SEISMIC RESPONSE AND PREVENTION OF LIQUEFACTION FAILURE OF SANDS PARTIALLY SATURATED THROUGH INTRODUCTION OF GAS BUBBLES

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ABSTRACT

Liquefaction of sands is an earthquake-induced phenomenon that can cause devastating damages to our built environment. Liquefaction mostly occurs in loose saturated sands when subjected to repeated or seismic loading, due to excess pressure generation of incompressible pore water. This research reports on developing a new liquefaction mitigating technique referred to as Induced Partial Saturation (IPS) which will be a cost-effective and practical solution for new as well as existing structures. The liquefaction mitigation measure that was explored improves earthquake resistance of loose saturated sands by introducing some amount of gas/air in the voids of the sand. Preliminary research performed by Eseller (2004) and Yegian et al. (2007) demonstrated that IPS can be a potential mitigation measure against liquefaction. This research further evaluates in depth the seismic response and liquefaction benefit of sands mitigated by IPS as well as it investigates the sustainability of IPS in sands under various conditions in nature through an integrated experimental and analytical research program.

The short- and long-term sustainability of entrapped gas/air bubbles was tested in large scale experimental setups under hydrostatic, and upward, downward, and lateral hydraulic gradients as well as under horizontal excitation. A cyclic simple shear liquefaction box (CSSLB) was designed and manufactured which can induce uniform shear strains in a sand specimen through the use of a shaking table. The CSSLB is versatile and accommodates the insertion of various types of transducers needed in this research. A new laboratory IPS technique was developed to prepare partially saturated sand specimens with uniformly distributed gas bubbles and at controllable degrees of saturations. The new technique involved mixing dry sand with a dental product
"Efferdent" (main ingredient being an oxygen source: Sodium Perborate) and raining the mixture in water leading to generation of oxygen gas bubbles in the voids. Then the uniformity of the soil characteristics (relative density and degree of saturation) of sand specimens prepared by IPS was evaluated using a multiple P and S wave measurement facility that was developed for use in large soil specimens. The P-S wave measurement set-up was used to investigate the potential use of P-wave measurements as a means for estimating partial degree of saturation in sands. After evaluating the soil characteristics, cyclic simple shear strain tests were performed on fully and partially saturated sand specimens prepared by IPS. The effect of important parameters including relative density, degree of saturation, induced shear strain levels, initial effective stresses, and applied number of cycles, on excess pore water pressure ratio ($r_u$) generated in sand specimens was investigated through a series of cyclic simple shear strain tests. Finally, an empirical model (RuPSS, excess pore water pressure ratio ($r_u$) in Partially Saturated Sands) was developed to predict excess pore water pressure ratios in partially saturated sands. The model was based on the experimental data obtained from the cyclic simple shear strain tests.

The results from this research show that gas/air bubbles entrapped in sand specimens by IPS remain in the voids even under various flow and ground shaking conditions encountered in nature. It was confirmed by the P and S wave measurements that the new IPS laboratory technique is capable of achieving uniform partially saturated sand specimens. Also P wave measurements performed in partially saturated sand specimens at several degrees of saturation demonstrated that contrary to published information, P wave velocity can not provide estimate of degree of saturation ($S$) for $S<$
90%. According to cyclic simple shear strain tests, IPS prevents sands from liquefying by limiting maximum $r_u$ ($r_{umax}$) to less than 1. Also, IPS leads to increased number of cycles required to reach $r_{umax}$, thus resulting in smaller values of excess pore pressures during small to moderate earthquakes. Also, excess pore water pressure ratio is reduced in specimens with lower degrees of saturation, higher relative densities and under lower strain amplitudes. The developed empirical model RuPSS predicts excess pore water pressure ratio in partially saturated sands during earthquakes by incorporating the effects of degree of saturation ($S$), relative density ($D_r$), initial effective stress ($\sigma_v'$), the induced shear strain amplitude ($\gamma$) and earthquake magnitude ($M$). The developed model can be used in field applications of IPS, by providing predictions of either $r_u$ for a design level $S$, or $S$ for a limiting value of $r_u$. 
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1. Introduction

Liquefaction-induced damages have been observed in every moderate to large earthquakes, most recently in 1995 Hyogoken Nanbu (Kobe), 1999 Adapazari and 1999 Duzce Earthquakes in Turkey. Liquefaction is the loss of shear strength in fully saturated loose sands due to excess pore water pressure build-up during a repeated loading or dynamic excitation, such as an earthquake. Intensive efforts have been made to understand the mechanism of liquefaction, and to develop procedures for analyzing the liquefaction potential at a site during a given seismic event. While research on liquefaction continues, the geotechnical engineering practice has developed various techniques for site improvement that can mitigate the potential effects of liquefaction. Existing mitigation measures are expensive and often are applicable only for a new project. Mitigating the liquefaction-induced damages to an existing structure in an urban community and/or to special structures remains to be a major challenge.

The research reported in this dissertation addresses that major challenge by exploring the potential beneficial effect of introducing small amounts of gas/air in liquefaction-susceptible soils as a measure for liquefaction mitigation. The technique involves generation of gas bubbles in loose saturated sand thus inducing partial saturation (IPS) leading to not only strength gain against liquefaction, but also potentially eliminating the occurrence of liquefaction under any size earthquake. The proposed technique, induced partial saturation (IPS), was first introduced in a preliminary research funded by NSF through the small grant for exploratory research program (SGER) under award No CMS-0234365 and reported in Eseller (2004) and Yegian et al. (2007). Two IPS techniques were implemented in the laboratory specimens and the specimens
partially saturated through IPS techniques were tested under cyclic simple shear strain tests. The test results demonstrated that gas/air entrapped specimens never liquefied. Therefore, the preliminary research results validated that IPS can be a potential mitigation measure against liquefaction.

After achieving encouraging results from the preliminary research, a major question arose that whether or not the induced gas/air will remain entrapped in sands under varying field conditions including hydrostatics, horizontal and vertical hydraulic gradients, and under horizontal excitations. The sustainability of gas/air under the specified conditions needed to be addressed.

On the other hand, the liquefaction response of sands mitigated by IPS was required to be investigated in depth before proceeding to the exploration of field implementation techniques. The benefit of IPS on liquefaction prevention as well as the desired level of IPS to be implemented for a particular site under a specific earthquake needs to be estimated analytically. In literature, many researchers (Chaney et al. (1978), Yang et al. (2003), Ishihara et al. (2004), Okamura et al. (2006)) introduced the liquefaction resistance of partially saturated sands through some analytical correlations. Their correlations were mostly for partially saturated sands at degrees of saturation greater than 95%. Moreover, Ishihara et al. (2004) demonstrated the liquefaction strength of partial saturation in terms of factor of safety, which can not be set for partially saturated sands since they never liquefy. Besides, even tough the liquefaction strength of partially saturated sands is improved during earthquakes, excess pore water pressures still generate which may be essential for the site. Therefore the liquefaction benefit of IPS would be best evaluated through an analytical model that estimates excess pore water
pressure generations under different soil conditions and at different degrees of induced partial saturation

In order to develop such an analytical model, the effects of various parameters on the excess pore water pressure generations in gas/air entrapped specimens need to be evaluated. Those effects can be investigated based on a series of cyclic tests performed on representative partially saturated specimens prepared by IPS. An enhanced experimental setup with all the integrated instruments is essential for these tests. Also, the current IPS laboratory techniques explored in the preliminary research were recognized to have some limitations when it comes to better evaluate the seismic response of partially saturated sand specimens in a more intensive study. A new IPS laboratory technique was crucial to explore in order to prepare specimens with uniformly distributed gas bubbles in a practical way and at controllable degrees of saturation.

The purpose of the reported research is to evaluate the potential feasibility and sustainability of IPS in liquefiable saturated sands as well as to develop an in-depth experimental and analytical understanding of the liquefaction benefit of gas/air entrapped sands. In order to achieve its goals, this research includes the following phases.

The first phase of the research includes an experimental investigation of the sustainability of entrapped gas/air in the sand specimens under varying conditions including hydrostatics, horizontal and vertical hydraulic gradients, and under horizontal excitations.

In the second phase of the research an enhanced experimental setup was developed in order to test fully and gas/air entrapped sand specimens under cyclic simple shear strains. The experimental setup includes a special liquefaction box called Cyclic
Simple Shear Liquefaction Box (CSSLB), miniature pore pressure transducers, linear variable displacement transducers and a multiple bender element and bending disk measurement facility which is developed in another phase of the research. A new laboratory IPS technique was explored that led to a much easier preparation of a more uniform partially saturated sand specimens at controllable degrees of saturation.

In the third phase of the research, a multiple bender element and bending disk measurement facility was established to identify that sand properties (density and degree of saturation) in the specimen prepared by the new IPS technique in the CSSLB are uniform. Also, it was investigated whether or not IPS can be qualified by P wave velocity measurements and consequently if the liquefaction benefit of IPS mitigation technique can be related to the P wave velocities. It was aimed that variations in the specimen density, stiffness and degree of saturation can be identified based on multiple S and P wave velocity measurements in large specimens.

In the forth phase of the research, an experimental research program was conducted on fully saturated sand specimens and partially saturated sand specimens prepared by the new IPS technique. The individual effects of various parameters (degree of saturation, relative density, cyclic shear strain amplitude and effective stresses) on liquefaction potential of air-entrapped sands were explored through 19 tests on fully saturated and 96 tests on air entrapped specimens.

In the fifth or final phase of the research, based on the results of the experimental program, an empirical model (RuPSS) was developed that predicts the excess pore water pressure ratios in partially saturated sand specimens whether naturally found in that condition or induced by the proposed liquefaction mitigation measure (IPS). The main
parameters which have significant effects on excess pore water pressure ratio model are determined to be degree of saturation, relative density, effective stresses, shear strain amplitude and earthquake magnitude. The developed empirical model incorporates all the effects of these parameters on the excess pore water pressure ratio in partially saturated sands.

This dissertation presents all the five phases of the research described. Chapter 2 reviews the liquefaction phenomenon and existing remediation measures against liquefaction. The concept of IPS, theoretical background and literature review on the resistance of partially saturated sands to liquefaction are introduced in Chapter 2. Also, the findings from the preliminary research are presented in this chapter.

Chapter 3 presents the experimental tests and the results of the sustainability of air-entrapped specimens under hydrostatic, short-and long term hydraulic gradients as well as lateral excitations.

Chapter 4 summarizes the experimental setup used for cyclic simple shear strain tests and introduces the new IPS laboratory technique.

Chapter 5 presents the details and establishing process of multiple bender element and bending disk measurement facility. This chapter also presents the results of multiple wave measurements for identifying the uniformity of sand specimen properties and for correlating the compression wave velocities to the degree of induced partial saturation.

Chapter 6 describes the procedures and the results of the cyclic simple shear strain tests performed on fully and partially saturated sand specimens.

Chapter 7 introduces the empirical RuPSS model which predicts excess pore water pressure ratios in partially saturated sands. The development of the model,
statistical analysis on the model goodness of fit and a numerical example are also presented in this chapter.

Finally in Chapter 8, the research presented is summarized and the conclusions are stated.
2. Concept of Induced-Partial Saturation (IPS) as a Liquefaction Mitigation Measure

2.1 Liquefaction Problem and Current Mitigation Techniques

Liquefaction is the loss of shear strength in fully saturated loose sands due to excess pore water pressure build-up during a repeated loading or dynamic excitation, such as an earthquake. Due to earthquake induced strains, a fully saturated sand tends to decrease in volume. However, this tendency is counteracted by an increase in pore water pressure since water in the pores is incompressible and drainage from the soil is momentarily impeded. Since total stress is constant through the loading, increase in pore water pressure results in a decrease in effective stress. This phenomenon can be explained in a schematic illustration of pore pressure generation during cyclic loading by Seed and Idriss (1982)

![Schematic illustration of mechanism of pore pressure generation during cyclic loading. Source: Seed and Idriss (1982).](image)

Figure 2-1 Schematic illustration of mechanism of pore pressure generation during cyclic loading. Seed and Idriss (1982)
Different definitions have been introduced for liquefaction in the literature. In this research, level ground liquefaction or initial liquefaction definition is employed. Initial liquefaction is defined as a state of zero effective stresses. When excess pore water pressure build up ($\Delta u$) in the pores becomes equal to the initial effective stress ($\sigma_0'$) of the soil, it is called initial liquefaction. It is usually defined by excess pore water pressure ratio $r_u$ being equal to 1. Initial liquefaction can be explained below:

Initial effective stress, $\sigma_0'$:

$$\sigma_0' = \sigma - u_0$$  \hspace{1cm} (2-1)

where $\sigma$ and $u_0$ are total stress and initial pore water pressure. Total stress doesn't change during loading. Due to the excess pore pressure generation $\Delta u$, final water pressure $u_f$ is expressed by:

$$u_f = u_0 + \Delta u$$

$$\sigma_f' = \sigma - (u_0 + \Delta u)$$

$$\sigma_f' = \sigma_0' - \Delta u$$

Initial liquefaction occurs when

$$\sigma_f' = \sigma_0' - \Delta u = 0$$

$$\sigma_f' = \sigma_0' (1 - \frac{\Delta u}{\sigma_0'}) = 0$$

$$r_u = \frac{\Delta u}{\sigma_0'} = 1$$  \hspace{1cm} (2-2)

Intensive efforts have been made to develop methods for evaluating liquefaction potential at a site during a given seismic event. Several approaches have been developed to understand the mechanism of liquefaction. These are based on field observations, experimental investigation, and theoretical studies. Current techniques to evaluate
liquefaction susceptibility of a particular site are: simplified procedure based on cyclic stress ratio by Seed et al. (1971), strain approach by Dobry et al. (1982), effective stress approach by Finn et al. (1977) and probabilistic approach by Yegian et al. (1978). Also, to date, significant progress has been made in the ability to assess consequences of liquefaction to the built environment. Liquefaction may result in expansive amount of damages. Major failure types are sand boils (Figure 2-2), lateral spread, flow failures, bearing capacity failure (Figure 2-3), and differential settlements and so on.

Figure 2-2 Sand boils near Niigata (photo by K. Steinbrugge, courtesy of EERC, Univ. of California)

Figure 2-3 Bearing capacity failure of a building after Izmit EQ, 1999
While extensive research is ongoing on analysis of liquefaction in sands and sands with fines, recent research has focused on use of soil remediation to reduce or eliminate risks and failures from liquefaction. Methods for ground improvement to mitigate liquefaction potential fall into four general categories those that densify loose soils, those that expedite drainage and dissipation of excess pore water pressure, those that provide confinement and limit lateral flow of the soil, and those that physically or chemically modify the soil to increase its strength. Table 2-1 summarizes general methods for mitigations of seismic soil liquefaction hazard. However, current soil improvement methods to safeguard against liquefaction are often very expensive and their applications are limited to new sites.
Table 2-1: General Methods for Mitigations of Seismic Soil Liquefaction Hazard (from Seed et al., 2003)

<table>
<thead>
<tr>
<th>General Category</th>
<th>Mitigation Methods</th>
<th>Notes</th>
</tr>
</thead>
</table>
| I. Excavation and/or compaction | (a) Excavation and disposal of liquefiable soils  
(b) Excavation and recompaction  
(c) Compaction (for new fill) |                                                                     |
| II. In-situ ground densification | (a) Compaction with vibratory probes (e.g.: Vibroflotation, Terraprobe, etc.)  
(b) Dynamic consolidation (Heavy tamping)  
(c) Compaction piles  
(d) Deep densification by blasting  
(e) Compaction grouting | - Can be coupled with installation of gravel columns  
- Can also provide reinforcement |
| III. Selected other types of ground treatment | (a) Permeation grouting  
(b) Jet grouting  
(c) Deep mixing  
(d) Drains  
- Gravel drains  
- Sand drains  
- Pre-fabricated strip drains  
(e) Surcharge pre-loading  
(f) Structural fills | - Many drain installation processes also provide in-situ densification. |
| IV. Berms, dikes, sea walls, and other edge containment structures/systems | (a) Structures and/or earth structures built to provide edge containment and thus to prevent large lateral spreading |                                                                      |
| V. Deep foundations | (a) Piles (installed by driving or vibration)  
(b) Piers (installed by drilling or excavation) | - Can also provide ground densification |
| VI. Reinforced shallow foundations | (a) Grade beams  
(b) Reinforced mat  
(c) Well-reinforced and/or post-tensioned mat  
(d) "Rigid" raft |                                                                      |
2.2 IPS–Proposed Liquefaction Mitigation Technique

As stated in the previous section, liquefaction occurs in fully saturated sands due to excess pressure build up of the incompressible fluid in the pores, which is water. Induced Partial Saturation (IPS) aims to prevent liquefaction by generating gas/air in the pores of fully saturated sands. The advantage of IPS over other mitigation techniques will be its cost effective implementation for new as well as existing structures.

Generation of gas/air in the voids reduces the excess pore water pressure owing to increased compressibility of the gas/air-water mixture in the voids. Figure 2-4 demonstrates the concept of IPS.

This section first presents the theoretical background for the reduction in excess pore water pressures in gas/air entrapped specimens. Then, some evidence from literature is reported on the liquefaction resistance of partially saturated sands. Finally, potential of IPS for liquefaction prevention is presented with preliminary experimental research findings.

Figure 2-4 Concept of liquefaction mitigation using entrapped gas/air.
2.2.1 Theoretical Effect of IPS on Excess Pore Water Pressure Reduction

The reduction in excess pore water pressure of gas/air entrapped sands during cyclic loading can be demonstrated theoretically. The momentary prevention of drainage of water in the field during liquefaction can be assessed in the laboratory by testing soils under undrained conditions. Excess pore water pressure development can be demonstrated theoretically in two different conditions: under static loading ($\Delta\sigma$) and under dynamic loading.

**Mechanism under static loading:**

Figure 2-5 shows a schematic representation of a partially saturated sand with water and air bubbles in the voids. The element of soil is under overall confining effective stress $\sigma_3'$. When a static load ($\Delta\sigma$) is applied to the soil, by Terzaghi's effective stress principle, change in effective stress will be:

$$\Delta\sigma' = \Delta\sigma - \Delta u$$  \hspace{1cm} (2-3)

Figure 2-5 shows a schematic representation of a partially saturated sand element: sand particles, water and air bubbles in the voids.
Volume change due to this change in effective stress $\Delta V_s$ can be obtained as below:

Compressibility of soil: $C_s = \frac{1}{\Delta \sigma'} \frac{\Delta V_s}{V_s}$

$\Delta V_s = C_s \Delta \sigma' V_s$ \hspace{1cm} (2-4)

where $V_s$ is the volume of soil.

Volume change of the soil is actually equal to the change in volume of voids ($\Delta V_v$), since soil solids cannot compress and the change in volume of voids will be equal to the total of the change in volume of water ($\Delta V_w$) and the change in volume of air ($\Delta V_a$).

$\Delta V_s = \Delta V_v = \Delta V_w + \Delta V_a$ \hspace{1cm} (2-5)

Under undrained conditions, this volume change ($\Delta V_v$) can be related to the pressure change of air and water in the pores separately and defined by the use of compressibility (Fredlund 1993):

$C = \frac{-1}{\frac{dV}{V}} \frac{dp}{dp}$ \hspace{1cm} (2-6)

Compressibility of water ($C_w$) and air ($C_a$) can be defined as below:

$C_w = \frac{1}{S n V_s} \frac{\Delta V_w}{\Delta u_w}$ \hspace{1cm} (2-7)

$C_a = \frac{1}{(1 - S) n V_s} \frac{\Delta V_a}{\Delta u_a}$ \hspace{1cm} (2-8)

where $n$ is the porosity and $S$ is the degree of saturation.
The excess air and water pressures generated in the voids will be all equal if the surface tension between air and water is neglected due to air being in bubble form (Fredlund 1993). That means that when an external load is applied, air and water in the pores will experience the same excess pressure $\Delta u$.

$$\Delta u_a = \Delta u_w = \Delta u$$  \hspace{1cm} 2-9

Therefore the compressibility of air and water mixture can be found as follows:

$$\Delta V_w + \Delta V_a = \left[SnV_s C_w + (1 - S)nV_s C_a\right] \Delta u$$

$$\frac{\Delta V_v}{nV_s} = \frac{\Delta V_w + \Delta V_a}{nV_s} = \left[SC_w + (1 - S)C_a\right] \Delta u$$

$$C_{aw} = SC_w + (1 - S)C_a$$

$$\Delta V_v = C_{aw}nV_s \Delta u$$  \hspace{1cm} 2-10

where $C_{aw}$ is the compressibility of air-water mixture.

Combining Eqns. 2-3, 2-4, 2-5 and 2-10 $\Delta u$ can be found as below:

$$\Delta u = \frac{C_s}{C_s + nC_{aw}} \Delta \sigma$$

or,

$$\Delta u = \frac{1}{1 + \frac{nC_{aw}}{C_v}} \Delta \sigma$$  \hspace{1cm} 2-11

$\Delta u/\Delta \sigma$ is defined also as B pore pressure parameter by Skempton (1954).

If $C_{aw}$ is introduced into the Eqn. 2-11, following expression is obtained

$$\Delta u = \frac{1}{1 + \frac{n[SC_w + (1 - S)C_a]}{C_s}} \Delta \sigma$$  \hspace{1cm} 2-12
Compressibility of air also can be derived further as below:

\[ C_a = -\frac{1}{V_a} \frac{dV_a}{du_a} \]

According to Boyle's Law:

\[ V_a = \frac{u_{a0} V_{a0}}{u_a} \quad 2-13 \]

where \( u_{a0} \) is initial air pressure and \( V_{a0} \) is initial volume of air.

Boyle's expression is differentiated and inserted in the compressibility equation:

\[ \frac{dV_a}{du_a} = -\frac{u_{a0} V_{a0}}{u_a^2} = \frac{u_a V_a}{u_a^2} \]

\[ C_a = -\frac{1}{V_a} (-\frac{u_a V_a}{u_a^2}) \]

\[ C_a = -\frac{1}{u_a} \quad 2-14 \]

Thus, Eqn. 2-12 becomes:

\[ \Delta u = \frac{1}{n} \frac{\Delta \sigma}{SC_w + (1-S) u_a} \quad \frac{1}{1 + \frac{C_a}{u_a}} \quad 2-15 \]

Therefore, Eqn. 2-15 implies that when \( S=1.0 \), since \( C_w \) is almost 0, \( \Delta u \) will be equal to the applied loading \( \Delta \sigma \). However when \( S<1.0 \), \( \Delta u \) will be less than applied loading due to \((1-S)/u_a\) term in the equation. On the other hand, the compressibility of air depends on the pressure of air in the pores.
**Mechanism under dynamic loading:**

Excess pore water pressure in fully saturated sand during one loading cycle of simple shear tests is modeled by Finn et al. (1976) as below. The derivation of the $\Delta u$ expression is very similar to the derivation explained above however this time the type of loading is dynamic:

$$\Delta u = \frac{\Delta \varepsilon_{vd}}{1 + \frac{n_p}{E_r K_w}}$$  \hspace{1cm} (2-16)

$\Delta u$: Excess pore pressure per load cycle
$\Delta \varepsilon_{vd}$: Net volumetric strain increment corresponding to the decrease in volume occurring during the load cycle in drained case
$E_r$: One dimensional rebound modulus of sand at $\sigma_v$
$n_p$: Porosity of the soil
$K_w$: Bulk Modulus of Water

Net volumetric strain increment $\Delta \varepsilon_{vd}$ is defined as to be a function of the total accumulated strain ($\varepsilon_{vd}$) and the strain cycle amplitude ($\gamma$). Rebound modulus is the soil skeleton characteristics. Therefore, they are not going to be changed when IPS is employed in the system. So the only part related to the fluid in the pores is the term of bulk modulus of the fluid. In gas/air entrapped specimens, that term will be exchanged with bulk modulus of water-air mixture, $K_{aw}$.

Eqn. 2-15 can be also expressed with compressibility of water–air mixture instead of bulk modulus:

$$C_{aw} = \frac{1}{K_{aw}}$$

$$\Delta u = \frac{\Delta \varepsilon_{vd}}{1 + \frac{n_p C_{aw}}{E_r}}$$
\[ \Delta u = \frac{\Delta \varepsilon_{vd}}{\frac{1}{E_r} + n_p \left[ S C_w + \frac{(1 - S)}{u_a} \right]} \]

Thus, Eqn. 2-17 shows that in air entrapped sands, the excess pore water pressure generated each loading cycle will be less than in fully saturated sands due to the increase in the compressibility expression of the pore fluid.

### 2.2.2 Literature Review on Liquefaction Strength of Partially Saturated Sands

Liquefaction strength and resistance of partially saturated sands has been reported by many researchers and published in the literature. Liquefaction response of partially saturated sands has been addressed for two different objectives. One group of researchers demonstrated the effect of undesirable air entrapment on the overestimation of the liquefaction strength of fully saturated sands. According to Martin, Fin and Seed (1978), explained that 1% reduction in the degree of saturation of a saturated sand specimen with 40% porosity can lead to 28% reduction in the pore water pressure increase per cycle, and stated: "Thus, the need for complete saturation during undrained cyclic tests is apparent if accurate experimental results are to be obtained".

Xia and Hu also demonstrated during liquefaction analysis tests on fully saturated sands that minute quantities of entrapped air can significantly increase the liquefaction strength. At a specific cyclic stress ratio, the number of cycles causing liquefaction in air entrapped specimens was found to be higher. Their laboratory data demonstrate that a reduction in the degree of saturation from 100% to 97.8% can lead to a greater than 30% increase in liquefaction strength (Figure 2-6).
The main consideration of other group of researchers is to evaluate and/or formulate the liquefaction resistance of partially saturated sands. According to the studies carried out by Chaney (1978) and Yoshimi et al. (1989), the resistance to liquefaction has been shown to increase roughly two times as much as that of fully saturated samples when degree of saturation drops to 90%. Tsukamoto et al. (2002) and Ishihara et al. (2004) stated that during P wave measurement tests in the field they observed imperfectly saturated zones. Then, they studied the resistance of partially saturated sand to the liquefaction and quantified degree of saturation (which is usually practically impossible to monitor in the field) in terms of P-wave velocity measurements, which can be measured in the field. They presented the liquefaction strength of partially saturated sands by normalizing it to the liquefaction strength of fully saturated as shown in Figure 2-7.
Yang et al. (2004) also demonstrated the resistance of partially saturated sands to liquefaction through a relationship that they developed between the liquefaction strength of partially saturated sand and P wave velocities. They also incorporated test data from other researchers, as shown in Figure 2-8. Then, they proposed an empirical correlation between the liquefaction strength of partially saturated sands and P wave velocity as below:

\[
(CSR)_{ps} = (CSR)_{fs} e^{\left\{\frac{0.71R}{(V_p/V_s)^2 - 4/3}\right\}}
\]

\[
R = \frac{2(1+\nu)}{3(1-2\nu)}
\]

where CSR is cyclic stress ratio, \(V_p\) and \(V_s\) are P and S wave velocities, \(\nu\) is the Poison's ratio.
2.2.3 Preliminary Research Findings

A preliminary research was performed to investigate if the higher strength of partially saturated sands can be potentially exploited to develop a new cost-effective liquefaction mitigation measure.

A special plexiglass liquefaction box (Northeastern University Liquefaction Box, NULB) was designed to allow preparation and testing of large size specimens (specimen size: 21cm x 33cm x 34cm) as well as testing them under cyclic motion induced on a shaking table. Figure 2-9 shows the sketch of the setup. The box walls and base are made of plexiglass material. Two sides of the box are designed to rotate about their bottom edges and the other two sides are fixed to the base of the box,
Figure 2-9 (a) Liquefaction box (NULB) and setup for testing of fully and partially saturated specimens (b) Plan view of the liquefaction box.

which in turn is fixed on the shaking table. The tops of the two movable sides are joined together at one end of an aluminum cross bar. The other end of the cross bar is fixed to a steel column in front of the shaking table. A special construction joint sealant (Sikaflex-
15LM) is used as a flexible watertight joint between the plexiglass sides. The sealant is a strong adhesive and acts as a flexible membrane allowing large deformations without rupture. Hence, when the shaking table is excited with cyclic displacements, through rotation around the bottom hinge, controlled simple shear strains were induced in the large specimens.

Two techniques were developed to induce gas/air in sand specimens prepared in the laboratory: 1) gas generation through electrolysis and 2) air entrapment through drainage-recharge of the pore water. In electrolysis, hydrogen and oxygen gases were generated in the saturated sand specimen when electrical current is applied at two electrode sheets placed inside the specimens. The degree of saturation can be significantly changed by controlling the current intensity. Figure 2-10 shows the test setup. In drainage-recharge, after preparing a fully saturated sand specimen, the pore water was slowly drained from the bottom of the liquefaction box and then the drained water was reintroduced from the top of the specimen at a slow rate. Figure 2-11 shows the test setup. After reintroducing all the drained water, a significant amount of water remained above the surface of the sand specimen indicating entrapment of air during recharge.

To evaluate the effect of partial saturation on liquefaction potential and pore pressure generation in sands, cyclic simple shear strains were applied on fully and partially saturated sand specimens prepared by electrolysis as well as drainage-recharge. Figure 2-12 (a) shows the cyclic simple shear strain history induced in all tests. The shear strain amplitude was 0.25% and the frequency was 4 Hz.
A comparison of the pore pressure generation in the fully and partially saturated specimens prepared through electrolysis and through drainage-recharge (D-R) are presented in Figure 2-12 (b) and (c) respectively. The test results revealed that the reduction in the degree of saturation led to significant reduction in the excess pore water
Figure 2-12 Shear strain time history applied (a). Comparison of excess pore water pressures generated in fully saturated Ottawa sand specimens and partially saturated Ottawa sand specimens prepared (b) through electrolysis (S = 96.3%) and (c) through drainage-recharge (S = 86.2%).
pressure. The maximum excess pore water pressure ratios and axial strains obtained in one fully and two partially saturated specimens tested are demonstrated in Table-2-2. Details of this preliminary research can be found in Eseller (2004) and Yegian et al. (2007).

