Describing Naturally Occurring Schedules:
Analysis of Feedback Functions for Shooting During Basketball Games

A Thesis Presented
by
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Table of Contents

A. Abstract ....................................................................................................................2
B. Introduction ..............................................................................................................3
C. Study 1 ...................................................................................................................10
   a. Method .......................................................................................................10
   b. Results and Discussion ..............................................................................11
D. Study 2 ...................................................................................................................13
   a. Participants .................................................................................................13
   b. Setting and Materials .................................................................................14
   c. Procedures ..................................................................................................14
   d. Results and Discussion ..............................................................................15
E. Discussion ..............................................................................................................16
F. References ..............................................................................................................20
G. Table 1 ...................................................................................................................22
H. Table 2 ...................................................................................................................23
I. Table 3 ...................................................................................................................24
J. Figure Captions ......................................................................................................25
K. Figure 1 ..................................................................................................................26
L. Figure 2 ..................................................................................................................27
M. Figure 3 ..................................................................................................................28
N. Figure 4 ..................................................................................................................29
Abstract

A number of recent studies have applied the matching law in analyzing behavior occurring in the context of sports including two- and three-point shot allocation by basketball players. This research contributes to our understanding of matching in that it suggests that the matching law may describe responding in naturally occurring human environments. However, because there is no experimental manipulation, the schedule in effect is unknown and it is unclear if obtained matching is a behavioral phenomenon (i.e., behavior conforming to relative reinforcement rates) or merely a relation between responding and reinforcement forced by ratio-like properties of the schedule. In the Study 1, both linear and hyperbolic feedback functions described the naturally occurring basketball shooting data; however data fell along a line which suggests responding was on a ratio schedule. In Study 2, an experiment was conducted in which the rate of basketball shooting was explicitly manipulated to generate a wider range of shooting rates than those observed in Study 1. For five of seven participants both equations described the data equally well and data fell along a linear path. For two participants the hyperbolic function more accurately described the data. Implications of these results for future descriptive analyses are discussed.
Describing Naturally Occurring Schedules:

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The Matching Law states that the relative rate of responding on two or more response alternatives will conform to match the relative rate of reinforcement available for responding on those alternatives. For example, if a variable interval (VI) 2-min schedule is arranged for a response key on the left and a VI 4 min is arranged for a response key on the right in an operant chamber, the matching law predicts that the organism will respond twice as often on the left key (VI 2 min) than the right key (VI 4 min). Baum (1974) proposed an equation which quantifies this relation and also describes bias and sensitivity to relative reinforcement rates. This equation is called the generalized matching law and is expressed quantitatively as:

\[ \log \left( \frac{B_1}{B_2} \right) = s \log \left( \frac{r_1}{r_2} \right) + \log b \]  

(1)

Where \( B_1 \) and \( B_2 \) are the response rates on two alternatives and \( r_1 \) and \( r_2 \) are the reinforcement rates for those alternatives. The variable \( s \) represents the slope of the line and refers to sensitivity to changes in reinforcement rate. When \( s \) is equal to 1, for each unit change in relative reinforcement rate there is an equal unit change in relative response rate. Most commonly, \( s \) is less than 1 (e.g., behavior is less sensitive to relative reinforcement rate than strict matching would predict) which is referred to as undermatching (Baum, 1979). The variable \( b \) represents bias which is an increase in response allocation toward one or the other alternative that is not due to different reinforcement schedules. \( \log b \) is the y intercept. If there is no bias, \( \log b \) equals zero and response bias will shift the line up or down, changing the y intercept.

