Autonomous Robotic Detection of Anti-Personnel Landmines using Ground-Penetrating Radar and On-Contact Antennas

A Dissertation Presented

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Dedicated to SSG Michael Ollis.

I will always treasure our friendship and continue to strive to be the kind of person you were.
Abstract

Ground-penetrating radar is a mature technology which has promise as a solution for humanitarian demining. The technology is fast, inexpensive, and capable of detecting both metallic and non-metallic landmine casings. However, the rough air-ground interface below which anti-personnel mines are buried, reduces the efficacy of air-coupled GPR by increasing clutter and masking target responses. Recent literature focuses on optimizing signal processing techniques to remove the effects of the surface and reliably extract the target reflection. Conversely, this work proposes the use of ground-contact antennas, which greatly improve signal penetration and are less affected by ground clutter, thereby simplifying data analysis. Achieving contact between the surface and the antennas is done by integrating the antennas onto the feet of the Walking Tri-Sphere, a non-articulated walking robotic platform designed by Square One Systems Design (Jackson, WY, USA). Rather than imaging the subsurface, localization of potential targets is achieved using a robust geometric analysis, minimizing the required number of GPR scans. Overall, by using fewer scans and simpler data processing techniques, this method is capable of increasing the surveying speed of traditional GPR methods.

The proposed detection system is evaluated experimentally using the P400 ultra-wideband impulse radar from Time Domain (Huntsville, AL, USA), and computationally using a 3D finite-difference time-domain model. Compact spiral antennas which operate from 3-6GHz were designed considering the application and desired coupling into the ground. The polarization and directivity of the antennas minimizes the direct signal, simplifying the identification of target reflections. Subsurface scans which satisfy both an amplitude and correlation threshold are then analyzed with a localization algorithm, which utilizes time-difference of arrivals to geometrically determine the target location. A minimum of four unique bistatic GPR scans are necessary to evaluate for the target’s position, and an increased number of GPR scans improves the accuracy and reliability of the results. Using the proposed localization method, metallic cylindrical targets are successfully located experimentally. Consideration of non-metallic targets is also addressed experimentally and more extensively computationally. Overall, the proposed method provides a viable solution for autonomous pre-screening of an area for humanitarian demining.
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December 13, 2014
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CHAPTER ONE

INTRODUCTION

Landmines are a truly unique weapon of war as they are inherently incapable of distinguishing between a soldier and a civilian. Additionally, they are the only existing type of weapon that remains active for decades after a conflict has ceased, causing a continual threat to civilians and denying access to economically valuable land. Current technologies which determine the positions of these hidden dangers are slow, costly, and in some cases ineffective. Despite decades of research in new humanitarian demining techniques spanning various fields of science and engineering, an inexpensive and effective technology that can address this global humanitarian issue has yet to emerge. This chapter discusses the motivation for further research developments in landmine detection, the international policies concerning humanitarian demining, and the overall objectives of the research presented in this dissertation.

1.1 Motivation and Background

Landmines were first used on a large scale in 1918 during World War I as an effective measure against the new assault tank technology. By World War II, variations of mines had been developed to specifically target either tanks or personnel. The U.S. army recorded that 2.5% of the fatalities and approximately 21% of the tank losses during WWII were due to mines [12]. Rarely were the mines removed after the conflict had ended, partly due to the lack of proper mine maps, the loss of maps, or a shift in the mine location due to soil shifts. Despite these difficulties, mines continued to be used during times of conflict throughout the 20th century. They provided a unique ability to strategically shape the battlefield by denying the enemy freedom to maneuver. During the Korean War, nearly ten different countries relied on anti-personnel mines
for defense. Even as recently as 2014, Russia used anti-personnel mines to create a divide between the Crimea peninsula and the rest of Ukraine [22, 40].

It has been estimated that landmines cause 15,000-20,000 casualties per year with approximately 40-50% of these victims being children [3, 39]. Figure 1.1 shows the number of recorded casualties for 2012 where the legend identifies the number of casualties per country. In 1995, Andersson et. al. conducted a

![Figure 1.1: World map showing the number of casualties due to landmines, explosive remnants of war, and cluster submunitions in 2012](image)

survey of the social cost of landmines in Afghanistan, Bosnia, Cambodia, and Mozambique. They found that the most frequent activity during an incident was agricultural or pastoral, and that one in three victims were killed. Those who did survive most often suffered from a loss of legs, and other common injuries included loss of arms, loss of fingers, and blindness. Andersson went on to analyze the economical costs of these injuries, and found that households with a landmine victim were 40% more likely to experience difficulty in providing food for the family. Many victims had to undergo multiple operations for their injuries, and more than half of all victims were admitted to a hospital for an average two month stay. In Cambodia 60% of the victims went into debt to pay for their medical attention, and 84% in Afghanistan. Furthermore, they determined that landmine presence greatly affected economic development for the entire country by preventing use of large areas of land for farming or natural resources. They estimated that if these areas
were not restricted, Afghanistan would be able to increase agricultural production by 88-200%, Cambodia by 135%, Bosnia by 11%, and Mozambique by 3-5%. Additionally, they noted that a total of 54,554 animals were lost due to landmines, with a minimum cash value of $6.5m, [6]. Ultimately, the presence of landmines not only poses a major humanitarian concern, but also a substantial barrier for economic growth.

The historical and current international policies surrounding landmines will be discussed in a subsequent section, but ultimately despite international law and significant effort, new mines are still laid by both governments and non-government groups. Figure 1.2 shows the extraordinary presence of landmines worldwide. Many government agencies and the United Nations have taken action against the use of landmines and have placed emphasis on the need for demining. These actions have decreased the production and use of landmines; however, they have not ridden the world of the already staggering presence of landmines, as the technology for detection and removal is still underwhelming. In 1994 it was estimated that 100,000 mines were found and destroyed per year (a more recent publication on the rate of mine removal could not be found); at this rate ridding all of these countries of mines would require hundreds of years [18]. It should also be noted that this estimate does not account for new landmines, which over the past 15 years have been laid by at least 27 countries [3]. Undoubtedly, new technologies offering higher accuracy, lower false alarm

![Mine Contamination as of October 2013](image)

Figure 1.2: Presence of landmines throughout the world as of October 2013
rates, and faster surveying speeds are necessary.

1.1.1 Types of Landmines

Every landmine consists of three components: the case, the explosive material, and the initiator. Originally, all mines were encased in metal, but as early as 1944, Germany began using low-metal mines in an effort to make them more difficult to detect. These included the Glasmine-43, which was an anti-personnel mine where the entire casing was made from glass; the topfmines, which were a series of low-metal anti-tank blast mines [74]; and the Schu-mine 42, which had a wooden case Fig. 1.3. Today, casing material varies between metal, wood, plastic, or a mixture of these components. The explosive material is typically either TNT, RDX, a mixture of the two, or Tetryl, and the initiator varies between pressure sensors, trip wires, tilt rods, acoustic and seismic fuses, or even light-influenced or magnetic-influenced fuses [36]. Landmines are classified by their design as either bounding, fragmentation, or blast. The variety of landmines is considerable with over 675 documented types of landmines [10], however most of these fall into one of two categories, anti-tank (AT) or anti-personnel (AP).

Figure 1.3: Photo of the earliest minimum-metal mine, the German Schu-mine 42
Anti-Tank

As the name suggests, anti-tank mines are designed to damage or destroy vehicles. AT mines are usually 15-30cm in diameter with a thickness of 5-9cm, and typically require 160-350kg of pressure to detonate. AT mines are less of a humanitarian concern as they are less numerous, and cannot be activated by a person’s body weight alone. Additionally, AT mines are typically buried deeper than AP mines and confined to battlefields [36]. For these reasons much of humanitarian demining focuses only on AP mines.

Figure 1.4: Photo of a Russian TM-46 anti-tank mine with a diameter of 30.5cm

Anti-Personnel

Anti-personnel mines are designed to be detonated by human weight, and the majority of civilian casualties are caused by AP mines. These mines are typically smaller and buried shallower than AT mines. The diameter can range from 5-15cm, the depth is usually no more than 20cm, and they typically require 5 - 10kg of pressure to detonate. Bounding types are launched into the air after being initiated such that the detonation causes injury to the person’s head and chest; fragmentation types release fragments in all directions causing injuries up to 200 meters away; and blast landmines are typically tripped by pressure and detonate underground. Blast types are usually designed not to kill, but to injure the person who initiated the weapon. One survey of 232 demining accidents found that 7% of victims of blast AP mines accidents were killed, whereas 38% of victims from fragmentations AP mines were killed [9].
1.1.2 The Difficulties of Landmine Detection

Ideally, a landmine detection system should quickly detect and locate a landmine with high precision, a 100% success rate, and a 0% false alarm rate. The device must also be practical, and as such, cannot be overly expensive. Unfortunately, there are many factors that prevent achieving this ideal system. First is that despite narrowing the detection target to solely AP mines, the numerous permutations of casing materials, explosive content, and geometrical shapes pose a difficult problem for a single technology. Additionally, the landmines can be buried at varying depths and angles. Another difficulty is the myriad of environmental conditions in which mines can be found. As seen in Figure 1.2, climatic conditions can range from deserts (i.e. Somalia, Kuwait), jungles (i.e. Cambodia, Vietnam), mountains (i.e. Afghanistan, El Salvador) and urban areas (i.e. Beirut, Former Yugoslavia), the topography of the ground surface can vary from smooth sand to rough soil, and the atmospheric temperature varies significantly from one country to the next [18, 36]. Similarly, rain or soil moisture can affect certain technologies’ capabilities of detection. The presence of scrap metal or residual explosive chemicals provides a challenge in decreasing the false alarm rate. Aside from these difficulties, the manner in which the demining is conducted is also under consideration. For instance, if a human operator is required, it is important that the operator be able to stand far enough away to avoid inadvertently triggering a mine. Collectively, the permutation of AP landmine designs, climate variations, clutter types, and implementation concerns, combine to form a very complicated problem.

Figure 1.5: Photos of anti-personnel mines from the Mine Action Programme in Sri Lanka; (a) VS50 diameter of 9cm, (b) Type72 diameter 7.8cm
1.1.3 Stages in Demining

There are various stages in humanitarian demining and technologies can be developed to improve and/or simplify one or more stages. The first stage is identifying which areas require surveying. The second is cutting the vegetation and collecting visible metal fragments to better examine the ground surface. The third, and the focus of this research, is the localization of all potential threats in an area. During this stage sensitivity to possible targets should be very high and all possible threats should be indicated. This stage may result in false positives which should be ruled out during stage four where the potential threats are confirmed and/or classified using another imaging modality or proding by an experienced deminer. Following this stage, the detected threats are removed or intentionally detonated. Finally, the area is rescanned to confirm clearance. Some technologies attempt to combine the third and fourth stages by providing detection and classification of the potential threat during the first sweep over an area.

1.1.4 Dangers in Demining

It has been estimated that one deminer is killed and three are injured for every 5000 mines cleared during humanitarian demining efforts [45]. In 2003 the Journal of Mine Action published a study conducted by Canadian and U.S. organizations, on the types and causes of injuries and deaths during 232 demining accidents, which resulted in 295 victims. For each of the accidents, they considered what stage of demining was taking place during the accident and found that excavation and missed mines accounted for 34% and 37% of all accidents respectively. The remaining 29% accounted for all other activities, including vegetation removal, trip wire, detection, handling, other and unknown [9]. The incidents for missed mines were a result of either a failure of the detection equipment or human error; by developing an automated vehicle for detection, these demining incidents may be reduced.

1.2 International Demining Policies

The overall political and legal framework is covered by the Mine Ban Treaty (also known as the Ottawa Treaty or Ottawa Convention and technically named the Convention on the Prohibition of the Use, Stockpiling, Production and Transfer of Anti-Personnel Mines and on their Deconstruction), which was signed in December 1997. All signatories of the treaty commit to never use, develop, produce, acquire, stockpile, retain or transfer anti-personnel mines, to destroy all mines in their stock within four years of signing, and to clear mined areas in their territory within 10 years. Additionally, in countries affected by mines, the
state agrees to conduct mine risk education and ensure that mine survivors, their families, and communities receive comprehensive assistance. In countries not affected by mine presence, the states agree to assist other locations with survivors and in clearance. Overall, eighty percent of the world’s states have joined the treaty (Fig. 1.6), and since 1997 the number of mine victims has decreased every year and over 46 million stocked AP mines have been destroyed. However, the treaty’s requirement that all mines be cleared within 10 years of signing often requires extensions due to the insufficient demining methods which are currently available and affordable.

Figure 1.6: Map of states that have signed the Mine Ban Treaty as of 2013

In 1997 the International Campaign to Ban Landmines (ICBL) and its founder Jody Williams were awarded the Nobel Peace Prize for their work towards the banning and clearing of anti-personnel mines, and the proposal of the Mine Ban Treaty. In 1998, the ICBL launched Landmine Monitor, which monitors and reports the implementation and compliance of the Mine Ban Treaty by its ratifiers, in addition to assessing the problems caused by landmines each year [3].
1.2.1 United Nations Policies

Each year since the development of the Mine Ban Treaty the United Nations General Assembly votes on a “Resolution on the Implementation of the 1997 Convention on the Prohibition of the Use, Stockpiling, Production and Transfer of Anti-personnel Mine and on Their Deconstruction.” This is a non-binding resolution, and provides an opportunity each year for all states (even those outside the Mine Ban Treaty) to reaffirm their support for the ban on anti-personnel mines and the objective of the treaty. Additionally, in the past 12 years only 1 country, Lebanon, has ever voted against the Mine Ban Treaty (though approximately 20 countries abstain from voting each year including the United States) [4]. In addition to the Mine Ban Treaty, international standards for humanitarian demining programs were first proposed at an international technical conference in Denmark in 1996. Criteria for all stages of demining were recommended, and a definition of “clearance” was determined. These principles were written into the first International Standards for Humanitarian Mine Clearance Operations, which was published in the first edition of the U.N. Mine Action Service (UNMAS) in March 1997. In 2001 the standards were redeveloped and renamed as the International Mine Action Standards (IMAS) [30].

Today, the UNMAS is the United Nations office responsible for the development and maintenance of IMAS, and the standards are produced with the assistance of the Geneva International Centre for Humanitarian Demining. Preparing, reviewing, and revising IMAS is done at least every three years by technical committees with the support of international, governmental, and non-governmental organizations. Section 5 of the IMAS defines the U.N. clearance operations, and currently states that,

"The area should be cleared of mines and UXOs to a standard and depth, which is agreed to be appropriate to the residual/planned use of the land and which is achievable in terms of the resources and time available. The contractor must achieve at least 99.6% of the agreed standard of clearance. The target for all U.N. sponsored clearance programmes is the removal of all mines and UXO to a depth of 200mm”

This high clearance requirement of 99.6% is one of the major differences between humanitarian and military demining, precluding the use of established military demining techniques and requiring new, more efficient technologies to be developed.

There are currently fourteen U.N. departments, agencies, programs, and funds that play a role in mine-action programs. The Mine Action and Effective Coordination guides the division of labor within the United Nations to assist affected states in reducing the threat and impact of mines, provide peace and security,
humanitarian relief, human rights, and socio-economic development. They find and destroy landmines, assist victims, teach people how to remain safe in mine-affected areas, and promote participation in the Mine Ban Treaty. The U.N. partners with civil society, the private sector, international and regional organizations, and donors. In a recent publication, [48], the U.N. stated that its ultimate goal is to secure levels of prevention and protection for individuals and communities to a point where the U.N. mine action assistance would no longer be requested.

1.2.2 United States Policies

In 1997 President Bill Clinton sent U.S. representatives to join the treaty negotiations in Oslo, and insisted that two provisions be included. The first was that an adequate transition period was needed to phase out the AP mines currently used by military troops and replace them with a new technology, particularly where they are used in the defense of Korea. The second was to preserve the use of self-destructing AT mines which, due to the phrasing of the treaty, would also be banned. Ultimately, these provisions were not included and President Clinton stated that “Unfortunately, as it is drafted, I cannot in good conscience add America’s name to that treaty. As Commander-in-Chief, I will not send our soldiers to defend the freedom of our people and the freedom of others without doing everything we can to make them as secure as possible [13].”

Although the Mine Ban Treaty was not signed during Clinton’s presidency, the Clinton administration did enact numerous initiatives to eradicate the issue of landmines for civilians. In 1996 President Clinton issued Presidential Decision Directive 48 which prohibited the use of non-self-destructing anti-personnel landmines by U.S. forces, except for the defense of Korea where landmines are a key component along the border between the North and South [14]. In addition, the directive also committed the U.S. to pursue a ban on anti-personnel landmine use, production, and transfer, and it directed the demilitarization of all non-self-destructing anti-personnel landmine stocks, except those required for Korea, by the end of 1999. Finally, it reserved the option to use self-destructing or self-deactivating anti-personnel landmines in military hostilities, if necessary, to safeguard American lives and hasten an end to any conflict. That same year the Secretary of Defense gave the U.S. Army the task of developing a humanitarian demining training program, which resulting in the development of the Humanitarian Demining Training Center (HDTC) and the Countermine Training Support Center (CTSC) [29]. In June 1998, the President signed another Presidential Decision Directive 64, which stated, among other things, that the United States would sign the Mine Ban Treaty by 2006 if suitable alternatives for mixed AT mines and AP mines could be found [15]. In 1999, the U.S. ratified the Convention on Certain Conventional Weapons (CCW) Amended Mines Protocol, which
requires that minefield containing non-self-destructing AP mines be marked and monitored, and that all AP mines be detectable using standard detection equipment [2]. This congressional committee which reviewed the protocol, concluded that “by restricting the use of long-duration anti-personnel landmines while allowing full military use of short-duration anti-personnel landmines, the protocol strikes an appropriate balance between humanitarian concerns and military requirements.” Finally, in 1998, the United States recognized that besides ending the placing of new mines, the major issue would be the removal of current mines. The U.S. then created the Demining 2010 Initiative, which had the ultimate goal of ending the threat of landmines to civilians by the year 2010.

Clinton’s presidency ended in 2000, and on February 27, 2004, the Bush administration announced a new policy which abandoned the objective of joining the Mine Ban Treaty by 2006, claiming that signing the treaty would require the U.S. to forgo a needed military capability which no other weapon could provide. The new policy also allowed for continued development of non-persistent AP and AT mines, but committed the U.S. to eliminate all persistent and non-detectable landmines (including AT mines) from its arsenal besides those stockpiled for use in the Republic of Korea [46]. The United States Campaign to Ban Landmines (USCBL), a coalition of people coordinated by the Human Rights Watch and affiliated with the ICBL, directly and vocally opposed these actions for “sending the wrong message” to mine-using countries [26]. For the past decade, the policies enacted by President Bush have remained active. However in 2014, the Obama administration began moving towards the goals of President Clinton. On June 27, the U.S. delegation announced that the U.S. would not produce or acquire any AP mines that are not compliant with the treaty [5] and on September 23, the Obama Administration announced that the U.S. would not use AP mines nor assist or encourage AP mine use except in the defense of Korea [47].

Ultimately, although the U.S. has not officially signed the Mine Ban Treaty, the U.S. does, in practice, abide by the major stipulations of the treaty. The United States has not placed new AP mines since 1991, nor has it exported them since 1992. Furthermore, the U.S. is currently the world’s largest contributor to humanitarian mine actions and since 1993 the U.S. has provided over $2.3 billion in aid to over 90 countries. This includes assisting in the clearance of mines, and providing medical rehabilitation to over 250,000 people [43].
1.3 Research Objective

The goal of this work is to develop a prototype for an autonomous walking robotic platform which detects potential anti-personnel mines using multi-bistatic ground-penetrating radar. As will be discussed in Chapter 3, the application of conventional air-coupled GPR for landmine detection is not a novel idea. However, the rough air-ground interface below which anti-personnel mines are buried, reduces the efficacy of air-coupled GPR by increasing clutter and masking target responses. Recent literature focuses on optimizing signal processing techniques to remove the effects of the surface and reliably extract the target reflection. Conversely, this work proposes the use of ground-contact antennas, which greatly improve signal penetration and are less affected by ground clutter, thereby simplifying data analysis. Achieving contact between the surface and the antennas is achieved by integrating the antennas onto the feet of the Walking Tri-Sphere, a non-articulated walking robotic platform designed by Square One Systems Design (Jackson, WY, USA). Rather than imaging the subsurface, localization of potential targets is achieved using a robust geometric analysis minimizing the required number of GPR scans. Overall, by using fewer scans and simpler data processing techniques, this method is capable of increasing the surveying speed of traditional GPR methods.

The remaining parts of this dissertation are divided into nine additional chapters. Chapter 2 discusses the history and basic physics of ground-penetrating radar and how it can be implemented for AP mine detection. Chapter 3 presents the past and current research concerning the use of GPR for demining. Chapter 4 will then discuss all of the hardware used to create the prototype, and some of the basic experiments and calibrations performed with the given system. Chapter 5 will discuss the finite-difference time-domain method and how it is used to model real world scenarios and the given radar equipment. Using the proposed system, Chapter 6 discusses the methods used to geometrically detect and locate a potential target using multiple bistatic GPR scans. This chapter also provides the algorithmic summaries for the proposed localization method. Next, Chapters 7 and 8 discuss the experimental characterization and experimental target localization results, respectively. Finally, Chapter 9, discusses the computational analysis for different soils and target types, as well as proposed improvements for future work.
CHAPTER TWO

SUBSURFACE IMAGING WITH GPR

Ground-penetrating radar (GPR) has been a developing technology since the early 1900s, after Heinrich Hertz demonstrated in 1886 that radio waves reflect off of solid objects. In essence, GPR utilizes the differences in electrical properties between objects and the surrounding medium for detection. An electromagnetic wave is emitted into the ground, and the changes in electrical impedance cause reflections of the wave. It can be used to image through soil, concrete, rock, wood, ice, and nearly any non-metallic material, and as such has been implemented for a wide array of applications including nondestructive testing of structures and pavements, studying of soil layers and bedrock, mapping of buried utility lines and pipes, archaeological investigations and in more recent years, has been applied to aid in forensic investigation. This chapter discusses GPR terminologies, the various types of GPR, and the physics behind GPR techniques. For references the reader is referred to the textbooks of Balanis, Ulaby, and Jol [7, 8, 37, 65]

2.1 Overview of Relevant Electromagnetic Properties

The fundamentals of GPR are rooted in electromagnetic field theory. This section does not attempt to detail all of the aspects of this large field of study, but rather introduces the relevant concepts and terminologies that will be used throughout the remainder of this dissertation.

2.1.1 Maxwell’s Equations

In the 19th century, James Clerk Maxwell unified the work of Faraday, Ampere, and Gauss into a set of partial differential equations that govern the relationship between electric and magnetic fields due to charges
and currents, now known as Maxwell’s Equations (Eq. 2.1).

\[
\nabla \times E = -\frac{\partial B}{\partial t} \quad (2.1a)
\]
\[
\nabla \times H = \frac{\partial D}{\partial t} + J_c + J_s \quad (2.1b)
\]
\[
\nabla \cdot B = 0 \quad (2.1c)
\]
\[
\nabla \cdot D = \rho \quad (2.1d)
\]

Here, \( E \) is the electric field intensity (V/m), \( H \) is the magnetic field intensity (A/m), \( J_s \) is the source current (A/m²), and \( \rho \) is the charge density (C/m³). The electric flux density \( D \) (C/m³), the magnetic flux density \( B \) (T), and the induced source current \( J_c \) (A/m²) are related to \( E \) and \( H \) by the constitutive relations (Eq. 2.2) which describe a material’s response to electromagnetic waves.

\[
D = \epsilon * E \quad (2.2a)
\]
\[
B = \mu * H \quad (2.2b)
\]
\[
J_c = \sigma * E \quad (2.2c)
\]

In Eq. 2.2, \( \mu \) is the magnetic permeability (H/m), \( \epsilon \) is the electric permittivity (F/m), \( \sigma \) is the conductivity (S/m) and * denotes convolution. In general these properties can be inhomogenous, nonlinear, anisotropic and dispersive.

2.1.2 Material Properties

The electrical properties of a material define the way in which electromagnetic waves propagate, reflect, and scatter and therefore dictate how a potential target responds to a GPR excitation and how the excitation propagates through the medium of interest. The meaning of these properties will be discussed in detail, and Table 2.1 summarizes the conductivity and relative permittivity values for various types of sands and soils that landmines can potentially be buried within.