Table 2-2 Effect of Induced Partial Saturation on Liquefaction Induced Excess Pore Water Pressure and Settlement

<table>
<thead>
<tr>
<th>Sand Specimen</th>
<th>Degree of Saturation, S,%</th>
<th>Max. Excess Pore Pressure Ratio, ( r_u )</th>
<th>Settlement, cm</th>
<th>Axial Strain, %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bottom Transducer</td>
<td>Top Transducer</td>
<td></td>
</tr>
<tr>
<td>Fully Saturated</td>
<td>99.7</td>
<td>1.00</td>
<td>1.04</td>
<td>1.71</td>
</tr>
<tr>
<td>Partially Saturated (1)</td>
<td>86.2</td>
<td>0.72</td>
<td>0.63</td>
<td>0.82</td>
</tr>
<tr>
<td>Partially Saturated (2)</td>
<td>86.5</td>
<td>0.68</td>
<td>0.66</td>
<td>0.65</td>
</tr>
</tbody>
</table>

In summary, two different laboratory methods were developed to evaluate the effectiveness and feasibility of Induced Partial Saturation (IPS). Preliminary research findings imply that Induced Partial Saturation (IPS) applied in laboratory specimens reduced significantly excess pore water pressures generated during cyclic shear strain tests. The research presented in this dissertation was carried out to better evaluate the endurance of entrapped gas/air in the sand specimens under different flow and dynamic conditions, the cyclic response of partially saturated sands experimentally with a more advanced integrated sensors and setup, and finally to develop an empirical model for predicting excess pore water pressures in partially saturated sands during earthquakes.
3. Short- and Long-Term Sustainability of Entrapped Gas/Air in Sands under Different Flow Conditions and Ground Shaking

3.1 Overview

After exploring the feasibility and potential of IPS technique to mitigate liquefaction in fully saturated sands, first task was to investigate whether or not gas/air induced in sands escape or diffuse under varying conditions including hydrostatics, horizontal and vertical hydraulic gradients, and under horizontal excitations.

Okamura et al. (2002) showed that during the improvement of a site by Soil Compaction Pile (SCP), where the compaction piles are pushed into the soil by pressurized air supplied from the top of the casing, some of the air percolated into the sand and some spouted from the ground surface. Based on this observation, Okamura et al. (2002) investigated another site improved 26 years ago by SCP and concluded that air bubbles survived for 26 years.

Having this supportive evidence, various conditions in nature that air entrapped in sands can be exposed to were determined and summarized in Figure 3-1. Large scale 1-D test setups were designed and fabricated in order to test gas/air entrapped specimens under hydrostatic, low to high hydraulic gradient flows, and lateral excitation. The flow tests were performed upward and downward vertical and lateral directions for short- and long-terms. Finally, a vertical sand column was tested under lateral excitation simulated on a small shaking table.
3.2 Sustainability of Gas/Air under Hydrostatic Conditions

First, sustainability of air under hydrostatic conditions was tested. In reality, in a deep soil layer the water pressure will be higher and may force the air out of the voids. On the other hand, one can argue that it would be more difficult for the air molecules to find a path and escape through a deep soil layer. A 180 cm plexiglass tube with an outer diameter of 10.12 cm (4-in) was rigidly fixed to a concrete column in the basement of the engineering building to minimize the effect of ambient vibrations. Figure 3-2 shows a photo of the experimental setup.

A 150 cm partially saturated sand specimen was prepared in the plexiglass tube by drainage-recharge (D-R) technique. Initial degree of saturation was calculated as 81.7% knowing the amount of water accumulated on top of the sand surface.
Figure 3-2 Setup for testing sustainability of air under hydrostatic conditions

Figure 3-3 Long-term monitoring of entrapped air under hydrostatic conditions

\[ S_{\text{initial}} = 81.7\%\]
\[ S_{\text{final}} = 83.8\% \]
Then, the top of the tube was tightly closed with a 5 cm NPT (national pipe thread) cap to eliminate any loss of evaporated water to the outside of the tube.

Accumulated free water above the sand surface was monitored for long-term. First 4-6 months, the data was recorded daily and after then it was recognized that the level of water was varying due to mostly evaporation and condensation process occurring depending on the building temperature. Therefore weekly measurements were performed for the rest of the observation period. Degree of saturation was evaluated based on the changes in the level of the soil height and the level of the water table recorded. Figure 3-3 shows the monitored and measured data for 115 weeks (almost 2.5 years). The results indicate that some of the air entrapped in the top 5-10 cm of the specimen had escaped the first week after the specimen was prepared through drainage-recharge. That was due to ongoing seepage of the free water into the specimen. After then the daily measurement data demonstrated that water level fluctuated depending on evaporation-condensation of water level. Then, weekly measurements demonstrated that the water level almost stayed steady. Therefore, it was concluded that initial degree of saturation of 81.7 % only slightly increased to 83.8 %, after 2.5 years of monitoring. The long-term monitoring of sustainability of induced partial saturation in a deep (150 cm) sand specimen led to the conclusion that under hydrostatic conditions, small well-distributed air bubbles can remain trapped for a long time.
3.3 Sustainability of Gas/Air under Hydraulic Gradient Flow

1-D large-scale flow test setups were designed and fabricated to investigate the sustainability and diffusion of entrapped air in specimens prepared by drainage-recharge. Tests were performed based on the principle of constant head permeability tests (Figure 3-4). The head difference between the two ends of the specimens was kept constant and flow rate of water travelled through the specimen was evaluated using the Darcy's 1-D flow equation:

\[ q = \frac{Q}{\Delta t} = A k \frac{\Delta h}{L} \]  

where \( Q \) is the volume of water collected, \( \Delta t \) is the time interval of flow, \( q \) is the flow rate, \( A \) is the cross sectional area of the specimen, \( k \) is the permeability of the soil, \( L \) is the soil length and \( \Delta h \) is the constant head difference between the two ends of the soil.

Figure 3-4 Schematic diagram of constant head permeability test setup (Bowles 1988)
3.3.1 Vertical Hydraulic Gradient Flow Tests

For testing air-entrapped specimens under vertical hydraulic gradient tests, a large-scale constant head flow test setup was designed and manufactured (Figure 3-5). A 165 cm long plexiglass tube with 7.6 cm (3 in) diameter was fixed to a concrete wall in the laboratory. Constant head water funnels were connected to the plexiglass tube through the top and bottom side holes. Since the bottom hole provided water flow from the side of the tube, it wouldn't cause 1-D flow in the bottom of the sand specimen. Also, it created piping just in the entrance of the valve due to a very small cross sectional area of the hole. Therefore, 1-D vertical flow on the sand specimens was provided by placing large to small size gravels in the bottom of the plexiglass tube.

![Experimental setup for testing short-term and long-term sustainability of air under downward and upward hydraulic gradients](image)

Figure 3-5 Experimental setup for testing short-term and long-term sustainability of air under downward and upward hydraulic gradients
Partially saturated sand columns (120 cm long) were prepared using the drainage recharge technique as performed for hydrostatic tests. After recording free amount of water accumulated on top of the specimen for calculation of degree of saturation, the plexiglass tube was filled all the way to the top with water fed from the top funnel. Then, the top of the plexiglass tube was tightly closed with the help of a 5 cm NPT cap without pressurizing water inside. With this arrangement, possible air escape was monitored from the top of the plexiglass tube. If air tends to escape it would travel upward in free water and accumulate at the top of the plexiglass tube by pushing water down. Then the volume of air accumulated at the top could be quantified.

Upward and downward gradient flow tests were performed by adjusting the relative height of the bottom and top funnels. Weight of outflow water was measured at specific time intervals to determine the flow rate.

**Upward Flow:**

First, vertical gradient flow tests were performed in upward direction. Upward gradient flow is recognized to be more critical for air to travel upward since it tends to reach to the atmospheric pressure, if it can find a path. Low and high hydraulic gradients were applied on a short and long term basis.

In short-term tests, a partially saturated sand specimen prepared at S=82.6% was tested under hydraulic gradients of 0.05, 0.1, 0.2, 0.3, 0.4 and 0.5, respectively. Each hydraulic gradient was applied for 3 hours. Air bubbles escaped especially from the 5-10 cm top portion of the specimen, in the beginning of the test. Although the degree of saturation changed just through the top portion, an average degree of saturation was determined for the whole specimen. Figure 3-6 shows the change in the degree of
saturation at consecutive gradients. The final degree of saturation went up to 83.6% only during the beginning of the test.

Figure 3-6 Variation of degree of saturation during short-term flow tests

Figure 3-7 Coefficient of permeability for fully and air-entrapped specimens

The permeability of the partially saturated sand specimen was evaluated and compared with the permeability of a fully saturated sand specimen which was also tested under constant head gradient flow. According to Fredlund (1993), Darcy's Law is still valid for partially saturated soils, however it is expected to be lower than fully saturated
and also depends on the degree of saturation or water content. It is also noted that since water flows through the pore space filled with water, lower degree of saturation causes water to flow through smaller pores with an increased tortuosity. This fact also confirms that water flow can not carry entrapped air with it and air behaves as a blockage along the flow path of water.

Figure 3-7 illustrates the comparison of coefficients of permeability of fully and air-entrapped sand specimens. The results imply that the coefficient of permeability of air-entrapped sand specimen was constant (k_{PS}=0.058 \text{ cm/s}) for the same degree of saturation (82.6-83.6 \% ) and lower than the fully saturated sand specimen's coefficient of permeability (k_{FS}=0.109 \text{ cm/s}), confirming what Fredlund (1993) reported. Also, Eqn. 3-2 is introduced by Fredlund (1993) to estimate the coefficient of permeability of partially saturated sands once the coefficient of permeability of fully saturated sand with the same void ratio is known. The coefficient of permeability of partially saturated sand tested was evaluated using Eqn. 3-2 and the coefficient of permeability of fully saturated sand specimen. The evaluated k_{PS} agreed with the measured value.

\[ k_{PS} = k_{FS} \times S^\delta \] 
\[ \delta = 3 \text{ for clean sands} \]
\[ k_{PS} = 0.11 \times (0.83)^3 = 0.06 \text{ cm/s} \]
\[ (k_{PS})_{measured} \approx (k_{PS})_{calculated} \]

In long term tests, hydraulic gradients of 0.05, 0.1, 0.2, 0.3, 0.41, and 0.52 were applied each for 30 hours. Low gradients represent natural flow and high gradients represent man-made flow conditions as shown in Figure 3-1. Soil water was replaced more than 300 times during the complete flow tests (Figure 3-8). Test results
demonstrated that small amount of air only escaped from the top 5-10 cm portion of the specimen during high gradient tests.

![Figure 3-8 Cumulative # of replacement of total water volume at the end of each applied hydraulic gradients](image)

**Figure 3-8** Cumulative # of replacement of total water volume at the end of each applied hydraulic gradients

![Figure 3-9 Variation of degree of saturation during long-term upward hydraulic gradient flow tests](image)

**Figure 3-9** Variation of degree of saturation during long-term upward hydraulic gradient flow tests
Figure 3-9 demonstrates the degrees of saturation evaluated at the end of each gradient flow test. The initial degree of saturation of 85.4% only increased to 86%, confirming no significant loss of entrapped air from the specimens.

Also, when flow rates from 1 to 30 hrs of flow were plotted as shown in Figure 3-10, it was observed that flow rates did not vary. This implies that the degree of saturation did not change significantly during the 30 hrs of flow.

![Flow rates during long-term upward hydraulic gradient flow tests](image)

**Downward Flow:**

After confirming the sustainability of air under short- and long-term upward flow, same hydraulic gradient tests were performed, this time in a downward direction, on a partially saturated sand specimen with initial degree of saturation of 86.4%. Test results demonstrated that entrapped air didn't escape or diffuse under even high gradient flows (Figure 3-11).
3.3.2 Lateral Hydraulic Gradient Flow Tests

Designing a 1-D lateral flow setup for preparing and testing air-entrapped specimens was a more challenging task. It was not possible to prepare specimens in a lateral tube. Therefore, a partially saturated sand specimen was prepared in a 192 cm long plexiglass tube with 7.6 cm diameter in vertical position and was laid down very slowly on a horizontal shelf. Figure 3-12 demonstrates a sketch and a photo of the lateral flow test setup. Gravel was used as a filter at both ends of the specimen. PVC elbows were tightened to the two ends of the plexiglass tube using plastic sleeves and circular metal clamps. Two vertical plexiglass tubes were tightly placed inside the plastic sleeves and also clamped to the elbows on both sides. The vertical plexiglass tubes were filled with water all the way to the top and tightly closed at the top with a 2 in NPT cap without pressurizing water. Constant head hydraulic gradients were applied by adjusting the level of water funnels at two sides.
Figure 3-12 Experimental setup for testing short-term and long-term sustainability of air under lateral hydraulic gradients
Lateral hydraulic gradients were applied gradually from 0.01 up to 0.8. At the end of the tests, there was no significant amount of air escaped in the vertical plexiglass tube. As shown in Figure 3-13 the degree of saturation stayed the same throughout the whole experiment, even under very high hydraulic gradients. At gradients higher than 0.5, it was recognized that water tended to flow faster especially through the top surface of the specimen. It was probably happening because flow velocity was increasing at the elbows. As a result, water created a closed flow channel between the specimen surface and the top of the tube. Although the rest of the specimen was tested under the applied flow, the lateral setup was not a practical and effective setup for high lateral gradients.

Figure 3-13 Variation of degree of saturation under lateral hydraulic gradient flow
3.4 Sustainability of Gas/Air under Ground Shaking

Further experiments were performed on a vertical column of a partially saturated sand to investigate the potential of air bubbles escaping from the sand specimen when subjected to horizontal excitations.

The vertical plexiglass tube used for upward and downward flow tests was placed on a mini shaking table. Since the specimen was intended to be tested under only lateral excitement, any possible rocking motion of the tube was eliminated by embedding it inside a trench opened on a wooden block. The tube was also secured with a clamp surrounding it at the top. Figure 3-14 demonstrates a photo of the test setup.
A partially saturated specimen was prepared at S=85.7 % by drainage-rescharge. Inside the tube from the soil surface all the way to the top was filled with water and closed with an NPT cap as performed in vertical flow tests.

Uniform cyclic motions at displacement amplitudes of 0.5 cm were induced on the shaking table at different frequencies in order to achieve different acceleration levels. The test started at 1 Hz and gradually the frequency was increased. The sustainability of air was observed as the number of cycles at the same time the accelerations increased. Figure 3-15 shows a typical accelerometer measurement obtained during 1 second of uniform cyclic motion at 5 Hz.

![Figure 3-15 A typical accelerometer measurement output during lateral shaking (f=5Hz)](image)

The amount of air escaped from the specimen during uniform cyclic motion was recorded by measuring the volume of air collected at the top of the tube. Small amount of air started escaping from the top portion of the specimen at 6 Hz, at the end of 700
cumulative numbers of cycles, which is unusually large number of cyclic shaking compared with typical earthquake motions. Figure 3-16 demonstrates the variation of existing volume of air normalized to the initial volume of air during uniform cyclic tests. The variation of degree of saturation was evaluated and presented in Figure 3-17. The escape of air was not reflected in the final degree of saturation owing to the significant settlements after 1000 cycles of lateral excitement under almost 1g acceleration (Figure 3-18). The settlements were not significantly related to excess pore water pressures, but they were related to dynamic effects of large accelerations.

Besides evaluating air escape from the top of the specimen, visual observations demonstrated that air bubbles tended to move upward and coagulate to form larger bubbles.

Finally, the results demonstrate that even under 1g horizontal excitation, and after 300,000 cycles, loss of air volume is less than 1% of the original air volume introduced in the sand specimen prior to shaking.
Figure 3-16 Normalized volume of air during uniform cyclic tests
Figure 3-17 Variation of degree of saturation during uniform cyclic tests
Figure 3-18 Settlement of the sand specimen during uniform cyclic tests
3.5 Conclusion

The sustainability of entrapped air in sand specimens was tested under different simulated field conditions including hydrostatics, horizontal and vertical hydraulic gradients, and horizontal excitations.

Large scale vertical and lateral flow test setups were designed and manufactured. Air entrapped soil Columns, (120 cm long) were prepared by the drainage-recharge method and were tested under low to high hydraulic gradients (i=0.1-0.5) applied in upward and downward directions. The sustainability of air in the specimens was tested for short- and long-term conditions. Also the coefficient of permeability of the partially saturated sand specimens were evaluated and compared with the coefficient of permeability of a fully saturated sand specimen with the same void ratio. Flow rates from 1 hr to 30 hr flow duration at each gradient were evaluated and compared with each other to assess the changes in degree of saturation. Also, a 192 cm long air-entrapped soil specimen was prepared by drainage-recharge and tested under low to high hydraulic gradients (i=0.01-0.8) applied in lateral direction.

Also, a vertical air-entrapped soil column was tested under lateral excitations using a mini shaking table. The frequency of the motion was gradually increased to induce accelerations from 0.02g to almost 1g. Escape and diffusion of air was monitored under uniform cyclic lateral displacements.

The observations and results of the tests led to the following conclusions:

1) Entrapped air in partially saturated sand specimens remains entrapped under small to very large horizontal and vertical hydraulic gradients. This conclusion was achieved by visual observation of any air accumulation at the top of the experimental
setup, as well as by comparing the coefficients of permeability in short and long term flow tests. Since the coefficient of permeability depends on the degree of saturation (Fredlund 1993), achieving same coefficient of permeability during the long term test was an important indicator that the air remained entrapped within the specimen.

2) Entrapped air remains in sand specimens even when the specimens are subjected to horizontal vibrations. The implication of this finding is that once a sand deposit is partially saturated, even under ground excitation either due to an earthquake or due to man or machine induced vibrations, the sand remains partially saturated and there is no loss of entrapped air.

Finally, the above experimental tests clearly demonstrated that once partial saturation is induced in a sand specimen by introducing gas bubbles, the bubbles remain in the specimen even under severe flow and shaking conditions. The results are very promising with respect to the practicality of the proposed liquefaction measure, IPS.
4. Experimental Setup for Cyclic Simple Shear Strain Tests

This chapter presents the details of the experimental setup for cyclic simple shear strain tests performed to evaluate the excess pore water pressure generations in fully and partially saturated sand specimens prepared by IPS. The experimental setup includes cyclic simple shear liquefaction box (CSSLB), data acquisition software LabVIEW, pore water pressure transducers Druck PDCR81, linear variable displacement transducers (LVDT); RDP DCTH400AG and bender elements and bending disks for S and P wave measurements. Also, a new IPS laboratory technique for preparing gas/air entrapped specimens is presented.

4.1 Cyclic Simple Shear Liquefaction Box (CSSLB)

In order to evaluate the cyclic response and the liquefaction behavior of fully and partially saturated sand specimens in the laboratory, an experimental setup was needed to simulate the state of shear strains induced in the field due to vertically propagating shear waves during earthquakes. In the preliminary research described in Chapter 2, a large scale liquefaction box (NULB) was designed and used on the shaking table to induce cyclic shear strains on the specimens. The NULB was determined to have boundary effects and it did not provide uniform shear strains along the rigid fixed sidewalls. Furthermore, it did not have an appropriate design for housing various types of transducers. A more versatile liquefaction box was needed for an intensive laboratory study of seismic response of partially saturated sands prepared by IPS. A new liquefaction box was designed and built to overcome the limitations stated above. The new liquefaction box was named Cyclic Simple Shear Liquefaction Box (CSSLB).
The CSSLB can:

1. induce uniform shear strains on large sand specimens,
2. minimize the sidewall boundary effects,
3. permit preparation and testing of large sand specimens,
4. accommodate special instrumentation: LVDT's, pore pressure transducers and bender elements and disks,
5. provide drainage control for undrained tests.

Figure 4-1 demonstrates the shearing mechanism of the CSSLB on a shaking table. The CSSLB has two rotating and two fixed walls (fixed walls being in the direction of shaking). The two rotating walls are hinged to the bottom plate, and also are connected to the two fixed walls and the bottom plate by a joint sealant. The sealant makes the joints water tight yet flexible allowing movements along the joints by compressing and elongating. Also, the tops of the two rotating walls are fixed to an outside steel beam located next to the shaking table, through two lateral aluminum side bars. The lateral side bars were bolted to other aluminum bars fixed on the outside surfaces of the rotating walls and on the outsider steel beam. Smooth bolts were used to provide rotation along the top joints. Hence, when the shaking table is excited with cyclic displacements, through rotation around the bottom hinge and the top bolts, controlled simple shear strains can be induced in the large specimens.

The design dimensions were determined based on various necessities. Having shorter length in the fixed side walls helped minimize boundary effects in the shaking direction. On the other hand, the rotating walls which are in the lateral direction
Figure 4-1 Schematic drawing of the simple-shear test setup (Ortakci 2007)

perpendicular to the shaking were designed longer to allow enough space in for pore pressure transducers, bender elements and bending disks. The height of the box was designed such that it would allow preparation of representative partially saturated specimens with different degrees of saturation in a practical way and also would allow enough space for instruments to be employed at different levels of the specimens. The design resulted with an inside plan dimensions of 19cm x 30 cm (7.5" x 12") and a height of 49 cm.

The adequacy of the design of the CSSLB with respect to eliminating boundary conditions was evaluated using numerical analysis. FLAC 5.0 (Itasca Consulting Group, Inc 2005) which is a two-dimensional explicit finite difference program for engineering mechanics calculations was used for numerical analysis. Plan and elevation sections of a sand model in the designed box were investigated under externally applied shear strains.
Also, sensitivity analyses were performed by Ortakci (2007) to validate the conclusion that the design of the box is optimal and boundary effects are negligible. The parameters used in the design are the shear modulus of the sand material, plexiglass sidewalls’ elastic modulus, shear modulus and Poisson’s ratio of the flexible joint sealant, and cohesion of interface elements to simulate the potential slip between the sand particles and the plexiglass sides of the box.

Figure 4-2 demonstrates X-displacements in the plan view model at the mid-height of the soil at nodes 6, 13 and 20 which correspond to locations of 4/21, 11/21, and 18/21 of the total distance within the soil specimen from the left rotating sidewall. The plan view model shearing simulation demonstrated that the fixed sidewalls have negligible effect on the displacement pattern within the sand specimen.

In elevation model, the displacements through the depth of the sand specimen were computed. Figure 4-3 shows the resulted shear strains along three vertical lines passing at 4/21, 11/21, and 18/21 of the total distances within the sand specimen, from the left rotating sidewall (nodes i = 6, 13, and 20). The model results demonstrated that shear strains are uniform from 8 to 10 cm from the base to the top of the specimen. So the pore pressure transducers need to be placed above 8-10 cm from the base.

CSSLB was built of plexiglass material based on the dimensions confirmed by numerical analysis. Sikaflex 15LM was selected as the joint sealant. Sikaflex used between the sidewalls and the rotating walls can compress and elongate by up to about 0.5 cm, leading to a 1% shear strain within a sand specimen. A plexiglass cover was fabricated with all the drainage controls for undrained tests. Final design and dimensions of CSSLB are shown in Figures 4-4 through 4-6.
Figure 4-2 X-displacements for the plan view model along three different lines at i = 6, 13, 20 (Ortakci 2007)
Figure 4-3 Shear strains of the free-surface elevation model along the lines at i = 6, 13, 20
(Ortakci 2007)
Figure 4-4 CSSLB front elevation view (a) sketch and (b) photo (Moving Sidewall) (Ortakci 2007)
Figure 4-5 CSSLB side elevation view (a) sketch and (b) photo (Fixed Sidewall) (Ortakci 2007)
Figure 4-6 CSSLB B-B section plan view (Ortakci 2007)
4.2 Measurement Equipment

4.2.1 Data Acquisition System

A special data acquisition card was used that provides communication between analog systems (the shaking table, various physical instruments: accelerometers, LVDT's, pore pressure transducers…) and digital systems (computer). The data acquisition card used is National Instruments multifunction data acquisition card (NI-DAQ) PCI-6052E. The card has a sampling rate of 333 kS/s, and can accommodate 16 analog input channels and 2 analog output channels. Therefore NI-DAQ can perform A/D (analog to digital) and D/A (digital to analog) functions. The NI-DAQ card was placed in one of the open slots of the back of the computer. The cables coming from the BNC connects of the instruments are collected in the terminals of an electrical board (CB-68LP) which is placed in a black connector box. CB-68LP is connected to the NI-DAQ card through a 68 pin SH68-68-EP cable.

National Instrument LabVIEW software program was used to control the instruments, and to analyze and monitor the test measurements. LabVIEW is a graphical programming environment to perform integrated measurements, tests, and control systems using intuitive graphical icons and wires that resemble a flowchart (www.Ni.com). For this research, a special LabVIEW worksheet was prepared for the cyclic simple shear strain tests as shown in Figure 4-7. The worksheet has two parts: 1) A combination of modules which reads the digital signal and sends it to the shaking table MTS console to induce cyclic displacements on the shaking table. The signal can also be plotted in a graph. 2) A combination of modules which receives the physical measurements from instruments and converts it to digital outputs. Mathematical function
modules can be used to calibrate and scale signals or to perform any mathematical function on the signals. Finally, the received signals can be written as LVM measurement file (a special file type of Microsoft Excel) and also displayed in a graph during the tests. The written files can be opened in Excel and modified for further analysis. The details of the module functions can be found in LabVIEW User Manual (2003).

Figure 4-7 The LabVIEW worksheet used for cyclic simple shear liquefaction tests
4.2.2 Instrumentation

4.2.2.1 Miniature Pore Pressure Transducers

A submersible transducer was needed that can measure rapid changes in water pressures during the cyclic simple shear strain tests. Miniature pore pressure transducer Druck PDCR 81 was the most suitable transducer for these purposes and was used in this research for measuring pore water pressures during the tests.

PDCR 81 is a very sensitive pressure transducer. It consists of a silicon diaphragm (sensor) supported on a glass cylinder and connected to a porous filter by a steel outer shell. Figure 4-8 demonstrates the details and the dimensions of the device provided from the manufacturer company General Electrics (GE).
PDCR 81 has different pressure range options depending on the sensitivity of the application. For this research, the smallest range which is 0-5 psi was used. A signal conditioner was needed since the output of the pore pressure transducers is too small (0-35mV range for 0-5 psi) and needs to be amplified in order to send the output signals to the data acquisition system. Also, ambient noise which is within the output range was filtered physically through signal conditioners. The signal conditioner used was Ultra–Simpak G448-0002 by Action Instruments (Figure 4-9). The procedure for the wire connections from PDCR 81 to the signal conditioner and from the signal conditioner to the data acquisition box can be found in Eseller (2004).

Calibration of the PDCR 81 was performed also using the signal conditioner screws available in front and monitoring the data on the LabVIEW screen. PDCR 81 was inserted in a water column and calibrated by adjusting the output signals at 0 psi to 0V and 2 psi (equivalent to 140.66 cm of water pressure) to 2 V. The output of the pore water pressure transducer (PPT) was scaled to height (cm) of water pressure instead of psi, by multiplying the conversion factor of 70.33 cm/psi. The details of the calibration process are reported by Eseller (2004) in details.

The only handicap with PDCR 81 was that they need to be saturated all the times. As the porous stone gets dry the PPT starts giving erroneous readings. Therefore it needs to be kept saturated even during the specimen preparation, which is explained later in this chapter. If there was any air intake and it got entrapped in the propos stone, the PPT was placed in a vacuum chamber shown in Figure 4-10 and air was vacuumed out of the porous stone under a vacuum of 50k-100Pa.
Figure 4-9 Signal conditioner Ultra–Simpak G448-0002 by Action Instruments

Figure 4-10 Vacuum chamber for saturating PPT porous stones
The PPT was passed through a plastic fitting which can be fixed into the CSSLB instrumentation hole (Figure 4-11). Silicon was used to secure the PPT cable inside the fitting as well as to eliminate any leakage from the specimen to the outside of the box. 3 PPT's were used; in some tests two of them at the same elevations to confirm PPT measurements whereas in some tests they were used at different elevations to estimate average excess pore water pressure ratios for the specimens prepared. All PPT's were placed 8-10 cm above the base since shear strains induced are uniform starting from that depth.

Figure 4-11 PDCR 81 pore water pressure transducer with a fitting used in cyclic simple shear strain tests

Pore water pressure measurement in partially saturated sand specimens using PDCR 81 was a crucial task. The details of this task are reported later in this chapter.
4.2.2.2 Linear Variable Displacement Transducers

A linear variable displacement transducer (LVDT) was used to measure relative displacements between the top and the bottom of the CSSLB, when the cyclic motions are induced on the shaking table. After seeking different types of LVDT's, a spring loaded LVDT was found to be best suited to the experimental setup (Ortakci 2007). The LVDT used was RDP DCTH400AG DC-DC with strokes of ±10mm (Figure 4-12). Two of them were used, one on each side of the CSSLB to confirm that the orientation of CSSLB along the shaking direction is symmetric. The LVDT was fixed to the aluminum side bar which is connected to the rotating wall. The tip of the LVDT was set on a small plexiglass plate which is glued to the fixed sidewall as shown in Figure 4-12. Shear strains induced on a specimen was computed by dividing the relative displacement measured by the height of the LVDT from the shaking table.

Figure 4-12 RDP DCTH400AG spring loaded LVDT fixed on the CSSLB lateral bar (Ortakci 2007)
4.2.2.3 Multiple Bender Element and Bending Disk Measurement Facility

A multiple bender element and bending disk measurement facility was established at Northeastern University to measure shear (S) and compressional (P) wave velocities in large sand specimens prepared in laboratory. S and P wave measurements were performed in fully and partially saturated sand specimens prepared to examine that the sand properties (density and degree of saturation) in the specimen in the CSSSLB are uniform, and if the presence and density of the bubbles can be detected.

Establishing such a facility required an intensive study and effort. The details of the multiple bender element and bending disk measurement facility with the measurement results and conclusions are reported in Chapter 5.
4.3 New IPS Laboratory Technique

The two previous laboratory techniques employed in the preliminary research to prepare partially saturated sand specimens were introduced in Chapter 2. One of these techniques was electrolysis which is generating oxygen and hydrogen gases in fully saturated sand specimens. It was noted that, although the degree of saturation was controllable by adjusting the current intensity in electrolysis, it wasn't effective enough to achieve uniform distribution of gas bubbles in the sands specimens. Gas bubbles were generated mostly around the electrodes, and as reported in Chapter 3, entrapped gas/air do not tend to diffuse in sand specimens. The other technique which is drainage-recharge (D-R) was a practical method of inducing uniform partial saturation, however the degree of saturation was not controllable. The range of degree of saturation achieved was 82-86%.

