FINITE ELEMENT MODELING OF LARGE DEFORMATION
RESPONSE OF REINFORCED CONCRETE BEAMS

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ABSTRACT

In a previous study, a 3/8 scaled physical model of a continuous two-span reinforced concrete (RC) beam was constructed and tested. The RC beam was subjected to a downward vertical displacement. In this study a detailed three dimensional finite element model of the RC beam is developed. Unlike a previous analytical study, the contribution of bond to the response of the beam is considered here by deploying interface elements between longitudinal reinforcement elements and concrete elements of the finite element model. Nonlinear material behavior such as concrete cracking and crushing and steel reinforcement yielding are also modeled. A geometrically nonlinear incremental static analysis using the computer program DIANA 9.2 is carried out. The model is analyzed under the applied deformations at the centerline. Analysis results include element forces, stresses and strains of the concrete and steel reinforcement and bond stresses and slippage along the beam. Stress-strain relationships of steel reinforcement and beam end rotations are compared with the corresponding experimental results. Overall, the analytical and experimental results are in good agreement.
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Chapter 1   Introduction

1.1   Overview

Understanding the behavior of reinforced concrete (RC) beams subjected to large deformations requires further analytical and experimental studies. In [1], analytical and experimental studies of an RC beam subjected to large deformations are presented using a detailed 3-D finite element model. Yet, the bond slip between steel reinforcement was not modeled. In this study, a different finite element model of an RC beam is developed, incorporating bond slip.

1.2   Objectives

The goals of this thesis are:

1) To develop a detailed three dimensional finite element model of a RC beam to be subjected to large deformations.

2) To evaluate the model by comparing the analytical results with the previously obtained experimental data.

1.3   Structure of the Thesis

In chapter 2, the spatial discretization of the RC beam is discussed. The topology of the elements used to model the concrete core and concrete cover, longitudinal and transverse steel reinforcement as well as the interface between concrete and steel are introduced.

In chapter 3, the constitutive modeling of concrete, steel reinforcement and interface steel-concrete is discussed. Using von Mises yield criteria, input stress-strain relationships for the transverse and longitudinal steel reinforcement are introduced. Utilizing the Modified Compression Field Theory, material modeling for concrete in compression and tension is presented. Evaluating existing bond-slip relationships and the corresponding experimental data, input bond-slip relationships for different regions in the RC beam are presented.
In chapter 4, the results of the analysis are presented. The variation of stresses and strains in reinforcement steel and concrete as well as of bond stresses and slippage along the beam is shown. The deformations of the RC beam sections are discussed and pull-out and push-in values of the anchored steel reinforcement are presented. The response of the RC beam at material, section and element level is presented.

In chapter 5, analytical steel reinforcement strains are evaluated and compared with the corresponding experimentally results. The analytical beam end rotations are also compared with the experimental results.

In chapter 6, concluding remarks are presented.
Chapter 2    Spatial Discretization and Element Topology

Figs. 2.1 to 2.3 show an RC beam reported in [1] and its cross sections. In order to capture the three dimensional state of stress and strain in the RC beam, a 3-D finite element model is developed. The interface between concrete and steel reinforcement is explicitly modeled using interface elements.

Figure 2.1: View of the RC beam reported in [1]

Figure 2.2: Cross section A-A

Figure 2.3: Cross section B-B
2.1 Cross Section Discretization

The discretization of beam section A-A is shown in Fig. 2.4. Concrete cover and core main areas are built of quadrilateral surfaces. The steel reinforcement cross sections comprise octagonal surfaces. The transition region from octagonal to quadrilateral surfaces is discretized using triangular surfaces. Given the symmetric cross section of the beam, only half of the beam cross section is modeled.

Figure 2.4: Discretization of the cross section of the RC beam reported in [1]

2.2 3-D Discretization

In order to save computational time, only a quarter of the RC beam is modeled. That is, one half of the cross section and one half of the two span beam is modeled. The finite element model of the experiment reported in [1] is composed of 5 Segments (see Figs. 2.1, 2.2, 2.3 and 2.5). Segment 1 includes a portion of the beam having the cross section shown in Fig. 2.3. Segments 2 and 3 have the cross section shown in Fig. 2.2. Segment 4 is the side support of the RC beam and its right hand side boundary nodes are fixed in all three translational degrees of freedom. Segment 5 is the center column part of the RC beam. At the left hand side boundary of segment 5, the experimental measured RC beam deformations will be imposed to the finite element model. Transverse reinforcement is modeled using the embedded reinforcement concept which is described later in this chapter. The top longitudinal reinforcement of the tested RC beam was anchored in the side support using a 90 degree hook. The top reinforcement of the finite element
model of the RC beam is anchored straight over a length of 11.0 in within the side support part in order to minimize discretization effort. Only small values of steel reinforcement longitudinal stress and strain are present at that location. Thus, it is believed that omitting to model the anchorage hook in the model will not significantly influence the results of the computation. 20-node isoparametric brick elements are selected to model the concrete cover and core (see Fig. 2.6). The longitudinal steel reinforcement of the RC beam is also modeled using 20-node isoparametric brick elements. This was primarily because of the need for an element to be compatible with the available interface elements in program DIANA 9.2 as discussed in the next section.

![Figure 2.5: 3-D finite element model of the RC beam reported in [1]](image)

![Figure 2.6: Topology of solid element](image)
2.2.1 Interface between Longitudinal Reinforcement and Concrete

The area between longitudinal steel reinforcement and concrete is modeled using 16-node plane quadrilateral isoparametric elements providing the capability to discretize curved surfaces (Fig. 2.7). The element comprises two overlaying planes. Each pair of nodes have coincident coordinates which implies zero thickness of the interface element.

![Figure 2.7: Topology of interface element and associated nodal displacements](image)

The set of variables describing the element state of deformation and the element forces contains nodal displacements $\mathbf{u}_e$, relative displacements $\Delta \mathbf{u}$ as well as normal traction $t_x$ and shear tractions $t_y$ and $t_z$. The location of the element nodes and the variables of the chosen interface element are shown in Fig.2.7.

\[
\begin{align*}
\mathbf{u}_e &= \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} && \mathbf{t} = \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} && \Delta \mathbf{u} = \begin{pmatrix} \Delta u_x \\ \Delta u_y \\ \Delta u_z \end{pmatrix}
\end{align*}
\] (2.1)

2.2.2 Transverse Reinforcement

Transverse reinforcement is modeled using embedded reinforcement. The strain of the reinforcement bar is computed from the displacement field of the concrete element in which the transverse reinforcement is embedded.
Chapter 3  Material Constitutive Models

3.1 Steel Reinforcement

3.1.1 Theoretical Description

The use of solid elements to discretize the longitudinal steel reinforcement of the RC beam (Section 2.2) requires a yield criteria to define the elastic limit of the steel reinforcement under combined state of stresses. The DIANA 9.2 program provides the Tresca and von Mises yield criteria which are both isotropic and independent of hydrostatic pressure and therefore suitable for modeling steel reinforcement represented by means of solid elements. Experimental tests on ductile steel have shown a particularly good agreement with von Mises criteria in describing the onset of yielding not only under one-dimensional state of stress but also under combined state of stresses [2]. Thus, the von Mises yield criteria is used in this study.

The von Mises yield criteria states that the onset of yielding depends on whether the octahedral shear stress or the maximum distortional energy has reached a critical value [3]

\[ f(\sigma_{ij}, \kappa) = J_2 - \kappa^2 = 0 \]  \hspace{1cm} (3.1)

where \( \kappa \) represents the yield stress in pure shear; \( \sigma_{ij} \) represents the state of stress and \( J_2 \) denotes the second deviatoric stress invariant.

In the literature, Eq. 3.1 is also referred to as yield surface in principal stress space [3]. If the von Mises criteria is calibrated to agree with the uniaxial tension yield stress \( \sigma_{yield} \), \( \kappa \) is

\[ \kappa = \frac{\sigma_{yield}}{\sqrt{3}} \]  \hspace{1cm} (3.2)

In program DIANA 9.2, the von Mises yield criteria is implemented such that it agrees with the uniaxial stress-strain relationship of steel reinforcement. Eq. 3.3 represents the von Mises yield surface in principal stress space calibrated to uniaxial tension test as it is available in program DIANA 9.2. It can be obtained by substituting Eq. 3.2 into Eq. 3.1.
If the von Mises yield criteria in Eq. 3.3 is used to describe steel reinforcement material behavior, only elastic-perfectly plastic material behavior can be modeled since plastic deformation are assumed to occur under a constant yield stress. Yet, steel reinforcement exhibits hardening behavior. In order to model this post-yield response, a hardening rule is used within the theory of incremental plastic flow. A hardening rule specifies the configuration of subsequent yield surfaces in stress space if a material is loaded beyond the initial yield surface represented by Eq. 3.3. Hardening rules can be categorized in isotropic, kinematic and mixed hardening rules.

In Program DIANA 9.2 an isotropic hardening rule is implemented in conjunction with the von Mises yield criteria. It is adopted in this study to model the hardening behavior of steel reinforcement and it is described subsequently.

\[ f(\sigma_{ij}, \sigma_{yield}) = \sqrt{3J_2} - \sigma_{yield} = 0 \] (3.3)

\[ \varepsilon_p(\sigma_{ij}, \sigma_{yield}, \varepsilon_p) = \sqrt{3J_2} - \sigma_{yield}(\varepsilon_p) = 0 \] (3.4)

\( \varepsilon_p \) (effective strain [3]), is an increasing function of the accumulated plastic strain increments \( d\varepsilon_{ij}^p \). There are two basic approaches within the theory of plastic flow to define the effective strain \( \varepsilon_p \), which are work and strain hardening approaches. In the Diana 9.2 program both approaches are implemented. Yet, in case of von Mises isotropic hardening material, both lead to the same scalar function for \( \varepsilon_p \) [3]. Within the strain hardening approach, the effective strain can be defined as a suitable combination of plastic strain increments \( d\varepsilon_{ij}^p \) [3]. In the DIANA 9.2 program the following definition is implemented.

\[ \varepsilon_p = \int \sqrt{\frac{2}{3} d\varepsilon_{ij}^p d\varepsilon_{ij}^p} \] (3.5)

Having \( \varepsilon_p, \sigma_{yield}(\varepsilon_p) \) can be found using uniaxial material test results. The corresponding associated flow rule implemented in the program DIANA 9.2 is used in this study.
3.1.2 Material Parameters of Steel Reinforcement

Experimental test results [1] as well as the data provided in [5] are used to determine the material parameters for longitudinal and transverse reinforcements, which are presented in Table 3.1. The modulus of elasticity $E$ is taken from [5]. The yield stress $\sigma_{\text{yield}}$ for both reinforcements has been determined from tension tests conducted in [1]. For the longitudinal steel reinforcement of the RC beam a trilinear idealization of the experimental uniaxial stress-strain relationship is adopted. For transverse reinforcement a bilinear idealization of the experimental uniaxial stress-strain relationship is used. Both uniaxial stress-strain relationships are shown in Fig. 3.1.

