Plasmonics and Metasurfaces for Infrared Wave Engineering

A DISSERTATION Presented

by

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Abstract

In this dissertation, we investigate several novel passive components composed of plasmonic materials at infrared regime. As an example for a passive plasmonic component, we present a bandpass filter integrated into a metal-insulator-metal (MIM) waveguide at mid-infrared range. Design techniques already developed in microwave and circuit theory used to realize the filter. The insulator is air and metal parts are silver where their loss considered in our simulations. The filter passband is from 27 THz to 33 THz (9.1 µm to 11.1 µm) and the simulated insertion loss is 1.7 dB. The filter length is 16.9 µm, almost 1.7 times center wavelength (10 µm).

Then, two bandpass filters operating at 27-33 THz and 36-41 THz bands are integrated with a power splitter to form a frequency diplexer. The performance of the designed diplexer is outstanding in terms of channel isolation (better than 30 dB) and loss. The proposed design method is scalable in the infrared and visible range and can also be used to realize frequency multiplexers.

In chapter 2, a reflectarray metasurfaces composed of rectangular metallic patches on top of a grounded dielectric layer is presented. The novelty of the reflectarray lies in its polarization dependent reflection angle. The reflection angle can be designed independently for either of the two incoming polarizations.

We demonstrate a 16λ×16λ birefringent reflectarray operating at 8.06 µm wavelength. The array reflects the two orthogonal polarizations into +30º and -30º directions. This birefringent metasurface can be used in polarimetry applications and waveplate working in reflection mode.

In chapter 3, the concept of plasmonic graded index material is introduced and investigated. A 15λ×15λ graded index metasurface is designed to collimate and steer the power emerging from an aperture into a narrow beam. The graded index pattern is obtained using holography technique. The presented metasurface works at 5.2 µm with fractional bandwidth of 8%. The metasurface steers the beam to θ=30º direction and the Half Power Beam Widths (HPBW) of the beam are 6º and 16º in elevation and azimuth planes, respectively. The graded index metasurface can be used to pattern the aperture of Quantum Cascade Laser (QCL) to enhance its beam collimation.

In chapter 4, we present a novel subwavelength double-layer unit cell constructed of L-shaped and concentric loop nanoantennas which can independently manipulate the amplitude and phase of an incident wave. The unit cell is made of two layers of scatterers, where the first can tune the amplitude and the second the desired phase. We show that metasurfaces composed of this unit cell can be used to achieve arbitrary transmission amplitude and phase profiles. Furthermore, we illustrate that these metasurfaces along with Fourier Transform (FT) blocks can be used to realize unique Linear Space Invariant (LSI) transfer functions. This approach opens opportunity for light processing on flat platforms.
To my beloved family
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1. Introduction

1.1. Motivation

During the last decades, semiconductor integrated circuits have dramatically revolutionized the information processing capacity of computers. This revolution is driven by both miniaturizing the transistor dimensions to nm scale and operation frequency to GHz (nanosecond pulse width). Currently, 16 nm CMOS technology is used to develop new IC’s for the next generation of processors. However, this revolution is approaching its end as further size reduction seems to be impossible due to quantum tunneling effect. Also, CMOS technology is not suitable for higher frequency operation as transistor gain decreases and interconnects delays become significant.

An encouraging solution to further augment processing speed is to use optical signals (electromagnetic waves) as information careers. The delay (relaxation) times in optical processes are in femtosecond order. However, due to diffraction limit, current miniaturization level in dielectric based optical circuits is far below their electronic counterparts. Dielectric lenses cannot localize light to an area smaller than half a wavelength ($\lambda/2$) and dielectric resonators volume must be at least ($\lambda/2)^3$ to confine electromagnetic energy ($\lambda$ is 1550 nm for optical communications).

Also, conventional optical components such as lenses and waveplates are bulky, hence they are not suitable for integration. Lenses are required to have a curved surface carefully designed based on Snell’s law of reflection and refraction to focus the beam in the focal point. In graded index lenses the wavefront is shaped by gradual phase accumulation along propagation distance for several wavelengths. The waveplates are also working based on different phase accumulation for two orthogonal polarizations.

Metals as Plasmonic materials with negative electric permittivity in optical frequency range can interact with light in subwavelength scales. Metal-Dielectric interface supports long range propagating surface plasmon polariton (SPP), electromagnetic waves coupled to the oscillation of free electrons in the metal. Also, local surface plasmons excited in metal nano-particles show strong absorption/scattering properties. This subwavelength interaction with light is a promising
solution for miniaturizing photonic devices and circuits. They can find applications both in thin surfaces for wavefront engineering as well as optical circuit design.

The local and strong interaction of subwavelength plasmonic particles can abruptly change light phase or polarization as desired. Therefore, it is possible to design a thin planar pattern of such particles, metasurface, to shape wavefront of an incoming light. Such surfaces are thin and flat, perfect for integration into photonic integrated circuits.

Furthermore, the subwavelength particles interacting with light can function as optical lumped elements. The notion of lumped element belongs to the circuit theory which is well developed at the low frequency range of electromagnetic spectrum (RF and Microwave range). Numerous analysis approaches and synthesize techniques are already studied and established in this area. The building blocks of circuits, lumped elements, are characterized by a voltage-current relation. For instance, this relation is a simple proportion for a resistor (V=RI). These lumped elements are functioning as modules of the circuit allowing simple and powerful design/synthesis approaches. One would imagine using the same powerful and developed techniques at higher frequency range of the spectrum (THz, IR and visible) to design advanced optical circuitry, provided lumped optical elements are available. Hence, the idea of optical lumped elements such as nanoresistors, nanoinductors and nanocapacitors promises the realization of optical nanocircuits.

1.2. Surface Plasmon Polaritons and Local Surface Plasmons

Surface plasmon is the coupled oscillation of electromagnetic waves and free electrons in the metal-dielectric interface. An unbound flat metal-dielectric interface (See Fig. 1.1 (a)) supports surface plasmon polaritons, surface electromagnetic waves coupled with electron oscillation along metal-dielectric interface. It is straightforward to solve Maxwell’s equations for the unbound flat metal-dielectric and obtain the propagation phase constant along the surface wave (1.1)

\[ k_{SPP} = k_0 \sqrt{\frac{\varepsilon_d \varepsilon_m}{\varepsilon_d + \varepsilon_m}} \] (1.1)
where $\varepsilon_d$ and $\varepsilon_m$ are relative electric permittivity of dielectric and metal, respectively and $k_0$ is free space wavenumber. Note that SPP propagates only when dielectric permittivities of the two media have **opposite signs**. Assuming lossless electric and metal, (the values of $\varepsilon_d$ and $\varepsilon_m$ are real) then $\varepsilon_m < 0$ will guarantee SPP propagation along the surface. Metals are considered plasmonic material at optical frequency range because of their free electrons. Therefore, their electric permittivity is described with Drude model\[3\].

$$\varepsilon_m(\omega) = 1 - \left(\frac{\omega_p}{\omega}\right)^2 \quad (1.2)$$

where $\omega_p$ is the plasma frequency of metal and oscillation damping of electrons is ignored. The plasma frequency of some noble metals such as gold and silver is in visible and ultraviolet range ($10^{15}$ Hz). According to Drude model, at frequencies below the plasma frequency, metal electric permittivity is negative and SPP propagation is possible. Fig. 1.1 (b) shows the dispersion diagram of the SPP mode propagating along the interface of a metal and a dielectric with $\varepsilon_d = 1.8$. Also shown is the dispersion diagram of the dielectric. According to Eq. (1.1), the SPP does not propagate at frequencies higher than $\omega_{sp}$ where $Re\left(\varepsilon_m(\omega_{sp})\right) = -\varepsilon_d$ ($\omega_{sp} = \omega_p / \sqrt{1 + \varepsilon_d}$).

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Fig. 1.1. (a) The unbound flat metal and dielectric interface supports SPP. (b) Dispersion diagrams for SPP propagating at the interface of a metal and dielectric.
Note that dielectric line does not intersect with SPP dispersion curve except at the origin. Therefore, the SPP is bound to the surface or equivalently free space radiation cannot directly couple to SPP. Also, at low frequencies the SPP is loosely bound to the surface and propagation constant is close to that of free space case. However, at high frequencies, the SPP bounds tightly to the surface and becomes a slow wave.

Let’s define the effective refractive index of SPP as

\[ n_{SPP} = \sqrt{n_d n_m \varepsilon_d \varepsilon_m} / \varepsilon_d + \varepsilon_m \]  

(1.3)

Then the SPP wavelength obtained from Eq. 1.1 and Eq. 1.3

\[ \lambda_{SPP} = 2\pi / k_{SPP} = \frac{\lambda_0}{n_{SPP}} \]  

(1.4)

where \( \lambda_0 \) is the free space wavelength. The metal permittivity, SPP effective index and wavelength are shown in Fig. 1.2. The same quantities are also shown for the case of free space filled with the dielectric. As demonstrated in these curves, when the frequency is small compared to \( \omega_p \), metal is essentially a perfect conductor. At frequencies close to the SPP cut off (\( \omega_{sp} \)), the SPP effective index becomes large and the SPP wavelength sharply decreases implying a highly confined slow wave.

SPP’s are propagating along the interface, whereas local surface plasmons (LSP’s) are plasmon oscillation of bound metal-dielectric geometries such as metallic nano-particles. The particle dimensions should be much smaller than the wavelength of the light (\( l \ll \lambda \)).

The light-matter interaction in a metallic nano-particle is similar to electrical mechanical energy conversion in an electromechanical resonator. An external electric field displaces electrons and positive ions to opposite sides of the particle. Then, the electric potential energy, stored in the electric field of positive ions and electron, transforms into kinetic energy when the charged particles attract each other. An external field with frequency equal to nano-particle resonance frequency (\( \omega_{sp} \)) can excite the oscillation and enhance the local field intensity (See Fig. 1.3). For example, the dipolar resonance of a small metallic sphere in air is excited when \( \varepsilon_m = -2\varepsilon_i \) [1].
The resonance frequency of the nanoparticles is independent of their size provided that $l \ll \lambda$. Nevertheless, particle shape and material can change the resonance frequency considerably.

![Graphs](image1.png)

Fig. 1.2. (a) The absolute value of permittivity. (b) Effective refraction index and (c) Propagation wavelength for the SPP and dielectric. When the frequency is small compared to $\omega_p$, metal is essentially a perfect conductor. At frequencies close to the SPP cut off ($\omega_{sp}$), the SPP effective index becomes large and the SPP wavelength sharply decreases implying a highly confined slow wave.

![Graphs](image2.png)

Fig. 1.3. External excitation and electron displacement in plasmonic nanoparticles. At the resonance wavelength, the oscillation intensify and absorb the incident field.
1.3. A Review of Plasmonics

Plasmonics focuses on exploring the interaction of light and metallic structures at infrared and visible part of spectrum. SPP propagation, localization and radiation are among the most interesting subjects of study in this field. These phenomena are widely investigated and developed in recent years. We briefly review some of the most important works in guiding, localizing and radiation of light in the scope of this dissertation.

1.3.1. Plasmonic Nanowaveguides and Circuits

The pioneer work in light nanoguiding beyond the diffraction limit was demonstrated in [4]. It was analytically proven that a metallic nanowire with negative dielectric permittivity \(\text{real}(\varepsilon_r) < 0\) supports a 1D optical wave with a diameter in nanometer range. This work then initiated the exploration of various plasmonic waveguides with different geometries such as nanowires [5, 6] nanoparticle chain [7], metal-insulator-metal [8, 9], strip [10], slot [11, 12], V-groove (channel) and wedge [13-15]. In these works, subwavelength confinement and guiding of light in metal-dielectric waveguides is presented. The interface of the plasmonic material and dielectric is designed to confine SPP in transverse plane. Propagation characteristics such as modes, dispersion diagram, and propagation length for these waveguides are thoroughly studied. For instance, a 50 nm gap etched in 50 nm-thick silver on a Silica substrate is studied in [12]. The fundamental mode of this deep subwavelength waveguide is quasi-TEM. The mode is confined in the slot region and especially close to wedges. The propagation length at optical communication wavelength \(\lambda=1.55 \mu m\) is \(\sim 20 \mu m\). Also, this waveguide offers a modal size of \(\sim 87 \text{ nm}\) at optical communication wavelength. Modal size is defined as the square root of the area where the mode power density is larger than \(1/e^2\) of the peak power density. The modal size of a Si based waveguide inside Silica is at least \(\sim 400 \text{ nm}\) at \(\lambda=1.55 \mu m\). Moreover, the modal size variation of the slot waveguide is small as we change the frequency.