The matching law has been well studied in the experimental laboratory with a variety of species, responses, and reinforcers. There is a growing body of applied literature as well. Researchers have applied matching theory to socially significant settings and behavior in two
ways to empirically validate its usefulness and generality. The first is by using the methods described in the experimental literature with socially significant behavior. In these studies, a reinforcer is identified and explicitly controlled by the experimenter to determine if response allocation matches reinforcer allocation. This method has been applied to social dynamics (Borrero, Crisolo, Tu, Rieland, Ross, Francisco, & Yamamoto, 2007), time spent working on math problems (e.g., Mace, Neef, Shade, & Mauro, 1994), and sensitivity to reinforcement in children with Attention Deficit Hyperactivity Disorder when taking methylphenidate compared to a placebo (Murray & Kollins, 2000). In all of these studies, the matching law accounted for much of the variance in response allocation. This suggests that in situations with two response alternatives, complex behavior, such as those involved in social dynamics, will conform to the relative rate of reinforcement obtained.

The second way in which applied researchers have examined the generality of the matching law is in describing naturally-occurring response and reinforcer allocations. For example, Borrero and Vollmer (2002) described allocation of problem behavior and an alternative appropriate response according to the matching law. In this study, the maintaining reinforcer for problem behavior was identified via a functional analysis. After identifying the reinforcer, experimenters observed the participants interacting with care providers. The experimenters collected data on appropriate and inappropriate behavior and the occurrence of consequent events including those identified to be maintaining inappropriate behavior. Borrero and Vollmer (2002) found that allocation of inappropriate and appropriate responding was well described by the matching law when the identified reinforcer was used. When the non-reinforcers were used in the analysis there was no or limited matching. This study, unlike those
described previously, demonstrates the occurrence of matching with socially significant behavior in a natural setting and without explicitly controlling the reinforcement rate.

Several studies have used sports as a context in which to evaluate the generality of matching in non-experimentally controlled environments. Examination of data from professional and collegiate sports, and basketball in particular, is convenient because the statistics are readily available, the responses (e.g., shooting) and reinforcers (e.g., points) are clearly defined, the reinforcer value stays consistent throughout the game, and the opportunity for two- or three-point shots at anytime approximates a concurrent choice arrangement (Vollmer & Bourret, 2000).

An example of a line of research in this vein is a series of studies examining allocation of two- and three-point shot taking in professional and collegiate basketball. In this research, the points gained by successful shots were used as the assumed reinforcer. Vollmer and Bourret (2000) examined allocation of two- and three-point shots by collegiate male and female basketball players from a Division I University. The authors found, when accounting for the difference in amount of points earned for two- and three-point shots, the matching equation predicted the relative response ratio of shots taken.

Romanowich, Bourret, and Vollmer (2007) replicated and extending these findings by examining three periods of time in professional basketball history in which the three-point line was moved closer and then moved back creating a naturally occurring reversal design. This type of manipulation might be expected to affect the reinforcement rate for three-point shots. When the authors compared the response ratios across the three time periods, they found that moving the line closer increased the relative rate of three-point shots compared to the first time period. When the line was moved further away again, the ratio of attempted three-point shots decreased again but not to the level observed during the first time period. For all three time periods, the
matching equation accounted for more than 95% of the variance in the observed data. This study provides further evidence suggesting matching between relative shooting rates and relative reinforcement rates in terms of the fit of the matching equation within each time period and the shift in responding and reinforcement rate across time periods is congruent with what an analysis of responding as a function of reinforcement rate would predict.

The previous studies analyzing basketball statistics only examined data from very good teams. Alferink, Critchfield, Hitt, and Higgins (2009) evaluated response allocation of two- and three-point basketball shots for a variety of collegiate basketball teams to determine if better teams or players would better match relative reinforcement rate of two- and three-point shots better than worse teams or players. The authors found that the matching equation accounted for more variance for better players and teams than worse players and teams. In addition, sensitivity to changes in relative reinforcement rate was higher (closer to 1) for better teams and players. In Study 3, Alferink and colleagues (2009) examined data for a single player across seasons. The authors found that the matching equation still accounted for much of the variability in the data for a single player.

