Carbon Nanotube Powered Regenerative Braking System

A Thesis Presented

by

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ABSTRACT

Regenerative braking systems capture kinetic energy and store it for later use, for example as electrochemical energy in a battery or as kinetic energy in a flywheel. There are currently no commercially available regenerative braking systems that convert kinetic energy into elastic strain energy for storage. Although elastically-deformed springs can permit rapid energy storage during braking and rapid energy release during acceleration, springs made of conventional materials do not have high enough energy densities for practical regenerative braking systems in vehicles. In contrast, carbon nanotubes have incredibly high tensile strength and maximum strain, so that they offer the potential for high energy density, high power density energy storage. CNTs assemblies can be twisted, straining their constituent CNTs and storing energy. This thesis uses that concept to power a regenerative braking system with energy storage in an assembly of CNTs (a CNT tape).

Preliminary experiments were performed on the CNT tape to determine its mechanical properties. Its typical specific stiffness was found to be 6.25 N/tex and its typical maximum elastic strain was found to be 2.4%. When loaded in torsion, single-ply CNT tape was found to be capable of releasing a maximum of 4.3 kJ/kg of strain energy and reaching an efficiency between stored and released energy of 60%. When similarly loaded in torsion, a 2-ply CNT tape released a maximum energy density of 8 kJ/kg, with a maximum efficiency of 50%. The data found in these preliminary experiments were used to benchmark mathematical models to predict the effect on performance of increasing the length or radius of the CNT tape.

Two table-top models systems were created to demonstrate the capabilities of CNT tape as an energy storage and actuating material for regenerative braking. The first model simply allowed for the conversation of energy between kinetic energy of the wheel and strain energy of a twisted CNT tape. With this model, work of up to 10.7 kJ/kg on a 2-ply CNT energy storage element was converted to up to
1.8 kJ/kg of rotational kinetic energy of the wheel. The second model added a regenerative braking component. A gear box permits the CNT tape’s twisting and untwisting to capture energy from rotation in one direction and to deliver it back to rotation in the same direction, a necessity for the braking system. A ratchet and pawl also allow for energy to be stored without being released.

The demonstrations of these mechanisms show that CNT tape is a viable option for energy storage in regenerative braking. With further modifications to this mechanism and improvements to CNT assemblies, a similar CNT powered system could be adapted to a full sized motor vehicle.
Acknowledgements

I would like to thank Dr. Carol Livermore, who was always available for guidance and support with this project. She introduced me to the world of carbon nanotubes and challenged me to become a better researcher and engineer. Without her, this project would not have been possible.

I would also like to thank Sanwei Liu for supporting me in the lab and being available at all hours to help with testing, design issues or any other problems I may have had.

Thank you to Jon Doughty in the Northeastern machine shop for creating parts necessary to create my design. And thank you to Nanocomp Technologies for providing the CNT.
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Chapter 1

Introduction

According to a recent study, the average American spends over $20,000 on gasoline over the lifetime of a car [1]. This cost is often more than the cost of the vehicle itself. These gasoline costs are so high in part because vehicles with traditional braking systems rely solely on gasoline to accelerate after braking or stopping. Traditional braking systems are the least efficient mechanisms ever designed. 100% of the kinetic energy in a moving vehicle is converted into heat through friction in the brakes and is essentially lost. In a world where so much emphasis is put on ‘being green’ and conserving energy, why is so much energy wasted through this inefficient mechanism?

Over the past decade, electric cars have grown immensely in popularity, partially due to their ability to regenerate some of their kinetic energy while braking. Energy may also be stored kinetically, in a flywheel. There have also been a handful of attempts to create regenerative braking systems using elastic energy storage, but none have been commercially successful, yet.
At the same time, carbon nanotubes have been at the forefront of the materials science world. Studies have demonstrated their incredible mechanical properties. Due to their very high tensile strength and elasticity, one potential application for them is energy storage.

This project uses the energy storage capabilities of CNTs to drive a regenerative braking system. With small, table-top models, the ability to convert kinetic energy of a rotating wheel into strain energy by twisting an assembly of CNTs called a CNT tape (and vice versa) is demonstrated.

1.1 Thesis Outline

In Chapter 2, background information is given about how traditional braking systems developed. Then, previous studies into the properties of CNTs are described.

In Chapter 3, preliminary experiments performed on CNT tape are presented, including the measured mechanical properties of the material.

In Chapter 4, the mechanical properties of CNT tape were used to predict its energy storage capabilities. Mathematical models were generated to predict the behavior of a twisted CNT tape. The effect of increasing the length and radius of the tape were also explored.

In Chapter 5, designs for a regenerative braking model are given. The first model simply depicts a method of converting rotational kinetic energy into strain energy and vice versa. The second model adds a gearbox and ratchet and pawl to the first model and shows how a regenerative braking system could work using CNT assemblies as the energy storage element.

In Chapter 6, the results of testing the models are described.

In Chapter 7, a summary of the thesis is presented and conclusions about the viability of the regenerative braking system are drawn.
Chapter 2

Background

2.1 Regenerative Braking

The goal of this thesis is to design a regenerative braking system. The first step in doing so is to explore how braking systems developed into what they are currently and what attempts have already been made to solve the problem. The wheel was invented sometime in the Neolithic period, over 6,500 years ago [2]. The wheel led to the invention of the plow and cart and in no time, the world was moving many times faster. Of course, the faster one moves, the quicker one needs to stop, so brakes were developed. During the Roman Empire, one of the earliest forms of braking systems was used: a wooden block [2]. By rubbing a slab of wood against the wheel, friction was created and the wheel slowed down.

For the next 2,000 years very little changed in the world of brakes. In 1886, Karl Benz invented the gasoline powered automobile. The single-cylinder four-stroke engine had an engine output of only 0.75 hp [3]. The brakes on the first automobile utilized the same wood block and lever mechanism seen before, only instead of a wood block, leather was used as it wore out less quickly. The Benz Patent-
Motorwagen would look like a child’s toy compared to today’s motor vehicles, but the braking system would look remarkably similar. Over the past century, there have been a handful of improvements to braking systems, but the driving concept remains largely unchanged: slow down a wheel by rubbing another surface against it.

The first major development to automotive brakes was the drum brake. Developed in the early 1900s, the drum brake uses a set of pads that rub against the drum, slowing the car through friction. Drum brakes use pistons to push the brake shoes outward, forcing the pads to make contact with the drum [4]. As the pads wear down, an adjusting mechanism pushes the brake shoes further away, allowing the pads to still reach the drum. The main disadvantage of the drum brake design is a failure mode known as fading. When braking from high speeds, drum brakes are prone to overheating and losing their effectiveness [4].

In the 1950s, the disc brake gained popularity. In essence, disc brakes work the same as drum brakes: push a pad against a rotating wheel to slow it down with friction [4]. The mechanism of the disc brake provides some advantages to the drum brake. A disc is able to dissipate heat more quickly than a drum, so fading is much less of a concern with disc brakes. Disc brakes are, however, more expensive than drum brakes and require hydraulic power [4]. Both types of brakes are still used today. Drum brakes are often used as emergency brakes on rear wheels and disc brakes are used for normal braking due to their better performance.

In terms of energy efficiency, drum and disc brakes are equal. They both convert all kinetic energy of the moving vehicle into heat, essentially wasting the energy. Regenerative brakes are the answer to this issue. Regenerative brakes convert kinetic energy into another form of useful energy (e.g. electric or mechanical). The converted energy can later be used to accelerate the vehicle.
Vehicles driven by electric motors are able to convert kinetic energy into electric energy by allowing their motors to run in reverse while braking. This creates drag on the wheels, slowing the car down while recharging the car’s battery. Typically, these systems can regenerate up to 25% of the kinetic energy removed from the moving vehicle [5].

The efficiency of regenerative brakes drops off at low and high speeds. At low speeds, not enough current is produced by the electric motor to slow the car. At higher rates of deceleration, regenerative brakes can account for a maximum of 75% the vehicle’s slowing capability [6].

The power supply of a regenerative braking system has both a maximum charge and maximum discharge rate. The discharge rate of an electric vehicle’s battery does not depend on whether its energy was regenerated, so its value does not hamper the effectiveness of the braking mechanism. The charge rate, however, can have an effect. The Chevy Volt and Nissan leaf, for example, have a maximum charging rate of 3.3 kW [7]. During rapid braking, the generator may rotate at speeds producing more than 3.3 kW of power, but the battery is limited to that charging rate.

The general consensus around the amount of energy this type of regenerative brake puts back into the system is, “not a lot—but it helps” [8]. Regenerative braking eliminates most of the losses due to friction in traditional brakes, but introduces other losses in converting between different forms of energy.

Mazda recently developed a “supercapacitor-type regenerative braking” system. Capacitors have higher power densities than batteries, but lower energy densities [9]. Therefore, this system could recharge the car’s battery at higher rates, but store less energy per unit mass.

There have also been a handful of attempts to convert kinetic energy into mechanical energy. This type of system is referred to as KERS (kinetic energy recovery system) and often involves
transferring the vehicle’s kinetic energy to a flywheel to be used later. KERS was introduced to Formula One racing in 2009, and automakers are attempting to develop it for consumer vehicles in the next few years [10].

### 2.2 Carbon Nanotubes

Carbon nanotubes (CNTs) are made of carbon atoms bonded in a sheet-like structure, which is rolled into a cylinder and capped. Individual CNTs are very small, with diameters on the nanometer scale. They were discovered in the late 20th century, and extensive testing has been done on their mechanical and conductive properties.

In one study, individual CNTs were demonstrated to have tensile strengths between 13-52 GPa and a Young’s Modulus of 320-1470 GPa [11]. In another study, they were show to also be able to achieve strains up to 13.7% before failure [12].

Applications for CNTs include structural supports, thermal lining on airplane components and lightweight electrical conductors. Most current applications involve the use of bulk assemblies of CNTs (many CNTs that are loosely held together). These materials are not able to achieve the same exceptional properties as single CNTs or ideal assemblies of CNTs, but they can nevertheless achieve properties that far exceed most other materials. The CNT assembly used in this project is tape-like structure that is grown by a gas phase synthesis process that incorporates a free-flowing catalyst. In its original form, the CNT tape does not include a twist as would be observed in a CNT yarn.

Dr. Weibang Lu wrote a review of current studies on the mechanical properties of CNT fibers. The tensile strength of CNT fibers was in the range of 0.23 GPa - 9.0 GPa. The Young’s Modulus was in the range of 70 GPa – 350 GPa [13]. For comparison, the Young’s Modulus of titanium alloys are typically within the range of 105-120 GPa [14]. The review also found that CNT fibers can also reach a
failure strain of at least 4.3%. [13]. Due to their high tensile strength and high maximum strain, CNTs make ideal candidates for mechanical energy storage via stretching or twisting.

Dr. Hill studied the mechanical properties of twisted CNT yarn. These specimens of CNT yarn were found to have a mean strength of 0.97 GPa, mean stiffness of 55.5 GPa, and elastic strain of 3.4%. The mean energy density of CNT is $4.9 \times 10^3$ kJ/m$^3$ or 4.2 kJ/kg. Dr. Hill concluded that, “in their current forms, macroscopic assemblies of CNTs into the fibers and yarn studied in this work are not yet ready for implementation as springs in practical energy storage applications” [15]. However, as CNT assemblies improve, this conclusion would likely change.

While energy storage with CNTs has been explored, using this property for regenerative braking has not yet been attempted. This project uses CNT assemblies in their current state to show how effective they can be when used for regenerative braking. With better CNT assemblies, the regenerative braking device designed in this project will be even more efficient.
Chapter 3

Characterization of the CNT energy storage material

The primary function of a regenerative braking system is to reduce the work that needs to be done by the primary fuel source, which in most traditional vehicles is gasoline. In order to do so, the system must not only be able to collect and store energy with acceptable efficiency, it must also release enough energy to be useful (i.e. energy released after losses such as heat or friction). Viscoelastic materials, such as cartilage, can often withstand significant applied loads by undergoing a deformation. Due to hysteresis and creep, however, the material may return to its original geometry without releasing an equal and opposite force. Such a material would be a poor candidate for a regenerative energy application.

For this project, carbon nanotube was chosen as the energy storage material due to its extremely high tensile strength and extendibility, as described in the Chapter 2. Ideal single-walled carbon nanotubes can have a Young’s Modulus up to 1 TPa and over 15% failure strain, with greater than 13% strain having been measured in practice [16]. However, these are ideal values and are not realistic for the CNT tape used for this application. Large CNT assemblies such as yarn and tape fall short of ideal CNT properties. These assemblies contain disordered individual CNTs, which affects how loads
are distributed. Repetitive loading also permanently reorganizes CNTs in the assembly, creating slack upon subsequent reloading. To prove that CNT tape has can store and release enough energy to be a viable option in regenerative braking, mathematical models were created and were benchmarked to mechanical properties found in the following preliminary tests.

In the first test, an Instron 5943 tensile tester was used to measure the stress-strain curve of CNT tape under tensile loading. From this test, mechanical properties, such as Young’s Modulus, elastic stress and strain limits, and failure stress and strain were found. Mechanical properties from this test were used in mathematical models to predict energy storage capabilities of the CNT tape.

In the second test, torsional loads were applied to samples of CNT tape. Torque, axial force and angle of rotation were recorded in this setup. A motor applied repeated cyclic loading patterns at increasing levels of deformation to samples until they broke at their failure strains. Both 1-ply and 2-ply samples were tested. These tests provided information on energy storage specifically in torsionally-loaded CNT tape, such as efficiency between work done and energy released as well as a relationship between the tape’s torsion and axial load. Further details of the experimental setup and results are presented below. With the data collected from these tests, a mechanism was designed to simulate a regenerative braking system powered by CNT tape.