**Conductivity**

Electrical conductivity is a measure of how easily electrons can travel through the material under the influence of an external electric field, i.e. how easily the material can carry an electric current. Materials are classified as either conductors or dielectrics based on the magnitudes of their conductivities. Free space is a perfect dielectric with \( \sigma_0 = 0 \) S/m, and a perfect conductor would have \( \sigma = \infty \). The conductivity can have a significant effect on the attenuation of a radar signal, and can also depend on frequency.
<table>
<thead>
<tr>
<th>Material</th>
<th>Conductivity (mS/m)</th>
<th>Relative Permittivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Dry Clay</td>
<td>1 - 100</td>
<td>2 - 20</td>
</tr>
<tr>
<td>Wet Clay</td>
<td>100-1000</td>
<td>15 - 40</td>
</tr>
<tr>
<td>Dry Sand</td>
<td>0.0001 - 1</td>
<td>3 - 6</td>
</tr>
<tr>
<td>Wet Sand</td>
<td>0.01 - 10</td>
<td>10 - 30</td>
</tr>
<tr>
<td>Dry Sandy Soil</td>
<td>0.1 - 100</td>
<td>4 - 6</td>
</tr>
<tr>
<td>Wet Sandy Soil</td>
<td>10 - 100</td>
<td>15 - 30</td>
</tr>
<tr>
<td>Dry Clayey Soil</td>
<td>0.1 - 100</td>
<td>4 - 6</td>
</tr>
<tr>
<td>Wet Clayey Soil</td>
<td>100 - 1000</td>
<td>10 - 15</td>
</tr>
<tr>
<td>Average Soil</td>
<td>5</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 2.1: Example permittivity and conductivity values for various ground types

**Permeability**

Magnetic permeability is a measure of the magnetization that a material obtains in response to an applied magnetic field. The permeability of free space is, $\mu_0 = 4\pi \times 10^{-7}$ H/m, and the relative permeability is a unitless quantity which is defined as the ratio of the material’s permeability to that of free space, $\mu_r = \mu/\mu_0$.

Diamagnetic and paramagnetic materials typically have a relative permeability similar to that of free space, i.e. $\mu_r \approx 1$, whereas ferromagnetic materials have a relative permeability much greater than 1. The materials considered in this work all have a $\mu_r \approx 1$, and in the computational studies the magnetic permeability of all materials is assumed to be one.

**Permittivity**

Electrical permittivity is a measure of the material’s ability to allow the formulation of an electric field within it. The permittivity of free space is, $\epsilon_0 = (c^2 \mu_0)^{-1} \approx 8.854 \times 10^{-12}$ F/m, and the relative permittivity is a unitless quantity which is defined as the ratio of the material’s permittivity to that of free space, $\epsilon_r = \epsilon/\epsilon_0$.

Relative permittivity is also referred to as the dielectric constant, however this can be misleading as the relative permittivity can be dependent on frequency.

**Intrinsic Impedance**

The intrinsic impedance of a material is the ratio of the electric field to the magnetic field and is measured in ohms. The general relation is,

$$\eta = \sqrt{\frac{j\omega \mu}{\sigma + j\omega \epsilon}}$$

(2.3)
For a lossless medium this reduces to $\eta = \sqrt{\mu/\epsilon}$, which means that for free space the intrinsic impedance is $\eta_0 = \sqrt{\mu_0/\epsilon_0} \approx 377\Omega$. As will be discussed, the change of impedance determines the reflections in a GPR signal.

### 2.1.3 Field Regions

In general, the properties of an electromagnetic wave radiating from an antenna vary according to the distance from the antenna, where the distance is related to the wavelength, $\lambda$ (Sec. 2.1.4). There are three main regions; the reactive near-field, the Fresnel region which is the radiating near-field, and the Fraunhofer region which is the far-field. Within the near-field, the reactive field dominates. The Fresnel region acts as the transition region between the near and far field. In this region, the radiation fields predominate and the angular field distribution is in general a function of the radial distance from the antenna. Finally, the far-field is the region where the angular and transverse field distributions are essentially independent of the distance from the antenna. Generally radiation patterns for an antenna are the resulting pattern in the far-field. For an electromagnetically short antenna, where the antenna is smaller than half of the wavelength of the radiation, the near field is the region where the radius from the antenna is, $r < \lambda/2\pi$, and the far field is the region where the radius is at least twice the wavelength. However, it is possible that the Fresnel region does not exist for small antennas. For larger antennas, the far field is defined as the Fraunhofer distance, $d_f = 2D^2/\lambda$, where $D$ is the largest dimension of the antenna, while the reactive near-field is defined as $r < 0.62\sqrt{D^3/\lambda}$. The remaining sections discuss electromagnetic field propagation in the far-field.

### 2.1.4 Wave Properties

The electric and magnetic fields can be decoupled by increasing the order of the differential equations to second order, which results in two wave equations, Eq. 2.4.

$$\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0 \quad (2.4a)$$

$$\nabla^2 H - \mu\sigma \frac{\partial H}{\partial t} - \mu\epsilon \frac{\partial^2 H}{\partial t^2} = 0 \quad (2.4b)$$

Note that these wave equations for source free regions (i.e. $J_c = 0$, $\rho = 0$) and using time-harmonic fields, the wave equations can be expressed as,
\[ \nabla^2 E = j\omega\mu\sigma E - \omega^2\mu\epsilon E = \gamma^2 E \]  
\[ \nabla^2 H = j\omega\mu\sigma H - \omega^2\mu\epsilon H = \gamma^2 H \]  

(2.5a)  
(2.5b)

where \( \gamma \) is the complex propagation constant, for which the real component is the attenuation constant, \( \alpha \), measured in Np/m and the imaginary component is the wavenumber, \( \beta \), in Rad/m.

**Wavenumber**

The wavenumber is the spacial frequency of a wave, measured in radians per unit distance and is also called the propagation constant or phase constant. Using the relations in Eq. 2.5, \( \beta \) can be expressed in terms of the frequency of the wave and the electrical properties of the material as,

\[ \beta = \omega \sqrt{\frac{\mu\epsilon}{2}} \left[ \sqrt{1 + \left( \frac{\sigma}{\omega\epsilon} \right)^2} - 1 \right] \]  

(2.6)

For a material with no conductivity, the wavenumber reduces to \( \beta = \omega\sqrt{\mu\epsilon} \).

**Attenuation**

The attenuation constant describes how much the magnitude of an electromagnetic wave will reduce while propagating through a medium. The general formulation for attenuation is shown in Eq. 2.7.

\[ \alpha = \omega \sqrt{\frac{\mu\epsilon}{2}} \left[ \sqrt{1 + \left( \frac{\sigma}{\omega\epsilon} \right)^2} - 1 \right] \text{ Np/m} \]  

(2.7)

Note that for a material with no conductivity, \( \alpha = 0 \) and there is no attenuation. Attenuation is the main component dictating the depth at which a target can be imaged. In certain materials, such as ice or sand, the attenuation of the signal is very low and imaging can be performed at hundreds of meters below ground. The skin depth of a medium is the distance that a wave can travel before the magnitude decreases by a factor of \( e^{-1} \approx 0.37 \). Skin depth is simply the inverse of the attenuation coefficient and is measured in meters.

**Wave Velocity**

The velocity of an electromagnetic wave is dictated by the wavenumber and the angular frequency, such that \( \nu = \omega/\beta \). For a material with no conductivity, the dependence on angular frequency cancels out and the
resulting relation is,
\[ \nu = \frac{\omega}{\omega \sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \] 
(2.8)

For the materials considered in this research (where \( \mu_r = 1 \)), the velocity is approximated as \( \nu = c/\sqrt{\varepsilon_r} \).

**Wavelength**

The wavelength of a signal is the particular distance at which the wave begins to repeat. Wavelength is measured in meters, and is inversely proportional to the wave number.

\[ \lambda = \frac{2\pi}{\beta} = \frac{2\pi \nu}{\omega} = \nu \] 
(2.9)

As will be discussed, in electromagnetics the absolute size of any object is irrelevant to how electromagnetic waves will behave. Instead, the size of all objects is relative compared to the wavelength. For instance, surface roughness is a relative measure and depends on the wavelength of the signal. A surface may appear rough at one wavelength, but would appear smooth to another, longer, wavelength [37].

**2.1.5 Wave Polarization**

The polarization of a radiated wave is defined as “that property of a radiated electromagnetic wave describing the time-varying direction and relative magnitude of the electric field vector; specifically, the figure traced as a function of time by the extremity of the vector at a fixed location in space, and the sense in which it is traced, as observed along the direction of propagation” [1]. In other words, the polarization of a uniform plane wave describes the curve traced out by the end point of the instantaneous electric field as it changes with time. The polarization of a wave depends on the phase of one \( E \) field component relative to that of the orthogonal \( E \) field component and there are three different polarization states; linear, circular, and elliptical.

In the linear polarization state, both \( E \) field components are either in phase, or out of phase, and therefore the vector describing the electric field is always along a line. For circular polarization, the \( E \) field components are \( \pm 90^\circ \) out of phase, and have the same magnitude. Depending on which component is leading, there are two different circular polarization states called left-hand polarization (LCP, also called counterclockwise) and right-hand circular polarization (RCP also called clockwise). Note that the direction of rotation is always determined by rotating the phase-leading component toward the phase-lagging component and observing the field rotation as the wave is traveling away from the observer. Finally, elliptical polarization is the most general case, of which circular and linear polarization are just specific cases. For
this case the $E$ field components are out of phase by some angle within the range, $-90^\circ \leq \delta \leq 90^\circ$, and the amplitudes of the field components can be equal or different. As with the circularly polarized waves, the sense of rotation is determined by the phase leading component.

The polarization of an antenna is defined by the polarization of the wave transmitted from the antenna. When operating as a receiver, the polarization of the antenna determines what percentage of the electromagnetic wave will be received, since only the components of the incident wave which are parallel to that of the antenna polarization can be extracted due to reciprocity. For instance, an RCP antenna can receive all of the power from an electromagnetic wave that is right-hand circularly polarized, but none of the power from an electromagnetic wave that is left-hand circularly polarized. If the incident wave is linearly polarized, only a portion of the power would be received by an RCP or LCP antenna. In general, the power loss due to a polarization mismatch is defined by the polarization loss factor (PLF).

### 2.1.6 Antenna Radiation Patterns

The antenna pattern, also referred to as the radiation pattern, describes the strength of the radiated wave in the three-dimensional space around the antenna. It is the far-field directional properties of the antenna measured at a fixed distance from the antenna. An isotropic antenna is one that radiates equal power in all directions; it is not physically realizable but it is used as a reference to calculate antenna properties. The antenna pattern is most commonly discussed in terms of the principal planes of the spherical coordinate system; the elevation plane (constant $\phi$) and the azimuth plane ($\theta = 90^\circ$).

Antenna radiation patterns can vary greatly depending on the application; however two general classes are omnidirectional antennas and directional antennas, Figure 2.1. Omnidirectional antennas radiate power uniformly in all directions in a single plane, usually defined as the azimuth plane, and then have decreasing power in the elevation plane. Conversely, directional antennas, also called beam antennas, radiate greater power in one or more directions. Directional antennas are often described in terms of their lobes, where the maximum power is confined to the main lobe, and additional power results in the undesirable side and back lobes. The directivity of an antenna, $D$, is defined as the ratio of the maximum power density radiated by the antenna compared to the power density radiated by an isotropic antenna at the same range and for the same input power. The radiation efficiency, $\xi$, is defined as the ratio of the power radiated from the antenna to the total power supplied to the antenna. Finally, the gain, $G$, which accounts for ohmic losses in the antenna material is defined as the product of the directivity and the radiation efficiency of the antenna, such that for a lossless antenna the gain and directivity are equivalent. Finally, since antennas are reciprocal devices, the
radiation pattern also defines the antenna’s receiving pattern.

Using these parameters, the power transfer between two antennas separated some distance $R$, can be determined by Friis transmission formula which states,

$$P_{rec} = G_t G_r \left( \frac{\lambda}{4\pi R} \right)^2 P_t$$

(2.10)

where $P_{rec}$ is the received power, $G_t$ is the gain of the transmitter, $G_r$ is the gain of the receiver, $\lambda$ is the wavelength of the field in the medium of interest, and $P_t$ is the transmitted power.

### 2.1.7 S-Parameters

A linear two-port microwave network is characterized by a number of equivalent circuit parameters, such as the transfer matrix and the impedance matrix. These can be described using the scattering parameters, also called the S-parameters, which are identified in Figure 2.2. Here, $S_{11}$ and $S_{22}$ are reflection coefficients while $S_{21}$ and $S_{12}$ are transmission coefficients. The S-parameters relate the magnitude of the incoming waves ($a_1$ and $a_2$) to the magnitude of the outgoing waves ($b_1$ and $b_2$).

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

(2.11)

The S-parameters are measured using a vector network analyzer (VNA), and typically a VNA will measure the reflection and transmission coefficients over a wide frequency range. The S-parameters are related to the incoming and outgoing waves’ voltages and currents, as well as the intrinsic impedances of
Figure 2.2: Two-port microwave junction and S-parameters

the two connections. The magnitude of the transmission coefficients is also called the insertion loss, while the magnitude of the reflection coefficients is also called the return loss. These values are typically expressed in decibels. Additionally, the return loss and insertion loss are related as, $|S_{11}|^2 + |S_{21}|^2 \leq 1$, such that the S-parameters of a passive device cannot exceed 1 (0dB). Ideally, the device will be lossless, and therefore, $|S_{11}|^2 + |S_{21}|^2 = 1$. For a reciprocal device, $S_{21} = S_{12}$.

2.2 GPR Basics

The previous section discussed the relevant electromagnetics principles that apply to GPR. This section discusses the specific terminology that is used throughout this dissertation and the different types of GPR modalities.

2.2.1 GPR Terminology

There are various terms that are used when discussing GPR. The system can either be ground-coupled, where the antennas are in direct contact with the ground, or it can be air-coupled, where the antennas are in air pointed towards the ground. Additionally, the antennas can be monostatic, meaning that the system uses one antenna to both transmit the excitation and receive the reflected signals; it can be bistatic, meaning it uses one antenna to transmit and one to receive; or it can be multistatic, meaning it uses one or more antennas to transmit and one or more antennas to receive.

The radar cross section (RCS) is a factor relating the power density of the scattered field in the direction of the receiver, to the power density of the radar’s transmitted field at the target’s range. It takes into account the absolute size of the target, the relative size of the target (compared with the incident field’s wavelength), the incident angle, the polarization of the incident wave, the material of the object, and the shape of the object. By manipulating these factors, the radar reflection from large objects can be reduced substantially,
such as with stealth aircrafts. RCS is measured in $m^2$, and the mathematical definition will be discussed in a subsequent section.

The dynamic range of a GPR system is the ratio of the largest receivable signal to the minimal detectable signal, defined as, \( DR = 20 \log \left( \frac{V_{\text{max}}}{V_{\text{min}}} \right) \), where $V_{\text{max}}$ is the maximum voltage that can be received and $V_{\text{min}}$ must be above the receiver noise. Since the amplitude of the radar signal decreases with distance, the dynamic range of a system affects the maximum range at which a target can be detected. Furthermore the weaker signals must surpass a minimum signal-to-noise ratio (SNR), which is the ratio of the power or energy of a received signal to the power or energy of the accompanying noise.

Finally, the FCC defines the bandwidth of a radar excitation as the frequency band bounded by a 10dB reduction from the highest radiated emission, based on the complete transmission system including the antenna. Here, the upper boundary is designated $f_H$, and the lower boundary is designated $f_L$. The center frequency is then $f_c = 0.5(f_H + f_L)$ and the fractional bandwidth is defined as,

\[
BW = \frac{2(f_H - f_L)}{(f_H + f_L)}
\]  

(2.12)

As will be discussed, the bandwidth of a system has a strong impact in the resolution of the GPR system.

### 2.2.2 Types of GPR

The main difference between types of radar is the manner in which the data is collected. Typically GPR systems are categorized either as impulse radar, which operates in the time domain, or continuous-wave (CW) radar, which operates in the frequency domain. In theory, given the same frequency specifications the type of data acquisition should have no effect on the resulting signal; however in practice significant differences occur and each method ultimately has its own advantages and disadvantages [37].

**Impulse GPR**

The first, and potentially most common type of GPR is the time-domain impulse GPR system. In this method, a short pulse is emitted into the ground and the reflected energy is received as a function of time, Figure 2.3. The received signal indicates the amplitude of energy scattered from subsurface objects, as well as any direct energy between the two antennas. The impulse excitation can vary from a monocycle pulse to an ultra-wideband (UWB) Gaussian pulse, and the center frequency can vary from MHz to GHz. The FCC defines a system as ultra-wideband (UWB) if the excitation has a fractional bandwidth equal to or greater
Impulse GPR is typically low-cost and generation of the excitation is simple. However, time-domain data acquisition can result in undesirable ringing, and inefficient use of transmit power. Additionally, as will be discussed in a subsequent section, the resolution is limited by the pulse width.

**Continuous Wave GPR**

The majority of GPR systems operate in the time domain, but in the last few decades the use of frequency domain based GPR systems has evolved. In the frequency domain there are two types of operation, either the continuous wave is frequency modulated with a linear sweep (FMCW GPR), or the frequency of the continuous wave is changed in fixed steps, called stepped frequency GPR. In a FMCW GPR, the received signal is mixed with the excitation signal and the difference in frequencies between the two is a function of the depth of the target. Similarly, for stepped-frequency GPR the transmitting frequency is stepped through in linear increments over the given bandwidth, and for each frequency the amplitude and phase of the received signal is compared with the transmitted signal to determine target information. For both methods, the received signal can be transformed into an equivalent time domain representation by an inverse Fourier transform, to create a synthesized pulse.

Advantages of CW GPR are the controlled transmission frequencies, the efficient use of power, the efficient sampling of wideband signals with simpler analog to digital converters, and an improved SNR. Additionally, the nature of the system allows for the collection of coherent data, and complex processing. However, CW radars require complex electronics increasing the cost, and suffer from slower acquisition.
times than impulse GPR since data is recorded for each individual frequency. Finally, time-varying gain cannot be applied to the return signal and the conversion from frequency to time can introduce sidelobes from strong signals that mask weak scatterers. The research implements an ultra-wideband GPR system, of which the physics and understanding is discussed in the next section.

2.3 Impulse GPR Scan Types

For impulse radar data can be collected into one-dimensional, two-dimensional, and three-dimensional raw data sets. These may also be processed using focused GPR.

2.3.1 A-Scans

Given a transmitter and a receiver, such as the bistatic pair shown in Figure 2.3, the simplest and quickest data acquisition method is to record a single scan of the subsurface, and consider the received signal. This received signal is termed an A-scan, and relates the signal amplitude with range. An example of two A-scans recorded using the Time Domain GPR are shown in Figure 2.4. Here the independent axis is time, which is related to the range, or in this case, target depth by the wave velocity \( (v = d/t) \), and the dependent axis is the amplitude of the received signals. Both A-scans were recorded using the same bistatic separation, but the first was recorded without anything buried below the antennas while the second was recorded above a metallic target. This comparison between A-scans, demonstrates how the presence of a target alters the

![Figure 2.4: Example of a comparison between two A-scans, one taken over a metallic cylindrical object and the other without the cylindrical object (i.e. the background).](image)
received signal, and the direct communication that occurs between the antennas. Using this reflection, the target depth can be determined and will be discussed in Section 6.1.

### 2.3.2 B-Scans

In the next data acquisition method, the antennas are moved in a straight line over an area, and at each incremental step a GPR scan is recorded, Fig. 2.5(a). This collection of A-scans creates a two-dimensional data set, called a B-scan such as the one shown in Figure 2.5(b). Here, it is clear that an object, with a reflection time of approximately 31ns, is buried below the GPR when the GPR is 25cm from its starting position. In a B-scan, reflections from a point scatterer located below the surface appear as hyperbolic structures where the hyperbola is a function of the antenna configuration, the depth of the scatterer, and the propagation velocity of the ground. This is due to the directivity of the antennas and the distances to the scatterer as the antennas are moved above the surface. If the radar is air-coupled, the hyperbolic shape is lost, and instead the target appears as a fourth order system which appears hyperbolic at the apex and towards the extents of the shape [52].

### 2.3.3 C-Scans

The slowest but most informative data collection technique is to collect a three-dimensional data set called a C-scan, which is essentially multiple parallel B-scans (Fig. 2.6). This means that the antennas are moved incrementally over a grid in two directions. C-scans can be represented in 3D, or as a series of 2-D slices with the slices usually taken at a given time (called time-slices), to show the reflections at a certain depth for
the given surveying grid. C-scans are advantageous in the amount of information that can be generated about the subsurface, however the data collection process is time-consuming and can be labor intensive. Additionally, the resulting data sets are large, and any post-processing techniques are necessarily computationally intensive.

![Figure 2.6: Relationship between B-scans and C-scans](image)

2.3.4 Synthetic Aperture Radar

The data acquisition methods for Synthetic Aperature Radar (SAR) can either be the same as recording a B-scan or C-scan, however the way the data is expressed differs. As was discussed, B-scans result in hyperbolic shapes where there are actually point scatterers, and similarly C-scans result in exaggerated ellipses. SAR is a post-processing technique that corrects for these approximations to create images that actually display the target’s size and position. SAR-GPR systems benefit from being easier to understand and presenting more information, however suffer from computational time and cost.

2.4 Impulse GPR Calculations

The radar used in this work is an impulse UWB GPR unit. This section discusses basics of impulse radar calculations and the physical limitations of the resolution.
2.4.1 Target Reflections

Target reflections are dependent on both the electrical properties of the object and the size of the object relative to the wavelength of the excitation. For relatively large boundaries, the incident wave reflects and transmits according to Fresnel’s equations, whereas for relatively small objects (where the size of the object is on the order of the excitation wavelength) targets are seen as scatterers.

Reflection and Transmission

When a plane wave is incident on a large and seemingly flat boundary, part of the energy is reflected and part is transmitted. The amount of signal which is reflected or transmitted is determined by the transmission coefficient $T$ and the reflection coefficient $\Gamma$, which are related as $T = 1 + \Gamma$. If the wave is normally incident, then the reflection coefficient is defined as,

$$\Gamma = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

where $\eta_2$ is the intrinsic impedance of the wave is propagating into, whereas $\eta_1$ is the intrinsic impedance of the medium that the wave is propagating from. Given this expression, the transmission coefficient is,

$$T = \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

If the wave is incident at an oblique angle then these coefficients incorporate Snell’s laws of reflection ($\theta_i = \theta_r$) and refraction ($\beta_1 \sin \theta_i = \beta_2 \sin \theta_t$). Additionally, at an oblique angle the reflection and transmission coefficients are dependent on the polarization of the incident wave, which can be described as the superposition of two orthogonally polarized waves; one which is parallel to the plane of incidence and one that is perpendicular to the plane of incidence. The reflection coefficients for the perpendicular and parallel components are shown in Eq. 2.15a and Eq. 2.15b respectively.

$$\Gamma_\perp = \frac{E_{\perp r}}{E_{\perp i}} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$$

$$\Gamma_\parallel = \frac{E_{\parallel r}}{E_{\parallel i}} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t}$$

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Scattering

Reflection and transmission of electromagnetic plane waves at semi-infinite planar interfaces is a highly simplified situation, particularly for mines buried below a rough surface. In general, wave propagation encounters structures of various size and shape. Rather than having incident, reflected, and transmitted fields, scattering field theory accounts for two fields; the incident field \((E_i, H_i)\) and the scattered field \((E_s, H_s)\). The mathematical calculations for scatterers is considerably more complicated, and beyond the scope of this electromagnetic summary. However a brief qualitative understanding is discussed.

Consider a spherical scatterer with radius, \(r\). Figure 2.7 shows the normalized monostatic RCS compared to the sphere radius in wavelengths, and there are three distinct regions. First, there is the Rayleigh region which corresponds to a radius much smaller than the wavelength, and in this region the RCS increases linearly. When the radius is much larger than the wavelength, the RCS is in the optical region and is independent of frequency and equal to the physical area of the cross section. The transitional region where the RCS oscillates is called the Mie or resonance region. The received power, \(P_r\), of radar system is related to the RCS, \(\sigma\), by

\[
P_r = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 r_t^2 r_r^2}
\]

where, \(P_t\) is the transmitted power (W), \(G_t\) and \(G_r\) are the gains of the transmitting and receiving antennas respectively, \(r_t\) and \(r_r\) are the distances from the target to the transmitter and receiver respectively, \(\lambda\) is the wavelength and \(\sigma\) is the RCS. If the system were monostatic, \(r_t\) and \(r_r\) would be equal.
2.4.2 GPR Resolution

Resolution defines the ability to distinguish targets and extract geometrical parameters of the target, such as size, shape, and thickness. There are two different resolutions to consider, range resolution (a.k.a longitudinal or depth resolution), and lateral resolution (a.k.a the angular or sideways resolution).

The range resolution of a radar system is defined in the IEEE St. 686-1990 as “the ability to distinguish between two targets solely by the measurement of their ranges (distance from the radar); usually expressed in terms of the minimum distance by which two targets of equal strength at the same azimuth and elevation angles must be spaced to be separately distinguishable.” Consider two targets located at distances \( D_1 \) and \( D_2 \), Figure 2.8.

Assuming a pulse excitation, the reflection from the first target would be at \( T_1 = 2D_1/v \), and will have the same length as the original excitation, \( \tau \). Similarly the return due to the second target will arrive at \( T_2 = 2D_2/v \). The two targets will be resolvable as long as \( T_2 \geq T_1 + \tau \). Therefore, the range resolution is \( \Delta D = D_2 - D_1 = v\tau/2 \).