The benefit of these descriptive analyses is that matching was found to occur in complex and uncontrolled natural environments. These data increase the robustness of the evidence supporting the matching law and suggest that its generality extends into these naturally occurring contexts. One problem with this research is that the schedule of reinforcement arranged for the responses is unknown unlike more controlled studies. In experimental research on the matching law, variable-interval (VI) schedules are typically used because the rate of behavior can vary widely without significantly affecting reinforcement rate. This is in comparison to variable-ratio (VR) schedules in which the rate of reinforcement is directly tied to the rate of behavior.
For example, if a pigeon pecks 50 times per minute on a VR 50 reinforcement schedule, then the pigeon will earn one reinforcer per minute. If the pigeon pecks 100 times per minute the pigeon will earn about two reinforcers per minute. However, on a VI 1-m schedule, if the pigeon pecks either 50 or 100 times per minute it will still only earn one reinforcer. In sports such as basketball, one might argue that the schedule of reinforcement for shooting baskets is, at least to some degree, ratio-like (Vollmer & Bourret, 2000). In other words, the more shots a player attempts, the more points the player will make. The concern, if basketball shooting is maintained on ratio schedule, is that the response rate would be determining the reinforcement rate and not vice versa. For example, if a player doubled the number of three point shots attempted in one game, tripled it in another, and quadrupled it in yet another, he or she might make double, triple, and quadruple the number of three point shots made in each game, respectively. This pattern of response and reinforcement rates may produce matching and it may appear as if behavior is conforming to changing reinforcement rates when, in fact, reinforcement rate is conforming to changes in shooting rates. If this were the case, there would only be matching in the sense of a mathematical relation between response and reinforcement rate but not in the behavioral sense of behavior being controlled by relative reinforcement rate.

Therefore, it is important to evaluate what types of schedules are in place in the natural environment. If the rate of reinforcement increases linearly as the rate of responding increases, matching may be forced. However, if rate of reinforcement is a negatively-accelerated function of response rate (i.e., the rate of increase in reinforcement rate slows as response rate increases) and variation in response rate can occur without having a large impact on reinforcement rate observed matching is not necessitated by the schedule and may be considered to be a property of behavior.
Applying feedback functions to data are one method used to determine which type of schedule is in place in the natural environment. A feedback function is a mathematical equation which states the relation between rate of responding and rate of reinforcement (Baum, 1973). The equation for a ratio-like schedule (Baum, 1981) is:

\[ r = \left( \frac{1}{c} \right) \times B \quad (2) \]

In this equation, \( r \) and \( B \) are reinforcement rate and response rate respectively. The value \( c \) is the average ratio value of the variable ratio schedule. Therefore, if the response rate is equal to 10 responses per minute, and the schedule arranged is a VR 2, then the rate of reinforcement (\( r \)) would be five reinforcers per min. The important property of this equation, especially when compared to the feedback function for interval schedules, is that there is a linear relation between response rate and the reinforcer rate. As response rate continues to increase, reinforcer rate will increase proportionally according to the schedule.

The feedback function for interval schedules suggests a hyperbolic relation between reinforcer rate and response rate. Baum (1992) suggests three theoretical and empirical requirements of interval schedule feedback functions. There must be 1) a negatively accelerated slope that 2) approaches an asymptote equal to the programmed rate of reinforcement and 3) the slope of the equation at low response rates should be equal to one. This produces the equation suggested by Prelec and Herrnstein (1978 in Baum 1992) which states:

\[ r = \frac{B}{tB + a} \quad (3) \]

In this equation, \( t \) is the value of the interval schedule and \( a \) is a free parameter which describes differences in the distribution of randomly selected interval values. This equation also produces a hyperbolic function such that, at response rates below the arranged reinforcer rate, the relation between responses and reinforcers is linear as in the ratio equation. For example, if a VI 1-min is
arranged, and the organism responds once every three min and earns one reinforcer every three minutes (.33 reinforcers per minute). During the next session, the organism responds more quickly at one response per minute there will be an increase in reinforcement rate to one reinforcer per minute. As response rates increase to the arranged schedule value, however, reinforcer rates asymptote at the average arranged interval value. For that same schedule, if the organism responds 10 times per min, the maximum rate of reinforcement will remain at one per min. More generally, at inter-response times (IRT) below the average inter-reinforcement interval (IRI), reinforcement rate increases quickly with increases in response rate. However, as the IRT increases to levels above the IRI, increases in reinforcement rate become smaller with increases in response rate.