Another interesting plasmonic waveguide, i.e. channel waveguide is theoretically studied in [13]. V shape grooves with depth of 1.1-1.3 \(\mu m\) and closing angle of 25\(^\circ\) are fabricated in a 1.8 \(\mu m\) thick gold film on Silica (The groove opening is 530 nm). Unlike the slot waveguide, the fundamental mode of the channel waveguide is a hybrid groove-wedge mode at optical communication range. The propagation lengths of the groove and wedge modes are 53 \(\mu m\) and
200 µm at 1.4 µm wavelength, respectively. Note that the propagation length of a SPP on flat gold surface is 300 µm at the same wavelength, hence light confinement in plasmonic waveguides is achieved at the expense of higher loss.

Plasmonic waveguides are used to realize some plasmonic waveguide based components. Several subwavelength waveguide components such as interferometers and ring resonators are fabricated in channel waveguides [16]. A ring resonator fabricated as wavelength filter at optical communication wavelength is with center wavelength of 1525 nm and insertion loss is smaller than 3 dB is reported in [16]. The ring resonator is realized inside a channel waveguide and its footprint is only 200 µm². The filter output versus wavelength shows a 40 nm bandwidth around 1525 nm.

Also, Engheta introduced the concept of optical nano-circuits using optical lumped elements realized with plasmonic and nonplasmonic particles [17, 18]. Optical electric field inside and outside of a deep subwavelength particle is obtained using quasi-static approximation of Maxwell’s equations. The optical “lumped impedance” is defined as the ratio of optical potential over displacement current. It should be noted that conductive current is small compared to displacement current when we are dealing with dielectrics and metals with plasmonic permittivity.

According to this definition, metallic nanoparticles (Ag, Au) exhibit negative nanocapacitors (effective inductive impedance) and conventional dielectric nanoparticles are essentially nanocapacitors at optical frequencies. When the imaginary part of dielectric permittivity is not zero, the material is lossy and equivalent circuit of the particle includes a nanoresistor as well (See Fig. 1.4 ).

![Fig. 1.4. Nanoparticles as lumped elements at optical frequencies.](image-url)
The values of these lumped components depend on size and material of the nanoparticle. Later, they experimentally demonstrated 2D optical lumped elements using a suspended array of Si\textsubscript{3}N\textsubscript{4} nanorods with deep subwavelength dimensions at infrared range [19]. Si\textsubscript{3}N\textsubscript{4} has Lorantzian electric permittivity model and real part of its dielectric permittivity is negative close to 11 \( \mu \)m wavelength. The experimental results agree well with the equivalent circuit model with lumped elements.

They have also offered the use of optical lumped elements inside plasmonic waveguides to realize resonators and filters. Equivalent circuit for plasmonic structures particles inside realistic 2D and 3D plasmonic waveguides are presented [20].

1.3.2. Optical Nanoantenna

The term optical nanoantenna refers to plasmonic components that convert optical radiation to confined optical energy and vice versa, analogues to microwave and RF antennas. However, optical nanoantenna has been used to refer to resonators with strong scattering or field localization by some authors. Although, optical nanoantennas and microwave antennas are similar in their role, they have considerable differences in material and physics. Unlike RF frequencies, at optical range field penetrates inside metal. Also, the polarization effects and displacement currents are not negligible. As a result, effective resonance length becomes smaller than half a wavelength and material loss becomes more prominent.

Several well-known nanoantenna geometries such as bowtie, sphere, core-shell and disk are demonstrated in [21-23]. Nanoantennas with rectangular, triangle and disk shapes operating in mid-infrared frequency range were investigated both numerically and experimentally in [21]. Surface current and charge density, calculated using FDTD, are demonstrating high intensity localized fields in the sharp edges and gaps. Also, farfield extinction measurement was used to obtain nanoantenna resonance wavelength.

Reference [22] reports the application of a 170 nm-long bowtie antenna to enhance the photoluminescence of a single semiconductor nanocrystal quantum dot. The bowtie antenna is fabricated on the apex of AFM pyramidal tip using focused ion beam milling. Also, nanoantennas can be utilized to couple energy to deep subwavelength waveguides[24]. In this
work, a dipole nanoantenna is connected to a slot antenna for efficient excitation. The antenna length and gap size are optimized to obtain maximum power transfer. Finally, references [25-27] offers recent reviews on nanoantenna literature.

Also, Nerkararyan used well-known phenomenon of charge concentration in sharp edges to propose a wedge and conical metal structure for strong local field enhancement[28, 29]. Their analytical treatment of the problem shows that the wavelength reduces to zero when as surface plasmon wave propagates towards the tip of the metal. At the same time phase and group velocity are reducing to zero and polariton is collimated in a very small area.

Large scale optical antennas such as reflectarrays and transmittarrays are introduced in 1.3.3 Plasmonic Metasurfaces plasmonic metasurfaces.

1.3.3. Plasmonic Metasurfaces

Conventionally, mirrors, lenses, polarizers and gratings are used to control free space optical radiation. Lenses are essentially 3D structures and require high precision fabrication of their surface curvature. They are not suitable for integrating into planar optical systems. Waveplate polarizers and graded index (GRIN) lenses must be several optical wavelengths thick to properly operate. In GRIN lenses, the wavefront phase is tailored with phase accumulation through long range propagation along the material. In birefringent waveplates, the required phase difference (90º or 180º) between the two orthogonal polarizations accumulates in several wavelengths.

Subwavelength plasmonic scatterers, sometimes referred to as nanoantennas, can locally interact with light and change phase, amplitude and polarization state abruptly. It is possible to design a planar pattern of such nanoantennas, metasurfaces, to shape the wavefront of the light as desired. Such metasurfaces have several advantages over their conventional counterparts. First, they can induce large phase delay and/or amplitude variation over a very thin surface while the light passes through them or reflects away from them. Also, polarizer metasurfaces abruptly change the polarization without the need for several wavelength phase accumulation. Such thin planar structures are extremely appealing for integration into planar optical circuits.

Moreover, metasurface provide subwavelength resolution for wavefront manipulation as desired. Such full control of phase and amplitude enables us to invent metasurface with unprecedented
functionalities. Also, compared with conventional optical diffractive elements with several orders of diffraction, metasurfaces are effectively collimating beam in a single direction. The field of metasurfaces initiated by the work demonstrating extraordinary transmission of light through a subwavelength hole in a metallic thin film surrounded by bull’s eye structure [30]. In this structure, the power emerging from the subwavelength hole excites the surface waves along the circular metal gratings whose period designed to collimate the beam in broadside direction. Focusing incident light beam into a focal point using such meta-lenses or beam scanners is demonstrated both in simulation and experiments [31-40]. Nanoslots, holes, dipoles, loops are the nanoantenna geometries usually used in such metasurfaces. One interesting application of such meta-lenses is shown in [33] where plasmonic gratings integrated into a quantum cascade laser (QCL) aperture for beam collimating and scanning. The power emanating from the aperture laser propagates along the corrugated plasmonic surface and leaks away into free space. Also, metasurfaces that bridge between surface waves and free space propagation are proposed and investigated in [37]. Here, the H-shaped nanoantennas are used to excite the leaky mode on the surface and give rise to free space radiation. Also, the concept of reflectarrays is borrowed from microwave range to propose novel meta-mirrors in optical range [41-45]. Such meta-mirrors can be used to design the reflection angle as desired in contrary with the predetermined reflection angle in conventional mirrors. Here, patch nanoantenna is used as the building block of the reflectarray metasurface. The patch dimension is designed for tailoring the phase front. Finally, v-dipoles are suggested for phase front engineering of cross polarized field [46]. Furthermore, waveplates and polarizer metasurfaces are suggested for polarization state manipulation on a thin surface [39, 47-52]. Nanoantennas with geometries that discriminate two orthogonal polarizations such as cross dipoles/slots, v-dipoles and meander lines are adopted as polarization dependent nanoantennas.

Metasurfaces proposed for applications such as beam steering and focusing are only tailoring the phase. A new generation of metasurfaces that manipulate both amplitude and phase of the incoming light are investigated [38, 39, 53, 54]. In such metasurfaces two or more layers are employed to enable both phase and amplitude engineering. Therefore, it is possible to realize a desired amplitude/phase profile. One can sandwich this metasurface between a Fourier transform and an inverse Fourier transform block to realize spatial functions such as derivation[53, 54]. It
is interesting to note that multilayer metasurfaces can be designed to be matched to free space. The key lies in interaction with both electric and magnetic field of light [38, 39].

### 1.4. Plasmonic Applications

Plasmonics has found interesting and exciting applications in diverse branches of science and technology from solar energy harvesting [55] to cancer diagnosis and therapy [56]. In this section, we briefly review some of the most interesting applications already reported in literature for current and future advanced optical devices.

#### 1.4.1. High Resolution Optical Imaging

One of the first applications of plasmonics was subwavelength optical microscopy. In conventional microscopy, the image resolution is determined by the wavelength of light. In early near-field scanning optical microscopy (NSOM) systems, apertures in metal coated optical probes were used for light monitoring. The resolution of such systems is then determined by the aperture size rather than the light wavelength. The optical throughput is very small in high resolution aperture based NSOM systems because the aperture size is small compared to the wavelength.

Apertureless near-field scanning optical microscopy (aNSOM) techniques where sharp metal tips or other plasmonic particles with high field localization are used as optical probes are then introduced. Particularly, an aNSOM system with deep subwavelength resolution ($\lambda/3000$) is reported in [57]. A sharp 1µm long metal cone is used to confine the incident field radiation. The operation wavelength is 118 µm and the 40 nm image resolution is obtained.

Also, a sharp high aspect ratio gold cone was used for high performance infrared nanoimaging of organic, molecular and biological materials [58].

Tip on aperture (TOA) systems improve high resolution imaging by taking advantage of both low optical background of aperture based systems and enhanced field localization of aperture less systems. In TOA systems, the metal tip is placed close to the aperture to improve aperture efficiency and field localization [59, 60]
Other than high resolution imaging, high field enhancement realized by plasmonic particles may find some applications in cutting edge technologies such as data storage [61, 62] and photolithography [63]. The sharp tip of metallic nanoantennas is heated up because of high field localization inside the lossy metal. One can integrate such nanoantennas with the writing head of thermally assisted magnetic recording system to enhance heat transfer and localization. For instance, storage areal density of 1.5 Tb/inch$^2$ with improved optical power efficiency is reported in [62] where the writing head is equipped with an E-shape plasmonic antenna.

Also, a plasmonic bull’s eye aperture lens is designed to integrate with the write head of a photolithography system and focus its ultraviolet light to the areas below 100 nm in diameter. This nanolithography method has the advantage of low cost and higher throughput compared with the other maskless nanolithography methods such as electron beam and scanning probe lithography [63].

1.4.2. Plasmonic for Photodetectors and Solar Cells

Miniaturized photodetectors offer higher speed, improved power efficiency and lower noise level. However, light coupling to small photodetectors is not efficient. One can use enhanced field localized by plasmonic structures to efficiently couple the optical power to the miniaturized photodetectors [64-69]. In Ref [65], a Si Schottkey photodiode with active area diameter of 300 nm is integrated to the aperture of a plasmonic bull’s eye structure. The laser is illuminating the bull’s eye at 840 nm and the photocurrent is increased several tenfold compared to bare detector. The high speed of 100 GHz is anticipated for Si photodiodes with integrated plasmonic antennas. Also, an active plasmonic antenna- diode structure designed for photodetection is reported in [68]. Rectangular gold nanorods are used as the metal part of a Schottky [68] diode and field enhancement devices.

Moreover, Ref [69] demonstrated a photodetector embedded inside deep subwavelength plasmonic gap waveguide. Integration of electro-optical devices inside plasmonic waveguide can lead to high speed active plasmonic devices and circuits.

