Theory Space of Supersymmetry in View of LHC Data
on Higgs and SUSY Mass Limits

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Abstract

An analysis of the theory space of supergravity unified models with and without non-universalities is carried out in view of the LHC data and the Higgs boson mass at $\sim 126$ GeV. Aside from the limits placed on the allowed parameter space of supergravity unified models due to the non-observation of supersymmetric particles thus far and the large loop correction needed to lift the tree value of the Higgs boson mass from below the $Z$ boson mass to the observed value, the data also have implications for the nature of radiative breaking of the electroweak symmetry. Thus the current data points to the radiative breaking occurring on the Hyperbolic Branch. This branch consists of three sub regions: These are the Focal Point region, Focal Curves, and Focal Surfaces. It is seen that most of the parameter space consistent with all the theoretical and experimental constraints lies on Focal Curves and Focal Surfaces while the Focal Point region is highly depleted. The LHC data on sparticle mass limits and the Higgs mass measurement also have important implications for the supersymmetric decay lifetime of the proton. It is shown that the proton decay lifetime $\tau(p \rightarrow \bar{\nu} + K^+)$ is strongly correlated with the Higgs boson mass and the measurement of the Higgs boson mass at $\sim 126$ GeV enhances the $p \rightarrow \bar{\nu}K^+$ lifetime removing significantly the tension in supersymmetric grand unified models due to the non-observation of this mode thus far.
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Chapter 1

Introduction

The Standard Model (SM) based on the symmetry gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ describes the strong and the electroweak interactions of quarks and leptons. Through the mechanism of spontaneous symmetry breaking in the Higgs sector, quarks, charged leptons and vector bosons gain mass and there is a residual neutral scalar field - the Higgs boson. The recently discovered boson at the CERN Larger Hadron Collider at a mass of $\sim 126$ GeV is largely believed to be the Higgs boson, and we shall assume this to be the case in this thesis. Further detailed fits with the LEP data show that the SM predictions are in excellent agreement with experiment.

However, despite the remarkable success of the SM, there remain weaknesses in the SM at the theoretical level. One of these is the so called hierarchy problem. One aspect of this problem relates to the quantum loop corrections to the Higgs boson mass. Thus at the loop level the Higgs boson mass receives a correction which is $\mathcal{O}(\Lambda^2)$ where $\Lambda$ is the cutoff. This correction arises from the exchange of quark-anti-quark loops in the self energy diagram of the Higgs boson mass. In a grand unified theory the natural scale of the cutoff is $\sim 10^{16}$ GeV, while for electroweak physics one needs the Higgs boson mass to be at $\mathcal{O}(100)$ GeV range. Thus a fine-tuning of one part in $10^{28}$ is necessary. Two of the main avenues
that have been proposed to overcome this problem are those based on technicolor and on supersymmetry. In supersymmetry the problem of quadratic divergence is resolved via the addition of squark loops which cancel the quadratic divergence. Supersymmetry is more attractive because of its elegance in that it allows one to make contact with string theory.

The minimal extension of the standard model which includes supersymmetry is the so called minimal supersymmetric standard model (MSSM). This model contains supersymmetric partners of all the SM particles and in addition contains two Higgs doublets, one of which gives mass to the up quarks and the other gives mass to the down quarks and the leptons. These lead to a total of 31 new particles in MSSM beyond those in the SM, including squarks, sleptons, charginos, neutralinos, the gluino and extra Higgs bosons. The MSSM, however, is based on global supersymmetry and global SUSY is difficult to break spontaneously which we need to do in order to give masses to the sparticles. One possibility is to give soft masses to the sparticles. However, in MSSM one may add more than a hundred such soft parameters to the Lagrangian which makes the model rather unpredictable. These problems are alleviated in supergravity unified models (SUGRA). The minimal version of SUGRA models is mSUGRA which contains just 5 parameters, and after radiative electroweak symmetry breaking this number reduces to 4 and the sign of Higgs mixing parameter $\mu$. Thus in mSUGRA one determines the low energy properties of sparticles starting with $4 + \frac{1}{2}$ parameters at the Grand Unification scale. In non-universal SUGRA models (NuSUGRA) this parameter space is enlarged to include non-universalties of gaugino masses and of the Higgs boson masses.

The focus of the work in this thesis is to investigate the implications of the
constraints on the theory space of supersymmetry, and specifically of SUGRA models in view of the mass limits on sparticles arising from the LHC and from the determination of Higgs boson mass at 126 GeV. In addition, in the analysis we will take into account a variety of other constraints such as constraints from flavor changing neutral current experiments and cosmological constraints specifically the relic density constraint. We will also investigate the implications of the LHC data regarding the nature of radiative breaking of the electroweak symmetry (REWSB). Thus it is known that REWSB contains two main branches: the ellipsoidal branch (EB) and the hyperbolic branch (HB). On HB some of the sparticle masses can get large up to several TeV even with small fine tuning. It is thus important to determine if the LHC data provides clues to on what branch of REWSB we live. Our analysis would indicate that HB prevails and scalar masses could be large. This result also has important implications for proton stability since large sfermion masses lead to larger lifetimes and tend to stabilize the proton against rapid proton decay arising from dimension five operators.

The outline of the rest of the thesis is as follows: in Chapter 2 we discuss the basics of the standard model including its gauge group, various interactions and the Higgs mechanism, as well as its success and theoretical drawbacks. In Chapter 3 we discuss supersymmetry and the minimal supersymmetric standard model, including a complete list of all the sparticles appearing in the framework of the latter. In Chapter 4 we discuss supergravity unified models, including the radiative electroweak symmetry breaking, sparticle masses, dark matter and a discussion of proton decay. In Chapter 5 we discuss constraints on unified models, including those from limits on sparticle masses, flavor-changing neutral current (FCNC) constraint, relic density constraint and the Higgs mass constraint. In Chapter 6 we discuss branches of radiative electroweak symmetry breaking, in the framework
of mSUGRA and NuSUGRA. Here the analysis is done first for mSUGRA and then extended to NuSUGRA. Chapter 7 is devoted to a discussion of naturalness and fine tuning where a comparison is given of some of the more prominent fine tuning criteria. Implications of SUGRA models for LHC and dark matter are discussed in Chapter 8. Here the analysis includes an interpretation of CMS and ATLAS data and its implications for the discovery of supersymmetry. Conclusions are given in Chapter 9.
The Standard Model (SM) of electroweak and strong interactions is based on the gauge group

\[ SU(3)_C \times SU(2)_L \times U(1)_Y, \]

where \( SU(3)_C \) is the gauge group of color interactions and the gauge group \( SU(2)_L \times U(1)_Y \) describes the electroweak interactions where \( U(1)_Y \) is the hypercharge gauge group. The matter content of the standard model consists of quarks and leptons which are grouped into \( SU(2)_L \) doublet and singlets. Thus the \( SU(2)_L \) doublets consist of

\[
q_i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}_i, \quad \ell_i = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_i, \quad (2.2)
\]

and \( SU(2)_L \) singlets consist of

\[
u_{Ri}, \quad d_{Ri}, \quad \text{and} \quad e_{Ri}, \quad (2.3)
\]
where \( i = 1, 2, 3 \) is a generation index and the left and right chiral fields are defined so that

\[
\Psi_{L,R} = P_{L,R} \Psi, \quad P_{L,R} = \frac{1 \pm \gamma_5}{2}.
\]  

(2.4)

In addition to quarks and leptons the matter content of the SM also includes an \( SU(2)_L \) doublet of scalar fields

\[
\phi = \begin{pmatrix} H^{(+)}/2 \hline H^{(0)} \end{pmatrix}.
\]  

(2.5)

The scalar doublet plays an important role in the standard model. It is responsible for the breaking of the electroweak symmetry \( SU(2)_L \times U(1)_Y \) down to the \( U(1)_{em} \) when \( < H^{(0)} > \) develops a vacuum expectation value, i.e., \( < H^{(0)} > \neq 0 \), i.e.,

\[
SU(2)_L \times U(1)_Y \to U(1)_{em}, \quad < H^{(0)} > \neq 0
\]  

(2.6)

The \( SU(2)_L \times U(1)_Y \) breaking gives masses to \( W^\pm \) and the Z gauge bosons. Quite remarkably the VEV \( < H^{(0)} > \) also gives masses to quarks and leptons. Thus the masses of both the gauge bosons corresponding to the broken gauges as well as the masses of the quarks and leptons all arise from the same common source, i.e., the VEV \( < H^{(0)} > \). After spontaneous breaking of the electroweak symmetry the spectrum of the gauge bosons consists of \( W^\pm, Z, \) and \( \gamma \). The exchange of \( \gamma \) is responsible for the electromagnetic interactions while the weak interactions arise from the exchange of \( W^\pm \) and \( Z \) bosons. The strong interactions arise from the exchange of color gluons. We give below some further details of the above.
The gauge covariant derivative $D_\mu$ is given by

$$D_\mu = \partial_\mu - i \left[ g_3 \sum_{a=1}^8 G_a \mathcal{T}_C^a + g_2 \sum_{i=1}^3 A_i \mathcal{T}_L^i + g' B_\mu \frac{Y}{2} \right]. \quad (2.7)$$

Here

$$T_C^a = \left( \frac{\lambda^a}{2}; 0 \right), \text{ for (quarks; leptons, Higgs)}, \quad (2.8)$$

where the $\lambda^a$ denote Gell-Mann matrices and

$$T_L^i = \left( \frac{\sigma^i}{2}; 0 \right), \text{ for SU}(2)_L \text{ (doublets; singlets)}, \quad (2.9)$$

and where $\sigma^i$ are the three Pauli matrices; and $Y$ is the hypercharge defined by

$$Q = T_L^3 + \frac{Y}{2}. \quad (2.10)$$

As mentioned above the gauge symmetry $SU(2)_L \times U(1)_Y$ needs to be broken and we need the Higgs boson to accomplish this. The simplest potential for the Higgs boson field has the form

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \quad (2.11)$$

where

$$\mu^2 > 0, \text{ and } \lambda > 0. \quad (2.12)$$

Here $\lambda > 0$ is needed to make certain that the potential is bounded from below and $\mu^2 > 0$ is needed to make sure the symmetric vacuum is not the minimum of the potential. With the above set up the spontaneous symmetry breaking can
occur giving a non-vanishing VEV to the Higgs field so that

$$
\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \text{ with } v = \mu/\sqrt{\lambda},
$$

(2.13)

and one has

$$
H^0 = (v + h)/\sqrt{2},
$$

(2.14)

where $h$ is the left over neutral dynamical Higgs boson field with a mass

$$
M_h^2 = 2\mu^2 > 0,
$$

(2.15)

at the tree level. Further, the three Goldstone bosons corresponding to the breaking $SU(2)_L \times U(1)_Y \to U(1)_{em}$ are absorbed by the massless gauge bosons of $SU(2)_L \times U(1)_Y$ so that three of the four gauge bosons become heavy by the Higgs mechanism. Thus one has

$$
\{A^1, A^2\} \to \{W^1, W^2\} \text{ and } \{A^3, B\} \to \{A, Z\},
$$

(2.16)

with $A$ being the massless photon field. Specifically the fields $A$ and $Z$ are related to the fields $A^3, B$ by

$$
Z = -\sin \theta_W B + \cos \theta_W A^3,
$$

(2.17)

$$
A = \cos \theta_W B + \sin \theta_W A^3,
$$

(2.18)
where $\theta_W$ is the weak angle and is defined so that

$$
\cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \quad \text{and} \quad \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}.
$$

(2.19)

The weak angle enters in the masses for the W and Z bosons so that

$$
M_W = \frac{g_2 v}{2} = \frac{g^2}{2\sqrt{2} \lambda} M_h, \quad M_Z = \frac{M_W}{\cos \theta_W}.
$$

(2.20)

As mentioned above the Higgs field is also responsible for the generation of masses for the quark and leptons fields. To illustrate this we exhibit below the Yukawa interactions for the quarks and leptons consistent with all the gauge symmetries so that

$$
V_Y = \lambda^{(e)}_{ij} \bar{\ell}_i \phi_{eRj} + \lambda^{(u)}_{ij} \bar{q}_i \phi_{uRj} + \lambda^{(d)}_{ij} \bar{q}_i \phi_{dRj} + \text{h.c.},
$$

(2.21)

with the complex conjugation of the Higgs doublet vector boson in Eq. (2.5) is $\bar{\phi} = i \sigma^2 \phi^*$ and $\lambda^{(e,u,d)}_{ij}$ are the the Yukawa couplings where $i, j = 1, 2, 3$ are the generation indices.

The Standard Model has been extremely successful in explaining a huge amount of electroweak data and specifically the data from the LEP I and LEP II experiments. The predictions of the standard model have been tested and they are in excellent agreement with experiment. However, in spite of its great success the Standard Model has some theoretical drawbacks. A crucial one concerns the fact that the loop corrections to the Higgs boson mass possess a quadratic divergence and requires a cut-off. Since the natural scale of the cut-off is the Planck mass $M_{Pl} = 1.2 \times 10^{18}$ GeV, a fine tuning of one part in $10^{32}$ is needed to get the Higgs mass to lie in the electroweak region. Further, one expects that the Standard...
Model to be remnant of a more unified theory and thus the three gauge coupling constants $g_1, g_2$ and $g_3$ for the electroweak and strong interactions should unify at a high scale. However, renormalization group analysis shows that using the particle spectrum of the Standard Model the gauge coupling constants do not unify. These features point to the possibility of new physics at the TeV scale. Several proposals have been made over the years to alleviate the difficulties of the standard model. These include most prominently technicolor and supersymmetry. Supersymmetry appears to be the more appealing possibility because it has several attractive features and we follow this possibility here.
Chapter 3

SUSY and MSSM

As mentioned in Chapter 1, supersymmetry provides a possible path to alleviate some of the problems of the standard model specifically those related to the quadratic divergence of the Higgs boson mass and to the unification of the gauge coupling constants. Supersymmetry in four space-time dimensions [5] connects two basic classes of elementary particles: bosons that have an integer-valued spin, and fermions that have a half-integer spin. It guarantees that in a multiplet, the number of bose helicity states is equal to that of fermi helicity states. Further, bosons and fermions in a supermultiplet can transform into each other via supersymmetry transformations (for reviews see [6, 7]). The generators of supersymmetry transformations $Q_\alpha$ satisfy a “graded” algebra [8] so that

$$\{Q_\alpha, Q_\beta^\dagger\} = -2(P_L \gamma^\mu \gamma^0)_{\alpha\beta} P_\mu,$$

$$\{Q_\alpha, Q_\beta\} = [Q_\alpha, P_\mu] = 0 = [P_\mu, P_\nu],$$

where $P_\mu$ is the energy momentum vector and $P_L = (1 - \gamma^5)/2$. Representations of supersymmetry fall in supermultiplets consisting of bosonic and fermionic components. The simplest supersymmetry multiplet is a chiral multiplet such as a left chiral multiplet consisting of one spin 1/2 left-handed Weyl spinor and its spin
zero super partner which is a complex scalar field. Such multiplets can accommodate quarks and leptons and their super partners, the squarks and sleptons. A chiral super multiplet can also consist of Higgs fields and its super partner the Higgsino fields. In addition to matter fields which belong to chiral super multiplet one also needs vector supermultiplets which contain spin 1 gauge bosons and spin 1/2 gaugino fields which are superpartners of the gauge bosons.

3.1 Minimal Supersymmetric Standard Model (MSSM)

The simplest and the most straightforward supersymmetric extension of the Standard Model is the so-called Minimal Supersymmetric Standard Model (MSSM). (for a review see [9]). This straightforward extension consists in promoting all the multiplets of the standard model into supermultiplets of $N = 1$ supergravity. In addition one includes an extra doublet of Higgs fields so that the two Higgs doublets can be labeled $\hat{H}_u$ and $\hat{H}_d$ where the hat refers to the fact that it is a supermultiplet. The reason for the two Higgs doublets is the following: The two Higgs doublets are needed since one needs one Higgs doublet $H_u$ to give mass to the up quarks and the other $H_d$ to give mass to the down quarks and the leptons. Further, one needs the Higgs doublets to cancel the anomaly. The particle content of MSSM is listed in Table 3.1 and in Table 3.2.

From Table 3.1 and Table 3.2 we see that there are 32 supersymmetric particles which we enumerate below.

- 12 squarks: denoted as $\{\tilde{u}_{L,R}, \tilde{c}_{L,R}, \tilde{t}_{L,R}, \tilde{d}_{L,R}, \tilde{s}_{L,R}, \tilde{b}_{L,R}\}$, which are the superpartners of $\{u_{L,R}, c_{L,R}, t_{L,R}, d_{L,R}, s_{L,R}, b_{L,R}\}$.

- 9 sleptons: denoted as $\{\tilde{e}_{L,R}, \tilde{\mu}_{L,R}, \tilde{\tau}_{L,R}, \tilde{\nu}_{eL}, \tilde{\nu}_{\mu L}, \tilde{\nu}_{\tau L}\}$, which are the super-
<table>
<thead>
<tr>
<th>Names</th>
<th>spin 0</th>
<th>spin 1/2</th>
<th>SU(3)$_C$, SU(2)$_L$, U(1)$_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>squarks, quarks (×3 families)</td>
<td>$Q$</td>
<td>$(\bar{u}_L \ d_L)$</td>
<td>$(u_L \ d_L)$</td>
</tr>
<tr>
<td></td>
<td>$\bar{u}$</td>
<td>$\bar{u}_R$</td>
<td>$u_R^\dagger$</td>
</tr>
<tr>
<td></td>
<td>$\bar{d}$</td>
<td>$\bar{d}_R$</td>
<td>$d_R^\dagger$</td>
</tr>
<tr>
<td>sleptons, leptons (×3 families)</td>
<td>$L$</td>
<td>$(\nu \ e_L)$</td>
<td>$(\nu \ e_L)$</td>
</tr>
<tr>
<td></td>
<td>$\bar{e}$</td>
<td>$\bar{e}_R$</td>
<td>$e_R^\dagger$</td>
</tr>
<tr>
<td>Higgs, higgsinos</td>
<td>$H_u$</td>
<td>$(H_u^+ \ H_u^0)$</td>
<td>$(H_u^+ \ H_u^0)$</td>
</tr>
<tr>
<td></td>
<td>$H_d$</td>
<td>$(H_d^0 \ H_d^-)$</td>
<td>$(H_d^0 \ H_d^-)$</td>
</tr>
</tbody>
</table>

Table 3.1: Chiral supermultiplets in the Minimal Supersymmetric Standard Model. The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions.

<table>
<thead>
<tr>
<th>Names</th>
<th>spin 1/2</th>
<th>spin 1</th>
<th>SU(3)$_C$, SU(2)$_L$, U(1)$_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>gluino, gluon</td>
<td>$\tilde{g}$</td>
<td>$g$</td>
<td>(8, 1, 0)</td>
</tr>
<tr>
<td>winos, W bosons</td>
<td>$W^\pm$</td>
<td>$W^0$</td>
<td>(1, 3, 0)</td>
</tr>
<tr>
<td>bino, B boson</td>
<td>$B^0$</td>
<td>$B^0$</td>
<td>(1, 1, 0)</td>
</tr>
</tbody>
</table>

Table 3.2: Gauge supermultiplets in the Minimal Supersymmetric Standard Model.

- 4 Higgs bosons: denoted as $h, H, A, H^\pm$, where $h$ and $H$ are neutral and CP even, $A$ is neutral and CP odd, and $H^\pm$ is charged.
- 2 charginos: denoted as $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^\pm$, which are the charged mass eigenstates in the gaugino-Higgsino sector.
- 4 neutralinos: denoted as $\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0$ and $\tilde{\chi}_4^0$, which are the neutral mass eigenstates in the gaugino-Higgsino sector.
- 1 gluino: denoted as $\tilde{g}$.

A useful symmetry in supersymmetry is R parity defined by

$$P_R = (-1)^{3(B-L)+2s}, \quad (3.3)$$

where $B$ and $L$ are the baryon number and the lepton number of a particle and $s$ is its spin. R parity as define above is +1 for all the standard model particles.
and is $-1$ for their supersymmetric partners. Thus the quarks and leptons, the Higgs fields, and the gauge bosons are all R parity even while the squarks, the sleptons, the Higgsinos and the gauginos are all R parity odd. We will assume the conservation of R parity. One consequence of this assumption is that the lowest mass supersymmetric particle is then stable and further since it is charge neutral it can be a possible candidate for dark matter. For a supermultiplet one defines matter parity as

$$P_M = (-1)^{3(B-L)},$$

(3.4)

so that particles of integer and half integer spin in the supermultiplet will have opposite R parity. A superpotential which conserves matter parity will automatically conserve R parity and vice versa. For MSSM an R parity conserving superpotential takes the form

$$W = \hat{U}^C Y_u \hat{Q} \hat{H}_u + \hat{D}^C Y_d \hat{Q} \hat{H}_d + \hat{E}^C Y_e \hat{L} \hat{H}_d + \mu \hat{H}_u \hat{H}_d,$$

(3.5)

where $u, d, e$ indicate particle types.

Supersymmetry if at all a symmetry of nature must be a broken symmetry since, for example, no supersymmetric particles have been observed thus far. This implies that we must generate extra masses for the supersymmetric partners. However to do this in global supersymmetry is difficult. Tentatively one may just add by hand a set of soft terms to give extra masses to scalars such as shown below

$$\mathcal{L}_{\text{soft}}^{(2)} = m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + \bar{Q}^\dagger M_Q^2 \bar{Q} + \bar{L}^\dagger M_L^2 \bar{L}$$

$$+ \tilde{u}^\dagger m_{\tilde{u}}^2 \tilde{u}^C + \tilde{d}^\dagger m_{\tilde{d}}^2 \tilde{d}^C + \tilde{e}^\dagger m_{\tilde{e}}^2 \tilde{e}^C.$$

(3.6)
In addition one can add soft trilinear terms so that

\[ \mathcal{L}_{\text{soft}}^{(3)} = \tilde{u}^C h_u \tilde{Q} H_u + \tilde{d}^C h_d \tilde{Q} H_d + \tilde{e}^C h_e \tilde{L} H_d + B H_u H_d + \text{h.c.}, \quad (3.7) \]

where \( M_Q^2, M_L^2, m_u^2, m_d^2, m_e^2 \) and \( h_u,d,e \) are all \( 3 \times 3 \) matrices.