Therefore, a new technique was explored to induce uniform partial saturation in the sand specimens at controllable degrees of saturation. It was recognized that a special chemical compound which is a main ingredient of the dental product "Efferdent" produces oxygen gases when it gets into reaction with water. This main ingredient is sodium perborate (48% of Efferdent) which crystallizes as the monohydrate or tetrahydrate in water. When sodium perborate monohydrate (NaBO$_3$.H$_2$O) gets into reaction with water, it generates hydrogen peroxide H$_2$O$_2$. H$_2$O$_2$ is a perfect oxygen gas source when it reacts with water. The chemical reactions are introduced below:

\[
2(\text{NaBO}_3.\text{H}_2\text{O}) + 2\text{H}_2\text{O} \rightarrow 2\text{H}_2\text{O}_2 + 2\text{BO}_3^{3-} + 2\text{Na}^+ + 4\text{H}^+ \\
2\text{H}_2\text{O}_2 \rightarrow 2\text{H}_2\text{O} + \text{O}_2
\]
4.3.1 Specimen Preparation

To prepare partially saturated sands, Efferdent tablets were powdered and mixed with dry sand. The sand used was Ottawa sand (ASTM graded silica sand). The gradation and soil characteristics of Ottawa sand are presented in Figure 4-13 and Table 4-1, respectively. Figure 4-14 shows a picture of the powdered Efferdent. Then partially saturated specimens were prepared by wet pluviation technique which is basically raining the Efferdent-dry sand mix in the CSSLB filled partially with water as shown in Figure 4-15. The details of wet pluviation technique can be found in Eseller (2004).

![Figure 4-13 Ottawa sand gradation curve](image)

Table 4-1 Ottawa Sand Index Properties (Holtz and Kovacs, 1981)

<table>
<thead>
<tr>
<th></th>
<th>Particle Size and Gradation</th>
<th>Voids</th>
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<td>Approx. D$_0$ (mm)</td>
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<td></td>
<td>$D_{\text{max}}$</td>
<td>$D_{\text{min}}$</td>
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<th></th>
<th>Density (Mg/m$^3$)</th>
<th>Dry Density, $\rho_d$</th>
<th>Wet Density, $\rho$</th>
<th>Submerged Density, $\rho'$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Min (Loose)</td>
<td>Max (Dense)</td>
<td>Min (Loose)</td>
</tr>
<tr>
<td>Standard Ottawa Sand</td>
<td></td>
<td>1.49</td>
<td>1.78</td>
<td>1.51</td>
</tr>
</tbody>
</table>
Figure 4-14 Powdered Efferdent

Figure 4-15 Preparation of partially saturated specimen with powdered Efferdent-dry sand mix
Small scale experiments were performed by Gokyer (2009) using various Efferdent to sand ratios. A correlation was obtained between Efferdent to sand ratio ($\frac{M_{\text{eff}}}{M_{\text{dry\,sand}}, g/kg}$) and the degree of saturation when the reaction is complete as shown in Figure 4-16. Therefore, the Efferdent to sand ratio necessitated for the intended degree of saturation in each test was determined from the correlation in Figure 4-16.

The degree of saturation of the specimens was calculated using phase relation equations, which are reported in Gokyer (2009). The presence of oxygen gases and their distribution within the sand specimens were also evaluated by taking micro pictures of the specimens with a professional camera (Gokyer 2009). Figure 4-17 illustrates a micro picture taken from a partially saturated specimen at a degree of saturation of 80%. The micro pictures confirmed that uniformly distributed gas bubbles were generated in the specimens prepared by the new technique.
4.3.2 Pore Water Pressure Measurements in Gas Entrapped Sand Specimens

The Druck PDCR-81 pore pressure transducer (PPT) has proven to be capable of measuring pore water pressures in unsaturated soils, where matric suction also exists (Muraleetharan and Granger, 1999). In this research, PPT measurements were performed in gas-entrapped sand specimens, which is different than measurements in unsaturated sands. In partially saturated sands prepared by gas generation with the chemical compound of Efferdent, two main issues were experienced during pore water pressure measurements with PDCR 81:

1. Sometimes, PPT’s in the Efferdent-sand-water mixture gave readings higher than hydrostatic water pressures after some time is elapsed after the initiation of Efferdent-water reaction.
2. PPTs did not measure any water pressures under hydrostatic values or negative water pressures even at degrees of saturation as low as 50-60%. This confirms that there is no matric suction in partially saturated sands prepared by gas generation.

Making measurements with PPTs that appear to give erroneous results was a concern in this research. Therefore, small scale tests were performed to investigate the reasons of such at times erroneous readings and solutions were sought.

First and second PPT's were inserted in a small scale flask first partially filled with water, and then partially saturated sand specimens were prepared with the Efferdent-sand mix. As the time elapsed, Efferdent started reacting with water and degree of saturation started decreasing. The PPT readings were monitored and plotted in Figure 4-18 and 4-19. The PPT readings \( u_{\text{ppt}} \) were normalized to hydrostatic water pressure \( u_{\text{hyd}} \) at the elevation of the PPT. As illustrated in the plots, the responses of PPT's were different. First PPT started giving erroneous readings by an error factor of 1.1 \( (u_{\text{ppt}}/u_{\text{hyd}}) \) after 47 hours whereas PPT 3 started giving erroneous readings after 10 hours and by an error factor of 1.6. It was concluded that the responses depended on the starting degree of saturation (or the Efferdent amount) and how long the PPT stayed in the specimen. Also the different responses of PPTs were related to the possible differences in the structure of the porous stones.

Two hypotheses were suggested for the reasons of the erroneous readings of PPTs in Efferdent-sand-water mixtures: First hypothesis was that the Efferdent particles or Efferdent solution travels inside the porous stone and generates air in the porous stone
or in the gap between the diaphragm and the porous stone. The air entrapped inside the PPT increases the pressure exerted on the diaphragm. Second hypothesis was that gas bubbles generated in front of the porous stone exerts pressure and increases the pressure reading of the PPT. To investigate the first hypothesis, the PPTs were tested in an Efferdent water solution. The PPT readings in the solution were monitored and it was observed that the readings started giving higher than hydrostatic pressures. Then they were taken out and then left under vacuum for a while. It was observed that bubbles were coming out of the PPTs. When the PPT readings were tested they were back to
hydrostatic pressure. To prove the first hypothesis with another test, the second PPT was isolated with a membrane (Figure 4-20) and measurements were taken in a specimen prepared with Efferdent. The results demonstrated that PPT 3 did not measure higher than hydrostatic pressures (Figure 4-21). Therefore the first hypothesis holds based on the experimental results. Second hypothesis was difficult to test however it can also have an effect on the pressure increase.

![Second PPT covered with a membrane](image)

Figure 4-20 Second PPT covered with a membrane

![Second PPT response in gas-entrapped specimen prepared with Efferdent when covered with a membrane](image)

Figure 4-21 Second PPT response in gas-entrapped specimen prepared with Efferdent when covered with a membrane

The PPTs could not be utilized with a membrane since water needs to get passed the porous stone in order to measure water pressures. The use of membrane only proved
the hypothesis about the excessive water pressure readings however it did not solve the problem.

While different solutions were being sought, it was recognized that the excess pore water pressures were measured accurately even though the hydrostatic pressures were measured higher. Therefore, the hydrostatic pressures were measured with an offset and the PPT was still capable of reading the changes in the water pressures accurately. Figure 4-22 demonstrates the PPT response to water pressure changes created by increasing and decreasing the amount of water in the flask. This was a very encouraging result because the primary interest was to measure excess pore water pressures in partially saturated sands during cyclic simple shear strain tests.

![PPT responses to hydrostatic water pressure changes](image)

Figure 4-22 PPT responses to hydrostatic water pressure changes
Therefore, although the PPT PDCR 81 sometimes gets out of calibration and gives an offset in gas-entrapped specimens prepared by Efferdent-sand-water mixtures, it is capable of measuring excess pore water pressures in both cases. Also, when a PPT does not get an offset, it was observed that matric suction was not observed in gas generated sand specimens even at degrees of saturation as low as 50-60%.
5. Multiple Bender Element and Bending Disk Measurement Facility for Evaluating Soil Characteristics in Large Shaking Table Specimens

5.1 Introduction

Measurement of Shear (S) and compressional (P) wave velocities have been widely carried out in the field or laboratory in geotechnical engineering, especially in problems of soil dynamics (soil-structure interaction, soil behavior during earthquakes), liquefaction analysis, as well as in sample quality assessment. Measurement of shear wave velocity is a convenient non-destructive way of estimating a very important soil parameter in geotechnical engineering, which is the small strain shear modulus, $G_{\text{max}}$. Also, shear wave velocities can be employed to assess soil sample disturbance. P wave velocity is related to small strain constrained modulus $M_{\text{max}}$ and also is recognized as an indicator of saturation level of soils. Measurement of P wave velocity is not as common as of shear wave velocity; however it is becoming a common technique, especially for liquefaction analyses of imperfectly saturated sands.

Recently, piezoelectric transducers have become popular and promoted as a better way of measuring wave velocities in the laboratory. Piezoelectric transducers can generate waves by producing small deformations in soils. Velocities can be evaluated by measuring the wave travel time. Bender element, which consists of a special configuration of piezoelectric ceramics, was proposed by many researchers (Shirley and Hampton (1978), Dyvik and Madshus (1985)) for generating and receiving shear waves. Bending disk has a different configuration of piezoelectric ceramics and is suggested for measurement of P waves. P and S wave measurements using these devices are more
advantageous over other tests owing to their ease of computation, adequacy of one simple
measurement and providing valuable data during the continuation of other soil testing.

Despite its simplicity and convenience in computation, the use of piezoelectric
transducers for S and P wave velocity measurements through piezoelectric transducers
still hold uncertainties in the application and interpretation of test data. Many researchers
explored their uses in different setups and reported on the improvement of signal
interpretation techniques. Various challenges and difficulties expressed by other
scientists together with the problems faced in this research are all reported in this chapter.

In this task of the research, multiple bender element and bending disk
measurement system was established to evaluate shear and compressional wave velocities
in large sand specimen setups, such as shaking table specimens. Designing and
manufacturing a complete bender element and bending disk setup is a challenging
process and requires various tasks. An intensive literature study was performed to learn
from bender element and bending disk setups used for other soil testing apparatuses.
After evaluating other studies, new bender element and bending disk setups which can be
placed in the instrumentation holes of the CSSLB, were devised. All the challenges faced
during the preparation and manufacturing of the complete bender element and bending
disk setups are reported. Special equipment was prepared for generating S and P waves
in the source transducer (bender element or bending disk) and receiving the waves
traveled through the soil at the transducers located at several locations of the CSSLB.
The effect of numerous factors on the signal interpretation; such as frequency and
amplitude of the source signal, wave travel path and distance, type of signal, ambient or
electrical noise were all evaluated. Using first arrival method, the wave velocities were calculated by dividing the travel distance by the travel time of the wave.

Besides establishing a new wave measurement system for large laboratory specimens, one of the primary goals of this research task was to evaluate the uniformity of soil characteristics in large fully saturated specimens as well as in partially saturated specimens prepared by the new mitigation technique IPS. Based on multiple S wave velocity measurements throughout large specimens, variations in the specimen density or stiffness can be identified. Also, the multiple bender element system was tested in loose and dense sand specimens under different effective stresses. The results were confirmed with theoretical correlations.

Another goal of this research task was to investigate whether or not IPS technique can be quantified by P wave velocity measurements and consequently if the liquefaction benefit of IPS mitigation technique can be related to the P wave velocities. To explore this task, P wave measurements were performed in specimens prepared by IPS at several saturation levels. The results were also compared with some other works reported in the literature.

This chapter first describes characteristics of piezoelectric materials, properties of bender elements and bending disks, and the measurement equipment details. Second, manufacturing of a complete bender element setup, multiple shear wave measurements through the new system and various factors affecting the signal interpretation are presented. Evaluation of localized soil characteristics in specimens prepared by IPS and the effect of density and effective stress on the shear wave velocities are reported next.
Third, manufacturing of bending disks, and results of P wave measurements, and interpretation and evaluation of IPS through P wave velocities are presented.

5.2 Piezoelectric Ceramics

Piezoelectric material is a crystal which can convert electrical energy to mechanical deformation or mechanical deformation to electrical energy. In other words piezoelectric material deforms by electrical polarity or creates electrical polarity when it deforms. They are categorized as motors or generators depending on the function. Piezo actuators or motors convert electrical energy to mechanical and piezo sensors or generators convert mechanical energy to electrical energy.

Piezoelectric behavior was first discovered in certain crystalline minerals in nature by Jacques and Pierre Curie in 1880. Now, piezoelectric ceramics can be manufactured in the industry and modified according to the application area. The most popular and strong piezoelectric ceramic manufactured is a composition of lead zirconate titanate which is also the ceramic used in this research. The piezoelectric ceramics used in this research are manufactured by Piezo Systems Inc. located in Cambridge Massachusetts. The lead zirconate titanate type ceramic is designated as PSI-5A-4E.

During the manufacturing process of piezo electric ceramics, they are polarized permanently at high temperatures by exposing the element to direct electric field (Figure 5-1). Then the electric field is removed and the ceramic remains as it is polarized. The direction of the field is called poling voltage. So every piezo piece has a permanent polarization direction which is demonstrated with an arrow "P" pointing from the positive to the negative poling electrode (in the direction of poling voltage).
Figure 5.2 demonstrates a piezoelectric ceramic disk with upward polarization. When a piezoelectric ceramic is compressed in a direction parallel to the polarization, a voltage difference is generated in the same direction of polarization, which forces the piece to pull back to the original length. A reverse action creates a voltage generated in the reverse direction of polarization. These types of ceramics are called piezo generators. When a voltage is applied in the direction of polarization (in the direction of poling voltage), the ceramic elongates in that direction. If it is excited with a reverse voltage it compresses. These ceramics are called piezo motors.

Figure 5-1 Permanent polarization of piezo ceramics during manufacturing (APC International, Ltd, 2002)

Figure 5-2 Generator and motor actions of a piezoelectric ceramic (APC International, Ltd, 2002)
5.2.1 Bender Elements for Shear Wave Measurements

Bender elements are thin 2-layer piezoelectric ceramics with a thin metal shim in between. Figure 5-3 demonstrates a typical bender element. Two piezoceramics are bonded to a reinforcement material in between and covered with thin electrodes on the surfaces so that wires can be connected.

Bender elements can be used as generators or motors depending on their functions. For shear wave measurements in soils, 2 cantilever type mounted bender elements (Figure 5-4) are embedded in the soil in a longitudinal alignment, while first one (shear wave source) acts as a motor and the second one (shear wave receiver) acts as a generator. As reported in the previous section, when a motor piezoelectric ceramic is excited with a voltage difference, it elongates or shortens depending on the applied voltage direction. Motor bender elements are designed such that when a voltage difference is supplied, one layer piezoceramic elongates and the other layer shortens, which results in a bending motion. Bending motion in the soil causes soil particles to move perpendicular to the surface of the bender element and creates a shear (S) wave which travels through the soil and hits the receiver (generator) bender element facing it. Generator bender element deforms by the wave received and generates an electrical field which can be measured as an electrical signal.

Reverse action (one elongating the other shortening) in the piezo layers of a bender element is provided by either using reverse polarized sheets (called X-poled) connected in series or same polarized sheets (Y-poled) connected in parallel. As shown in Figure 5-4, in series connection, one of the outer electrodes is connected to (-) voltage supply and the other is connected to (+). Since polarization of the layers is reverse, under
the same electric field, one elongates and the other shortens resulting in bending. In parallel connection, outer electrodes are connected to the same (+ or -) voltage and the metal shim in between the layers is connected to the opposite voltage (- or +). In that case, bending will develop if the two layers are of the same polarization since they experience reverse electrical field. Even tough they both provide bending, one is more advantageous over the other depending on their functions. A parallel connected bender element is more efficient when used as a motor. Because it can create the same amount of bending displacement at half of the voltage amplitude required for the bender element connected in series. Similarly, a series connected bender element is used more efficiently as a generator, since the same deflection will result in a double amplitude signal when compared with the signal received from the parallel connected Y-pole.

Figure 5-3 A typical bender element configuration (side view) (Piezo Systems, Inc.)

Figure 5-4 X poled series connection bender (a) and Y poled parallel connection bender (b) (from Piezo Systems Inc.)
Depending on the application, different size bender elements with different stiffnesses are used. As the size and stiffness of bender elements increase, it provides more deflection and resists higher blocking forces (Refer to Deniz 2008 for details). One of the ultimate goals of this research task is to establish a shear wave measurement system for large scale specimens. For that purpose after evaluating different type and size of bender elements, an optimum size bender element was found, which will provide an adequate amount of deflection for the wave to reach the receiver before dying, at the same time which is feasible enough to mount in the experimental setup. Finally, PSI-5A-4E type bender elements were found to be the best suitable one for this research. The bender element used is 3.18 cm long, 1.27 cm wide 0.051 cm thick, and made of lead zirconate titanate. Both x poled and y poled were used depending on their functions.

![Bender Elements used in the research](image)

<table>
<thead>
<tr>
<th>Type:</th>
<th>PSI-5A4E</th>
</tr>
</thead>
<tbody>
<tr>
<td>P/N:</td>
<td>T220-A4-303X or Y</td>
</tr>
<tr>
<td>Size:</td>
<td>0.51Tx12.7Wx31.8L in mm's</td>
</tr>
<tr>
<td>Polarization:</td>
<td>x- or y- poled</td>
</tr>
</tbody>
</table>

Figure 5-5 Bender Elements used in the research
5.2.2 Bending Disks for Compressional Wave Measurements

Bending disk also consists of two thin layers of piezoelectric ceramics attached to each other however without a thin metal shim in between, since bending disk doesn't require that much reinforcement. Figure 5-6 demonstrates top and side views of different size disks. They function based on the same principle as in bender elements. When the motor disk (source) is excited with an electrical field, one ceramic disk enlarges and the other shrinks, causing the disk to bow in and out. When embedded in soils this bowing motion pushes the soil particles further and creates compressional (P) waves. If the wave is strong enough to reach the receiver without dying, it compresses the generator disk (receiver) and produces electrical field which can be recorded as an electrical signal.

The disk is fixed to a mounting all around from its circumference as shown in Figure 5-7. The bowing impact amplifies as the disk diameter gets larger. Hence, the disk should be large enough to provide high amplitude wave which can be caught in the receiver before it dies. At the same time, the disk has to be fixed to a reasonable size mounting which can be properly inserted inside the experimental soil setup. Therefore, a 3.18 cm diameter bending disk was determined to be the best proper size for this research. T216-A4NO-373 is the part number of the disk whose real size is shown in Figure 5-6. The disks are only manufactured as X-pole and connected in series. Figure 5-8 shows the bending disk used in this research. Please refer to Deniz (2008) for more details about bending disks.
Figure 5-6  Typical bending disk configuration (Piezo Systems, Inc.)

Figure 5-7  Bending disk mounted all around the circumference (Piezo Systems, Inc.)
5.2.3 **Bender Element and Bending Disk Measurement Equipment**

Special equipment was used to send an electrical signal to the source bender elements and bending disks as well as to record electrical signals generated at the receivers. Figure 5-9 demonstrates the measurement setup diagram in a sequence. A function generator sends a signal which is amplified through a power amplifier. The amplified signal which is recorded in a digital oscilloscope actuates the source bender element or bending disk. The wave created by the source is transmitted to the receiver and the received wave is recorded in the digital oscilloscope. The source and received signals recorded in real time can be then downloaded in a computer.

Figure 5-10 demonstrates the measurement equipment. The function generator used is Hewlett Packard 15 MHz Waveform Generator Model 33120A. The function generator can send different types of signals such as sinusoidal, square, triangle, sawtooth
etc. as a single pulse or a continuous signal. After evaluating various types of signals, the best signal for P and S wave measurements was determined to be one pulse sinusoidal signal. Single sine pulse is the safest signal that does not cause any damage to piezoceramic materials under high voltages. Also data is better interpreted with a sine function.

In this research, a new experimental wave measurement setup was intended to be developed in large sand specimens that will be effective for long wave traveling distances. Therefore, the equipment should send enough energy so that the wave energy can be transmitted to the longest possible distance. To get the maximum deflection in the bender element and bending disks high amplitude signals need to be sent. X-pole bender elements and bending disks connected in series and in parallel provide largest deflection at amplitude of ±180 V and ±90 V, respectively. However the function generator can provide up to ±10 V. A power amplifier was utilized to intensify the output signal from the function generator. It is "Piezo Linear Amplifier Model EPA-104-115" from Piezo Systems, Inc. It can boost incoming voltages up to ±200 V.

The received signals at the generator bender elements or bending disks are displayed and recorded in a digital oscilloscope which has a high sampling rate. A high sampling rate was needed since high frequency signals are sent through bender elements and disks. It is a multichannel (up to 12 channels) Yokogawa DL750 Scopeocorder which has 6 digital channels and 6 digital signal processing (DSP) channels. It offers advantages over other scopes since it has higher vertical resolution, filtering, stacking, signal averaging and also all the analysis tools like cursors, waveform parameter calculations, math and DSP channels, and Fourier transforms (FFT's).
Figure 5-9 Bender element and bending disk measurement equipment diagram

Figure 5-10 Bender element and bending disk measurement equipment
Most of the math functions and FFT’s can be performed during data processing.

The data acquisition modules are provided separately according to the required measurement specifications. In this research, a high sensitive module 701251 was used, which has a high speed sampling rate (1MS/s). The module has a high sensitivity amplitude range of peak-to-peak 10mV with very low noise levels. It is a very well suited digitizing module for this research since the received signal amplitudes will be as low as 1 mV. The measured data in the scope can be downloaded in a computer through USB port and displayed on the screen using special software "DLWave". The measured data can be also downloaded as an Excel or a picture file in DLWave. Further details on the specifications of the equipment can be found in Deniz (2008).

5.2.4 Use of Bender Elements and Bending Disks in Geotechnical Applications and in the Literature

Over the past three decades, use of piezoceramics in geotechnical engineering has become very popular since they provide non-destructive soil testing for soil properties through measurement of shear and compressional wave velocities.
Shear wave velocity measured in soils is related to small strain shear modulus $G_{\text{max}}$ through the following equation:

$$G_{\text{max}} = V_s^2 \rho$$

$G_{\text{max}}$ is the maximum shear modulus corresponding to small shear strains in the soil.

$V_s$ is the shear wave velocity measured in the soil.

$\rho$ is the density of the soil.

Small strain shear modulus ($G_{\text{max}}$) is a very important soil parameter in geotechnical engineering problems especially in dynamic problems (soil-structure interaction, soil behavior during earthquakes) and liquefaction assessment, as well as in sample quality assessment. $G_{\text{max}}$ has been commonly measured in the laboratory by resonant column tests. In resonant column tests, harmonic loads are applied to hollow shape soil specimens at different frequencies to determine the fundamental frequency of the soil which is the frequency of motion causing the highest strain. Shear wave velocity is related to the fundamental frequency and can be evaluated through wave propagation equations of soils. The disadvantage of the resonant column tests is that the soil is loaded several times and the same soil specimen cannot be used for other soil testing. However bender element measurements eliminate soil disturbance and generates very small displacements. It also facilitates the testing procedure to estimate the shear modulus through a single measurement.

The use of bender elements for shear wave measurements was first introduced by Shirley and Hampton (1978). Dyvik and Madshus (1985) adapted their use in laboratory setups and developed bender element test setups in triaxial, oedometer and simple shear test apparatuses and compared with the results from resonant column tests. Gohl and Finn (1991) implemented them in centrifuge and shaking table samples. Brignoli et al.
(1996), Lee and Santamarina (2005) and Leong et al. (2005) improved the use of bender elements in laboratory specimens and provided valuable information on the best design and manufacturing of the bender element setups as well as the interpretation of the signals. Landon et al. (2007) implemented wave velocity measurement with bender elements in sample quality assessment and developed a portable bender element measurement device.

In order to develop a new shear wave measurement system for large sand specimens, an intensive study was performed on other researchers' work. Types of bender elements, testing apparatus, equipments used, type of soil and its saturation level, bender elements mounting, electrical wiring design and signal interpretation techniques suggested and used by some of the other researchers were tabulated in Table 5-1. All findings by other researchers were examined and a multiple bender element measurement system to be applicable in large sand specimens was developed at Northeastern University. Advantages and disadvantages of other researchers' techniques were also reported in the next section of this chapter.

P-wave velocity is related to elastic properties of the soil at the same time it is considered as an indication of saturation level. For P wave measurements, various types of transducers are explored in the literature. Hydrophones or geophones and pulse generators (acoustic transducers) are widely used for P wave velocity measurements in laboratory tests (Tamura et. al 2002) and Emerson and Foray (2006). The use of piezo electric ceramics for P wave measurements has become common after 1990's. Gohl and Finn (1991) used bender elements and reported that they detected P waves at the receiver bender elements. Agarwal and Ishibashi (1991) also used bender elements however in a
different configuration. In their case, bender elements were embedded in soils in a face to face alignment instead of tip to tip. With this configuration, P waves were produced by compressing the soil with the bending motion. Extender elements were proposed and used by Lings and Greening (2001) to measure P wave velocities. Extenders have exactly the same configuration as bender elements except the type of connection. When x-pole bender element is connected in parallel or y-pole bender element is connected in series, the two attached layers of piezoceramics both lengthen or shrink and they become extenders. De Alba et al. (1984) also used piezoceramic transducers however they only could detect P wave in specimens with S=100%. Ishihara et al. (2002) utilized large size piezoelectric disks (1.5 cm diameter and 4.2 cm long) to trigger P waves in triaxial specimens. Brignoli et al. (1996) used one layer circular piezoelectric transducer and were capable to detect P waves in triaxial specimens.

The proposed bender element configurations and extenders by the researchers mentioned above did not work for P wave velocity measurements in large sand specimens. Bender element designs are not intended to detect P waves. The waves arriving to the receiver bender element before the S wave are distorted P waves. Therefore it can not tell the exact arrival time of the P wave. Extenders are capable to detect P waves in fully saturated sand specimens; however they are not effective enough to work in large distances with partially saturated sand specimens. Very large size disks were not desired since it is not practical to mount in shaking table setups. On the other hand, the intention of this research is to develop an array of transducers at various depths. Hence, a new setup was explored to measure P wave velocities in large sand specimens by means of piezoceramic bending disks at suitable sizes.
<table>
<thead>
<tr>
<th>Reference</th>
<th>Test Setup</th>
<th>BE type</th>
<th>BE Dimensions, mm</th>
<th>Protrusion Length, cm</th>
<th>Water proofing</th>
<th>Grounding for series</th>
<th>Tip-to-tip Distance, cm</th>
<th>Soil Tested</th>
<th>DAQ</th>
<th>Noise Control</th>
<th>Signal Generated</th>
<th>Amp. of the source signal, ±V</th>
<th>Frequency of the signal</th>
<th>Amplitude of the received signal, mV</th>
<th>Arrival Time, meth. of interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gohl and Finn (1991)</td>
<td>Centrifuge</td>
<td>R205-S code G-1195 Picco. Prod. Inc of Metuchen, NJ. Series</td>
<td>Not specified</td>
<td>Not specified</td>
<td>Dry tests</td>
<td>Entire BE coated with an electrical insulant</td>
<td>1.5-4</td>
<td>20 cm deep Nevada 120 sand (loose and dense)</td>
<td>PC based. Output connected to an amplifier. 1MS/s combined, 250kS/s per channel</td>
<td>No</td>
<td>Square</td>
<td>10</td>
<td>30 Hz</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Gohl and Finn (1991)</td>
<td>Shake Table</td>
<td>R205-S code G-1195 Picco. Prod. Inc of Metuchen, NJ. Series</td>
<td>Not specified</td>
<td>Not specified</td>
<td>Dry tests</td>
<td>Entire BE coated with an electrical insulant</td>
<td>2.5-15</td>
<td>61 cm deep C-109 Ottawa Sand e=0.72</td>
<td>(FG-OS)-BE-soil-AMP.-OS.</td>
<td>No</td>
<td>Square</td>
<td>30</td>
<td>0.5-30 Hz (vel. measured insensitive of freq)</td>
<td>0-10</td>
<td>NA</td>
</tr>
<tr>
<td>Agarwal, Tarun and Ishibashi, Isao (1991)</td>
<td>Cubical triaxial test (102 X102 X 102) mm</td>
<td>Vernitron</td>
<td>S waves: PZT-5A 6.35 X 6.35 X 0.61 P waves: same different configuration</td>
<td>0.5-0.6</td>
<td>Dry tests</td>
<td>All the cables (RG58/U coaxial) are grounded</td>
<td>9.5 cm for S wave, less than 9 cm for P wave</td>
<td>Dry sand</td>
<td>FG-PA-Step up Transformer-(OS-Box)-BE-soil-Box-OS-Printer.</td>
<td>Switch box to minimize noise.</td>
<td>Sine pulse</td>
<td>400-600 for P and 80-150V for S</td>
<td>Around natural freq of BE: 13.56kHz</td>
<td>5-100</td>
<td>Peak-to-peak</td>
</tr>
<tr>
<td>Brignoli, E., G., M. Gotti, M., and Stokoe, K., H. II (1996)</td>
<td>Triaxial</td>
<td>Vernitron</td>
<td>S waves: PZT-5H 20 X 10 X 0.5 P waves: compression transducer: PZT-5A cylindrical</td>
<td>1.5% of specimen length (= 0.15 cm)</td>
<td>2-component epoxy glue</td>
<td>All the cables are coaxial.</td>
<td>10-14 cm</td>
<td>Saturated sand and clay</td>
<td>Dry sand</td>
<td>FG(PA-OS)-BE-soil-BE-SA-OS-PC.</td>
<td>No filter</td>
<td>Single sine pulse</td>
<td>10 for S and 130 for P</td>
<td>5-1000</td>
<td>First arrival (***they calibrated zero time. t=tfa-tc (tc changes with f (2.5-15 μs))</td>
</tr>
<tr>
<td>Christopher Baxter (1999)</td>
<td>Rigid wall cells(14.5 cm diam.)</td>
<td>Piezo Systems</td>
<td>10 X 7 X 6</td>
<td>0.5</td>
<td>2 part epoxy made by 3M Scotchcast electrical Resin No 5</td>
<td>All the equipments and BE's are connected to a common ground</td>
<td>10-12</td>
<td>Saturated sand</td>
<td>FG(PA-OS)-BE-soil-OS-PC.</td>
<td>No</td>
<td>Half sine wave</td>
<td>20</td>
<td>NA</td>
<td>0.1-3</td>
<td>Peak-to-peak</td>
</tr>
<tr>
<td>Lee and Santamarina (2005)</td>
<td>Triaxial</td>
<td>PZT(lead-zirconate-titanate)</td>
<td>12.7 X 8.0 X 0.6</td>
<td>0.62</td>
<td>Polyurethane</td>
<td>Silver conductive paint</td>
<td>3.2, 6.05, 10, 10.5, 15, 10</td>
<td>Dry-partially and fully saturated</td>
<td>(FG-OS)-BE-soil-BE-OS.</td>
<td>NA</td>
<td>Step signal, impulse, sinusoidal</td>
<td>2.5 for L=3.2 cm</td>
<td>0.5-40 kHz</td>
<td>Best response is at 4kHz (f=3.0kHz)</td>
<td>0-1</td>
</tr>
<tr>
<td>Leong et al. (2005)</td>
<td>Triaxial</td>
<td>x poled PZT 5A Piezo PS15A T220-A4-103X</td>
<td>15.9 X 3.2 X 0.51</td>
<td>0.5</td>
<td>Epoxy glue</td>
<td>NA</td>
<td>10</td>
<td>Dry sand, mudstone residual soil (partially sat.) fully sat. kaolin</td>
<td>FG(PA-OS)-BE-soil-OS-PC.</td>
<td>Hanning Window</td>
<td>Sinusoidal (most)-square</td>
<td>10</td>
<td>0.5, 4, 16 kHz (Lin/2, &gt;3.33)</td>
<td>10-200</td>
<td>First arrival (0.2-0.3 ms)</td>
</tr>
</tbody>
</table>

**Table 5-1 Summary of Bender Element Measurements (Setups and Signal interpretations) by Some Other Researchers**
5.3 Shear (S) Wave Measurements in Large Sand Specimens

This section presents design and manufacturing process of complete bender element measurement setup, evaluation of shear wave signals, various effects on the signal interpretation, and finally assessment of soil uniformity and soil characteristics using shear wave velocity measurements in large sand specimens.