The schedule of reinforcement can be described by fitting these equations to the data. If the observed data fall along a straight line, and the linear or ratio feedback function fits the data better, then the schedule in the environment is ratio-like. However, if the increase in reinforcement rate slows at higher response rates and the hyperbolic or interval feedback function fits the data better, then the schedule in the environment is interval-like.

Given the importance of the schedule in interpreting matching data, the purpose of this study is to use feedback functions to describe the schedule in effect for basketball shooting. In Study 1, archival data from the National Basketball Association (NBA) were analyzed. In Study 2, an experimental manipulation was conducted in which the rate of basketball shooting was controlled for volunteer participants in order to analyze the schedule of reinforcement across a greater range of shooting values.
Study 1

Method

Archival data for eight professional male basketball players were obtained from the National Basketball Association website (www.nba.com). Data were obtained from the 2008-2009 regular season. Data from the first eight players listed on the NBA website when the data were sorted according the three-point shot percentage that met inclusion criteria were selected. The participant must have attempted a 3-point shot and played at least 10 min in more than half of the games of the regular season to be included in the study.

The years of professional play and position varied between participants. Participant A was a Guard and had played professionally for 13 years. Participants B and C were both in the NBA for six years and played Forward and Guard respectively. Participants D, E, F, G, and H had played professionally for two years. Participants D and H played Forward and Participants E, F, and G played Guard.

Data for each player and all players combined were plotted with shots taken per min on the x-axis and points made per hour on the y-axis. Points made were calculated by multiplying the number of 2- and 3-point shots made by 2 and 3 respectively. The rates for shooting and points made were calculated by dividing the total 2- or 3-point shots taken or the points gained in a game divided by the total minutes played during that game. To determine whether the linear (Equation 2) or hyperbolic (Equation 3) feedback function more accurately described the data, lines of best fit were created using least-squares regression for each equation (“Coefficient of determination,” 2010). The coefficient of determination was calculated for each line. The coefficient of determination ($R^2$), also often called variance accounted for, is a statistical measure used to describe the goodness of fit of an equation. However, $R^2$ may be inflated for the linear
function because this function is forced through the origin. Therefore, in addition to \( R^2 \), the sum-of-squares residual (\( SS_{res} \)) is provided for each line. While \( SS_{res} \) is not a normalized measure, it provides a more direct description of the relative goodness of fit for these two functions fit to the same data. \( SS_{res} \) is calculated by subtracting the value obtained by the value predicted by the equation and squaring the sum of differences.

**Results and Discussion**

Table 1 displays the coefficient of determination and sum-of-squares residual for 2-point shots for all players. Comparisons of best fit between the linear and hyperbolic function indicate that for all eight of the participants both the linear and hyperbolic feedback functions produced similar fits; however, a visual inspection reveals that the data fell along a linear path. These data suggest that a more ratio-like schedule may be occurring in the natural environment for basketball shooting for two point shots. An analysis of the season averages for 2-point shooting (Figure 1, bottom-right panel) results in a linear function form. Again, a visual analysis suggests data patterns which appear more linear than hyperbolic for two point shots. Figure 1 displays the data from three participants and an aggregation of data for all participants. The three individual data sets were chosen as representative of the three pattern types which were obtained across all participants. The top-left panel of Figure 1 exemplifies a pattern in which the function form for the linear and hyperbolic best fit line are nearly identical. This pattern occurred for five of the eight participants. All of the data falls along the linear portion of the hyperbolic function. The top-right panel of Figure 1 indicates a data pattern which resulted in identical fits for both the linear and hyperbolic function which was observed for two participants. This occurs when \( t \) in the hyperbolic function equals zero. In this case, \( c \) equals \( a \) which makes the two equations mathematically identical. The only case in which the \( R^2 \) for the hyperbolic function was more
than .1 different than the $R^2$ for the linear function was for Participant G (Figure 1, bottom left panel). However, because $R^2$ was very low (less than .5) for both equations, the difference between the two $R^2$ values was not meaningful.