Supersymmetric grand unified models have at least two benefits over the non-supersymmetric ones. First the problems of quadratic divergence in the Higgs boson mass is resolved due to cancellation between quark loops and squarks loops. Second because of the appearance of sparticle masses the evolution of the gauge coupling constants allows the gauge coupling unification to occur.
Chapter 4

SUGRA Unified Models

As discussed in Chapter 3, supersymmetry if at all a valid symmetry of nature must be a broken symmetry with masses of the superpartners of the normal standard model particles much larger than the masses of the standard model particles. However, in global supersymmetry it is difficult to achieve spontaneous breaking of supersymmetry in a phenomenologically acceptable fashion. Part of the reason is that the potential of globally supersymmetric theories is positive definite and thus the symmetric vacuum is the minimum of the potential. This makes the breaking of supersymmetry difficult. A promising theory which overcomes this problem is local supersymmetry or supergravity \cite{5,10,14}.

Specifically supergravity grand unification \cite{15,17} allows for a phenomenologically viable breaking of supersymmetry. Supergravity grand unification is based on applied supergravity which couples \( N = 1 \) matter fields and gauge fields with the supergravity multiplet \cite{15,16,18,19}. To be specific, \( N = 1 \) supergravity is coupled with \( N = 1 \) matter which contains quarks, leptons and Higgs fields which are in anomaly free combinations of representations of the gauge group. In the meantime, \( N = 1 \) supergravity is also coupled with \( N = 1 \) Yang-Mills fields in the adjoint representation of the gauge group \( G \).
The effective Lagrangian of applied supergravity that is constructed is of the most general form and depends on three arbitrary functions: the Kahler potential $K(\phi, \phi^\dagger)$, the superpotential $W(\phi)$, and the gauge kinetic function $f_{\alpha\beta}(\phi)$. Here $\phi$ are the spin zero components of the left handed chiral multiplets, and $\alpha, \beta$ denote the gauge indices of the adjoint representation.

In the effective theory, $W$ and $K$ only appear in a fixed combination combination so that

$$G = \kappa^2 K + \log[\kappa^6 WW^\dagger],$$

with

$$\kappa = 1/M_{\text{Pl}},$$

where $M_{\text{Pl}}$ is the Planck mass defined by

$$M_{\text{Pl}} = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}\text{GeV},$$

where $G_N$ is Newton’s constant. Thus the effective supergravity Lagrangian depends solely on $f_{\alpha\beta}$ and $G$. The effective theory also exhibits invariance under the following transformations

$$K \rightarrow K - h(\phi) - h^\dagger(\phi), \quad W \rightarrow e^{\kappa h} W,$$

which is called the Kahler transformation. In model building one needs to make
assumptions regarding the Kahler potential or the Kahler metric defined by

$$K^i_j = K^j_i \equiv \frac{\partial^2 K}{\partial \phi_i \phi_j^\dagger} = \kappa^{-2} G^j_i.$$  \hfill (4.5)

The simplest assumption on the Kahler metric is to assume that it is flat, i.e., that

$$K^i_j = \delta^i_j$$ which corresponds to the assumption $$K = \sum_i \phi_i \phi_i^\dagger$$. However, more generally the Kahler potential is not flat and often phenomenology requires a non-flat Kahler potential, and in string models most often this is indeed the case.

We discuss now the supergravity scalar potential. It is here that one has perhaps the most crucial difference between global supersymmetry and local supersymmetry. Thus while in global supersymmetry the scalar potential is positive definite, this is not the case for $$N = 1$$ supergravity. Here one finds that the effective potential is given by \[ \textbf{[15, 18, 19]} \]

$$V = \kappa^{-4} e^{-G} \left[ (G^{-1})^i_j G^i_j - 3 \right] + \frac{g^2}{2} \left[ \text{Re}(f^{-1})_{\alpha\beta} \right] D_\alpha D_\beta,$$ \hfill (4.6)

where $$(f^{-1})_{\alpha\beta}$$ and $$(G^{-1})^i_j$$ are the inverse of matrix $$f_{\alpha\beta}$$, $$g$$ is the gauge coupling constant and the term $$D_\alpha$$ is given by

$$D_\alpha = \kappa^{-2} G^i_j (T^\alpha)_{ij} z_j,$$ \hfill (4.7)

with $$T^\alpha$$ being the generators of the group. One could also explicitly write the scalar potential as a function of $$W$$ and $$K$$ so that

$$V = e^{\kappa K} \left[ (K^{-1})^i_j \left( \frac{\partial W}{\partial z_i} + \kappa^2 K_i W \right) \left( \frac{\partial W}{\partial z_j} + \kappa^2 K_j W \right)^\dagger - 3\kappa^2 |W|^2 \right] + V_D.$$(4.8)
The scalar field has a kinetic energy of the form

\[ -K_{i,j}^{\mu}(D^\mu \phi_i) (D_\mu \phi_j)^\dagger, \]  

(4.9)

where \( D_\mu \) is the gauge covariant derivative.

In the last chapter we saw that the superpotential of MSSM includes a term of the form \( \mu H_1 H_2 \). For phenomenological reasons one expects this term to be of the size of the electroweak scale. However, how this comes about appears to be rather mysterious. One approach to how such a term come about is to assume that it arises from the Kahler potential. Thus in the Kahler potential which is a dimension 2 operator one can write

\[ K = K_0 (\phi_i \phi_i^\dagger) + c_0 H_1 H_2, \]  

(4.10)

with \( \phi_i \) being the scalar in MSSM and \( c_0 \) a dimensionless coefficient. Next suppose we make a Kahler transformation to move this term from the Kahler potential to the superpotential. In this case the superpotential takes the form

\[ W e^{\kappa^2 f} = W + c_0 \kappa^2 W H_1 H_2 + \cdots. \]  

(4.11)

Now in gravity mediating breaking of supersymmetry to be discussed later, the superpotential develops a non-vanishing VEV for \( W \) and one gets a non-vanishing \( \mu_0 \) so that

\[ \mu_0 = c_0 \kappa^2 \langle W \rangle, \]  

(4.12)

and we shall see below that the term \( \kappa^2 \langle W \rangle \) is on the order of the electroweak...
scale. We note here that a term of the form $c_0 H_1 H_2$ does arise quite naturally in string models \[21\].

As mentioned already the breaking of supersymmetry breaking within globally supersymmetric models is problematic since in such models, the potential is positive-definite and thus the symmetric vacuum is the true minimum of the potential. This means that the breaking of supersymmetry, would result in a large non-vanishing vacuum energy. In addition of course there are other unphysical features such as a massless Goldstino field which is not realized in nature. Supergravity unified models overcome these drawbacks of the spontaneous breaking of supersymmetry in global supersymmetry. Thus Eq. (4.8), exhibits a negative sign in the scalar potential which means that the vacuum energy could be fine tuned to zero. Further, one finds that the massless Goldstino field can be absorbed by the gravitino field making the gravitino field massive. We discuss now spontaneous breaking of supersymmetry within supergravity grand unification. There are two key assumptions regarding supersymmetry breaking in supergravity: first, supersymmetry is broken in a hidden sector; second, the supersymmetry breaking is passed on to visible sector via gravitational interactions. To see this explicitly \[15\] \[22\], we write the superpotential so that

$$W = W_{\text{vis}} + W_{\text{hid}},$$

(4.13)

where $W_{\text{vis}}$ depends only on the MSSM fields containing quarks, leptons and Higgs fields, and $W_{\text{hid}}$ contains fields in the hidden sector where supersymmetry breaks, which give rise to a non-vanishing vacuum expectation VEV, i.e.,

$$\langle W \rangle = \langle W_{\text{hid}} \rangle,$$

(4.14)
where \( \langle W_{\text{hid}} \rangle \) is assumed of size \( m^2 M_{\text{Pl}} \), and \( m \sim 10^{11} \) GeV is an intermediate scale. The VEV formation above leads to breaking of supersymmetry in the hidden sector. It is then passed on to the visible sector via gravitational interactions. First the breaking of supersymmetry gives mass to the gravitino

\[
M_{3/2} = \kappa^2 e^{K/2} |W|, \tag{4.15}
\]

Gravity mediation leads to a generation of soft breaking terms. They consist of scalar masses, gaugino masses, and trilinear terms in the scalar potential. Thus the scalar masses are typically of size

\[
m_0 \sim (\kappa^2 \langle W_{\text{hid}} \rangle), \tag{4.16}
\]

If one assumes \( m \sim 10^{11} \) GeV, then \( m_0 \sim 10^3 \) GeV. One of the remarkable features of supergravity grand unification is that the low energy region does not depend on the GUT scale \( M_G \) \cite{15, 23, 24}.

Next we discuss the gauge-gaugino sector of supergravity models which is given by

\[
\mathcal{L}_{\text{gauge}} = -\frac{1}{4} \Re \left[ f_{\alpha \beta} F_{\mu \nu}^\alpha F_{\beta \mu \nu}^\beta \right] + \frac{1}{4} i \Im \left[ f_{\alpha \beta} F_{\mu \nu}^\alpha \tilde{F}_{\beta \mu \nu}^\beta \right] + \frac{1}{2} \Re \left[ f_{\alpha \beta} \left( -\frac{1}{2} \tilde{\lambda}^\alpha \varphi \lambda^\beta \right) \right] - \frac{1}{8} i \Im \left[ f_{\alpha \beta} e^{-1} D_\mu (\varepsilon^{\alpha \beta \gamma \delta} \tilde{\gamma}_5 \lambda^\delta) \right] + \frac{1}{4} e^{G/2} G^{a} (G^{-1})_{b}^{a} (\partial f_{\alpha \beta}/\partial x^{b} \lambda^{a} \lambda^{\beta}) + \text{h.c.}
\]

The function \( f_{\alpha \beta} \) has explicit dependence on fields which transform in the underlying gauge group as an irreducible non-singlet representation or simply a singlet, both of which give rise to gaugino masses after the spontaneous breaking of supersymmetry and gauge symmetry. The assumption that \( f_{\alpha \beta} \) transform like a singlet of the underlying gauge group leads to a universal gaugino mass term \( m_{1/2} \tilde{\lambda}_\alpha \lambda_\alpha \).
Implications of supergravity models have been investigated in a number of works both for the universal and for the non-universal case and the literature is rather vast [6, 8, 9, 25–51].

As discussed already there are 32 supersymmetric particles, and a priori they can have an arbitrary mass hierarchy leading to a huge number of hierarchical patterns, i.e., as many as $O(10^{25})$. This number is reduced considerably in supergravity models. Further, the number of hierarchical patterns decrease in a significant way by inclusion of constraints both experimental and theoretical. One of the most important theoretical constraints is radiative breaking of the electroweak symmetry which we discuss below.

### 4.1 Radiative Electroweak Symmetry Breaking

The potential that leads to the soft breaking of supersymmetry in minimal supergravity (mSUGRA) is

$$ V = \sum_a |\frac{\partial W}{\partial \phi_a}|^2 + m_0^2 \sum_i \phi_i^\dagger \phi_i + A_0 W^{(3)} + B_0 W^{(2)} + m_{1/2} \sum_{\alpha=3,2,1} \tilde{\lambda}_\alpha \lambda_\alpha, \quad (4.17) $$

For MSSM one has

$$ W^{(2)} = \mu_0 \epsilon_{ij} H_1^i H_2^j, \quad (4.18) $$

$$ W^{(3)} = \lambda_{ij}^{(u)} q_i H_2 u^C_j + \lambda_{ij}^{(d)} q_i H_1 d^C_j + \lambda_{ij}^{(e)} \ell_i H_1 e^C_j, \quad (4.19) $$

where $i, j = 1, 2, 3$ are the generation indices, $H_1 \equiv H_d$ and $H_2 \equiv H_u$ are the MSSM Higgs doublets, and $\lambda_{ij}^{(u,d,e)}$ are the Yukawa couplings.
The soft breaking of mSUGRA model is then determined by the soft parameters

\[ m_0, \ m_{1/2}, \ A_0, \ B_0. \]  \hfill (4.20)

To link the experiments at the electroweak scale with mSUGRA at the GUT scale, one utilizes renormalization group evolution (RGE) [40]. This involves evolving the Higgs potential from the GUT scale down to the electroweak scale. Thus one writes

\[ V = V_0 + \Delta V_1, \]  \hfill (4.21)

where \( V_0 \) is the tree level potential

\[ V_0 = m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2(H_1H_2 + \text{h.c.}) + \frac{1}{8}(g_2^2 + g_Y^2)(|H_1|^2 - |H_2|^2)^2, \]  \hfill (4.22)

and \( \Delta V_1 \) is the correction at the one-loop level [52, 53]

\[ \Delta V_1 = \frac{1}{64\pi^2} \sum_a (-1)^{2s_a}n_a M_a^4 \ln \left[ \frac{M_a^2}{e^{3/2}Q^2} \right]. \]  \hfill (4.23)

In the above expressions, \( m_i(t), \ g_2(t), \ g_Y(t) \) are parameters at scale \( Q \) where \( t = \ln(M_G^2/Q^2) \) and \( m_i \) are given by [25]

\[ \begin{align*}
    m_i^2(t) &= m_{H_i}^2(t) + \mu^2(t), & i &= 1, 2, \quad (4.24) \\
    m_3^2(t) &= -B(t)\mu(t), \quad (4.25)
\end{align*} \]

where

\[ \begin{align*}
    m_i^2(0) &= m_0^2 + \mu_0^2, & i &= 1, 2; \\
    m_3^2(0) &= -B_0\mu_0; \quad (4.27)
\end{align*} \]
are the boundary conditions effectively at the GUT scale, i.e.,

$$Q = M_G(t = 0).$$  \hfill (4.28)

In Eq. (4.23), $M_a \equiv M_a(v_1, v_2)$ is the mass of particle $a$ evaluated at the tree level as a function of $v_i = \langle H_i \rangle$, and $s_a$ and $n_a$ are the corresponding spin and number of helicity states of $a$.

As we evolve the Higgs potential from the GUT scale to the electroweak scale, the determinant of the Higgs mass square matrix turns negative leading to spontaneous breaking of the electroweak symmetry. More appropriately one needs the following conditions for generating the breakdown of the electroweak symmetry

$$D = m_{11}^2 m_{22}^2 - m_3^4 < 0, \quad (4.29)$$

$$L = m_1^2 + m_2^2 - 2|m_3^2| > 0. \quad (4.30)$$

In the renomalization group analysis and in the breakdown of the electroweak symmetry, the top (stop) Yukawa coupling plays a central role. Indeed it is due to the renormalization group evolution of the stops and the $H_2$ Higgs boson masses that $m_{H_2}^2$ turns negative as one evolves down to electroweak scale from the GUT scale, leading to electroweak symmetry breaking. This symmetry breaking mechanism is referred to as the radiative electroweak symmetry breaking (REWSB) since it is the RG evolution which drives the breaking.

The constraints of electroweak symmetry breaking can be obtained by mini-
minizing the Higgs potential, which lead to the following two constraints

\[ \mu_1^2 - m_3^2 \tan \beta + \frac{1}{2} M_Z^2 \cos(2\beta) = 0, \]  
\[ \mu_2^2 - m_3^2 \cot \beta - \frac{1}{2} M_Z^2 \cos(2\beta) = 0, \]

where

\[ \tan \beta \equiv \frac{v_2}{v_1}, \]  

and

\[ \mu_i^2 = m_i^2 + \Sigma_i, \quad i = 1, 2, \]  

and where \( \Sigma_i \) being the corrections at the loop level as a result of the Higgs potential \( \Delta V_1 \) at the loop level. If one treats \( \mu_i^2 \) and \( m_3^2 \) as free inputs one finds,

\[ \sin(2\beta) = \frac{2m_3^2}{\mu_1^2 + \mu_2^2}, \]

and

\[ \frac{1}{2} M_Z^2 = \frac{\mu_1^2 - \mu_2^2 \tan^2 \beta}{\tan^2 \beta - 1}. \]

One can use Eq. (4.35) to replace \( B_0 \) by \( \tan \beta \) as a free parameter and use Eq. (4.36) to determine \( \mu \) except for a sign. In this case the low energy theory is then determined by the following set of parameters

\[ m_0, \ m_{1/2}, \ A_0, \ \tan \beta, \ \text{sgn}(\mu), \]

This is the mSUGRA model in the form used in the phenomenological analyses of data at colliders and elsewhere.
We discuss now issues related to naturalness and fine tuning. To discuss this issue we rewrite the symmetry breaking equation Eq. (4.36) as follows [54–57]

\[ \Phi = \frac{1}{4} + \frac{\mu^2}{M_Z^2}, \tag{4.38} \]

with

\[ \Phi^{-1} \equiv 4 \frac{\lambda^2 - \mu^2}{\lambda^2 + \mu^2}. \tag{4.39} \]

Here \( \Phi \) quantifies the measurement of naturalness. Applying the constraints of REWSB and not considering the \( b \)-quark couplings, one will have

\[ \Phi = -\frac{1}{4} + \left( \frac{m_0}{M_Z} \right)^2 C_1 + \left( \frac{A_0}{M_Z} \right)^2 C_2 + \left( \frac{m_{1/2}}{M_Z} \right)^2 C_3 + \left( \frac{m_{1/2} A_0}{M_Z^2} \right) C_4 + \frac{\Delta \mu^2_{\text{loop}}}{M_Z^2}, \tag{4.40} \]

with

\[ C_1 = \frac{1}{\tan^2 \beta - 1} \left( 1 - \frac{3D_0 - 1}{2} \tan \beta \right), \tag{4.41} \]
\[ C_2 = \frac{\tan^2 \beta}{\tan^2 \beta - 1} k, \tag{4.42} \]
\[ C_3 = \frac{1}{\tan^2 \beta - 1} (g - e \tan^2 \beta), \tag{4.43} \]
\[ C_4 = -\frac{\tan^2 \beta}{\tan^2 \beta - 1} f, \tag{4.44} \]
\[ \Delta \mu^2_{\text{loop}} = \frac{\Sigma_1 - \Sigma_2 \tan^2 \beta}{\tan^2 \beta - 1}. \tag{4.45} \]

where

\[ D_0 = 1 - \left( m_t / m_f \right)^2, \tag{4.46} \]

with \( m_f \sim 200 \sin \beta \) and \( e, f, g, k \) defined in [58]. These results will serve as a
starting point of the discussion in Ch[7].

4.2 Sparticle Masses

The SUSY masses we discuss below are the quantities that are defined at the electroweak scale so that they are accessible experimentally. As discussed above these masses can be given for the mSUGRA case in terms of 4 + 1 parameters, i.e., $m_0$, $m_{1/2}$, $A_0$, $\tan\beta$ and the sign of $\mu$, $\text{sgn}(\mu)$. More generally they will depend on additional parameters if one includes non-universalities. In this section explicit forms of the sparticles mass matrices are given. To compact notation we use $s_W = \sin\theta_W$, $c_W = \cos\theta_W$, $s_\beta = \sin\beta$, and $c_\beta = \cos\beta$. With this notation the mass (or mass$^2$) matrices are as below:

- **Up squarks**: Up squarks mass squared matrix takes the form

$$
M_u^2 = \begin{pmatrix}
M_Q^2 + m_u^2 + M_Z^2(\frac{1}{2} - Q_u s_W^2) \cos 2\beta & m_u (A_u^* - \mu \cot\beta) \\
m_u (A_u - \mu^* \cot\beta) & m_u^2 + m_u^2 + M_Z^2 Q_u s_W^2 \cos 2\beta
\end{pmatrix},
$$

where $Q$ denotes the electric charge and $Q_u = \frac{2}{3}$.

- **Down squarks**: Down squarks mass squared matrix takes the form

$$
M_d^2 = \begin{pmatrix}
M_Q^2 + m_d^2 - M_Z^2(\frac{1}{2} + Q_d s_W^2) \cos 2\beta & m_d (A_d^* - \mu \tan\beta) \\
m_d (A_d - \mu^* \tan\beta) & m_d^2 + m_d^2 + M_Z^2 Q_d s_W^2 \cos 2\beta
\end{pmatrix},
$$

where $Q$ denotes the electric charge and $Q_d = -\frac{1}{3}$.
• Sleptons: Sleptons mass squared matrix takes the form

\[
M^2_L = \begin{pmatrix}
M^2_L + m_e^2 - M_Z^2 s_W^2 & m_e (A_e - \mu \tan \beta) \\
m_e (A_e - \mu^* \tan \beta) & m_e^2 + m_e^2 - M^2_Z s_W^2 \cos 2\beta
\end{pmatrix},
\]

(4.49)

• Neutralinos: Neutralinos are mixtures of gauginos and Higgsinos and their mass matrix takes the form

\[
M_{\tilde{\chi}^0} = \begin{pmatrix}
M_1 & 0 & -M_W s c \beta & M_Z s W s \beta \\
0 & M_2 & M_Z c W c \beta & -M_Z c W s \beta \\
-M_Z s W c \beta & M_Z c W c \beta & 0 & -\mu \\
M_Z s W s \beta & -M_Z c W s \beta & -\mu & 0
\end{pmatrix},
\]

(4.50)

• Charginos: Charginos are mixtures of charged electroweak gauginos and charged Higgsinos and their mass matrix takes the form

\[
M_{\tilde{\chi}^\pm} = \begin{pmatrix}
M_2 & \sqrt{2} M_W s \beta \\
\sqrt{2} M_W c \beta & \mu
\end{pmatrix}.
\]

(4.51)

4.3 Dark Matter in Supergravity

In supergravity unified models over a large part of the allowed parameter space the lightest neutralino turns out to be the lightest supersymmetric particle and with R parity conservation it becomes a candidate for cold dark matter. It is also quite remarkable that the neutralino can produce just the right amount of dark matter over a significant region of the parameter space of supergravity models. Below we give a brief description of its relic density in supergravity unified models.
Neutralino relic density is defined by

$$\Omega_{\chi} \equiv \rho_{\chi}/\rho_c$$

(4.52)

where $\rho_{\chi}$ is the matter density due to neutralinos, and $\rho_c$ is the critical mass density needed to close the universe where $\rho_c$ is given by

$$\rho_c = \frac{3H_0^2}{8\pi G_N},$$

(4.53)

where $H_0$ is the Hubble parameter. The current value of $\rho_c$ is

$$\rho_c = 1.9h_0^2 \times 10^{-29}\text{gm/cm}^3,$$

(4.54)

where the Hubble parameter $h_0$ presently has a numerical value of $h_0 = 0.7 \pm 0.013$ in units of 100 km/secMpc. The neutralino number density in the early universe obeys the Boltzmann equation [59]

$$\frac{dn}{dt} = -3Hn - \langle \sigma v \rangle (n^2 - n_0^2),$$

(4.55)

where $n_0$ is the equilibrium number density, $\langle \sigma v \rangle$ the thermal average of the cross section of neutralino annihilation $\sigma(\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow X)$, $v$ the relative velocity of $\tilde{\chi}_1^0$, and $H$ the Hubble parameter at time $t$. The thermal average $\langle \sigma v \rangle$ can be expressed as

$$\langle \sigma v \rangle = \frac{\int_0^\infty dv v^2(\sigma v)e^{-v^2/4x}}{\int_0^\infty dv v^2e^{-v^2/4x}},$$

(4.56)

with $x = kT/m_\chi$. A solution to the Boltzmann equation yields [25]

$$\Omega_{\chi}h_0^2 = 2.5 \times 10^{-11} \left(\frac{T_\chi}{T_\gamma}\right)^3 \left(\frac{T_\gamma}{2.75}\right)^3 \frac{N_f^{1/2}}{J(x_f)},$$

(4.57)
where $T_\gamma$ is the current microwave background temperature, $x_f$ is the “freeze out” temperature corresponding to the temperature where the annihilation rate becomes smaller than the expansion rate, so that $\tilde{\chi}_1^0$ decouples from the background. $x_f$ is typically small with a value $x_f \sim 0.04$. $N_f$ is the number of degrees of freedom at freeze out and typically $N_f \simeq 289.5/8$ [60]. Finally $J(x_f)$ is given by

$$J(x_f) = \int_0^{x_f} dx \langle \sigma v \rangle x \ (\text{GeV}^{-2}).$$

The relic density in a realistic model is much more complicated [61–65]. For example, in the co-annihilation region, one needs to consider several coupled channels and the total number density for various specifies of particles obeys the equation

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{eff}} v_{\text{rel}} \rangle (n^2 - n_{\text{eq}}^2),$$

with

$$\sigma_{\text{eff}} = \sum_{ij} \sigma_{ij} \gamma_i \gamma_j,$$

where $\sigma_{ij}$ denotes the cross-section of annihilation of particles $i$ and $j$, and $\gamma_i = n_{\text{eq}}^i / n_{\text{eq}}$ where $n_{\text{eq}}^i$ denotes the sparticle number density of $i$ at equilibrium state. Here a numerical analysis is needed and often one uses micrOMEGAs [66, 67] for relic density analyses. We will discuss this issue further in a later section.