<table>
<thead>
<tr>
<th></th>
<th>Longitudinal Reinforcement</th>
<th>Transverse Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ [ksi]</td>
<td>29,000</td>
<td>29,000</td>
</tr>
<tr>
<td>$\sigma_{\text{yield}}$ [ksi]</td>
<td>75</td>
<td>60</td>
</tr>
<tr>
<td>$\epsilon_{\text{yield}}$</td>
<td>0.0026</td>
<td>0.0021</td>
</tr>
<tr>
<td>$E_{\text{postyield}}$ [ksi]</td>
<td>287</td>
<td>306</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$\sigma_{\text{ult}}$ [ksi]</td>
<td>105</td>
<td>90</td>
</tr>
<tr>
<td>$\epsilon_{\text{ult}}$</td>
<td>0.13</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Table 3.1: Material parameters and characteristic values of uniaxial stress-strain relationships of longitudinal and transverse steel reinforcement

Figure 3.1: Uniaxial stress-strain relationships of longitudinal and transverse steel reinforcements
3.2 Concrete

In program DIANA 9.2, the three approaches to establish constitutive relations for concrete in compression are: plasticity, elasticity and damage based approach.

Three-dimensional material models based on plasticity approach, namely the Drucker-Prager model and the Modified Mohr-Coulomb model, are available in program DIANA 9.2. Within the plasticity approach the total concrete strain $\epsilon_c$ is decomposed in an elastic part $\epsilon_p^e$ and in a plastic part $\epsilon_p^p$. The concept of strain decomposition appeals the most to the physical nature of cracked concrete especially if combined with the smeared crack approach where the total strain is decomposed to the “crack strain” and the strain of the solid concrete [6]. Yet, preliminary simulations using Drucker-Prager model in compression combined with a multi-directional smeared crack model in tension revealed numerical difficulties (convergence problems). According to the manual of program DIANA 9.2 [4], such difficulties are likely to occur when in one integration point simultaneously concrete compressive softening is accompanied by several active cracks in the remaining lateral directions.

An elasticity based concept is the Modified Compression Field Theory. It was originally developed within a two-dimensional framework [7]. The purpose was to predict the load-deformation response of plane cracked reinforced concrete elements subjected to in-plane shear and normal stresses. Subsequently, the concept was extended and enhanced, respectively, in order to describe the load-deformation characteristic of reinforced concrete solids subjected to a general three-dimensional state of stress [8]. This enhanced three-dimensional model constitutes the basis for the elasticity based total strain concrete material model available in the program DIANA 9.2.

Lastly, a damage based material model, namely the Modified Maekawa Model, is implemented in the program DIANA 9.2. This model is based on two-dimensional and three-dimensional cyclic loading data. The damage based approach was not considered in this study.

3.2.1 2-D Modified Compression Field Theory

Using a total strain based approach, concrete behavior in compression and tension is modeled in this study. The total strain based approach for concrete material modeling in the program DIANA 9.2 is based on the Modified Compression Field Theory [7]. In the Modified Compression
Field Theory, stress-strain relationships are evaluated in terms of average stresses and strains. Figs. 3.2 and 3.3 show a reinforced concrete element and the corresponding two-dimensional states of stress and strain.

Figure 3.2: 2-D state of stress in a reinforced concrete element [7]

Figure 3.3: 2-d state of strain in a reinforced element [7]

In order to relate the principal average stresses $\sigma_{c,1}$ and $\sigma_{c,2}$ to principal average strains $\epsilon_1$ and $\epsilon_2$, an average stress-strain constitutive relationship is required. Such an average stress-strain relationship can differ from local stress-strain relationships determined from standard material tests.

An extensive series of reinforced concrete panels were tested in [7] in order to obtain information with regard to characteristics of such an concrete average stress-strain constitutive relationship. For details with regard to the set up the reader is referred to [7]. In these tests, in-plane normal stresses $\sigma_x$, $\sigma_y$ and shear stresses $\tau_{xy}$, depicted in Fig. 3.2, were applied to the reinforced concrete panels. The corresponding average strains $\epsilon_x$, $\epsilon_y$ and $\gamma_{xy}$, shown in Fig. 3.3, were measured. Based on these measured quantities, the following average stress-strain constitutive relationship for concrete in compression was developed:

$$\sigma_{c,2} = \sigma_{c,2,max} \cdot \left[ 2 \left( \frac{\epsilon_2}{\epsilon_c} \right) - \left( \frac{\epsilon_2}{\epsilon_c} \right)^2 \right]$$

(3.6)

where $\sigma_{c,2}$ represents the second principal stress in concrete (maximum compression); $\epsilon_2$ represents the second principal strain in concrete and
\[ \frac{\sigma_{c2,max}}{f_c'} = \frac{1}{0.8 - 0.34 \cdot \epsilon_1'/\epsilon_c'} \]  

(3.7)

where \( f_c' \) represents the concrete compressive strength; \( \epsilon_c' \) represents the concrete strain at \( f_c' \) and \( \epsilon_1 \) represents the first principal strain in concrete.

Eq. 3.6 and 3.7 reflects the main findings of the study with regard to the compressive load-deformation characteristics of reinforced concrete panels. As one can see, looking at the constitutive relationships presented above, the principal average compressive stress \( \sigma_{c2} \) is not only a function of the principal compressive strain \( \epsilon_2 \) but it also depends on the principal tensile strain \( \epsilon_1 \). In other words, if cracked concrete is subjected to high tensile strains normal to the compressive direction, it shows softer and weaker characteristic in compression.

### 3.2.2 3-D Modified Compression Field Theory

Section 3.2.1 was focused on the load-deformation response of plane reinforced concrete elements subjected to plane state of stress and strain incorporating compressive strength degradation of concrete if tensile strain in the lateral direction prevails. However, the enhancement of compressive strength and ductility of concrete due to confining effects has not been considered. The 3-D extension of the Modified Compression Field Theory [8] and the corresponding proposed orthotropic secant material matrix for a concrete solid element, summarized in the following section, accounts for the aforementioned concrete material behavior.

The three dimensional secant material matrix of an orthotropic concrete material in principal directions is [8]

\[
[D] = \frac{1}{\phi} \begin{bmatrix}
E_{c1}(1 - v_{32}v_{23}) & E_{c1}(v_{12} + v_{13}v_{32}) & E_{c1}(v_{13} + v_{12}v_{23}) & 0 & 0 & 0 \\
E_{c2}(v_{21} + v_{31}v_{23}) & E_{c2}(1 - v_{31}v_{13}) & E_{c2}(v_{23} + v_{21}v_{13}) & 0 & 0 & 0 \\
E_{c3}(v_{31} + v_{21}v_{32}) & E_{c3}(v_{32} + v_{12}v_{31}) & E_{c3}(1 - v_{21}v_{12}) & 0 & 0 & 0 \\
0 & 0 & 0 & \phi G_{c12} & 0 & 0 \\
0 & 0 & 0 & 0 & \phi G_{c23} & 0 \\
0 & 0 & 0 & 0 & 0 & \phi G_{c31}
\end{bmatrix}
\]  

(3.8)

where

\[ \phi = 1 - v_{23}v_{32} - v_{21}v_{12} - v_{31}v_{13} - v_{21}v_{32}v_{13} - v_{31}v_{12}v_{23} \]  

(3.9)
The term \( \nu_{ij} \) stands for the i-th strain component due to a stress in the j-th direction or simply Poisson’s ratios. The shear moduli are given \[8\]

\[
G_{c12} = \frac{E_{c1}E_{c2}}{E_{c1}(1 + \nu_{12}) + E_{c2}(1 + \nu_{12})}
\]  

(3.10)

\[
G_{c23} = \frac{E_{c2}E_{c3}}{E_{c2}(1 + \nu_{23}) + E_{c3}(1 + \nu_{32})}
\]  

(3.11)

\[
G_{c13} = \frac{E_{c1}E_{c3}}{E_{c1}(1 + \nu_{13}) + E_{c3}(1 + \nu_{31})}
\]  

(3.12)

where \( E_{c1} \), \( E_{c2} \) and \( E_{c3} \) represent secant elastic moduli of concrete.

For the definition of the secant elastic moduli the reader is referred to \[8\]. Within a simulation, the state of stress and strain is transformed into the corresponding principal axis system where the uniaxial stress-strain relationships are evaluated.

**Concrete Constitutive Laws within the 3-D Modified Compression Field Theory**

Within the 3-D Modified Compression Field Theory, an uniaxial compressive stress-strain relationship based on the work in \[9\] is implemented. It reads \[8\]

\[
\sigma_{c3} = -f_p \frac{\epsilon_{c3}}{\epsilon_p} \frac{n}{n - 1 + (\epsilon_{c3}/\epsilon_p)^nk}
\]  

(3.13)

where \( \sigma_{c3} \) represents the third principal stress in concrete (compression); \( \epsilon_{c3} \) represents the third principal strain in concrete; \( f_p \) represents the uniaxial peak compressive stress of concrete; \( \epsilon_p \) represents the strain in the concrete at \( f_p \); \( n = 0.80 \); \( k = 1 \) for the ascending branch of Eq. 3.13 and \( k = 0.67 + \frac{f_p}{62} \) for the descending branch of Eq. 3.13.

The uniaxial peak stress \( f_p \) as well strain \( \epsilon_p \) at peak stress increase if confining stresses exist. In order to account for confinement effects, a failure criteria is implemented in the 3-D Modified Compression Field Theory \[8\]

\[
f = 2.0108 \frac{J_2}{J'_2} + 0.9714 \sqrt{J_2} f'_c + 9.1412 \frac{\sigma_{c1}}{f'_c} + 0.2312 \frac{I_1}{f'_c} - 1 = 0
\]  

(3.14)
where \( I_1 \) represents the first invariant of the stress tensor; \( J_2 \) represent the second deviatoric stress invariant; \( \sigma_{c1} \) represents the first principal stress in concrete and \( f'_c \) represents the compressive strength of concrete.

