A BI-DIRECTIONAL LOAD RATING MODEL
OF THE FLEXURAL RESPONSE OF A
PRESTRESSED CONCRETE BRIDGE BEAM ELEMENT

A Dissertation Presented

by

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ABSTRACT

It is essential that load rating capacity of bridges and their elements be monitored to preserve their structural integrity and ensure safety of the public. Load rating analysis, based on current design codes, is a basis used to identify bridges that should be earmarked for rehabilitation. Routine bridge inspections, which can identify a decrease in structural condition, are performed independently from load rating analyses. Thus, this research is motivated by a vision in which the inspections and load rating processes are integrated into a bridge management system.

The objective of the presented research is to develop a bi-directional element condition model that uses data typically obtained during routine bridge inspections. This model, developed for a prestressed concrete beam because it is the critical element in girder-supported bridge systems, has three main components. (1) The prognostic component is used to calculate bridge load ratings probabilistically considering random variables representative of load and resistance effects. (2) The deterioration component is used to evaluate the effects of deterioration on relevant random variables. (3) The diagnostic component helps to identify possible deficiencies that result in an undesirable bridge load rating.

Bayesian networks are used to implement prognostic and diagnostic features of the bi-directional model. Parent variables (input variables to the network) are assumed to be random and obtained from different data sources (random variables available in the literature and expert solicitation). Monte Carlo simulations are used to evaluate the conditional probabilities for use in the Bayesian network. Time-dependent corrosion effects are implemented as an example of considering deterioration in the bi-directional model.
model. The model is applied to the critical limit state (flexure at midspan) of the critical element (simply-supported prestressed concrete girder) of a prestressed concrete bridge at ten-year intervals throughout its service life.
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CHAPTER I
INTRODUCTION

1.1 Overview

There are over 600,000 bridges within our nation’s infrastructure (FHWA 2007), which must be properly maintained. This poses a challenge as the bridge population is made up of different aged bridges of various material types. According to the National Bridge Inventory (NBI), the average age of the United States bridge population is 50 years (FHWA 2007), which happens to be the normal design life of a bridge. All bridges deteriorate, although each bridge type is susceptible to particular deterioration mechanisms. For example, a steel girder bridge is subject to fatigue from the cyclic traffic loads. Reinforced and prestressed concrete bridges suffer from corrosion due to chloride infiltration from a marine environment or deicing salts. Bridge owners are expected to monitor these different bridge deteriorations and to carry out the necessary rehabilitation in order to preserve structural system integrity.

This research focuses on the development of a condition assessment model for the critical element of the prestressed concrete bridge type – the prestressed concrete I girder. Approximately 22% of all the bridges in the United States are of this material type (FHWA 2007). The first prestressed concrete bridge, the Walnut Lane Memorial Bridge, was built in 1949 in Philadelphia, Pennsylvania. From there, the 1950’s witnessed a rapid growth of the prestressing industry (PCI 2003). It is no wonder that prestressed concrete has become such a popular building material. Prestressed concrete combines the tensile strength of high strength steel and the compressive strength of high performance concrete. Prestressed concrete has its advantages over other bridge
materials. It has a lower initial cost, less maintenance and a longer life expectancy. It also is quickly erected and aesthetically pleasing (PCI 2003).

There are two types of prestressed concrete: post-tensioned and pre-tensioned. In the post-tensioning process, the tension is applied to the steel after the concrete has hardened. In the pretensioning process, the steel is pretensioned within the forms, then the concrete is poured. Once the concrete has hardened, the anchored ends (shown in Figure 1.1) of the steel are released transferring the force in the strands to the concrete through the bond between the two materials (PCI 2003).

**FIGURE 1.1.** Pretensioning of a Prestressed Concrete Beam (adapted from PCI 2003)

Of all material types, prestressed concrete has the least percentage of structurally deficient bridges as shown in Figure 1.2 (FHWA 2007). Bridges are considered structurally deficient when a component (deck, superstructure or substructure) is determined to be in poor condition exhibiting advanced section loss, deterioration, spalling or scour. Bridges are eligible for rehabilitation using federal funds if they are deemed structurally deficient and they have a sufficiency rating of 80% or less. Further, if structurally deficient bridges have a sufficiency rating of 50% or less, federal funds are available for replacement. The sufficiency rating formula scores each bridge from 0 to
100% in order to properly categorize the bridge. The factors that make up the sufficiency rating are: 1) Structural Adequacy and Safety (up to 55%), 2) Serviceability and Functional Obsolescence (up to 30%), and 3) Essentiality for Public Use (up to 15%). The sufficiency rating formula is based on information particular to each bridge: inventory data, inspection data and the load rating.

**FIGURE 1.2.** Percentage of Structurally Deficient Bridges per Material Type

The sufficiency rating is the primary factor used by the federal government in determining eligibility for federal funding for each rehabilitation or replacement project. In turn, with confirmation of the availability of federal funding for certain projects, highway departments are aided in their prioritization of bridge maintenance, repair, and rehabilitation. Nearly half of the sufficiency rating (45%) is based on factors not directly related to the structural integrity or the safety of the system rated. While it can be debated whether or not this is the most appropriate approach to ranking bridges, it is the
practice employed. The remainder of the sufficiency rating (55%) does represent the structural condition of the bridge being based on the structural deficiencies observed in inspection and the calculated load rating.

Data from a number of different sources are used in the evaluation of the sufficiency rating formula. These sources include initial design and as-built information, referred to as “inventory” data, data collected as part of the mandated bi-annual bridge “inspection” process, and information obtained through load rating analyses performed on an as needed basis. Following is a description of each of these data sources with a particular emphasis, in each case, on how the data contributes to the overall assessment of a bridge.

1.1.1 Inventory Data

The inventory data of a bridge is the basic information particular to a bridge. It includes the following (AASHTO 2003):

- structure number
- name
- year built
- year reconstructed
- highway system
- location,
- description of structure (superstructure type, substructure type, etc.)
- skew
- spans (# & length(s))
- structure length
- bridge roadway width
- deck width
- clearances
- wearing surface and deck protective system
- curb and sidewalk widths
- railings and parapets
- bridge approach alignment
- lanes on or under structure
- average daily traffic (ADT) & average daily truck traffic (ADTT)
- design load
- features intersected
- plans and dimensions
- critical features (such as scour critical locations or fatigue prone details)

The inventory data of a bridge typically remains the same through the life of the bridge. If the bridge is significantly modified, for example by widening, the inventory data is updated accordingly.
1.1.2 Inspection Data

It is federally mandated that each bridge receive a routine inspection once every two years (FHWA 1995b). During a typical inspection, data (much of which is visual) is collected that defines the condition of the bridge. It is referred to as condition rating data. The National Bridge Inspection Standards (NBIS) have set forth the condition rating descriptions shown in Table 1.1 (FHWA 1995b).

Table 1.1 Condition Rating Descriptions (adapted from FHWA 1995b)

<table>
<thead>
<tr>
<th>Inspection Rating</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>NOT APPLICABLE</td>
</tr>
<tr>
<td>9</td>
<td>EXCELLENT CONDITION</td>
</tr>
<tr>
<td>8</td>
<td>VERY GOOD CONDITION - no problems noted.</td>
</tr>
<tr>
<td>7</td>
<td>GOOD CONDITION - some minor problems.</td>
</tr>
<tr>
<td></td>
<td>SATISFACTORY CONDITION - structural elements show some minor deterioration</td>
</tr>
<tr>
<td>6</td>
<td>FAIR CONDITION - all primary structural elements are sound but may have minor section loss, cracking salling or scour.</td>
</tr>
<tr>
<td>5</td>
<td>POOR CONDITION - advanced section loss, deterioration, spalling, or scour.</td>
</tr>
<tr>
<td>4</td>
<td>SERIOUS CONDITION - loss of section, deterioration, spalling, or scour have seriously affected primary structural components. Local failures are possible. Fatigue cracks in steel or shear cracks in concrete may be present.</td>
</tr>
<tr>
<td>3</td>
<td>CRITICAL CONDITION - advanced deterioration of primary structural elements. Fatigue cracks in steel or shear cracks in concrete may be present or scour may have removed substructure support. Unless closely monitored, it may be necessary to close the bridge until corrective action is taken.</td>
</tr>
<tr>
<td></td>
<td>&quot;IMMINENT&quot; FAILURE CONDITION - major deterioration or section loss present in critical structural components or obvious vertical or horizontal movement affecting structural stability. Bridge is closed to traffic but corrective action may put it back into light service.</td>
</tr>
<tr>
<td>2</td>
<td>FAILED CONDITION - out of service - beyond corrective action</td>
</tr>
</tbody>
</table>

These ratings are assigned for each element for a typical two-span bridge system. Example elements are individual: joints, bearings, and beams. Bridge inspectors are
directed to rate each element of a bridge according to the descriptions shown in Table 1.1. For example, in order to arrive at the condition rating of the superstructure of a two-span, multi-girder, steel stringer bridge, each span is looked at separately. Within the span, each beam and each bearing receives a condition rating. Other miscellaneous categories are rated as well such as member alignment, and deflection under load. The lowest rating of all the elements within a span, determine the superstructure condition rating for that span. Likewise, the lowest superstructure span rating determines the overall or total superstructure rating for the bridge. This follows the critical element concept, where the element having the lowest condition rating conservatively determines the component condition rating that applies to the overall bridge.

While the rating descriptions shown in Table 1.1 offer general guidelines, many states have adopted their own condition rating systems that offer more descriptive detail. This is all in an effort to achieve more consistent ratings from inspector to inspector. Each bridge inspector is human and possesses his own interpretation of the descriptive guidelines. There are acknowledged factors that have bearing on this subjectivity ranging from number of years experience to physical abilities. The basis of the routine inspection is visual observation. Aside from the related inspector bias, a question is raised regarding what cannot be seen. For example, fatigue fractures begin as microscopic cracks, and corrosion activity of a reinforced concrete beam is initiated internally at the steel. These ratings based on visual inspections lead to quantifiable metrics that are based on qualitative interpretation.

Nondestructive technology has been emerging as a valuable asset in assessing material conditions within bridge members. However its application remains limited and
is not tied to the condition rating system. There have been several coordinated efforts to introduce nondestructive sensing methods into the regular inspection process. There is a correlation between nondestructive measurements and the condition ratings described in Table 1.1 suggesting the merit in a combined approach to bridge rating (Yanev 2007).

1.1.3 Load Rating

Load ratings, coupled with routine inspections described in 1.1.2, are used in assessing the condition of a bridge. A load rating is performed on every new bridge as it opens to traffic. This serves as the baseline load rating to which all future load ratings will be compared. Over the lifetime of the bridge, a routine inspection occurs every two years. An obvious change in load-carrying capacity indicated by the routine inspection, for example, warrants the re-rating of the bridge in order to evaluate the effect of collision damage, additional deadload, or improvements that may alter structural performance. Also, knowledge of current load ratings is essential when issuing overload permits to trucks. At this time, while the bi-annual inspection may trigger a load rating analysis, the processes performed to implement the load rating are done relatively independently.

A load rating consists of a series of code-based calculations that are carried out on main bridge elements considering critical limit states to determine the safe liveload capacity of a bridge where liveload is defined as loads resulting from vehicular and pedestrian traffic. The calculated load rating is a ratio of capacity to demand having an obvious threshold of one. For example, a rating factor less than one is considered a failed rating. The load rating process, like the bridge inspection process, follows the critical
element concept, in that the bridge element having the lowest rating factor governs the load rating of the entire bridge. A failed bridge rating prompts the bridge owner to consider posting of the bridge to limit the weight of vehicles allowed to cross. Bridge closure is mandatory if the load rating shows that the bridge is unable to carry 3 tons, a load equivalent to a pick-up truck. Posted and closed bridges cripple infrastructure systems forcing detours on truck traffic, emergency vehicles and school buses. The result is a financial burden on the users in terms of the additional mileage and inconvenience. Bridge owners are also fiscally affected by the calculated load rating as the Federal Highway Administration (FHWA) determines eligibility of federal funding through a sufficiency rating formula that incorporates the most current load rating of a bridge.

The calculated load factor is an important indicator of the load that a bridge may safely carry. Over the years, the bridge community has become more aware of the uncertainties of load and resistance effects and how they impact the structural integrity of a bridge. Load rating methods that are used today differ in the manner of accounting for the uncertainty that surrounds the load and resistance variables. There are three different approaches allowed by the code today: Allowable Stress, Load Factor, and Load and Resistance Factor Rating.

**Allowable Stress (AS)** is a design method that was presented by American Association of State Highway and Transportation Officials (AASHTO) in the 1930s. Allowable Stress uses one factor of safety and is considered a deterministic method. AS is based on the premise that the resisting material behaves elastically and is isotropic and homogeneous. Allowable limits are set at the threshold beyond which the material no longer behaves elastically. This point of failure is not clear cut and not as easy to define
as ultimate capacity (or failure). Also, in AS, the loads are used without factors. Loads are combined in an attempt to represent actual loading situations. The fact that different loadings have a different probability of occurrence is not recognized. Uncertainty is handled in the factor of safety that is applied to the resistance. This factor of safety has been determined through engineering judgment, as opposed to reliability methods, and does not easily translate into a probability of failure (Barker and Puckett 1997).

The Load Factor (LF) method, introduced in the 1970’s by AASHTO, incorporates a factor for each load (live, dead, snow, etc.). The basic concept of LF is that provided resistance must exceed required strength. Two limit states are considered, serviceability and ultimate. This method recognizes inelastic strength as well as loss of stability. Although an improvement from AS, LF falls short in that it does not properly account for the probabilistic nature of the resistance variables. Uncertainty in resistance is generally accounted for through a strength reduction factor, accounting for possibilities such as members that are undersized or lacking strength, inaccuracies in the equations. This resistance factor has been developed separately from the load factors not realistically portraying their simultaneous occurrence (BEH 1999).

Bridge code based on Load and Resistance Factor Design (LRFD), has been developed using reliability methods and yields probabilities of failures directly translated from reliability indexes commonly referred to as $\beta$. The greater the value of $\beta$, the safer the structure. This method has been calibrated with the existing design practice, LF. Researchers concluded that a $\beta = 2.5$ provides an acceptable level of safety for current bridge evaluation. This translates into a 6 out of 1000 chance that loading will exceed resistance during the lifetime of the bridge (Jaramilla and Huo 2005).
LRFD offers uniform level of safety. Load factors are set so that all loads have the same probability of being exceeded during design life. In addition, LRFD includes site-specific load factors as well as separate factors that consider ductility, redundancy and operational importance. The live loading used in LRFD (HL-93) represents current highway loading and varying span lengths. This is an improvement from the previous design methods that used 3 separate legal load models to address short, medium and long spans. Also, LRFD is designed to be used in the same manner as previous codes allowing Engineers to make a smooth transition from LF to LRFD. The Federal Highway Administration set an October 1, 2007 deadline after which all new designs must be done using the LRFD code (FHWA 2006a).

FHWA directs that this new design be rated according to the rating component in LRFD, or **Load and Resistance Factor Rating (LRFR)**. LRFR shows great promise in its representation of the uncertainties of both the load and resistances. However, the choice of which method to use in the rating calculations is at the discretion of the Engineer, as the LRFR methodology will not be mandated for existing bridges previously rated according to AS or LF (FHWA 2006b). Typically, the code with which the bridge was designed is that which is used to allow comparison with the initial load rating. As long as this practice is allowed, the random nature of the loads and resistances of a bridge will not be adequately represented in the load rating. This misrepresentation is unfortunate due to the threshold nature of the load rating. Therefore, the majority of bridges will continue to be rated using methods that are lacking in the representation of the uncertainty of the load and resistance variables.
In summary, the load rating factor is an important number considering aspects of safety and finances. It is a measure of the load carrying capacity of the bridge that will ensure the safety of the public. It is a main contributor to the sufficiency rating formula used to calculate eligibility for federally funded rehabilitation or replacement. Improvement is needed within the load rating process. Automatic calculation based on results of the biennial inspection would remove the burden of time-consuming calculations from the engineer and provide an up-to-date bridge load rating. Also, uniformity in basic design methodology is necessary to properly account for randomness. As explained before, a bridge tends to be load rated using the methodology under which it was designed and if that happens to be AS or LF, then the representation of randomness is lacking. The adoption of LRFR for all bridges will allow fair comparison between bridges. Tools that calculate using all three methodologies will be valuable in educating the engineer through the relative comparisons of the resulting rating factors.

1.2 Identified Needs

The discussion above demonstrates the current accepted processes for inspection and load rating of bridges and their elements. Following are identified needs related to the development of a rational model for assessing the condition of critical bridge elements.

1.2.1 Need for Combined Load Rating and Inspection Processes

Currently bridges (broken down to the element level) are assessed through the two processes, load rating and inspection. The load rating of a bridge determines the safe live
load carrying capacity of the bridge. The inspection of the bridge determines the condition of its elements. While the inspection of a bridge is mandated to occur once every two years, a load rating analysis is performed on an as-needed basis. The two processes are carried out separately: at different times, by different personnel. A comprehensive model should combine the two processes allowing the load rating analysis to be carried out automatically stemming from the data from every inspection cycle. This will allow the two processes to function harmoniously while keeping the bridge load rating information current.

1.2.2 Need to Incorporate Randomness

Any realistic portrayal of bridges and their behavior must account for contributing uncertainties. There are aleatory uncertainties that are underlying and inherent. For example, some physical uncertainties include variation in steel yield strength, dimensions of a structural element, or variability of wind loading. Uncertainties such as these may be represented by random variables, which introduces epistemic uncertainties associated with modeling. There is a statistical uncertainty in random variables as to how well the intrinsic uncertainty is represented. In modeling, there is an uncertainty associated with the portrayal of the behavior or performance of the bridge element. This uncertainty is increased when the model predicts future behavior, attempting to estimate the strength of the element and the loads that it will be experiencing. Epistemic uncertainties such as these can be reduced with more information and knowledge in order to improve the model [Melchers 1999]. Thus, there is a need to properly account for both aleatory and epistemic uncertainties associated with bridge element performance.
1.2.3 Need to Use All Load Rating Analysis Methods (AS, LF, LRFR)

Since the current codes still permit load rating analysis methods based on AS, LF, and LRFR, load rating model development should incorporate the three analysis methods currently used in the United States for the design and analysis of bridges and their components: Allowable Stress (AS), Load Factor (LF) and Load and Resistance Factor Rating (LRFR). By calculating the load ratings using all three, the results may be compared. This will familiarize the Engineer with the different expected outcomes of each of the methods. This will facilitate the implementation of the LRFR method by educating the Engineer through the presentation of the LRFR load rating alongside the load ratings of the well-known AS and LF.

1.2.4 Need to Accept All Data Formats

As described previously there are multiple sources of data available to assess bridge condition. Thus, any new development should accept all data formats. The input data required for a load rating analysis is often numerical and representative of resistance and loading effects. On the other hand, inspection data is often in the form qualitative descriptions. Also, the model will incorporate both random variables and constants. Random variables are ideal for modeling particular bridge element attributes such as live loads. Other bridge element parameters, such as beam length, are more appropriately modeled using a constant.
1.2.5 Need to Be Both Predictive and Diagnostic

It is important to determine if a load rating fails, and if so, the likely causes of failure. A predictive model will be able to determine the future condition of the bridge element (and subsequently, the bridge) along with the corresponding load ratings. There is also a need to diagnose symptoms observed during inspection for the underlying deterioration mechanism. This combination of prediction and diagnosis calls for a bi-directional model.

1.2.6 Need to Be Useful to All Members of the Bridge Community

The bridge engineering community needs a tool that can encapsulate information from different inspections and load rating analyses that provides physically meaningful information about condition. Students could be introduced to the bridge inspection and load rating and learn how the two processes work together to determine the condition of a bridge element. The Designer could troubleshoot different design options in order to determine the best alternative to prolong the life of the bridge. The Inspector could use a model such as this to diagnose symptoms in the field and to help direct additional tests that should be performed to do a thorough examination of a suspect bridge member. The Engineer could use this model to help in putting together a repair plan. Once the underlying deterioration has been diagnosed, effective repair and maintenance plans could be recommended. Finally, this model will be an asset to the Owner/Manager of a bridge, i.e. the decision-maker.
1.3 Dissertation Objectives

The objective of this research is to address inadequacies in current bridge inspection and load rating processes to create a load rating Bayesian network–based model that has the ability to assess the condition of a bridge element. This is done through the development of a load rating model for the critical element (prestressed girder) of a prestressed concrete bridge system. In practice, the overall rating for a prestressed concrete bridge system is limited to the lowest rating for any critical element. The model developed as part of this dissertation will

- combine the two currently used processes of assessment: bridge load rating and bridge inspection;
- represent appropriate randomness of resisting and loading effects;
- perform calculations using: AS, LF and LRFR;
- accept all data formats ranging from numerical measurements to qualitative descriptions;
- possess predictive and diagnostic capabilities.

The fundamental objective of this research is to provide a bridge condition assessment tool that will contribute to the preservation of our nation’s infrastructure and ultimately protect public safety. Through the automation of all of these features, the model will be helpful to all members of the bridge community: Student, Designer, Inspector, Engineer and Owner/Manager. Following is a brief description of the scope considered in the Bayesian network–based model development.
1.4 Dissertation Scope

The scope of this dissertation is to develop a load rating Bayesian network–based model for the critical element (an interior prestressed girder) of a prestressed concrete bridge. The network will consider the three current code–based approaches to evaluating load factors for the prestressed girder and address the objectives specified in Section 1.3. A deterioration mechanism for corrosion is introduced as well as factors that can be used to determinate the initiation of deterioration.

1.5 Organization of the Thesis

Chapter 1 serves as an introduction to the research conducted in this thesis.

Chapter 2 begins with an in-depth explanation of the load rating process and continues with full descriptions of the load rating methods, Allowable Stress (AS), Load Factor (LF) and Load and Resistance Factor Rating (LRFR). The load rating model is introduced modeling the flexural response of a simply-supported prestressed concrete bridge beam. The deterioration component, probabilistic prognostic component and the diagnostic Bayesian network component are described. The prestressed concrete bridge beam upon which the model is developed is described in detail. The chapter ends with a review and comparison of alternative methods that were considered for developing the model. These include fault tree analysis, Markov decision process, structural reliability and artificial intelligence.

Chapter 3 introduces the deterioration component of the model. The most common form of deterioration of prestressed concrete members is corrosion of prestressing steel through chloride infiltration. The results of the deterioration
component are applied to the random variables that serve as input to the prognostic component and diagnostic BN component of the prestress concrete beam load rating model introduced in Chapter 4 and Chapter 5, respectively. The probabilistic prognostic component and the diagnostic BN component are implemented at time-stepped intervals.

**Chapter 4** focuses on the probabilistic prognostic component of the model. This component calculates the load ratings using Monte Carlo simulation of the equations within the three codes: Allowable Stress (AS), Load Factor (LF) and Load and Resistance Factor Rating (LRFR). The entire process of development of the component is explained and includes the source of randomness of the root variables and the incorporation of judgment. The results of a sensitivity study of the input random variables are presented. An investigation is conducted into the correlation of the input variables. The results of the probabilistic prognostic component applied to the prestressed concrete bridge beam presented in Chapter II are presented and discussed.

**Chapter 5** presents a Bayesian network as the primary computational tool used in the model. Bayesian networks offer a graphical modeling language for representing uncertain relationships (Heckerman et al, 1995). In the chapter, the history and previous applications are presented. The underlying theory is explained and the method of modeling is described. The advantages and disadvantages of using a Bayesian network are discussed as well as the modeling logistics. The diagnostic BN component is applied to the same prestressed concrete bridge beam example used in Chapters 3 and 4. The component is run at time-stepped intervals and these results are presented and discussed.

**Chapter 6** the model is verified in detail from three aspects: 1) input, 2) output and 3) sensitivity studies. Specifically, input to the main components of the model
(probabilistic prognostic and diagnostic BN) are presented and explained in detail. The outputs of these two components are reviewed and discussed. The following sensitivity studies are summarized and discussed: input design variables, discretization thresholds of variables, functionality of inspection cluster.

Chapter 7 demonstrates the application of the model to an actual bridge. The bridge, built in 1965, is slated for necessary replacement due to the extensive deterioration caused by a corrosive marine environment. The results of the model, both prognostic and diagnostic, for the critical beam element are compared to the actual inspection data and rating calculations over the life of the bridge. The beneficial uses of the model are explored and discussed in a retrospective manner, with regards as to how the results could have helped to avoid this costly bridge replacement.

Chapter 8 presents summary, conclusions and recommendations for future research. The specific contributions of this research are presented. Future research is envisioned including expanding the element model with additional critical limit states and other forms of deterioration. The framework necessary for the implementation of this contribution to a broader scope is provided. The development of the modeling concept from this application to a beam element to that of an entire bridge structure is discussed.
CHAPTER II
LOAD RATING MODEL OF A BRIDGE BEAM ELEMENT

2.1 Bridge Load Ratings

Bridge load ratings are performed to determine the safe load carrying capacity of a bridge. The calculations are based on information particular to each bridge such as the compressive strength of concrete assumed for design among other data found on the plans. The results of the most recent field inspection are also included within the rating analysis in order to verify information shown on the original or as-built plans or incorporate any observed changes that impact the load carrying capacity of the bridge. Bridge load rating calculations are carried out on an element-by-element basis. In other words, each load-carrying element is rated, such as a beam. Typically, the lowest element rating governs the load rating for the entire bridge.

Load ratings are done initially when a bridge is newly built and then later during service life when there is an obvious change that may affect its load carrying capacity. Bridges are inspected once every two years, and the bridge load rating should be updated to reflect any relevant changes observed during an inspection. Bridge load ratings are also part of the permitting process of overload vehicles. It is the responsibility of the Bridge Owner to have an up-to-date bridge load rating reflective of current conditions. Typical Bridge Owners are comprised of the federal government, states, and municipalities. As the majority of bridges are owned at the state level, most state departments of transportation have in place a program dedicated to the systematic
completion of bridge load ratings. These calculations are carried out by Bridge Engineers employed at the state or subbed out to Consultants.

In a bridge load rating, two types of load ratings are calculated, inventory and operating. An inventory load rating determines the live load that can safely utilize a bridge for an indefinite period of time. An operating load rating determines the maximum permissible live load (AASHTO 2003).

In the end, load rating factors are calculated upon which recommendations may be made to the Bridge Owner. The critical threshold of a load rating factor is 1.00. If the load rating factor calculates below 1.00, posting and possible closure is advised. However, this is a policy decision that is left to the Owner. Several factors must be taken into consideration with a possible posting or closure, such as the routine use of the bridge and weighing the economic hardship imposed by a restriction against jeopardizing the safety of the public. The protocol used in making these recommendations is not consistent among states where some use the inventory rating factor as a basis and others the operating rating factor.

The series of calculations that make up a load rating are based on analytical methods used for design. These techniques have developed over the years in an attempt to accurately portray the loading and resistance effects of a bridge and provide a satisfactory level of safety. These are simplified evaluation procedures that tend to be conservative. More detailed analysis is justified when the results indicate that a larger capacity is necessary (AASHTO 2003).

Randomness is an important consideration in regards to bridge load rating and design. At the design stage, the uncertainty lies in the amount of loading the bridge will
experience in service. At the rating stage, the uncertainty is associated with the actual condition of the bridge and its strength resistance. There seems to be a greater amount of variability considering the strength resistance of existing bridges as opposed to the design stage. Failure to account for the spectrum of possibilities of structural performance could result in a public safety risk. On the other hand, an overly conservative evaluation could result in expensive and unnecessary restrictions, rehabilitation or replacement (AASHTO 2003).

### 2.2 Bridge Load Rating Methodologies

Three methodologies are currently used in the design and rating of bridges, each accounting for the load and resistance uncertainties differently. Allowable Stress (AS), a design method presented by the American Association of State Highway and Transportation Officials (AASHTO) in the 1930s, uses one factor of safety and is considered a deterministic method. Load Factor (LF), introduced in the 1970’s by AASHTO, incorporates a factor for each load (live, dead, snow, etc.). Although an improvement from AS, LF falls short in that it does not account for the probabilistic nature of the resistance variables. Currently, AASHTO in conjunction with FHWA is working on the implementation of Load and Resistance Factor Design (LRFD) to incorporate the variability of both load and resistance. Following October 1st, 2007, all new design must be LRFD (FHWA 2006a). Within three years following that deadline (October 1st 2010), all new bridges designed according to LRFD must be rated according to Load and Resistance Factor Rating (LRFR). However, current bridges could be rated with LF and in some cases AS (FHWA 2006b).
2.2.1 Allowable Stress

Allowable Stress (AS) is a design method that was presented by AASHTO in the 1930s (FHWA 2006a). Allowable Stress uses one factor of safety and is considered a deterministic method. AS is based on the premise that the resisting material behaves elastically and is isotropic and homogeneous. Allowable limits are set at the threshold beyond which the material no longer behaves elastically. This point of failure is not clear cut and not as easy to define as ultimate capacity (or failure). Also, in AS, the loads are used without factors. Loads are combined in an attempt to represent actual loading situations. The fact that different loadings have a different probability of occurrence is not recognized. Uncertainty is taken up in the factor of safety that is applied to the resistance. This factor of safety has been determined through engineering judgment, as opposed to reliability methods, and does not easily translate into a probability of failure (Barker and Puckett 1997).

The following general expression should be used in determining the AS load rating of the elements of a bridge (AASHTO 2003):

\[
RF = \frac{C - A_1 D}{A_2 L (1 + I)}
\]  

(2.1)

where:

RF = the rating factor for the liveload carrying capacity. The rating factor multiplied by the rating vehicle in tons gives the rating of the structure.

C = the capacity of the element (depends on the structural material and the rating level desired with the higher value for C used for the operating level)

D = the deadload effect on the element

L = the liveload effect on the element

I = the impact factor to be used with the liveload effect

A_1 = factor for deadload (A_1 = 1.0 for AS method)

A_2 = factor for liveload (A_2 = 1.0 for AS method)
It should be noted that the load effects employed within the formula may be those typically applied, i.e. axial force, vertical shear force, bending moment, axial stress, shear stress, and bending stresses. The form of the load effect chosen by the Engineer to use in the analysis determines the corresponding capacity of the bridge element to resist that applied load. In working with the AS method, the load effect selected is typically a form of stress.

### 2.2.2 Load Factor

The Load Factor (LF) method, introduced in the 1970’s by AASHTO, incorporates a factor for each load (live, dead, snow, etc.) (FHWA 2006a). The basic concept of LF is that provided resistance must exceed required strength. Conventional elastic procedures are used to calculate the load effects. Specific load factors are then applied to obtain the strength demand (PCI 2003). Two limit states are considered, serviceability and ultimate. This method recognizes inelastic strength as well as loss of stability. Nominal capacities are based on ultimate strength theory and are dictated by the guidelines presented in the AASHTO Standard Specifications (AASHTO 2003). Although an improvement from AS, LF falls short in that it does not properly account for the probabilistic nature of the resistance variables. Uncertainty in resistance is generally accounted for through a strength reduction factor, accounting for possibilities such as members that are undersized or lacking strength, inaccuracies in the equations. This resistance factor has been developed separately from the load factors not realistically portraying their simultaneous occurrence (BEH 1999).
The following general expression should be used in determining the LF load rating of the elements of a bridge (AASHTO 2003):

\[
RF = \frac{C - A_1D}{A_2L(1+I)}
\]  

(2.2)

where:

- \(RF\) = the rating factor for the liveload carrying capacity. The rating factor multiplied by the rating vehicle in tons gives the rating of the structure.
- \(C\) = the capacity of the member (same regardless of the rating level desired)
- \(D\) = the deadload effect on the element
- \(L\) = the liveload effect on the element
- \(I\) = the impact factor to be used with the liveload effect
- \(A_1\) = factor for deadload (\(A_1 = 1.3\))
- \(A_2\) = factor for liveload (\(A_2 = 2.17\) for inventory and \(A_2 = 1.3\) for operating)

It should be noted that the load effects employed within the formula may be those typically applied, i.e. axial force, vertical shear force, bending moment, axial stress, shear stress, and bending stresses. The form of the load effect chosen by the Engineer to use in the analysis determines the corresponding capacity of bridge element to resist that applied load.

2.2.3 Load and Resistance Factor Rating

Currently, AASHTO in conjunction with FHWA is working on the implementation of Load and Resistance Factor Design (LRFD) to incorporate the variability of both load and resistance. In LRFD, the load and resistance factors are based on probability and have been developed using reliability methods. The load factors consider the following uncertainties: magnitudes, positions, and combinations. Similarly, the resistance factors account for the following uncertainties: material properties,
strength-predicting equations, workmanship, quality control, and consequence of failure (Barker and Puckett 1997). In LRFD, the factors have been developed to satisfy the requirement that resistance exceeds load demand allowing only a very small overlap as deemed tolerable by the code writing experts (BEH 1999).

LRFD, being based on reliability methods, yields probabilities of failures directly translated from reliability indexes commonly referred to as $\beta$. The greater the value of $\beta$, the safer is the structure. This method has been calibrated with the existing design practice, LF. While the design of new structures calls for a $\beta = 3.5$, researchers have balanced economics with safety and concluded that a lesser $\beta = 2.5$ provides an acceptable level of safety for current bridge evaluation. This translates into a 6 out of 1000 chance that loading will exceed resistance during the remaining lifetime of the bridge (Jaramilla and Huo 2005).

LRFD offers uniform level of safety. Load factors are set so that all loads have the same probability of being exceeded during design life. In addition, LRFD includes site-specific load factors as well as separate factors that consider ductility, redundancy and operational importance. The live loading used in LRFD (HL-93) represents current highway loading and varying span lengths. This is an improvement from the previous design methods that used 3 separate legal load models to address short, medium and long spans. Also, LRFD is set up to be user-friendly to allow Engineers to make a smooth transition from LF to LRFD. Since the October 1st, 2007 deadline, all new design must be LRFD (FHWA 2006a). FHWA directs that this new design be rated according to the rating component in LRFD, or Load and Resistance Factor Rating (LRFR) with a
deadline of October 1st, 2010 (FHWA 2006b). LRFR shows great promise in its representation of the uncertainties of both the load and resistances.

LRFR may be applied to the following limit states: service, strength and fatigue. It should be noted that the aforementioned calibration based upon structural reliability theory applies only to the strength limit state, which addresses strength and stability. However, guidance is provided regarding the service limit state, which deals with cracking, deformations and stresses, the cracking stress being taken as the modulus of rupture (AASHTO 1998). The fatigue limit state, although presented in the LRFR code, is not included in this research. Extreme events, such as earthquake or vessel collision are beyond the methods of evaluation presented in this Code. The analytical model presented assumes a linear response, i.e. the load effect is proportional to the load applied (AASHTO 2003). On the other hand, the resistance portion of the model assumes a nonlinear response. The justification for combining linear load response and nonlinear resistance response is based on the “lower bound theorem” as stated in the Manual (AASHTO 2003):

The lower bound theorem states that for a structure that behaves in a ductile manner the collapse load computed on the basis of an assumed equilibrium moment diagram is less than or equal to the true ultimate collapse load. Restated in simpler terms, the theorem implies that as long as the requirements of ductility and equilibrium are satisfied, the exact distribution of internal force effects is not required at the strength limit state. The lower bound theorem does not apply in cases where buckling may occur prior to yielding and redistribution of force effects.

The lower bound theorem is best suited to the strength limit state. Other limits states, service or fatigue, address non-ductile failure modes and loads below the level that
require load redistribution. Also, as previously stated, the Code is not applicable to extreme events such as a hurricane or earthquake.

As presented in the LRFR Manual (AASHTO 2003), the general load rating equation applicable to each element and connection subjected to a single force effect (i.e. axial force, flexure, shear) is as follows:

\[
RF = \frac{C - (\gamma_{DC})(DC) - (\gamma_{DW})(DW)\pm(\gamma_p)(P)}{\gamma_L(LL + IM)}
\]  

(2.3)

where:

\(RF\) = Rating factor

\(C\) = Capacity

\(DC\) = dead load effect due to structural elements and attachments

\(DW\) = dead load effect due to wearing surface and utilities

\(P\) = Permanent loads other than dead loads

\(LL\) = Live load effect (1.75, 0.80; 1.35)*

\(IM\) = Dynamic load allowance

\(\gamma_{DC}\) = LRFD load factor for structural elements (1.25, 1.00)*

\(\gamma_{DW}\) = LRFD load factor for wearing surfaces and utilities (1.50, 1.00)***

\(\gamma_p\) = LRFD load factor for permanent loads other than dead loads = 1.0

\(\gamma_L\) = Evaluation live load factor

*Load factor depends on desired rating (inventory, operating) and the limit state.

**Load factor at strength limit state may be taken as 1.25 where thickness has been field measured.

The first assessment is a design load rating of bridge at its current condition. This evaluation serves as a screening process. In other words, if the bridge passes the design load, load ratings using the legal loads will also be satisfactory (AASHTO 1998).

The choice of which method to use in the rating calculations is at the discretion of the Engineer, as the LRFR methodology will not be mandated for existing bridges previously rated according to AS or LF (FHWA 2006b). However it is should be mentioned that FHWA requires that the load rating factor be reported in terms of LF or
LRFR methodology, not AS (FHWA 2006b). Typically, the code with which the bridge was designed is that which is used to allow comparison with the initial load rating. For example, Massachusetts calculates the load ratings for existing bridges in both AS, for load posting purposes, and LF, for federal reporting purposes. As long as this practice is allowed, the random nature of the loads and resistances of a bridge will not be adequately represented in the load rating. This misrepresentation is unfortunate due to the threshold nature of the load rating. Therefore, the majority of bridges will continue to be rated using methods that are lacking in the representation of the uncertainty of the load and resistance variables.

2.3 Load Rating of Prestressed Concrete Members

This research focuses on one bridge element, a prestressed concrete beam. AASHTO Specifications detail the procedure followed to calculate the inventory and operating ratings representing three codes recently used in the United States: AS, LF and LRFR). A prestressed concrete beam behaves differently under loading through time. Initially, the prestressed steel within the beam enables the entire cross-section to remain in compression. Thus cracking is less likely and the entire cross-section may be considered in calculation of the resistance to applied loads, as opposed to reinforced concrete beams that are subject to flexural cracking and once the concrete is cracked, is considered ineffective. However, even though cracking is less likely, it is still a possibility, and similar to reinforced concrete beams, once cracking occurs, the concrete in tension is considered ineffective. This presents a significant change in the behavior of
the beam and its response to loads. Because of this, it is necessary to consider both service and ultimate strength situations when designing and rating the beam.

**FIGURE 2.1.** Behavior of a Prestressed Concrete Beam

<table>
<thead>
<tr>
<th>Conditions at Service Load</th>
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<tbody>
<tr>
<td>Section</td>
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<td><img src="image" alt="Diagram" /></td>
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<table>
<thead>
<tr>
<th>Conditions at Ultimate</th>
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<tr>
<td>Section</td>
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<tr>
<td><img src="image" alt="Diagram" /></td>
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</tbody>
</table>

**LEGEND:**
- C = compression forces in concrete
- $\varepsilon_c = \text{compressive strain in concrete}$
- $\varepsilon_{py} = \text{strain in prestressing steel at yield}$
- $\varepsilon_s = \text{tensile strain in prestressing steel}$
- $f_{ps} = \text{stress in prestressing steel}$
- $f_s = \text{allowable stress in prestressing steel}$
- T = tensile forces in prestressing steel
- $T_c = \text{tensile forces in concrete}$

**FIGURE 2.1.** Behavior of a Prestressed Concrete Beam
For the service limit state, prestressed members are assumed to act as uncracked sections, bonded steel is treated as a transformed area. Strains in the concrete are assumed to vary linearly. For the strength limit state, conditions of equilibrium and strain compatibility are the basis of the calculated resistance of the members. Strain is directly proportional to the distance from the neutral axis. As ultimate capacity is approached, the rectangular stress distribution serves as a good approximation of the actual stress distribution in the compression zone. It is assumed that the curved compression diagram is replaced with a rectangular one with an average stress of $0.85f'_c$. The rectangular diagram of depth $a$ is assumed to have the same center of gravity and total magnitude as the curved diagram. This distance, $a$, is set equal to $\beta_1c$ where $\beta_1$ is a value determined by testing (McCormac 1998).

It is important that these two stages of behavior be adequately accounted for in the rating calculations. Inventory rating determines the load that may be safely carried under service conditions, and operating rating determines the maximum permissible load. Typically, in the design of a prestressed beam, Allowable Stress is used to size the beam and determine the reinforcement, while Load Factor validates the ultimate capacity. Following the design methodology, PCI directs those performing load ratings on prestressed concrete bridges to use Allowable Stress to calculate the inventory rating and Load Factor to calculate the operating rating. PCI also suggests that posting of prestressed concrete bridges be based on the inventory rating and that the issuance of truck permits be based on the operating rating (PCI 2003). As for Load and Resistance Factor Rating, AASHTO directs that Strength I and Service III load combinations are
appropriate for prestressed concrete components (AASHTO 2003). It should be noted that Service III is a check that is done at the design stage for crack control is optional. The Fatigue limit state is not normally checked as is not a concern until after cracking has begun, and most prestressed concrete members are designed to have no cracking under service loads.

2.4 Load Rating Model Description

The proposed model is probabilistic and is designed to provide prognostic and diagnostic information. The prognostic component calculates forward to obtain load rating factors of a prestressed concrete bridge beam element. The diagnostic component calculates backwards to help identify the causes of a low rating and their relative rankings. The diagnostic component is implemented with a Bayesian network integrated with results from the prognostic component, as described in Figure 2.2. The deterioration component models the response of the prestressed concrete beam element to the effects of corrosion of interior steel. The output of the deterioration component is incorporated into the input of the probabilistic prognostic component and the diagnostic Bayesian network component. The model introduced in this chapter is a prototype that represents one point in time. The model is bridge specific applied to a prestressed concrete beam.
Previous studies related to the calculation of bridge load ratings have considered probability but they do not have diagnostic capability. Sirca and Adeli offer a method of converting load ratings from Allowable Stress to Load Factor using case-based reasoning, an artificial intelligence approach (Sirca and Adeli 2005). Akgul and Frangopol (2004) investigate the relation between the reliability of a bridge and its load rating calculated using LF. While the results presented by these researchers are useful, the LRFR method is not considered. On the other hand, Bhattacharya et al. (2005) focus on the incorporation of site-specific data into the load rating procedure using the LRFR format. The comprehensive model proposed here not only has a probabilistic aspect but also allows the user to investigate the sensitivity of the various input parameters under the guidelines of all three rating methods.
2.4.1 Deterioration Component

The deterioration component models the most common form of deterioration for a prestressed concrete beam, corrosion of the interior steel due to chloride infiltration. This component serves as a catalyst for the temporal form of the model as it represents the response of the beam to this deterioration mechanism through time. The results of this component are applied as input to the probabilistic prognostic component and the diagnostic Bayesian network component.

2.4.2 Probabilistic Prognostic Component

The probabilistic prognostic component calculates the ratings using Monte Carlo simulation of the three codes: AS, LF, LRFR. Monte Carlo simulation is a method of estimation of the variability of a system that considers a combination of random variables. Each variable or parameter is generated randomly according to its specified statistics. All variables are input into the series of load rating equations simultaneously and computed several times rendering a multitude of values representing the variability of the system. From these values, standard statistical parameters, such as mean and standard deviation can be calculated. The probabilistic prognostic component has been encoded using Matlab authored by Mathworks (See Appendix A5). Matlab is an interactive and programming tool for technical computing and visualization (Hanselman and Littlefield 1997).
2.4.3 Diagnostic Bayesian Network Component

The diagnostic component consists of a Bayesian network (BN), which is a computational tool that allows reasoning under uncertainty. The graphical data structure of the BN captures the dependency relations between variables. It facilitates the combination of prior knowledge and new information, or evidence, in order to update variables. The computations supporting this capability are based on the theorem, Bayes’ Rule, hence the name *Bayesian network*.

Bayesian Networks offer a unique representation of the load rating process. The construction of the BN allows a proper representation of the inter-relationships of the random variables and the underlying rating methodology. The ability to combine prior knowledge with new information, or evidence, is directly applicable to the incorporation of data obtained from the periodic inspections throughout the life of the bridge. The diagnostic capabilities shed light on unobservable deterioration sources through observable inspection evidence. The bi-directional feature allows sensitivity ranking of the variables promoting improvements in the accuracy of the load rating process.

2.5 Model: Beam Element in a Prestressed Concrete Bridge

The model is developed based on the element that typically governs the load rating of a simply supported beam considering the critical limit state of the flexural response. Specifically, it applies to an interior prestressed concrete beam in a single span bridge.
2.5.1 Description

An AASHTO Type III prestressed concrete beam that is the interior beam of a newly built, two lane bridge carrying a road over a river is evaluated for the purpose of this example (See Figures 2.3 through 2.5). The beam is one element of a bridge with 5 simple spans, each 65 feet long. Six beams, spaced at 8’-2” on center, carry each span with an 8 inch concrete deck. The total width of the bridge is 46’-3”. The details and properties of this girder are adopted from an example problem found in the 2003 Prestressed Concrete Institute (PCI) Bridge Design Manual (PCI 2003). Typically, the rating of the beam element governs the overall load rating of the bridge. Therefore this example focuses on the beam and the critical limit state that considers the flexural strength at midspan and stress at the bottom of the beam (PCI 2003).

FIGURE 2.3. Elevation of Example Bridge (adapted from PCI 2003)
Inventory rating determines the load that may be safely carried under service conditions, and operating rating determines the maximum permissible load. In the design of a prestressed beam, Allowable Stress is used to size the beam and determine the reinforcement, while Load Factor validates the ultimate capacity. Following the design methodology, PCI directs those performing load ratings on prestressed concrete bridges to use Allowable Stress to calculate the inventory rating and Load Factor to calculate the operating rating. PCI also suggests that posting of prestressed concrete bridges be based on the inventory rating and that the issuance of truck permits be based on the operating rating (PCI 2003).

The model includes calculation of the inventory and operating ratings based on the LRFR code (AASHTO 1998, AASHTO 2003). An HL-93 live loading is considered. Two inventory ratings are calculated, one based on strength (Strength I), the other based on service conditions (Service III). AASHTO considers the calculation of the Service III case optional.

### 2.6 Alternative Modeling Methods

This section explores different methods that could be used to model a bridge and its performance. Those methods are the following: fault tree analysis, Markov decision process, structural reliability and artificial intelligence.

#### 2.6.1 Fault Tree Analysis

Fault tree analysis is one method that may be used to model a complex system for purposes of evaluation and decision-making (Bobbio et al. 2001, Vesely et al. 1981). A constructed fault tree has an event at the top that is undesirable for the system being
modeled. The causes of this top event are systematically deduced through logic gates and events that are more specifically defined than the top event. See Figure 2.6 for explanation of the symbols used in a fault tree.

![Fault Tree Diagram]

**FIGURE 2.6.** Symbols used in a Fault Tree (Ang and Tang 1984)

The fault tree may be used in a qualitative aspect as a graphical depiction of a system and its interrelating components and elements. A fault tree also has the advantage of being used in a quantitative aspect to obtain the probabilities of the failure events. Once the probabilities of the basic events are acquired, the Boolean algebra of the tree can be executed resulting in the probabilities of the intermediate events and ultimately, the failure event. Cut sets are the combinations of events, both basic and intermediate, that result in the failure event, in other words, the failure paths. Minimal cut sets are those cut sets that include the least amount of events. The probabilities of the basic events of fault trees could be elicited from experts, or calculated from existing data, or obtained from analytical reliability models.
A fault tree is a viable approach to modeling a bridge and provides a method to evaluate the performance of the system (Sianipar and Adams 1997, Johnson 1999). The fault tree describes each bridge as a system of interacting components and elements. An advantage is its ability to unveil logical interrelationships of the bridge system both visually through the layout of the tree and mathematically through the Boolean algebra. The bridge can be modeled in its entirety including element interactions, redundancy, deterioration mechanisms, such as corrosion and fatigue, and environmental factors.

Although there are many advantages of using a fault tree to model a bridge system, there are also disadvantages. The construction of a fault tree can be laborious and time consuming. The events have to be carefully chosen and worded. The relationships represented through the Boolean gates have to be well thought out. There is inflexibility associated with the modeling (Bobbio et al. 2001). The logic gates (AND, OR) are deterministic and binary. The events also are binary. They are assumed statistically independent. Also, the fault tree is not a temporal model that can be used in a predictive manner.

A BN is superior over a fault tree in modeling a bridge as a system. A network is a better modeling structure than a tree. For example, in a fault tree, if there is a common cause, that particular node must be replicated on all affected branches. This replication is not necessary with the network format (Castillo et al. 1999). Modeling with a BN is more flexible than with a fault tree. Instead of deterministic binary logic gates, BNs are able to model with probabilistic gates (e.g. noisy gates, gates with leaks). BNs easily accommodate local dependencies that are not limited to deterministic. Non-root nodes are used in the BN to model those Boolean gates in the fault tree. Also, BNs are bi-
directional (forward and backward analysis), whereas the fault tree analyzes in one direction only. BNs are a more robust modeling tool than FT. They better represent uncertainty and complex interactions within a system (Bobbio et al. 2001).

2.6.2 Markovian Decision Process

Markov decision process (MDP) is an optimization tool that allows decision-making in an uncertain environment. It has dynamic predictive capabilities that make a well-suited application to infrastructure management programs (Russell and Norvig 1995, Scherer and Glagola 1994). Pontis, the bridge management software recommended by AASHTO is licensed by most states. The main analytical engine within Pontis follows the Markov decision process (Golabi et al. 1993).

In using the discrete time Markov decision process as a modeling tool, several steps must be followed (Scherer and Glagola 1994). First, the state variables must be determined. In other words, the fundamental components of the problem must be expressed as discrete states. For example, the deterioration of the resistance of a structural member could be considered as low, medium or high (Ellis et al. 1995). Any inspection, maintenance and repair actions must be discretized similarly. It is important that the states of the system be determined in order that the model is able to harness all the information necessary to foster effective decision-making. Often this leads to “state-space explosion” which leaves the model computationally intractable. However, there are classification methods that can be used to reduce the state-space (Scherer and Glagola 1994).

Second, the Markov chains (transition matrices) must be developed. The Markov chains hold probabilities that describe the transition of each state of each variable from its
current condition to a future condition. There are several constraints involved in this step. For example, the Markovian property states that “the condition distribution of a transition from the present state to any future state, given the past state, is independent of any past states and depends only on the present state” (Scherer and Glagola 1994). Also, in using a finite-state Markov chain, the transition probabilities are stationary. With the initial probabilities of each state variable distribution determined and a one-step transition matrix generated, n-step transition matrices can be generated offering predictive descriptions of the system being modeled (Scherer and Glagola 1994).

Third, decision or control variables must be determined. The definition of these variables depends on the system being modeled and the associated maintenance or repair actions (Scherer and Glagola 1994). In the fourth step, the rewards (or costs) associated with taking actions are determined. Typically, in reference to bridges, repair or maintenance costs are established. The fifth and final step is the selection of some objective function, which in most cases regarding bridge structures is minimizing the expected costs over the service life while ensuring an acceptable level of structural reliability (Ellis et al. 1995).

There are several advantages to using the Markov decision process in optimization problems. It is probabilistic and can handle multiple random variables (Morcous et al. 2002). It is applicable to large-scale systems. It is dynamic and predictive, allowing sequential decision-making (Scherer and Glagola 1994). In its discrete time form, the Markovian model is computationally tractable and may be solved using a linear program (Ellis et al. 1995). All of these characteristics make it well suited
to model a bridge as a system throughout its service life encompassing all of the dynamic processes of deterioration, maintenance and repair.

There are also several disadvantages associated with the Markov decision process. This modeling approach has a number of constraints including the Markov property where the future condition of a bridge element, for example, depends only on its current condition and not on its past history (Morcous et al. 2002, Russell and Norvig 1995, Scherer and Glagola 1994). The involved variables (deterioration, maintenance, repair) must be well thought out and discretized in order to properly represent the system. Hierarchical or other classification methods may have to be used to avoid “state-space explosion” (Scherer and Glagola 1994). In using the discrete time form in order for the model to remain computationally tractable, the performance of the bridge through its lifetime is represented in an unrealistic manner (Ellis et al. 1995). For example, the actual process of deterioration is nonstationary, whereas the Markov decision process uses stationary transition probabilities and discrete time intervals (Morcous et al. 2002). Also, the interaction between deterioration mechanisms is not effectively modeled (Morcous et al. 2002). The Markov decision process also assumes in regard to inspections, that a condition of an element is fully observable, it does not account for uncertainty or partial observance in its basic discrete time form (Ellis et al. 1995).

