A MONTE CARLO BASED METHOD FOR THE DYNAMIC PERFORMANCE
ANALYSIS OF TALL BUILDINGS UNDER TURBULENT WIND LOADING

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by

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ABSTRACT

The current state of the practice for the design of tall buildings, subjected to wind hazards, is prescriptive based (i.e., ASCE 7-05). However, the concept of using a non-prescriptive methodology, which would allow for design beyond the code criteria, seems preferable in the context of tall buildings from an economic point of view. This type of approach to design is referred to as performance-based engineering (PBE).

This thesis presents a procedure for the calculation of the dynamic response of tall buildings, using Monte Carlo (MC) integration methods, as a first attempt at a performance assessment. The proposed MC numerical procedure was used to compute the power spectral density of the buffeting loads, and to derive statistical information about the dynamic response in the presence of uncertainty in the wind loading (i.e., measurements errors and modeling deficiencies). Three structures were utilized to validate the proposed algorithm: a steel chimney, a concrete chimney, and the CAARC prototype building. The CAARC building had been selected by the Commonwealth Advisory Aeronautical Research Council of Australia as a benchmark high-rise structure for studying dynamic response affected by wind loading. This thesis computed the structural fragility curves of the CAARC building under extreme winds, and preliminary investigations were conducted to assess the applicability of the developed methodology in design utilizing PBE. The assessments concentrate on serviceability limit states (displacements) to ascertain, for example, human comfort and damage to non-structural elements on the façade due to an extreme wind event.

The results of these studies validated the appropriateness of the developed algorithm through comparison to reference values obtained from the literature. Furthermore, structural fragility curves were successfully derived, and a preliminary performance
analysis, based on simulated along-wind response serviceability limit states, was conducted for the CAARC building. The wind loading in this analysis was consistent with the New England coastal region.
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The graduate students in this department are some of the most wonderful people I have ever had the opportunity to work with. I am very appreciative for all the times you were able to answer my questions. I will greatly miss my ‘deskmates’.

This thesis is dedicated to my family. To my parents, Mark and Vicky, who have always believed in my abilities; to my brother, Shane, who inspires me; and last, but certainly not least, to my husband, David – I could not have reached this point without his loving support and encouragement.
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1 CHAPTER ONE: INTRODUCTION

1.1 RESEARCH OVERVIEW: EXPLORING PERFORMANCE-BASED ENGINEERING AND ITS APPLICATION TO WIND ENGINEERING

Current design codes call for the use of prescriptive methods, i.e., (ASCE, 2005), for the design of tall buildings against wind hazards; these methods utilize a set of simplified loading scenarios, and criteria based on demand vs. capacity, creating a “loss of freedom” in design and execution (Inokuma, 2002). However, in the context of tall buildings, which are extensive and costly projects, the idea of using a non-prescriptive methodology, to allow for design beyond the code criteria, has been recognized as an advantageous approach (Bashor and Kareem, 2007; Bracci, 2006; Jain et al., 2001; Norton et al., 2008; Rosowsky and Ellingwood, 2002). This approach is often referred to as “performance-based engineering” (PBE), and was originally introduced in earthquake engineering to overcome the limitations of traditional prescriptive structural engineering codes.

The basic concept behind PBE is to allow for the freedom of the design process (Inokuma, 2002). The idea is to ensure that a building, for example, subjected to different levels of hazards (as opposed to the largest foreseeable event), is capable of achieving a selected performance objective level (Ellingwood et al., 2004). The commonality among current PBE methodologies is that they go beyond the prescriptive codes’ specification by ensuring that life safety be preserved under “severe” events; they also prescribe that there shall not be collapse under “extreme” events, and that there shall be immediate occupancy under “moderate” events (the exact definitions of “severe”, “extreme”, and “moderate” are still under development) (Ellingwood et al., 2004). The overall concept of PBE provides an
attractive alternative for owners, since it can enable cost-effective design and can reduce planning in the aftermath of a catastrophic event.

PBE is not a recently developed approach to design, however. In fact, PBE was introduced in the late 1960’s as part of the Operation Breakthrough Project (a housing initiative) (Rosowsky and Ellingwood, 2002). The U.S. Department of Housing and Urban Development sponsored this project at the National Bureau of Standards, and in 1977 a set of reliability-based criteria for strength and serviceability limit states against earthquake hazards was published (NBSIR, 1977). At that time, PBE was viewed by the building code community as a liability (Rosowsky and Ellingwood, 2002). Furthermore, the computational systems were not available to conduct the type of analyses required in PBE (Ellingwood, 2000).

There has been a resurrection of this method, mostly in the area of fire-resistant design of building structural systems and in earthquake-resistant design with the SEAOC Vision 2000 activity (Rosowsky and Ellingwood, 2002). SEAOC (Structural Engineers Association of California) set about to design structures based on a correlation between earthquake recurrence intervals and performance levels in their Vision 2000 project, as shown in Figure 1.1 (Norton et al., 2008; SEAOC, 1995). Using this figure, a designer can combine a performance level with an excitation level to determine the design criteria (Norton et al., 2008). Other examples of publications, in the earthquake engineering community, which provide applications of the PBE methodology include ATC 33, FEMA 273 (later replaced by FEMA 356), ATC 40 (Norton et al., 2008), as well as additional publications by SEAOC, such as the “SEAOC Blue Book” (SEAOC, 1999). Figure 1.2, from (SEAOC, 1999) is an updated version (expansion) of Figure 1.1 from (SEAOC, 1995).
The previously mentioned publications are just a sampling of the comprehensive research activities in the recent past that have been undertaken in the area of risk-based assessment of structural integrity (PBE). However, many of these activities specifically focus on earthquake engineering. Less emphasis has been devoted to the problem of PBE in the presence of wind hazards, especially for sensitive systems, such as high-rise buildings (Bashor and Kareem, 2007; Bracci, 2006; Jain et al., 2001; Norton et al., 2008), where (apart
from structural integrity considerations) the presence of dynamic amplification of the vibration is of relevance for serviceability. Even though risk-based methods for seismic design are readily available to practicing engineers, a direct “technology transfer” to wind hazards is not possible due to the uniqueness of the excitation mechanism. For example, there is no "life safety" issue for extreme wind events, since buildings are usually evacuated prior to the arrival of a major hurricane. Furthermore, strength is not considered an issue because tall buildings are usually not severely damaged after a storm; however, non-structural elements typically are a problem (e.g., secondary systems may malfunction after the storm, the façade may need non-structural repairs, etc.). These aspects are often not considered in earthquake engineering since they are only secondary, but they are very important in wind engineering. These distinctions lead to different definitions of limit states. Even though the methodology based on "fragility curves" is applicable, the performance objectives are different; therefore, investigations focusing specifically on wind effects on tall buildings are necessary to bridge this gap. Moreover, a method to conduct systematic analyses, which allow for a structured development of fragility curves, is necessary.

A literature review of PBE in the field of wind engineering has revealed that most attention has been given to the study of the effects on low-rise buildings (Ellingwood et al., 2004; Ellingwood and Tekie, 1999; Li and Ellingwood, 2006; van de Lindt et al., 2005), where damage and collapse can be related to localized loss of capacity in key members or connections. Few studies are available on high-rise buildings, in which either a framework for the analysis of uncertainty is developed (Bashor and Kareem, 2007), or in which a methodology for the design of buildings is proposed (Jain et al., 2001; Norton et al., 2008).
In this thesis, the research activity is directed towards the development of an algorithm for the analysis of wind-induced response of a tall building, which may further enable an assessment of performance. As part of these investigations, this thesis focuses on the development of a numerical method for estimating the (linear) dynamic response of high-rise structures due to turbulence-induced loads. The significance of this approach, for example, is related to the possibility of replicating the “closed-form” analytical methods (based on random vibration techniques in the frequency domain) through numerical Monte Carlo (MC) algorithms. The advantage, from the computational standpoint, is that it further enables the subsequent implementation of numerical statistical analyses, as in the presence of uncertain wind loading scenarios, i.e., errors in the estimation of the aerodynamic actions.

Capitalizing on the computational advantages of this proposed procedure, this thesis will discuss the development of a “Two-Step MC Procedure”, which allows for the estimation of fragility curves associated with serviceability limit states (displacements) due to the uncertainty mainly associated with the input (e.g., experimental and measurements errors, modeling deficiencies, etc.). The overall objective of the developed approach aims to enable further implementation as a more comprehensive simulation technique, by potentially providing designers with a useful tool for practical application. The methodology will assist in the selection of objective levels (for displacements), which will not be based on prescriptive standards (based on strength).

An example of the anticipated future applications (considering the objective mentioned above) could be related to a condominium building in Miami, FL, which is still in the design stage. The owner of this tall residential building wants to know if his building will suffer damage during the next hurricane. With the application of the proposed design
methodologies, the type of damage (if any, mild to severe) may be assessed relative to the type of wind hazard (i.e., severe for the case of the hurricane). Ultimately, these different objective levels, based on strength, could potentially be associated with a cost so that the owner could make a decision (e.g., save money during construction by establishing a lower performance level, or reduce maintenance costs in the future with a higher level of performance).

Another scenario could be related to a tall office building in Boston, MA. The owner of this building might be less concerned with severe damage due to wind hazards (since the building’s location is not typically subjected to extreme wind events), and more concerned with the occupant comfort criteria. Recent studies have shown that occupant comfort criterion plays an important role in the design of tall buildings, since significant motion can cause occupants to experience extreme discomfort (Bashor and Kareem, 2007; Bashor et al., 2005; Burton et al., 2006; Kijewski-Correa and Pirnia, 2009). Again, the proposed methodology might possibly be employed by this owner to ensure the comfort of the building’s occupants by selecting an appropriate performance, based on the estimation of vibration levels; these levels would either need to be controlled or minimized to be undetectable by humans.

1.2 **Research Objectives**

The objectives of this research are as follows:

1) Develop a numerical procedure that accounts for the uncertainty in the characterization of aerodynamic loading to compute the dynamic response of high-rise buildings due to turbulence-induced wind loading. The procedure will
utilize the MC numerical method of integration for the input loading for calculation of the response dominated by the fundamental bending mode of the structure (Step 1). Loading effects and structural response estimation will be based on established methods for the analysis of a structure under turbulent winds, usually referred to as buffeting analysis (Davenport, 1971; Kareem, 1988).

2) Validate the developed numerical procedure by computer simulations, through the comparison of results available in the literature.

3) Implement the numerical procedure (Step 1) to derive statistical information about the dynamic response due to errors and variability in the characterization of wind loading (Step 2). Combine steps 1 and 2 to perform a preliminary analysis of dynamic response performance under the influence of high winds.

1.3 **Thesis Organization**

Chapter Two offers an overview of the formulation developed for the calculation of the dynamic response. A discussion of the employed simulation methods (“closed-form” and MC) is provided, as well as a description of the simulated buildings.

Chapter Three contains the results of the analyses conducted using the “closed-form” approach. The three main components discussed in this chapter are as follows: 1) the preliminary investigations conducted during the development stage of the algorithm; 2) the verification and updating of model and wind loading parameters; and 3) the validation that the results can be used as target values.

Chapter Four presents the results and analyses of the MC algorithm. The goal of this chapter is to demonstrate that the MC algorithm is capable of accurately estimating the modal
buffeting force. Additionally, this performance analysis is used to optimally select parameters related to the implementation of the algorithm.

Chapter Five discusses the implementation of the “Two-Step Monte Carlo Algorithm”. The results of the preliminary fragility analysis, based on performance, are presented, and an exploratory example is employed to show how the designer of a structure might possibly be able to utilize this algorithm.

Common wind engineering terms, which will be referred to in this thesis, are defined below in Table 1.1. Table 1.2 contains a listing of the nomenclature appearing in this thesis.

*Table 1.1 – List of terms*

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<tbody>
<tr>
<td>Aeroelastic Phenomena</td>
<td>Describes the significant interactions between the aerodynamic forces and structural motions.</td>
</tr>
<tr>
<td>Background Response</td>
<td>A result of the turbulent component of the wind velocity, whose frequencies do not coincide with the natural frequencies of the structure.</td>
</tr>
<tr>
<td>Boundary Layer</td>
<td>The layer where turbulent mixing decreases the horizontal drag force (exerted by the Earth’s surface) as a function of height. Within the boundary layer, the wind speed increases with elevation until the gradient speed is reached at the top of the boundary layer. The boundary layer typically ranges from a few hundred meters to several kilometers.</td>
</tr>
<tr>
<td>Buffeting</td>
<td>Unsteady loading of a structure by velocity fluctuations in the oncoming flow.</td>
</tr>
<tr>
<td>Logarithm Law</td>
<td>A relationship between the roughness length and friction velocity, which is used to determine the mean wind profile.</td>
</tr>
<tr>
<td>Mean Recurrence Interval</td>
<td>The characteristic time between consecutive wind events.</td>
</tr>
<tr>
<td>Power Law</td>
<td>A representation of the mean wind profile, in horizontally homogeneous terrain, whereby a ratio of heights is raised to some exponent (dependent on the roughness length).</td>
</tr>
<tr>
<td>Resonant Response</td>
<td>A result of the turbulent component of the wind velocity, whose frequencies coincide with the natural frequencies of the structure.</td>
</tr>
</tbody>
</table>
Roughness Length  A quantity used to describe the terrain roughness.
Turbulence Intensity  A quantity relating the turbulence fluctuations to the mean wind speed at an elevation.
Velocity Spectra  Represents the stationary, frequency-dependent dynamic characteristics of wind turbulence (and loading).
Vortex Shedding  The phenomena whereby alternating vortices are shed from the structure creating a trail of vortices downstream.
Wake  Region of disturbed flow (usually turbulent), which is downstream of a solid body, caused by the flow of the fluid around the body.
Wind  Motion of air with respect to the Earth’s surface (usually treated as an incompressible fluid).

Note [a]: Terms are listed alphabetically
Note [b]: Definitions are derived from (Simiu and Miyata, 2006; Simiu and Scanlan, 1996)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>building width</td>
</tr>
<tr>
<td>$c(z)$</td>
<td>simulated damping per unit length</td>
</tr>
<tr>
<td>$C_D$</td>
<td>drag coefficient per unit length</td>
</tr>
<tr>
<td>$\bar{\epsilon}_z$</td>
<td>modal damping</td>
</tr>
<tr>
<td>$C_L$</td>
<td>lift coefficient per unit length</td>
</tr>
<tr>
<td>$C_z$</td>
<td>exponential decay coefficient</td>
</tr>
<tr>
<td>$D$</td>
<td>building depth</td>
</tr>
<tr>
<td>$F$</td>
<td>reduced frequency</td>
</tr>
<tr>
<td>$\tilde{f}(n,z_1,z_2)$</td>
<td>generic surface integral</td>
</tr>
<tr>
<td>$F(z,t)$</td>
<td>wind force per unit length</td>
</tr>
<tr>
<td>$f_{v_z}$</td>
<td>probability density function of reference wind speed</td>
</tr>
<tr>
<td>$g$</td>
<td>mode designation</td>
</tr>
<tr>
<td>$h$</td>
<td>building height</td>
</tr>
<tr>
<td>$I_u$</td>
<td>longitudinal turbulence intensity</td>
</tr>
<tr>
<td>$I_v$</td>
<td>lateral turbulence intensity</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$k$</td>
<td>tolerance factor for normal distribution</td>
</tr>
<tr>
<td>$k(z)$</td>
<td>simulated stiffness per unit length</td>
</tr>
<tr>
<td>$\bar{k}_g$</td>
<td>modal stiffness</td>
</tr>
<tr>
<td>$m(z)$</td>
<td>simulated mass per unit length</td>
</tr>
<tr>
<td>$M_{gg}$</td>
<td>modal mass (mode $g$)</td>
</tr>
<tr>
<td>$n$</td>
<td>natural frequency</td>
</tr>
<tr>
<td>$N_{MC}$</td>
<td>number of Monte Carlo integration points</td>
</tr>
<tr>
<td>$n_p$</td>
<td>number of sampled data points</td>
</tr>
<tr>
<td>$n_0$</td>
<td>fundamental natural frequency</td>
</tr>
<tr>
<td>$P$</td>
<td>probability</td>
</tr>
<tr>
<td>$P_T$</td>
<td>probability of “loss of performance”</td>
</tr>
<tr>
<td>$S_c$</td>
<td>Scruton number</td>
</tr>
<tr>
<td>$s_n$</td>
<td>sample standard deviation</td>
</tr>
<tr>
<td>$S_u(n,z)$</td>
<td>longitudinal turbulence spectrum</td>
</tr>
<tr>
<td>$St$</td>
<td>Strouhal number</td>
</tr>
<tr>
<td>$T$</td>
<td>pre-selected threshold</td>
</tr>
<tr>
<td>$u(z,t)$</td>
<td>turbulence component of wind velocity (x-direction)</td>
</tr>
<tr>
<td>$U(z,t)$</td>
<td>velocity of the wind</td>
</tr>
<tr>
<td>$\bar{U}(z)$</td>
<td>mean component of wind velocity</td>
</tr>
<tr>
<td>$\bar{U}_c$</td>
<td>critical vortex shedding velocity</td>
</tr>
<tr>
<td>$v(z,t)$</td>
<td>turbulence component of wind velocity (y-direction)</td>
</tr>
<tr>
<td>$V_z$</td>
<td>mean wind velocity ($V_z=\bar{U}(h)$ at $z=h$)</td>
</tr>
<tr>
<td>$w(z,t)$</td>
<td>turbulence component of wind velocity (z-direction)</td>
</tr>
<tr>
<td>$X$</td>
<td>generic random variable</td>
</tr>
<tr>
<td>$\bar{X}$</td>
<td>mean or static response</td>
</tr>
<tr>
<td>$x(z,t)$</td>
<td>dynamic displacement (x-direction)</td>
</tr>
<tr>
<td>$\bar{x}_o$</td>
<td>mean of sample</td>
</tr>
<tr>
<td>$y(z,t)$</td>
<td>dynamic displacement (y-direction)</td>
</tr>
<tr>
<td>$z$</td>
<td>elevation</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$Z_A, Z_B$</td>
<td>upper bound integration limit</td>
</tr>
<tr>
<td>$z_0$</td>
<td>roughness length</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>lower bound integration limit</td>
</tr>
<tr>
<td>$z_1, z_2$</td>
<td>integration variables</td>
</tr>
<tr>
<td>$z^*$</td>
<td>represents the extent of the normal distribution</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>incidence angle of the wind</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>initial angle of attack</td>
</tr>
<tr>
<td>$\Delta_z$</td>
<td>Riemann sum partitions</td>
</tr>
<tr>
<td>$\Delta_f$</td>
<td>frequency axis divider</td>
</tr>
<tr>
<td>$\zeta_{str}$</td>
<td>structural damping</td>
</tr>
<tr>
<td>$\zeta_{aero}$</td>
<td>aerodynamic damping</td>
</tr>
<tr>
<td>$\mu$</td>
<td>true value of population mean</td>
</tr>
<tr>
<td>$\xi_g(t)$</td>
<td>dynamic generalized coordinate of displacement (mode $g$)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density of air</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>true value of standard deviation</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>root mean square of along-wind turbulence</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>root mean square of across-wind turbulence</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>root mean square (RMS) response in $x$-direction</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>root mean square (RMS) response in $y$-direction</td>
</tr>
<tr>
<td>$\Phi_g(z)$</td>
<td>mode shape (mode $g$)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>circular frequency</td>
</tr>
<tr>
<td>$\omega$</td>
<td>circular frequency</td>
</tr>
</tbody>
</table>
1.4 References

ASCE (2005). Minimum design loads for buildings and other structures (7-05). Reston, VA, USA, American Society of Civil Engineers.