Another promising application of plasmonics is in thin film photovoltaic cells where it can bridge between electronic and photonic length scales. Thin film semiconductor solar cells are
economical because of much lower material cost. Nevertheless, the absorption length of light is much longer than carrier diffusion length in most semiconductors deposited in thin film materials, leading to poor absorption efficiency. Plasmonic structures can improve light absorption in thin film solar cells through light trapping. Plasmonic particles on top of a thin film solar cell cause multiple scattering and a longer effective optical pass. Also, high field localization of such plasmonic particles increases solar cell effective cross section. Finally, plasmonic structures supporting SPP, gratings, are utilized to couple free space radiation of sunlight to SPP at the interface of metal and thin film solar cell. These methods are illustrated in [70]. An ultrathin (160 nm) a-Si:H solar cell with grating in the back metal is reported in [71]. The photocurrent is shown to increase 1.5 times in the nanopatterned solar cell compared with the flat back metal solar cell.

1.5. Dissertation structure

This dissertation is organized as follows. In Chapter 2, we focus on integrated photonic circuits. We show how design methods in circuit and transmission line theory can be applied to design a filter inside an MIM waveguide at infrared range. We then design a frequency diplexer using two such filters.

In Chapter 3, we focus on reflectarray metasurfaces. Particularly, we present a birefringent reflectarray for light beaming and focusing. The novel feature of this reflectarray is the polarization dependent reflection angle. The reflectarray discriminates the input polarization and reflect each polarization the designed angle.

In Chapter 4, the concept of plasmonic graded index materials is introduced. The effective index is engineered using plasmonic structures. A graded-index metasurface for antenna application at infrared frequency range is designed. The holography technique is use to obtain graded index pattern. Such metasurfaces can be used to bridge surface waves to radiating ones. For instance, the can be integrated into the aperture of a Quantum Cascade Laser to collimate and steer the beam.

Chapter 5 focuses on a more general type of metasurfaces for light manipulation where both phase and amplitude are controlled independently. A double layer metasurface is suggested for this purpose. The first layer is composed of L-shaped slot to tailor the amplitude and the second
layer is composed of concentric loop to engineer the phase. Cascading these two metasurfaces offers the opportunity to design desired mathematical functions, which can lead us to enable the idea of “flat optics” engineering. Novel examples have been demonstrated. Future works are discussed in Chapter 6.
2. MIM Plasmonic Frequency Diplexer

In this chapter, a plasmonic frequency diplexer in a metal-insulator-metal waveguide at mid-infrared range is presented. The diplexer guides the 30 THz (10 µm) and 38.5 THz (7.8 µm) frequency contents of the input wave into their separate outputs. The structure comprises two bandpass filters, with center frequencies at 30 THz and 38.5 THz, connected to a power splitter. A circuit-based model is used to design the configuration. The effect of metal loss is carefully considered in the simulations. The performance of the designed diplexer is outstanding in terms of the frequency selectivity and loss. Particularly, the insertion loss is 2.1 dB and 3.7 dB for low frequency and high frequency channels and the isolation is larger than 30 dBs.

2.1. Introduction

Plasmonic materials interact with light beyond diffraction limit and hence offer a promising solution for miniaturized photonic integrated circuits. Previously, a variety of Plasmonic waveguides such as nanowires [5, 6], nanoparticle chain [7], metal-insulator-metal [8, 9], strip [10], slot [11, 12], groove and wedge [13-15] are introduced and studied thoroughly. An MIM waveguide is essentially a layer of insulator sandwiched between two layers of metal. The MIM is considered as a prominent component for guiding optical waves because of its subwavelength size, high level of light confinement, simple manufacturing and integration. Many MIM based components such as bends and splitters [72], filters [73-76], couplers [77] and frequency multiplexers (color routers) [78] have been investigated as well. Frequency multiplexers are an essential part of optical systems with applications in spectral imaging, sensing, and communication. A variety of multiplexing structures for the abovementioned applications have been investigated. Laux et al. have introduced a photon sorter based on overlapping aperture-groove type structures [79], suitable for spectral imaging applications. A simple compact submicron dichroic splitter, composed of two metal grooves sorting normally incident photons, is reported in [80] as well. A non-periodic nanoslit array on metal film is also suggested as a multiple wavelength-focusing device [81]. Also, a wavelength add/drop multiplexer based on ring resonator structure in plasmonic groove waveguide is reported in [82].
Here we realize a multiplexing scheme using a combination of filter banks and a power splitter in MIM waveguide. This topology is most suitable for integrated optical circuits. A systematic filter synthesis method based on transmission line theory and impedance inverters is selected for filter design. The filters are implemented by introducing deliberately designed discontinuities that play the role of impedance inverters into the waveguide. The equivalent circuits of the discontinuities are obtained and used in the design method.

The effect of loss incurred by metal is carefully considered in our modeling. Two bandpass filters are designed at 10 \( \mu \text{m} \) and 8 \( \mu \text{m} \) and integrated with a T-shaped power splitter to realize a MIM frequency diplexer. This approach eliminates the need for cumbersome tuning and optimization procedures based on full wave simulators.

This chapter is organized as follows: In Section 2.2, we briefly review basic filter theory and its application to the design of a bandpass filter in an MIM waveguide. Section 2.3 continues with the design and simulation of a frequency diplexer achieved using the filters we obtained in Section 2.2.

### 2.2. Filter Design

Ladder type filters are a major type of filters constructed from series and parallel connections of capacitors (inductors) and inductors (capacitors). In waveguide geometry, it is often easier to have only parallel or series elements due to practical considerations. One can use impedance inverters to replace all parallel elements with series ones or vice versa. An impedance inverter is a two port circuit where the input impedance seen in port 1 is \( Z_{\text{in1}} = K^2/Z_{L2} \) provided that \( Z_{L2} \) is connected to port 2. For instance, a series inductor with an impedance inverter connected at each end can replace a shunt capacitor. In this section, we first briefly review the general scheme for designing a filter using impedance inverters, and then apply it to the design of a bandpass filter inside an MIM waveguide.
2.2.1. Filter design using impedance inverters

We show a step-by-step design procedure for a bandpass filter using impedance inverters in a waveguide. The filter transmission characteristic is presented in Fig. 2.1 (f_{pl} and f_{ph} determine filter lower and higher passband frequencies, f_{sl} and f_{sh} denote lower and higher stopband frequencies, R is the allowed ripple in the passband and A_s is the required attenuation in stopband).

![Transmission characteristic of a bandpass filter](image)

Fig. 2.1. Transmission (dB) characteristic of a bandpass filter.

We start with a normalized nth order lowpass ladder-type filter shown in Fig. 2.2(a). The component (L or C) values (g_i) are tabulated in [83] for different types of filter realizations (e.g. Chebychev and Butterworth) and filter orders.

As mentioned earlier, each shunt capacitor in Fig. 2.2.(a) can be replaced with a series inductor with an impedance inverter connected at each end as shown in Fig. 2.2.(b). Then the resulting filter is transformed from lowpass to bandpass by changing series inductors to series LC circuits resonating at the center frequency of the filter. The series LC resonators can be substituted by general reactance X(\omega) resonating at the same center frequency (See Fig. 2.2.(c)). The K values of the equivalent impedance inverters are then obtained using Eq. (2.1) [83]

\[
K_{i,i+1} = \frac{\pi \omega_f}{2 \sqrt{\beta_i \beta_{i+1}}}, i = 1..n - 1, K_{0,1} = \frac{\pi \omega_f}{\sqrt{2 \beta_1 \beta_0}}, K_{n,n+1} = \frac{\pi \omega_f}{\sqrt{2 \beta_n \beta_{n+1}}} \tag{2.1}
\]
In Eq. (2.1), \( \omega_f \) is the fractional bandwidth calculated from (2.2)

\[
\omega_f = \frac{f_{\text{HH}} - f_{\text{PL}}}{f_0} \tag{2.2}
\]

The next step is to create an appropriate discontinuity in a waveguide (See Fig. 2.3(a)) to function as an impedance inverter. We characterize the discontinuity with an equivalent circuit

![Equivalent Circuit Diagram](image)

and the waveguide with a transmission line to simplify the analysis. Here, we model the discontinuity with a lossless T-type equivalent circuit (See Fig. 2.3(b)). It has been shown that a T-type circuit with two identical segments of transmission lines added at each end (See Fig. 2.3 (c)) can function as a K-inverter with parameters given in (2.3) [83].

\[
K = Z_0 \left| \tan\left(\frac{\varphi}{2} + \tan^{-1}\left(\frac{X_a}{Z_0}\right)\right) \right| \tag{2.3a}
\]

\[
\varphi = -\tan^{-1}\left(\frac{2X_b}{Z_0} + \frac{X_a}{Z_0}\right) - \tan^{-1}\left(\frac{X_a}{Z_0}\right) \tag{2.3b}
\]
$Z_0$ is the wave impedance of the waveguide, or equivalently, the characteristic impedance of the transmission line.

Fig. 2.3. (a) A general discontinuity in a waveguide. (b) A T-type equivalent circuit representation. (c) A T-type circuit with two equal transmission lines at each end is equivalent with an impedance inverter with the parameters given in Eq. (2.3).

We need impedance inverters with several values of $K$ calculated from Eq. (2.1) to realize the filter. A range of $K$ values can be achieved by changing one or more geometry parameters of the discontinuity in the waveguide. A half wavelength series segment of transmission line can conveniently be used as $X(\omega)$. Then all one would need is to interleave the corresponding impedance inverters in series with a half wavelength transmission line to make the filter as shown in Fig. 2.2. (c). The distance between waveguide discontinuities can be obtained from (2.4)

\[
\begin{align*}
  l_i &= \frac{\lambda_{g0}\theta_i}{2\pi} \quad (2.4a) \\
  \theta_i &= \pi + \frac{1}{2}(\varphi_{i-1,i} + \varphi_{i,i+1}) \text{ radians, } i = 1, \ldots, n. \quad (2.4b)
\end{align*}
\]

where $\lambda_{g0}$ is guided wavelength at the center frequency of the filter. The $\pi$ in Eq. (2.4.b) accounts for the half wavelength transmission line resonator.

It is important to note that the design approach presented here assumes narrow band operation. Therefore, it is accurate for filter designs with fractional bandwidth of up to 20%. We should note that the accuracy will degrade as the bandwidth is increased.
2.2.2. An MIM filter

Here, a bandpass filter will be designed in an MIM waveguide based on the method reviewed in previous section. The MIM waveguide is composed of two layers of silver with air as insulator in between. The thickness of the MIM insulator layer is 0.8 μm, less than one tenth of the operating wavelength (10 μm). This results in deep subwavelength single mode operation.

The Chebychev realization is selected for the filter design with specifications given in Table 2-1. The filter order is calculated to be 3 from Table 2-1. The \( g_i \) values are found in [83] and then used in Eq. (2.1) to calculate the K values. These are given in Table 2-2.

<table>
<thead>
<tr>
<th>( A_s )</th>
<th>( R )</th>
<th>( f_{SL} )</th>
<th>( f_{PL} )</th>
<th>( f_0 )</th>
<th>( f_{pit} )</th>
<th>( f_{sh} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 dB</td>
<td>1 dB</td>
<td>24.75</td>
<td>27</td>
<td>29.8</td>
<td>33</td>
<td>36</td>
</tr>
</tbody>
</table>

Table 2-2 Value for normalized lumped elements and normalized K of equivalent impedance inverters

<table>
<thead>
<tr>
<th>( g_i )</th>
<th>( K_{i,i+1}/Z_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.38</td>
<td>0.21</td>
</tr>
<tr>
<td>0.21</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Next we create a discontinuity in the MIM waveguide to function as a K-inverter. The fundamental mode of an MIM waveguide is TM. The field lines are similar to a parallel plate waveguide, i.e. the transverse electric field (\( E_x \)) lines are perpendicular to the metal layers while the magnetic field (\( H_y \)) lines are parallel with the metal layers. Here we want to note that the field penetrates inside metal and that the electric field has a component in the direction of propagation. However, this longitudinal component is small enough to approximate the dominant TM mode with a quasi-TEM mode and analyze the MIM waveguide with an equivalent transmission line. Fig. 2.4 shows the electric field intensity of the quasi-TEM mode inside a 0.8μm wide MIM waveguide at 30 THz.
Considering the direction of the transverse electric field, if we locally decrease insulator thickness, the electric field intensity between the metals is increased, causing a parallel capacitive effect. This capacitor can work as the parallel reactance \( X_b \) in an equivalent T-type circuit. We utilize two identical indentations with width \( w_g \) and gap size in between \( d_g \) in an MIM waveguide with insulator thickness of 0.8 \( \mu m \) (See Fig. 2.5) to serve as the desired discontinuity. We use 2D FDTD simulator from Lumerical [84] to characterize these indentations. The structure depicted in Fig. 2.5 is setup in Lumerical environment and excited at the center frequency of the filter, \( f_0 = 30 \) THz, with the dominant TM mode.

