcher will most likely not know about the capacities of the network while making his decision. Based on his experience or a simple judgment he would pick one of the two supply yards, let say yard $s_2$, and will assign the two cars available in that yard to the demand node. What happens next is more interesting. Since arc $b_{2R}: g_{2R}$ has only one unit of capacity left, only one of the two cars will make it to its destination by Saturday night. The second car however will not be able to take $b_{2R}: g_{2R}$. So it will stay till there is an empty capacity available probably tomorrow! The day after? As long as this car arrives to the customer before the order expiration date, i.e. within three days delay, the performance of this dispatcher will beat our model's solution. Order fulfillment ratio is an important performance measure for the distribution
department and such an output of Scenario 3 is not acceptable! The reality is a pure allocation model that takes a single path provided by the trip planner as the face value is overseeing the dynamics of the service network and hence is not realistic. In the next section we provide a way to include this dynamic into the model.
4.2 Pure Allocation Model on a Bipartite Resource Constrained Graph

Figure 4-3 An arc-train network with only one path between any O-D pair (a-1) can be transformed to a resource-constrained bipartite graph (a-2), while the one with more than one path (b-1) is transformed to a bipartite multigraph (b-2).
4.3 Allocation-Timing Model on a Bipartite Resource Constrained Multigraph

In order to avoid the Pure Allocation models foible in ignoring the dynamics of the service network, and at the same time avoiding combining the decisions that are made in different departments, i.e. the car routing and car distribution decisions, we take the trip plan’s suggested path as a basis and try to develop new paths by adding inventory and delay arcs to it. In this research such paths that use the same physical path, i.e. blocks and trains, to the ones in the trip plan but with different timings are referred to as pseudo trip-plans or pseudo paths. The blue path and the green-red-blue path in Figure 4-3(b-1) are created using such a procedure. The bipartite transformation of the block-train network after introducing pseudo paths is a multigraph (Figure 4-3(b-2)).

After introducing the pseudo trip-plans, the optimal solution of Example 4-1 in Section 4.2 is to send two cars from $s_2$ to $d$, where one car is planned on path $b_{1T}: g_{1T}-b_{2R}: g_{2R}$ and another one is planned on path $b_{1T}: g_{1T}-b_{2F}: g_{2F}$.

This path based formulation of empty railcars in which the paths connecting feasible supply-demands are given by calling an external application (trip planner) but the car distribution model is in charge of the optimal supply to demand allocation and the timing of the movements, in this research is referred to as the Allocation-Timing model.

4.3.1 An Algorithm for Generating Pseudo-paths of a Trip plan

Before presenting our algorithm for finding all pseudo paths associated with a trip plan, we recall the concept of lattice path in combinatorics and see how it is connected to our algorithm. A lattice path $L$ in $\mathbb{Z}^d$ of length $k$ with steps in $S$ is a sequence $v_0, v_1, \ldots, v_k \in \mathbb{Z}^d$ such that each consecutive difference $v_i - v_{i-1}$ lies in $S$ (Stanley, 2011). An example
of a lattice path in $\mathbb{Z}^2$ of length 4 with steps in $S = \{(0,1), (1,0), (1,1)\}$ is $L = \{(1,0), (1,1), (2,1), (3,2), (3,3)\}$ and is illustrated in Figure 4-4.

![Figure 4-4 Lattice path of length four in $\mathbb{Z}^2$](image)

A North-Eastern (NE) lattice path is a path consisting of $(1, 0)$ and $(0, 1)$ steps, where $(1,0)$ steps are called East steps and denoted by E and $(0,1)$ steps are called North steps and denoted by N.

Figure 4-5 shows six NE paths originating from point $(0,0)$. In fact there is no more NE path with two E’s and two N’s originating from $(0,0)$ in addition to these six paths. And more strongly any NE path connecting $(0,0)$ to $(2,2)$ must have two N’s and two E’s.

NE lattice paths most commonly begin at the origin. This convention allows us to encode all the information about a NE lattice path $L$ in a single permutation word. The length of the word gives us the number of steps of the lattice path, $k$, and the order of the N's and E's communicates the sequence of $L$. Furthermore, the number of N's and the number of E's in the word determines the end point of $L$. ²

The set of all NE paths with $a$ steps of N and $b$ steps of E can be generated as the set of all permutation of $a + b$ items with $a$ identical items N and $b$ identical items E, or simply saying the set of all words that can be written using $a$ times letter N and $b$ times letter E. Size of such a set is $\binom{a + b}{a}$.

Now let’s consider a case where trip planner has indicated that path $b_{1T} - b_{2R}$ in Figure 4-6(a) is optimal path between $s$ and $d$. The problem of finding all pseudo paths between $s$ and $d$ is similar to finding all lattice paths between points $(0,0)$ and $(2,2)$. Path $b_{1T} - b_{2R}$ can be encoded as IBIB, where I stands for inventory arc and B stands for block. The five other paths can be listed as: IIBB, representing $b_{1W} - b_{2R}$; BBII, representing $b_{1M} - b_{2T}$; IBBI, representing $b_{1T} - b_{2W}$, BIBI representing $b_{1M} - b_{2W}$, and BIIB representing $b_{1M} - b_{2R}$. Furthermore let’s consider a case of Figure 4-6(b) where a maximum delay of two days is acceptable by customer at demand node $d$. Then the permutations should be generated using two more inventory arcs, IBIBII, resulting in 15 paths.

Next we develop a dynamic programming method to find all pseudo paths associated with a given trip plan (path). A pseudocode of the major components of the algorithm is presented here. The method returnAllPseudoPaths() takes a given path $l$ as input and translates it to a word using PathToString() method. Then using another method, returnListOfAllPermutations(), returns all words that are created using letters of this word. Finally all these words are translated back to their associated paths, using StringToPath() method, and the output is returned. Among the three inner methods (i.e. PathToString(), returnListOfAllPermutations(), and StringToPath()), method returnListOfAllPermutations as the main combinatorial part of the pseudo path generator is presented in detail.
4.3 Allocation-Timing Model on a Bipartite Resource Constrained Multigraph

The `returnListOfAllPermutations()` reads a given word letter-by-letter. After reading each letter, and before reading the next one, it generates all the words that can be created using the substring traversed so far and save them in a set called `currentSet`. Similar to other dynamic programming methods, at each step and after reading each letter, the algorithm uses the substrings created before to generate new substrings of one size more. The algorithm ends when all the letters are traversed. Example 4-2 illustrates how `returnListofAllPermutations()` works.
Pseudocode for generating all pseudo paths of a given path

```
returnAllPseudoPaths( Path l){
 Array<Path> pseudoPathsList = new array();
 String givenWord = PathToString(l);
 Array<String> wordList = returnListOfAllPermutations(givenWord);
 For (Each word In wordList){
 Path pseudoPath = StringToPath(word);
 pseudoPathsList.add(pseudoPath)
 }
 Return pseudoPathsList;
}
```

Pseudocode for generating all permutations of a given word

```
returnListOfAllPermutations( Array<Char> givenWord) {
 Set<Array<Char>> permutationsList = New Set();
 Array<Char> start = New Array();
 i = 1;
 start[i] = givenWord[i];
 permutationsList.Add(start);
 for (i = 2; i <= givenWord.Length; i++) {
    Set<Array<Char>> currentSet = new Set();
    for (Each permutation In permutationsList) {
        for (j = 1; j <= permutation.size() + 1; j++) {
            Array<Char> newPermutation = permutation;
            newPermutation.add(j, givenWord[i]);
            currentSet.add(newPermutation);
        }
    }
    permutationsList = currentSet;
 }
 return permutationsList;
}
Example 4-2) Generate all permutations of a givenWord = BIIB

givenWord = BIIB
permutationsList = {}, start = []

----------
i=1 \rightarrow \text{givenWord}[i] = B
start = [B]
permutationsList = [{B}]
----------
i=2 \rightarrow \text{givenWord}[i] = I
currentSet = {}
permutation = [B]
permutationsList = { [IB], [BI] }

permutation = [IB]
permutationsList = { [IB], [BI] }

----------
i=3 \rightarrow \text{givenWord}[i] = I
currentSet = {}
permutationsList = { [IB], [BI] }

permutation = [BI]
permutationsList = { [IB], [BI], [BI] }

----------
i=4 \rightarrow \text{givenWord}[i] = B
currentSet = {}
permutationsList = { [IB], [BI], [BI], [BI] }

permutation = [IIB]

permutation = [IBI]
permutationsList = { [IB], [IB], [IB], [IB] }

permutation = [BII]

permutationsList = { [IB], [IB], [IB], [IB], [IB], [IB] }

<table>
<thead>
<tr>
<th>j</th>
<th>newPerm</th>
<th>currentSet</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[IB]</td>
<td>{ [IB] }</td>
</tr>
<tr>
<td>1</td>
<td>[IB]</td>
<td>{ [IB], [BI] }</td>
</tr>
<tr>
<td>1</td>
<td>[IB]</td>
<td>{ [IB], [IB], [IB] }</td>
</tr>
<tr>
<td>1</td>
<td>[IB]</td>
<td>{ [IB], [IB], [IB], [IB] }</td>
</tr>
<tr>
<td>1</td>
<td>[IB]</td>
<td>{ [IB], [IB], [IB], [IB], [IB] }</td>
</tr>
<tr>
<td>2</td>
<td>[IB]</td>
<td>{ [IB], [IB], [IB], [IB] }</td>
</tr>
<tr>
<td>2</td>
<td>[IB]</td>
<td>{ [IB], [IB], [IB], [IB], [IB] }</td>
</tr>
<tr>
<td>2</td>
<td>[IB]</td>
<td>{ [IB], [IB], [IB], [IB], [IB], [IB] }</td>
</tr>
<tr>
<td>3</td>
<td>[IB]</td>
<td>{ [IB], [IB], [IB], [IB], [IB], [IB] }</td>
</tr>
<tr>
<td>3</td>
<td>[IB]</td>
<td>{ [IB], [IB], [IB], [IB], [IB], [IB], [IB] }</td>
</tr>
<tr>
<td>4</td>
<td>[IB]</td>
<td>{ [IB], [IB], [IB], [IB], [IB], [IB], [IB], [IB] }</td>
</tr>
</tbody>
</table>
Figure 4-6. A trip plan and its associated pseudo paths when a) no delay is permitted, and b) a maximum delay of two days is acceptable
4.3 Allocation-Timing Model on a Bipartite Resource Constrained Multigraph

4.3.2 A Lagrangian Heuristic

The path-based Allocation-Timing model is not easy to solve by integer programming methods. This is why developing heuristic methods is needed. One method could be a Lagrangian based heuristic by dualizing the capacity constraints and keeping the supply and demand constraints as the main constraints. The algorithm is similar to the one of Holmberg et al. (2008). The Lagrangian Relaxation is implemented by dualizing constraint sets (4-15) and (4-16) as follows:

\[ LR(\alpha, \beta) = \min \limits_{x,y,z} \left[ \sum_{b \in B} (f_b - \beta_b v_b) z_b + \sum_{l \in L} \sum_{k \in K, k \in \psi} c^k_l \right. \]

\[ \left. + \omega^k \sum_{g \in G_l, g_l = (g : l \in L_g)} \alpha_g + \tau^k \sum_{b \in B_l, B_l = (b : l \in L_B)} \beta_b \right] x^k_l \]

\[ + \sum_{n \in N_k, k \in \psi} c_n y_n - \sum_{g \in G} \alpha_g u_g \]

s.t. (4-13), (4-14), (4-17), (4-18), (4-19), (4-20)

The Lagrangian function (4-21) can be decomposed into three components in (4-22), among them one function, (4-23) to be optimized on \( z \), and another one, (4-24), to be optimized on \( x \) and \( y \). The third component in (4-22) is a constant for a given vector \( \alpha \) and does not need to be further considered.

\[ LR(\alpha, \beta) = LR_1(\beta) + LR_2(\alpha, \beta) - \sum_{g \in G} \alpha_g u_g \]  

(4-22)

\[ LR_1(\beta) = \min \limits_{z} (\sum_{b \in B} (f_b - \beta_b v_b) z_b ) \quad \text{s.t. (4-20)} \]  

(4-23)

\[ LR_2(\alpha, \beta) = \min \limits_{x,y} \left( \sum_{l \in L} \sum_{k \in K, k \in \psi} c^k_l \right. \]

\[ \left. + \omega^k \sum_{g \in G_l, g_l = (g : l \in L_g)} \alpha_g + \tau^k \sum_{b \in B_l, B_l = (b : l \in L_B)} \beta_b \right] x^k_l \]

\[ + \sum_{n \in N_k, k \in \psi} c_n y_n \]

s.t. (4-13), (4-14), (4-17), (4-18), (4-19)
The first sub-problem, $LR_1(\beta)$, has a trivial solution and the second one, $LR_2(\alpha, \beta)$, decomposes into a number of generalized transportation problems, $LR^K_2(\alpha, \beta)$, one for each car type. If $\mu^k_l = 1 \; \forall \; k \in K, l \in L$ (1:1 substitution case), the linear programming relaxation of $LR^K_2(\alpha, \beta)$ has an integral optimal solution.

The Lagrangian Dual problem associated with the original optimization problem is then formulated as maximizing the Lagrangian Relaxation sub-problem and is solved using subgradient optimization. The downside of the Lagrangian Relaxation is that the feasibility of the optimal solution of the relaxed problem in the original problem is not guaranteed. Hence we combine the subgradient method with a heuristic to find feasible solutions from the solutions of the Lagrangian Relaxation problem.

Next, we describe the Lagrangian heuristic. In this algorithm, $LB$ and $UB$ are the lower and the upper bound of the original problem. $\alpha^{(r)}$ and $\beta^{(r)}$ are the Lagrangian multipliers in iteration $r$ and $(x^{(r)}, y^{(r)}, z^{(r)})$ is the solution to the subproblem in that iteration. $\alpha^{(r)}, \beta^{(r)}, x^{(r)}, y^{(r)}, z^{(r)}$ each is a vector of its associated elements. For example $x^{(r)}$ is a vector of $x_k^{r}$ and $\alpha^{(r)}$ is a vector of $\alpha_g^{(r)}$. The Lagrangian heuristic is as follows:

```
Lagrangian heuristic() {
Step 0. (Initialization)
    Pick the termination parameter $t_{max}, \epsilon$ and $r_{max}$; Choose a starting point $\alpha^0 = 0; \beta^0 = 0; LB = -\infty; UB = initializeUB(); \lambda^0 = 2; \text{ Set iteration number } r = 0; t_{start} = \text{ current time};

    initializeUB() {
        return $\sum_{n \in N, K \subseteq \psi} c_n \; d_n + \sum_{n \in N, K \subseteq \psi} c'_n \; (d_n - y_{n_{max}}$)
    }