2 Chapter Two: General Formulation and Background Information

2.1 Introduction to the Methods for Solving the Dynamic Response of High-Rise Buildings Under Wind Excitation

In the early 1960s, approaches for simulating the dynamic response of buildings under wind excitations started to be investigated when it was realized that the aerodynamic effects should be considered in the design of modern tall buildings (Simiu and Scanlan, 1996). In (Davenport, 1961), the initial model for representing turbulent wind flow was developed, and the subsequent procedure for estimating the along-wind response was presented in (Davenport, 1967). After Davenport introduced this model, there were numerous procedures introduced to calculate the dynamic along-wind response of structures (Solari, 1982). In (Solari, 1982), a closed form approach was introduced, and has become the basis for many further developments (Piccardo and Solari, 1998b). Modern high-rise buildings are not limited to movement in the along-wind direction – dynamic wind loads induce oscillations in the along-wind, across-wind, and torsional directions (Kareem, 1988). Whereas along-wind oscillations (induced by turbulence) can be reasonably estimated using quasi-steady and strip theories, across-wind and torsional oscillations (partially caused by separation of the wake flow and vortex shedding) cannot (Cheng et al., 2002). Research is being conducted in this area, however, more formal approaches to determining the across-wind and torsional response are still under development and are not presently incorporated into current design practices (Chen and Kareem, 2005).
The following sections will highlight the approach employed in this thesis to calculate the dynamic response in the along-wind and across-wind directions.

2.2 **OVERVIEW OF FREQUENCY-DOMAIN PROCEDURE FOR COMPUTING DYNAMIC RESPONSE OF A TALL STRUCTURE**

Flexible structures, such as tall buildings, subjected to dynamic wind loading experience fluctuating deflections in primarily three directions: along-wind deflections, which are in the direction of the wind; across-wind deflections, which are in the direction normal to the wind; and torsional deflections, which are a result of the non-uniform wind pressures over the surface of the building and a result of different mass centers and elastic centers (Simiu and Miyata, 2006). Figure 2.1 shows a horizontal plane view of a tall structure. The along-wind response occurs in the x-direction, the across-wind response occurs in the y-direction, and the torsional response occurs about the z-axis (out of page).

![Figure 2.1 – Schematic of building orientation and wind directionality (horizontal plane view)](image)
The velocity of the wind, as shown in Figure 2.1, is given by the equation

\[ U(z,t) = \bar{U}(z) + u(z,t) \]  

(2.1)

where \( \bar{U}(z) \) is the mean component of the velocity and \( u(z,t) \) is the turbulence component.

The direction of the wind is given by \( \alpha \) ("angle of attack" of the wind). The turbulence components in the \( y \)-direction and \( z \)-direction (not shown) are denoted by \( v(z,t) \) and \( w(z,t) \), respectively.

In order to compute the total response of the structure, three parts must be considered. The first part is the mean response, or the static response; the static response is a result of the mean wind component of the velocity. The second part is the quasi-static or "background" portion of the response, which is a result of the turbulent component of the wind velocity whose frequencies do not coincide with the natural frequency of the structure. The third part is the dynamic or "resonant" portion of the response. This is also a result of the turbulent component of the wind velocity, but is for the wind force components whose frequencies coincide with the natural frequencies of the structure, which may induce resonant effects. (Simiu and Miyata, 2006)

In the along-wind direction, the dynamic response of tall structures subjected to severe wind loading is primarily governed by the turbulence. In the across-wind direction, the dynamic response is primarily governed by vortex shedding with a smaller contribution from the across-wind turbulence (Holmes, 2001). At the present time, effects of vortex shedding on the across-wind loading were neglected, but could be readily considered in the future.
The methodology proposed by this research utilizes the spectral approach of standard frequency-domain random-vibration analysis for the estimation of wind-induced loading and dynamic response on a high-rise building (Davenport, 1971; Kareem, 1988, 1992; Piccando and Solari, 1998b). The spectral approach has the benefit of being able to compute both the dynamic response and equivalent static response (Solari, 1989). The root mean square (RMS) response and mean response at the top of the structure will be considered, as well as the peak response. The RMS response will be determined for both the along-wind and across-wind directions, whereas the mean and peak responses will only be determined for the along-wind direction.

The procedure developed in this paper for calculation of the dynamic response assumes the following:

• The structure is linear-elastic.

It is assumed that the structure’s deformations are small and that the structure’s material follows a linear stress-strain relationship. Without this assumption, the principle of superposition is not valid. Additionally, since this thesis is primarily focused on serviceability limit states for high-rise buildings, this assumption is acceptable.

• Modal superposition is employed.

In order to solve the continuous system, the physical coordinates of the degrees of freedom can be transformed into a number of discrete, equivalent single degree of freedom (DOF) dynamic systems by modal superposition. Since solutions for single DOF systems are easily obtained through analytical procedures, modal
superposition allows complex systems to be analyzed through the superposition of equivalent single DOF systems (Kareem, 1992).

- Quasi-steady formulation of the wind loading is employed.

  The aerodynamic loading was based on equivalent quasi-steady formulation of wind forces (Davenport, 1971; Kareem, 1988; Solari and Kareem, 1998) since the main objective of the simulations was the validation of the proposed numerical procedure.

- Only a few modes of superposition are required.

  In theory, there are infinite mode shapes in a continuous structure, however, in practice, it is sufficient to consider the first few fundamental mode shapes only (Humar, 2002). Furthermore, for the case of wind loading (typified by low frequency harmonic content – usually below one to two Hertz), it is acceptable to only consider the fundamental modes (Piccardo and Solari, 1998b).

- Torsional effects can be neglected.

  The simplified building that is considered has a prismatic floor plan. The main wind force resisting system is composed of lateral frames, which are symmetric with respect to the primary axes of bending, with negligible simulated eccentricity between the elastic and mass centers. Torsional effects are typically more pronounced in buildings that have characteristics opposite of what is being considered (Tallin and Ellingwood, 1985). Therefore, torsional effects for the employed building type are viewed as secondary in importance, and are also neglected because of the exploratory nature of this study.
• Vortex shedding can be neglected.

In the along-wind direction and in practical loading applications, the dynamic forces as a result of vortex shedding are rarely significant as they are much smaller than the across-wind forces (ESDU, 2004). It is reasonable to neglect vortex shedding since the majority of the current analyses utilize the along-wind response, with less emphasis on the across-wind response. Additionally, vortex shedding may primarily be considered of importance at low to medium wind velocities, and is less critical for displacements than it is for accelerations.

2.2.1 ALONG-WIND AND ACROSS-WIND FORMULATIONS

For modern tall buildings – which are more flexible, have lower damping, and are lighter in weight than their shorter counterparts – the natural frequency of vibration may be in the same range as the exciting force, causing the structure to vibrate in resonance (Simiu and Scanlan, 1996). Modeling this type of structure is best done using a system with a continuously distributed mass per unit length, \( m(z) \). The dynamic response of these systems can be represented as a linear combination of coordinate dependent “shape functions”. These systems have what are called modes, or “modal shapes”, which correspond to each of the structure’s natural frequencies of vibration (Humar, 2002). As is justified by the hypotheses given in Section 2.2, this research only considers the first two bending modes about the principle axes of the building. These modes are designated as along-wind and across-wind because the mean wind direction coincides with one of the principle axes (see Figure 2.1).
Additionally, for the following formulation, the two modes – along-wind and across-wind – are considered as mechanically uncoupled.

The formulation to calculate the dynamic response begins with the dynamic equation of motion in the physical coordinate system. A description of the variables in this and subsequent chapters can be found in Table 1.2 (Section 1.3). Equation 2.2 represents the equation of motion in the along-wind direction and Equation 2.3 represents the equation of motion in the across-wind direction.

\[
m(z)x''(z,t) + c(z)x'(z,t) + k(z)x(z,t) = F_D(z,t)
\]
\[
m(z)y''(z,t) + c(z)y'(z,t) + k(z)y(z,t) = F_L(z,t)
\]

The parameters \(m(z), c(z),\) and \(k(z)\) simulate respectively the mass, damping, and stiffness properties of the structure as a function of the height, \(z\). The quantity \(F_D(z,t)\) is the force in the along-wind direction, and \(F_L(z,t)\) is the force in the across-wind direction (see Figure 2.2). The prime in Equations 2.2 and 2.3 indicates differentiation with respect to time.

While the dynamic analysis of a structure can be done in either the time domain (Equations 2.2 and 2.3) or the frequency domain, the frequency domain approach provides a computational advantage for linear systems – computations done in the frequency domain utilize the modal approach allowing the system of equations to be dynamically uncoupled and the solution to become algebraic (Kareem, 1988, 1992). Figure 2.2 depicts the bending displacements of the structure as a result of dynamic wind loading per unit height, \(F_d(z,t)\), with \(h\) being the total height of the building. The normalized mode shape, which is not
necessarily linear, is denoted by \( \Phi_{g,d}(z) \) where \( d=x \) when solving for the along-wind response, and \( d=y \) when solving for the across-wind response.

For example, when the direction of the wind loading is orthogonal to one of the faces, (such that the mean wind direction is approximately equal to an angle of attack \( \alpha_0=0 \) for wind perpendicular to the “D-z” or “D-h” face – Figure 2.1), the dynamic loading in the along-wind direction is a combination of the mean wind load, the fluctuating load from turbulence, and the aerodynamic damping load. Since \( \bar{U}(z) \) is typically much larger than the absolute values of \( u(z,t) \) and \( \xi'_{g,x}(t) \), Equation 2.4 reduces to Equation 2.5 (Dyrbye and Hansen, 1997).

\[
F_x(z,t) = \frac{1}{2} \rho D(z) \left[ (\bar{U}(z) + u(z,t)) - \sum_{g} \Phi_{g,x}(z) \xi'_{g,x}(t) \right]^2 C_D
\] (2.4)
\[ F_y(z,t) = \frac{1}{2} \rho C_D D(z) \overline{U}^2(z) + \rho C_L D(z) \overline{U}(z) u(z,t) - \rho C_D D(z) \overline{U}(z) \Phi_{g,x}(z) \xi_{g,x}(t) \] 

In the across-wind direction, the dynamic loading is a function of the turbulence buffeting from fluctuations in the x-direction and y-direction, and the aerodynamic loading.

\[ F_y(z,t) = \rho C_L D(z) \overline{U}(z) u(z,t) + \frac{1}{4} \rho \left( \frac{dC_L}{d\alpha} + C_D \right) D(z) \overline{U}(z) v(z,t) - \frac{1}{4} \rho \left( \frac{dC_L}{d\alpha} + C_D \right) D(z) \overline{U}(z) \sum_g \Phi_{g,y}(z) \xi_{g,y}(t) \] 

In the previous equations, \( C_D \) is the dimensionless drag coefficient per unit length referenced to the initial angle of attack (coincident with mean wind direction, \( \alpha_0 \)). Similarly, \( C_L \) is the lift coefficient per unit length. The quantity \( \left. \frac{dC_L}{d\alpha} \right|_{\alpha_0} \) is the derivative of the lift coefficient corresponding to the mean angle of attack, \( \alpha_0 \).

The total displacement resulting from the wind loading, \( x_{\text{total}}(z,t) \) or \( y_{\text{total}}(z,t) \) (Figure 2.2), is given by Equation 2.7, and the system coordinates are related to the modal coordinates by Equation 2.8. The total displacement, \( d_{\text{total}}(z,t) \), in terms of generalized modal coordinates is shown by Equation 2.9.

\[ d_{\text{total}}(z,t) = \bar{d}(z) + d(z,t) \]
\[ d(z,t) = \sum_g \Phi_{g,d}(z)\ddot{\xi}_{g,d}(t) \] (2.8)

\[ d_{\text{total}}(z,t) = \sum_g \Phi_{g,d}(z) \left[ \ddot{d}_g(z) + \ddot{\xi}_{g,d}(t) \right] \] (2.9)

where \( \ddot{d}_g(z) \) is the mean coordinate of displacement of mode \( g \) and \( \ddot{\xi}_{g,d}(t) \) is the dynamic generalized coordinate of displacement of mode \( g \), in the direction of \( d \). (Kareem, 1992)

The dynamic response in the along-wind and across-wind directions will be developed through the equation of motion in terms of generalized modal coordinates (Equation 2.10).

\[ \bar{M}_{gg,d}\dddot{\xi}(t) + \bar{c}_{g,d}\ddot{\xi}(t) + \bar{k}_{g,d}\dot{\xi}(t) = F_d(z,t) \] (2.10)

where the modal mass is \( \bar{M}_{gg,d} = \int_0^h \Phi_{g,d}^2(z)m(z)dz \); the modal damping, \( \bar{c}_{g,d} \), which is a combination of structural and aerodynamic damping, is \( \bar{c}_{g,d} = 2\bar{M}_{gg,d}\omega_{0,g,d} \left( \ddot{\xi}_{st,g,d} + \ddot{\xi}_{aero,g,d} \right) \); and the modal stiffness is \( \bar{k}_{g,d} = \bar{M}_{gg,d}\omega_{0,g,d}^2 \).

By taking the Fourier transform \( \hat{\xi}(\omega) = \int_{-\infty}^{\infty} \ddot{\xi}(t)e^{-i\omega t} dt \) of Equation 2.10, the equations are re-written in the frequency domain as opposed to the time domain. The main equations for the evaluation of the dynamic response in the along-wind direction are summarized below.
\[ S_{Q_x,Q_x} (\omega) = \rho^2 C_D^2 \int_0^h \int_0^h D(z)^2 \Phi_{g,x} (z_1) \Phi_{g,x} (z_2) \overline{U}(z_1) \overline{U}(z_2) S_{uu} (\omega, z_1, z_2) dz_1 dz_2 \] (2.11)

\[ S_{uu} (\omega, z_1, z_2) = \sqrt{S_u (\omega, z_1) S_u (\omega, z_2)} * \exp \left( \frac{\omega \sqrt{C_z (z_1 - z_2)^2}}{2\pi \sqrt{U(z_1) + U(z_2)}} \right) \] (2.11a)

\[ S_{g,x,g,x} (\omega) = \sum_g \Phi_{g,x} (z) \Phi_{g,x} (z) S_{Q_x,Q_x} (\omega) \left| \overline{H}_{g,x} (\omega) \right|^2 \] (2.12)

\[ S_{xx} (\omega, z) = \sum_g \Phi_{g,x} (z) \Phi_{g,x} (z) \frac{S_{Q_x,Q_x} (\omega)}{M_{g,x}^2 \omega_0^4_{g,x} \left[ 1 - \left( \frac{\omega}{\omega_0^g} \right)^2 \right] + 2 \left( \frac{2 \omega}{\omega_0^g} \right) \left( \xi^*_m + \xi_{avg,g,x} \right)^2} \] (2.13)

\[ \sigma_x (z) = \sqrt{\int S_{xx} (\omega, z) d\omega} \] (2.14)

\[ \overline{x}_g (z) = \sum_g \left[ \Phi_{g,x} (z = h) \frac{1}{M_{g,x} \omega_0^4_{g,x}} \int_0^h 1 \rho \overline{U^2} (z) \Phi_{g,x} (z) C_p D(z) dz \right] \] (2.15)

\[ X_{peak} (z) = \overline{X} (z) + g \sigma_x (z) \] (2.16)

\[ g = \sqrt{2 \log(\omega_0^g)} + \frac{0.577}{\sqrt{2 \log(\omega_0^g)}} \] (2.16a)

Equation 2.11 corresponds to the modal force spectra as a function of circular frequency, \( \omega \), with \( C_D \) being the drag coefficient normalized with respect to the \( B \) dimension of the building. Equation 2.11a, \( S_{uu} (\omega, z_1, z_2) \), is the cross-spectrum of the along-wind turbulence, \( u \), at \( z_1 \) and \( z_2 \). The quantity \( C_z \) is the exponential decay coefficient, which is a measure of the coherence in the turbulence pressure loading (e.g., larger values of \( C_z \) indicate more coherence of pressures and therefore a larger response) (Simiu and Scanlan, 1996). It has been suggested by (Vickery, 1970) that, based on wind tunnel tests, \( C_z \) can be taken as 10
(Simiu and Scanlan, 1996). Equation 2.13 is the expansion of Equation 2.12 where $H_{ss}(\omega)$
is the mechanic transfer function (Simiu and Miyata, 2006). It represents the total response
auto-spectra, without modal cross coupling, from which the RMS response is found
(Equation 2.14). The mean response (Equation 2.15) is a projection of the mean wind forces
onto the dynamic modes. The peak response (Equation 2.16) is calculated by Davenport’s
peak factor standard equation (Davenport, 1964, 1971).

Similar formulations were derived for the across-wind direction. The final equation
employed in the numerical algorithm (described in the following chapter) is given below.
The remaining equations to calculate the response in the $y$-direction can be adapted from the
along-wind formulations developed above.

$$S_{Q_x,Q_y}(\omega) = \frac{1}{4} \rho^2 \left( \frac{dC_L}{d\alpha} + C_D \right) \int_0^h \int_0^h (D(z)^2 \Phi_{g,x}(z) \Phi_{g,y}(z) \overline{U}(z) \overline{U}(z) S_{uu}(\omega, z_1, z_2) dz_1 dz_2 \right) +
\int_0^h \int_0^h (D(z)^2 \Phi_{g,x}(z) \Phi_{g,y}(z) \overline{U}(z) \overline{U}(z) S_{uu}(\omega, z_1, z_2) dz_1 dz_2 \right) $$

(2.17)

### 2.2.2 COUPLING FORMULATION

Thus far, it has been assumed that the modes $\Phi_{g,x}(z)$ and $\Phi_{g,y}(z)$ are mutually
orthogonal modes, meaning that they do not couple. However, coupling can occur if
aerodynamic effects are considered (Chen and Kareem, 2005). The coupling equations that
follow were derived by adapting equations available in the literature for tall slender
structures (Caracoglia, 2007; Simiu and Scanlan, 1996). Additionally, the equations to
compute the coupled dynamic response are only for the case of $g=1$ (first fundamental mode) in the primary directions, along-wind and across-wind ($x$ and $y$ directions, respectively).

As shown in Section 2.2.1 (Equation 2.12), $S_{g,g}$ is a function of the modal force and the transfer function. In the coupled formulation, assuming that the turbulence in the $x$-direction and $y$-direction are uncorrelated, the matrix $S_{g,g}$ is diagonal and the same functions that were previously derived are employed (Equations 2.11 and 2.17). The transfer matrix, denoted by $E$, is a 2x2 matrix (Equation 2.20), where $\chi_{aero,12}$ and $\chi_{aero,21}$ (Equation 2.20b) represent the equivalent aerodynamic damping. Matrix $E$ includes the contribution of the fundamental modes in each direction ($x$ and $y$). When the values for equivalent aerodynamic damping are equal to zero, the responses in the $x$-direction and $y$-direction are the same as for the uncoupled case.

\[
S_{g,g} (\omega) = \left( E(\omega)^{-1} \right) \left( S_{\omega,\omega} (\omega) \right) \left( E(\omega)^{-1} \right)^T
\]  
(2.18)

\[
S_{\omega,\omega} (\omega) = \begin{bmatrix}
S_{\omega_x,\omega_x} (\omega) & 0 \\
0 & S_{\omega_y,\omega_y} (\omega)
\end{bmatrix}
\]  
(2.19)

\[
E(\omega) = \begin{bmatrix}
E_{11}(\omega) & E_{12}(\omega) \\
E_{21}(\omega) & E_{22}(\omega)
\end{bmatrix}
\]  
(2.20)

\[
E_{11}(\omega) = M_{gg,x} \left( \omega_{0_{x,x}}^2 - \omega^2 \right) + 2i\omega M_{gg,x} \omega_{0_{x,x}} \left( \xi_{s_{x,x}} + \xi_{aero_{x,x}} \right)
\]

\[
E_{12}(\omega) = 2i\omega M_{gg,x} \omega_{0_{x,y}} \left( \chi_{aero,12} \right)
\]

\[
E_{21}(\omega) = 2i\omega M_{gg,y} \omega_{0_{y,x}} \left( \chi_{aero,21} \right)
\]

\[
E_{22}(\omega) = M_{gg,y} \left( \omega_{0_{y,y}}^2 - \omega^2 \right) + 2i\omega M_{gg,y} \omega_{0_{y,y}} \left( \xi_{s_{y,y}} + \xi_{aero_{y,y}} \right)
\]  
(2.20a)
The flow of wind around a building creates a number of phenomena, one of which is vortex shedding. While this study does not currently incorporate vortex shedding, it is important to note that vortex shedding is one of the contributors to the across-wind dynamic response of a structure.