The real part of the effective index \( (n_{\text{eff}}) \) is 1.022 and the imaginary part \( (n_{\text{eff}}) \) is 0.013. Also, \( Z_0 = \eta_l / n_{\text{eff}} \) where \( \eta_l \) is the intrinsic impedance of the insulator. From the Lumerical solution, we obtain network scattering parameters, and use Eq. (2.5.a-b) to calculate \( X_a \) and \( X_b \). Note that
we have de-embedded the extra length on each side of the indentations from the simulated S parameters. Values of $K$ and $\varphi$ can be obtained using Eq. (2.3.a-b).

\[
\frac{jX_b}{Z_0} = \frac{2S_{21}}{(1-S_{11})^2-S_{21}^2} \quad (2.5a)
\]

\[
\frac{jX_a}{Z_0} = \frac{1+S_{11}-S_{21}}{1-S_{11}+S_{21}} \quad (2.5b)
\]

At microwave frequency range, the metal loss is negligible and the values we obtain for $X_a$ and $X_b$ are real, implying pure reactive discontinuity, as desired. At infrared frequencies, metal cannot be treated as in the microwave range, and we need to carefully consider its material parameter in a design scheme. Here we use the material model from Lumerical material library. Lumerical uses data from Palik’s Handbook of Optical Constants [85]. $X_a$ and $X_b$ are complex valued, as expected. We will investigate these values after we have calculated $X_a$ and $X_b$.

For our design, we sweep the gap size and obtain T-type circuit component values, $X_a$ and $X_b$. Also, we calculate the parameters of the K-inverter realized with this T-type circuit and two segments of transmission line connected at each end (Fig. 2.3 (c)). These are gathered in Fig. 2.6. Fig. 2.6.(a) depicts the absolute value of scattering parameters for indentations shown in Fig. 2.5. As expected, increasing the gap improves the transmission through the indentations.

As mentioned before, the values obtained for $X_a$ and $X_b$ are complex. We denote the real part of the series and parallel impedances with $R_a$ and $R_b$, respectively. The calculated values of $R_a$, $X_a$, $R_b$, and $X_b$ are normalized to $Z_0$ and depicted in Fig. 2.6.(b). The value of $R_a$ is less than 5% of $X_a$ and $R_b$ is less than 1% of $X_b$. Therefore, these values can be neglected and we can use design equations already derived for the lossless case. We expect each pair of indentations to add 0.1 dB maximum insertion loss from Fig. 2.6.(a). As Fig. 2.6.(b) suggests, the series impedance is inductive and the parallel one is capacitive. Specifically, $X_b$ is a linear function of $d_g$, implying that the capacitance can be well approximated with the parallel plate capacitor formula, $C = \varepsilon_0 w/d_g$. Fig. 2.6.(c) shows the normalized values of $K$ calculated from Eq. (2.3a). The value of $K$ is small for smaller gaps and increases as the gap becomes wider. Finally, $\varphi$ is shown in Fig. 2.6. (d).
Using the obtained graphs for $K$ and $\varphi$ versus $d_g$, Fig. 2.6.(c-d), we can calculate the required indentation gap size and transmission line length that should be between them. Each length can be obtained from corresponding values of $\varphi$ inserted in Eq. (2.4).

Fig. 2.6. (a) Absolute values of S parameters for the discontinuity show in Fig. 4. Expectedly, as the gap size increase the transmission ($S_{12}$) also increases. (b) The real and imaginary part of normalized $X_a$ and $X_b$. The imaginary parts represent loss in the indentations. (c) Calculated $K$ for equivalent impedance inverter. (d) Calculated $\varphi$ for equivalent impedance inverter.

The designed filter parameters are summarized in Table 2-3. We expect a 0.4 dB loss due to the four indentions set in the filter structure and a calculated 1.3 dB transmission loss for the attenuation in entire length of the filter. The filter is simulated using Lumerical, where we obtained both the transmission and reflection curves for the device. The frequency response reveals a 3% shift to the right. Therefore, we adjust the frequencies in design specification accordingly to compensate for the frequency shift, and obtain the final filter parameters. These are also given in Table 2-3.
Table 2-3 Calculated values of gap sizes ($d_{gl}$) and distance between indentations ($l_i$)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>$d_{gl}$ ($\mu$m)</td>
<td>0.24</td>
<td>0.12</td>
<td>0.12</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>$l_i$ ($\mu$m)</td>
<td>5.2</td>
<td>5</td>
<td>5.2</td>
<td>-</td>
</tr>
<tr>
<td>Final</td>
<td>$d_{gl}$ ($\mu$m)</td>
<td>0.3</td>
<td>0.15</td>
<td>0.15</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>$l_i$ ($\mu$m)</td>
<td>5.7</td>
<td>5.5</td>
<td>5.7</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 2.7. (a) The 2D structure of the bandpass filter working at 30THz detailing the dimensions. The length of the filter is 20 $\mu$m ($2\lambda_0$). (b) A 3D view of the filter. (c) Transmission and reflection characteristics.

Fig. 2.7.(a) details filter dimensions and Fig. 2.7. (b) depicts a 3D view of the filter. Fig. 2.7. (c) depicts the reflection and transmission in dB versus frequency.

The obtained design specifications from simulation are compared with design goals in Table 2-4.

Table 2-4 Design specifications of the bandpass filter design goals compared to simulated values (Frequencies are in THz)

<table>
<thead>
<tr>
<th></th>
<th>$f_{sl}$</th>
<th>$f_{pl}$</th>
<th>$f_0$</th>
<th>$f_{PH}$</th>
<th>$f_{SH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design</td>
<td>20 dB</td>
<td>24.75</td>
<td>27</td>
<td>29.8</td>
<td>33</td>
</tr>
<tr>
<td>Sim.</td>
<td>20 dB</td>
<td>-</td>
<td>26.8</td>
<td>29.6</td>
<td>32.8</td>
</tr>
</tbody>
</table>

The 3 dB bandwidth is considered for calculating the filter passband. The values obtained from simulation match those of the design requirement. The transmission amplitude is normalized to the transmission of a thru MIM waveguide with the same length of the filter to show the effect of
the indentations in increasing the loss. This loss is around 0.5 dB, which is close to what is expected from a set of four indentations (0.4 dB). The transmission thru loss itself is around 1.2 dB. The reflection is small at the entire frequency range of filter operation. Here, it is important to note that the use of a narrow bandwidth filters dictates a smaller gap size, which is more lossy. The normalized intensity of the magnetic fields inside the filter are depicted in Fig. 2.8 at three frequencies, namely $f = 20, 30, 40$ THz. The field profiles clearly show the filtering property of the structure. While the wave passes through the filter at 30 THz, it is totally reflected at both 20 and 40 THz.

![Normalized intensity of the magnetic field](image)

The presented filter design scheme is very robust and can be used to design a bandpass filter at any desired optical frequency. One can use this approach for designing filters in 3D waveguides. Proper discontinuity and circuit model accuracy must be investigated in each case. Slot waveguide, as a 3D counterpart of MIM waveguide studied in [12], have very similar mode profile and propagation constants to those of MIM waveguide provided that slot width is much smaller than its height, or equally mode is majorly confined in slot area. Reference [86] illustrates application of the proposed filter design method in plasmonic slot waveguide with height over width ratio of 3 (See Fig. 2.7. (b))

Finally, we note that the air insulator can be replaced with a more desirable insulator for fabrication purposes. Either way, the design approach remains the same. The length of the filter will be reduced by a factor equal to the refractive index of the insulator providing the opportunity to enhance filter miniaturization.
2.3. Frequency Diplexer Design and Simulation

A frequency multiplexer can be made of a filter bank connected to the output ports of an n-way power divider. At the passband of one filter, the other filters reject the input wave. Therefore, one can route the frequency components of the input wave to the corresponding filters. In this section, we combine two filters of the type illustrated in Section II with a power splitter to make a frequency diplexer.

Fig. 2.9(a) depicts the frequency diplexer structure. We use a T-splitter introduced in [72] as a power divider component. Two filters with specifications shown in Table 2-5 are designed using the previously elaborated method. The filters are then inserted in each branch of the T-splitter. The main challenge is to find $s_1$ and $s_2$, the distance between the first filter indentations and the T-splitter output ports (See Fig. 2.9(a)). We choose $s_1$ and $s_2$ such that at the frequency passband of each filter, the other branch of the splitter becomes virtually blocked and the incoming wave encounters a bend instead of a splitter. If we had a perfect electric conductor instead of a metal, the solution would be to create a virtual short circuit boundary condition at each branch of the T-splitter to make a bend toward the other branch. In this case, however, because of loss, the short circuit boundary condition is neither feasible, nor desirable. Nevertheless, we may use it as the starting point. For instance, to find $s_2$, we calculate first the input reflection coefficient of filter II from cross section A-B ($\Gamma_{in II}$) versus various $s_2$ at 30 THz. We choose the initial $s_2$ such that the phase of the reflection coefficient is equal to 180 degrees. Next, we remove the indentations of filter I from the structure and adjust $s_2$ to maximize $T_1$ at $f = 30$ THz. As a result of this choice, the incoming wave at 30 THz goes into the desired branch slightly distorted. First, we calculate that the initial $s_2$ is equal to 0.15 μm. The optimized value of $s_2$ for maximum power at branch I is 0.8 μm. This physical length is equal to a 15 degrees phase shift from the initial short circuit. We obtain $s_1 = 3.8$ μm in the same manner.

The frequency diplexer is simulated in Lumerical to obtain the transmission characteristics through the output ports. Fig. 2.9(b) depicts the transmission through branch I and branch II together with reflection from the input port. As shown, the diplexer can perfectly separate the two frequency content of the incoming pulse at different wavelengths. The channel isolation is higher than 30 dB.
Table 2-5 The design specifications of the two filters designed to be embedded in the frequency diplexer. (The frequencies are in THz)

<table>
<thead>
<tr>
<th></th>
<th>$A_{sl} (dB)$</th>
<th>$f_{PL}$</th>
<th>$f_0$</th>
<th>$f_{PH}$</th>
<th>$f_{SH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filter I</td>
<td>Design</td>
<td>20</td>
<td>27</td>
<td>30</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>Sim.</td>
<td>20</td>
<td>26.6</td>
<td>29.5</td>
<td>32.8</td>
</tr>
<tr>
<td>Filter II</td>
<td>Design</td>
<td>20</td>
<td>36</td>
<td>38.5</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>Sim.</td>
<td>20</td>
<td>36.4</td>
<td>38.8</td>
<td>41.4</td>
</tr>
</tbody>
</table>

It should be noted that we did not change any filter design parameters when we integrated them with the T-splitter. The only design parameters we needed to adjust were $s_1$ and $s_2$.

The geometry and fabrication of the proposed frequency diplexer is practical and the design steps are easy to follow. Fig. 2.10 depicts the normalized intensity of the magnetic fields in the diplexer when we excite it at $f = 30$ THz and $f = 38.5$ THz respectively. The routing of the input waves, based on their frequency to the corresponding output port, is illustrated.

Fig. 2.9. (a) The structure of the frequency diplexer. (b) Transmission performance from input port to output ports I and II. Also shown is the reflection from input port.

