Fishing for New Physics with Massive Neutral Dibosons: Measurements of ZZ Production Cross Section and the Search for Invisible Higgs Boson Decays Beyond the Standard Model with the CMS Detector at the LHC

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Abstract of Dissertation

The Standard Model of particle physics is a theory describing the fundamental interactions and properties of subatomic particles. A key feature is its ability to explain particle mass through the Higgs mechanism, and a by-product of this mechanism is the Higgs boson. The discovery of the Higgs boson, in 2012 at CERN, completed the Standard Model particle zoo, but observed phenomena, like dark matter, remain unexplained.

The analyses presented explore proton-proton collision events resulting in a Z boson plus missing transverse energy (MET). The motivation for this is to investigate two processes: Standard Model (SM) ZZ production, and beyond Standard Model (BSM) ZH production, in particular the $\text{ZZ} \rightarrow \ell^+\ell^-\nu\bar{\nu}$ and $\text{ZH} \rightarrow \ell^+\ell^- + H_{\text{inv}}$ channels. The place-holder $H_{\text{inv}}$ is for all Higgs boson decay modes resulting in undetected “invisible” particles, which may branch to new physics, like dark matter particles. The data used are from Run 1 (2011–2012) of CMS, where proton-proton collisions at 7 TeV and 8 TeV were delivered by the LHC. The Compact Muon Solenoid (CMS) is a general-purpose detector located along the Large Hadron Collider (LHC), which is a particle accelerator at CERN in Geneva, Switzerland.

To extract these signals containing real MET from background containing fake mismeasured MET, a new “reduced MET” variable is constructed and optimized. This assists in the measurement of the ZZ production cross section. The results of the exclusive $\text{ZZ} \rightarrow \ell^+\ell^-\nu\bar{\nu}$ cross section measurement are $201^{+82}_{-69}$ fb and $264^{+81}_{-64}$ fb from
the 7 and 8 TeV portions of Run 1 data, respectively. Bayesian unfolding is used to measure a cross section of $224^{+68}_{-70}$ fb from the 8 TeV data. These results both agree with next-to-leading order predictions from the Standard Model. The differential cross section as a function of transverse momentum of the Z boson is also measured from unfolding, for the purpose of providing a way to compare data to new theories.

To distinguish $ZH \rightarrow \ell^+\ell^-+H_{inv}$ from $ZZ \rightarrow \ell^+\ell^-\nu\bar{\nu}$ a machine learning algorithm is used with physical variables as the input. A shape analysis is performed on the resulting distribution, and an upper limit is placed at 95% C.L. on the invisible branching fraction of the Higgs boson. For a Higgs boson with a Standard Model cross section and mass of 125 GeV, the observed limit on the branching fraction is 52% and the expected is 49%. Considering a mass spectrum of 115-200 GeV, a fully invisible Higgs is excluded for masses below 163 GeV.
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Chapter 1

Theory

1.1 The Standard Model

The Standard Model of particle physics is a theory of particle properties and interactions, and is the most successful theoretical description of the universe. The predictions from the model match experimental results to a high degree of precision. It is built from the gauge symmetry group $SU(3)_C \times SU(2)_L \times U(1)_Y$, which consists of unitary and special unitary groups, and describes electroweak and strong force interactions [1–3]. One recent success of the Standard Model is the discovery of the Higgs boson at both the CMS and ATLAS detectors at the LHC [4, 5]. It was the last major fundamental piece to be found in order to fulfill Standard Model predictions.

1.1.1 Particles and Interactions

All known elementary particles can be grouped in two categories: fermions and bosons. Bosons have integer spin, and fermions have half-integer spin. Fermions can then be grouped further into two families: quarks and leptons, which are each divided into three generations. Bosons are grouped into gauge bosons which are carriers of the
fundamental forces, and scalar bosons which enforce gauge invariance and generate mass. Most particles are unstable and will decay into lighter particles, except for the electron, up and down quarks (which constitute protons and neutrons), photons and neutrinos. The full zoo of Standard Model particles is shown in Fig. 1.1.

Each particle has a set of properties known as quantum numbers, which include spin, electric charge and parity. Fermions have the property of flavor. There are six flavors for quarks: up, down, charm, strange, top and bottom, and six flavors for leptons: electron, muon, tau, and three corresponding neutrino flavors. Flavor is not a conserved property, since it can be changed through weak interaction for quarks (via W boson exchange). It is conserved for leptons, however. Lepton number and baryon
number are another set of quantum numbers which are independently conserved in the Standard Model.

In the Standard Model, chirality is important for fermions in terms of interaction. Each generation of fermions has a left-handed doublet, and two right-handed singlets for quarks, but there is only one right-handed singlet for leptons: either there are no right-handed neutrinos or they do not interact. Due to this, neutrinos are predicted to be massless, which disagrees with observation and requires extensions beyond Standard Model. Table 1.1 describes the fermions in terms of their quantum numbers.

Table 1.1: List of fermions and quantum numbers [7]. Note: \( \ell \in \{e, \mu, \tau\} \), \( u \in \{u, c, t\} \), \( d \in \{d, s, b\} \).

<table>
<thead>
<tr>
<th>Particles</th>
<th>symbols</th>
<th>isospin ( T_3 )</th>
<th>hypercharge ( Y )</th>
<th>charge ( Q )</th>
<th>spin</th>
<th>lepton number</th>
<th>baryon number</th>
<th>color charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton doublet</td>
<td>( (\nu_{\ell})_L )</td>
<td>+1/2</td>
<td>-1</td>
<td>0</td>
<td>1/2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>( \ell_L )</td>
<td>-1/2</td>
<td>-1</td>
<td>-1</td>
<td>1/2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Lepton singlet</td>
<td>( \ell_R )</td>
<td>0</td>
<td>-2</td>
<td>-1</td>
<td>1/2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Quark doublet</td>
<td>( u_L )</td>
<td>+1/2</td>
<td>+1/3</td>
<td>+2/3</td>
<td>1/2</td>
<td>0</td>
<td>1/3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>( d_L )</td>
<td>-1/2</td>
<td>+1/3</td>
<td>-1/3</td>
<td>1/2</td>
<td>0</td>
<td>1/3</td>
<td>1</td>
</tr>
<tr>
<td>Quark singlets</td>
<td>( u_R )</td>
<td>0</td>
<td>+4/3</td>
<td>+2/3</td>
<td>1/2</td>
<td>0</td>
<td>1/3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>( d_R )</td>
<td>0</td>
<td>+2/3</td>
<td>-1/3</td>
<td>1/2</td>
<td>0</td>
<td>1/3</td>
<td>1</td>
</tr>
</tbody>
</table>

The quantum number called weak isospin \( T_3 \) describes how a particle interacts with the weak force. It plays the same role in weak interaction as electric charge \( Q \) does in electromagnetic interaction, and is a conserved quantity in any given interaction. Only left-handed fermions interact through the weak force, since the right-handed fermions have \( T_3 = 0 \). Weak isospin is the generator of the \( SU(2)_L \) component of electroweak gauge symmetry. Hypercharge \( Y \) is generator of the \( U(1)_Y \) component of electroweak gauge symmetry.

Before electroweak symmetry breaking, there are three (weak force) \( W \) bosons, a \( B \) boson, and two \( \phi \) complex scalar bosons. After symmetry breaking, the \( B \) and third
Table 1.2: List of bosons and quantum numbers. Note: $W^3$ and $B$ mix into $Z$ and $A$, and that $\phi^+$ does not survive electroweak symmetry breaking.

<table>
<thead>
<tr>
<th>Particles</th>
<th>symbols</th>
<th>$T_3$</th>
<th>hypercharge $Y$</th>
<th>charge $Q$</th>
<th>spin</th>
<th>lepton number</th>
<th>baryon number</th>
<th>color charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boson triplet</td>
<td>$W^+$</td>
<td>+1</td>
<td>0</td>
<td>+1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$W^3/Z$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$W^-$</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Boson singlet</td>
<td>$B/A$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Boson doublet</td>
<td>$\phi^+$</td>
<td>+1/2</td>
<td>+1</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\phi^0$</td>
<td>-1/2</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Boson octet</td>
<td>$g$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

$W$ bosons mix to give $Z$ and $A$ bosons, and only the neutral $\phi$ boson survives. This results in weak isospin and hypercharge mixing to form electric charge by:

$$Q = T_3 + \frac{Y}{2}$$ (1.1)

which is a simple result of the unification of electromagnetism and the weak force.

Table 1.2 lists boson quantum numbers. The electroweak force is mediated by the vector bosons: photons, $W^\pm$ and $Z$ bosons. The scalar Higgs boson doublet serves to enforce gauge invariance, and give particles mass. The range of the weak force is limited due to the massiveness of the $W$ and $Z$ bosons, unlike the EM force which has infinite range and zero mass.

The strong force is mediated by gluons, which are also vector bosons. Color charge is the $SU(3)_C$ generator for the strong force chromodynamics, and results in eight gluon fields. Only quarks and gluons have color charge, where there are three kinds of charges, where quarks have one charge, and gluons have two. Gluons are able to interact with each other, unlike photons. Single quarks cannot be isolated due to this force, so bare color charge is not an observable property. The strong force is not unified.
with the electroweak force in the Standard Model.

There is a finite list of interactions allowed in the Standard Model. Figure 1.2 shows how different particles couple. Fermions couple to bosons, and that is how the force between them is mediated. Bosons also couple to other bosons, and in some cases like W, Higgs and gluons couple to themselves. Interactions or couplings are described by interaction terms in the Lagrangian of the Standard Model.

Figure 1.2: Standard Model Interactions [8].

1.1.2 Fields and Lagrangians

A particle is a localized excitation, or quantum of a field. There are several types of fields that occupy spacetime, and they couple to one another in a set ways. A way to describe a theory of fields and how they couple, is to write a Lagrangian. A Lagrangian consists of two parts, a free component and an interaction component: \( \mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{coupling}} \), where \( \mathcal{L}_{\text{free}} \) describes the fields if they never interact, and \( \mathcal{L}_{\text{coupling}} \)
describes how the fields interact with each other in any given theory. The wavefunction describing a particle in quantum mechanics is promoted to a field operator consisting of creation and annihilation operators, which act on a state of a vacuum or set of particles.

A Lagrangian describing any number of fields is a function of those fields, and their spacetime derivatives: \( L = L(\phi_1, \partial_\mu \phi_1, \phi_2, \partial_\mu \phi_2, \ldots) \) So computing the 4-D version of the Euler-Lagrange equation:

\[
\frac{\partial L}{\partial \phi_i} = \partial_\mu \frac{\partial L}{\partial (\partial_\mu \phi_i)} \tag{1.2}
\]

will give the equation of motion for field \( \phi_i \).

The Standard Model Lagrangian consists of four main components, the Yang-Mills part, the Weyl-Dirac part, the Yukawa couplings part, and the Higgs part:

\[
L_{SM} = L_Y + L_{WD} + L_{Yu} + L_H. \tag{1.3}
\]

The Yang-Mills part (\( L_Y \)) describes the kinetic portion of the gauge boson fields, and how they interact with each other. The Weyl-Dirac part (\( L_{WD} \)) describes the kinetic portion of the fermion fields, and the fermion-gauge boson couplings. The Yukawa part (\( L_{Yu} \)) describes how the Higgs field couples to the charged fermions, generating their non-zero masses, and resolving hypercharge anomalies. The Higgs part (\( L_H \)) describes the kinetic portion of the Higgs field, the coupling to the gauge bosons, and the potential portion, allowing for spontaneous symmetry breaking and giving all coupled fields non-zero masses. These concepts are expanded on in the next sections.
1.1.3 Quantum Electrodynamics

The Dirac free-particle Lagrangian describes the behavior of a lone spin $\frac{1}{2}$ particle, a fermion.

$$\mathcal{L} = \bar{\Psi}(x)(i\gamma^\mu \partial_\mu - m)\Psi(x)$$  \hspace{1cm} (1.4)

Where $\Psi$ is a Dirac spinor describing the state of a fermion. However, it is not invariant to any local phase change, making the physics depend on the configuration of the fields, or gauge choice. To show this, a $U(1)_{EM}$ local phase change $\theta(x)$ is applied.

$$\Psi(x) \rightarrow \Psi'(x) = e^{-i\theta(x)}\Psi(x)$$  \hspace{1cm} (1.5)

$$\mathcal{L} \rightarrow \mathcal{L'} = \bar{\Psi}(x)(i\gamma^\mu \partial_\mu + \gamma^\mu \partial_\mu \theta(x) - m)\Psi(x)$$

Thus the Dirac free-particle Lagrangian is not invariant.

To impose $U(1)$ invariance, a gauge-covariant derivative is introduced to replace the ordinary derivative, so that: $\bar{\psi}D_\mu \psi \rightarrow \bar{\psi}' D_\mu \psi' = \bar{\psi}D_\mu \psi$. This is done by adding a vector potential $A_\mu$ to the derivative, which is physically motivated by how vector potentials displace momentum:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ieA_\mu$$  \hspace{1cm} (1.6)

where $A_\mu$ is a gauge field of a boson, in this case the photon field, or electromagnetic vector potential. To compensate for the phase change $\Psi(x) \rightarrow \Psi'(x) = e^{-i\theta(x)}\Psi(x)$, the gauge field must transform:

$$A_\mu \rightarrow A'_\mu = A_\mu + \frac{1}{e} \partial_\mu \theta(x)$$  \hspace{1cm} (1.7)

This is successful, because this kind of transformation on the vector potential leaves
the electric and magnetic fields unchanged. The modified Lagrangian is now invariant to local phase changes of $U(1)$ symmetry.

\[ \mathcal{L}' = \bar{\Psi}(x)(i\gamma^\mu D_\mu - m)\Psi(x) \]  \hspace{1cm} (1.8)

However, it still needs to fully include Maxwell’s equations, which are described by the Lagrangian:

\[ \mathcal{L}_{\text{Maxwell}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu \]  \hspace{1cm} (1.9)

Where the antisymmetric field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ gives values for electric and magnetic field strengths, and $J^\mu$ is a 4-dimensional current source. Only the kinetic term from $\mathcal{L}_{\text{Maxwell}}$ needs to be added, since it can be seen that the source term is already taken into account when $\mathcal{L}'$ is expanded.

\[ \mathcal{L}_{\text{coupling}} = J^\mu A_\nu = ie\bar{\Psi}(x)\gamma^\mu A_\mu \bar{\Psi} \]  \hspace{1cm} (1.10)

This is also known as the coupling or interaction term, since it couples charged particles to the electromagnetic field. The coupling strength $e$ is also the electric charge of an electron. It corresponds to the Feynman vertex diagram of two electrically charged fermions and one photon, shown in Fig. 1.3.

Figure 1.3: Electromagnetic interaction: Feynman diagram vertex.
Finally, the complete quantum electrodynamic Lagrangian can be written as:

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$  \hspace{1cm} (1.11)$$

A photon mass term $m_\gamma A_\mu A^\mu$ would not be gauge invariant. So $m_\gamma = 0$.

This example of $U(1)$ local phase transitions is relatively simple due to the generators commuting with the vector potential term in the covariant derivative, but in general this does not happen. In the general case where $\psi(x) \to \psi'(x) = \mathcal{G}(x)\psi(x)$, where $\mathcal{G}$ is some element of a symmetry group, the ordinary derivative of the field $\psi$ transforms like:

$$\partial_\mu \psi \to \partial_\mu \mathcal{G}\psi = \partial_\mu (\mathcal{G})\psi + \mathcal{G}\partial_\mu \psi.$$

The general covariant derivative necessarily replaces the ordinary derivative for gauge invariance, is:

$$\partial_\mu \to D_\mu = \partial_\mu + igA_\mu,$$

where $A_\mu$ is a vector potential of some kind containing matrices such that $[\mathcal{G}, A_\mu] \neq 0$, and $g$ is a coupling constant. In order for Lagrangians to be gauge invariant, the vector potential must transform in response to a change of phase. It can then be deduced that:

$$\text{if: } \psi \to \psi' = \mathcal{G}\psi$$

$$\text{then: } A_\mu \to A'_\mu = \mathcal{G}A_\mu \mathcal{G}^{-1} + \frac{i}{g}(\partial_\mu \mathcal{G})\mathcal{G}^{-1}$$  \hspace{1cm} (1.12)$$

If the phase change is unitary, then $\mathcal{G}^{-1} = \mathcal{G}^\dagger$. 

9
1.1.4 Quantum Chromodynamics

QCD arises from imposing local $SU(3)$ invariance on the Dirac free-particle Lagrangian, instead of local $U(1)$ invariance like in QED.

$$\Psi(x) \rightarrow \Psi'(x) = e^{-it^a \phi^a(x)} \Psi(x) \quad (1.13)$$

Where $t^a$ are the Gell-Mann matrices, the eight generators of the $SU(3)$ group that obeys the Lie algebra $[t^a, t^b] = if^{abc}t^c$, where $f^{abc}$ is the structure constant (sum over repeated indices). To impose $SU(3)$ invariance, a gauge-covariant derivative is built in the same fashion as that of QED:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ig_s t^a G_\mu^a \quad (1.14)$$

Where $G_\mu^a$ is a gauge field of a gluon or vector potential of the strong force, and $g_s$ is the strong coupling constant.

The QCD Lagrangian, which contains elements of the Weyl-Dirac and Yang-Mills terms, resembles that of QED, where the $F$ field strength tensor is replaced with a $G$ field strength tensor, and $\Psi$ is a color triplet field (quark).

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi \quad (1.15)$$

Where $G_{\mu\nu}^a$ is the field strength tensor for gluon $a$.

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu \quad (1.16)$$

A field strength tensor can be derived by commuting the covariant derivative operators: $[D_\mu, D_\nu]$. The first two terms are the same form as the electromagnetic field
strength tensor. The third term is a non-abelian mixing term that allows for gluon self-interaction. Expanding the terms of the Lagrangian:

\[ L^{\text{kinetic}} = \bar{\Psi}(x)(i\gamma^\mu \partial_\mu - m)\Psi(x) - \frac{1}{4}(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a)(\partial^{\mu} G^{a,\nu} - \partial^{\nu} G^{a,\mu}) \]

\[ L^{\text{gluon self}} = - \frac{g_s}{2}(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a) f^{abc} G_\mu^b G_\nu^c - \frac{g_s^2}{4} f^{abc} f^{ade} G_\mu^b G_\nu^c G_\rho^d G_\sigma^e \]

\[ L^{\text{quark gluon}} = - g_s \bar{\Psi} \gamma^\mu G_\mu^a t^a \Psi \] (1.17)

The kinetic portion is the same to that of the photon. The first and second terms of the gluon self-interacting portion correspond respectively to triple and quartic gluon vertices. The quark-gluon portion corresponds to the vertex of two quarks and one gluon, see Fig. 1.4. These interaction terms imply that quarks carry one color charge, and gluons carry two, and that all quarks and gluons couple with the same strength.

Figure 1.4: quark-gluon interaction vertex and gluon-self interaction vertices.

1.1.5 Electroweak Theory

Electroweak theory arises by imposing local \( U(1) \times SU(2) \) invariance.

\[ \Psi(x) \rightarrow \Psi'(x) = e^{-i\theta(x)} \times e^{-i\tau^i \alpha^i(x)} \Psi(x) \] (1.18)

Where \( \tau^i \) are the three Pauli spin matrices, the three generators of the\( SU(2) \) group, that obeys the Lie algebra \([\tau^i, \tau^j] = 2i\epsilon^{ijk} \tau^k\), where the structure constant \( \epsilon^{ijk} \) is
the Levi-Civita tensor. First, to impose \( U(1) \) invariance, introduce a gauge-covariant derivative just as in QED:

\[
\partial_\mu \rightarrow D_\mu = \partial_\mu + \frac{g_1}{2} B_\mu
\]

\[
B_\mu \rightarrow B'_\mu = B_\mu + \frac{2}{g_1} \partial_\mu \theta(x),
\]

Then, to impose \( SU(2) \) invariance, introduce another vector potential term \( (\tau^i W^i_\mu) \) to the gauge-covariant derivative:

\[
\tau^i W^i_\mu = \tau^1 W^1_\mu + \tau^2 W^2_\mu + \tau^3 W^3_\mu = \begin{pmatrix}
W^3_\mu & W^1_\mu - W^2_\mu \\
W^1_\mu + W^2_\mu & -W^3_\mu
\end{pmatrix} \quad (1.20)
\]

\[
\partial_\mu \rightarrow D_\mu = \partial_\mu + \frac{g_1}{2} B_\mu + \frac{g_2}{2} \tau^i W^i_\mu
\]

Where \( g_1 \) represents the \( U(1) \) hypercharge coupling, and \( g_2 \) represents the \( SU(2) \) isospin coupling. The related field strength tensors \( \propto [D_\mu, D_\nu] \) are:

\[
B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu
\]

\[
W_{\mu\nu}^i = \partial_\mu W_{\nu}^i - \partial_\nu W_{\mu}^i + g_2 \epsilon^{ijk} W_{\mu}^j W_{\nu}^k
\]

The third term in the field strength tensor \( W_{\mu\nu}^i \) is a non-abelian mixing term that allows for \( W \) self-interaction:

\[
\mathcal{L}_W = -\frac{1}{4} W_{\mu\nu}^i W^{i,\mu\nu} = -\frac{1}{4} (\partial_\mu W_{\nu}^i - \partial_\nu W_{\mu}^i)(\partial_\nu W_{\mu}^{i,\nu} - \partial_\mu W_{\nu}^{i,\mu}) + \mathcal{L}_{W-self} \quad (1.22)
\]

\[
\mathcal{L}_{W-self} = -\frac{g_2}{2} (\partial_\mu W_{\nu}^i - \partial_\nu W_{\mu}^i)\epsilon^{ijk} W_{\nu}^{j,\mu} W_{\mu}^{k,\nu} - \frac{g_2}{4} \epsilon^{ijk} \epsilon^{ilm} W_{\nu}^{j} W_{\mu}^{k} W_{\nu}^{l} W_{\mu}^{m,\nu}
\]

where the first and second terms of the \( W \) self-interacting portion, which will be revisited after symmetry breaking.

The fermion field \( \Psi \) is composed of left-handed doublet \( (L = P_L \Psi) \) and right-
handed singlet projections \((R = P_R \Psi)\), thus \(\Psi = L + R\). The doublet and singlet fields are defined as:

for leptons:
\[
L = \begin{pmatrix}
\nu_e \\
\ell^-
\end{pmatrix}_L, \quad R = \ell_R
\]

for quarks:
\[
L = \begin{pmatrix}
u_u \\
\bar{d}
\end{pmatrix}_L, \quad R = u_R + d_R
\]

From the \(SU(2) \times U(1)\) phase change, the \(SU(2)\) component does not occur in the right-handed projection:

\[
L \rightarrow L' = e^{-i\theta(x)} \times e^{-i\tau^i \alpha^i(x)} L
\]

\[
R \rightarrow R' = e^{-i\theta(x)} R
\]

and the kinetic portion of the fermion Lagrangian terms can be split into left and right-handed components, where the \(SU(2)\) vector potential is not retained in the gauge-covariant derivative acting on right-handed fields, meaning they do not interact with the W bosons:

\[
\mathcal{L}_{\text{EW}}^{\text{fermion}} = i \bar{\Psi} \gamma^\mu D^\mu \Psi \\
= i \bar{L} \gamma^\mu (\partial_\mu + i \frac{g_1}{2} B_\mu + i \frac{g_2}{2} \tau^i W^i_\mu) L + i \bar{R} \gamma^\mu (\partial_\mu + i \frac{g_1}{2} B_\mu) R
\]

The retention of the \(W\) fields coupling to left-handed fermions results in a flavor-changing interaction, shown in Fig. 1.5. The fermion mass terms are not gauge invariant, since \(m \bar{\Psi} \Psi = m (\bar{L}R + \bar{R}L)\), implying the fermions are massless, which disagrees with observation.

The \(W^{\pm}_\mu\) fields are charged, and it is convenient to define them as \(W^{\pm}_\mu = (W^1_\mu \mp i W^2_\mu)/\sqrt{2}\). The electroweak Lagrangian, which contains elements of the Weyl-
Dirac and Yang-Mills terms, then takes the form:

\[
\mathcal{L}_{EW} = -\frac{1}{2} W^{-\mu\nu} W^{\mu\nu} - \frac{1}{4} W^3_{\mu\nu} W^{3\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \mathcal{L}_{\text{fermion}}^F
\]  

(1.26)

The resulting \( W \) and \( B \) bosons are massless, which also disagrees with observation. So the option are to either drop \( SU(2) \) invariance from the theory, or introduce another field to give rise to both fermion and boson masses. This field will constitute the Yukawa and Higgs components of the Standard Model Lagrangian.

### 1.1.6 The Higgs Mechanism

To give the fermions and gauge bosons non-zero masses without violating gauge invariance, a scalar doublet field is introduced with two complex fields, giving four degrees of freedom [9–12]:

\[
\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \\ \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}
\]

and like the fermion doublets \( SU(2) \times U(1) \) invariance is imposed:

\[
\Phi(x) \rightarrow \Phi'(x) = e^{-i\theta(x)} \times e^{-i\tau^a t^a(x)} \Phi(x).
\]
The value of this field in its ground/vacuum state is determined by finding the minimum of a potential $V$.

$$V(\Phi) = \frac{m^2}{2\phi_0^2} (\Phi^\dagger \Phi - \phi_0^2)^2$$

(1.28)

In general, due to the presence of both quartic and quadratic terms in the potential, the minimum $V$ corresponds to a circular set of $\Phi$ vacuum states defined by $|\Phi|^2 = \phi_0^2$, shown in Fig. 1.6. Therefore, the vacuum state of the system is degenerate in the four-dimensional scalar field space. Choosing a vacuum from this set will break symmetry. A simple choice is $\Phi = (0, \phi_0)$. This choice of vacuum uses all degrees of freedom from $SU(2)$ symmetry to fix $\phi^i$ fields at zero, except for one of the real fields: $\phi^4 \equiv \phi_0$.

From global symmetry breaking, Goldstone bosons are generated, and then “eaten” by the gauge bosons as a consequence of local symmetry breaking, giving the gauge bosons longitudinal polarization (mass). Each Goldstone boson originates from a generator of the broken global symmetry.

Figure 1.6: Quartic + quadratic Higgs potential, allowing for non-zero vacuum expectation value [13]. Note: the field value at $A$ has a higher energy than at $B$. 
The entire Higgs Lagrangian, including kinetic terms is:

\[ \mathcal{L}_H = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \]  \hspace{1cm} (1.29)

where the gauge-covariant derivative is the same as the one on acting on fermions:

\[ D_\mu = \partial_\mu + i \frac{g_1}{2} B_\mu + i \frac{g_2}{2} \tau^i W^i_\mu. \]

The ground state is then built upon to describe excited states:

\[ \Phi_{\text{ground}} = \begin{pmatrix} 0 \\ \phi_0 \end{pmatrix} \rightarrow \Phi = \begin{pmatrix} 0 \\ \phi_0 + h(x)/\sqrt{2} \end{pmatrix} \]  \hspace{1cm} (1.30)

where the fluctuations \( h(x) \) around the vacuum expectation value is a real field called the “Higgs” field. The next step is to express the lagrangian in terms of \( h(x) \). The potential \( V \), then takes the form:

\[ V(\Phi) = m^2 \left( h^2 \right) + \frac{h^3}{\sqrt{2} \phi_0} + \frac{h^4}{8 \phi_0^2}. \]  \hspace{1cm} (1.31)

The covariant derivative of \( \Phi \) takes the form:

\[ D_\mu \Phi = \begin{pmatrix} 0 \\ \partial_\mu h/\sqrt{2} \end{pmatrix} + \frac{ig_1}{2} \begin{pmatrix} 0 \\ B_\mu (\phi_0 + h/\sqrt{2}) \end{pmatrix} + \frac{ig_2}{2} \begin{pmatrix} \sqrt{2} W^+_\mu (\phi_0 + h/\sqrt{2}) \\ -W^-_\mu (\phi_0 + h/\sqrt{2}) \end{pmatrix}. \]  \hspace{1cm} (1.32)

Plug equation 1.32 into equation 1.29 to arrive at the kinetic terms of the Lagrangian.
in terms of $h$:

$$
\mathcal{L}_H = (D^\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) = \frac{1}{2} \partial_\mu \partial^\mu h + \frac{g_2^2}{2} W^\mu_\mu (\phi_0 + h/\sqrt{2})^2 \\
+ \left( \frac{g_2^2}{4} W^3_\mu W^{3\mu} - \frac{g_1 g_2}{2} W^3_\mu B^\mu + \frac{g_1^2}{4} B_\mu B^\mu \right) (\phi_0 + h/\sqrt{2}) - V(\Phi)
$$

(1.33)

The bosons fields $W^3_\mu$ and $B_\mu$ in the second line of equation 1.33 take on masses, but the eigenstates are mixed in this basis. To unmix the eigenstates, this term can be expressed as an inner product of vectors with a matrix transforming the vectors, then diagonalize the matrix to arrive at the mass eigenstate basis of $Z$ and $A$ boson fields:

$$
g_2^2 W^3_\mu W^{3\mu} - 2 g_1 g_2 W^3_\mu B^\mu + g_1^2 B_\mu B^\mu \\
= (W^3_\mu, B_\mu) \begin{pmatrix} g_2^2 & -g_1 g_2 \\ -g_1 g_2 & g_1^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} \\
= (W^3_\mu, B_\mu) M^{-1} M \begin{pmatrix} g_2^2 & -g_1 g_2 \\ -g_1 g_2 & g_1^2 \end{pmatrix} M^{-1} M \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} \\
= (A_\mu, Z_\mu) \begin{pmatrix} 0 & 0 \\ 0 & g_1^2 + g_2^2 \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} \\
= (g_1^2 + g_2^2) Z_\mu Z^\mu + 0 \cdot A_\mu A^\mu.
$$

(1.34)

Transforming from the original basis to the mass basis is a type of rotation by an angle $\theta_W$, called the “Weinberg angle”. The fields $A$ and $Z$ can then be related to $W^3$ and $B$ by the equations:

$$
Z_\mu = W^3_\mu \cos(\theta_W) - B_\mu \sin(\theta_W) \\
A_\mu = W^3_\mu \sin(\theta_W) + B_\mu \cos(\theta_W)
$$

(1.35)
where the Weinberg angle can also be expressed in terms of coupling constants:

\[
\cos(\theta_W) = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}, \quad \sin(\theta_W) = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}.
\] (1.36)

The $Z$ and $A$ components of the covariant derivative can be expressed to show how electric charge is related to isospin and hypercharge. First, $g_1$ contains the hypercharge operator thus: $g_1 \rightarrow g_1 Y$. Second, the isospin operator is a multiple of the Pauli spin matrix: $\tau_3 \rightarrow 2T_3$. Thus you have:

\[
D_\mu = \partial_\mu + iA_\mu \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \left( T_3 + \frac{Y}{2} \right) + iZ_\mu \frac{1}{\sqrt{g_1^2 + g_2^2}} \left( g_2^2 T_3 - g_1^2 \frac{Y}{2} \right)
\] (1.37)

where the factors on the $A_\mu$ term result in coupling to the electromagnetic field by charge $Q$ from equation 1.1, describing the charge operator. The resulting Feynman vertices are shown in Fig. 1.7.