3.1 Instron Test

Useful energy released by CNT tape will always be less than work done and energy stored. Making these two values as close as possible will increase the efficiency of a regenerative braking system, and that will be discussed in the following sections. This section describes Instron tests that were used to measure the mechanical properties of CNT tape and prove that enough energy can be
stored to make CNT yarn a viable candidate for regenerative braking, regardless of how much energy
can be released in a useful way.

An Instron test was performed for samples of 127 μm x 1 mm CNT tape with various lengths
between 5-15 mm. CNT tape was provided by Nanocomp Technologies, Inc. Samples were prepared by
gluing a piece of CNT between two pieces of paper, seen in Figure 3.1. The Instron tester attached to
the edges of the paper to pull the sample apart.

![Prepared sample for an Instron tensile test](image)

**Figure 3.1- Prepared sample for an Instron tensile test**

The Instron machine pulled the ends of the sample apart, steadily increasing the sample’s strain.
The force exerted by the stretched sample was measured, and various properties of the sample were
calculated, including specific stress, as well as stress and strain limits. Figure 3.2 shows typical stress-
strain curves for CNT tape.
Figure 3.2- Stress-Strain Curve for CNT Tape. Young’s Modulus and elastic strain is constant between samples, though failure strain has variability.

Stresses were calculated by the Instron testing program using the dimensions of the rectangular tape. The tape is not a compact structure, so variations in the area under loading can render the stress measurements somewhat inaccurate. Instead, specific stresses and specific stiffness are normalized to the linear mass density of the material rather than to its area. The CNT tape has a measured linear mass density of 10.17 tex, where 1 tex = 1 mg/m.

In the following table, $\varepsilon$ represents strain (either at the elastic limit or at strain failure), $\sigma$ represents the specific stress (at the elastic limit or at strain failure), and $\overline{E}$ represents the specific stiffness of the elastic region.
Table 3.1- Mechanical Properties of CNT Tape

<table>
<thead>
<tr>
<th>Sample</th>
<th>Length</th>
<th>$\varepsilon_{\text{elastic}}$</th>
<th>$\varepsilon_{\text{failure}}$</th>
<th>$\text{Load}_{\text{elastic}}$</th>
<th>$\text{Load}_{\text{failure}}$</th>
<th>$\bar{\sigma}_{\text{elastic}}$</th>
<th>$\bar{\sigma}_{\text{failure}}$</th>
<th>$\bar{E}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mm</td>
<td>mm/mm</td>
<td>mm/mm</td>
<td>N</td>
<td>N</td>
<td>N/tex</td>
<td>N/tex</td>
<td>N/tex</td>
</tr>
<tr>
<td>1</td>
<td>8.3</td>
<td>0.03</td>
<td>0.198</td>
<td>1.41</td>
<td>3</td>
<td>0.139</td>
<td>0.295</td>
<td>4.621</td>
</tr>
<tr>
<td>2</td>
<td>12.1</td>
<td>0.025</td>
<td>0.173</td>
<td>1.44</td>
<td>3.55</td>
<td>0.142</td>
<td>0.349</td>
<td>5.664</td>
</tr>
<tr>
<td>3</td>
<td>6.58</td>
<td>0.017</td>
<td>0.05</td>
<td>1.06</td>
<td>1.35</td>
<td>0.104</td>
<td>0.133</td>
<td>6.131</td>
</tr>
<tr>
<td>4</td>
<td>6.1</td>
<td>0.022</td>
<td>0.159</td>
<td>1.81</td>
<td>4.49</td>
<td>0.178</td>
<td>0.441</td>
<td>8.090</td>
</tr>
<tr>
<td>5</td>
<td>6.85</td>
<td>0.02</td>
<td>0.1</td>
<td>1.19</td>
<td>1.91</td>
<td>0.117</td>
<td>0.188</td>
<td>5.851</td>
</tr>
<tr>
<td>6</td>
<td>10.61</td>
<td>0.02</td>
<td>0.154</td>
<td>1.76</td>
<td>3.52</td>
<td>0.173</td>
<td>0.346</td>
<td>8.653</td>
</tr>
<tr>
<td>7</td>
<td>14.16</td>
<td>0.024</td>
<td>0.114</td>
<td>1.69</td>
<td>2.89</td>
<td>0.166</td>
<td>0.284</td>
<td>6.924</td>
</tr>
<tr>
<td>8</td>
<td>9.98</td>
<td>0.026</td>
<td>0.177</td>
<td>1.38</td>
<td>2.93</td>
<td>0.136</td>
<td>0.288</td>
<td>5.219</td>
</tr>
<tr>
<td>9</td>
<td>11.14</td>
<td>0.03</td>
<td>0.112</td>
<td>1.66</td>
<td>2.78</td>
<td>0.163</td>
<td>0.273</td>
<td>5.441</td>
</tr>
<tr>
<td>10</td>
<td>10.9</td>
<td>0.024</td>
<td>0.168</td>
<td>1.72</td>
<td>3.61</td>
<td>0.169</td>
<td>0.355</td>
<td>7.047</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>0.0238</td>
<td>0.1405</td>
<td>1.512</td>
<td>3.003</td>
<td>0.149</td>
<td>0.295</td>
<td>6.247</td>
</tr>
</tbody>
</table>

There is clearly a large variance in the mechanical properties of different samples of CNT tape. There are two primary reasons for this. First, sample preparation has a very large effect. Gluing the samples at an angle or slightly misaligned can increase stress in one area of the tape and cause plastic deformation or failure in some strands of the tape before others. Samples 3 and 5 were both glued at angles, and results for these trials seem to confirm the negative effect of doing so. Second, the CNT tape is not homogenous so different sections may be stronger or weaker than others. The maximum failure strain is 19.8% while most samples can experience between 15-20% strain before failure. Plastic deformation occurs at a more reliable value, at an average strain of 2.4%, close to the average values found in previous studies. This value (2.4%) represents the elastic strain limit to which the yarn can be repeatedly loaded without plastic deformation. When using CNT material for energy storage, it will be kept below the elastic strain limit so that each sample can be used many times with minimal damage to the yarn’s structure.

The specific stiffness of CNT tape is about 6.25 N/tex on average (also within an acceptable range compared to previous studies), with a maximum value of 8.65 N/tex. Using the estimated cross
sectional area of the twisted CNT tape (described in Chapter 4), the Young’s Modulus is 7.21 GPa, much lower than the Young’s Modulus seen in individual CNTs (up to 1 TPa). In CNT tape, individual CNTs are disordered and most do not experience pure axial loading. CNTs are also being held together by being tightly wrapped against each other, so this specific stiffness more accurately represents a combination of the strength of these bonds and the stretching of individual CNTs. The average specific stress at the elastic limit is 0.15 N/tex.

These values are used to predict the amount of energy stored in torsionally loaded CNT tape in Chapter 4.

3.2 Torsion Tests

From the information gathered through the Instron tests, it is clear that CNT tape can store enough energy under an axial load to be a candidate for regenerative braking. However, the regenerative braking mechanism designed in this project utilizes torsionally-loaded CNTs, which have lower energy storage capabilities. To assess these capabilities, CNT tape was tested in a mechanism designed by Sanwei Liu. In the setup, a motor (Oriental Motor model DG85R-ARBA-3) rotates one end of a 1” (25.4 mm) piece of CNT tape, while the other end is fixed to a torque transducer (Vibrac Mini Series). The rotating end is attached to a force transducer (Strain Measurement Device model S251). Figure 3.3 shows the mechanism.
Figure 3.3- This mechanism (designed by Sanwei Liu) measures torque and axial force of a sample rotated at one end.

A prepared sample is seen in Figure 3.4. The same 127 μm x 1 mm CNT tape was used for this experiment. The samples were centered by wrapping the ends around the metal pins and gluing the CNT at the edge of the holding structure, resulting in a 1” (25.4 mm) sample length. Both single-ply and 2-ply samples were tested with this mechanism. 2-ply samples simply were prepared using two pieces of CNT tape glued in parallel. When twisted, they act similarly to a thicker single-ply sample. The holding structures were attached to the entire mechanism using screws and preloaded to remove slack using the positioner attached to the base.
3.2.1 Single-Ply Test

Two single-ply samples were tested with this mechanism. The motor twisted the samples at 60 degree/sec to 5 complete rotations. The motor then untwisted the samples back to their original starting point. These steps were repeated four times. Next, the motor twisted the samples to 10 complete rotations, and back to the starting point. This step was also repeated four times. This sequence (5x5 rotations, 5x10 rotations, 5x15 rotations, etc...) continued until the sample broke. Figure 3.5 graphically shows the pattern of loading for these tests.

Figure 3.4- Prepared sample for a torsion test
Figure 3.51- Procedure for the Torsion Tests. Each sample was loaded for 5 trials at increasing number of rotations (5, 10, 15...) until yarn failure.

Sample T2 broke between 30-35 full rotations, while T3 broke between 35-40 full rotations. A variance in the maximum number of rotations was expected, similar to the variance in failure strain observed in the Instron tests. As seen in the Instron tests, the failure strain of CNT tape is very dependent on any small defects in the sample and the arrangement of individuals CNTs in the sample (which cannot be well controlled). When the tape is twisted many times, it becomes more compact, so most defects in tape structure likely present less of an effect at higher rotations, and the variance is primarily due to the CNT alignment.

Figure 3.6 shows the post-processed data for sample T3. The raw data show two main features. The first is the cyclic loading structure shown here, and the second is a set of oscillations (more clearly
seen in Figures 3.7 and 3.8) that are superimposed on the basic cyclic loading structure. The oscillations reflect misalignment of the rotational axes in the experimental apparatus, which increases and decreases the axial force and the torque periodically as the specimen rotates. For clarity, Figure 3.6 plots only the midpoint of each oscillatory cycle. This torque is plotted against angle of rotation. The first rotation of each cycle is highlighted in red. After the first rotation, CNT realignment and permanent deformation causes low angles of rotation to produce less torque than before. With each subsequent cycle, the same angle of rotation produces less torque, while the load characteristics asymptotically approach a stable cycle. This effect is visually seen as the ‘bunching up’ of blue curves in Figure 3.6.

![Torque vs. Angle of Rotation](image)

**Figure 3.6- Torque vs. Angle of Rotation for a 2-ply sample of CNT tape.** Similar plots were generate for 1-ply tests. The red curves represent the first cycle to a given angle of rotation, and the blue lines represent subsequent trials to that angle. After loading to a high angle of rotation, strand realignment occurs, subsequent trials yield less energy storage. This effect decreases and appears to plateau after a few cycles.

The ultimate goal of these tests is to determine the work done on the CNT tape, as well as the useful energy released by the CNT tape. Work done and extracted energy can be calculated using Eq. 3.1.

\[
U = \int T d\phi
\]  
Eq. 3.1
The apparatus measures both torque and angle of rotation, so utilizing this relation becomes a simple matter of plotting the values against each other and integrating. Figure 3.7 shows the first instance of rotation from rest to 15 rotations and back to rest. Figure 3.8 shows the fifth consecutive cycle of rest to 15 rotations and back to rest. In both images, the black line represents the torque while load is being applied to the tape, while the red shows the torque during unloading. Therefore, the area under the black curve is work done during loading, and the area under the red curve is energy released during unloading. The difference between these areas represents the intrinsic energy loss of the unwinding tape, likely due to friction of the CNTs against each other. The oscillations in torque are due to the CNT sample being slightly off-center, so the CNT tape produces a cone effect when twisting. The peaks of oscillations are reached when the tape is most off-center (and consequently, most strained). Therefore, every pair full oscillation represents one full rotation. The actual torque lies somewhere between the peaks, so a running averaged was used to smooth the data.
Figure 3.7- Loop 11 represents the first trial to 15 rotations. The black line represents the torque during loading and the red line represents the torque during unloading.
Figure 3.8- Loop 15 represents the fifth and final trial to 15 rotations. The input torque rises significantly later than in Loop 11 (due to deformation and strand realignment), but the output torque remains mostly unchanged.

Loop 11 has a maximum torque of 3.1E-5 N*m, but an efficiency (defined as energy released divided by energy stored) of only 31%. Loop 15, meanwhile, only has a maximum torque of 2.2E-5 N*m, whereas its efficiency is 67%. More cycles are shown below that confirm these trends. There are two reasons for these changes. First, slack is forming in the tape with each consecutive cycle. As the tape experiences plastic deformation, it does not return to its original length. Clearly, this causes less overall torque (and consequentially, energy storage) given the same number of rotations. On the other hand, as the tape rotates, it is becoming more compact, approximating a cylindrical yarn more closely than a rectangular piece of tape. The elastic region also shifts after numerous loads. During the first cycle, the tape is experiencing significant permanent reorganization. By the fifth consecutive cycle, however, the deformation is mostly elastic. In practice, it may be worthwhile to reapply the yarn’s preload after the
first few cycles to remove any slack. That way, all rotations of the wheel during braking will produce torque in the yarn. After the fifth cycle, individual strands of CNT may have also realigned, causing the efficiency to increase and plateau.

These two effects result in higher efficiency as the tape undergoes more cycles, until both the maximum torque and the efficiency eventually reach a plateau. Figures 3.9 and 3.10 show these effects over the entire test for sample T2.

![Graph showing energy density of CNT Tape](image)

**Figure 3.9**-Output energy density of CNT Tape at different angles of rotation. In some cases, the energy has plateaued after 5 cycles. In other cases, it is expected to eventually plateau after more cycles.
From Figure 3.9, it is clear that increasing the number of rotations of the tape will increase the torque it experiences and the energy it releases. It is also evident that, while putting the tape under many consecutive cycles will reduce the maximum torque it experiences, the resulting torque begins to plateau after a few consecutive cycles. Sample T2 reached a maximum energy density of about 4.14 kJ/kg and decreased to around 3.0 kJ/kg. By comparison, sample T3 reached a maximum energy density of about 4.32 kJ/kg and decreased to around 3.4 kJ/kg. These higher values can be attributed to T3 reaching 40 complete rotations, therefore experiencing more torque and storing more energy.