Similarly, the lateral resolution determines the minimum spacing between targets at the same range, and is related to the wave velocity, the distance to the target, and the pulse width of the system, as

\[
\Delta l \geq \sqrt{\frac{v^2 \tau}{2}} \quad (2.17)
\]

Note, that as the distance of the target from the radar increases, the lateral resolution also increases, Figure 2.9.
2.4.3 Excitation Frequency

As discussed above, the scale at which GPR can detect objects is proportional to the wavelength of the excitation, which is dependent on the velocity and frequency of the excitation. Since the velocity is determined by the soil’s electrical properties, the only parameter that can be controlled is the operating frequency thereby making it the major design element of GPR. However, as frequency increases, the attenuation in soil increases, creating a trade-off between exploration depth and spatial resolution. Optimizing the excitation frequency therefore requires consideration of the object to be imaged, its potential location, and the soil type. In order to detect a subsurface landmine, the wavelength of the excitation in the soil must be on the order of the landmine radius, which for AP mines is between 3 and 7cm.
CHAPTER THREE

LITERATURE REVIEW OF GPR DEMINING TECHNIQUES

Despite decades of research, no one technology has yet to provide a fast, inexpensive, and reliable solution for AP landmine detection. Research is currently continuing in the biological, chemical, nuclear, mechanical, and electromagnetic fields, for which a basic overview is included in Hines, 2012 [33] and in [38, 39]. The different fields have various advantages and disadvantages, and in some cases researchers are considering combined technologies to overcome a single technology’s limitations. This chapter provides a thorough review of the application of GPR in both a standalone and combined setting for the use of humanitarian demining.

GPR is a promising technology for humanitarian demining, as it can potentially be used to identify AP mines independent of the target casing. However, performance is limited by extensive clutter and weak target reflections. Since AP mines are typically buried close to the surface, attenuation of the excitation signal is not typically a concern. However, the reflection from the target is masked by the reflection from the air-ground interface, and if the mine contains minimal metal extracting the reflection can be highly difficult. Additionally, despite substantial research in the field, and a vast database on radar responses, many results for landmines remain unavailable [71]. Typically, RCSs can be modeled to predict radar responses; but the complexities and variety of the geometries associated with landmine detection prevent any generalizations about radar responses for mines. Furthermore, in typical radar scenarios the target of interest exists in the far field, whereas in demining the target can lie in the near field. Finally, if the permittivity of the soil is high, the amount of power entering the soil will be low due to the impedance change from air, creating an even weaker target response. These unique aspects collectively form a new challenge for radar detection.
3.1 Surveying with GPR

The first question for GPR demining research is how to move the system to survey a large area. Many methods are handheld, meaning they require a deminer to canvas an area carrying the equipment, similar to that shown in Figure 3.1(a). These methods allow all areas to be surveyed, regardless of terrain, however, they also require that people be in the field during the initial scan of the area, which can result in deminer casualties. Additionally, the success of these methods depends on the user providing adequate coverage of the area and usually requires the user to form the final decision between potential targets and clutter. If the required equipment is too heavy to be carried by a single person, some research groups employ vehicular methods, Fig. 3.1(b). These methods can be manned or autonomous. The manned vehicular methods have the same downfalls as handheld equipment. Additionally, depending on the capabilities of the vehicles, many of these modalities may be limited in the types of terrain that can be surveyed. The major benefit of autonomous vehicular methods is that deminers are not required to be in the area during the initial surveying, which could potentially prevent deminers casualties. Additionally, the automated methods do not require user input to decipher between potential targets and clutter. Although this requires increased complexity in programming, the results can be more consistent.

The type of surveying method has an important impact in the types of detection algorithms that can be used and the ways in which they are implemented. For instance, a vehicle-based system will have accurate-positioning of the antennas, and could be capable of utilizing an antenna array. Conversely, for the majority of handheld systems, the position of the antennas is unknown, and the height of the antennas above the ground...
ground may vary. Therefore, simpler detection algorithms are typically used for handheld systems, whereas more advanced algorithms can be used for vehicle-based systems.

### 3.2 Target Detection and Classification

The signal processing of GPR for landmine detection is often divided into two distinctions: detection and classification. The detection stage should be fast, thorough, and have a 100% probability of detection regardless of false alarms. Ultimately the data processing should occur in real time. After an initial detection has been made, the next step is to determine if the potential target is indeed a threat. This step is necessary to lower the false alarm rate, and discern between non-metallic mines and safe clutter. There are also efforts to discern the type of mine buried below the surface, in order to better inform the deminer before excavation.

#### 3.2.1 Image based detection

Image-based detection is the most common approach in GPR research. A conventional detection method for ground-coupled GPR involves the detection of certain spatial distributions of the reflected energy. For a B-scan the reflection from a subsurface scatterer appears hyperbolic, and can typically be detected by removing background effects, detecting edges within the image, and then applying a randomized Hough transform to detect the hyperbolas from which the target’s location can be extracted ([41, 69]). However, since the antennas are usually air-coupled, the shape of the reflection from a subsurface scatterer is not exactly hyperbolic, but rather a fourth-order curve that can be approximated as a hyperbola near its top and at its far branches. Therefore, Milisavljevic and Yarovoy modified traditional hyperbola detection algorithms, and incorporated the fact that the targets of interest are flush or shallowly buried [42, 53]. This approach works well for strong scatterers and cases where the background response can be removed; however, these techniques can be compromised when the air-ground interface is considerably rough as the hyperbolic shape can be lost.

Many of the difficulties involved in the target detection within B-scans can be avoided by using C-scans, and the depth cross-sections. In these depth slices, the target response appears as an ellipse, which can be determined using similar image processing techniques, [72]. By detecting the ellipse through consecutive depth slices, the horizontal position and depth of the target can be determined, as well as the permittivity of the soil. This method is more robust than the hyperbola detection within B-scans; however, it comes at the added cost of increasing the required surveying time. Similarly, target detection from focused GPR
data is considerably simpler than in B-scans, as the focusing algorithm places all reflectors in their correct positions and reconstructs the reflector shape; however, accurately implementing a focusing algorithm can be computationally expensive. After focusing the data, image processing techniques can be used to detect potential targets. One major disadvantage of any projection technique is that the depth dimension is lost. Similar methods for focused C-scans have been suggested by Cosgrove [17]. Furthermore, if polarimetric information is available it can be used to suppress clutter and improve object detection [60, 63].

Since the rough-air soil interface can corrupt the spatial distributions within both B-scans and C-scans, there has been significant research in various statistical methods to detect potential targets within these scan types. Gader et al. have done research in both fuzzy logic [28] and hidden Markov models [27] which both attempt to detect any variations from models of the background soil. Similarly, Potin has experimented with abrupt change detection theory [51], Zoubir has considered Kalman filters [76], Xu has utilized HANOVA tests [70], and multiple groups have looked at generalized likelihood ratio tests [31, 35].

In image-based classification features of the potential target are extracted and analyzed. These features can be based on the target’s shape, its statistics, or its comparison with a template. Depending on the number of features analyzed, the data processing time can increase substantially and therefore significant research has been done in selecting the optimal features. Wilson et al., has described the performance variability of different classification algorithms when applied to a set of experimental data from NIITEK radar. This radar system collects data every 5cm and generates a wideband pulse of 200MHz to 7GHz. Wilson only applied these methods to AT mines, which are considerably larger than the larger humanitarian issue of AP mines. The first algorithm Wilson studied was the hidden Markov model (HMM), which models the time-varying behavior GPR using edge direction information and computes the likelihood that a sequence of measurements is consistent with a buried landmine. The second method extracts geometric features of the GPR data associated with the the given location and applies a feed-forward order-weighted average (FOWA) to discriminate between clutter and potential targets. The third algorithm uses the frequency spectrum of the received signal, extracts features associated with a given location, and formulated a confidence value based on similarity to a collection of features that characterize mine objects. The fourth and final algorithm computes edge histograms from the frequency of occurrence of edge orientations in the data, and then uses a fuzzy K-nearest neighbor (K-NN) algorithm to generate a target confidence level. Wilson’s results concluded that the two edge-based algorithms (EHD and HMM) had the best overall performance, however there was variability in the performances depending on the surveying soil type. Wilson also suggests that a fusion of algorithms could result in better overall performance [68]. However these classification based algorithms
already have an increased complexity for the overall detection system. They are typically researched due to their considerable increase in overall performance.

### 3.2.2 A-Scan Detection

As discussed in Chapter 2, A-Scans are the quickest and simplest form of GPR data, but as such, provide the least amount of information pertaining to the subsurface. However, if the expected response from an object of interest is known, it is possible to create a detection algorithm. Roth et al. considers decomposing the target reflection into its components, by extracting three parameters from each reflection; the magnitude of the reflection off of the top of the object, the magnitude of the reflection from the bottom of the object and the separation in time between these two reflections [56, 57, 58]. The templates for comparison are approximated from the integral representations of the scattered field. The scattering problem is linearized using the Born or Physical Optics approximation (depending on the target casing), and the back-scattered field is approximated with a half-space Green’s tensor [59]. FDTD simulations and measured GPR data in sand has shown successful results; however the experimental results rely on the use of two-dimensional imaging to determine the optimal A-scan for post-processing. Conversely, Kovalenko et al. have studied a searching algorithm which does not require deconvolution but instead bases detection on the minimal discrepancy between the received signal and a given template.

### 3.2.3 Clutter

Clutter is the major limiting factor on the detectability of landmines, since both extraneous scatterers may be buried near the target (i.e. metal fragments, shrapnel, rocks, etc), and the variations in the topology can act as clutter depending on the soil type and excitation frequencies. Soil properties have also been found to change the natural resonance of buried mines, limiting the effectiveness of frequency domain detection [11, 66].

Rappaport et al. have concentrated efforts in understanding and minimizes the effects of the rough air-ground interface. Computational simulations have demonstrated that the distribution of the air-ground interface can increase the difficulty in isolating shallow subsurface objects [23], and that statistics and hypothesis testing can be generated to enhance the detection of small, shallow, low contrast targets [54, 70, 75]. Alternatively, Morgenthaler looks at semi-analytic mode matching (SAMM) algorithm to more easily model and understand the effects of the rough surface on target identification [44]. Other efforts have attempt to minimize the topological clutter by altering the polarization of the excitation.
3.3 GPR Fusion with Other Technologies

In the last decade there has been interest in the idea of using multiple sensors in a single detection system. The majority of these multisensor systems combine GPR with a metal detector and either a visual-light or infrared camera. One disadvantage of fusing technologies is the increased cost of the equipment, making use in poverty stricken countries more difficult. However, this can be acceptable if both detection rates are improved and false alarms are decreased.

Currently there are three such systems in production, the AN/PSS-14, the MINEHOUND, and the Advanced Landmine Imaging System (ALIS); all of which use a metal detector for initial detection and GPR for classification and/or confirmation. The AN/PSS-14 is a handheld sensor which alerts the user with an audio cue when a mine is detected, while also providing visual information. This system generates a terrain model using Principle Component Analysis and adjusts its detection methods according to the varying soil profiles [55, 61]. It was originally called the Handheld Standoff Mine Detection System (HSTAMIDS), and was developed under United States military sponsorship; although there is desire to use the technology for humanitarian demining. However, human error has caused concern in the reliability of this system [32]. Personnel, who were able to perform the various techniques sufficiently well at the end of training, quickly forgot the skills after only 30 days, and most of the retention issues lied with the metal detector techniques. Additionally, this study showed that even at the top of their training, personnel were unable to reliably reject clutter. The MINEHOUND also provides audio cues to the operator, with information pertaining to the size, depth, and symmetry of the target. The output frequency range is 64Hz to 1024Hz, and the volume and frequency of the audio output corresponds to information about the target. The GPR has a bandwidth of 250MHz to 2.5GHz, and each A-scan is analyzed by applying drift compensation and background/clutter removal (the algorithms are proprietary and developed by ERA Technology Ltd.) [20]. Field trials have shown that the MINEHOUND typically reduces false alarm rates by a factor of five [19]. However, the current system performance is only able to reliably locate AP mines up to a depth of 15cm, which does not meet the United Nations standard of 20cm [19]. The ALIS system is also a handheld system which provides the operator with an image of the subsurface. By using a CCD camera, the sensor movement from the ground is tracked allowing for the the GPR scans to be adjusted with a stabilization algorithm. It then uses SAR techniques to provide the operator with an image of a subsurface plane with a potential target. Success has been demonstrated during field tests in mine affected countries. The ALIS system uses a stepped-frequency GPR, which allows for a considerably large bandwidth from 100MHz to 4GHz [62].
3.4 Literature Review Summary

To date the most utilized detection methods remain to be metal detectors and chemical sniffing dogs. However, the substantial number of low-metal and no-metal mines is rendering metal detectors insufficient, and dogs have been shown to be unreliable in clearance probability [50]. Over the past two decades GPR has emerged as a highly promising technique, but questions about how to survey an area, what type of data should be recorded, and how to decipher between threats and clutter still remain. Overall, researchers agree that GPR can provide a faster and more efficient detection modality, but there is plethora of on-going research on the optimal implementation methods. Recording images of the subsurface is the most informative method of GPR, but the added time and computational costs can be a detriment. Additionally, most of these methods ultimately depend on a user to make the final decisions about the subsurface, but this can be unreliable and skill erosion can occur. Furthermore, it requires that a deminer be present in the field during the initial pre-screening process, which can result in demining accidents. The methods presented in this dissertation differ significantly from alternative methods, in that the vehicle, detection, and classification algorithms are all autonomous. Furthermore, few GPR technologies have demonstrated AP mine detection in an experimental setting without the use of an additional technology.
As discussed in the previous chapter, AP mines are typically buried below a rough air-ground interface which presents a major difficulty in detection using GPR, and motivates the use of ground-coupled antennas. If the air-ground interface was flat, the GPR response at the surface would be predictable and consistent. However, since the surface is rough, the reflections become difficult to predict as numerous interactions between the features on the surface occur and cause the target signal to degrade. Macroscopically, one can consider the general shape of the electromagnetic waveform as it propagates through the soil. When a spherical waveform is incident upon an infinitely flat surface, the waveform propagating through the new medium would be approximately hyperbolic; whereas for ground-contact antennas, the expected waveform would be spherical. In the case of mine detection, the rough surface greatly affects the hyperbolic waveform for the air-coupled system, but has a less significant affect on the spherical waveform produced by the ground-coupled system, as seen in Figure 4.1. Additionally, for the same input power, the penetration depth is significantly greater for the ground-coupled case. This conservation of the spherical waveform ameliorates the challenges inherent in a rough surface analysis.

All of these benefits are well-known, but achieving ground-contact of the antennas on a rough terrain without damaging the equipment can be difficult. This chapter discusses how ground-contact is achieved by integrating custom spiral antennas onto the feet of a non-articulated walking robotic platform. Also included in this chapter are the initial testing, calibration techniques, and methods used to improve the received signal from the overall radar system.
Figure 4.1: Three-dimensional comparisons of subsurface wavefronts for a rough air-ground interface generated using FDTD (Chapter 5) when the system is either (a) air-coupled or (b) ground-coupled; Two-dimensional comparison for (c) an air-coupled and (d) ground-coupled system.
4.1 Walking Tri-Sphere

The Tri-Sphere Multi-Mode Mobility Platform seen in Figure 4.2, and referred to as the Walking Tri-Sphere (WTS), is a non-articulated walking robotic platform designed by Square One Systems Design in Jackson Wyoming. The WTS consists of two platforms called the feet, each with three fixed legs called the toes, joined by a manipulator. Each foot can translate and rotate and the WTS walks by lifting, translating, and lowering its two platforms in succession. The length of the robot’s step can be adjusted in real time and its center of mass can be shifted in all directions. Additionally, the WTS is designed with an “adaptive” walking algorithm which automatically adjusts the foot orientation and placement to accommodate obstacles and surface irregularities [21]. These features are ideal for the proposed automated landmine detection system.

![Figure 4.2: Photo of the Tri-Sphere Multi-Mode Mobility Platform designed by Square One Systems Design](image)

Ground-contact antennas can be achieved by integrating antennas into the bottom of each of the toes, and due to the way the robot travels, the antennas endure minimal damage. Furthermore, the non-articulated legs allows for normal contact with the ground. Finally, the ability of the WTS to adjust its step size allows for larger steps to be taken when no potential target is nearby, and for smaller, more cautious, steps to be taken as the robot comes upon a target.

In order to achieve bistatic GPR, the transmitting antenna and the receiving antenna are each incorporated onto a different toe of the WTS, where the distance between the toes determines the bistatic separation. Square One Systems Design is capable of manufacturing the WTS in various sizes, and the dimensions of the specific WTS used for this prototype are shown in Figure 4.3(a). Since the platforms are capable of rotating, the toe separation between adjacent toes can be altered. However, rotation is asymmetric, as seen
in Figure 4.3(b), where the dark green regions indicate the receiver range of the three sets of neighboring toes given that the transmitter is positioned on the platform shown in black. Here, the maximum bistatic separation is limited to 30cm. Due to the asymmetric movements, using antennas on all of the toes would have added complications in programming and two sets of antennas would have limited mobility compared to third. Instead, for this initial prototype, only the pair of toes with the largest range of mobility are utilized. Figure 4.4 shows the possible receiver locations for this pair of toes, given that the transmitter is positioned at the origin. The WTS’s movements are programmed in LabVIEW, and the positions of all of the antennas are known in reference to the center of the top platform. The ultimate goal would be to have the movement algorithms fully automated. However, for the initial experiments presented in this work, smaller programs were written and called by the user. These will be discussed in Section 6.6.
4.2 Antennas

The physical limitations of the antennas have a substantial effect on the overall radar system, and therefore are discussed first. The antennas need to be able to maintain full contact with the ground at each step of the WTS, but also must operate in a frequency range capable of imaging AP mines. This section discusses the antenna design, the initial testing with a vector network analyzer, and how the antennas are mounted onto the WTS.

4.2.1 Spiral Antenna Design

As discussed in Chapter 2, the main design parameter for impulse GPR is the center frequency and bandwidth of the system. Ultimately, a higher frequency yields better subsurface resolution but also results in a shallower imaging depth. As seen in the literature reviews, a typical center frequency for mine detection is near 2GHz; however for this ground-coupled project, a 2GHz center frequency would result in antennas too large to reliably have the entire antenna in contact with the ground. Instead the antennas were designed with a goal frequency of 3GHz to 5GHz, and a maximum diameter of 2cm. The antennas were designed by Dr. Borja Gonzalez-Valdes at Northeastern University, and are shown in Figure 4.5(a). Two-arm cavity backed spiral antennas were used to produce circular polarization and symmetric radiation in the azimuth plane. The main component is an archimedean spiral printed on a dielectric substrate. The spiral is backed...
Figure 4.5: (a) Basic schematic of the two-arm cavity backed spiral antennas; (b) Coaxial cable connection to the arms of the spiral antenna with a metallic cavity that is filled with an absorbent material, to prevent energy from reflecting off of the metallic cavity and corrupting the radiation pattern in the z-direction. A coaxial cable runs through the cavity and dielectric substrate and connects to the two arms of the spiral as shown in Figure 4.5(b). The outer conductor of the coaxial is also connected to the bottom of the metallic cavity. The main design parameters are presented in Figure 4.6 where $D$ is the antenna diameter, $H$ is the antenna height, $h_{\text{coax}}$ is the coaxial connection length, $h_{\text{cav}}$ is the cavity height, $h_{\text{sub}}$ is the substrate height, $h_{\text{rad}}$ is the radome height, and $t_{\text{cav}}$ is the cavity thickness. These parameters and their dimensions are tabulated in 4.1. For the construction, an RG179B/U coaxial cable is used, the absorber is ECCOSORB FDS, the dielectric substrate is Rogers RO3210, and the plastic radome is TBC.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>Value (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Antenna Diameter</td>
<td>20</td>
</tr>
<tr>
<td>$H$</td>
<td>Antenna Height</td>
<td>13.8</td>
</tr>
<tr>
<td>$h_{coax}$</td>
<td>Coaxial Connection Length</td>
<td>10</td>
</tr>
<tr>
<td>$h_{cav}$</td>
<td>Cavity Height</td>
<td>11.0</td>
</tr>
<tr>
<td>$h_{sub}$</td>
<td>Substrate Height</td>
<td>1.8</td>
</tr>
<tr>
<td>$h_{rad}$</td>
<td>Radome Height</td>
<td>1</td>
</tr>
<tr>
<td>$t_{cav}$</td>
<td>Cavity Thickness</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 4.1: Summary of antenna design parameters defined in Figure 4.6

Since the circular polarization of the excitation will change directions upon reflection, the transmitter and receiver must have different handedness. Figure 4.7 shows the two different spiral orientations for right-handed circular polarization (RCP) and left-handed circular polarization (LCP). It should be noted that the original concept for this project was to have spiral antennas which could both transmit and receive both right and left handed polarization, however limitation of the physical antenna diameter prevented this design and ultimately required a designated transmitter and receiver.

![Figure 4.7: Spiral orientations for (a) RCP and (b) LCP polarization](image)

Thus far, only the physical design parameters have been presented. Electrically, there were two things to consider; the radiation pattern, and the impedance. The impedance of the antennas determines the power transmission from the radar to the soil, and the simulated impedance is shown in Figure 4.8. Note that the impedance is not consistent across frequency, meaning that different frequencies will have different coupling into the soil. Additionally, since the antennas were designed to couple to soil, coupling into electrically different mediums, such as free space ($\eta_0 \approx 377$), will be poor.

Finally, Figure 4.9 shows the simulated radiation patterns of the antennas for four different frequencies.
Note that these are the radiation patterns in the elevation plane $\phi = 0$, and that the scale of the dependent axis changes between plots. For all frequencies, the strongest gains occurs at $\theta = 0$, with 1.5dB at 2GHz, 8dB at 3GHz, 9.5dB at 4GHz, and 10dB at 5GHz. Furthermore, in each case the gain decreases as the elevation angle increases, until the point where side lobes occur.

4.2.2 Spiral Antenna Fabrication

Two sets of spiral antennas were fabricated at Square One Systems Design, since mechanical wear at the solder joint between the coaxial cable and the spiral eventually led to failure of the first set of antennas. The cause of the breakdown was identified to be insufficient strain relief at the solder joint. The second set of antennas was manufactured using epoxy within the cable housing, as opposed to silicone caulking, to reduce wear. Variability in the manufacturing process between antenna sets caused a significant change in radiation pattern, with the second set exhibiting decreased anisotropy in the elevation plane. Experimental challenges resulting from this change in radiation pattern are discussed in detail in Chapter 7.

4.2.3 Mounting Antennas onto the WTS

In order to integrate the spiral antennas onto the toes of the WTS, custom mounts were designed and fabricated. Figure 4.10 shows the schematic of the mounting method and photos of the constructed mount.
Figure 4.9: Simulated radiation patterns versus elevation angle for (a) 2GHz, (b) 3GHz, (c) 4GHz, and (d) 5GHz
main body of the toe mount attaches to the WTS via a hollow threaded sleeve, allowing for easy assembly/disassemble. The antenna is then attached to the mount via a ball joint, which allows the antenna to remain in contact with the ground for angles up to 23°. This degree of freedom can be seen in Figure 4.10(c) where the antenna is at an angle compared to the toe mount. Mounted within the toe are three sensors used to detect contact with the ground. The first sensor is triggered once the antenna makes initial contact, as the pressure increases the internal spring (seen in the schematic Fig. 4.10(a)) compresses and activates the second and third sensors, Fig. 4.10(c). These sensors are essential for the WTS to adaptively walk over rugged terrain. The spring requires a force of 2.5lbs to initially move the toe, and 5lbs for full compression. Figure 4.11 also shows the mechanical connections of the WTS toe to the antenna with the cabling and switch locations identified.

Figure 4.10: Mounting design for spiral antennas; (a) Schematic of mounting method; (b) Photo of constructed mount without compression; (c) Photo of constructed mount with compression

Figure 4.11: Image showing the overall WTS toe mechanics and connections
Unfortunately, during field testing it became apparent that the weight of the current prototype caused the WTS toes to sink into the sand, rather than stopping upon contact with the surface. Ultimately, a smaller, lighter WTS would be used, but for these experiments a temporary solution was required. In order to circumvent this issue, weight distributors were designed to be fitted between the first joint and the top of the antenna housing. These distributors were designed in AutoCAD and fabricated on a 3D printer, Figure 4.12. The implementation of the distributors did not affect the antenna radiation characteristics, but did prevent the originally designed degree of angular freedom which allowed the antenna housing to rotate on the ball joint.

![Figure 4.12: 3D printed weight distributors designed to assist the WTS in traversing sand](image)

4.2.4 Test Antenna

In some experiments the commercially available antennas were used instead of the custom spiral antennas. These planar elliptical dipole antennas (Fig. 4.13) were purchased from Time Domain located in Huntsville Alabama. The Time Domain BroadSpec antennas are omni directional in the azimuth plane to within ±1.5dB and are rated with a return loss of approximately -12dB.
## 4.3 Radar System

A P400 ultra-wideband impulse radar system was also purchased from Time Domain, Fig. 4.14. This purchase was dictated by the operating frequencies of the antennas, and the necessity for a physically small system capable of battery operation. The P400 is lightweight with a physically small footprint (3in x 3in), operates from 3.1 to 5.3GHz, and is battery operated. Additionally, unlike some similar radar systems the P400 is designed with the following specifications:

<table>
<thead>
<tr>
<th>Specification</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions</td>
<td>76mm x 80mm x 16mm</td>
</tr>
<tr>
<td>Temperature Range</td>
<td>-10° to 60°</td>
</tr>
<tr>
<td>Power Requirements</td>
<td>5.75 - 30V @4.0 Watts max</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>3.1GHz to 5.3GHz</td>
</tr>
<tr>
<td>Center Frequency</td>
<td>4.3GHz</td>
</tr>
</tbody>
</table>

Figure 4.14: Photo of the P400 radar (courtesy of Time Domain) and specifications
P400 does not require an external data acquisition method. Instead it has two methods of interfacing with a computer directly, USB and Ethernet (Fig. 4.15).