![Figure 1.7: Vertex of fermion and “new” neutral gauge bosons.](image)

The new $Z$ and $A$ bosons do not interact with one another or themselves in the Standard Model. Their field strength tensors are:

\[
Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu
\]

\[
A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.
\] (1.38)

The total electroweak Lagrangian including the Higgs Lagrangian, expressed in
terms of the $W^\pm$, $Z$ and $A$ bosons is:

$$
\mathcal{L}_{EW+H} = \frac{1}{2} \partial_\mu h \partial^\mu h - m^2 h^2 - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{\phi^2_0 (g_1^2 + g_2^2)}{4} Z_\mu Z^\mu \\
- \frac{1}{2} (D_\mu W^\mp_\nu - D_\nu W^\mp_\mu)^* (D^\mu W^{+\nu} - D^{\nu} W^{+\mu}) + \frac{g_2^2 \phi^2_0}{2} W^\mu W^{-\mu} \\
- \frac{m^2}{\sqrt{2} \phi_0} h^3 - \frac{m^2}{8 \phi_0^2} h^4 + \left( \frac{1}{4} h^2 + \frac{\phi_0}{\sqrt{2}} h \right) \left( g_2^2 W^{-\mu}_\mu W^+ + \frac{g_1^2 + g_2^2}{2} Z_\mu Z^\mu \right) \\
+ \frac{g_2^2}{4} (W^{-\mu}_\mu W^+ - W^{-\nu}_\nu W^+)(W^{-\mu} W^{+\nu} - W^{-\nu} W^{+\mu}) \\
+ \frac{ig_2}{2} (A_{\mu\nu} \sin(\theta_W) + Z_{\mu\nu} \cos(\theta_W))(W^{-\mu} W^{+\nu} - W^{-\nu} W^{+\mu}) \\
- g_2^2 \cos^2(\theta_W) (Z_\mu Z^\mu W^{-\mu} W^+ - Z_\mu Z^\nu W^{-\nu} W^+ + W^\mu) \\
+ \frac{ig_2}{2} \cos(\theta_W) (Z_\mu W^{-\mu} - Z_\nu W^{-\nu}) (D^\mu W^{+\nu} - D^{\nu} W^{+\mu}) \\
- \frac{ig_2}{2} \cos(\theta_W) (Z_\mu W^+_\mu - Z_\nu W^+_\nu) (D^\mu W^{+\nu} - D^{\nu} W^{+\mu})^*
$$

(1.39)

where after all the substitutions the $W^\pm$ field acquires a coupling to the photon field through a gauge derivative:

$$
D_\mu W^\mp_\nu = (\partial_\mu + ig_2 \sin(\theta_W) A_\mu) W^\mp_\nu .
$$

This Lagrangian gives mass expressions for all bosons, except for the Higgs boson itself. The Higgs boson mass is not predicted by the Standard Model. Most of the $SU(2) \times U(1)$ symmetry is gone from the symmetry breaking. The interaction terms, containing three or more boson fields, have no trace of $SU(2)$ symmetry. The masses for bosons become non-zero except for the photon, with the $Z$ boson mass being greater than the $W$ boson mass by a predictable ratio, which agrees with observation. It also describes how bosons couple to one another in various 3-point and 4-point vertices, shown in Fig. 1.8.
The fermions acquire mass in a different way than the bosons. They acquire mass through Yukawa couplings. In the previous subsection, it was shown that the mass terms of fermions could not be $SU(2)$ invariant. However, if the mass terms have the Higgs doublet multiplying the left-handed fermion doublet, the $SU(2)$ symmetry is restored.

$$\mathcal{L}_{\text{Yu}} = -\lambda_{\psi}(\bar{L}\Phi R + \bar{R}\Phi^\dagger L)$$  \hspace{1cm} (1.40)

After symmetry breaking, for the case of leptons, the result is effectively a gauge
invariant mass term, and an interaction term with the Higgs field (Fig. 1.9):

\[
L_{\text{lepton}}^{\text{Yu}} = -\lambda_l (\bar{\nu}, \ell)_L \left( \begin{array}{c} 0 \\ \phi_0 + h/\sqrt{2} \end{array} \right) \ell_R + \bar{\ell}_R (0, \phi_0 + h/\sqrt{2}) \left( \begin{array}{c} \nu \\ \ell \end{array} \right)_L \\
= -\lambda_l \left( \phi_0 + \frac{h}{\sqrt{2}} \right) (\bar{\ell}_L \ell_R + \bar{\ell}_R \ell_L) \\
= -\lambda_l \phi_0 \bar{\ell} \ell - \frac{\lambda_l}{\sqrt{2}} h \bar{\ell} \ell
\] (1.41)

Figure 1.9: Yukawa coupling interaction term of a fermion to the Higgs boson.

The mass of the leptons are not predicted; they are only allowed to be non-zero by this mechanism. For quarks this procedure results in more complicated mass-type terms \((\bar{\Psi}_i y_{ij} \Phi \Psi_j)\) which imply mixing of mass eigenstates, i.e. the Cabibbo-Kobayashi-Maskawa (CKM) matrix, due to “up-type” quarks having mass, unlike their lepton counterparts neutrinos. The lepton and boson masses can be written in terms of coupling constants and the vacuum expectation value:

\[
m_{\Psi} = \lambda_{\Psi} \phi_0 \\
m_{\tilde{W}} = \frac{\phi_0 g_2}{\sqrt{2}} \\
m_{\tilde{Z}} = \frac{\phi_0 \sqrt{g_1^2 + g_2^2}}{\sqrt{2}} \\
m_{\gamma} = 0
\] (1.42)
\[
\frac{m_W}{m_Z} = \cos(\theta_W) \\
e = g_2 \sin(\theta_W) = g_1 \cos(\theta_W) \\
\sqrt{2} \phi_0 = \text{VEV} = 246 \text{ GeV}
\]

The total Standard Model Lagrangian can then be written as:

\[
\mathcal{L}_{SM} = -\frac{1}{4} F_{\mu \nu}^A F^{A, \mu \nu} + |D_\mu \Phi|^2 - V(\Phi) + i \bar{\Psi} \gamma^\mu D_\mu \Psi + \bar{\Psi} g \gamma^\mu \Phi \gamma^\mu \Psi + \text{h.c.} \tag{1.43}
\]

where \( F^A \) is a placeholder for all gauge fields, and \( A \) is the index summing over all 8 \( SU(3) \) gluon fields, and all 4 \( SU(2) \) and \( U(1) \) electroweak fields, the total gauge-covariant derivative is:

\[
D_\mu = \partial_\mu + \frac{g_1}{2} B_\mu + \frac{g_2}{2} \tau^i W^i_\mu + ig_s \tau^a G^a_\mu \tag{1.44}
\]

where the additional “h.c.” term stands for the Hermitian conjugates of the \( \Psi \) terms to account for both left and right-handed matter and antimatter fermion fields.

### 1.2 Beyond the Standard Model

The Standard Model has predictive power, but does not describe all known phenomena. It does not describe gravity, does not predict the existence of dark matter, does not predict neutrino oscillation and non-zero masses of neutrinos. Extensions can be made to the Standard Model to account for these phenomena beyond the Standard Model (BSM). These extensions can be tested in colliders and either confirmed or ruled out. In the next subsections, some relevant examples of BSM theories are explored.
1.2.1 Anomalous Triple Gauge Couplings

Neutral vector bosons $Z$ and $\gamma$ do not couple directly to themselves or each other in the Standard Model, as seen in equations 1.38. However in an extension of the Standard Model, described by the Lagrangian [14]:

$$
\mathcal{L}_{VZZ} = -\frac{e}{m_Z^2} \left( f_4^V (\partial_\mu A^{\mu \alpha}) + f_4^Z (\partial_\mu Z^{\mu \alpha}) \right) Z_\beta (\partial_\beta Z_\alpha)
+ \frac{e}{m_Z^2} \left( f_5^V (\partial_\mu A^{\mu \alpha}) + f_5^Z (\partial_\mu Z^{\mu \alpha}) \right) \tilde{Z}^{\alpha \beta} Z_\beta
$$

(1.45)

where the $V = Z, \gamma$, and the dual field strength tensor $\tilde{Z}^{\alpha \beta} = \frac{1}{2} \varepsilon^{\alpha \beta \sigma \rho} Z_{\sigma \rho}$, triple neutral couplings occur at tree-level, see Fig. 1.10. The coefficients $f_i^V$, and $f_i^Z$ correspond to the couplings $ZZ\gamma$ and $ZZZ$, respectively. All the operators in equation 1.45 are Lorentz-invariant, and $U(1)_{\text{EM}}$ gauge invariant, but are not $SU(2)_L \times U(1)_Y$ gauge invariant. In particular, the terms attached to the $f_4^V$ coefficients violate charge-parity ($CP$) symmetry, and the terms attached to the $f_5^V$ coefficients conserve $CP$.

To avoid unitarity violation at energies above the cut-off scale $\Lambda$, the Lagrangian can be modified with form-factors on each coupling constant, such that:

$$
f_i^V = \frac{f_i^{V,0}}{(1 + \hat{s}/\Lambda^2)^n}
$$

(1.46)

where $\hat{s}$ is the square of the invariant mass of the diboson system, $\Lambda$ is the scale where
new physics beyond the Standard Model appears, \( f_{i,0}^{V} \) are the low-energy approximations of the couplings, and \( n \) is the form-factor power, which is not used in the LHC limits. The presence of triple gauge couplings has a kinematic effect observable in the transverse momentum (\( p_T \)) of the detected Z boson. The difference in \( p_T \) spectrum between Standard Model Z and anomalous triple gauge coupling Z is shown in Fig. 1.11.

![Anomalous triple gauge Z compared to Standard Model Z in p_T spectrum](image)

**Figure 1.11:** “Anomalous” triple gauge Z compared to Standard Model Z in \( p_T \) spectrum, where \( f_{i}^{Z} = -0.02 \) (black), Standard Model (red), and the difference between the two (blue) representing signal.

### 1.2.2 Invisible Higgs Boson Decays

The newly discovered 125 GeV Higgs boson is thus far consistent in its properties with the predictions of the Standard Model. However, it is possible that the Higgs could exhibit some behavior connecting it to phenomena beyond the Standard Model. One
way to see if the Higgs extends beyond the Standard Model is to measure the invisible branching ratio. The Standard Model predicts a small invisible branching ratio of the Higgs boson \( BR(H \rightarrow ZZ^* \rightarrow \bar{\nu}\nu\bar{\nu}) \approx 10^{-4} \), so if an excess is observed in this decay mode it would imply new physics.

One possibility is that there is an additional scalar singlet field \( S \) that couples to the Higgs [15,16] by:

\[
\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2} \partial_\mu S \partial^\mu S - \frac{\mu_S^2}{2} S^2 - \frac{\lambda_S}{4} S^4 - \frac{\lambda_{HS}}{4} S^2 \Phi^\dagger \Phi - \frac{\kappa_1}{2} S \Phi^\dagger \Phi - \frac{\kappa_3}{3} S^3 \tag{1.47}
\]

It is required that the vacuum satisfy \( \langle S \rangle = 0 \), to prevent \( h-S \) mixing, where \( S \) is a stable dark matter candidate. After symmetry breaking, the couplings of Higgs to dark matter takes the form:

\[
\mathcal{L} = \frac{\lambda_{HS}}{4} \sqrt{2\phi_0} S^2 h + \frac{\lambda_{HS}}{8} S^2 h^2 \tag{1.48}
\]

where the first term allows for the Higgs boson decay into two dark matter singlets, shown in Fig. 1.12.

Figure 1.12: Vertex of Higgs boson decay into two singlet scalar dark matter candidates.

Another possibility comes from the fact that neutrinos have non-zero masses. To account for this a see-saw mechanism involving mixing between the light neutrinos and an undiscovered set of heavy neutrinos can give neutrinos non-zero masses [17]. A set
of right-handed heavy neutrinos $N_R$ are introduced to create neutrino mass-type terms in the Yukawa portion of the Lagrangian

$$\mathcal{L} = -\bar{N}_R y_{ij} \Phi^i \ell_{Lj} - \frac{1}{2} \bar{N}_R (M_N)_{ij} N^c_{Rj} + \text{h.c.} \hspace{1cm} (1.49)$$

where $y$ is the Yukawa coupling matrix, $M_N$ is the Majorana mass matrix, and $N^c_R$ is the charge conjugate of $N_R$. After symmetry breaking, a consequence is that the neutrinos couple to the Higgs boson:

$$\mathcal{L} = -\bar{N}_R y_{\nu} L \frac{h}{\sqrt{2}} + \text{h.c.} \hspace{1cm} (1.50)$$

These couplings allow the Higgs boson to decay into a pair of neutrinos, one heavy and one light, shown in Fig. 1.13. This theory also results in $h\nu\nu$ and $hNN$ couplings as well, but the cross section corresponding to $h\nu N$ is the largest.

In some versions of Supersymmetry there are stable neutral Lightest SUSY Particles (LSPs), and this can open up decays of the Higgs boson into pairs of LSPs, for example neutralinos [18]. Theories of extra-dimensional gravity predict that the Higgs boson decays into a pair of graviscalars, or oscillates into a graviscalar and then disappears from our brane [19, 20].
1.3 Massive Neutral Diboson Production

The Feynman vertices shown in the previous sections can be connecting by their legs (by propagators), as long as the legs represent the same type of particle [21]. The Feynman rules of a model, consisting of possible vertices and propagators, can be derived from the Lagrangian. From this, any combination is a possible process in the models the vertices belong to. The processes of interest in the following analyses are $pp \rightarrow ZZ$ and $pp \rightarrow ZH$, which are Standard Model processes. In terms of Mandelstam variables, the $ZZ$ scattering proceeds via $t$-channel and $u$-channel, and the $ZH$ scattering via $s$-channel, as shown in Fig. 1.14. In both cases, one $Z$ boson decays into a pair of charged leptons, and the other neutral boson decays invisibly, meaning the decay products escape direct detection. In the case of the invisible $Z$ boson, it is predicted to decay into neutrinos in the Standard Model. In the case of the invisible Higgs boson, it can decay into four neutrinos in the Standard Model, but this is very rare. So an observed significant invisible branching ratio of the Higgs boson would be the signature of non-standard behavior.

![Feynman diagrams](image-url)
For the ZZ Feynman diagram in Fig. 1.14, the corresponding amplitude is:

\[ M = \bar{u}(p_2)\gamma_\mu g_5 \epsilon^\mu(k_2) \frac{i(p_1 - k_1) + m}{(p_1 - k_1)^2 - m^2} \epsilon^\nu(k_1) \gamma_\nu g_5 u(p_1) \]  

(1.51)

where: \[ g_5 = i \frac{g_2}{\cos(\theta_W)} \left( \frac{1 - \gamma_5 T_3}{2} - \sin^2(\theta_W) Q \right). \]

The ZH Feynman diagram has the corresponding amplitude:

\[ M = h(k_2) \left( g_2 \frac{m_Z}{\cos(\theta_W)} g_{\mu\nu} \right) \epsilon^\nu(k_1) \frac{g^{\rho\nu} - (p_1^\rho + p_2^\rho)(p_1^\nu + p_2^\nu) \frac{1}{m^2}}{(p_1 + p_2)^2 + m_Z^2} \gamma_\nu g_5 \bar{u}(p_2) v(p_1) \]  

(1.52)

where \( \epsilon, h, u, \) and \( v \) represent the Z boson, the Higgs boson, and fermions, respectively, and where the initial momenta are \( p_1 \) and \( p_2 \) and the final momenta are \( k_1 \) and \( k_2 \).

Since the leptonic decays of the Z boson are of interest, to calculate those branching ratios, the following replacement is made:

\[ \epsilon^\nu(k_1) \rightarrow \frac{g^{\mu\nu} - k_1^\mu k_1^\nu \frac{1}{m_Z}}{k_1^2 + m_Z^2} \gamma_\nu g_5 \bar{a}(k_3) b(k_4) \]  

(1.53)

where \( k_1 = k_3 + k_4 \), and \( a \) and \( b \) represent either charged lepton or neutrino pairs. Then after summing over all the initial and final polarizations the center-of-mass (CM) cross section as a function of angle is then:

\[ \frac{d\sigma_{CM}}{d\Omega} \sim \frac{|M|^2}{E_{CM}^2} \]  

(1.54)

However, due to the fact that a proton is a composite object, a collision between two is not as clean as the Feynman diagrams suggest. The center-of-mass energy per collision, in general, is less than the total energy of initial protons. Also, the initial states are convoluted by parton superpositions within the proton, and this is
represented by parton distribution functions:

\[ \sum_i \int d\xi f_i(\xi) p_{z,i} = p_z \]  

(1.55)

where \( f_i(\xi) \) is the parton distribution function for parton \( i \), \( p_z \) and \( p_{z,i} \) are the longitudinal momenta of an initial proton and its parton constituents, respectively, and \( \xi \) is the parton fraction relating \( p_z \) and \( p_{z,i} \).

1.3.1 Corrections

A Feynman diagram of a process is built from propagators and vertices. However, the more vertices there are in a process, the more suppressed it tends to be. A diagram with no self-energy loops is called a tree-level or leading-order diagram (LO). A diagram with self-energy loops is a next-to-leading order (NLO) perturbation.

The \( Z \) boson couples to quarks with a strength of about \( g_2 \). So the LO Feynman amplitude in the case of \( ZZ \) production is of the order \( \mathcal{M} = O(g_2^2) \), since there are two \( Z \)-quark-quark vertices. The coupling constant can also be expressed in terms of a fine structure constant \( \alpha_W \) such that \( g_2 = \sqrt{4\pi\alpha_W} \). The leading order cross section for this process is of the order \( \sigma \sim O(\alpha_W^2) \).

Nature does not behave according to tree-level predictions, however. There is an infinite number of diagrams that can be constructed with the same particles going in (qq) and the same particles going out (ZZ). When calculating cross section, the NLO diagrams correspond to higher orders of perturbation and contribute their own cross sections, as well as interfering with the leading order diagram and each other, constructively and destructively. The additional scattering and interference terms in the cross section are called corrections. Since there is an infinite number of possible diagrams, there are conventional cut-offs: electroweak corrections go to NLO, and QCD
corrections go to NNLO.

Figure 1.15 shows some diagrams contributing to the Feynman amplitude. Diagrams containing loops with two electroweak-quark-quark vertices have NLO amplitudes of $\mathcal{M} = \mathcal{O}(\alpha_W^2)$. Diagrams containing loops with gluon-quark-quark and triple-gluon vertices have NLO and NNLO amplitudes of $\mathcal{M} = \mathcal{O}(\alpha_W \alpha_S) + \mathcal{O}(\alpha_W \alpha_S^2)$.

In the case of electroweak corrections, a cross section can be computed:

$$\sigma_{NLO} = \sigma_{LO}(\alpha_W^2) + \sigma(\alpha_W^4) + \text{interference}(\alpha_W^3) + ... = k \times \sigma_{LO}(\alpha_W^2)$$  \hspace{1cm} \text{(1.56)}$$

where contributions from other electroweak boson loops are accounted for, and the $k$ factor is used to scale LO cross sections to the desired order. The same is done for QCD corrections, but with more terms from NNLO contributions. For the analyses presented in following chapters, the electroweak and QCD corrections are computed numerically [22,23].

In general, corrections are energy-dependent. In the case of the diboson signal and
primary background samples, the electroweak $k$ factors are dependent on the transverse momentum of the visible $Z$ boson ($p_T$):

$$k = 1 + \kappa(p_T)$$

$$\kappa(p_T) = a + b \cdot p_T$$

(1.57)

where $a$, and $b$ are parameters dependent on the process being corrected, and if $p_T$ is below a given threshold then $\kappa(p_T) = 0$. Table 1.3 shows the diboson processes and their corresponding electroweak corrections in the following analyses.

<table>
<thead>
<tr>
<th>process</th>
<th>$a$</th>
<th>$b$ [GeV$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>WZ</td>
<td>$1.9 \times 10^{-2}$</td>
<td>$-3.7 \times 10^{-4}$</td>
</tr>
<tr>
<td>ZZ</td>
<td>$5.5 \times 10^{-3}$</td>
<td>$-7.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>ZH</td>
<td>$-4.4 \times 10^{-2}$</td>
<td>$-3.3 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 1.3: NLO EWK correction parameters with threshold of 50 GeV.
Chapter 2

Experimental Apparatus

This chapter describes the Large Hadron Collider (LHC) [24] and the Compact Muon Solenoid (CMS) [25] detector, which is located along the LHC. The LHC is a two-ring superconducting synchrotron proton (and ion) accelerator and collider. CMS is a general-purpose detector with specialized subdetector systems dedicated to measuring the different physical aspects of a collision. The collective goal of these machines is to explore physics beyond the Standard Model corresponding to collision energies up to 14 TeV (7 TeV for each beam). As of today, the LHC has run at collision energies of 7 and 8 TeV.

2.1 Large Hadron Collider

The LHC runs along the pre-existing LEP tunnel at CERN, and has a 27 km circumference [24]. The tunnel ranges between 45 m and 170 m below ground, with a tilt of 1.4% sloping toward Lake Geneva and a diameter of about 4 m.

The tunnel has eight octants comprised of eight arcs and eight straight sections. The straight sections are 528 m long and can house experiments or utilities, and are
called “insertions” and are located at “points”. Proton beam collisions take place at four of the points, where the beams cross and detectors are housed (see Fig. 2.2). The detectors CMS (Compact Muon Solenoid), ATLAS (A Toroidal LHC Apparatus), ALICE (A Large Ion Collider Experiment), and LHCb (LHC Beauty) are located where pp collisions take place, around the ring at points 5, 1, 2 and 8, respectively (see Fig. 2.1). The other four straight sections do not have beam crossings. Points 2 and 8 also include the injection systems for the two beams. The beams are injected from below the LHC. Two collimation systems are contained at points 3 and 7. Point 4 has independent radio frequency (RF) systems, one for each beam, designed to accelerate the protons. Point 6 has a beam dump for each beam, where the beam is extracted vertically.

The LHC is fed protons from the injector chain shown in Fig. 2.3. First, the protons
are extracted from hydrogen gas. Second, the protons are injected into Linac2 where they accelerate to 50 MeV. Third, they are injected into PSB (Proton Synchrotron Booster) where they accelerate to 1.4 GeV. Fourth, the PS (Proton Synchrotron) accelerates them to 25 GeV. Fifth, the SPS (Super Proton Synchrotron) accelerates them to 450 GeV before they are then injected into the LHC. The nominal luminosity at CMS (and ATLAS) is $10^{34} \text{ cm}^{-2}\text{s}^{-1}$, or $10^{-2} \text{ pb}^{-1}\text{s}^{-1}$ for beams of 2,808 bunches of $1.1 \times 10^{11}$ protons each. Protons are bunched with space between the bunches cor-

Figure 2.2: Schematic of LHC layout [24].
responding to 25 ns by design, but 50 ns in the 2012 run. The beams do not collide head-on, but veer slightly to collide at a small angle to prevent unwanted activity near the interaction point (IP).

The arcs are each about 2.5 km long, and contain 1232 superconducting dipole magnets in total, each with a length of 14.3 m. The dipole magnets are responsible for curving the beams around the rings. There are also 392 quadrupole magnets, which range in lengths 5-7 m. The quadrupole magnets focusing the beams. The magnets contain two separate bores for the two proton beams. Figure 2.4 shows the cross section of a dipole. The arc sections are made of 23 regular arc cells, which are 108.9 m long, and are made of two half cells which are 53.45 m long. Each half cell contains a cold mass 5.355 m long, with a cryostat which is 6.63 m long. The LHC magnet system uses NbTi (Niobium-Titanium) Rutherford cables, cooled to 2 K by superfluid Helium. The peak dipole field strength 8.3 T limits the maximum beam energy to 7 TeV.
2.1.1 Luminosity Parameters

Integrated luminosity $\mathcal{L}$ over a duration of time is described by the expression:

$$\mathcal{L} = \int_T Ldt$$  \hspace{1cm} (2.1)

where the machine (instantaneous) luminosity $L$ is effectively time dependent, and $T$ is a cumulative span of collision time. The total number of events $N(X)$ of a process $X$ in the LHC collisions is described by the expression:

$$N(X) = \int_T L\sigma_{\text{event}}(X)dt$$  \hspace{1cm} (2.2)
where $\sigma_{\text{event}}(X)$ is the cross section for the event of a type of process $X$.

Events from physical processes beyond the Standard Model are expected to be very rare, therefore in order to observe them, LHC collisions require high beam energy, intensities, and time. The dependence of machine luminosity on beam parameters is described by the expression:

$$L = \frac{N_b^2 n_b f_{\text{rev}} \gamma_r}{4\pi \epsilon_n \beta^*} F$$

(2.3)

where $N_b$ is the number of particles in a bunch, $n_b$ is the number of bunches in a beam, $f_{\text{rev}}$ is the revolution frequency of the beam, $\gamma_r$ is the relativistic gamma factor, $\epsilon_n$ is the normalized transverse beam emittance, $\beta^*$ is the beta function at a collision point, and $F$ is a factor of geometric luminosity reduction from the crossing angle at the IP. The expression describing $F$ is:

$$F = \left(1 + \left(\frac{\theta_c \sigma_z}{2\sigma^*}\right)^2\right)^{-\frac{1}{2}}$$

(2.4)

where $\theta_c$ is the full crossing angle at the IP, $\sigma_z$ is the RMS of the bunch length, and $\sigma^*$ is the RMS of the transverse beam size. This assumes that $\sigma_z \ll \beta^*$, and that the set of parameters for both beams are equal. The nonlinear beam-to-beam interaction limits the maximum particle density per bunch. Beam-to-beam interaction is measured by the linear tune shift $\xi$, which is given by the expression:

$$\xi = \frac{N_b r_p}{4\pi \epsilon_n}$$

(2.5)

where $r_p$ is the classical radius of the proton, $r_p = \frac{e^2}{4\pi\epsilon_0 m_p c^2}$. The values of beam parameters are in Table 2.1 for both the original design specifications and the 2012 run. The LHC beam energy was 3.5 TeV per beam in 2010 and 2011, and was increased to 4 TeV in 2012. The maximum instantaneous luminosity $L$ was only a fraction of
## 2.2 Compact Muon Solenoid Introduction

The CMS detector measures the products of proton-proton collisions from the LHC [25]. It is barrel-shaped with a 15 m diameter, has a length of 25 m, and weighs 12,500 tons, and is 100 m underground, between Lake Geneva and the Jura mountains. CMS has three main subdetector systems, an inner tracker, calorimeters, and a muon tracking system. Figure 2.6 shows a schematic of CMS.

At 14 TeV, the total proton-proton cross section is expected to be 100 mb. The design luminosity of the LHC corresponds to $10^9$ collisions per second. This rate is reduced for the sake of storage and processing by an online event selection. By using high timing resolution and high granularity detectors, the effect of pile-up collisions, where there are multiple inelastic proton-proton collisions per bunch crossing, can
be reduced. The particles from pile-up interactions would otherwise be mistaken for particles from the primary interaction region. The large number of particles produced in these interactions can also lead to radiation damage, which can impact the detector’s performance over time.