For 3-point shooting, linear feedback functions more accurately described the data for all eight of the eight participants and the data fall along a linear path. The best-fit lines for the aggregate data also suggest a linear function form for the data. Figure 2 displays the data from the same three players in Figure 1. In the bottom-right panel of Figure 2, each data point represents a season average for a player. Most players’ data conformed to patterns in the top-left and top-right panel of Figure 2. As response rate increased, reinforcement rate also increased for these players; however, variability in reinforcement rate also increased as response rate increased. This is demonstrated by the triangular pattern demonstrated by the data for these players.

Across both 2- and 3-point shooting analyses, the $R^2$ for all players and aggregate data was low as shown in Tables 1 and 2. For individual players, $R^2$ averaged .465 (range, .348 to .578) and .420 (range, .223 to .564) for 2-point shooting for linear and hyperbolic functions respectively. For individual player’s 3-point shots, $R^2$ averaged .402 (range, .341 to .497) and .396 (range, .315 to .497) for linear and hyperbolic functions respectively. These $R^2$ values are not uncommon for single player data evaluation by game (e.g., Reed, Critchfield, & Martens, 2006). However, in the bottom right panel of Figures 1 and 2, in which season means for each player are plotted, the fits are much better with $R^2$ values of .977 and .807 for the hyperbolic function for 2- and 3-point shooting respectively. These $R^2$ values are also consistent with other studies which utilized a similar level of analysis (e.g., Vollmer & Bourret, 2000).
While the fits for both equations were similar for the data collected in Study 1, a ratio-like schedule may be operating in the natural environment because the data fell along the linear portion of the hyperbolic function. However, this conclusion is tentative as the predominant difference between the two forms is the negatively-accelerated slope that reaches an asymptote for the hyperbolic function. At higher shooting rates, the linear function predicts increasing reinforcement rates, while the hyperbolic function predicts that reinforcement rate will remain nearly constant. Perhaps the rates of shooting obtained in Study 1 were not high enough to demonstrate the differences between the two equations. Study 2 was conducted to examine the schedule of reinforcement for basketball shooting at higher response rates to better determine if reinforcement rate continues to increase linearly or becomes negatively accelerated.

Study 2

In Study 1, most of the data available fell along the linear portion of a hyperbolic function. In Study 2, an experiment was conducted in which shooting rates were controlled to produce higher rates than those rates observed during professional basketball games.

Method

Participants. Seven adults, four women and three men, participated in Study 2. Participants were in their mid to late twenties. Participants’ basketball experience varied from no previous structured basketball experience to playing on basketball teams in high school. Participant I was 29-years-old and had never played on an organized basketball team. Participant J was 27-years-old and also had never played on an organized basketball team. Participant K was 29-years-old and had started playing basketball on organized basketball teams for seven years at the youth and high school level. Participant L was 26-years-old and participated in youth basketball teams for three years. Participant M was 29-years-old and played for six years total
(four years in youth basketball and two years in high school basketball). Participant N was 24-years-old and had never played on an organized basketball team. Participant O was 26-years-old and played on an organized basketball teams for ten years (nine years on youth leagues and one year at the high school level). Participants were recruited based on interest and availability.

Setting and Materials. All sessions took place in a small indoor gym with a basketball hoop. The basketball hoop was 3.05 m (10 ft) from the ground. Participants shot basketballs from a distance of 4.57 m (15 ft) from the far side of the basketball hoop. All participants, men and women, shot eight basketballs. The basketballs were 74.93 cm (29.5 in) in circumference and weighed approximately 22 oz.

Procedure. After participants arrived at the gym, the experimenter provided an informed consent form and asked the participants to review the form and ask any questions. The experimenter also verbally described the task and reminded participants that they could stop at any time. After the experimenter responded to any questions from the participant, the participant was encouraged to stretch, warm-up, and take some practice shots for approximately five min.