### 4.4 Proton Stability

In grand unified theories of particle interactions quarks and leptons belong to common multiplets and thus grand unified theories lead to proton decay. In non-supersymmetric theories proton decay arises from baryon and lepton number vio-
lating dimension six operators which is mediated by lepto-quark exchange and the most dominant decay mode is $p \rightarrow e^+ + \pi^0$. In supersymmetric theories proton decay can arise from R parity violating (RPV) dimension 4 operators of type

$$W_{RPV} = \lambda'_B u^c d^c + \lambda'_L Q d^c L + \lambda''_L L L e^c$$

(4.61)

The above interactions lead to fast proton decay and for consistency with experiment one needs to suppress them, i.e., one needs the constraint

$$\lambda'_B \lambda'_L < O(10^{-27}).$$

(4.62)

A more efficient way to suppress these terms is to impose R parity conservation which automatically eliminates the above terms in the superpotential. However, even with R parity conservation, baryon and lepton number violating dimension five operators can arise in supersymmetric theories. These are of the type

$$QQQL \quad LLLL - \text{type}$$

(4.63)

$$u^c d^c d^c e^c \quad RRRR - \text{type}$$

(4.64)

Figure 4.1: Diagrams that contribute to proton decay arising from $LLLL$ and $RRRR$ lepton and baryon number violating dimension 5 operators.

When dressed with sfermion and gaugino exchange diagrams these operators lead to proton decay. Thus in supersymmetric GUTs proton decay from dimension
five operators depends very sensitively on the sparticle spectrum since the sparticle spectrum enters in the dressing loop diagrams which involve the exchange of squarks and sleptons, gluinos, charginos, and neutralinos \cite{68-72} (for recent reviews see \cite{73-75}). Thus low values of sfermion masses can lead to too rapid a proton decay for the mode $p \to \bar{\nu}K^+$ in conflict with the current experimental limit \cite{75}, i.e.,

$$\tau_{\text{exp}}(p \to \bar{\nu}K^+) > 4 \times 10^{33} \text{ yr.}$$ (4.65)

Since a heavy Higgs boson mass in the vicinity of $\sim 126$ GeV implies relatively large values of sfermion masses it is pertinent to investigate proton stability within the constraint of the experimentally observed large Higgs boson mass. We will limit ourselves to generic $SU(5)$ type models. Further, while chargino $\tilde{\chi}^\pm$, gluino $\tilde{g}$ and neutralino $\tilde{\chi}^0$ exchange diagrams all contribute to the decay width, the dominant contribution comes from the chargino exchange diagram and we will limit ourselves to considerations for decay with this exchange. Thus here the decay width is given by \cite{76},

$$\Gamma(p \to \bar{\nu}K^+) = \left( \frac{\beta_p}{M_{H_3}} \right)^2 |A|^2 |B_i|^2 C,$$ (4.66)

where $M_{H_3}$ is the Higgsino triplet mass and $\beta_p$ is the matrix element between the proton and the vacuum state of the 3 quark operator so that

$$\beta_p U_L^\gamma = \epsilon_{abc} \epsilon_{a\alpha} \delta_{\gamma} < 0|d^a_{uL} u^\beta_{bL} c_{cL}|p >,$$ (4.67)

where $U_L^\gamma$ is the proton spinor. The most reliable evaluation of $\beta_p$ comes from
lattice gauge calculations and is given \[77\] as

\[ \beta_p = 0.0118 \text{ GeV}^3. \] (4.68)

Other factors that appear in Eq.(4.66) have the following meaning: \( A \) contains the quark mass and CKM factors, \( B_i \) are the functions that describe the dressing loop diagrams, and \( C \) contains chiral Lagrangian factors which convert the Lagrangian involving quark fields to the effective Lagrangian involving mesons and baryons. Individually these functions are given by

\[ A = \frac{\alpha_s^2}{2 M_W^2} m_s m_c V_{21} V_{21} A_L A_S, \] (4.69)

where \( m_s \) is the strange quark mass, \( m_c \) is the charm quark mass, \( V_{ij} \) are the CKM factors, and \( A_L \) and \( A_S \) are the long distance and the short distance renormalization group suppression factors as one evolves the operators from the GUT scale down to the electro-weak scale and then from the electroweak scale down to 1 GeV \cite{69 78 81}, and \( B_i \) are given by

\[ B_i = \frac{1}{\sin 2\beta} \frac{m_t^d V_{21}^i}{m_s V_{21}^i} [P_2 B_{2i} + \frac{m_t V_{31} V_{32}}{m_c V_{21} V_{22}} P_3 B_{3i}], \] (4.70)

where \( m_t^d \) is the down quark mass for flavor \( i \) and \( m_t \) is the top quark mass. Here the first term in the bracket is the contribution from the second generation and the second term is the contribution from the third generation and \( P_2, P_3 \) with values \((\pm 1)\) are the relative parities of the second and the third generation contributions.

The functions \( B_{ji} \) are the loop integrals defined by

\[ B_{ji} = F(\bar{u}_i, \bar{d}_j, \chi^\pm) + (\bar{d}_j \to \bar{e}_j), \] (4.71)
where

\[
F(\tilde{u}_i, \tilde{d}_j, \tilde{\chi}^\pm) = [E \cos \gamma_- \sin \gamma_+ \tilde{f}(\tilde{u}_i, \tilde{d}_j, \tilde{\chi}^\pm) + \cos \gamma_+ \sin \gamma_- \tilde{f}(\tilde{u}_i, \tilde{d}_j, \tilde{\chi}^\pm)]
\]

\[
- \frac{1}{2} \frac{\delta_{ij} \mu^i}{2 \sqrt{2} M_W} \sin \beta [E \sin \gamma_- \sin \gamma_+ \tilde{f}(\tilde{u}_{i1}, \tilde{d}_j, \tilde{\chi}^\pm) - \cos \gamma_- \cos \gamma_+ \tilde{f}(\tilde{u}_{i1}, \tilde{d}_j, \tilde{\chi}^\pm)
- (\tilde{u}_{i1} \rightarrow \tilde{u}_{i2})],
\]

and where \( \tilde{f} \) appearing in Eq.(4.72) is given by

\[
\tilde{f}(\tilde{u}_i, \tilde{d}_j, \tilde{\chi}^\pm) = \sin^2 \delta_{ui} f(\tilde{u}_{i1}, \tilde{d}_j, \tilde{\chi}^\pm) + \cos^2 \delta_{ui} f(\tilde{u}_{i2}, \tilde{d}_j, \tilde{\chi}^\pm).
\]

Here the tilde quantities in the arguments are the sparticle masses, i.e., \( \tilde{u}_i \) are the up squark masses for flavor \( i \) and \( \tilde{d}_j \) are the down squark masses for flavor \( j \) and the function \( f \) is defined by

\[
f(a, b, c) = \frac{m_c}{m_b - m_c} \frac{m_b^2}{m_a^2 - m_b^2} \ln \frac{m_a^2}{m_b^2} - (m_a \rightarrow m_c).
\]

Further in Eq.(4.72)

\[
\gamma_\pm = \beta_+ \pm \beta_-,
\]

where

\[
\sin 2\beta_\pm = (\mu \pm m_2)/[4\nu_{\pm}^2 + (\mu \pm m_2)^2]^{1/2},
\]

with

\[
\sqrt{2}\nu_{\pm} = M_W (\sin \beta \pm \cos \beta),
\]
and

\[ \sin 2\delta_{u3} = -2(A_t + \mu \cot \beta) m_t / (m_{t1}^2 - m_{t2}^2). \] (4.78)

The value of \( E \) is determined as follows,

\[ E = 1 \quad \text{when} \quad \sin 2\beta > \mu m_2 / M_W^2, \]

\[ E = -1 \quad \text{when} \quad \sin 2\beta < \mu m_2 / M_W^2. \] (4.79)

Finally \( C \) is given by

\[ C = \frac{m_N}{32\pi f^2} \left(1 + \frac{m_N(D + F)}{m_B} \right) \left(1 - \frac{m_K}{m_N^2} \right)^2, \] (4.80)

where \( \tilde{t}_i \) are the stop masses and \( f_\pi, D, F, \ldots \) etc are the chiral Lagrangian factors and we use the numerical values

\[ f_\pi = 0.131 \text{ GeV}, \]
\[ D = 0.8, \]
\[ F = 0.47, \]
\[ m_N = 0.94 \text{ GeV}, \]
\[ m_K = 0.495 \text{ GeV}, \]
\[ m_B = 1.15 \text{ GeV}, \] (4.81)

and we choose \( P_2 = 1 \) and \( P_3 = -1 \). The partial decay lifetime of the proton into \( p \to \bar{\nu} + K^+ \) mode is given by

\[ \tau(p \to \bar{\nu} + K^+) = \hbar / \Gamma(p \to \bar{\nu} + K^+). \] (4.82)
These results will be used in the quantification of proton stability Ch.7.3.
Chapter 5

Constraints on Unified Models

To confront unified models of particle interactions with experiment we impose a number of experimental constraints on the allowed parameter space of models. These constraints are listed below.

- Higgs Mass Constraints:

Over the past year the ATLAS and the CMS Collaborations have identified a signal for a boson around $\sim 126$ GeV. Thus the ATLAS Collaboration finds a signal at

$$m_h = 126.0 \pm 0.4\text{(stat)} \pm 0.4\text{(sys)} \text{ GeV}, \quad (5.1)$$

which is at the 5.0$\sigma$ level \[82\] while the CMS Collaboration finds a signal at

$$m_h = 125.3 \pm 0.4\text{(stat)} \pm 0.5\text{(sys)} \text{ GeV}, \quad (5.2)$$

at the 5.0$\sigma$ level \[83\]. In our analysis we consider the following window on the Higgs boson mass for all model points

$$124.5 \text{ (GeV)} \leq m_h \leq 126.8 \text{ (GeV)}. \quad (5.3)$$
• Relic Density Constraints from Planck Measurement:

Within the context of the standard spatially-flat six-parameter CDM cosmology, the cosmological parameters are determined from Planck data to high precision, and the relic density of cold dark matter (CDM) is determined to be

\[ \Omega_\chi h^2 = (0.1199 \pm 0.0027), \]  

with an error of 68% as in [84]. In the analysis we use the following criteria for relic density

\[ \Omega_\chi < 0.13. \]  

This is to allow for the possibility of a multi-component dark matter where part of the relic density is made up of other possible species of dark matter.

• \( b \to s\gamma \) Constraint:

The above flavor changing neutral current (FCNC) process arises only at the loop level both in the standard model as well as in supersymmetric models. The current experimental limits on this process are given by [85],

\[ (2.77 \times 10^{-4}) \leq Br (b \to s\gamma) \leq (4.37 \times 10^{-4}). \]  

• \( B_s \to \mu^+\mu^- \) Constraint:

In supersymmetric theories \( B_s \to \mu^+\mu^- \) has a \( \tan^6 \beta \) dependence. Thus, models with large \( \tan \beta \) could be severely constrained by experiment. The experimental situation for this decay is the following: The CDF Collaboration [86] using 7 fb\(^{-1} \) of integrated luminosity gives \( Br (B_s \to \mu^+\mu^-) = \)
(1.8^{+1.1}_{-0.9}) \times 10^{-8}$, and this in turn provides an upper limit of

$$\mathcal{B} r \left( B_s \rightarrow \mu^+ \mu^- \right) \leq 4.0 \times 10^{-8}, \quad (5.7)$$

at 95\% confidence level. The DØ experiment [87] using 6.1 fb$^{-1}$ of integrated luminosity gives an upper limit on the branching fraction so that

$$\mathcal{B} r \left( B_s \rightarrow \mu^+ \mu^- \right) \leq 5.1 \times 10^{-8}, \quad (5.8)$$

at the 95\% C.L. In our analysis a more conservative value is used, i.e.,

$$\mathcal{B} r \left( B_s \rightarrow \mu^+ \mu^- \right) \leq 1.1 \times 10^{-8}. \quad (5.9)$$

• Sparticle Mass Constraints:

In the analysis we use the experimental constraints on sparticle masses from the Large Electron Position (LEP) collider [88] as follows: $m_{\tilde{\tau}_1} > 81.9$ GeV for the lighter stau, $m_{\tilde{\chi}_1^\pm} > 103.5$ GeV for lighter chargino, $m_{\tilde{t}_1} > 95.7$ GeV for the lighter stop, $m_{\tilde{b}_1} > 89$ GeV for the lighter sbottom, $m_{\tilde{e}_R} > 107$ GeV for the right-handed selectron, and $m_{\tilde{\mu}_R} > 94$ GeV for the right-handed smuon.

• $g_\mu - 2$ Constraints:

In supersymmetric theories, the supersymmetric electroweak corrections [89–97] to the muon anomalous magnetic moment $g_\mu - 2$ can be as large or larger than the standard model electroweak corrections [98, 99]. The current experimental status of $g_\mu - 2$ can be found in [100]. In our analysis we take the following range for the supersymmetric contribution

$$(-11.4 \times 10^{-10}) \leq \delta (g_\mu - 2) \leq (9.4 \times 10^{-9}). \quad (5.10)$$
Several naturalness, hierarchy, and fine-tuning problems exist in particle physics: some big and some small. The most severe one relates to the smallness of the vacuum energy in units of the Planck mass, followed by the smallness of the ratio $M_W/M_{Pl}$. There are several other small-to-moderate size hierarchies such as the ratio $M_{\text{GUT}}/M_{Pl}$ and the ratios in the fermion mass spectra such as $m_u/m_t$.

Also, there are hierarchy problems of a more technical nature, such as in the Higgs sector of the standard model, where the Higgs boson mass receives a loop correction which is quadratically dependent on the cutoff. This problem is resolved in supersymmetric models with a cancellation between the fermionic and super-fermionic loops which results in the quadratic dependence on the cut-off being replaced by a logarithmic dependence. A similar problem at a much smaller scale often called the little hierarchy problem appears for supersymmetric models if the scalar masses turn out to be large. In fact, in certain models of soft breaking the scalar masses can get large, as is the case in supergravity grand unified models [8, 15, 17] with hierarchical breaking of supersymmetry [42, 101, 102] and for certain string motivated models [103]. Large scalar masses have also been considered in other contexts [104, 105].

The little hierarchy problem can be simply described as follows: in the radiative
electroweak symmetry breaking (REWSB) one has

\[ \frac{1}{2} M_Z^2 \simeq -\mu^2 - m_{H_2}^2, \]  

where \( \mu \) is the Higgs mixing parameter and \( m_{H_2} \) is the mass of the Higgs boson that couples to the top quark. Naively \( m_{H_2} \) gets large as the universal scalar mass \( m_0 \) gets large and a large cancellation is needed between \( \mu \) and \( m_{H_2} \) to get a small \( M_Z \). A more practical approach is to view the REWSB relation as a determination of \( \mu \) which is the viewpoint adopted here. From this perspective, if \( m_0 \) is large the accessibility of sparticles at the LHC rests on the size of \( m_{1/2} \) and \( \mu \) and thus a small \( \mu \) (and a small \( m_{H_2} \)) is desirable. One note in passing that if \( m_0 \) is indeed large, the LHC would turn into a gaugino factory with the sparticles produced being gluinos [106, 107], charginos and neutralinos. One can also note that this region gives a significant enhancement to proton lifetime [73] because of the smallness of the gaugino masses and relative heaviness of the squark masses.

The question then is how one may achieve a small \( \mu \) for the above class of models in the context of radiative electroweak symmetry breaking. The basic mechanism for achieving the above was first realized in [54] (for further works see [29, 40, 108]). In the analysis of [54] it was found that there exist two natural regions of radiative breaking, one where there is an upper bound on the soft parameters \( m_0, m_{1/2}, A_0 \) for a fixed \( \mu \) (the Ellipsoidal Branch, EB), and the other where one or more soft parameters can get very large for fixed \( \mu \) (the Hyperbolic Branch, HB). In a later work [55], it was shown that there exists a region where the value of the Higgs mass squared, \( m_{H_2}^2 \), becomes essentially independent of the values of the input parameter \( m_0 \) at the GUT scale. Such a region was then labeled the Focus Point.

Here solutions of the Hyperbolic Branch are classified in Sec. 6.1.1 and showed that it contains three main regions:
• Focal Points (HB/FP): This region lies at the boundary between the Ellipsoidal and the Hyperbolic Branches where $\mu^2$ becomes independent of $m_0^2$ and thus $m_0$ can get large while $\mu$ remains fixed with the other soft parameters being held fixed. In this definition Focal Point is not included on the EB. The Focal Point is technically different from the Focus Point [55] but for $\tan \beta \gg 1$ they are essentially the same as will be made clear in Sec. 6.1.1 and Sec. 6.1.2. The HB/FP region, however, is only a small part of HB and the larger parts of HB are Focal Curves and Focal Surfaces as discussed below, and in detail in Sec. 6.1.3 and Sec. 6.1.4.

• Focal Curves (HB/FC): Focal Curves are where two soft parameters are comparable and can get large while $\mu$ is fixed. One can define HB/FC such that the HB/FP region is excluded.

• Focal Surfaces (HB/FS): Here one may have a fixed (and small) $\mu$ while the three dimensional soft parameters may get large. The HB/FS region is the set of all Focal Curves and thus does not include the HB/FP region. In all these regions one or more soft parameters can get large while $\mu$ is small.

6.1 mSUGRA

In this section I will discuss in detail the classification of HB into the three broad regions mentioned in the last section. I begin with the equation for the radiative breaking of the electroweak symmetry

$$\mu^2 + \frac{1}{2} M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1}, \quad (6.2)$$

where one have

$$m_{H_i}^2 = m_{H_i}^2 + \Sigma_i, \quad (6.3)$$
and $\Sigma_i$ is the contribution arising from the loop corrections to the effective potential for $i = 1, 2$ [53]. In the analysis here I will focus on the supergravity grand unification model with universal boundary conditions [15, 23, 24] whose soft breaking sector is described by

$$\left( m_0, m_{1/2}, A_0, \tan \beta, \text{sgn}(\mu) \right), \quad (6.4)$$

where $m_0$ is the universal scalar mass, $m_{1/2}$ is the universal gaugino mass, $A_0$ is the universal trilinear coupling and $\mu$ is the Higgs mixing parameter in the superpotential. The model of Eq. (6.4) is referred to as mSUGRA or sometimes as the constrained minimal supersymmetric model, CMSSM. The analysis is done using the techniques given in [109, 110] where one starts with universal boundary conditions given by Eq. (6.4) for the soft parameters at the GUT scale and evolves the sparticle masses downwards using renormalization group equations. For illustration in the text, I consider one loop evolution where I neglect the Yukawa couplings except for the top quark. The simulations presented later are done using numerical codes which include the effects of the $b$ and $\tau$ Yukawa couplings. As discussed in previous section, the radiative electroweak symmetry breaking allows for a determination of $\mu^2$ in terms of the soft parameters as [41, 54]

$$\mu^2 = \frac{1}{2} M_Z^2 + m_0^2 C_1 + A_0^2 C_2 + m_{1/2}^2 C_3 + m_{1/2} A_0 C_4 + \Delta \mu^2_{\text{loop}}, \quad (6.5)$$

or,

$$\mu^2 + \frac{1}{2} M_Z^2 = m_0^2 C_1 + A_0^2 C_2 + m_{1/2}^2 C_3' + \Delta \mu^2_{\text{loop}}, \quad (6.6)$$
where

\[ A'_o = A_0 + \frac{C_4}{2C_2} m_{1/2}, \quad (6.7) \]

and

\[
\begin{align*}
C_1 &= \frac{1}{\tan^2 \beta - 1} \left( 1 - \frac{3D_0 - 1}{2} \tan^2 \beta \right), \\
C_2 &= \frac{\tan^2 \beta}{\tan^2 \beta - 1} k, \\
C_3 &= \frac{1}{\tan^2 \beta - 1} (g - e \tan^2 \beta), \\
C_3' &= C_3 - \frac{C_4}{4C_2}, \\
C_4 &= -\frac{\tan^2 \beta}{\tan^2 \beta - 1} f,
\end{align*}
\]

and \( e, f, g, k \) are as defined in [58, 111]. \( D_0(t) \) is defined by

\[ D_0(t) = (1 + 6Y_0 F(t))^{-1}. \quad (6.13) \]

Here

\[ Y_0 = \frac{h_t(0)^2}{16\pi^2}, \quad (6.14) \]

where \( h_t(0) \) is the top Yukawa coupling at the GUT scale, \( M_G \approx 2 \times 10^{16}\text{GeV} \).