The process of shedding vortices causes forces to be exerted on a structure, which can lead to considerable oscillations in the across-wind direction when the natural frequency of the structure coincides with the frequency of the vortex shedding (ESDU, 2004). When this occurs, the across-wind and torsional motions cannot be accurately predicted because the quasi-steady theory becomes invalid (Cheng et al., 2002). For high-rise buildings, the synchronization of the frequencies is considered to produce “lock-in” motion, where the flow of the wind and the motion of the building adversely affect each other (Simiu and Miyata, 2006). If the across-wind response is found exclusively from the turbulence (lift forces), it is likely to underestimate the response since the phenomenon of lock-in will intensify the building’s motion (Cheng et al., 2002). Future developments of the proposed numerical algorithm should include the effects of vortex shedding on the response, and, in particular, the across-wind response. This thesis will present analyses mainly related to the along-wind response.
and across-wind response, induced by turbulent pressures at high wind velocity regimes, where vortex shedding has a less significant impact.

2.3 DESCRIPTION OF SIMULATIONS

The following section provides background information for the types of numerical simulations that are conducted in this research. The methods on which the simulations are based, as well as the purpose behind the simulations, is included.

2.3.1 “CLOSED-FORM” SIMULATIONS

The “closed-form” (CF) simulations employ a discrete integration scheme for the derivation of the modal force. The modal force is subsequently utilized to determine the dynamic RMS response of the building at the top floor, $z=h$, by integration (frequency by frequency) of the power spectral density of the response. The desired outcomes of these simulations are as follows (objective 2 of research, Section 1.2):

- To verify and update model and wind loading parameters, for each of the simulated building types, to be compatible with the wind tunnel response data (Melbourne, 1980; Piccardo and Solari, 2002) and/or with calculations using accepted mathematical models (Solari, 1982).
- To obtain results that justify their use as “exact” target values for use in the performance analysis of the Monte Carlo algorithm (see Section 2.3.2).
There are a number of challenges with the CF approach. Perhaps the most important, in regards to this research, is the fact that uncertainty cannot be directly incorporated. In order for a fragility analysis to be conducted, the effects of uncertainty in the characterization of the aerodynamic loading must be simulated. Another challenge of the CF approach is the computational time it takes to run a single simulation (i.e., run time). To accurately estimate the double integral in the equation for modal force (Equations 2.11 and 2.17) in a numerical form, the number of partitions in the Riemann sum must be large enough to produce accurate results. It is assumed that $\Delta z=1\,\text{m}$, making the number of partitions equal to $h/\Delta z$. When $h$ is large (i.e., for a tall building), the time that it takes to run these simulations does not rationalize its use in practical applications as a tool for the designer. A third challenge, since the integration is done frequency by frequency, is determining a suitable amount and spacing for these points, which need to be capable of “sensing” relevant local variations in the integrand function over the whole frequency domain. The number of frequency points is increased (typically by a ratio of 3:1 or 4:1) in the very low frequency range (where the effects of “background” response may play a role), and near the natural frequency (where resonance is critical).

2.3.2 Monte Carlo Simulations

In general, Monte Carlo (MC) algorithms are useful when replicating phenomena with uncertainty in the inputs (i.e., stochastic problems). As described in (Grigoriu, 2002), there are three steps to solve a stochastic problem using Monte Carlo methods. The first step is to generate a random sample of values within the definition of the stochastic problem. The
second step is to deterministically solve the function (i.e., both the modal force integral and the turbulence-induced dynamic response of the building) using the random sample generated in the first step. Finally, the third step is to statistically analyze the solutions, which are the output from the previous step. (Grigoriu, 2002)

In this research, the effects of uncertainty in the characterization of aerodynamic loading and their influence on the dynamic response are simulated through MC algorithms. The significance of this approach is related to the possibility of replicating the CF analytical methods through equivalent MC algorithms. The proposed methodology utilizes MC methods to accomplish the following:

- To accurately compute the power spectral density of the modal buffeting force (Equations 2.11 and 2.17) as a function of frequency, for each mode of the structure, by numerical integration (objective 1 of the research, Section 1.2).

- To derive statistical information about the dynamic response of a high-rise building due to the uncertainty in the characterization of dynamic loading (objective 3 of the research, Section 1.2).

By combining these two steps, we are using what we have termed a “Two-Step Monte Carlo Algorithm” to estimate the dynamic response of high-rise buildings due to turbulence-induced loading.

The MC approximation of a generic surface integral, \( \hat{f}(n, z_1, z_2) \), is found by utilizing a set of randomly generated and uniformly distributed numbers (integration points) to numerically estimate the statistical expectation of the function (Robert and Casella, 2004). In other words, the objective is to convert the integral into a problem of expected value.
calculations (Tempo et al., 2005). In the case of a double integral (as in Equations 2.11 and 2.17), there are two randomly generated samples of size \( N_{MC} \) defined on the intervals \([Z_0, Z_A]\) and \([Z_0, Z_B]\), where the variables \(Z_0\), \(Z_A\), and \(Z_B\) are the integration limits along the height of the structure. Equation 2.21 gives the general form of the equation used to approximate the double integral in Equations 2.11 and 2.17.

\[
\int_{Z_0}^{Z_A} \int_{Z_0}^{Z_B} \tilde{f}(n, z_1, z_2) \, dz_1 \, dz_2 = E\left[\tilde{f}(n, z_1, z_2)\right]\left(Z_A - Z_0\right)\left(Z_B - Z_0\right)
\]  

(2.21)

2.4 DESCRIPTIONS OF MODELED STRUCTURES

Three structures were employed in this research to achieve the objectives outlined in Chapter One: a steel chimney, a concrete chimney, and the CAARC building. While all three structures were used to complete the second objective (validation), the CAARC building was exclusively employed for the third objective (implementation). The following section provides a detailed description of each structure, including model and wind loading parameters.

2.4.1 STEEL CHIMNEY

The steel chimney is an adaptation from a numerical example in (Solari, 1988) of an unlined, welded steel stack with a variable diameter. It is assumed that this structure is in Western Massachusetts and that the terrain is fairly homogeneous and low without any considerable obstructions. This assumption gives a relatively small value for the roughness
length, \( z_0 \) (see Table 1.1 for definition). Figure 2.3 is a schematic view of the structure, which shows the tapering of the chimney from \( z=0 \text{m} \) to \( z=z_d \) according to the equation in Table 2.1. Since this is the case, the diameter is defined as a function of height (where height is denoted by \( z \)), as is the mass (see Table 2.1 for relationships). The mode shape has been defined as normalized to one at \( z=h \).

\[ D(z < z_d) \]

\[ D(z = z_d) \]

\[ D(z > z_d) \]

\[ \text{Wind} \]

\[ h \]

\[ z_d \]

\[ \text{Figure 2.3 – Schematic view of the steel chimney} \]

The direction of the wind, as shown in Figure 2.3, corresponds to the along-wind direction. The velocity power spectrum density (PSD) is given by Equation 2.22, and is hereinto referred to as the Kaimal PSD (Simiu and Scanlan, 1996). \( S_n(n, z) \) denotes the Kaimal PSD, \( n \) is the frequency, and \( f' \) is a reduced frequency (Equation 2.22a). The 3-second design wind speed for this region, at an elevation \( z=10 \text{m} \), is 45m/s (ASCE, 2005). The velocity profile, \( \bar{U}(z) \), and the friction velocity, \( u_* \), were derived using the Logarithm
Law (see Table 1.1 for definition) and the design wind speed (Simiu and Miyata, 2006). The along-wind response was only considered (for validation purposes) in the study of this structure.

\[
\frac{nS_u(n, z)}{u^*} = \frac{200 f}{\left(1 + 50 f^2\right)^{5/3}}
\]  \hspace{1cm} (2.22)

\[
f = \frac{nz}{U(z)}
\]  \hspace{1cm} (2.22a)

Table 2.1 – Model and wind loading parameters for steel chimney

<table>
<thead>
<tr>
<th>Variable[^a]</th>
<th>Value Assigned</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h) [m]</td>
<td>100</td>
</tr>
<tr>
<td>(z_d) [m]</td>
<td>32</td>
</tr>
<tr>
<td>(\Delta z) [m] (Sect. 2.2.1)</td>
<td>1</td>
</tr>
<tr>
<td>(m(z&lt;z_d)) [kg/m]</td>
<td>5000-78.125z</td>
</tr>
<tr>
<td>(m(z\geq z_d)) [kg/m]</td>
<td>2500</td>
</tr>
<tr>
<td>(D(z&lt;z_d)) [m]</td>
<td>(-3z/z_d + 7)</td>
</tr>
<tr>
<td>(D(z\geq z_d)) [m]</td>
<td>4</td>
</tr>
<tr>
<td>(\bar{v}^\ast), (z)</td>
<td>0.01</td>
</tr>
<tr>
<td>(n_{0_{v\ast}}) [Hz], (g=1)</td>
<td>0.49</td>
</tr>
<tr>
<td>(\Phi_{v\ast}) (z), (g=1)</td>
<td>((z/h)^\gamma; \gamma = 2.2)</td>
</tr>
<tr>
<td>(Z_0) [m] (Eqn. 2.21)</td>
<td>0</td>
</tr>
<tr>
<td>(Z_A = Z_B) [m] (Eqn. 2.21)</td>
<td>(h)</td>
</tr>
<tr>
<td>(\rho) [kg/m(^3)], air</td>
<td>1.25</td>
</tr>
<tr>
<td>Roughness Length, (z_0) [m]</td>
<td>0.06</td>
</tr>
<tr>
<td>(\bar{U}(z)) [m/s], Log Law (Table 1.1)</td>
<td>(2.5 u, \log(z/z_0))</td>
</tr>
<tr>
<td>(C_z)</td>
<td>10</td>
</tr>
<tr>
<td>(C_D)</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Note [^a]: Descriptions of the variables can be found in Table 1.2 (Section 1.3); some definitions can also be found in Table 1.1 (Section 1.3).
2.4.2 CONCRETE CHIMNEY

The concrete chimney was chosen because full-scale data was available (Müller and Nieser, 1975/76), and it was extensively analyzed in studies conducted by (Piccardo and Solari, 1998a, 2000, 2002; Solari and Piccardo, 2001). The chimney is located in Southern Germany in a relatively flat region, with prevailing winds from the southwest. It has a circular cross-section and continuously tapers from $z=0m$ to $z=h$. (Müller and Nieser, 1975/76). Figure 2.4 is a dimensional schematic, which illustrates that an average value of the diameter was employed in the simulations as opposed to a height dependent diameter ($D=D_{avg}=5.6m$).

![Figure 2.4 – Schematic view of the concrete chimney](image)

The data from full-scale measurements was collected and analyzed in (Müller and Nieser, 1975/76), from which the wind field parameters (Table 2.2) were derived. The
parameters employed in this research were taken from Tables 5 and 6 in (Piccardo and Solari, 2002). Initially, the Kaimal PSD (Equation 2.22) was utilized to model the wind field; however, analysis (see Section 3.4.1) showed that the Solari PSD (Solari, 1987) produced more accurate results, and was therefore employed (Equation 2.23). In Equation 2.23, $\beta_u$ is a turbulence intensity factor (Solari and Tubino, 2002). As with the chimney, the along-wind response was only considered (for validation purposes) in the study of this structure.

$$\frac{nS_u(n,z)}{u^*} = \frac{2.21 \beta_u^{2.5} f}{(1 + 3.31 \beta_u^{1.5} f)^{3/5}}$$  \hspace{1cm} (2.23)

$$f = \frac{nz}{\bar{U}(z)}$$  \hspace{1cm} (2.23a)

$$\beta_u = 6 - 1.1 \arctan \left[ \ln(z_0) + 1.75 \right]$$  \hspace{1cm} (2.23b)

**Table 2.2 - Model and wind loading parameters for concrete chimney (Piccardo and Solari, 2002)**

<table>
<thead>
<tr>
<th>Variable[a]</th>
<th>Value Assigned</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$ [m]</td>
<td>180</td>
</tr>
<tr>
<td>$\Delta z$ [m] (Sect. 2.2.1)</td>
<td>1</td>
</tr>
<tr>
<td>$m(z)$ [kg/m]</td>
<td>10,686</td>
</tr>
<tr>
<td>$D$ [m]</td>
<td>5.6</td>
</tr>
<tr>
<td>$\varepsilon_{st,e}$, $g=1$</td>
<td>0.005</td>
</tr>
<tr>
<td>$n_{0x}$ [Hz], $g=1$</td>
<td>0.26</td>
</tr>
<tr>
<td>$\Phi_{6,x}(z)$, $g=1$</td>
<td>$(z/h)^\gamma$; $\gamma = 2.15$</td>
</tr>
<tr>
<td>$Z_0$ [m] (Eqn. 2.21)</td>
<td>0</td>
</tr>
<tr>
<td>$Z_A = Z_B$ [m] (Eqn. 2.21)</td>
<td>$h$</td>
</tr>
<tr>
<td>$\rho$ [kg/m$^3$], air</td>
<td>1.25</td>
</tr>
<tr>
<td>Roughness Length, $z_0$ [m]</td>
<td>0.1</td>
</tr>
<tr>
<td>$\bar{U}(z)$ [m/s], Power Law (Table 1.1)</td>
<td>$\bar{U}(h)(z/h)^{\beta}$; $\beta = 0.15$</td>
</tr>
<tr>
<td>$C_z$</td>
<td>7</td>
</tr>
<tr>
<td>$C_D$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Note [a]: Descriptions of the variables can be found in Table 1.2 (Section 1.3); some definitions can also be found in Table 1.1 (Section 1.3).
2.4.3 CAARC BUILDING

The CAARC building is analyzed as a prototype structure in this research. This prototype had been selected by the Commonwealth Advisory Aeronautical Research Council of Australia in the early Eighties as a benchmark high-rise structure to be employed for studying its dynamic response affected by wind loading (Melbourne, 1980). Figure 2.5 shows a schematic view of the CAARC building with an indication of the main dimensions (depth $D$, width $B$, height $h$).

![Figure 2.5 – Schematic view of the CAARC building](image)

The building is a prismatic structure with a rectangular floor plan of dimensions $B=30.5m$ and $D=45.7m$. The reference wind direction, corresponding to a zero angle of incidence, coincides with the direction orthogonal to the vertical face designated as “$D-h$” in Figure 2.5. Table 2.3 summarizes the CAARC building full-scale model parameters that
were employed in the simulations. Dynamic response was derived from wind tunnel-based aeroelastic model experiments conducted in a series of laboratories (Melbourne, 1980). First mode response in the along-wind and across-wind directions were simulated by considering the information available in the literature (Melbourne, 1980). Building mass per unit length, \( m(z) \), structural frequencies, and modal damping were calibrated to represent the full-scale response of the prototype. Mode shapes (Table 2.3) were approximated as linear.

Boundary layer wind loading was simulated by analyzing the approaching wind characteristics selected as reference in (Melbourne, 1980) for the tests. Turbulence intensities (Table 2.3) in the direction parallel to the along-wind \( B \) direction \( (u) \) and parallel to the across-wind \( D \) direction \( (v) \) were also derived by calibration of the values employed in the wind tunnel (Melbourne, 1980) to model full-scale behavior. In the absence of more detailed information from wind tunnel tests, a reduction of the across-wind turbulence intensity, \( I_v \), in comparison with the along-wind turbulence intensity, \( I_u \), was postulated (Dyrbye and Hansen, 1997). The longitudinal velocity spectrum employed to represent the stationary time-dependent dynamic loading was derived by fitting experimentally observed data in the wind tunnel (Melbourne, 1980), as indicated in Equation 2.24.

\[
\frac{nS_u(n,z)}{U(z)^2I_u^2} = \frac{0.6f}{(2 + f^2)^{5/6}} 
\]

\[
f = \frac{1600n}{U(z)}
\]  

Only static coefficients of lift and drag and their first derivatives were considered in the analyses, while also neglecting any eccentricity in the wind loading, which is plausible.
for mean wind direction orthogonal to the “D-h” face. Reference values (Table 2.3) of $C_D$, $C_L$, etc. per unit length at initial angle of attack, $\alpha_0=0$ (Figure 2.1), were derived by integration of mean surface pressure coefficients measured in the wind tunnel (Melbourne, 1980) around the building along a reference strip of unit height taken at approximately two-thirds of the total height. These values were held constant along the $z$-direction for the purpose of the simulations, even though the algorithm readily allows for the parameters to locally vary with $z$.

Table 2.3 - Model and wind loading parameters for CAARC building (Melbourne, 1980)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value Assigned</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$ [m]</td>
<td>183</td>
</tr>
<tr>
<td>$\Delta z$ [m] (Sect. 2.2.1)</td>
<td>1</td>
</tr>
<tr>
<td>$m(z)$ [kg/m]</td>
<td>220,800</td>
</tr>
<tr>
<td>$D$ [m]</td>
<td>45.7</td>
</tr>
<tr>
<td>$B$ [m]</td>
<td>30.5</td>
</tr>
<tr>
<td>$\xi_{m,x}^* = \xi_{m,y}^*$, $g=1$</td>
<td>0.01</td>
</tr>
<tr>
<td>$n_{g,x} = n_{g,y}$ [Hz], $g=1$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\Phi_{g,z}(z) = \Phi_{g,y}(z)$, $g=1$</td>
<td>$(z/h)^\gamma$; $\gamma = 1.0$</td>
</tr>
<tr>
<td>$Z_0$ [m] (Eqn. 2.21)</td>
<td>0</td>
</tr>
<tr>
<td>$Z_A = Z_B$ [m] (Eqn. 2.21)</td>
<td>$h$</td>
</tr>
<tr>
<td>$\rho$ [kg/m$^3$], air</td>
<td>1.25</td>
</tr>
<tr>
<td>Roughness Length, $z_0$ [m]</td>
<td>0.5</td>
</tr>
<tr>
<td>$\bar{U}(z)$ [m/s], Power Law</td>
<td>$\bar{U}(h)(z/h)^\beta$; $\beta = 0.25$</td>
</tr>
<tr>
<td>$I_8(z)$</td>
<td>-0.055$(z/h) + 0.1533$</td>
</tr>
<tr>
<td>$I_9(z)$</td>
<td>0.75$I_8(z)$</td>
</tr>
<tr>
<td>$C_{z,u}$</td>
<td>10</td>
</tr>
<tr>
<td>$C_{z,v}$</td>
<td>0.667$C_{z,u}$</td>
</tr>
<tr>
<td>$C_D$</td>
<td>1</td>
</tr>
<tr>
<td>$C_L$</td>
<td>-0.1</td>
</tr>
<tr>
<td>$dC_D/d\alpha$</td>
<td>-1.1</td>
</tr>
<tr>
<td>$dC_L/d\alpha$</td>
<td>-2.2</td>
</tr>
</tbody>
</table>

Note [a]: Descriptions of the variables can be found in Table 1.2 (Section 1.3); some definitions can also be found in Table 1.1 (Section 1.3).

Note [b]: Derived from wind tunnel measurements; Figure 2 in (Melbourne, 1980)
2.5 References

ASCE (2005). Minimum design loads for buildings and other structures (7-05). Reston, VA, USA, American Society of Civil Engineers.


3 Chapter Three: Validation of Numerical Algorithm ("Closed-Form" Simulations)

3.1 Overview of Analyses

The simulations and analyses summarized in this chapter are related to the "closed-form" (CF) approach to determine the dynamic structural response. These were done for the purposes listed below, which were previously stated in Section 2.3.1 (objective 2 of research, Section 1.2):

- To verify and update model and wind loading parameters, for each of the simulated building types, to be compatible with the wind tunnel response data (Melbourne, 1980; Piccardo and Solari, 2002) and/or with calculations using accepted mathematical models (Solari, 1982).
- To obtain results that justify their use as "exact" target values for use in the performance analysis of the Monte Carlo algorithm (see Section 2.3.2).

The results of the preliminary investigations for the CF algorithm (briefly introduced in Section 2.3.1) will be presented in Section 3.1.1, and analyzed in Section 3.2. Objectives 1 and 2 of the CF numerical simulations (above) will be discussed according to the three building types described in the previous chapter. Each selected structure had data available for the validations of objective 2. There was no full-scale nor wind tunnel data available for the steel chimney (Section 3.3); therefore, validation was accomplished by use of a mathematical model (Simiu and Scanlan, 1996; Solari, 1982). In Section 3.4, the validation of the simulated concrete chimney was done using the available full-scale data and the
studies conducted by Piccardo and Solari (2002). Wind tunnel data, provided for the CAARC building, was used in Section 3.5. Sections 3.4 and 3.5 will be divided into two parts – the first part will address objective 1 of the CF simulations, and the second part will address objective 2. For the steel chimney, discussed in Section 3.3, only objective 2 will be addressed.

