Seismic Response Reconstruction and Evaluation of Nonlinear Static Analysis Procedures Using Data from Instrumented Buildings

A Dissertation Presented

by

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In Nonlinear Static Procedures (NSP) the maximum structural responses are predicted from the result of pushover analyses in which the structure reaches a “target displacement” determined from the response of an” equivalent” SDOF system to the seismic loading, defined as an elastic spectrum. This study evaluates the predictive capability of NSP using seismic data recorded on instrumented buildings and examines the issue of seismic response reconstruction. In connection with the second item it is shown that commonly used interpolation schemes are tantamount to fitting the response into a basis whose span is dictated by the number and location of sensors. A scheme, designated as the Minimum Norm Response Corrector (MIRC), which estimates the response as that of a nominal model subjected to the seismic excitation plus a set of pseudo forces that enforce the measurements is presented. It is found that MIRC provides estimates of the response that are significantly more accurate than that from interpolation schemes, especially for shears and drifts.

The single and multi-mode NSPs are evaluated by contrasting the predictions with the “true” values inferred from the measurements using MIRC. It is shown that the single-mode procedures provide reasonably accurate estimates of all responses in the short building but poor prediction of responses when the contribution from higher modes is substantial, i.e., shears and drifts in the tall buildings. An energy consistent (EC) SDOF is developed and is shown to result in more accurate predictions of roof (target) displacement than those used in the typical NSP. Incorporation of the EC SDOF into single and multi-mode procedures is presented as an alternative NSP and is evaluated. It
is shown that while multi-mode procedures significantly improve the predictions in the tall buildings, a single-mode procedure using the EC SDOF and two load patterns, namely, one from the maximum linear story shears and the other from the code loading, gives prediction that are on par with those from multi-mode procedures.
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CHAPTER 1

INTRODUCTION

1.1 Overview

Many existing buildings in seismically active regions were designed and constructed before modern understanding of earthquake engineering and advanced analysis tools became available. Many of these buildings may pose economic and life safety threats in seismic areas like California, therefore, it is of public interest to identify them and take appropriate corrective action. An assessment based on satisfying current design codes, which are intended to be conservative for new construction, imposes unnecessarily tough requirements on the existing buildings. Since demolition and rebuilding of all the stock is not economically feasible, a case by case in-depth performance assessment is necessary. An ideal performance assessment would be achieved by carrying out a series of nonlinear dynamic analysis (NDA) under an ensemble of ground motions that represent the hazard. Such undertaking, even with the present state of analytical tools, is considered impractical for routine applications.

The procedures that have emerged as a compromise between code-based linear analysis and sophisticated nonlinear dynamic analysis are the Nonlinear Static Procedures (NSP). The NSP has been central to performance-based design strategies in which
performance is expressed in terms of anticipated damage under given level of seismic hazard and has been widely used by practitioners since the late 90’s. In the NSP’s the requirement of a detailed modeling of structural nonlinear behavior is retained but the dynamic analysis is replaced by monotonic pushover analyses. The fundamental assumption in development of NSP is that the maximum response of the structure to a given ground shaking can be obtained from configurations where the structure, pushed laterally under some prescribed load patterns, reaches a certain displacement known as the “target displacement” or the “performance point”. The “target displacement” is an estimate of the maximum roof displacement and is determined from a SDOF system often called the “Equivalent SDOF”. The NSP that have been developed differ primarily in the procedure to determine the target displacement and in how to perform the pushover analysis. Two prominent NSPs, namely, the Capacity Spectrum Method (ATC-40, 1996) and the Coefficient Method (FEMA 356, 2000) received considerable attention and their publication as seismic rehabilitation guidelines lead to a substantial increase in the use of NSP.

Despite widespread use, the simplified underlying assumptions in the development of NSP have long raised question about their accuracy and have made them subject to a number of evaluation studies. Various researchers that have looked into the accuracy of NSP have noted limitations which, needless to say, stem from the fact that the nature of the NSP is static and the dynamic response does not fit in any static analysis (Krawinkler and Seneviratna, 1998; Neim and Lobo, 1999; Antoniou and Pinho, 2004a and Chopra and Goel, 2006). In light of reports of significant discrepancies in the response prediction of NSPs when applied to the same condition (Aschheim et al. 1998, Chopra and Goel
the Applied Technology Council (ATC) carried out an extensive study to examine the two NSP in vogue, namely, CSM (ATC 40, 1996) and CM (FEMA 356, 200) and proposed the improved versions thereof in FEMA 440 (2006). Moreover, some researchers have developed advanced NSP that attempt to offer improvements to the early methods at the expense of increasing complexity (Chopra and Goel 2002, Chopra et. al. 2004, Kalkan and Kunnath 2006).

1.2 Fundamentals and Classification of Nonlinear Static Procedures

The fundamental underpinning of NSP is that the response of a structure to monotonically increasing lateral loads, when it reaches a configuration defined by “target displacement”, is a prediction of the maximum inelastic response under earthquake excitation. The “target displacement” is determined by subjecting the “equivalent” SDOF system to the expected ground motion hazard which, in NSP, is typically represented with response spectra. Engineering demand parameters such as story displacements, inter-story drifts, story shears and other local responses are subsequently determined by pushing the structure to reach the predicted maximum roof displacement.

The NSP that rely on only one SDOF reduction to predict the structural responses are known as single-mode NSP. Multi-mode NSPs involve a number of pushover analyses with loads proportional to the mode shapes. For each pushover analysis a separate SDOF reduction and target displacement is determined. Responses from each pushover analysis at the corresponding target displacement are combined to give the total predictions of the
structural responses. Multi-mode NSP operates on the assumption that the classical modal analysis can be extended to inelastic systems (Chopra and Goel, 2002).

With respect to the load patterns used in the pushover analysis, the NSP are classified into conventional and adaptive methods. Conventional methods are those in which the pattern of the lateral loads is unchanged while the magnitude monotonically increases. Adaptive methods are those in which the load pattern changes as inelasticity propagates. The variant loads are determined based on “instantaneous mode shapes” of the structure determined from eigenvalue analysis.

1.3 Motivation

In spite of the growing number of instrumented buildings and ample recorded motions, there are few studies to evaluate the effectiveness of NSP in light of observations from real buildings. Most of the work that has been done to evaluate the current NSP or to develop and validate the new NSP is based purely on analytical studies where the nonlinear dynamic analysis (NDA) is taken as the benchmark with which the NSP is contrasted. Analytical studies using NDA, however, does not give a clear indication of the effectiveness of the NSP in real applications. The reason is that the uncertainties that exist in real situations regarding structural modeling are not accounted for. Instrumented buildings that have already experienced some strong motions offer useful information and provide a great opportunity for evaluation of the effectiveness of NSP in practice. The information from instrumented buildings allow the real situation to be emulated by applying the NSP to the building, taking the recorded motion as the future design earthquake, and comparing the NSP predictions with recorded data. Motivated by
availability of seismic recordings and detailed information of buildings, this study utilizes the data obtained at 5 reinforced concrete buildings in California to make observations about the predictive capability of standard NSPs (adopted by codes and guidelines) and to assess the effectiveness of some advanced NSPs that have been proposed to offer some improvements.

Using measured seismic response to evaluate the NSP is not as straightforward as it may seem. Instrumentation is sparse along the height of the building and the response quantities of interest are not directly measured; it is necessary, therefore, to resort to estimations and infer the unmeasured responses from the measurements. Estimation theory is a mature field in control but research pertaining to application in structural engineering problems is limited. The traditional approach to this problem in earthquake engineering has been to estimate the response of unmeasured floors by interpolation between the measurements (Lui et al. 1990, De la Llera and Chopra 1995, Naeim 1998, Limongelli 2003, Naeim et. al., 2006). A perception that error from interpolation is always small is sometimes reflected in the literature through references to “measured base-shear” or “measured inter-story drift” even though it is evident that these quantities are not measured but are inferred from the measurements. Bernal and Nasseri (2007 and 2009) and Goel and Nishimoto (2009) have shown that error from interpolation in many situations can be large enough to make the estimated responses misleading. In this study response estimation is carried out using procedure adopted from estimation theory.
1.4 Objective, Scope, and Approach of the Study

Objective

The main objective is to evaluate the effectiveness of nonlinear static analysis procedures in predicting various responses of structures in light of the real seismic data recorded at five instrumented buildings. Another objective is to identify or develop an estimation scheme that best suits the problem of seismic response reconstruction.

Scope

The study consists of two major parts:

1. Reconstruction of the full response of instrumented buildings from available measurements.
2. Application of NSP to the instrumented buildings and evaluation of their predictive capability by comparing with the reconstructed response.

To achieve the main objective, namely, evaluation of the predictive capability of NSP using real seismic data, this study examines a wide range of standard and advanced NSP. Emphasis is placed on the procedures that are more likely to be endorsed for practical applications. Justification for this decision is made clear by recalling that the primary incentive of development of NSP was to avoid complexity of the nonlinear dynamic analysis in routine applications. Needless to say if the level of sophistication in application of a NSP is too high one may prefer to do a nonlinear dynamic analysis.

Moreover, the study looks into the fundamental assumptions and components of NSP to identify the possibility of further improvements. An energy-consistent SDOF reduction is developed and is incorporated into single-mode and multi-mode procedures (called EC
NSP throughout this report). Current options for lateral load patterns and alternatives to offer possible improvement in NSP are examined.

Instrumental evaluation involves comparison of the most desired engineering demand parameters, namely story displacements, inter story drifts, story shears and overturning moment with their benchmark values inferred from real seismic measurements. The following non-adaptive NSP are included in the evaluations:

- Equivalent Linearization (ATC 40 and FEMA 440 methods)
- Coefficient Method (FEMA 356 and FEMA 440 methods)
- Modal Pushover (Chopra and Goel 2002)
- Modified Modal Pushover (Chopra and Goel 2004)
- EC NSP (Hernandez-Montes, 2004 and the versions proposed in this study)

To achieve the objective of finding the best estimation scheme for response reconstruction a major part of this study is dedicated to the estimation problem. State estimation, known as a problem of finding the “best estimate” of the state trajectory of a dynamic system, is looked into. In control engineering the system is represented by a model whose dynamic response to a given input can be determined mathematically but due to the presence of unmeasured disturbances and/or unknown initial conditions the analytically computed state does not match the state of the real system. A primary need for state estimation in control arises in design of a state feedback controller which is used to enhance the performance or stabilize a system. Since only a subset of the states are available (measured), an “estimator” is used to provide an estimate of the full state using the measurement of the input and some outputs of the system. The estimator is often
called an “observer” (Luenberger, 1964); many variation of the observer have been developed for different purposes. A particular form of the observer that best applies when the main source of uncertainty in the state is due to unmeasured noise is the Kalman Filter (Kalman, 1960) which gives optimal estimates of the state provided that the noise is zero mean and white and its covariance is known. The situation that prevails in structural engineering is one where the main source for error between a model’s estimate and the real system is due to the error in the model, a premise that is different from those in the original observer development. This study examines the use of the classical observers and some variations of them in estimation of the seismic response from the measurements and explores the possibility of adapting the classical methods for the problem at hand. An estimation method that applies the state estimation formulation in the context of structural engineering is developed which is based on determination of a set of corrective forces that drive the response of the nominal model to the measured responses.

In summary this study presents an estimation method for response reconstruction and uses the best estimates of the response from available measurements to critically evaluate the predictive capability of NSPs.

1.5 Organization of the Dissertation

This document is organized as follows. Chapter 1 provides an introduction including the objectives, scope and approach and presents a literature review. Chapter 2 extensively discusses the SDOF reduction where the procedures used by NSP are examined and an “energy consistent” SDOF reduction is developed. Chapter 3 presents the details and
illustrative examples for the single and multi-mode NSPs. Also, the “EC NSP” that incorporates the energy consistent SDOF reduction in the single and multi-mode procedures are introduced and illustrated in Chapter 3. Seismic response reconstruction is presented in Chapter 4 where data driven basis fitting and model based estimation schemes are studied. Furthermore, the estimation method developed in this study designated as the Minimum Norm Response Corrector (MIRC) is presented in Chapter 4. The chapter ends with a simulation study to identify the best applicable scheme for seismic response reconstruction. Chapter 5 presents application of the NSP to 5 instrumented buildings in California. Performance of the procedure is examined in the individual buildings and the aggregate of the results are quantified statistically. Chapter 6 addresses the possible approaches for improving the NSP. Current options for load patterns and possible alternatives are examined in a simulation study with nonlinear dynamic analysis. Based on the observations a suggested approach for improving the NSP is examined in the instrumented buildings. The document closes in Chapter 7 with a summary and conclusion of the research.

1.6 Literature Review

History and Development of NSP

Inception of nonlinear static analysis is often attributed to the work of Freeman et al. (1975) in developing a rapid seismic evaluation method which is considered as the origin of the “Capacity Spectrum Method”. Saiidi and Sozen (1981), Fajfar and Fischinger (1989) proposed a method in which the response of a MDOF system was determined from nonlinear dynamic analysis of an equivalent SDOF system. A breakthrough in the
development of simplified nonlinear analysis approaches occurred in the late 90’s with introduction of the two prominent NSP namely, “Capacity Spectrum Method” (CSM) and “Coefficient Method” (CM). The Capacity Spectrum Method was first introduced in the seismic evaluation guideline ATC-40 (1996) and is based on equivalent linearization idea in which response of a nonlinear oscillator is approximated by response of a linear SDOF with longer period and larger damping. The Coefficient Method was first introduced in FEMA 273 (1997) and was further developed and published as a pre-standard for seismic rehabilitation of buildings in FEMA 356 (2000). In CM, the maximum nonlinear displacement is obtained by adjusting the equal displacement rule. The two methods were later updated in FEMA 440 (2006). Fajfar (1999 and 2002) proposed the “N2” method which is similar to the CSM but differs in using inelastic response spectra instead of the elastic response spectra.

Recognizing the inadequacy of single-mode NSP in predicting the responses of structures that are dominated by higher modes, researchers attempted to account for these effects in the NSP. One of the earliest contributions to incorporate higher modes in NSP was made by Paret et al. (1996) and Sasaki et al. (1998) who proposed “Multi-Modal Pushover” (MMP) that requires performing several pushover analyses with load patterns proportional to the different mode shapes. Moghadam and Tso (2002) developed a method of “Pushover Result Combination” (PRC) that uses several pushover analyses under load patterns proportional to mode shapes and takes the total response as weighted sum of the results for each modal pushover analysis. Chopra and Goel (2002) proposed “Modal Pushover Analysis” based on the assumption of extending modal analysis to inelastic system and neglecting the coupling. In MPA the “$n^{th}$ mode” response is obtained
from response of a pushover analysis (under load patterns proportional to the mode shapes) at a roof displacement determined from nonlinear time history analysis of the “$n^{th}$ mode” SDOF. Total responses are obtained from SRSS modal combination rule. Possible reversal in pushover curve corresponding to higher mode sometimes cause problems. Chopra et al. (2004) proposed a modified version of the MPA (called MMPA) in which responses at higher modes are considered linear and therefore the reversal problem is avoided. Hernandez-Montes et al. (2004) proposed an energy-based method that follows the MPA procedures but uses an energy method to establish the force-displacement relationship of the SDOF system.

Some researchers have also proposed adaptive pushover methods in which the lateral loads during pushover analysis are recalculated using the “instantaneous mode shapes” that are determined from eigen-value analysis at each step that nonlinearity occurs. Bracci et al. (1997) is among the first who proposed an adaptive pushover method. Gupta and Kunnath (2000) developed an adaptive pushover method in which the load pattern is constantly updated with changes in the instantaneous dynamic properties and scaled using a site specific response spectrum. Elnashai (2001) and later Antoniou and Pinho (2004a) proposed an approach in which the adaptive load pattern at each step is computed from a normalized shape obtained from the combined modal story forces and scaled with spectral acceleration for the modal period at that step. Another adaptive modal pushover called “Adaptive Modal Combination” (AMC) was introduced by Kalkan and Kunnath (2006) that integrates the concepts of the CSM, MPA and the adaptive method of Gupta and Kunnath (2000). Evaluation studies of conventional and adaptive pushover methods with nonlinear response history analysis have shown that the adaptive variants does not
provide major advantages over the conventional methods except for some minor improvement in global responses (Papanikolaou et al., 2005).

**Review of the Past Evaluations of NSP using Instrumental Data**

Goel (2003) evaluated the coefficient method of FEMA 356 and the MPA method using data recorded at 4 buildings that demonstrated nonlinear behavior during Northridge 1994 earthquake. He concluded that FEMA 356 method significantly underestimated the drifts at upper floors and overestimated the drifts at lower floors and added that the uniform load distribution lead to the largest errors in the prediction of drifts. The study also showed that the MPA provided much better predictions of drift. Goel (2004) used the same buildings to evaluate coefficient method of FEMA 356, MPA and the adaptive spectra-based method of Gupta and Kunnath (2000). The study made the same conclusions regarding FEMA 356 and MPA but added that the adaptive method compared to the FEMA 356 method resulted in smaller underestimation of the upper floors drift but larger overestimation on the lower level drifts. Goel and Chadwell (2007) evaluated ATC-40 and FEMA 356 methods as well as their modification in FEMA 440 using data recorded at 5 buildings. They concluded that NSP sometimes overestimated and sometime underestimated the peak roof displacement and reported that the modified methods in FEMA 440 did not necessarily provide improvements. Cardone (2007) experimentally evaluated the CSM, CM and the N2 method using a 1:3.3 scale model of a 3 story building in a shaking table and reported that all the methods successfully predicted the maximum responses and observed little influence of load patterns on predictions. All instrumental studies have employed interpolation to estimate the benchmark responses (except Cardone, 2007 which used experimental data).
CHAPTER 2

MDOF TO SDOF REDUCTION

2.1 Overview

Many simplified inelastic analyses that have been proposed (Pique, 1976; Saiidi and Sozen, 1981; Fajfar and Fischinger, 1989) are based on using a nonlinear SDOF system to estimate the response of the real structure. The NSP also relies on formulation of a SDOF representation of the structure often called in the literature the “Equivalent SDOF”. This chapter addresses the problem of SDOF reduction; the approach used by typical NSP is looked into and an energy based approach for SDOF reduction is developed and examined through comparison with nonlinear dynamic analysis.

2.2 MDOF to SDOF Reduction in Nonlinear Static Procedures

The SDOF reduction used in simplified procedures including the NSP is based on assuming a displacement pattern and subsequently replacing the consistent restoring force with a prescribed load distribution. To begin, let’s recall the equation of motion for an inelastic MDOF system (for simplicity the damping term is not included)

\[ M\ddot{u} + R(u) = -Mra_g \]  (2.1)
where $R(u)$ is the restoring force of the inelastic system which is a function of the displacement vector. Displacements of the MDOF can be fully described by a matrix of $N$ (size of the system) independent vectors, $S$, and $N$ generalized displacements, $Q$, namely $u = S Q$. By prescribing the displacement using only a single shape, $s$, one gets

$$u = s q + \varepsilon$$

(2.2)

where $\varepsilon$ is a vector showing the error due to truncation of the other $n-1$ vectors of the $S$. Substituting 2.2 in 2.1 and remembering that the derivatives of the error vector with respect to time are $\dot{\varepsilon}$ and $\ddot{\varepsilon}$ one gets

$$M s \ddot{q} + R(q) = -M r a_g - (M \ddot{\varepsilon} + g(\varepsilon))$$

(2.3a)

$$M s \ddot{q} + R(q) = -M r a_g - E(t)$$

(2.3b)

$g(\varepsilon)$ represents the error in the restoring force due to the truncation and definition of $E$ is evident. Pre-multiplying Eq.2.3b by a weighting row vector $\varphi'$ for which $\varphi'E = 0$ gives

$$\varphi' M s \ddot{q} + \varphi' R(q) = -\varphi' M r a_g$$

(2.4)

$$\ddot{q} + \frac{\varphi' R(q)}{\varphi' M s} = -\frac{\varphi' M r a_g}{\varphi' M s}$$

(2.5)

In standard NSP the imposed shape to convert the MDOF to SDOF is taken as the 1st mode shape namely $u = \phi_q q$ and the weighting vector $\varphi'$ is taken as $\phi_1'$. Recalling the
definition of modal mass and participation factor from classical modal analysis (Eq.2.6)

Eq.2.5 can be written as Eq.2.7

\[ M_n = \phi_n^i M \phi_n \]
\[ L_n = \phi_n^i Mr \]
\[ \Gamma_n = \frac{L_n}{M_n} \]

\[ \ddot{q} + \frac{\phi^i_n R(q)}{M_1} = -\Gamma_1 a_g(t) \]  \hspace{1cm} (2.7a)

\[ \ddot{D} + \frac{\phi^i_n R(q)}{L_1} = -a_g(t) \]  \hspace{1cm} (2.7b)

where \( D \) is the generalized displacement of the equivalent SDOF and is equal to \( \frac{q}{\Gamma_1} \). The term \( R(q) \) is a nonlinear function of \( q \) and to obtain an explicit expression for the second term one needs to examine the relation between \( R(q) \) and \( q \) using a displacement control nonlinear analysis in which the \( q \) is incrementally increased. Such displacement control analysis, however, is too cumbersome and one could consider the possibility of re-framing the problem in terms of a load control analysis. Of course, if one could determine the loading that gives the displacement shape assumed in derivation of Eq.2.7 then the displacement and the load control alternatives would give identical results. In a typical NSP, \( R(q) \) is approximated by the applied loads \( (P) \) in a load control pushover analysis which are taken proportional to the 1\(^{st}\) mode shape namely \( P = pM\phi_1 \) with \( p \) being an incrementally increasing scalar.
Following this assumption a relationship between the resisting force term, 

\[ F_r = \frac{\phi^R(q)}{L_1} \]

and \( D \) can be established by multiplying and dividing the resisting force by \( L_1 \) (using Eq. 2.6); one gets

\[ F_r = \frac{\phi^r (pM \phi)(\phi^r M_r)}{L_1 (\phi^r M_r)} \tag{2.8} \]

Recognizing that \( p \phi^r M_r \) is the base shear in the pushover analysis, it follows that

\[ F_r = \frac{V_b M_1}{L_1^2} \quad \text{or} \quad F_r = \frac{V_b}{EMM_1} \tag{2.9} \]

where the \( EMM_1 \) is the first mode effective modal mass. The generalized SDOF displacement, \( D \), and the roof displacement of the MDOF system are related by Eq.2.10

\[ D = \frac{u_{\text{roof}}}{\Gamma_1 \phi_1, \text{roof}} \tag{2.10} \]

In summary the relationship between the restoring force and the generalized displacement of the equivalent SDOF is established by converting the pushover curve of the MDOF system \((V_b-u_{\text{roof}})\) using Eqs.2.9 and 2.10. Damping term can be conveniently added using a damping ratio \( \zeta \) and frequency, \( w \) defined by the \( F_r-D \) relationship i.e. at the yield point of the bilinear representation of \( F_r-D \) curve.

\[ w = \frac{\sqrt{F_{ry}}}{\sqrt{D_y}} \tag{2.11} \]
The final form of the equation of motion for the “equivalent” SDOF system is

\[ \ddot{D} + 2\zeta \omega \dot{D} + F_r = -a_g(t) \]  

(2.12)

### 2.3 An Energy Consistent MDOF to SDOF Reduction

The MDOF to SDOF reduction described in the previous section relies on the assumption that displacement and lateral load are proportional to \( \phi_1 \). Inconsistency in these assumptions is realized by noting that in pushover analysis other load patterns e.g. code based or triangular could be used and that the displacement shape continuously changes. In this section a MDOF to SDOF reduction is developed, on the basis of maintaining equal energy, which avoids presumption on displacement shape and load patterns. The fundamental assumption in developing the energy consistent SDOF is that the displacement profile of the structure in the dynamic response has a one to one correspondence with a displacement shape of the pushover analysis under known loads.

#### 2.3.1 Development and Formulation

Assume that a MDOF system has a displacement configuration that is fully specified by a single parameter \( Y \) through \( u(t) = \Psi(Y)Y(t) \), where the shape \( \Psi \) varies with \( Y \). The SDOF system is characterized by \( Y \) and has the property such that the work, \( W_s \), defined by the area under the force-deformation relationship at \( Y \) is equal to the work of the lateral loads in the pushover analysis, \( W_m \) at the corresponding displacements. Moreover the kinetic energy of the SDOF system, \( E_s^k \), is equal to that of the MDOF system, \( E_m^k \),
which is independent of the path. Equation of motion of such SDOF system is written as follows.

\[ M_e \ddot{Y} + R_e = -L_e a_g(t) \]  \hspace{1cm} (2.13)

where the coefficients in the equation \( M_e, R_e \) and \( L_e \) are respectively the equivalent mass, equivalent restoring force and participation factor.

2.3.1.1 Equivalent Restoring Force, \( R_e \) and the Generalized Displacement \( Y \)

Let \( P \) be the pushover load vector and \( u \) the displacement vector of the MDOF system. The total work done by pushover loads is

\[ W_m = \int P^t du \]  \hspace{1cm} (2.14)

and the work of the SDOF system is

\[ W_s = \int R_e dY \]  \hspace{1cm} (2.15)

We take \( R_e \) to be equal to the base shear of the MDOF system, \( V_b \), and determine the generalized displacement \( Y \).

\[ R_e = V_b \]  \hspace{1cm} (2.16)

Substituting Eq.2.16 in Eq.2.15 and setting it equal to Eq.2.14 gives the generalized displacement, \( Y \), as follows. For simplicity the subscript is dropped.

\[ \int V_b dY = W \]  \hspace{1cm} (2.17)
Taking the derivative of Eq.2.17 with respect to $Y$ one gets

$$V_b = \frac{dW}{dY} \quad \Rightarrow \quad dY = \frac{dW}{V_b} \quad (2.18)$$

$$Y = \int \frac{dW}{V_b} \quad (2.19)$$

Eqs.2.16 and 2.19 establish the generalized force - displacement relationship of the equivalent system. The tangent stiffness of this equivalent system, $K_t$, which is equal to $\frac{dV_b}{dY}$ can be obtained as follows

$$\frac{dW}{dY} = \frac{dW}{dV_b} \frac{dV_b}{dY} \quad (2.20)$$

from 2.18 we know that $\frac{dW}{dY} = V_b$ therefore

$$K_t = \frac{V_b}{\frac{dW}{dV_b}} \quad (2.21)$$

Eqs.2.19 and 2.21 can be evaluated numerically at each step of the pushover analysis by replacing $dW$ and $dY$ with incremental values, $\Delta W$ and $\Delta Y$. Using Eqs.2.18 and 2.19 one gets

$$\Delta Y(k) = \frac{W(k+1) - W(k)}{V_b} \quad (2.22a)$$

$$Y(k+1) = Y(k) + \Delta Y(k) \quad (2.22b)$$
Where \( \bar{V}_b \) is the average value of \( V_b \) between step \( k \) and \( k+1 \) with the exception of the 1st step in which it is equal to \( V_b(1) \).