Figure 3.10 shows the same trend in efficiency as discussed above, over the entire test. As the tape becomes more compact and the elastic region shifts, its efficiency increases since there is less reorganization of the arrangement of individual CNTs. Energy efficiency for 1-ply tape increases to
around 60% after numerous cycles. There is, however, a caveat to this. After shifting the elastic region, a sample would experience less torque at fewer rotations because there is now an inherent slack. Because of this, a regenerative braking system must keep the CNT tape in the elastic region to prevent performance reduction due to slack in the tape.

Sample T2 reached a maximum output torque of 5.6E-5 N*m and a maximum output energy of 1.07 mJ. Sample T3 reached a maximum output torque of 7.9E-5 N*m and a maximum output energy of 1.11 mJ. Considering that the sample length is only 1” (25.4 mm), this is a useful amount of energy, but the question remains: is it enough to accelerate a wheel?

The wheel that will be used to model a car braking system has a moment of inertia of 4.01E-5 N*m. Assuming all energy and torque will be used to rotate the wheel and no additional losses, a 1” (25.4 mm) single-ply sample of CNT tape would accelerate the wheel at 1.9 rad/s² and have a maximum angular velocity of 7.4 rad/s. When accounting for additional losses, it does not appear that this small amount of CNT tape could accelerate the model wheel at a significant speed. However, given thicker or longer tape, it may be able to do so.

During this test, the axial force from the stretched CNT tape was also recorded. When plotted with torque, there is a clear relationship between the two values.
Comparing peak values for torque and force, the ratio for force to torque is found to be about 60,000:1. The same ratio was found for sample T2. Using this ratio, the torque in a CNT sample can be estimated from the force, simplifying the design of the mechanism in Chapter 5. Another ratio was found using 2-ply tape.

3.2.2 2-Ply Test

As seen above, a 1” (25.4 mm) piece of single-ply CNT tape may not provide enough torque and stored energy to significantly accelerate a wheel. There are two simple methods to remedy this: change the mechanism, or generate more torque and energy by using more CNT material. The second method

Figure 3.11- Torque and force plotted on top of each other. There is a linear relationship between the two and a proportionality constant was found to relate them.
is easier to implement and would be required for a full-sized braking system, so it was explored first. The mechanism was also adjusted to reduce energy losses, which is discussed in Chapter 5.

In Chapter 4, there is a discussion on the effect of increasing the length or ply of CNT tape. According to these models, doubling the ply of CNT tape would increase the torque 2.82 times, reduce the maximum number of rotations by a factor of .707 and increase the total energy stored by a factor of 2. Table 3 shows the maximum values for single-ply CNT tape, along with both expected and actual values for 2-ply tape.

Table 3.2- Expected vs. Actual values for 2-ply torsion test

<table>
<thead>
<tr>
<th></th>
<th>1-Ply</th>
<th>2-Ply</th>
<th>Expected</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (mJ)</td>
<td>1.11</td>
<td>2.22</td>
<td>4.1</td>
<td>4.1</td>
</tr>
<tr>
<td>Torque (N*m)</td>
<td>7.90E-05</td>
<td>2.23E-04</td>
<td>2.70E-04</td>
<td></td>
</tr>
<tr>
<td>Rotations</td>
<td>40</td>
<td>28.3</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

The model holds up well in predicting the maximum torque and number of rotations, however it is off by a factor of 2 in predicting the maximum energy released. It is off by the same factor for maximum energy stored. This discrepancy is largely due to the geometry of the ideal model versus the actual geometry of the 2-ply tape. If the tape were to act as a single cylinder (as the model assumes), the inner most strands would experience no strain, while the outermost strands would experience a maximum strain. In reality, the two pieces of tape wrap around each other, so all strands of tape are experiencing some, but not equal, amounts of strain. In addition, the twisted strands re not expected to form a perfect cylinder when they are twisted together. Deviations from a perfect cylinder will increase the proportion of the material at large radius, where its effectiveness in storing energy is enhanced. This model may not be accurate enough to predict energy storage in the CNT tape, but it can be used to estimate a safe torque and number of rotations for later tests.
Figures 3.12 and 3.13 show similar trends in 2-ply tape as seen in the 1-ply tests. Energy density increases fairly linearly with angle of rotation, decreasing with subsequent cycles as reorganization occurs and the elastic region shifts. At some loading levels, the energy density appears to begin to plateau within the five cycles at that loading level. The maximum output energy density for 2-ply CNT is 8.0 kJ/kg, decreasing to around 5.7 kJ/kg after five cycles (again, larger than the energy density for 1-ply by about a factor of 2). Efficiency also follows the same trend as 1-ply tape, but reaches a lower value (around 50% compared to 60% for 1-ply). The lower efficiency can be attributed to the plies undergoing slightly different strains. If one ply had slightly more slack when glued, it would produce less torque per rotation.

![Graph showing output energy density for 2-ply CNT tape.](image)

*Figure 3.12- Output energy density for 2-ply CNT tape. The energy density is about twice the density of 1-ply CNT tape.*
The force to torque ratio for 2-ply tape was found to be about 26,000:1. This ratio is substantially lower than the ratio found for 1-ply CNT. This means that, for a given torque, the axial force is about twice as large for 1-ply tape as 2-ply. This can again be attributed to the geometry of two strands of tape wrapping around each other. Since more strands of the tape are being stretched in the 2-ply setup, more torque is produced. This effect is also shown in mathematical models in Chapter 4.
Wheel dynamics can again be calculated to see if 2-ply CNT tape is expected to provide enough torque and energy for a regenerative braking system. A 1” (25.4 mm) sample of 2-ply tape could accelerate the wheel at 6.7 rad/s² and reach a maximum angular velocity of 14.3 rad/s, again assuming no additional losses. These are much more promising values for a model braking system than obtained with a 1-ply sample. The wheel dynamics could be improved even further by using a longer sample of tape, or an additional ply.

3.3 Summary

The CNT tape available for this project is a viable candidate for energy storage in regenerative braking. A 1” (25.4 mm) sample of single-ply CNT tape produced a useful amount of energy and torque,
though it may not be enough to accelerate a wheel and reach a significant speed. By using multiple ply yarn and longer samples, the energy and torque available can both be increased. When testing the regenerative braking system, it is important to keep the tape in an elastic region to allow for many consecutive load cycles without a loss of efficiency. The highest amount of energy is produced at high numbers of rotation, close to the failure strain. So, a preload must be reapplied to the CNT tape to reduce slack formed while reaching that angle of rotation. This will cause lower angles of rotation to produce substantially less energy and torque, so the braking mechanism is designed to keep the angle of rotation high and reach maximum strain energy quickly.
Chapter 4

Ideal Strain Energy Calculations

Three models were created to calculate the theoretical maximum strain energy in a yarn made of carbon nanotubes. The first model calculates the energy associated with stretching the yarn. The second model calculates the energy from twisting the yarn, assuming the yarn is made of many infinitesimal strings. The third model calculates the energy from twisting the yarn when treating it like a solid. All models use constant values given below in Table 4.1. Young’s modulus and the maximum elastic strain were found in preliminary experiments.

The tape’s radius after twisting into a compact, yarn-like structure was estimated by comparing the linear mass density of the CNT tape (10.17 tex) to the density of a more compact CNT yarn with known diameter (47 µm) and linear mass density (2 tex). The cross section of tape has 5.085 more material than the cross section of yarn. The tape’s compact diameter was calculated to be 106 µm. The outer diameter is likely slightly higher in reality because it will not compact into as uniform a cylindrical shape as the CNT yarn used for comparison.
Table 4.1- CNT Tape properties as determined in Chapter 3

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Stiffness ($E'$)</td>
<td>6.25 N/tex</td>
</tr>
<tr>
<td>Maximum Elastic Strain ($\epsilon$)</td>
<td>0.024</td>
</tr>
<tr>
<td>Yarn Diameter</td>
<td>106 µm</td>
</tr>
<tr>
<td>Yarn Length</td>
<td>0.0254 m</td>
</tr>
<tr>
<td>Linear Mass Density ($\lambda$)</td>
<td>10.17 tex</td>
</tr>
</tbody>
</table>

4.1 Stretched Model

The stretched model assumes a constant and uniform force is being applied to one end of the yarn. This model does not provide an accurate assessment of the yarn’s behavior under torsional load, but it provides an upper bound estimate of the maximum energy storage. Below are calculations used to find the maximum strain energy under such conditions assuming the mechanical properties remain linear with increasing load.

\[
E' = \frac{\sigma}{\epsilon} \quad \text{Eq. 4.1}
\]

\[
\sigma = \frac{F}{\lambda} \quad \text{Eq. 4.2}
\]

Equations 4.1 and 4.2 are combined to give equation 4.3.

\[
F = E' \epsilon \lambda \quad \text{Eq. 4.3}
\]

Equations 4.4 and 4.5 represent the relation between force and potential energy.

\[
\frac{dU}{dm} = F(m) \quad \text{Eq. 4.4}
\]

\[
U = \int F(m) dm + U(m_0) \quad \text{Eq. 4.5}
\]

Using the values listed, the maximum force applied to one end of the yarn is 1.53 N and the specific stress under that force is 0.15 N/tex. Both of these values are very similar to the values found in the Instron test in Chapter 3. This similarity is expected, given that measurements of tensile stress and strain are the source of the numbers used in the present calculations. However, the results serve to validate the appropriateness of the models.
Substituting equation 4.3 into equation 4.5 yields equation 4.6, solving for which gives a simple solution for strain energy, equations 4.7. Alternatively, equation 5.8 yields energy density.

\[
U = \int E \varepsilon \bar{\varepsilon} dm \quad \text{Eq. 4.6}
\]

\[
U = \frac{1}{2} E \varepsilon^2 m \quad \text{Eq. 4.7}
\]

\[
\bar{U} = \frac{1}{2} E \varepsilon^2 \quad \text{Eq. 4.8}
\]

The maximum strain energy stored by 0.0254 m of tape is predicted to be 0.465 mJ. The maximum strain energy density is predicted to be 1.8 kJ/kg.

### 4.2 Twisted Strings Model

The first twisting model treats the yarn as many infinitesimal strings being twisted in a helix formation. The helix radius is equal to the yarn radius for the outermost strings. At the center of the yarn, the helix radius is zero. The outer strings will reach maximum strain first and will therefore contribute most of the strain energy. The inner strings will not be stretched as far due to the small helix radius and will store less energy. Below are calculations used to find the maximum strain energy under such conditions.
Figure 4.1 - Diagram of the model yarn used in these calculations

As seen in Figure 4.1, $\theta$ represents the angle of rotation, $\varphi$ represents the helix angle and $R$ represents the outer radius of the twisted yarn. To determine the maximum angle of twist and the helix angle, simple geometry can be used in combination with the maximum elastic strain obtained from tensile tests. The twisted yarn can be represented by the triangle, below.

Figure 4.2 - Simple geometry is used to find the maximum angle of rotation

The untwisted string is length $L$, and the maximum strain is 0.024. Thus, the maximum twisted length is $1.024L$. $R\varphi$ is the arc length of all rotations. Using $R$ and $L$ given in Table 4.1, the maximum angle of twist is found to be 105.6 radians (or 16.8 rotations).
The radial force is found by multiplying the total force by the sine of the helix angle. It is then multiplied by the quantity \( r/R \) (where \( r \) is radius and \( R \) is maximum radius) to account for higher forces in the outer strings than the inner strings.

\[
dF_\theta(r) = dF_{\text{max}} \left( \frac{r}{R} \right) \sin \gamma \quad \text{Eq. 4.9}
\]

Torque is found by multiplying the radial force by the distance from the axis. The total torque is equal to the sum of all torques.

\[
dT = rdF_\theta(r) \quad \text{Eq. 4.10}
\]

\[
T = \sum_A dT \quad \text{Eq. 4.11}
\]

Equation 4.12 gives the definition of axial stress.

\[
dF_{\text{max}} = \sigma_{xx}^{\text{max}} dA \quad \text{Eq. 4.12}
\]

The maximum axial stress is simply equal to the maximum specific stress, found in the previous section, multiplied by the volumetric density (1152.8 kg/m\(^3\), from Chapter 2). It is equal to 172.9 MPa. For small angles, a sine approximation is valid. Using the maximum value of \( R \), \( \gamma \) is equal to roughly 0.21 radians, making the sine approximation valid.

\[
\sin \gamma = \gamma = \frac{\varphi}{1.024L} \quad \text{Eq. 4.13}
\]

Combining equations 4.9, 4.10, 4.11, 4.12 and 4.13 gives the following equation for torque,

\[
T = \int \int r \sigma_{xx} \left( \frac{r}{R} \right) \frac{\varphi}{1.024L} r dr d\theta = \int_0^{2\pi} \int_0^R r^4 \sigma_{xx}^{\text{max}} \frac{\varphi}{1.024L} dr d\theta = \frac{2\pi}{5} R^4 \sigma_{xx}^{\text{max}} \frac{\varphi^2}{1.024L} \quad \text{Eq. 4.14}
\]

The maximum torque produced by a 1” (25.4 mm) sample of this yarn is therefore predicted to be 6.96E-6 N*m.

Integrating Eq. 4.14 with respect to \( \varphi \) gives the maximum strain energy of the yarn.

\[
U = \int T d\varphi = \frac{\pi R^4}{4L} \sigma_{xx}^{\text{max}} \frac{\varphi^2}{1.024} \quad \text{Eq. 4.14}
\]

The maximum strain energy stored by a 1” (25.4 mm) sample of this yarn is therefore 0.459 mJ. This equation for strain energy can be divided by mass (m) to give energy density.
The maximum strain energy density predicted using the twisted strings model is 1.78 kJ/kg.