3.1.1 Preliminary Investigations

Prior to meeting the CF numerical simulation objectives, two investigations were conducted regarding the development of the algorithm. The first investigation was used to select an appropriate value for $\Delta z$. The quantity $\Delta z$ is the value used to discretize the height of the structure, which is subsequently used in the computation of the double integral (Equations 2.11 and 2.17). The selection of $\Delta z$ is important because it affects the calculation of this integral, and therefore, the response of the structure. Since the answer that is obtained through the CF simulations (for the response) will later be used as an “exact” target value for the MC simulations (second goal of CF simulations), the accuracy of the result should be considered. This investigation was only conducted for the numerical model of the steel chimney.

A second investigation was conducted to determine the number of divisions for the frequency axis. In general, the total dynamic (single-mode) response of a structure to wind excitation can be considered as the sum of the “background” response and the “resonant” response (Section 2.2). The “background” part of the response is typically of concern in the low frequency range where the natural frequencies of the structure are very different from the frequency of the wind force components; the “resonant” part of the response occurs when the
structure’s natural frequency of vibration (when single-mode truncated response is acceptable) is coincident with the frequency of the wind force components (Simiu and Miyata, 2006). Since the total RMS dynamic response, numerically computed by integration over the frequency axis \((n>0)\), is a sum of these two parts, it seemed logical to provide enhanced resolution in the ranges where “background” and “resonant” response are most significant (i.e., low frequency range and near the fundamental natural frequency of the structure). It was determined that these ranges would be defined as a function of the structure’s fundamental natural frequency \((n_0)\). The four intervals, designated by Roman numerals, are given in Table 3.1 and illustrated in Figure 3.1. Intervals I and III are divided into a larger number of discrete divisions than intervals II and IV, as they are considered to be the ranges where the “background” and “resonant” response are of concern. The exact number of divisions for each interval (indicated in Figure 3.1 by \(\Delta f_{1}\) and \(\Delta f_{2}\)) will be analyzed and discussed in Section 3.2.2. This second investigation was conducted for all three structures (the steel chimney, the concrete chimney, and the CAARC building).

**Table 3.1 – Frequency intervals employed in CF numerical simulations**

<table>
<thead>
<tr>
<th>Frequency Interval</th>
<th>Frequency Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>(n &lt; 0.5n_0)</td>
</tr>
<tr>
<td>II</td>
<td>(0.5n_0 &lt; n &lt; 0.9n_0)</td>
</tr>
<tr>
<td>III</td>
<td>(0.9n_0 &lt; n &lt; 1.1n_0)</td>
</tr>
<tr>
<td>IV</td>
<td>(1.1n_0 &lt; n &lt; 2.0n_0)</td>
</tr>
</tbody>
</table>
3.1.2 OBJECTIVE 1 OF CF SIMULATIONS: VERIFICATION AND UPDATING OF MODEL AND WIND LOADING PARAMETERS

The first objective of the CF numerical analyses, to update model and wind loading parameters, was accomplished through repeated simulations as a function of each building type. Studies focused on the concrete chimney and CAARC building. For the concrete chimney, a single study was conducted to examine the effects of the velocity spectrum on the response (Kaimal vs. Solari). In the case of the CAARC building, the studies included the derivation of reference values $C_D$, $C_L$, etc., the curve fitting of turbulence intensity data, and
the calibration of turbulence intensity in the $x$-direction to the turbulence intensity in the $y$-
direction for subsequent comparison with wind tunnel data.

### 3.1.3 **Objective 2 of CF Simulations: Result Justification**

The second objective, to justify the use of the CF results as “exact” target values, will be addressed by the analysis of the RMS response in the $x$ and $y$ directions (Sections 3.3 through 3.5). The effects of vortex shedding on the $y$-direction response will be addressed since there was discrepancy between the published data and simulated results.

### 3.2 **Results of Preliminary Investigations**

These preliminary investigations were necessary to ensure that the results obtained through the CF simulations were accurate, and that the equations employed for the numerical calculation of the dynamic response were correctly implemented. Section 3.2.1 will discuss the spatial discretization along the structure’s height (employed in the $z$-direction), and Section 3.2.2 will discuss the division of the frequency axis.

#### 3.2.1 **Discretization of the Building Height ($z$-direction)**

The first investigation determined the appropriate value for the number of partitions, $\Delta z$, used in the Riemann sum of the CF simulations; this investigation was only conducted for the steel chimney. Five simulations were performed, identical in all aspects except for $\Delta z$ ($\Delta z=0.2m, 0.5m, 1.0m, 2.0m$, and $5.0m$). Table 3.2 lists the values for the RMS response and the mean response ($\sigma_x$ and $\bar{X}$, respectively), as well as the relative run time and the relative
response, for this set of simulations. Figure 3.2 shows the total response auto-spectra, $S_{xx}$, versus frequency, $n$.

**Table 3.2 – Comparison of results for the steel chimney based on variable values of $\Delta z$**

<table>
<thead>
<tr>
<th>$\Delta z$ [m]</th>
<th>$\sigma_x$ [m]</th>
<th>$\bar{X}$ [m]</th>
<th>Relative Run Time$^{[a]}$</th>
<th>Relative Response$^{[b]}$, $\sigma_{x,rel}$</th>
<th>Relative Response$^{[b]}$, $\sigma_{x,rel}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.1336</td>
<td>0.3980</td>
<td>115</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1331</td>
<td>0.3969</td>
<td>8.10</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td>1.0</td>
<td>0.1323</td>
<td>0.3950</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>2.0</td>
<td>0.1308</td>
<td>0.3913</td>
<td>0.12</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>5.0</td>
<td>0.1267</td>
<td>0.3808</td>
<td>0.04</td>
<td>0.96</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Note$^{[a]}$: Run time is relative to results for $\Delta z=1.0$ m
Note$^{[b]}$: Error is relative to results for $\Delta z=1.0$ m

*Figure 3.2 – Comparison of $S_{xx}$ for variable values of $\Delta z$ (measured in meters) at the top of the steel chimney ($z=h$)*

It has been suggested in (Piccardo and Solari, 2002) that $\Delta z$ be taken as less than or equal to one; therefore, the run time and error were related to the results obtained for
\( \Delta z=1.0\text{m} \) (Table 3.2). For \( \Delta z<1.0\text{m} \), there is a dramatic increase in run time (compared to \( \Delta z=1.0\text{m} \)), with effectively no change (less than 1%) between the results for \( \sigma_\epsilon \) and \( \bar{X} \) (Figure 3.2 and Table 3.2). On the other hand, the relative run time for \( \Delta z=5.0\text{m} \) is incredibly small, but the relative error for both \( \sigma_\epsilon \) and \( \bar{X} \) dramatically change (in comparison to the other cases). Based on these results, the two cases that seem most desirable are for \( \Delta z=1.0\text{m} \) and \( \Delta z=2.0\text{m} \). The case of \( \Delta z=2.0\text{m} \) may appear to be the best choice because it is approximately ten times faster, with very little relative error (compared to \( \Delta z=1.0\text{m} \)), however, it was decided that \( \Delta z \) would be taken as 1.0m. This decision was primarily based on the increased resolution of the results (Figure 3.2). In addition, the discretization will only be constant throughout the height if the value of the height is divisible by \( \Delta z \) (i.e., \( h/\Delta z \)); therefore, to enhance the ease of numerical computation and implementation for any height (e.g., \( h=183\text{m} \) for the CAARC building), \( \Delta z \) was taken as 1.0m.

### 3.2.2 Discretization of the Frequency Axis

The second investigation involves the discretization of the frequency axis to derive the response PSD. The first aspect of this investigation was defining the frequency ranges that would subsequently be associated with one of the two predetermined divisions (\( \Delta f_{1} \) and \( \Delta f_{2} \)). As discussed in Section 3.1.1, the frequency intervals were defined based on reasonable assumptions of the location (along the frequency axis) of the “background” and “resonant” portions of the response. Once these four intervals were created (Table 3.1), subsequent divisions were made (denoted by \( \Delta f_{1} \) and \( \Delta f_{2} \)) to discretize each of the intervals. This is the second aspect of the investigation, which will be discussed in this section.
study was conducted for each of the three structures, and the response auto-spectra, for each structure, are shown in Table 3.3 and Figures 3.3 to 3.5.

**Table 3.3 – Comparison of results for the determination of the frequency axis discretization**

<table>
<thead>
<tr>
<th>Investigated Cases</th>
<th>Steel Chimney at ( \bar{U}(h)=45\text{m/s} )</th>
<th>Concrete Chimney at ( \bar{U}(h)=40\text{m/s} )</th>
<th>CAARC Building at ( \bar{U}(h)=50\text{m/s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case No. ( \Delta f_1, \Delta f_2 )</td>
<td>Relative ( \sigma_c ) Relative Run Time</td>
<td>Relative ( \sigma_c ) Relative Run Time</td>
<td>Relative ( \sigma_c ) Relative Run Time</td>
</tr>
<tr>
<td>(i) 20 5</td>
<td>0.96 0.14</td>
<td>0.98 0.13</td>
<td>1.00 0.13</td>
</tr>
<tr>
<td>(ii) 30 10</td>
<td>0.98 0.32</td>
<td>0.99 0.33</td>
<td>1.00 0.33</td>
</tr>
<tr>
<td>(iii) 50 20</td>
<td>1.00 1.00</td>
<td>1.00 1.00</td>
<td>1.00 1.00</td>
</tr>
</tbody>
</table>

Note: All values for RMS response and run time are relative to case (iii).

**Figure 3.3 – Comparison of \( S_{xx} \) for variable \( \Delta f_1 \) and \( \Delta f_2 \), at the top of the steel chimney \((z=h)\) with \( \bar{U}(h)=45\text{m/s} \)**
Figure 3.4 – Comparison of $S_{xx}$ for variable $\Delta f_1$ and $\Delta f_2$, at the top of the concrete chimney ($z=h$) with $U(h)=40\text{m/s}$

Figure 3.5 – Comparison of $S_{xx}$ for variable $\Delta f_1$ and $\Delta f_2$, at the top of the CAARC building ($z=h$) with $U(h)=50\text{m/s}$
Based on the results from these simulations, case (ii) was chosen with $\Delta f_1=30$ and $\Delta f_2=10$. These simulations ran approximately three times faster than case (iii) where $\Delta f_1=50$ and $\Delta f_2=20$, with an insignificant change in the RMS response. While there was an even greater gain, in respect to run time, when using case (i) ($\Delta f_1=20$ and $\Delta f_2=5$), the difference in the response was significant. This is especially true for the steel chimney, for which the “background” response plays a larger role (Figure 3.3) in comparison to the resonant component. This fact was anticipated and can be related to the structure’s much longer natural frequency, which can be associated with a lower spectral ordinate in the input turbulence spectrum (i.e., a reduced excitation level in comparison with the other structures at resonance).

Moreover, the divisions, $\Delta f_1$ and $\Delta f_2$, are not independent of the four intervals (defined in Table 3.1). These four intervals are a function of the structure’s natural frequency; therefore, as the natural frequency of the structure increases, the divisions become proportionally less over the interval. Consider the steel chimney, which has a natural frequency about two times greater than the natural frequency of the other structures. This characteristic makes the length of interval I longer for the steel chimney compared to the other structures. As an example, interval I ($n<0.5n_0$) for the steel chimney is $n<0.245$Hz, and $n<0.1$Hz for the CAARC building. For the CAARC building, the frequency steps ($0.5n_0/\Delta f_1$) are in increments of 0.0033, whereas, for the steel chimney, the frequency steps are in increments of 0.0082. As this example illustrates, there is a considerable reduction in the resolution of the auto-spectra, and subsequent calculation of the RMS response, due to a larger frequency step. This calibration was important so that the CF algorithm could be generally extended to any type of structure.
3.3 **Validation With Simulated Mathematical Model (Literature): Steel Chimney**

This section discusses the results of the simulations related to the steel chimney, and will only address objective 2. As stated in Section 3.1, there is no need to update the simulated model and wind loading parameters since they were pre-determined, and the validation does not depend on full-scale or wind tunnel data. To meet objective 2, a simulated mathematical model (i.e., closed-form equivalent structural response method) was utilized. This model estimates the along-wind response for tall structures, such as buildings, with an approximately linear fundamental mode shape. The procedure was developed by (Solari, 1982) and summarized in (Simiu and Scanlan, 1996). Solari’s mathematical model (equations not shown for brevity) was employed to evaluate the CF simulation developed in this thesis for the steel chimney, and to justify its use in the performance evaluation of the MC algorithm (Chapter 4).

Some assumptions had to be made in order to apply the known information for the steel chimney (see Section 2.4.1) to the procedure presented in (Simiu and Scanlan, 1996) for estimating the along-wind response. The equations from (Solari, 1982) employ constant dimensions; therefore, the first assumption was that the diameter, and subsequently the mass, would be constant over the height (as opposed to variable throughout the height as was defined in Section 2.4.1).

Since $C_l$ and $C_w$ (the leeward and windward pressure coefficients) were unknown, the second assumption was that the square of the reduced drag coefficient, $C_{Df}$, was approximately equal to the square of the drag coefficient, $C_D$ ($C_D \approx C_l + C_w$). Equation 3.1 shows Solari’s (1982) equation for the reduced drag coefficient, $C_{Df}$. The dimensionless
quantity $\chi \neq 0$, is a function of the building’s dimensions, roughness length, natural frequency, and friction velocity, and controls how accurate this approximation will be ($\chi = 1$ was considered).

$$C_{Df}^2 = C_w^2 + 2C_wC_i\chi + C_i^2$$ (3.1)

The third assumption was that the results of the CF simulations with a linear mode shape would be closer to the mathematical model than the results using a non-linear mode shape. This is a reasonable assumption since the equations developed by (Solari, 1982) are based on a linear mode shape.

To validate the CF simulation, it was necessary to understand the effects of a non-linear mode shape on Solari’s (1982) mathematical model. Simulations were conducted using the modeled steel chimney to compare the results from the CF simulation and Solari’s method (see Figures 3.6 and 3.7). Figures 3.6(a) and 3.6(b) show the RMS response and mean response for a non-linear mode shape, $\Phi = (z/h)^\gamma$, $\gamma = 2.2$; Figures 3.7(a) and 3.7(b) show the RMS response and mean response for a linear mode shape, $\gamma = 1.0$. It can be seen in Figures 3.6(a) and 3.7(a) that the difference between the RMS response for the CF algorithm and the mathematical model is greater for a non-linear mode shape (Figure 3.6(a)). The difference is less obvious for the mean response (Figures 3.6(b) and 3.7(b)), however, the same conclusion can be made. These conclusions are also presented in Table 3.4, which shows that for the velocities greater than 15m/s, the absolute error of the RMS response is consistently smaller for the simulations using a linear mode shape. The absolute error of the mean response, over the entire range of velocities, is also less for the simulations using a
linear mode shape. These results were anticipated since the CF simulations employing a linear mode shape would be more closely related to the mathematical model. It can be concluded that since the simulations employing a linear mode shape produced results very closely related to the mathematical model, that the results using a non-linear mode are also acceptable.

Due to the excellent correlation between the CF simulations and the mathematical model (as indicated by Figures 3.6 and 3.7, and Table 3.4), these results confirm that the estimations by the CF algorithm can be taken as “exact” target values in the performance evaluation of the MC algorithm, thus satisfying objective 2 (Section 3.1).

**Table 3.4 – Absolute error (expressed as a percentage) between the CF numerical simulations and mathematical model for the RMS response and mean response**

<table>
<thead>
<tr>
<th>$\bar{U}(h)$ [m/s]</th>
<th>Absolute Error for $\sigma_u$, ($\gamma=2.2$)</th>
<th>Absolute Error for $\sigma_u$, ($\gamma=1.0)$</th>
<th>Absolute Error for $\bar{X}$, ($\gamma=2.2$)</th>
<th>Absolute Error for $\bar{X}$, ($\gamma=1.0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.04%</td>
<td>0.05%</td>
<td>0.04%</td>
<td>0.03%</td>
</tr>
<tr>
<td>10</td>
<td>0.05%</td>
<td>0.12%</td>
<td>0.15%</td>
<td>0.13%</td>
</tr>
<tr>
<td>15</td>
<td>0.02%</td>
<td>0.14%</td>
<td>0.33%</td>
<td>0.30%</td>
</tr>
<tr>
<td>20</td>
<td>0.22%</td>
<td>0.10%</td>
<td>0.59%</td>
<td>0.54%</td>
</tr>
<tr>
<td>25</td>
<td>0.54%</td>
<td>0.00%</td>
<td>0.93%</td>
<td>0.84%</td>
</tr>
<tr>
<td>30</td>
<td>0.97%</td>
<td>0.17%</td>
<td>1.34%</td>
<td>1.21%</td>
</tr>
<tr>
<td>35</td>
<td>1.52%</td>
<td>0.41%</td>
<td>1.82%</td>
<td>1.65%</td>
</tr>
<tr>
<td>40</td>
<td>2.16%</td>
<td>0.69%</td>
<td>2.38%</td>
<td>2.16%</td>
</tr>
<tr>
<td>45</td>
<td>2.88%</td>
<td>1.01%</td>
<td>3.01%</td>
<td>2.73%</td>
</tr>
</tbody>
</table>
Figure 3.6 – Comparison of the CF numerical simulation to the mathematical model (Simiu and Scanlan, 1996) for the steel chimney with a non-linear mode shape, $\gamma=2.2$: (a) along-wind RMS response vs. velocity; (b) mean response vs. velocity

Figure 3.7 – Comparison of the CF numerical simulation to the mathematical model (Simiu and Scanlan, 1996) for the steel chimney with a linear mode shape, $\gamma=1.0$: (a) along-wind RMS response vs. velocity; (b) mean response vs. velocity
3.4  VALIDATION WITH FULL-SCALE DATA: CONCRETE CHIMNEY

3.4.1  OBJECTIVE 1 OF CF SIMULATIONS

Most of the model and wind loading parameters, which were necessary for computing the response of the concrete chimney, were derived from (Piccardo and Solari, 2002); however, the velocity power spectrum density (PSD) was not directly available. Therefore, the Kaimal PSD was used in the CF algorithm developed for the steel chimney, even though initial simulations using the Kaimal PSD had indicated that better results might be obtained if another spectrum was utilized. Based on available literature containing analyses of a nearly identical structure (Piccardo and Solari, 1998a, b, 2000), the Solari PSD (Solari, 1987) was employed. To verify this assumption, an analysis was conducted to compare the two spectra and the RMS response in the x-direction. Figure 3.8 shows the two normalized velocity PSD for $\bar{U}(h)=40\text{m/s}$, and Figure 3.9 shows the two response auto-spectra, as well as the values of the RMS response for each spectrum. Figure 3.8 indicates that there is a larger discrepancy between the two spectra at lower frequencies, which is reflected in Figure 3.9 by the difference between the response spectra. Figure 3.9 also gives the values for the RMS response corresponding to each of the spectra, and further indicates that the Solari PSD performs better than the Kaimal PSD for this structure even though, in terms of the RMS response, the differences appear relatively small. Based on this analysis, the Solari PSD was chosen for the remaining simulations in this thesis pertaining to the concrete chimney.
Figure 3.8 – Normalized Kaimal PSD vs. Normalized Solari PSD for the concrete chimney at $U(h)=40\text{m/s}$

Figure 3.9 – Comparison of the response auto-spectra using the Kaimal PSD and Solari PSD at $U(h)=40\text{m/s}$
3.4.2 Objective 2 of CF Simulations

The validity of the CF numerical simulations for the concrete chimney was based on results for both the $x$-direction (along-wind) and $y$-direction (across-wind) RMS response. Figures 3.10 and 3.11 show the full-scale data points and a trendline derived from (Piccardo and Solari, 2002), which were used to quantify the accuracy to which the CF algorithm performed. Figure 3.10 indicates that the CF algorithm is consistent with full-scale data and other investigations. The correspondence is very good in the along-wind direction, with a maximum absolute error of 5% at the highest velocity ($\bar{U}(h)=40\text{m/s}$) and an absolute error of only 1.7% at $\bar{U}(h)=30\text{m/s}$. In the across-wind direction (Figure 3.11), the results of the CF simulations are similarly correlated (less than 2% absolute error) when outside of the velocity range of 5m/s to 15m/s. The clear discrepancy in this range can be attributed to the effects of vortex shedding, which were not simulated in the CF algorithm developed in this thesis. A brief investigation of vortex shedding effects on the across-wind response of this structure was conducted to substantiate this claim and provide proper justification that the CF algorithm can be used in future aspects of this research.