In comparing Bayesian networks to a model based on Markov methods, there are some advantages that are obvious. The structure of the BN allows a compact representation of all of the variables though localized network clusters, thereby avoiding the “state space explosion” characteristic of Markov decision processes. Also, interaction between variables is better represented in BNs. Also, a BN is able to account for
uncertainty and partial observance as opposed to the discrete time form of Markov decision process. (Hahn et al. 2002) proposes that BNs can complement Markov methods by providing information that fosters efficient data collection through the BN’s ability to rank the importance of evidence.

However, the Markov decision process is well suited to sequential decision-making, which a Bayesian network is not, due to BNs representing one point in time. It should be mentioned, however, that Dynamic Bayesian Networks (DBNs) are directed graphical models that can be used as temporal models (Murphy 1998). It is an interesting coincidence that DBNs make use of hidden Markov models (HMMs). DBNs have been applied in the area of transportation to model travel routes (Gogate et al. 2006) or to simulate driving behavior (Miska et al. 2006). However, DBNs have yet to be used to model structure-related processes.

2.6.3 Reliability Methods

Reliability methods applied to structures (structural reliability) study situations where capacity exceeds demand. In bridges, capacity refers to strength and stiffness while demand refers to the types and magnitudes of loads. Structural reliability offers formalized calculations of the probability of failure in these cases. Structural reliability considers random variables to represent the bridge loads and resistances. Failure is defined through limit state equations, which may represent any level of failure from ultimate (collapse) down to disruption of normal use (Melchers 1999, Ghosn and Frangopol 1999).

The basic limit state equation includes the loads (type, magnitude) a structure experiences and the response of the structure (strength, stiffness) to those loads.
Equation 2.4 (Melchers 1999) shows the basic reliability equation where the probability of failure is the probability that the loads, $S$, are greater than the resistance, $R$, expressed through a limit state function, $G$.

$$ p_f = P(R - S \leq 0) = P(G(R, S \leq 0)) = \int_{-\infty}^{\infty} F_R(x)f_s(x)dx $$

(2.4)

where

- $p_f$ = probability of failure
- $R$ = resistance (strength, stiffness)
- $S$ = load effects (type, magnitude)
- $G$ = limit state function
- $F_R(x)$ = probability that the actual resistance $R$ is less than some value $x$ (representing failure)
- $f_s(x)$ = probability that the load effect $S$ acting in the member has a value between $x$ and $x + \Delta x$ in the limit as $\Delta x$ approaches 0.

By taking the integral over all $x$, the total probability of failure is obtained. Closed form integration is not always possible. Approximate methods are typically used such as First or Second Order Reliability Methods (FORM/SORM) or simulation techniques (Melchers 1999). FORM is a numerical technique where the failure surface is approximated by a tangent at the most likely failure point. In other words, the limit state function is linearized using first order Taylor Series Expansion about the point where the failure surface is closest to the mean of the random variables. This point is not initially known. Therefore, it is an iterative process that eventually finds the point and the minimum distance between the mean point and the failure surface. This minimum distance is the safety index from which the probability of failure is calculated (Ghosn and Frangopol 1999).
FORM has a number of variations that are used depending on whether the involved variables are Normal or non-Normal, correlated or uncorrelated. SORM is similar to FORM but improves in accuracy as the second order Taylor Series expansion is used. To consider the lifetime reliability of a structure, the reliability of each component must be computed separately. The possible failure (shear, moment) or deterioration of each component must be expressed as a limit state function. The component reliabilities are combined to model the structure as a system. For example, a bridge could be modeled as three girders in parallel. Further, considering the limit state functions over time requires complex numerical integration (Frangopol 1999).

A great amount of research effort in recent decades has been devoted to applying reliability methods to bridge structures. Reliability is the basis of many structural design codes including Load and Resistance Factor Design (LRFD). By implementing this approach, uniform or consistent reliability levels will be achieved for all bridge types. While current bridge management systems are based on condition states, researchers are confident that future bridge management systems will be based on time-dependent reliability (Kong and Frangopol 2005). Much of the current research is working towards this goal. Researchers are performing reliability assessments and generating reliability profiles for different bridge types (Val and Melchers 1996, Thoft-Christensen 1996a). Other researchers are developing optimization methods based on structural reliability that encompasses deterioration, maintenance and failure costs (Kong and Frangopol 2005).

While reliability methods show a lot of promise in their application to bridge structures, there are some associated difficulties. (Melchers 1999) finds the estimation of failure probabilities such as loss of life to be a difficult task. (Guan and Melchers 2000)
point out that the limit state in FORM is required to be somewhat linear and that might not be a realistic portrayal of the failure. Also, it may be difficult and tedious to develop a limit state function for every type of failure (shear, moment) and deterioration, etc. for every bridge element.

In making comparisons between Bayesian networks and reliability methods it becomes evident that the differences between the two make them hard to compare. Reliability methods solve limit state equations for probabilities of failure of components or elements. However, in representing a system, the modeling is simplified as basic parallel or series element sets. On the other hand, Bayesian networks represent complex systems of inter-related elements with varying degrees of dependency. However, it is necessary to obtain failure probabilities for root variables. Also, Bayesian networks possess the bi-directional feature of forward and back propagation of uncertainties, useful in the updating that results from observed evidence. Reliability methods do not have this capability. However, researchers recognize the powerful modeling tool that can be achieved through the combination of reliability methods and Bayesian networks (Mahadevan and Smith 2001, Castillo et al. 1999).

2.6.4 Artificial Intelligence

Artificial intelligence (AI) is the science that utilizes computer techniques in order to automate intelligent behavior (Morcous et al. 2002). A combination of engineering and mathematics is used to design systems that think or act rationally, like humans (Russell and Norvig 1992). Some AI methods included genetic algorithms, knowledge-based expert systems, neural networks and Bayesian networks. All of these methods
assist in decision-making. Each method has been applied to bridge structures with varying degrees of success.

Genetic algorithms surfaced in the 1960’s from experiments in machine evolution supported by the ever-increasing ability of computational power. In a genetic algorithm, an individual or group of individuals is randomly selected from a population. A reproduction operator is applied to the individual. The success of the evolvement of the individual is measured by a fitness function. The whole process is repeated a number of times until an individual is found successful according to the fitness function. This type of optimization method is generally applicable to a wide range of problems (Russell and Norvig 1995). Only recently has the research community shown interest applying it in the area of civil engineering. In (Matsuho and Shiraki 1996), a genetic algorithm is applied to traffic congestion. In (Liu and Frangopol 2006), a genetic algorithm is used to solve an optimization problem that prioritizes maintenance resources for deteriorating bridges.

The genetic algorithm finds a fit individual using simulated environment

**Function** Genetic Algorithm *(population, Fitness-Fn)* **returns** an individual

**Inputs:** population, a set of individuals

FITTNESS-Fn, a function that measures the fitness of an individual

**Repeat**

Parents – selection(population, Fitness-Fn)

Population – reproduction(parents)

**Until** some individual is fit enough

**Return** the best individual in population, according to Fitness-Fn

**FIGURE 2.7.** Genetic Algorithm (Russell and Norvig 1995)
Comparing genetic algorithms with BNs, both do not require historical data in order to be developed, as is necessary with neural networks. However, in regards to genetic algorithms, the formulation of functions that are generic and objective involves a considerable effort and resources (McCabe et al. 1998). This is not the case with BNs which may simply be developed using expert opinion.

The 1970’s brought a new probabilistic reasoning system, a knowledge-based system, also know as an expert system or a rule-based system. The expertise of a knowledge-based system is based on a large number of carefully crafted rules. These rules are obtained from elicitation of a wide range of experts. One drawback of a knowledge-based system is that the rules are not based on any general theoretical model. The designer of the model must have extensive knowledge of the subject system including its domain in order to achieve proper representation of the system and all of its uncertainty through the rules and certainty factors that incorporate a calculus of uncertainty (Russell and Norvig 1995). While knowledge-based systems have been successfully applied in the in the area of medicine, the use of this type of AI has been limited in bridge management. Denmark uses knowledge-based systems for optimal reliability-based inspection and maintenance of reinforced concrete bridges (Thoft-Christensen 1996b). (Furuta et al. 1996) has proposed the application of a knowledge-based system in steel bridge management.

One feature of the BN is that it is able to rank possible causes when used in the diagnostic manner. Knowledge-based expert systems cannot do this (McCabe and Raimondi 2000). Also, BNs are bi-directional. They can perform forward or back propagation. In a knowledge-based expert system, the rules would have to be all re-
written (McCabe and Raimondi 2000, Sahely and Bagley 2001). Evidence may be entered at any variable location in the network. This is not the case for knowledge-based expert systems where specific points are designated for this purpose (McCabe and Raimondi 2000). BNs have more modeling flexibility than knowledge-based expert systems where everything must be expressed as a rule. BNs have the ability to model rules implicitly or explicitly according to their degree of importance (McCabe et al. 1998). Bayesian networks provide a probabilistic technique that is computationally tractable. The representation of uncertainty in BNs is an improvement over that of knowledge-based expert systems where uncertainty factors combined with probability calculus are crudely applied to certain rules (Heckerman et al. 1995). Researchers have found that BNs have better performance that knowledge-based expert systems because “it had more parameters and was better tuned to the domain expert’s subjective assessments” (Pradhan et al. 1996).

According to (Russell and Norvig 1995), “a neural network is a computational model that shares some of the properties of brains: it consists of many simple units working in parallel with no central control. The connections between units have numeric weights that can be modified by the learning element.”

Within the past two decades, the neural network has been applied to bridges and other infrastructure components. (Yun and Bahng 1996) propose a neural network in order to estimate parameters of complex structural systems such as a truss bridge. (Martinelli at al. 1995) uses neural networks in combination with expert systems in application to nondestructive testing techniques of highway bridges. The neural network is used for pattern recognition and sensor data reduction. The expert system is used not
only for its knowledge base but also serves as a user interface. (Sirca and Adeli 2004) introduce a neural network that is trained to convert section properties used in Working Stress design to the properties necessary for Load Factor design. The model is applied to steel girders.

A neural network is superior to traditional computation methods where mathematical relationships are difficult to derive because it can recognize complicated patterns through its automated process of finding the polynomial that best fits data points (Martinelli et al. 1995, Morcous et al. 2002, Sirca and Adeli 2004).

There are some disadvantages associated with the use of neural networks. A large amount of data is necessary from which the neural network can be trained (Martinelli et al. 1995). Uncertainty associated with bridge deterioration is not fully represented, specifically inherent uncertainty and the uncertainty attached to unobserved deficiencies. Neural networks have an inherent parallelism that does not lend itself to proper representation of interaction between deterioration mechanisms and bridge elements. Updating the model with new inspection data is difficult (Morcous et al. 2002). Also, humans cannot construct or understand neural network representations because the calculations carried out by the network do not do so in a semantically meaningful way (Russell and Norvig 1995).

Comparing neural networks to BNs, one principle difference is that Bayesian networks are localized representations and neural networks are distributed (Russell and Norvig 1995). A Bayesian network can be and often is constructed by a human. A neural network is constructed automatically (Russell and Norvig 1995). Both can handle discrete and continuous input. Although, the BN can handle both formats at the same
time, while a neural network cannot (Russell and Norvig 1995). Neural networks have less modeling flexibility than BNs. In BNs, variables can be easily added to the model. Also, evidence may be entered in at any variable located within the network. However, with neural networks, changes such as these would require an entirely new network (McCabe and Raimondi 2000). BNs are bi-directional, neural networks are not (Sahely and Bagley 2001). Sufficient historical data is required to train the neural network. A BN may simply be developed using expert opinion (McCabe et al. 1998).

2.7 Summary

This chapter explains bridge load ratings in detail including the methodologies used: Allowable Stress (AS), Load Factor (LF) and Load and Resistance Factor Rating (LRFR). The particular application to prestressed concrete members is described. The proposed model is presented. The main computational components are explained: deterioration component, probabilistic prognostic component, and diagnostic BN component. The model is developed with a particular application to the governing element, an interior beam of a prestressed concrete bridge. The chapter ends with a review and comparison of alternative methods that were considered for developing the model. These include fault tree analysis, Markov decision process, structural reliability and artificial intelligence.
CHAPTER III
DETERIORATION COMPONENT

3.1 Introduction

This chapter introduces the deterioration component of the load rating model that considers deterioration of a prestressed concrete bridge girder. The most common form of deterioration of prestressed concrete members is corrosion of prestressing steel through chloride infiltration. The deterioration component models this corrosion process. The results of the deterioration component are applied to the random variables that serve as input to the prognostic and diagnostic components of the load rating model. As will be explained in detail in Chapter IV, the prognostic component calculates bridge load ratings probabilistically according to the methodology of the following design codes: Allowable Stress (AS), Load Factor (LF), Load and Resistance Factor Rating (LRFR). In turn as explained in Chapter V, results of the prognostic component contribute to the preparation

FIGURE 3.1. Model Layout Highlighting Deterioration Component
of the input for the diagnostic component, a Bayesian network (BN). The BN helps identify possible deficiencies that result in an undesirable bridge load rating.

3.2 Deterioration of a Prestressed Concrete Beam

Chloride contamination is considered the most threatening source of deterioration to a prestressed concrete beam (PB 1993, Richardson 2002). Therefore, it is the deterioration mechanism considered within this deterioration component. The chloride attacks are initiated through environmental exposure, such as deicing salts or salt spray of seawater. Salt laden moisture infiltrates through the porous concrete and is accelerated when in the presence of cracks and initiates corrosion of the steel, which leads to concrete delamination, cracking and finally spalling. The steps of deterioration of a prestressed concrete beam induced by chloride contamination are laid out in Figure 3.2 and further explained in the following sections.

FIGURE 3.2. Deterioration of a Prestressed Concrete Beam due to Chloride Contamination
3.2.1 Chloride Contamination

Chloride contamination is the presence of recrystallized soluble salts within the concrete. Chloride ions exist in the environment in deicing salts, marine spray and industrial pollutants. Chloride ions diffuse through concrete pores and cracks and eventually reach the steel within the concrete. In the presence of chloride ions along with water and oxygen, corrosion of steel begins (FHWA 1995a, PB 1993).

3.2.2 Steel Corrosion

Sufficient amounts of water, oxygen and chloride ions must be present for corrosion to begin, for example the moisture content must be 3.5% or higher (PB 1993). The steel is attacked and corrosion products form, such as iron oxide or rust.

3.2.3 Delamination

Delamination occurs when the layers of concrete separate, typically at the outermost layer of reinforcing steel. The corrosion product (rust) increases the volume of the corroded steel up to 10 times. This induces stress in the surrounding concrete leading to delamination (FHWA 1995a).

3.2.4 Cracking

Cracking is a vulnerable characteristic of concrete. Cracks may develop early in the fabrication stage due to thermal forces induced in the hydration process. As long as cracks such as this are limited to a width dictated in the design codes (0.004 inch (0.10 mm) for concrete exposed to seawater and 0.01 inch (0.25 mm) for concrete in moderate
climates (PB 1993)), the fabricated beam achieves quality control approval. Throughout the life of a concrete beam, cracks occur stemming from freeze-thaw cycles and traffic loads. Pre-existing cracks such as these enable the corrosion process by providing a pathway of infiltration of the chloride ions. Also, new cracks are created and old cracks are enlarged due to the forces brought on by the delamination of concrete. Cracks may be further enlarged by stresses within the beam from overloads. Large cracks eventually turn into spalls. The basic design theory of a prestressed concrete beam ensures a minimal presence of cracks through the compressive force of the prestressing steel. For this reason, the presence of cracks is more significant within a prestressed concrete beam as opposed to a reinforced concrete beam and could be indicative of structural distress.

3.2.5 Spalling

A large crack is not the only precursor of a spall. Tensile forces induced by the corrosion products and friction of thermal movement cause the separation of outer layers of delaminated concrete. The resulting spall is a depression that is circular or oval in shape and may expose reinforcing or prestressing steel (FHWA 1995a).

3.3 Description of Deterioration Component

The deterioration component models the corrosion of prestressing steel. Two stages are considered: 1) corrosion initiation and 2) corrosion propagation. The deterioration component is explained through the use of the example problem introduced in Chapter II. An AASHTO Type III prestressed concrete beam that is the interior beam of a newly built, two lane bridge carrying a road over a river is evaluated for the purpose
of this example (See Figures 2.3 through 2.5). The beam is one element of a bridge with 5 simple spans, each 65 feet long. Six beams, spaced at 8’-2” on center, carry each span with an 8 inch concrete deck. The total width of the bridge is 46’-3”. The details and properties of this girder are adopted from an example problem found in the 2003 Prestressed Concrete Institute (PCI) Bridge Design Manual (PCI 2003).

3.3.1 Corrosion Initiation

The first stage, corrosion initiation, involves the diffusion of the chloride ions through the concrete. Chloride diffusion is commonly modeled using Fick’s second law, which is a partial differential equation (Stanish et al. 1997). Crank’s solution may be applied to this partial differential equation (Crank 1975). John Crank was a 20th century mathematical physicist made famous through his numerical solutions of partial differential equations (Wikipedia 2007). Crank’s solution to Fick’s second law has long been accepted as sound engineering (Richardson 2002). The following equation gives the time required to reach the threshold (critical) level of chloride concentration \( C(x,t) \) at a distance \( x \) from the surface. This is the time at which the corrosion process will start, i.e. corrosion initiation.

\[
T_i = \frac{x^2}{4D_c} \left[ \operatorname{erf}^{-1}\left(1 - \frac{C_{cr}}{C_o}\right) \right]^2
\]

(3.1)

where

- \( T_i \) = time to reach the threshold (critical) level of chloride concentration (time, yr)
- \( x \) = distance from outer surface of the solid (length, in)
- \( D_c \) = effective diffusion coefficient (area/time, in\(^2\)/yr)
- \( C_{cr} \) = critical concentration of chloride ions (mass/volume, lb/in\(^3\))
- \( C_o \) = chloride concentration on the surface of concrete (% by weight of concrete)
This model of chloride diffusion through concrete is well known and used by many researchers (Akgul 2002, Estes 1997, Enright 1998, Thoft-Christensen et al. 1997). The major assumptions are 1) chloride intrusion occurs through diffusion only, 2) concrete cracking is not specifically modeled, and 3) diffusion is uniform around beam perimeter. The values used for $D_a$, $C_{cr}$ and $C_o$ are based on those found in literature. The distance, $x$, from the outer surface of the solid is beam specific. All possible distances of diffusion were considered for the beam of this example. Figure 3.3 shows that the shortest distance of diffusion to a prestressing strand for this example beam is 2.95 in. Related figures showing distances of diffusion are shown in Appendix B1 in addition to all details regarding this model.

**FIGURE 3.3.** Minimum Distance of Diffusion to Prestressing Strand for Example Beam
3.3.2 Corrosion Propagation

The second stage, corrosion propagation, involves the attack of the chloride ions upon the prestressing steel. The strand area is assumed to decrease through time once corrosion is initiated. The corrosion rate used in this research is estimated at 0.00225 in/yr (2.25mils/yr). This rate is based on that of conventional reinforced concrete and estimated at 75% of that of reinforced steel to account for the high mechanical strength and other metallurgical properties of prestressed strands (Akgul 2002). The corrosion rate is applied in a calculated reduction of area. Specifically, the cross-section of the 7-wire strand is approximated as a circular strand area, as shown in Figure 3.4. From there, a linear reduction of the diameter is calculated assuming corrosion uniformly around the perimeter of the bar.

![Diagram showing the approximation of a 7-wire prestressing steel strand as a circular area.](image)

**FIGURE 3.4** Approximation of 7-Wire Prestressing Steel Strand

3.3.3 Results

In the end, the deterioration component results in the average strand area through time, see Figure 3.5. The deterioration process introduced in this section and explained in detail in Appendix B1 has been coded in Matlab (Appendix B2). The program
calculates the time to reach the critical level of chloride concentration based on minimum distances of diffusion. This program also calculates the reduction in area of prestressed strands due to corrosion. As a result, the program plots the average strand area through time, see Figure 3.5. These values are used as input in the time model being applied to both the prognostic component and the diagnostic BN, as explained in Chapters IV and V.

![Figure 3.5. Average Area of Prestressing Strand through Time](image)

**FIGURE 3.5.** Average Area of Prestressing Strand through Time

### 3.4 Summary

This chapter presents a deterioration component of the load rating model of a prestressed concrete bridge girder where the primary mechanism is corrosion of the prestressed strands due to infiltration of chloride ions. The conducted research summarized within this chapter paves the way for the modeling of the beam behavior through time. In other words, the results of the deterioration component feed into the other components (probabilistic prognostic component and diagnostic BN component) as input. This enables the entire model to offer temporal results representing the flexural
response of a prestressed concrete girder to this deterioration in addition to anticipated design loads.
CHAPTER IV
PROBABILISTIC PROGNOSTIC COMPONENT

4.1 Description

The probabilistic prognostic component calculates the load ratings using Monte Carlo simulation of the three codes: AS, LF, LRFR. This component is prognostic in that it is a generative model. The input consists of random variables and the output reflects the influence of these input variables. Monte Carlo simulation is a method of estimation of the variability of a system that considers a combination of random variables. Each variable or parameter is generated randomly according to its specified statistics. All variables are input into a series of equations simultaneously and computed several times rendering a multitude of values representing the variability of the system. From these values, standard statistical parameters, such as mean and standard deviation can be calculated. The probabilistic prognostic component has been encoded using Matlab authored by Mathworks (See Appendix A5). Matlab is an interactive and programming tool for technical computing and visualization (Hanselman and Littlefield 1997).

FIGURE 4.1. Model Layout Highlighting Probabilistic Prognostic Component
4.2 Development of Component

The probabilistic prognostic component is developed by coding in Matlab the series of load rating equations. These equations follow the three design and analysis methodologies that are currently used: AS, LF, LRFR. The load rating calculations include evaluating the moments caused by deadloads and liveloads. The prestress losses are computed also in the AS methodology. In all methodologies, stresses on the composite and non-composite sections are determined due to deadloads, liveloads, and prestress forces. The nominal moment capacity is also computed for the strength methods, LF and LRFR. The resulting load rating factor in each methodology is the ratio of available live load capacity over the live load demand.

4.2.1 Monte Carlo Simulation

Monte Carlo simulation (MCS) is a tool used to estimate the variability of a system made up of multiple random variables. The flexural response of the beam element of a bridge in the load rating process is viewed as a system where uncertainties arise due to loading and resistance effects as well as those inherent within the equations. Typically, in a system considering many variables, a global distribution cannot be found in a closed form approach. Monte Carlo simulation is useful in application to the load rating process. Based on the statistical constraints prescribed for each input variable, random numbers are generated to simulate its distribution. With each generation of a set of input variables, the output variable is calculated through the equation that represents the relationship between the input variables and output variable. This is repeated for a
number of trials. An increased number of trials results in a decreased margin of error, but also an increased computer time. In specifying the number of trials or simulations, the goal is to attain a level of certainty and minimize the amount of error associated with the approximation. A reasonable sample size is such that the change in size of the mean of a resulting variable is \((3\sigma/2)/\sqrt{N}\) where \(\sigma\) is the variance of the resulting variable and \(N\) is the number of simulations (Davies and Goldsmith 1976). For example, considering 10,000 runs, the change in size of the mean of the resulting variable is on order of 0.001, an acceptable level of error.

4.2.2 Beam Element in a Prestressed Concrete Bridge

The AASHTO Type III prestressed concrete beam (introduced in Chapter II) that is the interior beam of a newly built, two lane bridge carrying a road over a river is used for the purpose of model development of the probabilistic prognostic component (See Figures 4.2 through 4.4). The beam is one element of a bridge with 5 simple spans, each 65 feet long. Six beams, spaced at 8’-2” on center, carry each span with an 8 inch concrete deck. The total width of the bridge is 46’-3”. The details and properties of this girder are adopted from an example problem found in the 2003 Prestressed Concrete Institute (PCI) Bridge Design Manual (PCI 2003). Typically, the rating of the beam element governs the overall load rating of the bridge. Therefore, the focus is on the beam and the critical limit state that considers the flexural strength at midspan and stress at the bottom of the beam (PCI 2003).
FIGURE 4.2. Elevation of Bridge (adapted from PCI 2003)

8” thick slab continuous over supports

5 Simple Spans @ 65’-0” (center-to-center of bearings)

FIGURE 4.3. Cross-section of Bridge (adapted from PCI 2003)

46’-3”

43’-6”

AASHTO Type III Beams

5 spaces @ 8’-2”
4.2.3 Procedure

AASHTO Specifications detail the procedure followed to calculate the inventory and operating ratings representing three codes currently used in the United States: AS, LF and LRFR (AASHTO 1994, AASHTO 1996, AASHTO 1998, AASHTO 2003). Inventory rating determines the load that may be safely carried under service conditions, and operating rating determines the maximum permissible load. In the design of a prestressed beam, Allowable Stress is used to size the beam and determine the reinforcement, while Load Factor validates the ultimate capacity. Following the design methodology, PCI directs those performing load ratings on prestressed concrete bridges to use Allowable Stress to calculate the inventory rating and Load Factor to calculate the operating rating. PCI also suggests that posting of prestressed concrete bridges be based on the inventory rating and that the issuance of truck permits be based on the operating rating (PCI 2003).
The probabilistic prognostic component includes calculation of the inventory and operating ratings based on the LRFR code (AASHTO 1998, AASHTO 2003). An HL-93 live loading is considered. Two inventory ratings are calculated, one based on strength (Strength I), the other based on service conditions (Service III). AASHTO considers the calculation of the Service III case optional.

The following sections further detail the model development of the probabilistic prognostic component. Two scenarios are explained in detail: an as-built case at time, t=0 years and then a temporal case. The constraints of the temporal case differ from the as-built case in that deterioration and its effects are incorporated into the component.

4.3 Input Variables, As-Built Case

Table 4.1 lists the input or root variables that are used as input. These are the basic variables representing the resistance and loading effects. It should be mentioned that the variability considered in the probabilistic prognostic component is rooted within these variables. Not all possible sources of uncertainty are assigned a variable, such as load distribution, and the approximations inherent within the analysis methods and capacity formulations. The root variables considered in prognostic component focus on the material, geometrical and loading properties an Engineer would use in design and performing the rating calculations. The statistical properties (bias (mean/nominal), coefficient of variation (COV=standard deviation/mean) and distribution) of these variables are based on those found in the literature (Mirza and MacGregor 1979, Mirza et al. 1979, Mirza et al. 1980, Naaman and Siriaksorn 1982, MacGregor et al. 1983, Hamann and Bulleit 1987, Al-Harthy and Frangopol 1994, Steinberg 1995, El-Tawil and
Okeil 2002, Gilbertson and Ahlborn 2004). Earlier researchers performed extensive work in the area of reliability of basic material properties, mostly in efforts to develop a more rational design specification based on the probabilities of loads and resistances (Mirza and MacGregor 1979, Mirza et al. 1979, Mirza et al. 1979, Naaman and Siriaksorn 1982, MacGregor et al. 1983, Hamann and Bulleit 1987). Researchers more recently have used these basic published statistics within their research specifically applied to bridges (Steinberg 1995, El-Tawil and Okeil 2002, Gilbertson and Ahlborn 2004).

### TABLE 4.1. Input Variables, As-Built Case

<table>
<thead>
<tr>
<th>Variable Description</th>
<th>Units</th>
<th>Bias</th>
<th>COV</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sectional Area of Beam (A)</td>
<td>in²</td>
<td>1.00</td>
<td>0.020</td>
<td>Normal</td>
</tr>
<tr>
<td>Prestressing Strand Area (As)</td>
<td>in²</td>
<td>1.01</td>
<td>0.013</td>
<td>Normal</td>
</tr>
<tr>
<td>Concrete Strength of Beam (f'cb)</td>
<td>psi</td>
<td>1.10</td>
<td>0.180</td>
<td>Normal</td>
</tr>
<tr>
<td>Concrete Strength of Deck (f'cd)</td>
<td>psi</td>
<td>1.00</td>
<td>0.200</td>
<td>Normal</td>
</tr>
<tr>
<td>Prestressing Strand Strength (f's)</td>
<td>ksi</td>
<td>1.04</td>
<td>0.020</td>
<td>Normal</td>
</tr>
<tr>
<td>Modulus of Elasticity of Prestressing Strands (Es)</td>
<td>ksi</td>
<td>1.01</td>
<td>0.010</td>
<td>Normal</td>
</tr>
<tr>
<td>Moment of Inertia of Beam Cross-section (I)</td>
<td>in⁴</td>
<td>1.00</td>
<td>0.030</td>
<td>Normal</td>
</tr>
<tr>
<td>Moment of Inertia of Composite Section (Ic)</td>
<td>in⁴</td>
<td>1.00</td>
<td>0.035</td>
<td>Normal</td>
</tr>
<tr>
<td>Relative Humidity (RH)</td>
<td>%</td>
<td>1.00</td>
<td>0.120</td>
<td>Normal</td>
</tr>
<tr>
<td>Unit Weight of Concrete (wc)</td>
<td>pcf</td>
<td>1.00</td>
<td>0.030</td>
<td>Normal</td>
</tr>
<tr>
<td>Wearing Surface Thickness (wst)</td>
<td>psf(in)</td>
<td>1.10</td>
<td>0.200</td>
<td>Normal</td>
</tr>
</tbody>
</table>

**Cross-sectional Area of Beam (A, in²)**

**Bias = 1.0  COV = 0.02**

The cross-sectional area of a beam is the area of the geometrical shape that lies in the plane of section cut perpendicular to the longitudinal axis of the beam. Uncertainties arise at the as-built stage due to dimensional variances that occur during fabrication. Other researchers (Al-Harthy and Frangopol 1994) have recognized this randomness and have used statistics available in the literature (Mirza and MacGregor 1979) regarding this
variable. The mean appears to be taken as the nominal value. The coefficient of variation (COV) is 0.017. Mirza’s statistics concern prestressed concrete beams in general, not as specifically applied to bridges (Mirza et al. 1979, PCI 1999). Mirza’s study reviews the effects of the following variabilities upon the ultimate flexural strength of bonded prestressed concrete beams: variabilities of the strength and geometry of concrete and steel, the location of the reinforcement and the strength model itself. The study focuses on simply supported pretensioned factory precast and post-tensioned cast-in-place concrete. Cross-sections are similar to those of the building industry however the beams of larger depths could be representative of highway bridge girders. Fifty-six beams of excellent quality of construction are considered.

Gilbertson and Ahlborn (Gilbertson and Ahlborn 2004) estimated the statistical properties of this variable using the PCI guidelines that apply to dimensional tolerances of fabrication. Two beam cross-sections were considered in their study. The estimated mean of the variable was the nominal value in one case of the small beam, and adjusted to be the mean of the estimated hi and low values in the other case of the large beam. For the larger beam the maximum and minimum values were set up roughly at a 4% difference from the mean. For the smaller beam, 3% was used, i.e. maximum value was set to 1.03*mean and minimum was set to 0.97*mean. The coefficient of variation was calculated based on the previous estimations and assumptions. For the smaller beam it was 0.018 and understandably higher for the larger beam, 0.026.

Based on these two literature sources, the statistics of the random variable, $A$, used in this study were estimated to be the nominal value for the mean and a coefficient of variation of 0.02.
**Prestressing Strand Area**  \( (A_s, \text{ in}^2) \)

**Bias** = 1.01176  \hspace{1cm} **COV** = 0.0125

The prestressing strand area variable represents the cross-sectional area of a seven wire low-relaxation strand having a diameter of ½ inch. Variability arises at this initial stage due to fabrication practices. The probabilistic studies found in the literature (Al-Harthy and Frangopol 1994, Gilbertson and Ahlborn 2004) used the statistics provided by Naaman and Siriaksorn (Naaman and Siriaksorn 1982). Naaman and Siriaksorn conducted a study on the average ranges of the reliability index considering the serviceability limit state for the precast prestressed industry in general. The goal of the research was to provide results that could be used in the setting the load and resistance factors of future codes. Sixty-four beam cross-sections of various types of prestressed concrete beams were considered in the study. Statistical properties of both resistance and loading effects are collected and used in the Monte Carlo simulation of a number of limit state equations. Gilbertson and Ahlborn (Gilbertson and Ahlborn 2004) applied these statistics from Naaman and Siriaksorn to prestressed concrete beams used in bridges. Those statistics were also used in this study. The bias is 1.01176. Therefore the mean is 0.1548 in\(^2\) for a nominal value of 0.153 in\(^2\). The coefficient of variation is 0.0125.

**Concrete Strength of Beam**  \( (f'_{cb}, \text{ psi}) \)

**Bias** = 1.10  \hspace{1cm} **COV** = 0.18

Engineers specify a minimum compressive strength of concrete to be met at 28 days. There are many factors that contribute to this being a variable including the concrete strength at the time of release, as well as environmental conditions. There are several values found in the literature for the nominal compressive strength = 5000 psi.
Gilbertson and Ahlborn (Gilbertson and Ahlborn 2004) used a coefficient of variation (COV = 0.174) taken from the literature (Ellingwood 1978). However, they estimated the mean (mean = 1.1*nominal) based on direct communication with PCI certified plant managers who claimed that the concrete strength is typically 20% higher than nominal. To be conservative, Gilbertson and Ahlborn used only a 10% increase.

MacGregor (MacGregor et al. 1983) estimated these statistics from a collection of statistical distributions obtained from literature. His study considered variability of the material properties and dimensions that stem from average quality construction. Deterioration was not accounted for in his study. The value offered for the mean was 4028 psi for a beam of 5000 psi nominal strength. This translated to a bias of 0.81. The coefficient of variation was 0.15.

El-Tawil and Okeil (El-Tawil and Okeil 2002) estimated the statistics for concrete compressive strength of prestressed girders based on those found in the literature. The sources varied from (0.81 to 1.25)*Nominal for the mean. The range for the COV was 0.09 to 0.21. Both Normal and Lognormal distributions were used. From the collected information, the authors estimated a mean of 1.1*nominal and a COV of 0.18. A Normal distribution was assumed.

Steinberg (Steinberg 1995) based these statistics on two separate references, literature and experimental results. For the strength of concrete, this study estimated the mean at 4750 psi (bias=0.95) and the coefficient of variation at 0.18 for a prestressed concrete beam having a nominal strength of 5000 psi. These statistics were based on the research of MacGregor (MacGregor 1976) and Hamann and Bulleit (Hamann and Bulleit 1987). In turn, Hamann and Bulleit (Hamann and Bulleit 1987) used the statistics
presented in (Naaman and Siriaksorn 1982). Naaman and Siriaksorn (Naaman and Siriaksorn 1982) based their statistics (MEAN = (0.67-1.17)*nominal, COV = 0.1-0.25) on literature.

Based on the above references, primarily (Gilbertson and Ahlborn 2004), the statistics for $f'_{cb}$ in this investigation were estimated with a bias of 1.10 and a coefficient of variation of 0.18.

**Concrete Strength of Deck ($f'_{cd}$, psi)**

<table>
<thead>
<tr>
<th>Bias</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Engineers also specify a minimum compressive strength for the concrete deck. In review of the statistics available in the literature including (MacGregor et al. 1983 and Mirza et al. 1979), the properties were estimated: Bias = 1.0 and COV = 0.20. A Normal distribution was assumed. MacGregor’s statistics were based on a collection of statistical distributions obtained from literature. Their study considered variability of the material properties and dimensions that stem from average quality construction. Deterioration was not accounted for in his study. Mirza’s statistics were among those sources used for MacGregor’s estimated statistics and considered previous investigations of concrete cylindrical specimens. One observance was that the COV was higher for lower strength concretes (under 4000 psi) because greater quality control was required with higher strength concrete.

**Prestressing Strand Strength ($f'_s$, ksi)**

<table>
<thead>
<tr>
<th>Bias</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.04</td>
<td>0.02</td>
</tr>
</tbody>
</table>

This variable represents the minimum ultimate tensile strength of the prestressed strands. There are two grades of strands that are generally used, Grades 250 and 270. In this investigation, the nominal value for this property is 270 ksi, reflective of the Grade
270 that is normally used in bridges. However, the statistics found in literature support that more strength is typically provided in these strands.

MacGregor’s (MacGregor et al. 1983) estimate of this statistic (MEAN = 281 ksi, COV = 0.025) is based on the results found in a static test that was included in his research with Mirza (Mirza et al. 1980). These statistics originated from material test records of Con-Force products, Edmonton, Alberta, Canada for the years 1976-1977, 200 samples, half of them are stress relieved and the other half stabilized. Two strand diameters were represented in the samples, 7/16 and ½ inch. El-Tawil and Okeil (El-Tawil and Okeil 2002) use a mean =1.04*270 ksi and a COV = 0.02 based on statistics found in the literature. Naaman and Siriaksorn (Naaman and Siriaksorn 1982) estimate this statistic (MEAN=1.0387*270, COV=0.0142) based on those statistics found in the literature. Based on these statistics, a Bias of 1.04 and a COV of 0.02 was used in this study.

**Modulus of Elasticity of Prestressing Strand (\(E_s\), ksi)**

Bias = 1.01     COV = 0.01

The modulus of elasticity is the coefficient that relates stress to strain in the elastic region of a stress-strain diagram. The modulus of elasticity represents the ability of the strand to resist deformation within the elastic range. The nominal value is 28,500 ksi. The statistics for this variable can be traced back to (Naaman and Siriaksorn 1982). These researchers did extensive study into the reliability of prestressed concrete beams.
Moment of Inertia of Beam Cross-section ($I$, in$^4$)
Bias = 1.0    COV = 0.03

The moment of inertia is a geometrical property of an area about a particular reference axis. It has a mathematical definition and cannot be visualized in the same way as for example, a centroid of a section (Fitzgerald 1982). Moment of inertia plays an important role in the calculation of flexural stresses. Uncertainty arises in this property at this initial stage due to dimensional variances of the beam cross-section that occur during fabrication.

The statistics used in this study (bias = 1.0, COV = 0.03) are based upon those estimated for moment of inertia by Gilbertson and Ahlborn (Gilbertson and Ahlborn 2004). They estimated these statistics by applying allowable tolerances to the known nominal value.

Moment of Inertia of Composite Section ($I_c$, in$^4$)
Bias = 1.0    COV = 0.035

The moment of inertia of the composite section considers not only the beam itself but the concrete deck it supports, assuming effective composite action between the two causing them to act as one unit.

The statistic used in this study is based upon the one estimated for moment of inertia by Gilbertson and Ahlborn (Gilbertson and Ahlborn 2004). They estimated these statistics by applying allowable tolerances to the known nominal value. In this study the COV has been increased slightly to account for dimensional variance related to the deck, which is poured in the field.
Relative Humidity ($RH, \%$)
Bias = 1.0    COV = 0.12

Ambient relative humidity varies day to day, season to season and by location as well. This variable influences the curing process as well as the amount of loss of prestress within the strands. In this study, the nominal value for relative humidity is 70%.

Gilbertson and Ahlborn estimated statistics (MEAN = 75%, COV=0.118) based on calculations using the tolerances set forth in the PCI Manual. Based on these statistics, a Bias of 1.00 and a COV of 0.12 was used in this study.

Unit Weight of Concrete ($w_c$, pcf)
Bias = 1.0    COV = 0.03

The unit weight of concrete is a measure of its density. It is a variable that is primarily influenced by mix proportions. Based on the statistics found in literature, a Bias of 1.0 and a COV of 0.03 was used in this study. This statistic has been used by many researchers (Steinberg 1995, Gilbertson and Ahlborn 2004) and is referenced back to (Hamann and Bulleit 1987, Naaman and Siriaksorn 1982). Hamann and Bulleit performed Monte Carlo simulations to determine the reliability index pertaining to the flexural response of under-reinforced prestressed high strength concrete beams.

Wearing Surface Thickness ($wst$, psf)
Bias = 1.1    COV = 0.20

The thickness of the wearing surface, which may also be expressed as a surface area load, is considered a variable. At the beginning of the service life of a bridge, the uncertainty in the actual thickness of the wearing surface arises as it is applied in the field due to issues of quality control. In this study, the nominal value for the wearing surface thickness is 25 psf (2 in).
El-Tawil and Okeil (El-Tawil and Okeil 2002) used a mean of 1.1*nominal and a COV=0.2. This statistic is based on those found in the literature by these researchers (bias: (1.00-1.44), COV: (0.08-53.2)). They conducted a study into the LRFD provisions for prestressed concrete bridge girders strengthened with Carbon Fiber-Reinforced Polymer Laminates. In their study, they apply this statistic to wearing surface load, as opposed to thickness. Based on these statistics, a Bias of 1.1 and a COV of 0.2 was used in this study.

4.4 Sensitivity Study, As-Built Case

A sensitivity study was conducted that tested the influence of each input variable upon the resulting inventory load rating variables. Monte Carlo simulation of the load rating equations according to the three methodologies was used. The deterministic nominal value was taken for each variable except for one in order to evaluate that particular variable’s influence upon the resulting load rating variables. The results are shown in Figures 4.5 and 4.6.

Figure 4.5 includes the results presented graphically of each of the three methodologies. The inventory load rating factor calculated deterministically is shown in the red box for AS, LF and LRFR. The bar graphs demonstrate percentage of difference between the deterministic value and the mean of the inventory load rating factor calculated as a variable considering one input variable at a time with all others held constant. For example, in reviewing the AS plot, the input variable, Prestressing Strand Strength, $f'$, appears to be the most influential upon the AS inventory load rating factor. The calculated mean exceeds the deterministic value by 7.9%. This variable is most influential in all three methodologies at this time, $t = 0$. Prestressing strand area, $A_s$, is
second most influential showing an increase of 1.8% of the mean of the inventory load rating factor calculated as a variable over the deterministic value.

Figure 4.6 shows three plots, each pertaining to one of the methodologies: AS, LF, LRFR. Each curve represents the inventory load rating variable calculated as a continuous random variable. Each colored line represents the inventory load rating variable calculated considering just one the input variable with which it is labeled as random. The black curve considers all input variables as random. The vertical line considers all input as deterministic.

In all three cases, considering the input variables as random offers a final mean value that is higher than the deterministic load rating factor. This is primarily due to the fact that the most influential variables \((A_s, f'_s)\), among others, have a bias that is greater than 1.00.
Figure 4.5. Influence of Each Input Variable upon the Inventory Load Rating Factors, As-Built Case

Legend:
A: cross-sectional area of beam
As: prestressing strand area
fcb: compressive concrete strength of beam
fcd: compressive concrete strength of deck
fs: prestressing strand strength
Es: modulus of elasticity of prestressing strands
I: moment of inertia of beam cross-sectional area
Ic: moment of inertia of composite section
RH: relative humidity
wc: unit weight of concrete
wst: wearing surface thickness
FIGURE 4.6. Influence of Input Variables upon the Inventory Load Rating Variables, As-Built Case
4.5 Correlation

Table 4.1 lists the root variables that are used as input. These root variables were taken from literature sources that applied specifically to bridges (Gilbertson and Ahlborn 2004, Steinberg 1995, El-Tawil and Okeil 2002). The three journal papers (Gilbertson and Ahlborn 2004, Steinberg 1995, El-Tawil and Okeil 2002) gathered statistical data from many sources (Al-Harthy and Frangopol 1994, Ellingwood et al. 1980, Hamann and Bulleit 1987, MacGregor 1976, MacGregor et al. 1983, Mirza et al. 1979, Mirza and MacGregor 1979, Mirza et al. 1980, Naaman and Siriaksorn 1982) and set precedent by applying them to bridges. Two of these journal papers specifically discuss correlation among the variables.

Gilbertson and Ahlborn conducted a probabilistic comparison between methods that calculate prestress losses in prestressed concrete beams (Gilbertson and Ahlborn 2004). This study uses statistical data gathered from many sources. Data presented by several references were traced back to the original source and presented. In reference to the correlation of variables, these authors performed a test to assess the effects of correlating several of the variables. The analysis showed that the coefficient of variation changed slightly but showed no significant change in the mean total prestress losses. They observed that the general trends remained. Therefore, they assumed all variables independent in their studies.

Steinberg completed a probabilistic assessment of prestress loss in prestressed concrete bridge beams (Steinberg 1995). This study uses several publications that have documented statistical information used in the analysis of prestressed or reinforced concrete members. These statistics are not specifically dedicated to prestressed concrete bridge beams and no issue is made of this. In addressing correlation between the variables, the author acknowledges source publications that have provided statistical information on the variables. He states, “these derived statistics were found by first order estimates assuming no correlation between variables (Ang and Tang 1975).” Therefore, he assumes the variables are independent.

4.5.1 Correlation Matrix

The following correlation matrix has been assumed for the input variables of the probabilistic prognostic component for the as-built case. At this point in time, variability
arises from uncertainties associated with the fabrication and construction of the beam. This correlation matrix was developed rationally. A correlation matrix offers the relations between variables in terms of correlation coefficients. Correlation coefficients range from -1.0 to +1.0. A +1.0 value for a correlation coefficient represents the perfect positive correlation between two variables. As one variable increases, the other variable increases also, both deviating from their respective means in the same way. A -1.0 correlation coefficient represents the perfect negative correlation between two variables. As one variable increases, another variable decreases, both deviating from their respective means in the same way, but opposite directions. Care was taken in estimating the value of each correlation coefficient. Common misinterpretations particular to correlation coefficients were avoided. The correlation coefficient does not represent a cause and effect relationship. A correlation coefficient near zero may represent no relation between the variables or one that is not linear. Also, the values of the correlation coefficients are not on a linear scale. In other words, a correlation coefficient of 0.8 is not indicative of a relationship that is twice as strong as one having a correlation coefficient of 0.4. A more correct interpretation is that a correlation coefficient of 0.4 explains 16% \((100*(0.4)^2)\) of the variation while the coefficient of 0.8 explains 64% \((100*(0.8)^2)\) of the variation within a sample (Bernstein and Bernstein 1999). The relationships between input variables were considered, two variables at a time. Some variables were obviously independent of one another. The following relationships were rationalized:
Cross-sectional Area of Beam ($A$) and Prestressing Strand Area, ($A_s$)

According to the sensitivity analysis, $A_s$ is influential upon the resulting inventory load ratings of all three methodologies. $A$ is not influential, at the as-built stage, at time $t = 0$ years. How are these two variables related at this point in time when the variations arise due to uncertainties in fabrication and lack of quality control as represented by the statistics obtained through literature? It could be reasoned out that the prestressing strand area being slightly smaller than nominal value would allow a greater cross-sectional area of concrete. In other words, in the fabrication process, more concrete could “fit” in the form and between the strands if the strands were slightly smaller. However, this would provide an insignificant gain in cross-sectional area. Also nullifying this reasoning, the specifications particular to each standard beam configuration provide a nominal cross-sectional area that disregards (does not subtract out) strand areas. Therefore, the correlation coefficient representing the relationship between $A$ and $A_s$ at time $t = 0$ years is assumed zero.

Cross-sectional Area of Beam ($A$) and Moment of Inertia of Beam Cross-section ($I$)

According to the sensitivity analysis, neither $A$ or $I$ are influential upon the resulting inventory load ratings considering all three methodologies. Therefore, the correlation coefficient representing the relationship between $A$ and $I$ at the as-built stage when time $t = 0$ is assumed zero.
Cross-sectional Area of Beam ($A$) and Moment of Inertia of Composite Section ($I_c$)

According to the sensitivity analysis, neither $A$ or $I_c$ are influential upon the resulting inventory load rating considering all three methodologies. Therefore, the correlation coefficient representing the relationship between $A$ and $I_c$ at the as-built stage is assumed zero.

Compressive Concrete Strength of Beam ($f'_{cb}$) and Unit Weight of Concrete ($w_c$)

According to the sensitivity analysis, $f'_{cb}$ is influential upon the resulting inventory load rating calculated using the AS methodology. Taken alone as a variable, it shows an increase of the mean value of the AS inventory load rating of 1.65% over the deterministic value. As this may be an insignificant increase, the relationship between these two variables will be rationalized.

The PCI Bridge Design Manual states the following (PCI 2003):

*The unit weight of normal weight concrete is generally in the range from 140-150 pcf. For concrete with compressive strengths in excess of 10,000 psi, the unit weight may be as high as 155 pcf. The unit weight will vary depending on the amount and density of the aggregate and the air, water and cement contents. In the design of reinforced or prestressed concrete structures, the combination of normal weight concrete and reinforcement is commonly assumed to weigh 150 pcf but may be assumed as high as 160 pcf.*

From this a positive relation between $f'_{cb}$ and $w_c$ is acknowledged and a coefficient of correlation of 0.6 is assumed. In other words, 36% ($100 \times (0.6)^2$) of the variation of these two variables within a sample of 1000 bridges of this specific make-up could be explained by their relationship. It is not a complete correlation because the other sources of uncertainties must be considered with these two variables separately having to do with the curing process, quality control, among others.
Prestressing Strand Strength ($f'_s$) and Prestressing Strand Area ($A_s$)

According to the results of the sensitivity study for the as-built case, prestressing strand strength, $f'_s$, was the most influential variable upon the resulting inventory load rating calculated in all three methodologies. $A_s$ was the second most influential variable upon the resulting inventory load rating calculated in all three methodologies. Comparing properties and design strengths shown in the PCI Bridge Design Manual, for a nominal diameter strand of $\frac{1}{2}$ inch, a Grade 270 strand shows a nominal area of 0.153 in$^2$ while a Grade 250 strand shows a nominal area of 0.144 in$^2$. Based on this, the assumption can be made that a strand having greater strength will have a slightly greater area. The correlation coefficient is set at 0.6. In other words, 36% ($100\times(0.6)^2$) of the variation of these two variables within a sample of 1000 bridges of this specific make-up could be explained by their relationship. The prestressing strand strength may have variations due to the level of pretensioning that is actually achieved during fabrication considering the type of anchorage assembly that is used and also any anchorage seating losses that may occur. Also, indentations may be added to the prestressing strands decreasing the strand area in order to increase the bond between the concrete and steel and thereby decrease the transfer and development length. These are other variations not accounted for by this correlation coefficient between $f'_s$ and $A_s$. 
Prestressing Strand Strength ($f_s'$) and Modulus of Elasticity of Prestressing Strands ($E_s$)

According to the results of the sensitivity study for the as-built case, prestressing strand strength, $f_s'$, was the most influential variable upon the resulting inventory load rating calculated in all three methodologies. Modulus of elasticity, $E_s$, when considered as a sole variable resulted in a decrease of the mean of the AS inventory load rating of less than 0.2%, which could be deemed insignificant. Prestressing strand strength specifically refers to the ultimate strand strength of nominal value, 270,000 psi in this bridge specific case, and more commonly referred to as Grade 270. The modulus of elasticity, $E_s$, is the coefficient that relates stress to strain in the elastic region of a stress-strain diagram. The modulus of elasticity represents the ability of the strand to resist deformation within the elastic range. For a seven-wire, low relaxation prestressing strand, the PCI Manual shows that $E_s$ is the same regardless of the $f_s'$, as long as it is within the elastic range, which happens to be realized for greater strains with higher strength strands (Grade 270 vs. Grade 250). So, a zero correlation is assumed.

<table>
<thead>
<tr>
<th>A</th>
<th>As</th>
<th>fcb</th>
<th>fcd</th>
<th>fs</th>
<th>Es</th>
<th>l</th>
<th>Ic</th>
<th>RH</th>
<th>wc</th>
<th>wst</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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<td>0</td>
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<td>0</td>
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<td>0</td>
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</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<td>0</td>
<td>0</td>
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<tr>
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<td>0</td>
<td>1</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**FIGURE 4.7.** Correlation Matrix of Input Variables at Time, As-Built Case
This correlation matrix was incorporated into the probabilistic prognostic component Matlab program. Normal random variables were generated that reflected the correlated relationships. Cholesky decomposition was employed as a tool in creating the matrix of the correlated input Normal random variables. A check was performed to verify the accuracy of the simulation by calculating the sample covariance matrix. This test confirmed an acceptable simulation of correlated Normal random variables for input into the probabilistic prognostic component.

Figure 4.8 compares calculated inventory rating factor variables considering correlated and uncorrelated input variables. In all three methodologies, the means and the coefficients of variations (COV) are the same. The plots themselves show a slight variation between the curves representing correlated and uncorrelated input variables, as the plots considered more significant digits pertaining to the COVs than the two digits beyond the decimal point as shown in the labels.

This plot shows that the correlation of input variables at time for the as-built case has an insignificant effect upon the resulting inventory rating variables. This conclusion confirms the same conclusion of other researchers (Gilbertson and Ahlborn 2004, Steinberg 1995) as previously discussed.
**FIGURE 4.8** Comparison of Results Considering Correlation of Input Variables, As-Built Case
4.6 Probabilistic Prognostic Component, Temporal Case

The prognostic component incorporates the results of the deterioration component in order to calculate the bridge load ratings through time. In more explanation, the prognostic component is run at time steps of ten years. The results of the deterioration component at these ten year intervals, specifically the decreasing area of prestressing steel, $A_s$, is entered as one of the input variables. All of the input variables considered in the prognostic component of the time model are listed in Table 4.2. These variables are taken from the as-built case (time $t = 0$ years) explained previously. All variables remain of Normal distribution as a simplified approximation. The coefficients of variation of certain variables are increased in order to account for the uncertainty that occurs through time. The changes to input variables are summarized and then followed by a detailed explanation.