    While (stopping criteria is not met) Do {

        Step 1. (solve the subproblems)
            solve the subproblems, $LR_1(\beta^{(r)})$ and $LR_2(\alpha^{(r)}, \beta^{(r)})$ to find:
```

77
\[(x^{(r)}, y^{(r)}) \in \arg\min_{x, y} LR_2(\alpha^{(r)}, \beta^{(r)}) \text{ and}
(z^{(r)}) \in \arg\min_{z} LR_1(\beta^{(r)}).
\]
Then calculate:
\[LR(\alpha^{(r)}, \beta^{(r)}) = LR_1(\beta^{(r)}) + LR_2(\alpha^{(r)}, \beta^{(r)}) - \sum_{g \in G} \alpha_g^{(r)} u_g\]
\[LB = \max\{LB, LR(\alpha^{(r)}, \beta^{(r)})\}\]

**Step 2. (Determine the subgradient vector)**
\[\gamma_{\alpha, g}^{(r)} = \sum_{l \in L_g} \sum_{k \in K, \kappa \in \psi} \omega^k x_{l}^{k, (r)} - u_g, \forall g \in G\]
\[\gamma_{\beta, b}^{(r)} = \sum_{l \in L_b} \sum_{k \in K, \kappa \in \psi} \tau^k x_{l}^{k, (r)} - v_b z_{b}^{(r)}, \forall b \in B\]
\[\gamma^{(r)} = (\gamma_{\alpha}^{(r)}, \gamma_{\beta}^{(r)})\]

**Step 3. (Feasibility test)**
If \((\gamma^{(r)} \leq 0\) then \((x^{(r)}, y^{(r)}, z^{(r)})\) is feasible to the original problem)
Update the upper bound: \(UB = \min(UB, f(x^{(r)}, y^{(r)}, z^{(r)}))\)
Else
\[
\text{makeItFeasible( )}
\]
Use a heuristic to find a feasible solution \((\bar{x}^{(r)}, \bar{y}^{(r)}, \bar{z}^{(r)})\) to the original problem.
If (such a solution is found)
Update the upper bound: \(UB = \min(UB, f(\bar{x}^{(r)}, \bar{y}^{(r)}, \bar{z}^{(r)}))\)

**Step 4. (Stopping criteria - Optimality test):**
\[epGap = \text{Abs}\left(\frac{UB - LB}{1e - 10 + UB}\right)\]
\[t_{\text{run}} = \text{current time - t}_{\text{start}}\]
If \((t_{\text{run}} \geq t_{\text{max}} \text{ OR epGap} \leq \varepsilon \text{ OR } r \geq r_{\text{max}})\)
Return the best \(LB, UB, \) and the best feasible solution found, if any, to the original problem
Terminate

**Step 5. (Update the Lagrangian multipliers)**
\[\theta^{(r)} = \text{stepSizeFunction()} \text{ is the step size in iteration } r\]
\[\alpha_g^{(r+1)} = \max\left(0, \alpha_g^{(r)} + \theta^{(r)} \gamma_{\alpha, g}^{(r)}\right), \forall g \in G,\]
\[\beta_b^{(n+1)} = \max\left(0, \beta_b^{(r)} + \theta^{(r)} \gamma_{\beta, b}^{(r)}\right), \forall b \in B,\]

**Step 6. (Update the iteration parameters)**
4.3 Allocation-Timing Model on a Bipartite Resource Constrained Multigraph

The Algorithm starts with initializing the parameters and setting up the termination rules in Step 0. In Step 1, subproblems are solved and the lower and upper bounds on the original problem are updated accordingly. In Step 2, the amount of residual capacity of blocks and trains or their violation level, if overbooked, are calculated and the subgradient vector is reported. If all elements of the subgradient vector are nonnegative, Step 3 identifies the current solution as a feasible solution; otherwise, it uses another heuristic, called `makeItFeasible` which is presented latter in this section, to create a feasible solution out of this infeasible solution. Any time a feasible solution is identified or created the upper bound is updated. If none of the stopping criteria are met in Step 4, the Lagrangian multipliers will be calculated in Step 5 and a new iteration will be performed starting from Step 1. Parameter $\theta^{(r)}$ in Step 5 is called step size in $r$th iteration and a trade-off between the convergence time and solution precision should be considered while appointing a value to it. A too small value for the step size might result in a long running time since the algorithm might stuck in the current solution and does not converge, while a too large value for it might increase the chance of overshooting the optimal solution or it may result in a zigzag behavior between non optimal solutions. As mentioned in Ahuja et al. (1993) a balance between the time and precision can be achieved if using a nonsummable diminishing strategy:

$$\theta^{(r)} \geq 0, \lim_{r \to \infty} \theta^{(r)} = 0, \sum_{r=0}^{\infty} \theta^{(r)} = \infty \quad (4-25)$$

\[ updateLambda(h) \]
\[ r = r + 1 \]
Two step size setup strategies are presented here and both satisfy these conditions (Ahuja et al., 1993). The first strategy, called a constant step size strategy, uses the reverse of the iteration parameter \( r \geq 1 \) as step size and is presented as:

\[
\text{stepSizeFunction1}() \{
\theta^{(r)} = \frac{1}{r}
\}
\]

The second strategy adapted from “Newton’s Method” for solving systems of nonlinear equations employs the current bounds of the original problem as well as the Euclidean norm of the subgradient vector to assign the step size.

\[
\text{stepSizeFunction2}() \{
\theta^{(r)} = \frac{\lambda^{(r)} (UB - LR(\alpha^{(r)}, \beta^{(r)}))}{\|\gamma\|^2}
\}
\]

where \( \lambda^{(r)} \) is a scalar strictly between 0 and 2, initially set to 2 and updated, using \( \text{updateLambda}(h) \) procedure, if the lower bound is not improved for a given number of \( h \) iterations:

\[
\text{updateLambda}(h) \{
\text{If (LB has not improved for } h \text{ iterations)}
\lambda^{n+1} = \lambda^n / 2
\text{Else}
\lambda^{n+1} = \lambda^n
\}
\]

As it comes from its name, \( \text{makeItFeasible}() \) Algorithm takes an infeasible solution as an input and makes it feasible! The only constraints expected to be violated are the block and train capacity constraints as we have relaxed them in the Lagrangian Relaxation.
method. The violation level, or the amount by which the capacity is violated, are already on hand, \( y_{a,g}^{(r)} > 0 \) for train \( g \) and \( y_{\beta,b}^{(r)} > 0 \) for block \( b \). There might be many ways to alter the current solution in order to obtain a feasible solution. Here we use an optimization approach for this purpose.

\[
\text{makeItFeasible}(x^{(r)}, y^{(r)}, z^{(r)}) \{
\]

1) calculate the flow reductions

Find the minimum number of changes in assignment variables that resolve the violation of capacity constraints

\[
L K = \{(l,k) : l \in L, k \in K, K \subseteq \Psi, x_l^k > 0 \text{ in current infeasible solution of } LR\}
\]

\[
\begin{align*}
\text{min} & \sum_{(l,k) \in L K} \Delta x_l^k \ast c^k_l \\
\sum_{(l,k) \in L K} \Delta x_l^k \ast \tau^k & \geq y_{\beta,b}^{(r)} \quad \forall \ b \in B, y_{\beta,b}^{(r)} > 0 \quad (4-27) \\
\sum_{(l,k) \in L K} \Delta x_l^k \ast \omega^k & \geq y_{a,g}^{(r)} \quad \forall \ g \in G, y_{a,g}^{(r)} > 0 \quad (4-28) \\
0 & \leq \Delta x_l^k \leq x_l^k \quad \forall \ (l,k) \in L K \quad (4-29)
\end{align*}
\]

2) Update the decision variables

Update assignment variables

\[
x_l^k = x_l^k - \Delta x_l^k
\]

Then update the shortage variables and block construction variables accordingly

\[
y_n = d_n - \sum_{l \in L_n} \sum_{k \in K_n} \mu_l^k x_l^k \quad \forall \ n \in N_K, \forall \ K \subseteq \Psi
\]

\[
y'n = \max(y_n - y_n^{\max}, 0) \quad \forall \ n \in N_K, \forall \ K \subseteq \Psi
\]

If \( \exists (l,k), l \in L_b \text{ and } x_l^k > 0 \) then \( z_b = 1 \); else \( z_b = 0 \), \( \forall \ b \in B \)

3) Return the feasible solution \((\bar{x}^{(r)}, \bar{y}^{(r)}, \bar{z}^{(r)})\)

Let’s assume that for a given \((l,k)\) the assignment variable \( x_l^k \) has a positive value and since the flow variables are integral \( x_l^k \geq 1 \). Furthermore let’s assume that the coefficients, \( \omega^k \) or \( \tau^k \), of this variable in some of the violated constraints is nonzero.
Apparently reducing $x^k_l$ by a value of one will help the violated constraints to be closer to becoming a feasible solution. For example, if there are only three violated constraints all with a violation level of one and the coefficient of $x^k_l$ in two of them is one, then one unit reduction in $x^k_l$ will save two constraints from infeasibility. In general we prefer to make as less change as possible during `makeItFeasible()` process assuring that demand fulfillment ratio will not be dramatically affected.

The decision variable $\Delta x^k_l$ is the number of cars to be deducted from current value $x^k_l$. The objective function (4-26) tends to minimize the total deduction amount. The cost coefficients of the objective function here have reverse relations with the ones in the original problem. For example $c^r_l = \frac{1}{\log c_l}$ will assure that small cost coefficients $c^r_l$ are assigned to the paths with high assignment costs $c^k_l$ in the original problem, practically making their flow reduction more likely. While such a function might look intuitive, it oversees the trade-off between assignment cost and unmet demand penalty and might increase the overall cost. This is why we use a constant coefficient $c^r_l = 1$ that simply minimizes the total flow change. Constraints (4-27) and (4-28) insure that the total change in the flow is at least equal to the violation level while Constraint (4-29) states that the change in flow cannot be more than the flow!

Another way to find the Lagrangian multipliers is to use Dantzig-Wolfe Decomposition technique by defining the master problem on the bundle resource consumption constraints (4-15) and (4-16) and defining one sub-problem for each commodity over the supply-demand conservation constraint sets (4-13) and (4-14) for that commodity. One important advantage of the Dantzig-Wolfe decomposition which distinguishes it from other Lagrangian-based algorithms is that: In each iteration in addition
4.3 Allocation-Timing Model on a Bipartite Resource Constrained Multigraph

to providing a lower bound, it always maintains a feasible solution for the problem. However, this algorithm is usually not as time-efficient as the subgradient method (Ahuja et al., 1993).

4.3.3 An Iterative Relaxation and Rounding Heuristic

The decision variables of the PathBased formulation developed at Section 4.1 have some interesting properties:

**Property 4-1** The decision variables associated with the shortage (i.e. $y_n$) are dependent variables and given the value of assignment variables (i.e. $x_i^k$) their values can be calculated.

**Property 4-2** The decision variables associated with the critical shortage (i.e. $y'_n$) are dependent variables and given the value of shortage variables (i.e. $y_n$) their values can be calculated.

**Property 4-3** The design variables associated with the block setups (i.e. $z_b$) are dependent variables and given the value of assignment variables (i.e. $x_i^k$) their values can be calculated.

Based on Property 4-1 to Property 4-3 if the values of the assignment variables (i.e. $x_i^k$) are given, the values of the rest of the variables can be calculated and the objective function can be evaluated. Another observation from this model is that:

**Property 4-4** The optimal solution, and more generally any feasible solution of the LP Relaxation of the PathBased formulation, after dropping the fractional parts of the decision variables and updating the rest of variables, is feasible to the original PathBased model.
Property 4.4 always holds since a decision of even not satisfying any order is feasible to the model, although not desirable and definitely not optimal because of high shortage penalties. Hence any decrease in the assignment variables, as long as doesn’t make them negative, is feasible to the model but only after updating the shortage variables and block setup variables according to the changes. This interesting property opens up a variety of approximation techniques to us.

An easy, and probably the most common, approach to the IP problems is the Rounding technique. The rounding approach solves the linear relaxation of the IP problem and attempts to round the solution to an integer one by dropping all the fractional parts of the decision variables or by finding satisfactory solutions wherein the variable values are adjusted to close larger or smaller integer values.

Another way of dealing with Integer programs is Iterative Algorithm first introduced by Jain (2001) in solving the survivable network design problem. This technique benefits from properties of extreme point solutions through an iterative algorithm. At each iteration a subset of variables is rounded. Then, the constraints are modified to reflect the residual problem based on the rounded variables. Such an updated instance is passed to the next iteration for further processing.

In this research we develop an approximation iterative method based on consecutive relaxation and rounding as follows:

**Iterative Relaxation and Rounding Heuristic (IRRH)**

1. Call the PathBased formulation as Original IP problem (OIP) with decision variables \((x, y, y', z)\), where each element is a vector by itself, e.g. \(x\) is a vector of assignment variables \(x_i^k\). Let \((x^*, y^*, y'^*, z^*)\) be OIP’s optimal solution to be discovered. Set \((x^*, y^*, y'^*, z^*)\) to \((0,0,0,0)\)
2. \(\text{CurrentIPProblem} = \text{OIP}\)
3. Relax the integrality constraints of \(\text{CurrentIPProblem}\) and solve the LP
Relaxation (LPR) to optimality. Let \((X^*, Y^*, Y'^*, Z^*)\) be the optimal solution of LPR.

4- Update the assignment variables of OIP using \(x^*_l^k += \text{RoundDown}(X^*_l^k)\)

5- Update the rest of OIP variables based on (4-30), (4-31), and (4-32):

\[
y^*_n = d_n - \sum_{l \in L_n} \sum_{k \in K_n} \mu_l^k x^*_l^k \tag{4-30}
\]

\[
y'^*_n = \max(0, y^*_n - y^*_n^{\text{max}}) \tag{4-31}
\]

\[
z^*_b = \begin{cases} 
1 & \text{if } \exists l \in L_b, x^*_l^k > 0 \\
0 & \text{otherwise}
\end{cases} \tag{4-32}
\]

6- If stopping criteria is met

   Return the optimal solution \((x^*, y^*, y'^*, z^*)\) and objective value

Else

Introduce a new problem called IP Residual (IPR). IPR has the same formulation of OIP but on residual parameters noted by CAPITAL case letters (e.g. \(F_b\) in IPR is the block setup cost similar to \(f_b\) in OIP). Calculate the parameters of IPR based on equations (4-33) to (4-38).

\[
F_b = \begin{cases} 
0 & \text{if } z^*_b = 1 \\
f_b & \text{otherwise}
\end{cases} \tag{4-33}
\]

\[
S^k_m = s^k_m - \sum_{l \in L_m} x^*_l^k \tag{4-34}
\]

\[
D_n = y^*_n \tag{4-35}
\]

\[
Y_n^{\text{max}} = \begin{cases} 
y^*_n & \text{if } y^*_n \leq y_n^{\text{max}} \\
y_n^{\text{max}} & \text{otherwise}
\end{cases} \tag{4-36}
\]

\[
U_g = u_g - \sum_{l \in L_g} \sum_{k \in K \subseteq \psi} \omega^k x^*_l^k \tag{4-37}
\]

\[
V_b = v_b - \sum_{l \in L_b} \sum_{k \in K \subseteq \psi} \tau^k x^*_l^k \tag{4-38}
\]

CurrentIPProblem = IPR

Go to step 3

The stopping criteria might be time or number of iterations. The relaxation in Step 3 of the iterative algorithm can be performed in two ways: a) we can relax only the integer variables and keep the binary ones as it is (IRRH1) or b) we can relax all integer variables to general non negative variables and relax all binary variables to continuous variables between 0 and 1 (IRRH2).
4.4 Noncapacitated Model and its Optimistic Behavior on a Capacitated Network

As part of our validation procedure, and as it will be discussed later in Chapter 6, we compare the performance of our model with the noncapacitated model’s performance. However in order to be able to benchmark our model against the noncapacitated model, we need to recalculate the performance measure of this model in a capacitated network, simply saying to compare apples with apples and not with oranges!

First let us start with the formulation of a noncapacitated model. The objective of current noncapacitated models is usually to minimize the total assignment costs plus the shortage cost without considering the block setup costs. Regarding the constraints, everything is the same as the path-based Pure Allocation model except that capacity constraints for blocks and trains are relaxed. Objective function (4-39) along with the supply limitation (4-40), demand satisfaction (4-41), soft service level (4-42) and the variable type constraints (4-43)-(4-44) constitute the noncapacitated model formulation as follows (see Table 4-1 for the notation and Section 4.1 for the details on the constraints):

NC (see Table 4-1 for the notations):

\[
\begin{align*}
\text{Min} & \quad \sum_{l \in L} \sum_{k \in K, K \subseteq \Psi} c^k_l x^k_l + \sum_{n \in N_K, K \subseteq \Psi} c_n y_n + \sum_{n \in N_K, K \subseteq \Psi} c'_n y'_n \\
\sum_{l \in L_m} x^k_l & \leq s^k_m \quad \forall m \in M, \forall k \in K \subseteq \Psi \\
\sum_{l \in L_n} \mu_l^k x^k_l & + y_n = d_n \quad \forall n \in N_K, K \subseteq \Psi \\
y'_n & \geq y_n - y_n^{\max} \quad \forall n \in N_K, \forall K \subseteq \Psi \\
x^k_l & \geq 0 \text{ and integer} \quad \forall l \in L, \forall k \in K, K \subseteq \Psi
\end{align*}
\]
\[ y_n, y'_n \geq 0 \quad \forall n \in N_k, \forall K \subseteq \Psi \] (4-44)

**Property 4-5** The objective value of noncapacitated model NC is always at least as good as the Pure Allocation PA and the Allocation-Timing AT models.

Property 4-5 always holds since the feasible region of a NC, as a result of relaxing the capacity constraints, is a superset of the PA’s. Feasible region of NC and AT models can’t be compared directly since one has more arcs and the other one has more capacities. However since the single arc connecting a given supply-order pair in NC has the shortest cost among all pseudo arcs connecting these nodes in AT and at the same time has an infinite capacity on it; the objective value of NC will always be at least as good as AT, and as a result at least as good as PA. In addition to the feasible region size and capacities, absence of block setup costs in NC will potentially make the cost even lesser.

A direct conclusion of Property 4-5 is that benchmarking AT and PA models against NC is pointless since the latter always holds a lesser objective value. Now let us consider a case that the NC’s output has been used for car distribution planning in a network that is actually capacitated. Since NC oversees the capacity limits while planning, it is expected to see a gap between the actual implementation and the plan. The gap we are addressing here is the gap solely caused as a result of ignoring the capacities while the stochasticity of other parameters such as travel time, etc. is an unrelated issue. In order to compare the performance of capacitated models AT and PA against current uncapacitated model NC, we should design our benchmark based on the behavior of the uncapacitated model in a capacitated network which will be denoted as NCActual. Such behavior can be quantified, on average, by generating random problem instances and comparing the results.