Using the failure criteria Eq. 3.14, the compressive stress \( \sigma_{c3,f} \) that causes failure in presence of stresses \( \sigma_{c1}, \sigma_{c2} \) can be found. The stress \( \sigma_{c3,f} \) is used to compute the peak stress factor

\[
K_{\sigma} = \frac{\sigma_{c3,f}}{f'_c}
\]  

which is used to modify the uniaxial peak strength \( f_p \) of concrete (Eq. 3.13).

\[
f_p = K_{\sigma} f'_c
\]

Due to confinement effects, concrete shows, along with an enhanced uniaxial peak compressive stress \( f_p \), an increasing strain \( \epsilon_p \) at \( f_p \). This behavior is captured by mean of the peak strain factor \( K_{\epsilon} \). For low confining stresses \( (K_{\sigma} < 3) \):

\[
K_{\epsilon} = 0.2036K_{\sigma}^4 - 2.819K_{\sigma}^3 + 13.313K_{\sigma}^2 - 24.42K_{\sigma} + 13.718\sqrt{K_{\sigma}} + 1
\]

and for larger confining stresses:

\[
K_{\epsilon} = 5K_{\sigma} - 4
\]

The strain \( \epsilon_p \) in Eq. 3.13 is then modified:

\[
\epsilon_p = \epsilon_0 \left\{ K_{\sigma} \left( 1 - \frac{\sigma_{c3}}{\sigma_{c3,f}} \right) + K_{\epsilon} \left( \frac{\sigma_{c3}}{\sigma_{c3,f}} \right) \right\}
\]

where \( \sigma_{c3} \) represents the current stress in the third principal direction and with the peak strain under uniaxial compression and the strain at peak stress under uniaxial compression is

\[
\epsilon_0 = \frac{n}{n - 1} \frac{f'_c}{E_c}
\]

where \( n = 0.80 \) and \( E_c \) represents the concrete modulus of elasticity.
3.2.3 Concrete in Tension

Cracks in concrete can be modeled using discrete or smeared cracks. Within the discrete crack approach concrete cracking is modeled by means of a displacement discontinuity at the interface between concrete elements. The smeared crack concept takes a different approach in which the cracked material is assumed to be a continuum and the effect of cracking is described by means of an appropriate tensile stress-strain relationship. The crack is therefore smeared out over the effected elements. A smeared crack model will be used in this study to model cracking of concrete.

Within the original smeared crack concept implemented in program DIANA 9.2, the total strain of a cracked concrete solid $\epsilon$ is decomposed into a part $\epsilon_{cr}$ of the crack and a part $\epsilon_{co}$ of the solid material. This allows for implementing constitutive laws incorporating crack dilatancy which is essentially a coupling between the crack normal direction $n$ and tangential directions $s, t$ [6]. However, using the total strain based material model for concrete in compression it is conceptually not possible to combine the approach of decomposed strain in tension with the total strain approach of concrete in compression. Therefore, a total strain based smeared crack model [4], available in program DIANA 9.2, is used to model concrete in tension. The tensile strength of concrete $f_{ct}$ is deployed as conditional detection. Once a crack is initiated, two distinctive approaches as how to handle the crack subsequently exist: the single rotating crack approach and the single fixed crack approach. In program DIANA 9.2, both approaches are implemented within the context of total strain based concrete constitutive modeling. In the single fixed crack approach, the orientation of the crack remains unaltered during the entire computation process. In the total strain based material models, the stress-strain relationships of concrete are evaluated in principal stress space. Thus, a disadvantage arises when using the total strain single fixed crack model. At incipient of cracking, the element principal axes of strain are replaced by crack directions. Subsequently, the uniaxial constitutive stress-strain relationships are evaluated in a coordinate system constrained to the crack direction. This yield a misalignment of the principal directions of stresses and the principal directions of strains in subsequent load steps. This can lead to spurious high tensile stresses that might exceed the tensile strength $f_{ct}$ multiple times [6]. The misalignment of principal strain and crack direction can be avoided if a rotating crack model is used. The orientation of the crack rotates with the axes of principal strain. In conjunction with a tensile cut-off criteria, tensile stresses which exceed the tensile strength of concrete are then avoided. Thus, the rotating crack model is used in this study.
The post cracking response of reinforced concrete differs from that of plain concrete. While plain concrete is a brittle material which exhibits softening behavior upon crack initiation if subjected to uniaxial tensile stress, concrete in RC structures continues to carry tensile stress between the cracks due to the transfer of forces from the tensile reinforcement to the concrete through bond. In order to include this "tension stiffening" effect a material model proposed in [10] is used. The affiliated material parameters and tensile stress-strain relationship is presented in section 3.2.4.

### 3.2.4 Concrete Material Parameters

#### Concrete Compressive Stress-Strain Relationships

In the side support, center column and the RC beam cover, the compressive relationship presented in Eq. 3.13 is adopted. The compressive cylinder strength $f'_c$, listed in table 3.2, has been determined in compression tests [1]. The modulus of elasticity for concrete is estimated using

$$E_c = 57,000 \sqrt{f'_c} \quad [\text{psi}] \quad (3.21)$$

The strain at peak stress under uniaxial compression $\epsilon_0$ is determined using Eq. 3.20. The resulting uniaxial stress-strain relationship is shown in Fig. 3.4.

<table>
<thead>
<tr>
<th>Region</th>
<th>$E_c$ [ksi]</th>
<th>$\nu$</th>
<th>$f'_c$ [ksi]</th>
<th>$\epsilon_0$</th>
<th>$\epsilon_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core Concrete</td>
<td>4400</td>
<td>0.15</td>
<td>6.6</td>
<td>0.0022</td>
<td>0.02</td>
</tr>
<tr>
<td>Rest of Specimen</td>
<td>4400</td>
<td>0.15</td>
<td>6.0</td>
<td>0.002</td>
<td>0.006</td>
</tr>
</tbody>
</table>

The total strain based material model, as it is implemented in program DIANA 9.2, proved to be non suitable for the core elements. The approach to modify the peak compressive stress $f_p$ if confining stresses exist, taken in the 3-D Modified Compression Field Theory (Section 3.2.2), has been completely adopted. Yet, the strain at peak compressive stress $\epsilon_p$ is modified using a different approach. The peak strain factor $K_\epsilon$ (Eq. 3.17 and 3.18) is set to be equal to peak stress factor $K_\sigma$ (Eq. 3.15). However, setting $K_\epsilon = K_\sigma$ is suitable for unconfined concrete or the cover concrete of the RC beam [8]. For confined concrete, the ratio of strain at peak compressive stress $\epsilon_p$ to strain at peak stress under uniaxial compression $\epsilon_0$ increases much.
faster than the ratio of peak stress $f_p$ to compressive strength of concrete $f'_c$ [8]. Furthermore, for the enhanced ductility of confined concrete is accounted for using

$$
\sigma_{c3} = -f_p \left( 1 - (1 - r) \frac{\epsilon_{c3} - \epsilon_p}{\epsilon_u - \epsilon_p} \right)
$$

(3.22)

where $r$ represents a factor which models the residual strength and $\epsilon_u$ denotes the ultimate strain of concrete

$$
\epsilon_u = \left( \frac{f_p}{f_{cc}} \right)^\gamma \epsilon_p
$$

(3.23)

and $\gamma = 3.0$.

The ultimate strain $\epsilon_u$, predicted using Eq. 3.23, does not lead to sufficient ductility of the concrete core in the presence of the amount of confinement as is provided by the transverse reinforcement of the RC beam. This can be shown by comparing the existing model in program DIANA 9.2 with Manders model [11], summarized below.
The uniaxial compressive stress-strain relationship developed in [11] is

\[ \sigma_{c3} = \frac{f'_{cc}x_r}{r - 1 + x_r} \]  

(3.24)

with

\[ x = \frac{\epsilon_{c3}}{\epsilon_{cc}} \]

(3.25)

and

\[ \epsilon_{cc} = \epsilon_c \left[ 1 + 5 \left( \frac{f'_{cc}}{f'_{c0}} - 1 \right) \right] \]

(3.26)

where \( f'_{c0} \) represents the compressive strength of unconfined concrete; \( \epsilon_{c0} \) represents the corresponding concrete strain (taken to 0.002); \( f'_{cc} \) represents the compressive strength of confined concrete and \( \epsilon_{cc} \) represents the corresponding concrete strain.

In order to estimate the compressive strength \( f'_{cc} \) of confined concrete, the five parameter William-Warnke failure surface is used [11].

\[ f'_{cc} = f'_{c0} \left( -1.2456 + 2.254 \sqrt{1 + \frac{7.94 f'_{l}}{f'_{c0}} - 2 \frac{f'_{l}}{f'_{c0}}} \right) \]

(3.27)

where \( f'_{l} \) represents the effective lateral confining stress.

The effective lateral confining stress is a function of the yield stress \( f_{yh} \), of the transverse reinforcement, the confinement effectiveness coefficient \( k_e \) and the ratio \( \rho \) of the volume of transverse confining steel to the volume of the RC beam core.

\[ f'_{l} = k_e \rho f_{yh} \]

(3.28)

The confinement effectiveness coefficient is

\[ k_e = \frac{A_e}{A_{cc}} \]

(3.29)

with

\[ A_e = \left( 1 - \sum_{i=1}^{n} \frac{(w_i')^2}{6b_o d_c} \right) \left( 1 - \frac{s'}{2b_c} \right) \left( 1 - \frac{s'}{2d_c} \right) \]

(3.30)
\[ A_{cc} = A_c(1 - \rho_{cc}) \]  

(3.31)

where \( A_e \) represents the area [in\(^2\)] of the effectively confined concrete core; \( A_{cc} \) represents the area [in\(^2\)] of the core of the RC beam enclosed by the center lines of the perimeter hoop and \( \rho_{cc} \) represents the ratio of the area of the longitudinal reinforcement to the area of the core of the RC beam section.

In Fig. 3.5, the cross section of the RC beam at the face of the center column is depicted. The height of the compressive zone at peak compressive stress is estimated to a third of the section height. The strain at the top of the core section is assumed to be at \( \epsilon_{co} = 0.002 \).

The resulting uniaxial compressive stress-strain relation of the concrete core and the trilinear approximation, which is used in the finite element model, are shown in Fig. 3.6. The material parameters for the concrete core and the characteristic values of the uniaxial compressive stress-strain relationship are listed in Table 3.2.

**Concrete Tensile Stress-Strain Relationship**

A rotating crack model, outlined in section 3.2.3, is adopted in order to incorporate cracking of the concrete in the RC beam. In order to account for tension stiffening effects of reinforced concrete in tension, which contributes to the overall flexural stiffness of the beam, a piecewise linear tensile stress-strain relationship developed in [10] is adopted. The model is depicted in Fig. 3.7 and the characteristic values are listed in Table 3.3. The tensile strength used as tension cut-off condition is

\[ f_{ct}' = 6.45 \cdot \sqrt{f_c'} \quad [\text{psi}] \]  

(3.32)
Figure 3.6: Uniaxial compressive stress-strain relationship for concrete core [11] and the trilinear approximation

Figure 3.7: Uniaxial tensile stress-strain relationship of reinforced concrete [10]

Table 3.3: Parameters of tensile stress-strain relationship for reinforced concrete

<table>
<thead>
<tr>
<th>$P_t$</th>
<th>$R_t$</th>
<th>$S_t$</th>
<th>$F_t$</th>
<th>$\epsilon_{ct}$</th>
<th>$f_{ct}$ [ksi]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0.45</td>
<td>4</td>
<td>10</td>
<td>1.4e-4</td>
<td>0.5</td>
</tr>
</tbody>
</table>
3.3 Interface between Concrete and Steel Reinforcement

In order to capture the contribution of bond to the flexibility and deformation capacity of the beam, the interface between concrete and steel reinforcement elements is explicitly modeled by means of three-dimensional interface elements.