The reasoning behind adjusting the statistics of some of the input variables is to allow for the changes that occur and the level of uncertainty that rises due to temporal effects. The cross-sectional area of the beam ($A$), is input with the same mean as the as-built case. However, the coefficient of variation (COV) is increased to allow for an increase in the possibility of spalling that typically occurs with time. The prestressing strand area ($A_s$) also is set with a higher COV reflective of the uncertainty associated with determining the area of prestressing steel that remains effective. The mean of the variable $A_s$ decreases through time and this value is obtained from the deterioration component. The compressive strength of concrete typically increases through time. However, this increase in strength is not taken for granted and Engineers are directed to use design values in their load rating analyses. This is most likely due to the uncertainty
of how much strength the concrete has gained and also to account for the possibility of unsound or deteriorated concrete. This conservative approach is taken also with the variables representing concrete strength in the beam ($f'_{cb}$) and the deck ($f'_{cd}$). The statistics of variables that pertain to the strength ($f'_s$) and elasticity ($E_s$) of the prestressing strands remain the same as in the as-built case due to the fact that studies show these variables remain fairly constant through time. The moment of inertia of both the beam cross-section ($I$) and the composite section ($I_c$) retain the same mean as the as-built case. However, the coefficient of variation increases for both, with the COV of $I_c$ having a slightly greater increase than $I$ due to the added uncertainty of the deck dimensions through time. The thresholds of both moment of inertias are adjusted to coordinate with the thresholds set for $A$ facilitating the representation of their correlation within the BN diagnostic component. The following variables are assumed to be unaffected by time: relative humidity ($RH$), unit weight of concrete ($w_c$) and wearing surface thickness ($wst$). The uncertainty associated with these variables is adequately represented in the statistics carried over from as-built case representing time at year 0.

4.7 Input Variables, Temporal Case

Using the input or root variables shown in Table 4.2 and explained in detail following, the prognostic component calculates the bridge load ratings probabilistically according to the following codes: Allowable Stress (AS), Load Factor (LF), Load and Resistance Factor Rating (LRFR). Monte Carlo simulation of the series of equations renders an array of values that approximates a distribution that represents a load rating factor as a variable.
TABLE 4.2 Input Variables, Temporal Case

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
<th>Bias</th>
<th>COV</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Cross-sectional Area of Beam</td>
<td>in²</td>
<td>1.00</td>
<td>0.10</td>
<td>Normal</td>
</tr>
<tr>
<td>Aₚ</td>
<td>Prestressing Strand Area</td>
<td>in²</td>
<td>1.01</td>
<td>0.10</td>
<td>Normal</td>
</tr>
<tr>
<td>f'ₑᵇ</td>
<td>Concrete Strength of Beam</td>
<td>psi</td>
<td>1.10</td>
<td>0.18</td>
<td>Normal</td>
</tr>
<tr>
<td>f'ₑᵈ</td>
<td>Concrete Strength of Deck</td>
<td>psi</td>
<td>1.00</td>
<td>0.20</td>
<td>Normal</td>
</tr>
<tr>
<td>fₑ</td>
<td>Prestressing Strand Strength</td>
<td>ksi</td>
<td>1.04</td>
<td>0.02</td>
<td>Normal</td>
</tr>
<tr>
<td>Eₛ</td>
<td>Modulus of Elasticity of Prestressing Strands</td>
<td>ksi</td>
<td>1.01</td>
<td>0.01</td>
<td>Normal</td>
</tr>
<tr>
<td>l</td>
<td>Moment of Inertia of Beam Cross-section</td>
<td>in⁴</td>
<td>1.00</td>
<td>0.05</td>
<td>Normal</td>
</tr>
<tr>
<td>lₑ</td>
<td>Moment of Inertia of Composite Section</td>
<td>in⁴</td>
<td>1.00</td>
<td>0.06</td>
<td>Normal</td>
</tr>
<tr>
<td>RH</td>
<td>Relative Humidity</td>
<td>%</td>
<td>1.00</td>
<td>0.12</td>
<td>Normal</td>
</tr>
<tr>
<td>wₑ</td>
<td>Unit Weight of Concrete</td>
<td>pcf</td>
<td>1.00</td>
<td>0.03</td>
<td>Normal</td>
</tr>
<tr>
<td>wst</td>
<td>Wearing Surface Thickness</td>
<td>psf(in)</td>
<td>1.10</td>
<td>0.20</td>
<td>Normal</td>
</tr>
</tbody>
</table>

Cross-sectional Area of Beam ($A$, in²)

As-Built: Bias = 1.0 COV = 0.02
Temporal: Bias = 1.0 COV = 0.10

The cross-sectional area of the beam ($A$), is input with the same mean as the as-built case. However, the coefficient of variation (COV) is increased to allow for an increase in the possibility of spalling that typically occurs with time. The cross-sectional area, $A$, of a beam decreases with time. Concrete spalls due to interior corrosion. Also, collision damage (scrapes, nicks, gouges) causes a reduction in area. This model focuses on deterioration due to chloride attack. Chloride infiltrates through the concrete and once the steel within is reached, corrosion begins. The corrosion causes the steel to increase in size and exert pressure on the surrounding concrete causing delamination, cracking and eventually spalling. Incorporating the decrease in cross-sectional area over time, it is assumed that spalling is an “after-effect” of the corrosion of interior steel.
Prestressing Strand Area ($A_s$, in$^2$)

As-Built:  Bias = 1.01176    COV = 0.0125
Temporal:   Bias = 1.01176    COV = 0.10

The prestressing strand area ($A_s$) is set with a higher COV reflective of the uncertainty associated with determining the area of prestressing steel that remains effective. The mean of the variable $A_s$ decreases through time and this value is obtained from the deterioration component. The deterioration component considers that the area of the prestressing strand decreases with time due to corrosion. Each strand has its own time of corrosion initiation, based on its proximity to the outer surface. The deterioration component assumes that chloride diffusion occurs uniformly around the perimeter of the beam cross-section. Therefore, the chloride ions reach those strands that are closer to the surface of the beam. Details of the deterioration component may be found within Chapter III and Appendix B.

Concrete Strength of Beam ($f'_{cb}$, psi)

As-Built:  Bias = 1.10    COV = 0.18
Temporal:   Bias = 1.10    COV = 0.18

Concrete compressive strength typically increases with time. Several researchers have demonstrated that compressive concrete strength in prestressed beams increases with time. (Czaderski and Motavalli 2006) conducted a study to determine the remaining tendon force of a large-scale, 38-year-old prestressed concrete bridge girder. In this study they also took cores from the concrete beam to determine the strength. They found that the concrete strength increased over time, as expected. (Rabbat 1984) conducted a study on 25-year-old prestressed concrete bridge girders and found that the concrete compressive strength was 10,100 psi, where the specifications called for a 28-day strength of 5000 psi. (Pessiki et al. 1996) performed an evaluation of effective prestress
force in a 28-year-old prestressed concrete bridge beam. They found that the average compressive strength (8440 psi) from the cores was 65% greater than the 28-day strength specified (5100 psi). However, in the LRFR Manual (AASHTO 2003), the Engineer is directed to use the values tabulated below in the event that the design compressive strength is unknown. Note that the expected increase in strength over time is not considered, obviously for reasons of conservatism.

Concrete Strength of Deck ($f'_{cd}$, psi)

| As-Built: | Bias = 1.0 | COV = 0.20 |
| Temporal: | Bias = 1.0 | COV = 0.20 |

The compressive concrete strength of deck is assumed to increase with time, as with the concrete of the beams. However, the Manual (AASHTO 2003) directs that in the rating calculations that the design value for $f'_{cd}$ be used. In the event that the plans are not available, the below tabulated values should be used. Note that these values are of a conservative nature. Only if cores are taken and the actual compressive test is measured, can the anticipated increase in compressive strength be incorporated into the rating calculations.

**TABLE 4.3 Compressive Strength of Deck Concrete**

<table>
<thead>
<tr>
<th>Year of Construction</th>
<th>Compressive Strength, ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 1959</td>
<td>2.500</td>
</tr>
<tr>
<td>≥ 1959</td>
<td>3.000</td>
</tr>
</tbody>
</table>
**Prestressing Strand Strength ($f'_s$, ksi)**
- **As-Built:** Bias = 1.04  COV = 0.02
- **Temporal:** Bias = 1.04  COV = 0.02

Recent research has shown that prestressing strand strength, or the ultimate prestressing stress, remains fairly constant through time. (Czaderski and Motavalli 2006) conducted a study to determine the remaining tendon force of a large-scale, 38-year-old prestressed concrete bridge girder. In this study they measure the remaining force in prestressing tendons by cutting wires. “Material tests on the prestressing steel showed that 38 years of constant stress had no discernable effect on its tensile strength. That is, the tensile strength is still higher than the nominal values. Furthermore, the wires even exceeded the present minimum elongation requirement a maximum load, meaning that the ductility did not decrease noticeably.” Therefore, the statistics for this variable remain as in the as-built case.

**Modulus of Elasticity of Prestressing Strand ($E_s$, ksi)**
- **As-Built:** Bias = 1.011  COV = 0.01
- **Temporal:** Bias = 1.011  COV = 0.01

Similar to the prestressing strand strength, the modulus of elasticity of the prestressing strand is also assumed to remain fairly constant. Therefore the statistics that were used for this variable in the as-built case are used in the temporal case, also.

**Moment of Inertia of Beam Cross-section ($I$, in$^4$)**
- **As-Built:** Bias = 1.0  COV = 0.03
- **Temporal:** Bias = 1.0  COV = 0.05

The moment of inertia is reflective of the cross-sectional area of the beam, $A$. In the prognostic component of the time model, the area is assumed to remain intact. In other words, the possible decrease in area due to spalling is not directly modeled through
a decreasing mean value. However, the possibility of spalling is represented through an increased COV, similar to $A$.

**Moment of Inertia of Composite Section ($I_c$, in$^4$)**
- **As-Built:** Bias = 1.0, COV = 0.035
- **Temporal:** Bias = 1.0, COV = 0.06

The moment of inertia of the composite section is reflective of the cross-sectional area of the beam, $A$. Initially in the temporal case, the area is assumed to remain intact. In other words, the possible decrease in area due to spalling is not directly modeled through a decreasing mean value. However, the possibility of spalling is represented through an increased COV, similar to $A$ and $I$.

**Relative Humidity ($RH$, %)**
- **As-Built:** Bias = 1.0, COV = 0.12
- **Temporal:** Bias = 1.0, COV = 0.12

Ambient relative humidity is assumed the same as it was in the as-built case. This variable experiences daily and seasonal fluctuations but no steady increase or decrease through the years.

**Unit Weight of Concrete ($w_c$, pcf)**
- **As-Built:** Bias = 1.0, COV = 0.03
- **Temporal:** Bias = 1.0, COV = 0.03

Unit weight of concrete, $w_c$, is considered the same in this case as in the as-built case. A considerable amount of research was completed that focused on the effect of time upon the variable, unit weight of concrete ($w_c$). In the end, it was assumed that the variable, $w_c$, is independent of time. The nominal value for $w_c$ (150 pcf) is typically used. Variations on this value enter in at the design stage and stem from quality control issues.
However, through time, as concrete hardens, although density may change, the variation in the unit weight of concrete is deemed insignificant and is overlooked as a property to be measured. The temporal case of the probabilistic prognostic component reflects this perceived lack of dependency.

**Wearing Surface Thickness (wst, psf)**

As-Built: Bias = 1.1  COV = 0.20  
Temporal: Bias = 1.1  COV = 0.20

This variable, wst, as expressed in the time as-built case, sufficiently reflects the variability it will experience over a lifetime. Therefore, for the temporal case, it will remain the same.

**4.8 Sensitivity Study, Temporal Case**

Figure 4.9 includes three plots, each representing the results of one of the three methodologies. The inventory load rating factor calculated deterministically is shown in the red box for AS, LF and LRFR. The bar plots demonstrate percentage of difference between the deterministic value and the mean of the inventory load rating factor calculated as a variable considering one input variable at a time. For example, in reviewing the AS plot, the input variable, Prestressing Strand Area (As) appears to be the most influential upon the AS inventory load rating factor. The calculated mean is less than the deterministic value by 20%. This variable is most influential in all three methodologies in the temporal model. Prestressing strand strength (f’s) is second most influential showing an average increase of approximately 7% of the mean of the inventory load rating factor calculated as a variable over the deterministic value.
Figure 4.10 shows three plots, each pertaining to one of the methodologies: AS, LF, LRFR. Each curve represents the inventory load rating variable calculated as a continuous random variable. Each colored line represents the inventory load rating variable calculated considering just one the input variable with which it is labeled as random. The black curve considers all input variables as random. The vertical line considers all input as deterministic. In all three cases, considering the input variables as random offers a final mean value that is lower than the deterministic load rating factor. This is primarily due to the fact that the most influential variable, $A_s$, is representative of the deterioration mechanism.
FIGURE 4.9. Influence of Each Input Variable upon the Inventory Load Rating Factors, Temporal Case
FIGURE 4.10. Influence of Input Variables upon the Inventory Load Rating Variables, Temporal Case
4.8.1 Correlation Matrix, Temporal Version

The following correlation matrix has been assumed for the input variables of the probabilistic prognostic component adapted as a temporal case. In other words, this model considers the correlation of the variables during Service Life. During this time, variability arises from uncertainties associated with the deterioration that affects the beam, specifically corrosion of the interior steel due to chloride infiltration. This correlation matrix was developed rationally. The following relationships were considered:

**Cross-sectional Area of Beam \((A)\) and Prestressing Strand Area \((A_s)\)**

According to the sensitivity analysis, \(A_s\) is the most influential input variable upon the resulting inventory load ratings of all three methodologies. The inventory load rating decreases approximately 19% from its deterministic value for all three methodologies when \(A_s\) is the only considered variable. \(A\) is not as influential. The AS inventory load rating increases about 0.8% when \(A\) is the only considered variable and far less for the other methodologies. How are these two variables related during the Service Life when the variations arise due to deterioration in addition to the uncertainties in fabrication and lack of quality control as represented by the statistics obtained through literature? It could be reasoned out that the prestressing strand area decreases through time due to effects of corrosion. A reduction of the cross-sectional area of the beam due to spalling is also an effect of corrosion. From this a positive relation between \(A\) and \(A_s\) is acknowledged and a coefficient of correlation of 0.8 is assumed. In other words, 64% \((100 \times (0.8)^2)\) of the variation of these two variables within a sample of 1000 bridges of
this specific make-up could be explained by their relationship. It is not a complete correlation because other sources of uncertainties must be considered with these two variables separately such as the reduction of cross-sectional area due to collision damage.

**Cross-sectional Area of Beam \((A)\) and Moment of Inertia of Beam Cross-section \((I)\)**

According to the sensitivity analysis, neither \(A\) nor \(I\) are influential upon the resulting inventory load rating considering all three methodologies. However, as explained previously, the decrease of \(A\) is correlated with the decrease of \(A_s\), due to effects of corrosion. Cross-sectional area of a beam is also correlated with its moment of inertia. Therefore, the correlation coefficient representing the relationship between \(A\) and \(I\) at the as-built stage is assumed at 0.4. A correlation of 1.0 is not assumed because although these two variables are geometric in nature, they are not linearly related.

**Cross-sectional Area of Beam \((A)\) and Moment of Inertia of Composite Section \((I_c)\)**

According to the sensitivity analysis, neither \(A\) nor \(I_c\) are influential upon the resulting inventory load rating considering all three methodologies. However, because of the assumed correlation between \(A\) and \(A_s\), a correlation is assumed between \(A\) and \(I_c\), as explained above. However, more uncertainty is considered outside of this relationship due to variability in the deck properties. Therefore, the correlation coefficient representing the relationship between \(A\) and \(I_c\) in the temporal case is assumed 0.3.
Compressive Concrete Strength of Beam ($f'_{cb}$) and Unit Weight of Concrete ($w_c$)

The temporal case assumes that the compressive concrete strength ($f'_{cb}$) and the unit weight of concrete ($w_c$) do not change through time. Therefore the correlation coefficient of 0.6 that represents their relationship in the as-built case is retained.

Prestressing Strand Strength ($f'_s$) and Prestressing Strand Area ($A_s$)

According to the results of the sensitivity study for the temporal case, prestressing strand area ($A_s$) was the most influential variable upon the resulting inventory load rating calculated in all three methodologies. Prestressing strand strength ($f'_s$) was the second most influential variable upon the resulting inventory load rating calculated in all three methodologies. While these two variables were assumed correlated at the as-built stage, that is not the situation in the temporal case. Prestressing strand area, ($A_s$), decreases primarily due to the deterioration mechanism, corrosion. However, prestressing strand strength (ultimate strand strength of nominal value, 270,000 psi in this bridge specific case) is assumed to be unaffected by the underlying deterioration mechanism. Therefore, the correlation coefficient is set at 0.25 to account for the previous relation these two variables had at the as-built stage. In other words, 6.25% ($100*(0.25)^2$) of the variation of these two variables within a sample of 1000 bridges of this specific make-up could be explained by their relationship.
Prestressing Strand Strength ($f_s'$) and Modulus of Elasticity of Prestressing Strands ($E_s$)

According to the results of the sensitivity study for the temporal case, prestressing strand strength ($f_s'$) was the second most influential variable upon the resulting inventory load rating calculated in all three methodologies. Considering $f_s'$ as the only variable, the mean of the resulting inventory load rating factor showed an increase of about 6.4% with all three methodologies. Modulus of elasticity ($E_s$), when considered as a sole variable gave a decrease of the mean of the AS inventory load rating of less than 0.2%, which is deemed insignificant. Prestressing strand strength specifically refers to the ultimate strand strength of nominal value, 270,000 psi in this bridge specific case, and more commonly referred to as Grade 270. The modulus of elasticity ($E_s$) is the coefficient that relates stress to strain in the elastic region of a stress-strain diagram. The modulus of elasticity represents the ability of the strand to resist deformation within the elastic range. While corrosion does affect the area of prestressed strands, the prestressing strand strength is taken as a property particular to the strand type that is set during fabrication. In other words, the prestressing strand strength (Grade 270) is assumed to remain the same through time and that deterioration due to corrosion is represented in the decrease in strand area. Therefore, as with the as-built case, zero correlation is assumed.
The correlation matrix was incorporated into the probabilistic prognostic component encoded in Matlab in the same manner as described previously for the as-built case representing time at \( t = 0 \) years.

Figure 4.12 compares calculated inventory rating factor variables considering correlated and uncorrelated input variables. In all three methodologies, the means are the same. The coefficients of variation for all three methodologies show a slight increase when correlated input variables are considered.

As shown in Figure 4.12, the correlation of the input variables shows no effect upon the resulting inventory rating factor variables. A thorough inspection of the coding used in probabilistic prognostic component sheds light on this situation. The program begins with the generation of correlated Normal random variables that serve as input. However, the bulk of the program manipulates the 10,000 generated values of each variable in order prepare the input for the Bayesian network. Specifically, the BN software used is limited to discrete distributions and the translation from continuous to
discrete distribution for all involved variables occurs within the probabilistic prognostic components. With the calculation of each equation within the series that makes up the load rating process, the data is grouped in such a way to represent the every possible instantiation of input variable states (lo, med, hi). This quantization serves to filter out any effects that correlation of the input variables has upon the output variables. This aspect of the probabilistic prognostic component is acknowledged. All input variables are assumed independent in the continuing research.
FIGURE 4.12. Comparison of Results Considering Correlation of Input Variables, Temporal Case
4.9 Results

Figure 4.13 shows the inventory load ratings calculated as continuous random variables through the probabilistic prognostic component for the as-built stage. The means of the inventory load rating variables have the following order from lowest to highest: LRFR, AS, LF. The AS inventory load rating factor shows the greatest range of possible values of all three methodologies. All possible values are greater than one, i.e. a passing load rating, as would be expected at the beginning of Service Life.

![Probability Distribution](image)

**FIGURE 4.13.** Inventory Load Rating Variables, As-Built Case

Figure 4.14 shows the inventory load rating variables calculated using the temporal case of the probabilistic prognostic component along with the appropriate input variables for the year 60. All three curves show ranges of possible values that are below 1.00, i.e. failed load ratings. The AS methodology shows a 53.3% probability of a failed load rating while LF shows a 19.0% probability. The variability shown within the results follow the trends shown at the as-built stage, i.e. the AS methodology has the largest
distribution. All three curves display lower rating values than those of the as-built case, as expected considering deterioration and the represented bridge age of 60 years.

![Probability Distribution](image)

**FIGURE 4.14.** Inventory Load Rating Variables, Temporal Case

4.10 Summary

This chapter focused on the probabilistic prognostic component of the model. This component calculates the load ratings using Monte Carlo simulation of the equations within the three codes: Allowable Stress (AS), Load Factor (LF) and Load and Resistance Factor Rating (LRFR). The entire process of development of the component was explained including the source of randomness of the root variables and the incorporation of judgment. The results of a sensitivity study of the input random variables were presented. An investigation was conducted into the correlation of the input variables. The results of the probabilistic prognostic component applied to the prestressed concrete bridge beam presented in Chapter II were presented and discussed.
CHAPTER V
DIAGNOSTIC BAYESIAN NETWORK

5.1 Introduction

This chapter explains in detail the diagnostic component of the model, a Bayesian network (BN).

FIGURE 5.1. Model Layout Highlighting Diagnostic Bayesian Network

5.1.1 Definition

Bayesian networks (BNs), otherwise known as belief nets, causal networks, and probabilistic dependence graphs present a graphical modeling language for representing uncertain relationships (Heckerman et al. 1995). In other words, BNs provide a way to reason under uncertainty (Reed 1993, Pradhan et al. 1996) that ultimately leads to improved decision-making.
5.1.2 History and Applications

Wright, a statistician, introduced the BN in 1921 to implement crop failure analysis (Heckerman et al. 1995). In the 1970’s it was developed further at Stanford University and in the years following, it evolved into the area of artificial intelligence as a decision support tool intended as an alternative to the rule-based systems (McCabe et al. 1998). BNs have since been applied in many areas spanning domains such as medical diagnostics, software development and civil engineering. Hahn et al. (2002) used a BN in a model that prioritized the inspection of sewers. Sahely and Bagley used a BN to diagnose upsets in anaerobic wastewater treatment (Sahely and Bagley 2001). Borsuk et al. (2003) applied a BN to water quality and developed an estuarine response model to determine the total maximum daily load for nitrogen in a specific river. McCabe et al. (1998) integrated a BN within a simulation program to form an automated method of improving simulated construction operations (McCabe et al. 1998).

5.1.3 Description

BNs are ideal for modeling systems or processes that involve a number of interrelated variables. Each BN is a network of variables that are linked together according to dependency. Figure 5.2 is an example of a BN. The circles are variables. Although BNs are capable of handling both continuous and discrete variables, discrete variables are typically used. Therefore, each variable is divided into states such as Yes and No, or Low, Medium and High, etc. In BNs, a particular familial terminology is used. Parent variables cause child variables. These dependency relations are shown through arrows between the variables called arcs.
5.1.4 Theory

BNs are called Bayesian because the theoretical calculations of inference, or model evaluation, are based on Bayes’ Rule. For example, probability of the child node shown in Figure 5.2 takes the form:

\[
p(c_k) = \sum_i^n \sum_j^m p(c_k | a_i, b_j) p(a_i) p(b_j)
\]  \hspace{1cm} (5.1)

where

- \( a_i = \) parent variable \( a \) \hspace{0.5cm} (state \( i \) of \( n \) states)
- \( b_j = \) parent variable \( b \) \hspace{0.5cm} (state \( j \) of \( m \) states)
- \( c_k = \) child variable \( c \) \hspace{0.5cm} (state \( k \) of \( n \times m \) states)

The variables and arcs of a BN form a structure that is mathematically called Directed Acyclic Graph (DAG). The term Directed is used to refer to the arrows or directed arcs between the variables. The word Acyclic is used because the arrows or arcs may not form a directed loop within the network. According to (Jensen 1996), feedback cycles are difficult to model quantitatively and no calculus has been developed that can handle these cycles. BNs not only offer a graphical representation of the interlinking variables of a process, but a numerical one as well. One of the achievements of a BN is
the manner in which the joint distribution of all the variables is modeled in the network. Nodes that are not connected represent independency between variables or an asserted conditional independence. Conditional independence is also referred to as d-separation.

D-separation is defined as follows (Jensen 1996):

Two variables A and B in a causal network are d-separated if for all paths between A and B there is an intermediate variable V such that either – the connection is serial or diverging and the state of V is known or – the connection is converging and neither V nor any of V’s descendants have received evidence.

D-separation occurs within the network when two connected nodes become direction dependent separated through a blocking or intermediate node. The probability associated with the blocking node is known with certainty rendering the d-separated nodes conditionally independent of one another. This type of connectivity allows the joint probability distribution of all the variables of the system to be represented in a compact fashion through networked clusters based on local conditional distributions, enabling greater efficiency in computation and reduced dimensionality of the problem (Heckerman et al. 1995).

There is an ongoing effort to improve this modeling technique. Facilitating the computations of BNs is a continuing process. Many methods have been developed to increase the efficiency of evaluation of the networks including variational methods, sampling methods, cutset conditioning, and parametric approximation (Murphy 1998). Computer scientists are continually developing the BN to promote its application to real-world situations.
5.1.5 Features and Capabilities

The BN is an attractive modeling tool for applications that can benefit from both its diagnostic and predictive capabilities. The BN provides a graphical data structure that captures the dependency between variables, offering the user visual aid in understanding the inner workings of the process being modeled. In addition, it functions as a model that “learns”. In other words, given data, BNs are able to learn parameters and structure of the network (Murphy 1998).

BNs are bi-directional. A BN can diagnose causes to specific problems given information about output variables or predict outcomes given information about input variables. Logic is inherently reversible within a BN due to the calculations being based on Bayes’ Rule.

BNs facilitate the combination of prior knowledge and new information. Evidence (new information) may be entered into the BN in reference to any variable (at any point in the network and at any point in time, in no particular order), whether it be input or output or within the network. Once the evaluation is completed, the updated distributions of all variables are available to the user. This includes those variables that are not observable and embedded within the network. In summary, the BN evaluation renders probabilities of events before or after introduction of evidence and updates diagnosis or prediction, offering information on all variables within the network.

5.1.6 Disadvantage

As described by (Reed 1993), a realistic BN that includes an exhaustive list of variables having linking arcs representing all levels of dependency no matter what the
significance quickly becomes an intractable combinatorial explosion. With this in mind, care must be taken in laying out the network to include the significant variables and to declare the strongest dependencies. Also, computer scientists have developed approximating algorithms to simplify the calculations.

5.2 Modeling with Bayesian Networks

5.2.1 Modeling Logistics

BNs have two aspects, qualitative and quantitative. The structure of the network is set up qualitatively. The significant variables are included and the dependencies incorporated through arcs between the variables. This descriptive presentation facilitates communication between a user and the model (Heckerman et al. 1995). The structure of the network dictates the numerical effort required for the network to evolve into a probabilistic model that offers estimates of distributions of all model variables.

The modeler has the freedom to make the BN as simple or complicated as desired. An exhaustive collection of variables may be included within the network. On the other hand, the network may be limited to a handful of the most significant variables of interest. While BNs can handle both discrete and continuous variables, discrete variables are more typical. Each variable is divided into states, typically two, such as Yes or No. However, a greater number of states may be specified for each variable raising the level of detail (and difficulty of computation) within the network. For example, in dividing a continuous variable into discrete states, thresholds must be carefully chosen. One school of thought is to use thresholds that are suited to the characteristics of the variable.
Another is to use thresholds that lead to uniform discrete distributions for all variables in hopes that this will gain more accurate results.

The degree of complexity applies in the use of the arcs, also. Numerous arcs may be used to represent all dependencies, even the slightest. Or the child may be linked to the parents that have the greatest influence upon its value. The absence of an arc between two variables indicates conditional independence. In other words, there is no direct dependency between variables considering all possible states. A simplifying assumption such as this must be carefully made, weighing all of its ramifications.

Modeling with a BN, like all modeling, is an art in defining the scope, size, and degree of complexity of the model. It is a delicate balance between simplifying the model without compromising the representation of the system or process being modeled. In determining the variables along with their interdependencies, the modeler must be familiar with the system or process being modeled. Sensitivity studies are helpful in determining which basic input variables have the greatest influence on the output variables. As a whole, the model should be validated through comparison of alternate models, experimental results or case histories.

5.2.2 Conditional Probabilities

Each network requires a large amount of probabilities in order to function numerically and render a global distribution of the system or process being modeled. The basis of each probability may differ in epistemological status (Jensen 1996), some sources being literature survey, expert elicitation, experimental results and Monte Carlo simulation. For example, Hahn et al. (2002) employs experts to obtain the conditional
probabilities in their model that prioritizes the inspection of sewers. Sahely and Bagley (2001) use simulation to supply the conditional probabilities for their BN that diagnoses upsets in anaerobic wastewater treatment.

Obtaining the necessary probabilities is a difficult task. Statistics found in the literature may not exactly represent the variable within the network. For example, a network that models a prestressed concrete beam requires statistics on the material properties of prestressed concrete. While prestressed concrete has been frequently used since the 1950s, there has not been a great deal of statistical studies done. Other probabilistic models involving prestressed concrete base estimates for these variables on the statistics of reinforced concrete. Another source of statistics, expert elicitation, has its own drawbacks. The questions must be carefully administered in order to avoid bias. Also, the answers given by the experts are subjective. Considering a third source, experiments yielding results that can be used for estimates of the statistics of the variables may be expensive and involve a great deal of work and effort to obtain only a handful of statistics. Also, Monte Carlo simulation may be used to obtain statistics. However, there must be a formula or equation that models the relationship between the variables that also must have known statistics.

The statistics, or numerical distributions, of the two types of variables, root and child, in the BN are required. Root or input variables are those having no parents. For example, a root variable having two discrete states (yes and no) has the probability of being in either state (yes - 60%, no - 40%). Note that the discrete states for each variable are considered mutually exclusive and collectively exhaustive. The numerical representation of the child variables is more complicated. Typically, a conditional
probability table is used, the size of which is determined by the number of parents of the child, as well as the number of discrete states of each variable (parents and child). For example, the conditional probability table of a child variable with three states having two parents, each with three states, contains 27 conditional probabilities that represent each instantiation of the parents along with each of the three states of the child. Defining an exceptional amount of probabilities for a network is a definite drawback of the BN. However, this disadvantage does not outweigh the advantages as a modeling tool. Also, this obstacle has been overcome with various techniques and methods of approximation.

5.2.3 Software

BNs typically possess several nodes having numerous paths. Once there exists multiple paths between variables, in other words, more than one path from variable $A$ to variable $B$, the calculations become NP-hard (i.e. it is extremely unlikely that a deterministic polynomial time algorithm is available) (Charniak 1991) and applied software is necessary to approximate a solution. There are many packages available both professionally and academically that perform these calculations. The BN evaluated using “shell” software requires specific input. A new model is created within the shell through the development of a graphical structure. Typical BN software packages automatically set up the ‘empty’ conditional probability tables (Kadie et al. 2001). The user supplies the numerical values of the discrete distributions obtained through methods described previously.
5.2.4 Running the Model

With the process being numerically represented in this way, the BN allows bi-directional calculations, i.e. prediction or diagnosis. The basis of this is the symmetry of Bayes’ Rule. In a prognostic exercise (prediction), input variables are declared to be in a specific state, the network is evaluated, and the impact upon the output variables or other variables of interest is observed. On the other hand, a diagnostic exercise sheds light upon the variables that may be the sources of an undesirable outcome. Output variables are declared to be in a specific “failure” state, the network is evaluated and the variable distributions are updated. In arriving at a diagnosis, the possible source variables are relatively compared in the amount of change that occurred between the initial distribution and the updated distribution. The variable that shows the most dramatic change initiated by the declaration of the “problem” to be diagnosed, is the most likely source.

5.3 Development of Diagnostic BN, As-Built Case

The Bayesian network shown in Figure 5.3 represents the load rating processes defined by the AS, LF and LRFR codes. Various constructs of this network were explored and found inadequate as shown in the Appendix A3. The series of load rating equations presented in the codes represent the relations between variables (See Appendix A4). Although the variables of interest in this research are primarily those representing the input design parameters and the resulting load factors, intermediate variables are included to add structure to the network. The inclusion of these variables and the connecting arcs that represent the series of load rating equations allow the model to
remain manageable. It should be made clear that this BN does not “learn” the parameters and structure from data. This BN is developed rationally as part of this research.

The Bayesian network shown in Figure 5.3 represents one point in time, the as-built case at year 0. It applies to a prestressed concrete bridge beam and evaluates the load rating process considering the critical limit state of the flexural moment at midspan, stress at the bottom of the beam. All design or input variables (shown in grey) are assumed independent based on the results of the sensitivity and correlation study explained in Chapter IV.

**FIGURE 5.3.** Bayesian Network of Load Rating Process of Prestressed Concrete Bridge Beam, As-Built Case
5.4 Input for Diagnostic BN, As-Built Case

5.4.1 Root Variables, As-Built Case

As shown in Figure 5.4, the required input of this BN is prepared in the probabilistic prognostic component. The root variables of Table 4.1 shown shaded in Figure 5.3 are discretized into Low, Medium and High states according to sensible thresholds that have been set consistent with the characteristics of each variable.

FIGURE 5.4. Model Layout Highlighting Preparation of Input for BN
Cross-sectional Area of Beam \((A, \text{ in}^2)\)

The thresholds are set at 3% above and below the mean representing maximum and minimum values of \(A\). The 3% difference is based roughly on the allowable dimensional tolerances for prestressed concrete products (PCI 1999). The thresholds (545 in\(^2\) and 575 in\(^2\)) also approximate the 10% tails of the distribution.

**FIGURE 5.5.** BN Input of Cross-sectional Area of Beam, \(A\), As-Built Case
Prestressing Strand Area \((A_s, \text{in}^2)\)

In setting the thresholds that break this continuous variable into three states of \(Lo, Med\) and \(Hi\), the areas of the neighboring strand sizes (diameters 7/16 and 9/16 inch) were initially considered. However, the distribution of the variable was not large enough, the thresholds did not fall within the range. Therefore, thresholds were arbitrarily chosen centered around the nominal value of 0.153 in\(^2\). The prestressing strand area was considered to be in the \(Lo\) state if it fell below 0.151 in\(^2\). The prestressing steel area was considered to be in the \(Hi\) state if it was greater than 0.155 in\(^2\).

FIGURE 5.6. BN Input of Prestressing Strand Area, \(A_s\), As-Built Case
Concrete Strength of Beam ($f'_{cb}$, psi)

In setting the thresholds that divide this continuous distribution into $Lo$, $Med$ and $Hi$ states, the relationship of this variable with the initial concrete strength was considered. In previous versions of the model, the initial concrete strength was considered a variable that was correlated to the concrete strength of the beam through set thresholds. In later versions of the model (including this one), the effect of the initial concrete strength was absorbed into the concrete strength variable. The thresholds for $f'_{cb}$ were retained as they were reasonable.

![Prognostic Component discretizes Input Variables for diagnostic BN based on set thresholds](image)

**FIGURE 5.7.** BN Input of Compressive Concrete Strength of Beam, $f'_{cb}$, As-Built Case
Concrete Strength of Deck ($f'_{cd}$, psi)

The nominal value for concrete strength of deck is 3400 psi. The concrete deck is poured in the field; this increases the level of randomness. Typically, the strength for a deck is specified to be 4000 psi (MHD 2005). A concrete deck would be considered high strength if it surpassed a compressive strength of 5000 psi. On the other hand, a compressive strength below 3000 psi would be considered low and possibly not capable of providing the necessary strength. Therefore, the upper and lower thresholds for this variable are 5000 and 3000 psi, respectively.

**FIGURE 5.8.** BN Input of Compressive Concrete Strength of Deck, $f'_{cd}$, As-Built Case
Prestressing Strand Strength ($f'_s$, ksi)

In setting the lower threshold, 250 ksi initially seemed reasonable, based on observation of an idealized stress-strain curve for the seven-wire low-relaxation prestressing strand (PCI 2003). However, the distribution based on the published statistics did not spread to include this lower threshold. Therefore, a lower threshold of 270 ksi was set. While this is also the nominal value for the Prestressing Strand Strength, the published statistics indicate that more strength is generally provided so that 270 ksi would in fact be a reasonable low. As for the higher threshold, 290 ksi was estimated considering similar reasoning that although 270 ksi is the nominal ultimate, published statistics suggest that more strength is provided. This also suggests that the idealized stress-strain curve levels off at a higher strength that 270 ksi.

**FIGURE 5.9.** BN Input of Prestressing Strand Strength, $f'_s$, As-Built Case
Modulus of Elasticity of Prestressing Strand ($E_s$, ksi)

The $Lo$ and $Hi$ thresholds are arbitrarily set at 28,000 and 29,000 ksi.

**FIGURE 5.10.** BN Input of Modulus of Elasticity of Prestressing Strand, $E_s$, As-Built Case
Moment of Inertia of Beam Cross-section ($I$, in$^4$)

In this investigation, the nominal value for moment of inertia is 125,390 in$^4$. The thresholds were set at the 10% tail points (120,570 in$^4$ and 130,210 in$^4$). These thresholds coordinate with the thresholds set for $A$ and $I_c$ facilitating the representation of their correlation. Correlation between these variables is assumed zero at the as-built stage, based on results from the sensitivity and correlation study. The thresholds are retained because they are sensible and also to account for correlation of these variables in other model versions.

**FIGURE 5.11.** BN Input of Moment of Inertia of Beam, $I$, As-Built Case
Moment of Inertia of Composite Section ($I_c$, in$^4$)

In this investigation, the nominal value for composite moment of inertia is 364,324 in$^4$. The thresholds were set at the 10% tail points (347,980 in$^4$ and 380,670 in$^4$). These thresholds coordinate with the thresholds set for $A$ and $I$ facilitating the representation of their correlation. Correlation between these variables is assumed zero at the as-built stage based on results from the sensitivity and correlation study. The thresholds are retained because they are sensible and also to account for correlation of these variables in other model versions.

**FIGURE 5.12.** BN Input of Moment of Inertia of Composite Section, $I_c$, As-Built Case
Relative Humidity ($RH$, %)

In determining the thresholds for relative humidity, a map of the United States showing the annual ambient relative humidity was reviewed (PCI 2003). Relative humidity was as low as 30% in the southwest desert region and as high as 80% in some coastal regions. Assuming that this example bridge is in New England, the $Lo$ and $Hi$ thresholds were set at 60% and 80% relative humidity.

![Diagram showing the distribution of relative humidity and threshold levels](image)

**FIGURE 5.13.** BN Input of Relative Humidity, $RH$, As-Built Case
Unit Weight of Concrete ($w_c$, pcf)

As for setting the upper and lower thresholds, according to PCI Bridge Design Manual (PCI 2003), normal weight concrete is generally in the range of 140 to 150 pcf. Although concrete with high strength, greater than 10,000 psi, may have unit weight as high as 155 pcf. The varying weight is subject to the amount and density of aggregate, air, water and cement content. Therefore, the lower threshold is estimated at 145 pcf and the upper threshold is estimated at 155 pcf.

**FIGURE 5.14.** BN Input of Unit Weight of Concrete, $w_c$, As-Built Case
Wearing Surface Thickness \( (wst, \text{ psf}) \)

In setting the thresholds, it was noted that wearing surface thickness tends to be greater than set forth in design, as reflected by the bias found in literature. Therefore, the thresholds were set at 1.5 in. (18.75 psf) for Lo and 3 in. (37.5 psf) for Hi.

FIGURE 5.15. BN Input of Wearing Surface Thickness, \( wst \), As-Built Case
5.4.2 Conditional Probabilities

For the time as-built case, the prognostic component calculates the conditional probabilities within the Monte Carlo simulations. Monte Carlo simulation may be applied in the obtaining of necessary input of BNs. If the relationship between the child and the parents can be expressed as a mathematical formula, Monte Carlo simulation of the equation using the statistics of the parent variables may be used to obtain the conditional probabilities of the child variable. Based on the statistical constraints prescribed for each parent variable, random numbers are generated to simulate its distribution. With each generation of a set of parent variables, the child variable is calculated through the equation that represents the relationship between the parents that yields the child.

Considering further the 10,000 trials, there would be 10,000 resulting values of the child variable, generating an array of data from which a mean and standard deviation may be calculated. A histogram may also be generated and then a statistical distribution fitted to approximate a continuous distribution. In order to apply these results to a BN, this so called continuous distribution must be transformed into a discrete distribution by setting thresholds that divide. Setting the thresholds of each variable requires consideration of its characteristics. For example, a design variable describing the compressive strength of concrete should have thresholds that consider the stress strain curve of concrete and the strength at the point of rupture. In other words, significant thresholds unique to each variable may be obvious when its characteristics are considered, as explained in detail in the previous section.
As mentioned previously in Section 5.2.3, BNs are typically evaluated within a shell software that requires specific input. A new model is created within the shell through the development of a graphical structure as in Figure 5.3. MSBNx automatically sets up the conditional probability tables (Kadie et al. 2001). The user supplies the numerical values of the discrete distributions that, in this case, have been previously calculated by the prognostic component.

As an explanatory example, the following details the procedure used to fill a conditional probabilities table using Monte Carlo simulation. As previously stated, in order to use Monte Carlo simulation, the relationship between the parent variables must be expressed as an equation. In reference to Figure 5.16, the causal relationship is adopted from basic mechanics of materials theory:

\[ M_b = \frac{Aw_cL^2}{8} \]  

(5.2)

where

- \( M_b \) = beam moment
- \( A \) = cross-sectional area of beam
- \( w_c \) = weight of concrete
- \( L \) = length of beam

As shown in Figure 5.16, for two parents each defined by three states, and the child also defined by three states, twenty-seven conditional probabilities are required. (Note the conditional probability table in Fig. 5.16 is intentionally left blank to demonstrate how MSBNx sets up the empty table for the user to fill.) For example, if the cross-sectional area of the beam is low and the weight of concrete is low, what is the probability that the beam moment is low? Within the Monte Carlo simulations, the conditional probabilities are calculated. This requires an understanding of the
characteristics of the child variable in order to set appropriate thresholds that determine the low, medium and high states.

**FIGURE 5.16.** Conditional Probability Tables

### 5.5 Output for Diagnostic BN, As-Built Case

This version of the diagnostic BN of the bridge load rating process was applied to the example previously used within this research and repeated here for informational purposes. An AASHTO Type III prestressed concrete beam that is the interior beam of a newly built, two lane bridge carrying a road over a river is evaluated for the purpose of this example (See Figures 4.2 through 4.4). The beam is one element of a bridge with 5 simple spans, each 65 feet long. Six beams, spaced at 8’-2” on center, carry each span with an 8 inch concrete deck. The total width of the bridge is 46’-3”. The details and properties of this girder are adopted from an example problem found in the 2003 Prestressed Concrete Institute (PCI) Bridge Design Manual (PCI 2003).
5.5.1 Diagnostic Exercise of Diagnostic BN, As-Built Case

Three diagnostic investigations were conducted where the inventory load rating factor was declared to be less than one in each of the three methodologies (AS, LF, LRFR). No results were obtained for the LF investigation because the probability of failure was zero. In other words, at the beginning of service life, the probability of the load rating factor calculated according to the LF methodology being less than one was zero. This is due to several factors: 1) the beam is new, 2) the bias of the input variables renders a load factor with greater magnitude, 3) the calculations inherent in the LF methodology calculate a load rating factor greater than the other methodologies. In reviewing the results, comparisons were made between assumed distributions and updated distributions of the input parameters. The parameter showing the most dramatic change is the most likely source of the problem. In the two investigations pertaining to the other methodologies (AS, LRFR), the prestressing strand area (\(A_s\)) showed the most significant change in distribution, specifically pointing to a smaller size of strand than originally assumed as the root of the failed load rating (See Figure 5.17). Some other sources of a failed inventory load rating may be that the strength of the prestressed strands may be too low or the compressive concrete strength of the beam may not be high enough due to faulty fabrication. Figure 5.17 shows the assumed distributions of each of the input parameters in the left column. The second column shows the updated distributions of those input variables if the AS load rating factor is declared less than one. The third column shows no results for the updated LF load rating factor as explained above. The fourth column shows the updated distributions of those input variables if the LRFR load rating factor is declared less than one.
<table>
<thead>
<tr>
<th>Assumed Distribution</th>
<th>Updated Distribution</th>
<th>Inventory Rating &lt; 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AS</td>
<td>LF</td>
</tr>
<tr>
<td>p(A)</td>
<td>0.80</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>0.10  0.10</td>
<td>0.04  0.22</td>
</tr>
<tr>
<td>p(A_b)</td>
<td>0.51  0.46</td>
<td>0.64  0.30</td>
</tr>
<tr>
<td></td>
<td>0.03  0.06  0.15</td>
<td>0.06  0.30  0.46</td>
</tr>
<tr>
<td>p(f'_{cb})</td>
<td>0.73</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>0.09  0.18</td>
<td>0.09  0.01</td>
</tr>
<tr>
<td>p(f'_{cd})</td>
<td>0.28</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>0.28  0.01  0.02</td>
<td>0.27  0.01  0.02</td>
</tr>
<tr>
<td>p(f'_{s})</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>0.03  0.05</td>
<td>0.07  0.01</td>
</tr>
<tr>
<td>p(w_{c})</td>
<td>0.74</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>0.13  0.13</td>
<td>0.03  0.31</td>
</tr>
<tr>
<td>p(wst)</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>0.06  0.04</td>
<td>0.02  0.08</td>
</tr>
</tbody>
</table>

FIGURE 5.17. Comparison Between Assumed and Updated Distributions of Input Variables
The results were further dissected to make relative comparisons between the three methodologies. Specifically, the results pertaining to the variables that represent the randomness of the beam properties were compared regarding their degrees of influence over the final load rating factor (See Figure 5.17). Cross-sectional area ($A$) had little influence over the failed load rating factor. Both the AS and LRFR methodologies indicated that the cross-sectional area may have been larger than assumed. Prestressing steel area ($A_s$) showed a shift in distribution for both AS and LRFR, showing a decrease in the probability that $A_s$ is higher than assumed and an increase in the probability that $A_s$ is smaller than assumed.

Strength of beam concrete ($f'_{cb}$) is again most influential on a failed load rating when it comes to AS as opposed to LRFR. This is due to the fact that $f'_{cb}$ does not enter into the calculation for the LRFR inventory load rating. The updated AS distribution indicates that the strength of beam concrete may not be as high as originally assumed. Likewise, $f'_{cd}$ does not enter into the calculation for the AS inventory load rating and the updated distribution of strength of deck concrete ($f'_{cd}$) show no change from the assumed distribution. For the LRFR method, the probability that the strength of concrete is low increases.

Ultimate prestressing stress ($f'_s$) shows slight sensitivity considering the AS methodology, the updated distribution showing a decrease in the probability that $f'_s$ is higher than anticipated. This input parameter appears to have more of an influence over a failed LRFR inventory rating in a more significant shift that shows an increase in probability that the prestressing strand strength is lower than anticipated. Unit weight of concrete ($w_c$) has an effect on the inventory load rating factor in both methodologies.
The trend in all three suggests that a load rating factor less than one may have the contributing cause of a unit weight of concrete being higher than assumed. Regarding wearing surface thickness \((w_{st})\), the updated distributions pertaining to all three methods show the same trend, that the wearing surface thickness is most likely greater than assumed. The LRFR method revealed the greatest sensitivity of this parameter. This is understandable given that wearing surface thickness is applied with its own dedicated load factor in the LRFR calculations.

Comparisons were made between the updated and the assumed distributions of seven input parameters given a failed inventory load rating. The increase in probability of the occurrence of the problematic states of each variable was extracted and plotted for observation (See Figure 5.18). Considering AS, all input parameters show some sensitivity, except cross-sectional area of the beam \((A)\) and concrete strength of deck \((f'_{cd})\). Cross-sectional area of the beam \((A)\) actually showed a shift in the distribution that it was larger than assumed, contributing to a greater deadload moment and ultimately a lesser inventory rating factor. Concrete strength of deck \((f'_{cd})\) is not included within the calculations for AS inventory rating. However, the deficient states of the other variables show possible contribution to the failed load rating. Concrete strength of beam \((f'_{cb})\) appears to have the greatest influence.

The results using the LRFR method show a greater sensitivity than the AS method considering the deficient state of each variable. Like AS, the cross-sectional area \((A)\) of the beam shows an increase in probability that the size is greater than assumed. Concrete strength of beam \((f'_{cb})\) is not included within the calculations for LRFR inventory rating.
The temporal case of the diagnostic BN models the response of the prestressed concrete beam through time with the incorporation of deterioration. Corrosion of prestressing steel through chloride infiltration is the most common form of deterioration of prestressed concrete members, as discussed previously. This section develops the diagnostic BN to include variables and their inter-relations in order to adequately represent this response of a prestressed concrete bridge beam through time.

The load rating process includes an inspection of the bridge to perform a general assessment of its physical condition and the effects of deterioration. The type of inspection could range from a basic visual assessment to destructive material testing in order to determine necessary current information. Among other things, the inspector needs to collect member condition data and note material losses due to deterioration. A prestressed concrete beam exhibits particular symptoms of deterioration that may be seen by the inspector. The following section details these symptoms and the manner in which the diagnostic BN is developed with the inclusion of inspection variables.
The majority of inspections performed are primarily visual. FHWA’s Nondestructive Evaluation Validation Center (NDEVC) conducted a study into the reliability of these visual inspections. Among the findings were that routine inspections were completed with significant variability in assignment of condition ratings, inspection documentation, etc. (Moore et al. 2001).

It is well known within the bridge community that inspections are inconsistent and subjective. Improvements to this process have been initiated such as bridge training courses, a more detailed coding guide for condition rating and introducing new inspection techniques such as nondestructive testing (NDT). The diagnostic BN introduced here accounts for the randomness associated with an inspection and includes traditional forms of inspection also considering nondestructive testing. It is not a replacement of the human inspector, although it can give results without inspector input. It actually allows the inspector to observe and test as usual and then transforms the inspection data into an instantaneous assessment of the load carrying capacity. It also has the capability of leading a novice inspector through the visit by ranking the symptoms that should be observed first in order to get a clear diagnosis of deterioration.

The inspection of a prestressed concrete beam includes observances of its alignment, liveload effects, approaches and condition of drainage system (joints and scuppers). Also, dimensions shown on plans should be verified and the actual pavement thickness should be measured. Deterioration should be noted such as cracking, spalling, damage to prestressed steel and any other visible damage (AASHTO 2003).

Chloride contamination is the source of deterioration considered within the deterioration component of this model (See Chapter III). Through environmental
exposure, such as deicing salts or salt spray of seawater, the chloride attack begins. Salt laden moisture infiltrates through the porous concrete and is accelerated when in the presence of cracks and initiates corrosion of the steel that leads to concrete delamination, cracking and finally spalling. Figure 3.2 is repeated here as Figure 5.19 for informational purposes.

![Diagram of deterioration process](image)

**FIGURE 5.19.** Deterioration of a Prestressed Concrete Beam due to Chloride Contamination

The expanded diagnostic BN to consider deterioration over time is shown in Figure 5.20. The inspection cluster shown within the red dashed oval shows variables that represent deterioration mechanisms, observable symptoms of deterioration, as well as the design variables that are affected by the deterioration. This inspection cluster was developed based on the deterioration process shown in Figure 5.19. The root variables and conditional probability tables were estimated based on expert opinion and elicitation. For example, given an environment in *severe* state (marine salt exposure, deicing salts), the probability of chloride contamination is estimated at 80%. Details of the BN expansion as well as the input variables are in the following sections.
5.6.1 Chloride Contamination

Chloride contamination is the presence of recrystalized soluble salts within the concrete. Chloride ions exist in the environment in deicing salts, marine spray and industrial pollutants. Chloride ions diffuse through concrete pores and cracks, eventually reaching the steel within the concrete. In the presence of chloride ions along with water and oxygen, corrosion of steel begins (FHWA 1995a, PB 1993).
Evidence of chloride contamination is visible to a bridge inspector by efflorescence or dirty white surface deposits. Efflorescence is a combination of calcium carbonate leached out of the cement paste (FHWA 1995a). Chloride Contamination (CC) is added to the BN as a deterioration mechanism variable. It is assigned two states, *yes* and *no*. It is the parent variable of deterioration symptom variable, Efflorescence (*Eff*), which is also assigned the two states, *yes* and *no*. The decision to consider Efflorescence a binary variable as opposed to one having multiple states was in keeping with the initial attempt to simplify the modeling of the inspection cluster. The Condition Probability Table (CPT) is shown in Figure 5.21. The probabilities shown are estimated based on expert opinion and elicitation. If there is no chloride contamination, there is no efflorescence. On the other hand, the process of chloride contamination may have begun without the formation of efflorescence. Also, the inspector may not observe the efflorescence. A 30% probability accounts for these uncertainties among others.