### 2.3.1.2 Equivalent Mass, \( M_e \)

Equivalent mass, \( M_e \), is determined by setting the kinetic energy of the MDOF and SDOF systems equal. In a general derivation, regardless of the displacement shapes of the MDOF system, kinetic energy of the MDOF and SDOF systems namely \( E_m^k \) and \( E_s^k \) can be written as follows

\[
E_m^k = \frac{1}{2} \ddot{u}' M \ddot{u} \quad (2.23a)
\]

\[
E_s^k = \frac{1}{2} M_e \dot{Y}^2 \quad (2.23b)
\]

Using the chain rule the velocity term, \( \ddot{u} \) can be written as

\[
\frac{du}{dt} = \frac{du}{dY} \frac{dY}{dt} \quad (2.24a)
\]

\[
\ddot{u} = \frac{du}{dY} \dot{Y} \quad (2.24b)
\]

Replacing 2.24b in 2.23a and setting it equal to 2.23b one gets

\[
\frac{1}{2} \left( \frac{du}{dY} \right)' M \left( \frac{du}{dY} \right) \dot{Y}^2 = \frac{1}{2} M_e \dot{Y}^2 \quad (2.25)
\]

which gives the equivalent mass as
Eq. 2.26a is the general form for the energy consistent mass of the equivalent SDOF system. Note that in a pushover analysis the displacement vector $u$ is known at each step and the generalized displacement $Y$ can be determined from 2.22 and therefore the equivalent mass $M_e$ can be numerically computed by replacing the differential values $du$ and $dY$ with incremental values $\Delta u$ and $\Delta Y$ in (Eq.2.26b).

If the relationship between the displacement vector $u$ and the generalized displacement $Y$ through the variant shape vector $\Psi(Y)$ is accounted for explicitly, one can alternatively express the previous equations as follows

$$
\frac{du}{dY} = \frac{d\Psi}{dY} Y + \Psi
$$

(2.27)

Plugging Eq.2.27 into Eq.2.26 leads to the following equation for the equivalent mass

$$
M_e = \left(\frac{d\Psi}{dY} Y + \Psi\right)' M \left(\frac{d\Psi}{dY} Y + \Psi\right)
$$

(2.28)

2.3.1.3 Equivalent Participation Factor, $L_e$

To determine $L_e$, the work done by the external load over an incremental displacement in the MDOF and SDOF systems at given time, should be set equal. These works are respectively
\[ \Delta W_{ext}^m = \Delta u' M_r a_g \] (2.29)

\[ \Delta W_{ext}^s = L_e a_g \Delta Y \] (2.30)

setting the two equations equal one get

\[ L_e = -\frac{\Delta u' M_r}{\Delta Y} \] (2.31)

All the terms in Eq. 2.31 including the incremental displacements are either known or can be obtained from previous equations. Hence, \( L_e \) can be numerically computed at each step of the pushover analysis. If the relation of the generalized displacement and the displacement vector of the MDOF is explicitly accounted for, one can also write

\[ \Delta u = (\Psi + \frac{d\Psi}{dY}) \Delta Y (Y + \Delta Y) - Y \Psi \] (2.32a)

\[ \Delta u = \Delta Y (\Psi + \frac{d\Psi}{dY} Y) \] (2.32b)

Substituting Eq. 2.32b in Eq. 2.31 leads to the following expression for \( L_e \)

\[ L_e = -(\Psi + \frac{d\Psi}{dY}) Y' M_r \] (2.33)

To compare with the SDOF system in typical NSP, equation of motion of the “energy consistent” SDOF should be written in the form of Eq. 2.12. Dividing Eq. 2.13 by \( M_e \)

\[ \ddot{y} + \frac{V_s}{M_e} = -\frac{L_e}{M_e} a_s(t) \] (2.34)

and defining
\[ D = \frac{Y}{(\frac{L_c}{M_c})} \]  

(2.35a)

\[ F_{re} = \frac{V_h}{M_c(\frac{L_c}{M_c})} = \frac{V_h}{L_c} \]  

(2.35b)

one gets

\[ \ddot{D} + \frac{V_h}{L_c} = -a_g(t) \]  

(2.36)

A damping term can also be added using a damping ratio \( \zeta \) and a frequency \( \omega \) defined in Eq.2.37 at the yield point of a bilinear representation for \( F_{re} - D \) relationship.

\[ w = \sqrt{\frac{F_{re}}{D_y}} \]  

(2.37)

the final form of the SDOF reduction in the same form of Eq.2.12 is

\[ \ddot{D} + 2\zeta \omega \dot{D} + F_{re} = -a_g(t) \]  

(2.38)

2.3.2 Illustrative Example

The example is the 13 story Sherman Oaks building, one of instrumented buildings used in Chapter 5. The energy consistent SDOF system (EC SDOF) is shown for pushover analysis using 1st mode and code-based load patterns. Fig.2.1 shows the pushover curve in terms of base shear vs. roof displacement together with the generalized force (base shear) vs. generalized displacement \( (Y) \) of the energy consistent SDOF system. Different initial slope (and period) is due the fact that the generalized
displacement $Y$ does not necessarily coincides with the roof displacement. The energy consistent formulation implies that the physical location of the $Y$ along the height (assuming the structure is continuous) varies during the pushover analysis. This is illustrated in Fig.2.2a where the hypothetical location of the generalized displacement as a fraction of the total height is shown at different stages of the pushover analysis. Fig.2.2 also shows the variation of $M_e$ as a fraction of total mass and also variation of the ratio of $L_e/M_e$ for the two load patterns used in pushover analyses.

![Graphs showing pushover versus generalized force displacement and variation of energy consistent parameters.]

Fig.2.1. Pushover versus generalized force displacement curve a) Inverted triangular load pattern b) first mode shape proportional load pattern.

![Graphs showing variation of energy consistent parameters.]

Fig.2.2. Variation of energy consistent parameters during the pushover analysis a) Location of the generalized displacement along the height b) Effective mass b) Ratio of effective participation factor to mass.
The noteworthy point in Fig.2.2 is that the parameters of the energy consistent SDOF do not change significantly. Therefore, in solving Eq.2.38 or 2.13, with good approximation, these values can be taken constant i.e. equal to their average values. It should be noted that solution of Eq.2.47, even if variation of these parameters are taken into account, is not computationally cumbersome. The effect of variation can be incorporated into the $F_{re} - D$ relationship (see Fig.2.3). To compare the energy consistent SDOF with the “equivalent” SDOF used in typical NSP the equivalent restoring force versus generalized displacement is shown in Fig.2.3.

![Comparison of the equivalent restoring force in Energy Consistent (EC) and NSP equivalent SDOF](image)

**Fig.2.3.** Comparison of the equivalent restoring force in Energy Consistent (EC) and NSP equivalent SDOF a) code based load pattern b) first mode proportional load pattern.

While there is a notable difference in Fig.2.3a between EC and NSP for the code based, the plots are almost identical when the lateral load is proportional to the 1st mode shapes. The reason is that in this example when the loads are proportional to the 1st mode shape, the assumptions used in NSP to develop the SDOF system are not far from true. Therefore an energy consistent approach and NSP result in almost the same equivalent SDOF systems.
2.4 Evaluation of “Equivalent” SDOF Systems

The SDOF systems are evaluated in a simulation study with two 3 and 9 story frames. The roof displacements of the MDOF systems determined from nonlinear dynamic analysis for a suit of ground motions are taken as the benchmark and predictions of the roof displacement from SDOF systems whose force-deformation relationship is established from pushover analysis under predefined load patterns are compared with the benchmarks. In addition to the energy consistent SDOF (Eq.2.38), the “equivalent” SDOF systems conforming to formulations of typical simplified inelastic methods (Eq.2.12) are also considered. Note that for the latter the SDOF defined by ATC-40 (1996) which is identical to procedure that led to Eq.2.12 and the SDOF defined by FEMA 356 (2000), which only differs in using Eq.2.39 instead of Eq.2.10 in defining the restoring force of the SDOF, are used.

\[ F_r = \frac{V_b}{W/g} \Gamma_1 \]  
(2.39)

2.4.1 Specification of the Example Structures and Analyses Conditions

The examples are 2D frames of a 3 story and a 9 story steel structures. The two structures are intermediate frames of two frequently used model buildings originally developed in the SAC project (FEMA335C, 2000). Figs.2.4 and 2.5 show the specification of the frames. Effective Seismic mass and gravity loads on the frames shown in Table 2.1 and a typical 5% damping are used in the analyses. The 1st natural period of the 3 and 9 story structures are respectively 0.8 and 1.4 seconds. Models of the
structures are developed in Perform 3D (CSI, 2006) and the nonlinear behavior is modeled by elasto plastic hinges determined with a yield stress of 50ksi. The structures are simulated with the E-W component of the 30 ground motions (Table 5.1 for ground motion info) that are scaled to result in preset roof drifts of 1% and 2%. The two drift ratios are chosen to represent low and moderate drifts. Pushover curves for the two examples are shown in Fig.2.6.

Table 2.1. Gravity loads and effective seismic mass in the example buildings.

<table>
<thead>
<tr>
<th>Building</th>
<th>Floors</th>
<th>dead load (psf)</th>
<th>live load (psf)</th>
<th>total W (kips)</th>
<th>Mass (k-in/s²)</th>
</tr>
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<tr>
<td>3 Story</td>
<td>1</td>
<td>86</td>
<td>20</td>
<td>381.6</td>
<td>0.91</td>
</tr>
<tr>
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<td>2</td>
<td>86</td>
<td>20</td>
<td>381.6</td>
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</tr>
<tr>
<td></td>
<td>3</td>
<td>86</td>
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<td>381.6</td>
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<td>total</td>
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<td>86</td>
<td>20</td>
<td>477</td>
<td>0.19</td>
</tr>
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<tr>
<td></td>
<td>9</td>
<td>86</td>
<td>20</td>
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<tr>
<td></td>
<td>total</td>
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</table>

Fig.2.4. Elevation and element properties of the 3 story frame (Taken from FEMA 440, 2005).
Fig. 2.5. Elevation and element properties of the 9 story frame (Taken from FEMA 440, 2005).

Fig. 2.6. Pushover curves for the two structures.
2.4.2 Predicting Roof Displacement with “Equivalent” SDOF

Ratio of the roof displacement predicted by equivalent SDOF systems to the true values from NDA are computed for all the ground motions. Statistical distributions of the ratios for each lateral load pattern and for two levels of total roof drift ratios are shown in Figs.2.7 and 2.8 respectively for the 3 and 9 story structures. The statistical distributions include the mean, the 1st quartile (Q1), the 3rd quartile (Q3) and the max/min values. The 1st and 3rd quartile respectively denotes the 25 and 75 percentile values; the interquartile (range defined by Q1 and Q3) is an indication of dispersion of the data and since data contains the ratios, closeness to 1.0 indicates better accuracy. It can be clearly observed in Figs.2.7 and 2.8 that:

- Energy consistent equivalent SDOF gives the most accurate prediction of the MDOF roof displacement. The mean ratio observed in the two structures and for all combinations of roof drift and load pattern (except uniform) is between 0.97 and 1.02 with the smallest dispersion.

- Equivalent SDOF formulated by ATC 40 which is consistent with the assumption that displacements are in the 1st mode, gives good predictions with the 1st mode load pattern at small drifts. This is not surprising since in the foregoing circumstances the assumptions of the ATC-40 equivalent SDOF are almost true.

- Equivalent SDOF of FEMA gives the least accurate predictions with the largest dispersion.
Fig. 2.7. Statistics of the roof displacement ratio from Equivalent SDOF systems in the 3 story example.

Fig. 2.8. Statistics of the roof displacement ratio from Equivalent SDOF systems in the 9 story example.
CHAPTER 3

NONLINEAR STATIC PROCEDURES

3.1 Overview

This chapter provides the details of the NSPs considered in this study and elaborates their application through illustrative examples. In addition to the single-mode (CM and CSM) and multi-mode procedures (MPA and MMPA) an alternative NSP is introduced by incorporating the energy-consistent SDOF reduction to the current procedures. The main steps in a typical NSP whose framework is shown in Fig3.1 can be summarized as:

1. Performing a pushover analysis under a certain load pattern and obtaining a pushover curve (base shear versus roof displacement).
2. Guessing the target displacement e.g. using equal displacement rule (determining the damping and period corresponding to the target displacement if needed)
3. Estimating the roof displacement of MDOF system using a SDOF system.
4. If the estimated roof displacement is different than the initial target displacement repeating the process from step 2 until convergence is reached.
5. Obtaining the responses of interest from the pushover analysis corresponding to the target displacement determined in step 4.
3.2 Components of Typical Nonlinear Static Analysis Procedures

Three key components of NSP are a) selection of lateral load patterns for pushover analysis b) MDOF to SDOF reduction and c) determination of target displacement.

3.2.1 Selection of Lateral Load Patterns for Pushover Analysis

Lateral loads used in pushover analysis represent the inertial forces imposed by the earthquake. Their role in NSP is signified in the process of pushing the structure to the target displacement and in formulation of the “equivalent” SDOF. As discussed in
Chapter 2, properties of the SDOF system are determined from the capacity curve. Shape of the capacity curve on the other hand indirectly depends on the lateral loads used in pushover analysis as they incur different patterns of nonlinearity. Moreover, prediction of the maximum responses in NSP perspective closely depends on the lateral loads that drive the structure to the target displacement.

Distribution of the inertial forces during a dynamic action varies in time with the response of the structure. Since the maximum of various responses occur at different times, instantaneous distribution of inertial forces corresponding to the maximum responses is not the same (Fig.3.2). Shape of these distributions depends on the structure and the excitation. It is evident that there is no single load pattern that results in all the maximum responses.

![Fig.3.2. Distribution of inertia forces along the height of the 13 story Sherman Oaks building in Northridge 1994 earthquake a) at maximum base shear b) at maximum 5th story shear.](image-url)
The standard NSP guidelines, therefore, require that more than one load pattern be selected depending on the dynamic properties, and the final prediction be taken as the envelope of results. In FEMA 356 one load pattern is selected from code based and 1st mode (if more than 75% of the mass participates in the 1st mode) or SRSS of modal story shears and another load is selected from uniform and adaptive loads. In ATC 40 while the primary recommendation is the 1st mode other options such as code based, SRSS and adaptive are also permitted.

The idea of adaptive loads is not particularly superior to the conventional fixed load patterns. Use of the adaptive loads is based on the notion that the response of the structure and the inertial loads are governed by the instantaneous dynamic properties (i.e. instantaneous mode shapes). However, when structure yields during the dynamic action, displacement and velocity profiles do not change instantaneously. For the instantaneous dynamic properties to take over the response the extent of the change in the dynamic properties should be substantial and there should be sufficient time during which the structure retains those particular dynamic properties. The adaptive loads do not consider this matter; no solid connection can be established between the varying adaptive load and the varying inertial forces that the structure experiences in an earthquake. While significantly increasing the complexity and computational efforts (invoking several Eigen value analyses) the adaptive option do not provide any apparent advantage over the fixed loads and, as mentioned previously, are not included in this study.

3.2.2 Characterization of the “Equivalent” SDOF System

As was extensively discussed in Chapter 2, the standard NSPs formulate the SDOF system by postulating that the response is entirely in the 1st mode and the load patterns
used in the pushover analysis are proportional to the 1st mode shape. In ATC-40, for example, the procedure to establish the force-deformation relationship of the SDOF system is the same procedure presented in Chapter 2 (Eqs.2.18 and 2.19), namely,

\[ D = \frac{u_{\text{roof}}}{\Gamma_1 \phi_1} \quad F_r = \frac{V_b}{EMM_1} \]  

(3.1a,b)

In FEMA 356, conversion of the roof displacement to the generalized displacement of the SDOF is the same as in Eq.3.1 but to convert the base shear to the \( F_r \) the following equation is used.

\[ F_r = \frac{V_b}{W/g} \Gamma_1 \]  

(3.2)

where \( W \) is the total weight and \( g \) is the acceleration of gravity. For structures that are governed by the 1st mode the term \( \frac{V_b}{W/g} \) is almost equal to the \( EMM_1 \) and Eqs.3.2 and 3.1b are equivalent.

3.2.3 Determination of the “Target Displacement”

The target displacement can be directly determined by solving the nonlinear equation of motion for the SDOF system or by empirical methods. In standard NSP where the seismic hazard is represented in terms of smooth response spectra, the maximum displacement of the SDOF is obtained through modification of the response spectra analysis and introduction of some approximations. Details of these procedures are presented next.
3.3 Standard Single-Mode NSP

The single-mode NSP that are published in guidelines and standards are termed “standard” NSP and are presented next.

3.3.1 Capacity Spectrum Method of ATC 40

Capacity Spectrum Method (CSM) which is based on “Equivalent Linearization” is founded on the basic assumption that the maximum inelastic displacement of a nonlinear SDOF system can be approximated by maximum displacement of a linear elastic SDOF system with an equivalent damping and period larger than those of the original nonlinear SDOF system. The target displacement in the context of the CSM is called the “Performance Point” and is obtained at the intersection of the capacity curve and an elastic response spectrum for a longer period and a higher damping value (in ADRS format). The abscissa and ordinate of ADRS coordinate system respectively correspond to spectral displacement and spectral acceleration while the radial lines represent the period. To convert the capacity curve from the base shear–roof displacement coordinate to ADRS format the following equations are used.

\[ S_d = \frac{u_{\text{roof}}}{\Gamma \phi_{\text{roof}}} \quad \text{and} \quad S_a = \frac{V_b}{EMM_t g} \quad (3.3a,b) \]

In CSM the equivalent damping is determined from the area enclosed by the capacity curve as shown in Fig.3.3. The equivalent damping is taken as the sum of the initial damping (5%) and a viscous damping associated with the area of the hysteresis and the equivalent period is taken as the secant period at the performance point. Since
determination of the equivalent damping and period needs the knowledge of the performance point as a priori and the performance point determination requires the equivalent damping and period, the procedure is iterative. It begins with guessing the location of the performance point. Using the equal displacement rule, the spectral displacement of the linear system is often a proper guess.

Fig.3.3. Equivalent damping and period in CSM.

The equation that defines the equivalent damping is given in ATC-40 in terms of the coordinate of the performance point \((d_p,a_p)\) and the yield point \((d_y,a_y)\) of the bilinear representation of the capacity curve in ADRS format as follows:

\[
\beta_{eq} = 5 + \beta_v
\]  
(3.4)

\[
\beta_v = \frac{1}{4\pi} \frac{E_D}{E_{so}}
\]  
(3.5a)

\[
\beta_v = \frac{63.7 \kappa (a_y d_p - a_p d_y)}{a_p d_p}
\]  
(3.5b)
where $E_D$ is the energy dissipated by damping and $E_{so}$ is the maximum strain energy at the performance point. $\kappa$ accounts for the deviation of the actual hysteresis loop from the parallelogram assumed in derivation and depending on the type the structure and duration of shaking varies between 1.0 and 0.33 for stable to poor hysteresis loops.

3.3.2 Equivalent Linearization of FEMA 440

FEMA 440 showed that displacement response at short periods ($T \leq 0.5$ s) is significantly over predicted by CSM and at larger periods depending on the hysteresis type it could be underestimated or overestimated. The proposed modifications include new equations for the effective damping and period. Different equations are developed for different ranges of ductility and the constant coefficients of each equation depend on the different hysteresis types. Simpler equations that do not depend on the hysteresis shapes are also given by FEMA 440 and are as follows

For $1.0 < \mu < 4.0$

\[
\beta_{eff} = \beta_0 + 4.9(\mu - 1)^2 - 1.1(\mu - 1)^3 \tag{3.6}
\]

\[
T_{eff} = \left\{0.2(\mu - 1)^2 - 0.038(\mu - 1) + 1\right\}T_0 \tag{3.7}
\]

For $4.0 \leq \mu \leq 6.5$

\[
\beta_{eff} = \beta_0 + 14 + 0.32(\mu - 1) \tag{3.8}
\]

\[
T_{eff} = [0.28 + 0.13(\mu - 1) + 1]T_0 \tag{3.9}
\]

For $\mu > 6.5$
\[
\beta_{\text{eff}} = \beta_0 + 19 \left[ \frac{0.64(\mu - 1) - 1}{0.64(\mu - 1)^2} \right] \frac{T_{\text{eff}}}{T_0}
\]

(3.10)

\[
T_{\text{eff}} = \left\{ 0.89 \left[ \frac{\mu - 1}{\sqrt{1 + 0.05(\mu - 2)}} - 1 \right] + 1 \right\} T_0
\]

(3.11)

In the above equations \(\beta_0\) and \(T_0\) are initial damping (5%) and initial period of the equivalent SDOF system. It should be noted that the above equations are calibrated only for \(T_0\) between 0.2 and 2 seconds.

**Illustrative Example**

Application of the Equivalent Linearization methods to the 13 story Sherman Oaks building is shown for illustrations using the N-S component of the Northridge 1994 earthquake recorded at this station. Capacity curve in the format of base shear vs. roof displacement for a load pattern proportional to the 1st mode shape is shown in Fig.3.4a. The capacity curve converted to ADRS format using Eq.3.3 is shown in Fig.3.4b. Dynamic properties of the 1st mode \((T = 2.67\ \text{sec})\) used in Eq.3.3 for the conversion are \(\Gamma_1 = 7.68, \ \phi_{\text{roof}}^1 = 0.17, \ \text{EMM}_1 = 59.06\).

![Fig.3.4. Pushover curves in the N-S direction of the Sherman Oaks building a) base shear versus roof displacement b) \(S_a\) versus \(S_d\) (ADRS).](image)
Taking initial performance point at the intersection of the 5% damped elastic response spectrum and the capacity curve (Fig.3.5), the equivalent damping and period using ATC-40 and FEMA 440 expressions are:

**ATC-40:**  
\[ T_{eq} = 3.07 \text{ sec.} \quad \beta_{eq} = 10.86 \% \]

**FEMA 440:**  
\[ T_0 = 2.73 \text{ sec.} \quad \mu = 1.73 \quad T_{eq} = 2.98 \text{ sec.} \quad \beta_{eq} = 7.16 \% \]

![Fig.3.5. Illustration of the trial performance point.](image)

As can be seen, in the 1\textsuperscript{st} trial the equivalent damping is larger than the assumed 5\% and therefore one needs to repeat the procedure using a response spectrum corresponding to the equivalent damping. After a few iterations the process converges to the final performance point as shown in Fig 3.6. Coordinate of the performance point and the equivalent damping and periods are:

**ATC-40:**  
\[ S_a = 0.093g \quad (V_b = 2117 \text{ kips}) \quad S_d = 7.54'' \quad (u_{roof} = 9.55'') \quad T_{eq} = 2.89\text{ s.} \quad \beta_{eq} = 7.54 \% \]

**FEMA 440:**  
\[ S_a = 0.095g \quad (V_b = 2231\text{ kips}) \quad S_d = 7.67'' \quad (u_{roof} = 10.42'') \quad T_{eq} = 2.88\text{ s.} \quad \beta_{eq} = 6.55 \% \]
3.3.3 Coefficient Method of FEMA 356

In “Coefficient Method” the maximum inelastic displacement of an MDOF system is determined by modifying the elastic displacement of the “equivalent” SDOF with an effective period $T_e$. This idea is essentially an adjustment to the equal displacement rule. In CM the target displacement is obtained from:

$$\delta_t = C_0 C_1 C_2 C_3 \frac{T_e}{4\pi^2} g$$  \hspace{1cm} (3.12)

where $C_0$ accounts for the conversion of the spectral displacement to MDOF roof displacement and could be taken as the 1st modal participation factor at the roof level and can be alternatively computed using a shape vector corresponding to the deformation of the MDOF at the target displacement. $C_I$ is a modification factor that relates the maximum inelastic displacement and maximum elastic displacement and is given by

$$C_I = 1 \quad T_e \geq T_s$$  \hspace{1cm} (3.13a)
\[ C_1 = \left[ 1 + (R-1) \frac{T_s}{T_e} \right] / R \quad T_e < T_s \quad (3.13b) \]

where \( T_s \) is the characteristic period of the response spectrum defined as the transition from constant acceleration region to constant velocity region. \( T_e \) and \( R \) are the effective fundamental period and the ratio of elastic to yield strength of the structure defined below:

\[ T_e = T_i \sqrt{\frac{K_i}{K_e}} \quad (3.14) \]

\[ R = \frac{S_a}{V_x} \frac{C_m}{W} \quad (3.15) \]

\( C_m \) is the effective modal mass of the 1st mode normalized by the total mass. \( C_2 \) is a modification factor that accounts for deviation from an elastic perfectly plastic hysteresis. This coefficient represents the effect of pinched hysteresis, stiffness degradation and strength deterioration and is given in table 3.1 for various framing type and expected performance level it can alternatively be taken as 1.0. \( C_3 \) is an amplification factor to account for \( P-\Delta \) effects defined below and is taken 1.0 if the bilinear representation of the SDOF system demonstrates positive post yield stiffness.

\[ C_3 = 1 + \frac{\alpha (R-1)^{1.5}}{T_e} \quad (3.16) \]

where \( \alpha \) is the ratio of the post yield stiffness to effective elastic stiffness. Schematic summary of the Coefficient Method is shown in Fig.3.7.
Table 3.1. Values of coefficient $C_2$.

<table>
<thead>
<tr>
<th>Structural Performance</th>
<th>$T \leq 0.1$ sec.</th>
<th>$T &gt; = T_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Framing type 1 $^1$</td>
<td>Framing type 2 $^2$</td>
</tr>
<tr>
<td>Immediate Occupancy</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Life Safety</td>
<td>1.3</td>
<td>1.0</td>
</tr>
<tr>
<td>Collapse Prevention</td>
<td>1.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

1. structures that more than 30% of story shears at any level is resisted by a combination of: ordinary moment resisting frames, concentrically braced frames, frames with partially restrained connections, tension only braces, unreinforced masonry walls, shear critical, piers and spandrels of reinforced concrete or masonry.

2. All frames not assigned Farming 1.

\[
\delta_i = C_2 C_1 C_2 C_3 S_a \frac{T_e^2}{4\pi^2 g}
\]

Fig. 3.7. Summary of CM.

### 3.3.4 Coefficient Method of FEMA 440

FEMA 440 identified some drawbacks in the coefficients of Eq. 3.12 and as a remedy suggested modification to $C_1$ and $C_2$ and eliminated $C_3$. The study has shown that the value of $C_1$ that relates the displacement of an inelastic SDOF to its elastic counterpart is significantly underestimated at short periods. It also shows that the transition period above which the equal displacement rule holds is larger than what used in FEMA 356 namely, $T_s$. FEMA 440 proposed a new equation for $C_1$ as follows
\[ C_1 = 1 + \frac{R - 1}{\alpha T_e^2} \]  

(3.17)

\( R \) and \( T_e \) were described before. \( \alpha \) is a constant that depends on site class and is equal to 130, 90 and 60 for site classes B, C and D. For periods less that 0.2, \( C_1 \) at 0.2 seconds should be used and for periods larger than 1.0 second \( C_1 \) may be assumed to be 1.0.

FEMA 440 suggested that coefficient \( C_2 \) only represent the effect of stiffness degradations and the effect of strength degradation and \( P-\Delta \) effects be considered through limitations on the strength. The proposed equation for \( C_2 \) is given in Eq.3.18. For periods less that 0.2, \( C_2 \) at 0.2 seconds should be used and for periods larger than 0.7 second the value may be assumed to be 1.0.

\[ C_2 = 1 + \frac{1}{800} \left( \frac{R - 1}{T_e} \right)^2 \]  

(3.18)

As a replacement for coefficient \( C_3 \) and compensation for not considering strength degradation in \( C_2 \), FEMA 440 proposes a limitation on the strength for in-cycle strength degradation including the \( P-\Delta \) effects. This limitation given in terms of the maximum reduction of strength from elastic strength demand and is as follows

\[ R_{\text{max}} = \frac{\Delta_d}{\Delta_y} + \frac{\left| \alpha_e \right|}{4} \]  

(3.19)

\[ t = 1 + 0.15 Ln(T) \]  

(3.20)

\[ \alpha_e = \alpha_{p-\Delta} + \lambda (\alpha_2 - \alpha_{p-\Delta}) \]  

(3.21)
\( \lambda \) is between 0 and 1.0 and is suggested for 0.2 for sites subjected to near field and 0.8 for others. \( \Delta_d \) and \( \Delta_y \) are shown in the Fig. 3.8.