Further,

\[ F(t) = \int_0^t E(t') dt', \quad (6.15) \]

where

\[ E(t) = (1 + \beta_3 t)^{16/3\beta_3} (1 + \beta_2 t)^{3/\beta_2} (1 + \beta_1 t)^{13/9\beta_1}. \quad (6.16) \]
Here

$$\beta_i = \alpha_i(0)b_i/(4\pi), \ b_i = (-3, 1, 11), \quad (6.17)$$

for $SU(3), SU(2)$ and $U(1)$ and $t = \ln (M_2^2/Q^2)$ where $Q$ is the renormalization group point. Our normalizations are such that

$$\alpha_3(0) = \alpha_2(0) = \frac{5}{3}\alpha_1(0) = \alpha_G(0). \quad (6.18)$$

Further, $\Delta\mu_{\text{loop}}^2$ is the loop correction [53].

### 6.1.1 Focal Regions of Hyperbolic Branch (HB)

To understand the origin of the branches of radiative breaking it is useful to choose a renormalization group scale $Q$ where the loop correction $\Delta\mu_{\text{loop}}^2$ is minimized. In this circumstance if all the coefficients $C_1, C_2, C'_3$ are positive, the right hand side of Eq.(6.6) is a positive sum of squares which leads to an upper limit on each of soft parameters determined by the size of $\mu^2 + \frac{1}{2}M_Z^2$ on the left hand side. This is the so called Ellipsoidal Branch (EB) where $\mu$ sets an upper limit on the soft parameters and thus on the size of the sparticle masses. This is typically the case if the loop correction $\Delta\mu_{\text{loop}}^2$ is small. However, the situation changes drastically if the loop correction $\Delta\mu_{\text{loop}}^2$ is large. This is so because $C_i$ are functions of the renormalization group (RG) scale $Q$ and for the case when the loop correction $\Delta\mu_{\text{loop}}^2$ is large the RG dependence of $C_i$ can become significant. Indeed as we change the renormalization group scale $Q$, there is a rapid change in $\Delta\mu_{\text{loop}}^2$, and a rapid compensating change also in the remaining terms on the right hand side of Eq.(6.6) so that $\mu^2$ does not exhibit any rapid dependence on $Q$. Now it turns out that there are regions of the parameter space where one or more of the $C_i$
may turn negative as $Q$ varies. For the supergravity unified models with universal boundary conditions this is the case for $C_1$, i.e., in certain regions of the parameter space $C_1$ can turn negative while the remainder on the right hand side of Eq. (6.6) remains positive. In this case it is useful to write Eq. (6.6) in the following form

$$\mu^2 = \begin{pmatrix} +1 & \text{(EB)} \\ 0 & \text{(FP)} \\ -1 & \text{(HB)} \end{pmatrix} m_0^2 |C_1| + \Delta^2, \quad (6.19)$$

where $\Delta^2$ stands for the rest of the terms in Eq. (6.6). In Eq. (6.19) +1 corresponds to the Ellipsoidal Branch (EB), −1 corresponds to the Hyperbolic Branch (HB) and $C_1 = 0$ is the boundary point between the two which we call Focal Point (FP). Its approximate form when $\tan \beta >> 1$ is the Focus Point [55]. $C_1 = 0$ is achieved when $D_0 = 1/3$ We wish now to identify the allowed regions of the mSUGRA parameter space in terms of the branch on which they reside, i.e., EB, HB or FP.

Explicitly speaking, the three regions that HB of REWSB contains are:

- **The Focal Point (HB/FP):** I define the points where $C_1$ vanishes as Focal Points. From Eq. (6.5) and Eq. (6.8) one find that when $C_1 = 0$, $m_0$ can get large without affecting $\mu$. For practical purposes, for a fixed $\tan \beta$, I will take a small region around $C_1 = 0$, and call it the Focal Point region, specifically

$$|C_1| < \delta(Q,m_t), \quad \delta(Q,m_t) \ll 1. \quad (6.20)$$

In determining $\delta(Q,m_t)$ one is guided by the experimental error in the top quark mass from $m_t = (173.1 \pm 1.3)$ GeV. Now, for a fixed $\tan \beta$, $C_1 = C_1(m_t, Q)$ where $Q \sim \mathcal{O}(\sqrt{M_{\tilde{t}_1}M_{\tilde{t}_2}})$ and thus, $Q$ depends on the top mass via the dependence of the stop masses on $m_t$. However, this implicit
dependence on $m_t$ via $Q$ is rather weak and effectively

$$\delta C_1 = \frac{\delta C_1}{\delta m_t} \delta m_t.$$  \hspace{1cm} (6.21)

A direct analysis gives the following approximate result

$$\delta C_1 \simeq 3 (1 - D_0) \frac{\delta m_t}{m_t}.$$  \hspace{1cm} (6.22)

This result agrees with the one loop analysis in Fig. 6.1 and Fig. 6.2 where $\delta C_1$ can be interpreted as the vertical spacing between the curves in Fig. 6.2.

In the full numerical analysis presented later in identifying the parameter points that lie in the Focal Point region, one can calculate $\delta C_1$ numerically for each point by calculating the variation in $C_1$ for variations in $m_t$. 

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Focal Curves and Focal Surface: \( C_1(Q) < 0 \)

Figure 6.2: A display of the sensitivity of \( C_1(Q) \) to the top quark mass. The blue lines correspond to \( \pm 1\sigma \) in the top pole mass around the black line which corresponds to the central value, where the pole mass is taken to be \( m_t = (173.1 \pm 1.3) \) GeV.

- **Focal Curves (HB/FC):** The region where \( C_1 < 0 \) allows for two soft parameters to get large while \( \mu \) remains small is the Focal Curve region. In fact, in this case there are two general possibilities: HB/FC1 and HB/FC2. In the case of HB/FC1 (HB/FC2), one have \( C_1 < 0 \) and \( m_{1/2} (A_0) \) as well as \( \mu \) are held fixed with \( m_0 \) and \( A_0 \) (\( m_{1/2} \)) allowed to vary. These two cases can be combined into a single form HB/FC\( \alpha \) defined by \( C_1 < 0 \) and the constraint 
  \[
  (1 - \alpha) m_{1/2} = \alpha |A_0| \quad \text{where} \quad 0 < \alpha < 1.
  \]  
  One can note that HB/FC\( \alpha \) reduces to HB/FC1 when \( \alpha \sim 0 \) and reduces to HB/FC2 when \( \alpha \sim 1 \).

- **The Focal Surface (HB/FS):** is the region of HB where \( C_1 < 0 \) while all the soft parameters (except \( \tan \beta \)), i.e, \( m_0, m_{1/2}, A_0 \) vary and may get large while \( \mu \) remains fixed. In terms of HB/FC\( \alpha \), varying \( \alpha \) creates a Focal Surface.
6.1.2 Focal Point (FP) Region of HB

While the Hyperbolic Branch [54] and the Focus Point [35] both allow for large values of $m_0$ while $\mu$ remains small, the exact relationship of the Hyperbolic Branch and of the Focus Point has not been elucidated in the literature. In this section I establish a direct connection between the two. I will show that the Focus Point is the boundary point of a Focal Curve on the Hyperbolic Branch. Again for illustration I will consider one loop evolution, and among the Yukawa couplings retain only the top quark coupling. Here the scalar masses $m_{H_2}^2$, $m_\tilde{U}^2$ and $m_\tilde{Q}^2$ satisfy the following set of coupled equations

\[
\frac{dm_{H_2}^2}{dt} = -3Y_t \Sigma - 3Y_t A_t^2 + \left(3\tilde{\alpha}_2 M_2^2 + \tilde{\alpha}_1 M_1^2\right),
\]

\[
\frac{dm_{\tilde{U}}^2}{dt} = -2Y_t \Sigma - 2Y_t A_t^2 + \left(\frac{16}{3} \tilde{\alpha}_3 M_3^2 + \frac{16}{9} \tilde{\alpha}_1 M_1^2\right),
\]

\[
\frac{dm_{\tilde{Q}}^2}{dt} = -Y_t \Sigma - Y_t A_t + \left(\frac{16}{3} \tilde{\alpha}_3 M_3^2 + 3\tilde{\alpha}_2 M_2^2 + \frac{1}{9} \tilde{\alpha}_1 M_1^2\right),
\]

(6.23)

where

\[
\Sigma = (m_{H_2}^2 + m_{\tilde{Q}}^2 + m_{\tilde{U}}^2),
\]

(6.24)

and

\[
Y_t = \frac{h_t^2}{16\pi^2},
\]

(6.25)

and where $h_t$ is the Yukawa coupling at scale $Q$. The analysis of [55] made the observation that the solution to Eq. (6.23), can be written in the form

\[
m_t^2 = (m_t^2)_p + \delta m_t^2,
\]

(6.26)
where \((m^2_i)_p\) is the particular solution and the \(\delta m^2_i\) obey the homogeneous equation

\[
\frac{d}{dt} \begin{bmatrix} \delta m^2_{H_2} \\ \delta m^2_U \\ \delta m^2_Q \end{bmatrix} = -Y_t \begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \delta m^2_{H_2} \\ \delta m^2_U \\ \delta m^2_Q \end{bmatrix}.
\tag{6.27}
\]

The solution to the above with the universal boundary conditions at the GUT scale is given by

\[
\begin{bmatrix} \delta m^2_{H_2} \\ \delta m^2_U \\ \delta m^2_Q \end{bmatrix} = \frac{m^2_0}{2} \begin{bmatrix} 3J(t) - 1 \\ 2J(t) \\ J(t) + 1 \end{bmatrix},
\tag{6.28}
\]

where \(J\) is an integration factor defined by

\[
J(t) \equiv \exp \left[ -6 \int_0^t Y_t(t')dt' \right].
\tag{6.29}
\]

As \(Q \to M_G\), one has \(J(t) \to 1\) and the universality of the masses is recovered at the GUT scale. Noting that \(Y(t)\) at the one loop level satisfies the equation

\[
\frac{dY_t}{dt} = \left( \frac{16}{3} \tilde{\alpha}_3 + 3\tilde{\alpha}_3 + \frac{13}{9} \tilde{\alpha}_1 \right) Y_t - 6Y_t^2,
\tag{6.30}
\]

one finds \(Y_t\) so that

\[
Y_t(t) = \frac{Y(0)E(t)}{1 + 6Y(0)F(t)},
\tag{6.31}
\]

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where $F(t)$ and $E(t)$ are defined after Eq. (6.13), one can inspect $J(t)$ to find that $J(t) = D_0(t)$, where $D_0(t)$ is defined by Eq. (6.13). Thus $\delta m_{H_2}^2$ takes the form

$$\delta m_{H_2}^2 \equiv \frac{\delta m_{H_2}^2}{m_0^2} = \frac{1}{2} (3D_0 - 1),$$  

(6.32)

and $C_1$ can be expressed in terms of $\delta m_{H_2}^2$

$$C_1 = \frac{1}{\tan^2 \beta - 1} (1 - \delta m_{H_2}^2 \tan^2 \beta) \simeq -\delta m_{H_2}^2.$$

(6.33)

At the Focal Point $\mu^2$ essentially becomes independent of $m_0$. From Eq. (6.32) one can see that the correction $\delta m_{H_2}^2$ becomes independent of $m_0$ when $D_0 = 1/3$, which corresponds to the so called Focus Point region [55], and from Eq. (6.33) one finds that $\delta m_{H_2}^2 \to 0$ implies that $C_1$ also vanishes, for $\tan \beta \gg 1$. Thus for large $\tan \beta$, i.e. $\tan \beta \gtrsim 5$, the Focal Point and the Focus Point essentially merge. More explicitly, the Focus Point implies the vanishing of $\delta m_{H_2}^2$ while the Focal Point requires the vanishing of $C_1$. A numerical analysis of the behavior of $C_1$ as a function of $Q$ for a set of fixed $\tan \beta$’s is given in Fig. 6.2 as well as a graphical representation of the different branches. Fig. 6.2 shows that the Focal Point is the boundary point of HB or, in other words, the transition point between EB and HB, i.e., the Focal Point defined by $C_1 = 0$ is just the boundary point between EB defined by $C_1 > 0$ and HB define by $C_1 < 0$.

### 6.1.3 Focal Curves

To exhibit the emergence of a Focal Curve one may rewrite Eq. (6.2) in the following form

$$\mu^2 = -\frac{1}{2} M_Z^2 + m_0^2 C_1 + \bar{A}_0^2 C_2 + m_{1/2}^2 C_3 + \Delta \mu_{\text{loop}}^2,$$

(6.34)
with
\[ \bar{A}_0 \equiv A_0 + \frac{C_4}{2C_2} m_{1/2}, \quad C_3 \equiv C_3 - \frac{C_4^2}{4C_2}. \tag{6.35} \]

Now, suppose one go to the renormalization group point \( Q \) where the loop corrections are minimized and, further, one is in a region of \( \tan \beta \) and \( Q \) where \( C_1 \) is negative. In this case one finds that there exist curves where \( m_0 \) and \( A_0 \) get large while \( m_{1/2} \) is held fixed and \( \mu \) is relatively small compared to \( m_0 \) and \( A_0 \). Thus one can rewrite Eq. (6.34) in the form
\[ \left( \frac{A_0}{\sqrt{C_2}} \right)^2 - \left( \sqrt{|C_1|m_0} \right)^2 = \pm |\mu_1|^2 \quad \text{for HB/FC1} , \tag{6.36} \]

where
\[ \pm |\mu_1|^2 \equiv \mu^2 + \frac{1}{2} M_Z^2 - m_{1/2}^2 C_3 - \Delta \mu^2_{\text{loop}}, \tag{6.37} \]

where \( \pm \) indicates the overall sign of the right hand side. Thus one has two branches corresponding to the two signs. One can interpret Eq. (6.36) as an equation of a Focal Curve in the \( m_0 - \bar{A}_0 \) plane (or in the \( m_0 - A_0 \) plane around a shifted origin in \( A_0 \)) such that as \( m_0 \) and \( A_0 \) get large, \( \mu \) remains fixed for fixed \( m_{1/2} \) (this is Focal Curve HB/FC1 as defined in Sec. 6.1.3). In the limit when \( m_0, |A_0| \) (and \( Q \)) are much larger than \( \mu \) and \( m_{1/2} \) one gets the result
\[ \frac{\bar{A}_0}{m_0} \to \frac{A_0}{m_0} \to \pm \sqrt{\frac{|C_1|}{C_2}} \to \sim \pm 1 , \tag{6.38} \]

where the last entry in Eq. (6.38) arises from a numerical evaluation of \( C_1 \) and \( C_2 \) as given by Eq. (6.8) and Eq. (6.9) as shown in Fig. 6.3.

In order to identify which points lie on Focal Curves one can compute the \( C_i \)
Evolution of $\sqrt{|C_1|/C_2}$ and $\sqrt{|C_1|/C_3}$ with $Q$

\begin{figure}
\centering
\includegraphics[width=\textwidth]{focal_curves}
\caption{A numerical analysis of the evolution of $\sqrt{|C_1|/C_2}$ and $\sqrt{|C_1|/C_3}$ using Eq. (6.8), Eq. (6.9), and Eq. (6.10). Here one finds that $\sqrt{|C_1|/C_2}$ tends to $\sim 1$ and $\sqrt{|C_1|/C_3}$ tends to $\sim 0.4$ as $Q$ becomes large. The analysis is shown for $\tan \beta = 10$ and $\tan \beta = 45$.}
\end{figure}

for each point and then subject them to the conditions necessary for them to lie on a Focal Curve. Thus for the case presented above one can consider $m_{1/2}$ fixed while $m_0$ and $A_0$ vary with $C_1 < 0$ and outside the Focal Point region. An analysis illustrating Focal Curves in this case is given in Table 6.1. For this analysis and subsequent figures and tables I use both SUSPECT \cite{3} and SOFTSUSY \cite{4} which include the two loop renormalization group equations and the two loop corrections to the Higgs sector. The analysis is done for the case when $m_0$ lies in the range 500 GeV to 4000 GeV and $A_0$ lies in the range $-500$ GeV to $-3000$ GeV with $\tan \beta = 15$ and $\mu$ remaining within 10% of 600 GeV. A similar analysis is shown pictorially in Fig. 6.4 where I have displayed the Focal Curves for $m_{1/2} = 500$ GeV, $\tan \beta = 45$ and $\mu = (465 \pm 35)$ GeV. One can see that for $m_0$ and $|A_0|$ large, there is good agreement with Eq. (6.38), i.e., one finds $A_0/m_0 \rightarrow \pm 1$ asymptotically for
large $m_0$. One can note that the limit $A_0/m_0 \sim 1$ consistent with small $\mu$ was noticed and discussed recently in the analysis of [112] in the context of a string motivated model. From Fig. 6.4 one can note that this limit is part of HB and is specifically the end point of the Focal Curve HB/FC1. The left panel of Fig. 6.6 shows model points with $m_{1/2} < 1$ TeV and $m_0 > 10$ TeV with $\mu < 2$ TeV. The result of $m_0$ up to 10 TeV were exhibited in [54], and up to 30 TeV in [112], and here one can exhibit $m_0$ up to 50 TeV and beyond for $\mu < 2$ TeV, i.e., $\mu/m_0 \ll 1$.

Now there is also another possibility of achieving a Focal Curve which can be illustrated by writing Eq. (6.5) in the form

$$\mu^2 + \frac{1}{2}M_Z^2 = m_0^2C_1 + A_0^2C_2 + \overline{m}_{1/2}^2C_3 + \Delta \mu_{\text{loop}}^2.$$  \hfill (6.39)

$$\overline{m}_{1/2} \equiv m_{1/2} + \frac{C_4}{2C_3} A_0, \quad \overline{C}_2 \equiv C_2 - \frac{C_4^2}{4C_3^2}.$$  \hfill (6.40)

As before, one can write this equation in the form

$$\left(\sqrt{C_3 \overline{m}_{1/2}}\right)^2 - \left(\sqrt{|C_1|m_0}\right)^2 = \pm |\mu|^2 \quad \text{HB/FC2},$$  \hfill (6.41)
Figure 6.4: Exhibition of Focal Curves HB/FC1 with $m_{1/2} = 0.5$ TeV and $\tan \beta = 45$ where $\mu$ lies in the range $\mu = (0.465 \pm 0.035)$ TeV. Points are displayed by $\mu$ value.

where

$$\pm |\mu_2|^2 \equiv \mu^2 + \frac{1}{2} M_Z^2 - A_0^2 C_2 - \Delta \mu_{\text{loop}}^2 .$$

Thus again one has two branches depending on the sign. Here one keeps $A_0$ fixed while $m_0$ and $m_{1/2}$ get large and $\mu$ is relatively small (this is Focal Curve HB/FC2 as defined in Sec. 6.1.1). For the case when $|\mu_2|$ is small relative to $m_0$ and $\overline{m}_{1/2}$ one finds the following relationship asymptotically

$$\frac{m_{1/2}}{m_0} \rightarrow \frac{m_{1/2}}{m_0} \rightarrow \sqrt{\frac{|C_1|}{C_3}} \rightarrow \approx 0.4,$$

where the last entry in Eq. (6.43) is obtained by using Eq. (6.8) and Eq. (6.10) as shown in Fig. 6.3. An illustration of this case is given in Fig. 6.5 where $m_{1/2}$
Figure 6.5: An illustration of Focal Curves HB/FC2 which arise when $m_0$ and $m_{1/2}$ are free to vary while $A_0$ is fixed and $\mu$ is held relatively constant. The analysis is for $\tan \beta = 45$ and for four values of $A_0$ which are $A_0 = 0.7$ TeV (red), $A_0 = 1.2$ TeV (blue), $A_0 = 5.0$ TeV (cyan) and $A_0 = 2.5$ TeV (black). The analysis above shows that on the Focal Curve HB/FC1 and HB/FC2 one has good agreement with the asymptotic behavior as predicted by Eq. (6.38) and Eq. (6.43).

gets very large. For these curves one can see that there can still have models with $\mu$ small ($\mu \lesssim 450$ GeV) and $m_{1/2}$ large ($m_{1/2} \gtrsim 1500$ GeV), which leads to the gluino mass being on the order of a few TeV or larger.

To show that there exists a larger set of Focal Curves than the cases have been discussed above one can exhibit a whole set of parametric Focal Curves which could be labelled as HB/FC$\alpha$. To do this, define

$$ (1 - \alpha) m_{1/2} = \alpha |A_0|, \quad (6.44) $$
where $0 < \alpha < 1$. This allows us to rewrite Eq. (6.5) as

$$
\pm |\mu_\alpha|^2 = - \left( \sqrt{|C_1|} \right)^2 m_0^2 + C_\alpha A_0^2 .
$$

(6.45)

where

$$
\pm |\mu_\alpha|^2 = \mu^2 + \frac{1}{2} M_Z^2 - \Delta \mu_{\text{loop}}^2 .
$$

(6.46)

Further,

$$
C_\alpha = C_2 + \frac{\alpha^2}{(1 - \alpha)^2} C_3 + \frac{\alpha}{1 - \alpha} C_4 \text{ sgn} (A_0) ,
$$

(6.47)
Figure 6.7: Exhibition of Focal Curves $HB/FC_\alpha$ using $m_{1/2} = \frac{\alpha}{|\epsilon_\alpha|} |A_0|$ for $\tan \beta = 45$ and $\mu = (0.465 \pm 0.035)$ TeV with $m_0$ between 10 GeV and 10 TeV and $A_0$ between $-8m_0$ and $8m_0$. One could display the cases where $\alpha = 0.01, 0.05, 0.15, 0.25, 0.50$ and notice that for smaller $\alpha$ the asymptotic behavior is more steep.