The Strouhal number, $St$, relates the dominant frequency of vortex shedding, $n_s$, to the critical velocity, $\bar{U}_{c_s}$, measured at the top of the chimney, with $D$ being a reference diameter (Equation 3.2). When the critical vortex shedding velocity is reached, and $n_s=n_0$ (where $n_0$ is the natural frequency of the structure), a considerable increase in the across-wind oscillations can occur (ESDU, 1998) because of aeroelastic amplification. In the case of vortex shedding, this synchronization is referred to as “lock-in” (Simiu and Scanlan, 1996).
The Strouhal number was given for the concrete chimney in (Piccardo and Solari, 2002) as $St=0.19$. With this value known, the critical velocity was designated as $\bar{U}_c=7.7\text{m/s}$, and the range of significant inaccuracies (see Figure 3.11), in terms of the critical velocity, was defined as $0.7\bar{U}_c$ to $2.0\bar{U}_c$ from the analysis of the figure. A set of criteria was developed and presented in (ESDU, 1998) for calculating the lower and upper limits for the “lock-in” region. This criterion suggests a lower limit of $0.9\bar{U}_c$ and an upper limit of $1.1\bar{U}_c$ for this structure. The lower limit calculated from the criteria in (ESDU, 1998) was compatible with the lower limit derived from the CF simulation. The upper limit was conservatively estimated as $2.0\bar{U}_c$, but is still acceptable.

The Scruton number, $S_c$, is another indicator of dynamic amplification in this velocity range due to the effects of vortex shedding (i.e., “lock-in”). The quantity $S_c$ (Equation 3.3) is a non-dimensional parameter, which relates the structural mass, structural damping, and structural dimensions, and measures the likelihood of a structure to experience aeroelastic dynamic response (Holmes, 2001).

$$S_c = \frac{M\zeta}{\rho D^2}$$  \hspace{1cm} (3.3)

The Scruton number for this structure is approximately 1.4. For low values of $S_c$ (i.e., less than about 5), the structure experiences “lock-in”, which greatly increases the motion of the structure (Cheng et al., 2002).
Figure 3.10 – Comparison of the CF numerical simulation to the response based on full-scale data (Piccardo and Solari, 2002) for the concrete chimney (along-wind or x-direction)

Figure 3.11 – Comparison of the CF numerical simulation to the response based on full-scale data (Piccardo and Solari, 2002) for the concrete chimney (across-wind or y-direction)
Based on the near equivalence of the CF simulations with the full-scale data and trendline from (Piccardo and Solari, 2002), it can be concluded that these results qualify for use as “exact” target values.

3.5 Validation With Wind Tunnel Data: CAARC Building

3.5.1 Objective 1 of CF Simulations

The CAARC building is a prototype structure that has been selected in this study as a benchmark building because of the availability of experimental data from wind tunnel tests conducted at numerous institutions. The wind tunnel data from six research centers was reported in (Melbourne, 1980). The model and wind loading parameters, employed in the simulations of this thesis, were deduced from the results of their studies. The majority of these parameters were derived from the tests at Monash University (Australia); however, the data from other institutions was also utilized. This section includes the preliminary examinations of the unknown parameters for the CAARC building.

The first study was to determine the reference values of $C_D$ and $C_L$ (and their first derivatives) per unit length at initial angle of attack, $\alpha_0=0$ (see Figure 2.1). They were derived by integration of mean surface pressure coefficients as measured in the wind tunnel at three out of the six research centers – National Physical Laboratory (NPL, England), National Aeronautical Establishment (NAE, Canada), and The City University (England) (Melbourne, 1980). These three institutions were chosen because pressure coefficients were measured for smaller increments of the angle of attack (relative to the data from other universities). The effect of the angle of attack is important in the calculation of the first
derivatives of $C_D$ and $C_L$ since they should be referenced to the initial angle of attack, $\alpha_0=0$.

The secant method was used at an angle of attack equal to 15 degrees because additional information was not available; therefore, the first derivatives are a crude approximation and are most likely larger than the actual values. Using 15 degrees (as opposed to 30 degrees, which was the smallest increment at other institutions) reduced some of this error.

Table 3.5 – Reference values derived from various research centers

<table>
<thead>
<tr>
<th>Research Center</th>
<th>Geometric Model</th>
<th>$C_D$ [-]</th>
<th>$C_L$ [-]</th>
<th>$dC_D/,d\alpha$ [rad$^{-1}$]</th>
<th>$dC_L/,d\alpha$ [rad$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPL</td>
<td>1/240</td>
<td>0.861</td>
<td>-0.001</td>
<td>-0.6</td>
<td>-1.9</td>
</tr>
<tr>
<td>NAE</td>
<td>1/400</td>
<td>1.063</td>
<td>-0.015</td>
<td>-1.3</td>
<td>-2.6</td>
</tr>
<tr>
<td>City</td>
<td>1/690</td>
<td>0.892</td>
<td>-0.255</td>
<td>-1.5</td>
<td>-2.0</td>
</tr>
<tr>
<td>Averages</td>
<td></td>
<td>1</td>
<td>-0.1</td>
<td>-1.1</td>
<td>-2.2</td>
</tr>
</tbody>
</table>

A second study was conducted to derive a relationship between the turbulence intensity and the elevation ($z/h$) for the prototype building, parallel to the along-wind $B$ direction ($u$), through calibration of the values employed in the wind tunnel (Melbourne, 1980) to model full-scale behavior. The values were extracted from Figure 2 in (Melbourne, 1980), and plotted in Figure 3.12. Equation 3.4 defines the turbulence intensity, where $\alpha_u$ is the RMS response of the turbulent velocity fluctuations at $z$.

$$I_u(z) = \frac{\sigma_u(z)}{U(z)} \quad (3.4)$$

A regression analysis was performed using various trendlines – exponential, logarithmic, power, and linear. The results from the wind tunnel tests, provided in
(Melbourne, 1980) for the RMS response in the along-wind direction, were used as the target for this analysis (at $\bar{U}(h)=50\text{m/s}$). The linear trendline was selected because it provided results closest to this data set. In addition, the “coefficient of determination”, $R^2$, is very near to one indicating that this equation is capable of accurately approximating the data points. The equation of the trendline in Figure 3.12 was employed in the CF algorithm for calculation of $I_u$ for the prototype structure, which is needed to calculate the velocity PSD (Equation 2.24). The use of this equation for full-scale building response estimation is acceptable because of similarity scaling (laws) between full-scale and wind tunnel models (Simiu and Miyata, 2006).

![Figure 3.12](image)

**Figure 3.12 – Results of the regression analysis for the determination of the along-wind turbulence intensity ($I_u$), as a function of $z/h$**

In the absence of more detailed information from the wind tunnel tests, a third study was performed to define the across-wind turbulence intensity, $I_v$, and the across-wind force correlation decay coefficient, $C_{z,v}$. Both quantities are needed for computation of the modal force in the across-wind direction (Equation 2.17, Section 2.2.1).
A relationship was postulated in (Dyrbye and Hansen, 1997), which compared the along-wind turbulence intensity, $I_u$, with the across-wind turbulence intensity, $I_v$ (Equation 3.2). This relationship was included in the CF algorithm for the computation of the RMS response in the across-wind direction (Equation 3.5).

$$\sigma_v = 0.75 \sigma_u = 0.75 \left[ I_u(z) \overline{U}(z) \right]$$  \hspace{1cm} (3.5)

It was hypothesized in (Simiu and Scanlan, 1996), based on various wind tunnel test results, that the coefficient of decay for across-wind buffeting response determination, $C_{z,v}$, be taken as two-thirds the coefficient of decay for along-wind buffeting response determination, $C_{z,u}$. Therefore, the quantity $C_{z,v} = 0.667 C_{z,u}$ was used in the CF simulations.

### 3.5.2 **Objective 2 of CF Simulations**

Prior to the validation of the CF simulations for the CAARC building, a brief study was conducted to determine the effects of aerodynamic coupling between the $x$-direction and $y$-direction modal response (fundamental modes only) on the results (see Section 2.2.2 for the details). The findings of this study are presented in Figure 3.13. Based on this figure, it can be concluded that coupling (for the case of the CAARC building) has an insignificant effect on the results – the difference between the coupled and uncoupled curves is hardly detectable. The negligible effects of coupling might be related to an inaccurate derivation of the reference values, which play a significant role in the coupling equations. It was shown in Table 3.5 that the reference values derived from wind tunnel data, particularly for $C_L$ and $dC_L/d\alpha$, fell over a broad range. The averages of these values, which were employed in the
simulations, might not be indicative of the simulated wind effects experienced by the prototype building.

![Figure 3.13 – Comparison of the RMS response for coupled and uncoupled CF numerical simulations in the along-wind and across-wind direction](image)

Equations for the along-wind RMS, the along-wind mean response, and the across-wind RMS response were provided in (Melbourne, 1980). The equations were derived using the data from model tests conducted on the prototype building. Each of these equations was plotted against the results of the CF simulations. An example of the results is shown in Figures 3.14, 3.15, and 3.16, in which the numerically simulated response is compared to wind tunnel measurements as a function of the reduced velocity, $\bar{U}(h)/(n_0B)$. The building response in the along-wind direction was normalized with respect to the dimension $B$; in the across-wind direction, the building response was normalized with respect to the dimension $D$ (see Figure 2.5). Excellent correspondence was found in the along-wind direction, as evident from the analysis of Figure 3.14, with an absolute error of no more than 2% between the two curves. Similar results were observed in the case of the mean wind response (Figure 3.15), with a maximum absolute error of 10% at the highest analyzed velocity. Figure 3.16 shows
that there was more discrepancy when the comparison was extended to the across-wind
direction RMS response.

The large difference in the across-wind direction is most likely attributed to the
effects of vortex shedding, which was not simulated in the wind loading model employed in
this research. As mentioned in Section 3.4.2 for the concrete chimney, the Strouhal number
is a function of the critical vortex shedding velocity, \( \bar{U}_c \). Using the ratio of the building
dimensions and Figure 1 in (ESDU, 2004), the Strouhal number for the CAARC building
was estimated as \( St=0.134 \); therefore, the critical vortex shedding velocity for the CAARC
building is very high, \( \bar{U}_c=68.2 \text{m/s} \). At these velocities and Reynolds numbers, “lock-in” is
usually not possible since turbulence has the tendency of disrupting the periodic shedding of
vortices (non-simultaneous, spatially irregular vortices are shed). However, since the
Scrubton number for the CAARC building is small (estimated as \( S_c=0.85 \)), it can be postulated
that the structure may still experience some increase in building motion near the critical
velocity (Holmes, 2001), mostly because of an increment in the across-wind modal spectrum
ordinates in the proximity of the frequency, \( n_0 \). While there is not a direct correlation
between the concrete chimney and CAARC building, the vortex shedding criterion developed
for the concrete chimney (Section 3.4.2) was applied to the CAARC building. Based on this
criterion, the effects of vortex shedding should be approximately significant from 48m/s to
136m/s (corresponding to reduced velocities, \( \bar{U}_c/(n_0B) \), of 7.8 to 22.3), measured at \( z=h \).
Figure 3.16 only shows the results up to a reduced velocity of 8.2 since the maximum
velocity evaluated by the CF simulations from wind tunnel tests was \( \bar{U}(h)=50 \text{m/s} \)
(\( \bar{U}(h)/(n_0 B)=8.2 \)). In the reduced velocity range of 7.8 to 8.2, there is an approximate
absolute error of 14%, which will possibly be present until some point beyond the calculated
critical velocity of $\bar{U}_c=68.2\text{m/s}$ ($\bar{U}_c/(n_0B)=11.2$). These observations are believed to substantiate the differences that are apparent in Figure 3.16; however, further analyses would be necessary to confirm this observation, which is beyond the scope of this thesis.

**Figure 3.14** – Comparison of the CF simulation to the normalized RMS response ($\sigma_x$), for the CAARC building, derived from wind tunnel tests (Melbourne, 1980) with wind orthogonal to the “D-h” face (Figure 2.5), $B=30.5\text{m}$

**Figure 3.15** – Comparison of CF simulation to the CAARC building mean response ($\bar{X}$) derived from wind tunnel tests (Melbourne, 1980) for wind orthogonal to the “D-h” face (Figure 2.5)
In spite of the observed differences, it was concluded that the results presented in this section justify the use of the CF numerical simulation results as “exact” target values in the analysis of the MC algorithm, which will be described in the following chapters.
3.6 REFERENCES


4 CHAPTER FOUR: VALIDATION OF NUMERICAL ALGORITHM (MONTE CARLO SIMULATIONS)

4.1 OVERVIEW OF ANALYSES

The simulations and analyses provided in this chapter of the Monte Carlo (MC) numerical algorithm were performed to accomplish the following task (previously stated in Section 2.3.2):

- To accurately compute the power spectral density of the modal buffeting force (Equations 2.11 and 2.17) as a function of frequency, for each mode of the structure, by numerical integration (objective 1 of the research, Section 1.2). This task also corresponds to “Step 1” of the proposed numerical algorithm.

The investigations conducted in the following sections will serve to validate the MC algorithm, and will demonstrate that the algorithm is capable of accomplishing the above task. A number of studies were conducted for each of the three structures, which will be introduced in this section. Section 4.2 will present and describe the results of these studies, and Section 4.3 will provide an analysis of the results as they relate to the performance and implementation of the MC algorithm.

The capability of the MC algorithm was assessed through repeated MC estimations of the integral in Equation 2.11 (modal force for along-wind direction), as a function of the sample size, $N_{MC}$ (i.e., number of randomly generated MC integration points). The sensitivity study for the validation of the MC algorithm employed the “exact” target value in various numerical analyses. These assessments were used to select the optimal number of
integration points, $N_{MC}$, to minimize error and maximize effectiveness. The practical implications associated with the selection of $N_{MC}$ were also evaluated. The use of the target value in these investigations, which was obtained through the CF simulations, was substantiated by the analyses presented in Chapter 3.

There were a number of studies conducted to evaluate the performance of the MC algorithm, and the results will be presented in the form of figures and tables in Section 4.2. The first type of investigation compared the along-wind RMS response, in the form of a scatter plot, for 100 consecutive MC simulations as a function of $N_{MC}$. This figure was useful for determining the maximum relative error (corresponding to each value of $N_{MC}$), which was subsequently utilized to validate the MC algorithm. A second part of the overall study was to determine the coefficient of variation (CoV), which was used as a statistical indicator for the selection of the optimal $N_{MC}$ value. The estimator bias, another statistical indicator, was also employed in the selection of $N_{MC}$; the results of this study are presented in tables for each building type. Finally, the concepts of confidence intervals and tolerance intervals were considered to support the interpretation of the results of the MC simulations. These indicators were used to evaluate the fidelity of the MC algorithm and aid in the selection of $N_{MC}$. In the last set of tables, the average relative run times (for each value of $N_{MC}$) were used to evaluate the MC algorithm in terms of the practical implications for future implementations.
4.2 **VALIDATION OF MONTE CARLO ALGORITHM: RESULTS OF PERFORMANCE ANALYSIS**

In this section, the results of the MC simulations are presented. The CAARC building is the only structure later considered in the evaluation of Step 2 of the “Two-Step MC Algorithm” (Section 1.2 and Chapter 5); therefore, the discussion of the results tends to focus on the CAARC building, with supplementary and supporting discussion for the other structures.

Figures 4.1 through 4.3 show the main results of this study. The “scatter plots” (Figures 4.1 through 4.3) correspond to 100 consecutive MC simulations. The along-wind RMS response of the top floor of the CAARC building (Figure 4.1), computed by the MC method at velocity $\bar{U}(h)=50$ m/s (related to a reduced velocity $\bar{U}(h)/n_0B=8.2$), is compared to the “exact” value (see Figure 3.14). The maximum relative error (relative to the CF target value) varies between 7% for $N_{MC}=1,000$ and 2% for $N_{MC}=10,000$, while it is almost negligible for larger sample sizes.

Similar results were observed for the other simulated structures – the maximum relative error decreases from 8% to 4% for the steel chimney and from 7% to 3% for the concrete chimney, at $N_{MC}=1,000$ and $N_{MC}=10,000$, respectively. Again, for $N_{MC}$ greater than 10,000, the error becomes almost negligible for the two chimneys.
Figure 4.1 – Along-wind RMS response of the CAARC building at $\bar{U}(h) = 50\text{m/s}$ (Section 3.5), computed by the MC algorithm: Scatter plot of 100 consecutive MC simulations as a function of the number of MC integration points, $N_{MC}$.

Figure 4.2 – Along-wind RMS response of the steel chimney at $\bar{U}(h) = 45\text{m/s}$ (Section 3.3), computed by the MC algorithm: Scatter plot of 100 consecutive MC simulations as a function of the number of MC integration points, $N_{MC}$.
Figure 4.3 – Along-wind RMS response of the concrete chimney at $U(h)=40\text{m/s}$ (Section 3.4), computed by the MC algorithm: Scatter plot of 100 consecutive MC simulations as a function of the number of MC integration points, $N_{MC}$

The coefficient of variation (CoV), calculated for each sample population corresponding to a value of $N_{MC}$, was employed as a statistical indicator in the selection of the optimal $N_{MC}$ value (Figures 4.4 through 4.6). The CoV is the sample standard deviation, $s_n$, normalized by the sample mean, $\bar{x}_n$, where sample indicates the 100 consecutive MC simulations. As shown in Figure 4.4, the CoV for the CAARC building was very small, generally less than 2% in all cases, and almost negligible for $N_{MC}$ beyond 10,000. Moreover, there was a consistent reduction in the CoV, but the decrement was not proportional to the number of MC integration points. The same can be said of the two chimneys (Figures 4.5 and 4.6). These figures show that past $N_{MC}=10,000$, the CoV is essentially equal, independent of the building type and simulated wind scenario, and was very small for all cases (always less than 3%).
Figure 4.4 – Coefficient of variation of the along-wind RMS response of the CAARC building at $\bar{U}(h)=50\text{m/s}$ (computed by the MC algorithm) vs. the number of MC integration points, $N_{\text{MC}}$

Figure 4.5 – Coefficient of variation of the along-wind RMS response of the steel chimney at $\bar{U}(h)=45\text{m/s}$ (computed by the MC algorithm) vs. the number of MC integration points, $N_{\text{MC}}$
Figure 4.6 – Coefficient of variation of the along-wind RMS response of the concrete chimney at $\bar{U}(h)=40\text{m/s}$ (computed by the MC algorithm) vs. the number of MC integration points, $N_{MC}$

The estimator bias, another approach employed in this study, is the difference between the expected value of the estimator and the true value of the quantity that is being estimated. In this case, it is the difference between the “exact” target value (from CF simulations) and the mean of the sample, $\bar{x}_r$. Tables 4.1 through 4.3 show the bias, which was computed for each value of $N_{MC}$. In Table 4.1 (CAARC building), it can be seen that the bias for $N_{MC}=1,000$ is considerably larger than for any other sample size. On the other side of the spectrum, for large values of $N_{MC}$, the bias is also larger (relative to a middle range value such as $N_{MC}=10,000$). The assertions made for the CAARC building can also be applied to the case of the steel chimney (Table 4.2); however, the bias calculated for the concrete chimney (Table 4.3) does not behave the same for small values of $N_{MC}$ (i.e., $N_{MC}=1,000$). Unlike the CoV, a single statement (covering all the structures) cannot be made
to describe the trend of the bias; however, it can be said that the bias is relatively larger for higher values of \( N_{MC} \) (i.e., greater than \( N_{MC}=10,000 \)). Some dependence on the simulated structure and wind-loading scenario was deduced. The relationship between the bias and CoV, as it relates to the optimal selection of the number of integration points, will be discussed in the conclusions (Section 4.3).