![Diagram showing base shear and displacement](image)

**Fig.3.8.** Accounting for strength degradation in FEMA 440 (taken from FEMA 440, 2005).

**Illustrative Example**

Application of the CM to the same example is illustrated here for a pushover analysis with the 1st mode shape proportional load pattern. Figure 3.9 shows the capacity curve and construction of the bilinear representation.

![Trial target displacement and bilinear representation](image)

**Fig.3.9.** Trial target displacement and the bilinear representation of the capacity curve.
A trial target displacement is arbitrarily selected at $u_{\text{roof}} = 21.3$ inch with the corresponding base shear of 3018 kips. For this trial point the parameters of the bilinear representation (Eqs. 3.14 and 3.15) are

$$T_\nu = 2.69 \text{ sec.} \quad \alpha = 0.26 \quad \mu = 2.41 \quad T_e = 3.7 \text{ sec.} \quad T_s = 0.24 \text{ sec.} \quad \text{and} \quad R = 1.36$$

With these parameters all the CM coefficients are 1.0 except for the $C_0$ which is 1.3. In this particular example both the original CM (FEMA 356) and the modified version give the same coefficients. Spectral acceleration and displacements for a 5% damped elastic spectrum at the effective period of $T_e = 3.7$ sec are respectively 0.07g and 9.4”. The target displacement using Eq. 3.12 is 12.22” which is quite far from the initial guess. After a few iterations the target displacement converges to 11.86” which corresponds to a base shear of 2420kips and an equivalent period of 3 seconds.

### 3.4 Advanced NSP

In this study, two multi-mode methods, namely, Modal Pushover Analysis (Chopra and Goel, 2002) and Modified Modal Pushover (Chopra et al. 2004) are considered. In addition, combination of the energy-consistent SDOF reduction with both single and multi-mode pushover analysis is presented as an alternative NSP and is included in evaluations.

#### 3.4.1 Modal Pushover Analysis (MPA)

The Modal Pushover Analysis has been developed to account for higher mode effects in NSP. It is based on the assumption that the classical modal analysis theories
can be extended to inelastic systems. Seismic responses due to individual terms in the modal expansion of the effective earthquake forces are determined by a pushover analysis using the inertia force distribution for each mode. Prediction of the total response of the structure is made by combining these “modal” responses using SRSS modal combination rule. Recalling the equation of motion of a nonlinear system subjected to ground acceleration

\[ M\ddot{u} + C\dot{u} + R(u) = -M r a_g \]  

(3.22)

taking \( u = \Phi q \) and making use of the orthogonally of mode shapes with respect to \( M \) and \( C \) the equation of motion in the \( n^{th} \) mode can be obtained as

\[ \ddot{q}_n + 2\zeta_n \omega_n \dot{q}_n + \frac{\Phi_n^T R(q)}{M_n} = -\Gamma_n a_g \]  

(3.23)

The term \( R(q) \) is in reality a function of all the modal amplitudes \( q \) but in MPA it is assumed to be only a function of \( q_n \). Defining \( D_n = \frac{q_n}{\Gamma_n} \), the final form of the equation of motion for the \( n^{th} \) mode SDOF is

\[ \ddot{D}_n + 2\zeta_n \omega_n ^* \dot{D}_n + F_m = -a_g \]  

(3.24)

Force-deformation relationship of the \( n^{th} \) mode SDOF system defined by \( F_m \) and \( D_n \) is determined from the pushover analysis under a lateral load proportional to the mode shapes, namely, \( s_n^* = M \phi_n \). Converting the plot of base shear \( (V_b) \) vs. roof displacement \( (u_r) \) gives the force-deformation relationship of the SDOF system as follows
\[ F_{rn} = \frac{V_{bn}}{EMM_n} \]  
(3.25)

\[ D_n = \frac{u_{rn}}{\Gamma_n \phi_{rn}} \]  
(3.26)

where \( \phi_{rn} \) is the \( n^{th} \) mode shape at the roof. The \( n^{th} \) mode response of the MDOF system is obtained from the pushover analysis under \( S_n^* \) at a roof displacement \( u_m = \Gamma_n \Phi_n D_n \) in which \( D_n \) is the solution of Eq.3.24. In solving Eq.3.23 a bilinear idealization of the \( F_{rn} \- D_n \) relationship is used and the term \( w_n^* \) is computed at the yield point of the bilinear representation (Eq.3.27) which may not be equal to the \( w_n \).

\[ w_n^* = \sqrt{\frac{F_{ry}}{D_y}} \]  
(3.27)

In summary steps of the MPA are as follows:

1- For each mode \( n \), perform a pushover analysis under \( S_n^* = M\phi_n \) and obtain a capacity curve in terms of base shear \( (V_b) \) versus roof displacement \( (u_r) \).

2- Obtain a bilinear representation of the capacity curve and convert it to the force-displacement relationship of the equivalent SDOF system using Eqs.3.25 and 3.26.

3- Solve the equation of motion of the \( n^{th} \) mode SDOF in Eq.3.24 and obtain the roof displacement from \( u_{rn} = \Gamma_n \Phi_n D_n \).

4- Get the responses, \( r_n \), from the pushover analysis at a roof displacement equal to \( u_{rn} \).

5- Repeat steps 1 to 4 for as many modes as it is needed and obtain the total response from \( r = \left( \sum r_n^2 \right)^{0.5} \)
Remark

Pushover analysis with loads that are proportional to the higher mode shapes may result in irregular shapes or even reversals in the plots of base shear versus roof displacement. While this issue supports the assertion that roof displacement is not always the best quantity for representing the pushover curve, it causes problems in application of MPA. Since characterization of the force-deformation of the modal SDOF system relies on the base shear versus roof displacement relationship, appearance of reversal or any irregularity impedes the normal MPA procedures and requires some judgments. Fig.3.10 shows the pushover curves of the 3 story SAC building used in Chapter 2. As can be seen in the 3rd mode pushover curve, the roof displacement reverses. To proceed with the MPA procedure in this situation one either uses the 1st portion of the curve to define the SDOF properties or simply takes it linear as in the MMPA procedure.

![Fig.3.10. Modal pushover curves of the 3 story SAC building.](image)

### 3.4.2 Modified Modal Pushover Analysis (MMPA)

Modified version of the modal pushover analysis is developed based on the assumption that higher mode responses are elastic. The procedure therefore is identical to MPA for the 1st mode but for the other modes the SDOF response, $D_n$, is simply
determined from either an elastic response spectra or an elastic time history analysis. Since the higher mode responses are considered linear, there is no need for pushover analysis at higher modes and therefore the potential reversal problem is avoided.

**Illustrative Example**

The 13 story Sherman Oaks building is revisited to illustrate the application of MPA and MMPA. Fig.3.13 shows the force displacement of the 1\textsuperscript{st} mode SDOF converted from the pushover curve (Fig.3.6a) using Eqs.3.25 and 3.26. Modal properties of the structure are also shown in Table 3.2.

![Fig.3.13. Force displacement relationship the “modal” SDOF.](image)

**Table 3.2. Modal properties of the Sherman Oaks (in N-S direction).**

<table>
<thead>
<tr>
<th>mode</th>
<th>wn</th>
<th>w*</th>
<th>Ln</th>
<th>Mn</th>
<th>(\Gamma_n)</th>
<th>EMMn</th>
<th>(\phi_m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.34</td>
<td>2.35</td>
<td>7.68</td>
<td>1.00</td>
<td>7.68</td>
<td>59.06</td>
<td>0.17</td>
</tr>
<tr>
<td>2</td>
<td>7.30</td>
<td>8.41</td>
<td>2.97</td>
<td>1.00</td>
<td>2.97</td>
<td>8.82</td>
<td>0.16</td>
</tr>
<tr>
<td>3</td>
<td>12.82</td>
<td>12.69</td>
<td>-1.89</td>
<td>1.00</td>
<td>1.89</td>
<td>3.60</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Maximum displacement of the modal SDOF determined by solving Eq.3.24 and the corresponding modal roof displacement is given in Table 3.3. Responses from MMPA in which the second and third mode are considered linear are also shown.
Table 3.3. SDOF response and the roof displacement in MPA and MMPA.

<table>
<thead>
<tr>
<th>mode</th>
<th>MPA</th>
<th>MMPA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dn</td>
<td>u roof</td>
</tr>
<tr>
<td>1</td>
<td>9.26</td>
<td>10.89</td>
</tr>
<tr>
<td>2</td>
<td>4.61</td>
<td>-2.24</td>
</tr>
<tr>
<td>3</td>
<td>3.14</td>
<td>0.93</td>
</tr>
<tr>
<td>SRSS</td>
<td>11.16</td>
<td>11.31</td>
</tr>
</tbody>
</table>

“Modal” responses for drifts, shears etc. are then taken from the pushover at the corresponding modal roof displacement and the total estimate are the SRSS combination.

3.4.3 Energy-Based Methods

It was shown in Chapter 2 that the energy-consistent SDOF reduction improved the consistency in the assumptions and increases the accuracy in predicting roof (target) displacement. An alternative approach to the current NSP is examined in this study by replacing the original SDOF reduction with the energy-consistent variant. The approach can be applied to both single-mode and multi-mode methods.

3.4.3.1 Implementation in Single-Mode NSP

Implementation of the energy consistent SDOF in the single-mode NSP is straightforward and involves replacing the empirical procedures to compute the target displacement with a response history analysis of the nonlinear energy-consistent SDOF system in the following equation

\[ \ddot{D} + 2\zeta\omega D + F_{re} = -a_g(t) \]  \hspace{1cm} (3.28)

where

\[ D = \frac{Y}{(\frac{L_c}{M_c})} \]  \hspace{1cm} \[ F_{re} = \frac{V_b}{L_c} \]  \hspace{1cm} (3.29)
Maximum displacement of the Eq.3.28 when converted to the roof displacement in the MDOF system using the implicit relationship between the global displacement $Y$ and the MDOF displacements is taken as the target displacement. Responses from the pushover analysis corresponding to the target displacement give the predictions of the quantities of the interests.

### 3.4.3.2 Implementation in Multi-Mode NSP

To examine the possibility of implementing the suggested energy consistent SDOF reduction in the context of multi-mode NSP it is necessary to remember that multi-mode NSP does not rely on only one “equivalent” SDOF system but on as many SDOF system as the number of modes included in the analysis. Since displacement in each mode is by definition proportional to shape of that mode, namely $u_n = \phi_n q_n$, the equation of motion of each “modal” SDOF system is explicitly defined (Eq.3.32).

$$M_n \ddot{q}_n + 2 \zeta \omega_n M_n \dot{q}_n + \phi_n^T R(q) = -L_n a_g$$  \hspace{1cm} (3.32)

Eq.3.32 leads to the final form of the modal SDOF system given by Eq.3.24, namely, $\ddot{D}_n + 2 \zeta \omega_n \dot{D}_n + F_{rn} = -a_g$. In solving this equation relationship between $F_{rn}$ and $D_n$ is defined by converting the base shear versus roof displacement using Eq.3.25 and 3.26. One can realize by inspection of Eq.3.32 that implementation of the proposed energy
consistent approach to the “modal” SDOF equations is only possible in the resisting form term $F_{rn}$ since by taking $u_n = \phi_n q_n$ the equivalent terms $M_n$ and $L_n$ are automatically defined as $M_n$ and $L_n$.

In the original multi-mode NSP (i.e. MPA) the $F_{rn}$ is defined, consistent with the assumption of $u_n = \phi_n q_n$, by Eq.3.25 and 3.26. Since the only approximation in derivations of Eq.3.25 and 3.26 is due to neglecting the coupling, one reason for seeking an alternative way to define $F_{rn}$ is the fact that pushover analysis under loads proportional to higher modes is likely to result in badly shape or even reversals in the base shear versus roof displacement plots (Fig.3.12). This matter also poses a fundamental question about the appropriateness of representing the capacity curve in terms of base shear versus roof displacement while it can be equally represented by plotting base shear or any other story shear with a different floor displacement. The shape of the capacity curve and consequently the properties of the SDOF system can significantly change by the choice of quantities to represent the capacity curve. The subjective nature of this selection compels one to pursue the idea of incorporating the energy consistent SDOF system in the multi-mode pushover methods.

Application of the suggested energy consistent approach in the multi-mode pushover coincides with the procedure proposed by Hernandez-Montes et al. (2004) in which the generalized displacement of the SDOF system is determined by equating the work of the pushover loads and the SDOF system (the same procedure explained in Chapter 2). The force displacement relationship of the modal SDOF system is then defined by the relationship between $F_{rn}$ defined by Eq.3.25 and the energy consistent generalize displacement designated by $Y$ in Eq.3.31. The rest is the same as original MPA.
Illustrative Example

Application of the energy-consistent NSP in the form of a single-mode procedure is shown for pushover analysis with loads proportional to the 1st mode shape. The original capacity curve was shown previously in Figs. 3.6 and 3.10. Fig. 3.14a shows the computed generalized displacement, \( Y \), (Eq. 3.31) and Fig. 3.14b shows variation of \( M_e \) and \( L_e \) (Eq. 3.30) with \( Y \). Bilinear idealization of the force-displacement of the EC SDOF is represented with \( F_r \) of 44kips, \( w \) of 2.4 and post yield slope of 0.07 (Fig. 3.15). Solution of Eq. 3.28 gives the maximum displacement of \( D = 5.9'' \) which maps to a roof displacement of 11.5" using the corresponding step of the pushover analysis.

Fig. 3.14. a) generalized displacement vs. roof displacement b) variation of \( M_e \) and \( L_e \) with \( Y \).

Fig. 3.14. Force-deformation of the energy consistent SDOF system.
In application of the energy consistent SDOF to the multi-mode NSP the generalized displacement is determined from Eq.3.31 and $F_r$ from Eq.3.25. In Fig.3.15 the generalized displacement for each “mode” is shown versus the roof displacement. Force-displacement relationship of the “modal” SDOF and its bilinear idealization is shown in Fig.3.16. Finally the solution of the equation of motion for each mode and the corresponding roof displacement is summarized in Table.3.4.

![Fig.3.15. Generalized displacement using Eq.3.31 versus roof displacement.](image)

![Fig.3.16. Force displacement relationship the energy-consistent “modal” SDOF.](image)

Table 3.4. SDOF response and the roof displacement in EC MPA.

<table>
<thead>
<tr>
<th>&quot;mode&quot;</th>
<th>wn</th>
<th>w</th>
<th>Dn</th>
<th>u roof</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.34</td>
<td>2.27</td>
<td>9.56</td>
<td>11.01</td>
</tr>
<tr>
<td>2</td>
<td>7.30</td>
<td>7.34</td>
<td>6.16</td>
<td>2.15</td>
</tr>
<tr>
<td>3</td>
<td>12.82</td>
<td>12.88</td>
<td>3.11</td>
<td>0.91</td>
</tr>
<tr>
<td>SRSS</td>
<td></td>
<td></td>
<td></td>
<td>11.25</td>
</tr>
</tbody>
</table>
CHAPTER 4

SEISMIC RESPONSE RECONSTRUCTION

4.1 General

“Seismic Response Reconstruction” is the collection of tasks by which the engineering demand parameters are obtained from sparsely measured seismic data (usually accelerations). The cornerstone of response reconstruction is estimation of the quantities at un-instrumented levels. This Chapter examines the data driven estimation method using basis fitting (interpolation) and model based estimation using estimation theories of control. An alternative model based estimation method is developed and a simulation study using 4 mid to high rise building and 30 ground motion is carried out to evaluate the estimation schemes.

The fundamental idea in model based estimation is to use the information in the difference between the response of the nominal model and the measurements to improve the open loop responses. Fig.4.1 illustrates a block diagram of a general model based estimator. The real system is under some measured excitation $u$ and some unmeasured disturbances. Noise contaminated measurements of some quantities of the system is available in $y_m$. Open loop estimates of the measured quantities using the nominal model are designated as $\hat{y}$. The difference between $y_m$ and $\hat{y}$ are fed back to the nominal model
through the gain in order to minimize some functions of the error $\varepsilon$. It should be noted that the procedure although similar, is not the same as model updating strategies (Friswell and Mottershead, 1995, Hernandez and Bernal, 2006) since no attempt is made in the process to update the nominal model.

The process hinges on selection of the gain. In different estimation schemes the gain is selected based on the nature of the source of discrepancy between the estimates and the measurements. In general sources of discrepancy could be that 1) the initial condition is not known deterministically 2) the excitations are not measured accurately 3) the model itself is not precise. In control application the main reasons for uncertainty in the model responses are due to the first two items, but in seismic response reconstruction the uncertainty in the excitation and initial condition is usually little compared to imperfection of the nominal model hence the bulk of the uncertainties are associated with the third item.

Fig.4.1. Block diagram of a general model based estimator.
4.2 Preliminaries

Estimation theories are mainly developed in state-space. To examine these theories for seismic response reconstruction it is necessary to represent the dynamic equation of motion of a structure in state-space format.

4.2.1 State-Space Formulation

States of a finite dimensional system are known as the smallest number of variables that provide a complete description of the status of the system at any given time. For instance displacement and velocity can be the states of a structural system. If the state at time $t_i$ is known, it can be projected to the future times $t_n$ given the knowledge about the model and the excitation in the window of $[t_1 t_n]$. The mathematical structure that describes the dynamic of the system in terms of the states is the state-space representation. An implicit requirement for state-space formulation is that the system should be finite dimensional. Discretization of the systems into a finite dimensional model, as long as the important features are not discarded, is commonly accepted in engineering. The state space representation of a linear time invariant system (LTI) is given in Eq.4.1a. Measurements as a linear combination of states are given by Eq.4.1b.

\[
\dot{x} = A_c x + B_c u \\
y = C_c x + D_c u
\]

In Eq.4.1, $x \in \mathbb{R}^{N \times 1}$ is the state vector, $u \in \mathbb{R}^{I \times 1}$ is the input, $y \in \mathbb{R}^{O \times 1}$ is the measurement vector and the quadruple \{ $A_c$, $B_c$, $C_c$ and $D_c$ \} are respectively the state transition, input to state, state to output and input to output matrices of appropriate dimension (Juang, 1994).
$N$ is twice the finite dimension of the system ($n$). The solution of the LTI state space equation is (Kailath, 1990; Antsaklis and Michel, 1997)

\[ x(t) = e^{At}x_0 + \int_0^t e^{A(t-\tau)}B\tau \mu(\tau)d\tau \]  

(4.2)

A particularly useful form of state-space representation can be obtained in discrete time by making assumptions about the variation of $u$ between the two time stations and solving the integral of Eq.4.2. Taking $t = k\Delta t$ and defining $A_d = e^{A\Delta t}$ the standard form of the discrete time state space equation is

\[ z_{k+1} = A_d z_k + B_d u_k \quad ; \quad y_k = C_d z_k + D_d u_k \]  

(4.3)

A general parameterization of the input that leads to the standard state space model is given in Eq.4.4 where $f_j$ are arbitrary basis functions (Bernal, 2007).

\[ u(\tau) = f_0(\tau)u_k + f_1(\tau)u_{k+1} \]  

(4.4)

Common choices for variation of the input are to take it constant between the samples (zero order hold) i.e. $f_0 = 1$ and $f_1 = 0$, or to assume linear variation (first order hold) i.e. $f_0 = 1 - \tau / \Delta t$ and $f_1 = \tau / \Delta t$. In the latter case, the discrete time matrices are

\[ A_d = e^{A\Delta t} \]  

(4.5a)

\[ B_d = (A_d - I)^2 A_c^2 B \frac{1}{\Delta t} \]  

(4.5b)

\[ C_d = C_c \]  

(4.5c)

\[ D_d = D_c + C_c A_c^{-1}(A_d^{0.5} - I)B_c \]  

(4.5d)
4.2.2 Equation of Motion in State-Space

The state-space form of the equation of motion is attained by reducing the order of differentiation from two to one at the expense of doubling the number of equations. Consider the equations of motion for the case that the input is a ground motion excitation

\[ M \ddot{q}(t) + C_{\text{dam}} \dot{q}(t) + Kq(t) = M a_g(t) \]  \hspace{1cm} (4.6)

where \( r \) is the pseudo-static influence vector. In a simple 2D set up when the excitation \( a_g(t) \) is only a single motion in the direction of the dofs, \( r \) is a column vector of ones whose dimension, \( n \), is equal to the number of floors. Taking \( q = x_1 \) and \( \dot{q} = x_2 \) one gets

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\end{bmatrix} =
\begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C_{\text{dam}} \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\end{bmatrix} +
\begin{bmatrix}
0 \\
r \\
\end{bmatrix} a_g \\
\hspace{1cm} (4.7)
\]

Eq.4.7 in which state is a collection of the relative displacement and velocities can be written as

\[
\dot{x} = A_c x + B_c a_g \quad ; \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\
\hspace{1cm} (4.8)
\]

where

\[
A_c =
\begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C_{\text{dam}} \\
\end{bmatrix} \\
\hspace{1cm} (4.9)
\]

\[
B_c =
\begin{bmatrix}
0 \\
M^{-1}b_2 \\
\end{bmatrix} \quad ; \quad B_e =
\begin{bmatrix}
0 \\
r \\
\end{bmatrix} \\
\hspace{1cm} (4.10a,b)
\]

In Eq.4.10 \( b_2 = M.r \). Finally using the continuous to discrete transformation of Eq.4.5 the state space equation of motion in discrete time is obtained as follows.
\[ x(k+1) = A_d x(k) + B_d a_g(k) \]  \hspace{1cm} (4.11)

**The Output Equation**

Seismic measurements are usually absolute accelerations. A connection between the measurements and the state can be established by rearranging the Eq.4.6.

\[ M(\ddot{q} + a_g) + C_{dam} \dot{q} + Kq = 0 \]  \hspace{1cm} (4.12)

\[ \ddot{q}_{abs} = -M^{-1}Kq - M^{-1}C_{dam}\dot{q} \]  \hspace{1cm} (4.13)

\[ \ddot{q}_{abs} = \begin{bmatrix} -M^{-1}K & -M^{-1}C_{dam} \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \]  \hspace{1cm} (4.14)

Taking the measured entries of the acceleration vector as \( y_m \) and recalling the definition of the state one gets

\[ y_m = C_c x \]  \hspace{1cm} (4.15a)

\[ C_c = \begin{bmatrix} -M^{-1}K & -M^{-1}C_{dam} \end{bmatrix} \]  \hspace{1cm} (4.15b)

where \( C_c \) is \( m \) by \( 2n \) with \( m \) being the number of measurements. If the seismic measurements are relative displacement or velocity the matrix \( C_c \) will respectively be

\[ C_c = \begin{bmatrix} L_{mn} & 0_{mn} \end{bmatrix} ; \hspace{1cm} C_c = \begin{bmatrix} 0_{mn} & L_{mn} \end{bmatrix} \]  \hspace{1cm} (4.16a,b)

where \( L \) is Boolean with ones at the instrumented levels and zero elsewhere. For example if in a four story structure displacement at the second and fourth floors is measured then, \( L \) and \( C \) will be

\[ L_{2\times4} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \hspace{1cm} C_{2\times8} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \]
Connection between the state and the measurements in discrete time can be shown as

\[
y_m(k) = C_d x(k) + D_d a_s(k) \tag{4.17}
\]

### 4.3 Estimation via Basis Fitting

The idea of basis fitting in estimation is to take the response of the entire structure at any time as a linear combination of a set of predefined independent vectors.

#### 4.3.1 Formulation of Basis Fitting

Let \( y_m \) and \( y_u \) be the measured and the unmeasured coordinates with \( m \) and \( u \) representing the number of coordinates in each set, the total number of coordinates, \( n \), is therefore \( m+u \). Note that the measurement at the base is treated the same as other locations. The response at all coordinates can be expressed as

\[
\begin{bmatrix}
  y_m(t) \\
  y_u(t)
\end{bmatrix} = \begin{bmatrix}
  \Phi_{mm} & \Phi_{mu} \\
  \Phi_{um} & \Phi_{uu}
\end{bmatrix} \begin{bmatrix}
  Y_m(t) \\
  Y_u(t)
\end{bmatrix} \tag{4.18}
\]

where \( Y(t) \) is the generalized amplitude. Eq.4.18 is valid for any coefficient matrix, provided that it is invertible. Response at the unmeasured coordinates can be written as follows

\[
y_m(t) = \Phi_{mm} Y_m(t) + \Phi_{mu} Y_u(t) \tag{4.19}
\]

\[
Y_m(t) = \Phi_{mm}^{-1} [y_m(t) - \Phi_{mu} Y_u(t)] \tag{4.20}
\]

\[
y_u(t) = \Phi_{um} \Phi_{nn}^{-1} y_m(t) + \left( \Phi_{uu} - \Phi_{um} \Phi_{mm}^{-1} \Phi_{mu} \right) Y_u(t) \tag{4.21}
\]
The basis fitting approximation of the unmeasured coordinated, \( y_u \), is obtained by neglecting the second term in Eq.4.21

\[
y_u(t) = \Phi_{um} \Phi_{mm}^{-1} y_m(t)
\]  

(4.22)

Error in the basis fitting estimation, therefore, is equal to the neglected term, namely

\[
\varepsilon(t) = \left( \Phi_{uu} - \Phi_{uu} \Phi_{mm}^{-1} \Phi_{mu} \right) Y_2(t)
\]  

(4.23)

Response at all coordinates can be written as

\[
y(t) = \left[ \begin{array}{c} I_{mm} \\ \Phi_{uu} \Phi_{mm}^{-1} \end{array} \right] y_m(t)
\]  

(4.24)

where \( I_{mm} \) is the identity matrix of size \( m \). Eq.4.24 is the general form of the basis fitting \( y(t) = \Psi y_m(t) \) in which the matrix \( \Psi \) is the basis.

### 4.3.2 Cubic Spline Interpolation (CS)

Although not clear at the outset, schemes that use prescribed interpolation functions between the measurements to estimate the unmeasured quantities are a particular form of basis fitting. This matter is illustrated here for a general two dimensional structure in \((y,z)\) whose response in \( y \) is to be estimated; the CS interpolation uses a cubic polynomial to predict the response at unmeasured locations, namely,

\[
y(z,t) = a_1(t) + a_2(t)z + a_3(t)z^2 + a_4(t)z^3
\]  

(4.25)
In Eq. 4.25, \( z \) is the distance from the bottom sensor in each segment and the quadruple \( \{a_1, a_2, a_3, a_4\} \) are time dependent coefficients. For \( m \) number of sensors measuring response in a given direction including the ground, the number of segments between sensors is then \( m-1 \) and the number of constants to be identified at each time station is \( 4(m-1) \). The number of available equations to solve for these constants can be established by considering measurements, continuity and boundary conditions. Imposing continuity up to the second derivative at interior points gives \( 3(m-2) \) constraints; adding the \( m \) constraints that are provided by the measurements gives a total of \( 4m-2 \) equations. The two remaining constraints are typically taken as no slope at the base and the roof, for shear type buildings with little rocking, or as no slope at the base and a zero second derivative at the roof, for flexure dominated structures. Since the third derivative is related to the shear forces, it cannot be considered continuous at interior nodes because of the abrupt jumps at story levels. Needless to say, the boundary conditions that prevail in real structures are never as simple as the extreme cases of a shear or flexure dominated structure but, however, the effect of the deviations from the actual boundary condition are restricted to the estimates near the bottom and the top of the structure.