In the previous sections, the mechanism of shearing soil particles by means of bender elements was described. The shear wave is generated at the tip of the source bender element, travels through the soil particles and hits the receiver bender element. Travel time of the wave is evaluated in the oscilloscope record and shear wave velocity can be calculated from the following equation which is first introduced by Dyvik and Madshus (1985):

\[ V_s = \frac{L_{tt}}{\Delta t} \]  

\( L_{tt} \) is the tip to tip distance also said effective length  
\( \Delta t \) is the travel time of the wave.

5.3.1 Manufacturing and Design of Bender Element Measurement Setup

Designing and manufacturing a bender element setup which will be applicable for large sand specimens was a major challenge. Building a complete working bender element set up requires various design steps. An intensive literature study was performed and some of the literature work was summarized in Table 5-1. After evaluating different researcher's techniques, the most efficient bender element setup was designed to be used in large sands specimen setups, here in CSSLB.
Preparing a complete bender element setup includes proper wiring, waterproofing for testing in saturated soils, grounding and fixing in a proper mounting. Please refer to Deniz 2008 for more details.

1. **Electrical Connections**: Wiring is a very critical task since the bender elements have two electrodes on the surfaces which are exposed to air. The right cable would be the one which would eliminate the interaction between the electrical field generated and ambient noise. A coaxial shielded cable which eliminates ambient noise was found as a suitable one. It is "Xtra-Guard Shielded Continuous Flex Data Cable Multiconductor" manufactured by Alpha Wire Company. Due to the ease of wiring, x-pole bender elements were used generally and connected in series. As described in the previous sections, parallel connection requires trimming of one of the piezoceramic layers to reach to the metal shim in between. It is a very tedious task and can give permanent damage to the bender element. However, some bender elements were prepared also in parallel connection since it reduces ambient noise better. In series connection, red and black shielded wires were soldered on each surface of the bender element. The other ends of the wires were soldered to a coaxial cable with a BNC connector at the end. Also the cable has a third twisted wire which is used for grounding purposes. Grounding cable was left outside to be connected later to the mount.

2. **Waterproofing**: Since the bender elements have sole naked electrodes on their surfaces, they need to be waterproofed when submerged in saturated soils. This is another important task to resolve since the material to be used may change the characteristics of the bender elements. Several coating materials were explored based on the available options from the literature. For triaxial specimens, Dyvik and Madshus
(1985) and Landon et al. (2007) cased the bender elements with a two component epoxy resin. However a rigid resin reduces the bending motion significantly, yet it requires a case mount to dry for 24 hours. Since they performed their measurements in short specimens, they were still able to catch the wave signals. Then, heat shrinkage tubing was also tried as a waterproofing coat. Bender element was covered with a heat shrinkage tubing and the empty tip was glued with an epoxy. It was an alternative technique however was not the best one. Finally an air drying polyurethane coating was used after contacting Prof. Carlos Santamarina research group at Georgia Institute of Technology. It is an M-Coat A air drying polyurethane coating. When the bender element was coated with the polyurethane, the bender element was flexible enough to use its bending capacity, at the same time rigid enough to resist any external impacts when embedded in the soil.

3. **Mounting Bender Elements:** A special mounting was designed and manufactured to be utilized in large sand specimen setups. The design of the mounting was aimed to be set in CSSLB, however the design can also be adapted to other shaking table setups. The bender elements were placed in a 1.9 cm brass threaded fitting which can be inserted in the CSSLB instrumentation holes. Figure 5-12 demonstrates two sides and front view details of the design of the bender element in the mounting. A special multipiece plexiglass mold was designed and manufactured to center and align the bender element properly in the brass fitting as shown in Figure 5-13. Inside of the fitting with the bender element in center was filled with Devcon 5 minute epoxy.
Figure 5-12 Sketch of the bender element in the brass fitting

Figure 5-13 Mold for mounting the bender element in the brass fitting
4. **Grounding for Electromagnetic Coupling:** In water or saturated soil specimens, if the same signal sent to the source is received at the receiver at 0 time in milivolts, there is an electromagnetic coupling in the system. This phenomenon was observed during the tests and the signal due to electromagnetic coupling was interfering with the arrival of the shear wave signal at the receiver. This fact is defined as crosstalk and very well explained by Lee and Santamarina (2005). Crosstalk is an output of an electromagnetic field generated in the source bender element due to the alternating electrical field. The electromagnetic field is transmitted to the receiver bender element and generates an alternating electrical field on the receiver. Since it is a very rapid transmission, it arrives before the shear wave signal and it interferes the first arrival. Proper grounding is needed to overcome this problem. Lee and Santamarina proposed the use of silver conductive paint but they didn't specify how. After contacting Professor DeGroot and former Ph.D candidate Melissa Landon at UMASS, a grounding procedure was developed for this research setup. Silver conductive paint was applied all round the bender element as a thin strip and was touched to the shielded cable whose other end is connected to the ground of the oscilloscope. Lee and Santamarina also suggest using parallel connected bender elements to solve crosstalk. They state that when the outer electrodes are connected to the ground and the inner metal shim to the (+) pole, the bender elements will be self grounded. That didn’t work in this research and still low amplitude crosstalk was observed. This difference may be attributed to using high voltage amplitudes in order to measure shear wave velocities in large sand specimens. They only used ±2.5 V in the source whereas ±180V was used in this setup. Therefore
using parallel connected bender elements will not be a solution for grounding in large scale setups.

Figure 5-14 demonstrates the final bender element setup which can be placed in CSSLB or in any other shaking table setup which has appropriate instrumentation holes. An array of 4 bender elements were placed in the bottom and an array of another 4 bender elements were placed in the middle of the CSSLB. Figure 5-15 shows a photo of the overall shear wave measurement setup in CSSLB.

Figure 5-14 Final bender element setup for S-Wave measurements in large sand specimens
5.3.2 **S Wave Measurement Results and Signal Interpretation**

Shear wave measurements were performed in fully and partially saturated sand specimens prepared in CSSLB, at lower and middle depths. The top view of the bender element array for each level is demonstrated in Figure 5-16. With this configuration, S wave measurements were performed between 2 tip to tip aligned bender elements at short and long distances (15.5 and 27 cm). Also, the effect of the wave path on wave velocity measurements was evaluated at the bender elements placed diagonally. Numbers from 1 to 4 are the channel numbers and will be referred throughout the text in the measurement outputs.
There are several factors affecting S wave measurements and signal interpretation. In the literature, researchers addressed these issues and some of them were already summarized in Table 5-1. However more studies will be mentioned of as specific issues are addressed here.

**Coupling Effect:** When bender elements are embedded in the soil, very good coupling between the soil and bender element need to be provided so that the soil particles can experience enough disturbance by the bending action. If the coupling is weak, the wave amplitude will be weak also and the wave may die out before reaching the receiver. This issue is more critical in large sand specimens rather than in the triaxial specimens. Because triaxial specimens are usually consolidated under high confining pressures, thus coupling is strong. During the specimen preparation in CSSLB, coupling between the bender element and soil was more provided by tapping the soil around the bender slightly.
**Frequency of the Source Signal:** The frequency of the source signal plays a significant role in two important issues. These are the effect of resonant frequency and the occurrence of near field effect. According to theory of dynamics, if the motion frequency is equal to the resonant frequency of the system, the response motion of the system is the maximum. Therefore when the frequency of the signal sent is equal to the resonant frequency of the bender element system, high amplitude waves are created. This is considerably important for especially large sand specimens since if the frequency is sent at resonance frequency, the amplitude of the wave will be highest and the measurements can be made at the largest possible distances. Resonant frequency of bender elements mounted in the brass fitting can be determined by creating free vibration in the air. In this research, the resonant frequency of the bender element mounted in a brass fitting was evaluated as 800 Hz in the air. However the resonant frequency of the system (bender element and the soil around it) is different than the resonant frequency of a bender element in the air. This issue was addressed and demonstrated theoretically by Lee and Santamarina (2005). Depending on the soil characteristics affecting the soil stiffness around the bender element (e.g. density, effective stress), resonant frequencies of the system vary. In this research, resonant frequencies were determined in a range of 300-600 Hz. It was determined by sending different frequencies (close to the resonant frequency in the air) and observing the highest amplitude signals at the receiver bender element.

Near field effect is recognized as an early component wave arriving before the shear wave and was addressed by many researchers (Sanchez-Salinero et al. (1986), Brignoli et al. (1996), Arulnathan et al. (1998), Leong et al. (2005)). It is a critical effect
since it creates ambiguity in determination of wave arrival time. Sanchez-Salinero et al. (1986) stated that near field effect is generated at the shear wave source and deteriorates with increasing number of wavelengths. They demonstrated that near field effect is more introduced when \(\frac{L_{in}}{\lambda}\) is less than 1, where \(\lambda\) is the wavelength. Arulnathan et al. (1998) associated the occurrence of the near field effect to the 3-D wave propagation in a restrained medium. They claimed that the source bender element generates actually a body wave in 3-D space. Since the wave propagation doesn't occur in an unbounded 3-D medium, before the direct S wave arrives, other components of the body wave (it can be also P waves) generated at the source travels through the restrained boundaries of the test setup (the box wall or triaxial apparatus cell) and hits the receiver bender element in a distorted manner.

Near field effects were also observed in this study (Figure 5-17). The results confirmed findings by Sanchez-Salinero et al. (1986) and near field effect diminished with decreasing wavelength or increasing frequency however it still existed. Therefore, findings by Arulnathan et al. also may explain the existing near field effects in the measurements.

**Determination of Wave Travel Time (\(\Delta t\))**: Different methods were proposed by researchers for the determination of the wave travel time. These are first arrival, cross correlation, first peak etc. First arrival was found to be the most convenient and easiest tool in this research. However the near field effect needs to be clarified for the determination of the first arrival. Besides visual inspection of the effect, a procedure was developed to verify the first arrival of the shear wave. Before setting up the bender elements in the CSSLB, bender elements were placed tip to tip in contact. The source
bender element was excited and the response at the receiver was observed. Then bender elements (facing each other) were placed and oriented in the box such that when the first peak of the source signal was positive voltage, the first peak of the received signal was also positive. That will help validate the first arrived signal by checking the positive peak. The time lag between the start of the source signal and the start of the positive voltage signal was determined to be the arrival time of the shear wave ($\Delta t$). The tip to tip method was also used to evaluate if there is any phase lag between the applied voltage and the received voltage in the equipment. No phase lag was observed in the measurement equipment. Arulnathan et al. (1998) and Landon (2007) reported that they experienced a phase lag in the shear wave measurement tests.

**Test Setup Wall Effect:** Bender elements can also create vibrations at the box walls where the brass fittings are fixed, if it is not isolated with any energy absorbing material. As already stated in Chapter 4, CSSLB has sikaflex (flexible material which would absorb vibrations) material between the walls. However the CSSLB also has a metal hinge in the bottom connecting the moving walls to the base. Measurements made in empty box showed waves traveling through the walls at the receiver bender elements. The frequency of the wave through the box was detected as relatively high. During the shear wave measurements in the soil, those high frequency waves disappeared. The reason may be attributed to deamplification of high frequency components at the receiver bender element system with low resonant frequency.

**Travel Distance:** The bender element setup established is capable of making measurement of shear wave velocities at short as well as long distances. Figure 5-17 demonstrates shear wave measurements when channel 1 is the source and channels 2, 3
and 4 are the receivers. At the front receiver (channel 3) the shear wave was received after 2.92 ms and the velocity was evaluated as 52.5 m/s. In the measurement shown in Figure 5-18, the signal was sent at channel 2 and channels 1, 3 and 4 were the receivers. The receiver at the channel 4 was at a longitudinal distance of 27 cm and the shear wave was caught after 5.47 ms, which corresponds to 50 m/s of shear wave velocity. Therefore, velocity measurements at short and long distances confirm each other.

Another observation is that depending on the setup boundaries, near field effect varies at long and short distances. Since the setup geometry is relatively narrower along the long distance wave path, an enhanced near field effect due to restricted boundaries was observed, confirming what Arulnathan et al. (1998) reported. Shear wave velocities evaluated at the diagonal bender elements were also the same as the velocities obtained from the front bender elements. The fact of getting reasonable measurements at different bender element orientations enables the assessment of soil uniformity and localized soil characteristics in large sand specimens.

**Noise Control:** Noise problem is addressed also by some other researchers (Leong et al. and Agarwal and Ishibashi (1991)). The use of a data acquisition module with very low noise level diminished noises in the signals. Therefore data filtering wasn't required. Also the use of a shielded cable reduced the ambient noise as stated at the bender element preparation section. However, clearer signals were obtained by using the signal averaging function of DL750 digital oscilloscope.
Figure 5-17 Typical shear wave measurement output (Short distance, $L_{tt}=15.5$ cm)

Figure 5-18 Typical shear wave measurement output (Long distance, $L_{tt}=27$ cm)
### 5.3.3 Interpretation of Localized Soil Characteristics by Using Multiple Shear Wave Measurements in Large Sand Specimens

The new bender element measurement setup established at Northeastern University offers evaluation of localized soil stiffness as well as assessment of soil uniformity in large sand specimens. Also, the effect of some important soil parameters (effective stress, relative density and degree of saturation) on the shear wave velocity measurements were investigated and compared with the theory.

An array of eight bender elements, four at the upper and other four at the lower level, were placed in a fully saturated sand specimen prepared in the CSSLB. Figure 5-15 shows the experimental setup. The elevations of the bender elements at the upper and lower levels, soil depth above them and initial effective stresses are given in Table 5-2.

#### Table 5-2 Details of Upper and Lower Bender Element Measurement Tests

<table>
<thead>
<tr>
<th>Elevaion m</th>
<th>Depth, m</th>
<th>$\sigma_{v0}$, kPa</th>
<th>$\sigma_{v0}'$, kPa</th>
<th>$\sigma_{v1}'$, kPa $1^{st}$ Set of Weight</th>
<th>$\sigma_{v2}'$, kPa $2^{nd}$ Set of Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>BE Lower</td>
<td>0.075</td>
<td>0.33</td>
<td>6.5</td>
<td>3.0</td>
<td>10.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>17.4</td>
</tr>
<tr>
<td>BE Upper</td>
<td>0.185</td>
<td>0.22</td>
<td>4.4</td>
<td>2.0</td>
<td>9.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16.4</td>
</tr>
</tbody>
</table>

#### Sand Specimen Uniformity Assessment:

In theory, soil stiffness is known to be related to the relative density of the soil and effective stresses. Therefore, the homogeneity of the soil stiffness or density can be checked by measurements taken at different locations of the same specimen using multiple bender elements. This can be a useful tool to verify the soil uniformity of large specimens prepared in the laboratory.
First, measurements were performed at the same levels of the soil specimen (upper or lower) prepared by wet pluviation technique. The configuration shown in Figure 5-16 demonstrates the wave paths considered for the uniformity assessment. Shear wave velocities measured at the receivers at diagonal, short and long distances from the source were all the same, confirming the planar uniformity of the soil stiffness at different depths. Figure 5-17 and Figure 5-18 illustrate the measurements at the lower level of the specimen through the short distance (15.5 cm) and the long distance (27 cm) wave paths. The shear wave velocities at the short (SD) and long distance (LD) wave paths were calculated as follows:

\[ V_s = \frac{L_n}{\Delta t} \]

\[(V_s)_{SD} = 15.5 \text{ cm} / 2.95 \text{ ms} = 52.5 \text{ m/s} \]

\[(V_s)_{LD} = 27 \text{ cm} / 5.46 \text{ ms} = 50 \text{ m/s} \]

Similar measurements were performed at the upper level bender elements and compared with the lower level measurements to assess soil uniformity throughout the depth. Figure 5-19 compares measurements form upper and lower level bender elements aligned at the short path. Shear wave velocity at the upper level was achieved slightly lower than the velocity at the lower level. The difference may be attributed to lower effective stresses at the upper level. Theoretically it is known and will be reported in details later in this section that:

\[
\frac{(V_s)_1}{(V_s)_2} = \sqrt{\frac{\sigma_{v1}}{\sigma_{v2}}} \]

5-3
Figure 5-19 S wave measurements to evaluate the uniformity of the specimen through the depth.
Therefore shear wave velocity at the upper level can be adjusted for the difference of the effective stress as follows:

\[
(V_s)_1 = 52.5 \text{ m/s at } \sigma_{v1}' = 3 \text{ kPa}
\]

\[
(V_s)_2 \text{ at } \sigma_{v2}' = 2 \text{ kPa}:
\]

\[
(V_s)_2 = (V_s)_1 \sqrt{\frac{\sigma_{v2}'}{\sigma_{v1}'}}
\]

\[
(V_s)_2 = 52.5 \sqrt{\frac{2}{3}}
\]

\[
(V_s)_2 = 47.5 \text{ m/s}
\]

Adjusted velocity for the upper level is very close to the measured value (48.1 m/s). To conclude, the soil specimen uniformity was confirmed in plane at different depths as well as throughout the depth.

**Shear Wave Measurements at Different Relative Densities**

Measurements were performed also at loose and dense sand specimens prepared in CSSLB and compared with each other based on the theoretical correlation. As shown in Figure 5-20, shear wave velocities were achieved as 52.5 m/s and 70 m/s at relative densities 21% and 70%, respectively. The relationship between the shear wave velocities measured at the loose and dense specimens was confirmed by the empirical shear modulus formula by Seed and Idriss (1970):

Empirical Shear Modulus Formula by Seed and Idriss (1970):

\[
G_{\max} = 1000(K_2)_{\max} \sqrt{\sigma_m'}
\]

\[
\sigma_m' \text{ is the mean effective stress in lb/ft}^2 \text{ and it is the same for both conditions}
\]

\[
\frac{(G_{\max})_2}{(G_{\max})_1} = \left[\frac{(K_2)_{\max}_2}{(K_2)_{\max}_1}\right]^{1/2}
\]

\[
(K_2)_{\max} = 20 \sqrt{(N_1)_{60}}
\]
According to Das (2006):

\((N_1)_{60}\) for \((D_r) = 21\%) = 6-8

\((N_1)_{60}\) for \((D_r) = 70\%) = 36

Then,

\[
\left[\frac{(K_2)_{\text{max}}}{(K_1)_{\text{max}}}\right]_2 = \sqrt{\frac{36}{7}} = \frac{(G_{\text{max}})_2}{(G_{\text{max}})_1}
\]

\[
\frac{(V_s)_2}{(V_s)_1} = \sqrt{\frac{(G_{\text{max}})_2}{(G_{\text{max}})_1} \cdot \frac{\rho_1}{\rho_2}} = 1.28
\]

5-4

where \(\rho_1 = 1.94\) g/cm\(^3\) and \(\rho_2 = 2.03\) g/cm\(^3\)

Therefore, the shear wave velocity measured in the dense specimen \((D_r = 70\%)\) should be 1.28 times of the shear wave velocity measured in the loose specimen \((D_r = 21\%)\). The shear wave velocities in the dense soil were calculated by multiplying shear wave velocities measured in the loose sand by the factor of 1.28 found in Equation 5-4 and tabulated in Table 5-3. The measured velocities at the dense material were close to the calculated ones. Therefore once the relative density of a specimen is known, any changes in the relative density of the soil can be correlated to the shear wave velocity measurements in the specimen.

### Table 5-3 Comparison of Shear Wave Velocity Measurements at Loose and Dense Soils

<table>
<thead>
<tr>
<th></th>
<th>(\sigma_{v0}',) kPa</th>
<th>(V_s) measured at (D_r = 21%), m/s</th>
<th>(V_s) measured at (D_r = 70%), m/s</th>
<th>(V_s) at (D_r = 70%) from Eqn 5-4, m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>BE Lower, SD</td>
<td>3.0</td>
<td>52.5</td>
<td>70</td>
<td>67.31</td>
</tr>
<tr>
<td>BE Lower, LD</td>
<td>3.0</td>
<td>51</td>
<td>69</td>
<td>65.38</td>
</tr>
</tbody>
</table>
Figure 5-20 Shear wave measurements in loose and dense sand specimens

Source: CH 1 Receiver: CH 3
\[ \sigma_v' = 3 \text{ kPa} \quad D_r = 21\% \quad \Delta t = 2.95 \text{ ms} \quad V_s = 52.5 \text{ m/s} \]

Source: CH 3 Receiver: CH 1
\[ \sigma_v' = 3 \text{ kPa} \quad D_r = 70\% \quad \Delta t = 2.22 \text{ ms} \quad V_s = 70 \text{ m/s} \]
Also, one can be revealed from the results is that coupling is much better in dense specimens and the intensity of the received wave was higher (Figure 5-20). Another confirmation from this test is about the resonant frequency of the system. As stated earlier, the resonant frequency depends on the stiffness so does the relative density. At a denser soil, a higher source frequency was used for the best signal since the resonant frequency of the system is now higher due to stiffer soil conditions.

Finally, the results led to the conclusion that shear wave velocity measurements with multiple bender elements can be performed accurately in large loose to dense sand specimens and the variation in relative densities of the specimen can be determined using the theoretical relationship between the shear wave velocities at those densities.

**Shear Wave Measurements at Different Effective Stresses**

Measurements were also performed at different effective stresses. Initial effective stresses were 2 kPa and 3 kPa, at upper and lower levels of the specimen, respectively. Then the specimen was loaded with lead weights to increase the effective stresses. Due to restrained boundaries of the experimental setup (CSSLB), the distribution of the load through the soil was assumed to be 1:1. The shear wave velocity measurements confirmed the assumption.

Table 5-4 demonstrates the shear wave velocity measurements at upper and lower levels of the specimen at original stresses \((\sigma'_{i}=2 \text{ and } 3 \text{ kPa})\), then at stresses with 1\(^{st}\) set of load \((\sigma'_{i}=9.6 \text{ and } 10.6 \text{ kPa})\) and finally at stresses with 2\(^{nd}\) set of load \((\sigma'_{i}=17.4 \text{ kPa})\). The wave measurements at different effective stresses were reasonable and verified by empirical shear modulus formula by Seed and Idriss (1970):
Empirical Shear Modulus Formula by Seed and Idriss (1970):

\[ G_{\text{max}} = 1000(K_2)_{\text{max}} \sqrt{\sigma_m^{'}} \]

\[ \sigma_m^{' } = \frac{1}{3} (2\sigma_H^{' } + \sigma_V^{' }) \]

\[ = \frac{1}{3} (2K_0 \sigma_V^{' } + \sigma_V^{' }) = \frac{(1+2K_0)}{3} \sigma_V^{' } \]

for \( K_0 = 0.3 \)

\[ \sigma_m^{' } = 0.53 \sigma_V^{' } \]

\( (K_2)_{\text{max}} = 34 - 40 \) for loose sands

For the same void ratio \( K_{2\text{max}} \) will be the same so;

\[ \frac{(G_{\text{max}})_2}{(G_{\text{max}})_1} = \sqrt{\frac{0.53\sigma_{v2}^{'}}{0.53\sigma_{v1}^{'}}} = \frac{(V_s)_2^2}{(V_s)_1^2} \]

\[ (V_s)_2 = (V_s)_1 \sqrt{\frac{\sigma_{v2}^{'}}{\sigma_{v1}^{'}}} \]

The ratio of the shear wave velocities at higher level of effective stresses to the shear wave velocities at initial effective stresses were calculated using equation 5-4 and compared with the measured velocities. The results are tabulated in Table 5-4. The calculation can be shown in the following example:

\[ (V_s)_1 = 52.5 \text{ m/s at } \sigma_{v1}^{' } = 3 \text{ kPa} \]

\[ (V_s)_2 \text{ at } \sigma_{v2}^{' } = 10.6 \text{ kPa:} \]

\[ (V_s)_2 = (V_s)_1 \sqrt{\frac{\sigma_{v2}^{'}}{\sigma_{v1}^{'}}} \]

\[ (V_s)_2 = 52.5 \sqrt{\frac{10.6}{3}} \]

\[ (V_s)_2 = 71.9 \text{ m/s and } (V_s)_2 \text{ measured is } 70 \text{ m/s} \]
Measured and calculated velocities at higher effective stresses were very close. Also, shear wave velocity measurement outputs at 3 kPa and 17.4 kPa are shown in Figure 5-21. Again it can be noted that at higher effective stress, resonant frequency was higher since the soil stiffness is higher.

Test results indicate that the relationship between shear wave velocities measured at different effective stress levels is in good agreement with the theoretical relationship. Therefore shear wave measurements with multiple bender elements can be also a useful tool to interpret effective stresses in large sand specimens.

<table>
<thead>
<tr>
<th>Table 5-4 Shear Wave Velocities at Different Effective Stresses</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Weight</td>
</tr>
<tr>
<td>BE Lower, SD</td>
</tr>
<tr>
<td>BE Lower, LD</td>
</tr>
<tr>
<td>BE Upper, SD</td>
</tr>
<tr>
<td>BE Upper, LD</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1st Set of Weight</th>
<th>$\sigma_v', \text{ kPa}$</th>
<th>$V_s$ measured, m/s</th>
<th>$V_s$ (m/s) evaluated by $(\sigma_v'/\sigma_v)^{(1/4)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BE Lower, SD</td>
<td>10.6</td>
<td>70.5</td>
<td>71.9</td>
</tr>
<tr>
<td>BE Lower, LD</td>
<td>10.6</td>
<td>65.1</td>
<td>69.9</td>
</tr>
<tr>
<td>BE Upper, SD</td>
<td>9.6</td>
<td>68.9</td>
<td>71.2</td>
</tr>
<tr>
<td>BE Upper, LD</td>
<td>9.6</td>
<td>70.1</td>
<td>68.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2nd Set of Weight</th>
<th>$\sigma_v', \text{ kPa}$</th>
<th>$V_s$ measured, m/s</th>
<th>$V_s$ (m/s) evaluated by $(\sigma_v'/\sigma_v)^{(1/4)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BE Lower, SD</td>
<td>17.4</td>
<td>77.5</td>
<td>79.8</td>
</tr>
<tr>
<td>BE Lower, LD</td>
<td>17.4</td>
<td>74</td>
<td>73.6</td>
</tr>
</tbody>
</table>
Figure 5-21 Shear wave measurements in sand specimens under different effective stresses

- **f=300 Hz**
  - Voltage=160 V
  - $\sigma_v'=3 \, \text{kPa}$, $D_r=21\%$, $\Delta t=2.95 \, \text{ms}$, $V_s=52.5 \, \text{m/s}$

- **f=600 Hz**
  - Voltage=160 V
  - $\sigma_v'=17.4 \, \text{kPa}$, $D_r=21\%$, $\Delta t=2.0 \, \text{ms}$, $V_s=77.5 \, \text{m/s}$
Shear wave propagates in soils by displacing soil particles perpendicular to the propagation path, which is basically by shearing the soil. Also, it is to be noted that since water can not be sheared, pore water doesn't have any effect on shear wave velocity. Therefore, shear wave velocity or shear modulus (G) of soils is only related to soil structure, density and effective stresses present in the soil. Since the degree of saturation is a pore parameter and shear wave only travels through the soil skeleton, shear wave velocity is expected to be the same in fully and partially saturated specimens at the same density and effective stress conditions.

Shear wave velocity measurements can be also a useful tool to check whether or not IPS alters relative density or more importantly effective stresses in the sand. Because IPS only aims to reduce the pore water in the voids by generating gas bubbles in the voids without changing the structure or effective stresses in the soil.

Figure 5-22 illustrates the shear wave measurements in fully and partially saturated sand specimens with S=77%. As can be seen from the outputs, the shear wave velocities under the same density and effective stress conditions were very close, confirming that the new IPS technique implemented in the lab doesn't change any other soil characteristics than the degree of saturation and degree of saturation has no effect on the shear wave velocity.