To meet the goals of the LHC physics program, CMS is required to perform the following list of tasks:

- For muons system:
  - Good identification and momentum resolution over a range of muon momenta and angles
  - Determine the charge of muons with $p < 1$ TeV
  - Good resolution of dimuon mass ($\approx 1\%$ at 100 GeV)
Figure 2.6: The CMS detector shown with humans for scale [25].

- For inner tracker:
  - Good momentum resolution and reconstruction efficiency for charged particles
  - Efficient triggering and offline $b$-jet and $\tau$ tagging

- For calorimeters:
  - Good resolution for electromagnetic energy
  - Good mass resolution ($\approx 1\%$ at 100 GeV) for dielectrons and diphotons
  - Efficient isolation of leptons and photons at high luminosities
  - Good $\pi^0$ rejection
  - Good resolution for missing transverse energy and dijet mass
The following sections discuss the subdetector systems and other machinery that comprise the CMS detector.

### 2.3 Superconducting Solenoid

The superconducting solenoid is a central feature of CMS, shown in Fig. 2.7. It is designed to reach 4 Tesla in field strength, but was set at 3.8 Tesla for longevity, with a stored energy of 2.3 GJ. Its function is to bend the trajectories of charged particles to measure electric charge sign and momentum. This field strength is necessary to determine the electric charge of muons with $p > 1$ TeV. The solenoid has a bore of diameter 6 m and length 12.5 m.

![Superconducting solenoid with sectioned cryostat](image)

Figure 2.7: Superconducting solenoid with sectioned cryostat [25].

The flux is returned through a iron yoke weighing 10,000 tons, and is composed of five wheels, and two endcaps, which are each composed of three disks. The solenoid
surrounds the inner tracker and the calorimeter systems, and is encased in a cryostat that is cooled to 4.5 K with liquid helium. There is also a vacuum system to provide insulation inside the cryostat, which has a volume of 40 m$^3$. The cold mass consists of five modules, is 220 tons, and has four-layers of wound NbTi conductor. The amount of stored energy per unit cold mass is 11.6 KJ/kg by design. The conductor is a thin Rutherford-type cable, extruded with aluminum and reinforced by an aluminum alloy.

2.4 Inner Tracker

At the center of the detector is the tracking system, which contains silicon pixel and microstrip detectors. The inner tracker reconstructs the trajectories of charged particles with $p_T > 1$ GeV to perform the following important tasks:

- Reconstruction of primary decay vertices, secondary decay vertices, and the impact parameter: the displacement of tracks from those vertices. This allows for the identification of several kinds of particles and provides necessary input to the high level trigger:
  - Reconstruction of electrons and muons (the electromagnetic calorimeter and muon system do this as well)
  - Reconstruction of taus by recognizing various decay topologies
  - Reconstruction of heavy flavor quarks (by using secondary decay vertices).

The inner tracker has a diameter of 2.5 m, a length of 5.8 m, and has a total of 200 $m^2$ of active silicon area. It surrounds the interaction point, and sits within the solenoid. The magnetic field is homogenous throughout the tracker. The tracker is radiation resistant, has high granularity, and fast response to reliably identify trajectories and
attribute them to the correct bunch crossing. Particle trajectories are reconstructed as “tracks”. The inner tracker is composed of two detector subsystems: the pixel tracker and the silicon strip tracker, which both cover a pseudorapidity range of $|\eta| < 2.5$. The inner tracker layout is shown in Fig. 2.8.

![Figure 2.8: Schematic of CMS tracker. Dashes represent a detector module. Double dashes represent back-to-back modules [25].](image)

The pixel tracker is composed of three barrel layers (BPix) and two endcap disks (FPix) placed on opposing sides of the interaction point. The radii of the three barrels are 4.4 cm, 7.3 cm, 10.2 cm, and each have a length of 53 cm. The two endcap disks, on each side, extend from 6 to 15 cm radius, and are placed at $z = \pm 34.5$ cm and $z = \pm 46.6$ cm. The pixel tracker consists of 66 million pixel cells (48 million in BPix, 18 million in FPix) and covers an area of about 1.06 m$^2$ (0.78 m$^2$ in BPix and 0.28 m$^2$ in FPix). Each pixel has an area of about $100 \times 150 \mu m^2$. The pixel tracker delivers three high-precision position measurements on each charged particle trajectory.

The silicon strip tracker occupies the radial region of 20 cm to 116 cm. The silicon strips are parallel to the beam in the barrel, and arranged radially in the end caps. It is composed of three subsystems: the Tracker Inner Barrel and Disks (TIB/TID),
the Tracker Outer Barrel (TOB), and the Tracker EndCaps (TEC). The TIB and TID span radially from 20 cm to 55 cm, and are each composed of four barrel layers, and three disks at each end. They deliver up to four $r$-$\phi$ measurements per trajectory. This is done by using silicon micro-strip sensors with a thickness of 320 $\mu$m, with strip pitch (which is inter-strip distance) [30] of 80 $\mu$m in the first two layers and a strip pitch of 120 $\mu$m in the last two layers. This results in a resolution of 23 $\mu$m and 35 $\mu$m, respectively.

The strip pitch in TID ranges from 100 $\mu$m to 141 $\mu$m. The TOB surrounds the TIB and TID, spans radially to 116 cm, and is composed of six barrel layers. It delivers six $r$-$\phi$ measurements per trajectory. This is done by using micro-strip sensors with a thickness of 500 $\mu$m and strip pitch of 183 $\mu$m on the first four layers and 122 $\mu$m on the last two layers. This results in a resolution of 53 $\mu$m in the first four layers and 35 $\mu$m in the last two layers.

The TID/TIB/TOB subdetectors span the $z$ range of ±118 cm. The two TECs cover the region beyond this range, 124 cm < $|z|$ < 282 cm, and 22.5 cm < $|r|$ < 113.5 cm. Each TEC is composed of nine disks, with up to seven rings of micro-strip detectors. The TECs deliver up to nine $\phi$ measurements per trajectory. The micro-strips have a pitch ranging from 97 $\mu$m to 184 $\mu$m, a thickness of 320 $\mu$m in the first four rings, and 500 $\mu$m in the last three rings.

### 2.5 Electromagnetic Calorimeter

The CMS electromagnetic calorimeter (ECAL) is hermetic, homogenous and built out of lead tungstate crystals (PbWO$_4$). The purpose is to provide a high resolution measurement of energy deposits from electromagnetic showers. Crystalized lead tungstate scintillates when electrons and photons traverse it, and the emission is proportional to
the energy of the traversing particle. This is important for Higgs boson detection as one of the postulated decay channels is to two photons.

The high density crystals (8.28 g/cm$^3$) allow for fast calorimetry, fine granularity, and for the calorimeter to be compact. The LHC bunch crossing time is within the same order of magnitude as scintillation decay time of these crystals. A large fraction of light emitted from scintillation, approximately 80%, is emitted within 25 ns. The scintillation light emitted from the crystals is blue-green ($\lambda \sim 420-430$ nm).

There are 61,200 crystals mounted in the barrel (EB), and 7324 mounted in each of the two end caps (EE). To identify neutral pions, a preshower detector (ES) made of silicon strips is placed in front of the end caps. Avalanche photodiodes (APDs) are placed in the barrel, and vacuum phototriodes (VPTs) in the end caps, and are used as photodetectors. A schematic cross section of the ECAL is shown in Fig. 2.9.

Figure 2.9: Longitudinal view of ECAL layout [31].
2.5.1 Barrel Region

The EB covers a pseudorapidity range $|\eta| < 1.479$. The granularity is 360-fold in $\phi$ and $(2 \times 85)$-fold in $\eta$. The crystals are tapered, and with shape varying with position in $\eta$. To avoid cracks where particle could escape detection, the crystals are arranged in a quasi-projective geometry. Crystals are angled (3°) with respect to the vector from the interaction point in both the $\phi$ and $\eta$ projections. The cross section of a crystal is $22 \times 22$ mm$^2$ at the front face and $26 \times 26$ mm$^2$ at the rear face, which equates to about $0.0174 \times 0.0174$ in $\eta$-$\phi$ space. The length of a crystal is 230 mm, which equates to 25.8 radiation lengths. The total barrel crystal weight is 67.4 tons, and volume is 8.14 m$^3$.

A submodule involves the crystals being contained within a thin-walled (0.1 mm) structure with a layer of aluminum facing the crystal. Modules are built from submodules such that they are arranged by their position in $\eta$. Each module contains 400 or 500 crystals. A supermodule is assembled from four modules, so the total assembly contains 1700 crystals.

2.5.2 Endcap Region

The EE covers a pseudorapidity range $1.479 < |\eta| < 3.0$. The endcap consists of supercrystals (SCs), which consist of crystals arranged in a $5 \times 5$ configuration, in a carbon-fibre structure. The endcaps are split into two halves called “Dees”. Each Dee contains 3,662 crystals in 138 standard SCs and 18 partial SCs. The crystals are oriented toward a focal point located on the opposing side of the interaction point by 1,300 mm. The cross section of a crystal is $28.62 \times 28.62$ mm$^2$ at the front face, and $30 \times 30$ mm$^2$ at the rear face. The length of a crystal is 220 mm, which equates to 24.7 radiation lengths. The total weight of the endcap crystals is 24.0 tons and the volume
is $2.90 \text{ m}^3$. The calorimeter layout is shown in Fig. 2.10.

Photodetectors are used to amplify the signal from crystals, since the light yield is relatively low. They also must be insensitive to other radiation. In the barrel, the photodetectors are avalanche photodiodes (APDs), developed specifically for CMS. A pair of APDs are mounted on each crystal, and each APD has an active area of $5 \times 5 \text{ mm}^2$. In the endcaps, the photodetectors are vacuum phototriodes (VPTs), developed specifically for CMS. Each VPT is 25 mm in diameter, with an active area of about 280 mm$^2$. One VPT is mounted to the back of each crystal.

### 2.5.3 Preshower

The ES is a two-layer sampling calorimeter which covers a pseudorapidity range of $1.653 < |\eta| < 2.6$, and sits in front of the EE. Photons and electrons interact with the
lead radiator layers and create electromagnetic showers. Silicon strip sensors are placed behind radiators to measure deposited energy and shower profiles. The first layer and second layers have a material thickness of 2 and 1 radiation lengths, respectively. The material thickness of the first layer causes 95% of incident photons to cause a shower. The total thickness of the ES is 20 cm. Each silicon sensor has an area of $63 \times 63 \text{ mm}^2$, with an active area of $61 \times 61 \text{ mm}^2$ which is divided into 32 strips, each with a pitch of 1.9 mm. Due to how thin the pitch is, the spatial resolution of the ES is higher than the EE, which makes it useful for detecting $\pi^0 \rightarrow \gamma \gamma$ decays.

2.6 Hadronic Calorimeter

The CMS hadronic calorimeter (HCAL) is a hermetic calorimeter made of repeating layers brass absorber and scintillating plastic. The purpose is to detect a wide range of final states from high-energy processes. When a hadronic particle interacts with the absorber it produces secondary particles in a shower that can interact with the scintillator to produce blue-violet light. It is particularly useful for the measurement of hadronic jets, neutrinos, and exotic particles which leave missing transverse energy in an event.

The hadronic calorimeter envelops the tracker and electromagnetic calorimeter. The amount of material that can be used to absorb hadronic showers in the barrel (HB) is limited by the presence of the ECAL and solenoid, such that the allowed radial range is $1.77 \text{ m} < R < 2.95 \text{ m}$. Due to this limitation, an additional hadron calorimeter (HO), also called a “tail catcher”, is placed outside the solenoid. The endcaps (HE) is complemented by forward hadronic calorimeter (HF). The HB, HE, and HO cover a pseudorapidity range of $|\eta| < 3.0$. The HF are located 11.2 m from the interaction point extending the pseudorapidity range to $|\eta| < 5.2$. The layout of the HCAL is
Figure 2.11: Longitudinal view highlighting HCAL subdetectors [25].

shown in figure 2.11.

2.6.1 Barrel Region

The HB covers a pseudorapidity range of $|\eta| < 1.3$. The two half-barrels in HB consist of 36 identical azimuthal wedges, shown in Fig. 2.12. The wedges are made of flat brass absorber plates, with a density of 8.53 g/cm$^3$. Wedges are segmented into four sectors along $\phi$, and aligned with the beam axis. The absorber plates are arranged such that there is no dead material in the radially projective extent of a wedge. The absorber consists of a front stainless steel plate, eight brass plates, six thicker brass plates, and another stainless steel plate, with respective thicknesses of 40 mm, 50.5 mm, 56.5 mm, and 75 mm, which in total equates to 5.82 interaction lengths. Thickness increases with polar angle ($\theta$), and equates to about 10.6 interaction lengths at $|\eta| = 1.3$. The ECAL adds about 1.1 interaction lengths of material.

The plastic scintillator is divided into 16 sectors in $\eta$ space. This results in a
segmentation of $0.087 \times 0.087$ in $\eta$-$\phi$ space. The smallest scintillator unit is called a “tile”. The active medium consists of 70,000 tiles of wavelength-shifting fibers (WLS), with a diameter less than 1 mm, that shift the blue-violet light into green light. The green light is then fed through optical cables to a decoding unit (ODU) to arrange the fibers, which are then fed to a hybrid photodiode (HPD).

Tiles are grouped in $\phi$ into a single mechanical scintillator tray unit, also called a “megatile”. Trays are then arranged in slots within the wedge, shown in Fig. 2.13.

The successive tile layers are summed optically to form “towers” in $\eta$, shown in Fig. 2.14. Optical summation covers the path of a particle through the HCAL, is a measure of energy, and can be used to identify the particle type.
Figure 2.13: HB wedge with hermetic scintillator design [25].

Figure 2.14: Longitudinal view of HCAL tower segmentation. The different colors represent optical grouping of scintillator layers into different read-outs [25].
2.6.2 EndCap Region

The HE covers a pseudorapidity range of $1.3 < |\eta| < 3$, a region containing about 34% final state particles. The high luminosity of the LHC requires the HE to handle high counting rates and to be radiation resistant. It must also be non-magnetic due to its proximity to the ends of the solenoid. To fully contain hadronic showers, the number of interaction lengths is maximized. Jet energy resolution is limited in HE due to parton fragmentation, pile-up, and magnetic field effects. The absorber has a hermetic design to minimize cracks between HE and HB. The material thickness of the calorimeter equates to about 10 interaction lengths.

The trapezoidally shaped scintillator trays, containing wavelength-shifting fibers to collect scintillation light, are inserted into the gaps of the absorber. Signals from scintillator trays are transferred via optical cable to photodetectors. The photodetectors are multipixel hybrid photodiodes (HPDs), due to having large dynamical range, and low sensitivity to magnetic fields. There are 20,916 tiles in both HE calorimeters, totalling to 1368 trays. The granularity of calorimeters is $0.17 \times 0.17$ in $\eta$-$\phi$ space for $|\eta| \geq 1.6$, and $0.087 \times 0.087$ for $|\eta| < 1.6$.

2.6.3 Tail Catcher

The HO has a pseudorapidity range of $|\eta| < 1.3$ and is designed to be a tail catcher of hadronic showers, since the stopping power of the EB and HB, in this region, does not fully contain hadron showers. The HO is placed outside the solenoid, before and within the iron yoke, and utilizes the solenoid coil as an absorber. It is used to identify showers with starting points far from the primary vertices and to measure energy deposited after HB. The geometry of the muon system constrains the HO.

HO is divided into rings like the muon system and are indexed as $-2, -1, 0, 1, 2$.
along the $z$-axis. The central positions of the rings are $-5.342 \text{ m}$, $-2.686 \text{ m}$, $0 \text{ m}$, $+2.686 \text{ m}$, and $+5.342 \text{ m}$, respectively. Each ring has 12 identical sectors in $\phi$ space, each sector has 6 slices in $\phi$ space, and each slice is divided along $\eta$. The tiles are positioned to allow for a tower granularity of $0.087 \times 0.087$ in $\eta$-$\phi$ space.

Ring 0 has two layers of tile planes on either side of a 19.5 cm thick iron layer iron at radial distances of 3.82 m and 4.07 m, respectively, since at $\eta = 0$, HB has minimal absorber depth. All other rings have one layer at a radial distance of 4.07 m. The tiles in each slice are mechanically held together as a tray. Scintillation light from the tiles is collected in WLS fibers and transported to photodetectors located on the iron yoke. Both layers of ring 0 are divided into 8 parts in $\eta$ space, and indexed as $-4, -3, -2, -1, +1, +2, +3, +4$. Rings $\pm 1$ are divided into 6 parts, and rings $\pm 2$ are divided into 5 parts in $\eta$ space. The full length of a tray ranges from 2119 mm to 2510 mm. The calorimeter system has a minimum total depth of 11.8 interaction lengths, except for the regions between the barrel and endcaps. The HO layers embedded in the muon system rings is shown in Fig. 2.15.

![Figure 2.15: Longitudinal (left), and transverse (right) views of HO layers [25].](image)
2.6.4 Forward Calorimeter

The HF has a pseudorapidity range of \(3.0 < |\eta| < 5.2\), where there is a high particle flux. The energy deposited in the HF per proton-proton collision is 760 GeV, compared to 100 GeV in the rest of the detector. So the active material is made of quartz fibers, as it is radiation resistant. A hermetic shield made of steel, lead, concrete and polyethylene protects PMTs and front-end electronics from radiation damage. The HF is a cylindrical steel structure, with a radius of 130 cm, with a central hole for the beam pipe with a radius of 12.5 cm. The front face is 11.2 m from the interaction point. It is divided into 36 \(20^\circ\) modular wedges, with 18 beyond each endcap. A read-out box (RBX), which services half a wedge, houses 24 PMTs, which collect light. A cross-sectioned view of HF is shown in Fig. 2.16.

![Figure 2.16: Cross sectional view of HF. All dimensions in mm [25].](image)

Charged particle showers generate Cherenkov light within quartz fibers, which makes the calorimeter mostly sensitive to electromagnetic components of showers. The
absorber is made of 5mm thick plates of steel. Half the fibers run over the full depth of the absorber, which is 165 cm, or 10 interaction lengths, and the other half starts at a depth of 22 cm from the front of the detector. This arrangement allows for discrimination between showers generated by electrons and photons, which deposit a large fraction energy in the first 22 cm, and hadronic showers, which produce an equal amount of signals in both calorimeter sets. The fibers are bundled to form towers of granularity of $0.175 \times 0.175$ in $\eta$-$\phi$ space.

2.7 Muon System

The CMS muon system has three functions: muon identification, momentum measurement, and triggering. Muons are less affected by the stopping power of the tracker material than electrons, so muons propagate beyond the central tracking system and calorimeters. The muon system is composed of 250 drift tubes (DTs), 540 cathode strip chambers (CSCs), 610 resistive plate chambers (RPCs), and covers a pseudorapidity range of $|\eta| < 2.4$. This detection system is of central importance, since muons are comparatively easier to detect, and mass resolution is best for muon final states. This is particularly useful for Higgs boson detection, since some decays result in to two or four muons.

The high uniform solenoidal magnetic field, mostly contained within the yoke, allows for good muon momentum resolution, charge measurement, and trigger capability. The return yoke also serves as a hadron absorber, assisting in the identification of muons. The muon system consists of 25,000 m$^2$ in total detection plane area, and has a barrel and two planar endcap regions. For muon identification, three types of gaseous particle detectors are used.
2.7.1 Drift Tubes

In the barrel region, background flux induced by neutrons is low, as well as muon rate. Barrel drift tubes (DT) are standard rectangular drift cells, and span a pseudorapidity range $|\eta| < 1.2$, and have an average size of $2\text{m} \times 2.5\text{m}$. There are four stations/layers within and around the yoke. These are arranged as concentric cylinders around the beam axis, in five wheels ($+2, +1, 0, -1, -2$). The inner-most stations contain twelve sets of chambers, eight (in two groups of four) which measure muon position in $r$-$\phi$ space (magnetic bending plane), and four which measure in the $z$ direction. Two additional layers are embedded within the yoke hull itself, due to space constraints inside and outside the yoke. The layers are shown in Fig. 2.17. In each of the twelve
sets of chambers, are four chambers labelled MB1, MB2, MB3, MB4, placed at radii 4.0 m, 4.9 m, 5.9 m, 7.0 m, respectively. The inner three layers have 60 drift chambers each, and the outer layer has 70. The number of chambers in each station and their orientation allow for good efficiency, good time resolution, connecting muon hits into a single track trajectory, rejecting background hits, and efficiently identifying standalone bunch crossings. There are approximately 172,000 sensitive wires, with lengths around 2.4 m in $r$-$\phi$ space, in the chambers. The yoke supports between the chambers leave twelve dead spots in $\phi$, but supports are placed to not overlap in $\phi$ projection.

Drift chambers can be used as tracking detectors, due to the low expected rate, and relatively low magnetic field. The basic drift unit is a tube, with a cross section $13 \times 42$ mm$^2$. The maximum path and time of drift in a drift cell is 21 mm, which corresponds to a drift time of 380 ns in a gas mixture of 85%Ar + 15%CO$_2$. This is negligible occupancy due to the short path and time. When a minimum ionizing particle (MIP) strikes gas atoms, they become ionized. The displaced electrons drift to the positively

Figure 2.18: Material thickness for muons, in units of interaction length as a function of pseudorapidity [25].
charged wires. By registering the position along the wire where the electrons hit, as well as calculating the initial distance of the muon from the wire, DTs give the two dimensional position of a muon hit.

A drift tube chamber consists of two to three superlayers (SL), which consist of four layers of rectangular drift cells each. Each drift cell in each chamber is offset by half the length of a cell with respect to neighboring cells to eliminate dead spots in efficiency. A drift cell is shown in Fig. 2.19. In the inner SL, wires run perpendicular
to the beam axis, and measure the track position in $z$. In the outer two SLs, wires run parallel to the beam axis, and measure track position in $r$-$\phi$ space. There is no inner SL in the fourth station, which can only take $\phi$ measurements. The anode wire is made of gold-plated stainless-steel with a diameter of 50 $\mu$m. Field electrode strips made of 50 $\mu$m thick aluminum tape, and sit above and below the anode wire. Cathodes are placed on either side of I-beams, are also made of aluminum tape, and are insulated by mylar tape from the I-beam. The way the strips shape the electric field along with the gas mixture allow for a linear relationship between drift time and path length in the presence of the magnetic field. Drift cell operating gas gain is $10^5$, and the voltage difference between wires and strips is within $1.75 \text{ kV} < V < 1.85 \text{ kV}$ to maintain the

Figure 2.19: Drift cell showing drift lines and crossing muon [25].
2.7.2 Cathode Strip Chambers

There are four stations in the Endcap Muon system covering a pseudorapidity range of $0.9 < |\eta| < 2.4$, shown in Fig. 2.20. Stations consist of cathode strip chambers (CSCs) positioned in groups of rings orthogonal to the beam axis, shown in Fig. 2.21. There are a total of 540 CSCs, which are labelled by their station and ring: there are 72 in ME1/1, 72 in ME1/2, 72 in ME1/3, 36 in ME2/1, 72 in ME2/2, 36 in ME3/1, 72 in ME3/2, 36 in ME4/1, and 72 in ME4/2. The largest chambers are from ME2/2 and ME3/2 and have a size of $3.2 \times 1.5$ m$^2$. All chambers overlap, except for those in the ME1/3 ring, providing continuous coverage in $\phi$. The rings are arranged such that a muon with a pseudorapidity $1.2 < |\eta| < 2.4$ is detected by three or four CSCs. If the muon has a pseudorapidity $0.9 < |\eta| < 1.2$ then it is detected with the CSCs and the DTs. Muons with $|\eta| > 1.6$ are detected by RPCs as well.

Chambers are trapezoidal, shown in Fig. 2.22 and cover a $\phi$ space of either $10^\circ$ or
20°. Each CSC has six anode wire planes alternating with seven cathode strip panels, and provides pattern recognition to match hits to those from other stations and from the inner tracker. This serves to reject non-muon backgrounds. The anode wires are arranged azimuthally and provide precise measurement of muon position in $\eta$ space, or radial coordinate, and the beam-crossing time of a muon. The cathode strips of each chamber are arranged radially, have a constant $\Delta \phi$ width, and provide precise measurement of muon position in $r$-$\phi$ space, and are approximately perpendicular to the anode wires. When a MIP strikes gas atoms, it ionizes them. The displaced electrons drift in the electric field to the anode wires, causing an avalanche, and the
ions drift to the cathode strip, and induce a charge pulse perpendicular to the wire direction, to give a precise measurement of the two dimensional position of the MIP.

In the endcap region, the background rate is high, muon rate is high, approaching 1 kHz/cm$^2$, and there is a large non-uniform magnetic field. The CSCs are ideal for this, since they can operate under these conditions. Plus, precise gas, temperature, or pressure control are not required. They have 99% efficiency for finding track stubs by the Level-1 trigger per chamber, and a 92% probability of identifying correct bunch crossings by the Level-1 trigger per chamber. They have an $r$-$\phi$ spatial resolution of 2 mm at the Level-1 trigger, and an offline $r$-$\phi$ spatial resolution for ME1/1 and ME1/2 chambers of 75 $\mu$m, and 150 $\mu$m for the other chambers.

Figure 2.22: Layout of a single CSC (left). Illustration of the principle of CSC operation: interpolating induced charges to obtain a precise localization (right) [25].
The CSCs seven trapezoidal panels have a thickness of 16.2 mm, shown in Fig. 2.23. The panels are made from 12.7 mm thick polycarbonate, with two 16.2 mm FR4 skins glued to both sides of every other panel (1,3,5,7), leaving six 9.5 mm gas gaps. The panels are wrapped in copper 36 µm thick, forming the cathode planes. A pattern consisting of 80 strips is milled on one of each panel. The strips are radial, so they have a pitch that goes from 8.4 mm to 16 mm, with a 0.5 mm gap between strips. The three anode panels (2,4,6) have anode wires wrapped around them. Wire spacing is approximately 3.2 mm, and run a length of up to 1.2 m. The wires are gold-plated tungsten with a diameter of 50 µm. Each wire plane is divided into five independently regulated high-voltage (HV) sections by spacer bars. The nominal HV is 3.6 kV, which corresponds to a gas gain of $7 \times 10^4$. Gas enters through an inlet in a cathode gap bar, and flows to other planes through special holes in the panels. The nominal gas mixture
is 40%Ar + 50%CO$_2$ + 10%CF$_4$. The leak rate is < 1% at 7.5 mbar of pressure. A MIP displaces approximately 100 electrons per gas gap. The charge per MIP is about 1 pC per avalanche, shown in Fig. 2.22.

The 72 ME1/1 chambers differ from the others. The anode wires are tilted at an angle of 29° (Fig. 2.24). This wire tilt compensates for the Lorentz force from the magnetic field, allowing the strips to be parallel to electron drift for precise measurement of the $r$-$\phi$ position.

Offline reconstruction efficiency of simulated single muons is 95–99% except where the DTs meet the CSCs ($|\eta| = 1.2$), and the space between the DT wheels ($|\eta| = 0.25, 0.8$). Independent from the rest of CMS, the DT and CSC subsystems have good $p_T$ trigger efficiency, and high background rejection. The $p_T$ resolution of the Level-1 trigger is about 15% and 25% for the barrel and endcap regions, respectively. The amount of material within the muon system is above 16 interaction lengths, so punchthrough is negligible. Material thickness for muons, as a function of $\eta$ is shown in Fig. 2.18.
2.7.3 Resistive Plate Chambers

The RPCs are a complementary trigger system that was placed in both barrel and endcap regions due to the uncertainty in background rates and beam-crossing time measuring capability at full LHC luminosity. The RPCs span a pseudorapidity range of $|\eta| < 1.6$ and allow for a highly-segmented, independent and fast trigger with a sharp $p_T$ threshold. RPCs are double-gap chambers with good time resolution but a lower position resolution than the DTs and CSCs. They consist of two parallel plates separated by gas, one is an anode and another is a cathode. The displaced electrons are picked up by strips after a small drift time. They serve to resolve ambiguity in making tracks from multiple hits in a chamber. There are six layers of RPCs in the barrel region (RB) total, two in each of the two inner stations, and one in each of the outer two stations, and they are arranged into four stations, shown in Fig. 2.25. In the first two muons stations, RPC chambers are located before and after the DTs. These
chambers are labelled: RB1in and RB2in, and RB1out and RB2out. In the last two muon stations, RPC chambers are located only on the before of the DTs. These are labelled: RB3+, RB3−, RB4+, and RB4−. RB4 in sector 4 consists of four chambers: RB4++, RB4+, RB4−, RB4−−. There is only one RB4 chamber in sectors 9 and 11. Each chamber consists of either two or three double-gap modules aligned in beam direction. Each double-gap module has up to 96 strips, and each strip spans 5° or 16° for projective coverage in \( \phi \). Strips always run in the direction of the beam, and are divided into two parts for chambers RB1, RB3, RB4, and some of RB2. The exception with RB2 chambers is that they have strips divided into three parts for RB2out in wheels +2 and −2, and in RB2in in wheels +1, 0, and −1.

There are also three RPC stations in the endcap region: RE1, RE2, and RE3. These allow for coincidences from multiple stations to be used for triggering. The double gaps for every station are trapezoidal (Fig. 2.26), and are arranged in three concentric rings. They are made of two aluminum panels of 6mm thickness and a \( 16 \times 16 \text{ mm}^2 \) spacer frame. Strip panels are sandwiched in between gas gaps, are arranged in a radial fashion, and are segmented into three trigger sections. To avoid dead zones, they overlap in \( \phi \), except for station 1. RE1 is placed on the side of the first endcap disk of the yoke, YE1, closest to the interaction point. RE2 is placed on

Figure 2.26: An endcap RPC [25].
the back side of YE1, and RE3 on the interaction point side of YE3.