3.3.1 Mechanics of Interaction between Steel Reinforcement and Concrete

In RC elements, at the interface between the rebars and concrete, three mechanisms are activated if the rebars are subjected to tensile or compressive stresses: chemical adhesion, mechanical interaction between the rebar ribs and the concrete keys as well as friction [12]. Fig. 3.8 schematically shows bond stress-slip relationships [13].

For bond stress values $\tau \leq \tau_I$, the concrete surrounding the rebar remains in the elastic stage and chemical adhesion governs the transfer of the rebar stress into the surrounding concrete. The slip between the concrete and the rebar remains small [13].

![Figure 3.8: Qualitative local bond stress-slip relationships [13]](image-url)

For bond stress values $\tau > \tau_I$, the capacity of the chemical adhesion to transfer the rebar stress into the concrete is exceeded and bearing stresses in front of the rebar ribs are generated (mechanical interaction). Transverse cracks, shown in Fig. 3.9, arise in the surrounding concrete.
causing the rebar to slip. As the bond stress $\tau$ increases, the concrete subjected to bearing stresses crushes, shown in Fig. 3.9, and the bond stiffness decreases (Fig. 3.8, Stage II).

![Diagram showing bond failure](image)

Figure 3.9: Transverse cracks originated at the tips of the steel reinforcement ribs and crushed concrete in front of the ribs of the rebar [12]

The outward component of the bond stress is balanced by ring tensile stresses [14], shown in Fig. 3.10. If the tensile stresses become large enough, longitudinal cracks develop along the rebar. If no or an insufficient amount of transverse reinforcement is provided, the longitudinal cracks reach the outer concrete surface and bond failure occurs. (Fig. 3.8, Stage III, Curve 1).

As the amount of transverse reinforcement increases, the spreading of the longitudinal cracks is increasingly restricted as shown in Fig 3.11. The longitudinal cracks do not reach the outer concrete surface and as a result, larger bond stresses can develop [13] (Fig. 3.8, Stage IVa, Curve 2). The development of longitudinal cracks remains within an small area around the rebar section if the confinement action, either provided by transverse reinforcement or transverse pressure, further increases. More transverse cracks in the surrounding concrete arise and the crushing of the concrete in front of the rebar ribs continues [12]. This bond mechanism provides the largest bond strength (Fig. 3.8, stage IVb, Curve 3). The magnitude of bond stress in stage IVb can be as high as $0.5f'_c$.

At maximum bond stress $\tau_{\text{max}}$, the concrete keys between the rebar ribs start to shear off. With increasing slip, an increasingly larger part of the concrete keys are sheared off and the bond stress decreases until only the frictional part of the bond resistance $\tau_{\text{residual}}$ remains when the concrete surface is smoothed out [12] (Fig. 3.8, Curve 2 and 3).
3.3.2 Review of Existing Bond Stress-Slip Relationships

In [12], pull-out tests have been conducted using a specimen aimed to simulate conditions within a beam-column joint. The specimen consisted of a single deformed rebar embedded in a concrete block over a length of $l_e = 5d_b$. The bond stress along the embedment length was assumed to be evenly distributed due to the short anchorage length. For the majority of the tests, the concrete block was reinforced with longitudinal (perpendicular to the axis of the anchored rebar) and transverse (parallel to the axis of the anchored rebar) steel reinforcement. For the remaining tests the concrete block was not reinforced in order to obtain reference data for unconfined concrete. One protruding end of the anchored rebar was subjected to load under displacement control while the slip was measured as the movement of the unloaded end of the rebar with respect to the concrete anchorage block. The load applied to the rebar was reacted as a compressive force at the face of the concrete block. Bond stress-slip relationships were deduced by taking applied forces at given slip value and converting them into bond stress using Eq. 3.33.

$$\tau = \frac{F}{\pi \cdot d_b \cdot l_e}$$

(3.33)

where $F$ represents the applied force; $d_b$ represents the rebar diameter and $l_e$ represents the embedment length.
The test program consisted of several series each designed to investigate a single varying parameter that influences bond characteristic while all other specimen parameter were kept constant. The longitudinal reinforcement varied from #2 to #8 rebars and the transverse reinforcement varied from #2 to #4 stirrups. Furthermore, the diameter of the anchored rebar varied from 0.5 in to 1.0 in in order to obtain information with regard to the influence of the rebar diameter on the bond characteristics. Additionally, a test series was conducted where the specific rib area varied from $f_r = 0.066$ to $f_r = 0.12$. The specific rib area is

$$f_r = \frac{A_r}{\pi \cdot d_b \cdot s_r}$$  \hspace{1cm} (3.34)

where $A_r$ represents the area of the projection of a single rib on the cross-section of the rebar; $s_r$ represents the rib spacing and $d_b$ represents the rebar diameter.

A test series with concrete compressive strength varying from $f'_c = 4350$ psi to $f'_c = 7975$ psi was also conducted. In all tests, a splitting crack developed in the plane of the longitudinal axis of the anchored rebar and the failure of bond was observed if the concrete was not confined by reinforcement. The concrete between the ribs of the rebar was intact. In case of confined concrete, the growth of the splitting crack was controlled by the longitudinal reinforcement. The load could be increased further and failure of bond was caused by pulling out of the anchored rebar. It is reported that the concrete between the rebar ribs was completely sheared off and almost pulverized. As a result of these tests, Eq. 3.35 was proposed to represent an average of experimentally obtained bond stress-slip curves for confined concrete.

$$\tau = \tau_1 \cdot \left( \frac{s}{s_1} \right)^{\alpha}; s \leq s_1$$ \hspace{1cm} (3.35)

where $\tau$ represents the bond stress; $s$ represents the slip of the rebar; $\tau_1 = 13.5$ N/mm$^2$; $s_1 = 1$ mm and $\alpha = 0.4$

According to [12], bond between rebars and concrete scatters. The values for $\tau_1$ and $\alpha$ varied between $\tau_1 \approx 11.5$ N/mm$^2$ to $\tau_1 \approx 15.5$ N/mm$^2$ and $\alpha \approx 0.33$ to $\alpha \approx 0.45$. It is reported that the influence of the the rebar diameter on bond characteristics was rather small. The values of $\tau_{\text{max}}$ increased approximately proportional to $\sqrt{f'_c}$ and the corresponding slip values decreased almost proportional to $\frac{1}{\sqrt{f'_c}}$. Varying the clear spacing between the rebars from a minimum value of $s_b = 1d_b$ to a maximum value of $s_b = 4d_b$ resulted in an increase of $\tau_{\text{max}}$ of about 20%. It is furthermore reported that the distance between the rebar ribs $s_r$ greatly influences
the characteristic of bond. The slip value \( s_1 \) at which \( \tau_{\text{max}} \) was reached increased if \( s_r \) was enlarged. Thus, modification factors are proposed which may be applied to Eq. 3.35 if it is intended to use the bond stress-slip relationship (Eq. 3.35) in specific cases where different confining reinforcement details, rebar diameter, deformation pattern and concrete compressive strength prevail. These modification factors are presented and applied later in this chapter.

In [15], three direct tension tests were conducted. The test set up was designed to simulate conditions prevailing in a region of a beam between flexural tensile cracks. The specimen consisted of a single #8 rebar centrally embedded over a length of \( 18d_b \) in a cross section concrete block of 5x5x18 in. The rebar was subjected to tensile forces at the projecting ends. A bond-stress slip relationship was determined using a different technique compared to aforementioned pull-out test. The rebar was equipped with internal strain gages which were distributed along the embedment length. Additionally strain gages were embedded in the concrete block at a distance of 0.25 in from the rebar surface. The concrete strain and the strain of the embedded rebar were measured and recorded at different locations along the embedment length. Subsequently, each strain function was integrated resulting in the displacement of the concrete and the rebar. The slip was then computed by subtracting the concrete displacements from the rebar displacements. The findings of this study show that the bond stress-slip relationship varies significantly along the embedment length. The maximum bond stress varied from \( 4\sqrt{f_c'} \) at a distance of 2 in from the loaded end of the rebar to \( 10\sqrt{f_c'} \) at a distance of 6 in from the loaded end of the rebar. The slip at which \( \tau_{\text{max}} \) was reached varied from 0.0005 in at 2 in to 0.0015 in at 6 in from the loaded end of the rebar. Using the test results, Eq. 3.36 was proposed in [16] to represent a best fit to the average of the experimental results.

\[
\tau = f_c' \cdot (16.7s - 8260s^2 + 1.12 \cdot 10^6s^3) \quad \text{ksi}
\]  

where \( f_c' \) represents concrete compressive strength and \( s \) represents the rebar slip.

In [17], a series of direct tension tests on concrete square prisms was conducted. Each specimen was axially reinforced with one #8 central rebar. The rebar was instrumented with internal strain gages. This technique allowed for obtaining the steel stress distribution along the length of the rebar. The length of the concrete prism was 16 in which yield an embedment length of \( 16d_b \). Cross sectional dimensions varied from 2x2 in to 6x6 in which led to different ratios of concrete cover to rebar diameter. The specific rib area \( f_r \) for all rebars was nearly constant at \( f_r = 0.14 \) to 0.15. The concrete compressive strength varied from \( f_c' = 4300\text{-}5000 \) psi. The slip
of the rebar was measured at 0.25 in from the rebar surface using a micrometer. Bond stress-slip relationships were deduced by using Eq. 3.33. The force \( F \) in Eq. 3.33 was derived as force transferred into the concrete using information provided by the stress distribution along the rebar. It is reported that in all tests no crushing of concrete in front of the rebar lugs occurred. Therefore, it is concluded that the observed slip is due to the internal cracking of the first layer of the concrete surrounding the rebars (see Fig. 3.15) and due to bending and/or cracking of the concrete keys near the rebar ribs. Furthermore, it was observed that the concrete compressive strength had an insignificant effect on measured slip values. Yet, the maximum bond stress was observed to be proportional to \( \sqrt{f_c} \). It is also reported that with increasing cover thickness, the slip at which maximum bond stress was reached decreased owing to the enhanced restraining capacity of the concrete mass. As a result of this study, Eq. 3.37 was proposed to represent a best fit to experimentally obtained data which were normalized to a concrete compressive strength of \( f'_c = 5000 \) psi.

\[
\tau = 1.95 \cdot 10^6 s - 2.35 \cdot 10^9 s^2 + 1.39 \cdot 10^{12} s^3 - 0.33 \cdot 10^{15} s^4 \quad [\text{psi}] \tag{3.37}
\]

where \( f'_c \) represents concrete compressive strength and \( s \) represents the slip of the rebar.