Let \( a(t) \) be the vector of all the coefficients of the cubic spline \( (a_1-a_4 \) for the first segment, \( a_5-a_8 \) for the second etc.). The \( m \) equations that relate these coefficients to the measurements can be written as

\[
A_1 a = y_m
\]  
(4.26)

where \( A_1 \) has as many rows as measurements and as many columns as there are constants. Continuity leads to equations of the form \( f(a) = 0 \). The \( 3(m-2) \) continuity equations can
be grouped and written as Eq.4.27. The boundary conditions are two equations of the form \( g(a) = 0 \), and can be written as Eq.4.28.

\[
A_2.a = 0 \\
A_3.a = 0
\] (4.27) (4.28)

Responses at all floors are linearly related to the coefficients in the vector \( a \) as follows

\[
y = A_4.a
\] (4.29)

Eqs.4.27 and 4.28 can be combined in Eq.4.30

\[
\begin{bmatrix}
A_2 \\
A_3
\end{bmatrix}
\{a\} = \{0\} \quad \longrightarrow \quad A_{2,3}.a = 0
\] (4.30)

it follows that

\[
a = Q.v \\
Q = \text{null}(A_{2,3})
\] (4.31a,b)

Substituting the result of Eq.4.31 into Eq.4.26 and solving for \( v \) one gets

\[
v = [A_4 \cdot Q]^{-1}.y_m
\] (4.32)

the matrix in brackets is full rank and square, with a dimension equal to the number of measurements, \( m \). Substituting Eqs.4.32 into Eq.4.31 and the result into Eq.4.29 yields

\[
y(t) = \Psi_{cs}y_m(t)
\] (4.33a)

where
\[ \Psi_{cs} = A_4 Q \left[ A_4 Q \right]^{-1} \] (4.33b)

Eq.4.33 proves that CS interpolation is in fact a basis fitting where \( \Psi_{cs} \) is the basis. It can be recognized that the basis of the CS is dictated by the number and position of the sensors and there is no adaptability in the CS scheme once the sensor arrangement is set.

### 4.3.2.1 Comparing the Basis of CS with the Mode Shapes

Since ground recording is treated the same as other measurements the basis of the CS as determined in Eq.4.33b include the rigid body motion. To compare the CS basis with mode shapes of an \( n \) dof structure one should either take the rigid body mode together with the modes shapes in a matrix of dimension \( n+1 \) and compare with the CS basis, or alternatively deflate the CS basis to dimension \( n \) by removing the rigid body component and then compare with the mode shapes of the structure. Columns of the basis, per se, do not have a physical meaning but pre multiplication of Eq.4.33b by \( \Phi_{mm} \) transforms the basis to the form of the actual mode shapes.

For illustration a two-span 24 story frame assuming to be instrumented at one third, two thirds and the full height (levels 8, 16 and 24) is considered as an example. Height of the floors is 10’ and length of each span is 20’. Beam and column properties are defined by \( I_{\text{beam}} = \alpha \cdot I_{\text{column}} \) and are chosen to give a period of 3 second. The parameter \( \alpha \) controls the behavior; larger values indicate shear behavior. Fig.4.2 compare the 1st two basis of the CS excluding the rigid body (transformed to mode shapes) with the actual mode shapes of the structure with shear dominated and flexure dominated behavior. It can be seen that, in the shear dominated model CS basis is close to the model shapes of the structure at the
upper levels (Fig.4.2a) and in the flexure dominated model, CS basis is close to the modes shapes at lower levels (Fig.4.2b).

![Comparison of the 1st two “modes” of the CS and the modes shapes of the 24 story example](image)

**Fig.4.2.** Comparison of the 1st two “modes” of the CS and the modes shapes of the 24 story example a) shear dominated b) flexure dominated.

### 4.3.3 Limitation of Basis Fitting Scheme

Although the basis of CS prescribed by \( m \) number of sensors, are related to the basis of \( m-1 \) modes shape plus the rigid body mode, responses from basis fitting are not the same as responses from a truncated modal analysis using the same mode shapes. The major difference is that in basis fitting responses at the measured coordinates are forced to be exact at the expense of adding error to the generalized amplitudes (Eqs.4.20 to 4.22). The consequence can reflect at the un-instrumented coordinates in terms of spurious high frequency and large amplitude responses. In a truncated modal analysis using \( m \) modes response at all coordinates, \( u \), is
where $\phi_i$ is the mode shape and $Y_i$ is the generalized modal amplitude and $\varepsilon$ is the error due to truncation. Eq.4.34 indicates that the response from a truncated modal analysis is not exact at any floor, but the error is limited to the truncation i.e. if the response is dominated by those $m$ modes Eq.4.34 will be an adequate approximation of the true response. In basis fitting perspective, however, contribution of the truncated modes to the response is included in the basis in such a way that the measurements are reproduced. Theoretically for the responses in Eq.4.18 the generalized amplitude should satisfy

$$ Y_1(t) = \Phi^{-1}_{mm} \left[ y_m(t) - \Phi_{mm} Y_2(t) \right]. $$

In derivation of the basis fitting to match the measurements, $Y_2$ is neglected and $Y_1$ is made equal to $\Phi^{-1}_{mm} y_m$. In this proposition responses at non-instrumented levels are $y_u(t) = \Phi_{um} \Phi^{-1}_{mm} y_m(t)$. If the measurements have high frequencies (e.g. acceleration in flexible buildings), it could be amplified by $\Phi_{um} \Phi^{-1}_{mm}$ and lead to large errors in the estimates.

To illustrate this matter the 24 story example is revisited. Member properties are selected to result in 3 second period and a combined shear-flexural behavior dominated by 3 modes. The model is assumed to be instrumentation at the 3rd, 9th and 24th and is subjected to the magnitude 6 Parkfield (2004) earthquake recorded at Fault Zone 14. This ground motion is one of the records used later in this chapter and is selected because of broad frequency content and large PGA (1.31g). The difference between the truncated modal analysis and a basis fitting using 3 modes is shown in Fig.4.3 for estimated acceleration at the 20th floor.
Fig. 4.3. Estimated acceleration at the 20th floor of the 24 story example.

The high frequency, large amplitude response in the basis fitting approach is apparent. An important observation from this discussion is that although in a truncated modal analysis use of \( m \) modes that dominate the response will give reasonable approximation, in basis fitting having a basis of size \( m \) (i.e. having \( m \) sensors plus ground) is necessary but not sufficient. Accuracy depends to large extent on nature of measurements and the location of sensors. Note that in the foregoing discussion the mode shapes were exact. In CS interpolation with the same number of sensors, basis (transformed to modes) is not exactly the same as mode shapes which can increase the inaccuracy. For the interpolation methods in particular CS to provide accurate estimates the following conditions should exist:

- The number of sensors (excluding the ground) is at least equal to the number of modes that significantly contribute to the desired response.
- Basis defined by the number and location of sensor are a proper representation of the governing mode shapes.
- Measurements do not have high frequency components (e.g. displacements)
4.3.4 Improving Basis Fitting Estimation with Low Pass Filtering

Since the large error in basis fitting is of high frequency nature, low pass filtering the measurements prior to application of the basis fitting is an appropriate way to reduce the errors. The main concern in filtering is selection of the cutoff frequencies. From the analogy between basis fitting using a basis of size \( m+1 \) and a truncated modal analysis using \( m \) mode shapes in which all the frequencies beyond the \( m^{th} \) mode are eliminated, an appropriate frequency cutoff is suggested as the frequency of the \( m^{th} \) mode (\( m \) is the number of sensors excluding the ground).

The effect of filtering is illustrated in the 24 story example with 3 second period instrumented at 8\(^{th} \), 16\(^{th} \) and 24\(^{th} \) floors. The frequency of the third mode is 1.6Hz and the cutoff frequency is taken as 2Hz. Fig.4.4 compares the estimates of the inertial base shear (\( V_B^I \)) using CS with and without filtering. Inertial base shear is shown for comparisons because it is not directly affected by proximity to the sensors.

![Fig.4.4. Estimates of the inertial base shear at the 24 story example a) CS original b) CS with filtered measurements.](image)

The estimates using filtered measurements are a substantial improvement over the unfiltered CS estimates. The low pass filtering can be considered an effective way to
avoid the potentially large errors in the CS interpolations. However, selection of the
cutoff frequency limits the applicability of the low pass filtering as a general remedy.

4.4 State Estimation

4.4.1 Observers

Observer is a mathematical structure that uses a model and measurements of a
system to provide estimates of the state of the system. The idea of using an observer was
originally developed in connection with designing a feedback control system to improve
the performance or provide stability to a system. Since a feedback control system requires
information about all the states and usually only a subset of stated are available
(measured), an observer is used to estimate the unavailable states. In developing the
observer, the operating assumption is one where the main reason that mathematical
model alone does not give accurate estimates is due to uncertainty in the initial condition
and in characterization of the excitation. A classical full-state observer (Luenberger, 1971) for a general system (in discrete time) defined by Eq.4.35 is given in Eq.4.36 and
4.37. A block diagram of the Luenberger observer is shown in Fig.4.5.

\[
\begin{align*}
\dot{x} &= Ax + Bu ; \quad y = Cx \\
\dot{\hat{x}} &= A\hat{x} + Bu + G [y - C\hat{x}] \\
\hat{x} &= (A - GC)\hat{x} + [B \quad G] \begin{bmatrix} u \\ y \end{bmatrix}
\end{align*}
\]

In the above equations \( x \) is the true state vector, \( \hat{x} \) is the state vector estimate, \( y \) is the
measured output and \( \hat{y} \) is the estimated output.
Goal of the observer is to provide an estimate of the state such that when \( t \) goes to infinity the estimate \( \hat{x} \) asymptotically reaches the true state \( x \). The equation for state error can be written by subtracting Eq.4.36 from the 1\(^{st}\) equation in Eq.4.35 while replacing \( y \) with \( Cx \).

\[
\dot{e} = (A - GC)e
\]  

(4.37)

where \( e = x - \hat{x} \) and \( \dot{e} = \dot{x} - \dot{\hat{x}} \). The objective of having zero steady state error (as \( t \to \infty \)) is achieved if the error dynamics is stable. This is realized if and only if all the eigenvalues of \((A-GC)\) lie in the left half plane. The observer design is therefore reduced to a pole placement problem in which \( G \) is selected such the poles of \((A-GC)\) are placed in the left half plane. All the poles of the matrix \((A-GC)\) can be placed arbitrarily using any appropriate pole placement algorithm (Ackerman, 1972; Godbout and Jordan, 1975, Ogata, 2002) if the system is completely observable i.e. if the observability matrix, \( P_o \), for the pair \((A,C)\) is full rank.

\[
P_o = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}
\]  

(4.38)
Equation of the observer in discrete time is given by Eq.4.39. Stability of the error dynamic is achieved if the eigenvalues of \((A_d - GC_d)\) lie inside a unit circle. The observability matrix is obtained from Eq.4.38 by replacing \(A\) with \(A_d\).

\[
\tilde{x}(k+1) = A_d \tilde{x}(k) + B_d u(k) + G \left[ y(k) - C_d \tilde{x}(k) \right] \tag{4.39}
\]

When in addition to the unknown initial condition the excitation and measurements are noise contaminated the Kalman Filter (a time varying optimal observer) solves the observer problem (section 4.4.2). The mathematical representation of the system and the observer in the presence of process noise \((w)\) and measurement noise \((v)\) is given in Eqs.4.40 and 4.41.

\[
x(k+1) = A x(k) + B u(k) + w(k) \quad ; \quad y(k) = C x(k) + v(k) \tag{4.40}
\]

\[
\tilde{x}(k+1) = A \tilde{x}(k) + B u(k) + G \left[ y(k) - C \tilde{x}(k) \right] \tag{4.41}
\]

### 4.4.1.1 Observers in Seismic Response Reconstruction Problem

In a recent work by Hernandez and Bernal (2008) an observer has been developed that applies to situations where the primary source of uncertainty is error in the model. The dynamic equation for the real system under this condition is shown by Eq.4.42 in which \(A\) and \(B\) belong to the nominal model and \(\delta A\) and \(\delta B\) are errors thereof.

\[
x(k+1) = (A + \delta A)x(k) + (B + \delta B)u(k) \tag{4.42}
\]

Comparing Eq.4.42 with Eq.4.40 one notices that the disturbance in this case is equivalent to \(w(k) = \delta A x(k) + \delta B u(k)\). The observer proposed by Hernandez and Bernal
which is in a form of reduced order observers (Astrom and Wittenmark, 1990, Basseville and Nikiforov, 1993) has the following form

\[
\ddot{x}(k + 1) = (I - C^T C) A \dot{x}(k) + (I - HC) Bu(k) + C^T y(k + 1)
\]  

(4.43)

An interpretation of Eq.4.43 in the context of structural engineering is that the observer imposes the motion at the instrumented coordinates. The estimation scheme developed in this study is a generalization of this observer that uses the corrective forces at all coordinates.

### 4.4.2 The Kalman Filter

The Kalman Filter (Kalman, 1960) is the most renowned estimation scheme with a wide range of applicability. The Kalman Filter in its original form operates on the situations that the dynamics of the system is described by Eq.4.40 and the main reason for discrepancy between the response of the model and actual system is that the input cannot be exactly characterized due to unmeasured disturbances. The Kalman Filter provides optimal estimate of the states when process and measurement noise \(w\) and \(v\) are white, Gaussian, uncorrelated, zero-mean and of known covariance i.e.

\[
\begin{align*}
E(w) &= 0 \\
E(v) &= 0 \\
E(ww^T) &= Q \\
E(vv^T) &= R \\
E(vw^T) &= 0
\end{align*}
\]  

(4.44)

Optimality of the estimates in the Kalman Filter is in the sense that the trace of the covariance of the state error is minimized. The state estimation using the Kalman Filter is presented in various formats; in the most popular format it involves a forecast step and an
updating step. The procedure has a variable gain that converges quickly to a constant. A complete derivation and discussion on the Kalman Filter properties can be found in numerous references e.g. Gelb (1996), Kailath et al. (2000) and Simon (2006). The steps in discrete time applications involve a recursive least square solution with propagation of the mean and covariance of the state error, \( P \), as listed below.

1. Initialization of the state as \( \hat{x}^- (0) \) and the covariance of the state error as \( P^- (0) \).

2. Calculation of the innovations
\[
e(k) = (y(k) - C\hat{x}^- (k))
\] (4.45)

3. Computation of the Kalman gain from
\[
K(k) = P^-(k)C^T (AP^- (k)A^T + R)^{-1}
\] (4.46)

4. Updating the state
\[
\hat{x}^+ (k) = \hat{x}^- (k) + K(k) e(k)
\] (4.47)

5. Updating the covariance of the state error
\[
P^+ (k) = (I - K(k)C) P^- (k)
\] (4.48)

6. Advancing the state
\[
\hat{x}^- (k+1) = A \hat{x}^+ (k) + B u(k)
\] (4.49)

7, 8. Advancing the covariance of the state error using Eq.4.50 and repeating from step 2.
\[
P^- (k+1) = A P^+ (k) A^T + Q
\] (4.50)
4.4.2.1  Kalman Filter in Seismic Response Reconstruction Problem

In structural engineering problems as mentioned previously modeling error constitutes the main source of uncertainty. State estimation problem of this type is cast deterministically which makes the standard Kalman Filter unfit since most of the underlying assumptions are not valid. Modified versions of the Kalman Filter to accommodate deviations from the original assumptions (to account for colored or correlated noise) do not exactly fit the structural problem either since they still require stochastic information about the noise. Although the conditions that prevail in seismic applications do not match the Kalman Filter assumptions, in this study the Kalman Filter is used as an alternative observer. Presuming that measurements noise are not the most important sources of error in seismic applications, we take \( R = 0 \) and as simple idea take \( Q = I \). This ad-hoc selection of \( Q \) and \( R \) makes the Kalman Filter on par with the classical observers in which the poles of the state error dynamic equation are placed based on the Kalman gain.

4.4.3  RTS Smoother

Classical observers and the Kalman Filter provide an estimate of the state at a given time using measurements up to that time. When operating offline, all the time history of the measurement is available and to estimate the state at a particular time station it is possible to use measurements at future time stations. The schemes that use not only past but also future measurements are called smoothers. Three are 3 types of smoothers, Fixed-Point, Fixed-Lag and Fixed-Interval. As the names suggest the Fixed-Point is when one is interested in improving the state at a fixed time \( t \) when new measurements become available at the future times. The Fixed-Lag is when one estimates the state at time \( t \)
using the measurements up to time $(t+T)$ in which $T$ is a fixed time window. And the Fixed-Interval is one that estimates the state at any time within an interval using all the measurements in that interval. The RTS smoother examined here is a Fixed-Interval smoother introduced by Rauch et al. (1965) that takes the entire history as the fixed interval. The algorithm of the process is as follows:

- Perform a standard Kalman filter estimation and store: $x_{k,f}^-, x_{k,f}^+, P_{k,f}^-$ and $P_{k,f}^+$, where the subscript $f$ is added to indicate that these are results from the forward pass of the Kalman filter.

- Compute the smooth estimate of the state, $x_{k,s}$, as

\[
x_{k,s} = x_{k,f}^+ + G_k (x_{k+1,f} - x_{k+1,f}^-) \quad (4.51)
\]

\[
G_k = P_{k,f}^+ A^T [P_{k+1,f}^-]^{-1} \quad (4.52)
\]

**Numerical Illustration**

Performance of the Kalman, Smoother and the HB-Observer is illustrated in the 24 story example instrumented at the 8th, 16th and 24th floors that was used previously. In implementations measurements are taken as relative velocities and the nominal model is formulated using a combined shear and flexural continuous beam (Miranda and Taghavi 2005; Alimoradi et al. 2006) that has the same 1st period of the actual model. Fig.4.6 shows the estimates of the inertial base together with true values. Estimates from cubic spline interpolation were shown previously in Fig.4.4. Estimates of the nominal model (in open loop) are also shown for comparison. Results indicate that all these state estimation schemes have similar qualities that when compared to cubic spline in Fig.4.4a.
demonstrates significantly higher accuracy. In this particular example as can be seen in estimates with filtered CS (Fig.4.4b) are as good as the state estimation schemes.

![Comparison of the estimated inertial base shears with the true values in the 24 story example with 3 sec. period. a) Nominal model b) HB Observer c) Kalman d) RTS Smoother.](image)

Fig.4.6. Comparison of the estimated inertial base shears with the true values in the 24 story example with 3 sec. period. a) Nominal model b) HB Observer c) Kalman d) RTS Smoother.

### 4.5 Minimum Norm Response Corrector (MIRC)

#### 4.5.1 Description of the MIRC

A deterministic scheme, designated as “Minimum Norm Response Corrector” (MIRC), is developed here by blending the ideas of model based estimation with the concepts of structural engineering. The MIRC takes the difference between the response of a nominal model and the measurements to determine corrective forces that when applied to the nominal model together with the ground excitation, reproduce the measurements at the instrumented coordinates. The method is developed based on the
assumption of linear response and is shown to be useful for nonlinear responses as well. To begin, let’s recall the equation of motion for an accurate representation of a structure with linear behavior under a general form of excitation.

\[(M + \Delta M)\ddot{q} + (C_{dam} + \Delta C_{dam})\dot{q} + (K + \Delta K)q = b_{2d}P_d(t) + b_{2u}P_u(t)\]  \hspace{1cm} (4.53)

where \(q\) is the displacement response of the system at all dofs. Matrices in the parenthesis are the true system parameters which are the sum of the nominal model parameters \((M, C_{dam} and K)\) and the errors thereof \((\Delta M, \Delta C_{dam} and \Delta K)\). Applied forces are expressed in general form as the sum of known forces, \(P_d(t)\), and unmeasured forces, \(P_u(t)\) with spatial distributions respectively defined by \(b_{2d}\) and \(b_{2u}\). Eq.4.53 can be written as

\[M\ddot{q} + C_{dam}\dot{q} + Kq = b_{2d}P_d(t) + g(t)\]  \hspace{1cm} (4.54)

where \(g(t)\) is

\[g(t) = b_{2u}P_u(t) - (\Delta M\ddot{q} + \Delta C_{dam}\dot{q} + \Delta Kq)\]  \hspace{1cm} (4.55)

Interpretation of the above equations is that the nominal model subjected to known forced \(P_d\) can reproduce the exact responses \((q)\) provided that a corrective force, \(g(t)\), defined by Eq.4.55 is also applied. The force in Eq.4.55 is the only set of forces that ensure the responses of the nominal model at all levels are equal to the true responses. Evidently the true corrective forces can never be identified since all the ingredients of Eq.4.55 are unknown. Nonetheless, it gives a clue about the nature of these forces i.e. the corrective forces are expected to act at all dofs. Information in terms of available measurements at some coordinates can be utilized to obtain an approximation of the true corrective forces, \(\hat{g}(t)\). The objective of the MIRC, therefore, is to determine an
approximation of the corrective forces that ensures the nominal model (at least) reproduces the measurements.

Let $q_1$ be the response of the nominal model to known forces $P_d$, $q_2$ the response of the nominal model to $\hat{g}(t)$ and $q_m$ the measurements. In the MIRC, response of the nominal model to the corrective forces at the instrumented coordinates should be equal to the difference between $q_m$ and $q_1$ that is $q_{2m} = q_m - q_{1m}$ (where the subscript $m$ denoted the instrumented coordinates). For a given nominal model the residual $q_{2m}$ is known and the goal is to find the forces $\hat{g}(t)$. Fig.4.7 schematically depicts the MIRC idea.

![Fig.4.7. Illustration of the MIRC’s framework.](image)

**Definition of the MIRC Problem in State Space**

Equation of motion that describes the MIRC problem is

$$M \ddot{q}_2 + C_{d_{12}} \dot{q}_2 + Kq = \hat{g}(t)$$  \hspace{1cm} (4.56)

Using the material presented in section 4.2, Eq.4.75 can be expressed in state space as

$$x(k+1) = A_d x(k) + B_d \hat{g}(k) \quad ; \quad y(k) = C_d x(k) + D_d \hat{g}(k)$$  \hspace{1cm} (4.57)
The known output $y_m$ pertains to the residuals depending on what quantity is assumed measured, for example if displacements are measured $y_m$ is related to $q_{2m}$ or if velocity is measured $y_m$ is related to $\dot{q}_{2m}$ through an appropriate $C_d$. The problem of MIRC in finding the corrective force $\hat{g}$ is akin to the problem of “tracking” in control in which one seeks an input such that the system follows a certain reference (Anderson and Moore, 1990). This problem can be stated as:

“Suppose an $n$-dimensional LTI system is described by Eq.4.76 in which vector $y$ represents $m$ outputs. Also suppose that measurements of $y$ are available in a vector $y_m$ at all times $0 \leq t \leq T_{end}$. The problem in the MIRC is to find the input $\hat{g}$ so that the output $y$ at all times is equal to the measured output $y_m$.”

### 4.5.2 Derivation of the MIRC

A direct solution for the MIRC is shown by reframing the problem in the general form of Eq.4.58 and using inversion methods (Devasia et al., 1996 and Brinkerhoff and Devasia, 2000). In this form, $y$ is the known output (residuals defined in section 4.5.1), $H$ is a matrix that depends on the system parameters and relates inputs to outputs, and $u$ is the input (corrective force) to be determined.

$$y = H.u$$  \hspace{1cm} (4.58)

Examination of propagation of the state and output in Eq.4.57 leads to a formula that directly relates the output sequence to the input sequence as follows
At \( k = 1 \)

\[
x(1) = A_d x(0) + B_d \tilde{g}(0)
\]

\[
y(1) = C_d x(1) + D_d \tilde{g}(1) = C_d A_d x(0) + C_d B_d \tilde{g}(0) + D_d u(1)
\]

At \( k = 2 \)

\[
x(2) = A_d x(1) + B_d \tilde{g}(1) = A^2_d x(0) + A_d B_d \tilde{g}(0) + B_d \tilde{g}(1)
\]

\[
y(2) = C_d x(2) + D_d \tilde{g}(2) = C_d A^2_d x(0) + C_d A_d B_d \tilde{g}(0) + C_d B_d \tilde{g}(1) + D_d \tilde{g}(2)
\]

and at an arbitrary step \( l \), state and output sequences can be written as

\[
x(l) = A^l_d x(0) + \sum_{j=1}^{l} A^{j-1}_d B \tilde{g}(j-l)
\]

\[
y(l) = C_d A^l_d x(0) + \sum_{j=1}^{l} C_d A^{j-1}_d B \tilde{g}(j-l) + D_d \tilde{g}(l)
\]

(4.59)

recalling the definition of Markov Parameters (impulse response of the state space model) in Eq.4.60, the output sequence in Eq.4.59 can be written as shown in Eq.4.61.

\[
Y_j = C_d A^{j-1}_d B_d \quad j = 1,2,\ldots l
\]

\[
Y_0 = D_d
\]

(4.60)

\[
y(l) = C_d A^l_d x(0) + \sum_{j=0}^{l} Y_j \tilde{g}(l-j)
\]

(4.61)

Taking initial condition as zero which is a reasonable assumption for structure under a ground excitation yields a direct relationship between the output \( y \) and the input \( u \) for a (sequence of length \( l \)) as follows which is of the same form of Eq.4.58.
\[ y_l = H_l \tilde{g}_l \]  

where \( y_l \) is the outputs of the \( l \) sequence stacked in a vector of \((ml \times 1)\), and \( \tilde{g}_l \) is the inputs in a vector of \((nl \times 1)\) and finally \( H_l \) is a lower triangular block matrix of

\[
H_l = \begin{bmatrix}
Y_0 & 0 & \cdots & 0 \\
Y_1 & Y_0 & \cdots & 0 \\
& & \ddots & \vdots \\
Y_l & Y_{l-1} & \cdots & Y_0
\end{bmatrix}
\]

Size of the input vector \( \tilde{g}_l \) is more than the size of output vector \( y_l \) and therefore the solution is not unique. A general solution is given by

\[
\tilde{g}_l = H_l^{-*} y_l + \text{Null}(H_l) z(t)
\]

where \(-^*\) stands for pseudo inversion and \( z(t) \) is an arbitrary vector of appropriate dimension. In the MIRC strategy the corrective loads are taken as the minimum norm solution by neglecting the second term in Eq.4.64.

\[
\tilde{g}_l^{\text{MIRC}} = H_l^{-*} y_l
\]

It should be reminded that \( y_l \) is a vector consisting of the sequence of the residuals \( y_m \) and \( \tilde{g}_l^{\text{MIRC}} \) is the vector consisting of sequence of the corrective force \( \tilde{g} \) as shown where \( \text{vec} \) is an operator that stacks all the column of a matrix in a single one.

\[
y_l = \text{vec}[y_m(0) \ y_m(1) \ y_m(2) \ldots y_m(l)]
\]

\[
\tilde{g}_l^{\text{MIRC}} = \text{vec}[\hat{g}_0 \ \hat{g}_1 \ \ldots \ \hat{g}_l]
\]
Computational steps can be summarized as follows

1) Form the matrices $A_c$ and $B_c$ respectively from Eqs.4.9 and 4.10 with $b_2$ as identity. Take $D_c$ as zero $C_c$ from Eqs.4.16 depending on what quantity is measured.

2) Compute the discrete time matrices $A_d$, $B_d$, $C_d$ and $D_d$ from Eq.4.5.

3) Compute the Markov Parameters $Y$ from Eq.4.60 and form the matrix $H$ using Eq.4.63.