**Table 4.1 – Bias of MC simulations for the CAARC building vs. the sample size, \( N_{MC} \)**

<table>
<thead>
<tr>
<th>( N_{MC} )</th>
<th>Bias ( \times 10^{-3} ) ( (\alpha_{x,\text{target}} - \bar{x}_d) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>1.00</td>
</tr>
<tr>
<td>5,000</td>
<td>0.64</td>
</tr>
<tr>
<td>10,000</td>
<td>0.66</td>
</tr>
<tr>
<td>50,000</td>
<td>0.77</td>
</tr>
<tr>
<td>100,000</td>
<td>0.74</td>
</tr>
</tbody>
</table>

**Table 4.2 – Bias of MC simulations for the steel chimney vs. the sample size, \( N_{MC} \)**

<table>
<thead>
<tr>
<th>( N_{MC} )</th>
<th>Bias ( \times 10^{-3} ) ( (\alpha_{x,\text{target}} - \bar{x}_d) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>2.24</td>
</tr>
<tr>
<td>5,000</td>
<td>1.79</td>
</tr>
<tr>
<td>10,000</td>
<td>1.79</td>
</tr>
<tr>
<td>50,000</td>
<td>1.79</td>
</tr>
<tr>
<td>100,000</td>
<td>1.89</td>
</tr>
</tbody>
</table>

**Table 4.3 – Bias of MC simulations for the concrete chimney vs. the sample size, \( N_{MC} \)**

<table>
<thead>
<tr>
<th>( N_{MC} )</th>
<th>Bias ( \times 10^{-3} ) ( (\alpha_{x,\text{target}} - \bar{x}_d) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>0.23</td>
</tr>
<tr>
<td>5,000</td>
<td>0.46</td>
</tr>
<tr>
<td>10,000</td>
<td>0.47</td>
</tr>
<tr>
<td>50,000</td>
<td>0.72</td>
</tr>
<tr>
<td>100,000</td>
<td>0.69</td>
</tr>
</tbody>
</table>
A final statistical investigation employed the concepts of confidence intervals and tolerance intervals. A confidence interval is used to estimate the population mean based on experimentally observed data. Equation 4.1 (below) was used to calculate the confidence limits, where $\mu$ is the population mean, $\bar{x}_o$ is the sample mean, $s_n$ is the sample standard deviation, $z^{*}$ is the extent of the normal distribution with a given probability, and $n_p$ is the number of data points that are sampled (i.e., $n_p=100$) (Walpole et al., 2002):

$$
\mu = \bar{x}_o \pm z^{*} \frac{s_n}{\sqrt{n_p}}
$$

Equation 4.1 is valid if the unknown error can be described by a normally distributed random variable. Furthermore, the standard confidence interval equation relies on the population standard deviation; however, since the latter is generally unknown, it is replaced in Equation 4.1 with the sample standard deviation. While this technically means that Equation 4.1 is an approximation of the confidence interval, for large samples (i.e., $n_p>30$) it is a fairly accurate approximation (Walpole et al., 2002). This is commonly referred to as the large-sample confidence interval (Walpole et al., 2002).

In this study, the confidence interval defined in Equation 4.1 was used to estimate the population mean for a degree of confidence equal to 95% (i.e., $z^{*}=1.96$), where $z^{*}=1.96$ comes from the fact that 95% of the total area under a standard normal distribution curve is included between $-1.96$ and $+1.96$. A single set of data points, for each value of $N_{MC}$, is used to estimate the confidence limits for the population mean. The probability that these calculated confidence limits will contain the parameter $\mu$ is 0.95. That is to say, the
confidence limits for this single, random sample are geometrically represented by the extents of the 95% confidence interval. Since the sample mean is rarely identical to the population mean, some error, $e$, will be introduced in the calculation of the interval; however, the error will be no more than $e = \frac{z^*s_n}{\sqrt{n_p}}$. The larger the sample, the less significant this error becomes. For example, when $N_{MC}=10,000$ (CAARC building), the sample standard deviation is $s_n=0.0009$, and the error is calculated as $e=0.0002$. In other words, we can be 95% confident that the true mean, $\mu$, does not differ from the sample mean by more than 0.0002. The confidence limits described in this example are shown in Figure 4.7(b).

Subsequently, calculations were done to find the tolerance intervals for each set of data points. Unlike confidence intervals, tolerance intervals are not concerned with an estimator. Rather, they are related to the definition of an interval (of values) where a single observation might be found (e.g., the result of a single assessment of the modal force PSD by MC based integrals) (Walpole et al., 2002). This is relevant to the “Two-Step MC Algorithm” because the integral associated with Step 1 is evaluated only once, and is not derived as the average of a large sample of repeated MC based integration simulations. This fact corresponds to conducting a single computation of the response. It is important that a single estimate of the integral of the modal force PSD at a given frequency, by the MC simulation, give a result as close to the “exact” value as possible; therefore, the careful selection of $N_{MC}$ becomes one of the most critical decisions in the implementation of Step 1.

To calculate the tolerance limits for a sample with a normal distribution and known population mean, $\mu$, and standard deviation, $\sigma$, the following limits can be used: $\mu \pm z^*\sigma$. If $z^* = 1.96$, these bounds (covering the middle 95% of the population of observations) are
essentially the confidence interval for a fixed proportion of the measurements (Walpole et al., 2002). When \( \mu \) and \( \sigma \) are unknown, which is typically the case, Equation 4.2 can be applied.

\[
\bar{x}_n \pm ks_n
\]  

(4.2)

The quantity \( k \) is the tolerance factor for a normal distribution. In this study, \( k \) is defined such that there is a 99% confidence that the calculated tolerance limits will contain at least 95% of the measurements, \( k=2.36 \). The limiting confidence interval (e.g., 99%) must be added to the statement since the bounds given by Equation 4.2 cannot be expected to contain any specified proportion (e.g., 95%) all of the time (Walpole et al., 2002).

The results of both studies are presented in Figures 4.7 through 4.9. The plots for \( N_{MC}=5,000 \), \( N_{MC}=10,000 \), and \( N_{MC}=50,000 \) are included for each structure. For all figures, the horizontal axis represents the number of simulations that were conducted (i.e., \( n_p=100 \)) for a given \( N_{MC} \), and the vertical axis is the along-wind RMS response. Plotted for each case are the confidence and tolerance intervals, the target value (from CF simulations), and the results for each of the 100 simulations. It can be seen in these figures that the tolerance interval is always larger than the confidence interval (with the same degree of confidence). Due to the definition of these intervals, this must always be the case (Walpole et al., 2002). All figures show a correlation between the number of integration points, \( N_{MC} \), and the tolerance intervals – as \( N_{MC} \) increases, the tolerance interval decreases. The extent of the decrease is analyzed in Tables 4.7 through 4.9. For the CAARC building, the tolerance interval is approximately 10 times longer for \( N_{MC}=1,000 \) than it is for \( N_{MC}=100,000 \). For this structure, the benefit of using more integration points is clearly shown by the results of this
investigation; however, a smaller tolerance interval should not be the only consideration employed for the selection of the optimal $N_{MC}$. Figure 4.7 also suggests that as the tolerance interval decreases, the bias error may increase as the difference between the CF target value and the estimated mean grows. Additionally, in the case of the steel chimney (Figure 4.8(c)), the tolerance limits no longer contain the CF target value. This must also be a consideration since the overall goal of the MC simulations is to obtain results close to the CF “exact” value. The results from the analyses of other two structures (Figures 4.8 and 4.9) are similar to the results of the CAARC building (Figure 4.7), and confirmed the previous observations.

In any case, an important aspect is to determine the absolute difference between the CF target value and the sample mean of its estimator (or the confidence limits); consequently, error analysis was used to further investigate this aspect (Tables 4.7 through 4.9). For the CAARC building, the maximum relative error decreases and then is essentially the same for $N_{MC} \geq 10,000$. In the case of the steel chimney, the maximum relative error consistently decreases until after $N_{MC} = 50,000$. The concrete chimney behaves similarly except the error begins to decrease after $N_{MC} = 10,000$. 

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Figure 4.7 – Tolerance and confidence intervals for the along-wind RMS response of 100 MC simulations (CAARC building): (a) $N_{MC}=5,000$; (b) $N_{MC}=10,000$; (c) $N_{MC}=50,000$
Figure 4.8 – Tolerance and confidence intervals for the along-wind RMS response of 100 MC simulations (steel chimney): (a) $N_{MC}=5,000$; (b) $N_{MC}=10,000$; (c) $N_{MC}=50,000$
Figure 4.9 – Tolerance and confidence intervals for the along-wind RMS response of 100 MC simulations (concrete chimney): (a) \( N_{MC}=5,000 \); (b) \( N_{MC}=10,000 \); (c) \( N_{MC}=50,000 \)
### Table 4.7 – Supplementary information from the computed tolerance and confidence intervals for varying sample size, $N_{MC}$ (CAARC building)

<table>
<thead>
<tr>
<th>$N_{MC}$</th>
<th>Relative Length of Tolerance Interval$^{[a]}$</th>
<th>Maximum Relative Error for Confidence Interval$^{[b]}$</th>
<th>Minimum Relative Error for Confidence Interval$^{[b]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>10.1</td>
<td>1.19%</td>
<td>0.32%</td>
</tr>
<tr>
<td>5,000</td>
<td>5.0</td>
<td>0.70%</td>
<td>0.27%</td>
</tr>
<tr>
<td>10,000</td>
<td>3.1</td>
<td>0.64%</td>
<td>0.37%</td>
</tr>
<tr>
<td>50,000</td>
<td>1.4</td>
<td>0.65%</td>
<td>0.53%</td>
</tr>
<tr>
<td>100,000</td>
<td>1.0</td>
<td>0.63%</td>
<td>0.49%</td>
</tr>
</tbody>
</table>

Note [a]: Interval length relative to interval length at $N_{MC}=100,000$.  
Note [b]: Error relative to CF target value.

### Table 4.8 – Supplementary information from the computed tolerance and confidence intervals for varying sample size, $N_{MC}$ (steel chimney)

<table>
<thead>
<tr>
<th>$N_{MC}$</th>
<th>Relative Length of Tolerance Interval$^{[a]}$</th>
<th>Maximum Relative Error for Confidence Interval$^{[b]}$</th>
<th>Minimum Relative Error for Confidence Interval$^{[b]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>10.6</td>
<td>2.26%</td>
<td>1.18%</td>
</tr>
<tr>
<td>5,000</td>
<td>5.5</td>
<td>1.66%</td>
<td>1.10%</td>
</tr>
<tr>
<td>10,000</td>
<td>3.6</td>
<td>1.56%</td>
<td>1.20%</td>
</tr>
<tr>
<td>50,000</td>
<td>1.5</td>
<td>1.46%</td>
<td>1.30%</td>
</tr>
<tr>
<td>100,000</td>
<td>1.0</td>
<td>1.53%</td>
<td>1.38%</td>
</tr>
</tbody>
</table>

Note [a]: Interval length relative to interval length at $N_{MC}=100,000$.  
Note [b]: Error relative to CF target value.

### Table 4.9 – Supplementary information from the computed tolerance and confidence intervals for varying sample size, $N_{MC}$ (concrete chimney)

<table>
<thead>
<tr>
<th>$N_{MC}$</th>
<th>Relative Length of Tolerance Interval$^{[a]}$</th>
<th>Maximum Relative Error for Confidence Interval$^{[b]}$</th>
<th>Minimum Relative Error for Confidence Interval$^{[b]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>6.3</td>
<td>0.77%</td>
<td>0.36%</td>
</tr>
<tr>
<td>5,000</td>
<td>2.9</td>
<td>0.68%</td>
<td>0.16%</td>
</tr>
<tr>
<td>10,000</td>
<td>2.2</td>
<td>0.62%</td>
<td>0.23%</td>
</tr>
<tr>
<td>50,000</td>
<td>1.0</td>
<td>0.74%</td>
<td>0.57%</td>
</tr>
<tr>
<td>100,000</td>
<td>1.0</td>
<td>0.77%</td>
<td>0.49%</td>
</tr>
</tbody>
</table>

Note [a]: Interval length relative to interval length at $N_{MC}=100,000$.  
Note [b]: Error relative to CF target value.
Practical implications, associated with the selection of $N_{MC}$, were a final issue analyzed by this study. Consequently, the run time of the algorithm was considered as a second indicator of performance so that future implementations, such as a numerical tool for the practicing engineer, are plausible. Tables 4.4 through 4.6 summarize the effects of $N_{MC}$ on the run time of the MC algorithm. Values of run time were normalized with respect to the time for $N_{MC}=100,000$, and, for comparison purposes, the relative time corresponding to the CF simulation is indicated. In the case of the CAARC building, the CF simulation took almost three times longer than the longest MC simulation, and, in particular, about 120 times longer than the MC simulation for $N_{MC}=10,000$. The concrete chimney simulations had similar results, with a CF simulation that took about 90 times longer than the MC simulation for $N_{MC}=10,000$. In contrast, the steel chimney’s run time was faster using the CF simulation than the MC simulations for $N_{MC}=50,000$ and $N_{MC}=100,000$. This discrepancy can be explained by the fact that the steel chimney is about half the height of the concrete chimney and CAARC building. The calculation of the integral (Equation 2.11) is dependent on $\Delta z$ and the height, $z$. When $\Delta z$ is constant, the height is the sole factor affecting the calculation run time, therefore, for a shorter structure (steel chimney), the run time will be significantly reduced.
### Table 4.10 – Average run time for the CAARC building (Section 2.4.3) vs. the sample size, N_{MC}

<table>
<thead>
<tr>
<th>N_{MC}</th>
<th>Average Relative Run Time^[a]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>0.002</td>
</tr>
<tr>
<td>5,000</td>
<td>0.012</td>
</tr>
<tr>
<td>10,000</td>
<td>0.024</td>
</tr>
<tr>
<td>50,000</td>
<td>0.265</td>
</tr>
<tr>
<td>100,000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Closed-form 2.775

Note \^[a]: Run time relative to N_{MC}=100,000.

### Table 4.11 – Average run time for the steel chimney (Section 2.4.1) vs. the sample size, N_{MC}

<table>
<thead>
<tr>
<th>N_{MC}</th>
<th>Average Relative Run Time^[a]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>0.001</td>
</tr>
<tr>
<td>5,000</td>
<td>0.007</td>
</tr>
<tr>
<td>10,000</td>
<td>0.016</td>
</tr>
<tr>
<td>50,000</td>
<td>0.212</td>
</tr>
<tr>
<td>100,000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Closed-form 0.191

Note \^[a]: Run time relative to N_{MC}=100,000.

### Table 4.12 – Average run time for the concrete chimney (Section 2.4.2) vs. the sample size, N_{MC}

<table>
<thead>
<tr>
<th>N_{MC}</th>
<th>Average Relative Run Time^[a]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>0.003</td>
</tr>
<tr>
<td>5,000</td>
<td>0.014</td>
</tr>
<tr>
<td>10,000</td>
<td>0.031</td>
</tr>
<tr>
<td>50,000</td>
<td>0.360</td>
</tr>
<tr>
<td>100,000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Closed-form 2.781

Note \^[a]: Run time relative to N_{MC}=100,000.
4.3 Validation of Monte Carlo Algorithm: Discussion of Results

In the analysis of the results, the main considerations were the validation of the MC algorithm and the selection of \( N_{\text{MC}} \). The validation of the MC algorithm is apparent in every analysis presented in Section 4.2. In particular, Figure 4.1 through 4.3 show that (with relatively negligible error), the MC algorithm is capable of computing the response within the same order of magnitude. The remainder of this section focuses on the selection of \( N_{\text{MC}} \). Since the selection of \( N_{\text{MC}} \) is dependent upon correct results, the following discussions cover both considerations of this study; therefore, further discussion, related solely to the validation of the results (Section 4.2), will not be conducted.

In order to select the best possible value for \( N_{\text{MC}} \), it was necessary to find the optimal balance between performance (i.e., accuracy) and practicality. Without optimal performance of the MC algorithm, the results would lack meaning in practical applications; however, the extensive computation times, that may be associated with seemingly optimal “performance” levels, might also make the algorithm null in practice. Finding the perfect combination of accuracy and functionality was necessary in determining the optimal value for \( N_{\text{MC}} \), and was the goal of the analyses in Section 4.2.

By combining all the results from the numerous studies, the optimal number of integration points for these three structures (and in particular, for the CAARC building) was chosen as \( N_{\text{MC}}=10,000 \). While the three structures analyzed in this thesis provided the same conclusion (\( N_{\text{MC}}=10,000 \)), this may not always be the case. The selection of \( N_{\text{MC}} \) should not be considered independent of the structure; for that reason, some preliminary studies ought to be conducted to determine specific information about the structure. The types of studies presented in this chapter could be used to determine the error and its importance (i.e., an
error analysis). If this error analysis were conducted, it would be reasonable to set a guideline, for example, that the maximum relative error for confidence intervals (Figures 4.7 through 4.9) should be less than about 2%. However, if an error analysis were not conducted, the studies of this chapter (on three structures with varying model and wind loading parameters) demonstrated that $N_{MC}=10,000$ is a good approximation and a viable starting point. Furthermore, this observation is compatible with the definition of the expectation, $E\left[ f(n,z_1,z_2) \right]$ (Equation 2.21), since this quantity appears to be marginally affected by the size of the interval of integration. In other words, this quantity seems to be independent of height because of a subsequent re-scaling by the effective volume in Equation 2.21, which corresponds to a normalization of the expectation. The following discussion presents the key factors used to arrive at this optimal value for $N_{MC}$.

The first set of figures (4.1 through 4.3) indicated that there might be some bias in the integration estimator results (a variation from the expected or target value). As shown in Tables 4.1 through 4.3, the bias is relatively larger for higher values of $N_{MC}$ (i.e., greater than $N_{MC}=10,000$). The increasing bias (for higher values of $N_{MC}$) could be attributed to the generation of the random sample. Since a computer program generates the integration points, the sample may not be perfectly random as its size increases (pseudo-random number generation algorithm). As the sample size becomes larger, there is an increased chance that the generated points may be replicated. This is especially true for a sample that has finite boundaries (i.e., the height of the structure). In contrast to the bias, Figures 4.4 through 4.7 showed that the coefficient of variation decreased as $N_{MC}$ increased, indicating that as the sample size increases, the CoV does not experience a proportional decrease. Subsequently, there was relatively no improvement for $N_{MC}$ greater than 10,000 (CoV for $N_{MC}=50,000$ and
$N_{MC}=100,000$ were essentially equal). Based on the interpretation of these two results, it seems that there is an inverse relationship between the CoV and the bias, and $N_{MC}=10,000$ satisfied the accuracy condition for both of them.

The next set of investigations (Figures 4.7 through 4.9 and Tables 4.7 through 4.9) builds upon the previous studies with more a thorough statistical analysis. The comparison of the tolerance and confidence intervals yielded similar conclusions to those made about the relationship between the bias and CoV. While the tolerance interval consistently becomes smaller with increasing $N_{MC}$, the same cannot be said of the relative error derived from the confidence intervals (Tables 4.7 through 4.9). It was concluded that the degree of resolution (the extents of the tolerance limits) holds as much importance as the precision of the results (closeness of results to CF simulation). Again, $N_{MC}=10,000$ provides the best combination of resolution and precision.

In the final study, which considered the simulation run time (Tables 4.10 through 4.12), it seems that $N_{MC}$ less than 10,000 would be ideal in practical applications because the computation time is greatly reduced. However, significant error and variance are introduced (Figures 4.1 through 4.6), and this does not satisfy the accuracy requirement for design purposes. Furthermore, the relative run times associated with $N_{MC}=10,000$ were always less than the relative run times of the corresponding CF simulations.

As the studies in this chapter have shown, the performance of the MC algorithm is optimized for $N_{MC}=10,000$. This value will be employed in Step 1 of the “Two-Step MC Algorithm” for the CAARC building.
4.4 References

5 CHAPTER FIVE: IMPLEMENTATION OF “TWO-STEP MONTE CARLO ALGORITHM”

5.1 OVERVIEW OF ANALYSES

This chapter will discuss the implementation of the “Two-Step Monte Carlo Algorithm”. The previous chapter introduced and validated the application of the MC numerical algorithm in the computation of the power spectral density of the modal buffeting force (Step 1). Step 2, which involves the derivation of statistical information about the dynamic response due to the uncertainty in the characterization of dynamic wind loading, is employed in this chapter. By combining Steps 1 and 2, a fragility analysis can be conducted on a tall building under the influence of high winds. A fragility analysis of the CAARC prototype building (objective 3 of the research, Section 1.2) will be the focus of the studies in this chapter.