Chloride Contamination (CC) is a child of the variable, Environment (*Env*). The likelihood of chloride contamination is influenced by the surrounding environment and exposure to marine spray, deicing salts, etc. The Environment variable is defined in three states: *mild*, *moderate* and *severe*. For example, a bridge in the severe state may cross a body of salt water or may carry a high volume of traffic necessitating the use of deicing salts. The Condition Probability Table (CPT) is shown in Figure 5.21. The probabilities shown are estimated and demonstrate that the probability of chloride contamination increases with the environmental exposure to chlorides.
5.6.2 Steel Corrosion

In the presence of sufficient amounts of water, oxygen and chloride ions, corrosion begins. The steel is attacked and corrosion products form, such as iron oxide or rust. The inspector may observe the presence of rust stains, indicating steel corrosion. Figure 5.22 shows the additions to the BN to model corrosion. Corrosion (Corr) is a deterioration mechanism variable having two states, yes or no. This variable is parent to Rust (Rst), the deterioration symptom variable having two states, yes or no. The CPT is shown in Figure 5.22. If there is no corrosion, there are no rust stains. However, if corrosion of a strand is present, the corrosion products may have not yet manifested as rust stains. Also, the inspector may overlook the rust stains. The probabilities shown allow for these uncertainties. Chloride Contamination (CC) is a parent variable to Corrosion, as this version of the model focuses on chloride ions as the source of deterioration. If there is no chloride contamination, the model allows a 2% chance of...
corrosion from an alternate source such as sulfates. If there is chloride contamination, the infiltration may not have reached the steel initiating the corrosion. The estimated probabilities shown in Figure 5.22 reflect these uncertainties.

<table>
<thead>
<tr>
<th>Corrosion, Corr</th>
<th>Rust, Rst</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>0.70</td>
</tr>
<tr>
<td>No</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**FIGURE 5.22.** Additions to BN Modeling Corrosion

Corrosion in this BN may be directly linked to the area of the prestressed strands, $A_s$. The deterioration component assumes that the effects of corrosion cause a decrease in the area of the strands. The probabilistic prognostic component originally has $A_s$ with a normal distribution having a mean of 0.1548 in$^2$, the nominal being 0.153 in$^2$. The thresholds are set at 0.125 in$^2$ and 0.140 in$^2$, representing the loss of 4 and 2 strands, respectively. With these thresholds, the distribution shows an 83% probability that the area of the average prestressed strand is greater than 0.140 in$^2$. However, through time, corrosion is initiated (here, after 30 years) and propagated. The distribution shifts and shows an increasing probability that the area of the average strand is less than 0.125 in$^2$, which in this model represents a loss of about 4 strands. Figure 5.23 shows the CPT.
representative of probabilities of prestressed strand areas that would occur at 50 years. This diagnostic BN represents deterioration of the strand due to chloride contamination through the decrease in area of prestressed strands through time. Specifically, the temporal case is time-stepped every 10 years having a BN with applicable probabilities pertaining to that point in time.

<table>
<thead>
<tr>
<th>Time, $t$</th>
<th>Corrosion, $\text{Corr}$</th>
<th>Area of Prestressed Strands, $A_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 years</td>
<td>$\text{yes}$</td>
<td>4 strand loss</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>$\text{no}$</td>
<td>0.03</td>
</tr>
</tbody>
</table>

FIGURE 5.23. Linking Corrosion to Area of Prestressed Strands

5.6.3 Delamination

Delamination occurs when the layers of concrete separate, typically at the outermost layer of reinforcing steel. The corrosion product (rust) increases the volume of the corroded reinforcing steel up to 10 times. This induces stress in the surrounding concrete leading to delamination (FHWA 1995a).

Detection of delamination is not possible in a visual inspection. Usually, when the inspection involves nondestructive testing, hammer sounding is used to find delaminated areas that give off a hollow sound when tapped with a hammer. By incorporating this variable into the model, the possible scope of the inspection is
increased considering symptom variables detectable by methods that go beyond visual. For this reason, the delamination variable shown in Figure 5.24 is a darker shade of blue than the other symptom variables observed through visual inspection. The CPT shows that if no corrosion is present, there is no delamination. If corrosion is present, there is a chance that delamination has not yet occurred. There is also uncertainty involved in the inspector’s ability to identify the hollow sound associated with the delamination.

<table>
<thead>
<tr>
<th>Corrosion, Corr</th>
<th>Delamination, Del</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

**FIGURE 5.24.** Additions to BN Modeling Delamination

### 5.6.4 Cracking

Cracking is a vulnerability characteristic of concrete. Cracks may develop early on in the fabrication stage due to thermal forces induced in the hydration process. As long as cracks such as this are limited to a width dictated in the design codes (0.004 in (0.10 mm) for concrete exposed to seawater and 0.01 inch (0.25 mm) for concrete in moderate climates (PB 1993)), the fabricated beams are granted quality control approval. Throughout the life of a concrete beam, cracks occur stemmin from freeze-thaw cycles and traffic loads. Pre-existing cracks such as these enable the corrosion process by providing a pathway of infiltration of the chloride ions. New cracks are created and old
cracks are enlarged due to the forces brought on by the delamination of concrete. Cracks may be further enlarged by stresses within the beam from overloads. Large cracks eventually turn into spalls.

The basic design theory of prestressed concrete beams ensures a minimal presence of cracks through the compressive force of the prestressing steel. For this reason, the presence of cracks is more significant within a prestressed concrete beam as opposed to a reinforced concrete beam and could be indicative of structural distress.

During inspection, the length, width, location, and orientation (horizontal, vertical, diagonal) of cracks should be noted. This particular example problem focuses on the critical area typical of a simply-supported single span prestressed concrete beam, the bottom surface of the bottom flange at the centerspan. Here the beam is subjected to tensile flexural stresses. All cracks 1/16 inch (0.0625 in.) (1.56 mm) should be recorded for future comparison (FHWA 1995a). Cracks such as these are indicative of corrosion in a tendon. Figure 5.25 shows the additions to the BN to account for corrosion-induced cracking. The probabilities shown allow a minor chance of a large crack that pre-exists the products of corrosion. If corrosion does exist, the probabilities of a large crack is 65% (as opposed to 100%) to account for the possibility that corrosion has not progressed to the stage that cracking occurs and also, any uncertainties associated with the inspector’s observances.
5.6.5 Spalling

A large crack is not the only precursor of a spall. Tensile forces induced by the corrosion products and friction of thermal movement cause the separation of outer layers of delaminated concrete. The resulting spall is a depression that is circular or oval in shape and may expose reinforcing or prestressing steel (PB 1993). In inspection, the location and size of a spall is recorded. A small spall is not more than 1 inch deep or approximately 6 inches in diameter. A large spall is more than 1 inch deep or greater than 6 inches in diameter.

Figure 5.26 shows the additions made to the BN to represent spalling due to corrosion. Three states of the variable (Spill) are shown: none, small, large. The CPT shows that a very minor amount of spalling is probable in the absence of corrosion. If corrosion is present, the probability of spalls increases, as they are later by-products. Also, as mentioned prior, cracking is a precursor to spalling. While a crack observed by an inspector could contribute to a spall at a later date, the BN, representing one point in time, accounts for the situation of a crack that has a spall.
Spalling is a symptom that is observable in a field inspection. It can be directly linked to the cross-sectional area of the beam. Originally defined within the probabilistic prognostic component with thresholds set at 545 and 575 \text{in}^2, the cross-sectional area having a mean value of 560 \text{in}^2 has a 39\% probability of being less than 545 \text{in}^2 and 22\% probability of being between 545 and 575 \text{in}^2 leaving a 39\% probability of being greater than 575 \text{in}^2. If a small spall is in existence, the distribution shifts having a greater probability that the cross-sectional area is less than 545 \text{in}^2 (Lo). If a large spall is in existence, the distribution is estimated with 75\% probability of being less than 545 \text{in}^2, 20\% probability of being between 545 and 575 \text{in}^2, and a 5\% probability of being greater than 575 \text{in}^2.

<table>
<thead>
<tr>
<th>Corrosion, Corr</th>
<th>Crack, Crk</th>
<th>Spall, Spill</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&gt; 0.0625 in.</td>
<td>0.95</td>
<td>0.03</td>
</tr>
<tr>
<td>&lt; 0.0625 in.</td>
<td>0.95</td>
<td>0.03</td>
</tr>
<tr>
<td>yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&gt; 0.0625 in.</td>
<td>0.60</td>
<td>0.25</td>
</tr>
<tr>
<td>&lt; 0.0625 in.</td>
<td>0.80</td>
<td>0.15</td>
</tr>
</tbody>
</table>

**FIGURE 5.26.** Additions to BN Modeling Spalling due to Corrosion

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**Corr:** Corrosion

**Crk:** Crack

**Spill:** Spall
5.6.6 Half Cell Potential Reading

Nondestructive testing (NDT) allows inspectors to observe deterioration symptoms using methods other than visual. There are various tests available ranging from ultrasonic to magnetic methods. The type of bridge material (concrete, steel, etc.) determines which test is adequate.

One test used to determine whether there is corrosion activity is the half cell potential reading \((HCP)\). This is an electrical method that measures the difference in electrical potential between the copper-copper sulfate half-cell on the surface of the concrete and the steel inside the concrete. This difference in electrical potential causes corrosion (AASHTO 2003).
The half cell potential reading variable, *HCP*, is easily added to the BN as shown in Figure 5.28. It has three states based on the interpretations set forth by AASHTO (AASHTO 2003):

1) less negative than –0.20 volts indicates a 90 percent probability of no corrosion
2) between –0.20 and –0.35 volts, corrosion activity is uncertain
3) more negative than –0.35 volts is indicative of greater than 90 percent probability that corrosion is occurring.

<table>
<thead>
<tr>
<th>Corrosion, Corr</th>
<th>Half Cell Potential Reading, HCP</th>
<th>&lt;=-0.20 V</th>
<th>&gt;-0.20 V and &lt; -0.35 V</th>
<th>&gt; -0.35 V</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>0.90</td>
<td>0.10</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>yes</td>
<td>0.00</td>
<td>0.10</td>
<td>0.90</td>
<td></td>
</tr>
</tbody>
</table>

**FIGURE 5.28.** Addition of NDT to the BN Component
5.7 Output for Diagnostic BN, Temporal Case

This temporal case of the diagnostic BN of the bridge load rating process was applied to the example problem previously used within this research and repeated here for informational purposes. An AASHTO Type III prestressed concrete beam that is the interior beam of a newly built, two lane bridge carrying a road over a river is evaluated for the purpose of this example (See Figures 4.2 through 4.4). The beam is one element of a bridge with 5 simple spans, each 65 feet long. Six beams, spaced at 8’-2” on center, carry each span with an 8 inch concrete deck. The total width of the bridge is 46’-3”. The details and properties of this girder are adopted from an example problem found in the 2003 Prestressed Concrete Institute (PCI) Bridge Design Manual (PCI 2003). Three investigations that were conducted are summarized along with the results presented in the following sections.

5.7.1 Diagnostic Exercise of Diagnostic BN, Temporal Case

Three diagnostic tests were conducted where the inventory load rating factor was declared to be less than one in each of the three methodologies. In reviewing the results, comparisons were made between assumed distributions and updated distributions of the input parameters. The parameter showing the most dramatic change is the most likely source of the problem. At all time steps, the variables showing the most change were prestressing steel area ($A_s$) and beam cross-sectional area ($A$). This was the anticipated result as the deterioration mechanism, corrosion, is most influential over these two variables. Another source of a failed inventory load rating may be that the cross-sectional area of the beam is too small due to spalling, or the compressive concrete strength of the beam may not be high enough due to faulty fabrication. Figure 5.29
shows the assumed distributions of each of the input parameters in the left column. The second column shows the updated distributions of those input variables if the AS load rating factor is declared less than one. The third and fourth columns show the results for the updated LF and LRFR load rating factors, respectively.

The results were further dissected to make relative comparisons between the updated distributions of other input parameters with respect to the three methodologies. Specifically, the variables that reflect the randomness of the beam properties were reviewed to compare the degree of influence over the final load rating factor (See Figure 5.29 and Appendix C). Regarding beam cross-sectional area ($A$), the LF methodology seemed most sensitive suggesting that loss of area was a large contributor to the failing load rating. The LRFR methodology showed similar results, although with less magnitude. However, with AS, the updated distribution showed an increase in the probability that $A$ was larger than assumed. A large area in the AS methodology computes to a greater deadload which contributes to a failed load rating.

Prestressing steel area ($A_s$) was the primary source of a failed load rating regarding all three methodologies. This was the expected result. For the failing LF inventory rating, $A_s$ showed the most dramatic changes until year 60 and beyond, then the most dramatic changes in the variable were through the AS methodology.

Strength of beam concrete ($f'_{cb}$) was only influential on a failed load rating when it came to AS, as opposed to LF and LRFR. The updated AS distribution indicated that the strength of beam concrete may not be as high as originally assumed.
Ultimate prestressing stress ($f'_s$) showed the same effect considering all three methodologies. The updated distribution showed a slight increase in the probability that $f'_s$ is lower than anticipated.

Unit weight of concrete ($w_c$) had an effect on the inventory load rating factor in all three methodologies. The updated distribution in the AS methodology suggested that a load rating factor less than one may have had the contributing cause of a unit weight of concrete being higher than assumed. However, the updated distributions using LF and LRFR indicated that a low unit weight might have been the contributor to the failed load rating.

The results pertaining to other input parameters, strength of deck concrete ($f'_{cd}$) and wearing surface thickness ($w_{st}$) were also reviewed. For AS, the updated distributions of strength of deck concrete ($f'_{cd}$) showed no change from the assumed distribution, except for an insignificant change at year 70. For the LF and LRFR methods, the probability that the strength of concrete was low increased.

Regarding wearing surface thickness ($w_{st}$), the updated distributions pertaining to the strength methods, LF and LRFR, showed the same trend, that the wearing surface thickness was most likely less than assumed.

In summary, it should be noted that the changes in distribution that occurred to the variables $A$ and $A_s$ were of the greatest magnitude, the updated changes to the other variables were insignificant by comparison. See Figure 5.29 and Appendix C.
FIGURE 5.29. Comparison Between Assumed and Updated Distributions of Input Variables, Temporal Case, t = 50 years

<table>
<thead>
<tr>
<th>Assumed Distribution</th>
<th>Updated Distribution</th>
<th>Inventory Rating &lt; 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AS</td>
<td>LF</td>
</tr>
<tr>
<td>$p(A)$</td>
<td>0.41</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>0.21</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>0.47</td>
<td>0.21</td>
</tr>
<tr>
<td>$p(A_s)$</td>
<td>0.11</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>0.16</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>0.38</td>
<td>0.48</td>
</tr>
<tr>
<td>$p(f'_cb)$</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>0.09</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>0.32</td>
<td>0.42</td>
</tr>
<tr>
<td>$p(f'_cd)$</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>0.28</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>0.71</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>0.32</td>
<td>0.42</td>
</tr>
<tr>
<td>$p(f'_s)$</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>0.04</td>
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<tr>
<td></td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>0.46</td>
<td>0.38</td>
</tr>
<tr>
<td>$p(w_c)$</td>
<td>0.13</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>0.11</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>0.17</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>$p(wst)$</td>
<td>0.06</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>0.90</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Legend: lo, med, hi
Figure 5.30 shows further interpretations of the results. Comparisons were made between the updated and the assumed distributions of seven input parameters given a failed inventory load rating. The increase in probability of the occurrence of the problematic states of each variable was extracted and plotted for observation. Figure 5.30 represents the bridge at 50 years of age, when some deterioration has occurred.

Considering all methodologies, the most sensitivity is shown with the area of prestressing strands \( A_s \), as expected. The strength methods, LF and LRFR show the influence of low cross-sectional area of the beam \( A \) upon a failed load rating. However, this is not the case with the AS method. This is due to the fact that \( A \) is integral in calculation of deadload as well as loss of prestress in such a way that an increase in \( A \) would lead to a failed AS inventory load rating.

This investigation into the possible source of a failed load rating considering material and loading effects was performed to test the BN. The BN was built considering the deterioration of the prestressing steel, specifically the decrease in area due to...
corrosion, as well as the decrease of the cross-sectional area of the beam due to spalling, a product of corrosive deterioration. As anticipated, in the comparison of the assumed distributions of the material and loading effects and the updated distributions once the failed load rating was declared, the results show the area of prestressing steel as the most likely source of the problem. Unfortunately, in the basic visual inspection of a bridge, the area of the prestressing steel is not directly observable, which leads to the next investigation.

5.7.2 Diagnostic Investigation of Decreasing Area of Prestressing Steel

The condition of the prestressing steel is not visible to the inspector. However, there are symptoms that may be observed by the naked eye such as efflorescence, rust stains, cracks and spalls. Further, if the inspector has access, hammer sounding could be performed in order to detect delamination. Also, performing non-destructive tests, such as measuring the half cell potential offers information on the presence of corrosive activity. In this investigation, the model is tested out as a tool to help direct the inspector in choosing the most effective way to conduct the inspection in order to determine the condition of the prestressing steel.

MSBNx has capabilities that direct the user in conducting a diagnostic investigation efficiently. In this study, the decreasing area of prestressing steel due to corrosion is the most likely source of the failed load rating. The visible symptoms of this corrosive activity are efflorescence, rust stains, delamination, cracks, and spalls. These symptom nodes are designated informational. The variable that is not observable, area of prestressing steel \( A \) is designated hypothesis. MSBNx performs an analysis and ranks
the symptom nodes based on their value of information, or amount of weight that their evidence would bring to the hypothesis node. This is a utility-based diagnosis based on mutual information (Kadie et al. 2001). The resulting output is a list of symptoms which when observed in the suggested order, would provide the most clear and effective diagnosis.

**FIGURE 5.31.** Value of Information of Observable Symptoms Through Time

Figure 5.31 shows the value of information the observance of each symptom offers in assessing the condition of the prestressing steel. The plot shown through time indicates that the value of information provided by each symptom increases as the beam deteriorates. Clearly, the most valuable test is the half cell potential. However, this type of test is not routinely used in most states and may have to be contracted out. A study conducted in 2001 (Moore et al. 2001) showed that only 11 states use the half cell potential test. Hammer-sounding for delamination is more fundamental, however it may
require special equipment to gain access as well as arranged traffic protection. As shown, the BN ranks the tests and observances by their value of information, yet the question remains of the effects they have on the resulting inventory load rating factors.

![Graph showing comparison of probability of failed load rating with HCP test](image)

**FIGURE 5.32.** Comparison of Probability of Failed Load Rating with HCP Test

Figure 5.32 plots out the probability of a failed inventory load rating over time. Each methodology is designated a color: AS (red), LF (blue) and LRFR (yellow). The inspector makes a field visit to the bridge. Symptoms of deterioration are looked for but not observed. In other words, the inspector sees no signs of efflorescence or rust stains. Cracking is minimal and there are no spalls. With these observations entered into the BN, the probabilities of failed load ratings result, as shown in Figure 5.32 as the dashed lines. The inspector goes on to perform the half cell potential test and finds that there is corrosive activity within the beam. These results are entered into the BN and the probabilities of failed load ratings increase as shown in Figure 5.32 as the solid lines. It is evident that the results of the HCP test have the most impact on the probability of the AS inventory rating being less than 1, compared to the other methodologies. In
calculating the inventory rating using AS, the area of the prestressing steel ($A_s$) plays a greater role than with the strength methods.

In summary, this investigation displays the capabilities of the BN in directing an effective diagnosis of unobservable deterioration. The results suggest that the half cell potential test is the most effective in determining the condition of the prestressing steel. Incorporating the results of this diagnostic test into the load rating process would be most influential on the AS inventory load rating.

5.7.3 Prognostic Exercise of Diagnostic BN, Temporal Case

Concrete compressive strength typically increases with time. Several researchers have demonstrated this phenomenon as it applies to prestressed concrete beams. (Rabbat 1984) conducted a study on 25 year old prestressed concrete bridge girders and found that the concrete compressive strength was 10,100 psi, where the specifications called for a 28-day strength of 5000 psi. (Pessiki et al. 1996) performed an evaluation of effective prestress force in a 28 year old prestressed concrete bridge beam. They found that the average compressive strength (8440 psi) from the cores was 65% greater than the 28-day strength specified (5100 psi). (Czaderski and Motavalli 2006) conducted a study to determine the remaining tendon force of a large-scale, 38 year old prestressed concrete bridge girder. In this study they also took cores from the concrete beam to determine the strength. They also found that the concrete strength increased over time, as expected.

The objective of this investigation is to use the BN prognostically to observe the effects of strengthening concrete over time upon the calculated load rating factors. Typically, the compressive concrete strength that is shown on the plans is that used
within the computations. If the plans are missing, Engineers are directed to conservatively estimate the strength based on values tabulated in the Manual (AASHTO 2003). Inspectors cannot visually observe an increase (or decrease) in the compressive concrete strength. The Windsor Probe and Schmidt Hammer are tools that help reveal a relative measure of the strength of concrete compared to other concrete. Limitations are that these tools work with the surface only (or close to surface (length of probe)), and provide a relative comparison only, good with only fairly new concrete components (manual states up to a year old) (AASHTO 2003). AASHTO states that cores may be taken when the strength of the concrete is uncertain and the initial load capacity based on the design concrete strength fails. A concrete core offers direct results. However, it is expensive, time-consuming and destructive. (Gannon and Cady 1992) conducted a survey of the states to determine the commonality of core sampling. Their findings showed that 14 states routinely use cores to determine strength, 9 states use cores sometimes.

In this investigation the BN is manipulated to observe the effects of an increased compressive concrete strength. There are two variables, compressive strength of beam concrete \((f'_{cb})\) and compressive strength of deck concrete \((f'_{cd})\). Each of these variables is divided into three states: Lo, Med and Hi. If the Hi state is declared, the model performs the evaluation with this assumption. The results rendered by the model reveal the anticipated improvement to the load rating. This helps aids in the decision of whether or not to perform the core sampling.

Figure 5.33 shows the probability of a failed AS inventory load rating considering core sampling results. Four lines are plotted representing the probability of the failed AS
rating through time considering the following: 1) no core, 2) beam core, 3) deck core, 4) both beam and deck cores. Of the four, only a beam core that shows an increase in strength of the beam is helpful in decreasing the probability of a failed AS load rating. This is expected, as the concrete strength of the deck does not enter into the AS calculations. Results show that the probability of the AS inventory rating being less than 1 fell approximately 10 percentage points.

FIGURE 5.33. Probability of Failed AS Inventory Load Rating Considering Core Sampling Results

Figure 5.34 shows the probability of a failed LF inventory rating in a similar manner. As for the LF inventory load rating, a core from the deck that showed an increase in compressive strength could be entered as evidence resulting in a decrease in probability of a failed LF inventory rating. The decrease does not seem significant.
FIGURE 5.34. Probability of Failed LF Inventory Load Rating Considering Core Sampling Results

Considering the LRFR method, a strength method like LF as opposed to service like AS, strength of deck concrete is more influential than strength of beam concrete. Strength of beam concrete is indirectly represented in the modulus of elasticity of the beam concrete being included in the calculations. Results show little impact in incorporating the results of core samples, as seen in Figure 5.35.
This investigation showed that there was little improvement with the evidence incorporated from concrete core samples. LF inventory rating slightly improved with the strength obtained in a deck core. AS inventory rating improved with a greater beam compressive strength. However, the core samples did not offer any lift to the LRFR inventory rating. Based on this investigation, core sampling may not be advantageous in carrying out a load rating of a beam. This was an expected conclusion as compressive concrete strength has little effect on the capacity of flexural members and a greater effect on axial members (AASHTO 2003).

5.8 Summary

This chapter presents the diagnostic component of the model, a Bayesian network of a prestressed concrete girder. Two cases are presented that apply to time, an as-built
case and a temporal case. It is comprehensive in that it includes representation of the load rating process and field inspections. The BN considers as the primary deterioration mechanism, corrosion of the prestressed strands due to infiltration of chloride ions. The input of this BN component is generated from the deterioration component of Chapter III, the prognostic component of Chapter IV as well as expert opinion and elicitation explained within this chapter. Details pertaining to the construction of this network are found within the body of the chapter and Appendix C. The diagnostic BN component is applied to a prestressed concrete bridge girder. The results stress the importance of the consideration of uncertainty in the rating calculations, thus supporting the implementation of the LRFR methodology. Quality control is also important, as the results indicate the possible sources of an unacceptable initial rating are the inconsistencies that may occur in design, at the fabrication plant or in the field during construction. Results also show how the load rating factors are affected by the deterioration mechanism, corrosion. As expected, the load rating factors decrease through time in correlation with the decreasing prestressed strand area due to corrosion.
CHAPTER VI
VERIFICATION OF BRIDGE BEAM ELEMENT LOAD RATING MODEL

6.1 Introduction

In this chapter, the load rating model of a beam element of a prestressed concrete bridge is verified from three aspects: 1) input, 2) output and 3) sensitivity studies. Specifically, input to the main components of the model (probabilistic prognostic and diagnostic BN) is discussed and defended. The output of these two components is evaluated for reasonableness. Sensitivity studies of input design variables, discretization thresholds of variables and functionality of inspection cluster are summarized. The results allow an assessment of the behavior of the model and its ability to adequately represent the flexural response of a prestressed concrete bridge beam to dead and live loads.

6.2 Verification of Input

In this section the input into the two main components of the model (probabilistic prognostic component and diagnostic Bayesian network) is explained in detail and defended. Included within the explanation are the literature sources and rationale behind any estimations or assumptions used. This section is built upon information found in Chapters IV and V. References are made to pertinent details found in these Chapters.
6.2.1 Verification of Input of Probabilistic Prognostic Component

In this section the input used in the probabilistic prognostic component is presented and the origin is explained in detail. This is carried out for two cases of the probabilistic prognostic component. The as-built case represents the point in time at the beginning of service life of the bridge to which the beam element being modeled belongs. The temporal case represents the behavior of the beam element through time. Both cases are presented because the constraints of the model differ for the two thus affecting the inputs (as well as responses). The deterioration that is considered in the temporal case calls for a different set of input than the as-built case.

Input Variables of Probabilistic Prognostic Component, As-Built Case

Table 6.1 lists the input (or root) variables that are used. These are the basic variables representing the resistance and loading effects. These root variables focus on the material, geometric and loading properties an Engineer would use in design and rating calculations. The statistical properties (bias (mean/nominal), coefficient of variation (COV=standard deviation/mean) and distribution) of these variables are based on those found in the literature (Mirza and MacGregor 1979, Mirza et al. 1979, Mirza et al. 1980, Naaman and Siriaksorn 1982, MacGregor et al. 1983, Hamann and Bulleit 1987, Al-Harthiy and Frangopol 1994, Steinberg 1995, El-Tawil and Okeil 2002, Gilbertson and Ahlborn 2004). Earlier researchers performed extensive work in the area of reliability of basic material properties, mostly in efforts to develop a more rational design specification based on the probabilities of loads and resistances. Descriptions of their research may be found in Section 4.3 (Ellingwood et al. 1980, MacGregor 1976, Mirza and MacGregor
1979, Mirza et al. 1979, Mirza et al. 1979, Naaman and Siriaksorn 1982, MacGregor et
recently have used these basic published statistics within their research specifically
applied to bridges (See Section 4.3) (Steinberg 1995, El-Tawil and Okeil 2002,
Gilbertson and Ahlborn 2004). All of these statistics have been published in the
following reputable journals and publications: *Journal of Structural Division, PCI

**TABLE 6.1. Input Variables, As-Built Case**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
<th>Bias</th>
<th>COV</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Cross-sectional Area of Beam</td>
<td>in²</td>
<td>1.00</td>
<td>0.020</td>
<td>Normal</td>
</tr>
<tr>
<td>Aₚ</td>
<td>Prestressing Strand Area</td>
<td>in²</td>
<td>1.01</td>
<td>0.013</td>
<td>Normal</td>
</tr>
<tr>
<td>fᶜᵇ</td>
<td>Concrete Strength of Beam</td>
<td>psi</td>
<td>1.10</td>
<td>0.180</td>
<td>Normal</td>
</tr>
<tr>
<td>fᶜᵈ</td>
<td>Concrete Strength of Deck</td>
<td>psi</td>
<td>1.00</td>
<td>0.200</td>
<td>Normal</td>
</tr>
<tr>
<td>fₚ</td>
<td>Prestressing Strand Strength</td>
<td>ksi</td>
<td>1.04</td>
<td>0.020</td>
<td>Normal</td>
</tr>
<tr>
<td>Eₛ</td>
<td>Modulus of Elasticity of Prestressing Strands</td>
<td>ksi</td>
<td>1.01</td>
<td>0.010</td>
<td>Normal</td>
</tr>
<tr>
<td>I</td>
<td>Moment of Inertia of Beam Cross-section</td>
<td>in⁴</td>
<td>1.00</td>
<td>0.030</td>
<td>Normal</td>
</tr>
<tr>
<td>Iₖ</td>
<td>Moment of Inertia of Composite Section</td>
<td>in⁴</td>
<td>1.00</td>
<td>0.035</td>
<td>Normal</td>
</tr>
<tr>
<td>RH</td>
<td>Relative Humidity</td>
<td>%</td>
<td>1.00</td>
<td>0.120</td>
<td>Normal</td>
</tr>
<tr>
<td>wₑ</td>
<td>Unit Weight of Concrete</td>
<td>pcf</td>
<td>1.00</td>
<td>0.030</td>
<td>Normal</td>
</tr>
<tr>
<td>wst</td>
<td>Wearing Surface Thickness</td>
<td>psf(in)</td>
<td>1.10</td>
<td>0.200</td>
<td>Normal</td>
</tr>
</tbody>
</table>

**Input Variables of Probabilistic Prognostic Component, Temporal Case**

All of the input variables considered in the prognostic component of the time
model are listed in Table 6.2. These variables are based on those of the as-built case
explained previously. The coefficients of variation of certain variables are increased in
order to account for the uncertainty that occurs through time. This is assumed as a
reasonable approach based on expert estimation. Justification for this increase in the
coefficients of variation to account for randomness was sought in an exhaustive literature
survey. However, all discovered statistics of the variables of interest applied to an initial time at the beginning of service life of the prestressed concrete beam.

The reasoning behind adjusting the statistics of some of the input variables is to allow for the changes that occur and the level of uncertainty that arises due to temporal effects. The cross-sectional area of the beam \( A \) is input with the same mean as the as-built case. However, the coefficient of variation (COV) is increased to allow for the higher probability of spalling that typically occurs with time. The prestressing strand area \( A_s \) also is set with a higher COV reflective of the uncertainty associated with determining the area of prestressing steel that remains effective. The mean of the variable \( A_s \) decreases through time and this value is obtained from the deterioration component (See Chapter III). The compressive strength of concrete \( f'_{c} \) typically increases through time. However, this increase in strength is not taken for granted and Engineers are directed to use design values in their load rating analyses (AASHTO 2003). This is most likely due to the uncertainty of how much strength the concrete has gained and also to account for the possibility of unsound or deteriorated concrete.

The same conservative approach is taken also with the variables representing concrete strength in the beam \( f'_{cb} \) and the deck \( f'_{cd} \). The statistics of variables that pertain to the strength \( f'_{s} \) and elasticity \( E_s \) of the prestressing strands remain the same as in the as-built case due to the fact that studies show that these variables remain fairly constant through time. The moment of inertia of both the beam cross-section \( I \) and the composite section \( I_c \) retain the same mean as the as-built case. However, the coefficient of variation increases for both, with the COV of \( I_c \) having a slightly greater increase than \( I \) due to the added uncertainty of the deck dimensions through time. The thresholds of
both moment of inertias are adjusted to coordinate with the thresholds set for $A$
facilitating the representation of their correlation within the BN diagnostic component.
Finally, relative humidity ($RH$), unit weight of concrete ($w_c$) and wearing surface
thickness ($wst$) are assumed to be unaffected by time. None of these variables show an
increasing or decreasing trend during service life. The uncertainty associated with these
variables is adequately represented in the statistics carried over from the probabilistic
prognostic component representing the as-built stage.

**TABLE 6.2 Input Variables, Temporal Case**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
<th>Bias</th>
<th>COV</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Cross-sectional Area of Beam</td>
<td>$in^2$</td>
<td>1.00</td>
<td>0.100</td>
<td>Normal</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Prestressing Strand Area</td>
<td>$in^2$</td>
<td>1.01</td>
<td>0.100</td>
<td>Normal</td>
</tr>
<tr>
<td>$f'_{cb}$</td>
<td>Concrete Strength of Beam</td>
<td>psi</td>
<td>1.10</td>
<td>0.180</td>
<td>Normal</td>
</tr>
<tr>
<td>$f'_{cd}$</td>
<td>Concrete Strength of Deck</td>
<td>psi</td>
<td>1.00</td>
<td>0.200</td>
<td>Normal</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Prestressing Strand Strength</td>
<td>ksi</td>
<td>1.04</td>
<td>0.020</td>
<td>Normal</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of Elasticity of Prestressing Strands</td>
<td>ksi</td>
<td>1.01</td>
<td>0.010</td>
<td>Normal</td>
</tr>
<tr>
<td>$I$</td>
<td>Moment of Inertia of Beam Cross-section</td>
<td>$in^4$</td>
<td>1.00</td>
<td>0.050</td>
<td>Normal</td>
</tr>
<tr>
<td>$I_c$</td>
<td>Moment of Inertia of Composite Section</td>
<td>$in^4$</td>
<td>1.00</td>
<td>0.060</td>
<td>Normal</td>
</tr>
<tr>
<td>$RH$</td>
<td>Relative Humidity</td>
<td>%</td>
<td>1.00</td>
<td>0.120</td>
<td>Normal</td>
</tr>
<tr>
<td>$w_c$</td>
<td>Unit Weight of Concrete</td>
<td>pcf</td>
<td>1.00</td>
<td>0.030</td>
<td>Normal</td>
</tr>
<tr>
<td>$wst$</td>
<td>Wearing Surface Thickness</td>
<td>psf(in)</td>
<td>1.10</td>
<td>0.200</td>
<td>Normal</td>
</tr>
</tbody>
</table>

**6.2.2 Verification of Input of Diagnostic BN Component**

In this section the input necessary for the diagnostic BN is verified. Three sub-
sections are included. First, the input variables used in the probabilistic prognostic
component are modified for input into the BN software. Second, the process of obtaining
the conditional probabilities that represent the numerical inter-relationships between
variables is defended. These first two sub-sections pertain to the input that is used in the
core of the network, which represents the load rating process. The third subsection is in
reference to the inspection cluster which is attached to the load rating core of the BN. This cluster is developed based on a process of deterioration. The manner with which the input is obtained for the cluster explained. All three sub-sections give reference to Chapter V, which offers exhaustive details on the development of the diagnostic BN.

**FIGURE 6.1.** Load Rating Core of Diagnostic BN

**Input Variables of Diagnostic BN from Probabilistic Prognostic Component**

The continuous input or root variables of Tables 6.1 and 6.2 are discretized into *Low*, *Medium* and *High* states using sensible thresholds that have been set according to the characteristics of each variable. As explained in detail in Chapter V, these variables
are used as input for the design variables of the load rating core of the diagnostic BN (portion of BN within red dashed circle of Figure 6.1). The sensible thresholds that are used for each variable are based on thorough literature survey of various design and inspection manuals, as well as expert elicitation (AASHTO 1994, AASHTO 1996, AASHTO 1998, AASHTO 2003, FHWA 1995a, MHD 2005, PB 1993, PCI 1999, PCI 2003). An expert suggested discretizing the continuous variables according to these thresholds as opposed to arbitrary thresholds that would divide the distribution (such as 25% Low, 50% Medium, 25% High). A sensitivity study comparing the impact of different thresholds is carried out later in this chapter.

**Conditional Probabilities of Diagnostic BN Component, Load Rating Core**

Input that is necessary for the diagnostic BN component are conditional probabilities that numerically represent the relations between variables. Acceptable sources of conditional probabilities are literature survey, expert elicitation, experimental results and Monte Carlo simulation. A thorough review of the approaches used by other researchers in obtaining these conditional probabilities was conducted. For example, Hahn et al. (2002) employs experts to obtain the conditional probabilities in their model that prioritizes the inspection of sewers. Expert elicitation is useful in obtaining conditional probabilities when no statistics or data are available. Sahely and Bagley (2001) use simulation to supply the conditional probabilities for their BN that diagnoses upsets in anaerobic wastewater treatment. In this research, similar to Sahely and Bagley (2001), it was decided to use simulation techniques to obtain the necessary conditional probabilities for the portion of the network that modeled the load rating process. This
seemed the logical approach because the input random variables as well as the series of load rating equations were available to simulate.

The probabilistic prognostic component calculates these conditional probabilities within the Monte Carlo simulations for the load rating core of the network. Based on the statistical constraints prescribed for each parent variable, random numbers are generated to simulate its distribution. With each generation of a set of parent variables, the child variable is calculated through the equation that represents the relationship between the parents that yields the child. Considering 10,000 trials, there would be 10,000 resulting values of the child variable, generating an array of data from which a mean and standard deviation may be calculated. A histogram is generated and then a statistical distribution fitted to approximate a continuous distribution. In order to apply these results to the BN, this so called continuous distribution is transformed into a discrete distribution by set thresholds that consider the characteristics of the variable. This approach based on that of other researchers (Sahely and Bagley 2001) is explained in detail in Chapter V.

**Input Variables and Conditional Probabilities, Inspection Cluster**

The temporal case of the diagnostic BN considers deterioration of a prestressed concrete bridge girder. The most common form is corrosion of prestressing steel through chloride infiltration, as discussed previously in Chapter III. The expanded diagnostic BN models this deterioration (See Figure 6.2). The inspection cluster (within the red dashed oval) shows variables that represent deterioration mechanisms, observable symptoms of deterioration, as well as the design variables that are affected by the deterioration. This inspection cluster was developed based on the deterioration process considered here, corrosion of interior steel. The root variables and conditional probability tables in this
case were estimated based on expert opinion and elicitation. The root variables were obtained in this manner as the literature survey offered published statistics of these variables of interest and experimental results were not available. As for obtaining the conditional probabilities, the inter-relations between the variables were not based on load rating equations that could easily be simulated as in the load rating core of the BN. Again, experimental data was not available. Expert opinion and elicitation seemed the best option at the time of completion of this research. As new data and information becomes available the input variables and conditional probabilities can easily be updated. Details of the BN expansion as well as the input variables are in explained in Chapter V.

FIGURE 6.2. Inspection Cluster Addition to Diagnostic BN
6.3 Verification of Output

In this section, the results (or output) of the two main components of the model, probabilistic prognostic component and diagnostic BN component are discussed. Specifically, the results of the two cases (as-built, temporal) of each of these two components are reviewed in terms of their reasonableness and viability.

6.3.1 Verification of Output of Probabilistic Prognostic Component

Output of Probabilistic Prognostic Component, As-Built Case

Figure 6.3 shows the inventory load rating variables calculated according to the as-built case of the probabilistic prognostic component along with the appropriate input variables representing the beginning of service life. Each curve represents the resulting inventory load rating calculated according to one of the three methodologies. All three curves show the frequency of possible values shown on the horizontal axis. The plot shows the inventory load rating factor calculated according to LF is greater than AS. Prestressed concrete beams are designed typically according to service loads (using AS) in order that internal stresses may not surpass a specified limiting stress. Satisfying the service limit state typically controls design and calls for additional strands and increased capacity of the beam. Load Factor (LF) is often used after design to verify ultimate capacity. However, using the LF methodology to calculate the inventory rating factor renders a falsely higher load rating factor due to the fact that it is not consistent with the original design assumptions (PCI 2003). Therefore, the probabilistic prognostic component renders the expected results in that the inventory load ratings calculated according to LF are greater than the AS results.
In comparing the LF curve to the LRFR curve, the LF results are also greater than those of LRFR. These findings are in agreement with previously conducted research by others who also concluded that LRFR method generally resulted in lower rating factors for flexure (Lichtenstein 2001, Rogers and Jauregui 2005). Researchers Rogers and Jauregui (2005) conducted a comparative analysis on rating factors calculated using LF vs. LRFR and attribute the live load effects as the primary factor in this difference.

Figure 6.3 also shows that the ranges of possible inventory load rating factors are 1.00 or greater. In other words, the probability of a failed load rating is zero according to all three methodologies as would be expected at the as-built stage.

**FIGURE 6.3.** Inventory Load Rating Variables, As-Built Case

*Output of Probabilistic Prognostic Component, Temporal Version*

Figure 6.4 shows the inventory load rating variables calculated in the temporal case of the probabilistic prognostic component along with the appropriate input variables for the year 60. All three curves show ranges of possible values that are below 1.00, i.e.
failed load ratings, as expected considering deterioration at the represented bridge age of 60 years. The AS methodology shows a 53.3% probability of a failed load rating while LF shows a 19.0% probability. The variability shown within the results follow the trends shown at in the as-built case, i.e. the AS methodology has the largest distribution. This dispersion of the AS inventory rating factor is attributed to the influence of a greater number of random variables used within the calculations compared to the other methodologies.

**FIGURE 6.4.** Inventory Load Rating Variables, Temporal Case

Figure 6.5 shows a decrease in magnitude of the inventory load rating variable for all three methodologies, beginning once corrosion has been initiated after 30 years. However, the decrease of the AS inventory rating factor is more pronounced than the others at an average rate of 16.5% every 10 years after deterioration begins. The LF and LRFR inventory rating factors both decrease at an average rate of 7.5% every 10 years.
once deterioration begins. In explanation, the calculation of the AS rating factor considers the stresses of the prestressing tendons, as well as the losses, such as those due to creep. The final stress within the tendons after losses is influential upon the resulting AS inventory rating. This sets it apart from the other two strength methods. The calculation of the moment capacity is most influential with the LF and LRFR methods. All three methods account for the decrease in area of prestressing steel. However, the prestress losses are not considered within the LF and LRFR methods, because they are nominal strength calculations.

![Figure 6.5: Means of Inventory Load Rating Variables through Time](image)

**FIGURE 6.5.** Means of Inventory Load Rating Variables through Time

Comparing the results of the three methods, the inventory rating factor calculated according to LF is greater than AS. Not only is this the expected result, but also follows the trend of the results from the as-built case. It is also observed that the inventory load rating factor calculated according to AS is greater than LRFR until year 50. The criss-cross of these two lines showing the decrease in inventory ratings is
explained in the previous paragraph. In comparing the LF curve to the LRFR curve, the
LF results are also greater than those of LRFR. These findings are in agreement with
previously conducted research by others who also concluded that LRFR method
generally resulted in lower rating factors for flexure (Lichtenstein 2001, Rogers and
analysis on rating factors calculated using LF vs. LRFR and attribute the live load effects
as the primary factor in this difference, as pointed out previously.

6.3.2 Verification of Output of Diagnostic BN Component

In this section, the results of the diagnostic BN component presented in Chapter V
are discussed. Specifically, the results of the two cases (as-built and temporal) of this
component are reviewed in terms of their reasonableness and viability.

Output of Diagnostic BN Component, As-Built

In testing the diagnostic BN at the as-built stage, three investigations were
conducted where the inventory load rating factor was declared to be less than one in each
of the three methodologies (AS, LF, LRFR). No results were obtained for the LF
investigation because the probability of failure was zero. In other words, the diagnosis of
failed LF inventory load rating was not possible because the probability of a failed LF
inventory load rating is zero at the as-built stage. The other two methodologies (AS,
LRFR) each had a small probability (less than 1%) of a failed load rating that were
diagnosed using the BN. In reviewing the results, comparisons were made between
assumed distributions and updated distributions of the input parameters. The parameter
showing the most dramatic change was the most likely source of the problem. In the two remaining investigations (AS, LRFR), the prestressing strand area ($A_s$) showed the most significant change in distribution, specifically pointing to a smaller size of strand than originally assumed as the root of the failed load rating (See Chapter V including Figure 5.17). In summary, the results of these diagnostic investigations show that in the event of a failed inventory load rating (AS or LF), as small a possibility as there is ($< 1\%$), the most likely source is that the area provided by the prestressed strands is less than expected. Possible explanations for this could be using the wrong size strand, damage during fabrication, or incorrect strand pattern. It is reasonable to find the most likely source of a failed inventory load rating at the as-built stage relates to the area of the prestressing strands. While quality control at prestressed concrete plants is typically of high standards, aberrations are still possible. These results reveal the influence that the area of prestressed steel has on the load carrying capacity of a prestressed concrete beam.

**Output of Diagnostic BN Component, Temporal Case**

In testing the temporal case of the diagnostic BN at time step, $t = 50$ years, the same three diagnostic tests were conducted where the inventory load rating factor was declared to be less than one in each of the three methodologies. At this time step (and all others of the temporal case beyond $t = 0$ years), the variables showing the most change were prestressing steel area ($A_s$) and beam cross-sectional area ($A$). This was the anticipated result as the deterioration mechanism, corrosion, is most influential over these two variables.
6.4 Sensitivity Studies of Input Design Variables

In this section, the sensitivity of the input design variables is investigated. Eleven variables are considered that represent resistance and loading effects. Both the probabilistic prognostic component and the diagnostic BN component are tested. The two cases are also considered, as-built and temporal.

6.4.1 Sensitivity Study of Input Design Variables of Probabilistic Prognostic Component

In this section the sensitivity study of the input variables of the probabilistic prognostic component are described and the results are presented. This is carried out for the two cases, as-built and temporal.

*Sensitivity of Input Variables of Probabilistic Prognostic Component, As-Built Case*

A sensitivity study was conducted that tested the influence of each input variable upon the resulting inventory load rating variables. Monte Carlo simulation of the load rating equations according to the three methodologies was carried out. The deterministic nominal value was taken for each variable except for one in order to evaluate that particular variable’s influence upon the resulting load rating variables. The results are shown in Figures 6.6 and 6.7.

Figure 6.6 includes three plots, each representing the results of one of the three methodologies. The inventory load rating factor calculated deterministically is shown in the red box for AS, LF and LRFR. The bar plots demonstrate percentage of difference between the deterministic value and the mean of the inventory load rating factor
calculated as a variable considering one input variable at a time. For example, in reviewing the plot showing the AS results, the input variable, prestressing strand strength ($f'_s$) appears to be the most influential upon the AS inventory load rating factor. The calculated mean exceeds the deterministic value by 7.9%. This variable is most influential in all three methodologies at this as-built stage where time, $t = 0$. Prestressing strand area, $A_s$, is second most influential showing an increase of 1.8% of the mean of the inventory load rating factor calculated as a variable over the deterministic value.

Figure 6.7 shows three plots, each pertaining to one of the methodologies: AS, LF, LRFR. Each curve represents the inventory load rating variable calculated as a continuous random variable. Each colored line represents the inventory load rating variable calculated considering just the one input variable with which it is labeled as random. The black curve considers all input variables as random. The vertical line considers all input as deterministic.

In all three cases, considering the input variables as random offers a final mean value that is higher than the deterministic load rating factor. This is due primarily to the fact that the most influential variables ($A_s, f'_s$), among others, have a bias that is greater than 1.00, as dictated by the statistics found in the literature.
FIGURE 6.6. Influence of Each Input Variable upon the Inventory Load Rating Factors, As-Built Case

Legend:
A: cross-sectional area of beam
As: prestressing strand area
fcb: compressive concrete strength of beam
fcd: compressive concrete strength of deck
fs: prestressing strand strength
Es: modulus of elasticity of prestressing strands
I: moment of inertia of beam cross-sectional area
lc: moment of inertia of composite section
RH: relative humidity
wc: unit weight of concrete
wst: wearing surface thickness
FIGURE 6.7. Influence of Input Variables upon the Inventory Load Rating Variables, As-Built Case
Sensitivity of Input Variables of Probabilistic Prognostic Component, Temporal Case

Figure 6.8 includes three plots, each representing the results of one of the three methodologies. The inventory load rating factor calculated deterministically is shown in the red box for AS, LF and LRFR. The bar plots demonstrate percentage of difference between the deterministic value and the mean of the inventory load rating factor calculated as a variable considering one input variable at a time. For example, in reviewing the AS plot, the input variable, prestressing strand area ($A_s$) appears to be the most influential upon the AS inventory load rating factor. The calculated mean is less than the deterministic value by 20%. This is due to the decreased prestressing strand area due to corrosive deterioration. This variable ($A_s$) is most influential in all three methodologies in the temporal case. This is the anticipated result as the deterioration mechanism of corrosion directly affects this variable. Prestressing strand strength ($f'_s$) is second most influential showing an average increase of approximately 7% of the mean of the inventory load rating factor calculated as a variable over the deterministic value.

Figure 6.9 shows three plots, each pertaining to one of the methodologies: AS, LF, LRFR. Each curve represents the inventory load rating variable calculated as a continuous random variable. Each colored line represents the inventory load rating variable calculated considering just the one input variable with which it is labeled as random. The black curve considers all input variables as random. The vertical line considers all input as deterministic. In all three cases, considering the input variables as random offers a final mean value that is lower than the deterministic load rating factor. This is primarily due to the fact that the most influential variable, $A_s$, is representative of the deterioration mechanism.
FIGURE 6.8. Influence of Each Input Variable upon the Inventory Load Rating Factors, Temporal Case
FIGURE 6.9. Influence of Input Variables upon the Inventory Load Rating Variables, Temporal Case
6.4.2 Sensitivity Study of Input Design Variables to Diagnostic BN Component

In this investigation the BN is tested in order to find out how sensitive the output is to each input variable. For each of the eleven input variables of the load rating core, the BN is evaluated with all other variables declared to be in the medium state. Comparisons are made of the probability of a failed inventory load rating in each of these eleven cases with the probability that results from all input variables declared in the medium state. This determines the effect that a particular variable has on the outcome. The results are presented in bar graphs with the difference between the probabilities of failure in the form of percentages. This is carried out using the BN having no inspection cluster in order to focus on the design variables as the input.

Sensitivity of Input Variables of Diagnostic BN Component, As-Built Case

A sensitivity study of this sort was not carried out for the as-built case as the probability of a failed load rating in each of the three methodologies is either zero or close to zero.

Sensitivity of Input Variables of Diagnostic BN Component, Temporal Case

The results of the sensitivity study of the input variables of the BN for the temporal case, time $t = 60$ years are presented in Figures 6.10 through 6.12.
As shown in Figure 6.10, the input variable that the probability of a failed AS inventory load rating is most sensitive to is the prestressed steel area, $A_s$. The plot shows a 13% decrease in the probability of failed load rating if $A_s$ is not declared in the medium state (2–4 strands deteriorated). Cross-sectional area of the beam, $A$, is the second most influential variable. The results show a decrease of 9.5% in the probability of a failed load rating if $A$ is not declared in the medium state ($545 \text{ in}^2 – 575 \text{ in}^2$). All of the variables collectively cause a 10.3% decrease in the probability of a failed load rating when there are none declared to be in any state. It is reasonable that the most influential variables are prestressed steel area, $A_s$ and cross-sectional area of the beam, $A$ as these are the variables that are affected both directly and indirectly by the deterioration mechanism, corrosion.

**FIGURE 6.10.** Influence of Each Input Variable upon the AS Inventory Load Rating Factor, Temporal Case of BN
In Figure 6.11 are results of the sensitivity study of the input variables of the BN for the temporal case, time $t = 60$ years. The input variable that the probability of a failed LF inventory load rating is most sensitive to is the prestressed steel area, $A_s$. The plot shows a 35% decrease in the probability of failed load rating if $A_s$ is not declared in the medium state (2-4 strands considered deteriorated). Cross-sectional area of the beam is the second most influential variable. The results show an increase of 21% in the probability of a failed load rating if $A$ is not declared in the medium state ($545 \text{ in}^2 - 575 \text{ in}^2$). All of the variables collectively cause a 9.6% decrease in the probability of a failed load rating when there are none declared to be in any state. In other words, when all of the input variables are left uninstantiated, the probability of a failed load rating is 9.6% less than when all variables are declared to be in the medium state. This result suggests that some variables being either in the low or high state improves the inventory load rating, as expected. For example, the high state of the prestressed steel area actually represents little or no loss, while the medium state represents a loss of 2-4 strands. It is
expected that the most influential variables are prestressed steel area, $A_s$, and cross-sectional area of the beam, $A$ as these are the variables that impacted by the deterioration, corrosion.