Also, the uniformity of the sand specimen prepared by the new IPS technique can be assessed by performing multiple S wave measurements at several bender element channels of CSSLB. S wave measurements were performed in a partially saturated sand specimen with S=77%, at the upper and lower level bender elements to assess soil
f=500 Hz
Voltage=160 V

Source: CH 1 Receiver: CH 3
$\sigma_{v}' = 9.6$ kPa, $D_r = 21\%$, $S = 100\%$  $\Delta t = 2.22$ ms $V_s = 69.8$ m/s

f=700 Hz
Voltage=160 V

Source: CH 1 Receiver: CH 3
$\sigma_{v}' = 9.6$ kPa, $D_r = 18\%$, $S = 77\%$  $\Delta t = 2.32$ ms $V_s = 66.8$ m/s

Figure 5-22 Shear wave measurements in fully and partially saturated sand specimens
uniformity throughout the depth. Figure 5-23 compares measurements from upper and lower level bender elements aligned at the short path. Shear wave velocity at the upper level \( (V_s = 32 \text{ m/s}) \) was again achieved slightly lower than the velocity at the lower level \( (V_s = 41 \text{ m/s}) \). As stated earlier, the difference may be related to lower effective stresses at the upper level. When the upper level shear wave velocity was adjusted for the effective stress of 3 kPa using the theoretical relationship between shear wave velocity and effective stress, the adjusted shear wave velocity was obtained as 35 m/s which is close to the lower level shear wave velocity \( (V_s = 41 \text{ m/s}) \). Therefore, partially saturated sand specimens with uniform relative densities can be prepared with the new IPS technique.

In conclusion, the results demonstrated that the new multiple bender element measurement system established at Northeastern University works successfully under different soil conditions, and measurements are in good agreement with the theory. It also provides identification of localized soil characteristics in large sand specimens. Multiple measurements of S waves in several locations of sand specimens prepared by IPS confirmed that the new IPS technique enables preparing uniform partially saturated sand specimens.
Lower Level Measurements in the CSSLB. Source: CH 1, Receiver :CH 3
\[ \sigma_v' = 3 \text{ kPa}, \ D_r = 21\%, \ S = 77\% \ \Delta t = 3.7 \text{ ms} \ V_s = 41 \text{ m/s} \]

Upper Level Measurements in the CSSLB. Source: CH 1, Receiver :CH 3
\[ \sigma_v' = 2 \text{ kPa}, \ D_r = 21\%, \ S = 77\% \ \Delta t = 4.8 \text{ ms} \ V_s = 32 \text{ m/s} \]

Figure 5-23 Shear wave measurements in a partially saturated sand specimen at S=77% for assessment of soil uniformity
5.4 Compressional (P) Wave Measurements in Large Sand Specimens

This section presents the new multiple bending disk wave measurement setup established at Northeastern University to be applicable in large sand specimens especially in shaking table specimens. A summary of the preparation and manufacturing of the bending disks is reported. The section follows with interpretation of the P wave velocity measurements and its correlation to degree of saturation.

P wave velocities were evaluated by the following equation:

\[ V_p = \frac{L}{\Delta t} \quad 5-6 \]

- \( L \) is the distance between the bending disks
- \( \Delta t \) is the travel time of the wave.

5.4.1 Manufacturing and Design of Bending Disks Measurement Setup

The design of the bending disk setup was more challenging than the bender element setup due to its size, high frequency and high amplitude signal requirements. In terms of wiring, waterproofing and grounding, the procedure and materials applied were the same as in bender elements. For mounting, a special housing was designed. As shown in Figure 5-24, a bending disk wired, waterproofed and grounded with silver conductive paint was laterally placed on top of the PVC housing and silicon was applied in between. Silicon is an easy to cure material and also provides flexible base which doesn't restrict the bowing action of the bending disk. Holes were drilled around the PVC housing so that soil can get inside and provide equal support from the back of the disk.

The most important issues to be considered for the best received P wave signals are the frequency and shape of the signal, and the noise level. Higher frequencies (5-15
kHz) worked better in detecting P wave velocities. The type of signal was again determined to be sinusoidal single pulse since other signals (especially square) which create rapid change of the electrical charges damage the disk. Owing to fast damping of the P waves, received P wave amplitudes in large distances are much lower than the S waves. Therefore the noise level should be minimized especially during P wave measurements. Providing secure grounding and using averaging tool of the DL750 oscilloscope helped minimizing the noise level better.

![Figure 5-24 Bending disk final setup](image)

Unlike in bender elements, the arrival time is considered to be the first disturbance observed in the signal. In fully saturated specimens, the first disturbance in the signal was clear to detect however as the degree of saturation reduces, it got harder to detect the P wave. Figure 5-25 demonstrates the P waves measured at the bending disk at 23cm distance from the source. P wave velocity was found to be 1755 m/s which is bigger then P wave velocity in water (1492 m/s). Ishihara (1971) reported similar issues in P wave measurements and the reason was associated to the fact that soil stiffness also comes into effect in restrained setups resulting 10-15% higher wave velocities. In this research, P wave velocities in fully saturated specimens were measured in a range of
1450-1755 m/s. Figure 5-26 demonstrates the P wave measurements in partially saturated sand with S=82%. The P wave arrival is better detected in a zoomed section of the signal. Again 8 bending disks were oriented in the CSSLB as in the case of bender elements (Figure 5-16) and P wave velocity data were collected at different degrees of saturation.

Source: CH 2 Receiver: CH 4
S=100% Δt= 130.6 μs L=23 cm V_p= 1755 m/s

Figure 5-25 P wave velocity measurements in fully saturated sand specimens
Figure 5-26 P wave measurements in a partially saturated sand specimen in (a) 10ms time window and (b) 1 ms time window (S=82\%)

Source: CH 2 Receiver: CH 4

$S=82\% \quad \Delta t=453.6 \mu s \quad L=23 \text{ cm} \quad V_p=507 \text{ m/s}$

(a)

Source: CH 2 Receiver: CH 4

$S=82\% \quad \Delta t=453.6 \mu s \quad L=23 \text{ cm} \quad V_p=507 \text{ m/s}$

(b)
5.4.2 Interpretation and Evaluation of IPS by Using P-Wave Velocity Measurements

As stated earlier, one of the primary goals of this research task was to investigate whether or not IPS technique can be quantified by P wave velocity measurements and consequently if the liquefaction benefit of IPS mitigation technique can be related to P wave velocities. Then, this would be a useful in-situ tool to determine the state of saturation during the implementation of IPS technique in the field and eventually to evaluate the in-situ liquefaction benefit. This relation was aimed to be established in the laboratory through P wave measurements at different degrees of saturation levels achieved by IPS. Another goal of this task was to investigate the uniformity of the gas bubbles through the specimens prepared by IPS with multiple P wave measurements at different channels in CSSLB.

In the literature, Yang (2002) and Tamura et al. (2002), Ishihara et al. (2004) presented their studies on the effect of air bubbles on P-wave velocity through triaxial laboratory tests and they developed various theoretical equations which evaluate P wave velocity for different pore pressure parameter B, which is a measure of degree of saturation in triaxial tests. Yang (2002) and Ishihara et al. (2004) did further research on the correlation and developed a relationship between the liquefaction resistance of sands and P wave velocity which is a function of B value. According to the results of the study by Yang (2002) shown in Figure 5-27, P wave velocity reduces down to 400 m/s when the degree of saturation is as low as 98-99%. Then using the liquefaction resistance equation based on pore pressure parameter B developed by Chaney (1978), Yang (2002) derived a relationship between liquefaction resistance and P wave velocity in terms of number of cycles to liquefaction for fully and partially saturated sands, which is shown in
Figure 5-27. Ishihara et al. (2004) developed a similar correlation and reported the liquefaction resistance of partially saturated sands in relation to P wave velocities in terms of cyclic resistance ratios ($\text{CRR}_{PS}/\text{CRR}_{PS}$) as shown in Figure 5-28. In both studies, P wave velocity approaches to a constant value which is close to the P wave velocity of dry sand when $S<98\%$. Thus, when a correlation is attempted to be established between liquefaction resistance and P wave velocities, partially saturated sands with $S<98\%$ which have the same P wave velocities of 500-400 m/s will all reflect the same resistance to liquefaction. However, the response of a partially saturated sands to liquefaction is different and the benefit from partial saturation increases as the degree of saturation decreases. Therefore, these liquefaction resistance relationships can only be applied to partially saturated sands with $S$ between $99.9-98\%$. P wave velocity does not provide any information in terms of liquefaction resistance below $S=98\%$, since P wave velocity is the same.

P wave velocities measured in partially saturated sand specimens prepared by IPS at different degrees of saturations are displayed in Figure 5-29. Unlike the results by Ishihara et al. (2004) and Yang (2002), P wave velocities did not show a dramatic drop at $S=98\%$, instead, P wave velocities reduced down to 500 m/s after $S=90-95\%$. The difference may be attributed to the type of experimental setup used. Their results are based on triaxial small specimens whereas this research is based on large-scale sand specimens. Furthermore, the techniques applied to create partially saturated specimens are not the same. Ishihara et al. (2004) reported that they used tap water and low back pressure to achieve partially saturated specimens which doesn't guarantee uniform
Figure 5-27 (a) Variation of B value with degree of saturation, (b) Relationship between P wave velocity and B value for Toyoura sand c) Normalized number of cycles to liquefaction as a function of P wave velocity. (Dr=60%) (Yang 2002)
distribution of gases. As reported in Chapter 4, IPS holds promise in creating uniformly distributed gas bubbles throughout the sand specimens based on visual inspection.

Besides visual inspection, this research task aimed to explore if the uniformity of the gas bubbles in the specimens prepared by the new IPS technique can be identified by the P wave measurements performed at different channels of the CSSLB. Figure 5-29 also demonstrates the P wave velocities measured at different channels for the same specimens. Although the results demonstrate similar measurement data at different channels, since P wave velocity is not an indicator of degree of saturation below 90-95%, such a correlation can not be drawn also about the uniformity of gas bubbles at those saturation levels.
Finally, P wave measurements in large scale specimens prepared at different degrees of saturation by IPS reveal that P wave velocity can not be recognized as a measure or indicator of degree of saturation below S=90-95%. Therefore, IPS in the field can not be quantified by P wave velocity tests and P wave velocities can not be correlated to liquefaction benefit of IPS mitigation technique.

Figure 5-29 P wave velocities measured in specimens prepared by IPS with different degrees of saturations
5.5 Summary of Results

A new shear (S) and compressional (P) wave measurement system with multiple bender elements and bending disks was developed for large sand specimens. Bender elements and bending disks are made of piezoelectric ceramics and create small displacements which generate shear and compressional waves in the soils. Complete bender element and bending disk designs suitable for large experimental setups were manufactured and prepared through various steps and all the challenges through the manufacturing processes were reported. A special set of equipment for wave measurements including a function generator, power amplifier and a digital oscilloscope was established.

Shear wave measurements were performed at an array of bender elements located at several locations of the large sand specimens prepared in CSSLB. The best wave signals were achieved after investigating the effects of source signal type, frequency and amplitude, wave travel path, the interaction between the bender element, soil and test setup, travel distance, and noise control on the interpretation of the wave signals. Localized soil characteristics and uniformity of the soil specimens, fully saturated or partially saturated prepared by IPS, were all evaluated and presented using S wave measurements obtained from multiple bender elements. Also, functionality of the shear wave measurement system was confirmed under different relative densities and effective stresses.

A P wave measurement system was developed primarily to investigate if P wave velocities can be employed to evaluate IPS and also the liquefaction benefit by IPS. The measurements were performed using bending disks located again at several locations in
CSSLB. Results from P wave measurements in fully and partially saturated specimens prepared by IPS demonstrated that P wave velocity cannot be recognized as a measure or indicator of degree of saturation below S=90-95%. Therefore, IPS in the field cannot be quantified by P wave velocity tests and P wave velocities cannot be correlated to the liquefaction benefit of IPS mitigation technique.

Finally, a working multi-channel bender element and bending disk measurement system was established for testing large sand specimens in a shaking table setup CSSLB. The system holds great promise for evaluating soil characteristics of large laboratory soil specimens practically and in a non-destructive way.
6. Cyclic Simple Shear Strain Tests on Fully and Partially Saturated Sands

6.1 Overview

This chapter presents a series of cyclic simple shear strain tests conducted on fully and partially saturated sand specimens. The primary goal of these tests was to investigate the benefit of Induced Partial Saturation (IPS) on excess pore water pressure generations in sand specimens. Subsequently, based on the test results, an empirical model was developed, which predicts excess pore water pressure ratios in partially saturated sand specimens, which is described in Chapter 7.

A large-scale integrated experimental setup was designed and presented in Chapter 4. The setup includes the liquefaction box (CSSLB) which can induce uniform simple shear strains when fixed on a shaking table, and special instrumentations (PPTs and LVDTs, bender elements and bending disks).

Initially, cyclic simple shear strain tests were performed on fully saturated sand specimens to evaluate the control parameters of the testing program such as relative density, shear strain amplitudes, frequency of the motion, as well as to get comparative results with partially saturated sand tests. Then, a series of tests were performed on partially saturated sand specimens prepared at different degrees of saturation and relative densities. Finally, test results were compared in order to identify fundamental effects of each parameter (degree of saturation, relative density and cyclic strain amplitude) on the excess pore water pressure ratio in partially saturated sands.
6.2 Cyclic Simple Shear Strain Tests on Fully Saturated Sands

This section presents the procedure and results of cyclic simple shear strain tests performed on fully saturated sand specimens. Loose to very dense sand specimens were tested under shear strain levels of 0.01 to 0.2%. A total of 19 tests were performed.

6.2.1 Test Procedure

In order to perform cyclic simple shear tests on fully saturated sands, the test procedure followed is presented below:

1. The CSSLB is aligned on the shaking table and bolted. The external aluminum side bars are fixed to an outsider beam next to the shaking table. As described in Chapter 4, when the shaking table is excited with lateral displacements, due to the flexible Sikaflex material between fixed and rotating walls of the CSSLB and hinges in the bottom of the rotating walls, the CSSLB induces relative displacements between the top and the bottom of the box. Relative displacements are measured with displacement transducers (LVDTs) which are calibrated and fixed one on each of the lateral side bars. The tip of the LVDT spring loaded rod is set on a plexiglass plate glued on the rotating wall. (Refer to Chapter 4 for display pictures). The lateral alignment of the liquefaction box with the shaking table is confirmed by checking the relative displacements from the two side LVDTs. Dividing this relative displacement by the height of the LVDT locations from the base of the box yields the simple shear strains induced.

2. Pore pressure transducers (PPTs) are calibrated and inserted in the CSSSLB through the instrumentation holes on the side walls in the center of the specimen at three different depths. The PPTs are placed at depths higher than 8-10 cm from the bottom since shear strains are uniform above that level according to FLAC analysis of the shear
strain distribution in CSSLB. PPT fittings are wrapped with a white tape in order to eliminate any water leakage. Since pore pressure transducers need to be saturated at all times, predetermined amount of water needed for the expected volume of specimen is already poured in the CSSLB and PPTs are kept in water.

3. Fully saturated sand specimens are prepared in the CSSLB through wet pluviation technique with Ottawa sand (ASTM graded silica sand). The gradation and soil characteristics of Ottawa sand are presented in Figure 4-13 and Table 4-1, respectively. In wet pluviation technique, sand is rained into predetermined amount of water from a distance. With wet pluviation, a relative density of around 20% is achieved as a loosest condition. Denser specimens are obtained by subjecting the specimens to cyclic strains. The amount of dry Ottawa sand needed is weighed and rained into the predetermined amount of water slowly through a large funnel whose height can be adjusted from a top beam of a frame built around the shaking table (Figure 6-1). When the soil height reaches the water level, raining process is stopped. The soil surface is leveled and the height of the specimen is measured to evaluate the soil parameters of void ratio (e) or relative density (D_r) and degree of saturation (S). The soil parameters are calculated using phase relation equations which are described by (Eseller 2004). The soil specimen ready to test is shown in Figure 6-2.

After the specimen was prepared following the procedure stated above, sinusoidal signals generated in LabVIEW were sent to the MTS console which controls the shaking table. It is known in the literature that frequency of the signal has no effect on liquefaction response of soils. For practical purposes and not to cause large dynamic forces on the liquefaction box, first 4 HZ signals were sent. The test results implied that
excess water pressures dissipated before reaching liquefaction through the specimen or through the walls of the box. So, the dissipation occurred before the completion of total excess pore pressure build-up. Hence, undrained conditions were needed to simulate the liquefaction phenomenon of fully saturated sands in a realistic way.

Figure 6-1 Specimen preparation through wet pluviation

Figure 6-2 Typical fully saturated sand specimen tested
During the design of the CSSLB, a plexiglass cover with drainage controls was devised to provide undrained testing conditions. However, it was observed that even tough there was no drainage of excess pressure to the outside of the experimental setup, excess pressures still were redistributed inside the specimen and dissipated from the bottom to the top. Therefore, unlike in small triaxial or simple shear test samples, in large specimens undrained conditions can not be controlled by preventing drainage of the excess pressures to the outside of the test setup.

A solution to the dissipation problem was determined to be the use of higher frequency signals. At the same time, the frequency should not be too high to cause significant dynamic effect on the CSSLB. Optimum frequencies were determined at a range of 10-20 Hz. Higher frequencies were used in denser specimens tested under low amplitude of shear strains since they required more number of cycles.

Small to large amplitude sinusoidal cyclic shear strains were induced on fully saturated sand specimens at relative densities between 30-90%, and excess pore water pressures were measured. Figure 6-4 shows a typical shear strain history applied on the specimens with 0.1% amplitude and 10Hz. Test results and discussions are presented in the next section.

6.2.2 Results and Discussions

Four sets of test were performed on fully saturated sand specimens. In each set, a fully saturated sand specimen was prepared at around 30% relative density and tested under constant cyclic shear strain amplitude. An advantage of the testing program was that the same specimen could be tested under the same shear strain history as it gets denser uniformly after each excitation. Before proceeding to the next test, the soil height
was measured to get the new relative density and sufficient amount of time was allowed to make sure that all of the excess water pressures had dissipated.

Shear strain amplitudes applied in test series 1 to 4 are 0.0055%, 0.01%, 0.05% and 0.1%. Some data is available also from cyclic simple shear tests performed by (Eseller 2004) at 0.2% shear strain amplitude. Figure 6-3 demonstrates a typical LabVIEW software output which displays the PPT measurements at three levels and the LVDT measurements. The output displays the excess pore water pressures measured at three PPTs in a fully saturated sand specimen with relative density of 40% tested under cyclic simple shear strains of 0.1% amplitude. Excess pore water pressure ratio \((r_u = \Delta u/\sigma_{v0}')\) time histories were evaluated from the PPT measurements. Figure 6-5 demonstrates a typical \(r_u\) time history for a fully saturated sand specimen at relative density of 40% tested under shear strain amplitude of 0.1%. Number of cycles required to reach \(r_u\) of 1 \((N_L)\) was also evaluated. The soil data and results of 20 total tests are presented in Table A-1 of Appendix A-1. Following findings and discussions are revealed from the test results:

1. At 0.0055% shear strain amplitude, excess pore water pressure ratio \(r_u\) has not reached 1, indicating no initial liquefaction. This confirms the threshold strain criterion of Dobry et. al. (1982). According to Dobry et al. (1982), there is a threshold strain level below which initial liquefaction never occurs. It was observed that sand specimens tested above 0.1% strain amplitude achieved initial liquefaction. Therefore, the threshold strain is lower than 0.1% and higher than 0.0055%.

2. In all tests performed under the shear strain levels above the threshold, maximum excess pore water pressure ratios \((r_{umax})\) was 1, causing initial liquefaction.
Figure 6-3 A typical LabVIEW measurement output for a fully saturated sand specimen with $D_r=40\%$ and tested under $\gamma=0.1\%$
3. As the soil gets denser more number of cycles is required to reach liquefaction.

4. Similarly, higher shear strain amplitudes caused sand specimens to liquefy faster i.e. in less number of cycles.

The observations stated in items 3 and 4 are also demonstrated in Figure 6-6.
Finally, liquefaction of fully saturated sands was achieved in the laboratory with the test setup developed. Also, the control parameters were determined to get comparative results with the partially saturated sand tests.

![Diagram of shear strain and cycles to liquefaction](image)

Figure 6-6 The effect of relative density and shear strain amplitude on number of cycles required to reach initial liquefaction

### 6.3 Cyclic Simple Shear Strain Tests on Partially Saturated Sands

This section presents the specimen preparation, procedure and results of cyclic simple shear strain tests performed on partially saturated sand specimens. Loose to dense sand specimens at different degrees of saturation were tested under shear strain levels of 0.01 to 0.2%. A total of 96 tests were performed.

#### 6.3.1 Test Procedure

In order to perform cyclic simple shear tests on partially saturated sands, a typical test procedure was followed as presented below:
1. The CSSLB and the instruments are set up as explained in fully saturated sand tests. The only difference in this setup is that the PPTs are inserted in a membrane piece full with water to keep them saturated. As the PPT meets with water in CSSLB the membranes were taken out.

2. Partially saturated sand specimens are prepared with dry sand and Efferdent mixed at different ratios as explained in Chapter 4. As performed in fully saturated sand tests, the soil parameters are estimated using phase relation equations. Figure 6-7 shows a photo of a partially saturated sand specimen prepared in the CSSLB just before the start of the test.

![Figure 6-7 A partially saturated sand specimen tested](image)

During the testing program, partially saturated sand specimens with a range of degrees of saturation from 40% to 90% and at a range of relative densities from 20% to 65% were obtained. Similar to fully saturated sand specimen tests, cyclic simple shear strains were induced on the partially saturated sand specimens. However, since partial saturation reduces the permeability of the soil (Chapter 3), the dissipation rate was
relatively lower. Hence, signals with frequencies from 4 to 10Hz were used in partially saturated sand tests.

6.3.2 Results and Discussions

A total of 96 tests were performed on partially saturated sand specimens. The first 24 tests were defined as preliminary tests which led to the development of a preliminary model to predict excess pore water pressure ratio ($r_u$). The remaining 72 tests were defined as secondary tests which were conducted to improve and validate the preliminary model.

Most of the preliminary tests were performed at 0.1% shear strain. Specimens with different degrees of saturation were prepared again at the lowest possible density. Then, as the specimen was subjected to the cyclic strains, a denser specimen was obtained. When it densified, the degree of saturation also decreased since change in volume of voids was mostly due to dissipation of water in the voids not air. Also this can be explained in the following phase relation expression:

$$\Delta V_v \approx \Delta V_w \Rightarrow V_{ai} \approx V_{af}$$

$$S_i = 1 - S_{ai} = 1 - \frac{V_{ai}}{V_{vi}}$$

$$S_f = 1 - S_{af} = 1 - \frac{V_{ai}}{V_{vi} - \Delta V_v}$$

$$\Rightarrow S_{af} > S_{ai} \Rightarrow S_f < S_i$$

Where $\Delta V_v$ and $\Delta V_w$ are the change in the volume of voids and the volume of water, respectively, $V_a$ is the volume of air, $S$ is the degree of saturation and $S_a$ is the degree of air saturation. "i" and "f" stand for initial and final conditions respectively. Therefore at the end of each test, a denser sand specimen with lower degree of saturation
was obtained. During secondary tests, each specimen was tested under a sequence of 4 shear strain levels starting from the small strains: 0.01%, 0.05%, 0.1% and 0.2%. The reason for starting from the small level of strain was to benefit from gradual increase in the relative density as the specimen tested. Preliminary and secondary sets of test data and results are tabulated in Table A-2 and A-3 of Appendix A-1.

Figure 6-8 demonstrates a typical LabVIEW output with the PPT and LVDT measurements. The output displays excess pore water pressure generations in a sand specimen with degree of saturation of 81% and relative density of 51%, tested under cyclic simple shear strains of 0.1% amplitude. From 96 test outputs, excess pore water pressure ratio time histories were evaluated and following observations and results were explored:

1. In partially saturated sand specimens, excess pore water pressures also build up and get steady at a maximum value expressed as $r_{\text{umax}}$ (Figure 6-9).

2. As the soil gets denser (higher $D_r$) and/or the degree of saturation (S) decreases, $r_{\text{umax}}$ reduces. Figure 6-9 and 6-10 demonstrate comparison of $r_u$ generations in loose and medium dense sands at two different degrees of saturation (80 % and 55-60 %).

3. Strain level plays also a significant role on $r_{\text{umax}}$ value. Figure 6-11 demonstrates 3 different cyclic shear strain time histories applied on sand specimens with the same soil characteristics. The results indicate that higher strain amplitude results in an increased $r_{\text{umax}}$. 
Figure 6-8 A typical LabVIEW measurement output for a specimen with $S=81\%$, $D_r=51\%$ tested under $\gamma=0.1\%$
4. It requires more number of strain cycles (time multiplied by frequency) to reach $r_{\text{u,\max}}$ in specimens with

- lower degree of saturation,
- and/or higher relative density,
- and/or tested under lower strain levels.

Typical behaviors of partially saturated sand specimens under different cyclic shear strain levels were presented. A more intensive parametric study was performed on these test results to estimate individual effect of all the parameters on $r_u$ as well as to investigate the interactions between these effects. Ultimately, a mathematical model was developed to predict excess pore water pressure ratios in partially saturated sands (Chapter 7).
Figure 6-9 Effect of relative density on excess pore water pressure ratio at S=80% (b) under cyclic shear strain level of 0.1% (a)
Figure 6-10 Effect of relative density on excess pore water pressure ratio at S=55-60% (b) under cyclic shear strain level of 0.1% (a)
Shear strain, $\gamma$, %

Time, sec

(a)
6.4 Comparison of Excess Pore Water Pressure Generation in Fully and Partially Saturated Sands

Cyclic simple shear strain tests were performed on fully saturated and partially saturated sand specimens with different soil characteristics. Test results demonstrated that partially saturated sands never liquefied i.e. $r_u$ never reached 1. Figure 6-12 demonstrates a comparison of the excess pore water pressure ratios in fully and partially saturated specimens when tested under cyclic simple shear strains of 0.1% amplitude. The comparative results led to the following conclusions:

1. In partially saturated sands, excess pore water pressure build up occurs and reaches a maximum value ($r_{umax}$), which is always less than $r_{umax}$ of 1 generated in fully saturated sands. This implies that partially saturated sands never liquefy. "$r_{umax}$" gets lower as partial saturation is more induced.
2. Number of cycles required to reach $r_{umax}$ ($N_{max}$) is also always more than number of cycles required to reach liquefaction ($N_L$) in fully saturated sands. $N_{max}$ increases as the degree of saturation reduces.

![Graph showing cyclic simple shear strain history and comparison of excess pore water pressure ratios](image_url)

Figure 6-12 (a) Cyclic simple shear strain history applied on the soil specimens. (b) Comparison of the excess pore water pressure ratios in fully and partially saturated specimens under the cyclic motion given at (a)
Therefore, the benefit of IPS on liquefaction is introduced by two factors: first by decreasing maximum excess pore water pressure ratio ($r_{umax}$), second by increasing number of cycles required to reach maximum value ($r_{umax}$). The next chapter presents an empirical model which incorporates all these effects to predict excess pore water pressure ratios in partially saturated sands with different soil characteristics and subjected to different earthquake-induced shear strains.
7. An Empirical Model for Predicting Excess Pore Water Pressure Ratios in Partially Saturated Sands during Earthquakes

7.1 Overview

A mathematical model was developed to predict excess pore water pressure ratios ($r_u$) in partially saturated sands, based on a series of experimental tests. Cyclic response of partially saturated sands is a complex behavior. In Chapter 2, excess pore water generation in fully and partially saturated sands is briefly explained using theoretical concepts. Cyclic behavior of partially saturated sands is not yet very well understood. Thus, as an initial effort, in this research, an empirical model based on experimental data was developed.

Excess pore water pressure generation in partially saturated sands was investigated and experimental results were presented in Chapter 6. Based on the experimental results, an empirical model was developed for the prediction of excess pore pressures in partially saturated sands, whether naturally found in that condition or induced by the proposed liquefaction mitigation measure (IPS).

Liquefaction susceptibility of a particular site can be evaluated using current liquefaction analysis techniques (simplified procedure based on cyclic stress ratio by Seed et al. (1971), strain approach by Dobry et al (1982), effective stress approach by Finn et al. (1977) and probabilistic approach by Yegian et al. (1978)) for a characteristic earthquake with magnitude M. If liquefaction analysis for a site results in a factor of safety (FS) less than 1 the site typically requires remediation. When the site is mitigated
by IPS, excess pore water pressure ratios \( (r_u) \) can be estimated with the proposed model for a design level partial degree of saturation, \( S \) or the required partial saturation can be determined for a limiting value of the \( r_u \). Figure 7-1 displays the schematic of the model application.

![Diagram showing the application of the proposed model.](image)

**Liquefaction Analysis** \( \rightarrow r_u = 1.0 \)  
**Model Proposed** \( \rightarrow r_u < 1 \)  
\[ r_u = f(S, D_r, \gamma, \sigma_v', M) \]

Figure 7-1 Schematic showing the application of the proposed model.

Liquefaction resistance of partially saturated sands has been investigated by a number of researchers (Chaney et al. (1978), Yang et al. (2003), Ishihara et al. (2002), Okamura et al. (2006)), as presented in Chapter 2. Their liquefaction criterion assumes that partially saturated sands also liquefy because they assume that a triaxial or simple shear specimen liquefies when the axial strain reaches 5% double amplitude under constant cyclic stresses. So, liquefaction resistance was correlated to increasing number of cycles to reach double amplitude 5% axial strain for the same cyclic stress applied. Also, Ishihara et al. (2002) demonstrated increased factor of safety due to partial
saturation of sands. However, a factor of safety concept can not be set for partially saturated sands since partially saturated sands never liquefy, if liquefaction is defined as $r_u = 1.0$. The proposed model addresses the benefit of partial saturation on liquefaction prevention in terms of reduced excess pore water pressure ratio ($r_u$) instead of factor of safety.