The transmission loss is expected and acceptable at this range of frequency for the metal structure; the extra loss comes from the splitter. Multiplexers can also be realized using the proposed design methods.
Compared to a previously reported design [87] for obtaining a frequency triplexer based on dielectric photonic crystals, our design has the benefit of being much smaller in size. Furthermore, we provide both comprehensive and robust scheme for design construction. In contrast to a dielectric-based design, our metallic construction has more loss. Another approach for design of a frequency diplexer based on programming RWGN has been reported [78]. The operating principle of this RWGN diplexer is based on constructive and destructive interferences resulting from different path lengths. The dimension of the diplexer designed using RWGN is on the same order as the one designed in this paper. However, there are two disadvantages with using a RWGN based design. First, the structure needs two input/output waveguide for each of its ports. Second, control over the bandwidth of each channel is limited by the nature of its operating principle.

Also, we compare the specifications obtained here with a waveguiding-ring resonator-based diplexer reported in [82]. The reported structure operates at communication wavelength range. The transmission to the first and second bands are %57 and %45, respectively (best case). The values of transmission for our design including waveguide loss is %61 and %42 which is comparable with the reported design. However, the reported channel isolation in [82] is very poor (Several dB’s) compared to high isolation offered by the diplexer design here.

Fig. 2.10. Normalized intensity of the magnetic fields inside the diplexer depicted in dB. At $f = 30 \text{THz}$ the structure routes the input field to the output port I. At $f = 38.5 \text{THz}$ the structure routes the input field to the output port II.
2.4. Conclusion

A method to design MIM bandpass filters and diplexers at THz and mid-infrared range is proposed. We start with the concept of filters in circuit theory and implement a novel scheme for filter realization in infrared range. The required circuit elements are created using transmission lines and waveguide indentations in MIM. A bandpass filter which operates at 10 μm with lower and upper edges at 9.1 μm and 11.1 μm is presented. The effect of the dispersion characteristic of the metal and its loss at this frequency spectrum is fully considered. We combine two filters with a power splitter in an MIM to design a diplexer, and successfully separate the waves of 10 μm and 8 μm wavelengths in the incoming port. The performance of the diplexer is promising in terms of low transmission loss and power leakage.
3. Reflectarray Metasurfaces

In this chapter, we introduce the reflective type of metasurfaces developed for beam focusing and steering. First, the reflectarray design method is reviewed with an example of a $16\lambda \times 16\lambda$ reflectarray that works for a single linear polarization. This reflectarray operates at 8.06 $\mu$m (37.22 THz) and reflects the normal incident beam into $\theta_0 = 45^\circ$ and $\varphi_0 = 0^\circ$ direction. Then a novel reflectarray with polarization dependent reflection angle is demonstrated. The array reflects the normal incident plane wave in different angles based on transverse electric field polarization. For instance, the x-polarized and y-polarized electric field are reflected to $\theta_0 = 30^\circ$ and $\theta_0 = -30^\circ$, respectively.

3.1. Introduction

The term metasurfaces refers to flat platforms patterned with plasmonic particles with different geometries to manipulate incident light properties such as amplitude, phase and polarization in unprecedented ways. The field of optical metasurfaces commenced with the pioneer work of Lezec illustrating the extraordinary transmission of light through a subwavelength hole in a thin metallic film with bull’s eye pattern [30]. Later, a variety of transmit-arrays [31, 32, 34, 35, 38, 39, 53] and reflectarrays [41-45, 88] for light focusing/steering are explored. Other types of metasurfaces manipulating light polarization were also investigated [39, 45, 48, 50, 51].

Reflectarrays, a planar array of sub wavelength resonant elements, enable reflected beam manipulation in a more sophisticated way compared to a simple mirror. The geometry of individual elements can be precisely tailored, thanks to nano fabrication techniques, to provide the desired reflected beam.

The most common resonant element in reflectarrays are rectangular metallic patches. Usually the patch dimension along the incident field polarization (patch length) is determined for the specific reflection phase. Patch width can be fixed to a single value. However, this arrangement only works for the same electric field direction. One can obtain the same reflection angle for the orthogonal field direction using square patches.
In Section 3.2, we demonstrate a novel and interesting feature of polarization dependent reflection angle. Namely, the incident field will be reflected and focused in different angles based on the incoming wave polarization direction. Using this feature, one can separate electric field components of an elliptically polarized incoming wave. This feature can be used in measurement setups for simultaneous measurement of both polarizations.

### 3.2. Birefringent Reflectarray

Let’s start with the reflectarray that works for only one polarization to review design process. Assume we have a reflectarray of patch unit cells placed in x-y plane and we wish to focus the incoming wave into $\theta_{0x}$ and $\varphi_{0x}$. A normal Gaussian beam is used as illuminating source without loss of generality. The reflection phase distribution on the surface should be

$$
\psi_x(x,y) = k_0 \sin\theta_{0x}(xcos(\varphi_{0x}) + ysin(\varphi_{0x})) \quad (3.1)
$$

$k_0$ is free space wave number. The patch located at each $(x,y)$ should delay the incoming phase with $\psi_x(x,y)$. We need a diagram relating the patch reflection phase to its dimensions to transform phase distribution in Eq. (3.1) to a patch size distribution. Usually, this information is contained in a graph called S-diagram [89].

The patch unit cell is shown in Fig. 3.1 as the reflectarray building block. The patch and the ground metals are gold and the stand-off layer dielectric is a 300 nm thick Silicon. The unit cell size is 1.2 $\mu$m and gold layers thickness is 50 nm. The working wavelength is 8.06 $\mu$m.

We use Lumerical to calculate the reflection phase of the patch unit cell. The illuminating wave is x polarized and periodic boundary conditions are applied. Fig. 3.2 shows the S-diagram for the patch illuminated with x-polarized electric field and $d_y$ as a parameter. As depicted, we can obtain a range of 300° phase delay by varying $d_x$. For larger values of $d_y$, the slope of the S-diagram is smoother and we can get a lower phase sensitivity versus $d_x$. Also, the reflection remains close to unity when $d_y$ is larger, implying higher reflection efficiency.
Fig. 3.1. The unit cell of the reflectarray. Gold metal patches on top of a Grounded silicon stand-off layer. \(a=1.2 \, \mu m\), \(h=300 \, nm\) and \(t=50 \, nm\).

Using Eq. (3.1) for \(\theta_{0x} = 45^\circ\) and \(\varphi_{0x} = 0^\circ\), we obtain the reflection phase at each \((x,y)\) and then find the associated patch size from the S-diagram. Fig. 3.3 depicts the \(16\lambda \times 16\lambda\) patch reflectarray obtained. Note that the reflectarray pattern is constant along \(y\) direction since we selected \(\varphi_{0x} = 0^\circ\).

Fig. 3.2. The S-diagram of the unit cell shown in Fig. 3.1. The S diagram is smoother for larger \(d_y\) and gives a better phase resolution for \(d_x\).

The reflectarray is simulated in Lumerical. We illuminate the array with a normal Gaussian beam with \(20 \, \mu m\) beam waist. Fig. 3.4.(a) shows the normalized reflected beam. The field is reflected back into \(\theta_{0x} = 45^\circ\) and \(\varphi_{0x} = 0^\circ\) as determined. The \(\varphi = 0\) cut is shown in Fig. 3.4. (b). The
Half Power Beam Width (HPBW) is 8°. The small sidelobes in 0° and -45° are due to higher order diffractions from the periodic array. Similarly, we can generalize this method to incoming y polarized beam. We should impose desired reflection phase distribution for y-directed fields on each patch to re-radiate the y-directed field into $\theta_{0y}$ and $\varphi_{0y}$.

$$\psi_y(x, y) = k_0 \sin \theta_{0y} \left( x \cos(\varphi_{0y}) + y \sin(\varphi_{0y}) \right)$$  (3.2)

Fig. 3.3. The $16\lambda \times 16\lambda$ patch reflectarray. The enlarged part details $d_x$ variations. $d_x$ is changed 400 nm to 1100 nm in 100 nm steps.

Fig. 3.4. (a) 2D reflected beam in $\theta_{0x} = 45°$ and $\varphi_{0x} = 0°$. (b) The $\varphi = 0$, xz, cut The Half Power Beam Width (HPBW) is 8°.
Both phase distribution for x and y directed filed should be satisfied at the same time to realize a polarization dependent reflectarray. In other words, we have to find a proper patch size at each (x,y) that reflects $E_x$ and $E_y$ with phase delay incurred $\psi_x(x,y)$ and $\psi_y(x,y)$, respectively.

The S-diagram in Fig. 3.2 can be used for $\psi_y(x,y)$ by interchanging dx and dy. One can see that several pair of patch sizes $(d_x, d_y)$ can provide the same $\psi_x(x,y)$. Among those pairs, we search for the one that satisfies $\psi_x(x,y)$ at the same time. We devise a search algorithm with a reasonable phase tolerance ($20^\circ$) to obtain those $(d_x, d_y)$ pairs. We choose to reflect the x directed field in $\theta_{0x} = 30^\circ$ and $\varphi_{0x} = 0^\circ$ and y-directed electric field into $\theta_{0y} = -30^\circ$ and $\varphi_{0y} = 0^\circ$. A part of the resulting polarization dependent reflectarray is shown in Fig. 3.5. Also, $d_x, d_y, \psi_x(x,y), \psi_y(x,y)$ are depicted versus x position for one period. In Fig. 3.5.(d) and (e) the solid lines show the reflection phase calculated from Eq. (3.1) and Eq. (3.2) and the square and circle marks show the realized reflection phase for $E_x$ and $E_y$, respectively. The designed patch array has successfully provided the required reflection phase for both polarization directions.

Fig. 3.6 (a) and (b) depicts the reflected beam when we illuminate the reflectarray with x and y polarized beams, respectively. The beam is reflected in $\theta_{0x} = 30^\circ$ when x-polarized electric field illuminate the array while it reflects in $\theta_{0y} = -30^\circ$ when illuminated by $E_y$. Fig. 3.6(c) depicts the xz cut of both reflected beams.

We calculated the efficiency of the reflector by averaging the reflections from each patch to be 92%.
Fig. 3.5. (a) A part of the polarization dependent patch array. $d_x$ and $d_y$ are changed in steps of 50 nm. (b) and (c) $d_x$ and $d_y$ versus $x$ for one period. (d) and (e) Theoretical and realized $\psi_x(x, y)$ and $\psi_y(x, y)$ for one period. Note that at $x=0$, the phase delays cannot be realized with any of the available patches.

Fig. 3.6. (a) Reflected beam in $\theta_{0x} = 30^\circ$ and $\varphi_{0x} = 0^\circ$ for $x$-polarized illumination (b) 2D reflected beam in $\theta_{0y} = -30^\circ$ and $\varphi_{0y} = 0^\circ$ for $y$-polarized illumination. (c) The $\varphi = 0$, $xz$, cut of both reflected beams.
3.3. Conclusion

In Section 3.2, we introduced the concept of a birefringent reflectarray and presented a simple design procedure to realize them. In such reflectarrays, the incident plane wave is reflected in different angles based on transverse electric field polarization. For instance, the x-polarized and y-polarized electric field are reflected to $\theta_0 = 30^\circ$ and $\theta_0 = -30^\circ$, respectively. The polarization dependent reflectarray structure has no more fabrication complexity compared to the ordinary ones. This type of reflectarrays can be useful in polarimetry application. Also, they can be utilized for realization of optical reflective waveplates.
4. Plasmonic Graded Index Material

In this chapter, we present the fundamental principle of plasmonic graded-index materials for antenna application at infrared range. The method to derive effective refractive index is presented and applied to comprehensive study of effective index engineering. As an illustrative example for radiating structure, a graded index metasurface is designed for collimating the power emanating from an aperture to a steered narrow beam. The center frequency of operation is 57.5 THz. The Half Power Beam Widths (HPBW) of the beam radiated from a $15\lambda \times 15\lambda$ metasurface to $\theta=30^\circ$ direction are $6^\circ$ and $16^\circ$ in elevation and azimuth planes, respectively. Radiation efficiency is calculated to be 86%.

4.1. Introduction

Graded index optical materials, materials with gradual change in refractive index, have introduced a new degree of freedom for designing optical devices. Previously, simple index contrast (step index fibers) and geometrical curvature (lenses) were main design parameters for optical devices. In graded index materials however; the refractive index profile determines the light rays trace inside the material. Such designer materials are used to realize flat optical devices such as lenses for many applications for instance focusing, imaging and routing. Conventional graded index materials are produced with ion exchange method when diffusion is engineered to manipulate refractive index. Metamaterial graded index materials are realized by introducing subwavelength diffractive elements such as holes and gratings into the bulk of dielectric material [36, 90-92]. Since the inclusions are subwavelength, the effective index can be used to characterize the artificial medium.