2.7.4 Opto-mechanical System

The optical alignment system measures the positions of muon detectors relative to one another and relative to the inner tracker for the purpose of optimizing the momentum resolution of muons. CMS uses both opto-mechanical measurements, and alignment algorithm results based on muon tracks to these ends. For muons of momentum < 1 TeV, the chambers must be aligned within 100 µm in $r$-$\phi$ space to achieve optimal performance. There are several sources for misalignment of components in the muon detector:

- Construction tolerances.
- Gravitational distortions, causing deformations.
- Solenoidal magnet effects.
- Time-dependent effects, e.g. thermal instabilities.

2.8 Trigger

There are 20 collisions per crossing at a luminosity of $\sim 10^{34}$ cm$^{-2}$s$^{-1}$. So a substantial reduction is necessary as it is not feasible to store and process this amount of data, especially for a large number of events. The trigger system performs this task in two main steps: Level-1 Trigger (L1) and High-Level Trigger (HLT). This is the beginning of the physics event selection. The L1 is housed in programmable electronics, while the HLT is filtering software implemented in a farm of 1000 processors. The rate reduction factor is about $10^6$ by design for combined L1 and HLT.
The design L1 output rate is 100 kHz, which is lowered to 30 kHz for safety. It is necessary that L1 stores high-resolution data in the pipelined memory in the front-end electronics, while using segmented data to eliminate dead time. Every bunch crossing is analyzed by L1 with a 3.2 µs delay between the bunch crossing and the trigger decision being communicated to the front-end electronics. L1 is composed of FPGA hardware (field programmable gate arrays), except for in high flux regions where ASIC (application specific integrated circuits) and LUT (largely programmable look-up tables) are used. After L1, HLT can perform calculations on the read-out data. HLT algorithms change over time, so there is no single model HLT algorithm. A simple HLT algorithm, for example, would be to select events with trigger objects that have transverse energy above a particular threshold.

The L1 has local, regional and global components. The Local Triggers are at the bottom, and are called Trigger Primitive Generators (TPG). The basis for TPG includes energy deposits in calorimeter towers, track segments and hit patterns in muon chambers. At the middle are the Regional Triggers, which merge this information with pattern recognition to rank and sort trigger objects, such as electron and muon candidates. Ranking is a function of energy, momentum and quality. At the top of L1 is the Global Trigger, which is built on the Global Calorimeter and Global Muon Triggers.

These lower two triggers determine which calorimeter and muon objects rank highest throughout the entire detector and transfer them to the Global Trigger, which decides to reject or accept an event for further evaluation by HLT. The decision is based on the Trigger Control System (TCS), which looks at the readiness of the detector systems and the DAQ. The Level-1 Accept (L1A) communicates its decision to the other detector systems through a Timing, Trigger and Control system (TTC). The architecture of L1 is shown in Fig. 2.27. Some L1 Trigger electronics are placed on the
detectors and the rest are placed 90 m from the cavern.
Chapter 3

Event Reconstruction

Event reconstruction is the process by which raw data from the detector is turned into physics objects [31]. Different particles interact with different subdetector systems in the CMS detector, as shown in Fig. 3.1. Digitized signals from all the subdetectors are interpreted by reconstruction to get a complete picture of an event. Reconstruction has three phases: local reconstruction within a subdetector module, global reconstruction within an entire subdetector, and Level-3 (higher-level) which combines reconstructed objects. The local reconstruction process takes the digitized signals (“digis”) from subdetectors as input. Digis can be from real or simulated electronics. The output of the local reconstruction algorithm are reconstructed hits (“rechits”). The global reconstruction algorithms then combine rechits from different modules of the same subdetector, but not from modules of another subdetector. Reconstruction is carried out using ROOT software, which is based in C++ and python languages.
Figure 3.1: Slice of the CMS detector showing how different particles interact with different subdetector systems [33].

3.1 Track Reconstruction

Charged particles in the tracker from proton-proton collisions are reconstructed as tracks, which are trajectories constructed from silicon pixel and silicon strip detector data. Track reconstruction algorithms rely on the beamspot, which is the estimated region where proton-proton interaction takes place [34]. The reconstruction of charged tracks is a three stage process. The first stage of track and vertex reconstruction is performed by the combinatorial track finder (CTF) using only pixel hits and the location of the beamspot as a constraint [35]. Tracks are “seeded” from triplets or pairs of hits with the beamspot or pixel vertex used as the constraint. Pixel layers have low occupancy, and provide seeding and 2-D position information. The first and second iterations find prompt tracks with $p_T > 0.9$ GeV using pixel triplets and pairs as seeds. The third iteration reconstructs low momentum prompt tracks using pixel triplet seeds. The fourth iteration reconstructs displaced tracks using combinations of pixel triplets and strip layer seeds. The fifth and sixth iterations use strip pair seeds to reconstruct tracks without associated pixel hits.
Seeding gives an initial estimate of track parameters and uncertainty. The seed is propagated outward in search of additional hits to associate with the track. This is based on the Kalman filter approach. The trajectory is extrapolated to the next layer, where compatible hits are selected based on their corresponding $\chi^2$ between the measured and extrapolated position. Compatible hits are then added to the track trajectories iteratively by updating parameters and uncertainties by accounting for the added hits. Each update results in a new set of candidate trajectories. If a pair of tracks share over 50% of hits, the track with lower $\chi^2$ is kept. This iterative search-and-add approach continues until there are no more compatible hits, or the tracker boundary is reached. For each iteration, hits already added to a track are removed from the available collection to shrink the collection for the next iteration. The set of reconstructed tracks are checked for fakes and the track quality is assessed at the end of each iteration. This search-and-add approach is repeated starting from the outermost hits and propagating inward. Track parameters can be recalculated by combining the results from the inward and outward extrapolations. “High purity” tracks pass the track filter, which uses the number of hits associated with the track, the normalized $\chi^2$ of the track, and compatibility to a track coming from a pixel vertex.

The expected traverse momentum resolution of single muons of traverse momenta of 1, 10, 100 GeV are shown in Fig. 3.2. The resolution reaches 10 $\mu$m for high momentum tracks, while for lower momentum track resolution degrades due to multiple scattering. The expected track reconstruction efficiency for single muons and pions are shown in Fig. 3.3. For muons the efficiency is about 99%, but decreases where there are gaps. Efficiency is lower for pions, since hadrons interact more with the tracker material.
Figure 3.2: Resolution of track parameters for single muons with momenta of 1, 10, and 100 GeV. From left to right, parameters are transverse momentum, transverse impact parameter, and longitudinal impact parameter [25].

Figure 3.3: Global track reconstruction efficiency for muons (left), and pions (right), of transverse momenta of 1, 10, 100 GeV [25]

### 3.2 Primary Vertex Reconstruction

Vertex reconstruction has two main steps: vertex finding and fitting. Vertex finding involves collecting vertex candidates by grouping tracks [35] and can vary depending on the type of vertex (primary, secondary, and reconstruction of exclusive decays). Vertex fitting determines the best estimates of vertex parameters for a given set of tracks (position, covariance matrix, track parameters, etc), and fit quality indicators.
\( \chi^2 \), number of degrees of freedom, track weights). The primary vertex finder performs reconstruction using all the reconstructed tracks in the event.

The first part is track preselection, which is based on the distance of closest approach to the beamspot, and the \( p_T \). In particular, the preselection uses transverse impact parameter significance \( \frac{IP}{\sigma(IP)} < 3 \), and track momentum \( p_T > 1.5 \text{ GeV} \). Second is the formation of track clusters using the distance of closest approach to the beamspot along the \( z \)-axis. Tracks that are less than 1 mm apart are grouped together in a cluster. Third is a fit of primary-vertex candidates for each of these clusters. Tracks that are incompatible with the vertex candidate are discarded. Fourth is removing bad fits and vertices incompatible with the beam line.

After finding the vertex candidates, they are sorted in decreasing order by the sum of transverse momenta of associated tracks, \( \sum_{\text{track}} p_T^{\text{track}} \). Vertex fits must account for possible contamination of associated tracks from other vertices. The use of fitting algorithms helps to reduce this effect.

### 3.3 Muon Reconstruction

Muon reconstruction is performed using data from two subsystems: the inner tracker and the muon system [31]. Tracks from the inner tracker can be joined with tracks from the muon system to form global muons and tracker muons. Tracks in the muon system are built from reconstructed hits (or “rechits”) and segments, see Fig. 3.4.

### 3.3.1 RecHits and Segments

The primary objects in the DT local reconstruction are hits in the cell volume. The positions of these hits, with respect to wires, are computed by converting drift times to drift distances. Segment reconstruction in the DTs works independently on the \( r-\phi \)
Figure 3.4: Event showing muon reconstruction using information from the tracker and muon system [36].

and $r$-$z$ projections. At the end, the projections are combined into a 3-D segment. A segment candidate is built from a set of aligned hits in a chamber. The best segment candidates among those sharing hits are selected. Finally the hit reconstruction is updated using the information from the segment, and the segment itself is refit.

The primary objects in the CSC local reconstruction are also reconstructed hits [37]. A pulse height in each strip is used to cluster neighboring strips to determine a probable muon hit position. Each of the six layers in a chamber are considered individually. A 2-D rechit is created from local $x$ and $y$ values, at the intersection of a three-strip cluster and wire group. Segments in the CSCs are build from the rechit information from the six chamber layers, shown in Fig. 3.5. The algorithm starts with the first and last rechits in a chamber and makes a line between them. These endpoints are
required to have separation of 1 cm at most in $r$-$\phi$ projection. In the intermediate layers, a compatibility test is run on the rechits. Hits within 2.5 mm in $r$-$\phi$ space are considered, with a passing updated $\chi^2$/NDOF. Per layer, only the hit giving the best fit is kept. A segment can only be defined if there are at least four hits that meet these requirements, shown in Fig. 3.6. These hits are then flagged as to not be double counted in further reconstruction.

For the RPCs, the results of local reconstruction are points in the plane of the detector, via strip clustering procedure. In the barrel, the point is simply the center of a rectangle. In the endcap, its more complicated due to the trapezoidal shape.

![Figure 3.5: CSC strip layers building a segment out of hits from the presence of a muon [38].](image)

![Figure 3.6: For CSC $-4/1$, the number of hits used to make a segment (left) and the corresponding normalized $\chi^2$/NDOF distribution (right).](image)
3.3.2 Standalone Muons

There are two types of independently reconstructed tracks in CMS: those in the silicon tracker (tracker tracks), and those in the muon system (standalone muon tracks). The standalone (Level-2) muon reconstruction uses data from the muon detectors only [36]. The DTs and CSCs, as well as the RPCs take part in standalone muon reconstruction. The silicon tracker is not used here. Even though the RPCs have a coarser spatial resolution, they complement the CSCs and DTs, especially where geometric coverage is sparse (where the barrel and endcaps overlap). The reconstruction is initiated by track segments from the local reconstruction in the muon chambers.

Muon trajectories are seeded by the track position, momentum, and direction (all these quantities are called a “state vector”) associated with the segments in the innermost chambers. This is done using the Kalman filter procedure starting from the inside and propagating outward. The state vector predicted at the next station is compared to what is actually measured and is then updated, provided the hits being added pass a $\chi^2$ cut. Energy loss in the material, multiple scattering effects, and the non-uniform magnetic field in the muon system are taken into account during propagation. In the barrel DTs segments are used in the fit, in the endcap CSCs the 3-D rechits are used, and hits from the RPCs are included as well. If no matching hits or segments are found in one station, the search is continues in the next. Missing hits along a trajectory can be caused by detector inefficiencies, geometric cracks, or hard showering. This procedure is iterated until it reaches the outermost measurement surface. A backward Kalman filter is then implemented, which works from the outside inward, where the track is then propagated toward the interaction point. The track is then fit using a vertex constraint.
3.3.3 Global Muons

The global muon reconstruction consists of propagating muon trajectories inward to include hits from the inner tracker, to get a more complete trajectory. This level of reconstruction uses information from both standalone muon tracks and tracker tracks. At large transverse momenta, the global muon fit improves momentum resolution. The global muon reconstruction starts with propagating the trajectory of a standalone reconstructed muon from the innermost station (seeding) to the outer tracking measurement surface. This propagation takes into account muon energy loss in the material (steel, coil and calorimeter) and multiple scattering effects.

A regional track reconstruction is performed, where a region within the inner tracker is determined to be compatible with a muon trajectory. This region definition impacts the fake rate, CPU time, and reconstruction efficiency (shown in Fig. 3.7). Regional seeds consist of pairs of tracker hits, and are selected as the initial candidates for the muon trajectory. The hits are required to come from different tracker layers. Another Kalman filter algorithm is applied to this track reconstruction. The first step is trajectory building (seeded pattern recognition). The trajectories are propagated

Figure 3.7: Muon reconstruction efficiency as a function of pseudorapidity for Standalone reconstruction on simulated muons (left), and global reconstruction (right) [25].
and updated with compatible hits, starting from the innermost layer. The second is trajectory cleaning (disambiguation), by selecting on number of hits and track fit $\chi^2$. The third step is trajectory smoothing (final fit). All reconstructed tracks are refit without a interaction point constraint, using hits from the tracker and hits associated with the standalone reconstruction. An additional cleaning step is performed which uses a cut on $\chi^2$ to select final muon candidates. The final muon candidates are refit excluding rechits and segments with high $\chi^2$ in muon stations with high hit occupancy. Also, the trajectories are refit using only the silicon tracker hits and the hits from the innermost muon station. These fits are compared to a tracker-only fit to detect muon Bremsstrahlung or any kind of significant energy loss. This is important for accurate momentum reconstruction for muon with $p_T \approx 1$ TeV.

An alternative to global muon reconstruction is tracker muon reconstruction, which is an outward propagation approach. Reconstructed tracks are used as seeds. Energy deposited in the ECAL and HCAL, and segments in the muon system crossed by the extrapolation are taken into account for propagation and fitting. Possible muon candidates here are any tracker tracks with $p_T > 0.5$ GeV and $p > 2.5$ GeV. These are extrapolated outward to the muon system, and if at least one muon segment matches the extrapolation, it is a tracker muon track. Tracker muon reconstruction is more efficient for low-$p_T$ muons ($p_T < 5$ GeV), since only one muon segment is required, as opposed to global muons that require at least two or more segments.

Most reconstructed muons fall into the global muon and/or tracker muon category. Sometimes a standalone muon is found that does not qualify as global or tracker. These are called “Standalone-muon track only”. Only 1% of muons enter this category.

### 3.3.4 Classification

From the algorithms described, three distinct classes of muons can be established:
• Soft muon selection: which requires the candidate be a tracker muon that has a matching segment in the outermost station along the extrapolated trajectory, considering both position and momentum. These requirements are optimized for low-$p_T$ muons.

• Global muon selection: the only requirement is that the candidate is a global muon.

• Tight muon selection: the candidate must be a tracker muon and a global muon, with the additional requirements:
  
  $- p_T > 3$ GeV
  
  $- p_T > 3$ GeV
  
  $- \text{the global muon track fit } \chi^2 < 10$
  
  $- \text{at least one muon chamber hit is in the final track fit}$
  
  $- \text{must match to segments in two or more muon stations}$
  
  $- \text{corresponding tracker track must have more than 10 silicon tracker hits, with at least one pixel hit, and an impact parameter } d_{xy} < 2 \text{ mm, with respect to primary vertex.}$

### 3.3.5 CSC Validation

To monitor the software and data quality, an automated CSC validation package was developed [39]. After each run of cosmic, proton-proton collision, and heavy ion collision data, the CSC validation code would produce plots of relevant CSC information and move them to a viewable web space [40]. This allowed the CMS collaboration to monitor several specific quantities for the entire endcap subsystem and for individual CSCs:
• Occupancies and Multiplicities, see Fig. 3.8 of RecHits, Segments, Strip Digis and Wire Digis

• Efficiencies based on extrapolated segments, see Fig. 3.9

• Segment and RecHit global positions, see Fig. 3.10

• Resolution, timing, and gas gains

• Segment quality, see Fig. 3.6

• Dead chambers

• Standalone muon quality, see Fig. 3.11

Figure 3.8: Multiplicities of Local Reco Objects (Strip and Wire Digis, RecHits and Segments) per Event
Figure 3.9: Strip, Wire, RecHit and Segment efficiency per chamber, based on extrapolated segments.

Figure 3.10: Global positions of segments in each ME ring.
Figure 3.11: Number of standalone muons tracks per event (left), normalized $\chi^2$/NDOF (right) of standalone muon tracks, with single muon trigger.

3.4 Electron Reconstruction

Electrons and photons deposit their energy in the ECAL crystals via showers. To reconstruct an electron a single tracker track from an interaction vertex is matched to an ECAL “supercluster” energy deposit.

3.4.1 Tracks and Superclusters

Electron reconstruction seeding uses two algorithms which use two kinds of input: inner tracker tracks and ECAL energy deposits. One algorithm is the tracker-driven seeding, which starts with tracks and matches them to energy clusters in the ECAL [41]. This is more suited for low-$p_T$ electrons, and has better performance for electrons inside jets. The other algorithm is ECAL-driven seeding algorithm, which starts with the reconstruction of ECAL “superclusters” with $E_T > 4$ GeV, and matches them to hits in the silicon tracker. This is optimized for isolated electrons with $p_T > 5$ GeV. A supercluster is a group of energy deposit clusters in the ECAL, which is constructed using two main algorithms. The two main superclustering algorithms “hybrid” and “island” are used in the ECAL barrel and endcaps, respectively. The purpose of these algorithms is to deal with the energy spread in $\phi$. To reconstruct high energy electrons in the barrel, the hybrid algorithm utilizes the geometry of the barrel crystals to get
a shower shape measurement in $\eta$, and to find separate energy deposits in $\phi$. The algorithm is tuned for electron showers of $p_T > 5$ GeV. The island algorithm seeds with crystals containing energy deposits above a certain threshold. Then it examines neighboring crystals, first in $\phi$ space, then in $\eta$, and adds them to the cluster until there is an increase in energy or if an unread crystal is encountered.

From the superclusters, pixel hits are predicted by propagating inward using the energy and position of the supercluster. The first compatible hit is looked for within a loose $\phi$-$z$ window at the innermost pixel layer. If a first hit is found, a new estimate for the primary track vertex position along the $z$-axis is calculated. The resulting trajectory is propagated in search of a second hit in the next pixel layers within a narrower $\phi$-$z$ window. These pixel hits serve as electron tracks seeds for the Kalman filter algorithm.

For high $p_T$ electrons, a tight $\chi^2$ cut is made, with emphasis on the innermost track information. Electrons traversing the tracker radiate Bremsstrahlung photons, so the initial momentum vector and primary vertex information is the most intact along the trajectory. To account for this, a dedicated modelling of electron energy loss is used, and tracks are fitted with a Gaussian Sum Filter (GSF) [42]. Electron track efficiency is shown in Fig. 3.12.

For the tracker-driven algorithm, a multivariate analysis (MVA) is used. For the ECAL-driven algorithm, matching between the GSF track and the supercluster in $\eta$-$\phi$ space is used. ECAL-driven candidates that do not pass preselection, but pass the MVA are kept. Further criteria are imposed to reduce the amount of fakes. These include limits on ECAL shower shape, track-ECAL cluster matching, and compatibility with the primary vertex. Additional criteria are imposed to remove electrons produced in photon conversions.
Figure 3.12: Electron track reconstruction efficiency on simulated electrons as a function of $p_T$ (left) and pseudorapidity (right) [31] for electrons with $5 \text{ GeV} < p_T < 50 \text{ GeV}$.

### 3.4.2 Classification

Tracking and calorimetry observables can be used for separating electron candidates into distinct classes. Golden electrons are the most precise and numerous of the “good” classes. They are least affected by radiation emission, have a well matched reconstructed track and supercluster. The definition is:

- single seed supercluster

- a Bremsstrahlung fraction of $f_{\text{Brem}} = \frac{p_{\text{in}} - p_{\text{out}}}{p_{\text{in}}} < 0.2$

- track extrapolation and supercluster $\phi$ matching within 0.15 rad.

- $\frac{E_{\text{SC}}}{p_{\text{in}}} > 0.9$.

Big Brem electrons are another class which have good matching between track and (well-behaved) supercluster, but lose a large portion of energy from Bremsstrahlung emmission. The definition is the same as golden electrons, except for:

- a Bremsstrahlung fraction of $f_{\text{Brem}} > 0.5$

- $0.9 < \frac{E_{\text{SC}}}{p_{\text{in}}} < 1.1$. 

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Narrow electrons are an intermediate class. They have large Bremsstrahlung emission, but lower than Big Brem electrons, and their matching between track and supercluster is more relaxed. Showering electrons are the “bad” class, as they do not meet the criteria of the other classes. The fractions of electron classes are shown in Fig. 3.13. To avoid “fake” electrons, different identification variables can be used for different classes of electrons. The variables used in the following analyses have discriminating power for all classes:

- the ratio between energy deposits in the HCAL tower behind the ECAL seed cluster $H/E$
- $\Delta \eta = |\eta_{SC} - \eta_{track}|$
- $\Delta \phi = |\phi_{SC} - \phi_{track}|$
- the shower width $\sigma_{\eta \eta} = \sum_i (\eta_i - \eta_{SC})^2 \frac{E_i}{E_{SC}}$

where the $i$ index represents a crystal within a seed cluster.

Figure 3.13: A comparison of different electron classes in simulation, in terms of population fraction as a function of pseudorapidity. The initial electron energy is between 5-100 GeV [41].
3.5 Photon Reconstruction

Photon objects are reconstructed from superclusters [43,44]. The energy of each photon candidate is estimated based on an observable named “$r_9$”, shown in Fig. 3.14. The $r_9$ variable is the ratio between the energy contained in the $3 \times 3$ crystal array centered on the seed crystal of the supercluster of the photon, and the total energy in the supercluster. This variable can be used to determine if the photon is converted or non-converted. If $r_9 > 0.94$ in the barrel or $r_9 > 0.95$ in the endcap, the energy of the $5 \times 5$ crystals around the highest energy crystal is used. The threshold is larger in the endcap, because the crystals are physically larger than the ones in the barrel. The incident energy from a single electron or photon is 94% contained in the $3 \times 3$ crystals, and 97% contained in the $5 \times 5$ crystals.

3.6 Particle Flow

The goal of particle flow event reconstruction is to identify and measure all of the stable particles in an event using all of the CMS subdetectors. Stable particles are
electrons, muons, photons, charged and neutral hadrons. Individual particles in an event are listed and used in a similar fashion to particles from Monte-Carlo event generators. The particle flow algorithm is capable of building jets, determining the $E_T$, reconstructing and identifying taus, tagging b jets, and quantifying lepton isolation. The fundamental pieces for particle flow are the charged particle tracks, calorimetric clusters and muon tracks [45]. These elements are linked together into “blocks”, and treated as particles.

### 3.6.1 Tracking and Clustering

To uncover phenomena buried in jets from SM processes, the ability to identify and reconstruct as many final state particles as possible is vital. With particle flow, an iterative tracking method is used. Tracks are seeded and reconstructed with tight requirements, which results in a moderate efficiency and low fake rate. Then hits are removed if they were assigned, unambiguously, to tracks in the previous iteration. The requirements on track seeding can be loosened to increase efficiency, and the removal of hits allows for a low fake rate from reduced combinatorics. Tracks originating near the beam axis are found in the first three iterations with an efficiency of 99.5% and 90% for isolated muons and charged hadron jets, respectively. The last two iterations have a looser vertex constraint, allowing for reconstruction of particles originating from interactions with detector material.

The clustering algorithm has four parts: detect and measure stable neutral particles, isolate neutral particles from charged hadron deposits, reconstruct and identify electrons and corresponding Bremsstrahlung photons, and assist in the measurement of charged hadrons. Clustering is performed in each calorimeter subsystem (EB, EE, PS, HB, HE), except for HF. “Cluster seeds” are identified as cell energy maxima above a given threshold. “Topological clusters” are grown from seeds as long as one side or
more are shared with a cluster cell with an aggregated energy above a given threshold. For ECAL, the threshold is two standard deviations above noise. For HCAL, the threshold is 800 MeV. Topological clusters result in “Particle flow clusters”. The high granularity of the calorimeters is useful here, as the energy of each cell is weighted by cell-cluster distance among all particle flow clusters to determine energy and position.

### 3.6.2 Linking

The track and cluster elements are linked to get a more complete picture. The linking algorithm fully reconstructs each single particle, without double counting with information from different subdetectors. The quality of a link is measured by the $\phi$-$\eta$ distance between elements. A group of one to three linked elements compose a “block”.

Track and cluster elements are linked if the propagated position of the track is within the boundaries of the cluster. Tracks are propagated according to the expected shape of an electron shower in ECAL, and the expected interaction length of a hadron shower in HCAL. To account for gaps in the calorimeters, the cluster boundaries can be extended by the size of one cell.

Linking a charged particle track with a muon track is done when a global muon fit between the two tracks satisfies a cut on $\chi^2$. If there is ambiguity from multiple global muon fits, the one with the lowest $\chi^2$ is accepted.

### 3.6.3 Particles

Each block of elements is put through the particle flow algorithm to reconstruct and identify their corresponding particles. This set of reconstructed particles is a global description of an event, and can be used for further physics analysis.

Global muons qualify as “particle flow muons” if the momentum is compatible with
the corresponding tracker element within three standard deviations. The track is then removed from the block to avoid double counting. There are three selection types: isolated, pf-tight, and pf-loose. A cone of size $R = 0.3$ is defined around the muon trajectory. A muon is isolated if the $p_T$ sum over tracks and calorimeter hits within the cone is below 10% of the muon $p_T$. After this, pf-tight or pf-loose selections are applied. The pf-tight selection occurs before other types of particles are identified. To qualify for pf-tight, there needs to be a minimum number of hits along the muon track, and compatibility with a muon segment and calorimeter deposits. The pf-loose selection can recover muons lost due to muon track momentum being much larger than the otherwise compatible calorimeter deposit. Figure 3.15 shows $p_T$ resolution before and after this muon ID is applied.

![Figure 3.15: The $p_T$ resolution of jets with and without the pf algorithm, where 100 GeV $< p_T < 150$ GeV [46].](image)

After muon identification, electron identification is performed using the combination of tracker track and calorimeter supercluster variables. An identified electron
becomes a “particle flow electron”. The tracker is treated as a preshower to account for Bremsstrahlung energy loss when identifying track elements of the block. After identification, the corresponding tracks and clusters are removed to avoid double counting.

The tracks left over in the block become “particle flow charged hadrons”. Tighter criteria were applied to them so that the uncertainty of $p_T$ is below the energy resolution for charged hadrons. Momentum and energy are measured under the assumption they are from a pion. If they are compatible with calorimeter energy deposits, their momenta are redefined by a fit. If there is an excess of energy in the calorimeters it can result in “particle flow photons” and “particle flow neutral hadrons”. The left over clusters without links to any remaining tracks in ECAL and HCAL become particle flow photons and neutral hadrons, respectively.

### 3.7 Missing Transverse Energy Reconstruction

The initial proton pair does not have any significant momentum in the transverse plane, so all the resulting scattering and decay products after the collision should have a transverse vector sum of zero, $\sum_i \vec{p}_T^i = 0$. However, there may be some final state particles that do not interact with any of the detector systems. For example, neutrinos only interact through the weak nuclear force and carry no electric charge, so there will be no tracks or calorimeter energy deposits associated with neutrinos (see Fig. 3.16). Other theoretical particles may also avoid detection, like a long-lived Higgs boson, or dark matter. An event containing particles like these will have a significant imbalance in the transverse vector sum $\vec{p}_T$, and this imbalance is called missing transverse energy.
"MET" with the scalar value $E_T$ [48].

$$\vec{\not{p}}_T = \sum_i \vec{p}^i_T$$

$$E_T = \left| \vec{\not{p}}_T \right|$$

(3.1)

There are different ways to calculate a missing transverse energy. One way is to use only calorimeter objects in the sum, and this is "caloMET". Another way is to use all particle flow candidates in the sum, and this is "PFMET" [49]. The resolution of PFMET is compared to caloMET in Fig. 3.17.

### 3.8 Jet Reconstruction

Jets are the detector signatures of partons (quarks and gluons), which are produced in high energy pp collisions. There are four types of jet reconstruction at CMS: calojet,
PFjet, Track jet, and Jet-plus-track [50]. They combine information from different subdetectors to form input to a jet clustering algorithm. An often used clustering algorithm is the Anti-$k_T$ algorithm with a cone size parameter of $R = 0.5$.

Calorimeter jets (calojets) are reconstructed from energy deposits in the ECAL and HCAL calorimeter cells, where the cells are combined to make towers. In the barrel region, a tower consists of a sum of an HCAL cell and $5 \times 5$ ECAL crystals. In the end-cap region, a tower is made between HCAL cells and ECAL crystals in a more complex way. Thresholds are applied to the energies of individual cells when building towers to suppress electronic noise, for reconstruction of jets and missing energy. Calorimeter towers with $E_T < 0.3$ GeV are not used in calojet reconstruction to suppress pile-up contributions.