It was shown by Dobry et al. (1982) that excess pore water pressure generation is more related to cyclic strains induced by an earthquake rather than cyclic stresses. Also, the laboratory evaluation of liquefaction based on cyclic simple shear strains reduces the influence of a number of factors (density, soil fabric, strain history, length of time under sustained pressure). Since the factors affecting the cyclic stresses induced also affect the shear modulus the influence on the strain diminishes ($\gamma = \tau / G$) (Kramer 1996). Based on this evidence, a cyclic-strain based model would best be suited to predict the benefit of IPS on $r_u$.

Experimental results of $r_u$ measurements in sand specimens at different degrees of saturation were presented at the end of Chapter 6, and are again shown in Figure 7-2. The figure only presents the results for a given relative density and shear strain amplitude. The results demonstrate that IPS provides benefit on liquefaction prevention not only by reducing the maximum excess pore water pressure ratio ($r_{umax}$) but also by increasing the number of cycles required to reach $r_{umax}$. This chapter presents the development of empirical model called RuPSS ($r_u$ for Partially Saturated Sands) for predicting $r_u$ in partially saturated loose to medium dense sands under different earthquake magnitudes and shear strain histories.
Figure 7-2 Typical experimental results evaluated in the development of the proposed RuPSS model

7.2 Model Framework

A typical site that is either naturally in a partially saturated state or is a candidate for IPS is shown in Figure 7-3. The soil and earthquake parameters that are needed to make a prediction of the excess pore water ratio that the site will experience are shown in the argument of the function $r_u = f(S, D_r, \gamma, \sigma_{v'}, M)$. In order to develop this function, in this research, laboratory cyclic simple shear tests were performed as shown on the right-hand side of Figure 7-3.

The main parameters which have significant effect on excess pore water pressure ratio model are introduced below:

$S$: Degree of Saturation in the field, either existing degree of saturation or induced degree of saturation by IPS.
\( r_{u} = f(S, D_r, \gamma, \sigma'v, M) \)

Figure 7-3 Schematic demonstrating the field and laboratory conditions for the proposed model

**Dr**: Relative density of the sand.

**\( \gamma_{\text{max}} \)**: Maximum shear strain that the soil profile experiences due to a particular earthquake. Shear strain time history can be obtained from ground response analysis. Equivalent uniform cyclic shear strain amplitude, which is applied in the lab, can be obtained by:

\[
\gamma = \gamma_{\text{cyc}} = \gamma_{\text{eqv}} = \gamma_{\text{max}} \times R
\]

\[
R = \frac{M - 1}{10}
\]

**M**: Magnitude of the earthquake.
**ru**: Excess pore water pressure ratio

**rumax**: Maximum excess pore water pressure ratio.

**N**: Number of cyclic shear strain cycles.

**N_{max}**: Number of cycles required to reach maximum excess pore water pressure ratio (rumax).

The model was developed in two stages. In the first stage, a function was developed to estimate the maximum excess pore water pressure ratio (rumax) at a given strain amplitude regardless of the earthquake magnitude, or number of cycles of strain application. In the second stage, the effect of earthquake magnitude or number of cycles of strain application was introduced to estimate the ru corresponding to the earthquake magnitude.

The algorithm of the model evaluating the excess pore water pressure in partially saturated sands is presented in the schematic shown in Figure 7-4.
7.3 Model for Maximum Excess Pore Water Pressure Ratio, \( r_{\text{umax}} \)

In this section, the development of the first portion of the model, \( r_u = f(S, D_r, \gamma, \sigma', M) \) is presented. Maximum excess pore water pressure ratio depends on the degree of saturation, relative density, and the shear strain amplitude applied. It is the maximum excess pressure that water in the pores would experience, regardless the size of the earthquake (or number of cycles of strain application).

A series of preliminary tests were performed to evaluate the effect of the different model parameters on \( r_{\text{umax}} \) and to develop a preliminary mathematical model. A statistical analysis was performed to evaluate the goodness of fit to the experimental data as well as the uncertainties in the model. Then a second set of tests were performed to expand the data base and to improve the model coefficients as well as to minimize the uncertainties.

7.3.1 Preliminary \( r_{\text{umax}} \) Model Development

Initially, twenty four cyclic shear strain tests were performed on specimens with degrees of saturation ranging from 90% to 50% and relative densities between 20% and 53% at shear strain amplitudes of 0.1% and 0.05%. The results of the tests were tabulated and can be found in Table A-2 of Appendix A-1. The variation of \( r_{\text{umax}} \) for loose to medium dense sands specimens with different degrees of saturation tested at cyclic shear strain amplitude of 0.1% is displayed in Figure 7-5. It was observed that the most dominant parameter is \( S \) (degree of saturation). Maximum excess pore water pressure ratio (\( r_{\text{umax}} \)) decreases with decreasing degree of saturation. Also, \( r_{\text{umax}} \) decreases with decreasing \( S \) more rapidly in a relatively denser material.
Figure 7-5 Maximum excess pore water pressure ratio $r_{\text{umax}}$ vs. degree of saturation ($S$) for loose to medium dense sand specimens tested at $\gamma=0.1\%$.

The $r_{\text{umax}}$ equation was first developed for the loosest sand specimen which has relative density ($D_r$) of about 20% and at shear strain amplitude ($\gamma$) of 0.1%: Base function $f$. Then, a scaling factor function was established to relate $r_{\text{umax}}$ to relative densities greater than 20%: $F_D(S,D_r)$. Similarly, a scaling factor function was developed to relate $r_{\text{umax}}$ at shear strain of 0.1% to other levels of shear strain amplitude: $F_\gamma(S,\gamma)$. Then final $r_{\text{umax}}$ model equation was obtained by the product of the base function $f$ and the scaling factor functions $F_D$ and $F_\gamma$ as shown in Eqn. 7-2.

$$r_{\text{umax}} = f(S,D_r = 20\%, \gamma = 0.1\%) \times F_D(S,D_r) \times F_\gamma(S,\gamma)$$  \hspace{1cm} 7-2

$f$ is the base function of $S$ at $D_r = 20\%$ and $\gamma=0.1\%$

$F_D$ is scaling factor function for $D_r$ 's other than $D_r = 20\%$

$F_\gamma$ is scaling factor function for $\gamma$'s other than $\gamma=0.1\%$
A base function \( f \) was aimed to be developed for the loosest condition (\( D_r=20\% \)) and at \( \gamma=0.1\% \). Since there wasn't a lot of data for exact \( D_r=20\% \), data for a range of \( D_r=20-30\% \) was plotted at \( \gamma=0.1\% \) (Figure 7-6). A mathematical function for the upper boundary of the data was intended to be developed for \( D_r=20\% \).

\[
\text{BC: } r_{\text{umax}} = 0 \text{ therefore } f=0 \text{ at } S = 0
\]
\[
r_{\text{umax}} = 1 \text{ therefore } f=1 \text{ at } S = 1
\]

Various mathematical functions were explored to find a model equation which would fit the data best. The curve fitting tool in Matlab was used for this purpose. A modified form of Gaussian equation was determined to be the best model function, to describe the behavior of base function \( f \):

\[
f = S^n \times e^{-\left[\frac{1-S}{a}\right]^{n_s}}
\]

where \( S \) is in decimals and coefficients are:

\[
\begin{align*}
  n &= 0.5 \\
  a &= 0.53 \\
  n_s &= 4
\end{align*}
\]

The coefficients \( n \), \( a \), and \( n_s \) were estimated based on the boundary conditions (BC) and the test data. At the end of the preliminary model development, a statistical analysis was performed based on the entire test data and the coefficients introduced above were determined as the best fit estimates. Figure 7-6 demonstrates that the \( f \) base function is in good agreement with the test data for \( D_r=20-30\% \).
Figure 7-6 $f$ Base Function: Maximum excess pore water pressure ratio ($r_{\text{umax}}$) versus degree of saturation ($S$) for $D_r = 20\%$ and $\gamma = 0.1\%$.

$F_D$ formulation

$F_D$ is a scaling factor function which pulls down the $f$ base function for relative densities greater than 20%. When the base function $f$ is multiplied by $F_D$, it will give the maximum excess pore water pressure ratios ($r_{\text{umax}}$) for various degrees of saturation and relative densities greater than 20% at strain level of 0.1%.

The effect of relative densities greater than 20% and up to 53% on $r_{\text{umax}}$ was examined (Figure 7-5), and the following observations were made:

1. Between $S = 40$-100%, for the same degree of saturation as relative density gets larger, $r_{\text{umax}}$ gets smaller, which is physically expected. The trend in this variation was carefully evaluated in order to develop the best scaling factor function.
2. For S smaller than 40% relative densities has little effect on $r_{umax}$

3. For the same relative density the variation of $r_{umax}$ from $f$ base function depends on the degree of saturation. Also, there is a unique degree of saturation ($S_{max}$) at which the variation of $r_{umax}$ from $f$ is maximum.

4. Boundary conditions are as follows:

$$BC: r_{umax} = 1 \text{ and/or } F_D = 1 \text{ at } S = 1, \text{ at all } D_t's$$

$$F_D = 1 \text{ at } D_t = 20\%$$

Based on the observations stated above, following steps were taken to develop the scaling factor function $F_D$.

Several mathematical model functions were examined and the best fit equation that met the observed variation of test data was found to be a modified form of Rayleigh distribution equation:

$$y = \frac{x}{\beta} \times e^{\left[\frac{-x^2}{2\beta^2}\right]}$$  \hspace{1cm} 7-4

According to the boundary conditions, Eqn. 7-4 is adapted for $F_D$ vs S as follows:

$$x = (1 - S)$$
$$BC: F_D = 1 \text{ when } S = 1$$

$$F_D = 1 - \frac{(1 - S)}{\beta} \times e^{\left[\frac{-(1-S)^2}{2\beta^2}\right]}$$  \hspace{1cm} 7-5

Further understanding of the equation and the effects of the terms "b" and "β" on the equation was needed in order to implement the parameter $D_t$ into the function $F_D$.

The effect of coefficients $β$ and b on the amplitude of the function were determined. It
was concluded that "b" term governs the location of S for the maximum amplitude of $F_D$ ($S_{\text{max}D}$), as well as it governs the amplitude of $F_D$. "1/β" term governs only the amplitude of $F_D$. Since the amplitude of the $F_D$ equation needed to be formulated with the parameter $D_r$, first $1/\beta$ was defined as a function of $D_r$. Then, it was also observed based on the test data that the location of $S_{\text{max}D}$ also varies with $D_r$. Therefore the b term was also written as a function of $D_r$. As a result, the crest shifts toward left as shown in Figure 7-7. Then the amplitude of the $F_D$ is governed by $1/\beta$ and also b functions.

$$\frac{1}{\beta} = c \times (D_r - 0.2)$$

$$b = 1 - S_{\text{max}D} \times \left(\frac{0.2}{D_r}\right)^{n_b}$$

where $D_r$ is in decimals.

Final $F_D$ scaling factor function becomes as in Eqn. 7-6. The coefficients are determined based on the statistical analysis of $r_{\text{umax}} = f \times F_D$ for $\gamma=0.1\%$ and also confirmed in overall $r_{\text{umax}}$ statistical analysis which is presented in the following section. The details of the $F_D$ derivation is presented in Appendix B.

$$F_D = 1 - c \times (D_r - 0.2) \times (1 - S) \times e^{- \left[ \frac{(1-S)^2}{2\gamma (1-S_{\text{max}D} \times (\frac{0.2}{D_r})^{n_b})^2} \right]}$$ 7-6

where $c = 8.88$

$S_{\text{max}D} = 0.84$

$n_b = 0.25$

Figure 7-7 demonstrates $F_D$ function for different relative densities. The equation plots demonstrate that $F_D$ is 1 at $D_r=20\%$ and the amplitude of the $F_D$ decreases as $D_r$ increases. However, the value of $F_D$ decreases since it should scale down the $f$ base.
function for higher relative densities. Also, the maximum amplitude of the function $F_D$ occurs at lower degrees of saturation as $D_r$ increases.

![Figure 7-7 Scaling factor function $F_D$](image)

The test data at $\gamma=0.1\%$ for three ranges of relative densities are plotted and compared with the predicted $r_{umax}$ model as shown in Figure 7-8. Model plots are for average relative densities of each density data range. Test results reveal that $r_{umax}$ at $\gamma=0.1\%$ is in a good agreement with the test data. Finally, $r_{umax}$ at $\gamma=0.1\%$ can be predicted by Eqn. 7-7. The overall statistical analysis of the function $r_{umax}$ prediction is presented in the next section.

$$r_{umax, \gamma=0.1\%} = f(S, D_r = 20\%, \gamma = 0.1\%) \times F_D(S, D_r)$$

$$r_{umax, \gamma=0.1\%} = \left[ S^{0.5} \times e^{\left[\frac{1-S}{0.55}\right]^4} \right] \times \left[ 1 - 8.88 \times (D_r - 0.2) \times \left(1 - S\right) \times e^{\left[\frac{(1-S)^2}{2 \times (1-0.84 \times \left(\frac{0.2}{D_r}\right)^{0.25})^2}\right]} \right]$$ 7-7
Figure 7-8 The predicted preliminary $r_{\text{umax}}$ model at $\gamma=0.1\%$ compared with the test data

$F_{\gamma}$ Formulation

In the preliminary test series, only a few tests were performed at strain amplitudes different than $\gamma = 0.1\%$. Based on this data it was observed that $r_{\text{umax}}$ decreases with decreasing shear strain. Hence the scaling factor function $F_{\gamma}$ needed to adjust $r_{\text{umax}}$ for strains different form $0.1\%$. $F_{\gamma}$ was developed in a similar manner as that described for $D_r$.

The modified form of the Rayleigh distribution function used for $F_D$ (Eqn. 7-5) was implemented here for the $F_{\gamma}$ scaling factor function

$$F_{\gamma} = 1 - \left(1 - S\right) \frac{(1-\phi \gamma)^2}{\phi} \right)$$
Boundary conditions are very similar to the boundary conditions in $F_D$:

$BC: \ r_{umax} = 1 \ and/or \ F_\gamma = 1 \ at \ S = 1, \ at \ all \ \gamma's$

\[ F_\gamma = 1 \ for \ \gamma = 0.1% \]

In this case the location of $S$ where $F_\gamma$ gets the maximum values is designated as $S_{maxg}$. Based on the test data the location of $S_{maxg}$ didn't vary as $\gamma$ changes. Therefore the amplitude of the $F_\gamma$ function is only governed by "1/\phi". "1/\phi" is defined as a function of the corresponding parameter, which is shear strain amplitude, $\gamma$. After examining and testing different mathematical functions in Matlab curve fitting tool, the best fit equation $1/\phi$ was found to be a logarithmic function of $\gamma$ normalized to 0.1% shear strain. Therefore, $p$ and $1/\phi$ are as below:

\[
p = (1-S_{maxg})
\]

\[
1/\phi = d \times \log\left(\frac{-\gamma}{0.001}\right)
\]

where $\gamma$ is in decimals.

Then $F_\gamma$ function becomes as follows:

\[
F_\gamma = 1 - d \times (-\log\frac{\gamma}{0.001}) \times (1 - S) \times e^{-\frac{(1-S)^2}{2\times(1-S_{maxg})^2}}
\]

where $d = 3.1$

$S_{maxg} = 0.78$

Coefficients "d" and "$S_{maxg}$" found above were determined based on the statistical analysis performed for overall function of $r_{umax}$, which is described in the next section. According to the results, the coefficients needed to be improved with more test data.

$F_\gamma$ is plotted in Figure 7-9 for different $\gamma$'s. The function $F_\gamma$ is 1 at $\gamma$ is equal to 0.1% and decreases as the shear strain amplitude $\gamma$ decreases.
7.3.2 Statistical Error Analysis of Preliminary $r_{\text{umax}}$ Model

The goodness of fit of the preliminary mathematical model obtained for $r_{\text{umax}}$ was evaluated and best fit model coefficients were estimated. The uncertainties in the preliminary model are examined to determine the scope of additional experiments required to finalize the $r_{\text{umax}}$ model. A preliminary estimate of the contribution of each model parameter to the uncertainty was investigated.

The best estimates of the coefficients were determined by testing the experimental data with the overall predicted model $r_{\text{umax}}$ and are already presented in the formulation of the base function $f$ and scaling factor functions $F_D$ and $F_\gamma$. Several combinations of the model coefficients were examined in Matlab and a combination of coefficients was estimated by providing the least mean square error between the model and experimental data.
The preliminary \( r_{umax} \) model then is summarized with all the coefficients estimated as below. The best estimates of the preliminary model coefficients are again tabulated in Table 7-1.

\[
\begin{align*}
  r_{umax} &= f(S, D_r = 20\%, \gamma = 0.1\%) \times F_D(S, D_r) \times F_{\gamma}(S, \gamma) \\
  f &= S^n \times e^{-\frac{1-S}{a}} \\
  F_D &= 1 - c \times (D_r - 0.2) \times (1 - S) \times e^{-\frac{(1-S)^2}{2 \times (1-S_{max})^2}} \\
  F_{\gamma} &= 1 - d \times (-\log \frac{\gamma}{0.001}) \times (1 - S) \times e^{-\frac{(1-S)^2}{2 \times (1-S_{max})^2}}
\end{align*}
\]

Table 7-1 Estimated Coefficients of Preliminary \( r_{umax} \) Model

<table>
<thead>
<tr>
<th>n</th>
<th>n_s</th>
<th>a</th>
<th>c</th>
<th>( S_{maxD} )</th>
<th>n_b</th>
<th>d</th>
<th>( S_{maxg} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>4</td>
<td>0.53</td>
<td>8.88</td>
<td>0.84</td>
<td>0.25</td>
<td>3.1</td>
<td>0.78</td>
</tr>
</tbody>
</table>

For the best estimate coefficients tabulated in Table 7-1, following statistical analysis results were obtained:

\[
\begin{align*}
  SSE &= \sum_{i=1}^{n} (r_{ai} - \bar{r}_a)^2 = 0.17 \\
  MSE &= \frac{SSE}{(n-2)} = 0.007 \\
  SST &= \sum_{i=1}^{n} (r_{ai} - \bar{r}_a)^2 = 0.89 \\
  R^2 &= 1 - \frac{SSE}{SST} = 0.81
\end{align*}
\]
\( r_{ai} \) = Experimental data of \( r_{umax} \)

\( r_{mi} \) = Model data of \( r_{umax} \)

SSE = Sum of the squared error

MSE = Mean squared error

SST = Total corrected sum of squares

\( R^2 \) = Coefficient of determination

The details of the statistical analysis results of the preliminary model are presented in Table A-3 of Appendix A-1.

The coefficient of determination of 0.81 was not really a good fit although there is a good agreement between the test data and the model graphically as observed in Figure 7-8. Therefore, the preliminary model equation describes the behavior of the \( r_{umax} \) however it needs to be improved with more experimental data. The lack of fit can be also observed with more details in residual (\( \varepsilon = r_{ai} - r_{mi} \)) plots.

Figure 7-10 and 7-11 illustrates the residual distribution of the preliminary \( r_{umax} \) model versus the parameters of degree of saturation and relative density. According to the results, the model shows lack of fit at degree of saturation higher than 80% and relative densities greater than 40%. Therefore, the model needs to be improved with additional experimental data at those parameter ranges.
Figure 7-10 Residual distributions of the preliminary $r_{\text{umax}}$ model vs degree of saturation, $S$

Figure 7-11 Residual distributions of the preliminary $r_{\text{umax}}$ model vs relative density $D_r$
7.3.3 Modification of $r_{\text{umax}}$ Model

A second set of tests were performed in order to improve the model coefficients as well as to validate the adequacy of the model functional form. The results of a total of 72 tests performed in the second phase tests are presented in Table A-4 of Appendix A-1.

A parametric study again was performed in Matlab to find the best estimates of the model coefficients using the combined first and second phase test results (a total of 96 test data). The preliminary coefficients determined earlier for the base function $f$ and scaling factor function $F_D$ were generally confirmed by the second set of data. Using the combined test data, improved estimates of the scaling factor function $F_\gamma$ coefficients were obtained.

The general form of the model function appeared to be still valid even after including the second series of test results. However using the entire set of test results, better estimates of the coefficients of the model were derived, as tabulated in Table 7-2.

Table 7-2 Finalized $r_{\text{umax}}$ Model Coefficients

<table>
<thead>
<tr>
<th>$n$</th>
<th>$n_s$</th>
<th>$a$</th>
<th>$c$</th>
<th>$S_{\text{maxD}}$</th>
<th>$n_b$</th>
<th>$d$</th>
<th>$S_{\text{maxg}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>4</td>
<td>0.54</td>
<td>8.75</td>
<td>0.84</td>
<td>0.25</td>
<td>1.75</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Then the final form of the $r_{\text{umax}}$ with the finalized coefficients and all the simplifications is presented as below:

$$
r_{\text{umax}} = f(S, D_r = 20\%, \gamma = 0.1\%) \times F_D(S, D_r) \times F_\gamma(S, \gamma)
$$

$$
f = S^{0.5} \times e^{\left[\frac{(1-S)^4}{0.54}\right]}
$$

$$
F_D = 1 - 8.75 \times (D_r-0.2) \times (1-S) \times e^{-2\times(1-0.84\times(0.2)^{0.25})^2}
$$

$$
F_\gamma = 1 - 1.75 \times (-\log\frac{\gamma}{0.001}) \times (1-S) \times e^{-3.1(1-S)^2}
$$
7.3.4 Adequacy and Validation of the Modified $r_{umax}$ Model

Model adequacy or the goodness of the model fit was again evaluated by calculating mean squared error and coefficient of determination for all test data (96 test data). The details are presented in Table A-5 of Appendix A-1.

\[
SSE = \sum_{i=1}^{n} (r_{ai} - r_{mi})^2 = 0.61
\]

\[
MSE = \frac{SSE}{(n-2)} = 0.0066
\]

\[
SST = \sum_{i=1}^{n} (r_{ai} - \overline{r_{a}})^2 = 7.2
\]

\[
R^2 = 1 - \frac{SSE}{SST} = 0.92
\]

The mean square error reduced slightly however the coefficient of determination improved significantly by increasing to 0.92 with the additional experimental data. In a good fit model the residuals should follow a normal distribution. As shown in Figure 7-12, the residuals follow a normal distribution. A normal distribution function (Figure 7-13) can be plotted with the mean and the standard deviation of the residuals, which are calculated as -0.00052 and 0.082, respectively. According to the normal distribution of the residuals ($\varepsilon$), the proposed $r_{umax}$ model predicts the actual $r_{umax}$ within an error range of

\[-0.082 < \varepsilon < 0.081\]

with 68% confidence interval.

Residuals ($\varepsilon = r_{ai} - r_{mi}$) were also examined for individual parameters and shown in Figure 7-14. When residuals examined for each parameter independently, it was observed that the model estimates the actual data with almost equal higher or lower
Figure 7-12 The distribution of the residuals

Figure 7-13 The normal distribution function of the residuals

\[ \bar{\varepsilon} = -0.00052 \]
\[ \sigma_{\varepsilon} = 0.082 \]
Figure 7-14 Residual Distributions of the modified $r_{\text{umax}}$ model for each individual parameter
probabilities for S, D_r and \( \gamma \). For \( \gamma \), it appears that the residuals are not equally distributed only around the limits.

Model validation is also assessed by comparing model results with experimental data in graphical form. The comparison of the experimental data with the predicted model for \( \gamma \)'s of 0.2\%, 0.1\%, 0.05\%, and 0.01\% are shown in Figure 7-15 through 7-18. The predicted \( r_{\text{umax}} \) values from the model are in good agreement with the experimental data.

![Figure 7-15 Comparison of experimental data with the predicted \( r_{\text{umax}} \) model for \( \gamma=0.2\% \)](image)

Figure 7-15 Comparison of experimental data with the predicted \( r_{\text{umax}} \) model for \( \gamma=0.2\% \)
Figure 7-16 Comparison of experimental data with the predicted $r_{\text{umax}}$ model for $\gamma=0.1\%$

Figure 7-17 Comparison of experimental data with the predicted $r_{\text{umax}}$ model for $\gamma=0.05\%$
The proposed $r_{\text{umax}}$ model function predicts the maximum excess pore water pressure ratios in partially saturated loose to medium dense sands within an acceptable error range. The model function was determined to be a valid mathematical model which predicts the maximum excess pore water pressure ratios in partially saturated sands with $S = 0-99\%$ and $D_r = 20-65\%$, when subjected to cyclic simple shear strain amplitudes of $\gamma = 0.01-0.3\%$. 

Figure 7-18 Comparison of experimental data with the predicted $r_{\text{umax}}$ model for $\gamma=0.01\%$
7.3.5 Graphical 2-D and 3-D Representation of the Final $r_{\text{umax}}$ Model

The final $r_{\text{umax}}$ model equation with its finalized coefficients was presented in section 7.3.3 and is recalled here:

$$r_{\text{umax}} = f(S, D_r = 20\%, \gamma = 0.1\%) \times F_D(S, D_r) \times F_\gamma(S, \gamma)$$

$$f = S^{0.5} \times e^{-\left[\frac{1-S}{0.34}\right]^4}$$

$$F_D = 1 - 8.75 \times (D_r - 0.2) \times (1 - S) \times e^{-\frac{(1-S)^2}{2(1-0.84\times\frac{0.27}{D_r})^2}}}$$

$$F_\gamma = 1 - 1.75 \times (-\log \frac{\gamma}{0.001}) \times (1 - S) \times e^{-3.1(1-S)^2}$$

The graphical outputs of the model are presented in 2-D and 3-D forms in Figure 7-19 through 7-23. Excess pore water pressure ratios in loose and medium dense sands for different cyclic shear strain levels are shown in Figure 7-19 and Figure 7-20 respectively. 3-D graphics provide a better representation of the coupled effect of two of the three parameters (Figure 7-21, Figure 7-22 and Figure 7-23). According to the 2-D and 3-D graphics the $r_{\text{umax}}$ model demonstrates that:

1. The $r_{\text{umax}}$ can be as large as 0.8 for loose sands with degree of saturation above 80%, at any strain levels.

2. The effect of cyclic shear strain level on $r_{\text{umax}}$ is more pronounced in looser sands.

3. Below S=50-60%, regardless of the relative density and the shear strain level applied, $r_{\text{umax}}$ is smaller than 0.4-0.3.
Figure 7-19 The $r_{\text{umax}}$ predicted by the proposed model function for loose sands

Figure 7-20 The $r_{\text{umax}}$ predicted by the proposed model function for medium dense sands
Figure 7-21 3-D representation of $r_{\text{umax}}$ predicted by the proposed model for loose and medium dense sands
Figure 7-22 3-D representation of $r_{\text{umax}}$ predicted by the proposed model at low shear strain levels ($\gamma=0.01-0.05\%$)
Figure 7-23 3-D representation of $r_{\text{umax}}$ predicted by the proposed model at high shear strain levels ($\gamma=0.1$-$0.2\%$)
7.3.6 Effect of Initial Effective Stress on $r_{\text{umax}}$

Maximum excess pore water pressure ratio ($r_{\text{umax}}$) is the ultimate value that a sand can experience when subjected to cyclic shear strains. The maximum value is commonly observed to be 1 in fully saturated loose sands tested at shear strains above the threshold shear strain level. However, the number of cycles requires to have $r_{\text{umax}} = 1$ depends on the initial effective stresses. At higher effective stresses, a fully saturated sand will require more number of cycles of shear strain to liquefy. So, initial effective stress has no effect on the value of $r_{\text{umax}}$ since it would reach the ultimate failure if the soil is cycled enough. This fact can be explained in cyclic torsional shear stress tests performed by Ishihara (1985) in Figure 7-24. The initial confining stress was 98 kPa. Due to the excess pore water pressure build up, effective stress reduces and eventually hits the liquefaction failure line (defined as line of phase transformation). If the initial effective stress were lower under the applied torsional shear stresses, the stress path would hit the line in less number of cycles.

Figure 7-24 Laboratory test data from cyclic torsional shear stress tests performed on Fuji river sand having a medium density ($D_r=47\%$) (Reproduced from Ishihara 1985) (Day, 2002)
In partially saturated sands, there is no such failure line since partially saturated sands do not liquefy ($r_{\text{umax}} < 1$). Hence, $r_{\text{umax}}$ is expected to be the same for soils with different effective stresses. However, in soils with higher effective stresses, more number of cycles would be needed to reach that same $r_{\text{umax}}$.

A series of experimental tests were performed to prove the hypothesis stated above. Partially saturated sand specimens were tested at 2 to 10 kPa vertical effective stresses under a constant cyclic simple shear strain level. The test data is shown in Table 7-3. Since the soil specimen settled at each test run, the relative density couldn't be kept constant. Therefore, measured $r_{\text{umax}}$ values were adjusted for different relative densities using the $r_{\text{umax}}$ model. Test results imply that maximum excess pore water pressure ratios do not depend on the initial effective stresses (Figure 7-25).