**FIGURE 6.12.** Influence of Each Input Variable upon the LRFR Inventory Load Rating Factor, Temporal Case of BN

In Figure 6.12 are results of the sensitivity study of the input variables of the BN for the temporal case, time $t = 60$ years. The input variable that the probability of a failed LRFR inventory load rating is most sensitive to is the prestressed steel area, $A_s$. This is the expected result. The plot shows an 18% decrease in the probability of failed load rating if $A_s$ is not declared in the medium state (2-4 strands considered deteriorated). Moment of inertia of the beam cross-sectional is the second most influential variable. The results show a decrease of 5.2% in the probability of a failed load rating if $I$ is not declared in the medium state (123,540 in$^4$ – 127,240 in$^4$). All of the variables collectively cause a 14% decrease in the probability of a failed load rating when there are none declared to be in any state (Low, Med or High). In other words, the probability of a
failed LRFR load rating is 14% less when all variables remain uninstantiated as opposed to all being declared in the medium state.

6.4.3 Comparing Results, Probabilistic Prognostic Component vs. Diagnostic BN Component

The results of the sensitivity tests of the effect of each input variable upon the output (the probability of a failed inventory load rating) are compared between the two components (the probabilistic prognostic component and the diagnostic BN component). In both, the output is most sensitive to the variable, prestressed steel area, $A_s$, for all three methodologies. However, with the probabilistic prognostic component, the second most influential variable in all three methodologies is prestressing strand strength, $f_s'$. This is not the case with the diagnostic BN. In the AS and LF methodologies, beam cross-sectional area, $A$, is the second most influential, and moment of inertia of the beam cross-section, $I$, is for LRFR. This difference between these two components is explained by the difference between the two sets of input variables, continuous in the probabilistic prognostic component and discrete in the diagnostic BN component. It is expected, also, that the thresholds, which set the discretizations of these variables, have an impact on the sensitivity of these results. This leads to the investigation involving the estimation of thresholds covered in the following section.

6.5 Sensitivity Tests of Thresholds used to Discretize Variables in Diagnostic BN

In this investigation, the sensitivity of the thresholds used to discretize the continuous input design variables is tested for the temporal version of the model.
Originally the thresholds are set sensibly according to the characteristics of each variable in order to divide the continuous distributions into the discrete states of Low, Medium and High. To compare, the thresholds are set arbitrarily at the quartile points in order to achieve this discretization. Table 6.3 shows the two sets of thresholds and the percentage of the continuous distribution that ends up within each state as a result.

**TABLE 6.3** Discretized Distributions of Input Variables, Practical vs. Quartile Thresholds

<table>
<thead>
<tr>
<th>Variable lower mean upper lo-med-hi (%)</th>
<th>lower mean upper lo-med-hi (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (in²) 545 560 575 39% - 22% - 39%</td>
<td>522 560 598 25% - 50% - 25%</td>
</tr>
<tr>
<td>Aₚ (in²) 0.125 0.133 0.14 30% - 43% - 27%</td>
<td>0.124 0.133 0.142 25% - 50% - 25%</td>
</tr>
<tr>
<td>f'ₚ (psi) 4200 5500 6400 9% - 73% - 18%</td>
<td>4832 5500 6168 25% - 50% - 25%</td>
</tr>
<tr>
<td>f'ₚ (psi) 3000 3400 5000 28% -71% -1%</td>
<td>2941 3400 3859 25% - 50% - 25%</td>
</tr>
<tr>
<td>fₚ (psi) 270 281 290 3% - 92% - 5%</td>
<td>277 281 285 25% - 50% - 25%</td>
</tr>
<tr>
<td>Eₚ (ksi) 28000 28810 29000 1% - 73% - 26%</td>
<td>28619 28810 29008 25% - 50% - 25%</td>
</tr>
<tr>
<td>I (in⁴) 123540 125390 127240 39% - 22% - 39%</td>
<td>121160 125390 129620 25% - 50% - 25%</td>
</tr>
<tr>
<td>Iₚ (in⁴) 357880 364324 370770 39% - 22% - 39%</td>
<td>349580 364324 379070 25% - 50% - 25%</td>
</tr>
<tr>
<td>RH (%) 60 70 80 12% - 76% - 12%</td>
<td>64 70 76 25% - 50% - 25%</td>
</tr>
<tr>
<td>wc (pcf) 145 150 155 13% - 74% - 13%</td>
<td>147 150 153 25% - 50% - 25%</td>
</tr>
<tr>
<td>wst (psf/in) 18.75 27.5 37.5 6% - 90% - 4%</td>
<td>23.8 27.5 31.2 25% - 50% - 25%</td>
</tr>
</tbody>
</table>

6.5.1 Comparison of Input Variables

This section describes the differences between the input variable discrete distributions of the two sets of thresholds being tested.

The cross-sectional area of the beam (A) originally had thresholds symmetrical about the mean, 560 in². These thresholds were set based on spalling patterns representing a 2% section loss. The 25% Lo, 50% Med, 25% Hi distribution results in thresholds where the Lo threshold (522 in²) represents a section loss of 5% or more.
The area of the prestressed strands ($A_s$) originally had $Lo$ and $Hi$ thresholds (0.125 (4 strands deficient) and 0.140 in$^2$ (2 strands deficient)) that rendered a distribution of 30% $Lo$, 43% $Med$ and 27% $Hi$. The 25% $Lo$, 50% $Med$ and 25% $Hi$ distribution, symmetrical about the mean, results in similar thresholds.

The compressive strength of the beam concrete ($f'_{cb}$) originally had $Lo$ and $Hi$ thresholds (4,200 and 6,400 psi) that rendered a distribution of 9% $Lo$, 73% $Med$ and 18% $Hi$. The 25% $Lo$, 50% $Med$ and 25% $Hi$ distribution, symmetrical about the mean, results in a similar $Hi$ threshold (6,400 vs. 6,168 psi). However the $Lo$ threshold is much greater than the original (4,832 vs. 4,200 psi).

The compressive strength of the deck concrete ($f'_{cd}$) originally had $Lo$ and $Hi$ thresholds (3,000 and 5,000 psi) that rendered a distribution of 28% $Lo$, 71% $Med$ and 1% $Hi$. The 25% $Lo$, 50% $Med$ and 25% $Hi$ distribution, symmetrical about the mean, results in a similar $Lo$ threshold (2,941 vs. 3,000 psi). However the $Hi$ threshold is much lower than the original (3,859 vs. 5,000 psi).

The strength of the prestressed steel strand ($f_s'$) originally had a distribution of 3% $Lo$, 92% $Med$ and 5% $Hi$ determined by the thresholds of 270 and 290 psi set around the mean of 281 psi. The 25% $Lo$, 50% $Med$ and 25% $Hi$ distribution renders thresholds (277 and 285 psi) closer to the mean (281 psi).

The modulus of elasticity of the prestressed steel ($E_s$) originally had a distribution of 1% $Lo$, 73% $Med$ and 26% $Hi$. This distribution was not symmetrical about the mean. The 25% $Lo$, 50% $Med$ and 25% $Hi$ distribution renders thresholds that are only slightly different from the original thresholds. This is primarily due to a low coefficient of variation.
The moment of inertias for the beam itself and the composite section \((I\) and \(I_c\)) originally had discretized distributions of 39% \(Lo\), 22% \(Med\) and 39% \(Hi\). This was set up in a previous version to facilitate the modeling of correlation between these variables and the cross-sectional area of the beam \((A)\). These thresholds are symmetrical about the mean. The quartile thresholds are also symmetrical about the mean and show a slight change from the original thresholds.

The relative humidity \((RH)\) originally had a discretized distribution of 12% \(Lo\), 76% \(Med\) and 12% \(Hi\). This was determined by the thresholds set symmetrically at 60 and 80% about the mean of 70%. The quartile thresholds are also symmetrical about the mean and show a slight change from the original with the thresholds resulting at 64 and 76%.

The unit weight of concrete \((w_c)\) originally had a discretized distribution of 13% \(Lo\), 74% \(Med\) and 13% \(Hi\). This was determined by the thresholds set symmetrically at 145 and 155 pcf about the mean of 150 pcf. The quartile thresholds are also symmetrical about the mean and show a slight change from the original with the thresholds resulting at 147 and 153 pcf.

The wearing surface thickness \((wst)\) originally had a discretized distribution of 6% \(Lo\), 90% \(Med\) and 4% \(Hi\). This meant that there was a 90% probability that the wearing surface was between 1.5 and 3 inches thickness (which converts to 18.75 and 37.5 psf). In this investigation, the thresholds set at the quartile points result in a 50% probability that the wearing surface thickness is between 1.9 and 2.5 inches (which converts to 23.8 and 31.2 psf).
In summary, dividing the continuous input variables by quartile thresholds leaves most of the design variables with discrete distributions that differ greatly from those defined by the practical thresholds. The next section compares the results stemming from these two sets of discrete distributions.

6.5.2 Sensitivity of Input Variables

Here, the BN is tested in the same manner as the previous investigation in order to determine the sensitivity of the output to each input variable. However, this is carried out for the distribution of 25\% \textit{Lo}, 50\% \textit{Med} and 25\% \textit{Hi} applied to each input variable. The results are compared to those of the previous sensitivity test where the input variables are discretized through practical thresholds. In the same manner used previously, for each of the eleven input variables, the model is evaluated with all other variables declared to be in the medium state. Comparisons are made of the probability of a failed inventory load rating in each of these eleven cases with that of the medium state of all variables declared. This determines the effect that particular variable has on the outcome (probability of a failed inventory load rating). The results are presented in bar graphs with the difference between the probabilities of failure in the form of percentages. This is carried out for the temporal case at time \( t = 60 \) years. A sensitivity study of this sort is not carried out for the as-built case, as the probability of a failed load rating in each of the three methodologies is either zero or close to zero. In this investigation the BN having no inspection cluster was used in order to focus on the design variables as the input.
As shown in Figure 6.13, the input variable that is most influential upon the probability of a failed AS inventory load rating is the moment of inertia of the beam cross-section, $I$. The plot shows an 8.1% increase in the probability of failed load rating if $I$ is not declared in the medium state ($121,160 \text{ in}^4 – 129,620 \text{ in}^4$). Cross-sectional area of the beam, $A$, is the second most influential variable. The results show an increase of 5.5% in the probability of a failed load rating if $A$ is not declared in the medium state ($522 \text{ in}^2 – 598 \text{ in}^2$). All of the variables collectively cause a 12% increase in the probability of a failed load rating when there are none declared to be in any state.

**FIGURE 6.13.** Influence of Each Input Variable upon the AS Inventory Load Rating Factor, Temporal Case of BN, Quartile Distribution
As shown in Figure 6.14, the input variable that is most influential upon the probability of a failed LF inventory load rating is the area of prestressed strands, $A_s$. The plot shows a 700% increase in the probability of failed load rating if $A_s$ is not declared in the medium state (loss of 2-4 strands (0.124 in$^2$ – 0.142 in$^4$)). This result seems to be an outrageously high increase in the probability of a failed load rating. However, with the comparison being made to the probability of failure equal to 0.026, a 700% increase would increase the probability of failure to 0.206. In other words, when $A_s$ is not instantiated, while all other input variables are declared to be in the medium state, the probability of a failed inventory load rating is 0.206 (20.6%) compared to the 0.026 (2.6%) probability when all eleven variables are declared medium. Prestressing strand strength, $f'_s$, is the second most influential variable. The results show an increase of 77% in the probability of a failed load rating if $f'_s$ is not declared in the medium state (277 - 285 psi). All of the variables collectively cause a 630% increase in the probability of a failed load rating when there are none declared to be in any state. Using this distribution
of 25% Lo, 50% Med and 25% Hi, with all of the input variables declared in the medium state, the probability of failure of the LF inventory rating is a much lower 0.026 (2.6%). Each set of results is compared against this probability (where all input variables are set in the medium state). This explains why the results show such a dramatic increase, up from the extremely low probability of failure of 2.6%. Looking further into the high sensitivity of the LF inventory rating, the declaration of \( A_s \) in the high state, with all other input variables held in the medium state renders a probability of failure of practically zero (0.05%). On the other hand, the declaration of \( A_s \) in the low state, with all other input variables held in the medium state renders a probability of failure equal to 0.77 (77%). These results demonstrate the influence of \( A_s \) upon the probability of failure of the LF inventory rating.

![Figure 6.15](image)

**FIGURE 6.15.** Influence of Each Input Variable upon the LRFR Inventory Load Rating Factor, Temporal Case of BN, Quartile Distribution

As shown in Figure 6.15, the input variable that is most influential upon the probability of a failed LRFR inventory load rating is the area of prestressed strands, \( A_s \). The plot shows an 11.1% increase in the probability of failed load rating if \( A_s \) is not declared in the medium state (loss of 2 - 4 strands (0.124 in\(^2\) – 0.142 in\(^4\))). Cross-
sectional area of the beam, \( A \), is the second most influential variable. The results show an increase of 1.7% in the probability of a failed load rating if \( A \) is not declared in the medium state (522 in\(^2\) – 598 in\(^2\)). All of the variables collectively cause a 4.8% increase in the probability of a failed load rating when there are none declared to be in any state.

6.5.2 Comparison of Results, Sensitivity of Input Variables

As previously shown in the version having practical thresholds, the input variable that the probability of a failed AS inventory load rating is most sensitive to is the prestressed steel area, \( A_s \). Cross-sectional area of the beam, \( A \), is the second most influential variable. Considering the results of the 25\% Lo, 50\% Med, 25\% Hi distribution of input variables, the probability of a failed AS inventory load rating is most sensitive to the moment of inertia of the beam cross-section, \( I \). Cross-sectional area of the beam, \( A \), is the second most influential variable. It should be pointed out that the two sets of results cannot be compared directly. In each set of results, the probability of a failed AS inventory load rating resulting from every input variable being declared in the medium state is the basis used for analyzing the sensitivity results. Using the practical thresholds, this probability of failure of the AS inventory rating is 69\%. However, using the distributions of 25\% Lo, 50\% Med and 25\% Hi, this probability of failure of the AS inventory rating is a much lower 53\%. This explains why the results pertaining to the practical thresholds show a decrease in probability of failure, down from 69\%, and the 25%-50%-25% distribution shows an increase, up from 53\%.

As shown previously for the practical thresholds, the input variable that the probability of a failed LF inventory load rating is most sensitive to is the prestressed steel...
area, $A_s$. Cross-sectional area of the beam is the second most influential variable. For the thresholds determined by the 25%-50%-25% distribution into $Lo$, $Med$ and $Hi$, the input variable that the probability of a failed LF inventory load rating is most sensitive to is the prestressed steel area, $A_s$. Prestressed strand strength ($f_s'$) is the second most influential variable. As pointed out previously, each set of results is compared to a different probability of failure of the LF inventory rating. Using the practical thresholds, the probability of failure of the LF inventory rating is 15.6%. However, using the distributions of 25% $Lo$, 50% $Med$ and 25% $Hi$, this probability of failure of the AS inventory rating is a much lower 2.6%.

Pertaining to the practical thresholds, the input variable that the probability of a failed LRFR inventory load rating is most sensitive to is the prestressed steel area, $A_s$. Moment of inertia of the beam cross-sectional is the second most influential variable. Pertaining to the thresholds determined by the 25% $Lo$, 50% $Med$, 25% $Hi$ distribution, the input variable that the probability of a failed LRFR inventory load rating is most sensitive to is also the prestressed steel area, $A_s$. Area of the beam cross-sectional is the second most influential variable. Using the practical thresholds, once all of the input variables have been declared to be in the medium state, the probability of failure of the LRFR inventory rating is 52%. However, using the distributions of 25% $Lo$, 50% $Med$ and 25% $Hi$, with all of the input variables declared in the medium state, the probability of failure of the AS inventory rating is a much lower 40%. Each set of results is compared against this probability (where all input variables are set in the medium state). This explains why most of the results pertaining to the practical thresholds show a
decrease in probability of failure, down from 53%, and the 25%-50%-25% distribution shows an increase, up from 40%.

**Comparison of Results between Components, Sensitivity of Input Variables**

Comparing the results of the two threshold versions for the AS methodology, the practical threshold version is most sensitive to prestressed strand area ($A_s$) and second most to cross-sectional area of the beam ($A$). The results differ with the quartile threshold version, the most influential input variable being moment of inertia ($I$) and then cross-sectional area of the beam ($A$). It is important to know how these results compare to the sensitivity test results of the probabilistic prognostic component. Here, the AS inventory rating variable is most sensitive to prestressed strand area ($A_s$) and prestressed strand strength ($f'_s$), respectively.

Comparing the results of the two threshold versions for the LF methodology, the practical threshold version is most sensitive to prestressed strand area ($A_s$) and second most to moment of inertia of the beam cross-section ($I$). The results are similar with the quartile threshold version, the most influential input variable also being prestressed strand area ($A_s$). However, the second most influential variable is prestressed strand strength ($f'_s$). It is important to know how these results compare to the sensitivity test results of the probabilistic prognostic component. Here, the LF inventory rating variable is most sensitive to prestressed strand area ($A_s$) and prestressed strand strength ($f'_s$), respectively. These results are in agreement with those of the quartile threshold version.

Comparing the results of the two threshold versions for the LRFR methodology, the practical threshold version is most sensitive to prestressed strand area ($A_s$) and second
most to moment of inertia of the beam cross-section ($I$). The results are similar with the quartile threshold version, the most influential input variable also being prestressed strand area ($A_s$). However, the second most influential variable is cross-sectional area of the beam ($A$). It is important to know how these results compare to the sensitivity test results of the probabilistic prognostic component. Here, the LRFR inventory rating variable is most sensitive to prestressed strand area ($A_s$) and prestressed strand strength ($f's$), respectively. These results are in agreement with both threshold versions as far as the most influential variable is concerned, that being prestressed strand area ($A_s$).

6.5.4 Comparison of Results, Probability of Failure

The main result considered is the probability of failure pertaining to the beam element where failure is defined as the inventory load rating factor of the prestressed beam being less than one. This is calculated for each of the three methodologies. The probabilistic prognostic component yields the following probabilities of failure at year 60: AS (53%), LF (20%), and LRFR (43%). The diagnostic BN component having the practical thresholds yields the following probabilities of failure: AS (62%), LF (14%), and LRFR (45%). The diagnostic BN component having the quartile distributions yields the following probabilities of failure: AS (59%), LF (19%), and LRFR (42%). It is assumed that the most accurate calculation of the probability of failure is that of the probabilistic prognostic component using Monte Carlo simulation of the continuous distributions. The two sets of results representing the practical and the quartile thresholds are compared to these simulated results. The probabilities of failure calculated using the quartile distributions are in greater agreement with the simulated results. This indicates
that the arbitrary and uniform discretization of the variables may offer more valid results than that stemming from the practical thresholds. Future continuation of this research should consider these discretized distributions resulting from quartile distributions.

6.6 Sensitivity Test of Inspection Cluster of Diagnostic BN

This investigation tests the inspection cluster through the comparison of the results of the two components: probabilistic prognostic component and diagnostic Bayesian network in order to understand the entire model and its behavior for better interpretation of results. The version of the BN used in this test considers the correlation between the cross-sectional area of the beam \((A)\) and the moment of inertia of the beam cross-section itself \((I)\) and the composite section \((I_c)\), too.

Figure 6.16 is duplicated from the previous Figure 5.1 for explanatory purposes. In review, the probabilistic prognostic component consists of a program written in Matlab that performs Monte Carlo simulation of the load rating equations using the three methodologies: AS, LF and LRFR. The prognostic component works with continuous variables and provides statistics of the load rating factors generated as continuous variables. The diagnostic Bayesian network (BN) models the load rating process using the same variables as those used within the probabilistic prognostic component, although in a discrete format. While the BN has bi-directional capabilities, this investigation tests the functionality of the BN including the inspection cluster by focusing on its prognostic results in comparison with those of the probabilistic prognostic component.
As explained previously, these two components (probabilistic prognostic component and diagnostic BN component) both model the load rating process using two different computational tools: Monte Carlo simulation (probabilistic prognostic component) and a Bayesian network (BN). The diagnostic BN is dependent upon the prognostic component for input. The software shell used for the BN is MSBNx, which at this time is constrained to discrete variables only. The prognostic component discretizes the input variables into three states, Low, Medium and High according to set thresholds characteristic of each variable. The prognostic component also determines the conditional probabilities of each child variable in conjunction with the Monte Carlo simulation. With the input of the diagnostic BN coming directly from the calculations of the prognostic component, one would expect the results of the two components to be similar. This investigation takes an in-depth look into this comparison.
6.6.1 Comparison of Input Variables

In comparing the input variables of the probabilistic prognostic component vs. the diagnostic Bayesian network, the most notable difference is the type of variable. The input variables for the probabilistic prognostic component are continuous. The input variables for the diagnostic BN are discrete. Also, the point of input differs in each model. In other words, the input or root variables for the Monte Carlo simulation are the resistance and loading effects \( (A, A_s, f'_{cb}, \text{etc}...) \) and these variables are assumed independent. Recall that the investigation into the correlation of certain input variables (Chapter IV) reveals an insignificant impact on the results. This is not the case for the BN model as the variables \( A \) and \( A_s \) are no longer independent root variables. Figure 6.17 shows this difference. The design variables shown in gray are considered independent within the probabilistic prognostic component. However, with the diagnostic BN shown here, the variables \( A \) and \( A_s \) are linked through the inspection cluster. \( A \) and \( A_s \) are no longer independent root variables.
Comparing distributions of $A_s$ at time $t = 10$ years

The inspection cluster attaches onto the load rating core of the BN at the two variables, $A$ and $A_s$. No longer root variables, the distributions of these variables ($A$ and $A_s$) are interpreted by the BN and differ from the corresponding discrete translation of the continuous variable used in the Monte Carlo simulation. Prestressing strand area ($A_s$) at time $t = 10$ years is shown in Figure 6.18. In the probabilistic prognostic component, $A_s$ is a normal variable with a bias of 1.00 and a coefficient of variation (COV) of 0.10 representing the variation that could arise in fabrication but more importantly in deterioration. The thresholds are set diving the states Low, Medium and High, the low threshold set according to the number of strands assumed to be deteriorated (Lo: 4 or more strands deteriorated, Med: 2-4 strands deteriorated, Hi: less than 2 strands
deteriorated). The probabilistic prognostic component discretizes the variable into 3% Lo, 14% Med and 83% Hi. However, in the BN component, this variable is embedded and influenced by other variables and the discrete distribution is interpreted as 7% Lo, 20% Med and 73% Hi, showing a lower probability that \( A_s \) has less than 2 deteriorated strands.

**FIGURE 6.18.** Comparison of Variable \( A_s \) in Prognostic Component vs. BN Component, Time, \( t = 10 \) years

In order to model deterioration, time was incorporated into the model. The BN represents one point in time, therefore, it was run at time steps of 10 years. The probabilistic prognostic component was also run at each of the corresponding time steps to provide the conditional probabilities. At each time step, the probabilistic prognostic...
component was adjusted through the mean of the input variable, $A_s$, in order to represent the decrease through time due to corrosion. The generated conditional probability tables were input into each time case of BN component. Figure 6.18 shows this process for the variable $A_s$ at time $t = 10$ years.

**Comparing distributions of $A_s$ at time $t = 70$ years**

At time step, $t = 70$ years, Figure 6.19 shows the discrete distribution translated by the Matlab simulation of the prognostic component to be 47% $L_o$, 40% $M_{ed}$, 13% $H_i$, reflecting the decreasing area of prestressed strands due to corrosion. However, at this time step, the BN interprets the distribution of $A_s$ to be 28% $L_o$, 28% $M_{ed}$, 44% $H_i$. This distribution does not represent the anticipated deterioration of $A_s$ due to corrosion. However, the variables with which the $A_s$ is dependent upon have not been declared. For example, if the following states of variables are declared: Environment as Severe, Efflorescence as Yes, Rust Stains as Yes, Cracking as $> 0.0625$ inches, Spall as Large, as conditions would be expected at 70 years, the distribution of $A_s$ is updated to 49% $L_o$, 41% $M_{ed}$, 10% $H_i$ which is more comparable to the distribution of $A_s$ shown in the probabilistic prognostic component.
FIGURE 6.19. Comparison of Variable $A_s$ in Prognostic Component vs. BN Component, Time, $t = 70$ years
Cross-sectional Area of Beam, $A$

In the Monte Carlo simulation of the prognostic component, $A$ is a Normal variable with a bias of 1.00 and a coefficient of variation (COV) of 0.10 representing the variation that could arise in fabrication but more importantly in deterioration. The thresholds divide the continuous variable into Lo, Med and Hi states, the low threshold set according to estimated spalling patterns. At the temporal time steps, $t = 10, 20, 30$ years, etc., the probabilistic prognostic component discretizes the variable into 39% Lo, 22% Med and 39% Hi. However, in the diagnostic BN, this variable is embedded and influenced by other variables and the discrete distribution is interpreted as 41% Lo, 21% Med and 38% Hi, slightly shifting the distribution so that there is an increase in the probability of $A$ being in the Lo state.
FIGURE 6.20. Comparison of Variable $A$ in Prognostic Component vs. BN Component, Time, $t = 70$ years

In the successive time step runs of the Monte Carlo simulation of the prognostic component, $A$ does not change and is considered independent of the other input variables.
In the BN component, however, $A$ is indirectly linked to the decreasing $A_s$ and its distribution through the years shows the effects. At time step, $t = 10$ years, $A$ is 41% $Lo$, 21% $Med$, 38% $Hi$. However, at time step, $t=70$ years, $A$ is 44% $Lo$, 25% $Med$, 31% $Hi$, showing a shift in the distribution with a decrease in the probability that $A$ is in the $Hi$ state (See Figure 6.20). If the states of variables in the inspection cluster are declared to reflect the deteriorated condition of a 70 year old bridge: $Environment$ as $Severe$, $Efflorescence$ as $Yes$, $Rust Stains$ as $Yes$, $Cracking$ as $> 0.0625$ inches, $Spall$ as $Large$, the distribution of $A$ is updated to 76% $Lo$, 19% $Med$, 5% $Hi$. Comparison of the discrete distribution of $A$ with and without the evidence of an inspection entered in, demonstrate how the BN component models the deterioration of $A$ as a product of corrosion, where this is not considered in the prognostic component. The remaining input variables representing the resistance and loading effects ($f'_{cb}$, $f'_{cd}$...etc.) are considered independent root variables in both the probabilistic prognostic component and the diagnostic BN.
6.6.2 Comparison of Results, Sensitivity of Inspection Cluster

In this section the results are compared between the two components, probabilistic prognostic component and the diagnostic BN component. Specifically, the inventory load rating calculated according to the three methodologies is focused on. Each plot shows the inventory load rating calculated as a continuous variable by the probabilistic prognostic component and then converted to a discrete variable. This discrete variable is compared to the same discrete variable in the diagnostic BN in two ways, with and without evidence. In other words, the states of inspection variables that represent symptoms of deterioration observed in the field are declared (evidence) or not declared (no evidence). See Figures 6.21 to 6.23. The objective of these comparisons is to test the sensitivity of the inspection cluster of the BN against the Monte Carlo simulation of probabilistic prognostic component. The numerical results of the two components are reasonable according to expert opinion. In future research, comparison to a number of case histories would validate this opinion.

**AS inventory rating, ASinv**

At time, \( t = 70 \) years, the results for the AS inventory rating variable as shown in Figure 6.21. The simulated result of the prognostic component shows 74% probability of a failed load rating, with 26% probability of a passing load rating. As for the BN results, there is only a 58% probability of a failed load rating and 42% probability of a passing load rating. However if the following variables are declared: Environment as Severe, Efflorescence as Yes, Rust Stains as Yes, Cracking as > 0.0625 inches, Spall as Large, as
conditions would be expected at 70 years there is a 68% probability of a failed load rating.

FIGURE 6.21 Comparison of Variable $ASinv$ in Prognostic Component vs. BN Component, Time, $t = 70$ years
**LF inventory rating, LFinv**

At $t = 70$ years, the simulated continuous distribution shows a 42% probability of a failed LF inventory rating. The BN shows only a 33% probability of the LF inventory rating being less than one. However if the following variables are declared: *Environment* as *Severe*, *Efflorescence* as *Yes*, *Rust Stains* as *Yes*, *Cracking* as $>0.0625$ inches, *Spall* as *Large*, as conditions would be expected at 70 years, there is a 44% probability of a failed load rating, as shown in Figure 6.22.
FIGURE 6.22 Comparison of Variable $L_{Finv}$ in Prognostic Component vs. BN Component, Time, $t = 70$ years
**LRFR inventory rating, \( LRFR_{inv} \)**

At \( t = 70 \) years, the prognostic component shows a 68% probability of a failed load rating while the BN offers a discrete distribution with a 76% probability of failed \( LRFR_{inv} \). If the following variables are declared as with the other methodologies (Environment as Severe, Efflorescence as Yes, Rust Stains as Yes, Cracking as \( > 0.0625 \) inches, Spall as Large), the probability of a failed LRFR inventory load rating is 83%.
FIGURE 6.23 Comparison of Variable $LRFR_{inv}$ in Prognostic Component vs. BN, Time, $t = 70$ years
At 70 years, the results show a large spread among the probabilities of failure of the inventory ratings, as shown in Table 6.4. This is true for both the probabilistic prognostic component and the diagnostic BN component. This variance in results stems from the increased variability of some of the input variables such as $A$ and $A_s$, their COV increasing to 0.10. One commonality is that the LF methodology renders the lowest probability of a failed load rating. This is an expected result, as explained previously. Typically, prestressed concrete beams are designed according to service loads in order that internal stresses may not surpass a specified limiting stress. Satisfying the service limit state typically controls design and calls for additional strands and increased capacity of the beam. Load Factor (LF) is often used as a follow up to verify ultimate capacity. However, using the LF methodology to calculate the inventory rating factor renders a falsely higher load rating factor due to the fact that it is not consistent with the original design assumptions (PCI 2003).

The prognostic component shows AS offering the highest probability of a failed load rating while the BN shows LRFR methodology with the highest (See Table 6.4). The probabilistic prognostic component shows the expected result in that the AS methodology is most sensitive of all three methodologies to a decreasing area of prestressed strands (Figure 5.29 shows AS has the greatest increase in probability of $A_s$ being low as a source of a failed load rating). This stems from AS being based on service limits while LF and

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**TABLE 6.4 Comparisons of Probabilities of Failure of Inventory Ratings, Year 70**
LRFR are based on strength limits. Further, there are two sources of explanation for the LRFR inventory rating having the highest probability of failure in the BN component. The prognostic component considers all root variables to be independent. In other words, the decreasing $A_s$ over time does not influence $A$. This is not the case in the BN, in addition to $A_s$ are also the influences of all the variables within the inspection cluster upon $A$. Secondly, the influence of the decreasing $A_s$ upon $A$ is carried into other variables in the LRFR methodology such as the longitudinal stiffness parameter, $K_g$, which includes the influenced $A$ within its computational formula.

Another general observation that may be made regarding the Bayesian network is the increase in the probabilities of a failed load rating considering the evidence of deteriorated conditions. These results are expected as the observations of a field visit add certainty and enable the various states of the inspection variables to be declared within the BN offering a better assessment of the load carrying capacity of the bridge.

6.7 Summary

In this chapter the model was verified in detail from three aspects: 1) input, 2) output and 3) sensitivity studies. Specifically, input to the main components of the model (probabilistic prognostic and diagnostic BN) were explained in detail and defended. The outputs of these two components were reviewed and their reasonableness discussed. The following sensitivity studies were summarized and discussed: input design variables, discretization thresholds of variables, functionality of inspection cluster.
CHAPTER VI

APPLICATION OF MODEL TO AN ACTUAL BRIDGE BEAM

7.1 Introduction

This chapter demonstrates the application of the model to an actual bridge, which shall remain unidentified (Literature sources available from author upon request). The bridge, built in 1965, is slated for necessary replacement due to the extensive deterioration caused by a corrosive marine environment. The results of the model, both prognostic and diagnostic, are compared to the actual inspection data and rating calculations over the life of the bridge. The beneficial uses of the model are explored and discussed in a retrospective manner, with regards as to how the results could have helped to avoid this costly bridge replacement.

FIGURE 7.1 Example Bridge
The example beam is part of a bridge located in New England. It carries a causeway over a harbor providing access to a luxury hotel from the city in which it is located. It has twenty-four simply-supported spans carried by prestressed concrete I beams, six AASHTO Type II girders spaced at 6’-6”. The reinforced concrete deck is composite with a 6 ½ inch thickness. The bridge roadway width (curb to curb) is twenty-five feet. It carries two-way east-west traffic. There is a 6’-0” sidewalk along the south side of the bridge and a 5’-0” foot sidewalk along the north side of the bridge. Both sidewalks have a 2 foot concrete parapet with a one bar railing. Each span length is 43’-0” measured center-to-center of bearings. The supporting substructure is made up of 2 concrete abutments and 23 concrete-filled steel bents. The total structure length is 1046’-6”. The following utilities are carried by the bridge: water (18” diameter), sanitary (12” diameter), (2) gas (both 4” diameter). This bridge was built in 1965.

![Elevation of Bridge](image)

**FIGURE 7.2** Elevation of Bridge
FIGURE 7.3 Cross-section of Bridge

FIGURE 7.4 Cross-section of Beam of Bridge
7.2 History of Rating and Inspection Reports

The following reports were available through the department of transportation of the state in which the bridge is located (Literature sources available from author upon request): a 1985 rating report, a 1995 rating report and a 2002 inspection report.

7.2.1 1985 Rating Report

In June 1985, a rating report was completed. The rating was based on original plans and design computations. Any information that was not available was obtained during a field inspection, which consisted of a visual assessment of the superstructure only. This inspection was performed in November and December of 1984, when the bridge was approximately twenty years old.

The inspectors observed that the end spans showed the most deterioration. There were large spalls and cracks located at the bottom flange that exposed stirrups and prestressed steel. They attributed the increased deterioration of these spans to the spray of salt water crashing into the bulkheads. Also observed was a large spall at the corner of the bottom of flange of Span 11 over the navigable waterway. The inspectors suspected marine collision(s) precipitated this spall. Detailed field notes have been reproduced in Appendix D1. Even though deterioration was observed, the load rating of the bridge calculated by the engineers was passing, or greater than 1.00.
7.2.2 1995 Rating Report

In 1995, a rating report was completed. This rating was based on original plans and design computations. The previous rating report completed in 1985 was referenced also. A field inspection was performed, the bridge being approximately 30 years old.

The engineers noted the most deteriorated area was located in the first span on the mainland side. The beam in the worst condition had a large diagonal spall approximately 15 feet long and half the depth of the flange. Three of the prestressing strands were exposed and completely corroded. Therefore, the engineers eliminated these 3 strands from the rating analysis. The remaining beams in this span were also observed to have a varying degree of spalling and deterioration.

The engineers attributed the great degree of deterioration of this span (as well as the other end span) to the “close proximity of the water line and the resulting ocean spray” (reference available upon request). They recommended that rehabilitation be scheduled “as soon as possible” (reference available upon request). They also recommended that prior to rehabilitation that the frequency of inspection be increased from the mandated two years to every year. Detailed field notes have been reproduced in Appendix D1. Despite the poor condition observed and the urgent recommendations given, the load rating of the bridge calculated above 1.00.

7.2.3 2002 Inspection Report

In 2002, the Maintenance Division of state department of transportation conducted a routine inspection. The inspectors observed advanced section loss, deterioration and spalling. In the problem span, first in from the mainland, the worst
beam had “midspan north and south flange heavy spalling and cracking with rebar (30’ x 18” x 3”) with section loss and broken stirrups” (reference available upon request). The inspectors also noted broken wire strands having heavy section loss. However the number of affected strands was not recorded. Detailed field notes with commentary can be found in Appendix D1. After this inspection the decision to replace the bridge was made.

7.2.4 2007 Field Visit

In 2007, a field visit was conducted by the author. A cursory inspection was performed of the end span that suffered the most extensive deterioration. The photos reveal the extensive spalling, section loss and corrosion of the exposed strands. Broken stirrups are visible and hanging down. See Figures 7.5 and 7.6, photographs taken during field visit in October 2007.

FIGURES 7.5 and 7.6  Deteriorated Beam of Bridge No. 697, South Face and North Face
In the same manner as the two previous, a rating analysis was conducted based upon the observed condition of the beam. Specifically, it was conservatively assumed that the ten exposed prestressing strands were no longer effective. The inventory rating fails and the operating rating passes. Typically in the state in which this bridge is located, bridges are posted once the operating rating fails. This bridge remains without a posted load limit. The replacement plans are in the design stage.

7.3 Application of the Model to the Example Bridge Beam

In order to complete this example problem, each component of the model (deterioration, probabilistic prognostic and diagnostic BN) had to be customized to the example bridge.

7.3.1 Deterioration Component

First, the component that models the deterioration due to corrosion was modified. This component programmed in Matlab by the author calculates the decrease in area of prestressing strands through time. The deterioration component is explained previously in Chapter III. Remaining the same within the deterioration component are the diffusion coefficient, $D_c$, critical chloride concentration, $C_c$, surface chloride concentration, $C_o$, and the corrosion rate, $r_{corr}$. The values used for these parameters were chosen previously based on the assumption of a highly corrosive environment. The example bridge crosses over saltwater and is also subjected seasonally to deicing salts. The physical geometric parameters changed, specifically, the original area of a prestressing strand and the layout pattern of the strands. The positioning of the strands dictates the distance the chloride
ions must diffuse through from the surface. Figure 7.7 shows the sixteen strands that are a distance of 2 inches from the surface of the beam. The rest of the strands are at varying distances from the surface as shown in Appendix D2.

Reinforcing steel within the beam is shown in the beam cross-section of Figure 7.8. It should be noted that the diffusion of chloride ions encounters the reinforcing steel before the prestressing steel. This induces delamination, cracking and spalling and ushers in the initiation of corrosion of the prestressing strands. The reinforcing steel is not directly accounted for within the model. However, the acknowledgement of the possible effects upon the prestressing steel helps to justify the conservative choices for parameters, such as the diffusion coefficient, $D_c$. 

**FIGURE 7.7.** Diffusion Distance, Group 1
The results of the deterioration component offer an approximate representation of the effects of the deterioration mechanism, corrosion. The reinforcing steel, being the closest to the surface begins the corrosion process at about 7 years. Figure 6.32 shows the effects of the corrosion in terms of a decreasing number of strands through time. The deterioration begins at approximately 14 years with a loss of 2 to 3 strands for every 10 years. Figure 6.33 presents the deterioration in terms of decreasing average strand area. The plot shows an average decrease in strand area of about 0.005 in$^2$ every ten years.

Comparing the results of this model against the inspection notes at 30 years three strands were observed to be entirely corroded and the corrosion prediction equation estimates a reduction to 30.5 strands from 34 at year 30, a comparable value. However, the most recent field visit in 2007, when the bridge was approximately 42 years old,
revealed ten exposed and corroded strands. Although some of these strands may still provide strength to the beam, it could be argued that they are no longer effective. The number of strands reduced to 24 occurred at the age of 42, whereas the model predicts a reduction to 27 strands at this age. The rate of deterioration of this particular bridge appears to accelerate as the bridge ages. The results of the component show an average rate of deterioration that is useful as a basic predictor especially in the early stages of a bridge service life when rehabilitation and preservation is possible.

**FIGURE 7.9.** Number of Prestressing Strands vs. Time
7.3.2 Probabilistic Prognostic Component

The probabilistic prognostic component was modified to reflect the geometry of an interior beam of the example bridge. The distributions of some of the root variables changed. The nominal value of the cross-sectional area of the beam ($A$) is 369 in$^2$. The coefficient of variation (COV) is set at 0.10 to account for uncertainty in the size of the beam that occurs through time due to different levels of deterioration due to exposure to a variety of climates. The mean is set equal to the nominal value. The thresholds are set at values below the mean based on deterioration patterns A and B as shown in Figures 7.12 and 7.13, respectively.
FIGURE 7.11. Cross-sectional Area of Beam, $A$

Cross-sectional Area of Beam, $A$
Nominal = 369 in$^2$
Mean = 369 in$^2$
COV = 0.10
Lo threshold = 350 in$^2$
Hi threshold = 360 in$^2$

FIGURE 7.12. Deterioration Pattern A

2 strands exposed
Spalls: 3” x 3”
Original Area: 369 in$^2$
Deteriorated: 360 in$^2$
$\sim$ 2% Section Loss
The distribution for the area of prestressing strands, $A_s$, changed also. There are 34 strands, each having a diameter of $3/8$ inch and area of $0.0799 \text{ in}^2$. At the initial time step, where $t = 0$ years, the strands are all assumed to be intact, the mean is set at the nominal value, $0.0799 \text{ in}^2$. Thresholds for the distribution are set at values lower than the nominal at $0.0682 \text{ in}^2$ and $0.0728 \text{ in}^2$. These thresholds represent the loss of 5 strands and 3 strands, respectively.

**FIGURE 7.13.** Deterioration Pattern B

6 strands exposed
Spall: 2” x 10”
Original Area: 369 in²
Deteriorated: 350 in²
~ 5% Section Loss
Based on results from the deterioration component, this distribution of $A_s$ shifts through time, the mean decreasing with the loss of strands, as shown in Figure 7.15. It is assumed that the strand loss occurs initially with those in the outermost layer of the bottom flange. The eccentricity of the strands is recalculated at each time step to reflect this loss pattern. The distributions of the remaining time steps are shown in Appendix D3.
FIGURE 7.15. Area of Prestressing Strand, \( A_s \), \( t = 20 \) Years

Within the prognostic component, a few changes were made to the thresholds of some of the calculated variables, such as the nominal moment strength of section, \( M_n \). Previously, the thresholds for this variable were set at particular values above and below the mean. However, the beam of this bridge has less capacity than the example beam used to develop the model and the thresholds had to be adjusted to be applicable. Most of the variables calculated within the prognostic component have thresholds that are calculated at some percentile, say 25\(^{th}\), of the distribution.
7.3.3 Diagnostic Component

The diagnostic component, or the BN, required no changes to its structure. The network represents the load rating of a prestressed concrete beam along with the inspection symptoms related to the typical deterioration mechanism, corrosion. However, the distributions of the root variables, such as $A$ and $A_s$, did change to fit the this particular bridge. Also the conditional probability tables produced from the prognostic component were input into the BN. The prognostic component was run at each time step, likewise a BN was loaded with the input, ending up with a BN at each 10 year time step.

7.4 Results

7.4.1 Probabilistic Prognostic Component

The results of the prognostic component are presented in this section. Figure 7.16 shows the means of the inventory rating factors plotted through time. A decrease in magnitude is shown for all three methodologies. However, the decrease of the AS inventory rating factor is more pronounced than the others at an average rate of 16.5% every 10 years after deterioration begins. The LF and LRFR inventory rating factors both decrease at an average rate of 12% every 10 years once deterioration begins.
In explanation, the calculation of the AS rating factor considers the stresses of the prestressing tendons, as well as the losses, such as those due to creep. The final stress within the tendons (after losses) is influential upon the resulting AS inventory rating. This sets it apart from the other two strength methods. The calculation of the moment capacity is most influential with the LF and LRFR methods. All three methods account for the decrease in area of prestressing steel. However, the prestress losses are not considered within the LF and LRFR methods, because they are nominal strength calculations. Figure 7.17 shows a comparison between the final prestress stress and the nominal moment. The final prestress stress shows a greater decline over the years than the nominal moment. These trends help explain the resulting values of the inventory rating factors through time.
FIGURE 7.17 Ratios of Final Prestress Stress and Nominal Moment Through Time

Comparing the results of the three methods, the inventory rating factor calculated according to AS is greater than LF until year 50. In most cases, this is not the expected result. (Typically prestressed concrete beams are designed according to service loads in order that internal stresses may not surpass a specified limiting stress. Satisfying the service limit state typically controls design and calls for additional strands and increased capacity of the beam. Load Factor (LF) is often used as a follow up to verify ultimate capacity. However, using the LF methodology to calculate the inventory rating factor renders a falsely higher load rating factor due to the fact that it is not consistent with the original design assumptions (PCI 2003).) In this particular beam, the dimensions as well as the strand pattern result in an AS inventory rating that is greater than the LF inventory rating. Also, the strength of the deck concrete, which has an impact on the LF inventory rating is lower than typically used. In comparing the LF curve to the LRFR curve, the LF results are also greater than those of LRFR. These findings are in agreement with previously conducted research by others who also concluded that LRFR method generally resulted in lower rating factors for flexure (Lichtenstein 2001, Rogers and
Researchers Rogers and Jauregui conducted a comparative analysis on rating factors calculated using LF vs. LRFR and attribute the live load effects as the primary factor in this difference.

The results of the prognostic component compare with the actual performance of the bridge in regards to inventory rating. Both the 1985 and 1995 load rating analyses calculated a passing inventory load rating according to the AS methodology. The results of the prognostic component are in agreement and in fact, at these bridge ages of 20 and 30 years, all three methodologies render a passing mean inventory load rating. However, based on the 2007 site visit, the load rating analysis at 42 years bridge age discounted the 10 exposed, corroded strands resulting in a failed AS inventory rating. According to the results of the prognostic component, there is a nine percent chance of the AS inventory load rating being below 1.00 at age 40. As pointed out previously, the results of the deterioration component show agreement with the performance of the example bridge for the first 30 years, after that the deterioration accelerates at a greater rate. However, the prognostic component incorporated with the results of the deterioration component still serves as a basic predictor that renders useful information that can be used while the beam is in a condition state worth preserving.

7.4.2 Diagnostic BN Component

Three diagnostic tests were conducted where the inventory load rating factor was declared to be less than one in each of the three methodologies. In reviewing the results, comparisons were made between assumed distributions and updated distributions of the input parameters. The parameter showing the most dramatic change is the most likely
source of the problem. At all time steps, the variable showing the most change was prestressing steel area ($A_s$). This was the anticipated result as the deterioration mechanism, corrosion, is most influential over this variable.

**FIGURE 7.18** Comparison of Influence of Deficient States of Input Variables, Year 40

Figure 7.18 shows interpretations of the results from the diagnostic BN. Comparisons were made between the updated and the assumed distributions of seven input parameters given a failed inventory load rating. The increase in probability of the occurrence of the problematic states of each variable was extracted and plotted for observation. Figure 7.18 represents the bridge at 40 years of age, which is approximately the current age. Considering all methodologies, the most sensitivity is shown with the area of prestressing strands ($A_s$), as expected.

In making a site visit to the bridge in 2007 when the bridge age was 42 years, comparisons were made between the results shown in Figure 7.18 and the deterioration that the bridge actually showed. The problematic states listed in Figure 7.18 list only
three that are observable visually. An inspector would be able to see if there was any loss in the cross-sectional area, \( A \). Also, it may be possible that an inspector would be able to at least partially observe any loss of prestressing steel area \( A_s \), given extensive spalling. Thirdly, the thickness of the wearing surface \( wst \) could be measured and compared with the design value.

Determining other properties and whether they are in a problematic state would require more effort, most likely destructive testing. Focusing on the prestressing steel area being not high, this was in fact observable and may be seen in Figures 7.5 and 7.6. Extensive spalling was also observed on the site visit. However, most of the section loss consists of the concrete that makes up the protective cover over the reinforcing and prestressing steel and is not considered structural in nature. The loss of the prestressing steel area is what is critical in the deterioration of the beam’s structural performance.

Further analysis was done of the results of the BN. Figure 7.19 shows comparison of the influence of deficient states of input variables at year 50, approximately the year 2015. Similar to Figure 7.18, at age 40, the most sensitivity is shown with the area of prestressing strands \( A_s \), as expected. However, the increase in % points of probability of the deficient state of prestressing steel is about 10 points less at age 50 than at age 40. It is also noted that there is a greater possibility of the cross-sectional area being too low as a contributor to a failed load rating for the strength methods, LF and LRFR. At each time step, the conditional probability tables as well as the distributions of some of the input variables change. This leads to an evaluation that renders results particular to each time step. In other words, the relative comparison between the input variables and the updated variables differs with each time step offering
slightly increasing degrees of culpability of each of the problematic states in causing a failing load rating. It is reasonable to conclude that while the loss in prestress is still the main source of a failed load rating, the increasing section loss due to spalling shows a greater contribution to the threat to structural integrity of the beam as time goes on.

**FIGURE 7.19.** Comparison of Influence of Deficient States of Input Variables, Year 50

The results at time step year 60 are shown in Figure 7.20. This figure shows comparisons of deficient input states similar to Figures 7.18 and 7.19. However, the particular deficient state, *Prestressing Steel Area not high*, no longer exists. Figure 7.21 shows the distribution used for the input of the variable $A_s$ at the time step, year 60. The deterioration component, along with the prognostic component predicted these statistics which show the distribution within only two states of the original three states as determined by the thresholds. Therefore, the input into the BN for the variable $A_s$ consisted of two states only, *low* and *medium*. Figure 7.21 also shows the overwhelming majority of the distribution of $A_s$ below the low threshold. This input affected the results,
as expected. Figure 7.20 compares the deficient input states. Note that in regards to the prestressing steel area, the deficient state considered is that of the steel area being too low as opposed to not high. It is clear that the deficient state involving the prestressing steel area is still the most likely source of the failed inventory load rating. However, in the scale of the percentage (%) points, probability is smaller by a factor of ten when compared to the results of time step year 40 or year 50. This is due to the fact that the input distribution of $A_s$ is down to two states from the original three and also almost the entire distribution is within the low state. The updated distribution shows minimal change from the input distribution, understandably, because the original distribution shows how deficient the area of prestressing steel is in the first place. The results of this time step demonstrate how important it is to accurately define the distribution of the input variables.

**FIGURE 7.20** Comparison of Influence of Deficient States of Input Variables, Year 60
This investigation into the possible source of a failed load rating considering material and loading effects was performed to test the BN. The BN was built considering the deterioration of the prestressing steel, specifically the decrease in area due to corrosion, as well as the decrease of the cross-sectional area of the beam due to spalling, a product of corrosive deterioration. As anticipated, in the comparison of the assumed distributions of the material and loading effects and the updated distributions once the failed load rating was declared, the results show the area of prestressing steel as the most likely source of the problem. Unfortunately, in the basic visual inspection of a bridge, the area of the prestressing steel is typically not observable, which leads to the next investigation.
7.5 Diagnostic BN as an Inspection Planner

On this particular bridge, the condition of some of the prestressing steel is visible to the inspector. The inspection reports indicate loss of prestressing steel. This is due to the fact that the inspections occurred once the deterioration was advanced. Typically, once deterioration of the prestressing steel begins, it is not observable by the inspector. However, there are symptoms that may be visible such as efflorescence, rust stains, cracks and spalls. Further, if the inspector has access, hammer sounding could be performed in order to detect delamination. Also, performing non-destructive tests, such as measuring the half cell potential offers information on the presence of corrosive activity. In this investigation, the model is tested out to determine its usefulness in devising an inspection plan that assesses the deterioration so that it can be addressed in order to avoid necessary bridge replacement that advanced deterioration warrants.

MSB$N_x$ has capabilities that direct the user in conducting a diagnostic investigation efficiently. In this study, the decreasing area of prestressing steel due to corrosion is the most likely source of the failed load rating. The visible symptoms of this corrosive activity are efflorescence, rust stains, delamination, cracks, and spalls. These symptom nodes are designated informational. The variable that is not observable, area of prestressing steel ($A_s$) is designated hypothesis. MSB$N_x$ performs an analysis and ranks the symptom nodes based on their value of information, or amount of weight that their evidence would bring to the hypothesis node. This is a utility-based diagnosis based on mutual information (Kadie et al. 2001). The resulting output is a list of symptoms which when observed in the suggested order, would provide the most clear and effective diagnosis.
Figure 7.22 shows the value of information the observance of each symptom offers in assessing the condition of the prestressing steel. The plot shown through time indicates that the value of information provided by each symptom increases as the beam deteriorates. Clearly, the most valuable test is the half cell potential. However, this type of test is not routinely used in the state in which the bridge is located. A study conducted in 2001 (Moore et al. 2001) showed that only 11 states use the half cell potential test.

Inspection programs did not come into full swing until the 1980s and from the available reports, it appears that the first inspection during the service life of the bridge occurred in 1985 when it was 20 years old. At this point, the end spans showed signs of deterioration with spalling that exposed the prestressing steel. In hindsight, had a two year inspection cycle been executed, making use of a half cell potential test as shown to be most valuable by the BN, the inspectors would have had an idea of the deterioration
that was occurring but not visually observable. At that point, measures could have been taken to arrest the development of any further deterioration. Some suggestions include more frequent inspection along with an aggressive seal coat program.