![Image of P400 interface](image)

Figure 4.15: Various inputs and LEDs on the P400 (Photo courtesy of Time Domain)

The P400 radar has coherent operation, which maintains the phase information for each pulse. Therefore, numerous responses (of the same scenario) can be summed, and since the received power is proportional to the square of the received voltage, summing pulse voltages means the power increases at the square of the number of pulses. Conversely, noise is incoherent and will only sum linearly, leading to an improved signal-to-noise ratio (SNR). Ultimately, each doubling of integration increases SNR by 3dB. One effect of increasing the SNR is that smaller signals can be reliably detected, and the dynamic range of the system is similarly increased. One downfall of coherent operation is the cost of increased circuit complexity. In order to maintain coherence, the system timing must be held to a small fraction of the waveform’s period, otherwise the signal will start to decorrelate and the benefits of integration will be lost. This requires that timing accuracy be held to better than 10 picoseconds. Another downfall is that the greater integration rates require longer amounts of time per scan [49]. Therefore, if surveying speed is of interest, the integration rate should be selected based on the lowest acceptable SNR. In order to determine the optimal integration factor for this project, a radar scan of a buried target using the spiral antennas was recorded at an integration factor of 64, Fig. 4.16(a), and at 16,383 Fig. 4.16(b). In both cases the strong signals are easily distinguishable, and at the weaker signal strengths the received signal using a lower integration factor, visibly has a lower SNR compared with the scan recorded using a higher integration factor. However, for this project, surveying speed is of high importance and the SNR at the lower integration is acceptable. Finally, it should be noted that in Fig. 4.16, as well as all of the figures related to GPR scans, the signal strength is shown in unscaled volts.
Included in the design of the P400 was a transmitter/receiver switch. The purpose of this switch was to allow the user to change which antenna is the transmitter, without physically switching the antennas. However, due to reciprocity and the nature of GPR the switch was unnecessary for this project. Additionally, the switch causes the transmitting and receiving ports to not be completely isolated, meaning that energy delivered to the transmitter is also leaked to the receiver and vice-versa (Fig. 4.17). Therefore, at request, the switch was removed by Time Domain.

Figure 4.16: Comparative example demonstrating the results of coherent operation for the same GPR scan; (a) pulse integration factor of 64; (b) pulse integration factor of 16,383

Figure 4.17: Comparison of the received signal with and without the T/R switch when the antennas were isolated from one another
4.3.1 Circulator

A circulator is a passive, nonreciprocal device, typically with 3 or 4 ports, in which microwave frequencies entering at one port are transmitted to the next port in rotation and no others. These devices are often used in radar to route outgoing and incoming signals between the transmitter and receiver. Isolation from alternative ports is achieved by using a magnetized ferrite in the center of the circulator. The ferrite material is essentially enclosed in a cavity forming a resonator. When power is applied from one of the ports a standing wave pattern is established. In the case of a 3-port circulator, when the ferrite is unmagnetized the resulting standing wave delivers power to the other ports equally. However, when the ferrite is magnetized, the result is a standing wave pattern which is rotated $30^\circ$, such that full power is delivered to one port and the other port is positioned on a null of the standing wave. The direction that the standing wave rotates is dictated by the relationship between the magnetic biasing field and the ferromagnetic resonance field.

Since the P400 does not have sufficient isolation between the antenna ports, a three-port circulator was used to prevent received energy from being coupled to the transmitting port. To do this, one port of the circulator was connected to the receiving port of the P400 and the coupled port was connected to the receiving antenna. The third port was terminated with a matched load ($50\Omega$). The specifications for the circulator used are listed in Table 4.2.

<table>
<thead>
<tr>
<th>CirculatorSpecifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Number</td>
</tr>
<tr>
<td>Dimensions</td>
</tr>
<tr>
<td>Temperature Range</td>
</tr>
<tr>
<td>Operational Frequency Range</td>
</tr>
<tr>
<td>Insertion Loss</td>
</tr>
<tr>
<td>Isolation</td>
</tr>
</tbody>
</table>

Table 4.2: Specifications for the 3-port circulator

measuring the transmission coefficients between ports. These measurements are shown in Figure 4.18.

4.3.2 Cables and Impedance Matching

Ultimately, any change in impedance causes a reflection, including changes in the cable impedances, which can cause difficulty in post-processing. To avoid superfluous reflections, the impedances of the cables, connectors, and antennas should be matched. However, the P400 transmitting and receiving ports have an impedance of $50\Omega$, while the antennas have an impedance of approximately $75\Omega$ for coupling into the soil, Fig. 4.8. Figure 4.19 shows a block diagram of the complete radar system and the various impedances.
Figure 4.18: Transmission coefficients between various ports of the circulator throughout. There are two RG174 coaxial cables ($C$ and $H$) which are used as extension cables from the radar location on the platform to the toes of the WTS, followed by two RG178 coaxial cables ($D$ and $G$) used in the antenna construction. All of the adapters are 50$\Omega$. Overall, there are known reflections at the SMA/SMB adapters and at the interfaces between the antennas and the ground. It is possible to match the radar and antennas using additional hardware, however after analyzing the experimental data it was determined that the strongest reflections could be removed via time-gating. Later reflections were weak and therefore did not affect the results for the data presented. It is noted, however, that future versions of the prototype should include a balun to limit these superfluous reflections. Finally, it should be noted that in early experiments, the required length of the extension cables was unknown and therefore varied between experiments. These changes in cable length shift the absolute time of the target reflection.

Figure 4.19: Block diagram of the complete radar system and the component impedance values, note all of the adapters have $\eta = 50\Omega$
4.4 Hardware Testing and Calibration

Before experimenting with target detection, various experiments were performed to gain a better understanding of the radar system and spiral antennas. From these results, small programs were created to improve the collected radar signals.

4.4.1 Time Gating On-Board Chatter

Despite adding a circulator to the receiving port of the P400, there remained some “chatter” between the transmitting port and the receiving port when the excitation pulse is transmitted. This chatter is strong in amplitude, as seen in Figure 4.20 where the received chatter and the received signal from a metallic plate have comparable amplitudes. However, since the chatter also occurs at the same absolute time for every scan, regardless of the type of cables and antennas connected, and regardless of the area being surveyed, it is simply removed through time-gating.

![Figure 4.20: Comparison of the on-board chatter between the transmitter and receiver and a metallic plate reflection](image)

4.4.2 Increasing Sampling Rate

The P400 uses a sampling rate of approximately 16.4GHz, and although this satisfies the Nyquist rate, the resulting received signals are not ideal for certain time-of-flight techniques which will be discussed in Chapter 6. Therefore, to increase the sampling rate, the precise system clock of the P400 is utilized. First, a single GPR scan starting at 0ns is recorded using the 16.4GHz sampling rate. Next, GPR scans are recorded starting at 6ps, 12ps, 18ps, 24ps, etc.; ultimately resulting in 10 different GPR scans which are interlaced and smoothed using Eq. 4.1 with $M = 3$.

$$x[i] = \frac{1}{2M+1} \sum_{j=-M}^{M} x[i+j]$$

(4.1)
Using this method, the sampling rate is improved by a factor of 10, to 164GHz. A comparison of the original signal, and the smooth signal is shown in Figure 4.21.

![Comparison between a received signal using the original sampling rate of the P400 (blue) and the programmed sampling rate (black)](image)

Figure 4.21: Comparison between a received signal using the original sampling rate of the P400 (blue) and the programmed sampling rate (black)

### 4.4.3 Optimizing the Transmit Gain

The P400 has 63 different transmit gain settings. For the lowest setting, the power delivered to the antenna port is -14.5dBm and at the maximum setting it delivers 2.1dBm. In order to more easily locate plastic targets, a higher gain is desired; however, in certain cases this could cause the received signal to clip resulting in a loss of information concerning the target reflection. In order to optimize the received signal, a gain-feedback loop was implemented. The feedback loop starts at the maximum transmit gain; if the received signal is clipped then the transmit gain is reduced and the area is rescanned. This process is repeated until the received signal no longer exhibits clipping or until the transmit gain is at the minimum setting. The final scan is then scaled to correspond with maximum gain setting. Note, that in order to not have clipping from a metallic target at the minimum transmit gain, a 3dB attenuator was required. Figure 4.22(a) shows the peak values of a target reflection using each gain setting and the attenuator, and Figure 4.22(b) shows the scaling factors to normalize all of the gain settings to the maximum setting. A comparison of the target response for each gain setting before and after correcting for the transmit gain is shown in Figure 4.23.
Figure 4.22: (a) Plot of the peak target responses verses gain index; (b) Plot of the scale values needed to adjust all gain index values to the maximum gain setting

Figure 4.23: Target reflection for different gain values; (a) Raw data; (b) Gain corrected data
4.4.4 Known Termination Study

In order to determine the behavior of the received signal in relation to impedance mismatch, impedance stubs from a VNA calibration kit were used to analyze known impedance terminations. Figures 4.24(a) and 4.24(b) show the results when the transmitting cable and receiving cable are each loaded with the same termination type (short circuit, open circuit, or matched load). Note that the circulator was not used and that the transmitting and receiving cable differed in length by 2 feet. As expected, the reflections from the matched loads are small in magnitude and the short circuit and open circuit terminations result in reflections which are equal in amplitude and 180 degrees out of phase with one another.

Furthermore, for each signal, there are four distinguishable reflections at 34, 36, 58 and 63ns. Interestingly, the earliest two reflections are related to the transmitting and receiving cable. This was verified by terminating the transmitting cable with a matched load and changing the termination on the receiving cable, thereby isolating the reflections caused by the load on the receiving cable (Fig. 4.24(c)). Therefore, the reflections in Figure 4.24(b) are from both the receiving cable termination and the transmitting cable termination and are of the same order of magnitude, indicating that the excitation is being delivered to both ports of the P400 demonstrating the necessity for the circulator. The later two reflections, simply correspond to a back-and-forth between the termination and the radar for both the transmitting and receiving cables.
Figure 4.24: (a) Entire received data when both cables are loaded with the terminations indicated in the legend; (b) Portion of received data when both cables are loaded with the terminations indicated in the legend; (c) Portion of received data when the transmitting cable is terminated with a matched load and the termination on the receiving cable is altered
4.4.5 P400 Excitation Signal

Lastly, the excitation signal was analyzed by connecting the transmitting port directly to the receiving port using the coaxial cables, attenuator, and circulator, such that only the effects of the antennas were not accounted. The received signal is shown in Figure 4.25(a), and the frequency characteristics of this signal are shown in Figure 4.25(b). Note that in 4.25(a) there is an initial received signal which occurs between 1.5 and 3ns, and then there is a secondary response occurring between 3.5 and 4.5ns. The first is considerably stronger in magnitude, but the second impulse must be considered as it will cause secondary reflections. Additionally, the first major response has a width of approximately 1.5ns resulting in an approximate range resolution of 22cm in air, 14cm in sand ($\epsilon_r = 2.7$, $\sigma = 0.01$), and 10cm in soil ($\epsilon_r = 5$, $\sigma = 0.2$).
CHAPTER FIVE

COMPUTATIONALLY MODELING GPR IN DISPERSIVE SOIL

Computational data can be very useful for quickly and effectively comparing different experimental scenarios. In this project both experimental and computational data is presented, where the computational data is acquired by use of a three-dimensional finite-difference time domain (FDTD) model. This chapter reviews Kane Yee’s original derivation of the FDTD model for nondispersive materials, and the various techniques used to accurately model GPR. At the time of Yee’s derivation in 1966, the FDTD method did not warrant much attention due to the inadequacy of computational power. However, in the following decades, as computational power improved, the FDTD method became substantially more common and research modeling of more complicated materials began. It wasn’t until the early 1990s, nearly three decades after Yee’s derivation, that methods of modeling dispersive media were realized. The earliest methods involved using either a Debye or Lorentz media. The Debye model defines the relative permittivity as a complex-valued frequency-domain function with one or more real poles at separate frequencies, whereas the Lorentz model defines the same parameter with one or more pairs of complex-conjugate poles. Both of these models require a piecewise-linear recursive-convolution method to resolve the fields [64]. In this research an alternative method is employed called the four-zero technique developed by Rappaport et. al [67].

5.1 Modeling Electromagnetic Propagation in Dispersive Materials

The basis of modeling electromagnetic phenomenon are Maxwell’s equations which were discussed in Chapter 2 and are reiterated below.
\[ \nabla \times E = -\frac{\partial B}{\partial t} \]  
\[ \nabla \times H = \frac{\partial D}{\partial t} + J_c + J_s \]  
\[ \nabla \cdot B = 0 \]  
\[ \nabla \cdot D = \rho \]  

where the electric flux density \( D \) (C/m³), magnetic flux density \( B \) (T), and induced source current \( J_c \) (A/m²) are related to \( E \) and \( H \) by the constitutive relations (Eq. 5.2) which describe a material’s response to electromagnetic waves.

\[ D = \epsilon \ast E \]  
\[ B = \mu \ast H \]  
\[ J_c = \sigma \ast E \]

5.1.1 The Finite-Difference Time Domain Method

After Maxwell’s work, many analytical methods were employed to fully understand electromagnetic propagation. However, for many practical problems the mathematics is considerably complex. Soon after the development of computers, scientists and engineers began to research methods of computationally analyzing electromagnetic fields. Today, the most prevalent method is the finite-difference time-domain (FDTD) method which is a direct solution to the differential form of Maxwell’s equations and was first introduced by Kane Yee in 1966 [73]. In the original derivation only nondispersive, linear, homogeneous, materials were considered for which the convolutions in the constitutive equations (Eq. 2.2) are simplified to multiplications. Using these relations, the curl in both Faraday’s law (Eq. 2.1a) and Ampere’s law (Eq. 2.1b) can be expanded into six differential equations shown in Eq. 5.3.
\[
\frac{\partial H_x}{\partial t} = \frac{1}{\mu} \left( \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \right) \tag{5.3a}
\]
\[
\frac{\partial H_y}{\partial t} = \frac{1}{\mu} \left( \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right) \tag{5.3b}
\]
\[
\frac{\partial H_z}{\partial t} = \frac{1}{\mu} \left( \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right) \tag{5.3c}
\]
\[
\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon} \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x \right) \tag{5.3d}
\]
\[
\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon} \left( \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y \right) \tag{5.3e}
\]
\[
\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon} \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z \right) \tag{5.3f}
\]

To discretize these equations Yee defined what is now referred to as the standard Yee Cube, Fig. 5.1. Here, the magnetic and electric fields are offset, such that every electric field component is surrounded by four circulating magnetic field components and vice versa. Yee used the notation in Eq. 5.4 to define a function

\[ u_{i,j,k}(\Delta x, \Delta y, \Delta z, n\Delta t) = u_{i,j,k} \]

Figure 5.1: Positions of the electric and magnetic field components on a unit cell of the Yee cube

In order to discretize the differential equations in Eq. 5.3, Yee used central finite-difference approximations in both space (Eq. 5.5a) and time (Eq. 5.5b).
\[
\frac{\partial U_{i,j,k}^n}{\partial x} = \frac{U_{i+1/2,j,k}^n - U_{i-1/2,j,k}^n}{\Delta x} + O(\Delta x^2) \quad (5.5a)
\]

\[
\frac{\partial U_{i,j,k}^n}{\partial t} = \frac{U_{i,j,k}^{n+1/2} - U_{i,j,k}^{n-1/2}}{\Delta t} + O(\Delta t^2) \quad (5.5b)
\]

Using the finite-difference approximations and the above notations, the six differential equations from Maxwell’s equations (Eq. 5.3) can be discretized. For simplicity, these discretized equations are shown in Eq. 5.6 assuming that the spatial step sizes in the x, y, and z directions are equal (\(\Delta x = \Delta y = \Delta z = \Delta\)), and that the material properties (\(\mu, \epsilon, \sigma\)) are homogeneous.

\[
H_{x(i+1/2,j,k+1/2)}^{n+1/2} = H_{x(i,j+1/2,k+1/2)}^{n-1/2} + \frac{\Delta t}{\mu \Delta} \left( E_{y(i,j+1/2,k+1)}^n - E_{y(i,j,k+1/2)}^n - E_{z(i,j,k+1/2)}^n - E_{z(i,j+1,k+1/2)}^n \right) \quad (5.6a)
\]

\[
H_{y(i,j+1/2,k+1/2)}^{n+1/2} = H_{y(i,j+1/2,k+1/2)}^{n-1/2} + \frac{\Delta t}{\mu \Delta} \left( E_{z(i+1,j,k+1/2)}^n - E_{z(i,j,k+1/2)}^n - E_{x(i+1/2,j,k)}^n - E_{x(i,j+1/2,k)}^n \right) \quad (5.6b)
\]

\[
H_{z(i+1/2,j,k+1/2)}^{n+1/2} = H_{z(i,j+1/2,k+1/2)}^{n-1/2} + \frac{\Delta t}{\mu \Delta} \left( E_{x(i+1/2,j,k+1)}^n - E_{x(i+1/2,j,k)}^n - E_{y(i,1/2,j,k)}^n - E_{y(i+1/2,j,k)}^n \right) \quad (5.6c)
\]

\[
E_{x(i+1/2,j,k)}^{n+1} = \left( 1 - \frac{\sigma \Delta t}{\epsilon} \right) E_{x(i+1/2,j,k)}^n + \frac{\Delta t}{\epsilon \Delta} \left( H_{x(i+1/2,j,k+1/2)}^{n+1/2} - H_{x(i+1/2,j,k)}^{n+1/2} - H_{x(i,j+1/2,k-1/2)}^{n+1/2} + H_{x(i,j+1/2,k+1/2)}^{n+1/2} \right) \quad (5.6d)
\]

\[
E_{y(i,j+1/2,k)}^{n+1} = \left( 1 - \frac{\sigma \Delta t}{\epsilon} \right) E_{y(i,j+1/2,k)}^n + \frac{\Delta t}{\epsilon \Delta} \left( H_{y(i,j+1/2,k+1/2)}^{n+1/2} - H_{y(i,j+1/2,k)}^{n+1/2} - H_{y(i,j+1/2,k-1/2)}^{n+1/2} + H_{y(i,j+1/2,k+1/2)}^{n+1/2} \right) \quad (5.6e)
\]

\[
E_{z(i,j+1/2,k)}^{n+1} = \left( 1 - \frac{\sigma \Delta t}{\epsilon} \right) E_{z(i,j+1/2,k)}^n + \frac{\Delta t}{\epsilon \Delta} \left( H_{z(i,j+1/2,k+1/2)}^{n+1/2} - H_{z(i,j+1/2,k)}^{n+1/2} - H_{z(i,j+1/2,k-1/2)}^{n+1/2} + H_{z(i,j+1/2,k+1/2)}^{n+1/2} \right) \quad (5.6f)
\]
5.1.2 Modeling Dispersive Soil

In Yee’s derivation, the convolution in the constitutive relations (Eq. 2.2) was simplified by assuming that the material properties were independent of frequency. However, certain materials are dispersive, and in order to obtain accurate results, the convolutions cannot be ignored. One method for modeling frequency-dependent materials is the Four-Zero Conductivity Model developed by Rappaport [67]. This method uses a Padé approximation of the complex frequency-dependent conductivity in Z-transform space, such that \( \sigma \) can be expressed as

\[
\sigma(Z) = Z^{1/2} \frac{b_0 + b_1 Z^{-1} + b_2 Z^{-2} + b_3 Z^{-3}}{1 + a_1 Z^{-1}} \tag{5.7}
\]

where \( Z = e^{j\omega \Delta t} \) and \( Z^{-1} \) is a time delay of \( \Delta t \). Using this equation, the difference representation of Eq. 2.2c can be written as

\[
J^n - \frac{1}{2} + a_1 J^{n-3/2} = b_0 E^n + b_1 E^{n-1} + b_2 E^{n-2} + b_3 E^{n-3} \tag{5.8}
\]

Similarly, the dispersive FDTD form of Ampere’s law becomes

\[
\nabla \times \left( H^n - \frac{1}{2} + a_1 H^{n-3/2} \right) = \left( \frac{\varepsilon_a}{\Delta t} + b_0 \right) E^n + \left( a_1 - 1 \right) \frac{\varepsilon_a}{\Delta t} + b_1 \right) E^{n-1} + \left( -a_1 \frac{\varepsilon_a}{\Delta t} + b_2 \right) E^{n-2} + b_3 E^{n-3} \tag{5.9a}
\]

To determine the coefficients in Eq. 5.7, the real part of \( \sigma \) is fitted to measured conductivity, and the imaginary part of the loss tangent (\( \frac{\sigma}{\omega\varepsilon} \)) is fitted to the measured dielectric constant. The model was then optimized [67] by equating the conductivity and dielectric constant at three representative frequencies to the measured data and using an initial guess for \( a_1 \). Note that although the relative permittivity was defined to be real and constant, the actual permittivity of the medium is dependent on frequency as the complex permittivity and the conductivity are related by Eq. 5.10

\[
\varepsilon(Z) = \varepsilon_0 \left( \varepsilon_{avg} - j \frac{\sigma(Z)}{\omega\varepsilon_0} \right) \tag{5.10}
\]

5.1.3 Stability Constraints

When implementing an FDTD algorithm, the spatial step size and temporal step size are limited by two factors. First in order to properly simulate wave propagation, the spatial increments must be small compared to the wavelength. Typically a minimum of 10 points per wavelength is used. Second, to ensure stability,
the permissible temporal step size and the spatial step size are related and limited by the speed of the
electromagnetic wave. Traditionally, these dimensions must satisfy the Courant condition shown in Eq. 5.11, where \( c_0 \) is the speed of light in free space.

\[
\frac{c_0 \Delta t}{\sqrt{\epsilon_r}} \leq \left( \sqrt{\left( \frac{1}{\Delta x} \right)^2 + \left( \frac{1}{\Delta y} \right)^2 + \left( \frac{1}{\Delta z} \right)^2} \right)^{-1}
\]  

(5.11)

However, this condition is not satisfactory for the Four-Zero Model of dispersive materials. In this case, the three dimensional dispersive stability condition, when \( \Delta x = \Delta y = \Delta z = \Delta \) is,

\[
\epsilon_{Av}(Z - 1) \left( e_0 + e_1 Z^{-1} + e_2 Z^{-2} + e_3 Z^{-3} \right) + \frac{12 c_0^2 \Delta t^2}{\epsilon_{Av} \Delta^2} = 0
\]

(5.12)

where,

\[
e_0 = 1 + \frac{b_0 \Delta t}{\epsilon_0 \epsilon_{Av}}
\]

\[
e_1 = a_1 - 1 + \frac{b_1 \Delta t}{\epsilon_0 \epsilon_{Av}}
\]

\[
e_2 = -a_1 + \frac{b_2 \Delta t}{\epsilon_0 \epsilon_{Av}}
\]

\[
e_3 = \frac{b_3 \Delta t}{\epsilon_0 \epsilon_{Av}}
\]

and \( b_0, b_1, b_2, b_3 \), and \( a_1 \) are the coefficients from Eq. 5.7. The numerator of this stability condition is a fourth order polynomial in \( Z^{-1} \), and the solution yields four roots \( Z_{\alpha} \), where \( \alpha = 1, 2, 3, 4 \). Numerical stability requires that all four roots lie within the unit circle, such that \( |Z_{\alpha}| < 1 \). For each dispersive material, the temporal step size, \( \Delta t \), is established when the coefficients \( b_0, b_1, b_2, b_3 \), and \( a_1 \) are fitted to the measured conductivity. Therefore, for each material, the minimum spatial step size, \( \Delta \), must be determined such that \( |Z_{\alpha}| < 1 \) is met. Generally, this minimum value is larger than the value obtained from the traditional Courant condition for the same medium and time step.

### 5.2 Modeling the WTS Prototype on a Rough Air-Ground Interface

The previous section discussed how electromagnetic waves in dispersive material can be modeled via a 3D FDTD algorithm. This section discusses the methods involved in modeling the designed spiral antennas, as well as a computational region which emulates a landmine buried below a rough surface.
5.2.1 Modeling Circular Polarization

In order to enhance the target reflection, the pulse is polarized with right-hand circular polarization (RCP) and the receiver is polarized with left-hand circular polarization (LCP). However, this cannot be achieved within the FDTD model directly. Instead, two out-of-phase orthogonal dipoles are used, as shown in Figure 5.2. The handedness of the polarization is determined by the leading component. For RCP the $\vec{S}_2$ component will lag the $\vec{S}_1$ component by 90°. The total transmitted field is then $\vec{E}_T = E_{Tx}\hat{x} + E_{Ty}\hat{y}$ where

$$E_{Tx} = (\vec{S}_1 - j\vec{S}_2) \cdot \hat{x} = S_{1x} - jS_{2x}$$

$$E_{Ty} = (\vec{S}_1 - j\vec{S}_2) \cdot \hat{y} = S_{1y} - jS_{2y}$$

Next in order to model the received data for either an RCP or LCP antenna, the total transmitted field is multiplied by the polarization loss factor. For RCP the result is,

$$R_{RCP} = \frac{\hat{x} + j\hat{y}}{\sqrt{2}} \cdot \vec{E}_T$$

$$R_{RCP} = \frac{1}{\sqrt{2}} (S_{1x} - jS_{2x} + jS_{1y} + S_{2y})$$

Similarly, for receiving with an LCP antenna, the result is

$$R_{LCP} = \frac{\hat{x} - j\hat{y}}{\sqrt{2}} \cdot \vec{E}_T$$

$$R_{LCP} = \frac{1}{\sqrt{2}} (S_{1x} - jS_{2x} - jS_{1y} - S_{2y})$$

To evaluate the efficacy of this post-processing technique, two test simulations were evaluated. The first was simply a transmitter and receiver in free-space, as shown in Figure 5.3(a). For this simulation, using a transmitted RCP pulse, the received data was expected to have full power when matched to an RCP receiver and to receive zero power when received by an LCP antenna.