However in this research we formulate a mathematical model that reports the optimistic behavior of noncapacitated model in a capacitated network. Our goal is to show that the performance of our model is better than the optimistic behavior of the NC in a capacitated network and as a result is better than its average case behavior. The optimistic behavior happens when the actual flow of cars will be in compliance with the assignment costs.

NCAActual:

\[
\begin{align*}
\text{Min} & \quad \sum_{b \in B} f_b z_b + \sum_{l \in \text{Lpseudo}} \sum_{k \in K, K \subseteq \Psi} c_{l}^{k} x_{l}^{k} + \sum_{n \in N, K \subseteq \Psi} \text{Big} c_n y_n + \\
\sum_{n \in N, K \subseteq \Psi} \text{Big} c'_n y'_n \\
\sum_{l \in \text{Lpseudo} (l)} x_{l}^{k} + x'_{l} = x^*_{l} & \quad \forall (l, k), \text{ where flow of } k \text{ on } l \text{ is feasible to NC} \quad (4-46) \\
\sum_{l \in \text{Lpseudo}_n} \sum_{k \in K_n} \mu_{l}^{k} x_{l}^{k} + y_n = d_n & \quad \forall n \in N_K, \forall K \subseteq \Psi \quad (4-47) \\
y'_n \geq y_n - y_n^{max} & \quad \forall n \in N_K, \forall K \subseteq \Psi \quad (4-48) \\
\sum_{l \in \text{Lpseudo}_g} \sum_{k \in K, K \subseteq \Psi} \omega_{l}^{k} x_{l}^{k} \leq u_g & \quad \forall g \in G \quad (4-49) \\
\sum_{l \in \text{Lpseudo}_b} \sum_{k \in K, K \subseteq \Psi} \tau_{l}^{k} x_{l}^{k} \leq v_b z_b & \quad \forall b \in B \quad (4-50) \\
x_{l}^{k} \geq 0 \text{ and integer} & \quad \forall l' \in \text{Lpseudo} , \forall k \in K, K \subseteq \Psi \quad (4-51) \\
x'_{l}^{k} \geq 0 & \quad \forall l \in L, \forall k \in K, \forall K \subseteq \Psi \quad (4-52) \\
y_n, y'_n \geq 0 & \quad \forall n \in N_K, \forall K \subseteq \Psi \quad (4-53) \\
z_b \in \{0,1\} & \quad \forall b \in B \quad (4-54)
\end{align*}
\]

In this formulation \(l \in L\) is the path between the associated supply-order pair in NC while \(L^{\text{pseudo}} (l)\) is a set containing all pseudo paths associated with \(l\). The elements of such set is denoted by \(l'\). Consequently, \(L^{\text{pseudo}}_g\), \(L^{\text{pseudo}}_b\), \(L^{\text{pseudo}}_d\), and \(L^{\text{pseudo}}\) represent...
the updated version of sets \( L_g, L_b, L'_n \), and \( L \) after adding the pseudo paths. Practically \( L \) here is equal to \( L \) in NC (and AP) while \( L^{\text{pseudo}} \) is equal to \( L \) in AT.

The decision variable of NCA\(\text{Actual} \) \( x^k_{l'} \) is the number of cars of pool \( k \) that will actually flow on pseudo path \( l' \). The definition of unmet demand variables \( y_n, y'_n \) are the same as the ones in NC. In addition to that, \( x'^{k}_{l} \) is a new variable representing the overflow of the NC output in a capacitated network. The shortage penalties, i.e. \( \text{Big} c_n \) and \( \text{Big} c'_n \), in NCA\(\text{Actual} \) are not necessarily equal to the ones in NC (or Pure Allocation and Allocation-Timing) model(s), i.e. \( c_n \) and \( c'_n \). NC, Pure Allocation and Allocation-Timing models, are designed to help the decision maker while making the car distribution planning decisions; hence the trade-off between assignment costs and shortage penalties are important and the penalties should be well designed to guarantee that high cost assignments are avoided if the shortage penalties are less than the assignment costs. On the other hand, the goal of NCA\(\text{Actual} \) is to simulate the output of a noncapacitated model in a capacitated network and not to make a decision. The trade-off between assignment costs and shortage penalties are already considered in NC model, which its output is used as input of NCA\(\text{Actual} \), and any extra shortage reported by NCA\(\text{Actual} \) must be solely occurred as a result of capacity limitations. This is why shortage penalty costs in this model should be high enough to guarantee that no assignment will be missed as a result of high cost. However benchmarking NCA\(\text{Actual} \) against capacitated models (Pure Allocation, Allocation-Timing, and Allocation-Routing) is not possible unless the cost coefficients used in all models are the same. In order to be able to be able to make a valid comparison, we use objective function (4.45) for finding the optimal solution of NCA\(\text{Actual} \) then instead of using the
objective value reported by the model, we explicitly calculate the objective value using the
same cost coefficients used for other models as in (4-55)

\[
\min \sum_{b \in B} f_b z_b + \sum_{l \in L_{\text{pseudo}}} \sum_{k \in K, K \subseteq \Psi} c_{l_k}^k x_{l_k}^k + \sum_{n \in N, K \subseteq \Psi} c_n y_n + \sum_{n \in N, K \subseteq \Psi} c'_n y'_n
\]  

(4-55)

Needless to say that \(x'_{l_k}^k\) is the extra unmet demand, resulted from considering
capacities, in addition to the ones reported by the output of NC. New values for unmet
demand variables \(y_n\) and \(y'_n\) reported by NCActual already contain \(x'_{l_k}^k\) in it and there is
no need for the addition of another component into the objective function. However, since
this overflow might create congestion in the network, introducing another term as
congestion penalty might be realistic. Such a component can be formulated as \(c^{\text{cong}} \cdot \sum_{l \in L} \sum_{k \in K, K \subseteq \Psi} x'_{l_k}^k\) where \(c^{\text{cong}}\) is the congestion penalty rate. In this research we assume
that congestion penalty rate is zero and practically eliminate the congestion component.
Other notations can be found in Table 4-1.

NCActual contains the demand satisfaction (4-47), service level (4-48) and capacity
constraints (4-49) and (4-50) similar to the ones in AT. However the supply limitation
constraints will be eliminated. Instead, another constraint is added to ensure that the
assignments given by the noncapacitated model NC do not violate the capacities of the
network. Constraint (4-46) for each pool \(k\) determines \(x'_{l_k}^k\), the overload of each path \(l\), as
the difference between \(x^{*k}_{l_k}\), the optimal flow of NC, and \(\sum_{l \in L_{\text{pseudo}}} x^{l_k}_{l_k}\) the total number
of cars flowing on all pseudo paths of path \(l\). The remaining constraints are similar to the
ones in AT where \(l\) is replaced by \(l'\).
4.5 Trip Planning Algorithm

Both allocation and allocation-timing formulations of the car distribution problem require an external application to determine the trip plan between the potential supply-order pairs. We will call such an application a trip planner. In this section, we present a trip planning model and develop a method to solve it. The objective of such a model is to find the optimal route for a supply-order pair that minimizes the transportation cost, satisfies the order expiration date, and respects the physical constraints such as: height clearance (for railroad segment), weight clearance (for train), and length clearance (for block) as well as operational constraints such as: the availability of train service, the existence of dedicated yards for different types of freights, and the availability of yard capacity for blocking and classification.

Luckily most of the trip planner constraints can be transformed into one of the following prohibition formulations.

*Prohibited arc*: certain type of cars might be prohibited from travelling on a certain arc. For example a tank car might be prohibited from arcs that travel through cities. Arc prohibition might also happen if the arc's capacity is full. Such a prohibition might be resolved by simply excluding that arc from the network.

*Prohibited maneuver/subpath*: sometimes, an arc is not a prohibited arc for the shipment, but it is prohibited to be selected as the immediate next arc in a path. This kind of prohibition also happens in road transportation; for example, when a left-turn is prohibited. Prohibited maneuvers are harder to deal with however they have been studied recently and efficient algorithms are developed to solve them (Villeneuve and Desaulniers,
As mentioned in Section 3.2.2, the service network can be represented as either a
block-train network or a resource constrained blocking network. Next we discuss the trip
planning algorithms for both of these networks.

### 4.5.1 Trip Planning on the Time-Space Block-Train Network

In addition to the above mentioned physical constraints, another type of constraint
is enforced on the trip planning model as a result of the limitations of the time-space block-
train network.

A path may not originate from an intermediate point of a block. In Figure 3-4 of
Chapter 3 a path can be started by either an inventory arc or an arc with solid circle shape
tail. Once a block starts, it continues until the end of the block (the triangle head). If the
current arc has an open arrow shape head, the next arc in the path must be carrying the
same block and all other arcs are prohibited. Once the car reaches the block destination, it
can travel on an inventory arc or an arc with solid circle shape tail and all other arcs are
prohibited. Hence for a shipment from yard 2 on day 2 to yard 7 on day 6, paths $b_{5,2}: C_{2,2} -
 b_{5,2}: C_{3,3}$, $b_{5,3}: C_{2,3} - b_{5,3}: C_{3,4}$ and $b_{1,2}: A_{2,2} - b_{2,4}: B_{4,4}$ are feasible but path $b_{1,2}: A_{2,2} -
 b_{6,3}: B_{4,4}$ is infeasible. Yard 6 at day 4 is the end of the block $b_{1,2}$ (arc head is triangle).
Next arc must be a new block (arc tail must be a solid circle) or an inventory arc (maneuver
from arc $b_{1,2}: A_{2,2}$ to arcs $b_{4,1}: B_{4,4}$ or $b_{6,3}: B_{4,4}$ is prohibited). In fact block $b_{6,3}$ is created
on day 3 at yard 5 and is not subject to any intermediate reclassification till its destination
at yard 7 on day 5, and as a result it cannot pickup any cars at yard 6 on day 4.
The trip planning for a shipment between two nodes \( s \) and \( d \) in a time-space network can be restated as finding a minimum cost path between \( s \) and \( d \) while respecting the arc prohibitions and the maneuver prohibitions. A preliminary and high-level procedure for finding a trip plan is as follows:

1. Transform the time-space block-train multigraph to simple graph by introducing virtual nodes.
2. Eliminate all prohibited arcs from the network.
3. Find the shortest path between \( s \) and \( d \) in presence of prohibited maneuvers (e.g. using the algorithm presented by Villeneuve and Desaulniers (2005)).

### 4.5.2 Trip Planning on the Time-Space Resource Constrained Blocking Network

Contrary to the block-train network where each arc represents a train segment (and a block carried on it), arcs in the resource constrained blocking network are blocks and trains carrying those blocks are listed as block resources. The resource constrained blocking network is not a multigraph and does not have prohibited maneuvers. Hence after deleting the prohibited arcs, a shortest path algorithm, such as Dijkstra’s method, can be used for the trip planning purposes.

1. Eliminate all prohibited arcs from the network.
2. Use Dijkstra’s Algorithm to find the shortest path between \( s \) and \( d \).

Since trip planning on the resource constrained blocking network is much easier than on the block-train network, in this research we use this network for trip planning purposes.
5.1 Allocation-Routing Model

Chapter 5

Arc-Based Formulation for the Car Distribution Problem

Contrary to the path-based formulation for the car distribution problem which was defined on a bipartite multigraph and required a trip planner to provide the arcs of the graph, practically separating the car routing and car distribution decisions, the arc-based formulation is defined on a time-space network and integrates car routing and car distribution decisions in a single Allocation-Routing model. The decision here is not only supply to demand assignment but also finding the route that the supply will travel from its origin to destination. As this route is defined on a time-space network, the timing of movements will be automatically part of the route.

5.1 Allocation-Routing Model

Table 5-1 shows the notation for the indices, sets, parameters and variables used in the mathematical model. We start our formulation from the objective function. The objective function (5-1) minimizes the total cost which consists of block setup costs, block variable costs, delay costs, inventory costs, and unmet demand penalty as well as the contra costs for customer priorities and preferences (rewards or bonus). Since in the scheduled
railroads, general trains are unlikely to be canceled at the operational planning level, train setup cost is not considered in this model. The details on the components of the cost function are found in Section 3.2.5.

\[
\begin{align*}
\text{Min} & \quad \sum_{b \in B} f_b z_b + \sum_{b \in B} \sum_{k \in K, K \subseteq \Psi} c_b^k t_b^k + \sum_{d \in D} \sum_{k \in K, K \subseteq \Psi} c_d^\text{delay} t_d^k + \\
& \quad \sum_{i \in I} \sum_{k \in K, K \subseteq \Psi} c_i^k t_i^k + \sum_{n \in N, K \subseteq \Psi} c_n y_n + \sum_{n \in N, K \subseteq \Psi} c'_n y'_n + \\
& \quad \sum_{n \in N, K \subseteq \Psi} \sum_{k \in K} c_n^k x_n^k
\end{align*}
\]

(5-1)

First constraint set to consider is the node balance constraints (5-2) in the time-space flow network. Since substitution is not permitted in this network, each commodity (pool) has its own balance constraint, resulting in one constraint for each node-pool pair.

\[
\begin{align*}
\sum_{b \in B_m^{\text{out}}} t_b^k + \sum_{n \in N_m, k \in K} x_n^k + t_{i_m}^k + \quad & \forall \ m \in M, \forall k \in K, \ K \subseteq \Psi \\
l_m^k - \sum_{b \in B_m^{\text{in}}} t_b^k - t_{i_m}^k = s_m^k
\end{align*}
\]

(5-2)
### 5.1 Allocation-Routing Model

#### Table 5-1 Notation for the arc-based formulation

<table>
<thead>
<tr>
<th>Indices and Sets</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k \in K \subseteq \Psi )</td>
<td>Index for pools that belong to a specific car type</td>
</tr>
<tr>
<td>( m \in M )</td>
<td>Index for nodes in a time-space flow network</td>
</tr>
<tr>
<td>( n \in N_K )</td>
<td>Index for demand nodes which belong to car type ( K )</td>
</tr>
<tr>
<td>( n \in N_m )</td>
<td>Index for demand nodes associated with node ( m ) in time-space flow network</td>
</tr>
<tr>
<td>( g \in G )</td>
<td>Index for trains</td>
</tr>
<tr>
<td>( b \in B )</td>
<td>Index for blocks</td>
</tr>
<tr>
<td>( i \in I )</td>
<td>Index for inventory arcs</td>
</tr>
<tr>
<td>( d \in D )</td>
<td>Index for delay arcs</td>
</tr>
<tr>
<td>( K_n \subseteq K )</td>
<td>Set of all pools eligible for demand node ( n )</td>
</tr>
<tr>
<td>( B_m^\text{in} (B_m^\text{out}) \subseteq B )</td>
<td>Set of blocks entering (leaving) node ( m )</td>
</tr>
<tr>
<td>( l_m^\text{in} (l_m^\text{out}) \in I )</td>
<td>The inventory arc entering (leaving) node ( m )</td>
</tr>
<tr>
<td>( D_m^\text{in} (D_m^\text{out}) \subseteq D )</td>
<td>Set of delay arcs entering (leaving) demand node ( n )</td>
</tr>
<tr>
<td>( B_g \subseteq B )</td>
<td>Set of blocks carried on train ( g )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_b )</td>
<td>Setup cost for block ( b )</td>
</tr>
<tr>
<td>( c^k_n )</td>
<td>Contra cost (reward or bonus) of fulfilling one unit of demand ( n ) using a car of pool ( k )</td>
</tr>
<tr>
<td>( c_n )</td>
<td>Regular penalty- unit cost of unmet demand for demand node ( n )</td>
</tr>
<tr>
<td>( c^\prime_n )</td>
<td>Extra penalty- charged on unmet demand over ( y_n^{\max} )</td>
</tr>
<tr>
<td>( c^k )</td>
<td>Unit cost associated with travel of one car of pool ( k ) on block ( b )</td>
</tr>
<tr>
<td>( c^i_k )</td>
<td>Unit cost associated with rolling one car of pool ( k ) over inventory arc ( i )</td>
</tr>
<tr>
<td>( c^\text{delay}_d )</td>
<td>Unit cost associated with delay arc ( d )</td>
</tr>
<tr>
<td>( u_g )</td>
<td>Capacity of train ( g ) (train tonnage)</td>
</tr>
<tr>
<td>( v_b )</td>
<td>Capacity of block ( b ) (block length)</td>
</tr>
<tr>
<td>( \mu^k_n )</td>
<td>Substitution ratio of a car of pool ( k ) eligible for demand node ( n ). For example ( \mu^k_n = 2 ) means that a car from pool ( k ) will fulfill 2 units of demand of node ( n ).</td>
</tr>
<tr>
<td>( \omega^k )</td>
<td>Weight of an empty car of pool ( k )</td>
</tr>
<tr>
<td>( \tau^k )</td>
<td>Length of an empty car of pool ( k )</td>
</tr>
<tr>
<td>( s_m^k )</td>
<td>Supply of cars of pool ( k ) in supply node ( m )</td>
</tr>
<tr>
<td>( d_n )</td>
<td>Demand of cars at node ( n )</td>
</tr>
<tr>
<td>( y_n^{\max} )</td>
<td>Maximum unmet demand with regular penalty ( (c_n) ). Unmet demand over this point will be penalized by an extra penalty ( (c^\prime_n) ) added to ( c_n ).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^k_n )</td>
<td>Direct flow- Number of cars of pool ( k ) to arrive at demand node ( n )</td>
</tr>
<tr>
<td>( t^k_b )</td>
<td>Block flow- Number of cars of pool ( k ) to travel on block ( b )</td>
</tr>
<tr>
<td>( t^i_k )</td>
<td>Inventory arc flow- Number of cars of pool ( k ) to roll over inventory arc ( i )</td>
</tr>
<tr>
<td>( t^d_k )</td>
<td>Delay arc flow- Number of cars of pool ( k ) to arrive at the associated customer by delay arc ( d )</td>
</tr>
<tr>
<td>( l_m^k )</td>
<td>Leftover supply- Number of cars of pool ( k ) left over in supply node ( m )</td>
</tr>
<tr>
<td>( z_b )</td>
<td>Block creation variable- Binary variable, 1 if block ( b ) is created, and 0 otherwise</td>
</tr>
<tr>
<td>( y_n )</td>
<td>Unmet demand- Part of demand of node ( n ) that is not satisfied even with delay</td>
</tr>
<tr>
<td>( y^\prime_n )</td>
<td>Critical unmet demand- unmet demand over ( y_n^{\max} ) where ( y^\prime_n = y_n - y_n^{\max} )</td>
</tr>
</tbody>
</table>
Balance constraints for demand nodes are formulated as (5-3). Since contrary to the
time-space network nodes substitution is permitted at customer level, balance constraints
here are not defined for node-pool pairs and there exists only one constraint for each
demand node. Constraint (5-4) is the soft constraint on the maximum permitted unmet
demand and defines the critical unmet demand (subject to extra penalty).

\[