In evaluating the possible cost savings of pursuing a thorough inspection and rehabilitation program as opposed to complete bridge replacement, the following cost comparison was made. First of all, to improve the quality of the inspection by incorporating the half cell potential test, this equipment would have to be purchased at a one time fee of approximately $5,000 (NDT James 2007) that could be spread out among other similar projects. Other costs to be considered would be the training of the inspection personnel on the use of the equipment, as well as the additional time necessary to complete the inspection. Another option would be to use a subcontractor to perform the test. There are no subcontractors available in the state in which the bridge is located that perform a half cell potential survey. Searching on the internet, one located in Pennsylvania gives an idea about the associated costs. The company literature states that it can complete a survey of 500-600 square foot per day. For this bridge, that means it would take 4 days to complete one span. The crew works at an hourly rate, with a principal, senior consultant and technician, the grand total would be $375 per hour or say $4,000 per day and this does not include the travel and lodging for the crew. The whole contract for a one time half cell potential survey of the two end spans only would cost over $30,000 (CMC 2007). Based on this information, it seems fiscally wise that a state department of transportation purchase its own equipment for its own inspection crews to use.
With results of the half cell potential test showing corrosive activity, a waterproof seal coat could be applied to the necessary spans. The cost of a new waterproof seal coat would be approximately $75,000 per span. Initially, only the two end spans would have to be treated. An additional application may be necessary sometime during the service life of the bridge (Amos 2007).

A new bridge would cost over six million dollars (RIDOT 2007). This rough estimate is based only on demolition of the current bridge with the rebuilding of a same size bridge. It does not include approach work or traffic control. The design of the new bridge should address the splashing breakwater that was so damaging to the end spans. There are possible solutions. Using the same profile of the bridge, the hydraulic opening could be decreased by filling in below the end spans with rip rap. The spans that are over the waterway would have more of a clearance than the current end spans. However, decreasing the hydraulic opening of the bridge would raise environmental and permitting issues. Another solution would be to raise the profile of the bridge, in order to offer the end spans more clearance. This would necessitate more approach work and costly landtaking in a densely population and historically preserved section of the city in which the bridge is located. Currently, there are many research efforts to find a building material for marine structures that will last in a marine environment. Cutting edge material technology is usually expensive without a proven performance of longevity.

7.6 Inspector’s Use of the BN

In this investigation, the diagnostic BN is used in a way that inspectors might use it, which is to obtain an immediate assessment of the bridge. During a field inspection,
the inspector makes note of the condition of the bridge members. This evidence may be entered into the BN. Instantaneously, the inspector could get a probability as to whether the bridge fails its inventory rating.

Based upon the inspection reports available (references available upon request), evidence was entered into the BN as shown in Table 6.5.

**TABLE 6.5** Evidence Input into BN

<table>
<thead>
<tr>
<th></th>
<th>1985 (20 years)</th>
<th>1995 (30 years)</th>
<th>2002 (40 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cracks</strong></td>
<td>large</td>
<td>large</td>
<td>large</td>
</tr>
<tr>
<td><strong>Delamination</strong></td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td><strong>Efflorescence</strong></td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td><strong>Rust Stains</strong></td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td><strong>Spalls</strong></td>
<td>large</td>
<td>large</td>
<td>large</td>
</tr>
<tr>
<td><strong>Environment</strong></td>
<td>severe</td>
<td>severe</td>
<td>severe</td>
</tr>
</tbody>
</table>

Table 6.5 lists the typical symptoms of deterioration associated with a prestressed concrete beam. As explained before, efflorescence, delamination and rust stains are precursors to cracks and spalls. However, with the inspection beginning at bridge age 20 years, the inspector already notes the cracks and spalls. The existence of the other symptoms are irrelevant at this point. If an inspection had been completed prior to 1985, the observance of delaminations and rust stains prior to cracking and spalling would have been significant.

With the evidence of Table 6.5 entered as declarations of certain states of the listed variables, the BN offers the probability of failure (a load rating less than 1). In 1985, at age 20 years, the BN gives a probability of a failed inventory load rating of less
than 1% for all methodologies. In concurrence, the consultant that completed the 1985 rating report calculated a passing load. At the age of 30 years in 1995, the BN still gives a probability of a failed inventory load rating of less than 1% for the AS and LF methodologies and almost 5% for the LRFR method. The consultant that completed the rating report in 1995 calculated a passing load rating. Both of these consultants used the AS method with an ultimate capacity check. Using the BN with the same inspection input for time steps 40 and 50 years, it predicts an increasing probability of a failed inventory load rating, as expected. In hindsight, this model could have been used by Engineers to predict the decline of structural capacity. The results may have served as an impetus to implement the recommended rehabilitation program.

**FIGURE 7.23.** Comparison of Probability of Failed Load Rating with Increasing Bridge Age

Based on the results of the 2002 inspection performed by the state department of transportation, the decision was made to replace the bridge. This was based of the
condition of the end spans. Currently, the replacement bridge is in the design phase. Realistically, this bridge will most likely reach the age of 50 years before it is replaced.

6.7.7 Summary

This chapter applies the entire model, the deterioration, prognostic and diagnostic components, to an actual bridge, located in the New England area. The bridge is made up of 24 simply-supported spans of prestressed concrete girders subject to the corrosion of a severe marine environment. The model combines two separate yet complimentary methods of bridge assessment: inspection and load rating. Variables within the network representing those symptoms typical of corrosive deterioration are linked to variables that represent internal strains and stresses. This allows the user to determine the impact of any distress found in the field upon the load rating. The model performs rating analyses using three methods: AS, LF and LRFR. It may be used as a tool for comparison in the mandated transition of all design over to the LRFR method.

The results spell out the importance of some fundamental bridge management philosophies. First of all, regular inspections of a bridge, beginning at the start of service life are critical to the health of the bridge. This bridge was exposed to a highly corrosive environment with the marine salt water constantly splashing at the end spans. This accelerated the deterioration so that spalling was noted at a young age of 20 years. If regular inspections had been conducted prior, the symptoms indicating the beginnings of corrosion could have been observed and treated. Second, nondestructive technology (NDT) is a resource that should be utilized. For example, if the state department of transportation had incorporated a half cell potential survey into the early bridge
inspections that should have taken place, inspectors would have discovered corrosive activity even before they observed anything visually. An aggressive seal coating maintenance plan could have been put into place to arrest the deterioration and lengthen the life of the bridge. Thirdly, the results show that preservation over replacement is the optimal maintenance plan. The replacement of this bridge will be over six million dollars. A rigorous inspection and maintenance plan over the lifetime of the bridge would have cost far less.
CHAPTER VIII
CONCLUSIONS, LIMITATIONS, FUTURE WORK

This final chapter begins with a summary of the objective of this research and a description of the presented model including its components: diagnostic Bayesian network (BN) component, probabilistic prognostic component, deterioration component. The contributions of this research are listed and explained. The limitations of the model and its components are discussed. A generic algorithm is presented that details the steps necessary to replicate this model for another bridge type. General conclusions are made. The chapter ends with recommendations for future research including the description of a framework for extension of the model.

8.1 Summary

The objective of this research was to develop a load rating model of the flexural response of a beam element of a prestressed concrete bridge. The implementation of this single limit state demonstrates the viability for the use of a Bayesian network (BN) for use in the condition assessment of bridge components and their limit states. The main components of this load rating model are the diagnostic Bayesian network (BN) component, the probabilistic prognostic component, and the deterioration component, which is incorporated into both the diagnostic BN and the probabilistic prognostic component.
8.1.1. Diagnostic Bayesian Network (BN) Component

The main computational tool used in this model is a Bayesian network (BN). Here, a BN is used to model the load rating process. Variables that represent resistance and loading effects are interconnected according to their dependency relations with conditional probabilities. Inspection variables are also included to combine the two processes that are typically separate. The format of the BN allows the input of new information, such as inspection data, to facilitate the model updating to reflect the ever-changing bridge element condition. The BN can also be used to predict and diagnose bringing the model to an interactive level with the user.

8.1.2 Probabilistic Prognostic Component

In addition to the diagnostic BN, the model includes a probabilistic prognostic component. In this component, load ratings of a prestressed concrete bridge beam are calculated according to three commonly used design methods: AS, LF and LRFR. The calculations are probabilistic. Random variables are input to a series of equations based on physics and mechanics of materials. These equations are then evaluated using Monte Carlo simulation. In the end, the load rating factors for the flexural limit state of the bridge beam element are presented as variables as opposed to the deterministic values currently used in practice. In addition to offering these results, the prognostic component prepares the input necessary for the diagnostic BN through the discretization of variables as well as the calculation of the conditional probabilities.
8.1.3 Deterioration Component

The deterioration component adds a temporal dimension to all model functions through time (annual time scale) with the deterioration component. Here, the most likely deterioration mechanism of prestressed concrete beams, corrosion of interior steel through chloride infiltration is represented through well accepted deterioration models found in the literature. The deterioration model results are presented in the form of decreasing area of prestressed steel. These results are, in turn, used as input into the prognostic component rendering load rating variables influenced by this deterioration. The effect of the deterioration is also carried to the diagnostic BN directly and through its input prepared in the prognostic component. The prognostic and diagnostic components of the model are evaluated at ten-year intervals to assimilate the model through time.

8.2 Contributions of Research

The main contribution of this research is the development of an analytical model of the flexural response of a simply-supported interior beam of a prestressed concrete bridge for use in the load rating process. Currently, the bridge load rating process is carried out either manually with hand calculations or through an AASHTO recommended software called *Virtis*. *Virtis* is the load rating program that belongs to a bridge software suite licensed by AASHTO including *Pontis* (bridge management) and *Opis* (bridge design) (AASHTOWare 2007). The model proposed in this research introduces an improved approach for use in the load rating process of a bridge beam element. Several unique features of this model offer specific contributions:
Comprehensive inclusion of two assessment processes: load rating and inspection

This model incorporates the inspection process into the load rating process for a comprehensive assessment of the condition of a bridge beam element for the limit states considered, here the flexural limit state. This is possible due to the flexible format of the main component of the model, the diagnostic BN. Bayesian networks allow the interconnection of variables of various types. Here inspection variables are linked with design variables. This is an improvement to the standard approach in which Engineers link these variables through subjective judgments based on experience that prove to be inconsistent from Engineer to Engineer. Also, the BN presented in this dissertation possesses the interconnection between inspection and design parameters that is lacking in Virtis (AASHTOWare 2007). Specifically, the condition rating data of bridges that is collected during inspection and stored within Pontis databases does not transfer over to the Virtis database. However, the model proposed here is developed to include inspection variables ranging from those observed subjectively under current inspection practices to those obtained through the more objective nondestructive technology.

Appropriate representation of the randomness of resisting and loading effects

This is accomplished through both the probabilistic prognostic component and the diagnostic BN component. The probabilistic prognostic component performs Monte Carlo simulation of the load rating process with continuous random variables for input. Bayesian networks are by design probabilistic being conceived as an improvement to expert systems by considering uncertainty. Currently in practice, Engineers use the series of equations presented in the design codes to calculate load ratings in a deterministic
fashion. While these equations have been developed based on probability concepts, the uncertainty of resistance and loading effects is not adequately represented.

*Multiple analysis methodologies implemented concurrently*

The three analysis methods currently used in the United States are incorporated in the model: Allowable Stress (AS), Load Factor (LF) and Load and Resistance Factor Rating (LRFR). The concurrent calculation of the three results allows a comparison that helps familiarize the engineer with each methodology and expected outcome. This exercise is too tedious and time consuming to perform by hand. And while *Virtis* is capable of automating the calculation of load ratings, only AS and LF methodologies are included (AASHTOWare 2007). The model of this research facilitates the implementation of the latest design code, LRFD and its rating component, LRFR. FHWA encourages all research endeavors that facilitate the implementation of LRFD.

*All data formats handled ranging from qualitative to quantitative*

The ability to accept all data formats ranging from numerical measurements to qualitative descriptions. This is a feature of the diagnostic BN component. Coming from the realm of Artificial Intelligence, the BN has the flexibility of incorporating variables that are well defined with states of distinct numerical ranges along with descriptive variables that use general classifications such as mild, moderate and severe. This allows the marriage between the mathematically based load rating process and the subjective visually based inspection process. Currently, these two processes are each performed separately, at different times by different personnel.
**Bi-directional capabilities**

This model possesses both predictive and diagnostic capabilities. All three components contribute to this predictive capability. Both the probabilistic prognostic component and the diagnostic BN component generate the probability of a failed load rating of a prestressed concrete bridge beam. With the results of the deterioration component included as input into these two components, it is possible to predict the probability of failure at future points in time. Current methods (hand calculations and *Virtis*) do not have predictive capabilities. The BN component provides the diagnostic capabilities. BNs being based on Bayes’ rule have this bi-directional feature. Current practice does not use models of this sort that diagnose symptoms observed in the field for deterioration that may or may not be hidden. Typically, it is left to the visual observation of an inspector. This is subjective and inconsistent and also inaccurate in many cases.

7.3 Limitations of the Model and its Components

In this section, the limitations of the model presented in this research are discussed.

**Limited to one response of one element of one bridge type**

The model is limited to the flexural response of a beam element of a precast prestressed concrete girder bridge. The load rating equations are tailored to a simple span bridge of this material, and the flexural response limit state is the only failure considered. Shear, which can be a brittle failure can be considered and added to the network. The
flexural limit state was modeled in this research because it was the critical limit state that typically governs a load rating analysis for a simply-supported prestressed concrete beam. Only an interior beam is modeled, there are other load-carrying elements. Other bridge types exist besides prestressed concrete. In order to cater to the entire bridge population, a model would have to be developed for each available bridge type.

**Limited to one deterioration**

For the purpose of initial implementation and development, the model is limited to deterioration due to corrosion of steel within the beam through chloride infiltration. There are other forms of deterioration that also occur concurrently to a precast prestressed beam, such as corrosion due to carbon sulfates. Also, prestressed beams are often subject to collision damage, which promotes further or accelerated deterioration. However, it must be considered that the model is limited to a linear response as dictated by the design theory of the codes. In summary, the results presented in this model are skewed because only one deterioration mechanism is modeled.

**Time model is not continuous**

The probabilistic prognostic component and the diagnostic BN component are constrained to represent one point in time. In order to model through time, these two main components are implemented in time steps at 10 year intervals. This sort of analysis prevents troubleshooting maintenance and repair decisions and how they affect future conditions. A dynamic Bayesian network is a modeling tool that may enable this feature. However, it is beyond the scope of this dissertation.
8.4 Generic Algorithm to Expand Model and its Components to other Bridge Types

As discussed in the previous section, one limitation of the model presented within this research is its application to only one element of one bridge type, precast prestressed concrete beam. In this section, a generic algorithm is presented listing the steps necessary to develop a similar model applicable to a beam of alternate bridge types.

**Determine random variables**

First, the resistance and loading effects of the beam according to its bridge type must be determined and modeled as random variables. The loading effects may be similar among all bridge types. However, the resistance of each is particular to its material makeup. For example, in a steel stringer bridge, the yield strength of steel ($F_y$) is an obvious resistance variable. For each variable, the following must be determined: the mean, the coefficient of variation, and the upper and lower thresholds for discretization. This sort of information may be available in literature or carefully estimated.

**Code probabilistic prognostic component**

Second, the probabilistic prognostic component must be coded in a programming language such as Matlab. Specifically, the series of equations of the load rating analysis will be programmed using Monte Carlo simulation with the random variables as input. All three methodologies (AS, LF, LRFR) should be included. The prognostic component will also calculate intermediate variables, such as the nominal moment of the section ($M_n$), and prepare the input necessary for the diagnostic Bayesian network through
discretization of variables based on the set thresholds and calculation of conditional probabilities.

**Layout diagnostic Bayesian network**

Third, the diagnostic Bayesian network is laid out using available software such as MSBNx. The structure of the network is based on the series of equations in the load rating analysis. In other words, with each equation, one or more random variables is used as input to render an output variable. Translating to BN terminology, the input variables are parents to the output or child variable. Once the network is developed, the root variable distributions may be assigned and the conditional probability tables filled using the data prepared by the prognostic component.

**Determine relevant deterioration mechanisms**

Fourth, a deterioration mechanism(s) must be determined to include within the model. Each bridge beam type is susceptible to a number of forms of deterioration. For example, the condition of a steel stringer can worsen due to fatigue and/or section loss from corrosion. One deterioration mechanism may be more critical than the other. In other words, fatigue is more prominent in a bridge carrying interstate highway traffic. On the other hand, a steel stringer bridge over marine water is highly susceptible to corrosion. The model designer must determine the objective of the model in selecting the deterioration mechanism(s) to incorporate.
**Simulate deterioration component**

Fifth, the deterioration mechanism must be coded using a programming language such as Matlab. Typical deteriorations of bridge types have been modeled by others and may be found in literature. Another option would be to use experimental data. The result of the deterioration component should be in a form that can be utilized by the prognostic component and diagnostic BN. For example, a decrease in cross-sectional area of a steel stringer due to corrosion calculated at determined time steps would be an ideal result format.

**Expand diagnostic BN with inspection variables**

Sixth, the BN should be further developed through the addition of inspection variables. These variables represent the symptoms of deterioration observed by the inspector. Also included are variables that are not observable but still pertinent to the deterioration process. The relevant variables are linked to those affected resistance variables and thereby incorporated into the model.

**Implement the model**

Seventh, the entire model is implemented at the appropriate time steps, say ten years. Specifically, each component (prognostic, diagnostic BN) is run at each time step. The deterioration component results in input for the respective prognostic and diagnostic components. The prognostic component results also serve as input for the diagnostic BN. At that point, the diagnostic BN component may be used interactively to perform many exercises including assessing condition or troubleshooting possible repair plans.
8.5 Conclusions

The load rating model presented in this research is helpful on all levels to those in the bridge community. Students are introduced to bridge inspection and load rating and learn how the two processes work together to determine the condition of a beam element of a prestressed concrete bridge. The Designer has an aid in troubleshooting different beam design options in order to determine the best alternative. The Inspector can use this model to diagnose symptoms in the field and to help direct additional tests that should be performed to do a thorough examination of a suspect bridge beam. The Engineer can use this model in putting together a repair plan for a prestressed concrete beam. Once the underlying deterioration has been diagnosed, effective repair and maintenance plans can be recommended. Finally, this model serves as an asset to the Owner/Manager of a bridge, i.e. the decision-maker. There are many decisions associated with a bridge and its performance. Through the use of this model of the critical beam element (that typically governs bridge load ratings), the Owner/Manager will be aided in making optimal decisions that promote preservation of the bridge and ensure safety of the public.

This research has shown the importance of the consideration of uncertainty in the rating calculations. Although the load rating factor is a deterministic number that is evaluated according to the threshold of 1.00, it is necessary to recognize the random effects of deterioration, loading and resistance that are underlying the one single value. This research has taken an in-depth look at these random variables and how they are accounted for in a different manner in each of the three load rating methodologies. This model offers an educational comparison between the three methods, giving Engineers a sense of how the results of the methods are relative to one another. The bridge
community recognizes the importance of properly accounting for randomness within all bridge calculations, including load ratings. This is demonstrated in the mandating of the LRFD design philosophy. The model proposed in this research supports the implementation of the LRFR methodology.

This research spells out the importance of some fundamental bridge management philosophies. Diligent quality control is key as shown in the results of the time \( t = 0 \) version of the model that indicate the possible sources of an unacceptable initial load rating are the inconsistencies that may occur in design, at the fabrication plant or in the field during construction. The results of the temporal version of the model that has been applied to an actual bridge shows that regular inspections of a bridge, beginning at the start of service life, are critical to the health of the bridge. This unidentified bridge was exposed to a highly corrosive environment with the marine salt water constantly splashing at the end spans. This accelerated the deterioration to such a degree that spalling was noted at a young age of 20 years. If regular inspections had been conducted prior, the symptoms indicating the beginnings of corrosion could have been observed and treated. Also, nondestructive technology (NDT) is a resource that should be utilized. For example, if the state transportation agency where the bridge is located had incorporated a half cell potential survey into the early bridge inspections that should have taken place, inspectors would have discovered corrosive activity even before they observed anything visually. An aggressive seal coating maintenance plan could have been put into place to arrest the deterioration and lengthen the life of the bridge. Further, the results show that preservation over replacement is the optimal maintenance plan. The replacement of this
bridge will be over six million dollars. A rigorous inspection and maintenance plan over the lifetime of the bridge would have cost far less.

**8.6 Recommendations for Future Research and a Framework for Extension**

This section discusses future research in two applications: 1) further development of the beam element model and 2) expansion of the beam element model to a systems level in representing the entire bridge structure.

**8.6.1 Further Development of Beam Element Model**

The element model presented herein could be further tested through application to a beam of other existing prestressed concrete bridges. Another type of deterioration characteristic of a prestressed concrete bridge beam could be incorporated into the model. This expanded model would be a better portrayal of the condition of a prestressed concrete beam and would offer more accurate load rating results.

The same modeling approach could be applied to other material types, such as a steel stringer bridge beam. A library of element models could be developed to service the entire bridge population.

A load rating bridge beam model could be developed using a Dynamic Bayesian Network (DBN). This would be an improved temporal model over the time-stepped method presented in this research.
8.6.2 Development of Bridge System Model

This research presents a load rating model that applies to the flexural response of a prestressed concrete bridge beam element. This model can be used as a building block in order to represent the response of an entire bridge system. While beams are typically the main load-carrying element of a bridge, there are other elements that contribute to this system. For example, piers and abutments support beams that in turn support decks. There are several approaches to achieve the expansion of this model to this broader scope as discussed herein.

Currently, AASHTO code directs the Engineer to load rate a bridge according to a critical limit state, that being the most likely point of failure. Therefore, the Engineer must identify the critical elements and locations of these failure points. For a simply supported prestressed concrete beam bridge, the critical limit state happens to be flexural strength at midspan and stress at the bottom of the beam. It could be argued that the rating of the beam provides the simplest, albeit crudest, representation of the rating of the entire bridge system. The Code gives direction in this sense with the understanding that many agencies perform bridge ratings manually and that efficiency in the calculations is of utmost importance. In line with this thinking, the Code also directs the Engineer to eliminate the rating of a concrete deck with the assumption that there is enough reserve strength within the deck (AASHTO 2003). Also, recommended for elimination in a bridge load rating are the substructure elements, unless they are observed to be in poor condition.

An improvement to the critical element approach that could be applied to a systems model is found with other researchers. (Akgul and Frangopol 2004) model a
simple span prestressed concrete bridge as a system by considering the following limit states: 1) slab, flexure, 2) girder, flexure at midspan, 3) girder, concrete tension at midspan. The lowest rating of the critical limit states is adopted as the overall bridge load rating.

(Estes 1997) uses a series model to represent a multi-span bridge with the girders of each span modeled in parallel. Also included in the series model are the following elements: deck, pier and footing. It should be pointed out that a total of three girders modeled in parallel are assumed to represent the superstructure span, each girder considering two critical limit states (flexure and shear). (Enright 1998) also considers a series and parallel model of girders using a fault tree format to represent a bridge system.

(Tabsh and Nowak 1991) model a bridge system using a two dimensional grid. This approach accounts for the redundancy and reserve strength of a multi-girder bridge. Similarly, the bridge load rating program *Virtis* considers a girder system incorporating the framework of all girders into the analysis. It is up to the Engineer to enter the points of interest to obtain the related results.

These approaches to modeling a bridge as a system are only starting points. Here the framework is discussed for expansion of the Bayesian network component to the systems level. BNs provide a format that allows the representation simultaneously at both the element level and the system level. However, it not as simple as linking three identical beam element BN models together to model three beams in parallel. There are issues that must be considered such as multiple failure sequences and correlations between element level limit states (Mahadevan and Smith 2001). Multiple failure sequences can be expressed through the definition of conditional system failure
probability in the conditional probability tables. Correlated variables may be linked or even grouped as one variable using their joint probability distribution to generate the conditional probability tables. Also to be considered with the expansion of the model is the numerous potential failure sequences along with the increasing number of computations and the possibility that the model become intractable. This issue may be dealt with by using some sort of filtering method such as branch-and-bound or truncated enumeration to eliminate insignificant failure sequences (Mahadevan and Smith 2001). This would improve the efficiency of the model.

Ideally, the verification of this bridge system model in its entirety would take place through comparison to an adequate number of case histories. Portions of the model could be verified by comparison to analytical models generated through simulation or reliability methods.
VIII. REFERENCES


Gamble, W. L. (1972). *Proposed amendment to the AASHTO Standard Specifications for Highway Bridges, Article 1.6.7(B), Prestressed Losses*. Department of Civil Engineering, University of Illinois, Urbana, IL.


Yanev, Bojidar (2007). Personal communication between Sara Wadia-Fascetti and Bojidar Yanev, Executive Director of the Bridge Inspection and Bridge Management Department of the New York City Department of Transportation, New York, NY.

University of Colorado, Boulder, Colorado.
This Appendix describes the root variables and their sources in detail.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
<th>Bias</th>
<th>COV</th>
<th>Distribution</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>Cross-sectional Area of Beam</td>
<td>in²</td>
<td>1.00</td>
<td>0.020</td>
<td>Normal</td>
</tr>
<tr>
<td>Aₛ</td>
<td>Prestressing Strand Area</td>
<td>in²</td>
<td>1.01</td>
<td>0.013</td>
<td>Normal</td>
</tr>
<tr>
<td>f'ₖb</td>
<td>Concrete Strength of Beam</td>
<td>psi</td>
<td>1.10</td>
<td>0.180</td>
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<tr>
<td>f'ₖd</td>
<td>Concrete Strength of Deck</td>
<td>psi</td>
<td>1.00</td>
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<td>fₛ</td>
<td>Prestressing Strand Strength</td>
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<td>Eₛ</td>
<td>Modulus of Elasticity of Prestressing Strands</td>
<td>ksi</td>
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<td>0.010</td>
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<td>I</td>
<td>Moment of Inertia of Beam Cross-section</td>
<td>in⁴</td>
<td>1.00</td>
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<td>Normal</td>
</tr>
<tr>
<td>Iₖ</td>
<td>Moment of Inertia of Composite Section</td>
<td>in⁴</td>
<td>1.00</td>
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<td>RH</td>
<td>Relative Humidity</td>
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<tr>
<td>wₑ</td>
<td>Unit Weight of Concrete</td>
<td>pcf</td>
<td>1.00</td>
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<td>Normal</td>
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<td>wₛ</td>
<td>Wearing Surface Thickness</td>
<td>psf(in)</td>
<td>1.10</td>
<td>0.200</td>
<td>Normal</td>
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</table>
A Cross-sectional Area of Beam (in\(^2\))

<table>
<thead>
<tr>
<th>Function</th>
<th>Probability Density Function</th>
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<tr>
<td>520</td>
<td></td>
</tr>
<tr>
<td>530</td>
<td></td>
</tr>
<tr>
<td>540</td>
<td></td>
</tr>
<tr>
<td>550</td>
<td>80% Med</td>
</tr>
<tr>
<td>560</td>
<td>10% Lo</td>
</tr>
<tr>
<td>570</td>
<td></td>
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<tr>
<td>580</td>
<td>10% Hi</td>
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<tr>
<td>590</td>
<td></td>
</tr>
<tr>
<td>600</td>
<td></td>
</tr>
</tbody>
</table>

Based on these two sources, the statistics of the random variable, \(A\), used in this study were estimated to be the nominal value for the mean and a coefficient of variation of 0.02. The thresholds are set at 3\% above and below the mean representing maximum and minimum values of \(A\). The 3\% difference is based roughly on the allowable dimensional tolerances for prestressed concrete products (PCI 1999). The thresholds (545 in\(^2\) and 575 in\(^2\)) also approximate the 10\% tails of the distribution.

The cross-sectional area of a beam is the area of the geometrical shape that lies in the plane of section cut perpendicular to the longitudinal axis of the beam. Uncertainties arise at the as-built stage due to dimensional variances that occur during fabrication. Other researchers (Al-Harthy and Frangopol 1994b) have recognized this randomness and have used statistics found in literature (Mirza et al. 1979) regarding this variable. The mean appears to be taken as the nominal value. The coefficient of variation is 0.017. Mirza’s statistics concern prestressed concrete beams in general, not as specifically applied to bridges (Mirza et al. 1979), (PCI 1999).

Gilbertson and Ahlborn (Gilbertson and Ahlborn 2004) estimated the statistical properties of this variable using the PCI guidelines. Two beam cross-sections were considered in their study. The estimated mean of the variable was the nominal value in one case of the small beam, and adjusted to be the mean of the estimated hi and low values in the other case of the large beam. For the larger beam the maximum and minimum values were set up roughly at a 4\% difference from the mean. For the smaller beam, 3\% was used, i.e. maximum value was set to 1.03*mean and minimum was set to 0.97*mean. The coefficient of variation was calculated based on the previous estimations and assumptions. For the smaller beam it was 0.018 and understandably higher for the larger beam 0.026.
The prestressing strand area variable represents the cross-sectional area of a seven wire low-relaxation strand having a diameter of ½ inch. Variability arises at this initial stage due to fabrication practices.

The probabilistic studies found in the literature (Al-Harthy and Frangopol 1994b, Gilbertson and Ahlborn 2004) used the statistics provided by Naaman and Siriaksorn (Naaman and Siriaksorn 1982). Naaman and Siriaksorn conducted a study on the average ranges of the reliability index considering the serviceability limit state for the precast prestressed industry in general. Gilbertson and Ahlborn (Gilbertson and Ahlborn 2004) applied these statistics to prestressed concrete beams used in bridges. Those statistics were also used in this study. The bias was 1.01176. Therefore the mean was 0.1548 in² for a nominal value of 0.153 in². The coefficient of variation was 0.0125.

In setting the thresholds that break this continuous variable into three states of Lo, Med and Hi, the areas of the neighboring strand sizes (diameters 7/16 and 9/16 inch) were initially considered. However, the distribution of the variable was not large enough, the thresholds did not fall within the range. Therefore, thresholds were arbitrarily chosen centered around the nominal value of 0.153 in². The prestressing strand area was considered to be in the Lo state if it fell below 0.151 in². The prestressing steel area was considered to be in the Hi state if it was greater than 0.155 in².
Engineers specify a minimum compressive strength at the time of release of the prestressing strands in order to control stresses (compressive and tensile) within the concrete both at this initial time and while in service. This release strength is selected so that the temporary concrete stresses in the beam before losses due to creep and shrinkage do not exceed 60% of the concrete compressive strength at time of release. Also, the strength is selected so that in tension areas with no bonded reinforcement, the tensile stress will not exceed 200 psi or $3\sqrt{f_{cbi}'}$ (PCI 2003).

The statistics used for Concrete Strength of Beam at Transfer is based on that presented by Gilbertson and Ahlborn (Gilbertson and Ahlborn 2004). Their statistics are based on discussions with PCI certified plant managers that assert that concrete strength is actually 20% higher than nominal. Gilbertson and Ahlborn use an increase of only 10% to be conservative. A high coefficient of variation (COV=0.15) is also conservatively estimated to account for randomness. Gilbertson and Ahlborn estimate a higher coefficient of variation at 0.20 than found in the literature to reflect more variation at early stages of concrete curing. However, another researcher, Steinberg, assumes a coefficient of 0.15 (Steinberg 1995). This study estimates the COV at 0.15 for this variable.

As for the thresholds, the PCI Manual was used as reference (PCI 2003). The PCI Manual (PCI 2003) offers the properties of six different concrete mixes having different compressive strengths. In this study, the concrete has a specified minimum strength at transfer of 4000 psi (like Mix B). Based on the concrete mixes presented in the PCI Manual, the thresholds for Lo and Hi are set at the minimum specified strengths of Mixes A and C, 3500 psi and 5000 psi, respectively. This variable is not used in the model presented here.

<table>
<thead>
<tr>
<th>Mix</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specified Strength, psi Release</td>
<td>3500</td>
<td>4000</td>
<td>5000</td>
<td>6000</td>
<td>6000</td>
<td>8800</td>
</tr>
</tbody>
</table>

**Concrete Strength of Beam at Transfer, $f_{cbi}$**
- Nominal = 4000 psi
- Mean = 4400 psi
- COV = 0.15
- Lo threshold = 3500 psi
- Hi threshold = 5000 psi
Concrete Strength of Beam (psi)

 Engineers specify a minimum compressive strength of concrete to be met at 28 days. There are many factors that contribute to this being a variable including the concrete strength at the time of release, as well as environmental conditions.

There are several values found in the literature for the nominal compressive strength = 5000 psi:

Gilbertson and Ahlborn (Gilbertson and Ahlborn 2004) use a coefficient of variation (COV = 0.174) taken from the literature (Ellingwood 1978). However, they estimated the mean (mean = 1.1*nominal) based on direct communication with PCI certified plant managers who claimed that the concrete strength is typically 20% higher than nominal. To be conservative, Gilbertson and Ahlborn used only a 10% increase.

MacGregor (MacGregor et al. 1983) estimated these statistics from a collection of statistical distributions obtained from literature. His study considered variability of the material properties and dimensions that stem from average quality construction. Deterioration was not accounted for in his study. The value offered for the mean was 4028 psi for a beam of 5000 psi nominal strength. This translated to a bias of 0.81. The coefficient was 0.15.

El-Tawil and Okeil (El-Tawil and Okeil 2002) estimated the statistics for concrete compressive strength of prestressed girders based on those found in the literature. The sources varied from (0.81 to 1.25)*Nominal for the mean. The range for the COV was 0.09 to 0.21. Both Normal and Lognormal distributions were used. From the collected information, the authors estimated a mean of 1.1*nominal and a COV of 0.18. A normal distribution was assumed.

Steinberg (Steinberg 1995) bases these statistics on two separate references, literature and experimental results. For the strength of concrete, this study estimated the mean at 4750 psi (bias=0.95) and the coefficient of variation at 0.18 for a prestressed concrete beam having a nominal strength of 5000 psi. These statistics were based on the research of MacGregor (MacGregor 1976) and Hamann and Bulleit (Hamann and Bulleit 1987). In turn, Hamann and Bulleit (Hamann and Bulleit 1987) used the statistics presented in (Naaman and Siriaksorn 1982). Naaman and Siriaksorn (Naaman and Siriaksorn, 1982) based their statistics (MEAN = (0.67-1.17)*nominal, COV = 0.1-0.25) on literature.

Based on the above references, primarily (Gilbertson and Ahlborn 2004), the statistics for \( f'_{cb} \) in this investigation was estimated at with a bias of 1.10 and a coefficient of variation of 0.18.

In setting the thresholds that divide this continuous distribution into Lo, Med and Hi states, the relationship of this variable with the initial concrete strength was considered. The thresholds were set to coordinate with the thresholds for \( f_{cbi} \) facilitating the model representation of their correlation. It should be noted that the LRFD Manual (AASHTO 1998) requires that the concrete compressive strength of a prestressed beam be no lower than 4000 psi, which is below the Low Threshold specified here, 4200 psi.
Concrete Strength of Beam, $f_{cb}$

Nominal = 5000 psi
Mean = 5500 psi
COV = 0.18
Lo threshold = 4200 psi
Hi threshold = 6400 psi
$f'_{cd}$  Concrete Strength of Deck (psi)
Bias = 1.0  COV = 0.20

Engineers also specify a minimum compressive strength for the concrete decks. In review of the statistics available in the literature including (MacGregor et al. 1983), the properties were estimated: Bias = 1.0 and COV = 0.20. A normal distribution was assumed. MacGregor’s statistic was based on a collection of statistical distributions obtained from literature. Their study considered variability of the material properties and dimensions that stem from average quality construction. Deterioration was not accounted for in his study.

The nominal value for concrete strength of deck is 3400 psi. The concrete deck is poured in the field, this increases the level of randomness. Typically, the strength for a deck is specified to be 4000 psi (MHD 2005). A concrete deck would be considered high strength if it surpassed a compressive strength of 5000 psi. On the other hand, a compressive strength below 3000 psi would be considered low and possibly not capable of providing the necessary strength. Therefore, the upper and lower thresholds for this variable are 5000 and 3000 psi, respectively.
This variable represents the minimum ultimate tensile strength of the prestressed strands. There are two grades of strands that are generally used, Grades 250 and 270. In this investigation, the nominal value for this property is 270 ksi, reflective of the Grade 270 that is normally used in bridges. However, the statistics found in literature support that more strength is typically provided in these strands.

MacGregor’s (MacGregor et al. 1983) estimate of this statistic (MEAN = 281 ksi, COV = 0.025) is based on the results found in a static test that was included in his research with Mirza (Mirza et al. 1980). These statistics originated from material test records of Con-Force products, Edmonton, Alberta, Canada for the years 1976-1977, 200 samples, half of them are stress relieved and the other half stabilized. El-Tawil and Okeil (El-Tawil and Okeil 2002) use a mean = 1.04*270 ksi and a COV = 0.02 based on statistics found in the literature. Naaman and Siriaksorn (Naaman and Siriaksorn 1982) estimate this statistic (MEAN=1.0387*270, COV=0.0142) based on those statistics found in the literature.

Based on these statistics, a Bias of 1.04 and a COV of 0.02 was used in this study.

In setting the lower threshold, 250 ksi initially seemed reasonable, based on observation of an idealized stress-strain curve for the seven-wire low-relaxation prestressing strand (PCI 2003). However, the distribution based on the published statistics did not spread to include this lower threshold. Therefore, a lower threshold of 270 ksi was set. While this is also the nominal value for the Prestressing Strand Strength, the published statistics indicate that more strength is generally provided so that 270 ksi would in fact be a reasonable low. As for the higher threshold, 290 ksi was estimated considering similar reasoning that although 270 ksi is the nominal ultimate, published statistics suggest that more strength is provided. This also suggests that the idealized stress-strain curve levels off at a higher strength that 270 ksi.
The modulus of elasticity is the coefficient that relates stress to strain in the elastic region of a stress-strain diagram. The modulus of elasticity represents the ability of the strand to resist deformation within the elastic range. The nominal value is 28,500 ksi. The statistics for this variable can be traced back to (Naaman and Siriaksorn 1982). These researchers did extensive study into the reliability of prestressed concrete beams.

The Lo and Hi thresholds are arbitrarily set at 28,000 and 29,000 ksi.
Moment of Inertia of Beam Cross-section (in$^4$)
Bias = 1.0    COV = 0.03

The moment of inertia is a geometrical property of an area about a particular reference axis. It has a mathematical definition and cannot be visualized in the same way as for example, a centroid of a section (Fitzgerald 1982). Moment of inertia plays an important role in the calculation of flexural stresses. Uncertainty arises in this property at this initial stage due to dimensional variances of the beam cross-section that occurs during fabrication.

The statistics used in this study (bias = 1.0, COV = 0.03) are based upon those estimated for moment of inertia by Gilbertson and Ahlborn (Gilbertson and Ahlborn 2004). They estimated these statistics by applying allowable tolerances to the known nominal value.

In this investigation, the nominal value for moment of inertia is 125,390 in$^4$. The thresholds were set at the 10% tail points (120.570 in$^4$ and 130.210 in$^4$). These thresholds coordinate with the thresholds set for $A$ and $I_c$ facilitating the model representation of their correlation.
I_c  Moment of Inertia of Composite Section (in^4)
Bias = 1.0  COV = 0.035

The moment of inertia of the composite section considers not only the beam itself but the concrete deck it supports, assuming effective composite action between the two causing them to act as one unit.

The statistic used in this study is based upon the one estimated for moment of inertia by Gilbertson and Ahlborn (Gilbertson and Ahlborn 2004). They estimated these statistics by applying allowable tolerances to the known nominal value. In this study the COV has been increased slightly to account for dimensional variance related to the deck, which is poured in the field.

In this investigation, the nominal value for composite moment of inertia is 364,324 in^4. The thresholds were set at the 10% tail points (347,980 in^4 and 380,670 in^4). These thresholds coordinate with the thresholds set for A and I facilitating the model representation of their correlation.
**$M_{LL}$ Liveload Moment (ft-kips)**  
**Bias = 1.2**  
**COV = 0.18**

Liveload moment represents the loading on a bridge that is the result of vehicles and pedestrians. It is considered a variable because the liveload can range from light pedestrian traffic to tandem trailer trucks.

Nowak (Nowak 1993) has done extensive work concerning the variability that arises in the design of bridges that has been instrumental in the development of the Load and Resistance Factor design philosophy. These statistics (Mean = 1.2*Nominal, COV = 0.18) have been adopted for this study.

In setting the thresholds, the quartile points were arbitrarily selected, as there is no obvious upper and lower value when this variable is considered separate from the other variables.

This variable is not used in the model version presented in this dissertation.
Ambient relative humidity varies day to day, season to season and by location as well. This variable influences the curing process as well as the amount of loss of prestress within the strands. In this study, the nominal value for relative humidity is 70%.

Gilbertson and Ahlborn estimated statistics (MEAN = 75%, COV=0.118) based on calculations using the tolerances set forth in the PCI Manual. Based on these statistics, a Bias of 1.00 and a COV of 0.12 was used in this study.

In determining the thresholds for relative humidity, a map of the United States showing the annual ambient relative humidity was reviewed (PCI 2003). Relative humidity was as low as 30% in the southwest desert region and as high as 80% in some coastal regions. Assuming that this example bridge is in New England, the Lo and Hi thresholds were set at 60% and 80% relative humidity.
The unit weight of concrete is a measure of its density. It is a variable that is primarily influenced by mix proportions.

Based on the statistics found in literature, a Bias of 1.0 and a COV of 0.03 was used in this study. This statistic has been used by many researchers (Steinberg 1995, Gilbertson and Ahlborn 2004) and is referenced back to (Hamann and Bulleit 1987, Naaman and Siriaksen 1982).

As for setting the upper and lower thresholds, according to PCI Bridge Design Manual (PCI 2003), normal weight concrete is generally in the range of 140 to 150 pcf. Although concrete with high strength, greater than 10,000 psi, may have unit weight as high as 155 pcf. The varying weight is subject to the amount and density of aggregate, air, water and cement content. Therefore, the lower threshold is estimated at 145 pcf and the upper threshold is estimated at 155 pcf.
The thickness of the wearing surface, which may also be expressed as a surface area load, is considered a variable. At the beginning of the service life of a bridge, the uncertainty in the actual thickness of the wearing surface arises as it is applied in the field due to issues of quality control.

In this study, the nominal value for the wearing surface thickness is 25 psf (2 in).

El-Tawil and Okeil (El-Tawil and Okeil 2002) used a mean of 1.1*nominal and a COV=0.2. This statistic is based on those found in the literature by these researchers (Bias: (1.00-1.44), COV: (0.08-53.2)). They conducted a study into the LRFD provisions for prestressed concrete bridge girders strengthened with Carbon Fiber-Reinforced Polymer Laminates. In their study, they apply this statistic to Wearing Surface Load, as opposed to thickness.

Based on these statistics, a Bias of 1.1 and a COV of 0.2 was used in this study.

In setting the thresholds, it was noted that wearing surface thickness tends to be greater than set forth in design, as reflected by the Bias found in literature. Therefore, the thresholds were set at 1.5 in. (18.75 psf) for Lo and 3 in. (37.5 psf) for Hi.

**Wearing Surface Thickness, wst**
- Nominal = 25 psf
- Mean = 27.5 psf
- COV = 0.20
- Lo threshold = 18.75 psf
- Hi threshold = 37.5 psf
APPENDIX A2

This Appendix offers details on how the correlation between variables is incorporated in the BN. These correlations where used in a previous BN version not presented in this dissertation.

DIMENSIONAL VARIABLES

Cross-sectional Area of Beam ($A$), Moment of Inertia of Beam Cross-section ($I$) and Moment of Inertia of Composite Section ($I_c$) are related through dimensional variances. These correlations are easily represented in the Bayesian Network in a basic simple fashion by adding a variable dedicated to dimensional variances. If any dimensions have exceeded the tolerances set forth by PCI (PCI 1999), $A$, $I$ and $I_c$ are likely to be in their respective Hi states and vice versa. A dimensional variance variable ($DV$) is a reflection of the quality control at the precast plant. This study has estimated $DV$ at 10% Lo, 80% OK, 10% Hi. In other words, there is an 80% chance that the dimensions will adequately satisfy the set tolerances and 10% chances dimensions will be exceeded or deficient. The thresholds of the correlated variables have been set to complement this estimated distribution. In other words, there is a 10% chance that the dimensional tolerances will be exceeded. The model then assumes that there is a 100% probability that the Area will be Hi given the $DV$ is Hi. This is in agreement with the original distribution of $A$ which assumed 10% probability of $A$ being Hi considering a threshold of 575 in$^2$. So essentially, the thresholds are set so that the probabilities of the related variables can match, ultimately ending with a simplified representation of their correlation.

<table>
<thead>
<tr>
<th>Lo</th>
<th>OK</th>
<th>Hi</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.80</td>
<td>0.10</td>
</tr>
</tbody>
</table>

A: beam cross-sectional area
$DV$: dimensional variances
$I$: moment of inertia of beam
$I_c$: moment of inertia of composite section
BEAM CONCRETE STRENGTH VARIABLES

Concrete Strength of Beam at Transfer ($f'_{cbh}$) and Concrete Strength of Beam ($f'_{cb}$) are also related. The initial concrete strength ($f'_{cbi}$) is a precursor to the 28-day concrete strength, ($f'_{cb}$). If the initial concrete strength is in the high state, it is likely that the 28-day strength will be in the high state, also. This correlation is easily incorporated into the model.

The initial concrete strength has set thresholds which leave the following probabilities: 9% Lo, 73% Med, 18% Hi. The thresholds for the concrete strength have been set to render the same numerical probabilities for the three states (9% Lo, 73% Med, 18% Hi) to allow the correlation between the two variables to be modeled.

<table>
<thead>
<tr>
<th>Lo $f'_{cbi}$</th>
<th>Med $f'_{cbi}$</th>
<th>Hi $f'_{cbi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>0.73</td>
<td>0.18</td>
</tr>
</tbody>
</table>

$f'_{cb}$: beam concrete strength
$f'_{cbi}$: initial beam concrete strength
APPENDIX A3

Various constructs of the network were explored in order to find the appropriate model that achieved the delicate balance between level of detail and computational effort.

MODEL 1

Model 1 is laid out in a great level of detail including several intermediate variables that fall within the cascade down from the root design variables to the ending rating factor variables. Each step of calculations in the load rating procedure is represented in the structure of this model. The inclusion of the intermediate variables aided in controlling the amount of conditional probabilities that were required. In other words, each child has no more than three parents, with the exception of the Allowable Stress Rating Factor variable (ASRF).

The causal relations between many of the variables are very simple. For example, three deadload variables, 1) beam moment, \( M_B \), 2) slab moment, \( M_S \), 3) superimposed deadload moment, \( M_{SDL} \), are the parents and the total deadload moment, \( M_D \), is the child. In this case, the child is simply the disjunction of the parents. The relationship is deterministic, in that the parents specify the value of the child with no uncertainty (Russell and Norvig 1995).

In the model shown below, seven of the nine root variables are colored yellow to indicate design variables. Some the design variables are also shaded blue indicating inspection variables. The inspection aspects of the model will be addressed in the next phase. Also, the variables representing Creep, CR, Shrinkage, SH, and Total Prestress Loss, \( TPL \), have been shaded purple as variables of interest that are not directly observable.

A: beam area
\( A_s \): strand area
ASRF: AS inventory rating
CR: creep
\( f_{cb} \): beam concrete strength
\( f_{cd} \): deck concrete strength
\( f_{cde} \): concrete stress all DL
\( f_{cir} \): concrete stress beam DL
\( f_{dl} \): deadload stress
\( f_{fl} \): liveload stress
\( f_{fde} \): strand stress
LFRF: LF operating rating
\( M_B \): beam selfweight moment
\( M_S \): slab moment
\( M_N \): resistant moment
\( M_{SDL} \): superimposed DL mom.
\( P_{se} \): effective prestress force
\( P_{sit} \): initial prestress force
RH: relative humidity
SH: shrinkage
TPL: total prestress loss
\( w_c \): weight of concrete
ws: wearing surface thickness
\( w_{SDL} \): superimposed deadload
MODEL 2

Model 2 is based on the framework of Model 1. The nine root variables are included as well as the ending load rating variables. The unobservable variables of interest shown in purple are Creep (Cr), Shrinkage (SH), and Effective Prestress Force (Pse), which reflects the total prestress loss. Model 2 attempts to eliminate most of the intermediate variables of Model 1 in order to simplify the model. However, in the absence of these variables, children such as Creep have six parents resulting in an enormous amount of necessary conditional probabilities. Specifically, six parents, each with three states (Lo Med Hi) and one child with three states also requires the definition of 2187 conditional probabilities \((3)^6(3)^6=2187\).

---

**Model 2**

- **A**: beam area
- **As**: strand area
- **ASRF**: AS inventory rating
- **CRc**: creep
- **fcb**: beam concrete strength
- **fcd**: deck concrete strength
- **fs**: strand strength
- **LFRF**: LF operating rating
- **MB**: beam selfweight moment
- **MD**: deck moment
- **MLL**: liveload moment
- **Mn**: resistant moment
- **MSDL**: superimposed DL mom.
- **Pse**: effective prestress force
- **RH**: relative humidity
- **SH**: shrinkage
- **wc**: weight of concrete
- **wst**: wearing surface thickness
- **wSDL**: superimposed deadload
MODEL 3

Model 3 has the exact layout of Model 2. However, in order to alleviate the amount of conditional probabilities required, the Conditional Independence (CI) distribution is used. Conditional independence assumes that each parent has an independent chance of causing the child. By employing the CI distribution, the number of conditional probabilities is greatly decreased. For example, as explained with Model 2, Creep having six parents, results in an enormous amount of necessary conditional probabilities. Specifically, six parents, each with three states (Lo Med Hi) and one child with three states also requires 2187 conditional probabilities \((3^6 \times 3^1 = 2187)\). The table below shows the breakdown of conditional probabilities needed in using the CI distribution, only 36 conditional probabilities. A leak term accounts for the probabilities that none of the parents are in an abnormal state (Kadie et al. 2001). It should be noted that the CI distribution is typically used with the binary variables common in Bayesian Networks allowing the number of required conditional probabilities to drop from \(2^k\) to \(k\) to account for \(k\) parents. By using three states with each variable instead of two, the CI distribution earmarks one state as normal (Med in this case), while the Lo and Hi states are treated as abnormal.

<table>
<thead>
<tr>
<th>Mb</th>
<th>Md</th>
<th>fs</th>
<th>A</th>
<th>As</th>
<th>Ms</th>
<th>loCR</th>
<th>medCR</th>
<th>hiCR</th>
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<tbody>
<tr>
<td>Cl Leak Term</td>
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Although this model requires significantly less conditional probabilities to be determined and inputted, the author found that the same level of computational effort was necessary in the programming of the prognostic component. In other words, although there were fewer combinations of parental states to consider, the same amount of nested loops had to be coded as in the standard distribution. The author sees the value in using the Conditional Independence distribution, but chooses instead to continue with the simulation considering all possible instantiations of parents.
MODEL 4

Model 4 is based on the structure of Model 2 and Model 3. However, in an attempt to simplify the programming effort behind obtaining the conditional probabilities, a three-parent limit was set for each child. The selection of parents for each child was based on the results of a sensitivity study that explored the effects that each parent variable had on the outcome of the child variable through Monte Carlo simulation. As a result, some variables of the model fell out, leaving a sparse version of the network. This model may provide adequate results but the loss of the level of detail may prove problematic for the future development of this model.

Model 4

In conclusion of the investigation, the authors have opted for Model 1 for continuation of the research. Model 2 was infeasible in the number of conditional probabilities required for input and the associated level of computation necessary. Model 3 tried to take advantage of the Conditional Independence assumption available within the MSBNx software. However, this proved to have virtually the same unreasonable level of required computations as Model 2. Model 4 was unsuitable because of the lack of detail of the model.
APPENDIX A4
This appendix details the load rating equations that are used within the prognostic component to express the relationships (the arcs) between the variables of the BN.

BEAM SELF-WEIGHT MOMENT

\[ M_g = \frac{(A/144)w_cL^2}{8} \]

where

- \( M_g \) = unfactored bending moment due to weight of beam, ft-kips
- \( A \) = area of cross-section of the precast beam, in²
- \( w_c \) = unit weight of concrete, kcf
- \( L \) = span length, ft

This equation is based on fundamental principles of statics. It calculates the maximum moment of the beam that is located at the midspan. The following is assumed within the equation: 1) the beam lies in a plane, 2) forces act perpendicular to longitudinal axis, 3) beam is simply supported, free to rotate at the supports with one support free to translate in order to eliminate the development of any horizontal forces.

The simply-supported assumption allows the beam to be classified as statically determinate, meaning all the external reactions can be calculated based on statics alone. Also, the self-weight of the beam is considered to be uniform load that lies in the plane and acts perpendicularly to the longitudinal axis along the entire length of the beam.