Recently, the high demand for miniaturized photonic integrated circuits has brought a lot of attention to plasmonic devices. Subwavelength plasmonic particles can interact with light and manipulate light intensity, phase or polarization state. Several planar plasmonic devices are reported for beam focusing and steering applications at infrared and visible range [31, 32, 35, 40, 45, 46, 88]. These planar surfaces work as transmit or reflectarrays and essentially function as meta-lenses. One approach to describe the effect of these patterned plasmonic particles in the context of optical devices is to use well-known refractive index terminology. Even, the term
“generalized laws of Reflection and refraction” is used to describe the effect of patterned plasmonic particles on a surface [46].
Flat optics or optics on the surface using surface plasmons has emerged as a promising solution for photonic integrated circuits [37, 93-95]. Use of plasmonic particles can revolutionize refractive index engineering since a variety of index profiles can be designed merely with patterning metals on top of dielectrics. For instance, numerical simulation has shown graphene layers with patterned conductivity for focusing and Fourier optics applications [95]. Also, a surface patterned with H-shape plasmonic particles is shown to link surface waves to propagating waves [37]. The promise of plasmonic graded index material is low profile, ease of fabrication and enhanced optical devices.
Here, we are employing effective refractive index concept to realize desired index profiles using metallic nanostructures. First, we comprehensively explore the effective index manipulation using plasmonic particles. Then we design some plasmonic graded index devices to show the versatility of our approach. We use holography theorem to devise a planar graded index hologram integrated into an aperture for beam forming applications. Metallic patches are used to manipulate the refractive index of the surface around the aperture. The loss and bandwidth performance of such metasurfaces are investigated.
The paper is organized as follows. In Section 0, we show how to obtain effective refractive index of a unit cell of plasmonic particles. We thoroughly investigate the parameters tuning the effective index. In Section 4.3, we present a graded index hologram for beam forming application.

4.2. Artificial Plasmonic Graded Index Material

The refractive index can be easily manipulated using subwavelength building blocks (plasmonic scatterers or nanoantennas [21]) to provide a fine, yet physically realizable resolution for effective index approximation. The metallic particles interact with the surface wave and decrease wave velocity.

We chose square metal patches on a dielectric substrate backed with a metal as building blocks for effective index engineering (Fig. 4.1).
In a conventional dielectric slab waveguide, the effective index depends on center dielectric thickness and the materials index. Here, effective index is dependent on the patch size as well. So, instead of changing thickness or material, one can simply pattern the patch dimension as desired. This approach offers a huge simplicity advantage from fabrication standpoint.

The effective index is calculated from dispersion diagram of a guiding structure from Eq. (4.1).

\[ n_{\text{eff}} = \frac{k_t}{k_0} \quad (4.1) \]

Where \( k_t \) and \( k_0 \) are guided and free space propagation constants. The dispersion diagram of 2-D dielectric slabs are simply obtainable in forms of transcendent equations. However, one need to use numerical techniques for more complex structures which involve metals with Drude model at infrared frequency to precisely calculate propagation characteristics.

The dispersion diagram of patch building blocks can be calculated assuming local periodicity. We use 3D FDTD solver from Lumerical Inc. [84] to obtain the dispersion diagram of a square metal patch unit cell (Fig. 1) in 45-70 THz band (4.3-6.7 µm). Metal patch and substrate back metal are made from gold. The gold Palik model from Lumerical material library is used for simulation (\( n_{\text{Au}} = 4.4 + i 33.4 \) @55THz). The substrate is SiO2 with refractive index (n) of 1.35 at the desired band. The substrate and gold layer thickness are 300 nm and 50 nm, respectively.

Fig. 4.1. The unit cell of the metallic patch on a grounded substrate. The effective index can be manipulated changing substrate material and thickness as well as the patch size.
First we investigate the effect of patch size (d) on the effective index. The fundamental mode calculated here is a TM mode with $E_x$, $E_z$ and $H_y$ components (x is the propagation direction). The dispersion diagram for different patch sizes is depicted in Fig. 4.2(a). The dispersion diagram of a dielectric slab with thickness of 300 nm is also depicted in the same graph. As shown, the effective index of the plasmonic unit cell changes sharply compared to the dielectric slab. Therefore, it is possible to achieve the desired index at the frequency range of interest and skip the limitations of an all-dielectric slab. This occurs at the expense of higher propagation loss as clear from Fig. 4.2 (c). Also, the larger the patch, the sharper are the index variations. Fig. 4.2 (b) depicts the normalized group velocity of the wave in the unit cell compared with the dielectric slab case. The group velocity is the energy travel velocity and indicates density of a dielectric. One can observe slow wave propagation in the plasmonic patch unit cells.

![Dispersion diagram, Normalized group velocity and Loss diagram of the unit cell for different patch sizes and a dielectric slab waveguide. Intensity of electric field distribution at three different frequencies depicting loosely and tightly bounded surface waves.](image)
Note that a frequency non-dispersive waveguide has constant effective index over frequency. Therefore, frequency dispersion of the plasmonic unit cell is much higher compared to the ordinary slab waveguide and this limits the operation bandwidth of the devices with plasmonic graded index building blocks. The intensity of electric field is shown at three frequencies on the dispersion diagram. The electric field intensity is higher at the gap between the two unit cells. At lower frequencies, the filed is loosely bounded to the surface and extends into free space while at higher frequency the field is tightly bounded to the surface. Expectedly, at higher index of refractions, the field is more confined to the substrate and metal gap. This will result in increased propagation loss. The loss diagram depicted in Fig. 4.2(c) confirms this statement. It should be noted that larger patches incur more propagation loss.

We further investigate the effect of dielectric substrate refractive index and thickness on the effective refractive index of the unit cells. We evaluate the range of available effective indices obtainable with a plasmonic inclusion.

First we change the substrate thickness. We use the same unit cell parameters as before. The patch size is fixed to 0.8 µm and sweep the thickness from 200 nm to 500 nm. From dielectric slab theory we expect higher values for refractive index as we increase the thickness. Our simulation confirms this as shown in Fig. 4.3. For instance, the value of refractive index increases from 1.04 to 1.19 as we increase the thickness from 200 nm to 500 nm at 70 THz. As a result, according to the required range of refractive index, one must choose proper substrate thickness. Note that higher values of refractive index means more confined waves and extra metal loss.

Substrate refractive index considerably affects the effective index. To show this effect, we simulate three different unit cells as follows: unit cell A: a=1000 nm, n=1.35, unit cell B: a=800 nm, n=2 and unit cell C: a=700 nm, n=3. In all cases, the gap size (a-d) is 0.2 µm (The patch size is 900 nm, 700 nm and 500 nm respectively). The unit cell size is determined to keep the effective patch length (physical length divided by n) fixed. Also, the substrate thickness is 200 nm for all unit cells. The result of this set of simulation is shown in Fig. 4.4. Expectedly, Unit cell C has higher effective index because of substrate higher index. One can observe three different regions in the dispersion diagrams shown in Fig. 4.4. These regions are more obvious in group velocity or loss diagrams. For unit cell A group velocity slightly decrease with frequency and propagation loss goes up subtly. In unit cell B case, there is a sudden drop in group velocity.
and at the same time the propagation loss plummets. In unit cell C, the group velocity reaches to its minimum and stays there while the loss is almost constant.

![Dispersion diagram, Normalized group velocity, and Loss diagram](image)

**Fig. 4.3.** (a) Dispersion diagram, (b) Normalized group velocity and (c) Loss diagram of the unit to cell of 800 nm patch on top of SiO2 with thickness (h) sweeping from 200 nm to 500 nm. Thicker substrates increase effective refractive index at the expense of higher loss.

These curves reveal very useful information about the range of available refractive indices for different materials and the associated propagation loss. For instance, one would infer high effective index of 2 is obtained with n=3 at the expense of intolerable loss. Therefore, using plasmonic particles with high index materials is not a reasonable solution because of extra loss. One should adopt other solutions such using grating, holes or composite of high index and low index material as illustrated in [91, 92, 96].

Finally, we observe the effect of cell size on the effective index. Here we choose a= 1 µm, 0.9 µm and 0.8 µm with substrate a 200 nm thick SiO2. The simulation results show that the important parameter determining n is gap size rather than unit cell size. For instance, a 800 nm patch in a unit cell of 1 µm (gap size 200 nm) offers smaller effective index than a 800 nm patch in a 900 nm unit cell (gap size 100 nm).
Considering the curves offered in this section, one can generally associate higher effective index values with higher metal loss. Therefore, the operation frequency must be chosen to avoid working in the tightly bounded part of the dispersion diagram where metal loss is intolerable.

Fig. 4.4. (a) Dispersion diagram (b) Normalized group velocity and (c) Loss diagram for three unit cells with different dielectrics. The refractive index of unit cell A, B and C are 1.35, 2 and 3 respectively. One need to choose proper dielectric to obtain the desired refractive index, however, higher values of n are obtained at the expense of more loss.

4.3. Grade Index Surfaces for Radiation

Optical antennas are required for efficient coupling of optical power into free space or the numerical aperture of optical fibers. An optical antenna radiates source power into free space in a desired direction. Use of leaky wave antennas and holograms in Gigahertz range has shown to be promising antenna solutions for applications requiring high gain from a flat antenna [97, 98]. Optical leaky wave antenna is also proposed for beam collimation applications. A Si based optical leaky wave antenna is working at 1550 nm investigated in [96]. A Silicon slab (n=3.4) is periodically perturbed with Si$_3$N$_4$ (n=1.67) to excite leaky mode. Here, we use holography design technique to obtain a graded index hologram operating as an optical antenna.
The main idea of holography is to produce a desired output wave function from scattering of a known input wave function with a designed hologram. The function describing the profile of the hologram is obtained based on the interference pattern of the known input and the desired output wave functions. We may modify a surface to represent the function of the hologram by manipulating some quantity related to the surface such as its refractive index. Suppose we start with a desired output beam and a known input source as wave functions, \( \Psi_{in} \) and \( \Psi_{out} \). The interference pattern (hologram) is then proportional to \( \Psi_{out} \Psi_{in}^* \) [97, 99]

Imagine one is interested to calculate a hologram pattern that transforms a surface wave emanating from a point source into a pencil beam at the direction defined by \( \theta_0 \) and \( \varphi_0 \) in the spherical coordinate system. Then the input scalar wave function is proportional to

\[
\Psi_{in} \propto \exp\left(-jk_0n_{eq}\rho\right) \quad (4.2)
\]

in (4.2), \( k_0 \) is the free space wave number, \( n_{eq} \) is the equivalent refractive index of the surface and \( \rho \) is the radial distance from the center. The desired output beam is a pencil beam at the direction defined by \( \theta_0 \) and \( \varphi_0 \). As a result, the output wave function is \( \Psi_{out} \propto \exp(-jC(\rho, \varphi)) \) where phase distribution is given by

\[
C(\rho, \varphi) = k_0\rho\sin\theta_0\cos(\varphi - \varphi_0) \quad (4.3)
\]

Here, \( \varphi \) is the azimuthal component of the cylindrical coordinate system. The hologram effective index should be proportional to the interference pattern of input and output wave functions. Then, one can obtain the required refractive index profile

\[
n(\rho, \varphi) = n_0 \left\{ 1 + n_M\cos\left(k_0\rho\left(\sin\theta_0\cos(\varphi - \varphi_0) - n_{eq}\right)\right) \right\} \quad (4.4)
\]

where \( n_0 \) is the mean value and \( n_M \) is the modulating factor incorporated to span the whole range of available effective index. This relation is similar to the one for sinusoidal modulated reactance surfaces analyzed in [100] for 1D cases. Likewise one can deduct that higher index variations, larger (smaller) \( n_M \), causes faster (slower) leakage of the surface wave into free space, smaller (larger) antenna size and wider (narrower) beamwidth.
As shown in Fig. 4.2(a), effective refractive index can be modulated with varying the patch sizes. We can realize the effective index profile in Eq. (4.4) with varying patch sizes according to the data provided in Fig. 4.2(a). Fig. 4.5 shows the effective index versus patch size obtained from Fig. 4.1(a) at 57.5 THz. The mean value \( n_0 = 1.053 \) and modulating factor \( n_M = 0.32 \) are obtained from maximum and minimum achievable effective index.