In Fig. 3.12 the bond stress-slip relationships are shown for two slip ranges. As can be seen, the bond stress-slip relationships differ considerably. For very small values of up to a slip of about 0.0001 in the bond stiffness is rather close. But with increasing slip values the difference in the tangent modulus is apparent. According to [12], a large scatter in the initial bond stiffness may be caused by inaccuracies in measuring the slip between the rebar and the concrete correctly. If slip values are deduced from measured concrete and rebar strains, as conducted in [15], the error even accumulates. Also, the bond stress-slip relationship for rebars embedded over a length \( > 5d_b \) cannot be considered as constant along the embedment length. It varies considerably at distinct locations along the rebar. Furthermore, it is reported that the position of the rebars during casting influences the initial bond stiffness [12]. Rebars cast horizontally show much smaller initial bond stiffness compared to rebars cast vertically.
Another potential source of the scatter in initial bond stiffness is the use of different test specimens with different stress conditions prevailing in the concrete surrounding the rebar. While in pull-out tests the concrete is subjected to longitudinal compression, in direct tension tests the concrete is subjected to tensile stresses. Experimental findings indicate that one of the main sources of scatter in initial bond stiffness is the use of rebars with different deformation pattern which lead to different specific rib area \( f_r \) for identical rebar diameter [12]. Larger specific rib area yield larger bearing area of the rebar ribs which in turn induces smaller bearing stresses on the concrete between the rebar ribs at equal rebar stress level. This reduces the strain of the concrete in the vicinity of the rebar which lead to smaller slip values at the steel-concrete interface. Additionally, it is reported that for identical specimens maximum bond stress \( \tau_{\text{max}} \) was reached at slip values of \( s_{\text{max}} = 0.028 \text{ in} \) and \( s_{\text{max}} = 0.054 \text{ in} \) for rebars having a specific rib area of \( f_r = 0.12 \) and \( f_r = 0.066 \), respectively [12]. This corresponds to an increase in initial bond stiffness of about 50% due to an increase in specific rib area of about 50%. The specific rib area reported in [12] and [17] varied from \( f_r = 0.065 \) to \( f_r = 0.14 - 0.15 \), respectively, whereas the diameter of the rebar in both test series was the same.

3.3.3 Parameters influencing Bond Performance

Geometry and Deformation Pattern of Steel Reinforcement

As summarized and discussed in the previous section, the rib geometry of the rebar has paramount importance among the other parameters that effect bond behavior. It has been found that the maximum bond stress \( \tau_{\text{max}} \) strongly depends on the specific rib area \( f_r \). Experimental results [12] indicate that the slip \( s_{\text{max}} \) at which \( \tau_{\text{max}} \) is reached or the initial bond stiffness, respectively,
increases as the specific rib area increases. Furthermore, according to [12], the rib spacing \( s_r \) has a significant effect on the characteristic values of a bond stress-slip relationship. With increasing values of \( s_r \), the slip \( s_{\text{max}} \) at which maximum bond stress \( \tau_{\text{max}} \) is reached, increases. Similarly, the slip at which the bond stress-slip relationship levels off to the frictional bond stress \( \tau_{\text{residual}} \), increase. Another important factor is the cover thickness. With an increasing cover thickness, the restraining effect on the rebar is enhanced [17]. The level of stress, present in the rebar, also affects the bond performance. As long as the rebar remains within the linear elastic range, the influence of steel reinforcement stress remains small. Yet, experimental tests show that yielding of the steel reinforcement has a negative effect on bond. It results in a sharp nonlinear descending branch in the bond stress-slip relationship once yielding of the rebar has occurred [13].

**Concrete Characteristics**

The maximum bond stress \( \tau_{\text{max}} \) varies with increasing concrete compressive strength \( f'_c \) [12]. It has been experimentally shown that \( \tau_{\text{max}} \) is proportional to \( \sqrt{f'c} \) [12] and [17]. This is because bond action results from the localized pressure induced into the concrete in front of the rebar ribs and the pressure is directly related to the shear component of bond stress [13]. Furthermore, experimental results indicate that slip values corresponding to \( \tau_{\text{max}} \) decrease approximately proportional to \( 1/\sqrt{f'c} \) which results in an increasing bond stiffness [12].

**Confinement Effects**

Transverse compressive stresses favor bond action independent on whether they result from active or passive confinement. Active confinement, resulting from a direct support or from a continuity of a column in a beam-column joint, is more efficient to prevent splitting failure since it does not depend on the bond stress itself. On the contrary, passive confinement developed by concrete cover and transverse reinforcement is less effective since it has its origin in the dilatancy of concrete cracks and the development of cracks is related to bond stress [13]. Passive confinement controls the spreading of the longitudinal splitting cracks and prevents pure splitting failure, as reported in [12]. Experimental tests show that if sufficient passive confinement is provided or the stirrup confinement index \( \Omega \) is within a certain range, maximum bond strength increases to a certain extent [13].

Additional factors influencing bond characteristic are the clear rebar spacing \( s_b \) as well as the rebar diameter. In [12], a decreasing maximum bond stress is reported if the clear rebar spacing \( s_b \) falls below \( 4d_b \). A slight decrease of the maximum bond stress of about 10% for increasing
rebar diameter and equal values of relative rib area \( f_r \) was observed in [12]. Furthermore, rebar corrosion, rusting and the loading also affect bond-slip [13].

### 3.3.4 Implemented Bond Stress-Slip Relationships

The constitutive law, available in program DIANA 9.2, which governs the behavior of the 3-dimensional interface elements is [8]

\[
\begin{pmatrix}
\tau_n \\
\tau_t
\end{pmatrix} = \begin{pmatrix}
k_n & 0 \\
0 & \frac{\partial \tau(s_t)}{\partial s_t}
\end{pmatrix} \cdot \begin{pmatrix}
\Delta s_n \\
\Delta s_t
\end{pmatrix}
\] (3.38)

where \( \tau_n, \tau_t \) represent the normal and the tangential component of the interface traction or the bond stress; \( \tau(s_t) \) represents the bond stress-slip relationship provided as input; \( k_n \) represents the normal stiffness of the interface between concrete and rebar and \( s_n, s_t \) represent the normal and the shear component of the interface relative displacement or the bond slip.

As can be seen, the shear and normal components are decoupled. Thus, the interaction between the shear component of bond and the lateral contraction and extension of the rebars due to Poisson’s effect cannot be modeled. Furthermore, no communication protocol between interface elements and concrete elements or steel reinforcement elements is implemented in program DIANA 9.2. Thus, bond deterioration due to reinforcement yielding and concrete crushing cannot be considered.

In the finite element model of the RC beam, two distinctive regions with regard to bond conditions are defined. Region A (Fig. 3.13) is defined well as confined. Given the existence of longitudinal and transverse reinforcement, it is assumed that the bond characteristic in region A is of a pull-out failure type [13]. The bond stress-slip relationship for the interface elements around the rebars subjected to compressive stress and for the interface elements around the rebars subjected to tensile stress are assumed to be equal which complies with experimental findings [12]. Table 3.4 provides the values of \( \tau_{\text{max}}, \tau_{\text{residual}} \) and of \( s_1, s_2 \) and \( s_3 \) in region A, which are adopted according to the test data obtained in [12]. Note that the pattern of the bond stress-slip relationship in both regions is the same. The pattern was proposed in [12] and is shown in Fig. 3.14 along with the designation of the characteristic points of the curve. The concrete compressive strength \( f'_c \), the clear bar spacing \( s_b \) and the rebar diameter \( d_b \) differ from those prevailing in the pull-out tests [12]. Furthermore, the deformation pattern of the rebars used in [1] is different from that of the rebars used in [12].
Figure 3.13: Defined regions of bond in the finite element model
Figure 3.14: Pattern of the bond stress-slip relationship in region A and B

Table 3.4: Initial characteristic values of the bond stress-slip relationships in region A and B

<table>
<thead>
<tr>
<th>Region</th>
<th>$\tau_{\text{max}}$ [psi]</th>
<th>$\tau_{\text{residual}}$ [psi]</th>
<th>$s_1$ [in]</th>
<th>$s_2$ [in]</th>
<th>$s_3$ [in]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1640</td>
<td>410</td>
<td>0.035</td>
<td>0.07</td>
<td>0.4</td>
</tr>
<tr>
<td>B</td>
<td>800</td>
<td>200</td>
<td>0.01</td>
<td>0.02</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Thus, the initial values of the characteristic points of the bond stress-slip relationship in region A, given in Table 3.4, are subsequently modified according to suggestions made in [12] in order to account for differences in aforementioned parameters.

Region B within the finite element model of the RC beam is defined. As discussed earlier, the values of $\tau_{\text{max}}$ deduced from direct tension tests are considerable smaller than the values for $\tau_{\text{max}}$ obtained in pull-out test in the confined concrete blocks. A bond model presented in [14] is used to approximate the initial value of $\tau_{\text{max}}$ in region B. In the model, a cracked concrete sleeve around the rebar is restraint by an outer solid sleeve, subjected to tensile hoop stresses, such that the rebar does not slide out of the concrete [14](Fig. 3.15). The initial value of $\tau_{\text{max}}$ is estimated using Eq. 3.39. The value of $\tau_{\text{max}}$ represents the bond stress at the incident of bond failure of a embedded rebar once the thickness of the inner sleeve reaches a critical value, namely $1.664 \, d_b$ (Fig. 3.15). The transverse reinforcement of the tested RC beam comprised with a diameter of 0.135 in placed at 2.7 in. This is considerably less than in region A. Thus, the restraining effect on the concrete surrounding the rebars is not as pronounced as in region A. However, it provides to some extent additional restraining effects yielding a descending branch of the bond stress-slip relationship and an increased maximum bond stress $\tau_{\text{max}}$. 
\[ \tau_{\text{max}} = f_{ct} \cdot \frac{c_y + \frac{d_b}{2}}{1.664 \cdot d_b} = 800 \text{ psi} \quad (3.39) \]

where

\[ c_y = 0.8175 \text{ in concrete cover of the RC beam} \]
\[ d_b = 0.375 \text{ in rebar diameter} \]
\[ f_{ct} = 500 \text{ psi concrete tensile strength} \]

The initial value of \( \tau_{\text{max}} = 800 \text{ psi} \) compares reasonable to experimental data presented in section 3.3.2 (Fig. 3.12), given the scatter in maximum bond stress. With regard to the initial values of slip in region B, data given in [15] and [17] vary significantly as presented earlier and do not seem to be reliable. Therefore, the initial value of \( s_1 \) in region B is adopted based on a pull-out test on concrete specimen without confining reinforcement [12]. Values for the slip \( s_2 \) and \( s_3 \) are also suggested. However, these values are based on data which were obtained in pull-out tests showing splitting failure of the specimen yielding a sharp descending branch of the bond stress-slip relationship (in Fig. 3.14 \( s_2 = s_1 \)). Yet, due to the confinement effect, exerted by the transverse reinforcement of the RC beam, the pattern of the bond stress-slip relationship in region B is equal to that of region A. Thus, the ratio of \( s_2 \) to \( s_1 \) and of \( s_3 \) to \( s_1 \) in region A, which computes to 2 and 11.5, respectively, is used in order to obtain initial values for \( s_2 \) and \( s_3 \) in region B. Following the same approach, the ratio of \( \tau_{\text{max}} \) to \( \tau_{\text{residual}} \) in region A, which computes to 4, is used to obtain an initial value of \( \tau_{\text{residual}} \) in region B.