Each participant shot twice at each of four arranged rates of shooting. During each session, the participant took 20 shots. The basketballs were located on a rack to the left of the participant. The participant would take a ball from the rack and then shoot. Balls were replaced into the rack by others in the gym to make sure a ball was always available to the participant. Shots were arranged to be taken at 7.5 per minute, 15 per minute, 30 per minute, and 60 per minute. The order of these sessions was determined randomly for each participant before the start of the session. To meet each of the shooting rates, the participant was informed to take a shot each time the whistle blew or the experimenter said, “Shoot”. For each session, the experimenter blew a whistle or said, “Shoot” at the rate prescribed for that session.
Data were collected on the session duration, the number of shots taken, and the number of shots made. The duration recording began at the first shot and ended after the twentieth shot was taken. To calculate the rate of shots and points, the total shots taken or points made was divided by the session duration. A shot was defined as a basketball leaving the participants hands. One point was counted for each basketball that passed through the basketball hoop after a shot was taken. Basketballs that entered and then bounced out of the basket but did not pass completely through the hoop were not counted as points.

Inter-observer agreement (IOA) for shots and points was obtained during 38% of sessions and was 100%. IOA was calculated by counting the number of 10-s intervals in which each observer agreed exactly on the number of shots taken or points made divided by the total number of 10-s intervals multiplied by 100%. In some cases, due to the variation in session length, the last interval was less than 10 s in duration.

Results and Discussion

In Study 2, reinforcement rates at very high response rates were examined to determine if differences between the linear and hyperbolic feedback functions would be obtained at response rates higher than those observed in the data analyzed in Study 1. Figure 3 displays the data for each of seven participants. Similar to Study 1, for five of the seven participants, the hyperbolic and linear feedback function produced similar fits according to $R^2$ and SS$_{res}$. As shown in Table 3, the average $R^2$ for the hyperbolic function was .601 (range, .025 to .918) compared to .646 (range, .180 to .915) for the linear function. However, for two participants (I and J), the hyperbolic equation better described the data. For these two participants, $R^2$ was higher and SS$_{res}$ was lower for the hyperbolic function than the linear function. A visual analysis of the data in Figure 3 confirms the conclusions of the coefficient of determination analysis. For five of the
seven participants, the data fall along a linear path. At higher response rates for participants I and J the data become negatively accelerated at higher response rates.

The rates at the far right of the panels in Figure 3 are likely the highest rates. None of the participants were able to shoot at the arranged rate of 60 shots per min. The fastest rate across participants was approximately 41 shots per min. The individual data suggest ratio-like schedules at higher shooting rates due to the small difference in $R^2$ and $SS_{res}$ and the linear function form for most of the participants.

Discussion

The purpose of this study was to evaluate the application of feedback functions to describe the type of schedule in effect for basketball shooting. In Study 1 both equations produced similar fits for all participants. However, all of the data fell along the linear portion of the feedback functions which suggests a ratio-like schedule may be operating in this environment. In Study 2, a very artificial form of basketball playing was arranged to examine potential differences between the linear and hyperbolic feedback functions at higher rates of shooting. Study 2 provided some evidence that at a higher rate of shooting, there is a linear relation between rate of shooting and rate of points made; however, for two participants, as response rate increased, the increase in reinforcement rate became negatively accelerated. This type of function approximates that of an interval schedule. However, the primary schedule functioning in this context may still be ratio-like even though for two participants in the current study the feedback function more closely approximated what would be expected from an interval schedule. For example, the hyperbolic feedback function contained two free parameters and the linear equation only had one and a larger percentage of variance would be expected from an equation with more free parameters. Additionally, the negative acceleration in the feedback
functions at high response rates may have been a function of something other than the schedule. For example, at very high rates of shooting, such as those observed in Study 2, some part of the response topography may be altered such that shooting becomes less effective or variables such as fatigue may cause reinforcement rate to decrease. For example, when shooting at a low rate, the form of shot taker is very good. However, at high arranged response rates, participants began to hurl the ball toward the hoop as quickly as possible without regard to correct shooting form. The response topography shift may have resulted in reduced reinforcement rates. Also, at higher rates, the inter-response time between shots may not be sufficient for the entire shooting topography to occur which would result in less effective shots taken. These types of problems suggest that if shooters were trained to shoot at very high rates and became efficient shooters, than the outcome of the analysis may have been different. Over time, the response topography of rapid shooting may be shaped by the reinforcement contingency such that a linear relation between shooting and reinforcement rate would develop even at high rates.