Particle flow jets (PFjets) are reconstructed from the resulting sets of PF particles. PFjet momentum and spatial resolutions are better than those of calojets, due to the addition of tracking, and the high granularity of the ECAL. The ECAL precisely measures charged hadrons and photons within a jet, which constitute about 90% of the jet energy.

Figure 3.17: The missing energy resolution for PFMET and caloMET, with simulated $t\bar{t}$ sample [45].
The Jet-Plus-Track algorithm also uses the tracking detector to improve resolution of calojets. First calojets are reconstructed, then charged tracks are associated with a calojet based on $\phi$-$\eta$ separation between the vectors of the jet and track momentum. The associated tracks are propagated to the calorimeter, and if they intersect with the jet cone, they are classified as “in-cone” tracks. If the magnetic field bends the track out of the jet cone it is classified as “out-of-cone”. Both in-cone and out-of-cone track momenta are added to the energy of the calojet. Then for in-cone tracks the mean energy in the calorimeters is subtracted based on track momentum, and the direction of the calojet is corrected.

The track jet algorithm reconstructs jets from tracks in the inner tracker. Only well-measured tracks are used by this algorithm. It is completely independent of calorimeter measurements and is used for cross-checks.

Jets arising from $b$ quark hadronization and decay are present in many physics processes, including top quark decay, and Higgs boson decay. A signature of $b$ quark decay is a secondary vertex in the tracker substantially displaced from the beamspot [51]. The variables associated with a secondary vertex are distance and direction with respect to the primary vertex, and other properties such as multiplicity of secondary tracks, mass and energy. Variables used to distinguish $b$ hadron decay products from ordinary prompt tracks are impact parameter (IP) of a track with respect to the primary vertex and secondary vertex, and kinematic variables associated with the vertices. Some simple algorithms like Track Counting, or Simple Secondary Vertex output discriminator variables. Other more complex algorithms use IP significance of several tracks in a jet, or combine secondary vertices with track-based lifetime information, which yield discriminator variables like Jet Probability, Jet B Probability, or Combined Secondary Vertex tagger (CSV).
3.9 Simulated Event Generation

Simulation of physics in the detector is vital for searches beyond the Standard Model. Simulated events are produced via Monte Carlo methods (MC). Events can be generated consisting of predicted Standard Model phenomena, as well as exotic hypothesized phenomena. This allows for processes to be studied separately or together in arbitrary combinations. It also allows for the study of unobserved signals from various hypotheses and parameter spaces, like that of mass or branching ratio, for example. Different signals and background can be compared and contrasted using different variables in an analysis. Pile-up interactions are modelled in the MC samples as well. However, since pile-up is dependent on LHC conditions, which change frequently, the pile-up distributions are reweighted to agree with data.

Detector response is modelled with the GEANT4 simulation toolkit [52], which is built into the framework of CMS. GEANT4 handles detector geometry, tracking, response, and run management, and is validated using collision or cosmic data. Figure 3.18 contains plots of muon efficiency and isolation, and figure 3.19 contains $E_T$ resolution plots showing good agreement between real and simulated 7 TeV data. This allows simulated events and real events to be put on an equal footing in terms of reconstruction algorithms and further analysis.

Different event generator methods are used to model different kinds of processes. The decay and hadronization of hard scattering particles is modelled by PYTHIA6 (v4.22 for 7 TeV, v4.26 for 8 TeV) [55], which is a LO event generator capable of simulating initial state and final state parton showers, parton distributions, and multiple interactions. POWHEG (v2.0) [56] interfaces NLO QCD calculations with parton shower generators, like PYTHIA. MADGRAPH (v5.1.3) [57] is a general purpose matrix-element based generator, which interfaces with parton showering and hadronization generators.
Figure 3.18: Muon efficiency and isolation as a function of muon $p_T$, comparing simulation to real data [53].

Figure 3.19: The resolution for different $\not{E}_T$ types, comparing simulation to real data [54].

to LO and NLO processes. SHERPA [58] is another matrix element generator used to model boson and jet final states. The parton distribution functions (PDF) are modelled through CTEQ6L [59] parameterization at leading order (LO), and CT10 [60] parameterization at next to leading order (NLO). Cross sections and $k$-factors for many NLO processes, using matrix elements, are computer using MCFM [61], which is a parton-level Monte Carlo generator.
Chapter 4

Analysis I: ZZ

4.1 Motivation

This analysis focuses on the production of ZZ final states, where one Z decays to a charged lepton anti-lepton pair, and the other Z decays to neutrino [62]. The branching fraction of this decay mode is approximately 6 times larger than that of the $4\ell$ final state. The production cross section of these states is measured from proton-proton collisions with data collected by CMS during 2011 at a center-of-mass energy of 7 TeV, corresponding to 5.1 fb$^{-1}$ of integrated luminosity, and during 2012 at 8 TeV corresponding to an integrated luminosity of 19.6 fb$^{-1}$.

Diboson production, particularly ZZ production, is of interest because it probes electroweak self-interactions, and is a background for searches for new physics, such as invisible Higgs boson decays and anomalous triple gauge couplings. Measuring these phenomena precisely is required to confirm or rule out these models [63].

The next to leading order (NLO) production cross section is expected to be $6.46^{+0.30}_{-0.21}$ pb at 7 TeV and $7.92^{+0.37}_{-0.24}$ pb at 8 TeV [61], where the uncertainty is due to higher order corrections in the computation. At tree level, ZZ final states are pri-
marily produced in the $t$ and $u$ channels from the annihilation of a quark-antiquark pair.

From the $4\ell$ channel, the $ZZ$ production cross section was measured to be:

$$\sigma(pp \to ZZ) = 6.24^{+0.86}_{-0.80}\,(\text{stat})\,^{+0.41}_{-0.32}\,(\text{syst})\,\pm\,0.14\,(\text{lumi})\,\text{pb} \quad (4.1)$$

at 7 TeV, and

$$\sigma(pp \to ZZ) = 8.4 \pm 1.0\,(\text{stat})\,\pm\,0.7\,(\text{syst})\,\pm\,0.4\,(\text{lumi})\,\text{pb}, \quad (4.2)$$

at 8 TeV [64], and is in good agreement with the Standard Model.

The event topology of the $ZZ \to \ell^+\ell^-\nu\bar{\nu}$ signal consists of an overall imbalance in the vector sum of the transverse momentum of the event, from undetected neutrinos from a $Z$ decay ($\vec{p}_T$). This imbalance expressed as a scalar is colloquially called “missing transverse energy” ($|\vec{p}_T| = E_T$).

The largest source of background is from the Drell-Yan (DY) process, which has a cross section about five orders of magnitude larger than that of $ZZ \to \ell^+\ell^-\nu\bar{\nu}$.

A possible discriminating variable between $ZZ$ and DY is $E_T$, since $ZZ$ events tend to occupy higher values and DY tends to occupy lower values. However, Drell-Yan events can have large mismeasured $E_T$ from hadronic recoil, which leads to a contamination of the signal region. This instrumentally-induced fake $E_T$ will allow DY to look like signal, but with a modification to the $E_T$ variable, this effect can be reduced.

There are more sources of background, including other diboson processes like $WW$ and $WZ$ with leptonic decays, and QCD processes like $t\bar{t}$ and electroweak single top and $W$ boson production, with leptonic and neutrino decays. Sources with no $Z$ production are called “non-resonant”.
4.2 Simulation

Several Monte Carlo event generators are used to simulate the signal and background processes. The $ZZ \rightarrow \ell^+\ell^-\nu\bar{\nu}$ signal, the WW, WZ, Z + jets, W + jets, and $t\bar{t}$ + jets are all simulated using MadGraph5 (v1.3 for 7 TeV, v1.3.27 for 8 TeV) [57]. Single top processes are simulated with POWHEG [56]. For all processes, the parton showering is simulated with PYTHIA6 (v4.22 for 7 TeV, v4.26 for 8 TeV) [55], with the Z2 (Z2*) tune for 7 TeV(8 TeV) simulations [65]. The ZZ signal cross section is computed with the next-to-leading order (NLO) program MCFM [61], which also includes gluon-gluon initial state contributions. The charged dilepton $p_T$ spectrum of $ZZ \rightarrow \ell^+\ell^-\nu\bar{\nu}$ generated by MadGraph is compared with the NLO prediction from MCFM. Good agreement is found between the two, therefore no rescaling of the $p_T$ spectrum is needed. The parton distribution functions (PDF) are modelled through the CTEQ6L [59] parametrization at leading order (LO), and the CT10 parametrization [60] at NLO. The detector response to the simulated events is modelled with GEANT4 [66,67]. Reconstruction and analysis of simulated events are performed using the same software used for data events.

4.3 Object Selection

A signal event is primarily characterized by the following topology:

- a pair of oppositely charged, isolated leptons, with invariant mass ($m(\ell_1, \ell_2) \approx m_Z$) close to that of the Z boson.
- no additional leptons, other than the original pair of muons or electrons from a Z decay.
- large missing transverse energy, consistent with the presence of a neutrino pair.
Additionally, since the pair of Z bosons in a signal event is produced after the collision of two hadrons, extra jets from initial state radiation (ISR) may be found in an event.

4.3.1 Trigger

For a dilepton event to be logged in the data, it must pass a set of trigger requirements. The trigger requires the presence of two electrons or two muons along with a minimum $p_T$ threshold for each type of lepton. These thresholds vary over the data-taking period due to changing LHC beam conditions. The trigger for dileptons is unprescaled, meaning no passing events are removed from the sample. For the photon control sample for DY modelling, a prescaled single photon trigger is used. For non-resonant backgrounds, an electron-muon cross-trigger is used. All triggers used in this analysis are summarized in Table 4.1.

Table 4.1: List of trigger thresholds used to select signal and control data samples during 2011 and 2012 data taking.

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<td>135, 150, 160, 250, 300</td>
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</table>

4.3.2 Primary Vertex Selection

At least one primary vertex is required per event, reconstructed with a deterministic annealing algorithm [68]. The vertex must also be flagged as not fake and the fit
must have at least four degrees of freedom. The distance from the vertex to the beam spot must be less than 2 mm in the transverse plane, and less than 24 cm along the longitudinal axis. If there is more than one primary vertex in an event, the one with the highest weight $w_{\text{vertex}}$ is chosen, where

$$w_{\text{vertex}} = \sum_{i=1}^{n} p_{T_i}^2,$$

and the index $i$ corresponds to the tracks used in the fit of that vertex.

### 4.3.3 Muon Identification and Isolation

The standard POG [69] criteria are used for muon isolation and identification. These criteria also vary depending on the data-taking period. In 2011, the muons are identified by both global and tracker muon algorithms. In 2012, the muons [70] are also required to be identified by the particle-flow (PF) algorithm [49]. Quality cuts are imposed on the muon tracks, which are based on the number of hits, the $\chi^2$ of the track fit, and relative $p_T$ uncertainty. The muons used for Z boson reconstruction are selected from the $|\eta| < 2.4$ region of the muon spectrometer, and must have a minimum transverse momentum of $p_T > 20$ GeV. The muon must also be isolated, and isolation is calculated by summing the momenta of the muon tracks and calorimeter energy deposits within a cone centered on the track shown in Fig. 4.1. Muon tracks must be within the range of a primary vertex according to transverse and longitudinal impact parameter cuts, to reduce the dependence of the isolation cut on the number of pile-up interactions. Only muon tracks coming from the primary vertex are considered. For 2011 muons, the relative isolation is defined as:

$$I_{\text{rel}} = \frac{1}{p_T}(I_{\text{tracker}} + I_{\text{cal}})$$

(4.4)
where $I_{\text{tracker}}$ is the sum of the traverse momenta of all the charged particle tracks inside a cone of size $\Delta R = \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2} = 0.3$, around but excluding the muon track, and $I_{\text{cal}}$ is the sum of ECAL and HCAL energy deposits inside that same cone. Calorimeter deposits are corrected for the average energy density $\rho$ in the event by FastJet correction [71]:

$$I_{\text{cal}} = \max(0, I_{\text{uncorr}}^{\text{cal}} - 0.3^2 \pi \rho).$$  \hspace{1cm} (4.5)

For the 2012 dataset, particle-flow based isolation is used, and is defined:

$$I_{\text{rel}} = \frac{1}{p_T} [I_{\text{CH}} + \max(0, I_{\text{NH}} + I_\gamma - 0.5I_{\text{CH:PU}})]$$  \hspace{1cm} (4.6)

where the isolation sums $I_{\text{CH}}$, $I_{\text{NH}}$, and $I_\gamma$ correspond respectively to the sums of charged hadrons, neutral hadrons and photon candidates reconstructed by the PF algorithm, in a cone of size $\Delta R = 0.4$. To subtract the contribution from pile-up interactions from the neutral particle isolation, the term $-0.5 \cdot I_{\text{CH:PU}}$ is added [49],

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Figure 4.1: Isolation cone for a muon track [31].
where \( I_{\text{CH,PU} } \) represents the charged hadron contribution from pile-up. The details of the muon selection are shown in Table 4.2.

Table 4.2: Muon identification for Z candidate selection.

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<th>2012</th>
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<td>(p_T)</td>
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<tr>
<td>Valid pixel hits</td>
<td>&gt;0</td>
<td></td>
</tr>
<tr>
<td>Global track (\chi^2)</td>
<td>&lt;10</td>
<td></td>
</tr>
<tr>
<td>(\sigma(p_T)/p_T)</td>
<td>&lt;0.1</td>
<td>—</td>
</tr>
<tr>
<td>Transverse IP</td>
<td>&lt;0.02 cm</td>
<td>&lt;0.2 cm</td>
</tr>
<tr>
<td>Longitudinal IP</td>
<td>&lt;0.1 cm</td>
<td>&lt;0.5 cm</td>
</tr>
<tr>
<td>Detector-based relative isolation</td>
<td>&lt;0.15</td>
<td>—</td>
</tr>
<tr>
<td>PF relative isolation</td>
<td>—</td>
<td>&lt;0.2</td>
</tr>
</tbody>
</table>

4.3.4 Electron Identification and Isolation

The standard POG [72, 73] criteria are used for electron isolation and identification. Just like muons, the criteria vary depending on the data-taking period. Quality cuts are imposed on electron candidates, which are based on shower shape, track quality, and cluster-track matching, to primarily accept "golden" electrons. The electrons used for Z boson reconstruction are selected from the \( |\eta| < 2.5 \) region of the ECAL, excluding the gap between the barrel and endcaps: \( 1.4442 < |\eta| < 1.566 \), and must have a transverse momentum of \( p_T > 20 \text{ GeV} \). Electron tracks must be within the range of a primary vertex. The way relative isolation is computed is similar to that of muons. The details of electron selection are shown in Table 4.3.
Table 4.3: Electron identification for Z candidate selection. EB=ECAL barrel and EE=ECAL endcaps.

<table>
<thead>
<tr>
<th>Variable</th>
<th>2011</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EB</td>
<td>EE</td>
</tr>
<tr>
<td>$p_T$</td>
<td>&gt;20 GeV</td>
<td>&lt;1.4442</td>
</tr>
<tr>
<td>$</td>
<td>\eta</td>
<td>$</td>
</tr>
<tr>
<td>$\sigma_{i\eta i\eta}$</td>
<td>&lt;0.06</td>
<td>&lt;0.04</td>
</tr>
<tr>
<td>$\Delta \phi(\text{track}, SC)$</td>
<td>&lt;0.004</td>
<td>&lt;0.007</td>
</tr>
<tr>
<td>$\Delta \eta(\text{track}, SC)$</td>
<td>&lt;0.04</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>$H/E$</td>
<td>&lt;0.04</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>Transverse IP</td>
<td>&lt;0.02 cm</td>
<td>&lt;0.01 cm</td>
</tr>
<tr>
<td>Longitudinal IP</td>
<td>&lt;0.1 cm</td>
<td>&lt;0.1 cm</td>
</tr>
<tr>
<td>Missing hits</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Conversion vertex fit prob.</td>
<td>$10^{-6}$</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>$</td>
<td>1/E - 1/p</td>
<td>$</td>
</tr>
<tr>
<td>$\Delta R$ from muon candidates</td>
<td>&gt;0.1</td>
<td>&gt;0.1</td>
</tr>
<tr>
<td>Detector-based relative isolation</td>
<td>&lt;0.1</td>
<td>—</td>
</tr>
<tr>
<td>PF relative isolation</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

### 4.3.5 Z Candidate Selection

The Z boson candidates are selected from the electron and muon pair events, which pass the identification and isolation criteria. No additional charge requirements are imposed, since it is expected they will have opposite charges when decaying from a Z boson. A cut is placed on the invariant mass of the lepton pair within the nominal Z mass window to suppress non-resonant backgrounds. Since our signal has a boosted Z from recoiling against another Z, a minimum cut is placed on the transverse momentum of the Z boson at 45 GeV. This is also convenient for data-driven modelling of DY using the photon sample, which has high prescaling below 30 GeV. Invariant mass and transverse momentum distributions of selected lepton pairs in both data periods are shown in Figures 4.2 and 4.3, respectively.

The Z boson is also used for lepton tag and probe. At every selection stage, there is a chance that a lepton is misidentified, as another type of particle, due to poor
reconstruction. To correct for these effects, an event with a good lepton candidate is chosen as a tag. Then another loosely identified lepton of the same flavor in the events is chosen as the probe. The invariant mass of these two leptons must be within the $Z$ mass window to reenforce the chance that the probe is truly a lepton. Efficiencies and corrections can then be calculated and applied to simulation after studying the probe.

Figure 4.2: Dilepton invariant mass in $\mu\mu$ (left) and $ee$ (right) channels for 7 TeV (top) and 8 TeV (bottom). For 7 TeV, the generator level cut on the invariant mass of the dilepton system is 40 GeV. The gray error bands show statistical uncertainties.
Figure 4.3: Dilepton transverse momentum in $\mu\mu$ (left) and ee (right) channels, per running period (7 TeV top, 8 TeV bottom), for events with dilepton mass within $(91 \pm 7)$ GeV. The gray error bands show statistical uncertainties.

### 4.3.6 Loose Lepton Selection

For additional leptons, besides the pair from the Z boson decay, thresholds are loosened to increase background rejection power. To suppress WZ production, events are required to have no more than two leptons. The loosened criteria are shown in Tables 4.4
and 4.5. Figure 4.4 shows the number of additional leptons (with $p_T > 10 \text{ GeV}$) per event.

Figure 4.4: Number of loose leptons (e and $\mu$ with $p_T > 10 \text{ GeV}$) per event, in $\mu\mu$ (left) and ee (right) channels, per running period (7 TeV top, 8 TeV bottom).

### 4.3.7 Jet Identification, Corrections and B-tagging

Many background processes contain decays that result in jets, such as single top, $t\bar{t}$, and other diboson events with one boson hadronically decaying. On top of that, jets
Table 4.4: Muon identification for WZ background suppression.

<table>
<thead>
<tr>
<th>Variable</th>
<th>2011: Tight OR Soft ID</th>
<th>2012: Loose OR Soft ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\eta</td>
<td>$, $p_T$</td>
</tr>
<tr>
<td>GlobalMuon</td>
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<td>—</td>
</tr>
<tr>
<td>TrackerMuon</td>
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<td>true</td>
</tr>
<tr>
<td>Muon ID</td>
<td>—</td>
<td>“TMOneStationTight”</td>
</tr>
<tr>
<td>Muon matched stations</td>
<td>$&gt;1$</td>
<td>—</td>
</tr>
<tr>
<td>Valid muon hits</td>
<td>$&gt;0$</td>
<td>—</td>
</tr>
<tr>
<td>Valid tracker hits</td>
<td>$&gt;10$</td>
<td>$&gt;10$</td>
</tr>
<tr>
<td>Valid pixel hits</td>
<td>$&gt;1$</td>
<td>—</td>
</tr>
<tr>
<td>Pixel layers with hits</td>
<td>—</td>
<td>$&gt;1$</td>
</tr>
<tr>
<td>Transverse IP</td>
<td>$&lt;0.2$ cm</td>
<td>$&lt;3$ cm</td>
</tr>
<tr>
<td>Longitudinal IP</td>
<td>—</td>
<td>$&lt;30$ cm</td>
</tr>
<tr>
<td>$\chi^2$/d.o.f. of inner track</td>
<td>—</td>
<td>$&lt;1.8$</td>
</tr>
<tr>
<td>$\chi^2$/d.o.f. of global track</td>
<td>$&lt;10$</td>
<td>—</td>
</tr>
<tr>
<td>Detector-based relative isolation</td>
<td>$&lt;0.15$</td>
<td>—</td>
</tr>
<tr>
<td>$</td>
<td>\eta</td>
<td>$, $p_T$</td>
</tr>
<tr>
<td>PFMuon</td>
<td>true</td>
<td>true</td>
</tr>
<tr>
<td>TrackerMuon</td>
<td>true (OR GlobalMuon)</td>
<td>true</td>
</tr>
<tr>
<td>Muon ID</td>
<td>—</td>
<td>“TMOneStationTight”</td>
</tr>
<tr>
<td>Tracker layers with hits</td>
<td>—</td>
<td>$&gt;5$</td>
</tr>
<tr>
<td>Pixel layers with hits</td>
<td>—</td>
<td>$&gt;1$</td>
</tr>
<tr>
<td>$\chi^2$/d.o.f. of inner track</td>
<td>—</td>
<td>$&lt;1.8$</td>
</tr>
<tr>
<td>Transverse IP</td>
<td>—</td>
<td>$&lt;3$ cm</td>
</tr>
<tr>
<td>Longitudinal IP</td>
<td>—</td>
<td>$&lt;30$ cm</td>
</tr>
<tr>
<td>PF relative isolation</td>
<td>$&lt;0.2$</td>
<td>—</td>
</tr>
</tbody>
</table>

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Table 4.5: Electron identification for WZ background suppression.

<table>
<thead>
<tr>
<th>Variable</th>
<th>2011 Barrel</th>
<th>2011 Endcap</th>
<th>2012 Barrel</th>
<th>2012 Endcap</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\eta</td>
<td>$</td>
<td>&lt;1.4442</td>
<td>&gt;1.566</td>
</tr>
<tr>
<td>$p_T$</td>
<td>&gt;10 GeV</td>
<td></td>
<td>&gt;10 GeV</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\eta\eta}$</td>
<td>&lt;0.01</td>
<td>&lt;0.03</td>
<td>&lt;0.01</td>
<td>&lt;0.03</td>
</tr>
<tr>
<td>$\Delta\phi$</td>
<td>&lt;0.06</td>
<td>&lt;0.04</td>
<td>&lt;0.8</td>
<td>&lt;0.7</td>
</tr>
<tr>
<td>$\Delta\eta$</td>
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<td>&lt;0.007</td>
<td>&lt;0.007</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>$H/E$</td>
<td>&lt;0.04</td>
<td>&lt;0.1</td>
<td>&lt;0.15</td>
<td></td>
</tr>
<tr>
<td>Transverse IP</td>
<td>&lt;0.02 cm</td>
<td>&lt;0.04 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal IP</td>
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<td></td>
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</tr>
<tr>
<td>$</td>
<td>1/E - 1/p</td>
<td>$</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Conversion vertex fit prob.</td>
<td>$10^{-6}$</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>$\Delta R$ from muon candidates</td>
<td>&gt;0.1</td>
<td>&gt;0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detector-based relative isolation</td>
<td>&lt;0.1</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>PF relative isolation</td>
<td>—</td>
<td></td>
<td>&lt;0.15</td>
<td></td>
</tr>
</tbody>
</table>

are produced due to pile-up, and radiative emissions, so jet information is important for background rejection.

In this analysis, PF jets are used [49], reconstructed within $|\eta| < 5.0$. Transverse energy is corrected according to JetMET POG prescriptions [74], and further corrections are applied to reduce pile-up effects using FastJet [71,75]. Extra corrections are applied to MC jets to match the resolution of data by adding a smearing factor to the $p_T$ of each jet, depending on its $p_T$ and $\eta$ [76]. Jets are also required to pass the loose id cut [77] like additional leptons. Events are then categorized by the number of selected jets with $p_T > 30$ GeV. So if an event contains one jet with $p_T > 30$ GeV, it is said to “have one jet”.

To suppress backgrounds from top quark decays, events with one or more b-tagged jet are vetoed. A jet can be b-tagged if it has $p_T > 20$ GeV and is within the tracker region of $|\eta| > 2.4$. If a b-jet candidate passes the loose working point of its Combined Secondary Vertex (CSV) discriminator, $\delta_{CSV} > 0.244$ [78], then it is b-tagged. The
number of b-tagged jets per event with $p_T > 20$ GeV is shown in Fig. 4.5. No corrections are applied here, since the signal contains no b-jets. To further reduce backgrounds from top decays and other hadronic activity, events with 1 or more jets are vetoed. Figure 4.6 shows the number of jets with $p_T > 30$ GeV per event.

Figure 4.5: Number of b-tagged jets (CSV discriminator < 0.244) with $p_T > 20$ GeV, in $\mu\mu$ (left) and ee (right) channels, per running period (7 TeV top, 8 TeV bottom). The gray error bands show statistical uncertainties.
Figure 4.6: Number of jets with $p_T$ above 30 GeV per event, in $\mu\mu$ (left) and ee (right) channels, per running period (7 TeV top, 8 TeV bottom), after the dilepton mass and $p_T$ selection, and the b-tagging veto.

4.3.8 Final Signal Selection

A final optimized set of selection cuts are obtained by varying the dilepton mass window, the $p_T(Z)$ threshold, the min($p_T(jet)$). The list of selection cuts are shown in Table 4.6. Cut-flow plots for 7 and 8 TeV show the composition and agreement between data and simulation cut by cut in Fig. 4.7. The construction of the balance
variable and reduced-$\vec{E}_T$ variables are described in the following sections.

Table 4.6: Summary of the optimal signal selection [62].

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dilepton invariant mass</td>
<td>$</td>
</tr>
<tr>
<td>Dilepton $p_T$</td>
<td>$p_T(Z) &gt; 45$ GeV</td>
</tr>
<tr>
<td>b-tag veto</td>
<td>based on vertex info. (for jet with $p_T &gt; 20$ GeV)</td>
</tr>
<tr>
<td>Jet veto</td>
<td>no jets with $p_T &gt; 30$ GeV</td>
</tr>
<tr>
<td>Reduced $\vec{E}_T$</td>
<td>$&gt; 65$ GeV</td>
</tr>
<tr>
<td>$\vec{E}_T$ balance</td>
<td>$0.4 &lt; B &lt; 1.8$</td>
</tr>
<tr>
<td>$\Delta \phi(\vec{E}_T, \text{jet})$</td>
<td>$&gt; 0.5$ rad</td>
</tr>
<tr>
<td>$\Delta \phi(\vec{E}_T, \text{lept.})$</td>
<td>$&gt; 0.2$ rad</td>
</tr>
<tr>
<td>Lepton veto</td>
<td>no additional leptons $(e/\mu)$ with $p_T &gt; 10/3$ GeV</td>
</tr>
</tbody>
</table>

4.4 Balance and Alignment

The transverse balance variable is defined as a ratio between the PF $\vec{E}_T$ and the transverse momentum of the leptonically decaying $Z$ boson $p_T(Z)$:

$$B = \frac{\vec{E}_T}{p_T(Z)}.$$  \hspace{1cm} (4.7)

This variable is a measure of how well-balanced an event is, and can distinguish signal with real neutrinos from background with fake $\vec{E}_T$ from hadronic recoil. A cut is placed on $B$ to select events where $p_T(Z)$ and $\vec{E}_T$ are not too different in value: $0.4 < B < 1.8$. Figure 4.8 shows distributions of the $B$ distribution.

A mismeasurement of the jet energy can produce large fake $\vec{E}_T$, where the direction of $\vec{p}_T$ is aligned with the direction of a jet within the transverse plane. These events are characterized by a small transverse angle between $\vec{p}_T$ and the closest jet $\vec{p}_{T(jet)}$: $\Delta \phi(\vec{p}_T, \vec{p}_{T(jet)})$. The distribution of $\min \Delta \phi(\vec{p}_T, \vec{p}_{T(jet)})$ is shown in Fig. 4.9. To suppress $Z$+jets production, events with $\min \Delta \phi(\vec{p}_T, \vec{p}_{T(jet)}) < 0.5$ are rejected.
A mismeasurement of the transverse momentum of leptons can also produce fake $\not{E}_T$. This effect is usually negligible due to good lepton momentum resolution, but some events with large missing energy may have a small angle between $p_T$ and the $\not{p}_T$ of the closest lepton. Figure 4.10 shows $\min \Delta \phi(p_T, \not{p}_T(\ell))$. Events with $\not{E}_T > 60$ GeV and $\min \Delta \phi(p_T, \not{p}_T(\ell)) < 0.2$ are rejected.
4.5 Reduced Missing Transverse Energy

Missing transverse energy ($E_T$) is computed from the magnitude of the negative vector sum ($\not{p}_T$) of the transverse momenta of all particles (photons, electrons, muons, charged hadrons, neutral hadrons) in the event reconstructed with the PF technique. The resolution of $E_T$ is critical for the extraction of the $ZZ \rightarrow \ell^+\ell^-\nu\bar{\nu}$ signal given that it is the distinctive hallmark with respect to DY background. Since the DY background is
Figure 4.9: Distributions, after full selection, of the $\Delta \phi$ angle between the $\vec{p}_T$ and the closest jet (with $p_T > 20$ GeV), in ee (left) and $\mu \mu$ (right) and per running period (7 TeV top, 8 TeV bottom). The gray error bands show statistical uncertainties.

orders of magnitude greater than $ZZ \rightarrow \ell^+\ell^−\nu\bar{\nu}$ across a large portion of the spectrum, it is not sufficient to simply cut on $\vec{E}_T$. The resulting yield will contain either too much DY, or lose too much signal. The approach taken here is to replace the “ordinary” $\vec{E}_T$ variable with a “reduced” $\vec{E}_T$ variable.