![Figure 5.2: Dipole excitations for modeling circular polarization](image)
The second simulation examined the same antenna placement in free space, but with a scatterer placed some arbitrary distance from the antennas, as shown in Figure 5.3(b). The goal of this simulation was to assure that the target had no effect on the received RCP data and only appeared in the LCP data, since a reflection causes the handedness of the polarization to change. The results for both test simulations are shown in Figure 5.4, and in both cases the expected performance was achieved. For the RCP receiver the background simulation and the target simulation are nearly indistinguishable, whereas with the LCP receiver a distinct reflection from the target is present. This example also demonstrates how using circular polarization enhances target reflection while suppressing direct signals. This technique will not be as effective when examining GPR data due to the roughness of the soil, but will assist in enhancing the target reflection.

5.2.2 Modeling the Antenna Radiation Pattern

In addition to modeling the polarization of the antennas, modeling the radiation pattern of the spiral antennas is imperative in order to have a consistent comparison to the experimental performance. Again this required two simulations which were then altered in post-processing. First, consider Figure 5.5, which shows the three signals received in a typical ground-coupled simulation. There is the direct signal through air, A; the direct signal through ground, B; and the target reflection, C. Since the spiral antennas are not omnidirectional in the elevation plane, the power radiating in the directions of signals A and B will be substantially weaker than the signal radiating toward the target. Therefore, these portions of the received signal must be treated differently. To do this, the exact background response is simulated so that the target reflection can be determined using background subtraction. Next, using the known positions of the antennas and the exact
Figure 5.4: Comparison of received data the geometries shown in Figure 5.3 when the transmitted pulse is RCP and the receiving antenna is polarized for RCP (a) versus LCP (b).

Figure 5.5: (a) Schematic overview of signal pathways in a typical GPR simulation; (b) Illustration depicting geometric variables used to determine the antenna gain in the direction of the target.
position of the target, the elevation angle from the transmitter to the center of the target \((\theta_E)\) and the angle from the receiver to the center of the target \((\theta_R)\) are determined, Fig. 5.6. These angles can then be used to determine the corresponding gains \((G_E\) and \(G_R)\) in the direction of the target using the simulated radiation pattern shown in Figure 4.9(c). In accordance with the Friis transmission equation, the extracted target signal is then multiplied by the product of the transmit and receive gains, resulting in a target reflection with a signal strength related to the antenna radiation patterns. Similarly, the angle between the antennas can be determined and the exact background response can then be multiplied by the product of the gains for each antenna, resulting in a background signal which has a strength corresponding to the antenna radiation patterns. The final received signal is the superposition of the new background signal and the new target signal. Comparisons of the original signals and those adjusted to account for the applied radiation pattern for different excitations are shown in Figure 5.7.

![Diagram showing the elevation angle from the transmitter to the center of the target and the angle from the receiver to the center of the target.](image)

Figure 5.6: Example demonstrating how \(\theta_E\) and \(\theta_R\) relate to the antenna radiation patterns to determine \(G_E\) and \(G_R\): Note that these depictions are meant to serve as visual aids, and as such, are not to scale. The actual radiation patterns are shown in Figure 4.9(c)

Figure 5.7: Example of results from modeled antenna radiation pattern for the PulsON excitation
5.2.3 Modeling a Rough Surface

Realistically, the topography of soil does not follow a single statistical pattern of roughness. The ground may vary more quickly at one location and less quickly at another, therefore two functions are used to fully characterize the distribution of the random rough surface. These functions are a probability density function and an auto-correlation function [24]. The random surface heights were created using the probability density function for a Gaussian distribution (Eq. 5.17) with zero mean and a standard deviation, $\sigma_h$.

$$p(z) = \frac{1}{\sigma_h \sqrt{2\pi}} \exp \left( -\frac{z^2}{2\sigma^2_h} \right) \quad (5.17)$$

This function provides information about the height of the hills and valleys along the surface, but does not provide information about the distances between the hills and the valleys. These distances are determined by the autocorrelation function (Eq. 5.18) where $l_c$ is the correlation distance for which $R(x_d)$ will drop to the value $e^{-1}$ and $x_d$ is the distance between random heights at two different points on the surface.

$$R(x_d) = \exp \left( -\frac{x^2_d}{l^2_c} \right) \quad (5.18)$$

Figure 5.8 shows a comparison between two rough surfaces generated with this method. In 5.8(a), $\sigma_h = 2cm$ and $l_c = 10cm$, whereas in 5.8(b), $\sigma_h = 3cm$ and $l_c = 3cm$. These examples help to visualize the effect of these parameters, where the correlation distance determines the heights between the peaks and troughs, and the standard deviation determines the sizes of the peaks and troughs. Note that these images also depict a buried cylindrical target corresponding to a potential AP mine position with a 10cm diameter.

![Figure 5.8: Comparison between two computationally modeled rough surfaces; (a) $\sigma_h = 2cm$ and $l_c = 10cm$; (b) $\sigma_h = 3cm$ and $l_c = 3cm$](image)
The various ways in which GPR is used for mine detection were discussed in Chapter 3. Many of these techniques rely on an educated human deminer to decipher between potential targets and clutter. Conversely, this work considers automated GPR algorithms in order to produce an unmanned detection vehicle. This automation was divided into two distinct stages; pre-screening and localization. The pre-screening stage, also referred to as the initial detection stage, is used for canvassing a large area and determining which parts require closer consideration. This needs to be done quickly and with a high sensitivity for potential threats. The algorithm thus requires an optimal search pattern which minimizes the number of GPR scans and WTS movements, but also does not miss a potential target. When a potential target is identified, the second algorithm which handles target localization is called. During this stage, additional scans are recorded in order to determine whether or not the target should be considered a potential threat, and to determine the target’s approximate location. Both of these programs require knowledge about a potential target’s reflection, and the localization algorithm requires a fast and simple technique for determining a target’s position from the time-of-flights. This chapter discusses these considerations.

6.1 A-scan Data Processing

As was discussed in Chapter 2, a typical A-scan from a ground-coupled GPR system is comprised of the direct signals through air and soil, and the target reflection. In the presence of additional scatterers, such as clutter, the scattered signals may interfere with the target reflection. Often, the first step in determining the time-of-flight to a target is to isolate the target reflection. Recall that the pulse width of the P400
is 1.5 ns, corresponding to approximately 28 cm in sand and 20 cm in soil. Given a maximum bistatic of 30 cm, and a potential target buried at 5 cm, the difference in travel path distance would be only \( \approx 16 \) cm. Therefore, interference between the direct communication and target reflection is inevitable. However, as will be discussed more in the following chapter, due to the antennas’ radiation patterns, the direct signals are considerably weaker compared to a well-positioned target and background removal is not necessarily required for accurate positioning.

### 6.1.1 Background Removal

In traditional GPR applications, background removal is one of the most effective techniques for removing the direct signals (and for air-coupled systems the initial reflections). Consider two different received signals, one in which the target is present, \( y(t) \), and one in which the target is not present, \( b(t) \), then the scattered signal created by the target of interest is simply \( s(t) = y(t) - b(t) \) as shown in Figure 6.1. Experimentally

![Figure 6.1: Background removal of a rough surface](image)

(a) Field data with target present (b) Background field data (c) Field obtained using background removal, (d,e,f) Corresponding receiver data

the exact background is usually not known, but the average of numerous scans from a representative area can often be used to create an approximate background. This is not the case for a rough air-ground interface where estimating the background signal can be extremely difficult; though previous computational work
demonstrated that the background for ground-contact antennas on a rough surface can be approximated statistically using an initial approximation [33]. For this project background removal was not necessary due to the radiation characteristics of the antennas; however this process is used in creating the computational received signals and in analyzing the computational data where the exact background can be simulated.

6.1.2 Deconvolution

Many landmine detection methods use inverse methods, or deconvolution methods, in order to isolate the reflection of the potential target. Given an excitation, \( x(t) \), and the received GPR signal, \( y(t) \), the reflectivity series, \( h(t) \), that is the distances and reflection strengths off all targets, are related as \( y(t) = x(t) \ast h(t) \). Using this information, the reflectivity series can be determined in the frequency domain as,

\[
H(\omega) = \frac{Y(\omega)}{X(\omega)}
\]  

(6.1)

Directly evaluating for \( H(\omega) \) is difficult, since the result will be undefined for frequencies not in the excitation \( (X(\omega_0) = 0) \), but there are various ways to cope with this issue. The two methods considered here are regularization and Weiner filtering.

Regularization

There are various regularization techniques, but all operate on a similar principle; for frequencies where \( X(\omega) \) is small, Eq. 6.1 is evaluated using a substitute equation. One such method is water level regularization which is often used in seismology and evaluates \( H(\omega) = Y(\omega) / \hat{X}(\omega) \) where,

\[
\hat{X}(\omega) = \begin{cases} 
X(\omega), & |X(\omega)| > w \\
\frac{wX(\omega)}{|X(\omega)|}, & 0 < |X(\omega)| \leq w \\
w, & X(\omega) = 0 
\end{cases}
\]  

(6.2)

and \( w \) is a user defined value related to the magnitude of the excitation in the frequency domain. Note that as \( w \to 0, \hat{X}(\omega) \to X(\omega) \).
**Weiner Deconvolution**

Another way to approach the inherent problems in deconvolution is to apply a Weiner filter. This method is often used in image processing, but is also viable for one-dimensional data sets. This method defines $\hat{H}(\omega)$ as,

$$\hat{H}(\omega) = Y(\omega) \frac{X^*(\omega)}{\alpha_w X(\omega) X^*(\omega) + (1 - \alpha_w)}$$

(6.3)

where $X^*(\omega)$ is the complex conjugate of $X(\omega)$, and $\alpha_w$ is a user defined term in the range from 0 to 1. Note that as $\alpha_w \to 1$, $\hat{H}(\omega) \to H(\omega)$.

**Deconvolution Example**

For this example, two delta functions were convolved with the measured P400 excitation signal (Sec. 4.4.5) to synthesize a received GPR signal. The two delta functions were designed to correspond with the direct signal between two antennas and the reflection from a target reflection centered between the antennas buried some distance. In the first example these parameters were selected such that the direct signal and target reflection are distinct. Since the antennas are directive, the delta function corresponding to the direct signal was 30% of the amplitude of the delta function corresponding to the target reflection. The result after convolution is shown in Figure 6.2(a). This figure also shows the result after deconvolution for just the background delta function and just the target delta function, demonstrating that the final result is simply the superposition of these two responses. Next, three different $w$ parameters were used to deconvolve the modeled received signal using water level regularization, Fig. 6.2(b). Note that the results are compared to the original delta functions used to create the modeled received signal. Here, as $w$ decreases the result approaches the original delta functions. Similarly, three different $\alpha_w$ values were used to deconvolve the modeled received signal using Weiner filtering, and are shown in Figure 6.2(c) in comparison to the original delta functions. In this case, the result approaches the original delta functions as $\alpha_w$ approaches 1. Next, a second example was created where the direct signal and target reflection were indistinguishable in the synthesized received signal, Figure 6.3(a), and the same deconvolution methods were applied. In this case, the Weiner filtering method had difficulty parsing the two delta functions for any $\alpha_w$ value, whereas the regularization method was able to isolate these functions if $w$ was small enough.

Finally, it is noted that ideally, the result after deconvolution would be the exact reflectivity series, where the delta functions correspond to the distance of the target and RCS; however this is not the case for these techniques. There are methods that can resolve this type of solution (such as Tihonov regularization to
minimize the L1 norm of the reflectivity series, or MUSIC), but they are not employed in this work and are considered a future improvement.

Figure 6.2: Deconvolution Example 1: (a) Synthesized GPR signal; (b) Comparison of reflectivity series after deconvolution using regularization; (c) Comparison of reflectivity series after deconvolution using Weiner filtering
Figure 6.3: Deconvolution Example 2: (a) Synthesized GPR signal; (b) Comparison of reflectivity series after deconvolution using regularization; (c) Comparison of reflectivity series after deconvolution using Weiner filtering
6.1.3 Demodulation

In some cases, it was necessary to analyze the received signal in terms of the envelope and phase. These were determined using the corresponding \( I \) and \( Q \) channels which can be obtained by modulating the received signal with the excitation’s center frequency and applying a low-pass filter. Consider an A-scan, \( r(t) \), which can be expressed as the product of an envelope, \( A(t) \) and a cosine of a carrier frequency, \( \omega_0 \) with phase \( \phi(t) \), where

\[
r(t) = A(t) \cos[\omega_0 t + \phi(t)]
\]

In order to determine the envelope and phase function, we first express \( r(t) \) as,

\[
r(t) = \text{Re}\left\{ A(t) e^{j(\omega_0 t + \phi(t))} \right\} = \text{Re}\left\{ A(t) \cos[\omega_0 t + \phi(t)] + jA(t) \sin[\omega_0 t + \phi(t)] \right\}
\]

Modulating \( r(t) \) with a cosine of the same carrier frequency results in,

\[
r(t) = \text{Re}\left\{ \frac{1}{2} A(t) \cos[\omega_0 t + \phi(t)] \cos[2\omega_0 t + \phi(t)] + jA(t) \sin[\omega_0 t + \phi(t)] \sin[2\omega_0 t + \phi(t)] \right\}
\]

which using the product-to-sum identities for both cosine and sine results in,

\[
r(t) = \text{Re}\left\{ \frac{1}{2} A(t) \cos \phi(t) \cos[2\omega_0 t + \phi(t)] + j \frac{1}{2} A(t) \sin \phi(t) \sin[2\omega_0 t + \phi(t)] \right\}
\]

Next, by low-pass filtering the \( \cos[2\omega_0 t + \phi(t)] \) and \( \sin[2\omega_0 t + \phi(t)] \) drop out, resulting with the \( I \) and \( Q \) channels.

\[
r(t) = \text{Re}\left\{ \frac{1}{2} A(t) \cos \phi(t) \cos[2\omega_0 t + \phi(t)] + j \frac{1}{2} A(t) \sin \phi(t) \sin[2\omega_0 t + \phi(t)] \right\} = \text{Re}\left\{ \frac{1}{2} I(t) + j \frac{1}{2} Q(t) \right\}
\]

The envelope and phase can then be recreated as \( A(t) = \sqrt{I(t)^2 + Q(t)^2} \) and \( \phi(t) = \tan^{-1}\left\{ \frac{Q(t)}{I(t)} \right\} \). Figure 6.4 shows a comparison between the original received signal, versus the envelope and phase of the signal. Note that the envelopes of the signals are also used for displaying the received GPR B-scans.
Figure 6.4: Demodulation example; (a) Original received signal; (b) Envelope of received signal; (c) Phase of received signal
6.2  Time-of-Flight Techniques

Once the target reflection has been identified, the time-of-flight corresponding to the full path travel distance to the target must be determined. There are various ways that this is done, and the analysis presented in this work considers two methods; peak response, and correlation-detection.

6.2.1  Peak Response

Peak response simply uses the time corresponding to the maximum signal strength for the time-range of interest. Using a Tukey window filter, the received signal is time-gated depending on the bistatic separation and the maximum target depth, which was defined as 20cm based on the U.N. definition for humanitarian demining. The windowed received signal is then deconvolved using either regularization or Weiner filtering, and the TOF is recorded as the peak response.

6.2.2  Cross-Correlation

Cross-correlation returns a sequence of correlation coefficients between two signals. The resultant sequence has \(2N + 1\) entries, where \(N\) is the length of the signal, \(x[n]\). Each entry is the summation of the product of \(x[n]\) and \(y[n]\) shifted by \(k\). The cross-correlation is often normalized such that the autocorrelations of the two signals when \(k = 0\) are unity, as seen in Eq. 6.4.

\[
R_{xy}(k) = \frac{\sum_{k=-N}^{N} x[n] y^*[n - k]}{(\sum_{n} x[n]^2 \sum_{n} y[n]^2)^{1/2}} \tag{6.4}
\]

This results in correlation coefficients which fall between -1 and 1, where 1 means the two signals are perfectly correlated and -1 means the signals are inversely correlated.

The TOF then corresponds to the necessary shift for maximum correlation between the target signal and a reference signal. This reference signal will be discussed more in Chapter 7. Figure 6.5 demonstrates how cross-correlation is used to obtain the TOF. First, Fig. 6.5(a) shows the received target signal (red) plotted in comparison to the reference signal (black), which has a peak at 2.0ns. Next, Fig. 6.5(b) shows the correlation coefficient sequence \(R_{xy}\) versus \(k\), where the maximum correlation occurs when the reference signal is shifted -1.17ns. Finally, Fig. 6.5(c) shows the reference signal shifted for maximum correlation with the target signal. The full-path travel time (3.17ns) is the shift (-1.17ns) subtracted from the reference signal peak (2.0ns). Note, that since the reference signal peak is constant, the necessary shift for maximum correlation and the TOF can be discussed interchangeably as they convey the same information.
Figure 6.5: Determining time-of-flight using cross-correlation with a reference signal; (a) Original target response (red) and reference signal (black); (b) Cross-correlation coefficients versus lag; (c) Original target response (red) and shifted reference signal for maximum correlation (red)
6.3 Scan Selection

Both the pre-screening algorithm and the localization algorithm depend on amplitude thresholding in order to determine when a potential target is in range. The amplitude threshold is a piecewise function dependent on bistatic separation. It was developed by first fitting an exponential curve to the peak direct signals for various background scans, Fig. 6.6(a), and then altering the curve so that all of the peak responses were weaker than the corresponding threshold value, Fig. 6.6(b). Note that the final result was a piecewise thresholding function which has a minimum value of 20.

For the localization algorithm, if a scan surpasses the amplitude threshold, then cross-correlation is used to determine the correlation coefficient of the reflection with the reference signal. If the coefficient surpasses a given correlation threshold, then the TOF is determined using one of the above techniques and the data is inputted to the localization algorithm.

6.4 Pre-Screening

The first stage for target detection is to pre-screen an area for any possible targets, meaning that the GPR must be highly sensitive to potential reflections and the entire area must be canvased. The optimum walking pattern for the WTS involves surveying smaller areas in circles, and then optimally packing these smaller areas to gain complete coverage of the total area. First, the WTS moves about the transmitting toe to cover a circle with radius \( r \), and the receiver records scans at some set increment around the circumference of the search circle. The number of scans required around the circumference of the circle, as well as the
maximum radius will be discussed in Chapter 8. Next, after completing the search for the given area, the WTS then moves the transmitter to begin a new circle. These surveying circles can be packed in different ways, as shown in Figure 6.4 for a $1m^2$ area. The first method, square packing, is shown in Figure 6.7(a),

Figure 6.7: Example of (a) square and (b) hexagonal circle packing for pre-screening a large area for mines but visually exhibits poor coverage of the area. Alternatively, a hexagonal packing technique, Figure 6.7(b), demonstrates a much better surveying technique with 91% coverage. This pre-screening survey method is referred to as the packed circle search pattern (PCS).

6.4.1 Comparison to Alternative Imaging Methods

Ultimately the PCS should be optimized such that there are few scans per square meter, while maintaining a high detection rate. For an individual circle, the number of scans is related to the angle of rotation between scans ($\phi$) as $n = 2\pi / \phi$ and the area the individual circle surveys is related to the selected bistatic separation ($s$) as, $A = \pi s^2$. Therefore, the number of scans that are recorded in one square meter is, $N_{PCS} = n / A = (\phi\pi s^2) / (2\pi)$.

As discussed, alternative GPR landmine detection methods typically include imaging the subsurface with B-scans or C-scans. The number of scans required to survey an area using B-scans depends on the bistatic separation and the distance between subsequent scans ($d$) such that for one square meter, $N_{bscan} = (s d)^{-1}$, assuming that once a B-scan is completed, the GPR surveying unit is shifted over a distance $s$ to begin a new B-scan. The number of scans required to survey an area using C-scans depends only on the distance between subsequent scans, such that for one square meter $N_{scans} = d^{-2}$. Figure 6.8(a) shows a
comparison of the number of scans required for surveying an area with C-scans, B-scans, or PCS for various bistatic separations assuming $d = 2cm$ and $\phi = 20^\circ$. A similar comparison is shown in Figure 6.8(b) with additional plots for $\phi = 30^\circ$ and $\phi = 40^\circ$. Note that for the PCS algorithm, when the bistatic separation is small, the angle of rotation has a substantial effect on the number of scans; but as the bistatic separation increases this difference is minimized. Additionally, only when both $s$ and $\phi$ are small is $N_{\text{bscan}} < N_{\text{PCS}}$; in all other cases, the PCS method will require fewer scans per area and therefore result in faster surveying speeds.

### 6.5 Geometric Localization Method

After a potential target has been detected from the pre-screening algorithm, the final goal is to determine whether or not the potential target is a threat, and if so, where it is located. To do this, additional information is needed and is obtained by varying the receiver position. This section discusses the inherent geometries associated with bistatic GPR and the geometric localization methods considered for this project.
6.5.1 Bistatic GPR Geometries

For each GPR scan, using Eq. 6.5, the velocity and TOF define the full-path travel distance, that is the distance corresponding to the distance from the transmitter, \( E \), to the target, \( T \), and back to the receiver, \( R \).

\[
D_i = v(\text{TOF}_i) \quad (6.5a)
\]
\[
= |E - T| + |T - R_i| \quad (6.5b)
\]

Although, for any single scan this does not specify the direction in which the target lies. Consider a two-dimensional problem, the sum distance from transmitter-to-target and target-to-receiver will remain constant if the target lies somewhere on an ellipse, where the sum distance defines the major axis and the foci are the antenna positions, Figure 6.9. In three dimensions, this is true of a prolate spheroid, an ellipsoid which has equal minor axes. Given three unique bistatic GPR scans, the unique target position could be determined by the point of intersection. A closed-form solution using this technique was studied in [34] and is briefly reiterated in Appendix A. Amongst various hardware limitations for implementing this method, using a closed-form solution also posed issues with experimental error and newer methods were explored.

Next, consider two GPR scans that have the same transmitter position and varying receiver positions. For simplicity, assume that the transmitter, both receivers and the target are on coplanar. Then the change in
The full-path travel distance between the two scans is defined as,

\[ d_{ij} = D_i - D_j \] (6.6a)
\[ = |T - R_i| - |T - R_j| \] (6.6b)
\[ = v(TDOA_{ij}) \] (6.6c)

where \( TDOA_{ij} \) is the time-difference of arrival defined as, \( TDOA_{ij} = TOF_i - TOF_j \). Note that the position of the transmitter is irrelevant as long as it does not change between scans. Also, the path difference from the target to the receivers will remain constant independent of where the target lies, which means that the receivers act as foci of a hyperbola, Figure 6.10. Here, the major axis is defined as half the path difference \( (a = d_{ij}/2) \), and the minor axis is defined by the major axis and the distance between the receivers \( (b^2 = c^2 - a^2) \). In three-dimensions these properties extend to hyperboloids. Again, using three unique receiver locations, a closed-form solution for the target location could be determined; however, the solution would rely heavily on TOF accuracy. Therefore alternate derivations are explored in the following section.

One benefit of using TDOA calculations is that accurate time-zero knowledge is unnecessary, as long as it does not change between the scans being compared. Time-zero, \( t_0 \), is the correction factor needed to determine an object’s distance from a GPR that accounts for the excitation’s travel time through the circuitry and cables, but when comparing scans from the same system, the offset of \( t_0 \) is removed.

\[ TDOA_{ij} = (TOF_i + t_0) - (TOF_j + t_0) = TOF_i - TOF_j \]
This is beneficial if cables or connectors need to be changed in the field, as the system will not require any additional calibration.