\sum_{k \in K_n} \left( x_{n}^{k} + \sum_{d \in D_n} \mu_{n}^{k} t_{d}^{k} \right) + y_{n} = y_{n}^{\prime} \geq y_{n} - y_{n}^{\max} \quad \forall \ n \in N_{K}, \ K \subseteq \Psi
\]

\[
\sum_{k \in K_n} \sum_{d \in D_n} \mu_{n}^{k} t_{d}^{k} = d_{n}
\]

Constraint sets (5-2) to (5-6) along with the binary constraint associated with the
block creations and non-negativity and integrality constraints for all other decision
variables will guarantee the supply availability, demand satisfaction and network
capacities; however they fail with respect to the order expiration dates.

Figure 5-1 shows five demand nodes associated with a single customer profile in
five different days with a maximum accepted delay of two days for each order-day. Each
node hosts the inflow of two backward arrows, associated with the flow that arrives by one
day delay and two-day delay. The reason we introduce two arcs, rather than only one arc,
is to emphasize that a delay of more than two days is not accepted. However, our current constraints and specifically constraint (5-3) which is associated with the demand nodes, will allow a flow $X_2$ arrived at day 5 and assigned to fulfill the backorder of day 3 be assigned to the backorder of day 2 or even 1 without violating any constraint. We call this situation a leak on the delay arcs that violates the customer order time windows.

![Figure 5-1 Delay arcs in the customer level](image)

Any arc-based formulation must respect the order time windows. In other words the flow on delay arcs must respect the anti-leak rule described as follows:

**Anti-leak rule for the delay arcs:** The inflow of delay arcs $\sum_{d \in D_n} \mu_n^k t_d$ to a demand node $n$ MUST be completely absorbed by that demand node. And the outflow of delay arcs $\sum_{d \in D_n} \mu_n^k t_d$ from a demand node $n$ MUST be originated solely from its direct inflow, $x_n^k$, i.e. cars to arrive to the customer’s serving yard exactly in the day that this demand node is associated with.

Figure 5-2 illustrates the leak situations involving delay arcs. As illustrated, the only permitted flow through the delay arc is the one that originates from the direct flow entering from the time-space flow network.

Table 5-2 contains some examples of demand node inflow and outflow and whether they guarantee the anti-leak condition or not. For example Case 8 is not anti-leak since the number of cars flowing out of $n$ to fulfill the backorders from previous days is more than
the direct flow to $n$. This means part of this backorder flow is coming either from the unmet demand, a total nonsense acquisition!, or from the inflow of delay arcs which is a clear violation of anti-leak rule. Similarly case 9 is violating anti-leak rule since its delay arcs’ inflow is more than its demand and based on demand node balance constraint, the extra units will flow through delay arcs flowing out of $n$, a leak situation by definition.

![Image of diagram](image_url)

**Figure 5-2 Permitted flow vs leak in customer level**

<table>
<thead>
<tr>
<th>Case #</th>
<th>$d_n$</th>
<th>$\sum_{d \in D_n^{in}} \mu_n^k t_d^k$</th>
<th>$x_n^k$</th>
<th>$y_n$</th>
<th>$\sum_{d \in D_n^{out}} \mu_n^k t_d^k$</th>
<th>Extra condition</th>
<th>Anti-leak?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td></td>
<td>YES</td>
</tr>
<tr>
<td>2</td>
<td>$d_n$</td>
<td>$0 \leq a \leq d_n$</td>
<td>$0 \leq b \leq d_n$</td>
<td>$0 \leq c \leq d_n$</td>
<td>0</td>
<td>$a + b + c = d_n$</td>
<td>YES</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>0</td>
<td>5</td>
<td>8</td>
<td>3</td>
<td></td>
<td>YES</td>
</tr>
<tr>
<td>4</td>
<td>$d_n$</td>
<td>0</td>
<td>0</td>
<td>$b$</td>
<td>$0 \leq c \leq d_n$</td>
<td>$0 \leq d \leq d_n$</td>
<td>$b + c - d = d_n$</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td></td>
<td>YES?</td>
</tr>
<tr>
<td>6</td>
<td>$d_n$</td>
<td>$0 \leq a$</td>
<td>$0 \leq b$</td>
<td>$0 \leq c \leq d_n$</td>
<td>$d \leq a$</td>
<td>$a + b + c - d = d_n$</td>
<td>YES?</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>6</td>
<td></td>
<td>NO</td>
</tr>
<tr>
<td>8</td>
<td>$d_n$</td>
<td>$d_n &lt; a$</td>
<td>$d &lt; b$</td>
<td>$d &gt; b$</td>
<td></td>
<td></td>
<td>NO</td>
</tr>
<tr>
<td>9</td>
<td>$d_n$</td>
<td>$d_n &lt; a$</td>
<td>$0 &lt; b$</td>
<td>$d &gt; b$</td>
<td></td>
<td></td>
<td>NO</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>$0 &lt; a$</td>
<td>0</td>
<td>$0 &lt; a$</td>
<td></td>
<td>YES</td>
</tr>
</tbody>
</table>
5.2 Formulating the Anti-Leak Rule using Extra Constraints

In order to formulate the anti-leak rule we introduced two extra constraints. Constraint (5-7) states that for each demand node, the total backorder and unmet demand should not be more than its demand. Constraint (5-8) states that total number of cars of pool \( k \) carried back from node \( n \) (associated to a specific customer in a specific day) to fulfill the backorder of previous days of the same customer cannot be more than the number of cars of the same pool that has entered \( n \) directly from the time-space flow network.

\[
\sum_{k \in K_n} \sum_{d \in D_{n}^{in}} \mu_{n}^{k} t_{d}^{k} + y_{n} \leq d_{n} \quad \forall n \in N_{K}, \forall K \subseteq \Psi \tag{5-7}
\]

\[
\sum_{d \in D_{n}^{out}} \mu_{n}^{k} t_{d}^{k} \leq x_{n}^{k} \quad \forall n \in N_{K}, \forall K \subseteq \Psi, k \in K_{n} \tag{5-8}
\]

We were hoping that these two constraints will guarantee the anti-leak rule for the delay arcs. However numerical results showed that these constraints are not sufficient to guarantee this rule. Furthermore it turned out that (5-7) is redundant in presence of (5-3) and (5-8). Instead of insisting on introducing new set of constraints to formulate the anti-leak condition, we decided to change our decision variables in such a way that this condition is satisfied. Developing such constraint set however could be a good theoretical work and is an area open to the researchers.

5.3 Formulating the Anti-Leak Rule with Redefining the Decision Variables

In order to guarantee anti-leak condition among delay arcs, as illustrated in Figure 5-3, we split the flow of cars arriving at demand node \( n \), i.e. \( x_{n}^{k} \), to two separate variables: one variable to represent the part of this flow that fulfills its own demand, and will liberally...
note it by $x^k_n$, and another variable to represent the part of the flow that will cover the 
backorder of the same customer from previous days, and will denote it by $x'_n$, where $x'_n = 
\sum_{d \in D^\text{out}_n} t^k_d$.

$$\sum_{d \in D^\text{out}_n} \mu^k_n t^k_d = x^k_n \quad \text{and} \quad y_n \quad \sum_{d \in D^\text{in}_n} \mu^k_n t^k_d$$

Figure 5-3 Dealing with delay arc flow using a new variable

This change affects objective function (5-1), node balance constraint (5-2) and 
demand node balance constraint (5-3) and transforms them to (5-9), (5-10), and (5-11), 
respectively.

$$\text{Min} \ \sum_{b \in B} f_b z_b + \sum_{b \in B} \sum_{k \in K} c^k_b t^k_b + \sum_{d \in D} \sum_{k \in K} c^k_d t^k_d + \sum_{i \in I} \sum_{k \in K} c^k_i t^k_i + \sum_{n \in N} \sum_{k \in K} c^k_n y_n + \sum_{n \in N} \sum_{k \in K} c'_n y'_n + \sum_{n \in N} \sum_{k \in K} c^k_n (x^k_n + x'_n)$$

$$\sum_{b \in B^\text{out}_m} t^k_b + \sum_{n \in N} \text{where } k \in K \left( x^k_n + x'_n \right)^m_k + \forall m \in M, \forall k \in K, \forall K \subseteq \Psi$$

$$t^k_{i=l_m} + l^k_m - \sum_{b \in B^\text{in}_m} t^k_b - t^k_{i=l_m} = s^k_m$$
5.3 Formulating the Anti-Leak Rule with Redefining the Decision Variables

\[
\sum_{k \in K_n} (x_n^k + x_n'^k + \sum_{d \in D_n^\text{in}} \mu_n^{k,k_d}) + y_n - \forall n \in N_K, \forall K \subseteq \Psi
\]

\[
\sum_{k \in K_n} \sum_{d \in D_n^\text{out}} \mu_n^{k,k_d} = d_n
\]

and adds following two constraints to the model as well.

\[
\sum_{d \in D_n^\text{out}} t_d^k = x_n'^k \quad \forall n \in N_K, \forall K \subseteq \Psi, k \in K_n
\]

\[
x_n'^k \geq 0 \text{ and integer} \quad \forall n \in N_K, \forall K \subseteq \Psi, k \in K_n
\]

The arc-based formulation of the car distribution problem is as follows:

\[
\text{Min } \sum_{b \in B} f_b z_b + \sum_{b \in B} \sum_{k \in K, K \subseteq \Psi} c_b^k t_b^k + \sum_{d \in D} \sum_{k \in K, K \subseteq \Psi} c_d^k t_d^k +
\]

\[
\sum_{i \in I} \sum_{k \in K, K \subseteq \Psi} c_i^k t_i^k + \sum_{n \in N_K, K \subseteq \Psi} c_n y_n + \sum_{n \in N_K, K \subseteq \Psi} c_n' y_n'
\]

\[
\sum_{n \in N_K, K \subseteq \Psi} \sum_{k \in K_n} (x_n^k + x_n'^k)
\]

\[
\sum_{b \in B_m^\text{out}} t_b^k + \sum_{m \in M} \text{where } k \in K_n (x_n^k + x_n'^k) +
\]

\[
t_i^k = \begin{array}{l}
t_b^k + t_m - \sum_{b \in B_m^\text{in}} t_b^k - t_i^k \quad \forall m \in M, \forall k \in K, k \subseteq \Psi
\end{array}
\]

\[
\sum_{k \in K_n} (x_n^k + x_n'^k + \sum_{d \in D_n^\text{in}} \mu_n^{k,k_d}) + y_n - \forall n \in N_K, \forall K \subseteq \Psi
\]

\[
\sum_{k \in K_n} \sum_{d \in D_n^\text{out}} \mu_n^{k,k_d} = d_n
\]

\[
y_n' \geq y_n - y_n^{\text{max}} \quad \forall n \in N_K, \forall K \subseteq \Psi
\]

\[
\sum_{d \in D_n^\text{out}} t_d^k = x_n'^k \quad \forall n \in N_K, \forall K \subseteq \Psi, k \in K_n
\]

\[
\sum_{b \in B_g} \sum_{k \in K, K \subseteq \Psi} \omega_k^k t_b^k \leq u_g \quad \forall g \in G
\]

\[
\sum_{k \in K, K \subseteq \Psi} t_b^k \leq v_b z_b \quad \forall b \in B
\]
5.3 Formulating the Anti-Leak Rule with Redefining the Decision Variables

\[ t_j^k \geq 0 \text{ and integer} \quad \forall j \in E = B | I | D, \quad \forall k \in (5-21) \]
\[ K, K \subseteq \Psi \]
\[ x_n^k, x_n' \geq 0 \text{ and integer} \quad \forall n \in N_K, \forall K \subseteq \Psi, k \in K_n \quad (5-22) \]
\[ y_n, y_n' \geq 0 \quad \forall n \in N_K, \forall K \subseteq \Psi \quad (5-23) \]
\[ z_b \in \{0,1\} \quad \forall b \in B \quad (5-24) \]

Furthermore we can see, easily, that constraints (5-11) and (5-12) are only defining the unmet demand \( y_n \) and backordered demand \( x_n' \), respectively. In other words we can simultaneously decrease the number of constraints and variables by replacing \( y_n \) by its equivalent term \( d_n - \sum_{k \in K_n} (\mu_n^k x_n^k + \sum_{d \in D_n^k} \mu_n^k t_d^k) \) and by replacing \( x_n' \) by \( \sum_{d \in D_n^{out}} t_d^k \).

The non-negativity and integrality constraint (5-13) is automatically satisfied since \( t_d^k \geq 0 \text{ and integer} \ \forall d \in D, k \in K, K \subseteq \Psi \); however \( y_n \geq 0 \) is not guaranteed and should be added explicitly as a constraint, i.e. \( \sum_{k \in K_n} (\mu_n^k x_n^k + \sum_{d \in D_n^k} \mu_n^k t_d^k) \leq d_n \).

Finally the \( \sum_{d \in D} \sum_{k \in K \subseteq \Psi} c_d^k t_d^k \) component in the objective function can be replaced by its equivalent term \( \sum_{n \in N_K, K \subseteq \Psi} \sum_{d \in D_n^{in}} \sum_{k \in K_n} c_d^k t_d^k \). The arc-based formulation of the multicommodity car distribution problem, after aggregating the coefficients, is as follows:

Allocation-Routing (AR)
5.3 Formulating the Anti-Leak Rule with Redefining the Decision Variables

\[ \text{Min } \sum_{b \in B} f_b z_b + \sum_{b \in B} \sum_{k \in K \subseteq \Psi} c_b^k t_b^k + \sum_{n \in N \subseteq \Psi} \sum_{k \in K} c_n (c_n^k - c_n^k \mu_n^k) x_n^k + c_n^k \sum_{d \in D_{\text{out}}} t_d^k + \sum_{d \in D_{\text{in}}} (c_d^k - c_n^k \mu_n^k) t_d^k \]

\[ \sum_{i \in I} \sum_{k \in K \subseteq \Psi} c_i^k t_i^k + \sum_{n \in N \subseteq \Psi} c_n y_n^f + \sum_{n \in N \subseteq \Psi} c_n d_n \]

Subject to

\[ \sum_{b \in B_{\text{out}}} t_b^k + \sum_{n \in N_m \text{ where } k \in K_n} (x_n^k + c_n^k \sum_{d \in D_{\text{out}}} t_d^k + c_n^k \sum_{d \in D_{\text{in}}} \mu_n^k t_d^k - l_m^k - \sum_{b \in B_{\text{in}}} t_b^k - t_i^k) = s_m^k \]

\[ y_n^f + \sum_{k \in K_n} (\mu_n^k x_n^k + c_n^k \sum_{d \in D_{\text{in}}} \mu_n^k t_d^k) \geq d_n - \]

\[ y_n^{\text{max}} \]

\[ \sum_{k \in K_n} (\mu_n^k x_n^k + c_n^k \sum_{d \in D_{\text{in}}} \mu_n^k t_d^k) \leq d_n \]

\[ \sum_{b \in B_g} \sum_{k \in K \subseteq \Psi} \omega^k t_b^k \leq u_g \]

\[ \sum_{k \in K \subseteq \Psi} t_b^k \leq v_b z_b \]

\[ t_j^k \geq 0 \text{ and integer} \]

\[ x_n^k \geq 0 \text{ and integer} \]

\[ y_n^f \geq 0 \]

\[ z_b \in \{0, 1\} \]
Chapter 6

Numerical Results

In this Chapter we evaluate the validity of the models presented in this Dissertation as well as the efficiency of the solution methods developed to solve them. The car distribution models are implemented in Java and CPLEX solver is called using Concert Technology. We could have used CPLEX Optimization Programming Language (OPL) instead, but the Java programming provided us the opportunity to have trip planning, pseudo path generation and car distribution models, as well as the random case example generator all in one place, without the need to switch between different software. Furthermore, integrating a java program into the ERP system, compared to a stand-alone software, is easier for the railroad IT departments since Java, compared to OPL, is more popular in the industry. Before proceeding we should define some keywords that we will use in this chapter extensively.

Factors set A: is a set of parameters that are used to generate the input sets of the models (e.g. factor set A = (networkName, numCarTypes, numDemandProfiles, numSupplyNodes, capacityFactor) ).
5.3 Formulating the Anti-Leak Rule with Redefining the Decision Variables

**Problem size B of factors set A:** a problem size is an instance of factors set (e.g. problem size B= (Small, 2, 2, 3, 1) means a problem which is defined on top of a network called Small, has 2 car types, 2 demand profiles and 3 supply nodes and the network capacities are equal to the ones in the Small network (multiplied by 1 )).

**Case example/ instance C (of size B):** a data set that contains the coefficients of the car distribution model or the information that can be used to generate those coefficients. (e.g. Case examples C1 = (Small, {carType1, carType2}, {Profile1, Profile2}, {node1, node2, node5}, 1) and C2 = (Small, {carType1, carType2}, {Profile3, Profile8}, {node3, node5, node8}, 1) both are from the problem size of B = (Small, 2, 2, 3, 1) of Factor set A = (networkName, numCarTypes, numDemandProfiles, numSupplyNodes, capacityFactor))

**Scenario D (on Case example C):** a relation that takes a case example C, and an element of Factor set A as input and generates multiple case scenarios. (e.g. \( f(C2, \text{capacityFactor}) \) = { C21, C22 } where the only difference between C21 = (Small, {carType1, carType2}, {Profile3, Profile8}, {node3, node5, node8}, 2) and C22 = (Small, {carType1, carType2}, {Profile3, Profile8}, {node3, node5, node8}, 6) is that the capacities of blocks and trains in C22 are triple of the capacities of the same blocks and trains in C21.

We continue this chapter with some techniques that we used to verify the accuracy of the computer programs and the validity of the models and solution techniques. Then we illustrate those models in a small numerical example and finally evaluate the performance of different solution methods and heuristics on large scale problems.
6.1 Verification and Validation

Car distribution planning is an operational decision frequently made by the railroad companies. The frequency of the decisions might change from one to several times per day. Besides the impact on the railroad costs and revenues, the car distribution planning decision affects multiple stakeholders inside and outside the industry. Each decision made at each time by the car distribution department affects their own future decisions because of the direct impact on the supply and demand. The work load of employees working at the classification yards is also affected by these decisions. The customer's operations are highly influenced by the railroad decisions and a plant might be temporally shut down as a result of car supply shortage. A poor car distribution model might develop unstable outputs and consequently result in the loss of customer goodwill. This is why the car distribution model needs to be verified and validated as part of its development process. Based on AIAA (1998), validation is defined as the process of determining the degree at which a model is an accurate representation of the real world from the perspective of the intended uses of the model. Verification is defined as the process of determining that a model implementation accurately represents the developer’s conceptual description of the model and the solution to the model.

Roughly speaking, validation can be expressed by the query "Are you building the right thing?" and verification by "Are you building it right?". We followed some steps to ensure that we have proposed the right models and solution methods and have implemented them correctly.
The conceptual model was presented in the INFORMS 2014 conference and was further discussed by the railroad experts to ensure that it represents the real world problem and hence is useful for the practitioners in the industry.

After validating the conceptual model, the mathematical model as well as the proposed solution methods were discussed and validated by the PhD Dissertation Committee members.

A computer program was developed in Java; and its accuracy was verified using debugging techniques, observational verification and analytical verification. For debugging we made sure the computer program is free of any compile or run time error. However debugging is not enough to make sure the computer program represents the mathematical model. In this research we used two sets of techniques to verify that the computer program reflects the mathematical model. First for a number of small size problems we explicitly compared the mathematical model generated by the computer program with the mathematical model written based on the proposed formulation. We called this technique an observational verification. Observational verification was powerful enough to enable us to fix some basic errors like typo of “+” instead of “-“ or “=“ instead of “>=”, etc. However, because of the small and simple nature of those case examples, it is not sufficient to completely verify the accuracy of the program. Writing the mathematical model for the larger size and more complex case examples to compare with the exported model from the computer program is not practical, because of huge number of constraints and variables. In order to verify the accuracy of models for the larger size problems we used analytical verification techniques. Besides the popular techniques such as extreme condition tests, etc. we developed another analytical technique as a set of rules