**4.3 Twisted Solid Model**

The second twisting model treats the yarn as a solid piece of material. For this model to be accurate, the tape would need to be made of one large, solid CNT rather than a large number of individual CNTs. Clearly this is not the case, although there are interactions between adjacent CNTs that the twisted strings model does not account for and that may be more accurately represented by the twisted solid model. In reality, the yarn has characteristics of both models and the actual strain energy and torque are likely somewhere between the predict values.

Basic shaft mechanics are used to calculate the maximum strain energy under torsion. The variables depicted in Figure 4.3 are used again for this model. The strain energy in a shaft under torsion is given by equation 4.16. This equation assumes a linear relationship between torque and angle of rotation, as shown Chapter 3.

$$U = \frac{\pi R^4}{4 L} x_{xx} \frac{\varphi^2}{1.024}$$  \hspace{1cm} Eq. 4.15

The variables depicted in Figure 4.3 are used again for this model. The strain energy in a shaft under torsion is given by equation 4.16. This equation assumes a linear relationship between torque and angle of rotation, as shown Chapter 3.

Figure 4.3- Preliminary experiments showed that torque increases linearly with angle of twist in the elastic region. Energy is equal to the area under this curve.

Given this relationship, the strain energy can be simplified to equation 4.17, where \( T \) and \( \varphi \) are the maximum torque and twist angle before failure.
The maximum shearing stress is found using Mohr’s circle with the maximum axial stress found in the previous sections, as shown below.

\[ U = \int T d\phi \]  
\[ U = \frac{1}{2} T \phi \]

Eq. 4.16  
Eq. 4.17

The maximum shearing stress in this case is equal to \( \frac{1}{2} \) the maximum axial stress (which was found to be 16.3 MPa). The maximum shearing stress is, therefore, 8.15 MPa.

**Maximum Torsion**

When a force is applied uniformly to the end of a shaft, the torque is given by the sum of the differential elements of force multiplied by their respective distances from the axis. Equation 4.18 shows this concept in integral form.

\[ T = \int r dF \]  
\[ T = \int r(\tau dA) \]

Eq. 4.18  
Eq. 4.20

Equation 4.19 is the definition of shearing stress. Substituting 4.19 into 4.19 yields equation 4.20.

\[ dF = \tau dA \]  
\[ T = \int r(\tau dA) \]

Eq. 4.19  
Eq. 4.20

Hooke’s law for shearing stress and strain is given by equation 4.21. The shear modulus is found using equation 4.22. Poisson’s ratio \( (\nu) \) is estimated to be 0.25.
The arc length of the rotated end of the shaft can be found two ways, as seen in the model above. It is calculated as $\gamma L$, as well as $\varphi \rho$. Setting these terms equal to each other yields equation 4.23.

$$\gamma = \frac{r \varphi}{L}$$  \hspace{1cm} \text{Eq. 4.23}

Equation 4.23 at maximum radius $R$ becomes equation 4.24.

$$\gamma_{max} = \frac{r \varphi}{L}$$  \hspace{1cm} \text{Eq. 4.24}

Eliminating $\varphi$ from 4.23 and 4.24 yields 4.25.

$$\gamma = \frac{r \gamma_{max}}{R}$$  \hspace{1cm} \text{Eq. 4.25}

Substituting 4.21 into 4.25 gives the following result.

$$\frac{I}{G} = \frac{r}{RG} \tau_{max}$$  \hspace{1cm} \text{Eq. 4.26}

Eliminating the shear modulus $G$ gives,

$$\tau = \frac{r}{R} \tau_{max}$$  \hspace{1cm} \text{Eq. 4.27}

Inserting 4.27 into 4.20 gives,

$$T = \frac{T_{max}}{R} \int r^2 dA$$  \hspace{1cm} \text{Eq. 4.28}

The integral term in 4.28 represents the polar moment of inertia, $J$, which is calculated using 4.29.

$$J = \frac{1}{2} \pi R^4$$  \hspace{1cm} \text{Eq. 4.29}

Finally, the maximum torque applied to the shaft is found using 4.30.

$$T = \frac{T_{max}}{R} J$$  \hspace{1cm} \text{Eq. 4.30}

Plugging in values, the maximum torque from the solid model is predicted to be $2.09 \times 10^{-5}$ N*m.

\textbf{Angle of Twist}
Combining equations 4.21, 4.24 and 4.30 gives equation 4.31, an expression for the total angle of twist.

\[
\varphi = \frac{T L}{J G} = \frac{\tau_{\text{max}} L}{R G}
\]

Eq. 4.31

This angle is found to be 14.39 radians, or 2.29 rotations.

**Strain Energy**

Substituting 4.30 and 4.31 into 4.17 gives the following expression for strain energy.

\[
U = \frac{1}{2} T \varphi = \frac{1}{2} \frac{\tau_{\text{max}} L}{R} + \frac{\tau_{\text{max}} L}{R^2 G} = \frac{1}{2} \frac{\tau_{\text{max}}^2 L}{R^2 G}
\]

Eq. 4.32

The predicted maximum strain energy in the tape according to the solid model is 0.15 mJ. The equation for strain energy can be divided by volume to give energy density.

\[
\bar{U} = \frac{U}{m} = \frac{\frac{1}{2} \tau_{\text{max}}^2 L}{2\pi R^2 G}
\]

Eq. 4.33

The predicted maximum strain energy density using the twisted solid model is 0.5 kJ/kg.

**4.4 Comparison of Models**

<table>
<thead>
<tr>
<th></th>
<th>Strain Energy for 1” (25.4 mm) Yarn</th>
<th>Strain Energy Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stretched</td>
<td>0.465 mJ</td>
<td>1.8 kJ/kg</td>
</tr>
<tr>
<td>Twisted Strings</td>
<td>0.459 mJ</td>
<td>1.78 kJ/kg</td>
</tr>
<tr>
<td>Twisted Solid</td>
<td>0.15 mJ</td>
<td>0.5 kJ/kg</td>
</tr>
</tbody>
</table>

Table 4.1- The stretched model yields higher maximum energy than the twisted models since all strands of yarn are stretched equally, unlike the twisted models.

As expected, the stretched model yields the highest strain energy (although not by much). In this model, all elements of the yarn are being pulled to their maximum strain, whereas the twisted models have specific elements reaching their limits earlier than others. The twisted solid model yields the most conservative results so it can be used to estimate the amount of CNT tape to ensure that there will be enough energy and torque for the regenerative braking model.
The actual measured energy that could be extracted from 1” (25.4 mm) of single-ply CNT tape was actually over 1 mJ, as compared with the less than 0.5 mJ values predicted above. There are a number of possible reasons for this. These models assume the tape becomes a perfect cylinder upon twisting, which it does not. In addition, the 2.4% strain limit is not a strict limit; although the slope of the stress-strain curve changes at around 2.4%, it is expected that continuing to increase the loading beyond this point will store some additional energy in the system. Other factors may reduce the energy stored in the system as compared with the models. For example, the models do not take into friction of the tape against itself and realignment of the strands of CNT after multiple cycles of loading and unloading. Overall, the models are on the same order of magnitude as the actual values, so they can be useful in predicting energy storage capabilities of CNT tape.

4.5 Extending the Length and Ply of CNT Tape

Preliminary experiments in Chapter 3 showed that a 1” (25.4 mm) piece of CNT yarn stores 1.1 mJ of useful strain energy (that is, energy that can be released) under torsional loading; further increase in torsional loading led to failure. At the moment before failure, the yarn was under a torque of 7.9E-5 N*m. To easily see the effect that these values have on a regenerative braking system, maximum values of angular acceleration and velocity were calculated for a wheel if all energy and torque from the CNT tape was used to accelerate it. A 55.25 g wheel with a diameter of 76.2 mm has a moment of inertia of approximately 4E-5 kg*m². Assuming an ideal setup with no losses (i.e. all of the stored energy is converted to rotational kinetic energy of the wheel), the CNT material’s energy would rotate the wheel at a maximum angular velocity of 7.4 rad/s. The CNT material’s torque would yield a maximum acceleration of about 1.9 rad/s². Of course, these values were calculated without considering any losses, such as losses due to friction in the bearings. Realistically, the wheel will only experience a fraction of that velocity and acceleration. In order to provide enough torque and energy to produce
significant movement of the wheel in the presence of losses, longer pieces of yarn will be used in parallel (i.e. multi-ply yarn) to drive the wheel. Calculations were performed to determine the best combinations of length and thickness yarn to use.

The mathematical models were used to study the effects of changing length and radius of the CNT tape. The effects of adjusting the length and the number of plies were studied separately. The results were then combined to determine a set of multiplication factors that convert the strain energy and torque for a single ply yarn of a given length to a multi-ply yarn of a different length.

4.5.1 Increasing Number of Yarns in Parallel

In order to increase the effective radius of the tape, multiple pieces of yarn can be used in parallel with each other. If the yarns have the same length and are twisted together, they will, in effect, act as a single yarn with an increased radius. When the yarns are untwisted, this is a poor approximation. However, once the yarns are twisted, they become intertwined and this single yarn approximation becomes valid.

By increasing the number of yarns \( n \) times (a multiplying factor), the total cross sectional area also increases \( n \) times. Using the single yarn approximation, a new effective radius is calculated to be \( \sqrt{n} \). The solid beam under torsion model (equation 4.34) is examined to find the effect of increasing radius by a factor of \( \sqrt{n} \).

\[
T = \frac{GJ\varphi}{L} \quad \text{Eq. 4.34}
\]

\( G \), the shear modulus, is a material property, and therefore independent of radius. \( J \), the second moment of area, is proportional to \( r^4 \), or \( n^2 \). \( \varphi \) is proportional to length divided by radius. Since length is kept constant, it is only proportional to \( r^{-1} \), or \( \frac{1}{\sqrt{n}} \). Combining these effects, as the number of yarns in parallel is increased \( n \) times, the maximum torque is increased by a factor of \( n^{3/2} \).
Energy is proportional to torque, $T$, multiplied by angle of rotation, $\phi$. Energy, therefore, increases by $n$ when the number of yarns is increased by $n$. Since the mass of yarn is also increasing by a factor of $n$, energy density should remain independent of yarn thickness.

4.5.2 Increasing Yarn Length

Radius is held constant, and the effect of increasing the length of yarn $m$ times is observed. G, shear modulus, and J, second moment of area, are both independent of length. Maximum angle of rotation, $\phi$, is proportional to length, or $m$. Therefore, torque is proportional to length over length $(\frac{m}{m})$, so it is independent of length. This is as expected, and it was confirmed in the preliminary experiments where various lengths of yarn experienced the same maximum torque.

Energy is proportional to torque, $T$, multiplied by angle of rotation, $\theta$, which, in this case, is equal to $m$. The mass of the yarn increases by a factor of $m$, so the energy density is also independent of yarn length.

4.5.3 Combined Effect

The results of increasing the number of yarns and length of yarn can be combined into the following table.

Table 4.2- Increasing the length of yarn will increase maximum energy, while also increasing the angle of rotation to get there. Increasing the yarn ply will increase both torque and energy, while decreasing the maximum angle of rotation.

<table>
<thead>
<tr>
<th>Multiplication Factors</th>
<th>Torque</th>
<th>Rotation</th>
<th>Energy</th>
<th>Energy Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of yarns increased $n$ times</td>
<td>$\frac{3}{n^2}$</td>
<td>$\frac{1}{\sqrt{n}}$</td>
<td>$n$</td>
<td>1</td>
</tr>
<tr>
<td>Length of yarn increased $m$ times</td>
<td>1</td>
<td>$m$</td>
<td>$m$</td>
<td>1</td>
</tr>
<tr>
<td>Total Effect</td>
<td>$\frac{3}{n^2}$</td>
<td>$\frac{1}{\sqrt{n} \cdot m}$</td>
<td>$n \cdot m$</td>
<td>1</td>
</tr>
</tbody>
</table>
Repeating the process of finding multiplication factors for the twisted strings model yields identical results as those reported in Table 4.3.

While these multiplication factors are very helpful in deciding how much CNT to use to achieve a given torque or strain energy, it is not immediately evident how these multiplication factors would affect a regenerative braking system, which is the end goal. To get an idea of how these two ideas relate, dynamics of a single wheel under various values of torque and energy were considered.

\[ \sum T = I \alpha \]  
Eq. 4.35

\[ U = \frac{1}{2} I \omega^2 \]  
Eq. 4.36

If all torque and strain energy of the yarn was applied to a wheel, equations 4.35 and 4.36 would yield angular acceleration and maximum angular velocity. Following the same process as above, multiplying factors for acceleration and velocity can be found for various lengths and thicknesses of CNT yarn. In these equations, \( I \) represents the moment of inertia of the wheel only. The moments of inertia of other components and energy losses are neglected.

Table 4.3- Changing the length and ply of yarn have equal effects on the maximum velocity achieved by a wheel. Only increasing the yarn ply will increase the wheel's angular acceleration.

<table>
<thead>
<tr>
<th>Multiplication Factors- Wheel Dynamics</th>
<th>Angular Acceleration</th>
<th>Angular Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of yarns increased ( n ) times</td>
<td>( \frac{3}{n^2} )</td>
<td>( \sqrt{n} )</td>
</tr>
<tr>
<td>Length of yarn increased ( m ) times</td>
<td>( \frac{1}{m} )</td>
<td>( \sqrt{m} )</td>
</tr>
<tr>
<td>Total Effect</td>
<td>( \frac{3}{n^2} )</td>
<td>( \sqrt{n \cdot m} )</td>
</tr>
</tbody>
</table>

Angular acceleration is clearly only dependent on the thickness of yarn, so the effect of changing the number of yarns can be plotted in a line graph, below.
Figure 4.5- Angular acceleration increases with the ply of yarn raised to the power of 3/2. The x and y axes both represent multiplication factors, and are therefore unit-less.