4) Compute the residual $y_m$ as the difference between the measurements ($q_m$ or $\dot{q}_m$) and the responses of the nominal model at the instrumented coordinates ($q_{1m}$ or $\dot{q}_{1m}$) and place them in a columns to form $y_l$, using Eq.4.66a.

5) Calculate the MIRC forces $u_l$ from Eq.4.65 and reorder them using Eq.4.66b.

6) Final estimates are the response of the nominal model to the ground excitation and the corrective forces found at step 5.

4.5.2.1 Discussion

On the MIRC Solution

A question that may arise is whether or not the MIRC solution is the best answer to the problem of finding corrective forces. The problem of Eq.4.62 is one of an underdetermined system in which the number of unknowns ($n_l$) is more that the number of equations ($m_l$) since $n > m$. The matrix $H$ is wide and consists of the blocks of Markov Parameters. For stable structures where the corrective forces are assumed to act at all coordinates the matrix $H$ is full row rank and there will be infinite solutions. From the mathematical point of view therefore the minimum norm solution namely $\hat{g}_i^{\text{MIRC}} = H_i^{-*} y_i$ is the best answer. However priori information about the form of the true corrective forces exists. As shown in section 4.4.1 the unique corrective force that ensures that the
response everywhere is exact is \( g(t) = b_{2u}P_u(t) - (\Delta M \ddot{q} + \Delta C_{\text{dam}} \dot{q} + \Delta K q) \). With simplifying assumption that there is no unmeasured force \( P_u \) and there is no error in mass and damping matrices, the true corrective forces are \( g = \Delta K q \). For a sequence of \( l \) time steps the true corrective forces are

\[
g^{\text{True}}_l = \text{vec}[\Delta K q(0) \quad \Delta K q(1) \ldots \Delta K q(l)]
\]  

(4.67)

For the minimum norm solution of the MIRC to be equal to the true corrective force a component from the null space of \( H_l \) should be added that satisfies the following relationship

\[
g^{\text{True}}_j = \hat{g}^{\text{MIRC}}_j + \text{Null}(H_l).z
\]  

(4.68)

Ideally one could take advantage of the information in the form of Eq.4.68 to improve the MIRC solution. However due to the fact the true corrective forces are never achievable outside simulations, this is not possible. Attempts to identify specific characteristics about the mathematical structure of the corrective forces from Eq.4.68 did not succeed. Therefore in this study the minimum norm solution of Eq.4.65 is taken as the best solution.

**On the MIRC Computation**

In Eq.4.62 namely \( y_j = H_l \hat{g}_j \) the matrix \( H_l \) is a lower triangular block matrix. This particular form of the matrix \( H \) suggests that the solution can be done sequentially at successive time steps in which \( n \) unknowns of vectors \( \hat{g}_j \) are determined by solving \( m \) equations described by each row block \( j \). For example the first row block leads to
\(y_0 = Y_0 \hat{g}_0\) and the second one to \(y_i = Y_i u_i + Y_i \hat{g}_0\) in which \(\hat{g}_0\) is determined as the minimum norm solution of the previous step. This step by step approach involves separate minimum norm problems at each step and uses all the sequence of the inputs from the previous steps \(\hat{g}_0\) to \(\hat{g}_{j-1}\). This approach in computation however is not the same as the procedure proposed in section 4.4.2 where all the \(l\) time steps are used simultaneously. In a step by step approach the uncertainty in \(\hat{g}_j\) due to neglecting null \(Y_0\) \(z\) in the minimum norm solution at each step, is carried over to the future steps which can add up to a large uncertainty in the \(\hat{g}_j\). The procedure described in 4.4.2 takes advantage of the fact that the estimation is being done in an offline fashion and all the history of the measurements is available. This makes the MIRC procedure akin to Smoothers and avoids accumulating the uncertainty.

Efficiency however can be hampered by computing limitations. When the size of the structure is large and ground motion has long duration the matrix \(H\) gets large and the pseudo inverse could be computationally intensive. The limitation can be overcome by dividing the entire time to a number of smaller time windows. In this case response at the end of each window should be taken as the initial condition for the next window and since the MIRC assumes zero initial condition a minor adjustment should be applied. The adjustment involves subtracting the unforced response of the nominal model to the initial condition from the residuals and taking the difference as the new residuals for that time window. The rest will be the same since now the initial conditions can be taken zero.
4.5.2.2 Numerical Illustration of the MIRC

To deliver an appreciation of the potential improvements from the MIRC the example of the 24 story building used earlier is revisited. Fig.4.8 compares the estimates of the inertial base shear, $V_B^I$, from CS and MIRC and also shows the estimates from the nominal model and filtered CS. It can be seen that the MIRC estimates are notably better. A thorough evaluation of the estimation methods is presented in section 4.6.

Fig.4.8. Comparison of the estimates of inertial base shear in the 24 story example. a) CS b) filtered CS filtered c) Nominal Model d) MIRC.

4.5.2.3 Dependency of the MIRC on the Nominal Model

Accuracy in the model based estimation schemes undoubtedly depends on the nominal model. While achieving a fully robust scheme that overcomes all the
imperfection of the nominal model is not feasible, considering a model based scheme “successful” requires that the dependency on accuracy of the nominal model be small.

Determination of corrective forces in the MIRC explicitly takes into account the error in the nominal model in terms of the difference between the model response and the measurements. Therefore it is expected that the methodology allow a rather large error margin for the nominal model. To verify this anticipation the effect of model error on MIRC estimates is examined through an example in which the modeling error can be quantified with a simple parameter. The example is a two story one bay portal frame instrumented at the 2nd floor (Fig.4.9). Stiffness matrix is assembled from the individual elements stiffness matrices; mass is considered uniform (1.0) and damping is considered classical with a ratio of 5% in all modes. Properties EI are selected to give a period of 3 seconds. Example is subjected to the Parkfield 2004 earthquake used previously. Error in the nominal model is introduced by multiplying the stiffness of the first story elements with an increasing scale. Error in the mass and damping matrices in the nominal model is neglected therefore the stiffness scale solely quantifies the error in the nominal model. The overall stiffness of the nominal model is then adjusted to maintain the 1st period of 3 seconds. The normalized root mean square of the error in the time history of the MIRC estimates (defined later as NRMS in section 4.6.2) represents the error measure. Fig.4.9 shows the variation error for the inertial base shear and the first floor IDI with the stiffness scale. For comparison, error from CS interpolation and an open loop nominal model are also shown. It can be seen in Fig.4.9 that although the error in the MIRC increases with the increasing error in the nominal model (stiffness scale), it is still smaller than the error in CS especially for the inertial base shear. An interesting observation is
that even with a rather large error in the model namely the stiffness of the 1st floor being 5 times larger than the real values, the MIRC is still effective in compensating for the nominal model imperfection and reducing the (open loop) error.

![Graph](image_url)

Fig.4.9. Variation of normalized root mean square error of the MIRC estimates with the change in the stiffness of the 1st floor. a) Inertial base shear b) first floor IDI.

Although dependency of the MIRC on accuracy of the nominal model cannot be denied, the margin of error that a nominal model could have before the MIRC estimates become worse than CS interpolation is large. The minimum requirement for the nominal model set in this study for creating the nominal model which requires the nominal model to have the 1st natural period of the real structure results in sufficient accuracy of the nominal model whereby the MIRC is expected to give reasonably accurate estimates. It should be noted that in Chapter 5 in reconstruction of the response of instrumented building using the MIRC, the nominal model is derived from a highly refined model of the structure which increases the reliability of the estimates.
4.6 Evaluation of the Reconstruction Schemes

In this section performance of various estimation methods in seismic response reconstruction is evaluated through a series of simulation and statistical analysis. The evaluation addresses the cubic spline interpolation, observers in the forms of a Kalman Filter and RTS Smoother with $Q = I$ and $R = 0$, and the MIRC.

4.6.1 Evaluation Procedure

To evaluate the accuracy of estimation schemes, a strategy that comes to mind is to pretend that one of the sensors is not available and apply the estimation schemes to get the missing sensor’s data using the rest of the sensors and compare it with the real recording. Although this may sound logical at first, such evaluation that involves removing a sensor is not reliable because it changes the operating condition. This is evident for CS interpolation scheme since the CS basis is directly defined by the number and location of sensors. An appropriate approach for evaluation is simulation where a refined model of the structure substitutes the real building and its responses to a bi-directional excitation represent the “true responses”. Taking the simulated responses at instrumented levels as the measurements one can carry out the estimations and compare the estimates of the quantities of interest with the “true responses” at other levels. This approach eliminates the errors connected with sensor removals and preserves the operating conditions. The important issue in application of model based estimation in a simulation study is that the nominal model should reflect the expected modeling error. In this study as noted previously the nominal model is formulated in 2D based on a
combined shear-flexural continuum model with properties selected such that the first mode period matches that of the true model.

Four mid to high rise buildings instrumented by CSMIP are used in simulations. Table 4.1 gives some information about the buildings (more information can be found in Bernal and Nasseri, 2009 and Bernal, 2010). Refined 3D models of the buildings created in Perform 3D (CSI Inc., 2006) serve as surrogates of the true buildings. An ensemble of ground motion containing 30 worldwide earthquakes with $M_L > 6$ that covers a wide range of frequency content is used in the study. The list of the ground motions and some more information about them is given in Table 4.2.

Table 4.1. Specification of the four buildings used in evaluation of estimation methods.

<table>
<thead>
<tr>
<th>Building</th>
<th>CSMIP Station</th>
<th>Total Levels</th>
<th>Instrumented Levels</th>
<th>Structural Type</th>
<th>1st Natural Period (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 Story North Hollywood</td>
<td>24464</td>
<td>21</td>
<td>2, 8, 15, 21</td>
<td>RC MRF</td>
<td>1.80 2.00</td>
</tr>
<tr>
<td>13 Story Sherman Oaks</td>
<td>24322</td>
<td>13</td>
<td>1, 7, 13</td>
<td>RC MRF</td>
<td>2.56 2.78</td>
</tr>
<tr>
<td>10 Story San Jose</td>
<td>57356</td>
<td>10</td>
<td>5, 10</td>
<td>RC Shear wall</td>
<td>0.43 0.64</td>
</tr>
<tr>
<td>6 Story Burbank</td>
<td>24370</td>
<td>6</td>
<td>1, 2, 6</td>
<td>Steel MRF + perimeter wall</td>
<td>1.27 1.28</td>
</tr>
</tbody>
</table>

Accuracy of estimation methods is evaluated by comparing the estimate of story displacements, inter story drifts and shears with the true values. Evaluation addresses both linear and nonlinear responses; simulations are carried out once using the actual ground motions (and allowing nonlinear responses) and once by scaling down the motions to create strictly linear responses.
Table 4.2 Ground motions used in this study

<table>
<thead>
<tr>
<th>Serial No.</th>
<th>Station Name</th>
<th>Earthquake</th>
<th>Magnitude</th>
<th>Epic. Dist. (km)</th>
<th>PGA (H1, H2, V) - g</th>
<th>PGV (H1, H2, V) - cm/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Parkfield - Fault Zone 1</td>
<td>Parkfield Earthquake of September 28, 2004</td>
<td>6.0</td>
<td>9</td>
<td>0.59, 0.82, 0.26</td>
<td>63, 81, 10</td>
</tr>
<tr>
<td>2</td>
<td>Parkfield - Fault Zone 1</td>
<td>Parkfield Earthquake of September 28, 2004</td>
<td>6.0</td>
<td>12</td>
<td>1.31, 0.54, 0.56</td>
<td>83, 42, 23</td>
</tr>
<tr>
<td>3</td>
<td>Templeton - 1-story Hospital Ground Floor</td>
<td>San Simeon Earthquake of December 22, 2003</td>
<td>6.5</td>
<td>38</td>
<td>0.42, 0.46, 0.26</td>
<td>33, 27, 16</td>
</tr>
<tr>
<td>4</td>
<td>Amboy</td>
<td>Hector Mine Earthquake of October 16, 1999</td>
<td>7.1</td>
<td>48</td>
<td>0.15, 0.18, 0.13</td>
<td>26, 27, 12</td>
</tr>
<tr>
<td>5</td>
<td>Taiwan - CHY028</td>
<td>Chi-Chi Earthquake of September 21, 1999</td>
<td>7.6</td>
<td>7 to fault</td>
<td>0.82, 0.65, 0.34</td>
<td>67, 72, 36</td>
</tr>
<tr>
<td>6</td>
<td>Taiwan - TOU129</td>
<td>Chi-Chi Earthquake of September 21, 1999</td>
<td>7.6</td>
<td>1 to fault</td>
<td>0.63, 1.01, 0.34</td>
<td>36, 60, 35</td>
</tr>
<tr>
<td>7</td>
<td>Taiwan - TCU068</td>
<td>Chi-Chi Earthquake of September 21, 1999</td>
<td>7.6</td>
<td>1 to fault</td>
<td>0.46, 0.56, 0.49</td>
<td>176, 263, 187</td>
</tr>
<tr>
<td>8</td>
<td>Taiwan - CHY028</td>
<td>Chi-Chi Earthquake of September 21, 1999</td>
<td>7.6</td>
<td>10 to fault</td>
<td>0.42, 1.16, 0.34</td>
<td>46, 115, 25</td>
</tr>
<tr>
<td>9</td>
<td>Sylmar - County Hospital Lot</td>
<td>Northridge Earthquake of January 17, 1994</td>
<td>6.7</td>
<td>16</td>
<td>0.59, 0.83, 0.53</td>
<td>77, 129, 19</td>
</tr>
<tr>
<td>10</td>
<td>Newhall - LA County Fire Station</td>
<td>Northridge Earthquake of January 17, 1994</td>
<td>6.7</td>
<td>20</td>
<td>0.57, 0.58, 0.54</td>
<td>75, 95, 31</td>
</tr>
<tr>
<td>11</td>
<td>Los Angeles Reservoir Rinaldi Receiving Station FF</td>
<td>Northridge Earthquake of January 17, 1994</td>
<td>6.7</td>
<td>9</td>
<td>0.47, 0.83, 0.83</td>
<td>166, 73, 51</td>
</tr>
<tr>
<td>12</td>
<td>Santa Monica - City Hall Grounds</td>
<td>Northridge Earthquake of January 17, 1994</td>
<td>6.7</td>
<td>23</td>
<td>0.88, 0.37, 0.23</td>
<td>42, 25, 14</td>
</tr>
<tr>
<td>13</td>
<td>Lucerne Valley</td>
<td>Landers Earthquake of June 28, 1992</td>
<td>7.4</td>
<td>1 to fault</td>
<td>0.72, 0.78, 0.82</td>
<td>98, 32, 46</td>
</tr>
<tr>
<td>14</td>
<td>Yermo - Fire Station</td>
<td>Landers Earthquake of June 28, 1992</td>
<td>7.4</td>
<td>84</td>
<td>0.15, 0.24, 0.13</td>
<td>28, 51, 13</td>
</tr>
<tr>
<td>15</td>
<td>Big Bear Lake - Civic Center Grounds</td>
<td>Big Bear Earthquake of June 28, 1992</td>
<td>6.6</td>
<td>35</td>
<td>0.59, 0.43, 0.15</td>
<td>61, 30, 13</td>
</tr>
<tr>
<td>16</td>
<td>Petrolia - Fire Station</td>
<td>Cape Mendocino Earthquake of April 26, 1992</td>
<td>7.1</td>
<td>8</td>
<td>0.65, 0.58, 0.16</td>
<td>90, 48, 21</td>
</tr>
<tr>
<td>17</td>
<td>Petrolia - Fire Station</td>
<td>Petrolia Earthquake of April 25, 1992</td>
<td>7.1</td>
<td>11</td>
<td>1.04, 1.50, 0.75</td>
<td>41, 126, 60</td>
</tr>
<tr>
<td>18</td>
<td>Cape Mendocino</td>
<td>Petrolia Earthquake of April 25, 1992</td>
<td>7.1</td>
<td>18</td>
<td>0.39, 0.55, 0.20</td>
<td>45, 43, 10</td>
</tr>
<tr>
<td>19</td>
<td>Rio Dell - Hey105/Painter Street Overpass FF</td>
<td>Petrolia Earthquake of April 25, 1992</td>
<td>7.1</td>
<td>7</td>
<td>0.48, 0.63, 0.44</td>
<td>48, 55, 19</td>
</tr>
<tr>
<td>20</td>
<td>Corralitos - Eureka Canyon Road</td>
<td>Loma Prieta Earthquake of October 17, 1989</td>
<td>7.0</td>
<td>9</td>
<td>0.46, 0.44, 0.13</td>
<td>95, 84, 26</td>
</tr>
<tr>
<td>21</td>
<td>Los Gatos - Lindaan Dam Left Abutment</td>
<td>Loma Prieta Earthquake of October 17, 1989</td>
<td>7.0</td>
<td>19</td>
<td>0.46, 0.44, 0.13</td>
<td>95, 84, 26</td>
</tr>
<tr>
<td>22</td>
<td>Saratoga - Aloha Ave.</td>
<td>Loma Prieta Earthquake of October 17, 1989</td>
<td>7.0</td>
<td>4</td>
<td>0.32, 0.49, 0.35</td>
<td>44, 41, 26</td>
</tr>
<tr>
<td>23</td>
<td>El Centro - Imperial County Center Grounds</td>
<td>Superstition Hills Earthquake of November 24, 1987</td>
<td>6.6</td>
<td>36</td>
<td>0.26, 0.34, 0.12</td>
<td>41, 47, 8</td>
</tr>
<tr>
<td>24</td>
<td>Los Angeles - Obregon Park</td>
<td>Whittier Earthquake of October 1, 1987</td>
<td>6.1</td>
<td>10</td>
<td>0.43, 0.41, 0.13</td>
<td>22, 13, 5</td>
</tr>
<tr>
<td>25</td>
<td>Chalmette - Zack Ranch</td>
<td>Chalmette Valley Earthquake of July 21, 1986</td>
<td>6.4</td>
<td>14</td>
<td>0.40, 0.44, 0.30</td>
<td>43, 36, 12</td>
</tr>
<tr>
<td>26</td>
<td>El Centro - Array #6</td>
<td>Imperial Valley Earthquake of October 15, 1979</td>
<td>6.6</td>
<td>1 to fault</td>
<td>0.43, 0.37, 0.17</td>
<td>106, 63, 56</td>
</tr>
<tr>
<td>27</td>
<td>El Centro - Array #7</td>
<td>Imperial Valley Earthquake of October 15, 1979</td>
<td>6.6</td>
<td>1 to fault</td>
<td>0.45, 0.33, 0.30</td>
<td>106, 45, 26</td>
</tr>
<tr>
<td>28</td>
<td>El Centro - Imperial County Center Grounds</td>
<td>Imperial Valley Earthquake of October 15, 1979</td>
<td>6.6</td>
<td>28</td>
<td>0.24, 0.21, 0.24</td>
<td>64, 36, 17</td>
</tr>
<tr>
<td>29</td>
<td>El Centro - HwyMeloland Freeway FF</td>
<td>Imperial Valley Earthquake of October 15, 1979</td>
<td>6.6</td>
<td>19</td>
<td>0.31, 0.29, 0.23</td>
<td>72, 91, 29</td>
</tr>
<tr>
<td>30</td>
<td>El Centro - Irrigation District</td>
<td>El Centro Earthquake of May 18, 1948</td>
<td>6.9</td>
<td>17</td>
<td>0.34, 0.21, 0.21</td>
<td>33, 37, 11</td>
</tr>
</tbody>
</table>
4.6.2 Error Metrics and Evaluation Criteria

Evaluation criteria should address both the time history and the peak values. In this study separate error metrics are defined for the entire time history and for the peak values and observation the statistics of the error metrics are examined to identify the scheme with the smallest error. The error metrics are defined next.

Peak Response Metrics

Let $y$ be the true and $y_e$ be the estimated response. The peak response error metrics are defined for the ratio of the estimated response to the true responses as follows

$$
PRP = \frac{\max(y_e)}{\max(y)} \quad PRN = \frac{\min(y_e)}{\min(y)} \quad PRA = \max(PRP, PRN)
$$

where PRP and PRN are respectively the ratio of the peak positive and peak negative values and PRA is the maximum absolute value.

Time History Metrics

Error metrics for the entire time history (Eq.4.71) is computed over an effective time earthquake duration defined as the time between $t_0$ and $t_1$ when $I_n$ (Eq.4.70) reaches 0.05 and 0.95 (Trifunac and Brady, 1975).

$$
I_n(t) = \frac{\int_{0}^{\tau} (\ddot{x}(\tau))^2 d\tau}{\int_{0}^{\tau_{max}} (\ddot{x}(\tau))^2 d\tau}
$$

(4.70)
The peak response metrics are ratios of the estimated to true response; each ratio individually shows overestimation or underestimation. The time history metrics, NRMS, is defined as a normalized RMS of error; smaller values indicate more accuracy, but an NRMS value for a particular estimation scheme individually does not give a direct appreciation of the performance of the scheme. NRMS should be used in relative sense to compared two or more estimation schemes. In the following sections the terms NRMS and PRA are interchangeably used for error in the time history and error in the peak values respectively.

Evaluation Criteria

The data set for each estimation scheme $\{M_{i,j,k}^r\}$ is the aggregate of the error metrics computed at all levels of the buildings for all the ground motion in the ensemble. $M$ is the error metric (NRMS or PRA), $i$ is the building, $j$ is the floor number, $k$ is the ground motion and $r$ is the response quantity. Since estimations are carried out in each direction separately, each building contributes two set of data to the $\{M_{i,j,k}^r\}$ for each response (a total of 60 times the number of floors).

Expected value and the standard deviation of the error metrics data sets $\{M_{i,j,k}^r\}$ are the primary indicators examined to evaluate the accuracy. Since having a ratio of 1.0 which means estimating the peak response perfectly correct has little probability, for PRA a range that contains central 80% of the values is also examined as a secondary
indicator. As shown in Fig.4.10 this range is defined by 10 and 90 percentile shown by points “a” and “b” on the cumulative probability curve. The closer the \{a, b\} range is to 1.0 the better the accuracy of the estimator.

![CDF](image)

**Fig.4.10. Definition of parameters a and b that define the central 80% of the probability distribution function.**

### 4.6.3 Evaluation Result

Evaluation of the estimations for displacement, drift and story shears are shown next for linear and nonlinear runs. Note that in nonlinear runs results for the ground motions that lead to failure or severe nonlinearity are discarded.

#### 4.6.3.1 Estimation of Displacements

The data set used in evaluation of displacement estimates include the error metrics computed at all levels except those levels that are instrumented which leaves 2280 samples. Statistics of the PRA and NRMS are shown respectively in Fig.4.11 and Table 4.3. As can be seen estimate of the peak displacement with all schemes is accurate especially for linear responses. Although in nonlinear case accuracy in Kalman and
Smoother has diminished, the central 80% of the data shows that the estimated peak values are within 10% of the true values. The error metric for the time history, as shown in Table 4.3, has small mean and dispersion which confirms the accuracy of the estimates. The smallest error and dispersion can be seen in the MIRC.

![Fig.4.11. Mean and central 80% range of the PRA for story displacements.](image)

Table 4.3. Statistics of NRMS for story displacements.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Linear $\mu$</th>
<th>Linear $\sigma$</th>
<th>Nonlinear $\mu$</th>
<th>Nonlinear $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS</td>
<td>0.08</td>
<td>0.09</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>MIRC</td>
<td>0.03</td>
<td>0.03</td>
<td>0.05</td>
<td>0.08</td>
</tr>
<tr>
<td>Kalman</td>
<td>0.06</td>
<td>0.04</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>Smoother</td>
<td>0.05</td>
<td>0.04</td>
<td>0.10</td>
<td>0.10</td>
</tr>
</tbody>
</table>

### 4.6.3.2 Estimation of Inter - Story Drifts

The data set for drifts excludes the levels where measurements are available at the top and bottom levels; this situation occurs in Burbank building where the 1st and 2nd floors are instrumented and on the 1st floor of Sherman Oaks building. That leaves 2520 samples in the linear data set. Statistics of the PRA and NRMS are shown in Fig.4.12 and Table 4.4.
Mean of the PRA of the drifts (peak ratios) with all schemes is close to 1.0 for linear responses and is within 10% of the true values for nonlinear responses. In both cases, however, dispersion is notably larger than for displacement estimates. As can be seen in Fig.4.12 while the dispersion in Kalman, Smoother and the MIRC are comparable in linear responses it is much larger in CS. The range of the central 80% of the data in the CS which is [0.73, 1.37] for linear and [0.69, 1.4] for nonlinear responses is respectively more than 3 and 2 times wider than the MIRC [0.9, 1.07] and [0.84, 1.15] and is farther from 1. Statistics of the NRMS shown in Table 4.4 confirms that the error in the time history is also the smallest in the MRC and has the smallest dispersion. Error from Kalman and Smoother is also small in linear responses but has increased in nonlinear responses. Mean and standard deviation of the NRMS in the CS is respectively 3 and 1.5 times larger than the MIRC for linear and nonlinear responses.
4.6.3.3 Estimation of Story Shears

The data set for story shears excludes the roof, since it is always instrumented, and has 2760 samples in linear runs. Fig.4.13 and Table 4.5 show the statistics of the PRA and NRMS of the story shears. In evaluations estimates from low pass filtered CS are also included.

CS estimates of story shears are notably biased towards overestimation. Mean of the peak shears with CS for linear and nonlinear response are respectively 33% and 47% larger than the true values and have a large dispersion. Boundaries of the central 80% of the data in CS which are [0.9, 1.82] in linear and [0.89, 2.17] in nonlinear response are respectively 4 and 5 times wider than in the MIRC [0.91, 1.10] and [0.91, 1.13]. As expected, low pass filtering has improved the CS estimates. In the filtered CS overestimation is reduced to 8% in linear and 21% in nonlinear (which is still large) and the dispersion is halved. Yet the range of central 80% of the data with filtered CS is more than twice wider than the MIRC in both linear and nonlinear runs. Judged by the mean and the range of the central 80% of the data, the MIRC and Smoother have provided the best estimates of the peak story shears.

![Fig.4.13. Mean and central 80% range of the PRA for story shears compared for the four estimation methods in linear runs.](image-url)
Statistics of the time history error (Table 4.5) supports the observations for the peak values and shows that CS has the largest error. The mean and standard deviation of the NRMS in CS are respectively 4 and 3 times larger than the MIRC for linear and nonlinear response. With the filtered CS the mean and standard deviations are still more than twice of the MIRC. Error in the Kalman and Smoother are also small and are comparable to the MIRC in linear response.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Linear</th>
<th>Nonlinear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>CS</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>CS Filtered</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>MIRC</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Kalman</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>Smoother</td>
<td>0.06</td>
<td>0.06</td>
</tr>
</tbody>
</table>

### 4.6.3.4 Summary of the Evaluations

From the evaluations it can be concluded that The MIRC provides the most accurate estimates (peaks and time history) of all response quantities both for linear and nonlinear runs. The accuracy in the linear response as expected is higher. Summary of the observations is listed below.

- All the estimation schemes provide good estimates of the displacement response both for the peak values and the time history. The dispersion in the peak values is larger when response is nonlinear.

- Mean of the peak drifts with all estimation schemes is close to true values (within 10\% in nonlinear responses) while the MIRC has the smallest spread. For linear response, the central 80\% of the data with CS is between 0.73 and 1.37 times the true
values which is 3 times wider than the MIRC [0.9, 1.07]. For nonlinear response his range in CS [0.69 and 1.4] is more than twice of the MIRC [0.84 and 1.15].