that must always hold based on the nature of the models presented in this research. The models are abbreviated as follows:

PA is the path based Pure Allocation model, AT is the path based Allocation Timing model, NC is the Noncapacitated path based model, and AR is the arc based Allocation-Routing model.

**Property 6-1** Superset-Subset Relations: The objective values associated with the optimal solutions of different models presented in this Dissertation, on a given case example (set of inputs), satisfy the following relations:

a- AT <= PA  
b- AR <= AT, PA  
c- NC <= PA, AT, AR  
d- AR (after relaxing capacities) = NC

where the model abbreviations represent the respective model objective values.

**Prof:** Property 6-1a always holds since the feasible region of AT is a superset of feasible region of PA. This comes from this fact that the set of pseudo paths of a trip plan generated by Pseudo Path Generator presented in Section 4.3.1 always contains the trip plan itself. For proving Property 6-1b, it is sufficient to prove that AR <= AT since AT <= PA holds based on Property 6-1a. AR <= AT always holds since the allocation and routing decisions are integrated in one model in AR while AT only deals with the allocation decision while routing decision is made by Trip Planner. The optimal solutions of integrated models are known to be better than the aggregated models.

Next we prove Property 6-1c in three steps: i) NC <= PA always holds since the path connecting a given supply-demand pair in NC is the same as the path connecting the same
pair in PA with the only difference that this path is capacitated in PA. ii) NC \leq AT also holds true since although a given supply-order pair might be connected with more than one paths in AT, NC uses the shortest path among them without any capacity limits. iii) NC \leq AR also always holds. Although AR integrates the car routing and allocation decisions and based on Property 6-1b always overperforms PA and AT, the comparison with NC model is different. If we convert the arc flow decisions made by AR to path flows from supply to demand, the best path that can be offered by AR, for a given supply-demand pair is the shortest path between them. However in the presence of capacities, in the optimal solution of AR, a supply-demand pair might not be necessarily connected using this best path. On the other hand NC always uses the shortest path for each given supply-demand pair. Hence from i, ii, and iii we conclude that Property 6-1c always holds.

Property 6-1d always holds since if capacity limits are relaxed, for a given supply-demand pair, the integrated car routing and allocation model AR, after transforming the arc flows to path flows, will pick the shortest path to connect them. Hence the arc based formulation of AR model after relaxing capacity limits is equivalent to the path based noncapacitated model NC.

Table 6-1 introduces four networks that are used in this chapter to verify and validate the models and their solution procedures. The first static network has only 5 yards, 6 blocks, 3 trains and 9 train segments, resulting in a time-space network of 45 nodes, 18 blocks, 9 train and 27 train segments in a 3-day time horizon. Because of its small size we called this network “Super Tiny Network”. The Super Tiny Network was used for observational verification.
The second network is called “Tiny Network” and has the same yards, blocks and trains but in an 18-day time horizon. Tiny Network is too large to be used for observational verification and was used for fast analytical verification only. All models were run for hundreds of times on randomly generated case examples on top of the Tiny Network. Anytime any of the rules in Property 6-1 were violated the execution was terminated and the cause of violation was investigated. For example, the output of one model was fed to another model by establishing those variables to the values in the other model and if the new formulation was infeasible the causes of infeasibility were investigated. CPLEX solver in interactive mode was extremely helpful in finding the conflicts among the constraints.

The size of Tiny Network was big enough to capture some of the issues in the computer programs and small enough to be able to generate and run a case example in a few seconds. However the final verification was performed on a network with larger size called “Medium Network”. This network has 78 yards, 239 blocks, 95 trains and 323 train segments. The datasets for the physical and blocking networks were acquired from the 2011 - RAS Problem Solving Competition in train design (https://www.informs.org/Community/RAS/Problem-Solving-Competition/2011-RAS-Problem-Solving-Competition). The train design data including the train routing and train makeup plans were generously provided by one of the award winners, Team OR at UNIMI. Execution of all models for more than 10000 times on case examples of different scales generated on top of the Medium Network, verified the accuracy of all formulations based on the analytical verification techniques. We also used the same network for the illustrative example that will be presented in Section 6.2.
6.1 Verification and Validation

After verifying that the computer program correctly represents the model, the next step was to validate the performance of different models, especially our path based Allocation-Timing model, by benchmarking them against each other and also against the noncapacitated model as the current practice of many railway industries. Case examples used for validation purposes were generated on a bigger network called “Large Network” with 440 yards, 1369 blocks, 624 trains and 2167 train segments which in an 18-day time horizon results in 34941 nodes, 24642 blocks, 11232 trains and 39005 train segments. The Large Network was created by duplicating and manipulating the Medium Network for 4 times and another medium size network available at RAS 2011 competition for 2 times and connecting them in such a way that all components of physical, block and train networks stay connected.

As part of the validation phase, four performance measures were defined: a) objective value and b) fulfillment ratio as measures of efficiency of the models, and c) run time and d) relative optimality gap, as measures of efficiency of the solution methods. For each model, the objective value is the value of the objective function in the optimal solution, or the best incumbent if the optimum is not met; the fulfillment ratio is the ratio of satisfied demand to the entire demand for all car types and all customers; run time is the CPU time for solving the problem; and the relative optimality gap is a gauge to measure the gap between the current incumbent and tightest known bound and is identical to EpGap in CPLEX solver which is calculated by (6-1):

\[
EpGap = \frac{|f(\text{best node}) - f(\text{best integer solution})|}{1e^{-10} + |f(\text{best integer solution})|} = \frac{|LB - UB|}{1e^{-10} + |UB|}
\]  

(6-1)

The results of the validation phase are presented in Section 6.3.
### Table 6-1 Networks used for verification and validation of the models and the solution procedures

<table>
<thead>
<tr>
<th>Network</th>
<th>Static Network (Y, B, T, TS)</th>
<th>Associated Figure</th>
<th>Time-Space Network (N, B, T, TS)</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Super Tiny</td>
<td>(5, 6, 3, 9)</td>
<td>Figure 6-1</td>
<td>(45, 18, 9, 27)</td>
<td>Observational Verification</td>
</tr>
<tr>
<td>Tiny</td>
<td>(5, 6, 3, 9)</td>
<td>Figure 6-1</td>
<td>(298, 107, 54, 162)</td>
<td>Fast Analytical Verification</td>
</tr>
<tr>
<td>Medium (RAS 11)</td>
<td>(78,239, 95, 323)</td>
<td>Figure 6-2</td>
<td>(5981, 4012, 1710, 5814)</td>
<td>Analytical Verification &amp; Numerical Example</td>
</tr>
<tr>
<td>Large (manipulated RAS 11)</td>
<td>(440,1369,624, 2167)</td>
<td>Figure 6-3</td>
<td>(34941,24642,11232, 39005)</td>
<td>Validation</td>
</tr>
</tbody>
</table>

N: #Nodes, Y: #Yards, B: #Blocks, T: #Trains, TS: #Train Segments

---

**Figure 6-1** Static networks associated with the Tiny and Supper Tiny Networks: a) physical rail network, b) train network, c) blocking plan and d) block to train assignment
Figure 6-2 Static networks associated with the Medium Network: a) physical rail network, b) train network and c) blocking plan
Figure 6-3 Static networks associated with the Large Network: a) Physical rail network, b) train network and c) blocking plan
6.2 Illustrative Example

To illustrate the different modes of the model execution and gain insights into the practical aspects of the empty railcar distribution problem we developed the following numerical example.

6.2.1 Input data

Network- The network used for illustrative example is the Medium Network (see Section 6.1 and Table 6-1 for the description of different networks used in this chapter). The Large Network in Figure 6-2 includes a railroad network, Figure 6-2(a), as well as its corresponding service network, i.e. train routes, Figure 6-2(b), and blocking plan, Figure 6-2(c). The figures presented in this example are created in Gephi platform (http://gephi.github.io/), and their main goal is to present a big picture of the numerical example, while the details can be accessed in the supplementary materials. Figure 6-2(a) shows the railroad network with 93 rail yards as nodes and 135 physical tracks as arcs of the network. 95 trains scheduled on this network are shown in Figure 6-2(b). Each train might consist of multiple train segments, and as a result some physical arcs might bear more than one train. The train network in total has 323 train segments. The thickness of the edges in train network is proportional to the number of trains passing through that arc. Cars are carried by blocks. Figure 6-2(c) shows the blocking network where the diameter of the nodes represents the number of blocks entering and exiting the associated yard. From a business perspective bigger nodes belong to the hubs where more operations take place at the yard. The blocking network has 239 blocks built in 78 yards. The other 15 yards might be crew change points for the trains or local yards, where the car routing decisions are
made locally and therefore are out of the scope of the optimization model. As it can be seen in Figure 6-2(b) and Figure 6-2(c), the service network here is static and has no time dimension. The car distribution problem addressed in this research, however, is dynamic. This will require a time-space service network and prior to that train timetables. The post processing of static row data was implemented in an Excel Macro Software that is provided as a supplementary material. The main function of this software is to generate train timetables; to derive the resource-constraint blocking network from block to train assignment plan; to transition the static resource-constraint blocking network to a time-space network, by replicating the same blocks and trains and their schedules in future days; and finally to make these datasets format compatible with the trip planning and car distribution models input requirements.

Fleet- The Car Distribution Department is planning to apply the optimization model as prototype in only two car types: car type 1 that comprises pool 1, 2, and 3; and car type 2 that comprises pool 4, 5, and 6. For the sake of simplicity in interpreting the results, in this example we assume all cars to be of the same length and weight (i.e. $\omega^k = \tau^k = 1$). This implies that the capacity of blocks and trains can be expressed in units of cars, rather than in feet for block length and in tons for train tonnage. As a result, each assignment will occupy one unit of the blocks and trains carrying the car. Furthermore, we assume that if two pools are substitutable for a customer order, the substitution rate will be always 1.

Demand- The Car Distribution Department has received a total of 284 car orders in two car types, consisted of 131 cars of car type 1 and 153 of car type 2, from four customers. Table 6-2 contains the list of customer orders per day, for a one-week time horizon, along with some other information regarding the customer itself and the demand.
6.2 Illustrative Example

For instance “Customer: 1, Yard:1954, K_n:{1,3}, Pref:(2,1)}” in this table represents customer 1, located at yard 1954, requesting cars from pools 1 and 3, both from car type 1 where pool 3 is preferred (preference of 1) over pool 1 (preference 2) and other pool in this car type, i.e. pool 2, is not accepted. Other data associated with this customer are daily demand as well as the unit shortage penalty for each day and the soft bounds on the unmet demand level. Violating the soft bound is permitted but with a cost of doubling the penalty cost.

Supply- Currently, there are only 145 cars available for assignments, including 71 cars of car type 1 at yards 4227, 8765, and 105888 and 74 cars of car type 2 located at yards 4227, 2474, and 8429. The details on number of cars of each pool available at these yards are given in Table 6-3.

Table 6-2 Customers’ demand for a one-week time horizon

<table>
<thead>
<tr>
<th>Yard</th>
<th>Pool (k)</th>
<th>Car Type 1 (131 cars)</th>
<th>Car Type 2 (153 cars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>Priority</td>
<td>Shortage penalty (c_n, c'_n)</td>
<td>Soft bound (y_n_max)</td>
</tr>
<tr>
<td>Yard</td>
<td>Pool (k)</td>
<td>Car Type 1 (131 cars)</td>
<td>Car Type 2 (153 cars)</td>
</tr>
<tr>
<td>Day</td>
<td>Priority</td>
<td>Shortage penalty (c_n, c'_n)</td>
<td>Soft bound (y_n_max)</td>
</tr>
</tbody>
</table>
Table 6-3 Available supply of different pools in each yard

<table>
<thead>
<tr>
<th>Yard</th>
<th>Pool (k)</th>
<th>Supply (s^k_m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4227</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>4227</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>8765</td>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>8765</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>10588</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>10588</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Car type 1</td>
<td>71</td>
</tr>
<tr>
<td>4227</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>4227</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>2474</td>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>2474</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>8429</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>8429</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Car type 2</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>Total Supply</td>
<td>145</td>
</tr>
</tbody>
</table>

### 6.2.2 Model Results, Interpretation, and Remarks

The goal of the model is to fulfill, as much as possible, the demand of 284 cars for the one-week time horizon using a total supply of 145 cars while satisfying the network capacities and the order time windows (order expiration dates) and considering customer priorities and pool preferences. The model was run in different modes yielding, as expected, different results. Beginning with the Pure Allocation model, as detailed in Figure 6-4(a) only 72 cars are assigned to the orders. This result might not be acceptable to the Car Distribution Department but was not unexpected. As mentioned in Section 4.2, the pure Allocation model takes the Trip Planner output as the only feasible path that connects a given supply-demand pair and since this single path is capacitated, the output of PA is not satisfactory. The Allocation-Timing model, on the other hand, takes advantage of the Trip Planner suggested path to build pseudo paths, by replicating the same physical path in different days, and as a result has more channels to connect a given supply-demand pair. As it is illustrated in detail in Figure 6-5(a) the Allocation-Timing model assigns 119 cars to orders, which is clearly a big improvement over the pure Allocation model, but still
leaves 26 cars unassigned while an extra demand of 165 cars is still not fulfilled. As the model was designed to be proposed to the railroad industry, we could not accept the optimizer’s output at face value, without a good justification of the unmatched supply and demand. It was necessary to perform further study on the causes of this gap. Potential causes were identified as: the capacity constraints, the low unmet demand penalty, or simply a mismatch between the leftover supply and demand.

The capacity constraint effect was investigated first. In order to do so, we relaxed all capacity constraints, for both blocks and trains, in Pure Allocation model and ran the resulted noncapacitated (NC) model again. As the block capacity constraints were relaxed, the block setup costs were also eliminated from the cost function, changing it to a minimization of assignment and unmet demand penalty costs only. The noncapacitated model fulfilled 145 car orders, practically the most possible order fulfillment ratio using 145 car supplies (Figure 6-6(a)). However as mentioned in Section 4.4 the output of NC cannot be compared with the capacitated models’ unless its optimistic behavior in a capacitated network is evaluated using NCActual model. The results of NCActual in Figure 6-7(a) shows that in presence of network capacities, only 101 cars out of 145 cars planned by NC will reach the customer facilities during the order time windows. Figure 6-8 shows the detail of how network capacities affect the actual behavior of the noncapacitated model’s output.

The assignments of all 145 available car supply by NC means by itself that there is no mismatch between supply and demand. However it does not mean that the only cause of the 26 car gap in the output of AT model is the capacities. The next step is to investigate the effect of unmet demand penalty costs on the optimal solution. Currently, the unmet