The general concept behind a reduced $\vec{E}_T$ is, on an event-by-event basis, to reduce
Figure 4.10: Distributions of the $\Delta \phi$ angle between the $E_T$ and the closest lepton, in $\mu\mu$ (left), and ee (right) channels, after a cut on $p_T$ ($p_T > 45$ GeV) and on the dilepton mass ($|m_{\mu\mu} - m_Z| < 7.5$ GeV) and a veto on the jets (over 30 GeV) and on a third lepton. The gray error bands show statistical uncertainties.

The $E_T$ by considering possible contributions to fake $E_T$. The ideal signature is two well-measured leptons with substantial net transverse momentum which is not balanced by any substantial activity in the opposite side of the detector where the recoiling particles are expected. The algorithm begins by considering the transverse momentum of lepton pair, and then reducing this if there is possible balancing contributions from jets or
unclustered energy. An event with high value for reduced $E_T$ is one which has a very robust $E_T$ signature that is unlikely to come from mismeasurement. The reduced $E_T$ can be thought of as a minimum feasible value for the $E_T$, and not as an unbiased estimator of the true $E_T$.

This approach was used effectively in the D0 experiment’s analysis of the same process [79, 80], and before that in the analysis of similar final states in the OPAL experiment [81] and studied at CMS [82].

4.5.1 Reduced $E_T$ Definition

The reduced $E_T$ for each event is calculated by decomposing $E_T$, jet and lepton $p_T$ information along an orthogonal set of axes within the transverse plane of the detector. The decomposition axes are oriented such that projection onto one is insensitive to hadronic recoil, and projection onto the other is insensitive to lepton resolution. Since the signal is characterized by events with two well-measured leptons, we are only concerned with constructing reduced $E_T$ for those events. Two-dimensional transverse decomposition vectors are defined for both leptons ($\vec{p}_T(\ell_1)$ and $\vec{p}_T(\ell_2)$, ordered by $p_T$), each jet ($\vec{p}_T(\text{jet})$) and missing energy ($\vec{p}_T$) to be projected on, within each event. The orthogonal set of decomposition axes are also defined as two unit vectors: a “perpendicular axis” $\hat{a}_t$ and a “longitudinal axis” $\hat{a}_l$, shown in Fig. 4.11. If the angle $\phi$ between the two leptons is less than $\frac{\pi}{2}$, then

$$\hat{a}_t = \frac{\vec{p}_T(\ell_1) + \vec{p}_T(\ell_2)}{|\vec{p}_T(\ell_1) + \vec{p}_T(\ell_2)|}$$  \hfill (4.8)$$

and $\hat{a}_l$ is defined such that $\hat{a}_l \cdot \hat{a}_t = 0$ and $\vec{p}_T(\ell_1) \cdot \hat{a}_l > 0$. However, if $\phi \geq \frac{\pi}{2}$, then

$$\hat{a}_l = \frac{\vec{p}_T(\ell_1) - \vec{p}_T(\ell_2)}{|\vec{p}_T(\ell_1) - \vec{p}_T(\ell_2)|}$$  \hfill (4.9)$$
which is a normalized thrust axis, and $\hat{a}_t$ is defined such that $\hat{a}_t \cdot \hat{a}_t = 0$ and $\vec{p}_T(\ell_1) \cdot \hat{a}_t > 0$.

To accommodate photon samples, since there are no dileptons in those events, an adjustment is made such that $\{l, t\} = \{\perp \vec{p}_T(\ell \ell), \parallel \vec{p}_T(\ell \ell)\}$.

Two components are then defined: dilepton $p_T$ projection, and hadronic recoil projection, each having two projections, one on each axis. Dilepton $p_T$ projection is defined as

$$a_{i \ell} = (\vec{p}_T(\ell_1) + \vec{p}_T(\ell_2)) \cdot \hat{a}_i,$$

for each axis where $i \in \{l, t\}$. Recoil is defined in two different ways: clustered recoil ($\vec{R}_{\text{clus}}$), which is a vectorial sum of the particle-flow jets reconstructed in the event, with the projections:

$$a_{i \text{jet}} = \vec{R}_{\text{clus}} \cdot \hat{a}_i = \sum_j \vec{p}_T(\text{jet}_j) \cdot \hat{a}_i,$$

and unclustered recoil ($\vec{R}_{\text{uncl}}$), which is the sum of all particle-flow candidates in the event, except for the two leptons, constructed as the vector sum of the particle-flow $E_T$ and the dilepton system $p_T$, with opposite sign, and the projections:

$$a_{i E_T} = \vec{R}_{\text{uncl}} \cdot \hat{a}_i = (\vec{p}_T + \vec{p}_T(\ell_1) + \vec{p}_T(\ell_2)) \cdot \hat{a}_i.$$

The two projections $a_{i E_T}$ and $a_{i \text{jet}}$ can be combined into a “reduced” recoil in a variety of ways. The main idea, however, is to find the minimum of the two to reduce the possibility that an otherwise well-balanced event receives a significant amount of missing energy from hadronic activity, or calorimeter noise. This can be done by comparing $a_{i E_T}$ and $a_{i \text{jet}}$ before or after being added to other components.

The lepton $\vec{p}_T$ is varied by the fractional momentum uncertainty ($\delta p_T/p_T$), restricted to a value 1 or less, to account for the possibility of mismeasured lepton $p_T$. 

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contributing to fake $\not{E}_T$:

$$\sigma(\ell) = \min\left(1, \frac{\delta p_T(\ell)}{p_T(\ell)}\right)$$

(4.13)

$$\vec{p}_T'(\ell) = (1 - \sigma(\ell)) \cdot \vec{p}_T(\ell).$$

(4.14)

This reduced $\vec{p}_T'$ for each lepton is then used to define new axes $\vec{a}_t'$ and $\vec{a}_l'$, following the same steps as the dilepton $p_T$ projection described above. The lepton uncertainty projections are then defined as

$$\delta a_{t}^{\ell\ell} = a_{t}^{\ell\ell} - a_t^{\ell}$$

(4.15)

$$\delta a_{l}^{\ell\ell} = (\sigma(\ell_2)\vec{p}_T(\ell_2) - \sigma(\ell_1)\vec{p}_T(\ell_1)) \cdot \vec{a}_l.$$

(4.16)

There are two known ways to build reduced $\not{E}_T$ from combining these components: one is similar to the original OPAL [81] and D0 [79,80] version referred to as "D0 reduced $\not{E}_T$", and the other is referred to as a "CMS reduced-$\not{E}_T$" variation. The original version minimizes the recoil components individually. Then the dilepton, recoil, and uncertainty components are summed, with weights applied to the recoil ($W_{rec}$) and uncertainty ($W_{unc}$) terms for the purpose of optimization.

$$C_i^{D0} = \max(0, a_i^{\ell\ell} + W_{rec} \cdot \min(a_i^{\text{jet}}, -a_i^{\not{E}_T}, 0) + W_{unc} \cdot \delta a_i^{\ell\ell})$$

(4.17)

The CMS reduced-$\not{E}_T$ variation minimizes the reduced $\not{E}_T$ after the components are summed with weights, without a minimization bound 0.

$$C_i^{CMS} = \min\left(\left|a_i^{\ell\ell} - W_{rec} \cdot a_i^{\not{E}_T} + W_{unc} \cdot \delta a_i^{\ell\ell}\right|, \left|a_i^{\ell\ell} + W_{rec} \cdot a_i^{\text{jet}} + W_{unc} \cdot \delta a_i^{\ell\ell}\right|\right)$$

(4.18)
Each version of reduced-$E_T$ is then the components added in quadrature with an additional optimization weight $W_t$ on the $C_t^2$ component:

$$\text{reduced-}E_T = \sqrt{C_t^2 + W_t \cdot C_t^2}.$$  \hspace{1cm} (4.19)

Figure 4.11: Left: simplified decomposition axes for accommodating photon samples. Right: decomposition axes of reduced-$E_T$ in optimization process [79].

4.5.2 Reduced $E_T$ Optimization

The weights introduced in the construction of reduced-$E_T$ optimize the degree to which recoil and uncertainty projection components subtract from dilepton $p_T$ projection, and also the degree to which longitudinal and perpendicular components sum in quadrature. This effectively makes the reduced-$E_T$ a function of the weights $\text{reduced-}E_T = f(W_{\text{rec}}, W_{\text{unc}}, W_t)$. A variable cut is implemented on the reduced-$E_T$,

$$f(W_{\text{rec}}, W_{\text{unc}}, W_t) > E_{\text{cut}}.$$  \hspace{1cm} (4.20)
The weights and $E_{\text{cut}}$ each occupy a test range of incremental values, which are swept through iteratively. For each combination of weight and cut values, the surviving signal and background events are counted, then efficiencies and significance are calculated (see Fig. 4.12).

Treating DY (MC) as the only background and ZZ as the only signal, a specified signal efficiency range and background rejection range, a maximum significance, $S \sqrt{B+S}$, are selected which gives the corresponding values of $W_{\text{rec}}, W_{\text{unc}}, W_t$, and $E_{\text{cut}}$ as a working point. This optimization process is done for each version of reduced-$\slashed{E}_T$, as their behavior differs by construction. A side by side comparison of ordinary PF $\slashed{E}_T$ and a reduced-$\slashed{E}_T$, after optimization, can be seen in Fig. 4.13. The most optimum working point chosen is with the CMS reduced-$\slashed{E}_T$ construction, where $W_{\text{rec}} = 1.0$, $W_{\text{unc}} = 0.0$, $W_t = 1.0$, and $E_{\text{cut}} = 65 \text{ GeV}$, where the electron and muon channels are combined for optimization. The performance at this working point is similar to the optimal D0 reduced-$\slashed{E}_T$ working point, where $W_{\text{rec}} = 1.25$, $W_{\text{unc}} = 0.0$, $W_t = 1.0$, and $E_{\text{cut}} = 45 \text{ GeV}$, where the electron and muon channels are combined as well. These results show that for an optimal working point, the effects of hadronic recoil play a significant role, the perpendicular and longitudinal components have equal influence, and the effects of lepton uncertainty are negligible.

Optimization was performed on electron and muon channels independently, since the physics and detector response differs between channels. For the CMS reduced-$\slashed{E}_T$ the optimal point for muons is $W_{\text{rec}} = 0.75$, $W_{\text{unc}} = 1.0$, $W_t = 1.5$, and $E_{\text{cut}} = 70 \text{ GeV}$. For electrons the optimal point is $W_{\text{rec}} = 0.75$, $W_{\text{unc}} = 0.25$, $W_t = 0.75$, and $E_{\text{cut}} = 55 \text{ GeV}$. However, the efficiencies were comparable to the simpler optimal point of the combined channel.
Figure 4.12: Signal efficiency vs Background rejection (top), Significance vs Signal Efficiency, showing chosen optimal point in red (bottom). Each point corresponds to a set of weights and a cut on the resulting reduced-$E_T$ variable.
4.5.3 Stability of Reduced $\not{E}_T$

Rejecting DY is important, and so $\not{E}_T$ plays an vital role in this analysis. Before an optimized selection can be reached, a version of $\not{E}_T$ is needed that can describe data well, provides good DY rejection, and is stable under pile-up and energy scale variations.

The different $\not{E}_T$ definitions described in the previous subsection are compared:

- standard Particle Flow $\not{E}_T$
• D0 reduced-$\not{E}_T$

• CMS reduced-$\not{E}_T$.

Signal efficiency as a function of background (DY) efficiency is shown in Fig. 4.14 for the three different types of $\not{E}_T$ at various cut points. Both versions of reduced-$\not{E}_T$ have lower background efficiency than the standard PF $\not{E}_T$. Efficiencies are computed at various cut points, which are made after all other selections. At high luminosity, it is important to check stability with respect to pile-up, for these various types of $\not{E}_T$. The efficiencies are computed after varying the pile-up weight and the uncertainty on the jet energy scale, for each cut point. The stability plots show that both definitions of reduced-$\not{E}_T$ out-perform ordinary PF $\not{E}_T$. The original D0 reduced-$\not{E}_T$ slightly out-performs the CMS reduced-$\not{E}_T$, however the difference is marginal at the chosen operation point, where efficiency of DY is $10^{-3}$. The optimized CMS reduced-$\not{E}_T$ is chosen to be consistent with the definition used in the CMS $H \rightarrow ZZ \rightarrow \ell\ell\nu\nu$ analysis [83]. Figure 4.15 shows distributions of the two variables in data and MC at preselection level.

4.6 Background Estimation

Sometimes Monte Carlo simulation may not fully model a kinematic region of interest. So in place of simulation, a sample of events taken from real data is used to model or estimate a source of background.

4.6.1 Photon Sample for Drell-Yan

In the case of Drell-Yan Monte Carlo, pile-up and detector effects in the tail of the missing energy distribution may not be fully modelled. Plus, simulation is statistically
limited in that region. So, instead of simulation, an orthogonal $\gamma$+jets data sample is used to model $Z$+jets. Events in the photon sample are topologically equivalent, except for the lack of leptonic decay, so the underlying hadronic activity and $E_T$ distribution is expected to resemble that of DY production.

Some corrections are applied to the photon sample to make sure it models the DY process well. First, it is rescaled to the observed dilepton yield as a function of $p_T$ after the jet-veto is applied to both samples. This accounts for differences in efficiency and corrects for prescaling. Figure 4.16 shows the MC photon $p_T$ distribution before it is also rescaled.

To select photon events, shower shape, tracker isolation, and calorimeter deposits are used. Only photons within the barrel region are selected, due to high purity and resolution. After this selection, several processes with fake $E_T$ contribute:

- single $\gamma$ events
Figure 4.15: Comparison between data and Monte Carlo for the original $E_T$ (left) and the optimized reduced $E_T$ (right), for 7 TeV (top) and 8 TeV (bottom). Disagreement between data and MC at 8 TeV is seen due to Drell-Yan contribution not being fully modelled.

- double $\gamma$ events, where one photon escapes detection or fails identification

- QCD events that fake a photon.

Several processes with real $E_T$ may also contaminate this sample. Even though they have lower cross sections they may contaminate the tail of the $E_T$ distribution.
Figure 4.16: Photon $p_T$ spectra in MC at 7 TeV (left) and 8 TeV (right), before reweighting. The contribution from EWK processes is found to be about 1%. In the 8 TeV plots, the W+jets, $\gamma + Z$ and $\gamma + W$ processes are grouped as “EWK”.

- $W+\gamma$, where $W \rightarrow \ell \nu$
- $Z+\gamma$, where $Z \rightarrow \ell\ell$ or $\nu\nu$
- $W+$jets, where $W \rightarrow \ell\nu$ but $\ell$ fakes a $\gamma$
- electron conversion within the tracker.

To reduce their contribution, specific selections are applied to events:

- must contain exactly one photon and zero leptons
- only jets with $\Delta R > 0.4$ from the photon are used for jet-related selections
- electron veto to remove conversions \[84\]

Reduced-$\hat{E}_T$ is adjusted for $\gamma+$jets events by defining the longitudinal and perpendicular components with respect to the direction of $\vec{p}_T(\gamma)$, see Fig. 4.11. Figure 4.17 shows the distribution of the two components of reduced-$\hat{E}_T$ after rescaling photon data and MC. It can be seen that the electroweak events still contaminate the tail of the distribution, so it is subtracted from the rescaled photon distributions bin-by-bin. Figure 4.18 shows the composite reduced-$\hat{E}_T$ distribution after rescaling.
Figure 4.17: Longitudinal (left) and transverse (right) components of the reduced-$E_T$ variable in the photon sample. In the 8 TeV plots (bottom), the W+jets, $\gamma + Z$ and $\gamma + W$ processes are grouped as “EWK”. The gray error bands show statistical uncertainties.

An estimate of systematic uncertainty is assigned, as the difference between the DY yield in data and simulation (shown in Fig. 4.19) in a control region defined by inverting the reduced-$E_T$ cut. This gives a systematic uncertainty of about 25% at 7 TeV and 40% at 8 TeV. A closure test [85] is also performed on the $\gamma$+jets prediction using 8 TeV MC samples of different jet multiplicities, and is successful for all cate-
Figure 4.18: Reduced-\(\not{E}_T\) spectrum in the photon sample after reweighting to match the ee (left) and the \(\mu\mu\) (right) \(p_T\) spectrum. In the 8 TeV plots (bottom), the W+jets, \(\gamma + Z\) and \(\gamma + W\) processes are grouped as “EWK”. The gray error bands show statistical uncertainties.

gories, especially the 0-jet category. Figure 4.20 shows the reduced-\(\not{E}_T\) distributions in dilepton data and simulation, using the photon sample to describe the DY background, after the subtraction of electroweak backgrounds. There is good agreement along the entire spectrum.

Before using the photon sample to model DY, another approach was tried to model
Figure 4.19: Reduced-$E_T$ distributions in data and simulation at 7 TeV (left) and 8 TeV (right), after the full selection except the reduced-$E_T$ cut. The gray error bands show statistical uncertainties.

Figure 4.20: Reduced-$E_T$ spectrum in the inclusive $\ell\ell$ channel at 7 TeV (left) and 8 TeV (right), using the DY-template derived from the photon sample at preselection level [62]. The gray error bands show statistical uncertainties.

The tail region. The reduced-$E_T$ component distributions in the DY MC were smeared by adding a random Gaussian number to them in each event. A range of widths were iterated over to find the minimum $\chi^2$ between DY and data in a control region.
The problem with this procedure is that this forcing of agreement between MC and data in one variable caused disagreement in another.

4.6.2 Non-Resonant Background

Another data-driven method is used to estimate the total number of non-resonant background (NRB) events, which includes WW and top production. A control sample is selected, by applying the previously described selection cuts to $e\mu$ data events to get the yields. From there, two invariant mass regions are defined, shown in Fig. 4.21:

- the side-bands (SB) $55 \text{ GeV} < m(\ell_1, \ell_2) < 70 \text{ GeV}$ and $110 \text{ GeV} < m(\ell_1, \ell_2) < 200 \text{ GeV}$
- the peak $70 \text{ GeV} < m(\ell_1, \ell_2) < 110 \text{ GeV}$.

The $e\mu$ sample is expected to be comprised of only NRB. So to compute the NRB contamination in the same-flavor channels, a ratio is taken between the event yields from the $e\mu$ and $\ell^+\ell^-$ side bands, and multiplied by the events yield from the $e\mu$ peak region to get the NRB contamination in the $\ell^+\ell^-$ peak region:

$$\alpha_{\ell\ell} = \frac{N_{\ell\ell}^{\text{SB}}}{N_{e\mu}^{\text{SB}}}$$
$$N_{\ell\ell}^{\text{peak}} = \alpha_{\ell\ell} \times N_{e\mu}^{\text{peak}} \quad (4.21)$$

where this is done for both the $\mu^+\mu^-$ and $e^+e^-$ channels. The measured values of $\alpha$ with their corresponding statistical uncertainties are:

$$\alpha_{\mu\mu}^{7 \text{ TeV}} = 0.64 \pm 0.06$$
$$\alpha_{ee}^{7 \text{ TeV}} = 0.42 \pm 0.04$$
$$\alpha_{\mu\mu}^{8 \text{ TeV}} = 0.69 \pm 0.03$$
$$\alpha_{ee}^{8 \text{ TeV}} = 0.43 \pm 0.02$$
Figure 4.21: The peak region is between the dashed lines, the side-band (SB) region is outside the dashed lines. NRB scaling factors are obtained from the side-band region, and applied to the peak region.

Figure 4.22 shows the distributions of invariant mass of the $\mu^+\mu^-$, $e^+e^-$, and $e\mu$ channels after the main selection. The agreement between data and simulation (dominated by WW) in the control region is good, so the MC is taken as a reliable estimate of NRB. A closure test is performed by comparing the yields of NRB backgrounds in simulation with the prediction from data, and a total uncertainty of 20% is assigned to this method from the resulting bias ($\frac{N_{pred} - N_{true}}{N_{true}}$) and statistical uncertainty.

4.7 Systematic Uncertainties

The measurement of the ZZ cross section has multiple sources of uncertainties, which affect the number of observed events. A summary of all the systematic uncertainties is in Table 4.7.
Figure 4.22: Dilepton invariant mass distributions in $\mu\mu$ (top), $ee$ (middle), and $e\mu$ (bottom) channels, and 7 TeV (left), and 8 TeV (right), after the full selection. The side-bands of the $Z$ peak in the first two plots constitute a good control region for the WW background. The gray error bands show statistical uncertainties.
Table 4.7: Systematic uncertainties on the final cross sections due to each source separately, after the maximum-likelihood fit to extract the ZZ cross section. The uncertainties marked with an asterisk (*) are used as shape errors in the fit [62].

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Systematic uncertainty [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7 TeV</td>
</tr>
<tr>
<td>(*) MC statistics: ZZ (ee)</td>
<td>0.8</td>
</tr>
<tr>
<td>(*) MC statistics: ZZ (μμ)</td>
<td>1.3</td>
</tr>
<tr>
<td>(*) MC statistics: WZ (ee)</td>
<td>1.6</td>
</tr>
<tr>
<td>(*) MC statistics: WZ (μμ)</td>
<td>1.7</td>
</tr>
<tr>
<td>(*) Ctrl. sample statistics: DY (ee)</td>
<td>6.5</td>
</tr>
<tr>
<td>(*) Ctrl. sample statistics: DY (μμ)</td>
<td>5.8</td>
</tr>
<tr>
<td>(*) Ctrl. sample statistics: NRB (ee)</td>
<td>6.3</td>
</tr>
<tr>
<td>(*) Ctrl. sample statistics: NRB (μμ)</td>
<td>8.1</td>
</tr>
<tr>
<td>ZZ, WZ cross section: PDF+αS</td>
<td>5.5</td>
</tr>
<tr>
<td>(*) ZZ, WZ cross section: QCD scales</td>
<td>7.3</td>
</tr>
<tr>
<td>(*) ZZ, WZ cross section: NLO EWK corr.</td>
<td>2.4</td>
</tr>
<tr>
<td>Signal acceptance</td>
<td>3.0</td>
</tr>
<tr>
<td>Luminosity</td>
<td>3.6</td>
</tr>
<tr>
<td>(*) Pile-Up</td>
<td>0.5</td>
</tr>
<tr>
<td>Muon Trigger, ID, Isolation</td>
<td>4.1</td>
</tr>
<tr>
<td>Electron Trigger, ID, Isolation</td>
<td>1.7</td>
</tr>
<tr>
<td>(*) Lepton Momentum Scale</td>
<td>2.7</td>
</tr>
<tr>
<td>(*) Jet Energy Scale</td>
<td>6.0</td>
</tr>
<tr>
<td>(*) Jet Energy Resolution</td>
<td>0.8</td>
</tr>
<tr>
<td>(*) Unclustered $\not{E}_T$</td>
<td>2.0</td>
</tr>
<tr>
<td>(*) b-jet veto</td>
<td>0.3</td>
</tr>
<tr>
<td>Drell-Yan Normalization</td>
<td>6.6</td>
</tr>
<tr>
<td>Top &amp; WW Normalization</td>
<td>7.7</td>
</tr>
<tr>
<td>Total systematic uncertainty</td>
<td>21.0</td>
</tr>
<tr>
<td>Statistical uncertainty</td>
<td>27.9</td>
</tr>
</tbody>
</table>
4.7.1 Monte Carlo and Control Sample Statistics

For processes estimated from simulation, like ZZ and WZ, the limited statistics of the MC sample affects the precision of the modelling. Therefore, the statistical uncertainty corresponding to the sample is taken as a systematic uncertainty in the shape of kinematic distributions used in cross-section measurement. The same treatment is given to backgrounds estimated from data, like DY and NRB, since they are limited by the size of the control samples.

4.7.2 ZZ and WZ Cross Sections and Acceptance

The cross sections of $pp \rightarrow ZZ + X \rightarrow \ell^+ \ell^- \bar{\nu} \nu + X$ and $pp \rightarrow WZ + X \rightarrow \ell^+ \ell^- \nu \nu' + X$ are calculated using mcfm, version 6.2 [61], and using parton density function from the Les Houches accord PDF program (LHAPDF), version 5.8.7 [86]. The PDF+$\alpha_S$ uncertainty is evaluated as the maximum difference between the computed cross sections of three PDF sets, including the corresponding errors from $1\sigma$ variations of the PDF parameters and value of $\alpha_S$, with the QCD renormalization and factorization scale set to $\mu = 90 \text{ GeV} \approx m_Z$. The uncertainty for the renormalization and factorization scales is evaluated as the maximum difference of the central values of the cross section, computed at different scales: $\mu = (\frac{1}{2}, 1, 2) \times m_Z$, for each PDF set. For ZZ, the prescription described in [87,88] is followed for the case of 0 jet production, to consider the jet-veto.

The cross section computation takes into account the acceptance cuts on lepton $p_T$ and $\eta$, $m(\ell\ell)$, $E_T$, and the jet veto. Cross sections with and without cuts on generator level leptons and neutrinos are compared. The acceptance is recomputed after varying the QCD scales, and the maximum difference is taken as systematic uncertainty.
4.7.3 Lepton Efficiency and Momentum Scales

The efficiencies of the lepton trigger and identification are determined in data, using the tag-and-probe method with $Z \rightarrow \ell \ell$ events [85], and is used to correct simulated samples. A systematic uncertainty is computed for lepton momentum scale by shifting the nominal momenta by $\pm 1\sigma$, and propagating the variation to the reduced-$E_T$. The uncertainties are propagated by computing a new $E_T$ for each shift, and recomputing reduced-$E_T$. The shapes of kinematic distributions are expected to vary, so they are used as shape errors in the fit of the cross section.

4.7.4 Jet Energy Scale and Resolution

The uncertainty in the calibration of the jet energy scale (JES) directly affects the jet veto, balance cut, and reduced-$E_T$ cut. It is estimated by smearing the 4-momentum of the jets by $\pm 1\sigma$ [74]:

$$ P_4 \rightarrow P_4 \cdot \left(1 \pm \frac{\sigma_{\text{JES}}(p_T, \eta)}{p_T} \right) \quad (4.22) $$

where $\sigma_{\text{JES}}$ is the absolute uncertainty on the jet energy scale, which is parameterized as a function of $p_T$ and $\eta$ of the jet. This variation affects the energy imbalance in an event, so it is propagated to $E_T$ by:

$$ \bar{p}_T \rightarrow \bar{p}_T - \Delta \bar{p}_T(\text{jet}) \quad (4.23) $$

where $\Delta \bar{p}_T(\text{jet})$ is the difference between $\bar{p}_T(\text{jet})$ before and after smearing. The new $\bar{p}_T$ and jets are then used to calculate a new reduced-$E_T$ and other relevant variables.

A systematic uncertainty in jet energy resolution (JER) is also computed. The energy of jets in MC is corrected to reproduce the same resolution as observed in data,
by smearing the jet transverse momenta:

\[
p_T \rightarrow p_T \cdot \max[0, p_T^\text{GEN} + \text{Gauss}(\text{JER}, \sigma_{\text{JER}}) \cdot (p_T^\text{GEN} - p_T)] / p_T.
\]  

(4.24)

The JER values and errors are taken from the official CMS recommendations for 2011 [74]. These corrections are propagated to all variables dependent on jets. Since the shape of the distributions are affected by these variations, the systematic uncertainties described here are treated as errors on the shape in the cross section fit.

### 4.7.5 B-jet Veto

The b-tagging efficiency for the Combined Secondary Vertex (CSV) discriminator is taken from [89]. The working point in simulation for this b-tagger is shifted to reproduce the same efficiency observed in data. The uncertainty in efficiency is propagated to the event yields of MC-driven processes by varying the working point.

### 4.7.6 Pile-Up

Monte Carlo samples are reweighted to reproduce the same pile-up conditions observed in data. To compute the uncertainty associated with this procedure, the mean of the distribution of real pile-up is shifted by 8% when reweighting simulated samples, and measuring the variation in the final yields. The shape of the kinematic distributions can vary from the shifts, so the varied distributions are used as shape errors in the fit of the cross section.
4.8 Cross Section Measurement of ZZ Production

After the final selection cuts, the ZZ production cross section is measured from a profile likelihood fit to the reduced-\(\not{E}_T\) distributions of the \(e^+e^-\) and \(\mu^+\mu^-\) channels individually, and to the combined channel distribution, shown in Fig. 4.23. The fit accounts for ZZ signal and all background processes, where each previously described systematic uncertainty is treated as a nuisance parameter.

Figure 4.23: Reduced-\(\not{E}_T\) distribution in \(\ell\ell\) (\(\ell = e, \mu\)) channels, at final selection, at 7 TeV (left) and 8 TeV (right) [62]. The gray error band includes statistical and systematic uncertainties.