- Error in the time history of drift estimates is the smallest in the MIRC and the largest in CS. The mean error in the CS is 3 time and 1.5 times larger than the MIRC respectively in linear and nonlinear responses.

- Mean of the peak shears with CS is overestimated by as large as of 47% in nonlinear and 33% in linear responses. Low pass filtering reduces the overestimation to 21% in nonlinear and 8% in linear responses. Mean of the peak shears in the MIRC are 1.03 and 1.08 times the true values in linear and nonlinear responses.

- Dispersion in the estimates of the peak shears in CS is large. Range of the central 80% of the data in CS for linear response is between 0.9 and 1.82 times the true values which is more than 4 times the MIRC [0.91, 1.1]. For nonlinear runs the range for CS [0.89, 2.17] is more than 5 times that of MIRC [0.91, 1.13]. With filtering the range in the CS is still more than twice the MIRC.

- Error in the time history of shear estimates is the smallest in the MIRC and Smoother and the largest in CS. The mean error in the CS is 4 time and 3 times larger than the MIRC respectively in linear and nonlinear responses. With filtering the mean error in the CS is still more than 2 times the MIRC.

### 4.7 Closing Remarks

It was shown that cubic spline interpolation is a particular form of the basis fitting in which the basis is defined by the number and location of sensors. It is shown that the necessary condition to obtain accurate estimates from interpolation schemes is that the
number of sensors be at least equal to the number of modes that contribute significantly to the response and that the basis of interpolation dictated by the sensor locations be as close to the governing modes. Moreover, the measurements should not have high frequency components. In tall buildings the number of sensors (state of the practice) is often insufficient for the CS to give good estimates of quantities that have a strong contribution from higher modes (e.g. story shears). In this case low-pass filtering the acceleration data prior to performing the interpolation may improve the accuracy. The cutoff frequency of the low pass filter can be selected as an estimate of the $m^{\text{th}}$ mode frequency, where $m$ is the number of sensors measuring in the direction of interest, not including the base.

To achieve higher accuracy especially for shears and drifts, model based estimators are shown to be useful. The MIRC approach, developed in this study, on the base of finding corrective forces that reproduce the measurements, proved the most accurate of the estimators tried and has the advantage of being based on structural engineering theory. The nominal model used in the MIRC can be rather simple, as long as the fundamental period is calibrated to that of the actual building.

In summary the MIRC is proved to be the most accurate scheme to estimate the seismic response from available measurements, and therefore the estimates of the MIRC are taken as the “true responses” with which the NSPs are evaluated in some instrumented buildings in the next chapters.
CHAPTER 5

EVALUATION OF NSP USING DATA FROM INSTRUMENTED BUILDINGS

5.1 Overview

This chapter contributes to evaluation of single-mode and multi-mode NSP using data from instrumented buildings by comparing predictions of responses with those reconstructed from measurements. To attain reliable conclusions, it is important to understand and address all the factors that affect the comparisons. As shown in Fig. 5.1, any discrepancy observed between the NSP predictions and the benchmarks is comprised of a) error in the model b) error in the benchmark c) error and uncertainty in the NSP formulation.

To evaluate the NSP using instrumental data, effort has been made in this study to minimize the errors associated with modeling and benchmark. In Chapter 4, seismic response reconstruction has been studied and the estimation method developed in this study - the MIRC- has been shown to provide the best estimates of the responses given the available measurements. To minimize modeling error (to a level that reflects the expected modeling error in reality) the structural properties are extracted from construction
drawings and the model is verified with the dynamic properties obtained from system identification and with the measured responses at instrumented levels. It is therefore contended that the errors due to the 1st two sources are minimized and evaluations reflect the error from the nature and formulation of the particular NSP being studied.

Fig.5.1. Sources of error in evaluation of NSP.

5.2 General Considerations in Modeling and Analysis

The following summarizes the general considerations in modeling and analysis.

5.2.1 Modeling

Models of the buildings, whose material properties and component details are extracted from structural drawings, are created and analyzed in Perform 3D (CSI. Inc., 2006). To ensure that each model is reasonably accurate (to the extent possible) models are updated in a two-step verification process. In the first step the linear properties are adjusted so that the dynamic properties (in particular the 1st mode period and possibly the 1st mode shape) matches those obtained from system identification using the available measurements. The second step of verification involves adjusting the nonlinear properties
of members so that the responses at instrumented levels reasonably match the measurements. Note that achieving a perfect model that closely reproduces the measurements (without the use of feedback loop) is impossible and the goal here is to arrive at a reasonable model that captures the significance of the structure’s behavior.

Nonlinear behavior for frame elements is generally modeled as lumped plasticity via plastic hinges. In all walls and some critical frame elements (e.g. 1st floor columns of Imperial County Services Building) nonlinear behavior is modeled with fiber elements which explicitly consider the nonlinearity in materials. Force deformation relationship of all nonlinear components considering stiffness and strength degradation follow the backbone model shown in Fig.5.2.

![Fig.5.2. Parametric representation of a general force-deformation relationship in FEMA 356 and Perform 3D.](image)

**Component Stiffness**

Component stiffness is considered based on cracked sections. Following the recommendations of FEMA 356 the effective flexural stiffness for beams and columns are computed using respectively 50% and 70% of the moment of inertia of the gross section. Note that where it is applicable the beam includes the effect of slab in the form a T or L shape cross section.
Component Strength

Component strength is determined from the principles of strength of materials and procedures of ACI-318 using yield stress of reinforcements \( (f_y) \) and compressive strength of concrete \( (f'c) \) with no strength reduction factor \( (\phi) \). An over-strength factor as recommended in NSP guidelines is applied to the material strength to compute the upper bound strengths for deformation control actions such as beam flexural behavior for beams the over-strength factors are taken as 1.5 for steel and 1.25 for concrete. Force controlled action (i.e. axial force in columns) have no over-strength factor. For flexural strength of columns a factor of 1.25 and 1.15 for steel and concrete is used.

Component Deformation Capacity

Deformation capacities depend on condition of transverse reinforcement, shear force on the member, percentage of tensile and compressive reinforcement and also axial force (in columns). The values corresponding to different stages in force – deformation relationships of Fig.5.2 are taken from appropriate tables of FEMA 356.

Foundation and Soil Structure Interaction

Foundation in general is modeled with linear springs to account for possible soil-structure interaction if applicable. It is also considered as a free parameter in model updating to match the identified dynamic properties.

5.2.2 Analysis

Nonlinear static analysis is done separately in orthogonal directions (E-W and N-S). The general considerations in analysis are explained below.
Lateral Loads

NSP guidelines define the lateral loads based on the dynamic properties of each building. For the sake of uniformity load patterns proportional to mode shape, code prescribed lateral load distribution, and uniform loads are considered for all buildings.

Diaphragms

Rigidity or flexibility of the diaphragm as determined based on the ratio of maximum in plane deformation of the diaphragm to average lateral displacement of the floor (in the same direction) is taken into account explicitly. Where diaphragms are considered rigid, horizontal movement of all nodes in the same diaphragm are assigned a rigid constraint.

P-Δ Effects

P-Δ effects are taken into account in all the nonlinear static and dynamic analyses. Consideration of P-Δ in the process of determination of the target displacement follows specific details for each NSP procedure (Chapter 3).

Soil-Structure Interaction

Where SSI effects are present (e.g. Watsonville building), in addition to the foundation springs modeled at the base of the columns, some NSP (e.g. FEMA 440) suggest using higher damping ratios caused by radiation and hysteretic soil damping in the process of determination of the target displacement.

Gravity Loads

Gravity loads acting on the structure together with lateral loads are considered in the analyses. Guidelines require consideration of an upper bound (1.1 Dead + 1.1 Live)
and a lower bound (0.9 Dead) when component actions are evaluated. Note that in this study, since only the global responses are addressed, the envelope of the results for the two gravity load combinations is used.

**Multidirectional Effects**

When multidirectional effects are important (when there is irregularity in plan or columns in the two intersecting primary lateral load resisting frames) the NSP guidelines require that in analysis for a given direction, 30% of the force actions corresponding to the orthogonal direction be added. In this study it means that when story forces in a particular NSP under a given lateral load is being computed in X direction, 30% of the story forces (in X direction) due to lateral loads in Y direction should be added.

**Ground Motion Elastic Response Spectra**

Since the objective is to compare of NSP prediction with measurements from a specific ground motion, unlike the standard application of the NSP a specific response spectrum for that ground motion should be used in lieu of the generalized response spectra that the NSP guidelines define.

### 5.3 The Instrumented Buildings

#### 5.3.1 3-Story Santa Barbara UCSB office Building (CSMIP Station 25213)

This 3 story office building of the University of California at Santa Barbara was designed in 1960 and strengthened in 1975 by adding shear walls in both directions. The gravity load resisting system consists of a 2-1/2" concrete slab supported by reinforced concrete joists spanning between floor beams running in transverse direction. The floor
beams are supported by 14"x10" concrete interior columns and masonry block exterior columns. Lateral load resisting system after rehabilitation consists of concrete shear walls in both directions. The plan is essentially symmetrical with respect to an axis parallel to N-S direction with modular spans of 12ft at the ends and 24ft in the middle in the E-W direction. The height of the 1st and 2nd stories are 12ft and that of the 3rd story, which has a pitched roof, varies from 9ft to almost 13ft at the center of the plan.

The plan dimensions of the building are 34’ x 240’ along the N-S and E-W directions. Due to the dimensions of the building in plan significant distortions of the floor diaphragm for excitation in the N-S direction is expected. To allow for possible diaphragm flexibility no constraint is used in the model and instead the diaphragm is modeled with appropriate shell elements. Instrumentation consists of 9 accelerometers installed at the ground, 2nd floor and the roof with the roof sensor measuring only the N-S movement. Fig.5.3 shows a view of the building and instrumentation layout.

The available ground motion at this building is the magnitude 5.1 (M L) Santa Barbara 1978 earthquake with PGA of 0.398g and 0.27g in the N-S and E-W direction. The maximum recorded acceleration on the structure in the N-S direction is 0.99g at roof. In the E-W direction the maximum measured acceleration is 0.57g at the 2nd level. The 5% damped elastic response spectra are shown in Fig.5.4 for the N-S and E-W component of the motion using the excitation recorded at the center of the plan (channels 1 and 3). The 1st natural period and damping obtained from system identification using measured data is given are [0.28 sec. and 3.77%] in N-S and [0.22 sec. and 4.22%] in E-W direction.
Material properties used in the modeling are $f'_{c}$ of 3ksi for all concrete work, $f_y$ of 30 ksi for all reinforcement in beams and columns and $f_y$ of 60ksi and 40ksi respectively for bars greater that #4 and smaller than #3 in walls. The masonry walls were also modeled with a compressive stress $f'_{m}$ of 2 ksi and a modulus of elasticity of $550f'_{m}$ . The effective weights and masses used in the analysis are listed in table 5.1. To get an
appreciation of the reasonableness of the model acceleration response of the model at the roof is compared with measurements in Fig. 5.5. The capacity curves from the pushover analysis under three load patterns in E-W and N-S directions are shown in Fig. 5.6.

Table 5.1. Effective seismic weight and mass in the UCSB building.

<table>
<thead>
<tr>
<th>Level</th>
<th>Story Height (ft)</th>
<th>Weight (k)</th>
<th>Mass (k-in/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>1451</td>
<td>3.76</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>1250</td>
<td>3.24</td>
</tr>
<tr>
<td>3 (roof)</td>
<td>~10.5</td>
<td>980</td>
<td>2.54</td>
</tr>
<tr>
<td>total</td>
<td>~34.5</td>
<td>3681</td>
<td>9.54</td>
</tr>
</tbody>
</table>

Fig. 5.5. NDA acceleration response at the roof vs. the measurements in Ch. 7.

Fig. 5.6. Capacity curves of Santa Barbara building a) E-W b) N-S.
5.3.2 4-Story Watsonville Telephone Building (CSMIP Station 47459)

The four story building in Watsonville designed in 1948 was initially built with three stories and a fourth floor was added in 1955. The vertical load resisting system comprises 2.5” concrete slabs supported by composite steel-concrete frames, with the exception of the fourth floor where the 7” slab is supported by RC beams spanning in the E-W direction and supported on RC columns. The main lateral load resisting system consists of a number of solid and perforated shear walls. Typical plan dimensions are 72’-9” × 68’. Height of floor are 17’-10” at the 1st floor 16’-6” at the second and third and 15’-6” at the top floor.

Instrumentation consists of 13 accelerometers placed at the ground, 2nd floor and the roof. An interesting feature is the fact that there are 4 vertical accelerometers at the base which allow for characterization of the SSI and foundation effects. Fig.5.7 shows a view of the building and the instrumentation layout.

Fig.5.7. Instrumentation lay out and a picture of the 4 story Watsonville building (www.strongmotioncenter.org).
Two recorded ground motion are available at this building, the magnitude 6.2 (ML) Morgan Hill 1984 and the magnitude 7.0 Loma Prieta 1989 earthquakes. The former caused little response in the building but the latter with a PGA of 0.25g and 0.36g in N-S and E-W caused a maximum acceleration of 0.78g and 1.20g at the roof of the building. 5% damped elastic response spectra for the two motions are shown in Fig.5.8.

![Fig.5.8. Elastic spectra for 5% damping for Morgan Hill and Loma Prieta earthquakes a) E-W b) N-S.](image)

From system identification using the Morgan Hill ground motion data which is the smaller of the two motions the natural period and damping are found to be [0.2 sec. and 9%] in the N-S and [0.28 sec. and 5.1%] in the E-W directions.

Material properties used in modeling as extracted from the drawing are $f'_c = 2.75\text{ksi}$ for concrete work up to the 44th floor and $f'_c = 3.25\text{ksi}$ for the 4th floor and $f_y = 40\text{ksi}$ for all rebars and 36 ksi for the structural steel sections. Following the examination of vertical motions recorded at the base modeling foundation flexibility and SSI effects proved necessary. The effective weights and masses used in the analyses are shown in Table 5.2. Adequacy of the model is demonstrated in Fig.5.9 where roof acceleration
from NDA in the E-W direction is compared with the measurements. Capacity curves from pushover analysis in the E-W and N-S directions are also shown in Fig.5.10.

Fig.5.9. NDA acceleration response at the roof (E-W) vs. the measurements in Ch.11.

Table 5.2. Effective seismic weight and mass of the Watsonville building.

<table>
<thead>
<tr>
<th>Level</th>
<th>Story Height (ft)</th>
<th>Weight (k)</th>
<th>Mass (k-in/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.83</td>
<td>1492</td>
<td>3.87</td>
</tr>
<tr>
<td>2</td>
<td>16.50</td>
<td>1450</td>
<td>3.76</td>
</tr>
<tr>
<td>3</td>
<td>16.50</td>
<td>1388</td>
<td>3.60</td>
</tr>
<tr>
<td>4 (roof)</td>
<td>15.50</td>
<td>955</td>
<td>2.47</td>
</tr>
<tr>
<td>total</td>
<td>66.33</td>
<td>5285</td>
<td>13.69</td>
</tr>
</tbody>
</table>

Fig.5.10. Capacity curves for Watsonville building a) E-W b) N-S.
5.3.3 6-Story Imperial County Services Building (CSMIP Station 1260)

The 6 story imperial county services building in El-Centro California was designed in 1968 and built in 1969. The plan dimensions are approximately 137ft in the longitudinal direction (E-W) and 85ft in the transverse (N-S). Vertical load resisting system of the structure consists of 5\(^{"}\) RC slabs supported by RC pan joists spanning in transverse direction and supported by RC frame. Lateral load resisting system comprises RC shear walls in transverse (N-S) direction and rigid frame in longitudinal (E-W) direction. For architectural reasons some of the shear walls were discontinuous at the 1\(^{st}\) floor, namely, there was a transition from two shear walls above the second floor to four one story shear walls at the 1\(^{st}\) floor offset from the above walls. Instrumentation includes 13 accelerometers measuring the ground, 1\(^{st}\) floor, 3\(^{rd}\) floor and the roof. Fig.5.11 shows a picture of the building and instrumentation layout.

![Fig.5.11. Instrumentation lay out and a picture of the ICS building (www.strongmotioncenter.org).](www.strongmotioncenter.org)
The only available ground motion record in this building is from the magnitude 6.5 (M$_{L}$) Imperial Valley Earthquake of 1979 in which the building suffered extensive damages that lead to its demolition. The major damage took place at the base of the columns on the exterior axis where four columns crushed in an “explosive manner” and lead to nearly 26” of downward displacement (ATC-09, 1984). Almost all other 1st story columns showed flexural cracking and shear diagonal cracking. The discontinuous shear walls above the 1st story also exhibited some diagonal and horizontal cracking. Fig.5.12 depicts some of the damaged columns. A complete description of the building and detailed information about damage can be found in ATC-09 (1984). Fig.5.13 also shows the elastic response spectra of the El Centro earthquake for 5% damping.

System identification using the early portion of the records shows that the first natural period and damping are [0.44 sec. and 8%] in the N-S and [0.8 sec. and 5%] in the E-W direction. Table 5.3 shows the effective weights and mass used in analyses.
Modeling of the nonlinear behavior of the components in this structure is of prime importance. Knowing that the structure has suffered extensive damage, to capture the significance of the structures behavior modeling of strength degradation is essential. Biaxial bending and axial force interaction was considered as well as nonlinearity in the shear behavior. Strength loss was considered by modeling crushing of the concrete and buckling of the steel bars. $f'_c$ is 5ksi for columns and 4ksi for beams and walls and $f_y$ is 40ksi for all the rebars.

Table 5.3. Effective seismic weight and mass of the ICS building.

<table>
<thead>
<tr>
<th>Level</th>
<th>Story Height (ft)</th>
<th>Weight (k)</th>
<th>Mass (k-in/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.50</td>
<td>2332</td>
<td>6.04</td>
</tr>
<tr>
<td>2</td>
<td>13.50</td>
<td>2183</td>
<td>5.65</td>
</tr>
<tr>
<td>3</td>
<td>13.50</td>
<td>2173</td>
<td>5.62</td>
</tr>
<tr>
<td>4</td>
<td>13.50</td>
<td>2188</td>
<td>5.66</td>
</tr>
<tr>
<td>5</td>
<td>13.50</td>
<td>2180</td>
<td>5.64</td>
</tr>
<tr>
<td>6 (roof)</td>
<td>13.20</td>
<td>1837</td>
<td>4.75</td>
</tr>
<tr>
<td>Total</td>
<td>81.7</td>
<td>12893</td>
<td>33.37</td>
</tr>
</tbody>
</table>

Fig.5.14 shows the NDA acceleration response of the roof in together with the measurements in both directions. The model is reasonably following the measurements.
up to the time of damage (about 7 sec.) beyond which the response is no longer reliable in
due to the extent of damage. The response of the model in the N-S direction, however,
still reasonably follows the measurements. Note the incorporation of strength loss in
terms of concrete crushing and rebar buckling in fiber elements enable the model to
successfully capture the damage at the failed columns. The capacity curves from the
pushover analyses under three load patterns are shown in Fig. 5.15.

![Fig. 5.14. NDA acceleration response at the roof measurements a) E-W b) N-S.](image)

![Fig. 5.15. Capacity curves of ICS building a) E-W b) N-S.](image)

**5.3.4 13-Story Sherman Oaks Commercial Building (CSMIP Station 24322)**

The Sherman Oaks building has 13 stories above ground and two underground
levels and was designed in 1964. Vertical load resisting system consists of concrete slabs
(typically 4.5 inch thick) supported on concrete beams and columns. The lateral load resisting system consists of moment resisting frame in both directions. Plan dimensions at the all floors are 193ft in the E-W and 75ft in the N-S. Height of the first floor above the ground is 23ft and the other floors are all 11’-9” high. The building was instrumented with a total number of 15 sensors at the base, ground level, 1st and 7th floor as well as the roof. Fig.5.16 shows a picture of the building and instrumentation layout.

Three sets of strong motion were available at this building namely the Whittier earthquake 1987 (ML 6.1), Landers, 1992 (ML 7.3) and the Northridge earthquake of 1994 (ML 6.4). The first two motions did not cause large response or any damage but the Northridge earthquake induced noticeable, yet repairable structural damage in the form of cracks in the beams, slabs, girders, and walls (Naeim 1998). The Northridge earthquake recorded at this station has a PGA of 0.46g and 0.21g in the N-S and E-W direction and the maximum recorded acceleration at the roof is 0.66g and 0.27g in the same directions. Elastic response spectra with 5% damping is shown in Fig.5.17.

Fig.5.16. Instrumentation lay out and a picture of the 13 story Sherman Oaks building (www.strongmotioncenter.org).
System identification using the Landers recording results in the first natural period and damping of [2.79 sec. and 5%] in the N-S and [2.59 sec. and 5.36%] in the E-W direction. Effective weight and mass of the structure are shown in table 5.4.

Table 5.4. Effective seismic weight and mass of Sherman Oaks building.

<table>
<thead>
<tr>
<th>Level</th>
<th>Story Height (ft)</th>
<th>Weight (k)</th>
<th>Mass (k-in/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.00</td>
<td>2439</td>
<td>6.31</td>
</tr>
<tr>
<td>2</td>
<td>11.75</td>
<td>2286</td>
<td>5.92</td>
</tr>
<tr>
<td>3-11</td>
<td>11.75</td>
<td>2143</td>
<td>5.55</td>
</tr>
<tr>
<td>12</td>
<td>11.75</td>
<td>2143</td>
<td>5.55</td>
</tr>
<tr>
<td>13(roof)</td>
<td>11.75</td>
<td>2291</td>
<td>5.93</td>
</tr>
<tr>
<td>Total</td>
<td>164</td>
<td>28447</td>
<td>73.62</td>
</tr>
</tbody>
</table>

Note that the model of the building is assumed to start from the ground level and the two underground levels were not explicitly modelled instead rotational springs are provided at the base of the columns to also serve as free parameters for model updating. The material properties are taken as $f'_{c}$ of 5ksi for columns and 3.5ksi for beams and $f_y$ of 60ksi for all rebars. A comparison of the NDA roof acceleration response in (E-W) direction with the measurements in Ch11 in Fig.5.18 gives an appreciation of the
reasonableness of the model. Capacity curves from the pushover analysis are also shown in Fig.5.19.

Fig.5.18. NDA acceleration response at the roof (N-S) vs. the measurements in Ch.2.

Fig.5.19. Capacity curves of Sherman Oaks building a) E-W b) N-S.

5.3.5 20-Story North Hollywood Hotel Building (CSMIP Station24464)

The North Hollywood building was designed in 1967 and constructed in 1968. The building has 20 stories above one story below the ground. The plan is rectangular with a dimension of 57’-10” (N-S) and 183’-6” (E-W) at the typical floors. The gravity load resisting system consists of a reinforced concrete slab, with thickness that varies between 4.5” and 7” at different floors, supported by reinforced concrete beams and columns. The
lateral load resisting system consists of ductile moment resisting frames for the above ground floors and shear walls surrounding the plan at the underground level. The instrumentation consists of 16 accelerometers placed at the base, 3rd, 8th, and 15th floor as well as the roof. Fig.5.20 shows a picture of the building and the instrumentation layout.

Fig.5.20. Instrumentation lay out and a picture of the 20 story North Hollywood building (www.strongmotioncenter.org).

Two ground motion was recorded at this building namely the Whittier earthquake of 1987 (ML 6.1) and the Northridge 1994 earthquake (ML 6.4). The Northridge motion induced the largest response in the building. The recorded PGA for the Northridge earthquake is 0.33g and 0.11g in the N-S and E-W directions respectively and the maximum recorded accelerations on the structure is 0.66g and 0.31g in the N-S and E-W directions at the roof. The elastic response spectra for 5% damping for Northridge earthquake at this building is shown in Fig.5.21.
System identification using the data recorded at the smaller motion (Whittier 1987) gives the first natural period and damping as [2.08 sec. and 6.3%] in E-W and [1.87 sec. and 7.5%] in the N-S direction. The effective weight and mass is shown in table 5.5. Material properties used in the modeling include $f'_c$ of 3ksi for beams and walls and columns above the 10th floor, and 4ksi for lower columns. $F_y$ for the rebars is 40ksi except for columns up to the 10th floor which is 60ksi.

Table 5.5. Effective seismic weight and mass of Sherman Oaks building.

<table>
<thead>
<tr>
<th>Level</th>
<th>Story Height (ft)</th>
<th>Weight (k)</th>
<th>Mass (k-in/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18</td>
<td>3516</td>
<td>9.11</td>
</tr>
<tr>
<td>2</td>
<td>15.5</td>
<td>2510</td>
<td>6.50</td>
</tr>
<tr>
<td>3</td>
<td>15.5</td>
<td>2039</td>
<td>5.28</td>
</tr>
<tr>
<td>4-18</td>
<td>8.75</td>
<td>1059</td>
<td>2.74</td>
</tr>
<tr>
<td>19</td>
<td>10.5</td>
<td>1360</td>
<td>3.52</td>
</tr>
<tr>
<td>20</td>
<td>11</td>
<td>470</td>
<td>1.22</td>
</tr>
<tr>
<td>roof</td>
<td>10</td>
<td>1031</td>
<td>2.67</td>
</tr>
<tr>
<td>Total</td>
<td>211.75</td>
<td>26629</td>
<td>68.99</td>
</tr>
</tbody>
</table>

NDA acceleration response at the roof (in E-W) plotted together with the measurements in Fig.5.22 show the reasonableness of the analytical model. Capacity curves from the pushover analysis are also shown in Fig.5.23.
Fig. 5.22. NDA acceleration response at the roof (E-W) vs. the measurements in Ch.10.

Fig. 5.23. Capacity curves of North Hollywood building.

5.4 NSP Evaluations

Evaluations of single-mode (standard) NSP and multi-mode NSP are presented in a series of comparison of predictions with the “true values” estimated using the MIRC. Except for the Imperial County Services building which had extensive damage in E-W direction for all other buildings there are separate results for N-S and E-W directions. It should be noted that since measurements in the ICS building could be affected by the extensive damage, uncertainty in the benchmarks may be large and consequently the
results could be less reliable. Hence, in NSP evaluation less weight is given to observations from the ICS buildings.

5.4.1 Single Mode NSPs

Four single mode NSP as mentioned in Chapter 2, namely, the original CSM in ATC 40 and its updated version as the equivalent linearization in FEMA 440, the original CM in FEMA 356 and its updated version in FEMA 440 are considered.

5.4.1.1 Evaluations for Story Displacements

Figs 5.24 to 5.28 show the NSP predictions of story displacements at the five buildings.

![Fig. 5.24. Story displacements in Santa Barbara building a) E-W b) N-S.](image)
Fig. 5.25. Story displacements in Watsonville building a) E-W b) N-S.

Fig. 5.26. Story displacements in ICS building (N-S).

Fig. 5.27. Story displacements in Sherman Oaks building a) E-W b) N-S.
Single-mode NSPs, especially the two methods of FEMA 440, have provided reasonably accurate prediction of story displacements in the short buildings (Santa Barbara and Watsonville). This is not surprising since these buildings with fundamental periods of less than 0.5 seconds are clearly dominated by the 1st mode and such behavior is the main assumption in development of single-mode NSP. In the two taller buildings (Sherman Oaks and North Hollywood), displacement predictions from single-mode NSP is almost accurate at lower floors, whereas at upper floors predictions are much less accurate. As can be seen in Figs.5.27 and 5.28 upper floor displacements are sometimes overestimated and sometimes underestimated. This observation rules out a systematic bias in the procedures. Contribution from higher modes, however, is large for the upper floor displacement responses in these two buildings and is indeed the main reason that the single-mode NSPs are not accurate at these floors. Predictions from multi-mode NSP which are shown later in 5.4.2 further proves that this is in fact the case. In the mid-rise ICS building, all single-mode NSP have overestimated the displacements. Similar to the
taller buildings, error in the predictions at the lower floors is smaller than in the upper floors.