In setting the thresholds that determine the Low Medium and High states for \( M_g \), the characteristics of \( M_g \) were considered. \( M_g \) represents the moment load of the weight of the beam. The moment is considered too high when the beam is unable to support its own weight. A moment would be considered too low when the beam is not large enough to support prescribed superimposed dead and live loads. However, whether the moment due to self-weight is too high or too low cannot be separated out from the entire structural analysis of the beam. Therefore, the thresholds for this variable are arbitrarily set at 25% Lo, 50% Med, and 25% Hi (Lo threshold = 300 ft-kips, Hi threshold = 315 ft-kips).
SLAB MOMENT

\[ M_s = \frac{(be/12)(ts/12)w_cL^2}{8} \]

where  
\( M_s \) = unfactored bending moment due to deck slab, ft-kips  
\( be \) = effective slab width, in  
\( ts \) = slab thickness, in  
\( w_c \) = unit weight of concrete, kcf  
\( L \) = span length, ft

This equation is based on fundamental principles of statics. It calculates the maximum moment at the midspan of the beam due to the supported load of the concrete deck slab. The following is assumed within the equation: 1) the beam lies in a plane, 2) forces act perpendicular to longitudinal axis, 3) beam is simply supported, free to rotate at the supports with one support free to translate in order to eliminate the development of any horizontal forces. The simply-supported assumption allows the beam to be classified as statically determinate, meaning all the external reactions can be calculated based on statics alone. Also, the concrete slab is considered to be uniform load that lies in the plane and acts perpendicularly to the longitudinal axis along the entire length of the beam. The equation assumes that the tributary area of the slab carried by the beam, defined through the effective width, is reflective of the actual loading conditions.

In setting the thresholds that determine the Low, Medium and High states for \( M_s \), the characteristics of \( M_s \) were considered. \( M_s \) represents the moment load of the concrete deck slab supported by the beam. A moment such as this is considered too high when the beam is unable to support this load. This moment could be considered too low reflective of the deck deficient in size. However, whether the moment due to the concrete slab is too high or too low cannot be separated out from the entire structural analysis of the beam. Therefore, the thresholds for this variable are arbitrarily set at 25% Lo, 50% Med, and 25% Hi (Lo threshold = 423 ft-kips, Hi threshold = 440 ft-kips).
WEARING SURFACE MOMENT

\[ M_{ws} = \frac{rw(wst/1000)w_cL^2}{8(nob)} \]

where  
\( M_{ws} \) = unfactored bending moment due to wearing surface, ft-kips  
\( rw \) = roadway width, ft  
\( wst \) = wearing surface load, psf  
\( w_c \) = unit weight of concrete, kcf  
\( L \) = span length, ft  
\( nob \) = number of beams

This equation is based on fundamental principles of statics. It calculates the maximum moment at the midspan of the beam due to the supported load of the wearing surface. The following is assumed within the equation: 1) the beam lies in a plane, 2) forces act perpendicular to longitudinal axis, 3) beam is simply supported, free to rotate at the supports with one support free to translate in order to eliminate the development of any horizontal forces. The simply supported assumption allows the beam to be classified as statically determinate, meaning all the external reactions can be calculated based on statics alone. Also, the wearing surface is considered to be a uniform load that lies in the plane and acts perpendicularly to the longitudinal axis along the entire length of the beam. The equation also assumes that the entire load of the wearing surface is carried equally by each beam.

In setting the thresholds that determine the Low, Medium and High states for Mws, the characteristics of Mws were considered. Mws represents the moment load of the wearing surface supported by the beam. Whether the moment due to the concrete slab is too high or too low cannot be separated out from the entire structural analysis of the beam. Therefore, the thresholds for this variable are arbitrarily set at 25% Lo, 50% Med, and 25% Hi (Lo threshold = 91 ft-kips, Hi threshold = 119 ft-kips).
SUPERIMPOSED DEADLOAD MOMENT

\[ M_{sdl} = M_{ws} + \frac{2(barr/1000)L^2}{8(nob)} \]

where
- \( M_{sdl} \) = unfactored bending moment due to superimposed deadloads, ft-kips
- \( M_{ws} \) = unfactored bending moment due to wearing surface, ft-kips
- \( barr \) = barrier load, plf
- \( L \) = span length, ft
- \( nob \) = number of beams

This equation is based on the principle of superposition. From (Beer and Johnston 1981):

The principle of superposition states that the effect of a given combined loading on a structure may be obtained by determining separately the effects of the various loads and combining the results obtained, providing that the following conditions are satisfied:
1) Each effect is linearly related to the load which produces it
2) The deformation resulting from any given load is small and does not affect the conditions of application of the other loads.

In this equation the effects of the load from the wearing surface are combined with the effects of the load from the two barriers to form the moment due to superimposed deadload.

This equation is also based on fundamental principles of statics. It calculates the maximum moment at the midspan of the beam due to the supported load of the barriers. The following is assumed within the equation: 1) the beam lies in a plane, 2) forces act perpendicular to longitudinal axis, 3) beam is simply supported, free to rotate at the supports with one support free to translate in order to eliminate the development of any horizontal forces. The simply supported assumption allows the beam to be classified as statically determinate, meaning all the external reactions can be calculated based on statics alone. Also, the barrier loading considered a uniform load that lies in the plane and acts perpendicularly to the longitudinal axis along the entire length of the beam. The equation also assumes that the barrier load is divided and carried equally by each beam.

In setting the thresholds that determine the Low, Medium and High states for Msdl, the characteristics of Msdl were considered. Msdl represents the combined moment load of the wearing surface and the barriers supported by the beam. Whether the moment due to these superimposed loads is too high or too low cannot be separated out from the entire structural analysis of the beam. Therefore, the thresholds for this variable are arbitrarily set at 25% Lo, 50% Med, and 25% Hi (Lo threshold = 163 ft-kips, Hi threshold = 192 ft-kips).
TOTAL DEADLOAD MOMENT

\[ M_d = M_g + M_s + M_{ndl} \]

where

- \( M_d \) = total unfactored bending moment due to deadloads, ft-kips
- \( M_g \) = unfactored bending moment due to beam self-weight, ft-kips
- \( M_s \) = unfactored bending moment due to concrete deck slab, ft-kips
- \( M_{ndl} \) = unfactored bending moment due to superimposed deadloads, ft-kips

This equation is based on the principle of superposition. From (Beer and Johnston 1981):

_The principle of superposition states that the effect of a given combined loading on a structure may be obtained by determining separately the effects of the various loads and combining the results obtained, providing that the following conditions are satisfied:
1) Each effect is linearly related to the load which produces it
2) The deformation resulting from any given load is small and does not affect the conditions of application of the other loads._

Monte Carlo simulation of the above equation yields the following distribution for \( M_d \):

In setting the thresholds that determine the Low, Medium and High states for \( M_d \), the characteristics of \( M_d \) were considered. \( M_d \) represents the combined moment load of the deadloads due to the beam itself, the concrete deck slab and the superimposed deadload, which is the wearing surface and the barriers supported by the beam. Whether the moment due to these deadloads is too high or too low cannot be separated out from the entire structural analysis of the beam. Therefore, the thresholds for this variable are arbitrarily set at 25% Lo, 50% Med, and 25% Hi (Lo threshold = 891 ft-kips, Hi threshold = 943 ft-kips).
LIVELOAD MOMENT

\[ M_{LL} = HS20 \times WDF \]

where
\[ M_{LL} = \text{liveload moment per beam, ft-kips} \]
\[ HS20 = \text{maximum wheel-load moment (including impact), ft-kips} \]
\[ WDF = \text{AASHTO wheel-load distribution factor} \]

This equation is taken from AASHTO Standard Specifications (AASHTO 1996). It represents the liveload moment per beam based on the minimum loading bridges are designed to carry, HS20-44. This loading, developed in 1944, consists of a tractor truck (gross weight = 20 tons) with semi-trailer or the corresponding lane load. The variable, HS20 (Nowak 1993), is reflective of the randomness of this load and it includes impact. Typically, the AASHTO Specifications require the design engineer to account for dynamic, vibratory and impact effects by multiplying the liveload moment by an impact factor. The impact factor was not included within this program so as not to doubly account for impact. AASHTO also recommends a factor to account for the distribution of loads to the beams. For a prestressed concrete I-girder bridge designed for two or more traffic lanes, the distribution factor is the lateral spacing of the beams divided by 5.0. This deterministic value was used in the above equation. The above equation was adopted from those found in AASHTO assuming that load distribution and other sources of uncertainty are adequately represented (AASHTO 1996).

In setting the thresholds that determine the Low, Medium and High states for \( M_{LL} \), the characteristics of \( M_{LL} \) were considered. \( M_{LL} \) represents the design liveload. Whether the moment due to this liveload is too high or too low cannot be separated out from the entire structural analysis of the beam. Therefore, the thresholds for this variable are arbitrarily set at 25% Lo, 50% Med, and 25% Hi (Lo threshold = 879 ft-kips, Hi threshold = 956 ft-kips).
INITIAL PRESTRESS FORCE

\[ P_{ai} = 0.69(nos)(As)(fs) \]

where

\[ P_{ai} = \text{initial prestress force immediately after transfer, kips} \]
\[ nos = \text{number of strands} \]
\[ As = \text{prestressing strand area, in}^2 \]
\[ Fs = \text{prestressing strand strength, ksi} \]

This equation is taken from AASHTO Standard Specifications (AASHTO 1996). It represents the initial prestress force in the strands immediately after transfer. According to AASHTO, at this stage, the initial stress in the tendon has been reduced by elastic shortening of the concrete and tendon relaxation during placing and curing the concrete for pretensioned members. The reductions to initial tendon stress can be taken as 0.69fs for low relaxation strand in typical pretensioned members (AASHTO 1996). This equation assumes that the force is uniformly distributed over the total strand area.

In setting the thresholds that determine the Low, Medium and High states for \( \Psi \), the variables that were included within the equation were considered. The thresholds for \( As \) are set at 0.151 and 0.155 in\(^2\) for Lo and Hi respectively. The thresholds for \( fs \) are set at 270 and 290 ksi for Lo and Hi, respectively. Inputting the respective Lo thresholds and Hi thresholds for these variables, a Lo and Hi threshold estimates for \( \Psi \) are 620 and 680 kips, respectively.
INITIAL CONCRETE STRESS

\[ f_{cir} = \frac{P_{st}}{A} + \frac{P_{se}e^2}{I} - \frac{M_g}{I} \]

where

- \( f_{cir} \) = average concrete stress at the center of gravity of the pretensioning steel due to the pretensioning force and deadload of beam immediately after transfer, ksi
- \( P_{st} \) = initial prestress force immediately after transfer, kips
- \( A \) = area of cross-section of the precast beam, in\(^2\)
- \( e \) = eccentricity of prestressing strands, in
- \( I \) = major axis moment of inertia of beam, in\(^4\)
- \( M_g \) = unfactored bending moment of beam due to self-weight, ft-kips

This equation is taken from AASHTO Standard Specifications (AASHTO 1996). It represents the initial prestress stress in the concrete immediately after transfer. According to AASHTO, \( f_{cir} \) shall be computed at the section of maximum moment, which is midspan in this case. This equation assumes that the force is uniformly distributed over the beam cross-sectional area. It also assumes that the stresses in the beam will remain within the elastic range. The equation computes the stress in the concrete as the summation of idealized stresses: normal axial stress due to the strands, bending stress due to the eccentric axial loading of strands and bending stress due to self-weight moment of beam.

Monte Carlo simulation of the above equation yields the following distribution for \( f_{cir} \):

In setting the thresholds that determine the Low, Medium and High states for \( f_{cir} \), the allowable compressive stresses of concrete were considered. The stress due to the prestressing strands exerts a compressive stress in the concrete at the center of gravity of the pretensioning steel while the deadload of the beam offsets this slightly with tensile stress. In setting the thresholds, only compressive stress was considered, not tensile stress. The allowable compressive stress in a prestress beam immediately after release is 0.60*fci. Considering the nominal value for initial compressive strength, an allowable of 2.4 ksi is rendered. This was considered in setting the upper threshold. However, the maximum value calculated for \( f_{cir} \) in the distribution is 2.31 ksi, somewhat below the 2.4 ksi allowable stress. Therefore, the thresholds were arbitrarily set at the quartile points: Lo threshold: 2.00 ksi, Hi threshold: 2.11 ksi.
CONCRETE STRESS DUE TO OTHER DEADLOAD

\[ f_{cds} = \frac{M_s e}{I} - \frac{M_{sdl} (y_{hc} - y_{bs})}{I_c} \]

where

- \( f_{cds} \) = average concrete stress at the center of gravity of the pretensioning steel due to all deadloads except the deadload present at the time the pretensioning force is applied, ksi
- \( M_s \) = unfactored bending moment due to concrete deck slab, ft-kips
- \( M_{sdl} \) = unfactored bending moment due to superimposed deadloads, ft-kips
- \( e \) = eccentricity of prestressing strands, in
- \( I \) = major axis moment of inertia of beam, in\(^4\)
- \( I_c \) = major axis moment of inertia of composite section, in\(^4\)
- \( y_{hc} \) = distance from the centroid to the bottom of beam of composite section, in
- \( y_{bs} \) = distance from the center of gravity of strands to the bottom fiber of beam, in

This equation is taken from AASHTO Standard Specifications (AASHTO 1996). It represents the stress in the concrete due to all the permanent deadloads: slab, wearing surface and barriers. Although the loading of each of these deadloads occurs at different stages, the equation accounts for all assuming all have been loaded. This equation assumes that the force is uniformly distributed over the beam cross-sectional area. It also assumes that the stresses in the beam will remain within the elastic range. The equation computes the stress in the concrete as the summation of idealized stresses: normal axial stress due to the strands, bending stress due to weight of the slab on the beam itself and bending stress due to the wearing surface and barriers on the composite section.

In setting the thresholds that determine the Low, Medium and High states for \( f_{cds} \), the allowable tensile stresses of concrete were considered. The stress due to these deadloads exerts a tensile stress in the concrete at the center of gravity of the pretensioning steel. In setting the thresholds, only tensile stress was considered, not compressive stress. The allowable tensile stress at service (midspan) is \( 6 \times \sqrt{f_c} \) or 0.424 ksi. This allowable stress is based on the nominal value of compressive strength of concrete, while throughout the rest of this analysis compressive strength is treated as a variable. According to AASHTO Standard Specifications (AASHTO 1996), this equation assumes that there is bonded reinforcement within the beam. The equation calculates \( f_{cds} \) to be greater than -0.424 ksi. The Lo threshold will be twice the allowable, -0.848 ksi. The Hi threshold will be set at the allowable, -0.424 ksi. Therefore, the stress that is considered allowable would be in the Hi State of \( f_{cds} \).
INITIAL MODULUS OF ELASTICITY OF CONCRETE

\[ E_{ci} = \left(33(w_c)^{1.5} \sqrt{f_{ci}}\right)/1000 \]

where

\[ E_{ci} = \text{initial modulus of elasticity of concrete, ksi} \]
\[ w_c = \text{unit weight of concrete, pcf} \]
\[ f_{ci} = \text{initial concrete compressive strength, psi} \]

The modulus of elasticity is the ratio of uni-axial normal stress to corresponding strain up to the proportional limit considering both tensile and compressive stresses (PCI 2003). This property helps determine the amount of deformation under load. In other words, it is included in the following calculations: camber, deflection, axial shortening and elongation as well as prestress losses. This equation is taken from AASHTO Standard Specifications (AASHTO 1996). Although this equation is specifically applied to initial conditions here, it applies to any point in time, dependent upon the compressive strength of the concrete.

In setting the thresholds that determine the Low, Medium and High states for \( E_{ci} \), the thresholds for the variables included within the equation were considered. Plugging in the Lo threshold values for \( w_c \) (140 pcf) and \( f_{ci} \) (3500 psi) yielded a Lo threshold of approximately 3230 ksi. Likewise, plugging in the Hi threshold values for \( w_c \) (160 pcf) and \( f_{ci} \) (5000 psi) yielded a Hi threshold of approximately 4720 ksi.
ELASTIC SHORTENING LOSS

\[ ES = \frac{E_s}{E_{ci}} f_{cir} \]

where

\[ ES = \text{elastic shortening loss, ksi} \]
\[ E_s = \text{modulus of elasticity of strand, ksi} \]
\[ E_{ci} = \text{initial modulus of elasticity of concrete, ksi} \]
\[ f_{cir} = \text{average concrete stress at the center of gravity of the pretensioning steel due to the pretensioning force and deadload of beam immediately after transfer, ksi} \]

Elastic shortening is an instantaneous occurrence of shortening of the member at transfer. Elastic shortening losses of prestress are considered partially recoverable with superimposed loads. This equation is presented in the AASHTO Standard Specifications (AASHTO 1996) and is considered an approximate value. This is due to the fact that the calculation of ES depends on \( f_{cir} \) the stress in the concrete that also occurs simultaneously at transfer.

In setting the thresholds that determine the Low, Medium and High states for ES, it was recognized that Elastic Shortening Losses must be considered with all other losses that occur, Creep, Shrinkage, etc. With no significant thresholds being obvious, the Lo and Hi thresholds are set at the quartile points of the distribution (Lo 13.9 ksi, Hi 15.9 ksi).
SHRINKAGE LOSS

\[ SH = 17 - (0.15)RH \]

where

- \( SH \) = shrinkage loss, ksi
- \( RH \) = relative humidity, %

Shrinkage loss is one component of the total loss of prestress. It is the decrease in volume of the concrete element when it loses moisture by evaporation. Relative humidity plays a great role in Shrinkage in that the rate of shrinkage is lower at high states of relative humidity (Nawy 1989). According to the PCI Manual (PCI 2003), the equation itself is based on assumptions and estimations.

The equation for estimating loss due to shrinkage of concrete, \( SH \), is roughly based on an ultimate concrete shrinkage strain of approximately \(-0.00042\) and a modulus of elasticity of approximately 28,000 ksi for prestressing strands. Correction factors for different average ambient relative humidities were applied resulting in three values of \( SH \), depending on regional average humidity. A straight line was drawn to best fit the three discrete values, resulting in the following equation for pretensioned members.

In setting the thresholds that determine the Low, Medium and High states for \( SH \), it was recognized that Shrinkage Losses must be considered with all other losses that occur, Creep, Elastic shortening, etc. With no significant thresholds being obvious, the Lo and Hi thresholds are set at the quartile points of the distribution (Lo 5.65 ksi, Hi 7.33 ksi).
CREEP LOSS

\[ CR_c = 12f_{cir} - 7f_{cds} \]

where

- \( CR_c \) = loss of pretension due to creep of concrete, ksi
- \( f_{cir} \) = average concrete stress at the center of gravity of the pretensioning steel due to the pretensioning force and deadload of beam immediately after transfer, ksi
- \( f_{cds} \) = average concrete stress at the center of gravity of the pretensioning steel due to all deadloads except the deadload present at the time the pretensioning force is applied, ksi

Creep is the increase in strain with time due to a sustained load. Creep loss is one component of the total loss of prestress. It is not directly observable, elastic strain and shrinkage must be deducted from total deformation (Nawy 1989). According to the PCI Manual (PCI 2003), the equation itself is based on assumptions and estimations.

This equation was developed by Gamble (1972) and was based on field measurements of a series of bridges in Illinois. The first term is based on a creep coefficient of approximately 1.7 and a modular ratio of 7. The second term in the equation represents the instantaneous elastic stress increase in the bonded tendons after application of additional dead loads. (It should be noted that creep due to \( f_{cds} \) would have a tendency to further increase the second term, resulting in lower predictions for \( CR_c \). This equation represents a conservative approach, i.e. assuming a creep coefficient of zero for these later permanent loads).

In setting the thresholds that determine the Low, Medium and High states for \( CR_c \), it was recognized that Creep Losses must be considered with all other losses that occur, Shrinkage, Elastic shortening, etc. With no significant thresholds being obvious, the Lo and Hi thresholds are set at the quartile points of the distribution (Lo 18.22 ksi, Hi 19.34 ksi).
TOTAL PRESTRESS LOSS

\[ TPL = SH + ES + CR_c + CR_s \]

where

\[ TPL = \text{total prestress loss, ksi} \]
\[ SH = \text{loss of pretension due to shrinkage, ksi} \]
\[ ES = \text{loss of pretension due to elastic shortening, ksi} \]
\[ CR_c = \text{loss of pretension due to creep of concrete, ksi} \]
\[ CR_s = \text{loss of pretension due to relaxation, ksi} \]

Total Prestress Loss (TPL) represents the combined effects of the effects of creep, shrinkage and the relaxation of the prestressing steel on the effective stress in the strands. The above equation was introduced in the AASHTO 1971 Interim Revision, where the total loss is represented as a sum of the individual components (PCI 2003). This is an approximate method, it does not account for the effects of friction. It assumes normal weight concrete and typical prestressed strands (250 or 270 ksi, seven-wire, stress-relieved or low-relaxation strand; 240 ksi stress-relieved wires; 145 to 160 ksi smooth or deformed bars) (AASHTO 1996).

Loss of prestress due to relaxation, \( CR_s \), is one of the components included within the above equation. Relaxation is the gradual loss of stress under constant strain exhibited by prestressing steel at strain levels typically experienced by concrete bridge members.

\[ CR_s = 5 - 0.1ES - 0.05(SH + CR_c) \]

where

\[ CR_s = \text{loss of pretension due to relaxation, ksi} \]
\[ SH = \text{loss of pretension due to shrinkage, ksi} \]
\[ ES = \text{loss of pretension due to elastic shortening, ksi} \]
\[ CR_c = \text{loss of pretension due to creep of concrete, ksi} \]

This equation assumes that the strands are low relaxation strands. It is based on the equation for stress-relieved strands "but is adjusted for intrinsic relaxation in low-relaxation strand of approximately 25% that of stress-relieved strand" (PCI 2003). The equation for stress-relieved strands has the following origin (PCI 2003):

This equation was also developed by Gamble (1972). The 0.4 multiplier on the ES term (note: 0.1 for low-relaxation strands) was based on analytical studies of total relaxation of stress-relieved strands with varying values of elastic shortening loss. Gamble reasoned that shrinkage and creep losses, occurring gradually and at a time when relaxation would occur at a reduced rate, would produce only approximately one-half as much reduction in relaxation as would elastic shortening loss that occurs very soon after the strand is stressed.

In incorporating these variables into the model, the relaxation equation was absorbed into the total prestress loss equation.

\[ TPL = 0.95SH + 0.99ES + 0.95CR_c + 5 \]

In setting the thresholds that determine the Low, Medium and High states for TPL, the Lo and Hi thresholds are arbitrarily set at the quartile points of the distribution (Lo 42.54 ksi, Hi 44.93 ksi).
EFFECTIVE FINAL PRESTRESS FORCE

\[ P_{se} = nos \times A_s \times (0.75 \times f_s - TPL) \]

where

\[ P_{se} = \text{effective final prestress force, kips} \]
\[ nos = \text{number of strands} \]
\[ A_s = \text{area of prestressing strand, in}^2 \]
\[ f_s = \text{ultimate stress of pretensioning reinforcement, ksi} \]
\[ TPL = \text{total prestress loss, ksi} \]

This equation calculates the effective prestress force. It assumes the effective final prestress is uniformly spread over the cross-sectional area of all the strands. Effective final prestress is the initial prestress minus the total prestress loss. This equation assumes that initial prestress is 75% of the ultimate stress of pretensioning reinforcement. This is a typical assumption used for a long history of successful designs (AASHTO 1996, PCI 2003).

In setting the thresholds that determine the Low, Medium and High states for \( P_{se} \), the Lo and Hi thresholds are arbitrarily set at the quartile points: (Lo 557.10 kips, Hi 579.96 kips).
DEADLOAD STRESS ON NON-COMPOSITE SECTION

\[ f_{\text{NDL}} = -\left(\frac{M_g + M_s}{I}\right)y_b \]

where

- \( f_{\text{NDL}} \) = deadload stress on non-composite section, ksi
- \( M_g \) = unfactored bending moment due to weight of beam, ft-kips
- \( M_s \) = unfactored bending moment due to deck slab, ft-kips
- \( I \) = major axis moment of inertia of beam, in\(^4\)
- \( y_b \) = distance from centroid to extreme bottom fiber of the non-composite beam, in

This equation calculates a portion of the total tensile stress at Service, specifically at the midspan and bottom of the beam. This is the location of the stress that typically governs the design (PCI 2003). This basic mechanics of materials equation assumes 1) the beam lies in a plane, 2) forces act perpendicular to longitudinal axis, 3) beam is simply supported. It assumes the deadload stress is uniformly spread over the cross-sectional area of the beam.

Monte Carlo simulation of the above equation yields the following distribution for \( f_{\text{NDL}} \):

Mean = -1.44 ksi, COV = 0.04

In setting the thresholds that determine the Low, Medium and High states for \( f_{\text{NDL}} \), the Lo and Hi thresholds are arbitrarily set at the quartile points: (Lo = -1.48 ksi, Hi = -1.40 ksi). The allowable tensile stress at service (midspan) is \( 6\sqrt{f_{ci}} = -0.398 \) ksi. This magnitude is well below that of the deadload stress on the non-composite section. According to PCI (PCI 2003), the maximum stress at this point should be below \( 7.5\sqrt{f_{ci}} = -0.498 \) ksi. However, it must be kept in mind that this is only a portion of the stress being applied and counteracted. Evaluation must consider the stresses due to prestressing as well.
TOTAL DEADLOAD STRESS ON SECTION

\[ f_{DL} = \frac{\left( M_{sdl} \right) y_{bc}}{I_c} = f_{NDL} \]

where

- \( f_{DL} \) = total deadload stress on section, ksi
- \( M_{sdl} \) = bending moment due to superimposed deadload moment, ft-kips
- \( I_c \) = major axis moment of inertia of composite section, in^4
- \( y_{bc} \) = distance from centroid to extreme bottom fiber of the non-composite beam, in
- \( f_{NDL} \) = deadload stress on non-composite section, ksi

This equation calculates the total tensile stress at Service due to deadload, specifically at the midspan and bottom of the beam. This is the location of the stress that typically governs the design (PCI 2003). This basic mechanics of materials equation assumes 1) the beam lies in a plane, 2) forces act perpendicular to longitudinal axis, 3) beam is simply supported. It assumes the deadload stress is uniformly spread over the cross-sectional area of the beam.

In setting the thresholds that determine the Low, Medium and High states for \( f_{DL} \), the Lo and Hi thresholds are arbitrarily set at the quartile points: (Lo = -1.69 ksi, Hi = -1.59 ksi). The allowable tensile stress at service (midspan) is \( 6*\sqrt{f_{c1}} = -0.398 \) ksi. This magnitude is well below that of the deadload stress on the composite section. According to PCI (PCI 2003), the maximum stress at this point should be below \( 7.5*\sqrt{f_{c1}} = -0.498 \) ksi. However, it must be kept in mind that this is only a portion of the stress being applied and counteracted. Evaluation must consider the stresses due to prestressing as well.
LIVELOAD STRESS ON SECTION

\[ f_{LL} = \left( \frac{M_{LL}}{I_c} \right) y_{hc} \]

where

- \( f_{LL} \) = total liveload stress on section, ksi
- \( M_{LL} \) = liveload moment per beam, ft-kips
- \( I_c \) = major axis moment of inertia of composite section, in\(^4\)
- \( y_{hc} \) = distance from centroid to extreme bottom fiber of the non-composite beam, in

This equation calculates the total tensile stress at Service due to liveload, specifically at the midspan and bottom of the beam. This is the location of the stress that typically governs the design (PCI 2003). This basic mechanics of materials equation assumes 1) the beam lies in a plane, 2) forces act perpendicular to longitudinal axis, 3) beam is simply supported. It assumes the liveload stress is uniformly spread over the cross-sectional area of the beam.

In setting the thresholds that determine the Low, Medium and High states for \( f_{LL} \), the Lo and Hi thresholds are arbitrarily set at the quartile points: (Lo –1.04 ksi, Hi –0.81 ksi). The allowable tensile stress at service (midspan) is \( 6*\text{sqrt}(f_{ci}) = 0.398 \) ksi. This magnitude is well below that of the liveload stress on the composite section. According to PCI (PCI 2003), the maximum stress at this point should be below \( 7.5*\text{sqrt}(f_{ci}) = -0.498 \) ksi. However, it must be kept in mind that this is only a portion of the stress being applied and counteracted. Evaluation must consider the stresses due to prestressing as well.
STRESS FROM PRESTRESS FORCE

\[ f_{pe} = \frac{P_{se}}{A} + \frac{P_{se}ey_b}{I} \]

where

- \( f_{pe} \) = compressive stress in concrete due to effective pretension forces only (after allowance of all pretension losses) at extreme fiber of section where tensile stress is caused by externally applied loads, ksi
- \( P_{se} \) = effective prestressing force after allowing for all losses, kips
- \( A \) = area of cross-section of the precast beam, in²
- \( e \) = eccentricity of prestressing strands, in
- \( y_b \) = distance from centroid to extreme bottom fiber of the non-composite beam, in
- \( I \) = major axis moment of inertia of beam, in⁴

This equation represents the effective compressive stress in the concrete after pretension losses. \( f_{pe} \) is computed at the section of maximum moment, which is midspan in this case. This equation assumes that the force is uniformly distributed over the beam cross-sectional area. It also assumes that the stresses in the beam will remain within the elastic range. The equation computes the stress in the concrete as the summation of idealized stresses: normal axial stress due to the strands and bending stress due to the eccentric axial loading of strands.

In setting the thresholds that determine the Low, Medium and High states for \( f_{pe} \), the Lo and Hi thresholds are arbitrarily set at the quartile points: (Lo 2.43 ksi, Hi 2.55 ksi). The allowable tensile stress at service (midspan) is \( 6*\sqrt{f_{ci}} = -0.398 \text{ ksi} \). This magnitude is well below that of the stress from prestress force on the composite section. According to PCI (PCI 2003), the maximum stress at this point should be below \( 7.5*\sqrt{f_{ci}} = -0.498 \text{ ksi} \). However, it must be kept in mind that this is only a portion of the stress being applied and counteracted. Evaluation must consider the stresses due to dead and live loads as well.
ALLOWABLE STRESS INVENTORY RATING

\[ AS_{\text{inv}} = \frac{f_{pe} - f_{dl} + 6\sqrt{f_c}}{f_{ll}} / 1000 \]

where

- \( AS_{\text{inv}} \) = Allowable Stress Inventory Rating
- \( f_{pe} \) = compressive stress in concrete due to effective pretension forces only (after allowance of all pretension losses) at extreme fiber of section where tensile stress is caused by externally applied loads, ksi
- \( f_{dl} \) = total deadload stress on section, ksi
- \( f_c \) = concrete compressive strength, psi
- \( f_{ll} \) = total liveload stress on section, ksi

This equation calculates the Inventory Rating Factory using the Allowable Stress methodology. First of all, the Rating Factor is the ratio of available liveload moment (or stress in this case) capacity to the moment (or stress) produced by the load being investigated (PCI 2003). The Inventory Rating determines the load that can safely utilize the bridge for an indefinite period. In the Allowable Stress methodology, instead of moment capacities, we work with allowable stresses. The numerator calculates the stress that is available for liveload. In other words, the tensile stress that is allowed along with the stresses due to deadload and pretensioning forces are summed, yielding the stress capacity that is available for liveload. The denominator is the liveload stress with which the beam is loaded. If the ratio is less than one, the available liveload stress capacity is insufficient to support the actual liveload.

In setting the thresholds that determine the Low, Medium and High states for \( AS_{\text{inv}} \), the Lo threshold was set at 0.15. This Load Rating represents that of a 3 ton truck (3 tons/20 tons). A bridge at this capacity should be closed. The Hi threshold was set at 1.00. A Load Rating less than 1.00 suggests that a bridge should be posted, this decision being ultimately made by the Owner.
NOMINAL MOMENT STRENGTH OF SECTION

\[ M_n = A_s f_{su} \left( d - \frac{a}{2} \right) \]

where

\( M_n = \) nominal moment strength of section, ft-kips
\( A_s = \) area of prestressed reinforcement, in\(^2\)
\( f_{su} = \) average stress in prestressing reinforcement at ultimate load, ksi
\( d = \) distance from top of slab to centroid of prestressing force, in
\( a = \) depth of stress block, in

This equation calculates the nominal moment strength of section (PCI 2003, AASHTO 1996). It is an approximate and simplified formula. The following assumptions are considered (PCI 2003):

1) The compression zone is either rectangular or T-shaped. Although in many cases with the composite section, the depth of the stress block is within the thickness of the flange (or deck) causing the assumption of rectangular beam behavior as valid.

2) The compression zone is within only one type of concrete; for composite members it is assumed to be within the deck concrete as discussed above.

3)

4) Effective pretension is not less than 50% of the ultimate strength of the strands.

5) Steel content must be below the amount that causes the steel stress being predicted to be lower than the yield strength. In other words, the formulas are only intended as an interpolation function between the yield and ultimate strengths of the steel.

6) Equivalent rectangular stress block with ultimate concrete strain of 0.003.

In setting the thresholds that determine the Low, Medium and High states for \( M_n \), the maximum and minimum reinforcement limits as dictated in the codes were first investigated (PCI 2003, AASHTO 1996). The maximum flexural strength, staying with the assumption of a rectangular stress block, is assumed to be:

\[ M_{n(max)} = \left( 0.36 \beta_1 - 0.08 \beta_1^2 \right) f'_c \ be \ d^2 \]

where

\( M_{n(max)} = \) maximum flexural strength that assumes the flexural strength is controlled by the capacity of the concrete compression part, ft-kips
\( \beta_1 = \) factor for concrete strength
\( f'_c = \) concrete deck strength, ksi
\( be = \) effective flange width, in
\( d = \) distance from extreme compression fiber to centroid of the pretensioning force, in
The minimum allowable moment is specified as $M_{n(min)} = 1.2*M_{cr}$:

$$M_{n(min)} = 1.2M_{cr} = 1.2 \left[ \left( f_r + f_{pe} \right) S_{bc} - \left( M_g + M_s \right) \left( \frac{S_{bc}}{S_b} - 1 \right) \right]$$

where

- $M_{cr}$ = cracking moment, ft-kips
- $f_r = 7.5*sqrt(f_{cd})$ = modulus of rupture of concrete, ksi
- $f_{pe}$ = effective prestress stress, ksi
- $S_{bc}$ = section modulus of composite section, in$^3$
- $M_g$ = unfactored modulus of composite section, ft-kips
- $M_s$ = unfactored bending moment due to concrete deck slab, ft-kips
- $S_b$ = section modulus of non-composite section, in$^3$

It was found that these thresholds are considerably too high (~200,000 ft-kips). The flexural strength of the beam as a minimum should counteract the unfactored dead and liveloads (~2000 ft-kips). Based on this, the lower threshold is estimated at 3500 ft-kips and the higher threshold could be set at 4000 ft-kips.
ALLOWABLE STRESS OPERATING RATING

\[ ASop = \frac{0.75M_n - M_d}{M_{LL}} \]

where

- \( ASinv \) = Allowable Stress Operating Rating
- \( M_n \) = nominal moment strength of section, ft-kips
- \( M_d \) = total unfactored bending moment due to deadloads, ft-kips
- \( M_{LL} \) = liveload moment per beam, ft-kips

This equation calculates the Operating Rating Factory using the Allowable Stress methodology. First of all, the Rating Factor is the ratio of available liveload moment (or stress in this case) capacity to the moment produced by the load being investigated (PCI 2003). The Operating Rating determines the absolute maximum permissible load to which the bridge can be subjected. According to AASHTO, for the operating rating, the allowable stresses should result in moments not to exceed 75 percent of the ultimate moment capacity of the member (PCI 2003).

In setting the thresholds that determine the Low, Medium and High states for ASop, those of the ASinv were used, as they are significant values concerning Load Ratings. The Lo threshold was set at 0.15. This Load Rating represents that of a 3 ton truck (3 tons/20 tons). A bridge at this capacity should be closed. The Hi threshold was set at 1.00. A Load Rating less than 1.00 suggests that a bridge should be posted, this decision being ultimately made by the Owner. Please note that some Bridge Owners post based on Inventory Rating and some post based on Operating Rating.
LOAD FACTOR INVENTORY RATING

\[ LFinv = \frac{M_n - 1.3M_d}{1.3 \times 1.67 \times M_{LL}} \]

where

- \( LFinv \) = Load Factor Inventory Rating
- \( M_n \) = nominal moment strength of section, ft-kips
- \( M_d \) = total unfactored bending moment due to deadloads, ft-kips
- \( M_{LL} \) = liveload moment per beam, ft-kips

This equation calculates the Inventory Rating Factor using the Load Factor methodology. First of all, the Rating Factor is the ratio of available liveload moment (or stress in this case) capacity to the moment produced by the load being investigated (PCI 2003). The Inventory Rating determines the load that can safely utilize the bridge for an indefinite period. In this equation a different factor is applied to each load to reflect the uncertainty inherent within the calculations.

In setting the thresholds that determine the Low, Medium and High states for \( LFinv \), those of the ASinv were used, as they are significant values concerning Load Ratings. The Lo threshold was set at 0.15. This Load Rating represents that of a 3 ton truck (3 tons/20 tons). A bridge at this capacity should be closed. The Hi threshold was set at 1.00. A Load Rating less than 1.00 suggests that a bridge should be posted, this decision being ultimately made by the Owner. Please note that some Bridge Owners post based on Inventory Rating and some post based on Operating Rating.
LOAD FACTOR OPERATING RATING

\[ LFop = \frac{M_n - 1.3M_d}{1.3M_{LL}} \]

where

\( LFop \) = Load Factor Operating Rating
\( M_n \) = nominal moment strength of section, ft-kips
\( M_d \) = total unfactored bending moment due to deadloads, ft-kips
\( M_{LL} \) = liveload moment per beam, ft-kips

This equation calculates the Operating Rating Factory using the Load Factor methodology. First of all, the Rating Factor is the ratio of available liveload moment (or stress in this case) capacity to the moment produced by the load being investigated (PCI 2003). The Operating Rating determines the absolute maximum permissible load to which the bridge can be subjected. In this equation a different factor is applied to each load to reflect the uncertainty inherent within the calculations. Notice that the factor applied to the liveload moment is less than that applied in the Inventory Rating calculation, thus rendering a lesser denominator and ultimately a higher rating, as is expected with Operating Ratings.

In setting the thresholds that determine the Low, Medium and High states for LFop, those of the ASinv were used, as they are significant values concerning Load Ratings. The Lo threshold was set at 0.15. This Load Rating represents that of a 3 ton truck (3 tons/20 tons). A bridge at this capacity should be closed. The Hi threshold was set at 1.00. A Load Rating less than 1.00 suggests that a bridge should be posted, this decision being ultimately made by the Owner. Please note that some Bridge Owners post based on Inventory Rating and some post based on Operating Rating.
MODULUS OF ELASTICITY OF BEAM CONCRETE

\[ E_{cb} = 33w_c^{1.5} \sqrt{f'_{cb}} \]

where

- \( E_{cb} \) = modulus of elasticity of beam concrete, ksi
- \( w_c \) = unit weight of concrete, kcf
- \( f'_{cb} \) = strength of beam concrete, psi

The above expression calculates the secant modulus of elasticity of concrete, \( E_{cb} \) (PCI 1999). The modulus of elasticity is the ratio of uniaxial normal stress to corresponding strain up to the proportional limit for both tensile and compressive stresses. It is the material property that determines the amount of deformation under load (PCI 2003). This value satisfies the practical assumption that strains occurring during loading can be considered basically elastic (completely recoverable on unloading), and that any subsequent strain due to the load is regarded as creep. Precast concrete components are required to achieve a minimum concrete strength for release and removal from their precasting bed at an early age (12 – 18 hours). This often results in a concrete that has a 28 day compressive strength in excess of the specified 28 day strength. Consequently, the concrete has a higher modulus of elasticity and less creep than would occur if the actual strength were equal to the specified strength (PCI 2003).

In setting the thresholds that determine the Low, Medium and High states for \( E_{cb} \), the quartile points were arbitrarily assumed giving Lo threshold = 4181 ksi and Hi threshold = 4802 ksi.
MODULUS OF ELASTICITY OF DECK CONCRETE

\[ E_{cd} = 33w_c^{1.5} \sqrt{f'_{cd}} \]

where

- \( E_{cd} \) = modulus of elasticity of deck concrete, ksi
- \( w_c \) = unit weight of concrete, kcf
- \( f'_{cd} \) = strength of beam concrete, psi

The above expression calculates the secant modulus of elasticity of deck concrete, \( E_{cd} \) (PCI 1999). The modulus of elasticity is the ratio of uniaxial normal stress to corresponding strain up to the proportional limit for both tensile and compressive stresses. It is the material property that determines the amount of deformation under load (PCI 2003). This value satisfies the practical assumption that strains occurring during loading can be considered basically elastic (completely recoverable on unloading), and that any subsequent strain due to the load is regarded as creep.

In setting the thresholds that determine the Low, Medium and High states for \( E_{cd} \), the quartile points were arbitrarily assumed giving Lo threshold = 3256 ksi and Hi threshold = 3788 ksi.
LONGITUDINAL STIFFNESS PARAMETER

\[ K_g = n \left( I + A e_g^2 \right) \]

where

- \( K_g \) = longitudinal stiffness parameter, in\(^4\)
- \( n \) = modular ratio between beam and deck materials, \((E_{cb}/E_{cd})\)
- \( I \) = moment of inertia of beam, in\(^4\)
- \( A \) = cross-sectional area of beam, in\(^2\)
- \( e_g \) = distance between the centers of gravity of the basic beam and deck, in

The above equation accounts for the longitudinal stiffness of the beam (LRFD EQ. 4.6.2.2.1-1)(AASHTO 1998). The longitudinal stiffness is considered when calculating the liveload.

In setting the thresholds that determine the Low, Medium and High states for \( K_g \), the quartile points were arbitrarily assumed giving Lo threshold = 681277 ksi and Hi threshold = 835213 ksi.
LRFD LIVELOAD MOMENT

\[ M_{LL(LRFD)} = LDF \left[ M_{HS20\text{lane}} + \max(M_{HS20\text{truck}}, M_{HS20\text{tandem}}) (1 + IM) \right] \]

where

\[ M_{LL(LRFD)} = \text{HL-93 liveload moment, ft-kips} \]
\[ LDF = \text{lane load distribution factor} \]
\[ M_{HS20\text{lane}} = \text{maximum lane moment per lane, ft-kips} \]
\[ M_{HS20\text{truck}} = \text{maximum truck moment per lane, ft-kips} \]
\[ M_{HS20\text{tandem}} = \text{maximum tandem moment per lane, ft-kips} \]
\[ IM = \text{dynamic load allowance} \]

The live loading used in LRFD (HL-93) represents current highway loading and varying span lengths. Previously in the Standard Specifications three hypothetical legal vehicles were used to account for short, medium and long spans (Type 3, Type 3S2, Type 3-3). HL-93 combines the use of these three trucks and accounts for varying span lengths and offers the uniform reliability characteristic of the LRFD methodology.

HL-93 combines an HS20 lane load (a 0.64 k/ft uniformly distributed lane load similar to the lane load of the AASHTO Standard Specs but without any of the associated concentrated loads) and a vehicle (the greater of an HS20 or a 50 kip design tandem). The superposition of these loads offers a representation of the force effects due to a 57 ton vehicle (SCDOT 2006). This truck being heavier than the 20 ton truck (HS20) used in the previous Specifications is a more realistic representation of the current live loadings. Consequently, in the formula, the HS20 truck load used is twice the HS20 wheel load used to compute the liveload in the AS and LF methodologies.

Please note, that the variable used to represent HS20 liveload moment includes the effects of impact (Nowak 1993). Therefore the dynamic load allowance is not included within these calculations.

The above equation includes AASHTO lane-load distribution factor for cross-section “type k” with 2 or more lanes loaded:

\[ LDF = 0.075 + \left( \frac{S}{9.5} \right)^{0.6} \left( \frac{S}{L} \right)^{0.2} \left( \frac{K_g}{12Lt_s^4} \right)^{0.1} \]

where

\[ S = \text{space between beams, ft} \]
\[ L = \text{beam length, ft} \]
\[ K_g = \text{longitudinal stiffness parameter, in}^4 \]
\[ t_s = \text{slab thickness, in} \]

This equation accounts for the liveload distribution for the beam (LRFD Table 4.6.2.2b-1)(AASHTO 1998). The above equation was adopted from those found in AASHTO assuming that load distribution and other sources of uncertainty are adequately represented (AASHTO 1998).

In setting the thresholds that determine the Low, Medium and High states for \( M_{LL(LRFD)} \), the characteristics of \( M_{LL} \) were considered. \( M_{LL} \) represents the HL-93 design liveload. Whether the moment due to this liveload is too high or too low cannot be separated out from the entire structural analysis of the beam. Therefore, the thresholds for this variable are arbitrarily set at 25% Lo, 50% Med, and 25% Hi (Lo threshold = 932 ft-kips, Hi threshold = 1192 ft-kips).
LRFD LIVELOAD STRESS ON SECTION

\[ f_{LL(LRFD)} = \frac{(M_{LL(LRFD)})y_{hc}}{I_c} \]

where

- \( f_{LL(LRFD)} \) = total LRFD liveload stress on section, ksi
- \( M_{LL(LRFD)} \) = LRFD liveload moment per beam, ft-kips
- \( I_c \) = major axis moment of inertia of composite section, in\(^4\)
- \( y_{hc} \) = distance from centroid to extreme bottom fiber of the non-composite beam, in

This equation calculates the total tensile stress at Service due to liveload calculated according to the LRFD methodology, specifically at the midspan and bottom of the beam. This is the location of the stress that typically governs the design (PCI 2003). This basic mechanics of materials equation assumes 1) the beam lies in a plane, 2) forces act perpendicular to longitudinal axis, 3) beam is simply supported. It assumes the liveload stress is uniformly spread over the cross-sectional area of the beam.

In setting the thresholds that determine the Low, Medium and High states for \( f_{LL} \), the Lo and Hi thresholds are arbitrarily set at the quartile points: (Lo 1.08 ksi, Hi 1.38 ksi). The allowable tensile stress at service (midspan) is \( 6*\sqrt{f_{ci}} = 0.398 \) ksi. This magnitude is well below that of the liveload stress on the composite section. According to PCI (PCI 2003), the maximum stress at this point should be below \( 7.5*\sqrt{f_{ci}} = 0.498 \) ksi. However, it must be kept in mind that this is only a portion of the stress being applied and counteracted. Evaluation must consider the stresses due to prestressing as well.
LRFD NOMINAL MOMENT STRENGTH OF SECTION

\[ M_{n(LRFD)} = A_s f_{ps} \left( d - \frac{a}{2} \right) \]

where

- \( M_n \) = nominal moment strength of section, ft-kips
- \( A_s \) = area of prestressed reinforcement, in\(^2\)
- \( f_{ps} \) = average stress in prestressing reinforcement, ksi
- \( d \) = distance from top of slab to centroid of prestressing force, in
- \( a \) = depth of stress block, in

This equation calculates the nominal moment strength of section (PCI 2003, AASHTO 1998). It is an approximate and simplified formula. In addition to the assumptions of the equation for the nominal moment resistance for the LFD and ASD methodologies previously discussed, this equation does not consider compression reinforcement or mild tension reinforcement. Also, the section is assumed to behave as rectangular.

In setting the thresholds that determine the Low, Medium and High states for \( M_{n(LRFD)} \), the same thresholds were used as with the LFD and ASD flexural moment resistance, \( M_n \). Based on this, the lower threshold is estimated at 3500 ft-kips and the higher threshold could be set at 4000 ft-kips.
LRFD INVENTORY RATING: STRENGTH I

\[ LRFD_{inv1} = \frac{M_{n(LRFD)} - 1.25M_d - 2.75M_{ws}}{1.75M_{LL(LRFD)}} \]

where

- \( LRFD_{inv1} \) = LRFD Inventory Rating, Strength I
- \( M_{n(LRFD)} \) = LRFD nominal moment strength of section, ft-kips
- \( M_d \) = total unfactored bending moment due to deadloads, ft-kips
- \( M_{ws} \) = unfactored bending moment due to wearing surface, ft-kips
- \( M_{LL(LRFD)} \) = LRFD liveload moment per beam, ft-kips

This equation calculates the Inventory Rating Factor using the LRFD methodology. The load combination Strength I is used for design at the strength limit state (PCI 2003). The Inventory Rating determines the load that can safely utilize the bridge for an indefinite period. In this equation a different factor is applied to each load to reflect the uncertainty inherent within the calculations.

In setting the thresholds that determine the Low, Medium and High states for LRFDinv1, only one threshold is significant, 1.00. The LRFD rating factor calculated does not correlate to a vehicular tonnage the way the rating factors calculated according to the ASD and LFD methodologies do.
LRFD INVENTORY RATING: SERVICE III

\[
LRFD_{inv2} = \frac{f_{pe} - f_{dl} + 6\sqrt{f_{c}}/1000}{f_{ll}(LRFD)}
\]

where

- \( LRFD_{inv2} \) = LRFD Inventory Rating, Service III
- \( f_{pe} \) = compressive stress in concrete due to effective pretension forces only (after allowance of all pretension losses) at extreme fiber of section where tensile stress is caused by externally applied loads, ksi
- \( f_{dl} \) = total deadload stress on section, ksi
- \( f_{c} \) = concrete compressive strength, psi
- \( f_{ll}(LRFD) \) = LRFD total liveload stress on section, ksi

This equation calculates the Inventory Rating Factory using the LRFD methodology. As explained before, the Rating Factor is the ratio of available liveload moment (or stress in this case) capacity to the moment (or stress) produced by the load being investigated (PCI 2003). The Inventory Rating determines the load that can safely utilize the bridge for an indefinite period. In LRFD Service III, instead of moment capacities, we work with allowable stresses. The numerator calculates the stress that is available for liveload. In other words, the tensile stress that is allowed along with the stresses due to deadload and pretensioning forces are summed, yielding the stress capacity that is available for liveload. The denominator is the liveload stress with which the beam is loaded. If the ratio is less than one, the available liveload stress capacity is insufficient to support the actual liveload. Service III was introduced to specifically apply to prestressed concrete bridges under service conditions because Strength I led to an over-conservative design. In Service III the liveload is given less of a influence with a smaller numerical factor. The use of Service III is justified because of the multitude of bridges that have been designed previously for a lesser liveload exhibit no significant cracking (PCI 2003).

In setting the thresholds that determine the Low, Medium and High states for LRFDinv2, only one threshold is significant, 1.00. The LRFD rating factor calculated does not correlate to a vehicular tonnage the way the rating factors calculated according to the AS and LF methodologies do.
LRFD OPERATING RATING: STRENGTH I

\[
LRFD_{op} = \frac{M_{n(LRFD)} - 1.25M_d - 2.75M_{ws}}{1.35M_{LL(LRFD)}}
\]

where

- \( LRFD_{op} \) = LRFD Inventory Rating, Strength I
- \( M_{n(LRFD)} \) = LRFD nominal moment strength of section, ft-kips
- \( M_d \) = total unfactored bending moment due to deadloads, ft-kips
- \( M_{ws} \) = unfactored bending moment due to wearing surface, ft-kips
- \( M_{LL(LRFD)} \) = LRFD liveload moment per beam, ft-kips

This equation calculates the Operating Rating Factor using the LRFD methodology. The load combination Strength I is used for design at the strength limit state (PCI 2003). The Operating Rating determines the absolute maximum permissible load to which the bridge can be subjected. In this equation a different factor is applied to each load to reflect the uncertainty inherent within the calculations. Notice that the factor applied to the liveload moment is less than that applied in the Inventory Rating calculation, thus rendering a lesser denominator and ultimately a higher rating, as is expected with Operating Ratings.

In setting the thresholds that determine the Low, Medium and High states for \( LRFD_{op} \), only one threshold is significant, 1.00. The LRFD rating factor calculated does not correlate to a vehicular tonnage the way the rating factors calculated according to the ASD and LFD methodologies do.
APPENDIX A5

The probabilistic prognostic component is a Matlab program coded by the author.

The program is over 10,000 lines and is not included here.

It is available upon request.
APPENDIX B1

In developing the model of the corrosion of a prestressed concrete girder, an existing model used by many researchers (Akgul 2002, Estes 1997, Enright 1998, Thoft-Christensen et al. 1997) was adopted. The model presented in (Akgul 2002) served as a primary reference. Akgul’s research on lifetime reliability of bridges included the creation of time variant resistance models for corrosion of structural materials of bridge members. Major assumptions are: 1) chloride intrusion occurs through diffusion only, 2) diffusion is uniform around beam perimeter, 3) concrete cracking is not specifically modeled, 4) the corrosion of the prestressed strand is represented as a uniform reduction of the strand area around the perimeter. These assumptions are discussed in more detail as the model is explained.

Fick’s first law is used to model chloride diffusion in concrete considering one dimension (Stanish et al. 1997):

\[ J = -D_c \frac{dC}{dx} \]  

(B.1)

where

- \( J \) = flux of chloride ions (mass/(area x time), lb/in\(^2\)yr)
- \( D_c \) = effective diffusion coefficient (area/time, in\(^2\)yr)
- \( C \) = concentration of chloride ions (mass/volume, lb/in\(^3\))
- \( x \) = distance from outer surface of the solid (length, in.)