**Modification of Initial Characteristic Values due to Concrete Compressive Strength**

The compressive strength of the concrete of the RC beam is \( f'_c = 6 \text{ ksi} \). The initial value for \( \tau_{\text{max}} \) in region A is based on pull-out tests in specimens having a concrete compressive strength of \( f'_c = 4.35 \text{ ksi} \). In order to convert the initial values to account for the concrete strength of the RC beam, \( \tau_{\text{max}} \) and \( \tau_{\text{residual}} \) in region A are increased by a factor of about \( \sqrt{\frac{6}{4.35}} = 1.17 \).
In region B, $\tau_{\text{max}}$ is not modified since Eq. 3.39 was developed based on pull-out tests in specimens having also a compressive strength of $f'_{c} = 6$ ksi. The initial value for $s_1$ in region A and B is reduced by a factor of $\frac{1}{1.17}$ based on experimental findings in [12].

Modification of Initial Characteristic Values due to Confining Reinforcement

In [12], the ratio

$$\frac{\sum A_{sv}}{\sum A_s}$$

(3.40)

where $A_{sv}$ represents the area of the confining reinforcement and $A_s$ represents the area of the anchored RC beam reinforcement, varied between 0.0 to 4.0. Eq. 3.40 gives a value of about 1.6 for the bottom reinforcement of the RC beam and a value of about 4.0 for the top reinforcement. These values compare favorable to the values in [12]. Thus, the initial characteristic values of $\tau_{\text{max}}$ and $\tau_{\text{residual}}$ in region A are not modified.

The stirrup confinement index $\Omega$ is [13]

$$\Omega = \frac{A_{st}}{A^*}$$

(3.41)

where $A_{st}$ represents the area of the transverse reinforcement; $A^* = n \cdot d_b \cdot \Delta z$ represents the area of the rebars in the splitting plane; $n$ represents the number of rebars; $d_b$ represents the rebar diameter and $\Delta z = 2.7$ in represent the distance at which the transverse reinforcement is placed along the RC beam (Fig. 2.5, Page 3).

For the RC beam, $\Omega$ is 0.006 and 0.015 for five #3 rebars (top of section) and two #3 rebars (bottom of section). Experimental results indicate that for $\Omega$ between 0.006 to 0.014 a maximum bond stress $\tau_{\text{max}}$ of 0.12 $f'_{c}$ to 0.18 $f'_{c}$ can be expected [13]. This yield a maximum bond stress of $\tau_{\text{max}} \approx 720 - 1080$ psi considering that $f'_{c} = 6000$ psi. The initially estimated value of $\tau_{\text{max}} = 800$ psi represents a lower bound of maximum bond stress [14]. Thus, the initial characteristic value of $\tau_{\text{max}}$ in region B is increased by 200 psi in order to account for confinement effects exerted by the transverse reinforcement of the RC beam.

Modification of Initial Characteristic Values due to Deformation Pattern

The specific rib area $f_R$ of the #3 rebars used in [1] is 0.1. $f_R$ of the rebars used in [12] is 0.11. Thus, no modification factor is applied to $\tau_{\text{max}}$ in region A.
The rib spacing $s_r$ of the rebars used in [1] is $s_r = 0.2\text{ in}$. Yet, the rib spacing of the rebars used in [12] is $s_r = 0.4\text{ in}$. Thus, the initial characteristic values for $s_1$, $s_2$ and $s_3$ in region A and B are modified by a factor of $\frac{0.2\text{ in}}{0.4\text{ in}} = 0.5$, as suggested in [12].

**Modification of Initial Characteristic Values due to Rebar Spacing**

In the RC beam, the clear rebar spacing $s_b$ is 0.9 in for the top rebars and 4.27 in for the bottom rebars. The ratio

$$\frac{s_b}{d_b}$$

where $d_b$ represents the rebar diameter, is computed to be 2.4 and 11.38 for the top and the bottom rebars, respectively. Experimental findings suggest a reduction of $\tau_{\text{max}}$ and $\tau_{\text{residual}}$ by 10% for values of 2.4 [12]. This reduction is applied to the initial values in region A and B.

The final values for the characteristic points of the bond stress-slip relationships, which are implemented in the finite element model, are listed in Table 3.5 and Fig. 3.16 shows the affiliated bond stress-slip relationships.

Table 3.5: Characteristic values of interface bond stress-slip relationships of region A and B as implemented in the finite element model

<table>
<thead>
<tr>
<th>Region</th>
<th>$\tau_{\text{max}}$ [psi]</th>
<th>$\tau_{\text{residual}}$ [psi]</th>
<th>$s_1$ [in]</th>
<th>$s_2$ [in]</th>
<th>$s_3$ [in]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1740</td>
<td>500</td>
<td>0.016</td>
<td>0.035</td>
<td>0.2</td>
</tr>
<tr>
<td>B</td>
<td>1000</td>
<td>250</td>
<td>0.004</td>
<td>0.008</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Figure 3.16: Bond stress-slip relationship of interface elements in region A and B as implemented in the finite element model
Chapter 4 Analytical Results

In this chapter, the variation of stress, strain, bond stress, and slippage along the beam are presented. Fig. 4.2 shows the locations where the results for concrete, steel and interface elements were obtained. Contour plots of two longitudinal profiles are also presented. Longitudinal profile 1 includes core and cover concrete whereas longitudinal profile 2 includes only cover concrete, as depicted in Fig. 4.1. The results in this chapter are presented for different load steps (LS). The vertical displacement associated with each load step is listed in Table 4.1.

![Figure 4.1: Designation of longitudinal views used for contour plots](image1)

![Figure 4.2: Locations of concrete C, steel S and interface I elements](image2)

Table 4.1: Analysis load steps and associated vertical displacement at the face of center column

<table>
<thead>
<tr>
<th>Load Step (LS)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Displacement [in]</td>
<td>0.15</td>
<td>0.3</td>
<td>0.48</td>
<td>0.76</td>
<td>1.0</td>
<td>1.2</td>
<td>1.45</td>
<td>1.68</td>
<td>1.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load Step (LS)</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Displacement [in]</td>
<td>2.13</td>
<td>2.35</td>
<td>2.58</td>
<td>2.8</td>
<td>3.21</td>
<td>3.62</td>
<td>4.44</td>
<td>5.26</td>
</tr>
</tbody>
</table>
4.1 Analytical Results at Material Level

In this section, analytical results along the beam are presented. Note that the center line of the two-span RC beam (where the vertical displacement is applied) is located at zero distance (see Fig. 2.5). The face of the side support is at a distance of 82 in.

Concrete in Tension

Behavior of concrete in tension is modeled using rotating cracks (Sec. 3.2.3). Thus, at every location within the beam, tensile stresses are expected to be less than the tensile strength of $f_{ct} = 500$ psi. In Fig. 4.3, the variation of concrete longitudinal stress $\sigma_{c,zz}$ along the beam at location C3 is shown for three load steps LS1, LS2 and LS3 (see Table 4.1). As can be seen, longitudinal tensile stresses $\sigma_{c,zz}$ do not exceed the tensile strength $f_{ct} = 500$ psi at any location. The distance between location of peak tensile stress of 500 psi and the face of the center column in Fig. 4.3 can be considered as the length over which the concrete tensile strain has exceeded the crack strain of $\epsilon_{cr} = 0.00014$ and the concrete elements go through the descending branch of uniaxial tensile stress-strain relationship (see Fig. 3.7). The tensile stress equal to zero indicates that concrete tensile strain has exceeded ultimate tensile strain $\epsilon_{u,cr} = 0.0014$ (see Fig. 3.7). Within the distance between the peak tensile stress of 500 psi and inflection point (zero bending moment), concrete elements are in the pre-cracking elastic stage. As can be seen in Fig. 4.4, for larger deformations, tensile stresses no longer reach the tensile strength of $f_{ct} = 500$ psi. This indicates that all concrete elements between the face of the center column and inflection point and top bar cut-off location at C3 have passed the crack strain of $\epsilon_{cr} = 0.00014$. Three contour plots of concrete first principal stress $\sigma_{c,1}$, shown in Fig. 4.5, also confirm that stress values have not exceeded the tensile strength of $f_{ct} = 500$ psi throughout the analysis.
ANALYTICAL RESULTS AT MATERIAL LEVEL

Figure 4.3: Variation of concrete longitudinal stress $\sigma_{c,zz}$ at cross sectional location C3 at 0.15, 0.3 and 0.48 in of vertical displacement

Figure 4.4: Variation of concrete longitudinal stress $\sigma_{c,zz}$ at cross sectional location C3 at 1.45, 1.68 and 1.9 in of vertical displacement
Figure 4.5: Longitudinal profile 1 of first principal stress $\sigma_{c,1}$ [psi] at three vertical displacements: 0.76 (Top); 1.0 (Middle) and 1.2 in (Bottom)

Concrete in Compression

Figs. 4.6 and 4.7 show the variation of concrete longitudinal stress $\sigma_{c,zz}$ along the beam at cross sectional locations C1 and C2 (see Fig. 4.2), respectively. Maximum concrete compressive stress of $\sigma_{c,zz} = -5227$ psi (Fig. 4.6) and $\sigma_{c,zz} = -5789$ psi (Fig. 4.7) are calculated to occur at 1.9 in of vertical displacement for location C1 and C2, respectively. For concrete elements at cross sectional location C3 and C4, maximum concrete compressive stress in longitudinal direction of $\sigma_{c,zz} = -5911$ psi (Fig. 4.8) and $\sigma_{c,zz} = -6278$ psi (Fig. 4.9) are calculated at a vertical displacement of 1.2 in, respectively. The peak compressive strengths of $f'_{c} = 6000$ psi for cover concrete and $f'_{c}=6600$ psi for core concrete have not been exceeded throughout the analysis. With regards to third principal stress, maximum values of $\sigma_{c,3} = 6000$ psi and $\sigma_{c,3} = 6600$ psi for cover concrete and core concrete, respectively, are calculated (Figs. 4.10 and 4.11). This is consistent with the Modified Compression Field Theory where the state of stress is evaluated in principal stress space. Values for the third principal stress larger than the strength of uniaxial compressive stress-strain relationship are not expected to occur.
Figure 4.6: Variation of concrete longitudinal stress $\sigma_{c,zz}$ at cross sectional location C1 at 1.45, 1.68 and 1.9 in of vertical displacement

Figure 4.7: Variation of concrete longitudinal stress $\sigma_{c,zz}$ at cross sectional location C2 at 1.45, 1.68 and 1.9 in of vertical displacement
Figure 4.8: Variation of concrete longitudinal stress $\sigma_{c,zz}$ at cross sectional location C4 at 0.76, 1.0 and 1.2 in of vertical displacement