demand unit costs (ranged between 757 and 1446 and averaged at 1043 for days that customers placed orders), compared to the assignment unit costs, (ranged between 50 and 1461 and averaged at 1135), are not high enough to enforce the assignment over shortage for high-cost supply-order assignments. In order to analyze the effect of penalty costs, we ran the PA and AT models again, but this time using higher penalty costs, where the least penalty cost was larger than the biggest assignment cost. The results changed considerably. In the new optimal solution of AT, 129 cars were assigned to the orders, lowering the gap from 26 to 16. Further increase in shortage penalties did not improve the fulfillment ratio, confirming that the only cause of the extra 19 car gap between the maximum possible fulfilments (145 cars) and the AT output (129 cars) is the capacities of the network. The details on the assignments suggested by all path based models using high penalty costs are presented in Figure 6-4(b) to Figure 6-7(b).

Among the three above mentioned causes, the penalty costs seem to be of more importance. The mismatch (not a cause here) comes from the nature of supply and demand and we are almost empty handed against it. The capacity constraints are enforced by the network and should not be relaxed especially after we saw that the optimistic behavior of NC in capacitated network NCActual (assigning 101 cars in both low or high shortage cost cases) is worse than the output of AT (assigning 119 cars in low shortage cost case and 129 cars in case of high shortage costs). The unmet demand penalties however are part of the model parameters and we have more control over them.

While fulfilling 129 orders, using the high penalty costs in the Allocation-Timing model, compared to fulfilling only 119 orders using the initial penalty costs, might seem more interesting to the Car Distribution Department and customers; it is important to assure
that these assignments are also profitable. Railroad carriers, in general, charge their customers based on the number of miles and days it takes to deliver the loaded cars from customer facility to the requested destination and not by the number of empty cars they sent to them nor the number of days and miles it takes the empty cars to reach the customer facility (similar to taxi drivers!). While customers will also benefit from a better fleet utilization, minimizing the empty miles is more in the railroad interest compared to the customers’ interest. As an extreme case, consider that there is only one empty car available in Albany, New York, and two customers requested it: a customer located at Jacksonville, Florida, and a local customer in Upstate New York. Is it profitable (with respect to short term or long term considerations) to ship the car all the way to the South to satisfy the customer demand? The answer depends on many factors, among them how important and how profitable the customer in Florida is for the railroad carrier (Please see Section 3.2.5 for the details on cost function).
6.2 Illustrative Example

<table>
<thead>
<tr>
<th>Day</th>
<th>Priority</th>
<th>Yard</th>
<th>Pool (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2000</td>
<td>1.4140</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2000</td>
<td>1.4100</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2000</td>
<td>1.4090</td>
</tr>
</tbody>
</table>

**Car Type 1 (131 cars)**

Customer: 1, Yard: 1954, $K_n = \{1, 3\}$, Pref: (2, 1)

Customer: 2, Yard: 1339, $K_n = \{1, 3\}$, Pref: (2, 1)

Customer: 3, Yard: 6872, $K_n = \{1, 3\}$, Pref: (2, 1)

**Car Type 2 (153 cars)**

Customer: 4, Yard: 2040, $K_n = \{1, 3\}$, Pref: (2, 1)

<table>
<thead>
<tr>
<th>Shortage penalty ($c_n, c'_n$)</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_n, c'_n)</td>
<td>Priority</td>
</tr>
</tbody>
</table>

**Demand (284 cars)**

<table>
<thead>
<tr>
<th>Yard</th>
<th>Pool (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2000</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
</tr>
<tr>
<td>3</td>
<td>2000</td>
</tr>
</tbody>
</table>

**Car Type 1**

Customer: 2, Yard: 1339, $K_n = \{1, 3\}$, Pref: (2, 1)

**Car Type 2**

Customer: 4, Yard: 2040, $K_n = \{1, 3\}$, Pref: (2, 1)

<table>
<thead>
<tr>
<th>Demand ($d_n$)</th>
<th>Satisfied</th>
<th>Shortage ($y_n$)</th>
<th>Soft bound ($y_{n, max}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Pure Allocation Model (PA)**

<table>
<thead>
<tr>
<th>Supply</th>
<th>Car Type 1</th>
<th>Car Type 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>4227</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4227</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>8765</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

**Demand (284 cars)**

<table>
<thead>
<tr>
<th>Yard</th>
<th>Pool (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2000</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
</tr>
<tr>
<td>3</td>
<td>2000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demand ($d_n$)</th>
<th>Satisfied</th>
<th>Shortage ($y_n$)</th>
<th>Soft bound ($y_{n, max}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Pure Allocation Model (PA)**

<table>
<thead>
<tr>
<th>Supply</th>
<th>Car Type 1</th>
<th>Car Type 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>4227</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4227</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>8765</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

**Demand (284 cars)**

<table>
<thead>
<tr>
<th>Yard</th>
<th>Pool (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2000</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
</tr>
<tr>
<td>3</td>
<td>2000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demand ($d_n$)</th>
<th>Satisfied</th>
<th>Shortage ($y_n$)</th>
<th>Soft bound ($y_{n, max}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 6-4 Detailed supply-order assignment suggested by PA with a) low shortage and b) high shortage penalties
### 6.2 Illustrative Example

#### (a)

<table>
<thead>
<tr>
<th>Car Type 1 (131 cars)</th>
<th>Car Type 2 (153 cars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer: 1, Yard: 1954, K_n={1,3}, Pref: (2,1)</td>
<td>Customer: 2, Yard: 1339, K_n={1,3}, Pref: (1,1)</td>
</tr>
<tr>
<td>Customer: 3, Yard: 6872, K_n={4,6}, Pref: (2,1)</td>
<td>Customer: 4, Yard: 2040, K_n={4,6}, Pref: (1,1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priority</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

| Shortage penalty | 1/10 |

<table>
<thead>
<tr>
<th>Allocation-Timing Model (AT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply (s_n-k)</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>2169</td>
</tr>
<tr>
<td>2169</td>
</tr>
<tr>
<td>9979</td>
</tr>
<tr>
<td>9979</td>
</tr>
<tr>
<td>2019</td>
</tr>
<tr>
<td>2019</td>
</tr>
<tr>
<td>1290</td>
</tr>
<tr>
<td>1290</td>
</tr>
</tbody>
</table>

#### (b)

<table>
<thead>
<tr>
<th>DEMAND (284 cars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pool (k)</td>
</tr>
<tr>
<td>Car Type 1 (131 cars)</td>
</tr>
<tr>
<td>Customer: 1, Yard: 1954, K_n={1,3}, Pref: (2,1)</td>
</tr>
<tr>
<td>Customer: 3, Yard: 6872, K_n={4,6}, Pref: (2,1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priority</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

| Shortage penalty | 1/10 |

<table>
<thead>
<tr>
<th>Allocation-Timing Model (AT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply (s_n-k)</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
</tbody>
</table>

Figure 6-5 Detailed supply-order assignment suggested by AT with a) low shortage and b) high shortage penalties

124
6.2 Illustrative Example

(a) Noncapacitated Model (NC)

<table>
<thead>
<tr>
<th>Car Type 1 (131 cars)</th>
<th>Car Type 2 (153 cars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>Priority</td>
</tr>
<tr>
<td>1 2 3 4 5 6 7 1 2 3 4 5 6 7</td>
<td>1 2 3 4 5 6 7 1 2 3 4 5 6 7</td>
</tr>
<tr>
<td>Priority</td>
<td>Day</td>
</tr>
<tr>
<td>5 5</td>
<td>20 20</td>
</tr>
<tr>
<td>24 24</td>
<td>4 4</td>
</tr>
<tr>
<td>3 3</td>
<td>15 15</td>
</tr>
<tr>
<td>Supply (s-m-k)</td>
<td>Assigned (s-m-k)</td>
</tr>
<tr>
<td>Demand (d_n)</td>
<td>Satisfied (y_n)</td>
</tr>
<tr>
<td>0 0 3 8 15 21 26 0 0 2 6 12 17 21</td>
<td>0 0 3 8 15 21 26 0 0 2 6 12 17 21</td>
</tr>
<tr>
<td>Satisfied (y_n)</td>
<td>Shortage (y_n_max)</td>
</tr>
<tr>
<td>4 3 3 8 15 21 26 0 0 2 6 12 17 21</td>
<td>4 3 3 8 15 21 26 0 0 2 6 12 17 21</td>
</tr>
<tr>
<td>Shortage (y_n_max)</td>
<td>Soft bound (y_n_max)</td>
</tr>
<tr>
<td>5 5</td>
<td>15 15</td>
</tr>
<tr>
<td>20 20</td>
<td>4 4</td>
</tr>
<tr>
<td>24 24</td>
<td>3 3</td>
</tr>
<tr>
<td>15 15</td>
<td>15 15</td>
</tr>
<tr>
<td>20 20</td>
<td>4 4</td>
</tr>
<tr>
<td>24 24</td>
<td>3 3</td>
</tr>
<tr>
<td>15 15</td>
<td>15 15</td>
</tr>
<tr>
<td>20 20</td>
<td>4 4</td>
</tr>
<tr>
<td>24 24</td>
<td>3 3</td>
</tr>
<tr>
<td>15 15</td>
<td>15 15</td>
</tr>
<tr>
<td>Demand (d_n)</td>
<td>Satisfied (y_n)</td>
</tr>
<tr>
<td>0 0 3 8 15 21 26 0 0 2 6 12 17 21</td>
<td>0 0 3 8 15 21 26 0 0 2 6 12 17 21</td>
</tr>
<tr>
<td>Satisfied (y_n)</td>
<td>Shortage (y_n_max)</td>
</tr>
<tr>
<td>0 0 3 8 15 21 26 0 0 2 6 12 17 21</td>
<td>0 0 3 8 15 21 26 0 0 2 6 12 17 21</td>
</tr>
<tr>
<td>Shortage (y_n_max)</td>
<td>Soft bound (y_n_max)</td>
</tr>
<tr>
<td>0 0 3 8 15 21 26 0 0 2 6 12 17 21</td>
<td>0 0 3 8 15 21 26 0 0 2 6 12 17 21</td>
</tr>
</tbody>
</table>

Figure 6-6 Detailed supply-order assignment suggested by NC with a) low shortage and b) high shortage penalties
6.2 Illustrative Example

Figure 6-7 Detailed supply-order assignment suggested by NCActual with a) low shortage and b) high shortage penalties

126
6.2 Illustrative Example

Figure 6-8 Optimal assignment from noncapacitated model and its optimistic behavior
A high level comparison of the model performance measures in two scenarios of low or high shortage penalties is presented in Table 6-4. The huge difference between the optimal objective values of the two scenarios for each model in Table 6-4 is because of the high weight of shortage costs in the objective function. Hence the optimal cost of scenario 2, with less shortages comparing to Scenario 1 is considerably higher as a result of its higher shortage penalty costs! This will justify the high cost of 441968 compared to 278653 for the same number of order fulfillment of 145 in NC. Another point to mention is the difference between the objective value of AR and NC in Scenario 2 (respectively 459178 and 441968) with a similar fulfillment of 145 cars. Such a gap is expectable for two reasons: 1) AR has capacity limitations practically having a smaller feasible region compared to NC; and 2) NC does not consider block setup costs, resulting in a lower objective value.

<table>
<thead>
<tr>
<th>Model</th>
<th>Sc. 1 Medium penalty on shortage</th>
<th>Sc. 2 High penalty on shortage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure Allocation Model (PA)</td>
<td>72; 351546</td>
<td>72; 648629</td>
</tr>
<tr>
<td>Allocation-Timing Model (AT)</td>
<td>119; 309354</td>
<td>129; 503206</td>
</tr>
<tr>
<td>Allocation-Routing Model (AR)</td>
<td>138; 296584</td>
<td>145; 459178</td>
</tr>
<tr>
<td>Noncapacitated Model (NC)</td>
<td>145; 278653</td>
<td>145; 441968</td>
</tr>
<tr>
<td>NC on capacitated network (NCActual)</td>
<td>101; 331910</td>
<td>101; 578649</td>
</tr>
</tbody>
</table>

6.3 Performance of Models and Solution Procedures in Larger Scale Problems

The illustrative example presented in Section 6.2 was small enough to be solved using MIP solver of CPLEX software in about a second. However as the number of supply nodes and demand profiles increased, CPLEX started to show the signs of fatigue! The run
time for a problem with 20 supply nodes and 20 demand profiles was about 2 minutes and it increased to more than 5 minutes after scaling the size to 40 supply nodes and 40 demand profiles. The blocking network presented in Section 6.2, the Medium Network, has only 78 yards, and it is too small to be used for measuring the efficiency of the models on larger scale. In this section we first explain the black box of the procedure that we use to generate case examples. Then using case examples generated by this procedure, we perform some sensitivity analysis tests and finally study the average performance of the proposed heuristics in a variety of problem sizes.

Table 6-5 Different models, variable setup and algorithms to solve them

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Problem Name</th>
<th>Solution Procedure</th>
<th>Flow var. (x)</th>
<th>Block setup var. (z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path-Based</td>
<td>Pure Allocation (PA)</td>
<td>NC-IP (CPLEX MIP Solver)</td>
<td>Integer</td>
<td>Binary</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NCActual</td>
<td>Integer</td>
<td>Binary</td>
</tr>
<tr>
<td></td>
<td>Allocation-Timing (AT)</td>
<td>PA-IP (CPLEX MIP Solver)</td>
<td>Integer</td>
<td>Binary</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AT-IP (CPLEX MIP Solver)</td>
<td>Integer</td>
<td>Binary</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AT-LRH (Lagrangian Heuristic)</td>
<td>Integer</td>
<td>Binary</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AT-MBP (Mixed Binary Programming)</td>
<td>Cont+</td>
<td>Binary</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AT-MBPRound (Rounding of MBP)</td>
<td>Integer</td>
<td>Binary</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AT-IRRH1 (Iterative Relaxation and Rounding Heuristic based on MBP setup)</td>
<td>Integer</td>
<td>Binary</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AT-LPRelax (Linear Programming Relaxation)</td>
<td>Cont+</td>
<td>Cont+</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AT-LPRound (Linear Programming Rounding)</td>
<td>Integer</td>
<td>Binary</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AT-IRRH2 (Iterative Relaxation and Rounding Heuristic based on LP setup)</td>
<td>Integer</td>
<td>Binary</td>
</tr>
<tr>
<td>Arc-Based</td>
<td>Allocation-Routing (AR)</td>
<td>AR-IP</td>
<td>Integer</td>
<td>Binary</td>
</tr>
</tbody>
</table>
Before we proceed with the next section, we need to introduce some notation that we will use in this chapter referring to different models and algorithms. Table 6-5 contains such notation. For example, in the first row NC stands for the Non-Capacitated model. NC-IP refers to this model when it is solved using CPLEX built-in IP and MIP algorithms. The last two columns state that the flow variables are integer while block setup variables are binary (and none of them are relaxed).

6.3.1 Generating case examples

In this section, we first describe the input data components and then present the case example generation procedure.

6.3.1.1 Input

Network- The efficiency of the models and algorithms were validated in a bigger network, “Large Network”, with 440 yards, 1369 blocks, 624 trains and 2167 train segments which in an 18-day time horizon results in 34941 nodes, 24642 blocks, 11232 trains and 39005 train segments (please see Section 6.1 for the characteristics of different networks used for verification and validation purposes). The block and train capacities come from RAS 2011 datasets. We use RAS block and train capacities as original capacities. However, at any point of time, there are many loaded and empty cars already planned for movement which occupy part of network’s original capacity. The residual capacity of block $b$ is expressed as ratio of available capacity to its original capacity. Knowing that the majority of capacity of a block for day 1 is expected to be already booked by the empty and loaded cars planned before the current run of model, this ratio is assumed to be a monotonically increasing function of time (for example $\text{ratio}(t) = \min(100, (5 + 5 \times t))/100$). The residual capacities of trains are calculated in a similar way.
\( v_b = v_{b\text{original}} \times \text{ratio}(t) \) \hspace{1cm} (6-2)

\( u_g = u_{g\text{original}} \times \text{ratio}(t) \) \hspace{1cm} (6-3)

where \( t \) is the block/train departure day. This network will be used as input to generate case examples. In a case example some blocks might have block setup costs. If a block has a setup cost, which also means it is not yet constructed, its residual capacity should be equal to its original capacity, regardless of the day it is associated with.

\[ v_b = \begin{cases} v_{b\text{original}} \times \text{ratio}(t), & f_b = 0 \\ v_{b\text{original}}, & f_b > 0 \end{cases} \] \hspace{1cm} (6-4)

**Demand**- Demand is generated based on a Logistic function which is a monotonically increasing bounded function and is defined by (6-5) and is illustrated in Figure 6-9. In this function \( d(t) \) is customer demand on day \( t \), \( d^{\text{max}} \) is the maximum daily demand for this customer, and \( a \) and \( t_0 \) are the Logistic function parameters representing the steepness of the curve and the day associated with demand midpoint, respectively.

\[ d(t) = \frac{d^{\text{max}}}{1 + e^{-a(t-t_0)}} \] \hspace{1cm} (6-5)

The car distribution models presented in this dissertation, suggest supply to order assignment decisions in a two-week time horizon and give higher priority to the earlier days compared to later days, meaning that a customer’s demand for Monday has higher priority than the same customer’s demand for Tuesday of the same week. Hence a demand that was originally submitted to be fulfilled for today is most likely to be already (partially or fully) fulfilled/planned by the previous runs of the model, while a demand for 10 days from now has less chance to be fulfilled before the current run. This is why at each point of time the unfulfilled (modified) demand so far that should be considered as the demand
for the current run is expected to be close to zero for day 1 and close to its original demand for day 14 and this is exactly what a logistic demand function generates.

![Logistic function for creating daily demand](image)

Finally the algorithm to generate $n$ number of demand profiles (customers) along with their attributes such as eligible pools and preferences and daily demands is presented below:

**Algorithm for generating demand**