Eq. (6.45) shows that there exists parametric Focal Curves, parameterized by $\alpha$, where one can get the same value of $\mu$ which can be taken to be small, while $\alpha$ can take on values in the range $(0,1)$. This phenomenon illustrated in the right panel of Fig. 6.7 displays several Focal Curves for constant $\mu$. One finds that as $\alpha$ decreases the asymptotic form of the curves in the $A_0 - m_0$ plane become more steep. This result is in agreement with the theoretical prediction at one loop for the asymptotic ratio $A_0/m_0$ which is

$$A_0/m_0 \to \pm \sqrt{|C_1|/C_\alpha}. \quad (6.48)$$
6.1.4 Focal Surfaces

Next consider the radiative breaking of the electroweak symmetry where all the three parameters \(m_0, m_{1/2},\) or \(A_0\) can get large while \(\mu\) remains small. This solution is again valid in the region of the parameter space where \(C_1\) turns negative at the value of renormalization group point which minimizes the loop correction. This is the Focal Surface \(\text{HB/FS}\) as defined in Sec. 6.1.1 and one can express it in the following two forms

\[
\pm |\mu_s|^2 = -\left(\sqrt{|C_1|m_0}\right)^2 + \left(A_0\sqrt{C_2}\right)^2 + \left(\sqrt{C_3m_{1/2}}\right)^2,
\]

where

\[
\pm |\mu_s|^2 = \mu^2 + \frac{1}{2}M_Z^2 - \Delta\mu_{\text{loop}}^2.
\]

A summary of focal regions is given in Table 6.2. An exhibition of a Focal Surface for the case \(\mu = (0.465 \pm 0.035)\) TeV is given in Fig. 6.8 and Fig. 6.9. One can note that on the Focal Surface shown in Fig. 6.8 and Fig. 6.9, \(m_0, m_{1/2},\) or \(A_0\) can all be seen to get large in certain regions while \(\mu\) remains relatively constant. One can note in passing that another way to generate a Focal Surface is to consider a...
Figure 6.8: Exhibition of a Focal Surface when $\tan \beta = 45$ and $\mu = (0.465 \pm 0.035)$ TeV while $m_0, m_{1/2}, A_0$ can all get large. The plot shows the same Focal Surface using an interpolation of the points presented in the left panel.

Focal Curve HB/FC$_\alpha$ and let $\alpha$ vary over its allowed range $0 \leq \alpha < 1$. Thus a Focal Surface can be viewed as a collection of Focal Curves as in the right panel of Fig. 6.7.

### 6.2 NuSUGRA

In the early analyses using radiative breaking of the electroweak symmetry (for a review see [113]) only the Ellipsoidal Branch was known, in that a fixed value of the $\mu$ (the Higgs mixing parameter) implied upper limits on sparticle masses. However, the situation changed drastically with the discovery of the Hyperbolic Branch [29 54] (for related work see [108 112]) when it was discovered that another branch of radiative breaking of the electroweak symmetry existed where the sparticle masses could lie in the several TeV region while $\mu$ could still be at
Figure 6.9: Exhibition of a Focal Surface when $\tan \beta = 45$ and $\mu = (0.465 \pm 0.035)$ TeV while $m_0, m_{1/2}, A_0$ can all get large. The plot shows the same Focal Surface using an interpolation of the points presented in the left panel.

the sub TeV scale. Specifically on this branch TeV size scalars can exist consistent with small $\mu$.

In Sec 6.1 a classification of radiative breaking of the electroweak symmetry is given in terms of various branches. Here one may extend the analysis to NuSUGRA and classify the branches through a similar approach.

6.2.1 Analysis of $C_i$

The presence of non-universalities in the gaugino sector affects the co-efficients $C_i$ and here a computation is given for these by inclusion of non-universalities in the $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ gaugino sectors.

Begin with the radiative electroweak symmetry breaking with inclusion of non-universalities in the gaugino sector, one have

$$\mu^2 = \frac{(m_{H_1}^2 - m_{H_2}^2 \tan \beta^2)}{(\tan \beta^2 - 1)} - \frac{1}{2} M_Z^2 + \Delta \mu^2, \quad (6.50)$$
where

\[ m_{H_1}^2 = m_0^2 + \left( \frac{3}{10} \tilde{f}_1 + \frac{3}{2} \tilde{f}_2 \right), \quad (6.51) \]

\[ m_{H_2}^2 = \tilde{c}(t) + A_0 \tilde{f}(t) + m_0^2 h(t) - A_0^2 k(t), \quad (6.52) \]

To write everything in terms of \( m_1, m_2 \) and \( m_3 \), first one need to look into the expression of \( m_{H_1}^2 \)

\[ \tilde{f}_i(t) = \frac{1}{\beta_i} \left( 1 - \frac{1}{1 + \beta_i t^2} \right) \tilde{\alpha}_i(0) m_i^2 \equiv Z_i^f m_i^2, \quad (6.53) \]

where

\[ Z_i^f = \frac{1}{\beta_i} \left( 1 - \frac{1}{1 + \beta_i t^2} \right) \tilde{\alpha}_i(0). \quad (6.54) \]

It is useful to introduce a matrix \( M_{m_{H_1}} \) such that

\[ m_{H_1}^2 = \bar{m}^T \cdot M_{m_{H_1}} \cdot \bar{m} = (M_{m_{H_1}})_{ij} m_i m_j, \quad (6.55) \]

where \( M_{H_1} \) is given by

\[
M_{m_{H_1}} = \begin{pmatrix}
\frac{3}{10} Z_1^f & 0 & 0 \\
0 & \frac{3}{2} Z_2^f & 0 \\
0 & 0 & 0
\end{pmatrix}. \quad (6.56)
\]

Thus we have

\[ m_{H_1}^2 = m_0^2 + (M_{m_{H_1}})_{ij} m_i m_j. \quad (6.57) \]

The above exhibits the gaugino mass dependence of \( m_{H_1}^2 \) explicitly.

Now let us look at \( m_{H_2}^2 \) given by Eq. (6.52) and try to write it in a form which which exhibits the gaugino mass dependence of it explicitly. Now \( m_{H_1}^2 \) contains
the functions \( \tilde{e}(t) \) and \( \tilde{f}(t) \), so that

\[
\tilde{e} = 3 \left[ \frac{\tilde{G}_1 + Y_0 \tilde{G}_2}{D(t)} + \frac{(\tilde{H}_2 + 6Y_0 \tilde{H}_4)^2}{3D(t)^2} + \tilde{H}_8 \right],
\]

and

\[
\tilde{f} = -\frac{6Y_0 \tilde{H}_3(t)}{D(t)^2},
\]

where the functions \( \tilde{H}_i(t) \) functions are defined as

\[
\begin{align*}
\tilde{H}_2 &= \frac{13}{15} \tilde{f}_1(t) + 3 \tilde{h}_2(t) + \frac{16}{3} \tilde{h}_3(t), \\
\tilde{H}_3 &= \int_0^t E(t') \tilde{H}_2(t') dt', \\
\tilde{H}_4 &= F(t) \tilde{H}_2(t) - \tilde{H}_3(t), \\
\tilde{H}_5 &= \left( -\frac{22}{15} \tilde{f}_1(t) + 6 \tilde{f}_2(t) - \frac{16}{3} \tilde{f}_3(t) \right), \\
\tilde{H}_6 &= \int_0^t E(t') \tilde{H}_2(t')^2 dt', \\
\tilde{H}_8 &= \tilde{\alpha}_G \left( -\frac{8}{3} \tilde{f}_1(t) + \tilde{f}_2(t) - \frac{1}{3} \tilde{f}_3(t) \right),
\end{align*}
\]

and \( \tilde{h}_i \) are defined as

\[
\tilde{h}_i = \frac{t}{1 + \beta_i t} \tilde{\alpha}_i(0) m_i \equiv Z_i^h m_i,
\]

with

\[
Z_i^h = \frac{t}{1 + \beta_i t} \tilde{\alpha}_i(0).
\]

\( \tilde{H}_2(t) \) then takes the form

\[
\tilde{H}_2 = \left( \frac{13}{15} Z_1^h m_1 + 3 Z_2^h m_2 + \frac{16}{3} Z_3^h m_3 \right),
\]

and in the matrix notation we can write

\[
\tilde{H}_2 \equiv M_{\tilde{H}_2} \cdot \vec{m} = \left( M_{\tilde{H}_2} \right)_i m_i,
\]
where $\vec{M}_{\tilde{H}_2}$ is a row vector and

$$
\vec{M}_{\tilde{H}_2} = \left( \frac{13}{15} Z_1^h, 3Z_2^h, \frac{16}{3} Z_3^h \right),
$$

and $\vec{m}$ a column vector

$$
\vec{m} = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}.
$$

Similarly, we may write all the $M_{\tilde{H}_i}(t)$ in matrix or vector forms so that

$$
\tilde{H}_3 = \vec{M}_{\tilde{H}_3} \cdot \vec{m} \equiv (\vec{M}_{\tilde{H}_3})_i m_i, 
$$

$$
\tilde{H}_4 = \vec{M}_{\tilde{H}_4} \cdot \vec{m} \equiv (\vec{M}_{\tilde{H}_4})_i m_i, 
$$

$$
\tilde{H}_5 = \vec{m}^T \cdot \vec{M}_{\tilde{H}_5} \cdot \vec{m} \equiv (M_{\tilde{H}_5})_{ij} m_i m_j, 
$$

$$
\tilde{H}_6 = \left( \vec{M}_{\tilde{H}_6} \cdot \vec{m} \right)^2 \equiv \left( \vec{M}_{\tilde{H}_6} \right)_i \left( \vec{M}_{\tilde{H}_6} \right)_j m_i m_j, 
$$

$$
\tilde{H}_8 = \vec{m}^T \cdot \vec{M}_{\tilde{H}_8} \cdot \vec{m} \equiv (M_{\tilde{H}_8})_{ij} m_i m_j, 
$$
where the matrices $\tilde{M}_{\tilde{H}_3}$ etc are given by

\[
\tilde{M}_{\tilde{H}_3} = \int_0^t E(t') \tilde{M}_{\tilde{H}_2}(t') dt',
\]

\[
\tilde{M}_{\tilde{H}_4} = \tilde{M}_{\tilde{H}_2}(t) \int_0^t E(t') dt' - \int_0^t \tilde{M}_{\tilde{H}_2}(t') E(t') dt',
\]

\[
M_{\tilde{H}_5} = \begin{pmatrix} -\frac{22}{15} Z_1^f & 0 & 0 \\ 0 & 6 Z_2^h & 0 \\ 0 & 0 & -\frac{16}{3} Z_3^h \end{pmatrix},
\]

\[
M_{\tilde{H}_6} = \int_0^t \left( \tilde{M}_{\tilde{H}_2}(t') \right)^T \left( \tilde{M}_{\tilde{H}_2}(t') \right) E(t') dt',
\]

\[
M_{\tilde{H}_8} = \begin{pmatrix} -\frac{1}{3} Z_1^f & 0 & 0 \\ 0 & Z_2^h & 0 \\ 0 & 0 & -\frac{8}{3} Z_3^h \end{pmatrix},
\]

Similarly the functions $\tilde{F}_i(t)$ defined as

\[
\tilde{F}_2 = \frac{8}{15} \tilde{f}_1 + \frac{8}{3} \tilde{f}_2,
\]

\[
\tilde{F}_3 = F(t) \tilde{F}_2(t) - \int_0^t E(t') \tilde{F}_2(t') dt',
\]

\[
\tilde{F}_4 = \int_0^t E(t') \tilde{H}_5(t') dt',
\]

can also be written in matrix forms so that

\[
\tilde{F}_2 = \vec{m}^T \cdot M_{\tilde{F}_2} \cdot \vec{m} \equiv (M_{\tilde{F}_2})_{ij} m_i m_j,
\]

\[
\tilde{F}_3 = \vec{m}^T \cdot M_{\tilde{F}_3} \cdot \vec{m} \equiv (M_{\tilde{F}_3})_{ij} m_i m_j,
\]

\[
\tilde{F}_4 = \vec{m}^T \cdot M_{\tilde{F}_4} \cdot \vec{m} \equiv (M_{\tilde{F}_4})_{ij} m_i m_j,
\]
with $M_{\tilde{F}_i}$ is defined by

$$M_{\tilde{F}_2}(t) = \begin{pmatrix} \frac{8}{15} Z_1 \iota & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{8}{3} Z_3 \iota \end{pmatrix},$$

(6.88)

$$M_{\tilde{F}_3}(t) = F(t) M_{\tilde{F}_2}(t) - \int_0^t E(t') M_{\tilde{F}_2}(t') dt',$$

(6.89)

$$M_{\tilde{F}_4}(t) = \int_0^t E(t') M_{\tilde{H}_5}(t') dt',$$

(6.90)

We repeat the same procedure with functions $\tilde{G}_i$ defined by

$$\tilde{G}_1 = \tilde{F}_2(t) - \frac{1}{3} \tilde{H}_2(t)^2,$$

(6.91)

$$\tilde{G}_2 = 6 \tilde{F}_3(t) - \tilde{F}_4(t) - 4 \tilde{H}_2(t) \tilde{H}_4(t) + 2 F(t) \tilde{H}_2(t)^2 - 2 \tilde{H}_6(t),$$

(6.92)

which could also been written as,

$$\tilde{G}_1(t) = \vec{m}^T \cdot M_{\tilde{G}_1} \cdot \vec{m} \equiv (M_{\tilde{G}_1})_{ij} m_i m_j,$$

(6.93)

$$\tilde{G}_2(t) = \vec{m}^T \cdot M_{\tilde{G}_2} \cdot \vec{m} \equiv (M_{\tilde{G}_2})_{ij} m_i m_j,$$

(6.94)

with $M_{\tilde{G}_i}$ is defined by

$$M_{\tilde{G}_1}(t) = M_{\tilde{F}_2} - \frac{1}{3} (\vec{M}_{\tilde{H}_2})^T \cdot \vec{M}_{\tilde{H}_2},$$

(6.95)

$$M_{\tilde{G}_2}(t) = 6M_{\tilde{F}_3} - M_{\tilde{F}_4} - 4 (\vec{M}_{\tilde{H}_2})^T \cdot \vec{M}_{\tilde{H}_4}$$

$$+ 2 F(\vec{M}_{\tilde{H}_2})^T \cdot \vec{M}_{\tilde{H}_2} - 2 M_{\tilde{H}_6},$$

(6.96)
We return now to \( \tilde{e}(t) \) and \( \tilde{f}(t) \) given by

\[
\tilde{e}(t) = \frac{3}{2D(t)^2} \left( 3D(t) \left[ \hat{G}_1 + Y_0 \hat{G}_2 \right] + \frac{1}{3} \left[ \hat{H}_2 + 6Y_0 \hat{H}_4 \right]^2 + D(t)^2 \hat{H}_8 \right), \quad \text{(6.97)}
\]

\[
\tilde{f}(t) = -\frac{6Y_0 \tilde{H}_3}{D(t)^2}. \quad \text{(6.98)}
\]

We can write these in the matrix form so that

\[
\tilde{e}(t) = \vec{m}^T \cdot M_{\tilde{e}} \cdot \vec{m} \equiv (M_{\tilde{e}})_{ij} m_i m_j, \quad \text{(6.99)}
\]

\[
\tilde{f}(t) = \vec{M}_{\tilde{f}} \cdot \vec{m} \equiv (\vec{M}_{\tilde{f}})_{i} m_i, \quad \text{(6.100)}
\]

with

\[
M_{\tilde{e}} = \frac{3}{2D(t)^2} \left( 3D(t) \left[ M_{\hat{G}_1} + Y_0 M_{\hat{G}_2} \right] + \frac{1}{3} \left[ M_{\hat{H}_2} + 6Y_0 M_{\hat{H}_4} \right]^2 + D(t)^2 M_{\hat{H}_8} \right)
\]

\[
= \frac{9}{2D(t)} \left( M_{\hat{G}_1} + Y_0 M_{\hat{G}_2} \right) + \frac{1}{2D(t)^2} \left( (M_{\hat{H}_2} + 6Y_0 M_{\hat{H}_4})^2 + \frac{3}{2} M_{\hat{H}_8} \right)
\]

\[
= M_{\tilde{e}_1} + M_{\tilde{e}_2} + M_{\tilde{e}_3}, \quad \text{(6.101)}
\]

and

\[
\vec{M}_{\tilde{f}} = -\frac{6Y_0 \vec{M}_{\tilde{H}_3}}{D(t)^2}, \quad \text{(6.102)}
\]
where

\[
M_{\tilde{e}_1} = \frac{9}{2D(t)} (M_{\tilde{G}_1} + Y_0 M_{\tilde{G}_2}), \tag{6.103}
\]

\[
M_{\tilde{e}_2} = \frac{1}{2D(t)^2} (M_{\tilde{H}_2} + 6Y_0 \tilde{M}_{\tilde{H}_4}) \equiv (\tilde{M}_{\tilde{e}_4})^T \cdot \tilde{M}_{\tilde{e}_4}, \tag{6.104}
\]

\[
M_{\tilde{e}_3} = \frac{3}{2} \tilde{M}_{\tilde{H}_4}, \tag{6.105}
\]

\[
\tilde{M}_{\tilde{e}_4} = \frac{1}{\sqrt{2}D(t)} (M_{\tilde{H}_2} + 6Y_0 \tilde{M}_{\tilde{H}_4}), \tag{6.106}
\]

Thus one has

\[
(M_{\tilde{e}})_{ij} = (M_{\tilde{e}_1})_{ij} + (M_{\tilde{e}_2})_{ij} + (M_{\tilde{e}_3})_{ij}
= (M_{\tilde{e}_1})_{ij} + (\tilde{M}_{\tilde{e}_4})_i (\tilde{M}_{\tilde{e}_4})_j + (M_{\tilde{e}_3})_{ij}, \tag{6.107}
\]

\[
(\tilde{M}_f)_i = -\frac{6Y_0}{D(t)^2} (\tilde{M}_{\tilde{H}_4})_i. \tag{6.108}
\]

Using the above we can write \(m_{\tilde{H}_2}^2\) in the form

\[
m_{\tilde{H}_2}^2 = (M_{\tilde{e}})_{ij} m_i m_j + A_0 (\tilde{M}_f)_i m_i + m_0^2 h(t) - A_0^2 k(t). \tag{6.109}
\]

Thus using Eq. (6.57) and Eq. (6.109) in Eq. (6.50), we finally have

\[
\mu^2 + \frac{1}{2} M_{\tilde{z}}^2 = \frac{1 - h(t) \tan \beta^2}{\tan \beta^2 - 1} m_0^2 + \frac{k(t) \tan \beta^2}{\tan \beta^2 - 1} A_0^2
+ \frac{(M_{m_{\tilde{t}_4}})_{ij} - \tan \beta^2 (M_{\tilde{e}})_{ij}}{\tan \beta^2 - 1} m_i m_j - \frac{\tilde{M}_f)_i}{\tan \beta^2 - 1} m_i A_0
\equiv C_1 m_0^2 + C_2 A_0^2 + \tilde{C}_3 m_i m_j + \tilde{C}_4 m_i A_0 \tag{6.110}
\]
where \( C_i \)'s are such that,

\[
C_1 = \frac{1 - h(t) \tan \beta^2}{\tan \beta^2 - 1},
\]

(6.111)

\[
C_2 = \frac{k(t) \tan \beta^2}{\tan \beta^2 - 1},
\]

(6.112)

\[
\tilde{C}_{ij}^3 = \frac{(M_{m_R})_{ij} - \tan \beta^2 (M_e)_{ij}}{\tan \beta^2 - 1},
\]

(6.113)

\[
\tilde{C}_4^i = -\frac{\tan \beta^2}{\tan \beta^2 - 1} \left( \tilde{M}_f^i \right),
\]

(6.114)

where \( i, j = 1, 2, 3 \). \( \tilde{C}_3 \) and \( \tilde{C}_4 \) in Eq. (6.114) reduce to the universal case when \( m_i = m_{1/2} \) and in this case one has

\[
C_3 = \sum_{i,j=1,2,3} \tilde{C}_{ij}^3, \quad C_4 = \sum_{i=1,2,3} \tilde{C}_4^i.
\]

(6.115)

Based on the analytical forms of \( C_i \)'s in Eq. (6.114) we give a numerical analysis of them as a function of \( Q \) for various values of \( \tan \beta \) in Fig. 6.10 and Fig. 6.11. Discussions in the following sections shall refer to these figures.

### 6.2.2 Focal Curves and Surfaces

With the analysis on \( C_i \) for NuSUGRA in Sec. 6.2.1 we discuss now non-universalities in the gaugino sector. Thus after the breaking of the gauge group at the unification scale one would get the gaugino masses at low scales so that

\[
\tilde{m}_a^{(0)} = m_{1/2} \left( 1 + \sum_R C_R n_a^R \right), \quad a = 1, 2, 3,
\]

(6.116)

where the first term in the brace arises from singlet breaking and the second term is from breaking with non-singlet representations, and \( a = 1, 2, 3 \) corresponds to the gauge groups \( U(1), SU(2) \) and \( SU(3) \). A simple way to parameterize the gaugino
mass non-universality is to write

\[
\tilde{m}_a^{(0)} = m_{1/2} \left( 1 + \delta_a^{GN} \right),
\]

(6.117)

where \( \delta_a^{GN} \) run over some appropriate range. Gaugino sector non-universalities at the GUT scale affect the evolution of the Higgs mass squares, and thus affect the
As shown in Sec. 6.2.1 that in this case the radiative electroweak symmetry breaking equation, with inclusion of the gaugino mass non-universalities $\mu^2$ takes the form Eq. (6.2) for the universal soft breaking case is replaced by

$$\mu^2 + \frac{1}{2} M_Z^2 = C_1 m_0^2 + C_2 A_0^2 + \tilde{C}_3^{ij} m_i m_j + \tilde{C}_4^i m_i A_0 + \Delta \mu^2, \quad (6.118)$$

where $C_1$ and $C_2$ are as defined by Eq. (6.112) and Eq. (6.113) while $\tilde{C}_3^{ij}$ and $\tilde{C}_4^i$ are given by

$$\tilde{C}_3^{ij} = \frac{(M_{m_{H_1}})_{ij} - \tan^2 \beta (M_{\tilde{e}})_{ij}}{\tan^2 \beta - 1}, \quad \tilde{C}_4^i = \frac{\tan^2 \beta}{\tan^2 \beta - 1} (M_{\tilde{f}})_{i}, \quad (6.119)$$

as shown in Sec. 6.2.1, where $M_{m_{H_1}}$, $M_{\tilde{e}}$ and $M_{\tilde{f}}$ are also defined. $\tilde{C}_3$ and $\tilde{C}_4$ in Eq. (6.114) reduce to the universal case when $m_i = m_{1/2}$ and in this case one has

$$C_3 = \sum_{i,j=1,2,3} \tilde{C}_3^{ij}, C_4 = \sum_{i=1,2,3} \tilde{C}_4^i. \quad (6.120)$$

In Fig. 6.10 and Fig. 6.121 we display the dependence of $\tilde{C}'s$ on the RG scale $Q$. Here one finds that in addition to $C_1$, $\tilde{C}_3^{11}$ and $\tilde{C}_3^{22}$ assume negative values which gives the possibility of new focal curves. These possibilities shall be discussed in details below.