It can be also seen from the comparisons that in all the buildings, ATC-40 typically gives the smallest predictions. The large discrepancy between the original version of capacity spectrum method and coefficient methods (ATC-40 and FEMA356) has been noticeably reduced by the revised versions in FEMA 440. Needless to say since the discrepancy between ATC-40 and FEMA 356 had been one of the reasons for development of the revised versions, this observation is much expected.

5.4.1.2 Evaluations for Inter-Story Drifts

Comparisons of NSP prediction of inter-story drift indices (IDI) with “true values” for the five buildings are shown in Fig.5.29 to 5.33.

Fig.5.29. Inter story drift index in Santa Barbara building a) E-W b) N-S.
Fig. 5.30. Inter story drift index in Watsonville building a) E-W b) N-S.

Fig. 5.31. Inter story drift index in ICS building (N-S).

Fig. 5.32. Inter story drift index in Sherman Oaks building a) E-W b) N-S.
Comparison of predictions with true values shows that single-mode NSP are less accurate for story drifts than for displacements. This is primarily due to the fact that story drifts, compared to story displacements, are more susceptible to higher mode effects. In the two short buildings in which the main lateral load resisting system is shear wall, the maximum story drift ratio is less than the limit for immediate occupancy (0.85 in FEMA 356) and predictions with single-mode NSP are reasonably accurate. In the taller buildings, the maximum inter story drifts are less than the limit for life safety (i.e. 2% in ATC-40) and in most cases less than the 1% limit for immediate occupancy, nevertheless predictions with single-mode NSP are not accurate. Inter story drifts in these buildings are generally underestimated. Significant underestimation at the upper floors of Sherman Oaks and North Hollywood render the single-mode NSP ineffective for predicting drifts in these buildings. A noteworthy observation is that despite large inaccuracy at upper levels, predictions at the lower levels are reasonably good. Performance of the single-mode NSP in the mid-rise ICS building is similar to that in the tall buildings; while lower floor responses are predicted well, upper floors are notably underestimated.
Clarifying Comments

Large inaccuracy in the story drifts at the intermediate and upper levels of North Hollywood and Sherman Oaks building, despite rather reasonable predictions of story displacements at the lower and intermediate levels, warrants a thorough investigation of the two sides of the comparison. Re-examination of the benchmark values and comparing with cubic spline estimates and NDA predictions (as shown in Fig.5.34a for North Hollywood), despite differences, confirms that the large drifts at the upper levels are in fact real and are not resulted from computation errors.

![Diagram](image)

Fig.5.34. Inter story drift index in North Hollywood building a) examination of benchmarks b) linear response spectrum analysis.

The 1st natural period in these two buildings in the directions where large drifts were observed is rather long (around 2.5 sec). Although not directly related to story drifts, examination of the effective modal mass also shows that one mode is not enough to capture at least 90% of total mass in the North Hollywood and N-S direction of Sherman Oaks. Response spectrum analysis of the buildings under the given ground motion clearly shows that the 1st mode response is not an adequate approximation of the true values. As illustrated in Fig.5.34b for North Hollywood building, responses based on
only one mode are significantly smaller than true values (MIRC). With more number of 
modes included in the analysis, responses improve especially in the lower and 
intermediate levels. The examination clearly demonstrates the significance of higher 
modes in the responses of these buildings.

Neglecting the higher modes in the single-mode procedures proves to be the main 
reason for the large inaccuracy observed in story drifts. It should be also noted that the 
standard NSP guidelines explicitly address this issue and suggest that the use of single-
mode NSP be limited to the structures with 1st mode dominance. Hence in the North 
Hollywood and Sherman Oaks buildings the single-mode NSP are actually pushed 
outside their envelope and the inaccuracy observed is not surprising. The multi-mode 
NSP as shall be seen later, result in better predictions in these buildings.

5.4.1.3 Evaluations for Story Shears and Overturning Moments

Comparisons of the NSP prediction of story shears with the true values are shown 
in Figs.5.35 to 5.39

![Diagram of Story Shears in Santa Barbara building](image)

Fig.5.35. Story shears in Santa Barbara building a) E-W b) N-S.
Fig. 5.36. Story shears in Watsonville building a) E-W b) N-S.

Fig. 5.37. Story shears in ICS building (N-S).

Fig. 5.38. Story shears in Sherman Oaks building a) E-W b) N-S.
Performance of the single-mode NSP in predicting story shears is in general similar to their performance in predicting story drifts. In the two short buildings predicted story shears are close to the true values. Small overestimation in Santa Barbara building and underestimation in Watsonville is negligible compared to rather large story shears in these two buildings. Note that the unusually large story shears, despite no report of significant structural damage, in these two buildings is justified by their conservative design (and retrofit) in use of several solid and perforated shear walls. Regardless of the value of the story shears, predictions with single-mode NSP are reasonably accurate.

The common observation in the taller buildings is that the single-mode procedures have underestimated the story shears at upper floors. Extent of inaccuracy is not as large as story drifts but the cause of the inaccuracy is the same i.e. not considering the effect of higher modes in single-mode NSP. Similar to story drifts, predictions of the story shears at lower floors are much more accurate than at upper floors. In the ICS buildings while base and lower floor shears are predicted well, intermediate and upper levels are underestimated. An exception to the common observation is the E-W direction of

![Fig. 5.39. Story shears in North Hollywood building a) E-W b) N-S.](image)
Sherman Oaks building where, apart from ATC-40, the other single-mode procedures give relatively accurate predictions at all floors. A linear response spectrum analysis of the Sherman Oaks for Northridge earthquake (Fig. 5.40) shows that although the fundamental period in both directions is large (more than 2.5 seconds) in the E-W direction contribution of higher modes to the story shears is small. The single-mode NSP, therefore, is able to provide reasonable predictions.

![Fig. 5.40. Story shears in Sherman Oaks from linear response spectrum analysis of Northridge earthquake using different number of modes.](image)

It can be inferred from the accumulated observations that consideration of higher modes i.e. through the multi-mode NSP can potentially improve the NSP predictions of story shears in the where single-mode NSP did not perform well.

Comparison of the predicted overturning moment (OTM) with true values is shown in Figs. 5.41 to 5.45. Examination of the figures shows that the overturning moments in the short buildings are generally predicted well with the single-mode NSP. Excluding predictions from ATC-40 which are generally smaller than the other single-mode procedures, in the Santa Barbara and Watsonville some small overestimation at the base is observed (in the N-S). Despite this overestimation, the overall prediction of the
overturning moments in the short buildings is adequate. However, large inaccuracy in the
taller buildings can be seen. In the ICS building, single-mode procedures consistently
underestimate the response at all floors. In Sherman Oaks and North Hollywood
predictions are much worse. While upper floor responses are underestimated (by as large
as 30%), lower floor responses are significantly overestimated (by more than 50%). The
substantial inaccuracy observed proves the poor performance of single-mode NSP in
predicting overturning moments in the tall buildings.

Fig.5.41. Story overturning moment in Santa Barbara building a) E-W b) N-S.

Fig.5.42. Story overturning moment in Watsonville building a) E-W b) N-S.
Fig. 5.43. Story overturning moment in ICS building (N-S).

Fig. 5.44. Story overturning moment in Sherman Oaks building a) E-W b) N-S.

Fig. 5.45. Story overturning moment in North Hollywood building a) E-W b) N-S.
Given that the story shears were almost always underestimated at the upper floors of Sherman Oaks and North Hollywood, the overestimation of the overturning moments at the lower levels is noticeable. The key to explain this is to remember that the OTM are cumulative quantities whose values at each level include the effect of upper levels. In the NSP, the story shears used to compute the OTM at each level are predictions of the maximum story shears at all floors which in reality do not occur simultaneously. The true benchmark values, however, are the maximum OTM at each level determined from the time history of the reconstructed responses that may not necessarily correspond to the maximum story shears. Hence, it is possible that the OTM computed from the story shears in NSP be larger than the true values.

5.4.2 Multi-Mode NSPs

The multi-mode NSPs are expected to provide better predictions than the single-mode procedures when contribution of higher modes in the response of interest is large. Effectiveness of the multi mode procedures (MPA and MMPA) is evaluated in this section for the ICS, Sherman Oaks and North Hollywood buildings.

5.4.2.1 Evaluations for Story Displacements

Figs.5.46 to 5.48 compare the predictions of the story displacement from MPA and MMPA with the true values.
Fig. 5.46. Multi mode NSP story displacements in ICS building (N-S).

Fig. 5.47. Multi mode NSP story displacements in Sherman Oaks building a) E-W b) N-S.

Fig. 5.48. Multi mode NSP story displacements in North Hollywood building a) E-W b) N-S.
The figures when compared to the ones from single-mode NSP clearly show that consideration of higher modes through multi-modal pushover methods has significantly improved the prediction of story displacements in the tall buildings. While improvements in Sherman Oaks and North Hollywood building are substantial, in ICS the improvements from multi-mode NSP are little.

5.4.2.2 Evaluations for Inter - Story Drifts

Figs. 5.49 to 5.51 compare the predictions of the IDI with the true values.

Fig. 5.49. Multi mode NSP IDI in ICS building (N-S).

Fig. 5.50. Multi mode NSP IDI in Sherman Oaks building a) E-W b) N-S.
The largest in the single-mode NSP was observed previously at upper floor drifts in the tall buildings. Comparison of Figs.5.49 to 5.51 with those from single-mode NSP demonstrates that the multi-mode procedures have captured the profile of the true response and have resulted in significant improvements. The most visible improvement is in N-S direction of the Sherman Oaks building. When predictions from multi-mode NSP are inspected independently (not in comparison with single-mode NSP), however, one notices that while the accuracy in the lower and intermediate floors are reasonable, there is still some underestimation at upper floors. This observation in particular can be seen in the N-S direction of North Hollywood building.

5.4.2.3 Evaluations for Story Shears and Overturning Moments

Examination of figs.5.52 and 5.53 that compare the predictions of story shears with true values indicates that the multi-mode NSP have given reasonable predictions in the tall buildings. Significant improvements achieved by multi-mode can be realized by comparing these figures with those from single-mode procedures. In particular, the correct profile of the response along the height has been captured and the rather large...
underestimation that was previously observed with single-mode NSP at the upper floors has been erased. In some cases responses are conservatively overestimated. Such overestimation with multi-mode procedures commonly occurs in force components due to the SRSS modal combination. In the ICS building, however, predictions from multi-mode NSP are about the same as those in single-mode NSP.

Fig.5.52. Multi mode NSP story shear in Sherman Oaks building a) E-W b) N-S.

Fig.5.53. Multi mode NSP story shear in North Hollywood building a) E-W b) N-S.
Multi-mode NSP generally results in larger predictions of the overturning moment compared to single-mode procedures. Recalling the underestimation at the upper levels with single-mode procedures, use of the multi-mode NSP as shown in Figs. 5.54 and 5.55 has improved the predictions at the upper levels in the Sherman Oaks and North Hollywood building but has increased the overestimation at the lower level. SRSS combination is the main reason for this overestimation. In the ICS building, predictions from the multi-mode methods is essentially the same as those from single-mode NSP and no improvements have been observed.

Fig. 5.54. Multi mode NSP story overturning moment in Sherman Oaks building a) E-W b) N-S.

Fig. 5.55. Multi mode NSP story overturning moment in North Hollywood building a)E-W b)N-S.
5.4.3 Quantitative Presentation of the Combined Observations

Sections 5.4.1 and 5.4.2 presented qualitative evaluations of single-mode and multi-mode NSP by comparing the predictions with the true values in each building. In this section observations are collectively presented in a quantitative manner. For this purpose, ratio of predictions to true values for each response quantity is taken as the primary metric and some statistical properties of a data set consisting of the metric computed at various floors are used to quantify the observations. The advantage of working with the ratios is that each number independently gives a perception of accuracy and directly indicates overestimation or underestimation in the predictions. Needless to say, the closer a ratio is to one the higher the accuracy of predictions. Although mean of the ratios can reveal the bias in the predictions, it is not a reliable indicator of accuracy since it clouds the information about the error. This occurs when response quantities are underestimated at some levels and overestimated at other levels or underestimated in one direction and overestimated in another. While the mean ratio in these situations is likely to be close to 1, it does not reflect the possible large errors in individual ratios. Hence, judging the quality of predictions solely based on the mean of the ratios is not appropriate. Although dispersion measures such as standard deviation could be used, we follow the approach of ATC-55 and introduce a metric that directly accounts for prediction error. This metric, "normalized error", is defined as the absolute value of the difference between the prediction and true values divided by the true value. Taking, $y_p$ and $y_t$ respectively as the predictions and true values of any response quantity the two metrics are given in Eq.5.1.
\[ r = \frac{y_p}{y_t} \quad e = \frac{|y_t - y_p|}{y_t} \]  

(5.1)

where \( r \) and \( e \) are respectively the metric for ratios and errors. Statistics of the metrics for each response quantity are computed for a set of data that includes the values determined at all the floors and another set in which data is segregated into lower half and upper half of the intermediate and tall buildings. Data sets for each building (except ICS) are the aggregate of the metrics in both E-W and N-S directions. The mean of the ratio \( (\mu_r) \) and the mean of the normalized errors \( (\mu_e) \) are used to quantify the observations.

It is important to note that since the number of samples in the dataset is not large enough (only one recorded ground motion in each building) the result cannot be used to generalize the observations. The statistical properties presented next are only intended to quantify the observations of this study which is limited by the number of instrumented buildings and recorded ground motions.

**Story Displacements**

Table 5.6 shows the statistics of the displacement predictions with single and multi mode procedures in all the five buildings. The main observations are as follows:

- Mean of the displacements predicted with single mode NSP in the short buildings (Santa Barbara and Watsonville) is between 0.95 and 1.05 of the true values. Mean of the normalized error is 0.06 ~ 0.08.

- In the tall buildings (Sherman Oaks and North Hollywood), mean of the displacements predicted with single mode NSP over the full height is between 0.97 and 1.02 of the true values. Mean of the normalized error is 0.07 ~ 0.12.
• Results for the data segregated in lower and upper floors of tall buildings shows that mean of the normalized error in the lower half (0.05~0.10) is smaller than the upper half (0.08~0.14) for single-mode NSP.

• With multi-mode NSP, mean of the normalized error over the full height of the tall buildings is reduced to 0.05~0.08. At the upper floors these values are 0.06~0.08 which are notably less than those of single-mode procedures.

• Multi-mode NSP is not useful in short buildings; with multi-mode NSP the mean normalized error (due to SRSS) is increased to 0.9~0.11.

• In the ICS, mean of the predicted displacements with single-mode NSP is about 1.2 times the true value and mean of the normalized error is ~0.22. With multi-mode NSP mean of the displacement is 1.13 times the true values and mean error is reduced to 0.16.

Table 5.6. Statistics of evaluation metrics for displacement.

<table>
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<tr>
<th>Building</th>
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<th>Lower levels</th>
<th>Upper levels</th>
</tr>
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<td>µe</td>
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</table>

µr: Mean of the ratios
µe: Mean of normalized errors
LN 440: FEMA 440 Linearization
CM 440: FEMA 440 CM
Based on the accumulated observations, NSP has given accurate predictions of story displacements. Except in ICS buildings, the mean displacement is within 10% of the true values. The smallest error between the predictions and true values is observed in the short buildings and lower levels of the tall building. Multi-mode NSP has helped reduce the error at the upper level of tall buildings.

Inter Story Drifts

Table 5.7 shows the statistics of the inter story drift predictions. The main observations are listed below.

- In the short buildings, mean of the IDI predictions with single-mode NSP is about 0.94 of the true values and the mean normalized error is 0.06~0.10.

- In the tall buildings mean of the predictions with single-mode NSP over the full height is between 0.73 and 0.85 of the true values and the mean normalized error is 0.2~0.3.

- Predictions at upper floors of the tall buildings with single-mode NSP are poor, the mean value is between 0.5 and 0.72 of the true values and mean normalized error is 0.28~0.46.

- At lower levels, predictions are substantially better; mean ratio of the prediction to true values is between 0.93 and 1.0 and the mean normalized error is 0.09~0.12.

- With multi-mode NSP, predictions in the taller buildings are improved. Mean of predictions over the full height is between 0.88 and 0.96 of the true values and the mean normalized error is reduced to 0.09~015.

- With multi-mode NSP the mean normalized error of 0.12~0.21 at upper floors is less than half the error with single-mode NSP and the mean ratio of predictions to true
values is between 0.79 and 0.89. Despite improvements, these values independently indicate that inaccuracy and underestimation still remains.

- In the ICS building, mean of the predictions with single-mode NSP over the full height is about 0.9 of the true values. At lower levels mean error (0.07) is notably less than upper levels (0.23). Predictions with multi-mode NSP are only slightly better than single-mode procedures.

Table 5.7. Statistics of evaluation metrics for inter-story drifts.

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<th>Lower levels</th>
<th>Upper levels</th>
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<td>(\mu_r)</td>
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\(\mu_r\): Mean of the ratios  \(\mu_e\): Mean of normalized errors
LN 440: FEMA 440 Linearization  CM 440: FEMA 440 CM

Observations point out that in the instrumented buildings the single-mode NSP have underestimated story drifts. While underestimation and inaccuracy in the short buildings is small enough to render the single-mode NSP effective, in the taller buildings large underestimation and inaccuracy especially at the upper levels demonstrate poor performance of the single-mode NSP. At the lower levels however, prediction are almost
accurate. Overall improvement from multi-mode procedures in the tall building is considerable, yet improvement at the upper floors particularly in the North Hollywood building has proven insufficient to eliminate the inaccuracy and underestimation.

**Story Shear and Overturning Moment**

Statistics of the story shear predictions in Table 5.8 show that errors in the prediction of story shears are typically larger than displacement and smaller than story drifts.

<table>
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<tr>
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<th>Lower levels</th>
<th>Upper levels</th>
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-\( \mu_r \): Mean of the ratios
-\( \mu_e \): Mean of normalized errors

LN 440: FEMA 440 Linearization
CM 440: FEMA 440 CM

- In the short buildings, mean of the story shear predictions with single-mode NSP is between 1.02 and 1.05 of the true values and the mean normalized error is 0.07~0.09.
• In the tall buildings mean of the predictions with single-mode NSP over the full height is between 0.87 and 0.92 of the true values and the mean normalized error is 0.13~0.15.

• Predictions at upper floors of the tall buildings with single-mode NSP are not accurate, the mean value is between 0.79 and 0.85 of the true values and mean normalized error is 0.19~0.22.

• At lower levels, however, predictions with single-mode NSP are reasonably accurate; mean of the predictions is between 0.92 and 1.04 of the true values and the mean normalized error is 0.05~0.09.

• Predictions with multi-mode NSP are notably improved in the taller buildings. Mean of predictions over the full height is between 1.01 and 1.05 of the true values and the mean normalized error is reduced to 0.06~011.

• Multi-mode NSP have considerably improved the predictions and erased the underestimation at upper floors. Mean of the predictions is between 1.04 and 1.07 of the true values with a mean normalized error of 0.07~0.15.

• In the ICS building while mean of the predictions at upper floors are underestimated (almost 0.65 of the true values) and has large error (0.33~0.36), at lower levels predictions are notably better i.e. mean of predictions is between 0.88 and 0.95 of the true values and mean errors (0.08~0.12) are one third of upper levels. Multi-mode NSP is only slightly better than single-mode NSP in this building.

It can be inferred from the overall observations in the instrumented buildings that prediction of story shears in the short buildings is reasonably accurate. In the taller buildings, however, story shears are typically underestimated with single-mode NSP. While underestimation and inaccuracy at the upper levels is rather large, at the lower
levels prediction are reasonably accurate. With the use of multi-mode procedures the error is significantly reduced.

Table 5.9 summarizes the statistics of the story overturning moment (OTM) predictions. The main observations are listed below.

Table 5.9. Statistics of evaluation metrics for story overturning moment.

<table>
<thead>
<tr>
<th>Building</th>
<th>NSPs</th>
<th>All Levels</th>
<th>Lower Levels</th>
<th>Upper Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(\mu_r)</td>
<td>(\mu_e)</td>
<td>(\mu_r)</td>
</tr>
<tr>
<td>Santa Barbara</td>
<td>Single LN 440</td>
<td>1.05</td>
<td>0.07</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Mode NSP CM 440</td>
<td>1.07</td>
<td>0.09</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Multi Mode MPA NSP MPA</td>
<td>1.14</td>
<td>0.14</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.12</td>
<td>0.12</td>
<td>-</td>
</tr>
<tr>
<td>Watsonville</td>
<td>Single LN 440</td>
<td>1.02</td>
<td>0.08</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Mode NSP CM 440</td>
<td>1.02</td>
<td>0.08</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Multi Mode MPA NSP MMPA</td>
<td>1.05</td>
<td>0.08</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.06</td>
<td>0.09</td>
<td>-</td>
</tr>
<tr>
<td>Imperial County</td>
<td>Single LN 440</td>
<td>0.72</td>
<td>0.28</td>
<td>0.80</td>
</tr>
<tr>
<td>Services</td>
<td>Mode NSP CM 440</td>
<td>0.75</td>
<td>0.25</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>Multi Mode MPA NSP MMPA</td>
<td>0.80</td>
<td>0.20</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.72</td>
<td>0.28</td>
<td>0.81</td>
</tr>
<tr>
<td>Sherman Oaks</td>
<td>Single LN 440</td>
<td>0.89</td>
<td>0.17</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>Mode NSP CM 440</td>
<td>0.91</td>
<td>0.17</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>Multi Mode MPA NSP MMPA</td>
<td>1.02</td>
<td>0.12</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.06</td>
<td>0.09</td>
<td>1.03</td>
</tr>
<tr>
<td>North Hollywood</td>
<td>Single LN 440</td>
<td>1.01</td>
<td>0.30</td>
<td>1.31</td>
</tr>
<tr>
<td></td>
<td>Mode NSP CM 440</td>
<td>1.03</td>
<td>0.31</td>
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<tr>
<td></td>
<td>Multi Mode MPA NSP MMPA</td>
<td>1.22</td>
<td>0.27</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.25</td>
<td>0.29</td>
<td>1.32</td>
</tr>
</tbody>
</table>

\(\mu_r\) : Mean of the ratios
\(\mu_e\) : Mean of normalized errors
LN 440: FEMA 440 Linearization
CM 440: FEMA 440 CM

- In the short buildings, mean of the OTM predictions with single-mode NSP is between 1.02 and 1.07 of the true values and the mean normalized error is 0.08~0.09.
- At upper levels of the tall buildings OTM predictions with single-mode NSP are inaccurate and largely underestimated. Mean of predictions is between 0.74 and 0.78 of the true values and mean of normalized error is 0.22~0.30.
• At lower levels of the tall buildings, single-mode NSP have given large overestimation and inaccuracy. Overestimation is as high as 33% mean of normalized error is 0.11~0.33.

• With multi-mode NSP mean of the predictions at upper levels is between 1.01 and 1.18 of the true values and the mean of error is 0.08~0.27. At lower levels predictions with multi-mode NSP is about the same as single-mode NSP.

• In the ICS building, large underestimation (37%) and inaccuracy (mean error of 0.37) exists at upper floors. At lower floors mean predictions is about 0.8 of the true values and mean errors is halved (0.15~0.2).

Observations collectively indicate that prediction of story overturning moments with the single-mode NSP is accurate in the short buildings, whereas in the taller building NSP have proven ineffective in predicting OTM. While large underestimation has been observed at the upper levels, prediction at the lower levels has been significantly overestimated. The multi-mode NSP erased the underestimation at the upper levels but did not reduce the errors. The multi-mode NSP has been essentially ineffective at the lower levels.

5.5 Summary and Final Remarks

Performance of standard single-mode and multi-mode NSP is studied in light of data obtained from five instrumented buildings. It has been observed that the single-mode NSPs are adequate for predicting all responses in the short buildings. This observation is much expected since in the short buildings response is dominated by the 1st mode and
therefore the assumptions on which the single-mode NSP have been developed are satisfied.

In the tall buildings however, the single-mode NSP have proven to be less effective in predicting story shears, inter-story drifts and overturning moments. Smaller inaccuracy has been observed in story displacements; in fact predictions of displacement at the lower floor of the tall buildings have been reasonably accurate. The largest inaccuracy with the single-mode NSP has been observed in the inter-story drifts. At the upper levels drifts have been significantly underestimated; mean of the prediction at the upper levels were between 50 to 72 % of the true values. In the story shears, although not as severe as in story drifts, large underestimation and inaccuracy at the upper levels of the tall buildings has been observed; the mean of prediction were as low as 80% of the true values.

Although application of single-mode NSP to the tall building is not recommended by their guidelines, a noteworthy observation is the drastic difference between the accuracy at the upper and lower levels of the tall buildings; while predictions at upper floors are typically inaccurate, at lower floors responses are notably better. For story shears and drifts mean of the predictions is within 5% of the true values and mean normalized error is small. It can be stated that based on this observations the single-mode procedures have accurately predicted the response at the lower levels. For overturning moments, the single-mode NSP has proven totally inaccurate, mean of the predictions has shown up to 26% underestimation at upper levels and 33% overestimation at the lower levels.

Examination of linear response spectra has clearly demonstrated that in those responses that are not successfully predicted with single-mode NSP (e.g. upper level shear and drift) contribution of higher modes is large. The multi-mode procedures that
incorporate the higher modes through pushover analyses are shown to have significantly improved the predictions of story shears and drifts in the tall buildings. With the multi-mode NSP, underestimation in the upper levels shears has been erased and predictions err on the conservative side. Despite substantial improvements, still some noticeable inaccuracy and underestimation remained in the upper level drifts of the North Hollywood building.

The observations we have made in this study with the real instrumented buildings agree with the previous observations by other researchers in simulation studies (Krawinkler and Seneviranta, 1998; Kunnath and Gupta, 2000; Chopra and Goel, 2002; Goel and Chopra 2004; FEMA400, 2005) and are consistent with the general intuition that expecting high fidelity results from NSP is not possible. However, the observations give some insight to the level of accuracy that one can anticipate when using the NSP in seismic evaluation and rehabilitation of existing buildings. Application of NSP in critical decision making about the safety of a building or the need for seismic upgrading necessitate some reasonable conservatism in the NSP predictions. With this regard, the observed accuracy in the standard single-mode NSP for tall buildings especially in inter story drift and story forces (shear and moments) was not satisfactory. The multi-modal NSP offered notable improvements over the single-mode NSP especially in story shears. In the next Chapter, we examine the possibility of improving the current NSP by taking a close look at the load patterns used in pushover analysis and computation of the target displacement.
CHAPTER 6

POSSIBLE IMPROVEMENTS TO NSP

6.1 Overview

This Chapter examines the possibility of improving the NSP. It is not difficult to realize that an attempt to improve the NSP through adding more complexity and sophistication is likely to end in situations where use of NDA would be preferred over NSP. Nevertheless, modifications for the purpose of adding some levels of conservatism so that the results could be safely used in performance assessment is prudent and justifies the continuous work of researchers on improving the NSP.