Since angular velocity is dependent on both thickness and length of yarn, the effects of changing these variables can be plotted on a contour plot, below.

Figure 4.6- Maximum angular velocity is dependent on both ply and length of yarn. A balance must be found between them to achieve high velocities and accelerations while maintaining a reasonable angle of rotation. The x and y axes both represent multiplication factors, and are therefore unit-less.
For any mechanism, once the moment of inertia is calculated, these plots can be used to determine how much CNT yarn to use from the desired wheel dynamics.

### 4.6 Comparison with Preliminary Experiments

As mentioned previously, the mathematical models shown in this chapter predict less overall strain energy and torque than preliminary experiments have shown. Table 4.5 shows a summary of these values.

**Table 4.4- Preliminary experiments yielded more maximum energy and torque in a twisted CNT yarn than the mathematical models predicted.**

<table>
<thead>
<tr>
<th></th>
<th>Energy Density (kJ/kg)</th>
<th>Torque (N*m)</th>
<th>Angle of Rotation (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stretched Model</td>
<td>1.8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Twisted Strings Model</td>
<td>1.78</td>
<td>2.09E-05</td>
<td>105.6</td>
</tr>
<tr>
<td>Twisted Solid Model</td>
<td>0.5</td>
<td>6.96E-06</td>
<td>14.39</td>
</tr>
<tr>
<td>Single-Ply Torsion Test</td>
<td>4.3</td>
<td>7.90E-05</td>
<td>-</td>
</tr>
</tbody>
</table>

Though the twisted strings model predicts significantly more energy than the twisted solid model, its torque is much lower. The higher energy, therefore, comes from the higher number of rotations (105.6 radians in the strings model compared to 14.39 radians in the solid model). The actual number of rotations to remain in the elastic region is unknown from the preliminary experiments, but it appears to be closer to the strings model than the solid model.

All three of the mathematical models predict lower maximum energy than the single-ply results. The models assume the tape acts like a perfect cylinder, which it does not. Individual CNTs inside the tape assembly are very disordered and may be experiencing some transverse loads, possibly increasing the energy storage capabilities. In a perfect cylinder, the middle strands of tape do not contribute any torque, but in reality, threads wrap around each other, so all strands are experiencing strain. As the threads wrap around each other, they experience high and low strains so on average, this effect may not
have a significant impact. Another possible source of error is estimating the radius of the compact tape. A change in radius directly impacts the total number of rotations and the torque produced by each rotation. Finally, in the experiments, some storage of energy may be observed between the end of the initial linear region (assumed to be the elastic region) and the onset of failure.

As discussed in Chapter 3, the expected increase in energy when adding a ply of tape is off by a factor of 2. One assumption that could contribute to this difference is the calculated radius. The calculations assume a sample of multiple ply tape will act as a single piece of tape, though this is not an entirely accurate assumption. Figure 4.7 show samples of 1, 2 and 3-ply CNT tape, from bottom to top, respectively. The measurements listed are number of pixels at the same magnification. With this data, radius ratios are calculated, which can be compared to the calculated change in radius from the previous section. Table 4.6 shows these results.

![Figure 4.7](image)

Figure 4.7- The two images show 1, 2, and 3-ply CNT tape (from bottom to top, respectively) after loading and unloading. The ratios of their radii were found using pixel length, rather than directly measuring.
Table 4.5: The measured radii ratios match up well with the predicted values.

<table>
<thead>
<tr>
<th>Change in Radius by Increasing Ply</th>
<th>Average Pixel Size</th>
<th>Ratio to 1-Ply</th>
<th>Calculated Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Ply</td>
<td>70.21</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2-Ply</td>
<td>100.02</td>
<td>1.42</td>
<td>1.414</td>
</tr>
<tr>
<td>3-Ply</td>
<td>131.79</td>
<td>1.88</td>
<td>1.732</td>
</tr>
</tbody>
</table>

The calculated and actual radius ratios match very closely, so it is unlikely this is a major source of error in the energy calculation. Though multiple pieces of tape do not actually mold together to form a single ply, they wrap very closely and there is very little open space between them. A second possible explanation may be the effects of slack. It is possible that a given amount of slack plays a larger role in limiting energy storage in a thinner yarn than in a thicker yarn. The effect of longer samples of CNT tape is examined with the results of the braking mechanism model.

Though the mathematical models have errors in predicting strain energy, they are accurate in predicting number of rotations and torque. This is helpful in testing the regenerative braking mechanism to design safe limits as to not overstretched CNT tape while bringing it close to its maximum strain.
Chapter 5

Design

5.1 Initial Design Concepts

In order to store mechanical energy in a CNT, it must be under a strain. There are a number of ways to induce strain in a CNT. To visualize how various mechanisms could do this, rudimentary designs were drafted for a simple, 4-wheel structure. These designs were not meant to model an actual automobile; instead, they are simply structures that carry kinetic energy. They were used to brainstorm ideas about how kinetic energy could be converted to strain energy, and vice versa.

There are two mechanisms involved in regenerative braking systems: capturing/storing energy and releasing the energy. The following models show methods of collecting and immediately releasing energy. They do not include many of the necessary components of a regenerative braking system, such as accelerating the wheel in the positive direction and the ability to store energy without releasing it.
Figure 5.1 - A stretched yarn around an axle can rotate a wheel. Similar devices are often seen in toy cares. A CNT yarn with a loop tied at one end can hook onto a ratchet, twisting around the axle and producing strain in the yarn. The yarn can then be released and accelerate the wheel.

Figures 5.1 and 5.2 depict mechanisms that stretch a CNT yarn. Figure 5.1 shows the simplest method using a rotating wheel to stretch a yarn. The yarn is attached to the edge of a ratchet and fixed at the other end. When the ratchet rotates, the yarn is stretched around the axle by an axial load. The yarn could then be released to accelerate an attached wheel. The CNT will reach its highest allowable strain, but it will experience this strain at a very small total deflection. Since the yarn is being stretched by a rotating wheel, it would likely reach its failure stain before one complete rotation. This would obviously be an undesirable characteristic for a braking system, as vehicles tend to brake over a substantial distance, rather than executing a sudden stop.
Figure 5.2 - How a stretched yarn mechanism may attach to a car. Multiple yarns are used to eliminate loads on the bearings.

In Figure 5.2, the rotation of the wheel is transferred 90 degrees, to a vertical shaft. This model is similar to Figure 5.1, though rather than attaching the yarn directly to the wheel, it is attached to the vertical shaft. With the addition of a gear box, the rate of rotation of the vertical shaft could be adjusted so that the yarn does not reach its maximum strain as quickly as in the first model, although this solution would introduce additional energy losses to the system which could otherwise be avoided. The other concern with this model is the transverse load applied to the vertical shaft by the stretching yarns. This issue is diminished, but not entirely removed, by including an additional yarn on the opposite side of the shaft so that the two loads cancel each other out.

Figures 5.3, 5.4 and 5.4 depict mechanisms that twist a CNT yarn. Figure 5.3 shows the simplest form of this mechanism: a yarn attached to the center of a wheel at one end and to a fixed support (a wall) at the other. In his type of mechanism, the CNT yarn is akin to a beam with a torsional load. As discussed in Chapter 4, some strands of CNT in the tape will not reach the same strain as the stretched model so the total strain energy will be less. However, the yarn in this model would require more rotations of a wheel to reach its maximum strain, which is preferable for a braking system.
Figure 5.3- A twisted model can also rotate a wheel. It will release energy more slowly than the stretched model.

While the model in Figure 5.3 solves the major issue involved in a stretching mechanism, it is difficult to imagine how it could be applied to a real automobile. One possible solution would be to place the yarns inside of hollow car axles. Though this may be a viable solution at some point, it would be difficult to model as it involves a redesign of fundamental components of a car. One possible solution is depicted in Figure 5.4. Using multiple yarns around the axle, this mechanism could, theoretically, store much more energy than a mechanism that uses only one yarn in the center of a rotating wheel. However, there would be considerable losses due to friction between the yarns and axle, which makes this model unviable. Additionally, allowing the CNT yarn to rub against the axle would damage the yarn, and could possibly break it.
Figure 5.4- Twisting multiple yarns around an axle will cause the yarns to make contact with the axle, making this model unviable.

Alternatively, rotation from the wheel can be transmitted to another axis via gears on the axle, as seen in Figure 5.5. Though this design would include some additional losses due to the gearing, they are expected to be insignificant enough for this model to still be viable. The added losses could also be calculated and accounted for in experiments with this model.

Figure 5.5- Twisted yarns do not need to be on the same axis as the wheels. Using a gear system, torque and energy from twisted CNT can be transferred to a wheel.
From these models, it is clear that stretching a CNT yarn would allow for more energy storage than twisting the yarn. However, stretching the yarn would also have a significantly higher power density and the yarn would reach its maximum strain too quickly for it to be a viable option in regenerative brakes. Twisting the yarn would require many rotations of the wheel and strain would build up at a slower rate. The slower rate is consistent with how brakes are used in real life: gradual braking rather than sudden stops. Mathematical models and preliminary experiments agree that the twisting mechanism is the most suitable method of energy storage for regenerative braking.

5.2 Design Criteria

Once a twisting mechanism was decided on, a detailed design was the next step. Before designing such a mechanism, the following design criteria were considered.

1. Collect energy:

   A regenerative braking system must take energy from a moving vehicle to use later. A CNT element attached to a rotating wheel will extract energy from the wheel, slowing it in the process. In most regenerative braking systems, traditional brakes supplement the regenerative brakes to remove low levels of kinetic energy from the system and bring the car to a complete stop. In the small models created for this project, the twisted CNT can extract all energy from the system and supplemental brakes are not needed. Supplemental braking should also be available for safety, of course.

2. Release energy:

   Once enough energy is stored in the regenerative braking system, that energy must also be released. Figure (3) showed that the same mechanism that extracts energy from the wheel can operate in reverse to put energy back into the wheel.

3. Not exceed failure strain:
Should the CNT be stretched past its failure strain, it would break and need to be replaced in the mechanism. Though this would be an easy fix in the lab, it would not be sustainable in a commercially available braking system. To prevent overstretching, a load cell must be attached to one end of the CNT to measure axial force and strain.

5.3 Generation 1 Design

![Figure 5.6](image)

Figure 5.6- A simple mechanism was designed similar to Figure 5.3. There is no axle to interfere with the CNT, so it can directly attach to the wheel. This model was used to confirm output energy found in Chapter 3.

A regenerative braking mechanism must first collect and store energy, and later release it. Preliminary experiments proved that CNT yarn was able to store enough energy to be a potential candidate for this application. These experiments did not, however, show how much of this stored energy would be available to be released and accelerate a wheel. The purpose of this first mechanism is to answer that question.

Only a portion of the stored energy will be available to accelerate the wheel. Preliminary experiments showed that there is some strain energy loss due to hysteresis (about 60% efficiency after
many cycles of loading and unloading). Additionally, in these experiments, a motor untwists the CNT tape at a set speed. Thus, no conclusion could be made about the power available from the yarn. In the present mechanism, the yarn unwinds at its own pace, rotating a wheel. By allowing the yarn to do so, more conclusions are drawn about its viability in regenerative braking.

The concept behind this first mechanism is to create the most basic system to allow a twisted yarn to rotate a wheel by unwinding. A yarn is fixed at one end and attached to a wheel at the other end. A small rectangular attachment was designed in SolidWorks to connect the yarn with the load cell. The load cell is attached to a micropositioner (Thorlabs model RB13M). The micropositioner allows for adjustment of the tape’s fixed end to maintain alignment with the other end. A 3” diameter wheel was designed in SolidWorks and machined by Jon Doughty at Northeastern University. The wheel is attached to a 3”, long 1/8” diameter shaft (McMaster-Carr model 1162K108). The shaft is held in place by two miniature ball bearings (NBM Technologies model DDRI-418ZZ). Two bearings were used to prevent transverse movement of the shaft. The bearing mounts were also designed in SolidWorks and machined by Mr. Doughty. The only strict dimension on the machined parts was the hole diameter in the bearing mounts to ensure that the bearings fit tightly.

Lastly, the base of the mechanism is designed to keep the entire mechanism stable and accommodate various placements of the shafts and bearings. This allows for different lengths of yarn to be tested, as well as the addition of other components in the second mechanism. The base was designed in SolidWorks and printed on a Stratasys 3D printer. Bases to hold the micropositioner and Philtec sensor (discussed in section 5.5) were also designed in SolidWorks and printed on a MakerBot Replicator 2 3D printer.

The following table includes all of the rotating parts for the simple mechanism and their calculated rotational moments of inertia.
The moment of inertia wheel dominates the system and dictates how fast it will rotate. The CNT yarn also has a moment of inertia, but it is negligible compared with any of the other pieces. If the model is driven by 1” (25.4 mm) of CNT yarn, there is roughly 1.1 mJ of available energy and a starting torque of 7.9E-5 N*m (as shown in Chapter 3). If all available energy and torque were converted to rotational kinetic energy (disregarding any additional losses such as friction in the bearings) the system would begin to rotate with an angular acceleration of 1.9 rad/s^2 from rest and reach a maximum angular velocity of 7.4 rad/s. These values would be reduced by the existence of energy losses such as friction. A maximum wheel speed of 10-15 rad/s at an initial acceleration of 2-3 rad/s^2 is desirable for the final mechanism. Longer multiple-ply yarn was used in experiments to reach these speeds.