```
INPUT: n = numDemandProfiles; Demand parameters $(a, t_0, d_l, d_u)$; $C = setOfCarTypes$

Iterate for $n$ times:
  - Uniformly pick a yard
```
• Uniformly assign a car type from \( C \)
• Uniformly pick a set of pools within this car type
• Uniformly assign pool preferences from \( \{1,2,3\} \)
• Uniformly pick customer priority for day 1 (priority(Day 1) = \( \text{uni}\{1,2,3\} \))
• Priority (Day \( t \)) = priority (Day \( t-1 \)) / \( \sqrt{2} \)
• Calculate daily demand based on the logistic function given parameters \( (a,t_0,d_l,d_u) \)
  - \( d_{\text{max}} = \text{uni}(d_l,d_u) = \text{uni}(\text{maxDemand}_{\text{lowerBound}},\text{maxDemand}_{\text{upperBound}}) \)
  - \( d(t) = \frac{d_{\text{max}}}{1 + e^{-a(t-t_0)}} \)
  - Calculate the shortage penalties (see Section 3.2.5)

**Supply**- Total supply of cars for each car type is not necessarily equal to the total demand for that car type. It is often less, since we usually want to allocate the current supply to the demand for a two-week time horizon. Given the total demand of a particular car type and the ratio between supply and demand, the total supply of the car type can be easily calculated. Given that \( m \) is the number of yards that have excess of supply of this car type, \( m \) yards are uniformly selected and the total supply of the car type is then distributed, uniformly, in the \( m \) yards and between all pools inside this car type. The algorithm is as follows:

**Algorithm for generating supply**

**INPUT:** \( C = \text{setOfCarTypes}; m = \text{numSupplyNodes} ; r = \text{supToDemRatio} ; \)

**For each car type \( c \) in \( C \):**
- \( \text{TotalDemandOfCarType} (c) = \sum_{1\leq t \leq 14} d(t) \) where requested pools are in car type \( c \)
- \( s = \text{totalSupplyOfCarType}(c) = \text{TotalDemandOfCarType} (c) * r \)
- uniformly pick \( m \) yards
- uniformly distribute \( s \) number of cars between pools in car type \( c \) in this \( m \) yards

### 6.3.1.2 Procedure for generating case examples

In this section we present a procedure to generate case examples for simulation purposes. Numbers inside parenthesis are only an example.
6.3 Performance of Models and Solution Procedures in Larger Scale Problems

INPUT:
Network = Large Network (time-space), along with its capacities both original \( (v_b^{original}, u_g^{original}) \) and residual \( (v_b, u_g) \); c_maxBlockFixedCost (= 3000); case parameters (a) to (k) and scenario parameters (l) and (m):
(a) double ratioOfBlocksWithSetupCost (= 0.2)
(b) int numCarTypes (= 2)
(c) int numPoolsInCarType (= 2)
(d) int numDemandProfiles (= 100)
(e) int maxDemand_lowerBound (= 30)
(f) int maxDemand_upperBound (= 50)
(g) int acceptedDelay (= 1)
(h) double steepnessLogistic (= 1)
(i) int midpointLogistic (= 5)
(j) int numSupplyNodes (= 150)
(k) double supToDemRatio (= 0.5)
(l) double shortagePenaltyCostFactor (= 1)
(m) double capFactor (= 1)

generateCaseExample (Network, c_maxBlockFixedCost, ratioOfBlocksWithSetupCost, numCarType, numPoolInCarType, numDemandProfiles, maxDemand_lowerBound, maxDemand_upperBound, acceptedDelay, steepnessLogistic, midpointLogistic, numSupplyNodes, supToDemRatio, shortagePenaltyCostFactor, capFactor)

STEP 1 use (a) to calculate block setup costs. If block setup cost is not zero, reset the block capacity to its original value based on equation (6-4).

\[
 f_b = \\
\begin{cases} 
  c_{maxBlockFixedCost} \ast (1 - r) , & r = uni (0, 1) < \text{ratioOfBlocksWithSetupCost} \\
  0 & \text{otherwise} 
\end{cases}
\]

\[ v_b = v_b^{original} \text{ if } f_b > 0 \]

STEP 2 use (b) to generate car type names (e.g. carType1, carType2)
STEP 3 use (c) to generate pools and assign them to car types (e.g. pools 1,2,3,4 were 1,2 are from carType1 and 3,4 are from carType2).
STEP 4 use parameters in (d-i) as parameters in “Algorithm for generating demand” in Section 6.3.1.1 to generate demand.
STEP 5 use (j) and (k) as parameters in “Algorithm for generating supply” in Section 6.3.1.1 to generate supply
STEP 6 use (l) and (m) as scenario parameters to create different scenarios on the generated case example

6.3.2 Sensitivity Analysis on Network Capacities

In addition to the number of nodes and arcs of the network and number of car types, the size of the feasible region in Allocation-Timing model also depends on network capacities. If network capacities are too high, compared to the supply and demand, they
might become redundant and we might be able to relax them, practically solving the Noncapacitated model. In this section the effect of network capacities on the objective values and order fulfillment is analyzed. In order to do so first we developed a case example using parameters listed in Table 6-6 and then applied our path-based models to this case example in 10 different scenarios on network capacities ranged between 0.5 and 9.5 with a step of 1 (everything else remains the same). The execution in each scenario was terminated if the run time exceed 600 seconds or optimality gap was less than 1e-5. Figure 6-10 shows the performance measures (objective value, fulfilment, and run time) of different path-based formulations for this case example. As one can see in Figure 6-10(a) the Lagrangian Heuristic (AT-LRH) is not performing well and has a higher cost compared to AT-IP which solves the same model using the built-in algorithms of CPLEX package. In fact AT-LRH has the highest objective value at termination time in almost all cases. The failure of AT-LRH might have two causes: a) the massive number of constraints, all train and block capacity constraints, that are relaxed makes the LR severely infeasible that the makeItFeasible() process is not able to find a feasible solution without dramatically affecting the objective value and b) the construction times of creating subproblems is high and the algorithm is terminated because of time limit without performing many iterations towards optimality. In general we do not include the model construction times while collecting the run times as a performance measure since it’s a onetime task and all models have almost similar number of constraints and variables. However the subproblem construction is part of LRH and the construction times cannot be excluded from the run time. This is the reason we accept that this algorithm is not good for our problem.

Table 6-6 Parameters used to generate case example for sensitivity analysis
While AT-IP performs better than LRH, its performance is not acceptable in general. If we compare this method with PA-IP whose feasible region is a subset of AT-IP, we expect a superior performance (see Property 6-1a), while Figure 6-10(a) shows the opposite! A short look at the run time in Figure 6-10(c) reveals that in all scenarios AT-IP has terminated after 600 seconds and by reaching the time limit and not the optimality gap. In other words Property 6-1a is not violated here since the optimal solution of AT-IP is not reached. The main takeaway here is that solving the Allocation-Timing model using the Lagrangian Heuristics or the CPLEX built-in algorithms, is not efficient enough to even beat the solution suggested by Pure-Allocation model using CPLEX built-in algorithms.

The comparison between AT-IP and PA-IP should be sufficient to lead us to find more efficient algorithms to solve the AT model. However to emphasize the importance of such a need, we should compare the best algorithm so far, i.e. PA-IP, with the current

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network</td>
<td>Large Network</td>
</tr>
<tr>
<td>ratioOfBlocksWithSetupCost</td>
<td>0.2</td>
</tr>
<tr>
<td>numCarType</td>
<td>5</td>
</tr>
<tr>
<td>numPoolInCarType</td>
<td>5</td>
</tr>
<tr>
<td>numDemandProfiles</td>
<td>100</td>
</tr>
<tr>
<td>maxDemand_lowerBound</td>
<td>20</td>
</tr>
<tr>
<td>maxDemand_upperBound</td>
<td>50</td>
</tr>
<tr>
<td>acceptedDelay</td>
<td>1</td>
</tr>
<tr>
<td>steepnessLogistic</td>
<td>1</td>
</tr>
<tr>
<td>midpointLogistic</td>
<td>5</td>
</tr>
<tr>
<td>numSupplyNodes</td>
<td>100</td>
</tr>
<tr>
<td>suptoDemRatio</td>
<td>0.25</td>
</tr>
<tr>
<td>shortagePenaltyCostFactor</td>
<td>1</td>
</tr>
<tr>
<td>capFactor</td>
<td>0.5-9.5</td>
</tr>
<tr>
<td>Stopping criteria</td>
<td>10min or epGap 1e-5</td>
</tr>
<tr>
<td>totalSupply</td>
<td>8538</td>
</tr>
<tr>
<td>totalDemand</td>
<td>36048</td>
</tr>
</tbody>
</table>
practice of the railways, i.e. Noncapacitated model (NC). As it was mentioned in Section 4.4 any comparison between capacitated and noncapacitated model performance is not valid, unless the behavior of noncapacitated model in a capacitated network (NCA\text{Actual}) is considered. Unfortunately the result of AT-IP is not able to compete with the result of NCA\text{Actual}. This means if we do not develop an algorithm that is able to outperform NCA\text{Actual}, there is no way to convince railway industries to consider capacities while making car distribution decisions.

The iterative relaxation and rounding algorithm for the Allocation-Timing problem (AT-IRIRH1) was able to outperform both PA-IP and NCA\text{Actual} by fulfilling more demands and suggesting a lower total cost. First the integrality constraints for assignment variables were relaxed, while binary block setup variables remained as binary and the resulted program was solved. Then the assignment variable values were rounded down to integrality and the rest of variables and resources were updated. The residual problem was solved again by relaxing integrality condition of assignment variables and rounding down the final solution. The summation of the assignments of the two programs was considered as the best solution. In order to respect the 600 second time limit, the hard time limit of 500 second was enforced as a termination criterion on the first iteration, while a softer time limit of 100 second was enforced for the residual program in the second iteration. The term softer means, if the run time of first iteration was less than 500 (let say 350), the left over time (of 150 second) was utilized for the second iteration (providing a hard limit of 250 seconds for the second iteration). Also the optimality gap for the first problem was set to $1e^{-5}$ while for the second one it was set to $1e^{-4}$. 
6.3 Performance of Models and Solution Procedures in Larger Scale Problems

Since the AT-IRIRH1 was stopped by reaching the time limit in either the first or the second iteration, its optimality is not guaranteed. In fact AT-IRIRH2 which uses the linear programming relaxation provided better results in less time.

As you can see in Figure 6-10(b), when the capacity increases and the capacity constraints become practically redundant, the AT-IRIRH algorithms for the capacitated Allocation-Timing model provides a solution close to the one provided by the noncapacitated model.
6.3 Performance of Models and Solution Procedures in Larger Scale Problems

a) Figure 6-10 Path-Based formulation performance measures at the termination time when capacity increases: a) objective value, b) order fulfillment, and c) run time
6.3.3 Sensitivity Analysis on Shortage Penalties

As we saw in the Illustrative Example of Section 6.2, the optimal solution of the car distribution models are influenced by the trade-off between assignment costs and the shortage penalties. A model with high shortage penalties focuses on lowering the total unmet demand and tries to not leave any supply unused while there is a demand for that, unless it is not a good match. A model with lower shortage penalties, however, assigns a car to an order only if its assignment cost is less than the penalty associated to the loss of that demand.

In order to study the effect of shortage penalties on the order fulfillment ratio, we consider the case problem of Section 6.3.2, but this time instead of changing the capacities, we fix the capacity factor to 2.5 and changed the shortage penalty factor from 0.25 to 6.5, with a step of 0.25 from 0.25 to 2.5 and a step of 1 from 2.5 to 6.5.\(^3\)

Figure 6-11(b) shows that the order fulfilment for all models except AT-IP increases when the shortage penalty factor increases from 0.25 to 1 and then remains almost the same. Non-improvement of the order fulfilment in Figure 6-11(b), when shortage penalty factor increases from 1 to 6.5, while dramatic increase in the objective value (cost) confirms that our procedure for assigning values to the shortage penalties is well designed, in such a way that the shortage penalties are big enough to guarantee the maximum order fulfilments and not too big to overshoot the other components of the objective function.

\(^3\) The shortage penalty factor was initially changed between 0.5 and 6.5 with a step of 1. However the results showed that the order fulfillment is not affected after 2.5. This is why the effect of shortage penalty factor was studied in more detail from 0.25 to 2.5 with a step of 0.25. The dramatic change of slope in Figure 6-11(a) is a result of change of step size and does not have any insight attached to it.
(like assignment costs, and block costs). It also shows how AT-IRIRH1 and AT-IRIRH2 outperform other model and algorithms.
6.3 Performance of Models and Solution Procedures in Larger Scale Problems

Figure 6-11 Path-Based formulation performance measures at the termination time when shortage penalties increase: a) objective value, b) order fulfillment, and c) run time

6.3.4 Average Time Complexity of the Proposed Heuristic

Although the path-based car distribution problem, Allocation-Timing, was formulated as an Integer Programming model, the Iterative Relaxation and Rounding Heuristic (AT-IRRH2) presented in Section 4.3.3, repeatedly, solves the LP relaxation of the problem and its residuals. The worst case complexity of simplex method is known to be exponential; however it has a polynomial-time average complexity, under various probability distributions (Spielman and Teng, 2004). In order to improve the efficiency of the algorithm, we use the optimal basis of Pure Allocation model as the initial basis for Allocation-Timing model. In this section we examine whether the average complexity of AT-IRRH2 with initial basis is polynomial.

In order to analyze the average complexity of the algorithm, a number of computational experiments were conducted. The number of car types, \( c \), number of demand profiles, \( n \), and number of supply nodes, \( m \), were used as variable input for generating random instances of problems and all other parameters maintained the same for all cases based on table Table 6-7 and all instances were generated on the same network: the Large Network.

The model was run for 5 and 10 car types while the number of demand profiles and number of supply nodes both varied from 10 to 200 with a step of 10 for 10 to 50 and a step of 50 for 50 to 200.