To demonstrate the suggested design scheme, we design a hologram to collimate the radiated beam of an aperture into a narrow steered beam. The design frequency is 57.5 THz and the refractive index profile is obtained from Eq. 4.4 with \( \theta_0 = 30^\circ \) and \( \phi_0 = 0^\circ \) and transformed to patch size profile using Fig. 4.5. The resulting graded index hologram pattern is depicted in Fig. 4.6(a). The hologram size is 78 µm × 78 µm. The hologram is excited from a 3 µm × 2 µm aperture at the center. The aperture is the open end of a rectangular waveguide extended into the ground. The intensity distribution of the aperture is shown in the inset of Fig. 4.6(a). Also, 2D cut of the electric field is overlaid on hologram in Fig. 4.6(a). As illustrated, the power emanating from the aperture propagates along the surface and leaks into free space radiation mode. The main beam is directed to 30º as designed. The other beams are for higher order radiations which are not carrying considerable power. Fig. 4.6(b) depicts the simulated 2-D far-field intensity of the graded index hologram radiation. As expected, the hologram has collimated the surface wave coupling from the aperture to a narrow concentrated beam in 30º. The HPBW is 6º and 16º in elevation and azimuth directions respectively.

Fig. 4.5 Variations of effective index versus patch size for the unit cell of Fig. 2 at 57.5 THz. As expected, effective index is larger for larger patch sizes.
To explore the bandwidth performance of the designed beam former, we obtain the farfield at frequency range of 54-60 THz. As expected, the beam peak steers away 30° at other frequencies and the amplitude also drops. Fig. 4.7 shows normalized intensity of the peak versus frequency. As depicted, the half power intensity band width is 55-60 THz (8.7%). Also, the beam steers from 26.5° to 33.5° at this frequency range.

Fig. 4.6 (a) The hologram (Not to scale) patterning the aperture and the near field electric field intensity illustrating beam steering. The inset shows the field intensity at the aperture. (b) The simulated 2D farfield radiation from the graded index hologram. The beam is steered into \( \theta_0=30^\circ \) and \( \phi_0=0^\circ \). (c) The \( \phi_0=0^\circ \) cut of the farfield pattern. The HPBW is 6°. (d) The \( \theta_0=30^\circ \) cut of the farfield pattern. The HPBW is 16°.
We also calculate the injected power from the aperture to the surface wave, the radiated power and the power absorbed in metal at the design frequency, 57.5 THz. The radiation efficiency defined as the ratio of the radiated power to the injected power and total efficiency is the ratio of the radiated power to the source power. The difference between the source power and injected power is the power reflected back to the source because of mismatch. Our simulation shown only 2% of the source power reflects back. Hence, radiation efficiency and total efficiency are very close. Simulation results show that 86% of the source power radiates away from the source.

We note an important design consideration here. According to the dispersion curves in Fig. 4.2, the variation of the effective index is small and loss is negligible at low frequency part of the dispersion curves. On high frequency part of the curves, the variation on the effective index is large and loss is not negligible. A design at low frequency part will be low loss, but with small modulation factor $n_M$. The surface wave will leak into free space very slowly and a large hologram is required. On the other hand, the design at high frequencies will result in increased metal loss but a faster leakage into free space which doesn’t require a large hologram. The trade-off between metal loss and modulation factor should be carefully considered in the design procedure. The unit cells size can be tuned to obtain the required modulation factor and loss at the desired frequency. Also, the aperture can be the opening of a Quantum Cascade laser (QCL).

![The farfield intensity versus $\theta$ at different frequencies. From 55 THz to 60 THz, the Half Power Bandwidth is 8.7%.](image)

Fig. 4.7 The farfield intensity versus $\theta$ at different frequencies. From 55 THz to 60 THz, the Half Power Bandwidth is 8.7%.
4.4. Conclusion

We developed and characterized a unit cell for realizing plasmonic graded index material. The unit cell is a metallic patch on a grounded substrate. We then demonstrated the application of such unit cell for design of graded index structures for radiation at infrared range. A high gain high efficiency flat graded index antenna is presented at infrared range. The graded index pattern is obtained using holography technique. The flat graded index surface is excited from an aperture and produces a narrow beam at the desired angle. The antenna operates at 57.5 THz with half power fractional bandwidth of 8%.
5. Metasurface for Light Processing

In this chapter, we present a novel subwavelength unit cell constructed of nanoantennas which can independently manipulate the amplitude and phase of an incident wave. The unit cell is made of two layers of scatterers, where the first can tune the amplitude and the second the desired phase. We show that metasurfaces composed of this unit cell can be used to achieve arbitrary transmission amplitude and phase profiles. Furthermore, we show that these metasurfaces along with Fourier Transform (FT) blocks can be used to realize unique Linear Space Invariant (LSI) transfer functions. This approach opens opportunity for light processing on flat platforms. Novel examples are presented.

5.1. Introduction

Metasurfaces controlling light at subwavelength scales have attracted a lot of attention during the last decade. Planar patterns of nanoantennas (metallic patches, dipoles, loops, slits, nano-holes and core-shells) have been used to control the phase; amplitude and polarization of an incident wave both in reflection and transmission modes. So far, the primary emphasis has been on manipulating phase delay for beam focusing/steering applications [30-32, 34, 35, 41-46, 88]. It is rewarding yet challenging to tailor the transmission amplitude and phase profiles independent of each other.

We will review the capabilities of a metasurface with arbitrarily defined amplitude and phase profiles. In general, the input and output beams of an LSI system are related by

\[ \varphi(x, y) = \imath(x, y) * \varphi_m(x, y) \]  \hspace{1cm} (5.1)

Where \( \varphi_m(x, y) \) and \( \varphi(x, y) \) are the profiles of the incident and transmitted beams, \( \imath(x, y) \) is the impulse response of the system, and * denotes convolution. Emphasizing that \( \imath(x, y) \) is space invariant, Eq. (5.1) transforms to Eq. (5.2) in the Fourier domain.
Eq. (5.2) is identical to the relation between incident beam, transmitted beam and the transfer function of a metasurface. Therefore, we can use FT and inverse FT (IFT) blocks, along with the appropriate metasurface, to realize any LSI system (See Fig. 5.1). Spatial FT and IFT blocks are achievable using flat Gradient Index (GRIN) lenses [95, 101].

As a result, manipulating the transmission amplitude and phase independent of each other enable us to realize a metasurface with arbitrary $T_r(\vec{x},\vec{y})$, allowing development of unique LSI operations.

Nanoantennas previously designed cannot be utilized to manipulate both amplitude and phase at the same time. However, metallic concentric loops on a dielectric substrate can tune transmission phase with a fixed amplitude [35]. By adding another nanoantenna capable of tuning the amplitude alone, it is possible to achieve independent amplitude and phase adjustments. The focus of this paper is to introduce and characterize such novel nanoantennas.

This chapter is organized as follows. In Section 5.2, we introduce a nanoantenna that enables the transmission amplitude control. Section 5.3 reviews the concentric loops that provide phase variation. Cascading these two nanoantennas to achieve a building block controlling both amplitude and phase is discussed in Section 5.4. Two examples illustrating the application of the proposed metasurface nanoantennas are offered in Section 5.5.
5.2. L-Shaped Slot: Tuning Transmission Amplitude

Consider a resonant, narrow rectangular slot in a thin metal film. At resonance, the transmission is maximum. As we change the slot length, both transmission phase and amplitude change at the same time. To bypass this inherent amplitude/phase dependency, we modify the geometry of the rectangular slot to a 45° rotated L-shaped slot as shown in Fig. 5.2(a). It is possible to achieve constant transmission phase by keeping the resonance length, $h = h_1 + h_2$, fixed. The transmission amplitude is then tuned, independent of the phase, by changing each $h_1$ and $h_2$ (the arms’ lengths).

We provide physical insight for the above statement by examining the induced currents and scattered fields in details. Assume the L-shaped slot is illuminated with a normal x-polarized electric field. The incident electric field induces a fictitious magnetic current

$$\vec{M}_{\text{ind}} = \vec{E}_{\text{ind}} \times \hat{n} \quad (5.3)$$

where $\vec{M}_{\text{ind}}$ is the induced magnetic current in the L-shaped slot arms and $\hat{n}$ is the unit vector normal to the surface ( $\hat{z}$ in Fig. 5.2). The scattered electric field encircles the magnetic current source similar to magnetic fields encircling the electric current. Therefore, a y-directed M will radiate an x-directed electric field in $x = 0$ plane and vice versa.

If $h_1 = h_2$ (symmetric arms), $M_x$ in the two arms completely cancel and $M_y$ will radiate $E_y$ (Fig. 5.2(b)). On the other hand, if $h_1 \neq h_2$, $M_x$ will not vanish and a y-directed electric field will be re-radiated (Fig. 5.2(c)). As we increase the asymmetry of the L-shaped slot, the amplitude of the radiated $E_x$ will decrease while the amplitude of $E_y$ will increase. Note that the transmission phase delay will be the same as we keep the total slot length fixed. As a result, we can sweep the amplitude of the transmitted wave (both y and x components) and keep the transmission phase delay constant.

As we change the L-shaped slot geometry between the symmetric arms and a 45° slanted slot, the transmitted $E_x$ value decreases to half and $E_y$ increases from zero to half of the incident $E_x$. Hence, the scattered $E_y$ offers a wider range of relative amplitude variations. The unwanted polarization can be filtered out by a suitable polarizer.
To demonstrate the control over amplitude, we assume local periodicity and simulate the unit cell of the L-shaped slot shown in Fig. 5.2 using a 3D Finite Difference Time Domain (FDTD) solver from Lumerical [84]. The unit cell size is 1 µm and the metal film is a 50 nm thick silver. Lumerical uses Palik’s experimental data [85] to model metals such as silver in the infrared range. The background dielectric refractive index is assumed to be 3.15 and the operation wavelength is 5 µm.

![Fig. 5.2.](image)

- (a) An L-shaped slot in a thin metal film. The total slot length is \( h = h_1 + h_2 \).
- (b) A symmetric L-shaped slot \((h_1 = h_2)\) only transmits \( E_x \) when illuminated by \( E_x \).
- (c) Asymmetric L-shaped slot \((h_1 \neq h_2)\) scatters a \( y \) directed component as well. (Superscript \( i \) and \( t \) stand for incident and transmitted, respectively and subscript \( \text{ind} \) stands for induced).

The structure is illuminated with a \( x \)-polarized plane wave and the scattered/transmitted fields are monitored. Fig. 5.3 depicts the amplitude and phase variations of the scattered \( E_y \) component versus frequency for different values of \( |h_1 - h_2| \). As shown, the relative amplitude of the scattered \( E_y \) changes from zero to 0.45, while the phase variations is around 10º (at 60 THz). We can observe similar behavior for the transmitted \( x \)-polarized field. Nonetheless, relative variations in its amplitude are not significant compared to the \( y \)-polarized field. Remarkably, a 180º transmission phase shift can be achieved by mirroring the L-shaped slot with respect to the \( x \)-axis (or, equally replacing \( h_1 \) with \( h_2 \) and vice versa). It should be emphasized that the fictitious...
magnetic current scatters $E_y$ equally to both sides of the thin metal film. At the same time, reflected $E_x$ is small (in average, reflected $E_x$ carries 10% of the incident power).

![Graphs showing scattered $E_y$ amplitude and phase](image)

Fig. 5.3. (a) Scattered $E_y$ amplitude. The resonance of the L-shaped slot is at 60 THz. The scattered $E_y$ amplitude changes considerably as we change $\Delta h = |h_1 - h_2|$ (b) Scattered $E_y$ phase. Since we have fixed the total length of the L-shaped slot, $h = h_1 + h_2 = 1 \mu m$, the phase variation is very small at resonant frequency (The right plot in (b) shows phase variation in details).