Recalling the fundamental assumption of NSP, one can recognize that the error in predicted responses can stem from error in the target displacement (i.e. wrong estimate of the maximum roof displacement) or from how the structure is pushed to the target displacement. Attempts to improve the NSP, while retaining the fundamental assumption, should therefore address the SDOF reduction and the lateral load patterns. The former was extensively studied in Chapter 2 where an energy consistent method was developed and was shown to be superior to the SDOF reduction used by typical NSP. Subsequently an alternative NSP by incorporating the energy consistent SDOF in single-mode and multi-mode pushover was proposed in Chapter 3 (named EC NSP in this document). The
EC NSP increases the accuracy in prediction of target (roof) displacement and is suggested as a possible way of improving the current methods.

This chapter also examines the current option for lateral loads and suggests load patterns that may lead to improved prediction of shear and drifts. It should be noted that since the limited number of the instrumented buildings and recorded ground motion does not provide enough information in the quest for possible improvement, we resort into simulations where NDA responses are taken as benchmarks.

6.2 Examination of Lateral Load Patterns

Non-adaptive load patterns currently used in NSP and alternatives that can offer some improvements are studied here.

6.2.1 Evaluating the Current Options for Fixed Load Patterns

A close look at the current options for fixed load patterns is taken thorough a simulation study with the 3 and 9 story frames previously introduced in Chapter 2. Nonlinear dynamic analysis of the structures under a suit of 30 ground motions (E-W components of the ground motions in Chapter 4), scaled to result in predefined roof drifts of 1% and 2%, are taken as the true responses. The two selected frames represent low rise and mid/high rise structures and the two roof drifts represent the level of nonlinearity in the response. Assuming that the NSP have accurately predicted the roof displacement, the adequacy of load patterns are examined by comparing the pushover responses corresponding to the true roof displacement with the NDA benchmarks.
Lateral Load Patterns

The following five common fixed lateral load patterns are considered in the study.

Loads proportional to the 1st mode shape \( F = M \phi \)

Inverted triangular loads

Code based loads; \( F = \frac{W_i h_i^k}{\sum W_i h_i^k} \)

where \( W_i \) is the seismic mass and \( h_i \) is the elevation of floor \( i \) (measured from the base) and exponent \( k \) varies linearly between 1 for periods less than 0.5 second and 2 for periods equal or greater than 2.5 seconds.

Uniform loads

SRSS combination of modal story shears. A number of modes that comprise 90% of the total mass are used.

Table 6.1 summarized the lateral load patterns for the two examples. Note that the SRSS loads are determined for an average response spectrum of the unscaled 30 ground motions.

<table>
<thead>
<tr>
<th>Building</th>
<th>Floors</th>
<th>Mode 1</th>
<th>Triangular</th>
<th>Code</th>
<th>Uniform</th>
<th>SRSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Story</td>
<td>1</td>
<td>0.31</td>
<td>0.33</td>
<td>0.25</td>
<td>1.00</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.66</td>
<td>0.67</td>
<td>0.60</td>
<td>1.00</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>9 Story</td>
<td>1</td>
<td>0.14</td>
<td>0.15</td>
<td>0.05</td>
<td>1.00</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.24</td>
<td>0.25</td>
<td>0.11</td>
<td>1.00</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>8</td>
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<td>0.89</td>
<td>0.78</td>
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</tr>
<tr>
<td></td>
<td>9</td>
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<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>
6.2.1.1 Story Displacements

Mean of the ratio of predictions to the true values for the 30 motions are shown in Fig.6.1 for the 3 and 9 story buildings. As can be seen, displacements in the 3 story building at both drift levels are predicted very well with all load patterns except the uniform load. In the 9 story at 1% drift, displacements are predicted well with the 1st mode, triangular and SRSS load patterns. At 2% drift however those load patterns overestimate the response whereas the code based load pattern gives reasonable predictions. A noticeable observation is that the uniform load pattern in both structures and at all drift levels results in poor prediction of story displacements.

Fig.6.1. Mean of story displacement ratios a) 3 story b) 9 story.
6.2.1.2 **Inter-Story Drifts**

Mean ratios of the predicted to true IDI for the two examples are shown in Fig.6.2. It can be seen that in the 3 story structure predictions with all the load patterns except the uniform load are very good.

![Fig.6.2. Mean of IDI ratios a) 3 story b) 9 story.](image)

Accuracy of the predictions has diminished in the 9 story structures; at 1% drift level, the 1st mode, triangular and SRSS load patterns give good predictions at the lower and intermediate stories, but underestimate the IDI at upper stories by almost 35%. At 2% drift level predictions are significantly worse. Despite some reasonable predictions with code-based load pattern at lower stories, large underestimation can be observed will all
the load patterns at upper stories. The uniform load pattern consistently results in poor predictions.

6.2.1.3  Story Shears

Mean of the ratio of predicted to true story shears for the 3 and 9 story buildings are shown in Fig.6.3.

Fig.6.3. Mean of story shear ratios a) 3 story b) 9 story.

In the 3 story building at both drift levels predictions are reasonably good with all load patterns except the uniform load. In the 9 story building, at 1% drift level again the 1st mode, triangular and SRSS load patterns lead to good predictions at lower and intermediate stories while underestimating at upper stories by almost 35%. At 2% drift
level story shears are consistently underestimated with all load patterns. Aside from the uniform load pattern that gives gross errors, others also result in up to 50% underestimation at upper stories.

6.2.1.4 Story Overturning Moments

Mean of the ratio of predicted to true overturning moments for the 3 and 9 story buildings are shown in Fig.6.4.

![Fig.6.4. Mean of overturning moment ratios a) 3 story b) 9 story.](image)

In the 3 story building at both drift levels, all the load patterns except the uniform give good prediction; SRSS load pattern shows the best results. In the 9 story building at 1% drift all load patterns except the uniform give reasonable prediction at the lower
floors but result in up to 40% underestimation at upper floors. At 2% drift overturning moments are systematically underestimated with all the load patterns. Accuracy of the predicted overturning moment with uniform load pattern, similar to the accuracy in other responses, is poor.

6.2.2 Observations - Possible Ways to Improve NSP

Examinations in 6.2.1 have shown that all the load patterns except the uniform are equally good for the 3 story example which represents low-rise buildings. Moreover, in the 9 story example (representing mid/high rise buildings), when low levels of nonlinearity is expected (1% roof drift) the 1st mode and triangular load patterns result in reasonably good predictions of story displacement. The two load pattern also result in good prediction of drifts, story shears and overturning moments at lower floors but notable underestimation at upper floors. When larger nonlinearity is expected (2% roof drift) while code-based load pattern has given reasonable prediction of displacements, prediction of other responses are substantially inaccurate with large underestimation at upper floors. Although no load pattern stands out as the “best fixed load pattern”, the uniform load has proven inadequate in all cases studied.

Given that in the examinations the exact roof displacement was known, an important observation regarding the fundamental assumption of NSP can be made. The observation is that whether the accuracy in target displacement (maximum roof displacement) leads to accurate prediction of all other responses depends on the structures. In the 3 story example, where the 1st mode is expected to govern the response, such correlation between the maximum of the roof displacement and other quantities is possible, however, in the 9 story example this argument is not valid. In the former case,
as long as the roof displacement is predicted accurately, the common load patterns (except uniform) can result in accurate prediction of all responses. On the contrary in the latter case, where the responses are not dominated by the 1st mode, even if the maximum roof displacement is predicted correctly, the accuracy in other responses still depends to large extent on the lateral load patterns.

In brief, observations from this examination show the possible avenues to improve the NSP. In short buildings (dominated by the 1st mode), it is expected that increasing the accuracy in estimating the target displacement leads to improvements in the NSP predictions. In taller buildings, however, this remedy is not sufficient and one also needs to identify the proper lateral load patterns. The proper load pattern for different response quantities, as seen in the examinations, is not the same, therefore the logical approach is to use different load patterns for different responses. None of the common load patterns that were studied have proven adequate for shears and drifts in the case of the 9 story example (at 2% drift level); in the next section an attempt to identify proper load patterns is made in light of the results from simulations.

6.3 Identification of Proper Load Patterns

We attempt to identify proper lateral loads through examination of the patterns observed in nonlinear dynamic analyses and inspiration from linear static analysis. In a linear structure the static loads that result in maximum response quantities can be explicitly determined. For instance, the loads that result in the exact maximum story displacements, $u_{max}$, are $K.u_{max}$ or the loads that give the maximum story drifts, $dr_{max}$, are determined by adding the successive rows of $dr_{max}$ and multiplying by stiffness
matrix, $K$. Similarly, maximum story shears directly define (via successive subtraction) a unique load pattern that reproduces the maximum story shears. In nonlinear analysis, whereas no direct connection between lateral loads and the maximum story displacements or drifts can be made, the maximum story shears still define a load pattern that, if applied statically to the structure, can reproduce the exact maximum story shears.

Since it is not possible to theoretically identify the load patterns that reproduce maximum nonlinear displacements and drifts, we focus on the loads that correspond to the maximum story shears. In many structures, story drifts and story shears are related and the load pattern that results in maximum of one can be similar to those that correspond to the maximum of the other one. This is a reliable assertion in structures with shear dominated behavior (frame structures). Fig. 6.5 shows the lateral load corresponding to maximum story shears and story drifts in a linear 9 story one-span frame with shear and flexural behavior. All beams and columns have uniform properties with $I_{beam} = \alpha I_{column}$ and the behavior is controlled via $\alpha$. As $\alpha$ increases structure moves towards shear dominated behavior. Load patterns illustrated in Fig. 6.5 pertain to two extremes when the structure behaves in pure flexure and shear. In both cases the two load patterns are similar especially when shear behavior is dominant. Although the observation from this linear example cannot be extended to nonlinear behavior, it shows that our focus on a proper load pattern for story shear, given the limitations, is likely to be appropriate for inter story drifts too.

It is worth noting that since the maximum story shears are not concurrent, such lateral load pattern does not correspond to real inertial forces and are, in fact,
fictitious. More importantly, the maximum story shears are not known in real applications and can only be determined in a simulation study; the objective therefore is to see if any pattern in the shape of these loads can be identified.

Fig. 6.5. Lateral forces corresponding to maximum linear story shears and drifts for a 9 story frame subjected to Parkfield 2004 record. a) Flexure dominated b) shear dominated.

### 6.3.1 Load Patterns Corresponding to Maximum Story Shears ($F_{v_{\text{max}}}$)

Suppose the maximum story shears are known. Subtracting the maximum story shears at consecutive floors gives a unique load pattern ($F_{v_{\text{max}}}$) that reproduces the maximum story shears in a linear static analysis of the structure. In a nonlinear static (pushover) analysis this load pattern can potentially result in the maximum story shears provided that the structure does not fail before the loads are fully applied. Note that since in reality the maximum story shears do not happen at the same time a load
pattern defined on the basis of concurrent maximum values may exceed the capacity of the structure and could result in failure or non-convergence.

To identify possible patterns from simulations, Fig.6.6a show the lateral loads corresponding to the maximum story shears (normalized to the maximum value which is often at the roof) of the 9 story at 2% drift level for three ground motions in the ensemble and Fig.6.6b show the average of the load patterns for all the 30 records.

As can be seen variation in the shape of the load patterns for different ground motions is substantial (Fig.6.6a) and no specific information about a generic load pattern can be drawn from the shape of the average load pattern in Fig.6.6b except that the load at the roof is the largest. Nevertheless, if the load patterns determined from maximum story shears \( F_{v_{\text{max}}} \) are used in the pushover analysis, as one expects, predictions of the story shears will be very close to the true values. While such load
patterns do not promise any improvement in displacement and overturning moments, as explained previously some improvements in predictions of story drifts is also expected. Fig.6.7 shows the ratio of the predicted responses using $F_{\text{vmax}}$ to the true values for the ground motion # 2 (Parkfield 2004). For comparison, responses from the 1st mode and code-based load patterns are also shown. Results confirm that pushover responses using $F_{\text{vmax}}$ are notably better than those from other load patterns for shears and story drifts.

Fig.6.7. Ratio of the predicted response from pushover analysis to the true response for Parkfield 2004 earthquake. a) shear b) IDI c) overturning moment d) displacement.

The results obtained with $F_{\text{vmax}}$ are promising, however, the true maximum story shears are only known if a nonlinear dynamic analysis is carried out. This condition contradicts the main reason for restoring to NSP which was in fact to avoid NDA. Hence,
we set out to use an approximation of the maximum story shears and examine whether or not the forces corresponding to this approximation can be a proper alternative for $F_{v_{max}}$.

### 6.3.2 Alternative $F_{v_{max}}$ from Linear Analysis

In lieu of the true nonlinear maximum story shears, the maximum linear story shears that can be easily computed from either a linear dynamic analysis (LDA) or response spectrum analysis are used to obtain an alternative $F_{v_{max}}$. In response spectrum analysis, modal story shears are computed by summing the story forces of $\Gamma_i M \phi_i S_a$ where $S_a$ is the spectral acceleration at the period of mode $i$, and the total story shears are determined by SRSS modal combination rule.

Responses predicted from pushover analyses using the load patterns obtained from linear story shears ($F_{v_{maxL}}$) for the 9 story example are compared with those from common load patterns in Fig.6.8. Results are shown in terms of the mean of the ratio of the predicted to true values for the entire ground motion ensemble. Not surprisingly, the extent of improvement attained from $F_{v_{maxL}}$ is not as much as those from $F_{v_{max}}$. Nevertheless, improvements in the upper story shears and drifts, which are of the most concern, and also in overturning moments, are noticeable. At lower stories the code-based load pattern has resulted in better prediction of the story shears, drifts and story displacements. Overall observation from this particular example suggest that for story shears and drifts, consideration of two load patterns one being the code-based and the other one the $F_{v_{maxL}}$ load patterns can give the best results. The results from the envelope of the code based and $F_{v_{maxL}}$ are shown in Fig.6.9.
Fig. 6.8. Ratio of the predicted response from pushover analysis (under various load patterns) to the true response. a) shear b) IDI c) overturning moment d) displacement.

Fig. 6.9. Mean of the ratio of the predictions from pushover analysis to true values a) shears b) IDI.

6.3.3 Recommended Load Patterns

From examination of possible load patterns and observation from the simulations (limited to the 3 and 9 story examples) the following load patterns are recommended.
For Short Buildings (expected to be dominated by the 1st mode)

Any of the 1st mode, inverted triangular and code-based load patterns

For Tall Buildings (expected to have large contribution from higher modes)

Code based load pattern for story displacements

\( F_{\text{vmaxL}} \) from SRSS modal combination for overturning moments

\( F_{\text{vmaxL}} \) and code-based load patterns for story shears and inter story drifts.

### 6.4 Suggested Approach to Improve the NSP

From the accumulated study of the SDOF reduction and the lateral load patterns it can be concluded that combination of improving the accuracy in determination of the target displacement through the energy consistent SDOF reduction, and using the load patterns recommended in 6.3.3 can improve the predictive capability of NSP. The suggested approaches to improve the NSP predictions are listed below.

- Using the single-mode EC NSP as described in Chapter 3 with the load patterns recommended in section 6.3.3.

- Using the multi-mode EC NSP as described in Chapter 3. (for tall buildings)

### 6.5 Examination of the Suggested Approach in Instrumented Buildings

In this section potential improvement in the prediction of story shears and drifts from the suggested approach is examined in the instrumented buildings.
6.5.1 Story Shears and Drifts in the Tall Buildings

Predictions from multi-mode energy consistent NSP (EC MPA) and the envelope of single-mode EC NSP with code based and $F_{v_{\text{max}L}}$ (designated EC-SM in the plots) in the North Hollywood and Sherman Oaks buildings are shown in Figs. 6.10 and 6.11 respectively for inter story drifts and story shears. For comparison, the prediction from FEMA 440 (single-mode) and MPA (multi-mode) are also shown. Tables 6.2 and 6.3 also show the statistics of the evaluation metrics described in Chapter 5.

The EC-SM despite being essentially a single-mode procedure gives results that are comparable to the multi-mode procedures; predicted IDI with EC-SM at the upper levels of North Hollywood are considerably better than both MPA and EC MPA. Recalling that the largest inaccuracy with the single-mode NSP had occurred at the upper drifts in the North Hollywood, and that the improvement from MPA was not sufficient, it can be realized from Fig.6.10 that the predictions from EC-envelope are significant improvements. Mean of the prediction with EC-SM is 0.89 of the true values and mean normalized error is reduced from 0.45 (standard NSP) to 0.11. Predictions from EC MPA while being significantly better than standard single-mode procedures are essentially on par with those from the original MPA.

<table>
<thead>
<tr>
<th>Building</th>
<th>NSPs</th>
<th>All</th>
<th>Lower levels</th>
<th>Upper levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\mu_r$</td>
<td>$\mu_e$</td>
<td>$\mu_r$</td>
</tr>
<tr>
<td>Sherman</td>
<td>Single Mode</td>
<td>LN 440</td>
<td>0.85</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>Multi Mode</td>
<td>MPA</td>
<td>0.95</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>Energy</td>
<td>Envelope</td>
<td>1.03</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>Consistent</td>
<td>MPA</td>
<td>0.97</td>
<td>0.07</td>
</tr>
<tr>
<td>North Hollywood</td>
<td>Single Mode</td>
<td>LN 440</td>
<td>0.75</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>Multi Mode</td>
<td>MPA</td>
<td>0.89</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>Energy</td>
<td>SM</td>
<td>0.94</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>Consistent</td>
<td>MPA</td>
<td>0.92</td>
<td>0.13</td>
</tr>
</tbody>
</table>

$\mu_r$: Mean of the ratios
$\mu_e$: Mean of normalized errors
CM 440: FEMA 440 CM
Fig. 6.10. Prediction of IDI with multi-mode EC NSP and envelope of single-mode EC NSP

Story shear predictions with EC MPA and EC-envelope as well as the original MPA are all reasonable and demonstrate similar quality. Between the EC-envelope and EC MPA, predictions from the latter are slightly better especially at the lower levels.

Table 6.2. Statistics of evaluation metrics for story shears.

<table>
<thead>
<tr>
<th>Building</th>
<th>NSPs</th>
<th>All</th>
<th>Lower levels</th>
<th>Upper levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(\mu)</td>
<td>(\mu_e)</td>
<td>(\mu)</td>
</tr>
<tr>
<td>Sherman</td>
<td>Single Mode LN 440</td>
<td>0.90</td>
<td>0.13</td>
<td>0.96</td>
</tr>
<tr>
<td>Oaks</td>
<td>Multi Mode MPA</td>
<td>1.01</td>
<td>0.06</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>Energy</td>
<td>0.99</td>
<td>0.10</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>Envelope MPA</td>
<td>0.99</td>
<td>0.06</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Consistent</td>
<td>0.92</td>
<td>0.14</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>MPA</td>
<td>1.04</td>
<td>0.10</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>SM</td>
<td>1.06</td>
<td>0.10</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>Consistent MPA</td>
<td>1.05</td>
<td>0.08</td>
<td>1.04</td>
</tr>
</tbody>
</table>

\(\mu\): Mean of the ratios \(\mu_e\): Mean of normalized errors

CM 440: FEMA 440 CM
The overall observation from the results in the two tall buildings is that the nonlinear static procedures based on the two suggested approaches will give substantial improvements in prediction of story shears and drifts compared to the standard methods (e.g. FEMA 440). Improvements over the original multi-mode methods (i.e. MPA) are less evident except for upper floor drifts in the North Hollywood building.

### 6.5.2 Story Shears and Drifts in the Short Buildings

All response in the two short buildings (Santa Barbara and Watsonville) were predicted well with the standard single-mode NSP. The only noticeable inaccuracy was observed in the E-W drifts and N-S shears of Santa Barbara building. In the short
buildings, the suggested approach includes application of the single-mode EC NSP with any of the common load patterns. Predictions of the story shears and drifts for Santa Barbara building using the energy consistent approach with code-based and 1st mode load patterns are shown in Fig.6.12. For comparison responses from FEMA 440 are also shown. As can be seen in the plots, the suggested energy consistent approach has succeeded in improving the shear predictions. For drifts, the error observed in predictions with suggested method is about the same as in standard procedures but the predictions are in the conservative side.

![Fig.6.11. Responses predicted with single-mode EC NSP in the Santa Barbara building a) IDI (E-W) b) Shear (N-S).](image)

**6.6 Final Remarks**

It has been recognized from examinations in this chapter that in the short buildings obtaining an accurate prediction of the target displacement (maximum roof displacement) is the key; as long as the target displacement is predicted accurately, all the common load pattern except uniform load are likely to give reasonably good prediction of structural
responses. Simulations have also shown that in the taller buildings, accurate prediction of the roof displacement is not sufficient and proper load patterns should be used for different response quantities.

Load patterns determined by subtracting the successive maximum story shears obtained from a linear modal analysis ($F_{v\text{max}L}$) improved the prediction of story shear and drifts at upper floor of the 9 story example used in simulations. Observations (limited to the 9 story example) have shown that taking the envelope of the predictions with the code-based and $F_{v\text{max}L}$ loads results in the least inaccuracy in story shears and drifts.

Use of energy consistent single-mode NSP with the recommended load pattern and the energy consistent multi-mode NSP (EC MPA) have been suggested as possible approaches to improve the current NSP. Application of the suggested NSP to the instrument tall buildings has lead to notable improvement in upper floor inter story drifts that were poorly predicted with standard NSP. In particular, the mean of prediction at upper floors of the North Hollywood building that was about 54% of true values with standard method is improved to 89% of the true values and the mean normalized error is reduced from 0.46 to 0.11. Improvements over the original multi-mode NSP, however, are not as impressive; in the aforementioned case mean of the predictions with MPA is about 80% of the true values and mean of the normalized error is 0.21.

Furthermore, it has been observed that although the EC NSP with the suggested load patterns ($F_{v\text{max}L}$ and code based) is a single-mode procedure, it results in predictions that are comparable to those of the multi-mode procedures including the EC MPA. Improved accuracy together with the ease of application as a single-mode NSP is a great advantage for this procedure.
CHAPTER 7

SUMMARY AND CONCLUSION

7.1 Summary

The main objective of the study was to evaluate the effectiveness of NSP in predicting global engineering demand parameters i.e. displacement, drifts, shear and overturning moments using data from instrumented buildings. Sparsity of seismic recording at instrumented buildings and that the quantities of interest are not directly measured, initiated another thrust in this study to examine the problem of reconstructing seismic response from available recordings. The second objective was to study estimation schemes and to examine their applicability for seismic response reconstruction.

For the latter objective, data driven approach using local interpolator that has been traditionally used in structural engineering was looked into and was shown to be equivalent to fitting the response into a basis that is fully defined by sensor arrangements. The basis fitting perspective has made clear that a necessary condition for accuracy is that the number of sensors be no less than the number of modes that contribute substantially to the estimated quantity. Estimation theories from control were studied and a model based estimation scheme was developed by blending these theories with structural
engineering concepts. The methodology, designated as “MIRC”, is based on finding the minimum norm corrective forces that when applied to the structure together with ground excitation reproduce the measurements. Evaluation of estimation methods in simulation with four mid to high rise buildings showed large inaccuracy in interpolation schemes. The MIRC was shown to provide the most accurate estimations and was used as benchmark in NSP evaluations.

To pursue the main objective fundamental assumptions and constituents of NSP were thoroughly studied. MDOF to SDOF reduction, a central part of all NSP, was looked into and an energy-consistent alternative for SDOF reduction was developed and was shown to be superior to the approach taken by typical NSP. The single-mode procedures of ATC-40, FEMA 356 and their updated versions in FEMA 440 along with multi-mode procedures of MPA and MMPA were evaluated using five instrumented RC buildings. Single-mode NSPs were seen to be adequate for the short buildings and lower floors of the tall buildings while leading to significant inaccuracy at upper floors especially in shear and drifts. Disparity in the quality of predictions along the height and lack of accuracy at upper floors was shown to be the result of neglecting higher mode effects in single-mode procedures. Consideration of higher modes through multi-mode NSP significantly improved the predictions especially for shear and displacements yet was not sufficient for the upper floor drifts. Current option for lateral load patterns were examined and through ideas from NDA simulations and static analyses a pattern for lateral loads based on the maximum linear story shears was shown to improve the NSP predictions for shear and drifts. Incorporation of the energy-consistent SDOF reduction to single-mode and multi-mode NSP and use of the recommended lateral load patterns for various
responses were suggested as possible improvements to current NSP and were examined in the instrumented buildings.

7.2 Conclusions

Conclusions regarding the study on seismic response reconstruction are:

- Interpolation schemes are the same as fitting the response into a fixed basis whose dimensions is dictated by the number of sensors and its span is determined by the location of sensors. There is no flexibility in these schemes.

- The necessary but not sufficient condition to obtain adequate estimate with interpolations is that the number of sensors (excluding the ground) be at least equal to the number of modes that significantly contribute to the response.

- In tall buildings the number of sensors is often insufficient for the CS to give accurate estimates of shear and drifts. Low-pass filtering the acceleration data, using the frequency of $m^{th}$ mode as cutoff frequency improves the accuracy. ($m$ is number of sensors excluding the ground). More accurate estimates can be attained via model based estimation.

- The model based scheme developed in this study, that uses the minimum norm corrective forces to reproduce the measurements, provides higher accuracy especially for shear and drift. The nominal model can be rather simple, as long as the fundamental period is calibrated to that of the actual building.

Conclusions regarding the NSP evaluations (restricted to the cases studied) are:

- In the short buildings, mainly dominated by the 1st mode, single-mode NSP were adequate to predict all the responses.

- In the tall buildings with substantial contribution from higher modes while inaccuracy in displacement was not large, poor predictions of overturning moments and large
underestimation in upper floors shear and drift rendered the single-mode NSP ineffective.

- In the tall buildings, despite large inaccuracy at upper floors predictions with single-mode NSP at the lower floors were accurate.

- Consideration of higher modes via multi-mode NSP improved the predictions at the upper floors of the tall buildings especially for displacement and shears. Underestimation in story shears was eliminated; drift predictions were also improved but some inaccuracy remained at the upper floors.

- Accurate prediction of target displacement was the key for the success of NSP in the short buildings. As long as the target displacement was determined accurately, all common load patterns (code based, 1st mode, triangular) except the uniform load lead to accurate predictions.

- The energy-consistent SDOF reduction developed in this study was shown to provide more accurate prediction of roof displacement than the approach in typical NSP. Incorporation of the SDOF reduction to single-mode NSP was shown to be a proper way for improving NSP predictions in the short buildings.

- Accurate prediction of the target displacement did not ensure accuracy in other responses of the tall buildings. Load patterns in pushover analysis were shown to have more effect on the accuracy of NSP predictions. Different load pattern were required to obtain the best prediction of different responses.

- A load pattern extracted from maximum linear story shear ($F_{v_{\text{max}}}$) was shown to be appropriate for shears, drifts and overturning moments. Code based load pattern appeared appropriate for story displacements.

- The energy consistent multi-mode procedures (EC MPA) and the single-mode energy consistent NSP with code based and $F_{v_{\text{max}}}$ loads were suggested as improvements for NSP. Both resulted in better predictions than standard NSP in the two tall buildings while the latter benefits from the convenient features of single-mode procedure.
Future Work

1 Examination of L1 minimization to determine the corrective forces in the model based estimation scheme presented in this study (MIRC).

2 Comprehensive validation of the energy consistent NSP and the load patterns suggested in this study through nonlinear dynamic analyses of structures with various construction and behavior.

3 NSP have been widely used in large scale loss estimation. Currently CSM of ATC-40 is the base of many vulnerability and risk assessment methodologies (HAZUS; RISK-UE, 2003). Observations of this study and similar studies have shown that ATC-40 often underestimates the response. Examination of other NSP in vulnerability assessment is in schedule for near future.
REFERENCES