### Table 7-3 Effective Stress Tests Data

<table>
<thead>
<tr>
<th>Effective Stress Test #</th>
<th>$\sigma'$ (kPa)</th>
<th>S</th>
<th>$D_r$</th>
<th>$\gamma$, %</th>
<th>Model $r_{\text{umax}} = r_m$</th>
<th>PPT $r_{\text{umax}} = r_a$</th>
<th>PPT $r_{\text{umax}}$ Modified to $S=65%$, $D_r=42%$, $\gamma=0.1%$</th>
<th>$\sigma'/2.4$</th>
<th>$\frac{r_{\text{umax}}\sigma'}{r_{\text{umax}}\sigma'=2.4\text{ kPa}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.35</td>
<td>65%</td>
<td>42%</td>
<td>0.1</td>
<td>0.44</td>
<td>0.38</td>
<td>0.38</td>
<td>0.98</td>
<td>1.00</td>
</tr>
<tr>
<td>1</td>
<td>2.18</td>
<td>65%</td>
<td>42%</td>
<td>0.1</td>
<td>0.44</td>
<td>0.4</td>
<td>0.4</td>
<td>0.91</td>
<td>1.05</td>
</tr>
<tr>
<td>1</td>
<td>1.44</td>
<td>65%</td>
<td>42%</td>
<td>0.1</td>
<td>0.44</td>
<td>0.43</td>
<td>0.43</td>
<td>0.60</td>
<td>1.13</td>
</tr>
<tr>
<td>2</td>
<td>3.94</td>
<td>65%</td>
<td>44%</td>
<td>0.1</td>
<td>0.41</td>
<td>0.37</td>
<td>0.40</td>
<td>1.64</td>
<td>1.04</td>
</tr>
<tr>
<td>2</td>
<td>3.77</td>
<td>65%</td>
<td>44%</td>
<td>0.1</td>
<td>0.41</td>
<td>0.38</td>
<td>0.41</td>
<td>1.57</td>
<td>1.07</td>
</tr>
<tr>
<td>2</td>
<td>3.00</td>
<td>65%</td>
<td>44%</td>
<td>0.1</td>
<td>0.41</td>
<td>0.4</td>
<td>0.43</td>
<td>1.25</td>
<td>1.13</td>
</tr>
<tr>
<td>3</td>
<td>7.72</td>
<td>64%</td>
<td>54%</td>
<td>0.11</td>
<td>0.25</td>
<td>0.2</td>
<td>0.35</td>
<td>3.22</td>
<td>0.92</td>
</tr>
<tr>
<td>3</td>
<td>7.55</td>
<td>64%</td>
<td>54%</td>
<td>0.11</td>
<td>0.25</td>
<td>0.2</td>
<td>0.35</td>
<td>3.15</td>
<td>0.92</td>
</tr>
<tr>
<td>3</td>
<td>7.55</td>
<td>64%</td>
<td>54%</td>
<td>0.11</td>
<td>0.25</td>
<td>0.2</td>
<td>0.35</td>
<td>3.15</td>
<td>0.92</td>
</tr>
<tr>
<td>4</td>
<td>9.86</td>
<td>63%</td>
<td>60%</td>
<td>0.1</td>
<td>0.15</td>
<td>0.12</td>
<td>0.36</td>
<td>4.11</td>
<td>0.94</td>
</tr>
<tr>
<td>4</td>
<td>9.69</td>
<td>63%</td>
<td>60%</td>
<td>0.1</td>
<td>0.15</td>
<td>0.12</td>
<td>0.36</td>
<td>4.04</td>
<td>0.94</td>
</tr>
<tr>
<td>4</td>
<td>8.93</td>
<td>63%</td>
<td>60%</td>
<td>0.1</td>
<td>0.15</td>
<td>0.12</td>
<td>0.36</td>
<td>3.72</td>
<td>0.94</td>
</tr>
</tbody>
</table>
Figure 7-25 Normalized $r_{\text{umax}}$ versus normalized effective stresses to 2.4 kPa
7.4 Model for Excess Pore Water Pressure Ratio, $r_u$

7.4.1 Overview

In Section 7.3 an empirical model was presented that can be used to predict maximum excess pore pressure ratio $r_{umax}$ in partially saturated sands. The model assumes that the number of cycles of application of strain is large enough to achieve the maximum value of the pore pressure ratio. However, it is well known that earthquakes with different magnitudes apply different number of cycles of load application. Hence, if the magnitude of a design event is small enough that the number of cycles of strain application is few and $r_{u}$ cannot be achieved, it is important to adjust $r_{umax}$ to yield smaller value of excess pore pressure ratio $r_u$ corresponding to the smaller magnitude earthquake.

This section presents the second stage of the model which estimates the excess pore water pressure ratio ($r_u$) generated in partially saturated sands that is consistent with a specific earthquake magnitude $M$.

Figure 7-4 presented an overall frame work of the empirical model that is developed. The components 2, 3, and 4 shown in Figure 7-4 relate to the portion of the model that reduces $r_{umax}$ to $r_u$ as a function of earthquake magnitude. Component 2 estimates the maximum number of cycles required to reach $r_{umax}$ ($N_{max}$) based on soil characteristics and shear strain amplitude. Component 3 expresses irregular strain history induced in the soil due to an earthquake with magnitude $M$ in terms of uniform strains with equivalent number of cycles $N$. Finally component 4 evaluates a general behavior of rate of $r_u$ ($r_u/r_{umax}$) vs normalized number of strain cycles ($N/N_{max}$).
7.4.2 Estimation of Number of Cycles Required to Reach Maximum Excess Pore Water Pressure Ratio $r_{\text{umax}}$: ($N_{\text{max}}$)

The number of cycles required to achieve $r_{\text{umax}}$ ($N_{\text{max}}$) were examined using the entire laboratory test. $N_{\text{max}}$ was observed to be influenced by parameters $S$, $D$ and $\gamma$ as well as initial vertical effective stresses. It has established that $S$, $D$ and $\gamma$ have effect on $r_{\text{umax}}$ and as indicated in Figure 7-26. The data also shows $N_{\text{max}}$ changes along with $r_{\text{umax}}$. Thus, instead of finding the effect of each parameter individually, $N_{\text{max}}$ was correlated to $r_{\text{umax}}$ at a constant vertical effective stress.

Test data for vertical effective stresses ($\sigma_v'$) at a range of 2.2-2.8 kPa were used in the estimation of $N_{\text{max}}$ and are presented in Table A-6 of Appendix A-1. For very low $r_{\text{umax}}$'s, $N_{\text{max}}$ was very difficult to identify in some test data since a distinct peak was not observed in the excess pore water pressure generation. A linear regression analysis was performed to correlate $N_{\text{max}}$ to $r_{\text{umax}}$ at an average effective stress of $\sigma_v'=2.5$ kPa and a fitted equation was developed:

![Figure 7-26 Excess pore water pressure generation in partially saturated sands tested](image-url)
\[ N_{\text{max}} = 88 \times e^{-3r_{\text{u,max}}} \]

The fitted regression curve and 68% prediction intervals are shown in Figure 7-27. Details of the regression analysis and results are presented in Appendix A-2.

Figure 7-27 \( N_{\text{max}} \) vs \( r_{\text{u,max}} \) at \( \sigma_v' = 2.5 \) kPa
Next, the effect of initial effective stress on $N_{\text{max}}$ was investigated. Evaluating the effect experimentally would require a numerous number of tests since tests are required to be performed in a various combination of ($S$, $D_r$, and $\gamma$). Also, with the current experimental setup, only effective stresses up to 10 kPa can be applied, so an extrapolation would be needed for higher effective stresses.

The effect of initial effective stress on number of cycles to initial liquefaction, $N_L$ at $r_{\text{umax}}=1$ in fully saturated sands has been well investigated by many researchers. In this research, the effect of effective stress on $N_{\text{max}}$ was developed by utilizing known relationship of $N_L$ with effective stress. The procedure adopted was as follows:

1. The relationship of $N_{\text{max}}$ vs. $r_{\text{umax}}$ developed for average $\sigma'_v$ of 2.5 kPa was normalized to $N_L$ ($N_{\text{max}}$ at $r_{\text{umax}} = 1$ in Figure 7-27). The relationship between $N_{\text{max}}$ and $N_L$ at 2.5 kPa is also demonstrated in Figure 7-28. This relationship was assumed to be the same for all other effective stresses.

\[
N_{\text{max}} = 88 \times e^{-3r_{\text{umax}}} \\
\frac{N_{\text{max}}}{N_{\text{max}-\sigma'_v=1}} = \frac{88 \times e^{-3r_{\text{umax}}}}{88 \times e^{-3\times1}} = 20 \times e^{-3r_{\text{umax}}}
\]

7-10
2. The effect of effective stress on \( N_L \) was determined using the experimental data from strain based liquefaction tests available in the literature (Dobry et al. 1982 (Figure 7-29), Hazirbaba et al. 2005 (Figure 7-30), Chang et al. 2007 (Figure 7-31)) and also using the data from fully saturated sand tests performed in this research. Numbers of cycles to liquefaction for different effective stresses at different strain levels for a range of relative densities of 35 to 45% were extracted from the plots in the literature and combined with the fully saturated sand test data from this research as tabulated in Table 7-4. Then the data was plotted for 3 different shear strains as demonstrated in Figure 7-32.

\[
\frac{N_{\text{max}}}{N_L} = 20 \times e^{-3\sigma_{v'}}
\]
Figure 7-29 Pore water pressure buildup as a function of number of cycles for Monterey No. 0 sand at $\sigma_3' = 2000$ psf ($\sigma_v' = 143$ kPa), $D_r = 45\%$ and various cyclic shear strains
Figure 7-30 Excess pore water pressure generation under $\sigma_v'=25$ kPa and $\sigma_v'=100$ kPa at $\gamma=0.294$ % (a) and $\gamma=0.1$ % (b) during cyclic direct simple shear tests (Hazirbaba 2005)
Figure 7-31 Excess pore water pressure generation from in-situ and cyclic direct simple shear (CDSS) tests (a) and as a function of cycle ratio from in-situ, and CDSS tests (b) (Chang et al. 2007)

Table 7-4 Number of Cycles to Liquefaction ($N_L$) for Different Strain Levels ($\gamma$) and Under Different Initial Vertical Effective Stresses ($\sigma'_v$)

<table>
<thead>
<tr>
<th>Reference</th>
<th>$\gamma$, %</th>
<th>$\sigma'_v$, kPa</th>
<th>$N_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This study</td>
<td>0.01</td>
<td>2.5</td>
<td>7</td>
</tr>
<tr>
<td>Chang et al.</td>
<td>0.01</td>
<td>12.7</td>
<td>74</td>
</tr>
<tr>
<td>Chang et al.</td>
<td>0.01</td>
<td>25</td>
<td>100</td>
</tr>
<tr>
<td>This Study</td>
<td>0.1</td>
<td>2.5</td>
<td>3</td>
</tr>
<tr>
<td>Chang et al.</td>
<td>0.1</td>
<td>25</td>
<td>11</td>
</tr>
<tr>
<td>Hazirbaba</td>
<td>0.1</td>
<td>100</td>
<td>74</td>
</tr>
<tr>
<td>Dobry et al.</td>
<td>0.1</td>
<td>143</td>
<td>100</td>
</tr>
<tr>
<td>Hazirbaba</td>
<td>0.3</td>
<td>25</td>
<td>2.6</td>
</tr>
<tr>
<td>Hazirbaba</td>
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<td>100</td>
<td>6.5</td>
</tr>
<tr>
<td>Dobry et al.</td>
<td>0.3</td>
<td>143</td>
<td>10</td>
</tr>
</tbody>
</table>
Figure 7-32 NL as a function of vertical effective stresses at 3 strain levels

Figure 7-32 indicates that there is a linear relationship between NL and vertical effective stress $\sigma_v'$. For each strain level, the following equations were fitted:

- For $\gamma=0.01\%$:
  \[ N_L = 4.36 \sigma_v' \]
- For $\gamma=0.1\%$:
  \[ N_L = 0.71 \sigma_v' \]
- For $\gamma=0.3\%$:
  \[ N_L = 0.07 \sigma_v' \]

Then, $N_L$ can be described as follows:

\[ N_L = g(\gamma) \times \sigma_v' \]  \hspace{1cm} 7-11\]

$g(\gamma)$ has a unit of $1/kPa$ and is shown in Figure 7-33. Final form of the $N_L$ function becomes as in Eqn. 7-12. Figure 7-34 demonstrates that $N_L$ function is in a good agreement with the experimental data.

\[ N_L = g(\gamma) \times \sigma_v' \]

\[ N_L = \left( 5.33 \times e^{-2011/\gamma} \right) \times \frac{1}{kPa} \times \sigma_v' \]  \hspace{1cm} \sigma_v' \text{ in kPa}  \hspace{1cm} 7-12\]
3. The relationship between $N_{\text{max}}$ and $N_L$ was obtained by normalizing $N_{\text{max}}$ function to $N_L$ at 2.5 kPa effective stress in step 1. It was assumed that this normalization is valid for other effective stresses. The assumption can be acceptable since it is a conservative assumption. The normalization may give higher $N_{\text{max}}$ values for higher effective stresses but is not expected to give lower values, physically.

Figure 7-34 Developed $N_L$ function and its comparison with experimental data
Along with the assumption, the \( N_L \) equation obtained as a function of effective stress is multiplied with the normalized function \( \frac{N_{\text{max}}}{N_L} \).

\[
\frac{N_{\text{max}}}{N_L} = \left(20 \times e^{-3r_{\text{max}}}ight)
\]

\[
N_L = \left(5.33 \times e^{-2011\gamma} \left(\frac{1}{kPa}\right)\right) \times \sigma_v',
\]

\[
N_{\text{max}} = \left(20 \times e^{-3r_{\text{max}}}ight) \left[5.33 \times e^{-2011\gamma} \left(\frac{1}{kPa}\right)\right] \times \sigma_v',
\]

\[
N_{\text{max}} = 107 \times e^{-\left(3r_{\text{max}} + 2011\gamma\right)} \left(\frac{1}{kPa}\right) \times \sigma_v', \quad \sigma_v' \text{ is in kPa.} \quad 7-13
\]

Eqn. 7-13 estimates \( N_{\text{max}} \) values at higher effective stresses. The unit of the vertical effective stress must be in kPa. Shear strain amplitude in the equation is related to \( N_L \), the effect of shear strain amplitude on \( N_{\text{max}} \) was already built in \( N_{\text{max}} \) versus \( r_{\text{umax}} \) relationship (because \( r_{\text{umax}} \) is a function of shear strain). Figure 7-35 shows the \( N_{\text{max}} \) values estimated for different effective stresses when \( \gamma = 0.1\% \).

This part of the model needs more research and test data to get a more accurate formulation of \( N_{\text{max}} \) at different effective stresses. Future research should focus on performing cyclic simple shear strain tests on partially saturated sands at high effective stresses.
Figure 7-35 $N_{\text{max}}$ versus $r_{\text{umax}}$ at different effective stresses
(Kramer, 1996)

\[ \gamma = \gamma_{cyc} = \gamma_{eqv} = R\gamma_{max} \]

where \( R = \frac{M-1}{10} \)  

(Idriss amd Sun, 1992)

So the equivalent strain level may be different than 0.65\( \gamma_{max} \) depending on the earthquake magnitude. Therefore, in this research, the concept of determining equivalent number of uniform cycles at stress (or strain) levels of 0.65\( \tau_{max} \) \((\gamma_{max})\) for an earthquake
with magnitude $M$ (Seed et al. 1975) was modified for also other strain levels which are determined in ground motion analysis to be equivalent strain levels based on the earthquake magnitude.

Figure 7-36 Concept of equivalent number of uniform cycles implemented in RuPSS model

The procedure can be summarized in three steps:

1. A fitted equation was derived for Seed's equivalent number of uniform stress cycles as a function of earthquake magnitude, which is $N_{eqv}(M)$ for $R=0.65$

2. A conversion equation $k(R)$ was derived to convert $N_{eqv}(M)$ for $R=0.65$ to $N_{eqv}(M)$ for other $R$'s. Also $R$, which is a function of $M$ in ground response analysis, is then inserted in the conversion equation $k(R): k((M-1)/10)$.

3. $N_γ$ is evaluated by the product of conversion equation $k((M-1)/10)$ with $N_{eqv}(M)$ for $R=0.65$
7.4.3.1 Seed's Equivalent Number of Uniform Stress Cycles for Liquefaction Analysis

Seed at al. (1975) determined the equivalent number of uniform stress cycles at a stress level of $0.65\tau_{\text{max}}$ from acceleration records of past earthquakes. The data is plotted in Figure 7-37. However they didn't define it in an equation form. An equation was required to perform further steps in the evaluation of $N_\gamma$. A linear regression analysis was performed on the natural logarithm of $N_{\text{eqv}}$ versus earthquake magnitude. A fitted equation (Eqn. 7-14) was obtained and is plotted in Figure 7-37.

$$N_{\text{eqv}} = 0.057e^{0.72M}$$  \hspace{1cm} 7-14

![Figure 7-37 Data of equivalent number of uniform stress cycles at 0.65τ_{max} stress level (Seed et al. 1975) and the fitted regression curve (current work)]
7.4.3.2 Equivalent Number of Uniform Stress Cycles at Other Stress Levels

The development of equivalent number of uniform cycles at $0.65\tau_{\text{max}}$ described in the previous section and also the conversion of equivalent number of cycles between different stress levels are obtained from the cyclic strength curve (Figure 7-38) from typical laboratory results on saturated sands for an average relative density of 65% as proposed by Seed et al. (1975)

![Cyclic Stress Ratio vs. Number of Cycles to Liquefaction](image)

Figure 7-38 Representative curve for relationship between cyclic stress ratio and number of cycles to liquefaction (Astunias and Dobry 1982, reproduced from Seed et al. 1975)

Astunias and Dobry (1982) modified the plot of Figure 7-38. They divided all the stress ratios by the stress ratio causing liquefaction in one cycle and they also stated that shear stress histories are equivalent to acceleration time histories.

$$\frac{\tau_p}{(\tau_p)_1} = \frac{a_p}{(a_p)_1}$$

7-15
Subsequently, they also defined a factor of safety in the relationship. So, the peak acceleration causing liquefaction in one cycle \((a_p)_1\) was multiplied by a factor of safety:

\[
(a_p)_{\text{max}} = \text{F.S.} \ (a_p)_1
\]

Figure 7-39 Representative relationship between \(a_p/(a_p)_{\text{max}}\) and number of cycles required to cause liquefaction (Astunias and Dobry, 1982)

Then, \(a_p/(a_p)_{\text{max}}\) curves were plotted for several factors of safety (Figure 7-39). For the development of equivalent number of uniform cycles at 0.65 \(\tau_{\text{max}}\), Seed et al. suggested and used factor of safety of 1.5. Hence, for the development of the conversion equation, the curve for factor of safety of 1.5 will be used herein.

The \(a_p/(a_p)_{\text{max}}\) curve for F.S=1.5 is used to get a conversion factor between different stress levels. For example, 2 cycles at \((\tau_p)_{\text{max}}\) is equivalent to 6 cycles at 0.65\((\tau_p)_{\text{max}}\). So conversion factor from \((\tau_p)_{\text{max}}\) to 0.65\((\tau_p)_{\text{max}}\) is 6/2=3.
As described in the beginning of this section, the ratios of maximum shear strain amplitude is defined with R. The same R can be also used to describe the stress ratio:

\[ R = \frac{\gamma_{\text{eqv}}}{\gamma_{\text{max}}} = \frac{\gamma_{\text{eqv}} \times G}{\gamma_{\text{max}} \times G} = \frac{\tau_{\text{eqv}}}{\tau_{\text{max}}} \]  

7-17

For R's from 0.4 to 1, the conversion factors from \( R\tau_{\text{max}} \) to 0.65\( \tau_{\text{max}} \) and from 0.65\( \tau_{\text{max}} \) to \( R\tau_{\text{max}} \), which is defined as k are determined and tabulated in Table 7-5.

**Table 7-5 Conversion Factors from Different Stress Levels to 0.65\( \tau_{\text{max}} \) and vice-versa**

<table>
<thead>
<tr>
<th>( R = \tau / \tau_{\text{max}} )</th>
<th>Conversion Factor from ( R\tau_{\text{max}} ) to 0.65( \tau_{\text{max}} )</th>
<th>Conversion Factor from 0.65( \tau_{\text{max}} ) to ( R\tau_{\text{max}} ), k</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.00</td>
<td>0.33</td>
</tr>
<tr>
<td>0.95</td>
<td>2.73</td>
<td>0.37</td>
</tr>
<tr>
<td>0.9</td>
<td>2.40</td>
<td>0.42</td>
</tr>
<tr>
<td>0.85</td>
<td>2.07</td>
<td>0.48</td>
</tr>
<tr>
<td>0.8</td>
<td>1.71</td>
<td>0.58</td>
</tr>
<tr>
<td>0.75</td>
<td>1.43</td>
<td>0.70</td>
</tr>
<tr>
<td>0.7</td>
<td>1.20</td>
<td>0.83</td>
</tr>
<tr>
<td>0.65</td>
<td>1.00</td>
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<td>0.6</td>
<td>0.68</td>
<td>1.47</td>
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<tr>
<td>0.55</td>
<td>0.38</td>
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</tr>
<tr>
<td>0.5</td>
<td>0.21</td>
<td>4.67</td>
</tr>
<tr>
<td>0.45</td>
<td>0.10</td>
<td>9.67</td>
</tr>
<tr>
<td>0.4</td>
<td>0.04</td>
<td>26.67</td>
</tr>
</tbody>
</table>

The relationship between conversion factors k and R is demonstrated in Figure 7-40 and an equation was fitted to the data as below:

\[ k(R) = 0.114 \times e^{\left(\frac{1}{R^{1.3}}\right)} \]  

7-18

Equation k(R) is the conversion equation to convert \( N_{\text{eqv}} \) for 0.65\( \tau_{\text{max}} \) to \( N_{\text{eqv}} \) for other R's.

\[ N_{\text{eqv-R}} = N_{\text{eqv-R=0.65}} \times k(R) \]  

7-19
Figure 7-40 Conversion factors versus stress (strain) ratio $R$

\[ k(R) = 0.114 \times e^{\left(\frac{1}{R^{0.7}}\right)} \]

Figure 7-41 Equivalent number of uniform cycles for stress (strain) ratios of $R=0.4-0.75$

$N_{eqv}$

Earthquake Magnitude, $M$
Eqn. 7-19 is shown in Figure 7-41 for R values of 0.4 to 0.75. When R=0.65, k(R) is 1.

7.4.3.3 Equivalent Number of Uniform Shear Strain Cycles Corresponding to the Given Earthquake Magnitude: (Nγ)

As stated earlier, in ground response analysis, an irregular strain record is characterized in terms of equivalent uniform cyclic shear strains whose amplitude is a ratio of the maximum shear strain of the irregular record. This ratio, which is basically the strain ratio that was defined previously as R, is related to earthquake magnitude M.

As stated previously, in ground response analysis, equivalent uniform cyclic shear strain of a given earthquake magnitude M is presented as below:

\[ \gamma = \gamma_{\text{cyc}} = \gamma_{\text{eqv}} = R \times \gamma_{\text{max}} \]

\[ R = \frac{\gamma_{\text{eqv}}}{\gamma_{\text{max}}} = \frac{M - 1}{10} \]

e.g. for M=8 \[ R = \frac{8 - 1}{10} = 0.7 \Rightarrow \gamma_{\text{eqv}} = 0.7 \times \gamma_{\text{max}} \]

In the previous section, k(R) conversion equation was introduced. R was the stress (strain) ratio and was not contributed to earthquake magnitude M. Now, R is introduced in the k(R) equation as a function of M:

\[ k(R) = 0.114 \times e^{\left(\frac{1}{4R}\right)} \quad \text{and} \quad R = \frac{M - 1}{10} \]

\[ k(M) = 0.114 \times e^{\left(\frac{10}{M-1}\right)} \]

Then equivalent number of uniform cyclic shear strains corresponding to an earthquake magnitude M is defined as \(N_\gamma\) and can be estimated by the product of \(N_{\text{eqv}}\) for R=0.65 and k(M) conversion equation:
\[
N_γ = N_{\text{equiv}} \times 0.65 \times k(M)
\]
\[
N_γ = 0.057 e^{0.72M} \times 0.114 \times e^{\left(\frac{10}{10 - M}\right)^{0.72}}
\]
\[
N_γ = 0.0065 \times e^{\left(\frac{10}{10 - M}\right)^{0.72} + 0.72M}
\]

The solid line in Figure 7-42 demonstrates \(N_γ\) versus earthquake magnitude \(M\). Dashed lines are \(N_{\text{equiv}}\) for different stress (strain) ratios \(R\) which can be used in liquefaction analysis. Since in ground response analysis, equivalent strain amplitude \((R\gamma_{\text{max}})\) is also related to the earthquake magnitude \(M\), the strain ratio \((R)\) is unique for that magnitude and there is only one equivalent number of uniform strain cycles corresponding to each earthquake magnitude.

Figure 7-42 \(N_γ\) versus magnitude \(M\)
Figure 7-43 Summary of $N_γ$ evaluation
Overall procedure of determining equivalent number of uniform strain cycles corresponding to an earthquake with magnitude M is summarized in Figure 7-43.

### 7.4.4 Rate of Excess Pore Water Pressure Ratio ($r_u/r_{u,max}$) versus $N_\gamma/N_{max}$

Previous discussion have been focused on the developed model providing estimates of maximum pore water pressure ratio, $r_{u,max}$, the number of cycles of shear strain that it would take to achieve $r_{u,max}$, $N_{max}$, and the number of uniform shear strains that a specific earthquake with magnitude M would generate, $N_\gamma$. What remains is to correlate $N_\gamma$ to development of $r_u$, or reduction in $r_{u,max}$ as a function of the ratio of $N_\gamma/N_{max}$ (Eqn 7-22). In Figure 7-44, a typical excess pore water pressure generation curve is shown, on which the definition of $N_\gamma$, $N_{max}$, $r_{u,max}$ and $r_u$ are indicated.

\[
\frac{r_u}{r_{u,max}} = v\left(\frac{N_\gamma}{N_{max}}\right)
\]

![Figure 7-44 A typical generation of excess pore water pressure ratio from experimental results](image)

Figure 7-44 A typical generation of excess pore water pressure ratio from experimental results
Using the preliminary experimental test data, \( r_u/r_{umax} \) vs. \( N/N_{max} \) data were plotted as shown in Figure 7-45.

![Graph showing ru/ru_max vs. N/N_max](image)

Figure 7-45 \( r_u/r_{umax} \) versus \( N/N_{max} \) evaluated from the preliminary experimental test data

The evaluated set of data shown in Figure 7-45 can be formulated with upper, lower and median boundaries. A trigonometric equation was determined to be best describing this formulation (Eqn. 7-22). The upper, lower and median boundaries are determined with the power of the function, \( \theta \). The relationship resulted in a band with median, upper and lower boundaries is illustrated in Figure 7-46.

\[
\frac{r_u}{r_{umax}} = v\left(\frac{N_{f}}{N_{max}}\right) = \left(\sin\left[\left(\frac{N_{f}}{N_{max}} - 0.5\right)\times\pi\right] + 1\right)^\theta
\]

7-23
where $\theta = 0.25$ for the lower boundary

$\theta = 0.54$ for the median

$\theta = 1.1$ for the upper boundary

Figure 7-46 Fitted median and 90% data interval curves for $r_u/r_{umax}$ vs $N_y/N_{max}$

The relationship developed for the prediction of $r_u$ was confirmed by the second stage set of test data, as shown in Figure 7-47.

It was interesting to note that even in shear strain controlled tests on fully saturated sands performed by Chang et al. (2007) in the field and laboratory, demonstrated analogous patterns, which are different than the $r_u$ model that Seed et al. (1975) proposed based on stress-controlled tests (Figure 7-48). In stress-controlled tests, strains induced in each loading cycle increases since increasing $r_u$ decreases the effective
Figure 7-47 Confirmation of the model for the rate of $r_u$ with data from the secondary set of test.

Figure 7-48 Pore pressure generation as a function of cycle ratio from in-situ and strain-controlled CDSS testing and Seed et al. (1975) model based on stress-controlled testing. (Chang et al. 2007)
stress, also decreases the stiffness G. It is known that the stiffness G is proportional to the square root of the mean effective stress, accordingly to the square root of \( r_u \). Then for lower \( r_u \)'s, since the reduction in G is not that significant, increasing rate of shear strains induced in each loading cycle is relatively small. However for higher values of \( r_u \), G decreases dramatically by causing increased rate in shear strains induced and \( r_u \). Since excess pore water pressure generation in the field is more related to or governed by the strain time histories (Dobry et al., Chang et al.), the rate of \( r_u \) should be related to strain-induced results.

### 7.5 Evaluation of Excess Pore Water Pressure Ratio, \( r_u \)

All the parts of the RuPSS model described in this chapter are combined to get the final \( r_u \) function as shown in Eqn 7-24. "\( r_u \)" will be reduced down from \( r_{umax} \) if \( N_\gamma \) is less than \( N_{max} \). How much it will reduce will be determined inserting \( N_\gamma/N_{max} \) ratio into the function \( v \) determined in the previous section. The ratio of \( N_\gamma/N_{max} \) can be obtained as in Eqn. 7-25. Knowing \( r_{umax} \), and the ratio \( N_\gamma/N_{max} \), excess pore water pressure ratio \( r_u \) can be determined from Eqn. 7-26.

\[
R_u = r_{umax} \times v\left(\frac{N_\gamma}{N_{max}}\right)
\]

\[
\frac{N_\gamma}{N_{max}} = \frac{0.0065 \times e^{\left(\frac{10}{M-1}\right)1.8+0.72M}}{107 \times e^{-(3r_{umax}+2011\gamma)} (1/kPa) \times \sigma_v' \times (1/kPa)}
\]

\[
\frac{N_\gamma}{N_{max}} = \frac{6.1 \times 10^{-5} \times e^{\left(\frac{10}{M-1}\right)1.8+0.72M+3r_{umax}+2011\gamma}}{\sigma_v' (1/kPa)}
\]

\[
r_u = r_{umax} \times \left(\sin\left(\frac{N_\gamma}{N_{max}} - 0.5 \times \pi\right) + 1\right)^{0.54}
\]
In the field, relative density of soils is typically determined with the means of standard penetration tests. Relative density is defined by SPT N value corrected for overburden stress, \((N_1)_{60}\).