This section provides background information related to the two-step MC algorithm. Section 5.2 presents the studies and analyses related to Step 2 of the procedure, as well as the results of the fragility analysis. Section 5.3 contains an exploratory example of a performance assessment, which applies the fragility curves developed using the two-step MC algorithm (Section 5.2).

5.1.1 FRAGILITY ANALYSIS

Fragility analysis is a standardized methodology, utilized for performance-based structural design. As a general statement, fragility curves measure (or quantify) the overall
structural vulnerability (Norton et al., 2008). The likelihood of structural damage due to different levels – velocity levels in the case of wind engineering – is usually expressed by a fragility curve (Saxena et al., 2000). A collection of these curves describes the (conditional) probability of exceedance of representative indicators (the capacity), corresponding to a specific feature of the dynamic response, at a given wind velocity (the demand) (Bashor and Kareem, 2007; Ellingwood, 2000; Filliben et al., 2002). A set of thresholds is usually selected to represent different levels of structural performance derived from such indicators. These indicators are either required or are prescribed by the designer, and can include inter-story drift ratios, maximum lateral drift, and acceleration levels for occupant comfort (Bashor and Kareem, 2007; Filliben et al., 2002).

The fragility curves were developed in this thesis by deriving (numerically) the histogram, and the subsequent probability density function (PDF), from the along-wind RMS results of the two-step MC algorithm. The cumulative probabilities were then used to find the discrete probability of exceedance, represented by Equation 5.1, which corresponds to the vertical axis of the fragility plots. Equation 5.1 relates the structural performance (e.g., displacement) to a pre-defined threshold, and reads as follows: The probability that X (the computed response – e.g., RMS and peak response) exceeds T (a pre-defined threshold or limit state), given that $V_z = \bar{U}(h)$, where $V_z$ is the mean velocity experienced by the building at height $z$.

$$F_z = P[X > T | V_z = \bar{U}(h)]$$

(5.1)
It is important to select realistic limit states since these values are directly related to the development of the fragility curves (Erberik and Elshai, 2004). The dimensional thresholds or limit states used in this research (T1 to T5) were defined as a function of the building width ($B=30.5\text{m}$, see Figure 2.5), variable between $0.10\%B$ to $1.50\%B$. These thresholds, listed in Table 5.1, were selected as a first attempt at defining appropriate and realistic limit states. Threshold T5 may be associated with the maximum lateral drift, usually acceptable in the context of dynamic full-scale analyses of building response under wind excitation (Chan et al., 2007); the total drift of threshold T5 also corresponds to an inter-story drift ratio of $1/400$ for the CAARC building (Chan et al., 2007).

**Table 5.1 – Thresholds for use in the fragility analysis of the CAARC building**

<table>
<thead>
<tr>
<th>Label</th>
<th>Threshold Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>$0.10%B = 0.0305\text{m}$</td>
</tr>
<tr>
<td>T2</td>
<td>$0.25%B = 0.0763\text{m}$</td>
</tr>
<tr>
<td>T3</td>
<td>$0.50%B = 0.1525\text{m}$</td>
</tr>
<tr>
<td>T4</td>
<td>$1.00%B = 0.3050\text{m}$</td>
</tr>
<tr>
<td>T5</td>
<td>$1.50%B = 0.4575\text{m}$</td>
</tr>
</tbody>
</table>

5.1.2 **Factors of Uncertainty**

In the analysis of the wind-induced response of tall buildings, there is considerable uncertainty as a result of the unpredictable modeling of the wind (Bashor and Kareem, 2007; Kareem, 1988). In this thesis, the effects of random aerodynamic static coefficients and random exponential decay coefficients were the uncertainty factors being analyzed; other factors of uncertainty were not analyzed at this stage. Emphasis was given to the drag
coefficient, $C_D$, and the exponential decay coefficient of the along-wind turbulence cross-correlation along the building height, $C_z$. The $C_z$ coefficient is an indirect measure of the imperfect correlation of the along-wind turbulence loading in the $z$-direction under quasi-steady hypothesis for the loading (Section 2.2.1).

In Section 3.5, it was shown that the derivation of the drag coefficient, $C_D$, and the turbulence intensity (which is indirectly related to $C_z$), for the CAARC building, was subject to assumptions. Furthermore, had another person been given the same data from (Melbourne, 1980), their interpretation could have produced different values for these coefficients. The derived values of the drag coefficient (shown in Table 3.5 for three out of the six university laboratories), indicates that the drag coefficient varies for each university. The $C_z$ coefficient is usually determined experimentally (Simiu and Scanlan, 1996); however, in the absence of a more specific investigation in (Melbourne, 1980), it was assumed. The $C_z$ estimation was based on literature results for line-like structure, such as tall buildings; however, it is recognized that the quantity $C_z$ is an inherent source of uncertainty in structural engineering applications because of a lack of documentation (Simiu and Scanlan, 1996).

### 5.1.3 Two-Step MC Algorithm

The two-step MC algorithm is initialized with the generation of a uniformly distributed random sample (i.e., $C_D$ and $C_z$). The sample is converted to a gamma distribution through the calculation of the dispersion parameters (namely the shape and rate parameters, where the rate parameter is the inverse of the scale parameter), by a predetermined mean and standard deviation. The derivation method used to find the mean and standard deviation differs for $C_D$ and $C_z$ (see Section 5.2.1). Each iteration, of the overall
simulation, calculates the PSD of the modal buffeting force using MC integration methods (Step 1) for a single realization of a random variable \((C_D\) or \(C_z\)) from the previously generated sample set (Step 2). The proposed two-step MC algorithm is employed for the derivation of the fragility curves (Section 5.2), which were ultimately used as support for the performance assessment of the CAARC building (Section 5.3).

5.2 Fragility Analysis Using the “Two-Step Monte Carlo Algorithm”

The primary analyses were related to the along-wind response estimation due to buffeting for wind direction perpendicular to the “D-h” face of the CAARC building (see Figure 2.5). While the main objective of the studies was to numerically compute the fragility curves (Section 5.2.2), another objective was the further analysis of the proposed numerical MC algorithm (Section 5.2.1).

5.2.1 Analysis of “Two-Step Monte Carlo Algorithm”

Studies were conducted for the following three scenarios of uncertainty: 1) randomized \(C_D\), 2) randomized \(C_z\), and, 3) concurrently randomized \(C_D\) and \(C_z\). Figures 5.1 through 5.4 show typical results of the analyses conducted on the CAARC prototype building. The two-step proposed MC algorithm is employed for numerically deriving the histogram (Figure 5.1), and the subsequent probability density function (PDF), of the along-wind response for a random \(C_D\) coefficient (Figures 5.2 and 5.3) and \(C_z\) coefficient (Figure 5.4). Results for the mean velocity \(\bar{U}(h)=50\text{m/s}\), referenced at \(z=h\), are shown. In the following descriptions, ‘input’ refers to the quantities (i.e., the pre-determined mean and
standard deviation) that were employed to generate random samples of $C_D$ and $C_z$; ‘output’ refers to the actual quantities that were generated as a part of the two-step MC procedure, and the results that were calculated (i.e., RMS response).

Figure 5.1 depicts, as an example, a histogram obtained from the output of the proposed two-step MC algorithm for a randomized $C_D$. The histogram was created using 20 bins. Selection of the number of bins was based on the continuity and regularity of the PDF, empirically extracted from the histogram. Increasing the number of bins usually corresponds to an increment in the resolution of the PDF, but also to an increment in the local variations of the PDF between consecutive points, leading to an unreasonable (physically unacceptable) distribution.

![Histogram of Drag Coefficient](image)

*Figure 5.1 – Example histogram for randomly generated drag coefficient, $C_D$ at $\bar{U}(h)=50\text{m/s}*$

Table 5.2 summarizes the input and output quantities of the mean and standard deviation for the generated random samples of reference values, as well as the output quantities for the RMS response. These output values were employed to calculate the
probability-based PDFs, which are labeled as “probabilistic model” in Figures 5.2 through 5.4. The shape and rate parameters, used to develop the gamma distribution, were calculated using the output mean and standard deviation. This probability-based PDF is compared to the MC-based PDF, which was derived from the histogram.

Table 5.2 – Inputs and outputs for randomized reference values

<table>
<thead>
<tr>
<th>Randomized Reference Value</th>
<th>Input for Reference Values</th>
<th>Output for Reference Values</th>
<th>Output for RMS Response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>St. Dev.</td>
<td>Mean</td>
</tr>
<tr>
<td>$C_D$</td>
<td>1.00</td>
<td>0.30</td>
<td>0.998</td>
</tr>
<tr>
<td>$C_z$</td>
<td>10.25</td>
<td>2.00</td>
<td>10.248</td>
</tr>
<tr>
<td>$C_D / C_z$ [a]</td>
<td>1.00 / 10.25</td>
<td>0.30 / 2.00</td>
<td>1.003 / 10.252</td>
</tr>
</tbody>
</table>

Note [a]: Since both $C_D$ and $C_z$ were randomized, the values for the mean and standard deviation, respectively, are separated using the forward slash.

Figure 5.2 analyzes the effects of a random static drag coefficient per unit height, assumed as a constant throughout the height, and simulated as a gamma-type random variable with input mean 1.0 and input standard deviation 0.3. The mean and the standard deviation were derived through comparison of the wind tunnel tests conducted at various research institutions and described in (Melbourne, 1980) (see Section 3.5).

Figure 5.3 shows the PDF of the RMS response, $\alpha$, at the top of the building, based on the simulations conducted for a random $C_D$. Since there is a linear dependence between $C_D$ and the modal force (Equation 2.11), the MC-based PDF of the RMS response should also follow a gamma distribution. As shown in Figure 5.3, there are minimal differences between the MC-based PDF and the gamma-type PDF (probabilistic model), which was derived from the output mean and standard deviation (Table 5.2). This excellent
correspondence between the two curves serves as further verification of the proposed methodology.

Figure 5.2 – Comparison of probability-based PDF to MC-based PDF (Step 2) for the drag coefficient, $C_D$, at $\bar{U}(h)=50\text{m/s}$

Figure 5.3 – Comparison of probability-based PDF to MC-based PDF (Step 2) for the along-wind RMS response, $\sigma_\alpha$, at $\bar{U}(h)=50\text{m/s}$
As mentioned in Section 5.1.2, there was no information provided in (Melbourne, 1980) regarding the exponential decay coefficient; consequently, a short investigation was conducted to determine the input mean and standard deviation. A number of research activities have shown that $C_z=10$ is appropriate (Simiu and Scanlan, 1996); therefore, a mean value of 10 was selected as a starting point in this investigation. Other studies have shown that $C_z$ can be as low as 7 (Simiu and Scanlan, 1996), and can also be larger than 10 (e.g., 14), so these values were used as the upper and lower limits of the cumulative probability, $P[7<C_z<14]$. It was decided that $P[7<C_z<14]$ should be at least 90%, and, using an iterative approach, the mean and standard deviation of the target random variable ($C_z$) were adjusted so that this condition was satisfied. As a result of this study, the input mean and standard deviation were selected as 10.25 and 2.0, respectively. Figure 5.3 give the PDFs related to this investigation.

![Figure 5.4](image_url)  
*Figure 5.4 – Comparison of probability-based PDF to MC-based PDF (Step 2) for the decay coefficient, $C_z$, at $\overline{U}(h)=50\text{m/s})*
Based on the results of these analyses, depicted in Figures 5.2 through 5.4, it was concluded that the proposed two-step MC algorithm is capable of generating appropriate random samples and of accurately computing the response.

### 5.2.2 Results of Fragility Analysis

As previously discussed, the fragility analysis of the CAARC building relates exceedance probabilities (vertical axis) to the wind velocity (horizontal axis), and are computed by incorporating the effects of uncertainty in the description of the aerodynamic loading. The thresholds or limit states (Table 5.1) were pre-selected based on available information, and the performance analysis (Section 5.3) will have to be used to determine their applicability to design. The fragility curves for each of the three scenarios of uncertainty ($C_D$, $C_z$, and combination of $C_D$ and $C_z$) are presented in this section.

An example of fragility curves for an uncertain distribution of $C_D$, as a function of velocity, $\bar{U}(h)$, is shown in Figure 5.5. Figure 5.5(a) shows the along-wind RMS response at the top of the building; Figure 5.5(b) shows the along-wind peak response derived from Figure 5.5(a) by application of Davenport’s peak factor standard equation (Davenport, 1964, 1971). The interval of velocities coincides with medium to severe wind velocities for the CAARC building between 25m/s and 60m/s, measured at the top floor ($z=h$). Similar fragility curves were developed for the other two scenarios: uncertain $C_z$ (Figure 5.6) and combination of uncertain $C_D$ and $C_z$ (Figure 5.7). In Figures 5.5 through 5.7, it is important to note that the fragility curves are shown as “splines”, created from discrete data points, which are highlighted by markers in each figure.
Figure 5.5 – Example of fragility curves for uncertain distribution of $C_D$ as a function of velocity $\bar{U}(h)$: (a) along-wind RMS response; (b) peak response. Note: $T1$ to $T5$ are defined in Table 5.1, and are the same for both figures.
Figure 5.6 – Example of fragility curves for uncertain distribution of $C_z$ as a function of velocity $U(h)$: (a) along-wind RMS response; (b) peak response. Note: $T1$ to $T5$ are defined in Table 5.1, and are the same for both figures.
Figure 5.7 – Example of fragility curves for uncertain distributions of both $C_D$ and $C_z$ as a function of velocity $\bar{U}(h)$: (a) along-wind RMS response; (b) peak response. Note: T1 to T5 are defined in Table 5.1, and are the same for both figures.
The slope of a fragility curve can be associated with a measure of the uncertainty in the structural behavior for that threshold (Ellingwood, 2000). In this example for the CAARC building, with an uncertain distribution of $C_z$, it seems that the structural uncertainty for thresholds T4 and T5 is very high (slope is nearly one) for the case of the peak response (Figure 5.6(b)). Thresholds T1 through T3, for this case, have very little structural uncertainty with a slope close to zero. Furthermore, for thresholds T1 through T3 (which represent extremely small vibrations), the probability of exceedance is essentially 100% over the entire range of velocities (25m/s to 60m/s).

A secondary study was conducted for $C_z$ based on these results. In this additional investigation on $C_z$, the input mean remained 10.25, but the input standard deviation was incremented and set to 5.0 (almost three times the original input standard deviation of 2.0, see Section 5.2.1). The results of this study (Figure 5.8) confirm that $C_z$ marginally affects the fragility curve of the displacement at the top of the building, even in the presence of very large perturbation.

In spite of the previous discussion regarding an uncertain $C_z$, it appears in Figure 5.9 that there are negligible contributions from $C_z$ on the combined fragility analysis, which incorporates the effects of random $C_D$ and $C_z$. However, in a general performance analysis, it is important to consider the randomization of both variables due to the inherent unpredictability of the dynamic response for some of the threshold levels (Figures 5.7 and 5.8). Therefore, the combination of uncertain $C_D$ and $C_z$ was later used in the performance analysis (Section 5.3) to account for both types of uncertainty in the characterization of the wind loading.
Figure 5.8 – Results of additional study (standard deviation of $C_z$ equal to 5.0) – fragility curves for uncertain distribution of $C_z$ as a function of velocity $\bar{U}(h)$: (a) along-wind RMS response; (b) peak response. Note: $T1$ to $T5$ are defined in Table 5.1, and are the same for both figures.
Figure 5.9 – Comparison of fragility curves for random $C_D$ (dashed lines) to fragility curves for uncertain distributions of both $C_z$ and $C_D$, as a function of velocity $\bar{U}(h)$: (a) along-wind RMS response; (b) peak response.
Utilizing Figure 5.9 to draw conclusions, it can be seen that if the RMS response is employed as a measure of performance, and T5 is selected as the reference threshold for design purposes, the probability of exceedance is very low – below 1% even during severe wind conditions for dynamic vibration. However, if the peak response is the criterion, which includes the effects of the mean drift, the probability of exceedance becomes large (about 90% at $\bar{U}(h)=50\text{m/s}$). This fact may possibly lead to unacceptable drift ratios, which are mainly associated with the mean wind pressure and force distribution in this example of the CAARC building.

Even though this example suggests a predominant effect by the static wind load on the probability of exceedance for the CAARC building, the contribution of the dynamic response and of $\sigma$ cannot be ruled out for tall flexible structures in general. Therefore, fragility curves, which also incorporate structural dynamic response, seem preferable in the context of design against high winds.

### 5.3 Exploratory Performance Analysis of the CAARC Building

A preliminary and exploratory example of a performance-based (risk-based) analysis methodology is presented in this section. Due to the nature of this study, the results are not intended to provide actual design recommendations, but instead are meant to show an example of an application of the proposed algorithm. This study focused on serviceability issues such as occupant comfort criteria, and damage to secondary, non-structural systems (i.e., building equipment or building façades). In general, risk analysis requires the computation of a probability indicating the “loss of performance” for a range of possible
wind speeds, which is determined by combing the fragility curves with the wind speed hazard curve (Equation 5.2) (Li and Ellingwood, 2006).

\[ P_T = \int_0^\infty F_T f_{V_z} \, dV_z \]  

(5.2)

In Equation 5.2, \( P_T \) is the probability that there will be a “loss of performance” for a threshold \( T \), \( F_T \) is the structural fragility curve of the structure’s limit state under investigation (see Equation 5.1), and \( f_{V_z} \) is the probability density function (PDF) of the wind speed. The wind speed, \( V_z \), can be expressed in terms of any reference period, depending on the purpose and intended outcomes of the performance (risk) analysis being conducted (Li and Ellingwood, 2006). The PDF of the wind speed is derived through probabilistic modeling methods.

The first step of this investigation was to obtain full-scale wind records. Wind data from a buoy off the coast of Boston, MA, which is part of the NOAA (National Oceanic and Atmospheric Administration) system and is monitored by the National Data Buoy Center (NOAA Station 44013, 42.346 N 70.561W), was utilized (NOAA, 2009). Figure 5.10 is a picture of the buoy taken from (NOAA, 2009). This particular location was selected because the objective of this preliminary performance analysis was to assess the vulnerability of New England’s tall buildings to wind hazards. Wind speeds at this buoy (both continuous and peak speeds) are measured using an anemometer located 5 meters above the site elevation. This investigation considered both continuous data and peak data for reasons that will be explained later in this section.
The “continuous data” was based on a record from a single year (2008) in which readings were recorded every 10 minutes, and the peak data was derived from peak velocities over a 25-year period (1984-2008). The wind speeds provided by the National Data Buoy Center are averaged over an 8-minute period. Since this paper employs a 10-minute average wind speed, the data was multiplied by a factor of 0.98 to obtain an equivalent 10-minute wind speed (Simiu and Scanlan, 1996). Furthermore, the Logarithm Law was employed to derive the mean wind speed at the top of the building \((z=h)\) using two roughness lengths – \(z_0=0.002\text{m}\) and \(z_0=0.1\text{m}\). The roughness length \(z_0=0.002\text{m}\) corresponds to a sea surface type, whereas \(z_0=0.1\text{m}\) corresponds to urban terrain (Tieleman, 2003). A relationship between the friction velocity and roughness length, given in (Simiu and Scanlan, 1996), was used to relate
the wind data to an urban terrain, even though it was collected in an open-sea environment, which is not directly applicable to surface winds on land. The wind data proved to be reasonable based on the current design wind speeds found in (ASCE, 2005) for the coastal region of Massachusetts, which prescribes a 3-sec gust wind speed of 49m/s (at an elevation of 10m), relating to a 10-min wind speed of 33.8m/s (at an elevation of 10m). Table 5.3 indicates that the design wind speeds derived from (ASCE, 2005), at an elevation \( z = h \), are closely associated to the wind data that was utilized.