4.8.1 Profile Likelihood Method

The profile likelihood is maximized to give a signal strength \(\mu\), which translates into a cross section by: \(\mu = \sigma/\sigma_{\text{theory}}\). This also results in optimized background yields due to adjustments made to the nuisance parameters in the process of maximizing likelihood.

The predicted signal and background yields, before and after the fit to real data, are shown in Table 4.8.

The resulting fitted yields for signal and background processes can be expressed
Table 4.8: Predicted signal and background yields at 7 and 8 TeV, and corresponding values after the maximum-likelihood fit. The uncertainties include both statistical and systematic components [62].

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Process</th>
<th>Channel</th>
<th>Predicted yield</th>
<th>Fitted yield</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$ZZ \rightarrow 2\ell 2\nu$</td>
<td>ee</td>
<td>$14.0 \pm 1.9$</td>
<td>$12.0 \pm 4.4$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\mu\mu$</td>
<td>$21.7 \pm 3.2$</td>
<td>$18.4 \pm 6.6$</td>
</tr>
<tr>
<td>7 TeV</td>
<td>$WZ \rightarrow 3\ell \nu$</td>
<td>ee</td>
<td>$7.7 \pm 0.9$</td>
<td>$7.9 \pm 0.8$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\mu\mu$</td>
<td>$11.5 \pm 1.6$</td>
<td>$11.5 \pm 1.0$</td>
</tr>
<tr>
<td></td>
<td>Z+ jets</td>
<td>ee</td>
<td>$5.0 \pm 2.7$</td>
<td>$4.8 \pm 2.1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\mu\mu$</td>
<td>$8.3 \pm 4.8$</td>
<td>$4.8 \pm 1.8$</td>
</tr>
<tr>
<td></td>
<td>Non Resonant</td>
<td>ee</td>
<td>$7.7 \pm 3.1$</td>
<td>$7.4 \pm 2.4$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\mu\mu$</td>
<td>$11.2 \pm 4.8$</td>
<td>$9.2 \pm 2.8$</td>
</tr>
<tr>
<td></td>
<td>$ZZ \rightarrow 2\ell 2\nu$</td>
<td>ee</td>
<td>$77 \pm 16$</td>
<td>$69 \pm 14$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\mu\mu$</td>
<td>$109 \pm 23$</td>
<td>$99 \pm 22$</td>
</tr>
<tr>
<td>8 TeV</td>
<td>$WZ \rightarrow 3\ell \nu$</td>
<td>ee</td>
<td>$45 \pm 6$</td>
<td>$44.6 \pm 3.9$</td>
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<td></td>
<td>$\mu\mu$</td>
<td>$64 \pm 8$</td>
<td>$64.9 \pm 5.4$</td>
</tr>
<tr>
<td></td>
<td>Z+ jets</td>
<td>ee</td>
<td>$36 \pm 12$</td>
<td>$27.4 \pm 8.3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\mu\mu$</td>
<td>$63 \pm 21$</td>
<td>$52 \pm 14$</td>
</tr>
<tr>
<td></td>
<td>Non Resonant</td>
<td>ee</td>
<td>$31 \pm 9$</td>
<td>$34.2 \pm 5.7$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\mu\mu$</td>
<td>$50 \pm 14$</td>
<td>$54.3 \pm 9.9$</td>
</tr>
</tbody>
</table>

as a function depending on predicted yields from simulation or data-driven methods, and nuisance parameters. Nuisance parameters come into play as factors adjusting the nominal predictions on a process. Each one is then assigned a log-normal PDF constraint for the fit, and are allowed to fluctuate within these constraints, which results in small adjustments to the yields.

### 4.8.2 Results

The measured exclusive $ZZ \rightarrow \ell^+\ell^-\nu\bar{\nu}$ production cross section is obtained by scaling the theoretical cross section by the resulting signal strength $\mu$. These theoretical predictions are computed with mcfm [61] at NLO in QCD, and corrected for NLO electroweak effects by $-4.1\%$:

- $239^{+11}_{-8}$ fb at 7 TeV
• $292^{+13}_{-9}$ fb at 8 TeV.

The results are shown in Table 4.9.

Table 4.9: $p p \rightarrow ZZ \rightarrow \ell^+\ell^-\nu\bar{\nu}$ cross sections at 7 and 8 TeV, measured in the ee and $\mu\mu$ channels separately, and combined [62].

<table>
<thead>
<tr>
<th>Channel</th>
<th>$7$ TeV</th>
<th>$\sigma$ [fb]</th>
<th>$8$ TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>ee</td>
<td>$297^{+104}<em>{-93}$ (stat) $^{+82}</em>{-68}$ (syst) $\pm 10$ (lumi)</td>
<td>$238^{+50}<em>{-47}$ (stat) $^{+77}</em>{-54}$ (syst) $\pm 10$ (lumi)</td>
<td></td>
</tr>
<tr>
<td>$\mu\mu$</td>
<td>$143^{+71}<em>{-64}$ (stat) $^{+60}</em>{-59}$ (syst) $\pm 5$ (lumi)</td>
<td>$292^{+43}<em>{-41}$ (stat) $^{+90}</em>{-68}$ (syst) $\pm 12$ (lumi)</td>
<td></td>
</tr>
<tr>
<td>Combined</td>
<td>$201^{+59}<em>{-54}$ (stat) $^{+56}</em>{-43}$ (syst) $\pm 7$ (lumi)</td>
<td>$264^{+32}<em>{-31}$ (stat) $^{+74}</em>{-55}$ (syst) $\pm 11$ (lumi)</td>
<td></td>
</tr>
</tbody>
</table>

The measured inclusive ZZ production cross section is obtained by scaling the theoretical inclusive cross section by the signal strength $\mu$. The theoretical cross section is computed with the zero-width approximation in mcfm, which accounts for virtual photon contribution to lepton pair production, by neglecting the interference between $\gamma$ and the $Z$ boson. The resulting inclusive ZZ cross sections are:

$$7 \text{ TeV} : \quad \sigma(pp \rightarrow ZZ) = 5.2^{+1.5}_{-1.4} \text{(stat)}^{+1.5}_{-1.1} \text{(syst)} \pm 0.2 \text{(lumi)} \text{ pb} \quad (4.25)$$

$$8 \text{ TeV} : \quad \sigma(pp \rightarrow ZZ) = 6.9^{+0.8}_{-0.8} \text{(stat)}^{+1.9}_{-1.4} \text{(syst)} \pm 0.3 \text{(lumi)} \text{ pb} \quad (4.26)$$

For each $p_T$ bin, shown in Fig. 4.24, the cross section is measured by:

$$\sigma = \frac{N_{\text{obs}} - N_{\text{bkgd}}}{\epsilon A \mathcal{L}} \quad (4.27)$$

where $N_{\text{obs}}$ is the number of observed events in a bin, $N_{\text{bkgd}}$ is the yield of all background events, $\epsilon$ is the signal efficiency at final selection, $A$ is the signal acceptance, and $\mathcal{L}$ is the integrated luminosity.
Figure 4.24: $pp \rightarrow ZZ \rightarrow 2\ell 2\nu$ cross section ($60\text{GeV} < m(\ell\ell) < 120\text{GeV}$), at $7\text{ TeV}$ (left) and $8\text{ TeV}$ (right), in different ranges of $p_T(\ell\ell)$, compared to NLO predictions from MCFM.

### 4.9 Unfolding Transverse Momentum Spectra

Any observed number and distribution of data can be considered a distortion of the true number and distribution. To get an idea what the kinematics of ZZ events truly look like, detector effects must be removed. One way to do this is to “unfold” data to deconvolve the detector effects [90].

A discretized Bayesian cause-and-effect network is built using simulated data, where generator level information is the “cause” and the reconstruction level information is the “effect”, as shown in Fig. 4.25. Linking cause to effect has a probabilistic nature; therefore linking effect to cause will also be probabilistic, giving an associated uncertainty to the unfolding process.

The number of events per bin at the generator level can be written as a vector $\mathbf{N}_C$. The same can be done for events are reconstruction level: $\mathbf{N}_E$. These two vectors can
be related by a smearing matrix $\Lambda$ with elements $\lambda_{ji}$:

$$N_C = \{N(C_1), N(C_2), ..., N(C_{nC})\}$$

$$N_E = \{N(E_1), N(E_2), ..., N(E_{nE})\}$$

$$\lambda_{ji} = P(E_j|C_i)$$

Then to get an “inverse” unfolding matrix $\Theta$, with elements $\theta_{ij} = P(C_i|E_j)$, Bayes theorem is applied:

$$\theta_{ij} = \frac{\lambda_{ji} \cdot P(C_i|I)}{\sum_l \lambda_{jl} \cdot P(C_l|I)}$$

(4.29)

where $I$ is a placeholder for surrounding conditions. The matrix $\Theta$ can multiply real data $N_E$ at reconstruction level to get a real “generator” level set of data:

$$N_C = \Theta \times N_E.$$
are said to be in phase space $\varphi_C$.

The physical variable distribution used to unfold for both phase spaces is the transverse momentum of the $Z$ boson, $p_T(Z)$. The one-dimensional $p_T(Z)$ distribution in reconstruction level phase space is binned as: $N_E(\varphi_E)$, where $N$ is a vector-like object with elements that are the content for each bin. The other one-dimensional $p_T(Z)$ distribution in generator level phase space is binned as: $N_C(\varphi_C)$. The two-dimensional $p_T(Z) \times p_T(Z)$ distribution is in the conjunction of both phase spaces, and is binned as: $M(\varphi_E \cap \varphi_C)$. This two-dimensional distribution dictates the mapping of reconstruction level to generator level and so its structure, shown in Fig. 4.26, is representative of the unfolding matrix.

The unfolding matrix $\Theta$ is a function of the three $p_T(Z)$ distributions, $M, N_E, N_C$:

\[
\Theta = \Theta(M(\varphi_E \cap \varphi_C), N_E(\varphi_E), N_C(\varphi_C)). \tag{4.30}
\]

Fakes and misses are accounted for in the construction of $\Theta$. The phase space $\varphi_C$ contains misses, $\varphi_E$ contains fakes, and $\varphi_E \cap \varphi_C$ contains neither. So projecting $M(\varphi_E \cap \varphi_C)$ onto its “cause” and “effect” axes results in the one-dimensional distributions $N_C(\varphi_E \cap \varphi_C)$ and $N_E(\varphi_E \cap \varphi_C)$, respectively. The fractions of fake ZZ events and missed ZZ events are then given by:

\[
f_{\text{fakes}} = 1 - \frac{\sum N_E(\varphi_E \cap \varphi_C)}{\sum N_E(\varphi_E)} \tag{4.31}
\]

\[
f_{\text{misses}} = 1 - \frac{\sum N_C(\varphi_E \cap \varphi_C)}{\sum N_C(\varphi_C)} \tag{4.32}
\]

where $\sum N$ is a sum of all the elements in $N$. 

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4.9.2 Closure Test

A closure test is performed by splitting the ZZ MC into two orthogonal samples: $a$ and $b$. The first sample $a$ is used to construct an unfolding matrix in the way previously
described. The resulting unfolding matrix is then used to unfold the second sample \( b \) distribution. The unfolding matrix will unfold the distribution used to make it back to its original values, so testing on an orthogonal sample tests its validity. The ratios between the generated yields and unfolded yields of the second sample are shown in the top plot of Fig. 4.27, and these indicate that this method passes the closure test.

\[
\Theta^a \times \frac{N^{ZZ,b}_E}{N^{ZZ,b}_C}:
\]

![Image](image.png)

Figure 4.27: Ratio between unfolded ZZ \( p_T \) distribution and generated ZZ \( p_T \) distribution, indicating the unfolding method passes the closure test.

### 4.9.3 Unfolded Cross Section

The event yields in the \( p_T(Z) \) distribution of real data is subtracted by the event yields of the \( p_T(Z) \) distributions of all backgrounds, to arrive at the distribution of observed ZZ events from real data:

\[
N^{obs,ZZ}_E(\varphi_E) = N^{obs}_E(\varphi_E) - N^{bkgd}_E(\varphi_E).
\]  

(4.33)

The unfolding matrix \( \Theta \), constructed in the previous section, is then multiplied to the real data ZZ distribution to unfold the data into the “generator level” and phase space \( \varphi_C \):

\[
N^{obs,ZZ}_C(\varphi_C) = \Theta \times N^{obs,ZZ}_E(\varphi_E).
\]  

(4.34)

Systematic variations are accounted for by varying the unfolding matrix and the subtracted background distribution in the real data ZZ distribution. For every sys-
systematic fluctuation there is a corresponding $p_T(Z)$ distribution in ZZ and background MC. This results in an unfolding matrix, a distribution of observed ZZ events, and therefore an unfolded distribution for each systematic variation. In the unfolded $p_T(Z)$ distribution, the uncertainty for each bin can then be computed by taking the difference between the varied and central unfolded value and adding them in quadrature, and can be expressed as a vector-like object as well:

$$\delta N_{obs,ZZ} = \left( \sum_i \left( \Theta_i \times N_{E,i}^{obs,ZZ} - \Theta \times N_{E}^{obs,ZZ} \right)^2 \right)^{\frac{1}{2}}$$

(4.35)

where $i$ represents a systematic variation, and the left-pointing arrows on the exponents indicate element-wise operation on vector-like $N$ objects, such that $(N_1, N_2)^x = (N_1^x, N_2^x)$.

To get a more representative “true” set of event yields, the unfolded yields are scaled by kinematic acceptance as a function of $p_T(Z)$ calculated in mcfm, shown in Table 4.10. This accounts for events that the MC generator does not generate for the sake of computational efficiency. The kinematic acceptance cuts in mcfm are:

- $p_T(\ell) > 20$ GeV
- $\eta(\ell) < 2.5$
- $\Delta R(\ell, \text{jet}) > 0.4$
- $\Delta R(\ell, \ell) > 0.4$
- $R_{\text{jet}} = 0.5$.

Kinematic acceptance scaling may also be applied to the ZZ generator level distribution $N_C(\varphi_C)$ before the unfolding matrix is constructed, effectively modifying the $\varphi_C$ phase space, increasing the number of misses. Changing the order in which kinematic
acceptance and unfolding are applied results in a difference of 0.03% in unfolded yield for all channels explored here. The order chosen, however, is unfolding before rescaling to preserve unitarity in the unfolding process.

Table 4.10: Kinematic acceptance rescaling factors as a function of $p_T(Z)$, calculated using McFM.

<table>
<thead>
<tr>
<th></th>
<th>$1/A_{45-80}$ GeV</th>
<th>$1/A_{80-100}$ GeV</th>
<th>$1/A_{100-200}$ GeV</th>
<th>$1/A_{200-400}$ GeV</th>
<th>$1/A_{400-1000}$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/\mathcal{A}$</td>
<td>1.99 ± 0.10</td>
<td>2.01 ± 0.10</td>
<td>1.87 ± 0.12</td>
<td>1.54 ± 0.07</td>
<td>1.87 ± 0.09</td>
</tr>
</tbody>
</table>

Dividing the unfolded yields, $N^{\text{obs}}_{CZZ}(\phi_C)$, by integrated luminosity results in an unfolded cross section as a function of $p_T(Z)$. Scaling by $3^2$ for the combined channel, and 3 for the separated electron and muon channels gives an estimated cross section distribution for $ZZ \rightarrow \ell^+\ell^-\nu\bar{\nu}$. Nominal unfolded cross section per bin are listed in Table 4.11, with statistical and systematic uncertainties.

Table 4.11: Unfolded $pp \rightarrow ZZ \rightarrow \ell^+\ell^-\nu\bar{\nu}$ cross sections with statistical and systematic uncertainties, per bin, at 8 TeV. Cross section shows ee and $\mu\mu$ channels separately, and combined.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma_{45-80}$ GeV</th>
<th>$\sigma_{80-100}$ GeV</th>
<th>$\sigma_{100-200}$ GeV</th>
<th>$\sigma_{200-400}$ GeV</th>
<th>$\sigma_{400-1000}$ GeV [fb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ee</td>
<td>66 ± 26$^{+17}_{-16}$</td>
<td>10.5 ± 5.7$^{+3.1}_{-3.1}$</td>
<td>17.1 ± 6.2$^{+5.5}_{-5.7}$</td>
<td>1.4 ± 2.9$^{+0.7}_{-0.9}$</td>
<td>0.0 ± 2.5$^{+0.0}_{-0.0}$</td>
</tr>
<tr>
<td>$\mu\mu$</td>
<td>58 ± 34$^{+26}_{-29}$</td>
<td>19.8 ± 8.0$^{+5.4}_{-5.3}$</td>
<td>34.4 ± 8.6$^{+8.3}_{-8.6}$</td>
<td>3.5 ± 3.3$^{+1.3}_{-1.6}$</td>
<td>0.0 ± 1.0$^{+0.0}_{-0.0}$</td>
</tr>
<tr>
<td>Combined</td>
<td>62 ± 21$^{+21}_{-21}$</td>
<td>15.2 ± 4.9$^{+1.4}_{-1.2}$</td>
<td>25.8 ± 5.3$^{+6.9}_{-7.1}$</td>
<td>2.4 ± 2.2$^{+1.0}_{-1.2}$</td>
<td>0.0 ± 1.4$^{+0.0}_{-0.0}$</td>
</tr>
</tbody>
</table>

These cross section distributions are shown in Fig. 4.28 for combined channel, Fig. 4.29 for the electron channel and Fig. 4.30 for the muon channel. Each figure shows a comparison ratio between the unfolded distribution and the generator level distribution, as well as an NLO theoretical prediction computed in McFM.

Summing the content of the bins in the cross section distributions gives the total unfolded cross sections. These cross sections are shown in Table 4.12 and are consistent
Figure 4.28: Cross section as a function of $p_T(Z)$, unfolded from 8 TeV data, using the ee and $\mu\mu$ channels combined. Generator level distribution and NLO theory (mcfm) shown for comparison in main plots and subplots. The gray error band includes statistical and systematic uncertainties on the unfolded yields. Bottom subplots show uncertainty and fractional uncertainty.
Figure 4.29: Cross section as a function of $p_T(Z)$, unfolded from 8 TeV data, using the ee channel. Generator level distribution and NLO theory (MCFM) shown for comparison in main plots and subplots. The gray error band includes statistical and systematic uncertainties on the unfolded yields. Bottom subplots show uncertainty and fractional uncertainty.
Figure 4.30: Cross section as a function of $p_T(Z)$, unfolded from 8 TeV data, using the $\mu\mu$ channel. Generator level distribution and NLO theory (mcfm) shown for comparison in main plots and subplots. The gray error band includes statistical and systematic uncertainties on the unfolded yields. Bottom subplots show uncertainty and fractional uncertainty.
with the measurements in Table 4.9, after an additional rescaling by $S$ to account for events in the $p_T(Z) < 45$ GeV region, where

$$S = \frac{N(\text{no } p_T(Z) \text{ cut})}{N(p_T(Z) > 45 \text{ GeV})} = 2.13 \pm 0.05$$

and is computed using MCFM.

Table 4.12: Unfolded $pp \to ZZ \to \ell^+ \ell^- \nu \bar{\nu}$ cross sections at 8 TeV, measured in the ee and $\mu\mu$ channels separately, and combined, rescaled using MCFM.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma$ [fb] 8 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>ee</td>
<td>$202 \pm 58$ (stat) $^{+38}_{-36}$ (syst) $\pm 11$ (unfold) $\pm 5$ (resc)</td>
</tr>
<tr>
<td>$\mu\mu$</td>
<td>$247 \pm 77$ (stat) $^{+60}_{-66}$ (syst) $\pm 9$ (unfold) $\pm 6$ (resc)</td>
</tr>
<tr>
<td>Combined</td>
<td>$224 \pm 49$ (stat) $^{+47}_{-49}$ (syst) $\pm 6$ (unfold) $\pm 5$ (resc)</td>
</tr>
</tbody>
</table>

4.9.4 Differential Cross Section

The cross section distributions are scaled again by the energy range corresponding to the bin width to give differential cross section with respect to $p_T(Z)$, shown in Figures 4.31, 4.32, 4.33. The purpose is to provide a way to compare data to theoretical predictions of $\frac{d\sigma}{dp_T(Z)}$ from any new models. Differential cross section distribution as a function of $p_T(Z)$ is particularly sensitive to new physics in the high $p_T$ tail region.

To measure the dependence on the electroweak corrections, they are turned on and off (ewk or 1) in the construction of the unfolding matrix $\Theta$ and the subtracted background events $N^{\text{bkgd}}$, together and independently. The largest fluctuation in the final unfolded yield is $-3\%$ of the nominal unfolded yield by turning electroweak corrections off in both $\Theta$ and $N^{\text{bkgd}}$. The plots in Fig. 4.34 show how the unfolded distributions compared to the generator level with and without electroweak corrections.
Figure 4.31: Differential cross section as a function of $p_T(Z)$, unfolded from 8 TeV data, using the $\mu\mu$ and ee channels combined. Generator level distribution and NLO theory (MCFM) shown for comparison. The gray error band includes statistical and systematic uncertainties on the unfolded yields.
Figure 4.32: Differential cross section as a function of $p_T(Z)$, unfolded from 8 TeV data, using the ee channel. Generator level distribution and NLO theory (MCFM) shown for comparison. The gray error band includes statistical and systematic uncertainties on the unfolded yields.
Figure 4.33: Differential cross section as a function of $p_T(Z)$, unfolded from 8 TeV data, using the $\mu\mu$ channel. Generator level distribution and NLO theory (MCFM) shown for comparison. The gray error band includes statistical and systematic uncertainties on the unfolded yields.
Figure 4.34: The EWK corrections are turned on and off for both the generated distribution, and the data—background distribution, independently. This results in eight comparison plots, where “Gen1” is the generated distribution used in the unfolding matrix, and “Gen2” is the other. The gray error band includes statistical and systematic uncertainties on the unfolded yields.

Good agreement is sustained while varying the unfolding matrix and background yield independently.
Chapter 5

Analysis II: ZH

5.1 Motivation

The observation of a Higgs boson at the LHC has prompted studies to search for any connection it may have to new physics. If a significant invisible branching fraction of the Higgs boson is observed, it would strongly suggest physics beyond the Standard Model.

The expected rate of invisible Higgs decay [91] in the Standard Model is very small ($\sim 10^{-4}$ branching fraction). Both CMS and ATLAS have set indirect constraints on invisible Higgs boson decays using measured rates of visible decay modes. The current limits on the invisible branching fraction are 64% for CMS indirect search [92], 60% for ATLAS indirect search [93], and 65% for ATLAS direct search [94]. A combined-channel analysis at CMS put a limit on the invisible branching fraction at 58% [95].

This analysis covers a direct search, done at CMS, for invisible Higgs decays, with full 7 and 8 TeV datasets [96], with an extended analysis on 8 TeV. The focus is on the Higgs mass of 125 GeV, but the search extends to other possible Higgs bosons in the mass range of 105-145 GeV and 115-200 GeV in the multivariate part of the analysis.

The signal in this search consists of a Z boson Higgs-strahlung, where the radiated
Higgs boson decays invisibly after recoiling from the Z, see Fig. 5.1 (left). Since the decay products are not detected, the search is largely independent of the type of invisible decay products from the Higgs boson. There are several theoretical invisible Higgs decay modes beyond the Standard Model, and some are discussed in subsection 1.2.2.

The main background of the search is Standard Model ZZ production, where one Z boson decays into charged leptons, and the other decays into a pair of neutrinos; see Fig. 5.1 (right). It has the same detector signature as the signal: two leptons and missing transverse energy. There are some kinematic differences to be expected, since the spin states between ZZ and ZH are different, and there is a mass difference between the Higgs and Z bosons. This irreducible ZZ contribution comprises approximately 70% of the backgrounds remaining at final selection.

![Figure 5.1: (Left) ZH Signal, (Right) ZZ Main background](image)

### 5.2 Simulation and Background Estimation

The same set of simulations and parton distribution function modelling from Analysis I, described in section 4.2 are used here, except for the $t\bar{t}$ sample, along with ZH samples for all masses from 105-145 GeV, which are produced in POWHEG (v2.0) [56]. However, the ZH samples used in the multivariate analysis, for 8 TeV are produced in
**PYTHIA6** [55] with the Z2* tune, with 5-10 times the number of events as the POWHEG samples, as they were too statistically limited for this purpose. Next to leading order cross sections of ZZ and WZ processes are computed using MCFM. The ZH cross section is computed at NNLO in QCD coupling and NLO electroweak coupling, by the LHC Higgs Cross Section Working Group [97].

Data-driven estimation methods are borrowed from Analysis I, and described in section 4.6. Some differences arise for Drell-Yan and non-resonant background estimations, due to the lower yields in this analysis. For the Drell-Yan estimation method described in subsection 4.6.1, a 100% uncertainty is assigned, but the absolute contribution after final selection is small. For the non-resonant background estimation method described in subsection 4.6.2, a 25% uncertainty is assigned by comparing the scale factors with ones calculated without a veto on jets tagged as originating from bottom quarks (“b-tagged”).

### 5.3 Event Selection

Events from \( ZH \rightarrow \ell^+ \ell^- + H_{\text{inv}} \) and \( ZZ \rightarrow \ell^+ \ell^- \nu \bar{\nu} \) are characterized by large \( \slashed{E}_T \) from neutrinos or non-standard Higgs decay products. The large potential background, prior to final selection, comes from Z+jets events with large mis-measured \( \slashed{E}_T \) from hadronic recoil. This can flood the signal region with Z+jets events since the cross section of Z+jets is five orders of magnitude greater than that of ZZ \( \rightarrow \ell^+ \ell^- \nu \bar{\nu} \).

Object selection is identical to that of the ZZ analysis, described in section 4.3. To reduce the number of mis-measured energy events in the signal, “reduced missing transverse energy” (reduced-\( \slashed{E}_T \)) is constructed and optimized, discussed in detail in subsections 4.5.1 and 4.5.2. The reason for preference of reduced-\( \slashed{E}_T \) over \( \slashed{E}_T \) is that reduced-\( \slashed{E}_T \) performs better in signal efficiency and Drell-Yan background suppression.
The selection of events is grouped into two categories, one being the set of main selection and the other being the set of optimized selection [98]. The main selection focuses on isolating events with a Z boson decaying into leptons, as shown in Table 5.1. Events are selected to have two, well-identified, isolated, and same-flavor leptons. The transverse momentum of each lepton is required to be above 20 GeV. The invariant mass of the two leptons is required to be within the Z mass window, $|m_{\ell\ell} - m_Z| < 15$ GeV. Events are rejected if there are additional leptons of $p_T > 10$ GeV, to suppress WZ events. Since the signal is not characterized by any hadronic activity, events with energetic jets, $p_T > 30$ GeV, are rejected. Also, if a jet that is b-tagged with $p_T > 20$ GeV and $|\eta| < 2.5$, or if there is a soft muon with $p_T > 3$ GeV, the event is rejected.

Table 5.1: Main selection.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dilepton invariant mass</td>
<td>$</td>
</tr>
<tr>
<td>lepton $p_T$</td>
<td>$p_T &gt; 20$ GeV</td>
</tr>
<tr>
<td>b-tag veto</td>
<td>based on vertex info. (for jet with $p_T &gt; 20$ GeV)</td>
</tr>
<tr>
<td>Jet veto</td>
<td>no jets with $p_T &gt; 30$ GeV</td>
</tr>
<tr>
<td>Lepton veto</td>
<td>no additional leptons with $p_T &gt; 10, 3$ GeV</td>
</tr>
</tbody>
</table>

From this point on, there are two analyses performed: the first one is a simpler shape analysis performed on a single transverse mass variable, and the second is a shape analysis performed on the output distribution of a multivariate machine learning algorithm (MVA). For the single-variable analysis, an optimized selection is implemented on a set of three variables, the angle between $\vec{p}_T(\ell\ell)$ and $\vec{p}_T$, the balance ratio $B$, and reduced-$\not{E}_T$. All three are optimized together to obtain the best expected exclusion limit at 95% confidence level, treating ZH with a Higgs mass of 125 GeV as signal. The resulting selection is: $\Delta\phi(\ell\ell-\vec{p}_T) > 2.6$, reduced-$\not{E}_T > 110$ GeV (see Fig. 5.2), and $0.8 < B < 1.2$, and (see Fig. 5.3). This optimized selection is applied to the other
Figure 5.2: Reduced missing transverse energy distribution (top). Distribution of angle between $\vec{p}_T$ and $\vec{p}_T$ (bottom). For both, optimal selection is shown with dark blue line [96].

ZH samples of different Higgs masses. A loosened selection is used for the multivariate analysis, shown in Table 5.2, to reintroduce any trainable signal behavior lost in the optimized selection.
5.4 Systematic Uncertainties

Uncertainties on the efficiencies of ZH, ZZ, and WZ signals are derived from varying QCD scale, $\alpha_s$, and PDF type. Since a shape analysis is performed for computing the limit, errors are propagated to the rate and shape, listed in Table 5.3. The combined relative uncertainty on signal efficiency is 12%, and is dominated by theoretical and PDF uncertainties. Total relative background estimation uncertainty is 15%, and is dominated by theoretical uncertainties of WZ and ZZ modelling.