5.3.1 Desired Outputs

There are two important pieces of information that can be obtained from this mechanism: torque and angle of rotation, from which strain energy and other relevant information can be calculated. One end of the CNT tape is attached to a load cell (described in detail in section 5.5). The load cell prevents rotation of the CNT at that end (in the same way a wall would) and records the axial force caused by stresses in the twisted yarn. Torque is estimated from the measured force using the proportionality constants found in Chapter 3. The load cell provides data in real-time, allowing the user to stop the yarn from twisting and ensure that the yarn does not over-twist and break. The number of rotations is measured with the Philtec optical displacement sensor (described in detail in section 5.5). On one surface of the wheel, there are alternating areas of high and low surface heights. The change in

Table 5.6- Moment of Inertia for Generation 1

<table>
<thead>
<tr>
<th>Part</th>
<th>Moment of Inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td>3” Shaft</td>
<td>5.859E-9 kg*m^2</td>
</tr>
<tr>
<td>Ball Bearings x2</td>
<td>5.647E-9 kg*m^2</td>
</tr>
<tr>
<td>Wheel</td>
<td>4.010E-5 kg*m^2</td>
</tr>
<tr>
<td>Total</td>
<td>4.012E-5 kg*m^2</td>
</tr>
</tbody>
</table>
these heights is detected, and the angle of rotation is easily found. The actual sensor readings are not important here, as long as there is a clear difference between the high and low points from which rotation rate can be determined.

By collecting torque and rotation data, the strain energy stored in the yarn and delivered to the wheel is found. The results of this experiment are similar to the results of the preliminary torque experiment, but show the effect of allowing the yarn to unwind on its own. Knowing how much energy is stored or released versus the angle of rotation helped in the design of a more complex design.

5.4 Generation 2 Design

Figure 5.7 - A more complex design allows for new design criteria to be met. This model more closely resembles a regenerative braking system than the first model.

The purpose of the second model is to more closely mimic a regenerative braking system. While the first model showed the viability of CNT yarn to store energy and release energy from a rotating wheel,
this model proves the viability of CNT yarn in an actual braking system. There are three main concepts that are addressed by this design:

1) Store energy without releasing it or continuing to collect it:

   In the simple model, every rotating piece is connected. If wheel is rotating, so is the yarn. In a real world situation, a person may not want to accelerate or brake (e.g. letting a car coast without using the brakes). In this case, the yarn needs to be disconnected from the wheel and prevented from untwisting.

2) Prevent re-twisting of the CNT yarn in the wrong direction:

   After the CNT yarn has released all of its stored energy, the wheel will continue to rotate and re-twist the yarn in the opposite direction, acting like a pendulum. This will cause unnecessary wear and tear on the yarn, provide additional energy losses when converting between kinetic energy in the wheel and strain energy in the yarn, and begin to slow the car again. To prevent this from happening, the yarn cannot be permanently attached to the wheel and must be able to disengage from the other rotating pieces.

3) Accelerate the wheel in the positive direction:

   In Generation 1, the yarn and wheel act as a pendulum. The wheel rotates in the positive direction and the yarn follows while collecting strain energy. Once the yarn’s torque is sufficiently large, it untwists and the system rotates in the negative direction. In a regenerative braking system, the yarn must load and unload in the positive direction. For this to occur, there must be a mechanism that changes the direction of rotation.

5.4.2 Additions to Generation 1

The most substantial addition to the simple mechanism is the gearbox that can reverse the direction of rotation. In order to accelerate the wheel in the positive direction, the energy must be
released in the opposite direction from that in which it is stored. For this to be possible, there must be two settings in the gearbox: one that does not reverse rotation and one that does. Because two settings are necessary, the yarn and wheel must be initially separate, with some optional method of connection. The starting point for this design is shown below.

Figure 5.8- This is the base concept of the 2nd generation design. The yarn cannot be permanently attached to the wheel. Between the two shafts, a structure must connect them and designate whether to transmit rotation directly, or reverse the rotation.

To reverse the direction of a rotation, three bevel gears are used (McMaster-Carr model 7297K11). The bevel gears have a 1/8” bore diameter to fit on the two primary shafts seen in Figure 38 (these shafts are the same size as those used in Generation 1). An additional bevel gear between them transmits rotation in the opposite direction. This middle gear is adjustable so that it is disengaged when the direction of rotation should not be reversed.

To keep the direction of a rotation constant, a coupling device was designed to fit over both gears. The coupling device, shown in Figure 41, has teeth built into it to prevent independent rotation of the gears. This results in the two shafts rotating together. It was designed in SolidWorks and printed
with a MakerBot Replicator 2 3D printer. This design is not very practical for a full-sized car, but it is helpful for modeling the concept.

![Image](image_url)

**Figure 5.9** This coupling device has teeth extruded along the opening. This prevents gears from rotating independently, resulting in rotation being transmitted directly between the two shafts.

This gear box also helps solve the other issues for the complex design. It provides a method of completely detaching the yarn from the wheel. By adding a ratchet and pawl that can be engaged or disengaged manually, the yarn can be stopped in place or allowed to rotate freely even when it is not attached to the wheel.
Figure 5.10- The adjustable bevel gear will engaged when direction of rotation should be reversed (during loading), and disengaged to keep the direction constant (during loading). The coupling device will engage only when keeping the direction constant.

The final piece needed for this design is a structure to hold all of the new components. The ratchet is placed on the shaft connected directly to the yarn. The pawl is held in place by a separate shaft held above yarn’s axis. The bevel gear is held in place by a shaft, which, is held in place by bearings. A small, sliding block was designed and 3D printed to hold the bearings of this shaft and allow for small movements of the shaft. Screws hold the block in place while engaged and can be removed to disengage the bevel gear. This design is shown below. The base structures, ratchet and pawl were all designed and 3D printed.

Other methods of accomplish same direction rotation were explored, but none were deemed practical for a small model. One method of accomplishing this is to simply push the two shafts together with friction clutches. The bearings in this mechanism are securely in place so this is not an option (although it may be a viable method in a full-sized system). A second option is to use two gears between
the shafts. This reverses the rotation twice, resulting in no overall change. This concept is shown below.

There are very few sizes of small gears without manufacturing custom gears, so this design cannot be used for the mechanism (although it is also a viable option for a full-sized system).

Figure 5.11- In this position, the large bevel gear is engaged and reverses the direction of rotation between the two shafts. The two small gears on the side are disengaged.

Figure 5.12- In this position, the large bevel gear is disengaged and the two small gears are engaged. These small gears reverse the direction of rotation twice. This results in the same direction of rotation between the two shafts, albeit with greater energy losses due to more friction between gears.
Table 5.2 lists the moments of inertia of the rotating pieces in the Generation 2 design.

<table>
<thead>
<tr>
<th>Part</th>
<th>Moment of Inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td>3” Shaft x3</td>
<td>5.859E-9 kg*m²</td>
</tr>
<tr>
<td>Bevel Gears x3</td>
<td>3.337E-9 kg*m²</td>
</tr>
<tr>
<td>Ratchet</td>
<td>8.065E-8 kg*m²</td>
</tr>
<tr>
<td>Ball Bearings x6</td>
<td>5.647E-9 kg*m²</td>
</tr>
<tr>
<td>Wheel</td>
<td>4.010E-5 kg*m²</td>
</tr>
<tr>
<td>Total</td>
<td>4.024E-5 kg*m²</td>
</tr>
</tbody>
</table>

The moment of inertia of the wheel still dominates the moment of the system. However, the gears will add significant resistance due to friction between them. Without considering these additional friction losses, a 1” (25.4 mm) piece of tape could again provide the system with an angular acceleration of 1.9 rad/s² from rest and a maximum angular velocity of 7.4 rad/s. However, it is expected that substantially greater amounts of CNT material will be required to drive the system in practice. Results from various combinations of length and ply tape in the first generation experiments were examined to determine the amount of tape to use for this setup.

5.4.3 Desired Outputs

The load cell and Philtec sensors take the same measurements as they did in the first generation. With this data, a clearer picture is painted about how CNTs will function in a regenerative braking system. The overarching question this mechanism answers is how efficient this braking system will be. Of course, this model does not account for some other major losses in a real automobile, such as friction with the road, but those can be approximated based on the present results.

5.5 Experiment Setup

A Philtec fiberoptic sensor (model RC171) measures distance from a target based on a signal that is reflected off the surface. Surfaces with low reflectivity appear even further away due to the
signal either diffusing or scattering. In tests, a car was modeled with a wheel containing 10 alternating high and low surfaces (seen in figure 5.13). The Philtec sensor measured distance from the wheel as it was rotating. At every instance of surface height change, a time point was collected. A simple MATLAB code counted these instances of significant height change, yielding position data with a resolution of 1/10 of a rotation, which was differentiated to find velocity and acceleration. The low surfaces of the wheel were covered in masking tape to create a greater distinction between the optical signals of the two surfaces at different heights. The sensor runs at a frequency of 62 Hz, thus being able to accurately measure an angular velocity of up to 6 revolutions per second.

Figure 5.13- This is the wheel used to simulate a vehicle. There are five raised and five lower surfaces with which rotation is measured. The low surfaces were covered in masking tape. This results in lower reflectivity, and an even higher Philtec reading.
A load cell (SMD Sensors model S251) measured axial force produced by the twisting CNT. Axial force was converted into torque using the proportionality constant found in Chapter 3 (the proportionality constant changes based on the number of plies of CNT tape). The load cell has a 50 N full scale and less than 1% full scale error. With torque estimated from the load cell measurements and angle of rotation determined by the Philtec’s measurements, all other relevant information (such as energy and power) was calculated.

5.5.1 Causes of Error

There were a number of factors that could skew the results of these experiments. Individually, most of these factors can be controlled with proper sample preparation and can help explain some irregularities in the data.

Two types of adhesive were used depending on the amount of CNT tape being used. A 5-minute epoxy was used for 1 and 2-ply samples. It was found to have a maximum strength under 7 N, not
strong enough for use with 3-ply samples. For 3-ply samples, a stronger adhesive with a longer cure

time was used. Both types of adhesive have the potential to cause failures or non-ideal behavior, but
careful sample preparation helps to curb them. Figure 3 shows a glued 3-ply sample. The three strands
were spread out at the point of gluing because of imperfections in the fabrication process. This caused
additional strain in the outside pieces of tape, potentially leading to premature failure. It also shortened
the twisted section of CNT tape, which provides the torque to the system. A shorter length provides less
material for torsional energy storage.

![Image](image.png)

Figure 5.13- A 3-ply sample glued together. The three strands spread out at the point of gluing, causing additional strain in the outside pieces of tape and potentially leading to premature strain failure.

In Figure 5.13, tape was glued at the edge of the shaft and exited the glue at an angle, creating a
misalignment. To alleviate this problem, careful preparation was employed in subsequent tests, as seen
in Figure 5.14. This was difficult to achieve with the stronger adhesive, due to its long cure time. In
some tests, glue also spread past the ends of tape, in effect reducing the length of one or more plies of
tape and causing slack in the others. Again, careful preparation minimizes these effects.
Figure 5.14- CNT was glued on the end of the shaft, but emerged from the glue at an angle. This resulted in the tape twisting in a cone shape, the same effect seen when attaching the tape with a hook.

Figure 5.15- Glue crept up the side of one strand of tape. This prevented that portion of CNT to twist, effectively shortening the length of the strand and causing higher strain in that strand.

When working with a multiple ply sample, preparing slightly unequal lengths of tape is unavoidable. This causes some plies to be in tension, creating torque and storing energy, while others are slack (no tension) and contribute less to the energy storage. This effect is seen in Figure 5.16. Finally, the CNT tape itself can have small defects, such as those seen in Figure 5.17. These defects often do not appear to have a significant effect on the system’s performance, especially when the tape is
compact from twisting, but such no uniformities can contribute to energy storage differences between samples.

Figure 5.16- 3-ply CNT tape attached with glue. Only one thread is taut, while the others have significant slack. This causes one ply to reach its maximum strain much faster than the others.

Figure 5.17- Every sample of CNT tape has some defects, like the ones seen here. Samples with large defects were not used. As samples became more compact with repeated loading and unloading, the defects become less noticeable.

The initial mechanism designs used small hooks to keep the CNT tape in place, instead of glue. The goal was to tie the CNT to the hook, ensuring a strong connection that could be well-centered on the outermost point of the hook. Initial testing with hooks on the simple mechanism exposed two problems that lead to hooks being replaced by 2-part epoxy. First, the CNT’s tape knot was difficult to
center on the hook, causing some misalignment between the two tape ends (similar to the effect seen in Figure 5.13). Because one side was rotated off-center, the tape followed a cone-like path during twisting and untwisting. This caused additional strain to the tape that did not contribute any torque, preventing the sample from reaching its maximum torque and useful energy output. A much larger issue with attaching CNT tape via hooks was slippage. At high torques, the tape’s knot slipped against the hook, instantly creating slack in the yarn and reducing any built up torque. In multiple trials, the knot came untied at high torques. Because of these factors, the hook method was discarded and the adhesive alternative was used.

In both mechanisms, an interesting phenomenon was observed during both loading and unloading of CNT tape. Kinks form in the tape during loading, causing a sudden decrease in torque when they are released. Once the kinks disappear, the axial force continues on the expected trend. During unloading, the opposite happens. The kinks reappear, but instead cause an increase in torque as they induce more tension in the tape. Once the tape continues to unwind, the kinks naturally disappear. These occurrences do not appear to have any lasting negative effects, but are noticeable in the data (seen in Figure 8). It is expected that the formation of kinks may be partially controlled by adjusting the slack in the CNT material.
During unloading, kinks formed in the CNT tape. These kinks are unavoidable and do not have any lasting negative impacts, but their presence is noticeable in the load cell data. As the kink forms, the force (and torque) increase until the kink vanishes, at which point normal behavior resumes.

Figure 5.18: During unloading, kinks formed in the CNT tape. These kinks are unavoidable and do not have any lasting negative impacts, but their presence is noticeable in the load cell data. As the kink forms, the force (and torque) increase until the kink vanishes, at which point normal behavior resumes.
Chapter 6

Data/Results

This chapter includes test results for both regenerative braking models and discussions on these results. Energy and power information about CNT tape driving a regenerative braking system was the ultimate goal for these tests. The Generation 1 model was used to determine the capability of a CNT to provide enough power to accelerate a wheel, and a complex mechanism more closely mimicked regenerative braking in a driving scenario. The data was then used to judge the viability of a regenerative braking system using twisted CNT as the energy storage medium.