### 6.5.2 Linearized Hyperbolic Positioning

The first localization method considered, is to linearize Eq. 6.6 and solve for the target location using an overdetermined error minimization technique, such as the Method of Least Squares. Note that, if the system were monostatic, linearizing the solution would be straight-forward since \(|E - T| = |T - R_i|\), but for the bistatic case, certain assumptions are required. Assume that the transmitter remains in a constant position and the receiver is moved between scans, Figure 6.11. If we first assign one of the receivers to be the origin,

![Figure 6.11: Geometry for geometric localization derivation](image)

then the path difference equation simplifies to,

\[
d_{i0} + |T| = |T - R_i|
\]  \hspace{1cm} (6.7)

Deciding on which receiver should be used as \(R_0\) will be discussed below. Next squaring both sides results in,

\[
d_{i0}^2 + |T|^2 + 2d_{i0}|T| = |T - R_i|^2
\]

Expanding the distance formulas,

\[
d_{i0}^2 + t_x^2 + t_y^2 + t_z^2 + 2d_{i0}\sqrt{t_x^2 + t_y^2 + t_z^2} = (t_x - r_{ix})^2 + (t_y - r_{iy})^2 + (t_z - r_{iz})^2
\]
Now, at this point, the equation cannot be linearized. However, since all of the receivers are located on the air-ground interface, \( r_{iz} \approx 0 \). Therefore,

\[
\begin{align*}
\sqrt{t_x^2 + t_y^2 + t_z^2} &= (t_x - r_{ix})^2 + (t_y - r_{iy})^2 \\
\frac{d_{i0}^2 - r_{ix}^2 - r_{iy}^2 + 2r_{ix}t_x + 2r_{iy}t_y}{2d_{i0}} &= \sqrt{t_x^2 + t_y^2 + t_z^2}
\end{align*}
\] (6.8)

In Eq. 6.8, the \( z \)-component of the target location has been removed from the left-hand side of the equation. Additionally, the right-side of the equation only contains components of the target location, and therefore would be the same for any receiver location, such that

\[
\frac{d_{i0}^2 - r_{ix}^2 - r_{iy}^2 + 2r_{ix}t_x + 2r_{iy}t_y}{2d_{i0}} = \frac{d_{j0}^2 - r_{jx}^2 - r_{jy}^2 + 2r_{jx}t_x + 2r_{jy}t_y}{2d_{j0}}
\]

which can be rearranged as,

\[
2(d_{i0}r_{jx} - d_{j0}r_{ix})t_x + 2(d_{i0}r_{jy} - d_{j0}r_{iy})t_y = d_{j0} \left( d_{i0}^2 - r_{ix}^2 - r_{iy}^2 \right) - d_{i0} \left( d_{j0}^2 - r_{jx}^2 - r_{jy}^2 \right)
\]

Finally, note that the above equation can be written as a linear system of equations as,

\[
2 \begin{bmatrix}
(d_{i0}r_{jx} - d_{j0}r_{ix}) & (d_{i0}r_{jy} - d_{j0}r_{iy}) \\
\vdots & \vdots \\
(d_{N0}r_{(N-1)x} - d_{(N-1)0}r_{Nx}) & (d_{N0}r_{(N-1)y} - d_{(N-1)0}r_{Ny})
\end{bmatrix} \begin{bmatrix}
t_y \\
t_x
\end{bmatrix} = \begin{bmatrix}
b_{ij} \\
\vdots \\
b_{N(N-1)}
\end{bmatrix}
\] (6.9a)

where

\[
b_{ij} = d_{j0} \left( d_{i0}^2 - r_{ix}^2 - r_{iy}^2 \right) - d_{i0} \left( d_{j0}^2 - r_{jx}^2 - r_{jy}^2 \right)
\] (6.9b)

and \( N = n \cdot C_2 \) with \( n \) corresponding to the number of receivers. Note, that at least four unique receiver locations are required for this expression to be determined. Next, the depth component of the target can be evaluated using Eq. 6.8, given the data for any of the GPR pairs and the lateral coordinates of the target. Therefore, the accuracy of the depth component relies on accurate evaluation of the lateral coordinates. The depth solution can be improved by evaluating Eq. 6.8 for every receiver, and using the median depth as the approximation. Furthermore, in Eq. 6.7, one receiver was selected as the origin, from which the
rest of the derivation was based. This selection is critical to securing an accurate localization. Since this entire algorithm is computationally fast, the algorithm is evaluated using each receiver location as the origin, resulting in a set of solutions. The final localization is the median result of the solution set.

6.5.3 Error Minimization

The previous method created a linear system of equations by excluding the depth components of both the target and the antennas. Another method considered evaluates for the target depth during the optimization stage. Recall the path difference relation using $R_i = 0$, Eq. 6.7, which can be expressed as,

$$e = |T - R_i| - |T| - \nu(TDOA_{i0})$$  \hspace{1cm} (6.10)

since the TDOA has some amount of error. Therefore, another approach would be to minimize the sum of the square of the errors, such that,

$$f = \sum_{i=1}^{N-1} (|T - R_i| - |T| - \nu(TDOA_{i0}))^2$$  \hspace{1cm} (6.11)

This error minimization can be done using established numerical optimization techniques, such as the Nelder-Mead Simplex Method. Furthermore, using the known antenna radiation patterns and antenna positions, the solution can be constrained and optimized using SQP. Finally, as with the least squares solution, each receiver can be used as $R_0$, resulting in a cluster of solutions.

6.6 Detection and Localization Summary

This chapter and Chapter 4 discussed the ways in which the P400 and the WTS are used to detect and localize potential targets. This section provides an overall summary of the algorithms used for both pre-screening and localization, as well as examples of the results for the geometric localization techniques.

6.6.1 Algorithm Summary

The first function that was developed, was a GPR scan collection program, Algorithm 1. Recall that for each scan the gain setting of the P400 is optimized using a feedback system (Sec. 4.4.3), and the sampling frequency is increased by interlacing ten individual scans and smoothing the result (Sec. 4.4.2). Additionally, as will be discussed in Sec. 7.3.1, the phase of the scan must be corrected for antenna rotation. Finally
the time range of interest is extracted using a Tewky window. Note that this function is called by both the
pre-screening algorithm and the localization algorithm.

Algorithm 1 Record Scan

<table>
<thead>
<tr>
<th>procedure RECORD SCAN</th>
<th>▶ Main function for recording a GPR scan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Record scan using P400</td>
<td></td>
</tr>
<tr>
<td>loop Scan Clipped AND Gain ≠ 0 ▶ Gain feedback loop to determine optimal gain setting</td>
<td></td>
</tr>
<tr>
<td>Reduce gain</td>
<td></td>
</tr>
<tr>
<td>Record new scan</td>
<td></td>
</tr>
<tr>
<td>Record GPR scan using P400</td>
<td></td>
</tr>
<tr>
<td>for 1 to 10 do ▶ Collect 10 scans to improve P400 resolution by a factor of 10</td>
<td></td>
</tr>
<tr>
<td>Record scan</td>
<td></td>
</tr>
<tr>
<td>Increase start time</td>
<td></td>
</tr>
<tr>
<td>Interlace the 10 scans</td>
<td></td>
</tr>
<tr>
<td>Apply 7-point smoothing filter</td>
<td></td>
</tr>
<tr>
<td>Scale data for the maximum gain setting</td>
<td></td>
</tr>
<tr>
<td>Phase-correct for antenna rotation</td>
<td></td>
</tr>
<tr>
<td>Time-gate using a Tewky window</td>
<td></td>
</tr>
</tbody>
</table>

The main program is the pre-screening algorithm summarized in Algorithm 2, which searches a given
area using the PCS pattern. In order to evaluate this algorithm with the full-scale prototype, two different

Algorithm 2 Pre-Screening Algorithm: Main Program

<table>
<thead>
<tr>
<th>loop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position receiver ▶ Example, Figure 6.12</td>
</tr>
<tr>
<td>CALL Record Scan</td>
</tr>
<tr>
<td>if Peak Response &gt; Threshold then CALL Localization Data Collection</td>
</tr>
<tr>
<td>else if Circle Complete then</td>
</tr>
<tr>
<td>Move transmitter to next position via WTS walk</td>
</tr>
<tr>
<td>else</td>
</tr>
<tr>
<td>Lift top platform and rotate about transmitter ( \phi ) degrees ▶ Example, Figure 6.13</td>
</tr>
<tr>
<td>Lift bottom platform and rotate about transmitter ( \phi ) degrees</td>
</tr>
</tbody>
</table>

movement algorithms for the WTS were written in LabVIEW. The first was the basic walking algorithm,
which was used to move the transmitting toe to the new surveying location. The second was for surveying
different points about the transmitter. Since the distance between the neighboring toes was too large for the
PCS pattern, the bottom platform is rotated to achieve the desired bistatic separation, Figure 6.12. Next,
since the rotation of the bottom platform is limited, the WTS had to be rotated about the transmitter in
order to complete the surveying circle. As seen in Figure 6.13, this required first rotating the top platform
about the transmitter (blue) and then rotating the bottom platform the same number of degrees. Ultimately,
the pre-screening algorithm will continue to traverse the PCS pattern until a GPR scan surpasses a given
amplitude threshold, at which point the Localization Data Collection function, Algorithm 3, is called.
Figure 6.12: Photo of the WTS with the bottom platform rotated to achieve the desired bistatic separation distance for the PCS algorithm

Figure 6.13: Photos of the WTS’s movements about the transmitter
The localization function also required unique WTS movements. In this case, the bottom platform had to be both rotated and translated, Figure 6.14, in order to gather multi-bistatic GPR data at varying bistatic separations. For each bistatic GPR pair, a scan is recorded, and the bistatic separation is calculated in order to determine the corresponding amplitude threshold. Next, the time-of-flight is determined for scans surpassing both the amplitude and correlation thresholds, by either cross-correlation, regularization, or Weiner filtering. Finally, the known antenna positions and corresponding TOFs are inputted into one of the three localization sub-functions; least squares, constrained error minimization, or unconstrained error minimization. The following sections explain how the solution sets are realized for each of these localization techniques.

Figure 6.14: Photos of the WTS’s movements for recording multi-bistatic GPR; (a) WTS standard toe separations; (b) Using WTS rotation to achieve smaller bistatic separation
Algorithm 3 Localization Algorithm

```
procedure LOCALIZATION DATA COLLECTION
loop
    Position receiver
    CALL Record Scan
    Determine amplitude threshold for given bistatic separation
    Determine correlation coefficient
    if Received signal satisfies thresholds then
        Determine TOF using either cross-correlation, regularization, or Weiner filtering
        Count + 1
    if Count >= 4 then
        Approximate target location using LocalizationV1, LocalizationV2, or LocalizationV3
    if Localization Completed then
        Store target location
        CALL Pre-Screening Algorithm

procedure LOCALIZATIONV1(RxLocations, TOFs)
for All Bistatic Pairs do
    Assign receiver as origin
    Evaluate Eq. 6.9 for $t_x$ and $t_y$
    for All Bistatic Pairs do
        Evaluate Eq. 6.8 for $t_{zi}$
        $t_z$ = median($t_{zi}$)
    Store $T_i$ in solutionSet
Final Scattering Point = median(solutionSet)
```

```
procedure LOCALIZATIONV2(RxLocations, TOFs)
for All Bistatic Pairs do
    Assign receiver as origin
    Determine solution constraints based on antenna positions
    Evaluate Eq. 6.11 for $t_x$, $t_y$, and $t_z$ using SQP
    Store $T_i$ in solutionSet
Final Scattering Point = median(solutionSet)
```

```
procedure LOCALIZATIONV3(RxLocations, TOFs)
for All Bistatic Pairs do
    Assign receiver as origin
    Evaluate Eq. 6.11 for $t_x$, $t_y$, and $t_z$ using Nelder Simplex
    if $T_i$ is acceptable then
        Store $T_i$ in solutionSet
Final Scattering Point = median(solutionSet)
```
6.6.2 Least Squares Solutions

Figure 6.15 shows an example of how the least squares localization method generates a solution set and approximates the target's position. Note that in all of these figures the nominal target position is shown, as well as the transmitter position (magenta). First, the program selects one receiver as the origin, and evaluates for $t_x$ and $t_y$ using Eq. 6.9. Figure 6.15(a) shows the result where the red marker indicates the identified lateral coordinates of the target. Next, using those coordinates, the program evaluates Eq. 6.8 for every bistatic GPR pair, resulting in a solution set for the depth component, $t_z$. Figure 6.15(b) shows an example of the depth solutions (green markers), and the final approximated depth which corresponds to the median result (red marker). This $T$ location is then stored, and the process is repeated after assigning the next receiver as the origin, such that $N$ receivers results in $N$ solutions for $T$. Figures 6.15(c) and 6.15(d),
show these $N$ solutions in red. The final target location is defined as the median of the entire solution set.

### 6.6.3 Constrained Error Minimization

For the constrained error minimization method, a receiver is selected as the origin and Eq. 6.11 is evaluated using SQP. This optimization technique is available through for MATLAB’s function `fmincon`, and the stopping criteria are summarized in Table 6.1. The initial guess is the midpoint between the antennas for the bistatic GPR pair resulting in the strongest target reflection, and the constraints on the solution are defined by the known region of detectability for the antennas at the given bistatic separation. This detectable range is discussed in Chapter 7. Next, as with the least squares method, selecting a single receiver results in a coordinate location for $T$. The process is then repeated for each receiver location, resulting in a solution set for $T$. The final target location is defined as the median of the entire solution set.

<table>
<thead>
<tr>
<th>Criteria Value</th>
<th>Max Iterations</th>
<th>Function Tolerance</th>
<th>Solution Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1000</td>
<td>1e-6</td>
<td>1e-6</td>
</tr>
</tbody>
</table>

Table 6.1: Summary of the stopping criteria used for both SQP and Nelder-Mead Simplex Method

### 6.6.4 Unconstrained Error Minimization

For the unconstrained error minimization method, a receiver is selected as the origin and Eq. 6.11 is evaluated using the Nelder Simplex method. This optimization technique is the default technique for MATLAB’s `fminsearch`, and the stopping criteria are the same as those summarized in Table 6.1. As with the above technique, the initial guess is determined by the bistatic GPR pair with the strongest received signal, and receivers are selected in succession with resulting in a solution set for $T$. However, since the solutions are not constrained, some results are infeasible given the region of interest. These points are then rejected according the constraints defined by the antenna radiation patterns. This results in fewer values for the solution set of $T$, but as will be shown in Chapter 8, the final results are more accurate than the results obtained through the SQP minimization.
This chapter discusses the initial experiments conducted using the complete GPR system. These experiments were used to analyze the detection capabilities of the GPR system, as well as to optimize various parameters for the pre-screening and localization algorithms. All of these tests were conducted in sand without the use of the WTS.

### 7.1 Experimental Setup

Two different sand pits were used during the experiments. The first was a 60 x 90 x 30cm polyethylene container, Fig. 7.1(a). The second was the SoilBED facility located at Northeastern University, which is 1.52 x 1.82 x 1.22m, Fig. 7.1(b). The increased size of SoilBED made it a more ideal testing facility, since superfluous reflections were minimized. However, access to SoilBED was limited, and therefore the container was used for experiments where the undesired reflections didn’t affect the experimental results. The same type of sand was used in both cases.

Three different target types are discussed in this section. The first is a 60 x 60 x 0.2cm thin metallic plate which, due to its size relative to the wavelength of the P400, has a spectral reflection that obeys Fresnel’s equations. The second target is a cylinder with a diameter of 8cm and a height of 2.5cm, which is slightly smaller than a typical AP mine. The cylindrical target was made of an approximately 5mm thick low-density molded PVC shell filled with air. In the metallic target tests, the cylindrical target was encased in aluminum foil.
Figure 7.1: Photos of the two sand pit experimental facilities; (a) Polyethylene container; (b) SoilBED

7.1.1 Measuring the Sand Characteristics

The electrical characteristics of the sand were approximated using transmission data at various distances using the PulsON antennas. The antennas were buried at equal depths and the distance between the antennas was increased in 2cm increments between scans. The result was a received direct signal through ground, which shifted in time according to the incremental change in bistatic separation. An example of the received direct signals for three different bistatic separations is shown in Figure 7.2. Next, the time corresponding to a particular peak was recorded for each GPR scan, and plotted against the known changes in separation distance. Using a line of best fit, Fig. 7.3(a), the velocity of the ground wave was determined to be 18.02cm/ns, resulting in a relative permittivity of approximately 2.77. Similarly the change in signal strength was plotted versus the change in distance and using an exponential fit the attenuation coefficient was approximated to be 7.27, resulting in a conductivity of 0.09 S/m, Fig. 7.3(b).
Figure 7.3: (a) Plot of the time-of-flights corresponding to the leading peak of the direct signal versus the change in distance, and the line of best fit; (b) Plot of the normalized signal strengths corresponding to the leading peak of the direct signal versus the measured change in distance, and the corresponding exponential curve fit

7.2  Antenna Evaluations using a VNA

A vector network analyzer (VNA) with a maximum test frequency of 3GHz was used to evaluate the return loss and transmission characteristics of the spiral antennas for various conditions. Although a higher frequency VNA would have been ideal, this VNA was the only unit available near the experimental facilities. For the transmission tests, the metal plate was buried below the antennas in order to reverse the polarization of the received wave. All of these experiments were conducted in the polyethylene container.

7.2.1  Return Loss

The return loss characteristics for an LCP antenna and an RCP antenna are shown in Figure 7.4. Typical antenna operation requires a return loss of less than -10dB. Using this threshold, it is evident that the antennas begin radiating above 2.4GHz. The return loss was also analyzed using a second VNA with a maximum
frequency of 10GHz, and the antennas displayed sufficient radiation from 2.4GHz to above 5GHz, which accounts for the P400 excitation frequencies.

### 7.2.2 Transmission Coefficient with Varying Travel Distance

This experiment studied how the transmission coefficient changed with bistatic separation. The metallic plate was buried at two different distances, 5cm and 10cm, and the transmission coefficient between two spiral antennas was measured for four distances: 4cm, 12cm, 20cm, and 28cm. The results are shown in Figure 7.5. Note that in this experiment the overall travel distance is changing, as well as the angle at which the spectral reflection will occur. Recall that as the elevation angle increases, the radiated power decreases. This is evident in both Figures 7.5(d) and 7.5(c), where the transmission coefficient for a plate buried at 10cm is slightly stronger than the transmission coefficient for a plate buried at 5cm. Even though the travel path is longer, the radiation pattern corresponding to an elevation angle for a spectral reflection is stronger resulting in a stronger signal.

![Figure 7.5](image-url)
7.2.3 Directivity

Next, the consistency of the transmission coefficient with azimuth angle was measured by rotating the transmitting antenna about its center axis in increments of 30 degrees. For these tests, the metal sheet was buried 10cm below the surface and two bistatic separations were considered, 4.5cm and 9cm. Figure 7.6, shows the mean transmission coefficients for each bistatic separation, in comparison to a one standard deviation offset. Overall, as designed, the performance is consistent at all angles for the optimal transmitting frequencies (above 2.4GHz).

[Graph showing transmission coefficients]

Figure 7.6: Mean and standard deviation of the measured transmission coefficients from 1 to 3GHz with a plate buried at a depth of 10cm when the transmitting antenna is rotated 30° between each scan (N = 11); (a) bistatic separation = 4.5cm; (b) bistatic separation = 9cm

7.2.4 Effects of Surface Roughness

The transmission coefficients were then compared for a smooth air-ground interface versus a rough air-ground interface. This was done using the same bistatic separation and a plate buried at 10cm, Figure 7.7 shows the comparison of $S_{21}$ for a very smooth surface versus a rougher surface. Overall the changes in transmission were minimal. This was also confirmed with later experiments.
7.2.5 Channel Transfer Function

Farid et al. developed a method to determine the transfer coefficient due only to the electromagnetic wave propagation through the medium of interest, in this case sand. This parameter, called the Channel Transfer Function (CTF) [25], is calculated from the measured transfer coefficients and return losses of the system, as

\[
CTF = C_{21} = S_{21,Soil} = \frac{(|S_{11}|^2 - 1) S_{21}}{(|S_{11}|^2 - 1)^2 e^{2j\phi_{21,Ant}} - |S_{11}|^2 S_{21}^2}.
\] (7.1)

Ideally, the CTF and the transfer function of the overall system should be the same, but due to losses in the antennas this may not be the case. Figure 7.8 shows a comparison between \( S_{21} \) and the CTF, \( C_{21} \) for two different bistatic separations. Note that above 2.4GHz, \( S_{21} \approx C_{21} \). This corresponds with the frequency ranges for which the spiral antennas have a return loss less than 10dB, whereas for frequencies below 2.4GHz, \( C_{21} > S_{21} \).
7.2.6 Direct Signal Strength

The major aspect of the overall design for this prototype was to simplify data analysis from other GPR mine sensing methods. As has been discussed, one major hurdle for alternative methods is the removal of the initial response from the air-ground interface. In the case of ground-coupled GPR, there is no response from the air-ground interface and as was shown there is very little variation in transmission power when the surface roughness changes. This is due in part to the on-contact antennas, and also the limited direct signals through air and ground. The spiral antennas are backed by an absorber and are more directive into the ground. Therefore, there is very little energy coupled directly to the receiving antenna. To test the strength of the direct signal through air, a metallic plate was placed orthogonal to the air-ground interface between the antennas. After scanning the ground, the metallic plate between the antennas was removed and another scan was repeated. Comparisons of these scans for two different bistatic separations are shown in Figure 7.9. Overall, the placement of the metallic plate had no discernible effect on the scan above 2.4GHz, meaning that there is no direct air coupling between the antennas at the frequencies of interest. Note that no scatterer was buried during these experiments.

7.3 Antenna Evaluations using the P400

These next experiments evaluated the spiral antennas using the P400 in order to further understand their radiation characteristics in terms of a time-domain received signal.
Figure 7.9: Comparisons of the measured transmission coefficient with and without a metal divider blocking the direct signal through air; (a) Bistatic separation of 12cm; (b) Bistatic separation of 24cm

7.3.1 Directivity

The previous section examined the transmission coefficient within the azimuth plane and demonstrated that the power received was constant. In this section, the variation in the received signal for both changes in the azimuth and elevation angle were explored.

Azimuth Plane

As demonstrated with the VNA, the antennas radiate equal power for all $\phi$ directions when $\theta = 90^\circ$; however, the phase of the received signal changes according to the angle offset between the transmitting and receiving antenna. This can be observed in Figure 7.10 which shows an example of two received GPR scans for the same antenna positions, where the two cases only vary by the rotation of the receiving antenna. Since the WTS rotates the platforms in order vary the bistatic separation, the angle between the transmitting and receiving antennas between scans will vary. If unaccounted for, this could result in inaccurate TOF
measurements. However, this is adjusted in post-processing using the known change in yaw of the WTS platform, and demodulation (Chapter 6.1.3). First, the received signal is decomposed into the envelope and phase. Next, the phase change corresponding to the amount of antenna rotation is added to the phase of the original received signal. Finally, the envelope and phase are recombined yielding the phase-corrected received signal. An example of the phase corrected signal is shown in Figure 7.11.

![Figure 7.11: Example of phase correction for varying antenna azimuth angles](image)

**Elevation Plane**

It was known through simulations that the antennas were not isotropic in the elevation plane, causing the aforementioned relationship between the unknown depth of the target and the bistatic separation. Experimentally the variation in the elevation plane was determined using a consistent bistatic separation and plate depth and then varying the angle at which the transmitter was in contact with the ground, Figure 7.12. First, the antenna was placed in the traditional position, orthogonal to the ground, Figure 7.12(a). Here, the angle at which the spectral reflection occurs is \( \theta_0 = \arctan \left( \frac{0.5s}{d} \right) \), where \( s \) is the bistatic separation and \( d \) is the target depth. Next, the antenna was angled away from the receiver in increments of 15°, Fig. 7.12(b), such that the spectral angle, \( \theta \), increased as \( \theta = \theta_0 + 15n \). The transmitting antenna was then returned to the vertical position and angled toward the receiving antenna in 15° increments, Fig. 7.12(c), such that \( \theta = \theta_0 - 15n \). Scans were recorded until \( \theta \) exceed the \( \theta = 0 \) position, Fig. 7.12(d).

The peak signal response for each scan was extracted and then plotted in decibels using the maximum response for normalization. Two sets of data for bistatic separations of 20cm and 25cm were recorded for a plate buried at 14cm. The results are shown in Figure 7.13. Although the experimental radiation patterns are not sampled highly, they do demonstrate agreement with the simulated radiation patterns from Figure 7.13. The strongest responses occur between -30 and 30°, and there is greater than a 3dB falloff in signal strength when the elevation angle exceeds these limits. The simulated radiation patterns have an \( \approx 8\)dB falloff when \( \theta \) is increased from 0 to 70° which corresponds with the results for a bistatic separation of 20cm. The results
Figure 7.12: Geometry for experimentally measuring the radiation pattern versus azimuth angle; (a) Transmitting antenna orthogonal with ground, elevation angle corresponds to $\theta_0$; (b) Transmitting antenna angled away from receiving antenna, elevation angle is increased; (c) Transmitting antenna angled toward receiving antenna, elevation angle is decreased; (d) elevation angle equal 0

Figure 7.13: Experimental radiation pattern versus elevation angle determined using a plate buried at 14cm and two different bistatic separations.
for a larger bistatic separation show a more significant falloff in the same range, and this could partly be attributed to the larger travel path.

### 7.3.2 Polarization

In order to evaluate the polarization of the antennas, two B-scans were taken over a metallic cylindrical target with a bistatic separation of 7.62 cm, Fig 7.14. In both cases an RCP antenna was used as the transmitter, and the polarization of the receiving antenna was either LCP or RCP. As expected, when using two antennas of opposite polarization the target reflection is easy to identify compared to the background, whereas when using two antennas of the same polarization the target reflection is highly difficult to identify.

![Figure 7.14: Comparison of B-scans over a metallic cylindrical target when the antenna polarizations are (a) different and (b) the same](image)

### 7.4 Target Detection

The final section in this chapter discusses the initial detection experiments that were conducted. These were done to evaluate the accuracy and reliability of the TOF techniques for A-scan target detection and localization. By recording repetitive scans of the same target, the reliability in TOF determination was analyzed. Similarly, by comparing A-scans for different target depths and various bistatic separations the accuracy in detecting changes in distance was analyzed. First, however, this section presents a series of B-scans which were recorded to analyze the variations in scans by visual inspection.
7.4.1 B-Scan Evaluations

Although B-scans were not used in the overall detection method, they were used to gain a better understanding of the GPR system performance. Note that all of the B-scans in this section are shown using the same colorbar axis and are created using the envelopes of the A-scans. In all cases the step size between scans was 2cm. Also, throughout this section two bistatic separations, 7.62cm and 15.24cm, were considered for each case. The distances between the antennas were kept constant by mounting the antennas onto the rail seen in Figure 7.15. The rail was metallic, but various experiments confirmed that it did not carry any additional signal between the antennas.