Figure 4.9: Variation of concrete longitudinal stress $\sigma_{c,zz}$ at cross sectional location C3 at 0.76, 1.0 and 1.2 in of vertical displacement
ANOY topical RESULTS AT MATERIAL LEVEL

Figure 4.10: Longitudinal profile 1 of third principal stress $\sigma_{c,3}$ at 1.2, 1.45, 1.68 and 1.9 in (plots from top to bottom) of vertical displacement (the face of the side support is located on the left, the face of the center column is located on the right)

Figure 4.11: Longitudinal profile 2 of third principal stress $\sigma_{c,3}$ at 1.2, 1.45, 1.68 and 1.9 in (plots from top to bottom) of vertical displacement (the face of the side support is located on the right, the face of the center column is located on the left)
Steel Reinforcement

Figs. 4.12 and 4.13 show the variation of longitudinal strain $\epsilon_{s_{zz}}$ and stress $\sigma_{s_{zz}}$, respectively, for steel reinforcement elements at cross sectional location S1 (see Fig. 4.2). At the vicinity of the side support, a maximum tensile strain of $\epsilon_{s_{zz}} = 0.0341$ is calculated at a vertical displacement of 5.26 in (LS17). At the vicinity of the center column, a maximum compressive strain of $\epsilon_{s_{zz}} = -0.0355$ is computed at the same level of vertical displacement. Maximum steel reinforcement stress values in longitudinal direction of $\sigma_{s_{zz}} = 92300$ psi in tension and $\sigma_{s_{zz}} = -94500$ psi in compression are calculated at a vertical displacement of 5.26 in.

Figure 4.12: Variation of steel longitudinal strain at cross sectional location S1 at 4.44 and 5.26 in of vertical displacement (horizontal dashed lines represent yield strains)

Fig. 4.14 shows the variation of longitudinal strain $\epsilon_{s_{zz}}$ for steel reinforcement elements at cross sectional location S2 (see Fig. 4.2). At a vertical displacement of 5.26 in, a maximum tensile strain of $\epsilon_{s_{zz}} = 0.13$ is computed at the vicinity of the center column and a maximum compressive strain of $\epsilon_{s_{zz}} = -0.0586$ at the vicinity of the side support. Fig. 4.15 shows the tensile stress-strain relationships of the first three steel reinforcement elements from the face of the center column. As can be seen, the bar fracture occurs in the second element at a vertical displacement of 5.26 in.
ANALYTICAL RESULTS AT MATERIAL LEVEL

Figure 4.13: Variation of steel longitudinal stress at cross sectional location S1 at 4.44 in and 5.26 in of vertical displacement (horizontal dashed lines represent yield stresses)

Figure 4.14: Variation of steel longitudinal strain at cross sectional location S2 at 4.44 and 5.26 in of vertical displacement (horizontal dashed lines represent yield strains)
Figure 4.15: Tensile stress-strain of the first three steel reinforcement elements from the face of the center column at cross sectional location S2. (Uniaxial stress-strain relationship of steel reinforcement as implemented in the finite element model is also shown.)

*Interface between Concrete and Steel Reinforcement*

Figs. 4.16 and 4.17 show the variation of bond stress and bond slip, respectively, for interface elements at location I1 (see Fig. 4.2). At the face of the center column a change in sign of bond stress and bond slip values, compared to the remaining interface elements along the beam, is observed. Steel reinforcement in compression is pushed into the center block. In order to satisfy the compatibility, the first element of the beam steel reinforcement has to move into the center block yielding the same sign of bond stress and bond slip values as for interface elements within the center column. After a transition, bond stress and bond slip variation is consistent with the change in bending moment along the beam. The same pattern of bond stress and bond slip variation can be observed for interface elements around steel reinforcement in tension in the vicinity of the side support. As opposed to steel reinforcement in compression, the anchored steel reinforcement in tension is pulled out of the side support. Again, for compatibility reasons, bond stress and bond slip values of interface elements around the first steel reinforcement element of the beam have the same sign as bond stress and bond slip values of interface elements within the side support. The maximum bond stress of $\tau_{max} = 1000$ psi is calculated at a vertical displacement of about 1.68 in at the vicinity of the center column.
Figure 4.16: Variation of bond slip at cross sectional location I1 at 1.45, 1.68 and 1.9 in of vertical displacement

Figure 4.17: Variation of bond stress at cross sectional location I1 at 1.45, 1.68 and 1.9 in of vertical displacement
The corresponding bond slip is larger than the yield slip of \( s_1 = 0.004 \) which is consistent with the implemented bond stress-slip relationship. Bond stress and bond slip values for interface elements further from the face of the center column remain below maximum bond stress \( \tau_{max} \) and yield slip \( s_1 \) throughout the analysis.

Similar results are obtained for the variation of bond stress and bond slip (see Figs. 4.18 and 4.19) for interface elements at cross sectional location I2 (see Fig. 4.2). As for interface elements at cross sectional location I1 (see Fig. 4.2), the maximum bond stress of \( \tau_{max} = 1000 \text{ psi} \) is calculated only for interface elements around compressive reinforcement at the vicinity of the face of the side support (Fig. 4.19). Also, the corresponding bond slip is larger than 0.004 which is again consistent with the implemented bond stress-slip relationship. Bond stress and bond slip values for interface elements more far from the side support remain below maximum bond stress \( \tau_{max} \) and yield slip \( s_1 \), respectively, throughout the analysis. However, as opposed to interface elements at cross sectional location I1, the maximum bond stress \( \tau_{max} \) is calculated to occur at a vertical displacement of about 2.13 in.

![I2 Slip Distribution](image)

Figure 4.18: Variation of bond slip at cross sectional location I2 at 2.1, 2.35 and 2.58 in of vertical displacement
Figure 4.19: Variation of bond stress at cross sectional location I2 at 2.13, 2.35 and 2.58 in of vertical displacement

4.2 Bond Slip and Section Deformation

In this section, computed nodal displacements at selected locations will be evaluated with regard to sectional deformation as well as steel reinforcement pull-out and push-in. Fig. 4.20 shows the cross sections along the beam for which vertical lines, V2 and V3, are defined. Along these lines nodal displacements were obtained from the program. The designations of sections, for instance SEC 1_V2, are subsequently used in this section.

Figure 4.20: Designation of beam sections

Interface elements were implemented in the finite element model. As a result, the anchored steel reinforcement is either pulled out (tension) or pushed into (compression) the side support block or the center column block, respectively. In table 4.2, the calculated maximum values of steel reinforcement pull-out and push-in at SEC 1 and SEC 4 are listed.
Table 4.2: Values of steel reinforcement pull-out and push-in at about 5.26 in of vertical displacement

<table>
<thead>
<tr>
<th></th>
<th>SEC1</th>
<th></th>
<th>SEC4</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V2</td>
<td>V3</td>
<td>V2</td>
<td>V3</td>
</tr>
<tr>
<td>Pull Out</td>
<td>0.027 in</td>
<td>0.0136 in</td>
<td>0.04 in</td>
<td>-</td>
</tr>
<tr>
<td>Push In</td>
<td>0.0167 in</td>
<td>-</td>
<td>0.0169 in</td>
<td>0.013 in</td>
</tr>
</tbody>
</table>

The maximum value of bar pull-out is calculated for steel reinforcement in tension anchored in the center column block (SEC 4\_V2). The calculated maximum value of bar pull-out at SEC 1\_V2 is 32% less. A comparison of pull-out values calculated for SEC 1\_V2 and SEC 1\_V3 shows a difference of about 50%. The calculated bar push-in values are within similar range at both beam end sections.

Figs. 4.21 and 4.22 show nodal displacements at SEC 2\_V2 and SEC 3\_V2 at a vertical displacement of 1.0 in and 1.9 in. Note that the horizontal axis represents the nodal displacements in longitudinal direction (z-direction) and the vertical axis represents the section height. As can be seen, calculated nodal displacements indicate that during the deformation, cross sections remain plain if only the nodal displacements of concrete elements are considered. However, as a result of the bond slip, the cross sections do not remain plain if the nodal displacements of steel reinforcement elements are also considered.

Figs. 4.23 and 4.24 show nodal displacements at SEC 1\_V2 and SEC 4\_V2 at a vertical displacement of 1.0 in and Figs. 4.25 and 4.26 show nodal displacements at SEC 1\_V2 and SEC 4\_V2 at a vertical displacement of 1.9 in. Similar to SEC 2\_V2 and SEC 3\_V2, the calculated nodal displacements of the concrete elements of SEC 4\_V2 indicate that the section remains plain throughout the analysis. Yet, as for SEC 2\_V2 and SEC 3\_V2, due to the bond slip cross section SEC 4 does not remain plain. The concrete part of SEC 4 shows a non-plain deformation due to the highly localized pressure at the face of the side support induced by the single compressive rebar.
Figure 4.21: Deformation [in] of SEC 2 at vertical displacements of 1.0 in (top) and 1.9 in (bottom)

Figure 4.22: Deformation [in] of SEC 3 at vertical displacements of 1.0 in (top) and 1.9 in (bottom)
Figure 4.23: Deformation [in] of SEC 1 at vertical displacement of 1.0 in

Figure 4.24: Deformation [in] of SEC 4 at vertical displacement of 1.0 in

Figure 4.25: Deformation [in] of SEC 1 at vertical displacement of 1.9 in
4.3 Element End Forces

In this section, element end forces are presented. In order to calculate the beam internal forces, nodal reaction forces were obtained at the boundaries of the finite element model (see Fig. 2.5). Figs. 4.27 and 4.28 show beam bending moment response at the boundary of the center column block and at the boundary of side support block, respectively, versus the imposed vertical displacement. The horizontal axis represents the vertical displacement imposed at the boundary of the center column block. The vertical axis represents the moment calculated at the boundary of the center column and at the boundary of the side support, respectively. The peak positive bending moment at the boundary of the center column is 175.1 kip-in at a vertical displacement of 1.9 in (Fig. 4.27). The calculated response shows a constantly diminishing slope up to 1.0 in of vertical displacement. Beyond this level of deformation, the slope of the bending moment increases. At 1.9 in of vertical displacement the bending moment shows a drop. Further increased boundary deformations result in decreasing values of the bending moment. At a vertical displacement of 5.26 in a bending moment of 85.3 kip-in is computed.
The peak negative bending moment at the boundary of the side support is 249.9 kip-in at a vertical displacement of 1.68 in (Fig. 4.28). As opposed to the bending moment at the boundary of the center column, no intermediate increase of the slope of the bending moment is observed. Instead, the slope of the bending moment is constantly diminishing up to the peak at 1.68 in of vertical displacement. Increasing boundary deformation beyond vertical displacement of 1.68 in results in a negative slope of the bending moment. At a vertical displacement of 1.9 in, a sharp drop of the bending moment is observed. As boundary deformations are further increased, the bending moment constantly decreases and a bending moment of 108.8 kip-in is calculated at a vertical displacement of 5.26 in.