To examine the focal curves and focal surfaces for NuSUGRA, it is useful to define

$$C_3^G m_{1/2}^2 = \tilde{C}_3^{ij} m_i m_j, \quad C_4^G m_{1/2} = \tilde{C}_4^i m_i. \quad (6.121)$$

Further, in order to classify various regions of the radiative electroweak symmetry breaking (REWSB) for the NuSUGRA case it useful to write the REWSB
Table 6.3: Classification of focal curves in NuSUGRA models. Here one has the possibility of several focal curves. The focal curve HB/FC1 is defined similar to the mSUGRA case except that \( m_1, m_2, m_3 \) are all kept fixed. As in mSUGRA here too \( m_0 \) and \( A_0 \) can get large while \( \mu \) remains fixed. The focal curve HB/FC2 splits into three sub cases because of the gaugino non-universalities. Thus the case HB/FC2\(^{01}\) corresponds to the case when \( A_0, m_2, m_3 \) are kept fixed while \( m_0 \) and \( m_1 \) can get large. The focal curves HB/FC2\(^{02}\) and HB/FC2\(^{03}\) are similarly defined. For the NuSUGRA case 4 new type of focal curves arise. These are HB/FC3\(^{13}\), HB/FC3\(^{23}\), HB/FC4\(^{1}\), HB/FC4\(^{2}\). Their definitions are obvious from the table.

\[
\mu^2 + \frac{1}{2} M_Z^2 = C_1 m_0^2 + C_2 A_0^2 + C_3^{(i)} \overline{m}_i^2, \tag{6.122}
\]

with

\[
\overline{A}_0^2 = (A_0 + \sum_{i=1}^3 a_i m_i)^2, \quad \overline{m}_i = \sum_{j=1}^3 a_{ij} m_j, \tag{6.123}
\]

where \( a_i \) and \( a_{ij} \) are co-efficients of linear combinations and they are functions of \( C_2, \tilde{C}_3^{(i)} \) and \( \tilde{C}_4^i \).

From the analysis in the Sec. 6.2.1 one finds that the elements \( \tilde{C}_3^{11} \) and \( \tilde{C}_3^{22} \) are negative, which allows for the possibility of new focal curves. Thus using these results we define the following set of focal curves defined by a fixed value of \( \mu \): FC1-FC4. The focal curve FC1 is for the case when \( C_1 < 0 \) and \( m_0 \) and \( A_0 \) get large with fixed values of \( m_1, m_2, m_3 \). Focal curve FC2 arises when \( m_0 \) and one

<table>
<thead>
<tr>
<th>Focal Curve</th>
<th>large soft parameters</th>
<th>small soft parameters</th>
</tr>
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<tbody>
<tr>
<td>HB/FC1</td>
<td>( m_0 - A_0 )</td>
<td>( m_1, m_2, m_3 )</td>
</tr>
<tr>
<td>HB/FC2(^{01})</td>
<td>( m_0 - m_1 )</td>
<td>( A_0, m_2, m_3 )</td>
</tr>
<tr>
<td>HB/FC2(^{02})</td>
<td>( m_0 - m_2 )</td>
<td>( A_0, m_1, m_3 )</td>
</tr>
<tr>
<td>HB/FC2(^{03})</td>
<td>( m_0 - m_3 )</td>
<td>( A_0, m_1, m_2 )</td>
</tr>
<tr>
<td>HB/FC3(^{13})</td>
<td>( m_1 - m_3 )</td>
<td>( m_0, A_0, m_2 )</td>
</tr>
<tr>
<td>HB/FC3(^{23})</td>
<td>( m_2 - m_3 )</td>
<td>( m_0, A_0, m_1 )</td>
</tr>
<tr>
<td>HB/FC4(^{1})</td>
<td>( A_0 - m_1 )</td>
<td>( m_0, m_2, m_3 )</td>
</tr>
<tr>
<td>HB/FC4(^{2})</td>
<td>( A_0 - m_2 )</td>
<td>( m_0, m_1, m_3 )</td>
</tr>
</tbody>
</table>
of the gaugino masses $m_i$ ($i=1,2,3$) can get large while $A_0$ and other two gaugino masses remain fixed. There are three possibilities here and they are summarized in Table (6.2.2).

A Classification of all these possibilities with respect to FC1 to FC4 are:

- HB/FC1 where $C_1 < 0$ and $m_0$ and $A_0$ get large while $m_1, m_2, m_3$, tan $\beta$ are fixed;
- HB/FC2$^{01}$ where $C_1 > 0, \tilde{C}_{31}^{11} < 0$ and $m_0$ and $m_1$ get large while $A_0, m_2, m_3$, tan $\beta$ are fixed;
- HB/FC2$^{02}$ where $C_1 > 0, \tilde{C}_{32}^{22} < 0$ and $m_0$ and $m_2$ get large while $A_0, m_1, m_3$, tan $\beta$ remains fixed, and
- HB/FC2$^{03}$ where $C_1 < 0, \tilde{C}_{33}^{33} > 0$ and $m_0$ and $m_3$ get large while $A_0, m_1, m_2$, tan $\beta$ remains fixed.

The focal curves FC3 arise when two of the gaugino masses can get large while other soft parameters remain fixed. There are two possibilities here:

- HB/FC3$^{13}$ where $m_1$ and $m_3$ get large while other parameters, i.e., $A_0, m_0, m_2$, tan $\beta$ remains fixed. This can happen when $C_1 > 0$ but $\tilde{C}_{31}^{11}$ is negative, and
- HB/FC3$^{23}$ where $m_2$ and $m_3$ get large while other parameters, i.e., $A_0, m_0, m_1$, tan $\beta$ remains fixed. This can happen when $C_1 > 0$ but $\tilde{C}_{32}^{22}$ is negative.

The focal curves FC4 arise when $A_0$ and one of the gaugino masses get large while the remaining soft parameters remain fixed. There are two possibilities here. These are

- HB/FC4$^1$ where $A_0$ and $m_1$ get large while other parameters, i.e., $m_0, m_2, m_3$, tan $\beta$ remains fixed. This can happen since $C_2 > 0$ but $\tilde{C}_{31}^{11}$ is negative, and
HB/FC4$^2$ where $A_0$ and $m_2$ get large while other parameters, i.e., $m_0, m_1, m_3,$ 
tan $\beta$ remains fixed. This can happen since $C_2 > 0$ but $\hat{C}_3^{22}$ is negative.

Based on the above itemization of different focal curves, one can then discuss 
details corresponding to each of those focal curves. Here one finds that the asymptotic value of $m_{1/2}/m_0$ for fixed $\mu$ as $A_0$ gets large is affected by non-universality, i.e., one gets

$$\frac{m_{1/2}}{m_0} \to \sqrt{\frac{|C_1|}{C_3^G}}.$$  \hspace{1cm} (6.124)

An illustration of the dependence of $m_{1/2}/m_0$ on non-universalities for FC2 will 
be exhibited shortly.

The focal curves FC3 arise when two of the gaugino masses get large while 
other soft parameters remain fixed. There are two possibilities here. The first 
one is HB/FC3$^{13}$ where $m_1$ and $m_3$ get large while $A_0, m_0, m_2$ and $\tan \beta$ remain fixed. This can happen when $C_1 > 0$ but $\hat{C}_3^{11}$ is negative. The second possibility is HB/FC3$^{23}$ where $m_2$ and $m_3$ get large while $A_0, m_0, m_1$ and $\tan \beta$ remain fixed. This can happen when $C_1 > 0$ but $\hat{C}_3^{22}$ is negative. The focal curves FC4 arise when $A_0$ and one of the gaugino masses get large while the remaining soft parameters remain fixed. There are two possibilities here. The first one is HB/FC4$^1$ where $A_0$ 
and $m_1$ get large while $m_0, m_2, m_3$ and $\tan \beta$ remain fixed. This can happen since $C_2 > 0$ but $\hat{C}_3^{11}$ is negative. The second possibility is HB/FC4$^2$ where $A_0$ and $m_2$ 
get large while $m_0, m_1, m_3$ and $\tan \beta$ remain fixed. This can happen when $C_2 > 0$ 
but $\hat{C}_3^{22}$ is negative. We note that HB/FC3$^{12}$ does not materialize since $\hat{C}_3^{11}$ and $\hat{C}_3^{22}$ are both negative. Similarly HB/FC4$^3$ does not occur since $C_2$ and $\hat{C}_3^{33}$ are both positive. Further, while in principle HB/FC2$^{03}$ can occur when $C_1 < 0$ and $\hat{C}_3^{33}$ is positive, the numerical sizes do not favor appearance of this branch. Thus 
as shown in the figures in Sec. 6.2.1, $\hat{C}_3^{33}$ satisfy $|\hat{C}_{3}^{11}| \ll |\hat{C}_{3}^{22}| \ll |\hat{C}_{3}^{33}|$, where each
step is roughly a factor of 10. Thus in practice the focal curve HB/FC2 does not materialize. Further, for any value of tan β, the coefficient $C_1$ begins positive and for $\tan \beta \lesssim 5$ it never becomes negative (for $Q \lesssim 10$ TeV). Because of the above reasons additional possibilities such as HB/FC3 etc are not materialized.

For NuSUGRA we give a numerical illustration of some of the focal curves in Fig. (6.12). The top panel in Fig. (6.12) gives an analysis of the Focal Curve FC1 in the $m_0 - A_0$ plane. Here one finds that $m_0$ and $A_0$ can get as large as 10 TeV while $\mu$ lies in the range $(0.465 \pm 0.035)$ TeV when $\tan \beta = 45$ and $m_{1/2} = 0.5$ TeV. We note that the ratio $A_0/m_0$ asymptotes to the same value irrespective of the non-universalities. A similar analysis for FC2 is given in the right panel of Fig. (6.12) in the $m_0 - m_{1/2}$ plane for $\tan \beta = 45$. Again a variety of non-universalities are discussed. One finds that while $m_0$ and $m_{1/2}$ can get very large, i.e., as large as 10 TeV for $m_0$ and 5 TeV for $m_{1/2}$, one still has a small $\mu$, i.e., a $\mu$ range $(0.465 \pm 0.035)$ TeV. An analysis for FC3 is given in Fig. (6.13). The left panel gives a display of the focal curve FC3 in the $m_0 - m_2$ plane for the case when $\tan \beta = 45$, $A_0 = 1.5$ TeV, $m_{1/2} = 2$ TeV and $\delta_1 = 0 = \delta_3$ and $\delta_2$ lies in the range $(-1, 1)$. One finds that $\mu$ lies in the narrow range $(0.465 \pm 0.035)$ TeV. A very similar analysis in the $m_0 - m_3$ plane is given in the middle panel in Fig. (6.13) where $\delta_1 = 0 = \delta_2$ and $\delta_3$ lies in the range $(-1, 1)$ while all other parameters are as in the left panel. This is the focal curve FC3. Finally the right panel gives an analysis of the focal curve FC3 in the $m_2 - m_3$ plane for the case when $m_0 = 1$ TeV, $m_{1/2} = 2$ TeV, $\tan \beta = 45$ and $\delta_0 = 0$, $\delta_2 = (-1, 1)$, and $\delta_3 = (-1, 1)$. Here again one finds that $\mu$ lies in the range $(0.465 \pm 0.035)$ TeV while $m_2, m_3$ get large. From a convolution of the focal curves one can generate focal surfaces where more than two soft parameters can vary while $\mu$ remains fixed.

In mSUGRA in Sec. 6.1 we were able to carry out a three dimensional scan on $m_0$, $m_{1/2}$ and $A_0$ and present the result at constant $\mu$ at fixed $\tan \beta$. There we
presented different projections of 3-D surface into 2-D and 1-D spaces which were focal curves and focal point, respectively.

Here in the NuSUGRA framework we follow a similar approach, though the scan is now on $m_0, m_1, m_2, m_3, A_0$ at fixed $\tan \beta$ and small $\mu$ and what we obtain is essentially a 5-D surface. We project this 5-D surface into 2-D space and look into the projections as focal curves showed in Fig.6.12 and Fig.6.13. Out of all the possible projections are we only able to present those on $m_0 - A_0, m_0 - m_{1/2}, m_0 - m_2, m_0 - m_3, m_2 - m_3$ planes, due to the limited possibilities of parities of the parameters that satisfy REWSB equation. Therefore they are only several 2-D projections available.

For those 2-D focal curves, we carry out calculations of their slopes based on both simulation results and analytical derivations and make comparisons afterwards. In determining the slopes from simulations results, i.e., focal curve plots, we make linear fits to the linear part of each focal curves and determine the slopes base accordingly. Meanwhile, we determine the average $Q$ values at the upper end of each focal curve and use it as the input to obtain the analytical slopes of their corresponding focal curves. Then we compare these two and to our expectations they do not quite agree with each quantitatively, thanks to the additional contributions from and two-loop corrections in SoftSUSY which are not taken into account in the analytical derivations.

However they do agree with each other qualitatively, as shown in Fig.6.12. As shown in Fig.6.11 that $|\tilde{C}_{3}^{11}| \ll |\tilde{C}_{3}^{22}| \ll |\tilde{C}_{3}^{33}|$, a small $\delta_3$ could cause large deviations from mSUGRA than large $\delta_1$ or $\delta_2$ would, and $\delta_2$ causes larger deviations from mSUGRA than an identical $\delta_1$ would. This should be owing to the differences in the order of magnitude of $\tilde{C}_{3}^{ii}$ terms. After rewriting Eq.(6.118) into Eq.(6.122), we could calculate the coefficients corresponding to $\mu_i$ terms, which a graphical illustration is shown in Fig.6.14.

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Figure 6.11: An exhibition of $\tilde{C}_3^{ii}$ at different $\tan \beta$. Top panel: $\tilde{C}_3^{11}$ at $\tan \beta = 5, 6, 10$ and 45. Middle panel: $\tilde{C}_3^{22}$ at $\tan \beta = 5, 6, 10$ and 45. Bottom panel: $\tilde{C}_3^{33}$ at $\tan \beta = 5, 6, 10$ and 45. It is seen that $\tilde{C}_3^{11}$ and $\tilde{C}_3^{22}$ are negative, which allows the possibility of new focal curves as discussed.
Figure 6.12: Top left panel: Exhibition of the Focal Curve HB/FC1 of Table II with non-universalities in the gaugino sector. Here and in the right panel $\tan\beta = 45$ with $\mu = (0.465 \pm 0.035)$ TeV. The plot shows that non-universalities in the gaugino sector do not affect the asymptotic behavior of $A_0/m_0$ which is unchanged from the mSUGRA case. Top right panel: Exhibition of the effect of non-universalities on focal curves FC2. The analysis shows that the non-universalities have significant effect on FC2 type focal curves. The asymptotic form of the FC2 curves with non-universalities fits well with the result of Eq.(6.124).
Figure 6.13: Bottom panels show the three variety of FC2 curves; left panel: An exhibition of the Focal Curve HB/FC2\textsuperscript{03} in the $m_0 - m_3$ plane when $m_1 = m_3 = m_{1/2} = 2$ TeV and $A_0 = 1.5$ TeV; middle panel: A display of the Focal Curve HB/FC2\textsuperscript{02} in the $m_0 - m_2$ plane when $m_1 = m_3 = m_{1/2} = 2$ TeV and $A_0 = 1.5$ TeV; right panel: An exhibition of the Focal Curve HB/FC3\textsuperscript{23} in the $m_2 - m_3$ plane when $m_1 = m_{1/2} = 2$ TeV, $m_0 = 1$ TeV and $|A_0/m_0| < 0.1$. The model points are colored by $\mu$ value in units of TeV.
Figure 6.14: Exhibition of some of coefficients of square terms in the reduced form of REWSB equation with non-universalities in the gaugino sector. The plot shows that only $C_1$, the coefficient of $m_0^2$ changes sign.

$	an \beta = 45$
Chapter 7

Naturalness and Fine Tuning

The exact definition of naturalness is a somewhat subjective issue. It is often given a measure through fine-tuning defined is such a that a larger fine-tuning is construed as more unnatural than a smaller one. Again, the criteria used for quantifying what is naturalness are rather subjective in the definition of fine-tuning, and various variants abound (for various variants see, e.g., [114][125]).

7.1 Fine Tuning

To look into the issue of fine tuning, let us assume for simplicity that one is computing a quantity \( f(a) \) where \( a \) is a parameter allowed to take a range of values. One definition of fine-tuning often used is the quantity

\[
F_a = a \frac{d \log f(a)}{d \log a}.
\]  

(7.1)

This quantity, however, is more a measure of sensitivity rather than how much we need to fine tune a theoretically computed quantity to agree with an experimentally
measured one. We propose a more direct measure

\[ F_r = \frac{f_{\text{theory}}}{f_{\text{exp}}}, \tag{7.2} \]

where the subscript \( r \) on \( F_r \) refers to the fact that we are considering a ratio of two quantities. The two definitions Eq. 7.1 and Eq. 7.2 are indeed identical for the case when \( f(a) \) depends linearly on \( a \). However, for more complicated dependence on \( a \) they are different as shown below:

\[
\begin{align*}
  f(a) &= f_0 a^k, F_r = k^{-1} F_a, \\
  f(a) &= f_0 e^a, F_r = a^{-1} F_a, \\
  f(a) &= f_0 \log a, F_r = (\log a) F_a. \tag{7.3}
\end{align*}
\]

However, they differ by small numerical factors so that is not a crucial difference. The main advantage of Eq. 7.2 is that first of all it directly connects the theoretical computation to the desired experimental value. Thus, for example, if the theoretically computed quantity is \( 10^2 \) larger than the experimental one, one needs to fine tune the theoretical computation by a factor of a hundred. A more immediate utility is when there are many parameters in the theory in which case one needs to compute many \( F_a \) while there is only \( F_r \) to compute. Further, in many situations the function \( f(a_i) \) may be very complicated such as for the case of proton decay from dimension 5 operators which involves many parameters and many separate diagrams with varying degrees of sensitivity to the parameters. In this Eq. (7.2) gives a direct and more direct way to arrive at the degree of fine tuning than Eq. (7.1).
7.2 Electroweak Fine Tuning in $m_{H_u}^2$

First we discuss fine tuning for radiative breaking of the electroweak symmetry. If we view $m_{H_u}^2$ as the free parameter and start from electroweak symmetry breaking equation

$$\frac{1}{2} M_z^2 = -\mu^2 + \frac{m_{H_u}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1}, \quad (7.4)$$

one can then define two parameters of fine tuning related to $m_{H_u}^2$ as

$$F_1 = 1 + \frac{4\mu^2}{M_z^2} \tan^2 \beta \frac{\tan^2 \beta}{\tan^2 \beta - 1}, \quad (7.5)$$

$$F_2 = 1 + \frac{4\mu^2}{M_z^2} \tan^2 \beta \frac{\tan^2 \beta}{\tan^2 \beta - 1} + \frac{2|m_{H_u}^2|}{M_z^2} \tan^2 \beta \frac{\tan^2 \beta}{\tan^2 \beta - 1}, \quad (7.6)$$

and for large $\tilde{t}_2$ these could be simplified into

$$F_1 = 1 + \frac{4\mu^2}{M_z^2}, \quad (7.7)$$

$$F_2 = 1 + \frac{4\mu^2}{M_z^2} + \frac{2|m_{H_u}^2|}{M_z^2}. \quad (7.8)$$

To address the above results, one can view them in two ways

- If one views $M_z^2$ as arising from the cancellation between $\mu^2$ term and the remainder on the right hand side, the $F_1$ in the above results would lead to a fine tuning

$$F \approx \frac{4\mu^2}{M_z^2}. \quad (7.9)$$

- An alternate criteria for fine tuning is given by the condition [114]

$$F_a' = \frac{a}{f'(a)} f'(a), \quad (7.10)$$
Figure 7.1: An exhibition of the sensitive dependence of the proton lifetime for the decay mode $p \rightarrow \bar{\nu}K^+$ as a function of the Higgs boson mass for the supergravity unified model with universal boundary conditions. Parameters for curves 1-3 are as follows: Curve 1: $m_{1/2} = 4207$ GeV, $A_0 = 20823$ GeV, $\tan \beta = 7.3$ while $m_0$ varies, $M_{H_u}/M_G = 50$ here and for other curves; Curve 2: $m_{1/2} = 2035$ GeV, $A_0 = 16336$ GeV, $\tan \beta = 8$ while $m_0$ and $A_0$ vary and Curve 3: $m_{1/2} = 3048$ GeV, $A_0/m_0 = -0.5$, $\tan \beta = 6.5$ while $m_0$ and $A_0$ vary.

where $a$ is the sensitive parameter on which the function $f(a)$ depends. Using $f(a) = M_Z^2$ and the sensitive parameter as $m_{H_u}^2$ one finds another fine tuning measure from $F_2$ that

$$F' \approx \frac{2|m_{H_u}^2|}{M_Z^2}.$$  \hfill(7.11)

Graphical illustration of these fine tunings will be presented in later sections.

for both mSUGRA and NuSUGRA.