6.1 Generation 1 Testing

The purpose of the simple model was to determine the amount of energy that is released by the CNT tape when it is allowed to freely unwind. In previous experiments, twisted CNT tape was unloaded by a motor at a prescribed speed. This mechanism does not model a regenerative braking system. Rather, it models the portion of a braking system that accelerates the wheel, but not the portion that gathers energy. The mechanism was designed to minimize all energy losses. To test it, the CNT tape was manually twisted either a set number of rotations or to a certain torque. When the tape was
released, the stored energy and torque rotated the wheel. Samples of various length and ply were used to test this mechanism.

Figure 6.1- Generation 1 setup. This model is used to observe the energy storage and release capabilities of CNT tape in a simple wheel driven mechanism. On the right, the Philtec sensor indirectly measures angle of rotation. On the left, CNT tape is attached to a load cell, which measures axial force of the twisted yarn. Note that the slack in the CNT tape has not yet been removed in this photograph. This sample is 3” (76.2 mm) long.

6.1.1 1-ply Results

Three samples of various lengths were tested, but none produced enough torque to rotate the system. The maximum torques before failure are listed in Table 1. The apparent linear relationship between length and torque is misleading. It was shown in Chapter 4 that maximum torque is independent of tape length. The 2” (50.8 mm) sample used the hook method of attachment. The tape
came untied before any rotation occurred. The 1” (25.4 mm) sample used the quick drying epoxy method of attachment. This glue allowed for some movement of the CNT tape at the glued area, possibly creating lower than maximum torque. The 3” (76.2 mm) sample used the strong adhesive, but the tape still did not produce enough torque to rotate the mechanism.

Table 6.8: Maximum torques before failures differed between trials for various reasons (listed above). These torques are on the same order of magnitude as seen in Chapter 3, indicating that 1-ply tape does not produce enough torque to accelerate the mechanism.

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>Force (N)</th>
<th>Torque (N*m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.4</td>
<td>1.8</td>
<td>3.00E-05</td>
</tr>
<tr>
<td>50.8</td>
<td>2.7</td>
<td>4.50E-05</td>
</tr>
<tr>
<td>76.2</td>
<td>3.5</td>
<td>5.83E-05</td>
</tr>
</tbody>
</table>

6.1.2 2-ply Results

Tests using samples of 2-ply CNT tape proved to be much more successful than testing using 1-ply. During these tests, a number of concepts were assessed, including validating the mathematical model for increasing tape length, the efficiency of staying within the elastic region, and the result of inducing overstretching the tape and creating slack.

From the angle of rotation data obtained using the Philtec optical sensor, time points at every 1/10 rotation were available, and angle of rotation was easily calculated. Angular velocity was calculated by differentiating angle of rotation with respect to time. Angular acceleration was calculated by differentiating angular velocity with respect to time. Each of these curves was smoothed in MATLAB using a running average over 25 data points. Both smoothed and raw data can be seen in the plots below. The following three plots show the wheel dynamics calculated for a single trial.
Figure 6.2- The wheel’s movement during unloading follows the expected path: high acceleration followed by maximum velocity at zero acceleration, followed by deceleration to rest.
Figure 6.3 - The angular velocity of the wheel was found by differentiating the angle of rotation (Figure 10) with respect to time. The red line represents the smoothed curve for the green data.
Figure 6.3 - The angular acceleration of the wheel was found by differentiating the angular velocity (Figure 11) with respect to time. The red line represents the smoothed curve for the green data.

Torque was calculated in two ways. First, the net accelerating torque on the wheel system was calculated using the angular acceleration and the moment of inertia (which was calculated in Chapter 5). This torque represents the net torque on the system (including resistive torque from the bearings). This torque starts as a positive, and drops to a negative value (due to a net negative acceleration comprising frictional deceleration and the remnants of the CNT’s acceleration) before returning to zero at rest. It does not represent the torque in the yarn. The yarn itself cannot have a negative torque unless it is winding in the opposite direction, which it did not do in these tests. This was confirmed by counting total numbers of rotations during data analysis after each test. Second, the torque was estimated using the load cell data and the proportionality constants found in Chapter 3. This torque obtained from the load cell is referred to as the yarn torque. The following plot shows the two torques from one trial.
plotted on top of each other for comparison purposes. Note that the torques follow the same shape with different values, as expected.

Figure 6.4- Two values of torque are plotted against time. The green line is the torque found with the proportionality constant and axial force (also shown in Figure 13). The blue line is calculated from the acceleration of the wheel and represents net torque on the system, rather than torque from the yarn.

The net wheel torque was easily plotted against angle of rotation because both sets of data are on the same time scale. The yarn torque was collected at a different frequency, but was interpolated in order to be plotted against angle of rotation. Energy was calculated by integrating torque over angle of rotation. The negative values of the Philtec torque were ignored for this calculation. The kinetic energy at maximum angular velocity was also calculated for. Power was calculated by differentiating energy with respect to time. Energy and power from load cell data were more important because they show the behavior of the CNT itself, and this project is analyzing the viability of CNT for regenerative braking.
Energy and power calculated from the Philtec data were helpful in determining the efficiency of the mechanism. Energy and power densities were found by dividing the energy or power by the mass of the CNT material. The linear mass density of the CNT tape was calculated by weighing a known sample length. The following plot shows the power output of both the yarn and wheel as a function of time for one trial.

![Plot showing power output as a function of time](image)

Figure 6.5 - Power is found by differentiating the torque from Figure 13 with respect to time.

2-ply CNT tape reached a maximum energy density of 10.7 kJ/kg during unloading, close to the value of 8 kJ/kg found in Chapter 3. This difference can be attributed to varying strength of samples. This value gets smaller as strand reorganization occurs and plateaus around 8-9 kJ/kg after repeated tests. The maximum torque (estimated from the axial force) ranged from 1.6E-4 to 2.3E-4 N*m, depending on the sample. This is also consistent with the values found in the preliminary experiments.
The two methods of calculating output energy density of the system (via rotational kinetic energy and net torque on the system as calculated by the acceleration the wheel) yielded the same results. The wheel’s extracted energy density corresponded to a maximum energy density of 1.8 kJ/kg. The output energy of the yarn (measured from the estimated torque) was consistently 5-10 times higher than the maximum output energy of the system.

By increasing the length $m$ times, both the maximum number of rotations and maximum strain energy should increase by a factor of $m$, while the strain energy density should remain constant. To reach the target maximum strain, the 2.4” (61.0 mm) sample was rotated between 25-28 times. The 4.4” (111.8 mm) sample was rotated between 48-50 times. The 2.4” (61.0 mm) sample reached a maximum strain energy density 9.0 kJ/kg (about 1 mJ strain energy), whereas the 4.4” (111.8 mm) sample reached 10.7 kJ/kg (about 2.4 mJ strain energy). Here, the length of tape is increased by a ratio of 1.8, and the maximum angle of rotation also increases by a factor of 1.8. This agrees with the multiplication factors found in Chapter 4. The maximum CNT output energy density increased slightly between the samples, but the difference is within the typical observed scatter among CNT specimens and may be due to differences in the sample used. The total maximum strain energy increased by a factor 2.4 when the sample length was increased. This higher factor may be due to differences in sample strength. It could also possibly be due to slight differences in sample preparation (such as gluing at an angle). The energy density and total energy of the system followed the same trends as the CNT energy.

**Loading to the Same Angle of Rotation**

In some cases, the tape was rotated multiple trials to a specific number of rotation. Because the tape is rotated the same amount in each cycle, minimal structural reorganization is expected between the cycles. The tape experienced nearly the same torque and angular velocity responses in each trial.
Figure 6.6 shows torque and angular velocity plots for each cycle, and Table 2 shows data for these cycles.

Figure 6.6- The torque and angular velocity results for four trials after loading the wheel for 26 rotations (not starting at 0 rotations) in 4 consecutive trials. The results are nearly identical, indicating the yarn stayed in the elastic region and no significant realignment of individual CNTs occurred.

Table 6.2- Results of repetitive loading to a specific number of rotations. The energy and power output of the four trials remains constant because there is no new plastic deformation or CNT realignment. Many more trials of identical results would be expected if the yarn continued to be loaded to 26 rotations.

<table>
<thead>
<tr>
<th></th>
<th>Rotations Loaded</th>
<th>Rotations Unloaded after Release</th>
<th>Torque (N*m)</th>
<th>Acceleration (rad/s²)</th>
<th>Max Velocity (rad/s)</th>
<th>Tape Energy Density (kJ/kg)</th>
<th>System Energy Density (kJ/kg)</th>
<th>Power Density (W/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>27.20</td>
<td>21.1</td>
<td>1.64E-04</td>
<td>1.962</td>
<td>8.312</td>
<td>5.75</td>
<td>0.61</td>
<td>466.35</td>
</tr>
<tr>
<td>R2</td>
<td>26.20</td>
<td>22.5</td>
<td>1.72E-04</td>
<td>1.435</td>
<td>7.480</td>
<td>6.25</td>
<td>0.49</td>
<td>444.35</td>
</tr>
<tr>
<td>R3</td>
<td>25.80</td>
<td>20.1</td>
<td>1.65E-04</td>
<td>1.652</td>
<td>8.086</td>
<td>5.53</td>
<td>0.58</td>
<td>430.71</td>
</tr>
<tr>
<td>R4</td>
<td>26.00</td>
<td>19.4</td>
<td>1.72E-04</td>
<td>1.815</td>
<td>6.876</td>
<td>5.59</td>
<td>0.42</td>
<td>475.15</td>
</tr>
</tbody>
</table>
In these four cycles, the tape was loaded 26 full rotations. There is some variation in the number of rotations due to difficulties measuring the angle of rotation in real-time. ‘Rotations unloaded’ refers to the number of rotations the wheel spun when released. The energy and power densities (seen in Table 6.2) remain very consistent between the four trials. Additionally, the wheel dynamics do not change significantly between trials, indicating that the mechanism is stable and the ratio of stored strain energy to kinetic energy remains constant.

**Loading to the same torque value**

Tape was also repeatedly loaded to a set torque value. To reach this torque, the tape required the same number of rotations each cycle, indicating the tape was still within an elastic region and no new slack was expected to form. In each trial, the tape was loaded until the load cell reached a specific axial force, which was used to estimate the tape torque. Figure 6.7 shows identical torques during unloading after release at the same torque. Table 6.3 shows near identical energy values and wheel dynamics for these cycles, as well. During these cycles, the rotations loaded were nearly equal to rotations unloaded.
Figure 6.7- This plot shows the torque and angular velocity response after loading the yarn to a specified torque in two trials. The both responses appear to follow the same path during unloading.

Table 6.9- Results of repetitive loading to a specified torque. The results are nearly identical between the two trials.

<table>
<thead>
<tr>
<th></th>
<th>Rotations Loaded</th>
<th>Rotations Unloaded after Release</th>
<th>Torque (N*m)</th>
<th>Acceleration (rad/s²)</th>
<th>Max Velocity (rad/s)</th>
<th>Tape Energy Density (kJ/kg)</th>
<th>System Energy Density (kJ/kg)</th>
<th>Power Density (W/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>27.40</td>
<td>24.1</td>
<td>1.49E-04</td>
<td>0.793</td>
<td>7.542</td>
<td>8.60</td>
<td>0.96</td>
<td>551.70</td>
</tr>
<tr>
<td>T2</td>
<td>27.10</td>
<td>24.3</td>
<td>1.54E-04</td>
<td>1.319</td>
<td>7.454</td>
<td>8.78E</td>
<td>0.94</td>
<td>658.17</td>
</tr>
</tbody>
</table>

Reaching the same torque was possible without additional CNT realignment with each consecutive loading. To reach 1.5E-4 N*m, the tape was loaded to the same angle of rotation. Additionally, the energy and power outputs of the yarn are very similar. The maximum acceleration of T1 is significantly lower than T2, but this is likely due to an imbalance in the mechanism. Once the
imbalance was overcome, both samples reached the maximum velocity. These results demonstrate that CNT material can be repeatedly loaded to a torque and achieve the same results.

**Reaching the same torque after CNT reorganization**

Another concept that was observed using this mechanism was the effect of reaching the same torque in the tape after significant CNT reorganization. Trials O1 and O2 were not back to back; there were multiple trials between them. In trials O1 and O2, the tape was loaded to a constant axial force, which corresponds to a torque of 1.37E-4 N*m (estimated with the proportionality constants found in Chapter 3). Trial O2, however, took place after loading the tape to 2.3E-4 N*m of torque in a separate (unlisted) trial, which would have created realignment in the tape. From Table 6.4, it appears that the energy and power remain relatively unchanged between the cycles. However, more rotations are required to reach the same torque to produce these results. The second trial requires two additional rotations to reach its initial position, from which it follows the same path as the previous trial.