Although these examples may explain some occasions in a basketball game, there may be other events which produce interval-like schedules for shooting. For example, if a player is rapidly shooting at the three-point line, the defense may change to block more three-point shots. This change in defense might result in a decrease in the reinforcement rate for attempting three point shots at high rates. The events that occur in a basketball game that may produce an interval-like schedule may not have been present in the experimental arrangement in Study 2. Therefore, finding predominantly ratio-like schedule performance in Study 2 may not be surprising. However, for two participants, a negatively accelerated function was observed demonstrating that the arrangement in Study 2 may have been sensitive to interval-like properties of a schedule for shooting only. An additional analysis was conducted which superimposed VI
schedules onto the obtained data to determine the data patterns that may have been produced by an interval-like schedule. This analysis is displayed for Participant O in Figure 4. The reinforcement obtained was altered to match the VI schedule indicated in the upper-right corner of each graph. The only points (i.e., shots made) that counted toward the total were ones which would have been arranged if a VI schedule were operating. All of the shots taken were still counted. For example, if a VI 4 s schedule were operating, a point could only be made after 4 s had elapsed. If the participant made a point 2 s after the last point was made, that point would not count toward the total; however, a point made 6 s after the last point would count toward the total. As shown in Figure 4, as the interval in the VI schedule increased, the data become more negatively accelerated as rate increases. The SS$_{\text{residual}}$ reduces for the interval feedback function as the data become more hyperbolic as expected. However, R$^2$ remains larger for the ratio feedback function as the data become more hyperbolic. This is due to R$^2$ becoming inflated for the linear function because that function is forced through the origin. This analysis is useful for two reasons. First, it demonstrates the data patterns which might occur had a more predominantly interval-like schedule been operating in a basketball game. Second, it demonstrates the usefulness of SS$_{\text{residual}}$ in cases for which a linear function is forced through the origin. However, this analysis may not demonstrate data patterns that may occur if the schedule operating was a blended schedule.

It may also be the case that the naturally occurring schedule is a blended schedule containing ratio- and interval-like properties. Even with a blended schedule it is likely that the dominant schedule type in the blended schedule would produce either a linear or hyperbolic function. For example, if the schedule operating were a Mixed VI VR schedule (i.e., the player waits for a clear shot to the hoop [VI] and then takes a shot [VR based on shot percentage]), the
hyperbolic feedback function would better describe the data. However, the majority of the current data suggest that the ratio-like properties of the schedule control most of the relation between responses and reinforcers due to the linear relation that occurs.

The current data, overall, suggest that during a typical basketball game, the relation between shooting and points made may be linear such that the ratio-like features operating in the natural environment may control much of the reinforcement obtained. A linear relation between shooting rates and reinforcement rates creates a situation in which matching may be an artifact of the schedule operating in the natural environment rather than the allocation of behavior conforming to reinforcement rates such that the matching relation may be forced. Even though the linear relation may become less apparent at very high response rates, these rates are not common during typical basketball games. This finding suggests that researchers should be cautious in interpreting relations obtained during naturally occurring basketball games as examples of the process of matching and also that further research is needed on schedules in effect in non-experimentally controlled preparations before definitive conclusions are drawn from these data about the generality of the matching law.
References