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Figure 7.15: Photo of the rail used to maintain antenna separation for recording B-scans
```

First, the metallic plate was buried at 9cm and 13cm, and B-scans were created for the two bistatic separations, Figure 7.16. The B-scans were initiated over a 10cm area where the plate was not buried below in order to see a comparison of the background signal strength. First, note that there is a primary plate reflection followed by weaker “copies” of this primary reflection. These correspond to the second smaller impulse from the P400 excitation (discussed in Section 4.4.5), and multipath reflections. These effects are removed through time-gating for the depth of interest. Next, note the reduction in amplitude for the different cases, and recall that the spectral angle changes for each case in addition to the travel distance. When the plate is buried at 9cm, the bistatic separations of 7.62cm and 15.24cm, correspond to elevation angles of 22° and 40° respectively. Whereas, when the plate is buried at 13cm, these separations correspond to 16° and 30°. When comparing a single plate depth (Fig. 7.16(a) and 7.16(b) or Fig. 7.16(c) and 7.16(d)), it cannot easily be determined whether the reduction in amplitude for the larger bistatic separation is due to the increased travel distance or the increased azimuth angle. However, when comparing plate depths for a
Figure 7.16: Comparison of B-scans from a metallic plate buried at; (a) 9cm for a bistatic separation of 7.62cm and (b) 15.24cm, and a plate buried at (c) 13cm for a bistatic separation of 7.62cm and (d) 15.24cm
bistatic separation of 15.24cm, Fig. 7.16(d) has a stronger reflection despite Figure 7.16(b) having a shorter full-path travel distance, demonstrating the detection limit relationship between bistatic separation and target depth due to the antennas’ radiation patterns.

Finally, note that the reflection from the metallic plate has the same temporal width as the excitation, and therefore occurs over a 1.5ns range, corresponding to 27cm in distance. This is depicted visually in Figure 7.17, where the image is focused on the primary plate reflection and the dependent axis is shown in centimeters using the velocity calculated earlier. Also shown in this image are markers corresponding to the peak of the reflection. This large temporal width can cause difficulties in determining accurate TOFs,

![Figure 7.17](image)

Figure 7.17: Enlarged comparisons of B-scans from a metallic plate buried at; (a) 9cm for a bistatic separation of 7.62cm and (b) 15.24cm, and a plate buried at (c) 13cm for a bistatic separation of 7.62cm and (d) 15.24cm with the peak values identified through markers

as is evident from the varying peak responses despite the plate being straight. (Note, the plate is buried at a slight angle in all cases, but the peak response should be increasing linearly.) Using inverse methods, the width of the target reflection can be reduced to provide a more accurate and consistent TOF. Examples of the resulting B-scans when Weiner filtering each of the time-traces individually is shown in Figure 7.18. Here $\alpha_w$ is varied from 0, to 1e-5, and 1e-4, and the linearity of the peak values improves each time $\alpha_w$ is increased; however there is a limit to the improvement. Figure 7.19(b) shows the results for the same plate depth and bistatic separation using $\alpha_w = 0.1$. In this case, the linearity of the peak responses is lost and each column of the B-scan varies significantly. One of the difficulties in automating the entire detection system using a technique such as Weiner filtering is determining an optimized value for $\alpha_w$, particularly since the optimal value may change for different scenarios as shown in Figure 7.19(a). Additionally, numerous similar target scans cannot be compared to one another since they are unavailable in the multi-bistatic geometric target localization method. Therefore, in future applications conservative, rather than optimal, values of $\alpha_w$
Figure 7.18: Comparison of B-scans from a metallic plate buried at 13cm using Weiner filtering; (a) Original B-scan; (b) $\alpha_w = 0$; (c) $\alpha_w = 1e^{-5}$; (d) $\alpha_w = 1e^{-4}$

Figure 7.19: Comparison of two B-scans using Weiner filtering and $\alpha_w = 0.1$ for a metallic plate buried at (a) 9cm and (b) 13cm. Note that for the same $\alpha_w$, the noise in (a) is significantly higher than (b).
are employed. Note that the Weiner filtering did shift the TOF of the plate reflection; however, this shift is consistent for a given $\alpha_w$ and was not corrected since the geometric target localization method relies on time-difference of arrivals and therefore does not need to be time-zero corrected.

Finally, B-scans were recorded over the metallic cylindrical target as shown in Figure 7.20. As with the metallic plates, a change in the received magnitude is apparent. Additionally, multipath effects are observed, as the target appears to occur numerous times in the time trace (e.g. between 40 and 45ns) and does not appear in the background signals at these times. As with the plate reflections, the target reflection occurs over a 1.5ns range and can be reduced by using Weiner filtering as shown in Figure 7.21.

![Figure 7.20](image1)

**Figure 7.20:** Comparison of B-scans from a metallic cylindrical target buried at 7.5cm for different bistatic separations; (a) Bistatic separation of 7.62cm; (b) Bistatic separation of 15.24cm

![Figure 7.21](image2)

**Figure 7.21:** Comparison of B-scans from a metallic cylindrical target buried at 7.5cm for different bistatic separations using Weiner filtering; (a) Bistatic separation of 7.62cm; (b) Bistatic separation of 15.24cm
7.4.2 Repeatability and Accuracy of TOF Measurements

The next goal was to analyze the repeatability and accuracy of the TOF techniques discussed in Chapter 6. To do this, a collection of scans were recorded for a variety of cases. For each case, the metallic cylindrical target was buried at a measured depth; then, using a fixed bistatic separation centered above the target, five scans were recorded. Between each scan, the antennas were manually lifted from the ground, rotated about their midpoint, and placed down again. Translational displacement of the midpoint was kept constant within experimental variability (less than 2cm). This was repeated for five different target depths and five different bistatic separations. Figure 7.22 depicts the basic experimental setup. Overall, 25 unique cases were considered, resulting in a total of 125 GPR scans. The five scans that had the same bistatic separation and target depth are referred to as replicates. Figure 7.23 shows an example of the replicate data for a target buried at 10cm, and a bistatic separation of 15.24cm. Note that the replicates show some minor variability in phase, and that the signal strength varies. In order to maintain a consistent target depth between scans,

![Experimental setup for collecting replicate data](image)

Figure 7.22: Experimental setup for collecting replicate data; (a) Example of the depth geometry, note that the topology of the sand caused uncertainty in the depth measurement; (b) Top-down view showing an example of two replicate scans, green antennas and blue antennas. Note that the bistatic separation is constant and the target center is located in approximately the same position.

![Example of replicate data scans](image)

Figure 7.23: Example of replicate data scans for a metallic cylindrical target buried at 10cm, with a bistatic separation of 15.24cm.
efforts were made to keep the target and the surface of the sand parallel, as well as to place the antennas onto the sand with equal pressure (i.e. not allowing the antennas to sink into the sand). Though experimental error is intrinsic in measured data, in this section the same scans are used to compare various TOF methods, thus the experimental error is equal for each method and is disregarded.

The first goal was to analyze the reliability of the TOF measurements, by comparing the measured full-path travel distance between replicates. Since the target and air-ground interface remained unchanged between scans, the physically measured change in full-path travel distance between any two replicates, $d_M$, is zero. Whereas, the calculated (electromagnetically measured) change in distance is defined as,

$$
\Delta d_E = (TOF_A - TOF_B) \nu
$$

(7.2)

where $TOF_A$ is the TOF for the first replicate scan, $TOF_B$ is the TOF for the second replicate scan, and $\nu$ is the measured velocity in sand. Therefore, the absolute error is, $|\text{Error}| = |\Delta d_M - \Delta d_E| = |\Delta d_E|$. Given the 25 unique scenarios, there were a total of 250 combinations of replicates to compare. For each combination, $\Delta d_E$, was calculated and the errors were plotted using histograms with 1cm bins.

Figure 7.24 shows the resulting repeatability histograms using the three different TOF methods. First, Fig. 7.24(a) shows the results using correlation detection with four different reference signals. Here, R1 is the loopback signal discussed in Section 4.4.5, R2 is the signal remaining from the loopback signal after subtracting the RMS value, R3 is a windowed version of the loopback signal, and R4 is the mean reflection from a plate buried at 10cm. Note that for all of the correlation methods, the second highest error bin is for errors between 4 and 5cm. This is due to the wavelength of the excitation. As seen in Figure 7.25, the distance between peaks of the received signal is approximately 4.6cm. Therefore, correlation with an incorrect peak typically causes errors in multiples of 4.6cm. Next, Fig. 7.24(b) shows the results using the peak response after water-level regularization for different values of $w$. Similarly, Fig. 7.24(c) shows the results using the peak response after Weiner filtering given $\alpha_w$. Note that both deconvolution demonstrate variability and strong dependence on the user defined parameter. In particular, the regularization technique demonstrated the worst overall results when $w = 1e^{-5}$, but also demonstrated the best overall results when $w = 1e^{-3}$. Finally, Fig. 7.24(d) shows a comparison of the best methods from the correlation detector, the regularization, and Weiner filtering.
Figure 7.24: Histograms of the absolute distance error between replicate GPR scans using varying parameters for (a) cross-correlation, (b) regularization, and (c) Weiner filtering; (d) Histogram comparing the three TOF techniques using the optimized parameters.

Figure 7.25: Example demonstrating the reasoning for the 4 - 5cm error generated when using the cross-correlation TOF measurement technique.
Next, the accuracy of the TOF was evaluated by analyzing the estimated time-zero correction factor. Recall that for each case the full-path travel distance is defined as,

\[ D_i = \nu (TOF - t_0) \]  

such that, for each case, the time-zero correction factor can be approximated using the measured full-path travel distance,

\[ t_0 \approx \frac{TOF - D_i}{\nu} \]  

\[ \approx \frac{TOF - 2\sqrt{(0.5s)^2 + d^2}}{\nu} \]

where \( s \) is the bistatic separation and \( d \) is the physically measured target depth. Since time-zero is independent of the area being scanned, the value should not vary between scans. However, using this method, time-zero may vary between TOF techniques. Therefore, comparisons are made by removing the mean value from each data set. The results are shown in Figure 7.26. Note that the standard deviation for each data set is shown in the legend.

First, in Fig. 7.26(a), three of the reference signals result in similar distributions for time-zero, whereas the fourth reference signal results in a standard deviation of 0.45ns (corresponding to \( > 8 \) cm). Interestingly, this was the most repeatable reference signal, demonstrating that repeatability does not imply accuracy. However, using the peak value after deconvolution with regularization (Fig. 7.26(b)), the best results are for \( w = 1e^{-3} \), which agreed with the repeatability analysis. Additionally, as with the repeatability analysis, both deconvolution techniques demonstrated an optimal parameter choice, which, when exceeded, results in decreased accuracy. This was also observed in the B-scans shown in Section 7.4.1 using the Weiner filtering technique. A comparison of the best results for each technique is shown in Fig. 7.26(d).

Ultimately, this analysis provided a method for optimizing the individual parameters for each TOF technique. Once optimized, the overall accuracy and reliabilities were similar across techniques, with the exception of the cross-correlation technique which provided the weakest results due to correlation with incorrect peaks causing large errors when errors occurred. However, this data was recorded with the antennas positioned directly over the known target location, which will not always be the case in localization. Therefore, in the next chapter, a comparison between the optimized TOF techniques is considered using the geometric localization method.
Figure 7.26: Histograms of the absolute time-zero approximation using (a) cross-correlation, (b) regularization, and (c) Weiner filtering; (d) Histogram comparing the three TOF techniques using the optimized parameters.
7.4.3 Reflection Mapping

All of the previous experiments considered targets which were centered below the transmitter and receiver. The final experiment was conducted to determine how far from this idealized location the target could be moved and still be detected. This was done by burying the metallic cylindrical target and moving the antennas systematically in a grid above the target. The end result was a set of scans that emulated stationary antennas with the center of the cylindrical target moving. Figure 7.27 shows an example of the resulting data set, where each scan represented a target located in each grid space and the transmitter and receiver are constant. Note, that only the center of the target is moved into each grid point which are 2x2cm, and therefore the complete target occupies more than one grid space. Next, for each scan, the peak signal strength of the target reflection was selected and mapped to the corresponding grid location. These “reflection maps” were then plotted using a color grid. Since there is an established relationship between bistatic separation and target depth, two bistatic separations, 15 and 25cm, and two target depths 5 and 16cm were considered. The results for all four reflection maps are shown in Figure 7.28.

Note that the ideal target location, centered below the antennas, consistently has the strongest target reflection and there is a symmetric falloff in amplitude from this point. As expected the worst results occur at the four corners of the maps. Additionally, the large bistatic separation demonstrates weaker target reflections for both target depths. Next, recall the amplitude thresholding function in Figure 6.6. For a bistatic separation of 15cm, the amplitude threshold for target detection is a signal strength of 40, whereas for a bistatic separation of 25cm, the amplitude threshold for target detection is only 20. Using these thresholds, Figure 7.29 shows a binary version of the response maps corresponding to whether or not the reflection
would surpass the given threshold. Understanding these detectable ranges, provides information regarding the target location which can then be utilized to reject inaccurate localization results and to constrain solutions during optimization.

Figure 7.28: Reflection maps for a target buried at 5cm and a bistatic separation of (a) 15cm and (b) 25cm, and a target buried at 16cm for a bistatic separation of (c) 15cm and (d) 25cm
Figure 7.29: Reflection maps displaying detectable target locations based on amplitude threshold function for a target buried at 5cm and a bistatic separation of (a) 15cm and (b) 25cm, and a target buried at 16cm for a bistatic separation of (c) 15cm and (d) 25cm.
7.5 Mechanical Antenna Difficulties

As mentioned in Chapter 4, two sets of antennas were fabricated at Square One with varying methods. All of the experiments presented in this chapter were done using the first set of antennas. The second set were re-evaluated on the VNA and demonstrated similar radiation characteristics. However, one fundamental difference was that the housings on the second set of antennas were not grounded. This issue did not affect the strength of target reflections, but rather affected the direct signal between the antennas, particularly when there was any mechanical connection between the housings.

For example, the metal rail shown in Figure 7.15 carried a signal between the second set of antennas but did not carry a signal for the first set of antennas. A comparison for a background A-scan recorded using the metal rail is shown in Figure 7.30. Similarly, this issue caused a stronger direct signal when the antennas were submerged in the sand. Figure 7.31 shows a comparison of the direct signals when the antennas were simply atop the sand (antenna housings were not in contact with the sand), versus when the antennas were submerged in the sand. To account for these differences, the amplitude threshold was altered from a constant value to the piecewise function discussed in Section 6.3. These effects also had a negative effect on the localization results due to the interference between the direct signal and target signal, which in some cases altered the TOF. In the following chapter the antenna type will be specified.
DETECTION AND LOCALIZATION RESULTS

This chapter presents the experimental pre-screening and localization results. Criteria for localization accuracy are discussed and used to determine the optimal localization and TOF techniques. Experimentally there were three distinct groups of localization experiments, which, using the optimal localization technique are discussed in detail with examples. Localization of non-metallic targets is addressed using the WTS for data collection. Finally, the experimental pre-screening results are presented.

8.1 Data Collection Methods for Localization Experiments

Initial localization experiments were conducted in the SoilBed facility described in Chapter 7. The positions of the antennas were measured using the grid in Figure 8.1. Each grid space is 2.5cm$^2$ (the diameter of the spiral antennas). Using this method, two sets of experiments were conducting. The first set used the original antennas which were thoroughly analyzed and had minimal cross-talk between the antennas. The second set used the second set of antennas, which had large direct signals when the antenna housings were in contact with the sand.

Field experiments were conducted in Jackson, Wyoming at the Square One Systems Design facility, Figure 8.2. A 1 x 1 x 0.4m sandpit was built outdoors, and the moisture content of the sand varied between experiments based on the weather and time of day. As discussed in Chapter 4, the size and weight of the WTS available for experiments was larger and heavier than desired. This resulted in the antenna diameters’ being too small to support the WTS’s weight on sand. Therefore, weight distributors were designed to prevent substantial sinking into the sand, Figure 8.2. However, it is noted that for all scans, the antenna housings
were submerged below the air-ground interface approximately 2cm. Additionally, for these experiments the actual target location was measured from the transmitting toe based on the WTS’s built in coordinate system. The approximate coordinate system was identified on the WTS, Figure 8.3(b).

Experimentally, there were three distinct groups of localization experiments, which are summarized in Table 8.1. The first set was collected by hand at the SoilBED facility using the first set of antennas which were discussed in the previous chapter. The second set of experiments were also conducted by hand in SoilBED, but instead used the second set of antennas which had the direct signal issue when the antennas were buried. Finally, the third set of experiments were conducted using the WTS prototype and the second set of antennas. For Group 1, the metallic target was buried at three different depths (7.5cm, 11cm, and 13cm) and each for each depth, the data was collected using three different transmitter positions. In Group 2, fourteen targets at various depths were analyzed, and fewer bistatic GPR pairs were used. The evaluated

<table>
<thead>
<tr>
<th>Experimental Group 1</th>
<th>Data Collection</th>
<th>Antenna Set</th>
<th># of Targets</th>
<th># of Depths</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hand</td>
<td>Set 1</td>
<td>9</td>
<td>3</td>
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<tr>
<td>Experimental Group 2</td>
<td>Hand</td>
<td>Set 2</td>
<td>14</td>
<td>5</td>
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<td>Experimental Group 3</td>
<td>WTS</td>
<td>Set 2</td>
<td>14</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 8.1: Summary of the localization experimental groups

Figure 8.1: Photo of the grid used for recording localization data and tracking the receiver positions
Figure 8.2: Photo of the field experiments conducted in Jackson, Wyoming

Figure 8.3: (a) Photo of the weight distributors mounted onto the WTS; (b) Photo of the WTS top platform indicating the on-board Cartesian coordinates
target depths and the number of targets for each depth is summarized in Table 8.2. As with Group 1, for targets with the same depth, the transmitter position relative to the target position was altered. Finally, data for Group 3 was collected in the field using the WTS. Thirteen targets were considered, with the majority buried at shallow depths, Table 8.3. Due to the low attenuation in sand, shallow targets were more difficult to detect given the radiation pattern. One major improvement of the WTS was the accuracy in antenna placement.

### 8.2 Localization Accuracy

As discussed in Chapter 6, two localization algorithms and three time-of-flight techniques were under consideration. Section 6.6 explained in detail the solution sets for each localization algorithm. Recall, that each method results in a solution set of potential scattering points on the volumetric scatterer, and the final result is the median of the cluster. The TOF techniques used in this chapter are the cross-correlation, regularization, and Weiner filtering, each using the optimized parameters from Section 7.4.2. Table 8.4 shows the abbreviations used throughout this chapter for the different localization analysis techniques.

<table>
<thead>
<tr>
<th>Localization Method</th>
<th>TOF Technique</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Least Squares</td>
<td>Correlation</td>
<td>LS-C</td>
</tr>
<tr>
<td></td>
<td>Regularization</td>
<td>LS-R</td>
</tr>
<tr>
<td></td>
<td>Filtering</td>
<td>LS-F</td>
</tr>
<tr>
<td>Constrained Error Minimization</td>
<td>Correlation</td>
<td>CEM-C</td>
</tr>
<tr>
<td></td>
<td>Regularization</td>
<td>CEM-R</td>
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<tr>
<td></td>
<td>Filtering</td>
<td>CEM-F</td>
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<tr>
<td>Unconstrained Error Minimization</td>
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<td>EM-R</td>
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<tr>
<td></td>
<td>Filtering</td>
<td>EM-F</td>
</tr>
</tbody>
</table>

Table 8.4: Summary of the solution set abbreviations
8.2.1 Evaluating Localization Results

There were two criteria for evaluating the accuracy of the localization algorithm; the lateral accuracy and the depth accuracy, which, using the proposed localization methods are dependent upon one another. The accuracy of the lateral coordinates is evaluated using a distance error and a cluster analysis. If the localized point lies anywhere on the volumetric scatterer, then the result has zero error distance, an example of this is shown in Figure 8.4(a). If the lateral coordinates do not lie within the boundary of the measured target location, then the error is the distance from the median coordinates to the nearest edge of the measured target, Fig. 8.4(b). One important thing to note is that for both the initial testing and field testing the results of the localization algorithm are being compared to the physically measured target location, which may also contain inaccuracies. Due to this ±2cm uncertainty in the measured position of the target, successful localization is defined as being within a 6cm radius, as opposed to the physical 4cm radius of the cylindrical target. Additionally, a second lateral accuracy analysis is considered, which analyzes the variability of the solution set. Given that the entire solution set results in a cluster of scattering points, it can be projected that a tight cluster, Fig. 8.4(c) is a more reliable solution than a dispersed cluster, Fig. 8.4(b). This is analyzed quantitatively using the mean distance between the cluster’s centroid and each solution.

Finally, the accuracy of the predicted depth is analyzed with an error distance similar to that for the lateral coordinates, Fig. 8.5. If the identified depth is within the range that the volumetric scatterer occupies, then the depth error is 0, Fig. 8.5(a). Otherwise, the error is the absolute distance from the predicted depth

![Figure 8.4: Examples for evaluating the accuracy of the localization lateral coordinates; (a) example of a tight solution cluster and 0 distance error; (b) example of distance error](image)

Figure 8.4: Examples for evaluating the accuracy of the localization lateral coordinates; (a) example of a tight solution cluster and 0 distance error; (b) example of distance error
to the nearest point of the actual target, Fig. 8.5(b). Again, using the experimental data there is some uncertainty in the actual target position. Additionally, due to the variability in the air-ground interface, and the differences in the heights of the antennas on placement, the measured depth is expected to have greater error compared to the lateral coordinates. Therefore, the depth accuracy is compared to a target with a 4.5cm height as opposed to the physical height of 2.5cm.

Figure 8.5: Examples for evaluating the error in predicted depth of the target; (a) Error = 0; (b) Error is the distance from the blue marker to the bottom of the measured target position

8.2.2 Method for Statistical Analysis

In order to determine the optimal localization method, the lateral distance errors were analyzed using a two-way analysis of variance (ANOVA) test. In general, this statistical test compares experimental groups to determine whether a statistically significant difference in means exists, by determining whether or not the null hypothesis can be rejected. The two-way ANOVA considers two variables, which in this case are the localization methods and the TOF technique. The null hypothesis used in these tests is that the localization method and TOF technique do not have an affect on lateral localization accuracy. For each variable, statistical significance is said to exist for a $p$-value $< 0.05$. The interaction between the variables is also analyzed using the same $p$-value threshold.
8.3 Localization Results

Using the three localization data sets summarized in Table 8.1, the accuracies of the proposed multi-bistatic localization methods were analyzed and compared.

8.3.1 Comparison of Localization Techniques

The first goal was to establish the optimal localization and TOF techniques. For each experimental group, the targets were analyzed and the three error parameters were determined. The error parameters for all of the targets in that group were then averaged. Figure 8.6 shows the three resulting bar graphs, each comparing these averaged error parameters for the nine techniques. The first observation is that there are clear differences between the results for different localization methods. This is supported by the ANOVA test which resulted in a statistically significant difference between the localization techniques for all three experimental data sets, with p-values of 0.002, 0.015, and 0.001 respectively. The constrained error minimization technique resulted in the least accurate target depth predictions, as well as the greatest cluster dispersion. Furthermore, the least squares method demonstrated the highest accuracy in regards to depth prediction and cluster dispersion. Finally, the lateral distance accuracy does not show consistency between localization groups. For Group 1, the best lateral performance clearly occurs using the least squares method; whereas for Group 2 it occurs for the unconstrained error minimization method. However, one issue not addressed in these displays are the number of accurately located targets, and, more importantly, the number of missed targets. These metrics are summarized in Table 8.5, where the qualification for a perfectly located target is a lateral distance error and depth error both equal to zero. Here it clear that the least squares method provides

<table>
<thead>
<tr>
<th>Localization Method</th>
<th>Found Targets</th>
<th>Missed Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS-C</td>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>LS-R</td>
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<td>0</td>
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<td>LS-F</td>
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<td>0</td>
</tr>
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<td>EMC-C</td>
<td>2</td>
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</tr>
<tr>
<td>EMC-R</td>
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<td>0</td>
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<tr>
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<td>EM-C</td>
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<tr>
<td>EM-R</td>
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</tr>
<tr>
<td>EM-F</td>
<td>4</td>
<td>1</td>
</tr>
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</table>

Table 8.5: Summary of accurate target localizations and missed targets for the nine localization techniques

the most accurate and most reliable localizations. Computationally, this method also provided the fastest solution.
Figure 8.6: Comparisons of the nine localization techniques for (a) experimental group 1, (b) experimental group 2, and (c) experimental group 3
Next, although these results show clear trends for the three different localization methods, there is not any evidence to support an optimal TOF technique. This agrees with the results discussed in Section 7.4.2, and is supported by the ANOVA tests which did not result in statistical significance for any of the three data sets. Figure 8.7 shows a comparison of the lateral predication accuracy for one of the targets recorded with the WTS, using the least squares method and the three TOF techniques. In this case, the three different TOF techniques all predicted the target location within 2cm of one another. This example is representative of the results for all of the target localizations, supporting the use of any of the three techniques. Since the correlation technique is computationally the simplest and fastest technique, this method was utilized for the rest of the data processing. Additionally, unlike both the regularization and Weiner filtering techniques, correlation detection does not have an adjustable parameter which must be selected depending on the soil content.

![Figure 8.7: Localization comparison for a target buried at 6cm using the least squares method and TOFs acquired with (a) correlation, (b) regularization, and (c) Weiner filtering.](image)