Fig. 4.29 shows the calculated shear force response of the beam. The vertical axis represents the shear force of the beam and the horizontal axis represents the imposed vertical displacement. The calculated peak shear force is 4.48 kips at a vertical displacement of 1.9 in. Beyond vertical displacement of 1.9 in, the shear force reduces as imposed boundary deformations are increased until maximum vertical displacement of 5.26 in is reached. This level of vertical deformation corresponds to a shear force of 2.05 kips.
Figure 4.28: Moment at the boundary of the side support vs. imposed vertical displacement

Figure 4.29: Shear force response vs. imposed vertical displacement
4.4 State of Strain at Beam Ends

In this section, beam response at the section level is presented. Note that only results at SEC 1 (Face of Side Support) and SEC 4 (Face of Center Column) are discussed (see 4.20). Note that strain values, used to describe the beam response at section level, are average element strains of the first elements from either the face of the center column or the face of the side support. Considering an element length of about 1.0 in, results were calculated at about 0.5 in from the face of the side support or the the face of the center column. In figures presented, this location is associated with the first marker (either from the face of the center column or from the face of the side support) of the curves illustrating the variation of strains. Furthermore, cross sectional locations defined in Fig. 4.2 are used in figure captions.

The bottom bar at SEC 4 (face of center column) shows a strain of about $\epsilon_s = 0.003$ at a vertical displacement of 0.48 in (see Fig. 4.30, LS3), which is larger than the yield strain. At the same vertical displacement, the concrete at the top of SEC 4 is at a strain of $\epsilon_c = -0.002$ (Fig. 4.31, LS3).

Figure 4.30: Variation of steel longitudinal strain $\epsilon_{s,zz}$ at cross sectional location S2 at 0.15, 0.3 and 0.48 in of vertical displacement (horizontal dashed line represents yield strain)
Figure 4.31: Variation of concrete longitudinal strain $\epsilon_{c,zz}$ at cross sectional location C1 at 0.15, 0.3 and 0.48 in of vertical displacement (horizontal dashed lines represent peak compressive strain and crack strain)

At a vertical displacement of 0.76 in, concrete at the bottom of SEC 1 (face of side support) shows a strain of about $\epsilon_c = -0.0025$ (Fig. 4.32, LS4). The top bar at SEC 1 is at a strain of $\epsilon_s = 0.002$ at this level of vertical displacement which indicates that it has not yielded (Fig. 4.33, LS4).

At 1.2 in of vertical displacement, concrete at the top of the core at SEC 4 is at a strain of $\epsilon_c = -0.0022$ (Fig. 4.34, LS6). Note that concrete at the top at SEC 4 is at a strain of $\epsilon_c = -0.015$ (Fig. 4.35, LS6) which indicates a significant variation of longitudinal strain over the section height. The top bar at SEC 4 shows a strain of $\epsilon_s = -0.00145$ (Fig. 4.33, LS6). The bottom bar at SEC 4 is at a strain of $\epsilon_s = 0.025$ (Fig. 4.36, LS6). Furthermore, at this level of vertical displacement, concrete at the bottom of the core at SEC 1 is at a strain of $\epsilon_c = -0.0025$ (Fig.4.37, LS6). Concrete at the bottom at SEC 1 shows a strain of $\epsilon_c = -0.008$ (Fig. 4.32, LS6). Concurrently, the top bar at SEC 1 has yielded with a strain of $\epsilon_s = 0.0043$ (Fig. 4.33, LS6). The bottom bar at SEC 1 is at a strain of $\epsilon_s = -0.0015$ and has not yielded (Fig. 4.36, LS6).
Figure 4.32: Variation of concrete longitudinal strain $\epsilon_{c,zz}$ at cross sectional location C4 at 0.76, 1.0 and 1.2 in of vertical displacement (horizontal dashed lines represent peak compressive strain and crack strain)

Figure 4.33: Variation of steel longitudinal strain $\epsilon_{s,zz}$ at cross sectional location S1 at 0.76, 1.0 and 1.2 in of vertical displacement (horizontal dashed line represents yield strain)
Figure 4.34: Variation of concrete longitudinal strain $\epsilon_{c,zz}$ at cross sectional location C2 at 0.76, 1.0 and 1.2 in of vertical displacement (horizontal dashed lines represent peak compressive strain and crack strain)

Figure 4.35: Variation of concrete longitudinal strain $\epsilon_{c,zz}$ at cross sectional location C1 at 0.76, 1.0 and 1.2 in of vertical displacement (horizontal dashed lines represent peak compressive strain and crack strain)
Figure 4.36: Variation of steel longitudinal strain $\epsilon_{s,zz}$ at cross sectional location S2 at 0.76, 1.0 and 1.2 in of vertical displacement (horizontal dashed line represents yield strain)

Figure 4.37: Variation of concrete longitudinal strain $\epsilon_{c,zz}$ at cross sectional location C3 at 0.76, 1.0 and 1.2 in of vertical displacement (horizontal dashed lines represent peak compressive strain and crack strain)
The top bar at SEC 4 is yielded with a strain of $\epsilon_s = 0.0026$ at a vertical displacement of 1.45 in (Fig. 4.38, LS7). Concrete at the top of the core at SEC 4 is at a strain of $\epsilon_c = -0.0057$ at this level of vertical displacement. At a vertical displacement of 1.68 in, the bottom bar at SEC 1 is yielded with a strain of $\epsilon_s = -0.0045$ (Fig. 4.39, LS8).

Figure 4.38: Variation of steel longitudinal strain $\epsilon_{s,zz}$ at cross sectional location S1 at about 1.45, 1.68 and 1.9 in of vertical displacement (horizontal dashed lines represent yield strains)

Figure 4.39: Variation of steel longitudinal strain $\epsilon_{s,zz}$ at cross sectional location S2 at 1.45, 1.68 and 1.9 in of vertical displacement (horizontal dashed lines represent yield strains)
Chapter 5  Comparison between Analytical and Experimental Results

In the chapter, analytical and experimental results are compared. The comparison includes strains of reinforcing bars and the beam end rotations.

5.1 Steel Reinforcement Strain

Fig. 5.1 compares analytical and experimental strains of the bottom rebar at the face of the center column versus the imposed vertical displacement. Note that experimental data are available only up to a strain of about 0.033.

![Graph showing analytical and experimental strain history](image)

Figure 5.1: Analytical and experimental strain history of bottom rebar at the face of the center column (SEC 4) vs. imposed vertical displacement

The analytical strain reaches tensile yield strain of $\epsilon_s = 0.0025$ at about 0.48 in of vertical displacement. The experimental strain reaches tensile yield strain at about 0.68 in of vertical displacement. After yielding, the slope of experimental strain changes drastically compared
to the slope of analytical strain. This behavior is related to the yield plateau exhibited by the uniaxial stress-strain relationship of the rebars used in the experimental study. In this region of the uniaxial stress-strain relationship, strain increases with little or no increase in stress. Therefore, material stiffness drops considerably leading to large strain increments as vertical displacement is increased. The yield plateau is not modeled by the uniaxial stress-strain relationship implemented in the model (see Fig. 3.1). This causes significantly different slopes of the experimental and analytical tensile strains above the yield displacement. After the yield plateau of the uniaxial stress-strain relationship, stress increases again with increasing strain yielding enhanced material stiffness. This leads to decreasing tensile strain increments as the vertical displacement is further increased. Beyond 0.78 in of vertical displacement the slope of experimental tensile strain approaches the slope of analytical tensile strain. Beyond a vertical displacement of about 1.05 in, the analytical tensile strain is in good agreement with the experimentally measured tensile strain until a vertical displacement of about 1.6 in is reached.

Fig. 5.2 shows analytical and experimental tensile strains of the top rebar at the face of the side support vs. the imposed vertical displacement. The analytical tensile strain reaches yield strain of $\epsilon_s = 0.0025$ at about 1.0 in of vertical displacement. The experimental tensile strain reaches yield strain 1.5 in of vertical displacement. As for the tensile strain history at the face of the center column, a diminished material stiffness (yield plateau of uniaxial stress-strain relationship is reached) causes large tensile strain increments in the post yield range of the experimental strain. After the yield plateau of uniaxial stress-strain relationship, the slope of experimental tensile strain decreases and the strain increases linearly as vertical displacement is increased and the analytical tensile strains approach experimental results as the imposed vertical displacement increases. The analytical tensile strains at the face of the center column and at the face of the side support reach the yield strain at a lower level of vertical displacement compared to the vertical displacement at which experimental tensile strains reach yield strain. The yield plateau of experimental uniaxial stress-strain relationship is not modeled by the stress-strain relationship implemented in the finite element model. This leads to a difference between the slope of the analytical and experimental strains at the vertical displacement level associated with the yielding strain. However, both analytical tensile strain histories are in good agreement with experimentally measured tensile strains.
5.2 Beam End Rotations

Fig. 5.3 shows analytical and experimental beam end rotations measured from the face of the center column over a length of 8 in. Except for the vertical displacements less than about 0.3 in, both experimental and analytical beam end rotations increase almost linearly. Analytical beam end rotation is in good agreement with measured beam end rotation considering difference of only about 13% at about 5.26 in of vertical displacement.

Fig. 5.4 compares the analytical and experimental beam end rotations measured from the face of the side support over a length of 8 in. Beyond a vertical displacement of about 1.0 in a change in the slope of analytical beam end rotation can be observed. The change of slope in the experimental beam end rotations starts later at about 1.6 in of vertical displacement. At about 5.26 in of vertical displacement analytical beam end rotation exceeds experimental beam end rotation by approximately 20%. As for beam end rotations at the face of center column, the analytical beam end rotation at the face of the side support is in good agreement with experimental beam end rotation.
Figure 5.3: Analytical and experimental beam rotations over 8 in from the face of center column

Figure 5.4: Analytical and experimental beam rotations over 8 in from the face of the side support over a length of 8 in
Chapter 6   Concluding Remarks

The detailed finite element model of the continuous beam developed in this study allowed estimating analytical results that in general were in good agreement with the previously obtained experimental responses of the beam subjected to large deformations. One of the important response measures in large deformation response of RC beams is the deformation capacity corresponding to the first bar fracture. The analytical vertical displacement associated with the first bar fracture underestimated the corresponding experiential value by about 12%.

The current version of the Diana program does not account for the interaction between the concrete damage (crushing and cracking) and the bond deterioration. The inclusion of such an interaction would extend the plastic zone, leading to a more ductile response, which in turn would reduce the difference between the analytical and experimental vertical displacements associated with the first bar fracture.

The element rotation is another important and practical measure of deformation response. The analytical beam rotations over eight inches at the beam ends were in good agreement with the experimental results. The finite element modeling described in this study can be used in estimating rotation capacities of RC elements.
Bibliography


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