<table>
<thead>
<tr>
<th></th>
<th>Rotations Loaded</th>
<th>Rotations Unloaded after Release</th>
<th>Torque (N*m)</th>
<th>Acceleration (rad/s²)</th>
<th>Max Velocity (rad/s)</th>
<th>Tape Energy Density (kJ/kg)</th>
<th>System Energy Density (kJ/kg)</th>
<th>Power Density (W/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O1</td>
<td>18.20</td>
<td>17.4</td>
<td>1.37E-04</td>
<td>1.387</td>
<td>6.837</td>
<td>4.58</td>
<td>0.41</td>
<td>333.04</td>
</tr>
<tr>
<td>O2</td>
<td>21.20</td>
<td>19.9</td>
<td>1.37E-04</td>
<td>1.845</td>
<td>7.841</td>
<td>4.93</td>
<td>0.54</td>
<td>341.40</td>
</tr>
</tbody>
</table>

CNT tape output power was also analyzed from the data in these tests. The 2.4” (61.0 mm) sample reached a maximum power density of 625 W/kg (for a total of 7.75E-4 W) and the 4.4” (111.8 mm) sample reached a maximum power of 990 W/kg (for a total of 2.25E-3 W). Power density is expected to be independent of sample length, so this discrepancy is likely due to sample-to-sample differences.
6.1.3 3-ply Results

As expected, 3-ply tape produced significantly more torque and energy than 2-ply tape, resulting in much faster wheel dynamics. Unfortunately, 3-ply tape is considerably more difficult to prepare than 2-ply. When gluing 3 pieces of tape together, one piece typically shifts in the glue, resulting in a shorter length. This puts one piece of the tape under the most strain, partially negating the effect of using 3-ply tape. Because of this, the benefits of using 3-ply tape rather than 2-ply tape were not fully observed.

During testing, 3-ply CNT tape stored up to 9.6 kJ/kg of energy and reached a maximum torque of 2.82E-4 kJ. Unfortunately, these values were realized during a test using the quick-drying epoxy as adhesive. Because of this, the glue broke well before the tape was stretched to failure; the results therefore represent lower bounds of the 3-ply tape’s maximum performance. During testing with the stronger adhesive, all samples suffered from the issues with off-center mounting and slack as described above. The strong adhesive has a cure time of 15-20 hours, giving the tape plenty of time to shift and become uneven. This also results in a significant misalignment, causing additional strain on the tape.

3-ply CNT was loaded to the maximum torque achieved in 2-ply tests. At these values, the wheel reached a maximum acceleration of 6.6 rad/s² and a maximum velocity of 15 rad/s. Given better sample preparation for 3-ply tape, even faster wheel dynamics could be expected.

6.2 Generation 2 Testing

The purpose of the more complex model system was to more closely mimic a regenerative braking system. During unloading, the simple model released stored energy to a wheel in the opposite direction from the direction that the wheel would have rotated during the original energy extraction. The second generation system was designed to collect energy from a rotating wheel, store that energy, and release that energy upon demand to accelerate the wheel in the same direction as its motion during
the original energy extraction. Multiple tests were performed for this model. First, the yarn was loaded and released with the bevel gears remaining in place during both loading and unloading. The bevel gears switch direction of rotation between the shafts. This was done to observe the energy lost to friction in the gears. Next, the wheel was spun from rest and allowed to collect energy and release it without interference (acting like a pendulum). This test showed the efficiency between the kinetic energy of the wheel before and after regenerative braking.

The yarn was also loaded and released with the coupling device in place. This was done to observe the energy losses associated with this portion of the design. Finally, a test was performed to demonstrate collecting energy in one direction of rotation, and releasing it in the opposite direction. To reduce the torque and energy needed to rotate this system, a wheel with identical geometry to the wheel used in Generation 1 was 3D printed in order to reduce its mass. The new moment of inertia for the system is equal to 1.02E-5, and it is still dominated by the wheel’s rotational inertia. The lower moment of inertia enables the system to more easily tolerate the increased frictional losses of the gearbox.
Figure 6.8- Generation 2 Model. In this picture, the yarns are not preloaded. Before testing, yarns were preloaded to minimal axial force to ensure every rotation of the yarn produces torque.

Figure 6.9- Gearbox of the Generation 2 model. In the picture, the 3 bevel gears are engaged, resulting in reversed direction of rotation. The ratchet and pawl can be seen in the foreground, disengaged.
The length of the sample of CNT tape for this mechanism was 3.25” (82.6 mm), between the two lengths used for the 2-ply samples for the simple mechanism. As such, calculated values for the new sample should be between the two, if the systems were otherwise identical and there were no additional losses. This is, of course, not the case. There is additional friction due to the gears, but there is also a smaller moment of inertia due to the lighter wheel material.

The maximum energy density of the CNT tape during unloading was 4.2 kJ/kg, half the maximum energy density from the first mechanism’s 2-ply tests. The maximum torque achieved was 1.4E-4 N*m. This is slightly below the torque values seen in the simple mechanism. This difference is likely due to different samples, as additional energy losses in the mechanism would not affect maximum torque experienced by the tape. Table 6.5 shows the results from these trials.

Table 6.11- Results from loading and unloading Generation 2 model with the bevel gears engaged. This system results in the greatest energy loss and will give the most conservative energy and power results.

<table>
<thead>
<tr>
<th></th>
<th>Rotations Loaded</th>
<th>Rotations Unloaded after Release</th>
<th>Torque (N*m)</th>
<th>Acceleration (rad/s²)</th>
<th>Max Velocity (rad/s)</th>
<th>Tape Energy Density (kJ/kg)</th>
<th>System Energy Density (kJ/kg)</th>
<th>Power Density (W/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65.2</td>
<td>17.4</td>
<td>1.03E-04</td>
<td>1.993623</td>
<td>9.8668</td>
<td>2.35</td>
<td>0.29</td>
<td>404.43</td>
</tr>
<tr>
<td>2</td>
<td>25.9</td>
<td>20.4</td>
<td>1.16E-04</td>
<td>4.317485</td>
<td>12.25471</td>
<td>2.78</td>
<td>0.46</td>
<td>547.38</td>
</tr>
<tr>
<td>3</td>
<td>25.9</td>
<td>23.6</td>
<td>1.18E-04</td>
<td>4.756598</td>
<td>13.6831</td>
<td>2.90</td>
<td>0.57</td>
<td>601.58</td>
</tr>
<tr>
<td>4</td>
<td>38.5</td>
<td>31</td>
<td>1.38E-04</td>
<td>5.770942</td>
<td>17.16325</td>
<td>3.99</td>
<td>0.89</td>
<td>798.14</td>
</tr>
<tr>
<td>5</td>
<td>35.8</td>
<td>32.9</td>
<td>1.44E-04</td>
<td>9.482585</td>
<td>18.16963</td>
<td>4.20</td>
<td>1.00</td>
<td>1042.35</td>
</tr>
</tbody>
</table>

In these experiments, the tape only reached a maximum energy density of 4.2 kJ/kg, significantly less than the 9-10 kJ/kg seen in Generation 1. The system reached a maximum energy density of 1.0 kJ/kg, also less than the 1.8 kJ/kg seen in Generation 1. The ratios between strain energy and kinetic energy are between 5-10, similar to those seen in Generation 1. This ratio seems to be decreasing with each subsequent trial, indicating that once strand reorganization has occurred, the mechanism may be getting more efficient. This was seen before in Chapter 3. In the preliminary
experiments, the efficiency between input and output energy was very low during the first cycles, but increased to a maximum efficiency of 50-60% after many cycles. Unlike the preliminary experiments, the yarn is driving an entire mechanism so it is unlikely to reach the same level of efficiency.

It is also important to note the difference between Trial 1 and the other trials in this test. Trial 1 was loaded 65 rotations, yet only unwound 17 rotations upon release. The other trials, meanwhile, rotate similar numbers of rotations during unloading as loading. This is, again, due to the permanent strand realignment after the first cycle, while subsequent trials stayed in the more elastic region, allowing greater efficiency.

Next, the coupling device was also used during both loading and unloading. The wheel was spun at a high speed from rest and the tape converted that kinetic energy into strain energy, bringing the wheel to a stop. The tape then released the strain energy, accelerating the wheel. In effect, the system acted as a pendulum, losing some energy with each conversion between kinetic and strain energy.

Figure 6.10 shows one cycle of loading and unloading. At 2.5 seconds, the wheel was spun rapidly. At 5 seconds, the wheel came to a stop, and from there until 11 seconds, the yarn released its strain energy and accelerated the wheel. Because the wheel was spun so quickly from rest, the Philtec sensor could not read every height change of the wheel. Because of this, the maximum velocity shown 3 seconds is a low-end estimate of the velocity (35 rad/s). This also explains why the velocity curve is very unsteady. After release, the velocity reaches a maximum of 20 rad/s. This indicates that the regenerative braking system has efficiency up to 60%.
Figure 2- The coupling device was used during loading and unloading. The wheel was spun from rest, and allowed to collect and then release energy.

Initial tests were also performed in which the bevel gears were used during loading and the coupling mechanism was used during unloading. The strain energy in the tape dissipated by about 10% due to creep while changing between the gearbox settings. Apart from this additional loss, the mechanism worked and proved the regenerative aspect of the braking system.

Finally, a test was performed which was meant to mimic various braking conditions (energy is not converted from kinetic to strain energy as in a real braking situation; the tape is manually loaded).
Figure 3: The Generation 2 model was put through periods of collecting, storing and releasing energy to demonstrate how different settings of the mechanism may be used when driving.

Prior to time period 1, the tape was being twisted during breaking. During time period 1, the tape was allowed to unload before a brief period of time, before being abruptly stopped by the ratchet and pawl. In a real situation, this corresponds to releasing the gas and coasting. After period 1, the braking occurred again and more energy was collected in the tape. During period 2, the tape unwound briefly, stopped for a few seconds, and unwound again. This would correlate to a period of accelerating, coasting, and accelerating again. It could also represent a succession of quick bursts of acceleration and stops (such as in traffic). After period 2, the tape collected more energy, and released it for a long period of acceleration.
Though this graph does not have much quantitative meaning, it shows the three concepts this complex model was attempting to solve. It can store energy without releasing it or continuing to collect it. During periods where the torque is constant, energy is being stored and the wheel is disengaged. The car may be coasting, stopped, or accelerating without gathering more energy. The ratchet and pawl can also prevent re-twisting of the tape in the wrong direction. Though that was not an issue during this test, a sensor could engage the pawl if the tape becomes completely unwound. Lastly, the gearbox design allows the braking system to accelerate the car in the positive direction. During energy collection, the set of bevel gears is active, twisting the CNT tape in the opposite direction as the wheel. When the tape needs to be released, the coupling device is engaged, and the CNT rotates in the same direction as the wheel.

6.3 Summary

These mechanisms prove that CNT tape is a viable candidate for energy storage in regenerative braking for a model car. The mechanisms in this chapter showed that there are significant losses in converting strain energy to kinetic. While the CNT tape itself has an efficiency of 50-60% between loading and unloading (from Chapter 3), there is only a 10-40% conversion between strain energy and kinetic (40% conversion seen in Table 6.5). These values are in the same area as the 30% efficiency of electric regenerative brakes described in Chapter 2. In future generations of this mechanism, the efficiency will likely be higher.
Chapter 7

Conclusion

In preliminary experiments, it was determined that single-ply CNT tape could release up to 4.3 kJ/kg of strain energy. This value drops after subsequent trials due to individual strand realignment, but this effect plateaus after a few cycles to about 3.4 kJ/kg. Increasing the radius of CNT tape was not expected to have an effect on the energy density, but larger energy densities were observed for multiple ply tapes in practice, perhaps because of reduced effects of slack in wider CNT tapes. 2-ply CNT tape reached a maximum energy density of 8 kJ/kg, before plateauing around 5.7 kJ/kg. It was also determined that the efficiency between stored and released energy in the CNT tape was between 50-60%.

Two table-top models were designed and built to model a regenerative braking system. The first model simply converted rotational kinetic energy into strain energy, and vice versa. As predicted, 1-ply CNT tape did not produce enough torque to rotate the system. 2-ply CNT tape stored a maximum strain energy of 10.7 kJ/kg; a maximum of 1.8 kJ/kg was converted into rotational kinetic energy. This gave a ratio between input and output energy of between 5 and 10.
Compared to an actual automobile, this value is reasonable. Gasoline has an energy density of about 46 MJ/kg. A 3,000 lbs automobile traveling at maximum speed of 128 mph has a kinetic energy of 2.2 MJ. This yields a ratio of fuel to kinetic energy of 20.7, even higher than the ratio for the yarn. A mechanical braking system should regenerate more energy than an electric braking system because converting kinetic energy into strain energy offers the potential for lower losses than converting mechanical energy to electrical energy does. This braking system stores energy already in the mechanical domain, unlike electric cars which convert to the electrochemical domain to recharge the battery.

The primary issue with the Generation 1 mechanism was sample preparation. As discussed in Chapter 5, gluing the sample at a slight angle created a cone effect during twisting, causing an additional periodic strain to the system. Gluing multiple plies of CNT also proved to be a challenge, as one ply often ended up being shorter than the others. To prevent this problem in the future, multiple-ply CNT should be pre-twisted together. This would help keep the lengths of tape equal.

The Generation 2 mechanism added the regenerative braking component to Generation 1. With this mechanism the CNT element was twisted in the negative direction by using a bevel gear to change the rotation between two shafts. During unloaded, the bevel gear was disengaged and a coupling device held the two shafts together, preventing independent rotation. This allowed the wheel to be accelerated in the positive direction. Results of Generation 2 testing were more qualitative than Generation 1 results, but the mechanism acts as a proof of concept of the regenerative braking design.

In the future, the model could be upgraded and more quantitative results could be obtained from Generation 2. The coupling device should be improved to allow for easier (and possibly automatic) switching between gear box settings. A better attachment method for the CNT tape should also be designed. Future testing should determine the efficiency of the regenerative braking system. To do so, the kinetic energy of the system before and after braking can be compared to the strain energy of the
twisted CNT. The power densities of the braking system should also be explored in more detail. Finally, this design could be expanded to a small model car, and eventually a full sized motor vehicle.

Regenerative braking using carbon nanotubes elements as the energy storage medium were shown to be a legitimate possibility for the future. The CNT tape used in this thesis consisted of a very large number of disorganized individual CNTs. As the technology to make CNT assemblies improves, the CNT material will be able to store more energy and the efficiency of the braking system will increase. A CNT-powered braking system should be able to outperform a similar kinetic to electric style regenerative braking system. The two systems could be used in conjunction with each other, possibly reducing the need for friction brakes in some situations.
WORKS CITED