### 7.3 Proton Stability

To carry out analysis on the fine tuning on proton stability, let us first look into the numerical results of proton stability discussed in Sec.4.4. Typically supersym-
metric models give too rapid a proton decay for the mode

\[ p \rightarrow \nu + K^+ \]  \hspace{1cm} (7.12)

from dimension five operators \[126\]. One possible way out is the cancellation mechanism for the reduction of proton decay arising from different Higgs triplet representations at the GUT scale \[127\]. This is equivalent to raising the value of the effective Higgs triplet mass \[128\]. Specification of the GUT physics allows one to determine the effective Higgs triplet mass (see, e.g.,\[127, 129\]). Here, however, we do not commit to a specific GUT structure but rather consider \(SU(5)\) like models where due to various Higgs representations that enter at the GUT scale one has a number of Higgs triplets/anti-triplets \(H_i, \bar{H}_i\). Suppose we choose the basis in which only \(H_1, \bar{H}_1\) couple to matter, i.e., one has couplings of the type \[128\]

\[ \bar{H}_1 J + \bar{K} H_1 + \bar{H}_1 M_{ij} H_j, \]  \hspace{1cm} (7.13)

where \(J\) and \(\bar{K}\) are bilinear in matter fields and \(M_{ij}\) is the superheavy Higgs mass matrix. Many grand unified models automatically lead to such a possibility \[130, 131\]. Specifically in models of the type discussed in \[130\] one has only one light doublet and several Higgs triplets/anti-triplets. On eliminating the superheavy fields one finds that the effective proton decay operator is of the form

\[ -\bar{K} (M'_{H_3})^{-1} J, \]  \hspace{1cm} (7.14)

where \[128\]

\[ M'_{H_3} = (M^{-1}_{11})^{-1}. \]  \hspace{1cm} (7.15)
This allows $M'_{H_3}$ to be much larger than the GUT scale. In the analysis here we will use the effective mass

\[ M'_{H_3}^\text{eff} = M'_{H_3}/A_L A_S, \]  

(7.16)

and we consider three cases that

\[ M'_{H_3}^\text{eff}/M_G = 10, 25, 50 \]  

(7.17)

for analysis in this work here.

In Fig.(7.1) we exhibit the dependence of the proton lifetime for the decay mode $p \rightarrow \bar{\nu} + K^+$ as a function of the Higgs boson mass under the constraints discussed in the caption of Fig.(7.1). The curves show a very sharp dependence of the proton lifetime on the Higgs boson mass which increases by up to two orders of magnitude with a shift in the mass of the Higgs boson in the range of 5-10 GeV.

In Fig.(7.2) we exhibit the proton lifetime for the decay mode $p \rightarrow \bar{\nu} + K^+$ as a function of $m_0$ for the three values of $M'_{H_3}^\text{eff}$ when all the parameters in the model are allowed to vary consistently with the radiative electroweak symmetry breaking constraints and the experimental constraints including those from the LHC and the Planck experiment. One finds that the parameters compatible with all the constraints clearly prefer values of $m_0$ in the several TeV region.

We will use both $F$ and $F'$ in the analysis for comparison. For proton decay we will use a measure of fine tuning defined by

\[ F_{pd} = \frac{4 \times 10^{33} \text{ yr}}{\tau(p \rightarrow \bar{\nu}K^+)} \text{ yr}. \]  

(7.18)

This measure gives the amount of fine tuning needed in the theory parameters to
Figure 7.2: An exhibition of the partial lifetime for the decay mode $p \rightarrow \nu + K^+$ given by blue squares as a function of $m_0$ over the parameter space of the supergravity model with universal boundary conditions over the allowed ranges consistent with all the experimental constraints. Left panel: The case when $M_{H_3}^{\text{eff}}/M_G = 10$. Middle panel: Same as the left panel except for the case $M_{H_3}^{\text{eff}}/M_G = 25$. Right panel: Same as the left panel except for the case $M_{H_3}^{\text{eff}}/M_G = 50$. The current experimental lower limit for this mode is given by the horizontal black line. The analysis given here is consistent with the Higgs boson mass within a $2\sigma$ range.
enhance the lifetime so that the theoretical prediction is brought just above the current experimental lower limit. If we use the very crude approximation on the proton lifetime, i.e.,

$$\tau(p \to \bar{\nu}K^+) \simeq C \cdot \left(\frac{m_{\tilde{\chi}^\pm}}{m_{\tilde{q}}^2 M_{H_d}^{\text{eff}}}\right)^{-2},$$

(7.19)

and use $m_{\tilde{q}}^2$ or $m_{\chi^\pm}$ as the sensitive parameters, we have

$$F'_{m_{\chi^\pm}} = F'_{m_{\tilde{q}}^2} = 2F_{pd}. \quad (7.20)$$

Thus the two ways of defining the fine tuning differ only by a small numerical factor.

### 7.3.1 High Gluino Model

A SUGRA model that naturally leads to the suppression of proton decay from dimension five operators is the $\tilde{g}$SUGRA model [132]. This model was proposed to accommodate both the high mass of the Higgs boson, i.e., the Higgs mass in the vicinity of around 126 GeV and at the same time have a significant supersymmetric correction to the muon anomalous magnetic moment. This is possible by assuming the following set of soft parameters

$$m_0, A_0, \tan \beta, m_1 = m_2 = m, m_3 >> m, \text{sign}\mu \quad (7.21)$$

Here we assume $m_0, m$ to be small, but the gluino mass is assumed to be large. The largeness of the gluino mass drives the squark masses high because the gluino mass enters in the one loop renormalization group equations for the squark masses. On the other hand the gluino mass does not enter in the renormalization group equations for the slepton masses and hence the slepton masses remain small. Since
the squark masses are large, the loop corrections to the Higgs boson can be large and one can explain the 126 GeV Higgs mass. And since the slepton masses and the electroweak gaugino masses are relatively small one can have a significant corrections to the muon anomalous magnetic moment.

Now the largeness of the squark masses and the smallness of the electroweak gaugino masses can also suppress proton decay from dimension five operators. An analysis of proton lifetime in $\tilde{g}$SUGRA model is given in Fig.(7.3). The relative effect on the proton lifetime by varying $m_3$ is shown in Fig.(7.4) while the effect of varying $m_2/m_1$ is shown in Fig.(7.5). Additionally it would be interesting to investigate the landscape [36, 38] of sparticle mass hierarchies in this class of models.

![Proton Lifetime vs $M_{H_3}/M_{GUT}$](image)

Figure 7.3: An exhibition of the proton lifetime as a function of $M_{H_3}^{\text{eff}}/M_G$. Here $m_0 = 341$ GeV, $m_1 = 429$ GeV, $m_2 = 429$ GeV, $m_3 = 4290$ GeV, $A_0 = 293$ GeV, $\tan \beta = 9.73$. 

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Figure 7.4: An exhibition of the proton lifetime as a function of the $M_{H_3}^{\text{eff}}/M_G$. Here $m_0 = 341$ GeV, $m_1 = 429$ GeV, $m_2 = 429$ GeV, $A_0 = 293$ GeV, $\tan \beta = 9.73$. The lower magenta curve has $m_3 = 3432$ GeV and the upper black curve has $m_3 = 4290$ GeV.

Figure 7.5: An exhibition of the proton lifetime as a function of $M_{H_3}^{\text{eff}}/M_G$ when $m_1$ and $m_2$ are not necessarily equal at the GUT scale. Here for all points on the curve, $m_0 = 341$ GeV, $m_1 = m = 429$ GeV, $m_3 = 4290$ GeV, $A_0 = 293$ GeV, $\tan \beta = 9.73$. The lower blue curve has a $m_2 = 858$ GeV, the middle red curve has a $m_2 = 634.5$ GeV, and the upper black curve has a $m_2 = 429$ GeV.
7.4 Composite Fine Tuning

Having identified individual fine tunings in Sec.7.2 and Sec.7.3, one should now consider how to take multiple fine tunings into account and evaluate their composite effect. Thus one can define a composite fine tuning by the geometric mean of the individual ones, i.e.,

$$\mathcal{F} = \left( \prod_{i=1}^{n} F_i \right)^{\frac{1}{n}}. \quad (7.22)$$

Here our viewpoint is similar to that of [123] (for a related work see [133]). For our case $n = 2$ consisting of the fine tuning in the radiative electroweak symmetry breaking sector and the fine tuning needed to control proton decay from dimension five operators. An analysis of the fine tunings as a function of $m_0$ is given in Fig.(7.6), Fig.(7.7) and Fig.(7.8), where Fig.(7.6) gives the analysis for the case of mSUGRA and Fig.(7.7) gives the analysis for NuSUGRA. In both Fig.(7.6) and Fig.(7.7), the upper panels give the analysis using Eq.(7.9) and the lower panels give the analysis using Eq.(7.11). The red points are the fine tunings for radiative electroweak symmetry breaking. The blue points give the fine tuning needed in the theory prediction of $\tau(p \to \bar{\nu}K^+)$ to bring the lifetime prediction just above the experimental lower limit, and the black points correspond to the composite fine tuning as defined by Eq.(7.22). One finds that typically there is a preference for larger values of $m_0$ for the combined fine tuning including fine tuning from the electroweak sector and the fine tuning needed from proton stability. This result is more explicitly exhibited in Fig.(7.8) which shows fine tuning prefers regions of larger $m_0$ when the electroweak symmetry breaking and proton stability criteria are combined. A similar conclusion was arrived at in the work of [123] which combined the electroweak symmetry breaking, FCNC and CP violation criteria.
Figure 7.6: A display of the fine tuning as defined by Eqs. (7.9-7.22) vs the scalar mass $m_0$ when $M_{\text{eff}}^2/M_G = 50$, for mSUGRA. The upper panel is when $F$, the fine tuning of Eq. (7.9) is used and the lower panel is when $F'$, the fine tuning of Eq. (7.11) is used for the electroweak sector. The red points are the fine tunings values for the REWSB sector, the blue points for $\tau(p \rightarrow \bar{\nu}K^+)$, and the black points are the averages of the red and the blue points.
Figure 7.7: Similar to Fig. (7.6) A display of the fine tuning as defined by Eqs. (7.9-7.22) vs the scalar mass $m_0$ when $M_{E}^{eff}/M_G = 50$, for NuSUGRA. The upper panel is when $F$, the fine tuning of Eq. (7.9) is used and the lower panel is when $F'$, the fine tuning of Eq. (7.11) is used for the electroweak sector. The red points are the fine tunings values for the REWSB sector, the blue points for $\tau (p \rightarrow \bar{\nu} K^+)$, and the black points are the averages of the red and the blue points.
Figure 7.8: A display of the combined fine tuning as a function of $m_0$ is given for mSUGRA (solid line) and for NuSUGRA (dashed line). Here we have taken the average of the left and right panels and drawn smooth curves showing the rapid decrease of the fine tunings as $m_0$ increases.
Chapter 8

LHC and Dark Matter

With data from the search for supersymmetry with 35 pb$^{-1}$ at CMS and ATLAS, we put independent limits on the parameter space of the supergravity unified model with universal boundary conditions at the GUT scale for soft breaking, i.e., the mSUGRA model. We extend this study by examining other regions of the mSUGRA parameter space in $A_0$ and $\tan \beta$. Further, we contrast the reach of CMS and ATLAS with 35 pb$^{-1}$ of data with the indirect constraints, i.e., the constraints from the Higgs boson mass limits, from flavor physics and from the dark matter limits from WMAP. Specifically it is found that a significant part of the parameter space excluded by CMS and ATLAS is essentially already excluded by the indirect constraints and the fertile region of parameter space has yet to be explored. We also emphasize that gluino masses as low as 400 GeV but for squark masses much larger than the gluino mass remain unconstrained and further that much of the hyperbolic branch of radiative electroweak symmetry breaking, with low values of the Higgs mixing parameter $\mu$, is essentially untouched by the recent LHC analysis.
8.1 Interpreting CMS and ATLAS SUSY Results

A candidate model for new physics is the $N = 1$ supergravity grand unified model \[15, 17\] which with universal boundary conditions for soft breaking at the unification scale is the model mSUGRA \[15, 23, 17, 109\] (for reviews see \[134, 135\]) defined by the parameter space $m_0, m_{1/2}, A_0, \tan \beta$ and the sign of $\mu$, as well as $M_G$ and $\alpha_G$ where $M_G$ is the grand unification scale and $\alpha_G$ is the common value of $\alpha_1, \alpha_2, \alpha_3$ ($\alpha_i = g_i^2/(4\pi)$ and $g_i$ is gauge coupling) for the gauge groups $U(1) \times SU(2)_L \times SU(3)_c$ at the unification scale. This model has recently been investigated at the LHC with R parity conservation, and constraints on the model have been set with 35 pb$^{-1}$ of data by the CMS and ATLAS collaborations \[1, 2, 136\]. These works, therefore, produce the first direct constraints on supergravity unified models at the LHC. Indeed the recent results of CMS and ATLAS \[1, 2, 136\] are encouraging as they report to surpass the parameter space probed in previous \[137\] direct searches by LEP and by the Tevatron.

8.1.1 REACH Plots with 35 pb$^{-1}$ of Data

The ATLAS collaboration has released two analyses, one with 1 lepton \[1\] and the other with 0 leptons \[2\] both of which are considered in our analysis. For the 1 lepton analysis we follow the selection requirements that ATLAS reports in \[1\]. The preselection requirements for events are that a jet must have $p_T > 20$ GeV and $|\eta| < 2.5$, electrons must have $p_T > 20$ GeV and $|\eta| < 2.47$ and muons must have $p_T > 20$ GeV and $|\eta| < 2.4$. Further, we veto the “medium” electrons in the electromagnetic calorimeter transition region, $1.37 < |\eta| < 1.52$. An event is considered if it has a single lepton with $p_T > 20$ GeV and its three hardest jets have $p_T > 30$ GeV, with the leading jet having $p_T > 60$ GeV. The distance,
\( \Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} \), between each jet with the lepton must satisfy \( \Delta R (j_i, \ell) > 0.4 \), and events are rejected if the reconstructed missing energy, \( \vec{E}_T \), points in the direction of any of the three leading jets, \( \Delta \phi (j_i, \vec{E}_T) > 0.2 \). Events are then classified into 2 channels, depending on whether the lepton is a muon or an electron. These are then further classified into four regions based on the missing energy and \( m_T \) cuts, where we reconstruct the missing transverse momentum using the selected lepton plus jets with \( p_T > 20 \text{ GeV} \) and \( |\eta| < 4.9 \) following ATLAS analysis, and

\[
 m_T = \sqrt{2p_T (\ell) \vec{E}_T \left(1 - \cos \left(\Delta \phi (\ell, \vec{E}_T)\right)\right)}
\]

is the transverse mass between the lepton and the missing transverse momentum vector. The four regions alluded to above are labeled the “signal region”, the “top region”, the “W region” and the “QCD region”. For the “signal region” events were required to pass the additional cuts of \( m_T > 100 \text{ GeV}, \vec{E}_T > 125 \text{ GeV}, \vec{E}_T > 0.25m_{\text{eff}} \) and \( m_{\text{eff}} > 500 \text{ GeV} \). Here the effective mass, \( m_{\text{eff}} \), is the scalar sum of the missing energy with the \( p_T \)’s of the selected visible objects (in this case the lepton and the 3 jets). The number of events were then compared to the 95% CL upper bounds that ATLAS found (\( N_e < 2.2 \) events and \( N_\mu < 2.5 \) events) \[1\]. The “top region” and “W region” are defined by events with \( 30 \text{ GeV} < \vec{E}_T < 80 \text{ GeV} \) and \( 40 \text{ GeV} < m_T < 80 \text{ GeV} \), where the “top region” requires at least one of the three hardest jets to be \( b \)-tagged and the “W region” requires none of the three hardest jets to be \( b \)-tagged. The “QCD region” was required to have \( m_T, \vec{E}_T < 40 \text{ GeV} \) and was purely data driven. For our analysis events were rejected if they contaminated the three control regions. Using the standard model background from \[138\] we reproduced the ATLAS results.

For the 0 lepton analysis we follow the selection requirements that ATLAS reports in \[2\] where the pre-event selection is the same as for the 1 lepton case except that leptons are identified to have \( p_T > 10 \text{ GeV} \). Here the events are classified into 4 regions “A”, “B”, “C” and “D”; where regions A and B have at
Figure 8.1: (color online) Top Panel: Reach plot with 35 pb$^{-1}$ of integrated luminosity using the ATLAS cuts [1, 2] with different tan $\beta$ and $A_0$: $A_0 = 0$ and tan $\beta = 3$ (dashed line); $A_0 = 0$ and tan $\beta = 45$ (solid green line); $A_0 = 2m_0$ and tan $\beta = 45$ (solid red line). For comparison we give the ATLAS observed limit ($A_0 = 0$ and tan $\beta = 3$) (solid blue line). Bottom Panel: Reach plot with 35 pb$^{-1}$ of integrated luminosity of data using the ATLAS 0 lepton cuts. For comparison we give the ATLAS observed limit (red dashed line).
least 2 jets and regions C and D have at least 3 jets. When referring to different
cuts in these regions we define cuts on the “selected” jets to mean that the bare
minimum number of jets in this region must satisfy the following requirement:
For regions A and B “selected” jets mean that they are the first two hardest jets
and for regions C and D “selected” jets mean that they are the first three hardest
jets. Events are required to have $\not{E}_T > 100$ GeV and the selected jets must each
have $p_T > 40$ GeV with the leading jet $p_T > 120$ GeV. As in the case with
1 lepton, events are rejected if the missing energy points in the direction of any
of the selected jets, $\Delta \phi (j_i, \not{E}_T) > 0.4$, where $i$ is over the selected jets. Region A
requires events to have $\not{E}_T > 0.3 m_{\text{eff}}$ and $m_{\text{eff}} > 500$ GeV and regions C and D
require events to have $\not{E}_T > 0.25 m_{\text{eff}}$ with region C requiring $m_{\text{eff}} > 500$ GeV and
region D requiring $m_{\text{eff}} > 1$ TeV. In this case $m_{\text{eff}}$ is defined in terms of selected
jets, i.e. for regions A and B it is the scalar sum of the first two hardest jets and
for regions C and D it is the scalar sum of the first three hardest jets. For the
analysis here we do not apply the cut for region B, i.e. $m_{T2} > 300$ GeV, since the
models excluded in this region are already excluded in region D \[2\].

Following the framework of the ATLAS Collaboration \[1\] we have carried out a
set of three parameter sweeps in the $m_0 - m_{1/2}$ plane taking $m_{1/2} \leq 500$ GeV and
$m_0 \leq 1$ TeV. Two of the parameter sweeps were a 10 GeV $\times$ 10 GeV grid scan in
the $m_0 - m_{1/2}$ plane having a fixed universal trilinear parameter, $A_0 = 0$, and fixed
tan $\beta$; one set with tan $\beta = 3$ and the other with tan $\beta = 45$. A third parameter
scan was done with $A_0 = 2 m_0$ and tan $\beta = 45$. Throughout the analysis we
take $\mu > 0$ and $m_{\text{pole}}^{\text{top}} = 173.1$ GeV. For the simulation of the mSUGRA models,
renormalization group evolution and computation of the physical masses of the
sparticles was performed using \texttt{SuSpect} \[3\] and we implement both \texttt{MadGraph}
and \texttt{Pythia} for event generation \[139, 140\]. A comparison of our reach to the reach
done by the ATLAS Collaboration is shown in Fig.\text{(8.1)}. 
Figure 8.2: Top left panel: Number of signal events in the $m_0 - m_{1/2}$ plane for the case $A_0 = 0$, $\tan \beta = 3$ using the 1 lepton ATLAS cuts in the $m_0 - m_{1/2}$ plane. The dark areas correspond to number of events greater than 2 with the actual numbers indicated along the vertical line to the right while the white areas are filled with models but have number of events less than 2. Top right panel: Same as the left panel except that the plot is $m_{\tilde{g}}$ (gluino) − $m_{\tilde{q}}$ (squark) mass plane for the lightest squark of the first 2 generations. The square region in the left panel becomes squeezed into the polygon-like region in the physical mass plane in the right panel. One may note that the ATLAS constraints do not rule out a low mass gluino on the scale of order 400 GeV for heavy squarks. Bottom left panel: The same as the top left panel except that the analysis is done using 0 lepton ATLAS cuts. Bottom right panel: Same as the top right panel except that the analysis is done using the 0 lepton ATLAS cuts. The (red) stars correspond to channel D. In channel D we find maximally 51 events over the space scanned after a requirement that the number of events be at least 15 before cuts. However, when only considering models not already excluded by channels A and C, the number of events in channel D is maximally 18.

In Figure 8.2 we plot the number of signal events for electrons in the $m_0 - m_{1/2}$ plane where the reach plot from ATLAS is also exhibited and where the ATLAS reach plot corresponds to the number of observed events and those that have a larger number predicted by the model. For the 1 lepton analysis, we first
present the models excluded by the muon channel, colored by $N_{\text{events}}^{\mu}$ (indicated by squares). Next, we overlay from the remaining models, those that have been excluded by the electron channel, and colored by $N_{\text{events}}^{e}$ (indicated by diamonds). Similarly for the 0 lepton analysis, we begin with models excluded by channel A, colored by $N_{\text{events}}^{A}$ (indicated by squares); overlay models excluded by C (but not A) and colored by $N_{\text{events}}^{C}$ (indicated by diamonds). Next, we overlay models excluded by channel D alone in a single color (stars), as $N_{\text{events}}^{D}$ are not comparable with $N_{\text{events}}^{A}$ or $N_{\text{events}}^{C}$. We also show the number of signal events for electrons in the $m_{\tilde{g}} - m_{\tilde{q}}$ plane. An ATLAS reach curve is also exhibited.

The upper left panel of Fig. (8.2) gives us a more quantitative description of the electron and muon channels in putting constraints on the $m_{0} - m_{1/2}$ parameter space with 35 pb$^{-1}$ of data. As expected the largest number of single $e$ and $\mu$ events arise at low mass scales, i.e., for low values of $m_{0}$ and of $m_{1/2}$ and the number of signal events decrease and we approach the boundary after which they fall below 2 for the 1 lepton ATLAS analysis. It is also instructive to examine the signal events in the gluino-squark mass plane where the squark mass corresponds to the average first two generation squark mass. This is done in the upper right panel of Fig. (8.2). Here the polygon shape of the region is a simple mapping of the allowed parameter in the $m_{0} - m_{1/2}$ plane of the upper left panel. The plot is useful as it directly correlates squark and gluino model points that are either excluded or allowed by the 1 lepton ATLAS analysis. The 0 lepton analysis of the lower panels in Fig. (8.2) is very similar to the analysis of the upper panels except for different array of cuts. There is a general consistency in the analysis of the 1 lepton and the 0 lepton analysis, although the 0 lepton cuts appear more constraining as they appear to exclude a somewhat larger region of the parameter space. Together the analysis of the upper and lower panels of Fig. (8.2) gives us a more analytical understanding of the relative strengths of the 1 lepton and 0
lepton cuts.

8.1.2 Implications of Constraints

In the analysis of the reach plots experimental constraints were not imposed beyond those that arise from the ATLAS analyses. Next we include these constraints and in our analysis we will consider the larger parameter space when all four parameters $m_0$, $m_{1/2}$, $A_0$, $\tan \beta$ are varied. In doing so, we apply various constraints from searches on the sparticle mass limits, B-physics and from $g_\mu - 2$. Next we explore the constraint from upper bound on the relic density from WMAP only, and then with combination of all of the above. These indirect constraints were calculated using MicrOmegas, with the Standard Model contribution in the $\mathcal{B}r (b \to s\gamma)$ corrected using the NNLO analysis of Misiak et al. We now describe this more general analysis.