An additional systematic uncertainty on Drell-Yan yield is accounted for in the
### Table 5.3: Systematic Uncertainties [96].

<table>
<thead>
<tr>
<th>Type</th>
<th>Source</th>
<th>Uncertainty(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate</td>
<td>PDF</td>
<td>4–5</td>
</tr>
<tr>
<td></td>
<td>QCD scale variation (ZH)</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>QCD scale variation (VV)</td>
<td>7–10</td>
</tr>
<tr>
<td></td>
<td>Luminosity</td>
<td>2.2–4.4</td>
</tr>
<tr>
<td></td>
<td>Lepton Trigger, Reco., Iso.</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Z/γ* → ℓℓ normalization</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Top, W±W±, W±+jets normalization</td>
<td>25–100</td>
</tr>
<tr>
<td>Shape and Rate</td>
<td>MC statistics ZH,ZZ,WZ</td>
<td>1–5</td>
</tr>
<tr>
<td></td>
<td>Control sample statistics Z/γ* → ℓℓ</td>
<td>12–24</td>
</tr>
<tr>
<td></td>
<td>Control sample statistics NRB</td>
<td>53–100</td>
</tr>
<tr>
<td></td>
<td>Pile-up</td>
<td>0.1–0.3</td>
</tr>
<tr>
<td></td>
<td>b-tagging Efficiency</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>Lepton Momentum Scale</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Jet Energy Scale, Resolution</td>
<td>1–3</td>
</tr>
<tr>
<td></td>
<td>Unclustered energy</td>
<td>1–4</td>
</tr>
</tbody>
</table>

MVA portion of the analysis. Since the photon sample does not contain the same structure as other samples, a rescaled Drell-Yan Monte Carlo is used instead. Using a control region of \( \Delta \phi(\ell\ell) > 2.2 \), the Drell-Yan is rescaled to data. The uncertainty assigned to this rescaling is 11% propagated from MC and data.

#### 5.5 Single Variable Shape Analysis

At final selection, no significant excess is observed (see Table 5.4). A shape analysis is performed on the \( m_T \) distribution, which is a “pseudo tranverse mass” (see Fig. 5.4) between the Z boson and the missing energy. It is defined as:

\[
m_T^2 = \left( \sqrt{p_T(\ell\ell)^2 + m(\ell\ell)^2} + \sqrt{E_T^2 + m(\ell\ell)^2} \right)^2 - \left( \vec{p}_T(\ell\ell) + \vec{p}_T \right)^2. \quad (5.1)
\]

The Z mass is used as a placeholder for the mass of the invisibly decaying particle. This variable exploits the kinematic differences between \( ZH \rightarrow \ell^+\ell^- + H_{\text{inv}} \) and \( ZZ \rightarrow \)
Figure 5.4: (Top) Transverse Mass for 7 TeV, (Bottom) Transverse Mass for 8 TeV [96].
$\ell^+\ell^-\nu\bar{\nu}$ to improve the limit. Both signals have missing energy, but the mass and spin of the missing particle is different. The shape analysis is done for all Higgs masses.

<table>
<thead>
<tr>
<th>Process</th>
<th>$\sqrt{s} = 7$ TeV</th>
<th>$\sqrt{s} = 8$ TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ee</td>
<td>$\mu\mu$</td>
</tr>
<tr>
<td>ZH($m_H = 125$ GeV)</td>
<td>$2.2 \pm 0.3$</td>
<td>$3.3 \pm 0.5$</td>
</tr>
<tr>
<td></td>
<td>$11.8 \pm 1.9$</td>
<td>$16.7 \pm 2.5$</td>
</tr>
<tr>
<td>$Z/\gamma* \rightarrow ll$</td>
<td>$0.3 \pm 0.3$</td>
<td>$0.7 \pm 0.7$</td>
</tr>
<tr>
<td>Top/WW/W + jets</td>
<td>$0.4 \pm 0.4$</td>
<td>$0.6 \pm 0.6$</td>
</tr>
<tr>
<td></td>
<td>$1.0 \pm 1.0$</td>
<td>$1.9 \pm 1.9$</td>
</tr>
<tr>
<td>WZ $\rightarrow 3\ell$</td>
<td>$2.0 \pm 0.3$</td>
<td>$2.3 \pm 0.3$</td>
</tr>
<tr>
<td></td>
<td>$11.0 \pm 1.6$</td>
<td>$14.8 \pm 2.1$</td>
</tr>
<tr>
<td>$ZZ \rightarrow \ell^+\ell^-\nu\bar{\nu}$</td>
<td>$5.1 \pm 0.6$</td>
<td>$7.3 \pm 0.8$</td>
</tr>
<tr>
<td></td>
<td>$29.8 \pm 3.6$</td>
<td>$40.8 \pm 4.5$</td>
</tr>
<tr>
<td>total bkgd</td>
<td>$7.8 \pm 0.8$</td>
<td>$11.0 \pm 1.3$</td>
</tr>
<tr>
<td>Data</td>
<td>$10$</td>
<td>$11$</td>
</tr>
<tr>
<td></td>
<td>$43.1 \pm 4.1$</td>
<td>$59.6 \pm 5.5$</td>
</tr>
</tbody>
</table>

Table 5.4: Final Yields of single-variable ZH Analysis [96].

A binned shape analysis is performed on the $m_T$ distribution after final selection for each mass point. This is the simplest construction, where signal, background and observed data distributions all have the same binning. Using an asymptotic CL$_S$ [88] modified frequentist approach, with profile-likelihood test statistics, an upper limit is set on the invisible Higgs production cross section (see Fig. 5.5, left). Log-normal probability priors are used to describe systematic uncertainties. A shape analysis uses the event yields, and the distribution of those events in a variable. This is equivalent to performing a counting experiment in each bin.

The variables $p_T(Z)$ and $E_T$ were considered as well, but $m_T$ has lowest expected limit for each mass point. If a Standard Model production rate is assumed for each hypothesized Higgs mass, the limit can be expressed as a limit on the branching fraction to invisible modes (see Fig. 5.5, bottom). For a Higgs of mass 125 GeV, the expected 95% C.L. upper limit on $BR(H \rightarrow\text{invisible})$ is 91%, the observed is 75%. The limits corresponding to all Higgs masses are listed in Table 5.5.
Figure 5.5: (Top) Limit on $\sigma_{\text{ZH}} \cdot \text{BR}_{Z\rightarrow \ell\ell} \cdot \text{BR}_{H\rightarrow \text{inv}}$, (Bottom) Limit on $\text{BR}_{H\rightarrow \text{inv}}$; both are with 7 and 8 TeV samples combined [96].
### Table 5.5: Limits on $BR(H \rightarrow \text{invisible})$ for each Higgs Mass.

<table>
<thead>
<tr>
<th>$m_H$ (GeV)</th>
<th>105</th>
<th>115</th>
<th>125</th>
<th>135</th>
<th>145</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs Lim(%)</td>
<td>60</td>
<td>63</td>
<td>75</td>
<td>82</td>
<td>85</td>
</tr>
<tr>
<td>Exp Lim(%)</td>
<td>73</td>
<td>79</td>
<td>91</td>
<td>97</td>
<td>105</td>
</tr>
</tbody>
</table>

5.6 Multivariate Analysis

In the case of distinguishing $ZH \rightarrow \ell^+\ell^- + H_{\text{inv}}$ signal from diboson background, especially $ZZ \rightarrow \ell^+\ell^- \nu\bar{\nu}$, a simple cut-and-count method is not optimal, since the signal is buried in the background in a non-trivial way. So instead of counting, the shape of a physical variable distribution can be used to examine data. However, this may not provide the optimum shape for measuring the presence of signal. Fig. 5.6 shows some differences between $ZZ \rightarrow \ell^+\ell^- \nu\bar{\nu}$ and $ZH \rightarrow \ell^+\ell^- + H_{\text{inv}}$ physics at generator level. One of the differences can be seen in the Collins-Soper frame, where angle of separation between leptons is defined by:

$$\cos(\theta_{CS}) = \frac{2}{|Q|\sqrt{Q^2 + Q_T^2}}(\varrho_1^+ \varrho_2^- - \varrho_1^- \varrho_2^+ + \varrho_1^+ \varrho_2^-)$$

$$\varrho_i^\pm = \frac{1}{\sqrt{2}}(E_i \pm p_{z,i}) \quad (5.2)$$

$$Q = P_1 + P_2$$

where $P_i$ is the 4-vector of the $i$-th lepton and $\theta_{CS}$ is the angle of separation in the Collins-Soper frame [99].

One approach is to look at the shape of $N$-dimensional distributions involving an $N$-variable phase space, but this is computationally heavy, difficult to represent if $N > 2$, and may be unable to estimate the target function, i.e. maximally extract signal. An alternative to the brute force method is to use multivariate analysis (MVA), which can define a hyper-surface in the $N$-dimensional phase space to separate signal from...
background. There are several to choose from, likelihood, support vectors, decision trees, etc. Ideally, the shape of the response distributions will perform as well as, or better than the physical variable distributions used as input.

For this analysis boosted decision trees were chosen due to providing the best shape limits among all algorithms tested. The choice of input variables is physically motivated by the kinematic difference between signal and background, which also reduces computation time. This part of the analysis is performed on the 8 TeV data only.

5.6.1 Boosted Decision Trees

A decision tree is a classifier, built from binary trees, implementing a range of sequential decisions, shown in Fig. 5.7. Decisions are made on one variable at a time, until a stop criterion is met. This results in a phase space split into many regions that are each classified as signal or background, effectively a projection of a separating hypersurface onto a 1-D spectrum. The classification of a region depends on the majority of training events that end up in the final leaf node.

Boosting stabilizes the response of the decision trees to fluctuations in the training sample, enhancing the classification performance. Boosting is the process of apply-
ing the decision tree classification algorithm repeatedly to reweighted versions of the training sample, and then taking a weighted average of the classifiers produced.

In adaptive boosting [100], the events that were misclassified during decision tree training are given higher event weights in the training of the next decision tree. The training sample has an initial set of event weights in the first decision tree training, but the subsequent trees are trained using an iteratively modified set of event weights. The weights $w_i$ of previously misclassified events are multiplied by a common boost weight $\alpha$ to give the new weights.

$$\alpha = \frac{1 - \epsilon}{\epsilon}$$

$w_i \rightarrow w_{i+1} = \alpha_i \cdot w_i \quad (5.3)$

The boost weight is derived from the misclassification rate $\epsilon$ of the previous decision tree. These weights are renormalized so that the sum remains constant throughout boosting.
The result of an individual decision tree per event is $h = -1$ for background and $h = +1$ for signal. The boosted classifier $h_{\text{boost}}$ is given by a weighted average:

$$h_{\text{boost}} = \frac{1}{N} \sum_{i}^{N} \ln(\alpha_{i})h_{i}$$

(5.4)

where the sum is over all the classifiers in the size $N$ collection. This creates a gradient where more negative values are background-like, and more positive values are signal-like. Performance can be enhanced by slowing the learning process by adding a parameter $\beta$ such that $\alpha \rightarrow \alpha^{\beta}$. In this analysis, the number of boosts is $N = 100$, a range of $\beta$ values were explored, and $\beta = 0.5$ was chosen as the best overall when taking a minimum on expected limits.

### 5.6.2 Input Variables and Parameters

The event selection is loosened to recover signal events that may be separable from background. See Table 5.2 for cuts, and Table 5.6 for the resulting sample yields. To remove any possible bias generated by over-training, the samples are split into two orthogonal portions by randomly splitting them event by event: $\frac{2}{3}$ of events are dedicated to training, and the remaining $\frac{1}{3}$ is dedicated to evaluation, which plays no role in training. The samples are then appropriately rescaled to compensate for splitting. The training sample is further randomly split into $\frac{4}{5}$ dedicated to actually training the algorithm and $\frac{1}{5}$ for testing, to gauge overtraining. Systematically varied distributions are not split at all, and are entirely dedicated to evaluation. The MVA algorithm is trained only on the non-varied distribution and is used to evaluate the systematically varied distributions.

The detector signatures of ZH and ZZ are nearly identical, with a lepton pair and missing transverse energy per event. There are, however, some key differences in their
\[ \sqrt{s} = 8 \text{ TeV} \]

<table>
<thead>
<tr>
<th>Process</th>
<th>$ee$</th>
<th>$\mu\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ZH(m_H = 125 \text{ GeV})$</td>
<td>$36.5\pm0.9$</td>
<td>$52.8\pm1.2$</td>
</tr>
<tr>
<td>DY</td>
<td>$260\pm120$</td>
<td>$320\pm160$</td>
</tr>
<tr>
<td>Top/WW/W + jets</td>
<td>$109.4\pm9.8$</td>
<td>$157\pm14$</td>
</tr>
<tr>
<td>$WZ \to 3\ell$</td>
<td>$67.5\pm4.6$</td>
<td>$99.1\pm6.7$</td>
</tr>
<tr>
<td>$ZZ \to \ell^+\ell^−\nu\bar{\nu}$</td>
<td>$113.4\pm5.7$</td>
<td>$165.0\pm7.9$</td>
</tr>
<tr>
<td>total bkgd</td>
<td>$550\pm120$</td>
<td>$750\pm160$</td>
</tr>
<tr>
<td>Data</td>
<td>$497$</td>
<td>$779$</td>
</tr>
</tbody>
</table>

Table 5.6: Final Yields of ZH Analysis for MVA input.

Physics that manifest in distribution shapes. Since the possible spin states of ZZ and ZH are different ($S_{ZZ} = 2, 1, 0$, $S_{ZH} = 1$), it is expected that the kinematics will differ, particularly scattering angles. This results in some discrimination power between the angle variable distributions of background and signal.

Another difference is the mass of the invisible Higgs particle responsible for missing energy in the event. Since the range of Higgs masses explored here is above the Z boson mass, there is some discrimination power in the momentum and missing tranverse energy distributions.

The ZZ, WZ, and WW samples are set as backgrounds to train against the ZH samples for each mass point, to focus the algorithm is discriminating diboson states. This naturally pushes the other NRB and DY events to the background regions in the evaluation stage.

Choosing an optimal set of input variables and parameters is not trivial. The initial bank of variables is motivated by physics related to spin states and mass. To find an optimum working point, several sets of input variables are tested, many of them overlapping, and ranging in size from 2-8 variables.
The initial bank of variables is:

\[ m_T = \left( \sqrt{p_T(Z)^2 + m(Z)^2} + \sqrt{\not{E}_T^2 + m(Z)^2} \right)^2 - \left( \not{p}_T(Z) + \not{\phi}_T \right)^2 \]

\[ G_T = \max(\not{E}_T, p_T(Z)) \]

\[ p_{T2} = p_T(\ell_2) \]

\[ \Upsilon_T = \frac{p_T(Z) + \not{E}_T}{p_T(\ell_1) + p_T(\ell_2)} \]

\[ \Delta R = \sqrt{\Delta \phi(\ell_1, \ell_2)^2 + \Delta \eta(\ell_1, \ell_2)^2} \]

\[ \Delta \phi_1 = \Delta \phi(\ell_1, \not{\phi}_T) \]

\[ \Delta \phi_2 = \Delta \phi(\ell_2, \not{\phi}_T) \]

\[ \Delta \Omega = \Delta \phi(\ell_1, \ell_2) \times \Delta \eta(\ell_1, \ell_2) \]

where \( m_T \) is the pseudo-transverse mass between \( Z \) and \( \not{\phi}_T \) seen previously, \( G_T \) is the maximum of \( p_T(Z) \) and \( \not{E}_T \) per event, \( p_{T2} \) is the transverse momentum of the second lepton, \( \Upsilon_T \) is a ratio between two transverse momentum scalar sums, \( \Delta R \) is the familiar 2-D angle variable between leading leptons, \( \Delta \phi_i \) is the angle in the transverse plane between the \( i \)th lepton and \( \not{\phi}_T \), and \( \Delta \Omega \) is the solid angle between leptons. These variables are shown with both signal and background training samples normalized and super-imposed in Fig. 5.8 for \( m_H = 125 \text{ GeV} \) and in Fig. 5.9 for \( m_H = 200 \text{ GeV} \).

Each combination of variables is trained in BDT in a range of tree numbers (\( T : 100-2000 \)) and tree depths (\( D : 2-5 \)). This effectively creates a 3-dimensional algorithm input space \( \mathcal{I}_{ijk} = X_i \otimes T_j \otimes D_k \) for BDT training. The resulting set of evaluation distributions \( \mathcal{E}_{ijk} \) of background and signal, orthogonal from the training distributions, are each used to compute a corresponding expected shape limit \( L(\mathcal{E}_{ijk}) \), without considering real data to avoid bias. For each individual mass point, a minimum is found by sweeping through the expected limit values, and the corresponding element of \( \mathcal{I}_{ijk} \).
Figure 5.8: Input variables for SM-diboson vs invisible Higgs boson of mass 125 GeV. Solid blue distribution is signal, and hashed red is all diboson backgrounds.
Figure 5.9: Input variables for SM-diboson vs invisible Higgs boson of mass 200 GeV. Solid blue distribution is signal, and hashed red is all diboson backgrounds.
is logged for possible use on other mass points, shown in Table 5.7.

\[
\mathcal{E}_{ijk} = \text{BDT}(I_{ijk})
\]

\[
\min(L(\mathcal{E}_{ijk})) \rightarrow I_{\text{optimum}}.
\]

### Table 5.7: MVA parameters, minimum expected limit on \(BR\) and variable checklist for each mass point.

<table>
<thead>
<tr>
<th>(m_H ) (GeV)</th>
<th>115</th>
<th>125</th>
<th>135</th>
<th>145</th>
<th>155</th>
<th>175</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min Exp Lim(%)</td>
<td>86</td>
<td>60</td>
<td>104</td>
<td>117</td>
<td>125</td>
<td>150</td>
<td>188</td>
</tr>
<tr>
<td>Trees</td>
<td>1000</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>1300</td>
<td>1000</td>
</tr>
<tr>
<td>Depth</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Variables correlate with each other to varying degrees, and having relatively uncorrelated input variables, as in this case, tends to imply more information can be extracted by the algorithm (see Fig. 5.10). The correlation coefficients here are defined by division of covariance by standard deviation:

\[
\rho(X,Y) = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y}
\]

(5.7)

where \(\{\rho| -1 \leq \rho \leq 1\}\), and an independent pair of variables \(X\) and \(Y\) results in \(\rho = 0\).

The output BDT classifiers corresponding to the individually minimized expected limits are shown in Fig. 5.11 for Higgs mass 125 GeV, and in Fig. 5.12 for other mass points. These distributions consist of the training and testing samples, and show discrimination between background and signal with little to no overtraining. The
signal BDT response distribution becomes more distinct from background as Higgs boson mass increases. These distributions are also normalized together to highlight the difference in shape.

A “layered” approach was tried, where one of set of MVA were trained with ZZ as signal and another with ZZ as the only background and ZH as signal. Then the output of these MVAs were used as input variables in third MVA. This method was prone to overtraining, and adjusting parameters was non-trivial.

## 5.7 Shape Analysis of MVA Output

The limit from $m_T$ distribution shape is used as a benchmark for BDT shape limits minimization. Figure 5.13 shows the $m_T$ distribution for all backgrounds and 125 GeV Higgs signal. A notable difference in the shapes of the distributions can be seen in the tail, and this is used to compute the limits shown in Fig. 5.14.

To include systematics in the shape analysis, each sample distribution has a cor-
responding set of distributions for each systematic variation (for example jet energy resolution, scale, etc). Statistical error is also accounted for by varying the distribution shape one bin at a time by one standard deviation. The shape analysis is performed and a limit is computed for each mass point, using the same CL$_S$ \cite{88} construction as with $m_T$.

### 5.7.1 Limits on Invisible Higgs using MVA

The BDT classifier distributions from the individual minimization of expected limits for each mass point are shown in Figures 5.15-5.21. The shape analysis is performed on the BDT shape, considering real data, and a set of limits are calculated for each mass point, which is shown in Fig. 5.22. There is an overall improvement with respect to the Higgs mass spectrum in terms of exclusion with the use of BDT distribution shape.
Figure 5.12: BDT output for Higgs boson signal with other masses (Top: 115, 135 GeV, Middle: 145, 155 GeV, Bottom: 175, 200 GeV). Solid blue distribution is signal, and hashed red is all diboson backgrounds.
Figure 5.13: $m_T$ distributions with 125 GeV Higgs mass as signal, left: ee, right: $\mu\mu$.

Figure 5.14: Limits on $BR_{H\rightarrow inv}$, from shape of $m_T$ distribution.
Figure 5.15: BDT distributions with Higgs signal with mass 115 GeV, left: ee, right: $\mu\mu$. Disagreement between data and MC in the background region is due to relatively low statistics in Drell-Yan sample.

Figure 5.16: BDT distributions with Higgs signal with mass 125 GeV, left: ee, right: $\mu\mu$. Disagreement between data and MC in the background region is due to relatively low statistics in Drell-Yan sample.
Figure 5.17: BDT distributions with Higgs signal with mass 135 GeV, left: ee, right: $\mu\mu$.

Figure 5.18: BDT distributions with Higgs signal with mass 145 GeV, left: ee, right: $\mu\mu$. 
Figure 5.19: BDT distributions with Higgs signal with mass 155 GeV, left: ee, right: $\mu\mu$.

Figure 5.20: BDT distributions with Higgs signal with mass 175 GeV, left: ee, right: $\mu\mu$. 
Figure 5.21: BDT distributions with Higgs signal with mass 200 GeV, left: ee, right: $\mu\mu$.

Figure 5.22: Limits on $BR_{H\rightarrow inv}$, from shape of BDT distributions.
Figure 5.23: Overlay comparing pre-fit expected limits on $BR_{H\rightarrow \text{inv}}$, from shape of $m_T$ and BDT distributions.

Without refitting to real data, a comparison can be made between $m_T$ shape limits, and BDT shape limits for each mass point, shown in Fig. 5.23. This is an unbiased way to show that BDT improves discrimination power between ZH and ZZ. The expected limit lowers by an absolute 5-10% across the mass spectrum.

The upper limit values from the individual minimization scheme are shown in Table 5.8. The observed limit excludes a SM-like invisible Higgs with a 100% branching ratio, in the mass range of 163 GeV and lower. An invisible Higgs boson, with the same mass as the SM Higgs discovered (125 GeV), has an observed limit of 52%, and

<table>
<thead>
<tr>
<th>$m_H$ (GeV)</th>
<th>115</th>
<th>125</th>
<th>135</th>
<th>145</th>
<th>155</th>
<th>175</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs Lim(%)</td>
<td>64</td>
<td>52</td>
<td>69</td>
<td>81</td>
<td>86</td>
<td>119</td>
<td>172</td>
</tr>
<tr>
<td>Exp Lim(%)</td>
<td>82</td>
<td>49</td>
<td>106</td>
<td>115</td>
<td>126</td>
<td>150</td>
<td>190</td>
</tr>
</tbody>
</table>

Table 5.8: Limits on $BR_{H\rightarrow \text{inv}}$ for each Higgs Mass in the individual minimization.
expected limit of 49% on the branching ratio.

To get an idea of what the limit may look like under more ideal conditions, systematic uncertainties are removed, leaving just statistical error in taking the limit for each mass point. The resulting limit plot is in Fig. 5.24, and the values on the upper limits are shown in Table 5.9. An overlayed plot, in Fig. 5.25, shows the expected limit curve with systematics on top of the curve without systematics.

![Limits from BDT shape (w/o Systematic Uncertainty)](image)

Table 5.9: Limits on $BR_{H\rightarrow \text{inv}}$ for each Higgs Mass in the individual minimization, with no systematic uncertainties introduced.

<table>
<thead>
<tr>
<th>$m_H$ (GeV)</th>
<th>115</th>
<th>125</th>
<th>135</th>
<th>145</th>
<th>155</th>
<th>175</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp Lim(%)</td>
<td>51</td>
<td>41</td>
<td>67</td>
<td>77</td>
<td>86</td>
<td>108</td>
<td>142</td>
</tr>
</tbody>
</table>

The removal of systematics lowers the upper limits across the mass spectrum.
Figure 5.25: Overlay of expected limits on $BR_{H \to \text{inv}}$, from shape of BDT distributions, with and without systematic uncertainties included.

5.8 MVA Cross-Check Analysis

As a cross-check, all mass points are trained on the same six variables, rather than differing sets, to test if the set of limits remain consistent with the resulting limits from minimization. For the mass points from 115 GeV to 145 GeV, the BDT algorithm has 500 trees with a depth of 4. For the mass points from 155 GeV to 200 GeV, the BDT algorithm has 1000 trees with a depth of 3. The set of input variables is: $m_T, \Delta R, \Delta \phi_1, \Delta \phi_2, \Upsilon_T, p_T^2$, and the correlation matrices for these variables are shown in Fig. 5.26. The resulting BDT distributions for the lower mass range are shown in Fig. 5.27, and for the higher mass range in Fig. 5.28.

The set of cross-check BDT distributions, used for shape analysis, are shown in Figures 5.30-5.35. Even though these use a different $\mathcal{I}$, the distributions have similar characteristics to their minimized expected limit counterparts.
Figure 5.26: Correlations between variables for SM backgrounds and invisible Higgs boson (Top left: background, top right: 125 GeV signal, bottom: 200 GeV signal).
Figure 5.27: BDT output for Higgs boson of “lower” masses (Top: 115, 125 GeV, Bottom: 135, 145 GeV). Solid blue distribution is signal, and hashed red is all diboson backgrounds.
Figure 5.28: BDT output for Higgs boson of "higher" masses (Top: 155, 175 GeV, Bottom: 200 GeV). Solid blue distribution is signal, and hashed red is all diboson backgrounds.

Figure 5.29: BDT distributions with Higgs signal with mass 115 GeV, left: ee, right: $\mu\mu$. 
Figure 5.30: BDT distributions with Higgs signal with mass 125 GeV, left: ee, right: $\mu\mu$.

Figure 5.31: BDT distributions with Higgs signal with mass 135 GeV, left: ee, right: $\mu\mu$. 
Figure 5.32: BDT distributions with Higgs signal with mass 145 GeV, left: ee, right: $\mu\mu$.

Figure 5.33: BDT distributions with Higgs signal with mass 155 GeV, left: ee, right: $\mu\mu$. 

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Figure 5.34: BDT distributions with Higgs signal with mass 175 GeV, left: ee, right: $\mu\mu$.

Figure 5.35: BDT distributions with Higgs signal with mass 200 GeV, left: ee, right: $\mu\mu$. 
The resulting cross-check limit plot in Fig. 5.36 shows some point-by-point variation from the limit plot in Fig. 5.22, but shows the same overall observed exclusion range of $< 163$ GeV. This reinforces that the observed exclusion range is not an artifact of the individualized minimization scheme. The limits from the cross-check are also shown in Table 5.10.

\begin{table}[h]
\centering
\begin{tabular}{|c|cccccccc|}
\hline
$m_H$ (GeV) & 115 & 125 & 135 & 145 & 155 & 175 & 200 \\
\hline
Obs Lim(%) & 59 & 62 & 86 & 81 & 88 & 114 & 166 \\
Exp Lim(%) & 83 & 59 & 107 & 115 & 126 & 150 & 190 \\
\hline
\end{tabular}
\caption{Limits on $BR_{H \to \text{inv}}$ for each Higgs Mass in the cross-check.}
\end{table}
Chapter 6

Conclusion and Outlook

The evidence collected at the CMS detector suggests that the Standard Model remains a robust description of nature. The production of ZZ events from data confirms the Standard Model prediction in terms of cross section as a function of transverse momentum. In the case of direct measurement and in the case of unfolding data, the results agree with next-to-leading order predictions. At each invisible Higgs mass point along the range of 105-200 GeV, no significant excess is observed.

For a Higgs boson with a similar mass to the one recently discovered (125 GeV) with an assumed Standard Model cross section, an observed 95% C.L. upper exclusion limit of 52% is placed on the branching ratio. This means that the Higgs boson decays invisibly, at most, 52% of the time. For the rest of the mass spectrum, the observed limit on branching ratio dips below 100% for Higgs masses below 163 GeV. This means fully invisible Higgs bosons with masses below 163 GeV are excluded. These results are from the primary boosted decision tree analysis. So the Standard Model remains a good description of both the Z boson and the Higgs boson.

There remains the possibility that a Higgs below that mass exists, likely with a lower branching ratio than its Standard Model counterpart. This also leaves the possibility
for a second, heavier Higgs above 163 GeV, with a 100% branching ratio. If an invisible
SM-like Higgs boson exists, it is more likely to be a second heavier Higgs, rather than
new behavior of the recently discovered Higgs.

With Run 2 starting at the LHC soon, at a center-of-mass energy of 13 TeV, the
cross section increases by nearly three times. So if the same integrated luminosity
is recorded as Run 1, the expected upper limit on branching ratio can be lowered to
approximately 30% for Higgs mass of 125 GeV. The expected upper limits on an invis-
ible Higgs with 100% branching ratio will exclude Higgs masses of about 170 GeV and
lower. If an excess is observed, it can imply a connection to any one of an abundant
array of phenomena, like dark matter scalars, heavy neutrinos or extra dimensions.
Given what we know through observation of galactic rotation and neutrino oscillation,
the Standard Model has to break at some point. It will be exciting to see what nature
has been hiding in the next stage of high energy exploration.
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