The above equation assumes that the diffusion is constant in time and changes with distance only such that it represents steady state conditions. Fick’s second law accounts for non-steady state condition where \( C(x,t) \) changes with time:

\[ \frac{\partial C}{\partial t} = D_c \frac{\partial^2 C}{\partial x^2} \]  

(B.2)

Considering the boundary condition \( C(x = 0, t > 0) = C_o \), initial condition \( C(x > 0, t = 0) = 0 \), and infinite point condition \( C(x = \infty, t > 0) = 0 \), Crank’s solution may be applied to this partial differential equation (Crank 1975). John Crank was a 20th century mathematical physicist made famous through his numerical solutions of partial differential equations (Wikipedia 2007). Crank’s solution to Fick’s second law has long been accepted as sound engineering (Richardson 2002).

\[ C(x,t) = C_o \left[ 1 - \text{erf} \left( \frac{x}{\sqrt{4D_c t}} \right) \right] \]  

(B.3)

where

- \( C \) = concentration of chloride ions
- \( C_o \) = chloride concentration on the surface of concrete (% by weight of concrete)
- \( \text{erf} \) = the error function given by:

\[ \text{erf}(x) = \frac{2}{\sqrt{\pi}} \left( \frac{1}{0!} - \frac{x}{1!} + \frac{x^3}{2!} - \frac{x^5}{3!} + \frac{x^7}{4!} - \cdots \right) \]  

(B.4)
writing $C(x,t)$ in terms of the time variable, $t$:

$$
t = \frac{x^2}{4D_c} \left[ \text{erf}^{-1}\left(1 - \frac{C(x,t)}{C_o}\right) \right]^{-2}
$$

The above equation gives the time required to reach the chloride concentration $C(x,t)$ at a distance $x$ from the surface. In order to determine the time $t = T_i$ to reach the threshold (critical) level of chloride concentration, $C_{cr}$, at which the corrosion process will start, the $C_{cr}$ value of interest is input into the equation.

$$
T_i = \frac{x^2}{4D_c} \left[ \text{erf}^{-1}\left(1 - \frac{C_{cr}}{C_o}\right) \right]^{-2}
$$

**Surface Chloride Concentration, $C_o$**

Surface chloride concentration, $C_o$, refers to the average chloride concentration that is found on the surface of prestressed concrete girders. The value used for the surface chloride concentration, $C_o$, is adopted from the model developed by Akgul (2002). In the Akgul model, the mean chloride surface concentration on reinforced concrete and prestressed concrete girders is assumed roughly as 0.15% by weight of concrete. This number is based upon experimental results of Coggins et al. (1990) who conducted tests on 20-year-old prestressed concrete bridge girders. Coggins determined the chloride concentration at various points using 195 samples taken from three girders. Akgul plotted the data from all three beams onto one graph and fitted a fourth order polynomial to the experimental data. From this, a value of $C_o = 0.15\%$ by weight of concrete was estimated.

**Diffusion Coefficient, $D_c$**

The diffusion coefficient indicates the amount of chloride that is able to pass through the concrete in a certain amount of time. The diffusion coefficient, $D_c$, assumed for this model is 0.0865 in$^2$/yr. This value was obtained through the use of a state of the art computer-integrated knowledge system (CIKS) developed by the National Institute Standards and Technology (NIST). This CIKS is one of many developed by the NIST’s Building and Fire Research Laboratory in Gaithersburg, Maryland and specifically predicts the chloride ion diffusivity among other properties (Bentz et al. 1996). This program is directly accessible on the internet (http://ciks.cbt.nist.gov/vcctl11/newdiff.html). This NIST program has assumptions that fall in line with those previously specified for modeling corrosion of a prestressed concrete beam of this research. It assumes that the concrete has experienced no cracking or other defects. Also, it assumes that the chloride ion diffusion is the sole activity within the concrete. This NIST program also assumes that the concrete is saturated. The developers present this program not only as a predictor of the diffusivity of concrete but to estimate the improvements regarding diffusivity that come with the addition of silica fume. According to the developers, results rendered using the programs appear to be in good agreement with experimental data of recent studies (Bentz et al. 1996). An earlier version of this program was used by Akgul (2002) in determining a value for the diffusion coefficient, $D_c$. In this research, the most recent version of the program is used to determine a suitable diffusion coefficient, $D_c$.

Diffusion in concrete is affected by different mix design parameters. As such, the program requires for input, four basic parameters of mix design information: 1) water/cement ratio, 2) silica fume (mass % of concrete), 3) volume % of aggregate, and 4) the cement degree of hydration. In using this program here, the value of these parameters were chosen in reference to the example bridge used in this research. The 28-day compressive concrete strength of this particular bridge is specified at 5000 psi. The Sample Production Concrete Mixes listed in the PCI Manual list Mix A having a corresponding strength (PCI 2003).

1) Water/Cement Ratio

Water/cement ratio is a measure of the quality of the paste that determines the quality of the concrete. The paste is made up of portland cement, water and entrapped air. Reduced water content increases concrete strength (both compressive and flexural), lowers permeability, increases resistance to weathering, etc.
However enough water must be used to allow the concrete to consolidate properly (Kosmatka and Panarese 1988). In the program, the recommended range for the water/cement ratio is [0.30-0.50] with a default value of 0.40. The water/cement ratio used here (corresponding to Mix A) is 0.38.

2) Silica Fume
Silica fume is an admixture that changes the properties of the concrete. One intention of the developers of this NIST program was to promote the use of silica fume to improve the concrete in regards to diffusivity. The input requires a mass % of cement of silica fume with [0-10%] being the recommended range, default=5%. In this research, the concrete mix A (PCI 2003), has no silica fume, therefore the value of 0% was entered.

3) Volume Percent of Aggregate
The two major components of concrete are the aggregates and paste. Aggregates are classified as fine or coarse depending on their size. Aggregates are selected based on their strength and resistance characteristics as well as the desired array of sizes needed to make up 60-75% of the total volume of concrete. The program recommends the volume % of aggregate be 62-70%, the default being 70%. Concrete mix A (PCI 2003) offers no information on volume percent of aggregate. Therefore, the volume % of aggregate is set at 62%. This lowest value conservatively renders the highest diffusion coefficient. In this way the most aggressive chloride intrusion is considered.

4) Cement Degree of Hydration
Hydration is the chemical reaction between cement and water. The amounts of contributing cement compounds as well as water within the mix determine properties of the concrete such as strength. There are four chemical compounds that make up at least 90% of the weight of portland cement: tricalcium silicate, dicalcium silicate, tricalcium aluminate, tetracalcium aluminoferrite. The two calcium silicates react with water to form calcium hydroxide and calcium silicate hydrate. Calcium silicate hydrate gel is considered the most important strength-determining component of concrete (Kosmatka and Panarese 1988). The program recommends a range of [0.6-0.9] for cement degree of hydration, the default being 0.75. The PCI Manual offers information on the parameters of various Concrete Mixes, however cement degree of hydration is not included (PCI 2003). In the same manner as the previous parameter, volume percent of aggregate is set at the lowest value, 0.6, as it conservatively renders the highest value for a diffusion coefficient.

**Critical Chloride Concentration, C_{cr}**
Critical chloride concentration is the threshold value of chloride concentration at the surface of the steel after which corrosion begins. It is measured in % by weight of concrete. Akgul (2002) performed an extensive review of different values found in the literature. Based on these values, Akgul estimated a mean value of critical chloride concentration at 0.037%. This value is used in this research also.

**Distance from Surface, x**
All possible distances of diffusion were considered for the beam of this example. Figure B1 shows that the shortest distance of diffusion to a prestressing strand for this example beam is 2.95 in. Related figures showing distances of diffusion are also shown.
FIGURE B1  Diffusion Distance, Group 1

FIGURE B2  Diffusion Distance, Group 2
FIGURE B3  Diffusion Distance, Group 3

FIGURE B4  Diffusion Distance, Group 4
Based on this minimum distance of diffusion, a Matlab program has been written that calculates the time to reach the critical level of chloride concentration (See Appendix B2).

*Reinforcing Steel*

Reinforcing steel within the beam is shown in the beam cross-section of Figure B6. It should be noted that the diffusion of chloride ions encounters the reinforcing steel before the prestressing steel. This induces delamination, cracking and spalling and ushers in the initiation of corrosion of the prestressing strands. The reinforcing steel is not directly accounted for within the deterioration component. However the acknowledgement of the possible effects upon the prestressing steel help to justify the conservative choices for parameters, such as the diffusion coefficient, $D_c$. 

**FIGURE B5** Diffusion Distance, Group 5

**FIGURE B6** Reinforcing Steel in Prestressed Beam
APPENDIX B2

% Deterioration Component
% Prestressed Steel Corrosion Model
coded by Keary LeBeau
Northeastern University

% Variables

% Astrand = area of strand before deterioration occurs (in^2)
% Co = surface chloride concentration (0.15% by weight of concrete (Akgul 2002))
% Cr = critical chloride concentration (0.037% by weight of concrete (Akgul 2002))
% Dc = diffusion coefficient (0.825x10^-12 m^2/s = 0.0403 in^2/yr)(CIKS 2007)
% Do = original diameter (in)
% Dred = reduced diameter due to corrosion (in)
% rcorr = corrosion rate (2.25 mils/yr = 0.00225 in/yr)
% t = time (yrs)
% Tt = time that threshold level of critical chloride concentration (yrs)
% x = distance from face of beam to prestressing strand (in)

Astrand = 0.1548;                 %(in^2) NOTE: this is the mean of the variable in master program
%Dc = 0.0403;                      % diffusion coefficient (in^2/yr) (average of possible values)
Dc = 0.0865;                         % diffusion coefficient (in^2/yr) (maximum of possible values)
Ccr = 0.037;                          % critical chloride concentration (% by weight of concrete)
Co = 0.15;                             % surface chloride concentration (% by weight of concrete)
Do = 2*(sqrt(Astrand/pi));   % approximate diameter as if 7-wire strand was circular
Rcorr  = 0.00225;

% initialize Dred and Ared with whole diameters and areas
Dred(1:100,1:6) = ones*Do;
Ared(1:100,1:6) = ones*Astrand;

% Calculate the time to reach the threshold level of critical chloride concentration
% Loop to consider all possible distances of diffusion
x=[1.5 2.95 3 4.74 5 7];% distance from face to prestressing strand (in)

for k=1:6
    Tt(1,k)=(((x(1,k)-0.25)^2)/(4*Dc))*(erfinv(1-(Ccr/Co))^2);
    Tt(1,k)=ceil(Tt(1,k));

    % Calculate the time to complete disappearance after the start of corrosion
    Tend(1,k)=Tt(1,k)+(Do/rcorr);
    Tend(1,k)=ceil(Tend(1,k));
end

% This loop calculates the reduction in diameter of prestress steel.
% corrosion of bar is linear reduction of bar diameter
% Please note...prestress strands have a nominal diameter, say 0.5 inch.
% However, the area of the strand is not a perfect circle, this model
% considers the seven wire strand. Therefore, this model will only
% offer an estimate of the reduction of area that takes place

% calculate deficient areas through time
for t = Tt(1,k):100
    Dred(t,k) = Do-reorr*(t-Tt(1,k));
    Ared(t,k) = (pi/4)*(Dred(t,k))^2;
% adjust reduced area to apply to non-circular strand
%Ared(t,k) = Astrand*(((Dred(t,k))^2)/(Do)^2);
%Ared(t,k) = Astrand*(Dred(t,k)/Do);
end

%**********************************************************************

end

% this loop calculates the total As as it decreases through time
% it also calculates the average As to be input into the master program
for y=1:100
    totAs(y,1) = 4*Ared(y,2)+8*Ared(y,3)+2*Ared(y,4)+6*Ared(y,5)+2*Ared(y,6);
    totstrand(y,1) = totAs(y,1)/Astrand;
    aveAs(y,1) = totAs(y,1)/22;
end

plot(totstrand)
xlabel('Time, years')
ylabel('Total Number of Strands')
figure
plot(aveAs)
xlabel('Time, years')
ylabel('Average Strand Area')
APPENDIX B3

This Appendix describes the root variables and any changes in their statistics from the As-Built version as well as modifications to the actual structure of the BN.

TABLE B1  Root Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
<th>Bias</th>
<th>COV</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Cross-sectional Area of Beam</td>
<td>in²</td>
<td>1.00</td>
<td>0.100</td>
<td>Normal</td>
</tr>
<tr>
<td>Aₚ</td>
<td>Prestressing Strand Area</td>
<td>in²</td>
<td>1.01</td>
<td>0.100</td>
<td>Normal</td>
</tr>
<tr>
<td>f'ₚₚ</td>
<td>Concrete Strength of Beam</td>
<td>psi</td>
<td>1.10</td>
<td>0.180</td>
<td>Normal</td>
</tr>
<tr>
<td>f'ₚₜ</td>
<td>Concrete Strength of Deck</td>
<td>psi</td>
<td>1.00</td>
<td>0.200</td>
<td>Normal</td>
</tr>
<tr>
<td>fₚ</td>
<td>Prestressing Strand Strength</td>
<td>ksi</td>
<td>1.04</td>
<td>0.020</td>
<td>Normal</td>
</tr>
<tr>
<td>Eₚ</td>
<td>Modulus of Elasticity of Prestressing Strands</td>
<td>ksi</td>
<td>1.01</td>
<td>0.010</td>
<td>Normal</td>
</tr>
<tr>
<td>I</td>
<td>Moment of Inertia of Beam Cross-section</td>
<td>in⁴</td>
<td>1.00</td>
<td>0.050</td>
<td>Normal</td>
</tr>
<tr>
<td>Iₛ</td>
<td>Moment of Inertia of Composite Section</td>
<td>in⁴</td>
<td>1.00</td>
<td>0.060</td>
<td>Normal</td>
</tr>
<tr>
<td>RH</td>
<td>Relative Humidity</td>
<td>%</td>
<td>1.00</td>
<td>0.120</td>
<td>Normal</td>
</tr>
<tr>
<td>wₙ</td>
<td>Unit Weight of Concrete</td>
<td>pcf</td>
<td>1.00</td>
<td>0.030</td>
<td>Normal</td>
</tr>
<tr>
<td>wst</td>
<td>Wearing Surface Thickness</td>
<td>psf(in)</td>
<td>1.10</td>
<td>0.200</td>
<td>Normal</td>
</tr>
</tbody>
</table>
The cross-sectional area, $A$, of a beam decreases with time. Concrete spalls due to interior corrosion. Also, collision damage (scraps, nicks, gouges) causes a reduction in area. This model focuses on deterioration due to chloride attack. Chloride infiltrates through the concrete and once the steel within is reached, corrosion begins. The structural integrity of the beam is already compromised (interior corrosion of the steel initiated and propagated) once the spalling occurs. Therefore, this model considers the deterioration of the prestressing steel as the governing factor. It also assumes that $A$ and $A_s$ are independent. However, for purposes of discussion the behavior of the cross-sectional area through time is explored below.

**FIGURE B7** Distribution of Cross-sectional Area of Beam, $A$
Bridge spalling is indicative of interior corrosive activity. As discussed before, corrosion causes the steel to increase in size and exert pressure on the surrounding concrete causing delamination, cracking and eventually spalling. Two spalling patterns are considered for modeling purposes. Pattern A shows a beam with spalling of the two bottom corners. This equates to approximately 2% Section Loss of the beam. It also assumes that two strands are exposed as a result of the spalling.

Pattern B shows a larger spall on the bottom face of the beam. This spall is assumed to expose 4 strands. The Section Loss is approximately 5% of the total cross-sectional area.

**FIGURE B8** Deterioration Pattern A

**FIGURE B9** Deterioration Pattern B
Incorporating the decrease in cross-sectional area over time could take place in two separate versions of the BN diagnostic component. In the first version, it is assumed that spalling is an “after-effect” of the corrosion of interior steel. Therefore, there is a dependency between the two variables, $A$ and $A_s$.

Hypothetically, with Deterioration Pattern $A$ we assume two corroded strands (9% $A_s$) and two spalls (2% $A$). However, the two strands could be each half corroded (half of 9% $A_s$ is approximately 5% $A_s$) or there could be a greater amount of corrosion that is within the beam (say 4 strands or 18% $A_s$). The condition probability table linking the two would look something like this:

**Table B2 Conditional Probability Table of $A$**

<table>
<thead>
<tr>
<th>$A_s$</th>
<th>$\leq 90%$</th>
<th>$&gt;95%$</th>
<th>$90.01-95%$</th>
<th>$&gt;95%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq 90%$</td>
<td>0.80</td>
<td>0.15</td>
<td>0.45</td>
<td>0.20</td>
</tr>
<tr>
<td>$&gt;95%$</td>
<td>0.10</td>
<td>0.20</td>
<td>0.70</td>
<td></td>
</tr>
<tr>
<td>$90.01-95%$</td>
<td>0.30</td>
<td>0.45</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

**FIGURE B10** Dependency of $A$ upon $A_s$
By declaring the dependency of $A$ upon $A_s$, it is necessary to eliminate the dimensional variance ($DV$) variable. The moment of inertias ($I$ and $I_c$) become directly dependent upon $A$.

In the second version, the BN diagnostic component opens the door to the possibility of collision damage. Therefore, section loss of the beam could occur independent of the internal corrosion. This is where the relationship between these two variables becomes complicated. For example, the gouge or spall caused by collision damage could expose the internal steel to the corrosive environment, by-passing the chloride diffusion process. This scenario suggests that the variable $A_s$ is dependent upon the variable $A$, opposite to the first version. However, it should be pointed out that the possibility of collision damage begins once the bridge is built and traffic passes underneath. The corrosion process by diffusion takes time to establish. Therefore, if the two scenarios were to be considered together, $A_s$ would be dependent upon $A$, however, the conditional probability table would have to reflect the reverse influence of $A_s$ upon $A$, even going so far as to adjust the conditional probability table through time. It is most likely that this version will be explored in future research. One drawback to this research endeavor is that the codes specifically state that the analysis methods assume no collision damage. This could be overcome by specifying that a collision causes no structural damage but only nicks the concrete to expose the underlying steel.

To reiterate, as a starting point, the time model presented here considers the two variables independent and that no spalling has occurred. The variable $A$ will remain as defined in the As-Built version. However, the COV has been increased to account for the greater uncertainty associated with $A$ through time.
As
Prestressing Strand Area (in²)
As-Built: Bias = 1.01176 COV = 0.0125
Time: Bias = 1.01176 COV = 0.10

The deterioration component considers that the area of the prestressing strand decreases with time due to corrosion. Each strand has its own time of corrosion initiation, based on its proximity to the outer surface. The deterioration component assumes that chloride diffusion occurs uniformly around the perimeter of the beam cross-section. Therefore, the chloride ions reach those strands that are closer to the surface of the beam.

Details of the deterioration component may be found within the main body of the chapter and this appendix.

Regarding the BN diagnostic component, the variables that are immediately affected by the prestressing steel area decreasing through time are $P_{si}$, $P_{se}$, $M_n$, $M_{nLRFD}$, and $c$ as shown in the Figure.

$P_{si}$ – initial prestress force immediately after transfer. This is an initial value, therefore the Matlab program of the prognostic component is modified to consider this a constant. The deteriorating $A_s$ has no impact on this variable, indicated by the arc being colored red.

$P_{se}$ – effective final prestress force.

$M_n$ – nominal moment strength of section calculated according to AS and LF methodology.

$M_{nLRFD}$ – nominal moment strength of section calculated according to LRFD methodology.

$c$ – distance from extreme compression fiber to the neutral axis.
The BN is modified to account for $P_{sl}$ as an initial value that does not change through time. $P_{sl}$ was considered a variable at the As-Built stage due to its parents $f_t$ and $A_s$. However, $P_{sl}$ will not change through time. Therefore $P_{sl}$ is eliminated from the BN. $P_{sl}$ is expressed through the following equation:

$$P_{sl} = nos \times A_{s(init)} \times 0.69 \times f_s$$

where

- $P_{sl}$ = initial prestress force immediately after transfer
- $nos$ = number of strands
- $A_{s(init)}$ = initial area of prestressing strand
- $f_s$ = prestressing strand strength

In the above equation, $f_s$ is the only considered variable. Through the elimination of $P_{sl}$, the influence of $f_t$ upon $f_{cir}$ is declared by extending an arc from $f_t$ to $f_{cir}$. The variable, $f_{cir}$, is the average concrete stress at the center of gravity of the pretensioning steel due to the pretensioning force and deadload of beam immediately after transfer, ksi. It is an initial variable that is not affected by the decrease in the area of prestressing steel.
For this version of the prognostic component, the same bias is used to calculate the mean value, as was used in the As-Built version. New thresholds are set at 0.125 in\(^2\) (roughly 80% \(A_s\) (4 strands)) and 0.140 in\(^2\) (roughly 90% \(A_s\) (2 strands))) and the COV is increased to 0.10 to account for the greater uncertainty. The thresholds in the As-Built version were arbitrarily set at 0.151 and 0.155 values above and below the nominal, 0.153. However, since the deterioration built into this model is represented through a decrease in prestressed strand area, it seemed more fitting to lower the thresholds. The thresholds in their new setting serve as demarcations of actual loss. Four strands deteriorated leaves only 80% of the original 22 strands. The average strand area is 0.125 in\(^2\). Likewise, two strands deteriorated leaves only 90% of the original 22 strands. The average strand area is 0.140 in\(^2\).

**FIGURE B14** Distribution of Prestressing Strand Area, \(A_s\)
$f_{\text{ch}}$
Concrete Strength of Beam at Transfer (psi)
As-Built: Bias = 1.1    COV = 0.15
Time: Constant = Bias*Nominal

The concrete strength of beam at transfer is an initial value that will not change through time. Transfer refers to the point in fabrication of the beam when the stress is transferred between the concrete and steel due to the release of the prestressing strands. Therefore, the prognostic component is modified so that $f_{\text{ch}}$ is no longer a variable but a constant set at the mean value (4400 psi).

**FIGURE B15** Elimination of Initial Concrete Strength of Beam, $f_{\text{ch}}$
Concrete compressive strength typically increases with time. Several researchers have demonstrated that compressive concrete strength in prestressed beams increases with time. [Czaderski and Motavalli 2006] conducted a study to determine the remaining tendon force of a large-scale, 38-year-old prestressed concrete bridge girder. In this study they also took cores from the concrete beam to determine the strength. They found that the concrete strength increased over time, as expected. [Rabbat 1984] conducted a study on 25 year old prestressed concrete bridge girders and found that the concrete compressive strength was 10,100 psi, where the specifications called for a 28-day strength of 5000 psi. [Pessiki et al. 1996] performed an evaluation of effective prestress force in a 28 year old prestressed concrete bridge beam. They found that the average compressive strength (8440 psi) from the cores was 65% greater than the 28-day strength specified (5100 psi).

However, in the LRFR Manual (AASHTO 2003), the Engineer is directed to use the values tabulated below in the event that the design compressive strength is unknown. Note that the expected increase in strength over time is not considered obviously for reasons of conservatism.

**TABLE B3 Compressive Strength of Prestressed Concrete**

<table>
<thead>
<tr>
<th>Year of Construction</th>
<th>Compressive Strength, ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 1959</td>
<td>3.125</td>
</tr>
<tr>
<td>≥ 1959</td>
<td>3.750</td>
</tr>
</tbody>
</table>

Based on the direction from AASHTO (AASHTO 2003), the statistics for $f_{cb}$ in this investigation remain the same as the As-Built version with a bias of 1.10 and a coefficient of variation of 0.18.
The compressive concrete strength of deck is assumed to increase with time, as with the concrete of the beams. However, the Manual (AASHTO 2003) directs that in the rating calculations that the design value for $f'_{cd}$ be used. In the event that the plans are not available, the below tabulated values should be used. Note that these values are of a conservative nature. Only if cores are taken and the actual compressive test is measured, can the anticipated increase in compressive strength be incorporated into the rating calculations.

**TABLE B4** Compressive Strength of Deck Concrete

<table>
<thead>
<tr>
<th>Year of Construction</th>
<th>Compressive Strength, ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 1959</td>
<td>2.500</td>
</tr>
<tr>
<td>≥ 1959</td>
<td>3.000</td>
</tr>
</tbody>
</table>

**FIGURE B17** Distribution of Concrete Strength of Deck, $f_{cd}$
Recent research has shown that prestressing strand strength, or the ultimate prestressing stress, remains fairly constant through time. [Czaderski and Motavalli 2006] conducted a study to determine the remaining tendon force of a large-scale, 38-year-old prestressed concrete bridge girder. In this study they measure the remaining force in prestressing tendons by cutting wires. “Material tests on the prestressing steel showed that 38 years of constant stress had no discernable effect on its tensile strength. That is, the tensile strength is still higher than the nominal values. Furthermore, the wires even exceeded the present minimum elongation requirement a maximum load, meaning that the ductility did not decrease noticeably.” Therefore, the statistics for this variable will remain as in the As-Built version.

**FIGURE B18** Distribution of Prestressing Strand Strength, $f_s$
$E_s$  Modulus of Elasticity of Prestressing Strand (ksi)

As-Built:  Bias = 1.011  COV = 0.01
Time:  Bias = 1.011  COV = 0.01

Similar to the prestressing strand strength, the modulus of elasticity of the prestressing strand is also assumed to remain fairly constant. Therefore the statistics that were used for this variable in the As-Built version are used in the Time version, also.

**FIGURE B19** Distribution of Modulus of Elasticity of Prestressing Strand, $E_s$
Moment of Inertia of Beam Cross-section (in$^4$)

As-Built:  Bias = 1.0  COV = 0.03  
Time:  Bias = 1.0  COV = 0.05

The Moment of Inertia is reflective of the Cross-sectional Area of the Beam, $A$. In the prognostic component of the time model, the Area is assumed to remain intact. In other words, it is assumed that spalling has not taken place. Therefore the statistics for this variable will be the same as in the As-Built version. However, the COV is increased slightly, similar to $A$.

In this investigation, the nominal value for moment of inertia is 125,390 in$^4$. The thresholds were set at the 39% tail points (123,540 in$^4$ and 127,240 in$^4$). These thresholds coordinate with the thresholds set for $A$ and $I_c$, facilitating the representation of their correlation within the BN diagnostic component.

FIGURE B20  Distribution of Moment of Inertia, $I$
Once the model is modified to account for possible spalling, \( A \) is considered dependent upon \( A_s \) and likewise \( I \) and \( I_c \) are considered dependent on \( A \), as explained previously. The following common deterioration patterns are considered.

![Diagram of common deterioration patterns]

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Deteriorated</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y ) (in)</td>
<td>20.27</td>
<td>20.70</td>
</tr>
<tr>
<td>( y_c ) (in)</td>
<td>35.08</td>
<td>35.44</td>
</tr>
<tr>
<td>( y_s ) (in)</td>
<td>4.27</td>
<td>4.40</td>
</tr>
<tr>
<td>( A ) (in^2)</td>
<td>560</td>
<td>547</td>
</tr>
<tr>
<td>( I ) (in^4)</td>
<td>125390</td>
<td>120810</td>
</tr>
<tr>
<td>( I_c ) (in^4)</td>
<td>364324</td>
<td>350040</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Original</th>
<th>Deteriorated</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y ) (in)</td>
<td>20.27</td>
<td>21.34</td>
</tr>
<tr>
<td>( y_c ) (in)</td>
<td>35.08</td>
<td>35.97</td>
</tr>
<tr>
<td>( y_s ) (in)</td>
<td>4.27</td>
<td>4.56</td>
</tr>
<tr>
<td>( A ) (in^2)</td>
<td>560</td>
<td>530</td>
</tr>
<tr>
<td>( I ) (in^4)</td>
<td>125390</td>
<td>114196</td>
</tr>
<tr>
<td>( I_c ) (in^4)</td>
<td>364324</td>
<td>329480</td>
</tr>
</tbody>
</table>

**FIGURE B21** Common Deterioration Patterns
$I_c$  Moment of Inertia of Composite Section (in$^4$)

\[\text{As-Built: } \text{Bias} = 1.0 \quad \text{COV} = 0.035\]
\[\text{Time: } \text{Bias} = 1.0 \quad \text{COV} = 0.06\]

The Moment of Inertia of the Composite Section is reflective of the Cross-sectional Area of the Beam, $A$. Initially in the time model, the Area is assumed to remain intact. In other words, it is assumed that spalling has not taken place. Therefore the statistics for this variable will be the same as in the As-Built version. However, the COV is increased slightly, similar to $A$ and $I$.

Once the model is modified to account for possible spalling, $A$ is considered dependent upon $A$, and likewise $I$ and $I_c$ are considered dependent on $A$, as explained previously. The deterioration patterns shown in the previous section (variable $I$) are considered.

In this investigation, the nominal value for composite moment of inertia is 364,324 in$^4$. The thresholds were set at the 39% tail points (357,880 in$^4$ and 370,770 in$^4$). These thresholds coordinate with the thresholds set for $A$ and $I$ facilitating the representation of their correlation.

![Graph of Moment of Inertia of Composite Section, $I_c$](image)

**FIGURE B22** Distribution of Moment of Inertia of Composite Section, $I_c$
For this time version of the model that incorporates the deterioration component, the liveload moment variable was treated as a deterministic value. The reasoning behind this modification is found in the results of the As-Built version of the model, in that the liveload moment had an overwhelming influence on the results. With this variable in place, the contributions of the other variables in the calculation of a load rating factor are difficult to observe. These variables of interest in this research pertain to design and are within the control of the Design Engineer. In other words, the Design Engineer chooses the strength of concrete, etc. However, the liveload that uses the bridge is not something that is chosen by the Design Engineer. Therefore, it was decided to represent the liveload moment as a deterministic value calculated as set forth in the design codes.

As explained above, previously in the As-Built version, three diagnostic tests were conducted where the inventory load rating factor was declared to be less than one in each of the three methodologies. In reviewing the results, comparisons were made between assumed distributions and updated distributions of the input parameters. The parameter showing the most dramatic change is the most likely source of the problem. In all three investigations (AS, LF, LRFR), the liveload moment showed the most significant change in distribution, specifically pointing to a higher liveload moment than originally assumed as the root of the failed load rating (See Figure B23).

![Figure B23](image_url)

**FIGURE B23** Comparison Between Assumed and Updated Distributions of Input Variable, $M_{ll}$
The same investigation was conducted, only liveload moment was treated as a deterministic value. As expected, the results changed. This is recognized as a disconnect from the As-Built investigation. However, analysis of behavior of the material effects will be facilitated by holding the loading effects (liveload moment) constant.

In comparing the two plots that show the probabilistic inventory load rating factors calculated according to the codes, there are several observations. The spread of the distributions in the liveload moment variable plot appear similar, while this is not the case in the liveload moment deterministic plot. This can be explained by the overwhelming influence the variable Mll (COV= 0.18) had on the results. The load rating factors rank in the same order in both plots, highest to lowest: LF, AS, LRFR 1 (Strength I) and LRFR 2 (Service III).

**FIGURE B24** Comparison of Probabilistic Inventory Load Rating Factors (Mll variable vs. deterministic)

<table>
<thead>
<tr>
<th>Mll, variable</th>
<th>Mll, deterministic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td><strong>COV</strong></td>
</tr>
<tr>
<td>AS</td>
<td>1.45</td>
</tr>
<tr>
<td></td>
<td>0.24</td>
</tr>
<tr>
<td>LF</td>
<td>1.52</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td>LRFR 1</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td>LRFR 2</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>0.24</td>
</tr>
</tbody>
</table>

**TABLE B5** Comparison of Calculated Inventory Ratings (Mll variable vs. deterministic)

However, as shown in Table B5, the calculated load rating factors have a lower mean when considering liveload moment as a deterministic value (8% on average). As explained previously, the variable representing liveload moment (Nowak 1993), although it shows a bias of 1.20, includes impact. Therefore the impact factor is removed from the model assuming that it is sufficiently represented in Nowak’s liveload variable. However, the value of this impact factor is typically greater than 0.20. Therefore, the liveload effects used in the previous model, are slightly less than the deterministic value that would be used in a deterministic analysis.
Figure B25 presents results of the diagnostic investigation considering liveload moment as deterministic. Shown are relative comparisons between the updated distributions of other input parameters with respect to the three methodologies. Specifically, the variables that reflect the randomness of the beam properties were reviewed to compare the degree of influence over the final load rating factor (See Figure B25). Most noticeable is the lack of results pertaining to the LF methodology. This is due to the fact that the diagnostic BN was “impossible” to run with the identified problem of “LF Inventory Rating < 1.00.” Figure B24 shows the distribution of the LF inventory rating factor and the entire spread is well above one, as compared to the other distributions which show values at or less than one.

Cross-sectional area ($A$) had little influence over the failed load rating factor, considering LRFR 1 (Strength I). However, with AS and LRFR 2 (Service III), the updated distribution showed an increase in the probability that $A$ was most likely at or around the nominal magnitude. As for prestressing steel area ($A_s$), the updated distribution showed a dramatic decrease in the probability that $A_s$ is larger than assumed when considering the LRFR 1 (Strength I) methodology. The AS and LRFR 2 (Service III) investigations showed the same decrease in the probability, although to a lesser degree. In other words, in all three cases, with the inventory rating factor less than one, it is less likely that the prestressing steel area is higher than specified. Strength of beam concrete ($f'_{cb}$) shows no influence on a failed load rating when it comes to LRFR 1 (Strength I) as opposed to AS and LRFR 2 (Service III). Those updated distributions show a shift indicating that the strength of beam concrete is not as high as originally assumed. Ultimate prestressing stress ($f'_{ps}$) is sensitive considering the all three methodologies, the updated distributions show a shift in the elimination of the probability that $f'_{ps}$ is higher than anticipated. This input parameter appears to have more relative influence over a failed LRFR 1 (Strength I) inventory rating. Unit weight of concrete ($w_c$) has an effect on the inventory load rating factor in all three methodologies. The trend in the LRFR distributions suggests that a load rating factor less than one may have the contributing cause of a unit weight of concrete being higher than assumed.
FIGURE B25 Comparison Between Assumed and Updated Distributions of Input Variables ($M_l$ deterministic)
The results pertaining to other input parameters, strength of deck concrete ($f'_{cd}$) and wearing surface thickness ($wst$) were also reviewed. For AS and LRFR 2 (Service III), the updated distributions of strength of deck concrete ($f'_{cd}$) show no change from the assumed distribution. For the LRFR 1 (Strength I) method, the probability that the strength of concrete is low is dramatically increased with no possibility that the strength of concrete is greater than assumed. Regarding wearing surface thickness ($wst$), the updated distributions pertaining to the AS and LRFR 2 (Service III) methods show little change from the initially assumed distribution. However, the LRFR 1 (Strength I) distribution indicates that the wearing surface thickness is most likely greater than assumed.

Figure B26 shows the further interpretations of the results. Comparisons were made between the updated and the assumed distributions of seven input parameters given a failed inventory load rating. The increase in probability of the occurrence of the problematic states of each variable was extracted and plotted for observation. Considering AS, the problem states concerning prestressing steel area ($A_s$), strength of beam concrete ($f'_{cb}$) and the ultimate prestressing stress ($f's$) in the strands show an increase in probability.

The LRFR 1 (Strength I) show the problem states to be strength of deck concrete ($f'_{cd}$), area of prestressing steel ($A_s$), ultimate prestressing stress ($f's$), wearing surface thickness ($wst$), and unit weight of concrete ($w_c$).

The LRFR 2 (Service III) methodology seems most sensitive to the deficient states of unit weight of concrete ($w_c$), area of prestressing steel ($A_s$), strength of beam concrete ($f'_{cb}$), ultimate prestressing stress ($f's$) and wearing surface thickness ($wst$).

In comparing the results among the three methods, none appear sensitive to the beam having a cross-sectional area that is deficient. However, all three show the variables pertaining to the strands (area ($A_s$) and ultimate prestressing stress (or strength) ($f's$)) are affected. The methods pertaining to service limits (AS, LRFR 2 (Service III)) are sensitive to the strength of beam concrete ($f'_{cb}$), while LRFR 1 (Strength I) is sensitive to the strength of deck concrete ($f'_{cd}$). As stated above, unit weight of concrete ($w_c$) is instrumental in determining the deadload moment of the beam which is used directly to calculate the inventory rating in both LRFR methods. Unit weight of concrete ($w_c$) is also used in calculating factors applied to the liveload moment in the LRFR method. Also in LRFR, the moment due to the wearing surface is treated separately from the other loads, having its own applied factor.

![FIGURE B26 Comparison of Influence of Deficient States of Input Variables](image-url)
Ambient relative humidity is assumed the same as it was in the As-Built version. This variable experiences daily and seasonal fluctuations but no steady increase or decrease through the years.

FIGURE B27 Distribution of Relative Humidity, RH
Unit Weight of Concrete, $w_c$, is considered the same in this version as in the As-Built version. This assumption is based on previous research.

It should be mentioned that a considerable amount of research was completed investigating the link between the two variables, Original Plans ($OP$) and Weight of Concrete ($w_c$). In the end, it was found that the variable, $w_c$, is not dependent upon $OP$. The nominal value for $w_c$ (150 pcf) is typically used. Variations on this value enter in at the design stage and stem from quality control issues. However, through time, as concrete hardens, although density may change, the variation in the unit weight of concrete is deemed insignificant and is overlooked as a property to be measured. The model reflects this perceived lack of dependency.

![FIGURE B28 Distribution of Unit Weight of Concrete, $w_c$](image)
wst  Wearing Surface Thickness (psf)
As-Built:  Bias = 1.1   COV = 0.20
Time:  Bias = 1.1   COV = 0.20

This variable, \( wst \), as expressed in the As-Built version, sufficiently reflects the variability it will experience over a lifetime. Therefore, for the Time version, it will remain the same.

In this study, the nominal value for the wearing surface thickness is 25 psf (2 in).

Based on these statistics, a Bias of 1.1 and a COV of 0.2 was used in this study.

In setting the thresholds, it was noted that wearing surface thickness tends to be greater than set forth in design, as reflected by the Bias found in literature. Therefore, the thresholds were set at 1.5 in. (18.75 psf) for Lo and 3 in. (37.5 psf) for Hi.

**FIGURE B29** Distribution of Wearing Surface, \( wst \)
APPENDIX C

FIGURE C1. Comparison Between Assumed and Updated Distributions of Input Variables, 30 Years
### FIGURE C2. Comparison Between Assumed and Updated Distributions of Input Variables, 40 Years

<table>
<thead>
<tr>
<th></th>
<th>Assumed Distribution</th>
<th>Updated Distribution</th>
<th>Inventory Rating &lt; 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AS</td>
<td>LF</td>
<td>LRFR</td>
</tr>
<tr>
<td>$p(A)$</td>
<td>0.39</td>
<td>0.32</td>
<td>0.76</td>
</tr>
<tr>
<td>$p(A_s)$</td>
<td>0.07</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$p(f_{cb})$</td>
<td>0.09</td>
<td>0.13</td>
<td>0.09</td>
</tr>
<tr>
<td>$p(f_{cd})$</td>
<td>0.28</td>
<td>0.28</td>
<td>0.31</td>
</tr>
<tr>
<td>$p(f_s)$</td>
<td>0.03</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>$p(w_c)$</td>
<td>0.13</td>
<td>0.11</td>
<td>0.27</td>
</tr>
<tr>
<td>$p(wst)$</td>
<td>0.06</td>
<td>0.03</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Legend: lo, med, hi
<table>
<thead>
<tr>
<th>Assumed Distribution</th>
<th>Updated Distribution</th>
<th>Inventory Rating &lt; 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AS</td>
<td>LF</td>
</tr>
<tr>
<td>p(A)</td>
<td>0.41 0.21 0.38</td>
<td>0.30 0.22 0.48</td>
</tr>
<tr>
<td>p(A_s)</td>
<td>0.11 0.26 0.63</td>
<td>0.25 0.42 0.33</td>
</tr>
<tr>
<td>p(f'cb)</td>
<td>0.09 0.18 0.73</td>
<td>0.11 0.13 0.76</td>
</tr>
<tr>
<td>p(f'cd)</td>
<td>0.28 0.01 0.71</td>
<td>0.28 0.01 0.71</td>
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<tr>
<td>p(f_s)</td>
<td>0.03 0.05 0.92</td>
<td>0.04 0.03 0.93</td>
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<tr>
<td>p(w_c)</td>
<td>0.13 0.13 0.74</td>
<td>0.11 0.16 0.73</td>
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<tr>
<td>p(wst)</td>
<td>0.06 0.04 0.90</td>
<td>0.03 0.04 0.93</td>
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</tbody>
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**FIGURE C3.** Comparison Between Assumed and Updated Distributions of Input Variables, 50 Years
### FIGURE C4. Comparison Between Assumed and Updated Distributions of Input Variables, 60 Years

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</thead>
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<td>AS</td>
<td>LF</td>
</tr>
<tr>
<td>p(A)</td>
<td>0.40</td>
<td>0.34</td>
</tr>
<tr>
<td>p(A&lt;sub&gt;s&lt;/sub&gt;)</td>
<td>0.18</td>
<td>0.39</td>
</tr>
<tr>
<td>p(f&lt;sub&gt;c&lt;/sub&gt;b)</td>
<td>0.73</td>
<td>0.74</td>
</tr>
<tr>
<td>p(f&lt;sub&gt;c&lt;/sub&gt;d)</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td>p(f&lt;sub&gt;s&lt;/sub&gt;)</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>p(w&lt;sub&gt;c&lt;/sub&gt;)</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>p(w&lt;sub&gt;t&lt;/sub&gt;)</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>p(w&lt;sub&gt;t&lt;/sub&gt;)</td>
<td>0.13</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Legend: | lo | med | hi |

<p>| lo | med | hi |
| 0.09 | 0.11 | 0.09 |
| 0.28 | 0.28 | 0.32 |
| 0.92 | 0.92 | 0.93 |
| 0.03 | 0.04 | 0.04 |
| 0.13 | 0.13 | 0.16 |
| 0.06 | 0.04 | 0.10 | 0.01 | 0.08 | 0.00 |</p>
<table>
<thead>
<tr>
<th>Assumed Distribution</th>
<th>Updated Distribution Inventory Rating &lt; 1</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>AS</td>
</tr>
<tr>
<td>p(A)</td>
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</tr>
<tr>
<td>p(A_s)</td>
<td>0.28</td>
</tr>
<tr>
<td>p(f'_{cb})</td>
<td>0.09</td>
</tr>
<tr>
<td>p(f'_{cd})</td>
<td>0.28</td>
</tr>
<tr>
<td>p(f_s)</td>
<td>0.03</td>
</tr>
<tr>
<td>p(w_c)</td>
<td>0.13</td>
</tr>
<tr>
<td>p(wst)</td>
<td>0.06</td>
</tr>
</tbody>
</table>

FIGURE C5. Comparison Between Assumed and Updated Distributions of Input Variables, 70 Years
FIGURE C6. Comparison of Influence of Deficient States of Input Variables, Year 30

FIGURE C7. Comparison of Influence of Deficient States of Input Variables, Year 40

FIGURE C8. Comparison of Influence of Deficient States of Input Variables, Year 50
FIGURE C9. Comparison of Influence of Deficient States of Input Variables, Year 60

FIGURE C10. Comparison of Influence of Deficient States of Input Variables, Year 70
There are random popouts on the bottoms of the I-beams from 1” to 3” in diameter and up to 1” deep throughout the bridge.

Span #1 (located on the Goat Island side) exhibits random spalls up to 1’ in diameter and up to 1” deep on bottom of I-beams.

The South Fascia beam in span #11 exhibits a 6” x 6” x 3” deep spall on its bottom corner exposing a stirrup and a prestressing strand. This span is over the navigation channel.

Spans 22 and 23 exhibit 1’ diameter spalls exposing the stirrups. This condition is found within 10’ of the bearing areas.

The I-beams in Span #24 exhibit large spalls and cracks on the bottom flange in the area of the bulkhead (approximately midspan). Many of the spalls expose the prestressing strands and stirrups.

Many of the deck joints leak water onto the pier caps as was observed during a period of rain.

The scupper in span #2 between the south fascia beam and the first interior beam leaked water onto the pier cap.

The steel piles exhibit light to moderate rust with barnacle growth at the waterline.

Commentary:
The deteriorated condition in Span 1 and especially span 24 appears to be from the spray caused by the salt water crashing into the bulkheads. In light of this we recommend cleaning and patching of all the spalls on the prestressed beams and applying an epoxy waterproofing coat to the beams in spans 1 and 24.

At the present time the deterioration of the beams do not effect the ratings. However, if the condition is left unattended complete corrosion of the exposed prestressing strands may occur.

It is also recommended that the leaking deck joints be rehabilitated.

The bridge should be inspected periodically to detect any distress or deterioration.
INSPECTION OBSERVATIONS AND COMMENTARY

1995 RATING REPORT

There are random spalls on the bottom flange of various beams throughout the structure. These spalls range from 1 inch to 4 inches in diameter and are approximately 1 inch deep.

The majority of the beam deterioration exists in the first span (mainland side). The third beam from the south fascia is in the worst condition. This beam has a large diagonal spall approximately 15 feet long and half the depth of the flange. As a result of this deterioration, three of the 3/8 inch diameter exposed prestressing strands have corroded in their entirety. Therefore only 31 of the 34 prestressing strands were considered effective for the rating analysis. The remaining beams in this span are also exhibiting a varying degree of spalling and deterioration.

The second span has minor spalls on the underside of all but the south fascia beam flange. The west end of the third beam from the south fascia exhibits a major spall of the underside of the beam flange exposing one of the prestressing strands.

The majority of the deck scuppers exhibit minor rusting and clogging.

The south fascia beam in span thirteen (from mainland) exhibits a 6 inch long spall on its bottom corner exposing a stirrup and one of the prestressing strands.

The concrete filled steel piles exhibit moderate rust with heavy barnacle growth at the high tide waterline.

The thirteenth pier from the mainland exhibits cracking around the most southerly steel pile and this cracking propagates out to the edge of the pier cap.

The steel bulkhead in front of the west abutment exhibits severe deterioration.

The stone masonry wall in front of the east abutment exhibits minor separations in the blocks.

Efflorescence is present on the overhang of the deck adjacent to the north fascia beams in spans nineteen and twenty. It appears to be the result of leaking deck and construction joints.

CONCLUSIONS
Not in its entirety...only applicable excerpts

The excessive deterioration in spans one and twenty four appear to be mainly due to the close proximity of the spans to the water line and the resulting ocean spray.

The deteriorated beams in both span one and 24 should be scheduled for rehabilitation as soon as possible. Furthermore, the bridge should be scheduled for a yearly (or more frequent) inspection to monitor the condition of these beams until the final rehabilitation is undertaken. The Bridge Rating should be updated if any change from the present “as-inspected” condition is noted.

In addition to the other repair work, such as roadway joint rehabilitation and repair of minor spalls, required on this bridge, the condition of the bulkhead at the west abutment should also be considered. The steel bulkhead in front of the west abutment exhibits a major deterioration and should be replaced or undergo major rehabilitation in order to prevent undermining of the west abutment.
INSPECTION OBSERVATIONS AND COMMENTARY

2002 INSPECTION REPORT

The superstructure got an overall condition rating of 4 which means POOR CONDITION – advanced section loss, deterioration, spalling, or scour.
This overall rating was dictated by the spans which received the lowest rating of 4: 1, 22, 23, 24

Span 24 actually had one beam that was given the condition rating of 3 which means SERIOUS CONDITION – loss of section, deterioration, spalling, or scour have seriously affected primary structural components. Local failures are possible. Fatigue cracks in steel or shear cracks in concrete may be present.

Midspan north and south flange heavy spalling and cracking with rebar (30’ x 18” x 3”) with section loss and broken stirrups. Wire strands are broken and have heavy section loss. (unfortunately, the notes are not specific as to how many strands are affected)
APPENDIX D2

%************************************************************************************
% Prestressed Steel Corrosion Model: Example Bridge
%************************************************************************************

% Variables

% Astrand = area of strand before deterioration occurs (in^2)
% Co = surface chloride concentration (0.15% by weight of concrete (Akgul 2001))
% Cr = critical chloride concentration (0.037% by weight of concrete (Akgul 2001))
% Dc = diffusion coefficient (0.825x10^-12 m^2/s = 0.0403 in^2/yr)(CIKS 2007)
% Do = original diameter (in)
% Dred = reduced diameter due to corrosion (in)
% rcorr = corrosion rate (2.25 mils/yr = 0.00225 in/yr)
% t = time (yrs)
% Tt = time that threshold level of critical chloride concentration (yrs)
% x = distance from face of beam to prestressing strand (in)

Astrand=0.0799; %(in^2) NOTE: this is the mean of the variable in master program
Dc=0.0865; % diffusion coefficient (in^2/yr) (maximum of possible values)
Ccr=0.037; % critical chloride concentration (% by weight of concrete)
Co=0.15; % surface chloride concentration (% by weight of concrete)
Do=2*(sqrt(Astrand/pi)); % approximate diameter as if 7-wire strand was circular
rcorr=0.00225;

% initialize Dred and Ared with whole diameters and areas
Dred(1:100,1:7)=ones*Do;
Ared(1:100,1:7)=ones*Astrand;

%************************************************************************************

% Calculate the time to reach the threshold level of critical
% chloride concentration

% Loop to consider all possible distances of diffusion
x=[1.5 2.83 4 4.24 5 5.67]; % distance from face to prestressing strand (in)
for k=1:7
    Tt(1,k)=(((x(1,k)-0.25)^2)/(4*Dc))*(erfinv(1-(Ccr/Co)))^-2;
    Tt(1,k)=ceil(Tt(1,k));
end

% Calculate the time to complete disappearance after the start of
% corrosion
Tend(1,k)=Tt(1,k)+(Do/rcorr);
Tend(1,k)=ceil(Tend(1,k));
% This loop calculates the reduction in diameter of prestress steel.

% corrosion of bar is linear reduction of bar diameter
% Please note...prestress strands have a nominal diameter, say 0.5 inch.
% However, the area of the strand is not a perfect circle, this model
% considers the seven wire strand. Therefore, this model will only
% offer an estimate of the reduction of area that takes place

% calculate deficient areas through time
for t=Tt(k,1):100
    Dred(t,k)=Do-rcorr*(t-Tt(k,1));
    Ared(t,k)=(pi/4)*(Dred(t,k))^2;
    % adjust reduced area to apply to non-circular strand
    %Ared(t,k)=Astrand*((Dred(t,k))^2)/(Do)^2;
    %Ared(t,k)=Astrand*(Dred(t,k)/Do);
end

% this loop calculates the total As as it decreases through time
% it also calculates the average As to be input into the master program
for y=1:100
    totAs(y,1)=16*Ared(y,2)+4*Ared(y,3)+6*Ared(y,4)+4*Ared(y,5)+2*Ared(y,6)+2*Ared(y,7);
    totstrand(y,1)=totAs(y,1)/Astrand;
    aveAs(y,1)=totAs(y,1)/34;
end

plot(totstrand)
xlabel('Time, years')
ylabel('Total Number of Strands')
figure
plot(aveAs)
xlabel('Time, years')
ylabel('Average Strand Area')
FIGURE D1. Diffusion Distance, Group 1

16 strands
Tt=14 years

FIGURE D2. Diffusion Distance, Group 2

4 strands
Tt=29 years
**FIGURE D3.** Diffusion Distance, Group 3

- $y = 4''$
- 6 strands
- $T_t = 61$ years

**FIGURE D4.** Diffusion Distance, Group 4

- $y = 4.2426''$
- 4 strands
- $T_t = 69$ years
  
  (typ.)
**FIGURE D5.** Diffusion Distance, Group 5

- $y = 5''$
- 2 strands
- $T_t = 98$ years

**FIGURE D6.** Diffusion Distance, Group 6

- $y = 5.6569''$
- 2 strands
- $T_t = 127$ years
APPENDIX D3

FIGURE D7. Area of Prestressing Strand, $A_s$, $t=30$ Years

FIGURE D8. Area of Prestressing Strand, $A_s$, $t=40$ Years
FIGURE D9. Area of Prestressing Strand, $A_s$, t=50 Years

Area of Prestressing Strand, $A_s$
- Nominal = 0.0799 in$^2$
- Mean = 0.0606 in$^2$
- COV = 0.10
- Lo threshold = 0.0682 in$^2$
- Hi threshold = 0.0728 in$^2$

FIGURE D10. Area of Prestressing Strand, $A_s$, t=60 Years

Area of Prestressing Strand, $A_s$
- Nominal = 0.0799 in$^2$
- Mean = 0.0558 in$^2$
- COV = 0.10
- Lo threshold = 0.0682 in$^2$
- Hi threshold = 0.0728 in$^2$