### 8.3.2 Discussion

Using the optimized localization method (LS-C), this section presents the localization examples for the various experimental groups. First, for Group 1, which had three target depths each with three receiver positions, the resulting error parameters are summarized in Table 8.6. The solution sets for each of the target depths are depicted in Figures 8.8. Note in these results the three transmitter positions are indicated with triangle markers, and the solution sets for each are shown in the corresponding color.

In most cases the lateral coordinates of the target were successfully located. Additionally, in all cases the average dispersion was less than the radius of the target, indicated high reliability in the target identification. In the first two cases, Figure 8.8(a) and 8.8(b), the variation in transmitter location was minimal and the
<table>
<thead>
<tr>
<th>Corresponding Figure</th>
<th>Figure Color</th>
<th>Measured Depth (cm)</th>
<th>Predicted Depth (cm)</th>
<th>Distance Error (cm)</th>
<th>Average Cluster Dispersion (cm)</th>
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<tr>
<td>Fig. 8.8(a)</td>
<td>m</td>
<td>{4.5, 7.5}</td>
<td>9.0</td>
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<td>1.0</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>{4.5, 7.5}</td>
<td>8.5</td>
<td>0</td>
<td>1.4</td>
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<tr>
<td></td>
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<td>{4.5, 7.5}</td>
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<td>1.1</td>
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<tr>
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<td>{8, 11}</td>
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<td></td>
<td>b</td>
<td>{8, 11}</td>
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<td>1.4</td>
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<td>2.2</td>
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<tr>
<td></td>
<td>b</td>
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<td>{10, 13}</td>
<td>10.8</td>
<td>0</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 8.6: Summary of localization results for the targets considered in Group 1

Solution sets for each of the transmitter locations overlap, whereas for Figure 8.8(c) the transmitter locations varied more substantially and the resulting solution sets are more distinct. For all of the experiments, the same receiver positions were used and scans were only omitted if the thresholds were not surpassed. For the case where the transmitter locations varied substantially, the bistatic GPR scans surpassing the thresholds varied accordingly, thereby varying the nearest scattering point of the volumetric scatterer. This is also demonstrated in Figure 8.9, for three linear transmitter locations.

Figure 8.8: Solution sets for different transmitter locations (triangle markers) and a target depth of (a) 5.5cm, (b) 9cm, and (c) 11cm
Figure 8.9: Comparison of solution sets for three linear transmitter locations

Next, Figure 8.10 shows the error parameters for each target in both Group 2 and Group 3. Recall that both of these groups utilized the second set of antennas which had the direct signal issues. In general, the errors for Group 2 are substantially larger than the errors for Group 3 (note the different scales on the dependent axis). This could be due to the increased uncertainty in the receiver positions. The WTS has millimeter accuracy regarding the antenna positions, whereas the experiments conducted by hand have approximately 1cm accuracy in the antenna positions. This is considered in the following chapter computationally, and shown to have a substantial effect on the localization results. Finally, Figure 8.11 shows the solution sets for the target buried at 4cm for Group 3. Finally, it is important to note that in all experiments, the lateral localization error is consistently in the direction of the receiver positions.
Figure 8.10: Bar graphs of the localization errors for data collection (a) Group 2 and (b) Group 3

Figure 8.11: Localization solution sets for WTS for a target buried at 4cm
8.3.3 Initial non-metallic target experiments

Non-metallic targets were considered computationally in previous work, [33], and experimental procedures were initialized. Due to the weaker target signals, higher precision for known information was necessary for detecting non-metallic targets. Therefore, these experiments could only be conducted using the WTS. Furthermore, due to the low-contrast between plastic and dry sand, solid plastic targets were not considered as the target reflection would not be detectable. However, given an increased contrast, such as sand-to-air or soil-to-plastic, this localization method proved successful in an initial trial. Figure 8.12 shows the localization solution set for the plastic air-filled cylindrical target which is 8cm in diameter and 2.5cm in height. Unfortunately, due to time and weather limitations in Jackson, WY, additional experiments could not be conducted. However, non-metallic targets are discussed using computational data in the following chapter.

Figure 8.12: Localization solution set using least square method and cross-correlation technique for an plastic air-filled cylindrical target

8.4 Pre-Screening Field Experiments

Finally, the pre-screening parameters, discussed in Section 6.4, were determined using the full-scale prototype. Recall that the parameters necessary to determine the PCS pattern were the radius, $s$, of each surveying circle and the angle between bistatic GPR scans, $\phi$. Figure 8.13 shows an example of the PCS (blue) when $\phi = 40^\circ$ and $s = 30cm$ (note that for each surveying circle, the transmitter is located at the center and the receiver positions are indicated in magenta). Experimentally, four potentially difficult target locations were considered in order to optimize these parameters; (1 & 2) located directly below an antenna; (3) located on
one of the surveying circles but not below a receiver; and (4) located within the gaps between circles.

Figure 8.13: Example of basic PCS where the packed surveying circles are shown in blue, the receiver locations are positioned every $40^\circ$ shown in magenta, and the potentially difficult target positions are shown.

When the target is located directly below an antenna, detection was limited by the depth of the target and the antenna separation due to the anisotropy of the antennas’ radiation pattern in the elevation plane. Essentially, the elevation angle and the probability of detection are inversely related. Since the depth of the target, and therefore the elevation angle, are unknown, the antenna separation must be selected based on the most difficult detection scenario. Therefore, GPR scans were recorded for various bistatic separations and a metallic target buried $<5$cm below the transmitter. The peak responses were then plotted versus the bistatic separation and the amplitude threshold function, Figure 8.14. Overall, the target surpassed the amplitude threshold until the bistatic separation exceeds 23cm. Based on these results a PCS circle radius of 20cm was selected.

Next, the angle of rotation between scans was determined. The maximum allowable $\phi$ depends on the ability to detect target (3) in Figure 8.13. Therefore, the target was buried 20cm from the transmitter, again at a depth of 5cm from the antennas, and the receiver was moved in $5^\circ$ increments around the transmitter, each time recording a scan. It was observed that the target could only be identified consistently when the receiver was directly above some part of the target. Using a target with an 8cm diameter, and the determined bistatic separation of 20cm, this resulted in an approximate angle of rotation of $23^\circ$, which was then rounded to $20^\circ$ so that the scans are equally spaced. The final target in Figure 8.13 to consider was target (4). However, using $s = 20$cm and $\phi = 20^\circ$, the gaps are smaller than the size of a target and therefore target (4) will at
least lie partially on one search pattern, Figure 8.15.

Figure 8.14: Peak responses for a target buried 5cm below the transmitter compared with the amplitude threshold function

Figure 8.15: Optimized pre-screening PCS pattern using $s = 20cm$ and $\phi = 20^\circ$
Due to time and experimental constraints, certain concepts and scenarios could not be tested experimentally. Instead, these parameters were analyzed computationally using the FDTD algorithm discussed in Chapter 5. This included the optimization of the receiver positions, and system capabilities for non-metallic targets buried in varying soils. Note that although the data is simulated, there are still four potential sources of localization error. First, the antennas are modeled on a rough air-ground interface; whereas the localization algorithms make the assumption that the antennas are located on a flat interface. Second, the interaction of the background signal can cause inaccuracies in the TOF. Third, the targets modeled are volumetric and therefore the scattering point for each bistatic GPR pair can vary. Finally, the dispersive soils can cause inaccuracies in determining the TOF for each GPR pair. This chapter also presents future improvements which were considered throughout the project. Note that throughout this chapter the data is analyzed using the error parameters and notations summarized in Section 8.2.

9.1 Localization in Dry Sand

The first simulation set attempted to replicate the experimental conditions in order to analyze the importance of receiver placement, receiver position accuracy, and background removal. To do this, 100 targets were simulated at varying positions and depths between 5 and 20cm. Each simulation had 150 receivers, which were positioned in feasible locations based on the WTS’s movements. These included a minimum bistatic separation of 10cm, and a maximum bistatic separation of 29cm. Figure 9.1(a) shows the transmitter position (magenta) as well as the receiver positions (red). The modeled targets were the same size as those used in
Figure 9.1: (a) Simulated receiver positions (red) compared to the transmitter position (magenta); (b) Simulated sand surface

the experiments, and the sand was modeled with the measured electrical parameters from Section 7.1.1. An example of the simulation geometry is shown in Figure 9.1(b).

In the previous chapter, three different localization and three different TOF techniques were analyzed using the experimental data, and the analysis showed the least squares solution to be the optimal localization method. This analysis was repeated using the computational data set and 20 random receiver locations. The results are summarized in Figure 9.2, and confirm the findings of the experimental results, where the performance of the least squares solution surpassed both error minimization methods. Next, using the least squares solution with the correlation based TOF technique, various parameters are analyzed.

Figure 9.2: Summary of average localization error parameters for different localization methods and TOF techniques
9.1.1 Selecting Antenna Placements

One important question that was not addressed in previous chapters, was whether or not the receiver placement would have a significant effect on the localization results and if so, how to optimize the receiver positions. In practice, it would be faster to use as few bistatic GPR pairs as possible, and therefore optimized antenna placement may be necessary to maintain accuracy. In this section, ten GPR pairs are used and five methods for selecting these pairs are analyzed. The first method was simply a randomized subset of all the receiver locations. The second method was a linear collection of receivers, which included the minimum and maximum bistatic separations, Fig. 9.3(a). In the third method, locations around the perimeter of the feasible receiver placement region were used to maximize the change in path differences, $d_{ij}$, Fig. 9.3(b). Conversely, in the fourth method, clusters of receiver positions were used in order to ensure scattering from the same point of the volumetric target, Fig. 9.3(c). In this method, the cluster location was decided by

Figure 9.3: Simulated receiver positions analyzed for localization accuracy
determining which, of four bistatic separations, resulted in the strongest target reflection and centering the
cluster at that separation. The fifth method used this same process to determine an optimal bistatic separa-
tion, but then used receivers located near the perimeter of a circle surrounding that point with a radius of
5cm, Fig. 9.3(d). The goal of this last method was to provide a compromise for maximizing the change in
path differences while ensuring scattering from the same part of the volumetric target.

Figure 9.4 shows the resulting error parameters for these five methods. Ultimately, there is significant
variation in the results depending on the receiver placement and using a one-way ANOVA test resulted in
\[ p < 0.001 \]. Overall, the maximum receiver separation and the second cluster method resulted in the greatest
accuracy, whereas the linear and tight cluster of receivers had the most significant inaccuracy. Comparing
only the two best methods using an ANOVA test did not have a statistically significant result.

Figures 9.5 and 9.6 show some examples of the localization clusters for different target locations using
these five methods (note, all targets are buried 10cm). Note that in these figures, for each example the
antenna positions are shown in one figure, and the corresponding localization results are shown in another.
These figures correspond in the marker colors, such that the black receiver locations correspond to the black
localization cluster. Additionally, although for each case ten receiver positions were selected, less than ten
receivers may be plotted, as only the receivers corresponding to GPR scans used in the localization algorithm
are shown. That is, the receivers corresponding to GPR scans which were rejected by the amplitude and
correlation thresholds are not shown.

![Figure 9.4: Comparison of simulated results using different receiver positioning methods](image)

Figure 9.4: Comparison of simulated results using different receiver positioning methods
Figure 9.5: Localization cluster comparisons for different GPR pairs where the antenna placements are shown in (a), (c), and (d), and the corresponding localization results are shown in (b), (d) and (f) respectively.
Figure 9.6: Localization cluster comparisons for different GPR pairs where the antenna placements are shown in (a), (c), and (d), and the corresponding localization results are shown in (b), (d) and (f) respectively.
9.1.2 Accuracy of Antenna Placement

In the previous chapter, it was hypothesized that the localization results using the WTS demonstrated greater accuracy compared to the experimental results conducted by hand, due to the higher accuracy in known information, i.e. the antenna positions. This hypothesis was tested using the computational data by adding random error to the known receiver locations. The standard deviation of the error was increased in 1mm increments, from 0 to 10mm. For each standard deviation, the 100 targets were analyzed using ten bistatic GPR pairs, and the average of the lateral distance error, cluster dispersion, and absolute depth error was recorded. The final results are graphed in Figure 9.7. As expected, it clear that the localization results are highly dependent on the precision of the known receiver locations. For the WTS, the standard deviation of the placement error is <1mm, whereas for the experiments conducted by hand the standard deviation is estimated at 1cm. In Figure 9.7 the results emulating the WTS’s precision hardly differ from the perfect data set. However, the results emulating the experiments recorded by hand demonstrate significant error for the lateral distance and depth error, thereby confirming the presented hypothesis. Furthermore, recall that the WTS experiments were conducted using the second set of antennas, which had large direct signals. It is believed that using the designed antennas, would also improve the WTS localization results.

Figure 9.7: Analysis of the dependence of the localization errors on the receiver location accuracy
9.1.3 Background Removal

Finally, using this set of computational experiments, the necessity for background removal was considered. As discussed in Chapter 5, the exact background responses were simulated in order to model the antenna radiation patterns. In this section, the errors in localization were compared for the results using the full received GPR signal and the results using the exact target reflections. Overall, the mean change in displacement error, cluster error, and depth error were 0.7, 0.5, and 1.6 mm respectively, which demonstrated that when using the spiral antennas, background removal has negligible results on the localization results.

9.2 System Performance in Soil

AP mines are buried worldwide, and the soils in which they are found vary substantially. Often, GPR demining systems succeed in sand but fail to translate to soils with varying water content, due to the added dispersion and increased surface roughness [71]. It is therefore desirable to test the proposed methods for numerous soil types, however with the available resources this was not feasible experimentally. Instead, two different soils were considered computationally. Figure 9.8 shows the electrical characteristics of two dispersive soils; Bosnian soil with 10% moisture content, and sandy soil with 4% moisture content. Note, that both of these soils have a higher permittivity and conductivity at the frequency range of interest, compared to the sand used experimentally. Although this increases the attenuation of the signals, these differences also

![Figure 9.8: Distributions of the electrical properties for the three dispersive soils over frequency using the four-zero model [67]; (a) Relative Permittivity (b) Conductivity](image)

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improve the contrast between plastic targets and the soil, thereby making plastic targets easier to detect.

For each soil, a rough surface was generated using \( l_c = 10 \text{cm} \) and \( \sigma_h = 3 \text{cm} \), and 15 target locations were simulated, each with a metallic and non-metallic casing. Non-metallic targets were simply modeled as cylinders of TNT with \( \epsilon_r = 2.7 \). Also, three depths were considered for each target location, 5, 10 and 15 cm, resulting in 90 unique targets for each soil. The same receiver locations discussed in the previous section were used; however, the first observation was that due to the increased attenuation, the larger bistatic separations failed to recognize even the metallic targets. Figure 9.9 shows the normalized peak target responses versus bistatic separation for the three different modeled soil types. In all cases, as the bistatic separation increases the reflection from the target decreases and is caused not only by attenuation but also the radiation patterns of the spiral antennas, which were incorporated into the model. However, both soils attenuate more rapidly compared to the dry sand, causing the larger bistatic separations to have very weak target reflections. Experimentally, these reflections wouldn’t be identifiable due to system noise. Therefore, a maximum bistatic separation needs to be established for each soil. This would be done experimentally by determining the point at which a target reflection no longer exceeds the noise. Computationally, since there isn’t any noise, the maximum separation was selected as distance where the peak target reflections stopped significantly changing. For the Bosnian soil this was at approximately 22 cm, whereas for the sandy soil this was at approximately 24 cm.

Figure 9.10 plots the average localization errors for both the metallic and non-metallic targets buried in Bosnian soil and sandy soil. As was expected, the least accurate results seem to occur for the plastic targets buried in sandy soil, since the target reflections were the weakest due to the lowest contrast between the soil and the target. Unexpectedly, the metallic targets buried in Bosnian soil seemed to have a similar

![](image)

Figure 9.9: Comparison of the attenuation of a target reflection versus bistatic separation for two soils and dry sand
Figure 9.10: Summary of average localization errors for both metallic and non-metallic targets buried in Bosnian soil and sandy soil

level of inaccuracy. However, comparing the data using a two-way ANOVA test, did not result in statistical significance for either variable, soil type or target casing. Furthermore, the mean lateral error for the non-metallic targets buried in sandy soil exceeded the mean lateral error for metallic targets buried in dry sand by <3cm, despite the dispersion, target casing material, and increased surface roughness. Figures 9.11 and 9.12 show some examples of the localization clusters for different target depths given these soils and target casings.
Figure 9.11: Localization results for metallic targets buried in (a) sandy soil and (b) Bosnian soil and non-metallic targets buried in (a) sandy soil and Bosnian soil
Figure 9.12: Localization results for metallic targets buried in (a) sandy soil and (b) Bosnian soil and non-metallic targets buried in (a) sandy soil and Bosnian soil.
9.3 Future Improvements

Two hardware improvements that should be considered in the future are increasing the bandwidth of the impulse radar system and minimizing the size of the WTS. These improvements would increase the viability of this system for humanitarian demining in more general applications.

9.3.1 Increased Bandwidth

Since the spiral antennas radiate as low as 2.4GHz, the first improvement would be to increase the bandwidth of the excitation frequency. Currently the P400 uses a lower bound of 3.1GHz due to FCC regulations, [16]; however, in a demining scenario, this would not be an issue. Therefore, the bandwidth could be increased and the center frequency could be lowered to create a monocycle excitation, which would improve TOF accuracies and decrease signal attenuation. Figure 9.13, shows an example of an impulse excitation for a center frequency of 3.3GHz, and bandwidth of 2GHz. The decreased pulse width would greatly simplify target identification as both the range and lateral resolutions would improve.

![Figure 9.13: Comparison of P400 excitation with an optimized monocycle excitation](image-url)
9.3.2 WTS Size

One necessary improvement would be to decrease the size of the WTS. As has been mentioned, the large size and accompanying weight of the WTS caused the antennas to submerge below the air-ground interface. Practically, this would be too much weight to survey a mine field. Furthermore, a smaller robot would be able to more quickly survey a minefield. Rather than using only two of the WTS’s toes for antennas, all six could be utilized; five receivers and one transmitter, Figure 9.14. Using this method would reduce the number of WTS movements to acquire scans for localization. In addition, it would provide a better distribution of antennas which, as has been shown in this chapter, improves the localization results. Appendix A has some preliminary computational results using five receivers for a slightly different localization method, and the proposed least squares solution could easily be extended for this geometry.

Figure 9.14: Schematic of the WTS with six antennas
The goal of this research was to develop an autonomous method for detecting and localizing anti-personnel landmines using a walking robotic platform, namely the Walking Tri-Sphere from Square One Systems Design. The uncertainty of landmine presence constrains a country’s economic development and results in numerous civilian causalities each year. Many attempts have been made in a wide range of fields to locate these subsurface weapons, but no method to date has provided a quick, efficient, and affordable solution. This dissertation has presented a new method for autonomously detecting anti-personnel landmines through GPR. The advantages of on-contact antennas when the air-ground interface is rough were discussed and demonstrated, and this simple change presented an ability to use simplified post-processing techniques for GPR A-scans. Circularly polarized antennas were designed and fabricated to couple into the ground, thereby resulting in greater signal strength from potential targets. Furthermore, the directivity of the antennas limited direct signals between the antennas, further minimizing the effects of the air-ground surface. Chapter 6 discussed two different geometric multi-bistatic localization methods, which were each capable of locating a subsurface scatter with a minimum of four unique bistatic GPR A-scans. These techniques were verified computationally (Ch. 9) and experimentally on the full-scale prototype in Wyoming (Ch. 8), and the optimal technique was found to be the linearized least squares solution. Using an overdetermined solution, as opposed to the closed form solutions presented in previous work, provided a greater accuracy as outlying errors did not have a significant impact on the results. Furthermore, these methods were verified for various soil properties (Ch. 9) by using the computational 3D FDTD model discussed in Chapter 5, and future optimizations such as optimal receiver placement were presented.
Overall, the proposed localization technique and final prototype are a strong candidate for humanitarian
demining. The potential for an autonomous robotic system would remove deminers from the field during the
initial scan of an area, and GPR ground-contact antennas has the potential to provide significant information
about underground targets since the effects of the air-ground interface are inherently removed. The WTS
provides a specifically advantageous design for this concept, in that the non-articulated legs results in mostly
normal contact between the antennas and the ground. Finally, the computational speed of the localization
algorithms are much quicker than the WTS’s mechanical movements and ultimately the system can be
designed such that all of the GPR processing occurs during the WTS’s movements. Overall, these methods
have exhibited promising results for a high clearance rate and low false alarm rate which are indiscriminate
to target casing, and independent of the surface topology, and the proposed hardware improvements stand
to improve both the speed and accuracy of the overall system.
The presented localization methods used the time-difference of arrivals to determine the unknown target position either by a linearized method of least squares or error minimization. These techniques however, were not the only localization methods considered during the course of this research. As mentioned in Chapter 6, another approach is to simply use the time-of-arrivals and the intersecting point of the generated ellipsoids. This trilateration technique was considered extensively through computational simulations [33] and briefly summarized below. A multiple trilateration technique was also explored computationally, but ultimately was not used due to the size of the WTS prototype. Finally, a different approach for determining the intersection of the hyperboloids generated by the time-difference of arrivals was considered both computationally and experimentally.

A.1 Trilateration

Previous work by the author explored the localization capabilities of three unique GPR scans using antennas which were capable of transmitting both right and left handed circular polarization. In this method, the three antennas were positioned on the fixated toes of the WTS as shown in Figure A.1(a), yielding three unique GPR scans. The time-of-flight for a single GPR scan was related to the major axis of a prolate spheroid formed by the antenna and unknown target locations, such that $TOF_i = \frac{2a_i}{c}$. The resulting system of equations was then,
Figure A.1: Geometry of the equidistant platform legs of the Tri-Sphere on which the antennas will be located

\[ TOF_1 \nu = 2a_1 = |L_1 - T| + |T - L_2| \]  
\[ TOF_2 \nu = 2a_2 = |L_1 - T| + |T - L_3| \]  
\[ TOF_3 \nu = 2a_3 = |L_2 - T| + |T - L_3| \]  

By defining \( L_1 \) as the origin, expanding, and substituting the above equations can be rewritten as

\[ 0 = t_x^2 + t_y^2 + t_z^2 - (a_1 + a_2 - a_3)^2 \]  
\[ 0 = \left( \frac{d}{2} - t_x \right)^2 + \left( \frac{d\sqrt{3}}{2} - t_y \right)^2 + (t_z - z_2)^2 - (a_1 + a_3 - a_2)^2 \]  
\[ 0 = (t_x - d)^2 + t_y^2 + (t_z - z_3)^2 - (a_3 + a_2 - a_1)^2 \]

which can then be evaluated for explicit relations for \( t_x, t_y, \) and \( t_z \). These equations are not included due to their length, however, note that for the \( z \) coordinate there will be two solutions, a positive result corresponding to a location in air, and a negative result corresponding to the desired subsurface location.

Computational simulations for 10% wet Bosnian soil verified this technique for varying degrees of air-ground roughness. In total, 16 different surface topologies were considered, and the resulting scattering points for each are shown in Figures A.3(a) and A.3(b), for a metallic and non-metallic target respectively.
A.2 Multiple Averaged Trilateration

In transitioning into the experimental phase, it was realized that the limited antenna dimensions would not permit transmission of both polarizations. To account for this, a multiple averaged trilateration technique was considered. In this method, all six toes of the WTS were to be utilized, with one designated transmitter and five designated receivers. In this method, similar to the previous method, closed form solutions for $T$ were determined for each combination of three receivers. The algorithm then evaluated for each of these solutions, and recorded the median result as the target location. An example of the resulting solution sets for a metallic and non-metallic target buried in Bosnian soil are shown in Figure A.3. Again the red circle indicates the WTS location, and the blue circle indicates the target location. The scattering points identify
each of the solutions, while the diamond markers indicate the final result.

### A.3 Intersecting Planes

The last localization method considered, but not implemented, was to use the time-difference of arrivals and approximate the hyperboloids as cones for a set of linear receivers, Figure A.4. This assumption is valid when the target position is not near the hyperboloid’s vertex, which lies between the receiver positions. Next, the vertical plane which would slice the cone corresponding to the hyperbola, passes through the air-ground interface between the receivers under considerations, \(0.5(R_i + R_j)\) and the slope of the plane is defined by the hyperbola parameters, \(m = b/a\). Recall that, \(a = d_{ij}/2\) where \(d_{ij}\) is the change in full path travel distance; next, \(b = \sqrt{c^2 - a^2}\) where \(2c = R_j - R_i\). Following this procedure for multiple receiver pairs, the target location can then be determined by the intersection of the planes, Figure A.5. In this example the receiver positions are shown in black, the transmitter position is shown in red, the actual target position is shown in magenta, and the air-ground intercepts are shown in cyan.
REFERENCES


[26] The Center for Arms Control and Non-Proliferation. Mine ban advocates denounce white house decision to retain landmines and abandon mine ban treaty.