**Table 5.3 – Wind speeds derived from buoy data (NOAA, 2009) and ASCE 7-05 as a function of roughness length**

<table>
<thead>
<tr>
<th>Wind Speed [m/s]</th>
<th>( z_0 = 0.002 \text{m} )</th>
<th>( z_0 = 0.1 \text{m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Continuous Data</td>
<td>Peak Data</td>
</tr>
<tr>
<td>Minimum (data)</td>
<td>0.0</td>
<td>25.2</td>
</tr>
<tr>
<td>Maximum (data)</td>
<td>26.8</td>
<td>36.8</td>
</tr>
<tr>
<td>Design (ASCE 7-05)</td>
<td>-</td>
<td>56.6</td>
</tr>
<tr>
<td></td>
<td>Continuous Data</td>
<td>Peak Data</td>
</tr>
<tr>
<td>Minimum (data)</td>
<td>0.0</td>
<td>21.8</td>
</tr>
<tr>
<td>Maximum (data)</td>
<td>23.2</td>
<td>31.9</td>
</tr>
<tr>
<td>Design (ASCE 7-05)</td>
<td>-</td>
<td>49.1</td>
</tr>
</tbody>
</table>

With these two sets of wind data (continuous and peak), statistical modeling was employed to define the PDFs of the wind speed. Since the continuous data is taken over the course of a year and incorporates all speeds (including a near zero wind speed), the continuous data was fit to a Weibull distribution. This distribution was chosen because it is typically used for less severe wind speed models (Simiu and Scanlan, 1996). The results of this distribution are referred to as a “marginal wind velocity distribution”. The peak data, obtained from the 25-year period, was fit to a Gumbel distribution, otherwise known as an Extreme Value (Type I) distribution. This method was selected since it is a commonly accepted probabilistic method to model peak wind data (Simiu and Scanlan, 1996). This distribution is referred to as an “annual peak wind velocity distribution”. Table 5.4
summarizes the parameters that were used to obtain the PDFs of the wind speed for both sets of data. Figure 5.11 shows an example Gumbel probability plot for $z_0=0.1\text{m}$, which is used to graphically assess the appropriateness of the Gumbel distribution for the given wind speed records (Murphy and Jackson, 1997). This particular plot was developed by calculating the empirical cumulative frequency, $C(U)=i/(N+1)$, where $U$ is a random variable consisting of the annual peak wind speeds and $i$ is the index (1,…,N). The $y$-axis of the plot (Figure 5.11) is a function of the empirical cumulative frequency, referred to as the reduced variate, and is given by $r.v. = -\ln \{ -\ln [C(U)] \}$ (Murphy and Jackson, 1997). When the plotted points closely follow a straight line, it indicates that the data could have come from a Gumbel distribution. This type of test was conducted for all sets of wind data (and both types of distributions); the tests all produced the same result – a straight line – indicating the data could be fit to the corresponding distribution.

**Table 5.4 – Weibull and Gumbel distribution parameters**

<table>
<thead>
<tr>
<th>Distribution Parameters</th>
<th>Weibull Distribution (continuous data)</th>
<th>Gumbel Distribution (peak data)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$z_0=0.002\text{m}$</td>
<td>$z_0=0.1\text{m}$</td>
</tr>
<tr>
<td>Scale</td>
<td>8.99</td>
<td>7.79</td>
</tr>
<tr>
<td>Shape</td>
<td>2.153</td>
<td>2.153</td>
</tr>
</tbody>
</table>
Figure 5.11 – Example of Gumbel probability plot used to graphically assess the appropriateness of the Gumbel distribution for the set of data points ($z_0=0.1m$)

With the velocity PDFs derived and the structural fragility curves defined (Section 5.2), the next step in the performance analysis is to find $P_T$ (Equation 5.2). The probability, $P_T$, is a function of two curves: 1) the fragility curve, which is directly related to the type of response, and 2) the wind speed PDF, which is based on a particular set of wind data. The following discussion will address the combination of these two functions – response and wind – in the computation of $P_T$.

In this investigative example of risk analysis, the RMS and peak responses in the along-wind direction were employed to measure different types of performance of the CAARC building. In particular, the issues pertaining to human comfort/occupancy conditions and effects on secondary building systems were considered. Generally speaking, the RMS response can be seen as a continuous effect (i.e., smaller, but constant, displacements), while the peak response is more concerned with extreme events (i.e.,
excessive and large displacements). In terms of human comfort and occupancy, the RMS response might be related to the issue of human tolerance or comfort, whereas the peak response could be seen as a descriptor of human perception (an occupancy issue) (Kijewski-Correa and Pirnia, 2009). In terms of the secondary/non-structural building systems of a tall structure (such as equipment within the building and the building façade – windows, etc.), the RMS response may be important to consider when sensitive structures are installed, and the peak response may be of concern for building elements that are predisposed to failure under sudden and large vibrations.

The previous paragraph discussed the associations between response and performance issues. Similar relationships were made for the two types of wind distributions considered in this study: marginal wind velocity distributions (from the continuous data), and annual peak wind velocity distributions (from the peak data). For the purposes of this investigation, the marginal distribution generally represents the day-to-day wind speeds that a building can experience (the average speeds). Alternatively, the annual peak distribution generally corresponds to the extreme events that occur (the maximum speeds).

With the response and wind data defined, the goal at this point is to create optimal combinations of the response (fragility curves) and the wind data (wind PDFs). The three configurations of the response and wind data that were used to calculate the probability $P_T$ (“loss of performance”) are shown in Table 5.5. The following examples, provided for each of the three cases, are meant to better illustrate the capabilities of the proposed methodology, and are not intended to represent established guidelines. Nevertheless, the subsequent case descriptions are plausible, and are based on educated assessments of the interaction between
buildings and wind forces; the previous discussions of this section can testify to this statement.

Case 1 is the combination of the RMS response with the continuous wind data, and is meant to describe the levels of human tolerance. This case might also be used to describe a temporary downtime of secondary building systems such as sensitive equipment. Case 2 combines the RMS response with the peak wind data. This case may relate to equipment (or other non-structural elements) that are damaged or adversely affected due to a progressive deterioration (from smaller displacements, but larger velocities). Lastly, Case 3 combines the peak response with the peak wind data, which can pertain to human perception and the subsequent question of occupancy (e.g., the perception of discomfort is so evident, that the building cannot be temporarily occupied). This case might also describe excessive vibrations that cause sudden (as opposed to progressive) damage to secondary building systems/equipment and/or to the façade (i.e., windows). As previously stated, this study is not intended to provide probabilistic analysis of structural failure; therefore, none of the cases describe effects on the building’s structural system. Furthermore, structural failures in tall buildings due to wind hazards are not readily observed; it is more likely that a building’s secondary systems will incur damage, making serviceability issues of greater importance in this analysis (Bracci, 2006).
Table 5.5 – Description of wind cases employed in the performance analysis

<table>
<thead>
<tr>
<th>Case</th>
<th>Type of Response</th>
<th>Distribution Type</th>
<th>Wind Data</th>
<th>$z_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>RMS Response</td>
<td>Marginal Velocity</td>
<td>1-year Continuous Data</td>
<td>0.002m</td>
</tr>
<tr>
<td>1b</td>
<td>RMS Response</td>
<td>Marginal Velocity</td>
<td>1-year Continuous Data</td>
<td>0.1m</td>
</tr>
<tr>
<td>2a</td>
<td>RMS Response</td>
<td>Annual Peak Velocity</td>
<td>25-year Peak Data</td>
<td>0.002m</td>
</tr>
<tr>
<td>2b</td>
<td>RMS Response</td>
<td>Annual Peak Velocity</td>
<td>25-year Peak Data</td>
<td>0.1m</td>
</tr>
<tr>
<td>3a</td>
<td>Peak Response</td>
<td>Annual Peak Velocity</td>
<td>25-year Peak Data</td>
<td>0.002m</td>
</tr>
<tr>
<td>3b</td>
<td>Peak Response</td>
<td>Annual Peak Velocity</td>
<td>25-year Peak Data</td>
<td>0.1m</td>
</tr>
</tbody>
</table>

The following tables (Tables 5.6 and 5.7) show the results of the performance analysis based on the formerly described cases. Table 5.6 gives the results of the probability function, $P_T$, for a given threshold $T$ (T1 to T5); in other words, $P_T$ is the probability that the building will experience a “loss of performance” in a given time period (i.e., every 10-minutes for the continuous data and every year for the annual data) for a pre-defined threshold, $T$.

Table 5.7 employs these probabilities (Table 5.6) to calculate the mean recurrence interval for a given threshold for each of the response/wind cases. As an example, if threshold T1 were selected in conjunction with Case 1b, the building would exceed T1 approximately every 7 months. As opposed to threshold T3 through T5, of the same case, where the threshold is not likely to ever be exceeded. Threshold T5, which is comparable to the prescribed maximum lateral drift for the CAARC building, is not likely to be exceeded for any wind hazard case, except for Case 3a, where there is a mean recurrence interval of approximately 1,100 years. If a practitioner wished to draw conclusions based on these results, the following, for example, could be said: In an urban environment (‘b’ cases) it is unlikely that any wind hazard (peak or continuous) in the New England coastal region will cause the maximum lateral drift to be exceeded.
Table 5.6 – Probability describing the “loss of performance” for a threshold $T$ of the CAARC building

<table>
<thead>
<tr>
<th>Case</th>
<th>$P_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_1$</td>
</tr>
<tr>
<td>1a</td>
<td>0.0%</td>
</tr>
<tr>
<td>1b</td>
<td>0.4%</td>
</tr>
<tr>
<td>2a</td>
<td>40.2%</td>
</tr>
<tr>
<td>2b</td>
<td>89.5%</td>
</tr>
<tr>
<td>3a</td>
<td>82.1%</td>
</tr>
<tr>
<td>3b</td>
<td>98.0%</td>
</tr>
</tbody>
</table>

Table 5.7 – Mean recurrence interval of a wind event (based on full-scale data) for a given threshold $T$ of the CAARC building

<table>
<thead>
<tr>
<th>Case</th>
<th>Mean Recurrence Interval (in years)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_1$</td>
</tr>
<tr>
<td>1a</td>
<td>0.3</td>
</tr>
<tr>
<td>1b</td>
<td>0.6</td>
</tr>
<tr>
<td>2a</td>
<td>2.5</td>
</tr>
<tr>
<td>2b</td>
<td>10.3</td>
</tr>
<tr>
<td>3a</td>
<td>1.2</td>
</tr>
<tr>
<td>3b</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Note: The dash (‘-’) in this table indicates that the calculated probability is equal to zero (i.e., below the accuracy/resolution of the numerical integration algorithm, corresponding to a practically unreasonable mean recurrence interval).

The results of this preliminary performance analysis demonstrate that the proposed methodology is acceptable, and the developed “Two-Step Monte Carlo Algorithm”, validated using wind tunnel and full-scale data, was proven a useful tool for the fulfillment of this investigation. Not only did this analysis demonstrate the appropriateness of the developed algorithm, it also points towards the future directions this research should take. The observations made in this section are related to a single structure, and its response in the along-wind direction only; therefore, one future investigation should be to determine if the thresholds ($T_1$ to $T_5$), which were assumed in this thesis, are appropriate. Future analyses
should also incorporate more rigorous structural analysis techniques, and possibly account for the across-wind and torsional responses in addition to the along-wind response. A more advanced analysis would also allow higher modes of vibration to be considered, important for comfort criteria of occupants in very tall buildings (Irwin, 2009). Another consideration, which was not addressed by the performance analysis in this thesis, is cost (i.e., maintenance after a major wind hazard, repairs, etc.). Performance objectives and the cost to meet these objectives are inseparable entities; future studies should consider the relationship between the performance of the building and the cost associated with obtaining these levels. Finally, the performance analysis should be able to incorporate all levels of performance, from the performance of the secondary building systems and human comfort (as this section discussed) to the performance of the overall structure (i.e., more severe damage during extreme events). In summary, the analyses conducted herein provide the basis for the future directions that this research can take.
5.4 References

ASCE (2005). Minimum design loads for buildings and other structures (7-05). Reston, VA, USA, American Society of Civil Engineers.


Murphy, B. and Jackson, P. L. (1997), "Extreme Value Analysis: Return periods of severe wind events in the central interior of british columbia".


6 CHAPTER SIX: CONCLUSIONS AND FUTURE DIRECTIONS

6.1 CONCLUSIONS OF RESEARCH

This thesis summarizes the research activity aimed at the development of an algorithm for the analysis of tall buildings subjected to wind hazards. The work was motivated by recent study activities at Northeastern University aimed at the analysis of performance-based methods for wind engineering applications. As part of these investigations, a numerical two-step Monte Carlo (MC) algorithm, for the derivation of the (linear) dynamic response of high-rise structures, which also accounts for uncertainty and measurement errors in the wind loading, was implemented. The significance of this approach may be related to the possibility of replicating the “closed-form” analytical methods (based on random vibration techniques in the frequency domain) through numerical MC algorithms. From a computational standpoint, this method advantageously enables the subsequent implementation of numerical statistical analyses, as in the presence of an uncertain wind loading scenarios. Even though the first step of the proposed procedure could be replaced with equivalent estimation techniques (Kareem, 1988; Piccardo and Solari, 1998; Solari, 1988), based on a direct solution of the modal force and response spectra, the proposed technique has the advantage of allowing for subsequent generalizations. Among the features that are currently not present in the existing estimation techniques, aspects such as two direction modal coupling and the influence of higher modes can be readily incorporated, and are planned for the future stages of the study. Ultimately, this thesis
presents a preliminary simulation technique, which will potentially provide designers with a useful tool for practical applications.

The developed procedure relied on the following assumptions: the structure is linear-elastic, modal superposition is employed, quasi-steady formulation of the wind loading is used, few fundamental structural modes through superposition are required, and torsional effects are neglected; turbulence induced wind loading was exclusively considered. While the along-wind response and across-wind response were both considered for some of the analyses, the along-wind response was the only motion considered for the fragility and performance analyses. In addition, a limited number of simulations employing modal coupling were conducted. The effects on the response were minimal (for this particular structure and loading scenario); therefore, it was not necessary to conduct the simulations under the assumption of coupled modes.

Three simulated structures were selected at the onset of this research to conduct the analyses: a steel chimney (Solari, 1988), a concrete chimney (Müller and Nieser, 1975/76), and the CAARC benchmark building (Melbourne, 1980). The CAARC building had been selected by the Commonwealth Advisory Aeronautical Research Council of Australia in the early Eighties as a benchmark high-rise structure to be employed for studying its dynamic response affected by wind loading (Melbourne, 1980). More rigorous analyses were conducted for the CAARC building since it was the sole structure employed in this research for “structural fragility” analysis (serviceability criteria) and a preliminary performance analysis.

In the initial set of investigations, relating to the “closed-form” approach for solving for the response, each simulated structure was used as a calibration tool to update and verify
model and wind loading parameters (i.e., to ensure compatibility with the available data from wind tunnel testing (Melbourne, 1980; Piccardo and Solari, 2002), and the results of calculations using acceptable mathematical models (Solari, 1982)). This preliminary set of calculations facilitated the implementation of the developed procedure. Subsequently, validation of the numerical procedure was conducted by comparing the simulated results to the reference values extracted from the literature. Excellent correspondence was found in the along-wind direction. However, larger variations were observed for the across-wind direction (especially in the case of the CAARC building) at all wind velocities. These discrepancies were explained by the effects of vortex shedding, which were not simulated in the wind-loading model employed in this research. More investigations would be necessary to determine the cause of the observed dissimilarities. In spite of the observed differences, it was concluded that the results from the “closed-form” simulations could justifiably be used as “exact” target values in the performance analysis of the MC algorithm.

The next stage of this research was related to the MC algorithms. In general, these algorithms are useful numerical methods when replicating phenomena with uncertainty in the inputs (i.e., problems driven by the errors in the modeling or based on experimental data affected by measurements). In the analysis of the results, the main considerations were the validation of the MC algorithm and the selection of $N_{MC}$ (number of MC integration points). There was relatively no error observed between the “exact” target response data points (from the “closed-form” analysis) and the MC results, indicating that the MC algorithm is capable of computing the response within the same order of magnitude. This fact was proof enough for the validation of the developed numerical procedure. To select the most advantageous value for $N_{MC}$, statistical analyses were conducted to find the optimal balance between
accuracy and practicality. Without an optimal “performance” of the MC algorithm, the results would lack meaning in practical applications. However, the extensive computation times, for very $N_{MC}$, that may be associated with seemingly better accuracy, might also make the algorithm null in practice. It was concluded that the performance of the MC algorithm is optimized for $N_{MC}=10,000$ for the three simulated structures; therefore, this value was employed in the “Two-Step MC Algorithm”. More studies would possibly be required to extend this result to other structures and different wind loading descriptions (e.g., exposure categories in boundary layer, loading and response dominated by across-wind forces, etc.).

Finally, preliminary fragility and performance analyses were conducted, recognizing that there is considerable uncertainty as a result of the unpredictable modeling of the wind. Structural fragility curves were numerically derived using the Two-Step MC Algorithm, and by calculating the probability of exceedance for pre-defined threshold levels. Five thresholds associated with maximum drift ratios, ranging from almost unperceivable values to $1/400$ of the total building height, were considered. These thresholds were applied to preliminary structural performance assessments pertaining to secondary non-structural systems and human comfort (i.e., serviceability issues). Fragility curves were developed for the RMS and peak response ($x$-direction only). It was observed that for a given threshold at more severe velocities, the probability of exceedance is very low for the RMS response, compared to a much larger probability for the peak response. For the case of the CAARC building, it was observed that this might possibly lead to unacceptable total drift ratios, which are mainly associated with the mean wind pressure and force distribution. However, it was also concluded that the contribution of the dynamic response, and of $\alpha$, could not be ruled out for a tall flexible structure in general, such as the one under investigation. Therefore, fragility
curves, incorporating structural dynamic response, were viewed as preferable in the context of design against high winds.

The structural fragility curves were then used in a preliminary performance analysis, conducted on the CAARC prototype building, employing wind data from a buoy off the coast of Boston, MA. This particular location was selected to assess the vulnerability of New England’s tall buildings to wind hazards. The mean recurrence interval (MRI) associated with serviceability limit states was evaluated. The MRI results were dependent upon the selected performance threshold, and indicated that the MRI was primarily of significance for response dominated by lateral peaks. From this analysis, an MRI approximately equal to 1,100 years was obtained for a performance objective level corresponding to a drift ratio of 1/400, and for an extreme event. For this particular example, the results suggest that the original pre-selected thresholds could be increased since the MRI exceeded the current value employed by the U.S. Standard (i.e., approximately 700 years (ASCE, 2005)) for extreme events in non-hurricane prone areas.

The results of this preliminary performance analysis demonstrated that the proposed methodology is acceptable, and the developed “Two-Step Monte Carlo Algorithm”, validated using wind tunnel and full-scale data, was a useful tool for the fulfillment of this investigation.

In summary, the objectives of this research were met through the analyses conducted in this thesis. Objective 1, which was to develop a numerical MC procedure to compute the response of a tall building under uncertain wind loading, was accomplished through the repeated MC simulations on the three structures. The validation of this procedure to literature results (Objective 2) was achieved with the simulations employing a “closed-form”
approach. Finally, the results of the MC numerical simulations were utilized in fragility and performance analyses (Objective 3), where the statistical information about the dynamic response due to errors and variability in the characterization of wind loading was derived.

6.2 **Future Research Directions**

The preliminary performance analysis, conducted on the CAARC prototype building, also pointed to the future directions that this research may take. The following are just some of these suggestions based on the results of the investigations presented in this thesis.

- Determine if the thresholds (T1 to T5), which were assumed in this thesis, are appropriate. The performance results are related to a single structure (CAARC building), and its response in a single direction (along-wind); therefore, the analysis of other benchmark buildings seems like the next logical step for defining generalized serviceability limit states (thresholds).

- Inclusion of more rigorous structural and dynamic analysis techniques, accounting for the across-wind and torsional responses in addition to the along-wind response. The analyses of this thesis seemed to demonstrate that vortex shedding, for example, should be incorporated in the procedure for the response calculation.

- Consideration of higher modes of vibration, important for comfort criteria of occupants in very tall buildings (Irwin, 2009).

- Address the issue of cost in the performance analysis (i.e., maintenance after a major wind hazard, repairs, etc.). Performance objectives and the cost to meet these objectives are inseparable entities; future studies should consider the
relationship between the performance of the building and the cost associated with obtaining these levels (Norton et al., 2008).

- Incorporate all levels of performance in the performance analysis, from the performance of the secondary building systems and human comfort (as this thesis discussed) to the performance of the overall structure (i.e., potential severe damage during extreme events).
6.3 References

ASCE (2005). Minimum design loads for buildings and other structures (7-05). Reston, VA, USA, American Society of Civil Engineers.