In the upper left panel of Fig.(8.3) we apply the following “collider/flavor constraints” $m_h > 93.5$ GeV, $m_{\tilde{\tau}_1} > 81.9$ GeV, $m_{\tilde{\chi}_1^\pm} > 103.5$ GeV, and $m_{\tilde{t}_1} > 100$ GeV, along with $(-11.4 \times 10^{-10}) \leq \delta (g_\mu - 2) \leq (9.4 \times 10^{-9})$, see [47], $\mathcal{B}r (B_s \to \mu^+ \mu^-) \leq 4.7 \times 10^{-8}$ (90% C.L.) [143], and $(2.77 \times 10^{-4}) \leq \mathcal{B}r (b \to s\gamma) \leq (4.27 \times 10^{-4})$ [85].

These collider/flavor constraints by themselves have an effect, but the effect is quite small in terms of reducing the density of models that are already constrained by the ATLAS results.

We note that our scans of the parameter are very dense with $10^6$ models after EWSB alone. In the $m_0 - m_{1/2}$ plane the collider/flavor cuts eliminate 12% of the models. However because $A_0$ and $\tan \beta$ are not fixed to specific values, but are allowed to run over their full natural ranges, a model point which is eliminated for say, large $\tan \beta$ by $b \to s\gamma$ or $B_s \to \mu^+ \mu^-$ at a specific point in the $m_0 - m_{1/2}$ plane can correspond to a model point with a smaller value of $\tan \beta$ for the same...
Figure 8.3: Upper left panel: An exhibition of the allowed models indicated by grey (dark) dots in the $m_0 - m_{1/2}$ plane when only flavor and collider constraints are imposed. The region excluded by ATLAS (as well as CMS) lies below the thick black curve in the left hand corner. Upper right panel: same as the left upper panel except that only an upper bound on relic density of $\Omega h^2 \leq 0.14$ is imposed. Lower left panel: Same as the upper left panel except that the relic density constraint as in the upper right panel is also applied. This panel exhibits that most of the parameter space excluded by ATLAS is already excluded by the collider/flavor and relic density constraints. The dark region below the ATLAS curve is the extra region excluded by ATLAS which was not previously excluded by the indirect constraints. Lower right panel: The analysis of this figure is similar to the lower left panel except that models with $|\mu| < 500$ GeV are exhibited in green.

$(m_0, m_{1/2})$ which is not eliminated. Thus the $m_0 - m_{1/2}$ plane appears densely filled. This is contrary to what one would observe for fixed values of $(A_0, \tan \beta)$. For example, for $(A_0, \tan \beta) = (0, 45)$ the $b \to s\gamma$ constraint would remove models at large $m_0$ up to close to 2 TeV and $m_{1/2}$ up to about 750 GeV. As another example, for $(A_0, \tan \beta) = (0, 3)$ (the space looked at by ATLAS, and in the previous section) a strict limit of $m_h < 102$ GeV for light CP even Higgs removes all model points
below the ATLAS limits. However because one is varying \((A_0, \tan \beta)\) the area below the ATLAS limit is filled in this case.

Continuing on we next consider the “cosmological constraint” in the upper right panel of Fig. (8.3) where we apply only an upper bound on the relic density of the thermally produced neutralino dark matter of \(\Omega h^2 \leq 0.14\) \cite{144,147}. The WMAP upper bound constraint removes 96.5% of the models alone, thus this cosmological constraint is very severe eliminating a large fraction of models, but again the ATLAS constraints remain quite strong.

Next we consider the “combined collider/flavor and cosmological constraints” and find that together these constraints are generally much more severe than the ATLAS constraints. This is shown in the lower left panel of Fig. (8.3). Here models that were separately allowed by previously known collider/flavor constraints, and models that were separately allowed by just the upper bound from WMAP, are now eliminated under the imposition of the combined constraints. There is, however, a new region that ATLAS appears to exclude above and beyond what the indirect constraints exclude and this region is a region for low \(m_0\) and for \(m_{1/2}\) around 350 GeV. Thus it would require a larger integrated luminosity to move past the barren region, which is above the ATLAS bound, to get into the fertile region of the parameter space, where the fertile region is the area above the white patch in the lower panel of Fig. (8.3).

Finally in lower right panel of Fig. (8.3) we show the value of \(\mu\) (at the electroweak symmetry breaking scale) in the \(m_{1/2} - m_0\) plane where \(\mu\) is the Higgsino mass parameter that enters in the Higgs bilinear term in the superpotential. The analysis is given under the “combined constraints” discussed in the lower left panel of Fig. (8.3). We note that essentially all of the natural region of the parameter space corresponding to small \(\mu\), most of which lies close to the hyperbolic branch (Focus point) (HB/FP) \cite{53,55} of radiative breaking of the electroweak symmetry
or near the vicinity of the light CP even Higgs pole region [148] remains untouched by the CMS and LHC exclusion limits as illustrated in the lower right panel of Fig. (8.3) and remains to be explored. Further, as pointed out in Ref. (1) of [149–151], low mass gluinos as low as even 420 GeV in mSUGRA are allowed for the region for large $m_0$ where relic density can be satisfied on the light CP even Higgs pole [148]. This can be seen from Fig. (8.3) as the gluino and squark masses are exhibited in the plots. Along the Higgs pole region, electroweak symmetry breaking can also be natural, i.e., one has a small $\mu$. It is also seen that this region is not constrained by CMS and ATLAS since their limits taper off at large $m_0$ as $m_{\text{squark}}$ gets heavy and the jets from squark production are depleted (see Ref. (1) of [53–55]).

8.2 LHC Data Implication for mSUGRA

With applicable experimental constraints defined in Chapter 5 one should now be ready to discuss the effect of those so-called general constraints on both mSUGRA and NuSUGRA. Here first let us look into the case in mSUGRA, and the case in NuSUGRA will be addressed in Chapter 8.3.

8.2.1 Constraints of LHC-7 Data on HB

Now let us investigate the implications of the LHC data [1, 2, 152–154] on the focal regions constituted of the Focal Point, Focal Curves and Focal Surfaces. To this end we first generate mSUGRA parameter points using a uniformly distributed
random scan over the soft parameters with

\[ m_0 < 4 \text{ TeV}, \]
\[ m_{1/2} < 2 \text{ TeV}, \]
\[ A_0/m_0 \in (-10, 10), \]
\[ \tan \beta \in (1, 60). \] 

(8.1)

After the constraint of REWSB roughly 22 million mSUGRA parameter points are collected. These are then subject to experimental constraints which include the LEP and Tevatron limits on the Higgs mass and on the sparticle masses as discussed in [155, 156] and \( Br(B_s \to \mu^+\mu^-) \leq 1.1 \times 10^{-8} \) [157]. These constraints will be referred to as the general constraints. In imposing these constraints we use MicrOMEGAs [66] for the computation of the relic density and SUSPECT for the computation of the sparticle mass spectrum and \( \mu \) at the scale at which electroweak symmetry breaks, \( Q_{\text{EWSB}} \). A more statistically rigorous procedure for the implementation of the constraints would be to use \( \chi^2 \) or maximum likelihoods, but for the purpose of this analysis it is unnecessary.

CMS and ATLAS have reported results for supersymmetry searches [1, 2, 152–154] based on about 1 fb\(^{-1}\) of data. The implications of these results (as well as dark matter results) have been considered for the parameter space of SUSY models in a number of works [149, 155, 158, 160] and some discussion on the collider implications on naturalness can be found in [161–164]. Here we use the constraint arising from the recent ATLAS 1 fb\(^{-1}\) search [154] and the CMS 1 fb\(^{-1}\) search [152] to explore their implications on the focal region. The implications of the LHC data for the Ellipsoidal Branch and for the Hyperbolic Branch are exhibited in Fig. 8.4. The top left panel gives the parameter space in EB and here one finds that most of the model points being constrained by LHC-7 lie in the low \( m_0 \) region. The
top right panel gives the corresponding analysis for HB/FP and HB/FC. In the
analysis here we have assumed that $m_{1/2}/m_0 \leq 0.1$ for HB/FC1 and $A_0/m_0 \leq 0.1$
for HB/FC2. The middle left panel exhibits the same set of parameter points on
HB/FP and HB/FC as the top left panel except that the regions are now labeled
according to the sparticle landscape picture by the next to lightest particle
(NLP) beyond the Standard Model in the mass hierarchy (note that this includes
all of the sparticles and Higgs sector particles, but omits the Standard Model-like
$h^0$). Here one finds that most of the region being constrained by the LHC-7 data
is the high $m_0$ region. The middle right panel exhibits the Focal Point region,
HB/FP. Here one finds that the Focal Point region HB/FP is highly depleted and
is further constrained by the LHC-7 data. The bottom panels of Fig. 8.4 show
the parameter points on HB/FS which is the entire HB region except the HB/FP
region. The left panel displays the parameter points where the NLP is either a $\tilde{\chi}^{\pm}_1$
or $\tilde{\tau}_1$, and the right hand panel shows the parameter points where the NLP is $\tilde{t}, A$
or $H$. Thus the analysis of Fig. 8.4 shows that the HB/FP is almost empty and
most of the parameter space remaining on HB lies in the region of Focal Curves
or Focal Surfaces, i.e., it lies on HB/FC and HB/FS.

8.2.2 LHC Signals on HB/FC1

We discuss now an important phenomenon related to HB/FC1, which arises from
the constraint that $m_{1/2}$ and $\mu$ are fixed even though $A_0$ and $m_0$ get large.
This can lead to observable leptonic signatures, specifically the trileptonic signa-
ture [165, 166], even when $m_0$ lies in the several TeV region (For a recent work
on the trileptonic signal see [167]). The reason for this is rather obvious, in that
the chargino and the neutralino masses are held relatively constant along the Focal
Curve HB/FC1. Thus the production cross-section for the charginos and neutralinos
will be essentially independent of $m_0$. We are specifically interested in the

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Figure 8.4: Top Left: The mSUGRA parameter points passing the general constraints in the $m_0-m_{1/2}$ plane that are a part of the Ellipsoidal Branch, labeled by the NLP where in the definition of EB we have excluded the HB/FP region. Top Right: The mSUGRA parameter points in the $m_0-m_{1/2}$ plane passing the general constraints that are a part of HB/FC1, HB/FC2, or HB/FP, labeled as such. Bottom Left: An exhibition of the mSUGRA parameter points passing general constraints that also lie on HB/FC1 or HB/FC2, labeled by the NLP. Bottom Right: The mSUGRA parameter points passing the general constraints that arise from the Focal Point (HB/FP) region.

production cross-section of the light chargino $\tilde{\chi}_1^\pm$ and the second lightest neutralino $\tilde{\chi}_2^0$, i.e., $\sigma_{\tilde{\chi}_1^\pm\tilde{\chi}_2^0}$ which can lead to a trileptonic signal from the decay of $\tilde{\chi}_1^\pm,\tilde{\chi}_2^0$ so that $\tilde{\chi}_1^\pm \to l^\pm + \nu_l + \tilde{\chi}_1^0$ and $\tilde{\chi}_2^0 \to l^+l^-\tilde{\chi}_1^0$ (important contributions can also arise from the production of $\tilde{\chi}_1^\pm\tilde{\chi}_i^0$ (i=3,4) depending on the part of the parameter space one is in).

The chargino and neutralino final state can arise at tree level from two main processes in $pp$ collisions. Thus, for example, $\tilde{\chi}_a^+\tilde{\chi}_i^0$ can arise from the s-channel fusion diagram $u+d \to W^{*}\to \tilde{\chi}_a^+\tilde{\chi}_i^0$ and from the t-channel exchange diagram.
of a $\tilde{d}_L$ squark. The latter diagram is suppressed when $m_0$ is large so that the main production cross-section proceeds via the s-channel off-shell $W^\pm$ production \cite{166}. Thus the $\tilde{\chi}_1^{\pm}\tilde{\chi}_1^0$ production cross-section is expected to be independent of $m_0$ for large $m_0$.

The constancy of $\sigma_{\tilde{\chi}_1^{\pm}\tilde{\chi}_1^0}/\sigma_{\text{total}}$ is exhibited in Fig. 8.6 for HB/FC1 defined by $m_{1/2} = 0.35$ TeV, $\tan \beta = 45$ and $\mu = (0.20 \pm 0.01)$ TeV. The branching ratio into trileptons is also computed.

In the analysis we use SUSY-HIT \cite{168} for the computation of decays, PYTHIA \cite{140} for event generation, and PGS \cite{169} for detector simulation.

For the case of models exhibited in Fig. 8.6 the $\tilde{\chi}_1^{\pm}\tilde{\chi}_1^0$ production cross-section is $(164.3 \pm 9.97)$ fb and the $\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0$ production cross-section is $(112.1 \pm 8.53)$ fb, which leads to roughly 50 raw trilepton events at 10 fb$^{-1}$ where we have included $\tau$s in the definition of leptons. The number of events will be reduced when off-line cuts are imposed and a more detailed analysis would require further knowledge of the cuts used in the experimental multileptonic search at that luminosity. Of course a much larger number of events is expected at higher $\sqrt{s} = 10$ TeV, or $\sqrt{s} = 14$ TeV at the same luminosity. Similarly, the $\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0$ and $\tilde{\chi}_1^{\pm}\tilde{\chi}_3^0$ production states can decay hadronically. For the hadronic analysis we use the cuts as outlined in Ref. 1 of \cite{154} by ATLAS and find that our effective cross-sections are $(5.2 \pm 0.15)$ fb, $(0.7 \pm 0.16)$ fb, $(1.6 \pm 0.33)$ fb, $(0.6 \pm 0.18)$ fb and $(0.5 \pm 0.15)$ fb which can be compared to the reported 95% C.L. upper bounds at 1.04 fb$^{-1}$ of 22 fb, 25 fb, 429 fb, 27 fb and 17 fb, respectively. Typically these points produce hard jet signatures, but with low jet multiplicity. Thus the hadronic signals on HB/FC1 may become visible if a luminosity in excess of 20 fb$^{-1}$ can be achieved at LHC-7. Another possible channel for discovery would be a combination of jets and leptons, but such an analysis is outside the scope of the current work.
8.3 LHC-7 Data Implication for NuSUGRA

In Fig.(8.9) we display the nature of radiative breaking of the electroweak symmetry for all the model points within the allowed ranges of the parameter space for NuSUGRA. The points in red are those that lie on HB, the points in blue lie on EB and the points in green lie in the FP region as defined by Eq.(6.19). As in the mSUGRA case here too one finds that most of the parameter points lie on HB and only a small fraction lie on EB and FP. In Fig.(6) we give an analysis of the sensitivity of the proton lifetime to the Higgs boson mass for NuSUGRA. As in the mSUGRA case here too one finds that the proton lifetime is very sensitive to the Higgs boson mass with the proton lifetime changing by over two orders of magnitude with a shift in the Higgs boson mass in the range of 5-10 GeV. In Fig.(7.7) an analysis of the proton lifetime for the mode $p \rightarrow \bar{\nu}K^+$ is given over the allowed parameter space of NuSUGRA within the assumed limits. The figure shows the dispersion in the proton lifetime as all the parameter points are varied but does show the general trend that $p \rightarrow \bar{\nu}K^+$ lifetime increases with a larger SUSY scale.
Figure 8.5: Top Panel: A display of the mSUGRA parameter points containing the $\tilde{\chi}_1^\pm$ and the $\tilde{\tau}_1$ NLPs passing the general constraints and including the parameters in HB/FS, i.e., the entire Hyperbolic Branch except for HB/FP. Bottom Panel: Same as bottom top except the NLPs displayed are $\tilde{t}_1$, $A$, $H$. 
Figure 8.6: Fraction of the total cross-section that is made up by $\tilde{\chi}^\pm_1 \tilde{\chi}^0_2$ production as a function of $m_0$ at $\sqrt{s} = 7$ TeV. The analysis shows that the production cross-section is rather insensitive to $m_0$ which implies the signatures from HB/FC1 such as the trileptonic signal could be visible even in the asymptotic region when $m_0$ and $A_0$ are very large.
Figure 8.7: Top Panel: A display for the mSUGRA model points in the $m_0 - m_{1/2}$ plane that pass the general constraints as discussed in the text. Bottom Panel: A display of the spin-independent neutralino-proton cross-section $\sigma_{\tilde{\chi}_1^0, p}^{SI}$ for the parameter points in the top left panel.
Figure 8.8: Top Panel: A display of the spin-independent neutralino-proton cross-section, $\sigma_{\chi_1^0 p}^{SI}$, for the EB region. Bottom Panel: Same as the top panel except that the analysis is for HB which contains the Focal Point as well as Focal Curves and Focal Surfaces.
Figure 8.9: Exhibition of HB (red), EB (blue), FP (green) parameter points for NuSUGRA using the inputs given in Chapter 6.2. All parameter points satisfy the general constraints along with a $2\sigma$ constraint on the Higgs boson mass. As in the supergravity unified models with universal boundary conditions here too one finds that most of the allowed parameter space lies on the HB branch while EB and FP regions are highly depleted.
Chapter 9

Conclusion

In this thesis we have investigated the implications of the recent experimental data from the Large Hadron Collider on the theory space of supersymmetry. We have also investigated the implications of the Higgs boson mass measurement at $\sim 126$ GeV on the supersymmetry parameter space and for the allowed branches of radiative breaking of the electroweak symmetry. Another topic investigated includes implications of the LHC data and the Higgs boson for proton stability in unified models of particle interactions in supersymmetry. We summarize below the analysis and the conclusions in each of the topics mentioned above.

Just after the CMS and ATLAS released their first results, we carried out an analysis to determine the constraints the data would place on the parameter space of supersymmetric models and specifically on the parameter space of supergravity grand unified models. Thus the first results from the CMS and ATLAS analyses on the search for supersymmetry were impressive in that with only $35 \text{ pb}^{-1}$ of data their reach plots already exceeded those from CDF and DØ experiments at the Tevatron. Both CMS and ATLAS had given reach plots in the $m_0 - m_{1/2}$ plane for the case $A_0 = 0, \tan \beta = 3$. Immediate to the announcement of the CMS and ATLAS results we embarked on a project to investigate how the exclusion
plots will be modified for the cases when $A_0 \neq 0$ and values of $\tan \beta$ different from $\tan \beta = 3$ were considered. In our analysis we adopted the ATLAS cuts since they were the more stringent ones. In our analysis we found consistency with the 1 lepton and 0 lepton results of ATLAS for the case analyzed by ATLAS, i.e., $A_0 = 0, \tan \beta = 3$. Beyond that we investigated reach plots for other values of $A_0, \tan \beta$, i.e., $A_0 = 0, \tan \beta = 45$ and $A_0 = 2m_0, \tan \beta = 45$. Another interesting question explored in our work was a relative study of the constraints on the $m_0 - m_{1/2}$ parameter space by the CMS and ATLAS experiments vs the constraints that arise from Higgs mass limits, flavor physics, and from the dark matter constraints from WMAP. It was found that the current CMS and ATLAS limits were consistent with such constraints. Specifically a significant part of the parameter space excluded by the CMS and ATLAS 35 pb$^{-1}$ data was already excluded by the indirect constraints. In our works we emphasized that low gluino masses (even as low as 400 GeV) remain unconstrained in mSUGRA, and this conclusion holds generically for other high scale models of soft breaking, for the case when the squark masses are significantly larger than the gluino mass.

Another topic discussed in this thesis concerns the implications of the LHC data for radiative breaking of the electroweak symmetry. Thus it is known that radiative breaking of the electroweak symmetry consists of two main branches: the ellipsoidal branch and the hyperbolic branch. Further, it is shown that the Hyperbolic Branch of radiative electroweak symmetry breaking consists of several regions of the parameter space where $\mu$ is small. These regions consist of the Focal Points, Focal Curves and Focal Surfaces. The Focal Point (HB/FP) region is where $m_0$ can get large with fixed $m_{1/2}$ and $A_0$ while $\mu$ remains small. A small $\mu$ can also be achieved on Focal Curves and on Focal Surfaces. There are two possible Focal Curves: HB/FC1 and HB/FC2 such that on HB/FC1, $m_0$ and $A_0$
both may get large, while \( m_{1/2} \) and \( \mu \) remain fixed, while on HB/FC2, \( m_0 \) and \( m_{1/2} \) may get large while \( A_0 \) remains fixed. These two general categories can be unified by the parameter \( \alpha \) defining the Focal Curve mode HB/FC\(_\alpha\). An explicit illustration of these regions is given for mSUGRA in this thesis and it is shown that the HB/FP region is significantly depleted when all the experimental constrains along with the current constraints from the LHC-7 data are applied. Thus the remaining parameter points in this region lie on Focal Curves (or more generally, on Focal Surfaces). Thus if \( m_0 \) is indeed large while the gaugino masses are light, the LHC would turn into a gaugino factory.

Finally in this thesis we have investigated the implications of the Higgs boson mass of size \( \sim 126 \) GeV for proton stability. Thus typically in supersymmetric theories the Higgs boson mass at the tree level falls below the \( Z \) boson mass and lifting it above the \( Z \) boson mass requires loop corrections. Specifically, for a Higgs mass of \( \sim 126 \) GeV, one needs a high SUSY scale, i.e., in the several TeV region. Now proton decay from baryon and lepton number violating dimension five operators is very sensitive to the sparticle spectrum and since the SUSY scale is constrained by the Higgs boson mass, a strong correlation exists between the proton lifetime from dimension five operators and the Higgs boson mass. Specifically a few GeV upwards shift in the Higgs boson mass can result in orders of magnitude suppression of the proton decay from baryon and lepton number violating dimension five operators and a corresponding enhancement of the proton lifetime. We carried out an analysis of this correlation in various settings with and without non-universalities. As expected one finds that a heavier Higgs boson mass tends to produce a longer lifetime for the proton decay mode \( p \to \bar{\nu}K^+ \). This alleviates the problem of supersymmetric models where the proton lifetime from baryon and lepton number violating dimension five operators tends to be larger than the ex-
Experimental limits, and requires additional measures for its suppression such as a cancellation mechanism. However, one finds that there is a significant amount of the parameter space which lies close to the upper limit and thus $p \rightarrow \bar{\nu}K^+$ decay mode is the most likely candidate to be discovered first in the next generation of proton decay experiments.
Bibliography