Table 1

*R² and Parameter Values for 2-Point Shots for All NBA Players in Study 1*

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Table 2

*R² and Parameter Values for 3-Point Shots for All NBA Players in Study 1*

<table>
<thead>
<tr>
<th>Player</th>
<th>Linear Function</th>
<th></th>
<th>Hyperbolic Function</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>R²</td>
<td>SS&lt;sub&gt;residual&lt;/sub&gt;</td>
<td>c</td>
<td>R²</td>
</tr>
<tr>
<td>A</td>
<td>.497</td>
<td>0.501</td>
<td>0.866</td>
<td>.497</td>
</tr>
<tr>
<td>B</td>
<td>.429</td>
<td>0.572</td>
<td>0.843</td>
<td>.429</td>
</tr>
<tr>
<td>C</td>
<td>.405</td>
<td>0.442</td>
<td>0.936</td>
<td>.405</td>
</tr>
<tr>
<td>D</td>
<td>.415</td>
<td>0.337</td>
<td>0.808</td>
<td>.415</td>
</tr>
<tr>
<td>E</td>
<td>.402</td>
<td>0.480</td>
<td>0.807</td>
<td>.402</td>
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<td>F</td>
<td>.341</td>
<td>0.859</td>
<td>0.971</td>
<td>.315</td>
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<td>G</td>
<td>.359</td>
<td>0.592</td>
<td>0.981</td>
<td>.334</td>
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<tr>
<td>H</td>
<td>.370</td>
<td>0.403</td>
<td>0.865</td>
<td>.370</td>
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<td>Average</td>
<td>.402</td>
<td>0.523</td>
<td>0.885</td>
<td>.396</td>
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Table 3

*R² and Parameter Values for All Participants in Study 2*

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<th>Participant</th>
<th>Linear Function</th>
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<th>Hyperbolic Function</th>
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<tbody>
<tr>
<td></td>
<td>R²</td>
<td>SS&lt;sub&gt;residual&lt;/sub&gt;</td>
<td>c</td>
<td>R²</td>
<td>SS&lt;sub&gt;residual&lt;/sub&gt;</td>
<td>t</td>
</tr>
<tr>
<td>I</td>
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<td>K</td>
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<td>9.870</td>
<td>0.032</td>
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<td>L</td>
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<td>.669</td>
<td>48.582</td>
<td>0.0035</td>
</tr>
<tr>
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<td>.888</td>
<td>17.893</td>
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<td>5.391</td>
<td>.601</td>
<td>26.035</td>
<td>0.101</td>
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</tbody>
</table>
Figure Captions

Figure 1. Points per hour plotted against shots taken per minute for 2-point shots for three players (top left, top right, and bottom left panels). Each data point represents data for one player from one game. In the bottom right panel, each data point indicates the season average for a single participant. The solid line indicates the best fit line for the hyperbolic feedback function and the dashed line indicates the best fit line for the linear feedback function. A single line indicates that the hyperbolic and linear functions lie on top of each other.

Figure 2. Points per hour plotted against shots taken per minute for 3-point shots for three players (top left, top right, and bottom left panels). Each data point represents data for one player from one game. In the bottom right panel, each data point indicates the season average for a single participant. The solid line indicates the best fit line for the hyperbolic feedback function and the dashed line indicates the best fit line for the linear feedback function. A single line indicates that the hyperbolic and linear functions lie on top of each other.

Figure 3. Points per hour plotted against shots taken per minute for participants in Study 2. Black diamonds and white squares indicate data from first and second attempts at an arranged shooting rate respectively. The solid line indicates the best fit line for the hyperbolic feedback function and the dashed line indicates the best fit line for the linear feedback function. A single line indicates that the hyperbolic and linear functions lie on top of each other.

Figure 4. Analysis of data from Participant O in Study 2 with different VI schedules superimposed upon the obtained data. The original data and fits are presented in the bottom-left panel of the figure with the VI schedule estimated by the hyperbolic feedback function displayed below the participant ID. The $R^2$ value for each equation is presented next to the respective function in the legend.