For each tuple of \((c, n, m)\) values, 20 random instances were generated. The models were applied to each random instance and in each run, performance measures were collected and the average run time of the 20 runs for each problem size \((c, n, m)\) was calculated.

### Table 6-7 Parameters used to generate case examples for computational complexity calculations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network</td>
<td>Large Network</td>
</tr>
<tr>
<td>ratioOfBlocksWithSetupCost</td>
<td>0.2</td>
</tr>
<tr>
<td>numCarType</td>
<td>5, 10</td>
</tr>
<tr>
<td>numPoolInCarType</td>
<td>5</td>
</tr>
<tr>
<td>numDemandProfiles</td>
<td>10 to 200</td>
</tr>
<tr>
<td>maxDemand_lowerBound</td>
<td>30</td>
</tr>
<tr>
<td>maxDemand_upperBound</td>
<td>60</td>
</tr>
<tr>
<td>acceptedDelay</td>
<td>1</td>
</tr>
<tr>
<td>steepnessLogistic</td>
<td>1</td>
</tr>
<tr>
<td>midpointLogistic</td>
<td>5</td>
</tr>
<tr>
<td>numSupplyNodes</td>
<td>10 to 200</td>
</tr>
<tr>
<td>suptoDemRatio</td>
<td>0.5</td>
</tr>
<tr>
<td>shortagePenaltyCostFactor</td>
<td>1</td>
</tr>
<tr>
<td>capFactor</td>
<td>1</td>
</tr>
<tr>
<td>Stopping criteria (10min or epGap 1e-5)</td>
<td>25 min (1500 s) or epGap 1e-4</td>
</tr>
</tbody>
</table>

The hypothetical regression model was defined as a power law function:

\[
 t = \lambda c^\alpha n^\beta m^\gamma, \tag{6-6}
\]

where the run time \(t\) is the response variable while the number of car types \(c\), number of demand profiles \(n\), and number of supply nodes \(m\) are the regressor variables. A logarithmic transformation of (6-6), results in a multiple linear regression model:

\[
 T = \Lambda + \alpha C + \gamma N + \beta M \tag{6-7}
\]
6.3 Performance of Models and Solution Procedures in Larger Scale Problems

Where \( T = \ln(t) \), \( C = \ln(c) \), \( M = \ln(m) \), \( N = \ln(n) \) and \( \Lambda = \ln(\lambda) \) (and equivalently \( \lambda = e^\Lambda \)).

By fitting the time data to a power trendline, the equation \( t = 0.008164c^{-0.3219}n^{1.3675}m^{1.1386} \) with \( R^2 = 0.9 \) and adjusted \( R^2 = 0.8834 \) was obtained which confirms the polynomial average complexity of the algorithm. Figure 6-12 confirms that the number of demand profiles and number of supply nodes have more effect on solution time.

![Figure 6-12](image)

Figure 6-12 Average solution time of AT-IRRH2 with initial basis (solid green circles) along with its estimation from the regression model (empty blue circles)

There is a negative power on the number of car types \( c \), which means the run time decreases as the number of car types increases, might seem nonsense at first look. However after investigating the observed data, we noticed a similar trend. Further investigation showed that the reverse relationship between the number of car types and the run time does exist, and is originated from the procedure that we propose in Section 6.3.1.2 to generate the case examples. When the number of supply nodes and the number of demand profiles
as well as the distribution of supply and demand in each node are given, an increase in the number of car types, without increasing the total supply or demand, will result in a higher chance of mismatch between supply-order pairs and as a result less number of eligible paths or equivalently less number of variables. For example, if there are only two customers, each ordering only one car of car type 1 and we have a supply of two cars of car type 1 located at two different yards, then we can potentially have 4 eligible supply-order pairs. While if we have the same number of customers, each ordering only one car but from two different car types, then we will have a maximum of two supply-order pairs only. In fact \( n \) can be written as \( n = r \times c \) where \( r \) is the average number of demand profile per car type. The estimated run time function is transformed to \( t = 0.008164c^{1.0456}r^{1.3675}m^{1.1386} \) which means including a new commodity, along with its demand profiles, to the model will increase the solution time (almost linearly). Figure 6-13 shows that the power law model is a good fit for the average run time of the problem.

![Figure 6-13 Average run time vs estimated run time from a power law model](image-url)
6.3.5 Solution Quality of the Proposed Heuristic

Besides its polynomial solution time, the quality of the optimal solution suggested by IRRH2 is also promising. Figure 6-14 (a) shows the EpGap (Relative MIP Gap) after each iteration of the heuristic for different sizes of the problem. EpGap is defined by equation (6-1) in Section 6.1, with the linear relaxation solution being considered as the lower bound. Figure 6-14 (b) shows the solution time of the algorithm per iteration for the same problems considered in Figure 6-14 (a). As one can see, EpGap ranged between 0.2% and 1.8% in the first iteration, and improves to be lower than 1.5% after the second iteration. It is also noteworthy that the second iteration offers more improvement when the problem size increases. Figure 6-14 (c) shows that for problems with as large as 50 supply nodes and 50 demand nodes the EpGap drops from 1.1%, on average, in the first iteration to 0.81%, on average, in the second iteration, resulting in 26% improvement by only costing 16% extra solution time.
6.3 Performance of Models and Solution Procedures in Larger Scale Problems

![Graph showing performance measures of IRRH2 per iteration when the problem size increases.](image)

**Table: Average EpGap (MIP Gap) and Run Time of AT-IRRH2 with initial basis**

<table>
<thead>
<tr>
<th></th>
<th>Iteration 1</th>
<th>Iteration 2</th>
<th>Improvement</th>
<th>Relational Improvement</th>
<th>Run Time</th>
<th>Iteration 1</th>
<th>Iteration 2</th>
<th>Relational Extra time</th>
</tr>
</thead>
<tbody>
<tr>
<td>AvgLargeScale</td>
<td>1.10%</td>
<td>0.81%</td>
<td>0.29%</td>
<td>26%</td>
<td>542</td>
<td>472</td>
<td>70</td>
<td>16%</td>
</tr>
</tbody>
</table>

Figure 6-14 The performance measures of IRRH2 per iteration when the problem size increases. a) EpGap, b) solution time, and c) their trade-off
7.1 Summary

Because of spatial and temporal imbalances in the supply and demand of empty railcars/cars in freight transportation systems, empty equipment repositioning is inevitable. As the owners of rail network, trains and railcars, railway carriers charge their customers based on the days and the miles that a railcar is under load. Hence the cost of shipping empty cars to customers as well as the opportunity cost of extra inventory in customer facilities is on the railway’s shoulder. This is the reason why railway carriers have invested a lot of money on optimizing their empty railcar distribution processes. For instance, CSX invested $5 million to lunch their real-time car distribution system in 1997 (Gorman et al., 2010), followed by BNSF's Equipment Distribution System which cost $3 million.

Despite the vast studies performed in the railroad optimization modeling, there is still a need for another realistic modeling and optimization of the empty railcar distribution problem. In particular, there is no previous work that optimizes the empty railcar distribution decisions while simultaneously considering the resource (capacity) availabilities in the service network and the car type substitutability for the customer.
demands as well as separating the car distribution and car routing decision in accordance to the US railroad practices.

In this research we developed two formulations for the Empty Railcar Distribution problem, both aiming to minimize the total setup costs, total transportation costs, and total shortage penalties under supply limitation, demand satisfaction, customer preferences and priorities, and network capacity constraints.

We first formulated the problem as a path-based capacitated network flow model. Contrary to the traditional path-based formulations, the path connecting each supply-demand pair was given by an external application called Trip Planner which is defined on top of a time-space network. Another application called Pseudo Path Generator then was developed to generate alternative paths for each supply-order pair. Then we formulated the problem as an arc-based capacitated multi-commodity network flow model where contrary to the path-based model, the car routing and car distribution decisions were integrated in a single model. Both models are complex and because of the huge number of constraints and integer decision variables are hard to solve. However, the path-based formulation is more practical for the United States railroads since in US railroad industry car routing and car distribution decisions are separated from each other and usually made by different departments.

Before prescribing the usage of our models, the models, computer programs and solution procedures needed to be verified and validated. A computer program was developed in Java and its accuracy was verified using debugging techniques, observational verification and analytical verification (see Section 6.1). After verifying that the computer program correctly represents the model, the next step was to validate the performance of
7.1 Summary

different models, especially our path based Allocation-Timing model, by benchmarking them against each other and also against the noncapacitated model as the current practice of many railway industries. Case examples used for validation purposes were generated on a network called “Large Network” with 440 yards, 1369 blocks, 624 trains and 2167 train segments which in an 18-day time horizon results in 34941 nodes, 24642 blocks, 11232 trains and 39005 train segments.

The models were implemented in Java and for small size problems solved using Concert Technology of the IBM CPLEX solver. However as the number of supply nodes and demand profiles increased, the solution time increased exponentially. The run time for a problem with 20 supply nodes and 20 demand profiles was about 2 minutes while it exceeded an hour after scaling the size to 100 supply nodes and 100 demand profiles. In order to decrease the solution time of the path-based model, first we developed a Lagrangian Heuristic (AT-LRH) and upon its failure we developed an Iterative Relaxation and Rounding Heuristic (IRRH). In each iteration of IRRH, a subset of integrality constraints was relaxed and the associated variables in the optimal solution of this relaxed problem were rounded down. Then, the constraints were modified to reflect the residual problem based on the rounded variables. Such an updated instance was passed to the next iteration for further processing. The relaxation was performed in two different ways resulting in two different heuristics: a) relaxing only the integer variables and keeping the binary ones as they are (IRRH1) or b) relaxing all integer variables to general non-negative variables and all binary variables to continuous variables between 0 and 1 (IRRH2).
7.2 Results

The comparison of path-based and arc-based formulations in both capacitated and uncapacitated modes confirmed the efficiency of the heuristic from both run time and solution quality perspectives. The experiments in large-scale problems showed that AT-LRH is not performing well and has a higher cost compared to AT-IP which solves the same model using the built-in algorithms of CPLEX package. However since AT-IP was terminated because of time limit and not because of reaching the optimal solution, it was outperformed by PA-IP. The iterative relaxation and rounding algorithm for the Allocation-Timing problem (AT-IRIRH1) outperformed PA-IP in both capacitated and noncapacitated modes by fulfilling more demands and suggesting a lower total cost. AT-IRIRH2 proved to be even more efficient than AT-IRIRH1. Furthermore, to improve the efficiency of AT-IRIRH2, we used the optimal basis of Pure Allocation model as the initial basis for Allocation-Timing model.

In order to analyze the average complexity of the algorithm, a number of computational experiments were conducted. The number of car types, \( c \), number of demand profiles, \( n \), and number of supply nodes, \( m \), were used as variable input size parameters for generating random instances of problems and the solution time for each instance was collected. By fitting the solution time data to a power trendline, the equation \( t = 0.008164c^{-0.3219}n^{1.3675}m^{1.1386} \) with \( R^2 = 0.9 \) and adjusted \( R^2 = 0.8834 \) was obtained which confirms the polynomial average complexity of the algorithm.

7.3 Future Research

Despite the above mentioned merits, the proposed models and algorithms point to a number of directions for future research. The path-based AT problem presented in this
7.3 Future Research

research is a general integer programming model and its solution time, using integer programming techniques, highly depends on the number of integer variable. In this research we transformed the problem of finding all pseudo paths of a given trip plan, to finding all permutations of a given word with two letters B and I. The number of paths and consequently the number of assignment variables increase exponentially when the number of blocks (B) in the trip plan or the number of days (I) in the planning horizon increases. A smarter algorithm which builds only those pseudo paths that have higher chance of being in the optimal solution, or at least fathoms those with almost no chance of being in the optimal solution might help to lower the solution time of the integer programming models and enable us to use the CPLEX solver to solve the problem directly.

Another issue regarding the Pseudo Path Generator is that it has been built under the assumption that all trains operate regularly and repeat in the exact same time during all the days of the week. In reality however, not all trains operate seven days a week nor they are all on-time. A pseudo path generation algorithm that uses the actual train schedules rather than simply repeating them in the same time during the week is more realistic. Another contribution to this work could be developing solution techniques that do not depend on licensed solver engines. This might be helpful in two ways. First, by saving the licensing cost and second, by avoiding the incompatibility of newer versions of the optimization software with the other applications interacting with solvers in the car distribution system. Finally, the efficiency of proposed models and heuristics should be verified on real work data in a pilot period, before implementing in railway industries.
References
7.3 Future Research


7.3 Future Research


7.3 Future Research


7.3 Future Research