Two of the common correlations between \((N_1)_{60}\) and relative density \(D_r\) provided in the literature are used in this study. Either of them can be used in estimating relative density parameter from the field data.

\[
D_r = 25\sigma'_v^{(-0.12)}(N_1)^{0.46}
\]

\(\sigma'_v\) is in kPa

Yoshida 1988

\[
D_r = 25\sigma'_v^{(-0.12)}(N_1)^{0.46}
\]

Table 7-6 Relative Density versus SPT N by (Das 2006)

<table>
<thead>
<tr>
<th>SPT ((N_1)_{60})</th>
<th>% Relative Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 5</td>
<td>0 – 5</td>
</tr>
<tr>
<td>5 – 10</td>
<td>5– 30</td>
</tr>
<tr>
<td>10 – 30</td>
<td>30– 60</td>
</tr>
<tr>
<td>30 – 50</td>
<td>60– 95</td>
</tr>
</tbody>
</table>
7.6 Numerical Example

The application of the model can be illustrated in an example problem:

Liquefaction analysis is performed on the site given above and the site was determined to be liquefiable. IPS mitigation is planned for the site and a design degree of saturation = 80% is selected. The magnitude of the earthquake is assumed to be 6.5. Ground response analysis is performed and the shear strain time history record at 5m depth (middle of the soil layer) is evaluated. The maximum shear strain is 0.25%. To ensure that the excess pore water pressures that may still be developed in the partially saturated sand are reasonable low, the pore pressure ratio \( r_u \) is required. The following steps describe how the proposed empirical model can be used to obtain \( r_u \).
1. Estimate relative density from SPT blow count \((N_1)_{60} = 10\):

According to Das:

\[
D_r = 30\% \text{ for } (N_1)_{60} = 10
\]

2. Estimate \(\gamma\) from the shear strain time history obtained in ground response analysis by Eqn 7-1:

\[
\gamma = \gamma_{max} \times R
\]

\[
R = \frac{M-1}{10} = \frac{6.5-1}{10} = 0.55
\]

\[
\gamma = 0.25\% \times 0.55
\]

\[
\gamma = 0.14\%
\]

3. Evaluate \(r_{\text{umax}}\) using \(S = 0.8\), \(D_r = 0.30\), and \(\gamma = 0.0014\)

\[
r_{\text{umax}} = f(S, D_r = 20\%, \gamma = 0.1\%) \times F_D(S, D_r) \times F_\gamma(S, \gamma)
\]

\[
f = (0.8)^{0.5} \times e^{-[1-0.8]^{4}} = 0.88
\]

\[
F_D = 1 - 8.75 \times (0.3 - 0.2) \times (1 - 0.8) \times e^{-\left(\frac{(1-0.8)^2}{2 \times (1-0.84 \times (0.2) \times 0.3)^{0.25}}\right)} = 0.88
\]

\[
F_\gamma = 1 - 1.75 \times (-\log\left[\frac{0.0014}{0.001}\right]) \times (1 - 0.8) \times e^{-\left[-3.1(1-0.8)^2\right]} = 1.05
\]

\[
r_{\text{umax}} = f \times F_D \times F_\gamma
\]

\[
= 0.8
\]
4. Estimate $N_{\text{max}}$ from Eqn. 7-13:

Calculate the initial effective stress:

$$\sigma'_{v} = (\gamma' - \gamma_{w}) \times 5m$$
$$\sigma'_{v} = (2 \times 9.81 - 9.81) \times 5m$$
$$\sigma'_{v} = 49 \text{ kPa}$$

Eqn. 7-13:

$$N_{\text{max}} = 107 \times e^{-(3r_{\text{umax}} + 2011 \gamma')} \times \left( \frac{1}{\text{kPa}} \right) \sigma'_{v}$$

For $r_{\text{umax}} = 0.8$ and $\sigma'_{v} = 49 \text{ kPa}$ and $\gamma' = 0.0014$

$$N_{\text{max}} = 107 \times e^{-(3(0.80) + 2011(0.0014))} \times \left( \frac{1}{\text{kPa}} \right) (49\text{kPa})$$

$$= 29 \text{ cycles}$$

5. Estimate $N_{\gamma}$ from Eqn. 7-21 for $M=6.5$:

$$N_{\gamma} = 0.0065 \times e^{-\left[ \frac{10}{M-1} \right]^{1.8} + 0.72M}$$

$$= 13.2 \text{ cycles}$$

6. Evaluate $N_{\gamma}/N_{\text{max}}$:

$$\frac{N_{\gamma}}{N_{\text{max}}} = \frac{13.2 \text{ cycles}}{29 \text{ cycles}} = 0.46$$

7. Evaluate $r_{u}/r_{\text{umax}} = v(N_{\gamma}/N_{\text{max}})$ from Eqn. 7-23:

$$v\left(\frac{N_{\gamma}}{N_{\text{max}}}ight) = \left( \sin \left[ \frac{N_{\gamma}}{N_{\text{max}} - 0.5} \times \pi \right] + 1 \right)^{0.54}$$
For $N_{r}/N_{\text{max}}=0.46$

$$v\left(\frac{N_{r}}{N_{\text{max}}}\right) = \left(\frac{\sin \left[ \frac{13.2}{29} - 0.5 \times \pi \right] + 1}{2}\right)^{0.54}$$

$$\frac{r_{u}}{r_{u\text{max}}} = v\left(\frac{N_{r}}{N_{\text{max}}}\right) = 0.63$$

8. Evaluate $r_{u}$ at the middle of the soil layer:

$$r_{u} = 0.63r_{u\text{max}}$$

$$r_{u} = 0.63 \times 0.8$$

$$r_{u} = 0.5$$

Therefore, when the given sand is mitigated by IPS by lowering the degree of saturation down to 80%, excess pore water pressure ratio will be at 0.5, and liquefaction is not likely to occur at the site.
7.7 Summary

An empirical model was developed to predict earthquake-induced excess pore water pressure ratios \( r_u \) in partially saturated sands. The model primarily was developed to demonstrate the benefit of Induced Partial Saturation (IPS) in preventing the occurrence of liquefaction. The model is applicable for fully saturated liquefiable sand sites mitigated by IPS as well as naturally occurring partially saturated sand sites.

The parameters affecting \( r_u \) are determined to be degree of saturation \( S \), relative density \( D_r \), vertical effective stress \( \sigma_v' \), shear strain amplitude \( \gamma \) and earthquake magnitude \( M \). The following steps were followed in the development of the proposed model:

1. An empirical equation was developed to predict maximum excess pore pressure ratio \( r_{umax} \) as a function of degree of saturation, \( S \) and relative density, \( D_r \), cyclic shear strain, \( \gamma \). The model was developed using extensive data base generated running cyclic simple shear tests using a shaking table. The equation for estimation of \( r_{umax} \) is;

\[
r_{umax} = f(S, D_r = 20\%, \gamma = 0.1\%) \times F_D(S, D_r) \times F_\gamma(S, \gamma)
\]

\[
f = (S)^{0.5} \times e^{-\left[1-\frac{-1}{0.54}\right]^4}
\]

\[
F_D = 1 - 8.75 \times (D_r - 0.2) \times (1 - S) \times e^{-\left[\frac{(1-S)^2}{2(1-0.84(0.2/D_r)^{0.25})^2}\right]}
\]

\[
F_\gamma = 1 - 1.75 \times (-\log \frac{\gamma}{0.001}) \times (1 - S) \times e^{-\left[\frac{-3.1(1-S)^2}{2}\right]}
\]
2. An extension of the model was introduced to permit the estimation of excess pore pressure that considers in addition to the parameters listed above, earthquake magnitude M. The magnitude of an earthquake will have an effect on the conversion of earthquake-induced maximum shear strains to equivalent uniform cyclic shear strains, as well as on the number of cycles of application of the shear strain.

First, the number of cycles required to reach $r_{\text{umax}} (N_{\text{max}})$ was related to $r_{\text{umax}}$. $N_{\text{max}}$ was controlled by parameters $S$, $D_r$, $\gamma$ and $\sigma_v'$. Since the effect of $S$, $D_r$, and $\gamma$ were already built in $r_{\text{umax}}$, $N_{\text{max}}$ was related to $r_{\text{umax}}$ for a constant initial effective stress of 2.5kPa. The variation of $N_{\text{max}}$ with effective stress was estimated by normalizing $N_{\text{max}}$ function to $N_L$, the number of cycles required to achieve $r_u = 1$ (initial liquefaction). Then, the effect of effective stress on $N_L$ was evaluated from cyclic shear strain tests on fully saturated sands and from available data in the literature. Finally, $N_{\text{max}}$ was estimated as below:

$$N_{\text{max}} = 107 \times e^{-(3r_{\text{umax}}+2011\gamma)} \times \left(\frac{1}{kPa}\right)\sigma_v'$$

3. Earthquake-induced irregular strain histories are represented by uniform strain cycles with an equivalent uniform cyclic strain amplitude ($\gamma_{\text{eqv}}$ or $\gamma_{\text{cyc}}$) and equivalent number of cycles ($N_\gamma$). In the evaluation of $N_\gamma$ two concepts were combined:

a. The concept of representative equivalent number of uniform cycles at $0.65\tau_{\text{max}}$ (or at R=0.65) : $N_{\text{eqv}}$ for liquefaction analysis by Seed et al. (1975)

b. Conversion of $N_{\text{eqv}}$ at R=0.65 to $N_{\text{eqv}}$ at other R$\gamma_{\text{max}}$ where R is related to the earthquake magnitude M in ground response analysis.
$N_\gamma$ was estimated as:

$$N_\gamma = 0.0065 \times e^{\left[\frac{10}{M-1} \times 1.8 + 0.72M\right]}$$

4. The ratio of $N_\gamma$ to $N_{\text{max}}$ is evaluated:

$$\frac{N_\gamma}{N_{\text{max}}} = \frac{6.1 \times 10^{-5} \times e^{\left[\frac{10}{M-1} \times 1.8 + 0.72M + 3r_{\text{umax}} + 2011\gamma\right]}}{\sigma_{v}'(1/\text{kPa})}$$

5. The rate of excess pore water pressure ratio ($r_u/r_{\text{umax}}$) is correlated to $N_\gamma/N_{\text{max}}$ through $v$ function:

$$\frac{r_u}{r_{\text{umax}}} = v\left(\frac{N_\gamma}{N_{\text{max}}}\right) = \left[\sin\left(\frac{N_\gamma - 0.5 \times \pi}{N_{\text{max}} - 0.5 \times \pi}\right) + 1\right]^{0.54}$$

6. Excess pore water pressure ratio $r_u$ is estimated by the product of function $v$ and $r_{\text{umax}}$.

$$r_u = r_{\text{umax}} \times v\left(\frac{N_\gamma}{N_{\text{max}}}\right)$$

$$r_u = r_{\text{umax}} \times \left[\sin\left(\frac{N_\gamma - 0.5 \times \pi}{N_{\text{max}} - 0.5 \times \pi}\right) + 1\right]^{0.54} \times 2$$

where,

$$\frac{N_\gamma}{N_{\text{max}}} = \frac{6.1 \times 10^{-5} \times e^{\left[\frac{10}{M-1} \times 1.8 + 0.72M + 3r_{\text{umax}} + 2011\gamma\right]}}{\sigma_{v}'(1/\text{kPa})}$$

and $r_{\text{umax}}$ is estimated in step 1.
In summary, the model was presented in two stages. First stage can be used to estimate the maximum excess pore water pressure ratio \((r_{umax})\) which is presented in section 7.3. If it is of interest to determine further reduction in excess pore water pressure ratio due to a specific design earthquake magnitude, second stage of the model that is presented in section 7.4 and 7.5 can be used.

Finally, \(r_{umax}\) or \(r_u\) can be evaluated using the proposed model in partially saturated sands either mitigated by IPS, or naturally occurring. The model can be also used to assess the required degree of partial saturation for a site based on an allowable excess pore water pressure generated at the time of a design earthquake.
8. Summary and Conclusions

This dissertation presented research conducted on exploring the feasibility and sustainability of a new liquefaction mitigation technique proposed as "Induced Partial Saturation" (IPS), and developing a new methodology for the estimation of the benefit of IPS in liquefaction prevention. IPS was introduced earlier in a preliminary research reported in Eseller (2004) and Yegian et al. (2007), and its potential for liquefaction prevention was demonstrated. The research reported in this dissertation aimed at implementing an advanced experimental and analytical research program to evaluate IPS as an effective measure for the prevention of liquefaction and for limiting the build up of excess pore pressures in loose sands. To achieve these research goals, the following investigations were completed:

1. Large-scale test setups were designed and manufactured in order to assess the short- and long-term sustainability of gas/air entrapped in sand specimens prepared by IPS. Air-entrapped vertical soil columns with 120-150 cm height were tested under hydrostatic, low to high upward and downward hydraulic gradients and horizontal excitation. Also, an air-entrapped 192 cm horizontal soil column was tested under low to high horizontal hydraulic gradients. The sustainability of entrapped air in the specimen was demonstrated through visual quantification of escaped air as well as by interpreting the coefficient of permeability of the specimen which is a function of degree of saturation.
2. An advanced integrated experimental setup was designed and built to perform cyclic simple shear strain tests on fully and partially saturated sands. A cyclic simple shear liquefaction box (CSSLB) was manufactured which can induce uniform shear strains in specimens with the use of a shaking table. The box accommodates various type of transducers used in this research. Miniature pore water pressure transducers PDCR 81 were used for measuring excess pore water pressures in sand specimens during cyclic tests. A linear variable displacement transducer (LVDT) was fixed in CSSLB in the direction of shaking to evaluate shear strains induced on the sand specimens.

3. A new laboratory IPS technique was explored to prepare uniform partially saturated sand specimens at controllable degrees of saturation. It was discovered that a special chemical compound (sodium perborate) which is a main ingredient of the dental product "Efferdent" produces oxygen gases when it gets into reaction with water. Partially saturated sand specimens were prepared by raining evenly dispersed dry sand and powdered Efferdent mixtures in predetermined amounts of water. Additionally, the effect of gas bubbles in the specimens on pore water pressures measurements was assessed.

4. A multiple bender element and bending disk facility was established to perform P and S wave measurements in large fully saturated sand specimens as well as in large partially saturated sand specimens prepared by IPS. A comprehensive literature research was performed and all the effects of various
parameters on receiving and interpreting quality wave signals were explored. Measurements were performed at multiple bender elements and bending disks located at various locations of CSSLB for examining the uniformity of soil properties of the specimens prepared, as well as to investigate whether or not the P wave velocity measurements can be correlated to the various degrees of saturation obtained by IPS. Such a correlation was intended to be developed since it would lead to the interpretation of IPS benefit in terms of P wave velocities, which can be a useful tool for field applications.

5. An experimental research program was conducted to investigate the excess pore water pressure generations in gas-entrapped specimens prepared by IPS and to understand the effect of various parameters on the excess pore water pressures generated in partially saturated sand specimens. Cyclic simple shear strain tests were performed on fully saturated and gas-entrapped specimens prepared in CSSLB. The individual effects of soil parameters, which are degree of saturation, relative density, and applied shear strain amplitudes, on the excess pore water pressure generation were explored based on the experimental results.

6. An empirical model (RuPSS) was developed to predict excess pore water pressure ratios in partially saturated sands based on the results obtained from the cyclic simple shear strain tests. RuPSS was developed as a mathematical model that incorporates five governing parameters which are: degree of saturation,
relative density, shear strain amplitude, initial effective stress, and earthquake magnitude.

The completed research tasks described above led to the following conclusions:

1. Experimental tests demonstrated that gas/air entrapped within saturated sand remains in the sand even under large horizontal and vertical hydraulic gradients, as well as large horizontal vibrations similar to earthquake ground motions. These results are very promising with respect to the practicality of the proposed liquefaction measure, IPS.

2. The new induced partial saturation technique explored by mixing Efferdent powder with sand and raining them into water yielded partially saturated sand specimens with uniformly distributed gas bubbles. Also different degrees of saturation can be achieved by introducing various Efferdent-to-sand ratios.

3. The uniformity of specific soil characteristics (relative density, degree of saturation) in large laboratory specimens can be assessed with the established multiple bender element and bending disk measurement experimental setup. Wave measurements confirmed that uniform sand specimens can be achieved with the new IPS technique devised.
4. Contrary to published information (Ishihara et al. (2004) and Yang (2002)), P wave velocity $V_p$ can not be used as a measure or indicator of degree of saturation below S=90-95%. P-wave velocities decrease dramatically from a fully saturated value (compressive velocity of water) to the compressive velocity of the soil skeleton, at about $S = 90\%$. Therefore, measurement of $V_p$ can not be an effective field monitoring technique to determine degree of saturation in sands, or to evaluate the liquefaction resistance of partially saturated sands, as is proposed by other researchers.

5. According to the cyclic simple shear strain tests, excess pore water pressures generated in partially saturated sands never reach to the level of initial liquefaction. In other words, maximum excess pore water pressure ratio ($r_{umax}$) never reaches 1, which is the case in fully saturated sands. Excess pore water pressure generated in a partially saturated sand reaches a maximum value and remains steady under continued application of shear strain cycles.

6. The benefit of IPS to liquefaction prevention is provided by two factors: 1) by decreasing maximum excess pore water pressure ratio ($r_{umax}$), 2) by increasing number of cycles required to reach maximum value ($r_{umax}$). Maximum excess pore water pressure ratio $r_{umax}$ in partially saturated sands depends on degree of saturation, relative density and induced cyclic simple shear strain amplitude. It decreases with:

- decreasing degree of saturation,
• increasing relative density,
• decreasing shear strain amplitude.

7. The effect of relative density and shear strain amplitudes are independent from each other, however their individual effects on \( r_{\text{umax}} \) are dependent on the level of degree of saturation. For example \( r_{\text{umax}} \) in a denser material reduces more quickly than in a loser material, as the degree of saturation decreases.

8. Number of cycles required to reach maximum excess pore water pressure ratio in partially saturated sands (\( N_{\text{max}} \)) depends also on the degree of saturation, relative density, shear strain amplitude applied and initial effective stress. \( N_{\text{max}} \) increases with:

• decreasing degree of saturation,
• increasing relative density,
• decreasing shear strain amplitude.
• increasing initial effective stress

9. The developed empirical model RuPSS can predict excess pore water pressure ratios in partially saturated sands either naturally occurring in that condition or mitigated by implementing IPS in a fully saturated liquefiable sand site. The model has two components. The first component provides estimate of maximum excess pore water pressure ratio (\( r_{\text{umax}} \)) within an error range of \( \pm 0.082 \) with 68% confidence interval. This parameter is a function of degree of saturation, \( S \),
relative density, $D_r$, and induced shear strain, $\gamma$. Maximum excess pore water pressure ratio $r_{umax}$ is the ultimate value that a partially saturated sand having a specific $S$, $D_r$, and $\gamma$ may experience, regardless of the number of cycles of application of shear strain, which typically is a function of earthquake magnitude. It is noted that for typical number of cycles of earthquake excitations, the excess pore water pressure ratios ($r_u$) in the partially saturated sands are much smaller than the maximum excess pore water pressure ratios ($r_{umax}$). The second component of the model incorporates the earthquake magnitude or information on the number of strain cycles and provides estimates of excess pore water pressure ratio ($r_u$), which will be equal or smaller than $r_{umax}$ depending on the earthquake magnitude.

10. The RuPSS model can be used for analysis of a real site to estimate the design level degree of saturation which needs to be induced by IPS for allowable excess pore water pressure ratios in a project site, or to predict excess pore water pressure ratios for a planned degree of saturation level. RuPSS is also applicable for naturally partially saturated sand sites to predict excess pore water pressure ratios that can be induced under a design earthquake.

The research reported in this dissertation has clearly demonstrated that IPS does prevent the occurrence of liquefaction in representative laboratory sand specimens. The empirical model developed provides predictions of excess pore pressures for different site conditions and soil properties, thus providing great benefit in the implementation of IPS.
Therefore, the IPS technique holds great promise for wide application at project sites as a cost-effective liquefaction mitigation measure. For IPS to be accepted in earthquake engineering practice as a liquefaction mitigation measure further research need to be performed on exploring a field delivery system of uniform gas bubbles in liquefiable sites, and a verification technique to ensure that the design of IPS measure is properly implemented.
References


Deniz, O., (2008) "Bender Elements and Bender Disks for Measurement of Shear and Compressional Wave Velocities in Large Sand Specimens", *Master’s Thesis*, Northeastern University, Boston, MA.

Dyvik, R., & Madshus, C., (1985) "Lab Measurements of $G_{\text{max}}$ using Bender Elements", 
*Advances in the Art of Testing Soils Under Cyclic Conditions; Proc. ASCE,* 

Emerson, M., & Foray, P., (2006) "Laboratory P-wave Measurements in Dry and 


Finn, W.D. L., Byrne, P. M., & Martin, G. R., (1976) "Seismic Response and 
Liquefaction of Sands", *Journal of the Geotechnical Engineering Division, ASCE,* 
Vol. 102, No. GT8, August.

Liquefaction", *Journal of the Geotech. Eng. And Div., ASCE,* Vol.103, No. GT6, 
pp.517-533.


Gokyer, S., (2009) "Inducing and Imaging Partial Degree of Saturation in Laboratory Sand Specimens", *Master’s Thesis*, Northeastern University, Boston, MA.


*NIST/SEMATECH e-Handbook of Statistical Methods*


Ortakci, A.E., (2007) "Design and Manufacturing of a Cyclic Simple Shear Liquefaction Box (CSSLB)", Master’s Thesis, Northeastern University, Boston, MA.


APPENDICES
APPENDIX A
# Appendix A-1 Tables of Test Data and Analysis Results

Table A-1 Table of Cyclic Simple Shear Strain Test Results of Fully Saturated Sands

<table>
<thead>
<tr>
<th>Test</th>
<th>Dr, %</th>
<th>$\gamma$, %</th>
<th>Frequency, Hz</th>
<th>$\sigma'$, kPa</th>
<th>$r_u$</th>
<th>$N_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS 1</td>
<td>33%</td>
<td>0.0055</td>
<td>10</td>
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<td>0.41</td>
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* Data not used in N_{max} estimation due to uncertainty in maximum value selection
Table A-5 Statistical Error Analysis of the Modified rumax Model
S

Dr

γ,%

f

FD

Fγ

75%
74%
73%
73%
73%
72%
72%
72%
71%
70%
70%
69%
69%
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68%
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41%
86%
84%
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47%
21%
31%
35%
38%
47%
51%
52%
55%
57%
59%
61%
64%
64%
66%
67%

0.055
0.016
0.1
0.2
0.011
0.0516
0.105
0.205
0.052
0.108
0.206
0.0106
0.0516
0.107
0.205
0.011
0.052
0.108
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0.015
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0.2
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0.1
0.2
0.013
0.046
0.09
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0.02
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0.2
0.052
0.1
0.2
0.011
0.051
0.1
0.2
0.01
0.05
0.1
0.2

0.82
0.82
0.81
0.80
0.80
0.80
0.79
0.79
0.77
0.76
0.76
0.74
0.74
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0.77
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0.88
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0.78
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0.99
1.08
0.92
1.00
1.09
0.70
0.91
1.00
1.09
0.69
0.91
1.00
1.10

250

Model
rumax=rm
0.71
0.52
0.73
0.77
0.40
0.53
0.56
0.61
0.40
0.44
0.44
0.21
0.31
0.34
0.31
0.05
0.08
0.09
0.10
0.21
0.25
0.16
0.20
0.23
0.24
0.12
0.17
0.19
0.18
0.09
0.12
0.13
0.13
0.77
0.75
0.74
0.76
0.51
0.50
0.53
0.30
0.36
0.36
0.37
0.20
0.27
0.24
0.25

Test
rumax=ra
0.76
0.37
0.77
0.78
0.30
0.41
0.51
0.64
0.24
0.31
0.45
0.10
0.14
0.17
0.26
0.01
0.03
0.05
0.08
0.16
0.25
0.13
0.16
0.22
0.28
0.06
0.09
0.17
0.22
0.02
0.06
0.09
0.14
0.80
0.85
0.84
0.89
0.68
0.63
0.69
0.30
0.39
0.39
0.44
0.14
0.20
0.20
0.28

(ra-rm)

(ra-rm)2

(ra-ȓa)2

0.05
-0.16
0.04
0.01
-0.10
-0.12
-0.06
0.03
-0.16
-0.13
0.01
-0.12
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0.00
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0.04
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-0.07
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0.03

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Appendix A-2 Linear Regression Analysis Results for $N_{\text{max}}$

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Fitted Regression Equation: $y = -3.05x + 4.48$

Equations used in Linear Regression Analysis

\[
S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2
\]

\[
SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2
\]

\[
MSE = \frac{SSE}{n-2}
\]

\[
\sigma = \frac{SSE}{n}
\]

\[
s = \frac{SSE}{n-2} = MSE
\]

\[
SST = \sum_{i=1}^{n} (y_i - \bar{y})^2
\]

\[
R^2 = 1 - \frac{SSE}{SST}
\]

\[
\sigma_{\hat{y}-y} = \sigma \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{S_{xx}}}
\]

Prediction Interval of $(1-\alpha)100\%$

\[
y - t_{\alpha/2} (\sigma_{\hat{y}-y}) < y < y + t_{\alpha/2} (\sigma_{\hat{y}-y})
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Mean of $x$, $\hat{x}$: 0.51
Mean of $y$, $\hat{y}$: 2.92

Sample Size, $n$: 61

Sum of the Square of Differences of $x_i$s from $\hat{x}$, $S_{xx}$: 5.54

Sum of Squared Errors, SSE: 14.5

Model Standard Deviation, $\sigma$: 0.49

Mean Squared Error, MSE or unbiased estimate of $\sigma$, $s$: 0.49

Total Corrected Sum of Squares, SST: 66

Coefficient of Determination, $R^2$: 0.78

Standard Deviation of predicted $y$ from the fitted $\hat{y}$: $\sigma_{\hat{y}}$
### Results for 68% Prediction Interval

$t_{α/2}$ for $n-2=47$ is $1$

\[ \hat{y} = \ln(N_{\text{max}}) \]

\[ N_{\text{max}} = e^{\hat{y}} \]

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Derivation of Scaling Factor Function $F_D$

Based on the observations stated in Chapter 7, following steps were taken to develop scaling factor function $F_D$

**Step 1:** Several mathematical model functions were examined and the best fit equation which meets the observed variation of test data was found to be the Rayleigh distribution equation:

\[ y = \frac{x}{b^2} e^{-\frac{x^2}{2b^2}} \]  

The Rayleigh distribution equation is a simple exponential equation which has coefficient "b" whose effect on the shape of the function will be explained in the following steps. Rayleigh equation is shown in Figure B-1 for a random value of $b=0.18$.

![Figure B-1 Rayleigh distribution equation for a random coefficient $b=0.18$]
Step 2: Rayleigh equation was modified and coefficient b in the denominator of x was replaced with another coefficient β:

\[ y = \frac{x}{\beta} \times e^{\frac{-x^2}{2b^2}} \]

Step 3: According to the boundary conditions, Eqn. B-2 is adapted for \( F_D \) vs S as follows:

\[ x = (1 - S) \]

BC: \( F_D = 1 \) when \( S = 1 \)

\[ F_D = 1 - \frac{(1 - S)}{\beta} \times e^{\frac{-(1-S)^2}{2b^2}} \]

The plot of \( F_D \) versus S is shown in Figure B-2 for random values of \( \beta = 0.03 \) and \( b = 0.18 \).

Step 4: Further understanding of the equation and its coefficients \( \beta \) and \( b \) was needed in order to implement the variable \( D_r \) into the function. As stated in the observations in Chapter 7, there is a value of degree of saturation (\( S_{\text{maxD}} \)) where the variation from base function \( f \) is maximum. To find \( S_{\text{maxD}} \) and \( F_{D_{\text{max}}} \), the derivative of the function "\( F_D \)" is calculated. This will also determine the role of coefficient "\( b \)" and \( \beta \) in the location of \( S_{\text{maxD}} \) and amplitude of \( F_{D_{\text{max}}} \).
Figure B-2 $F_D$ Eqn. B-3 versus $S$

$$F_D = 1 - \frac{(1 - S)}{\beta} \times e^{\left(\frac{-(1-S)^2}{2b^2}\right)}$$

$$F_D' = \frac{1}{\beta} \times e^{\left(\frac{-(1-S)^2}{2b^2}\right)} - \frac{(1-S)}{\beta} \times \frac{2(1-S)}{2b^2} \times e^{\left(\frac{-(1-S)^2}{2b^2}\right)} = 0$$

$$\frac{1}{\beta} \times e^{\left(\frac{-(1-S)^2}{2b^2}\right)} \times \left[1 - \frac{(1-S)^2}{b^2}\right] = 0$$

$$(1 - S)^2 = b^2$$

$$S_{maxD} = 1 - b$$

$$F_{D_{max}} = 1 - \frac{(1-(1-b))}{\beta} \times e^{\left(\frac{-(1-(1-b))^2}{2b^2}\right)}$$

$$= 1 - \frac{b}{\beta} \times e^{[-0.5]}$$

B- 4
The derivation results demonstrate that "b" term governs the location of S for the maximum amplitude of $F_D$ ($S_{max_D}$), as well as it governs the amplitude of the $F_D$. "1/β" term governs only the amplitude of $F_D$. Since the amplitude of the $F_D$ equation needed to be formulated with the parameter $D_\alpha$, first 1/β was defined as a function of $D_r$.

\[
b = (1 - S_{max_D})
\]

\[
F_D = 1 - \frac{(1 - S)}{\beta} \times \exp\left(\frac{-(1-S)^2}{2\beta^2}\right)
\]

1/β should be 0 at $D_r$ is equal to 0.2 also should increase as $D_r$ increases. So that $F_D$ eventually "rumax" decreases as $D_r$ increases. The form of the equation then becomes as in equation 7-11 and is also plotted in Figure B-3 for c is equal to 1.

\[
\frac{1}{\beta} = c \times (D_r - 0.2)^{nd}
\]

Figure B-3 1/β: as a function of relative density $D_r$
"nd" other than 1 was determined to give $r_{\text{umax}}$ out of the boundary conditions. So $nd=1$ gave the values within the boundaries. "c" is an unknown coefficient and was determined later through statistical analysis. Thus, $F_D$ function can be written as below:

$$F_D = 1 - c \times (D_r - 0.2) \times (1 - S) \times e^{-\frac{(1-S)^2}{2\times(1-S_{\text{max}})F_b^2}}$$

B-6

Figure B-4 $F_D$ as in Eqn. B-6

**Step 5:** Based on the observations of the test data, it was observed that $S_{\text{maxD}}$ is not the same for all relative densities. That's why $b$ term or in other words $(1-S_{\text{maxD}})$ term in the exponential part was also defined as a function of $D_r$. So the final form of $F_D$ equation becomes:

$$F_D = 1 - c \times (D_r - 0.2) \times (1 - S) \times e^{-\frac{(1-S)^2}{2\times(1-S_{\text{max}})F_b^2 \times D_r^{y_b} \times y_b}}$$

B-7
where $c = 8.88$

$S_{\text{maxD}} = 0.84$

$n_b = 0.25$

Figure B-5 Final form of scaling factor function $F_D$