### 5.3. Concentric Loops: Tuning Transmission Phase

In this section, we review how to control the transmission phase with fixed amplitude [35]. Fig. 5.4(a) shows a unit cell consisting of concentric loops. A range of transmission phase delays can be obtained by changing the size of the inner and outer loops ($L_i$ and $L_o$), leaving the transmission amplitude virtually unaffected. We use Lumerical to simulate a unit cell designed to work at 5 µm (60 THz). The unit cell size is 1 µm and the background material refractive index is 3.15. The simulated transmission amplitude and phase for different values of $L_i$ and $L_o$ are shown in Fig. 5.4 (b-c). With a 10% percent amplitude variation tolerance, the phase delay changes over 45º. We can cascade two or more concentric loop unit cells to produce higher phase delay variations, at the expense of more amplitude tolerance.
5.4. Hybrid cell: Tuning amplitude and phase independently

We want to evaluate the possibility of cascading the proposed L-shaped slot unit cell and the concentric loops to realize a double-layered unit cell that controls the amplitude and phase independently. The spacing between the two unit cells should be large enough to mitigate the mutual coupling effect.

A cascaded unit cell of the two aforementioned nanoantennas is shown in Fig. 5.5. Simulation results show the mutual coupling effect can be ignored with spacing of 500 nm (λ/10) or larger in a background material with n = 3.15.

Amplitude and phase of the transmitted beam is then controlled by three parameters, |h_i – h_o|, L_i and L_o. The phase variation is mainly determined by L_i and L_o and |h_i – h_o| dominantly controls the amplitude. The cascaded unit cell is simulated for different values of geometrical parameters, and the transmission amplitude and phase are observed. (See Fig. 5.5 (b-e)).

Fig. 5.5(b) illustrates amplitude changes as a function of |h_i – h_o| with L_i as a sweeping parameter. The transmission amplitude varies considerably with the L-shaped slot parameter, |h_i – h_o|. The curves for different values of L_i coincide. In Fig. 5.5(c), the changes in transmission phase delay are negligible against |h_i – h_o|, while it shifts considerably with L_i. The transmission amplitude is hardly sensitive to L_o and L_i as revealed in Fig. 5.5(d) while both loop parameters affect transmission phase as presented in Fig. 5.5(e).
This cascaded unit cell establishes a building block to control amplitude and phase independently. We can use an array of these building blocks to realize any desired transfer function. Since the proposed unit cells are designed based on local periodicity assumption, abrupt changes in either amplitude or phase profiles are not perfectly achievable.

### 5.5. Light Signal Processing Application

In this section, we demonstrate some novel applications of the proposed nanoantenna building block and the associated metasurfaces.

![Diagram](image)

Fig. 5.5. (a) A double-layered unit cell composed of the unit cells in Fig. 5.2(a) and Fig. 5.4(a). (b) and (c) The amplitude changes strongly versus $|h_1 - h_2|$ and phase remains virtually unchanged. (d) and (e) The amplitude remains almost constant when concentric loops parameters are swept while the phase undergoes considerable variations. ($L_i$ is the sweep variable in all curves)

Let us first present a motivating application of the L-shaped slot nanoantennas in realizing spatial derivation functionality. Engheta has shown the conceptual implementation of some signal processing operations like differentiation and convolution based on metamaterials [102]. Recall that spatial differentiation is as easy as multiplying by $\tilde{f}$ in Fourier domain. The unit cell of the L-shaped slot proposed here can simply perform this multiplication.
Here, we design a metasurface to perform multiplication by $\tilde{y}$. For simplicity, a 2D case is considered when the wave propagates along the z-direction and the structure is infinite along x-direction. A rectangular pulse is selected as the spatial distribution of the incident wave. The pulse spatial width is 30 $\mu$m. We arrange an array of 15 L-shaped slots along y-direction, from $y = 0$ to 15 $\mu$m. The values of $h_1$ and $h_2$ for each of these nanoantennas are obtained from Fig. 5.3(a) to realize $T_y(\tilde{y}) \propto \tilde{y}$. The structure is mirrored with respect to x for the negative values of $y$ and repeated along x-direction.

Fig. 5.6(a) shows a part of the designed metasurface sandwiched between the two dielectric layers.

Fig. 5.6. (a) Metasurface of L-shaped slots designed to multiply the incident wave by $T_y(\tilde{y}) \propto \tilde{y}$. (b) The input constant amplitude is re-formed into linear amplitude after passing the metasurface. (c) The phase is shifted 180º for negative values of $\tilde{y}$. (Solid lines are simulation results and dashed lines are ideal output profile).

The metasurface is setup in Lumerical and illuminated with the 30 $\mu$m wide rectangular pulse. The normalized output beam amplitude and phase are monitored and depicted in Fig. 5.6(b) and (c), compared to the theory.
The comparison reveals that the metasurface performs multiplication by $\bar{y}$. Despite the good agreement between simulation and theory in the middle area of the metasurface, the mismatch is large at the edges since the local periodicity assumption doesn’t hold anymore. For practical purposes, we can filter out the x-polarized component by a linear polarizer. Tailoring the amplitude with a passive structure inherently decreases the efficiency. The maximum transmitted y-polarized component is 42% as shown in Fig. 5.3(a).

Now, we can add FT and IFT blocks to the realized metasurface (See Fig. 5.1) to obtain the differentiation functionality. An FT block can be realized by GRIN flat lenses [101]. These flat lenses are easy to fabricate by drilling subwavelength air holes on a dielectric slab [91]. Fig. 5.7 shows a GRIN material with refractive index profile

$$n(y) = n_1 \sqrt{1 - \left(\frac{y}{h}\right)^2} \quad (5.4)$$

The structure is infinite in x-direction and the input beam travels in z direction. After traveling a distance of $z_p - z_0 = \pi h/2$, the beam profile $\psi(z_p, y)$ is proportional to spatially-scaled FT of the beam at $z_0$ ($\psi(z_0, \bar{y})$). The details are given in Fig. 5.7 and [101]. Note that $\bar{y}$ still represents a spatial position, though it denotes the Fourier domain variable. In other words, the FT of the input beam profile is formed after traveling a distance of $\pi h/2$, scaled in the y axis by $\alpha$.

The differentiator is shown in Fig. 5.8(a). The two GRIN parts implement FT and IFT and the metasurface in between performs multiplication by $\bar{y}$. For this case, $n_1 = 3.3$, $h = 50 \mu m$ which gives $z_p - z_0 = 67.5 \mu m$.

The FDTD mesh for the metasurface is fine ($d_x = d_y = 12.5 nm$) to resolve the small features in L-shaped slot unit cells. Due to small mesh size, the simulation becomes computationally prohibitive. We adopt a block-by-block approach for this simulation. The first FT block is excited with the input beam profile. The output of this block is monitored and used as excitation for the metasurface block. Then, the metasurface output is monitored and sourced into the third block.

Note that the incident beam in the interface of the FT block and metasurface spans only 20 $\mu m$ around $y = 0$ where the index changes from 3.3 to 3.23. The maximum reflection between GRIN
lenses with the profile defined by Eq. (5.4) and the background material of the metasurface with 
$n = 3.15$ interface is 0.023.

Here, the Sinc function is selected as the input beam profile. The first GRIN block in Fig. 5.8(a) 
gives a rectangular pulse, which is the FT of a Sinc function. The same metasurface in Fig. 6 is 
used here to perform $i\tilde{y}$ multiplication. Then the second GRIN performs the IFT, producing the 
derivative of the Sinc function at the output.

This structure is simulated in Lumerical and the output of each block is shown in Fig. 5.8(b). The 
simulated IFT output wave profile is in agreement with the derivative of a Sinc function for the 
main lobes in the center. Aside from the errors originating from the metasurface, the discrepancy 
between the two in the far side lobes, is due to the imperfect IFT block. (The GRIN lens works 
well when index variations are small over a wavelength. As the index profile in Fig. 5.7 reveals, 
this is not the case at the edges).

Note that we designed the unit cell of the L-shaped slots and concentric loop in a background 
material with refractive index equal to 3.15, easily embeddable in the GRIN blocks. 
Nevertheless, the design concept can be implemented standalone or on a dielectric substrate 
without any loss of generality.

As last example, we devise a beam synthesis metasurface. The metasurface is configured to 
synthesize triangle envelope amplitude and phase when illuminated with a rectangular pulse. It 
should be mentioned that for each unit cell configuration, the desired amplitude and phase are 
obtained independently from the data provided in Fig. 5.3 and Fig. 5.4. Concentric unit cells are 
tuned slightly to compensate for the undesired phase delays induced by the L-shaped slots where 
needed.
Fig. 5.7. The GRIN material taking FT. The GRIN material with the profile defined in (4.4) gives accurate FT as long as refractive index variations are small. Therefore, the beam spatial extent should be concentrated in the center of the GRIN material for the best performance. The refractive index variation is shown in the right curve.

\[
\varphi(z_p, y) \propto \psi(z_0, \bar{y}) = \psi(z_0, \frac{y}{\alpha}) = FT(\psi(z_0, y))
\]

\[
\bar{y} = \frac{y}{\alpha}, \quad \alpha = \frac{\lambda h}{2\pi n_1}
\]

Fig. 5.8. (a) FT-Metasurface-IFT cascaded to perform differentiation. (b) Profile monitors at each block. A Sinc function is sourced into the spatial differentiator and its derivation is obtained on the other side.

A part of the double-layer metasurface realizing triangle phase and amplitude is shown in Fig. 5.9(a). The metasurface is excited with a 24 µm wide rectangular pulse. We expect
triangular waveforms for both amplitude and phase of the output wave. Output amplitude and phase are depicted in Fig. 5.9(b) and (c), respectively, both matching the triangular envelope. The maximum error is less than 7° in the phase curve except for the unit cells at each end of the array. The discrepancy between the designed phase and simulated one at the edges can be attributed to both the small input amplitude and diffraction from the edge of the array.

![Diagram of metasurface array](image)

Fig. 5.9. (a) A cascaded metasurface array for controlling both amplitude and phase of the transmitted beam independent of each other. This metasurface reshapes the rectangular pulse profile of the input beam to a triangular pulse. The transmission phase is reformed to an inverse triangle shape at the same time. The top layer, L-shaped slots, control the amplitude and the bottom layer, loops, tailoring the phase. (b) Output amplitude and (c) phase. (The squared lines are the designed values obtained from the simulation of the separate unit cells (Data obtained from Fig. 5.3 and Fig. 5.4). The red solid line is from full wave FDTD with Lumerical solver and thick black line is the input beam amplitude)

The above example clearly validates the ability of our model to control both amplitude and phase locally and independently, using metasurfaces of nanoantennas. We can configure the double layered metasurface to synthesize other desired transmission amplitude and phase profiles, and achieve mathematical operations of interest, as required by application. We mention that these
functional metasurfaces may be realized by merely patterning the metallic films with the standard nanofabrication technology. This is for the first time metasurfaces for light processing have been introduced, to the best of authors’ knowledge. As one can see, tremendous opportunities for light processing can be enabled by this new design.

5.6. Conclusion

In this chapter, we demonstrated the concept of metasurfaces nanoantennas to control both amplitude and phase of an incident wave, locally, desirably, and independently. In our model, the L-shaped slot is proposed to tailor the amplitude and the concentric loop to engineer the phase. Cascading these two metasurfaces offers the opportunity to design desired mathematical functions, which can lead us to enable the idea of “flat optics” engineering. Novel examples have been demonstrated.
6. Future Works

In chapter 2, we presented a sample case of filter design in a generic plasmonic waveguide with the design methods borrowed from circuit and transmission line theory. It is very interesting to consider realizing other passive components such as couplers and multiplexers. Also, using more realistic plasmonic waveguides such as groove, slot and wedge waveguides can be challenging and at the same time rewarding. Also, providing a transmission line model for each of plasmonic waveguides based on their geometry and material parameters could be a great step in improving plasmonic passive components design and simulation.

In chapters 3 to 5 we focused on metasurfaces in infrared frequency range with different applications. We introduced birefringent reflectarray with interesting polarization dependent response. Also, a beam collimator as a metasurface integrated into an aperture is presented. In future, it is possible to enhance the performance of current metasurfaces or use them for some specific applications. Particularly, it is possible to use the birefringent reflectarray concept to realize a quarter waveplate.

In chapter 5, we introduced the double layer metasurface for light computation. In this regard, it is extremely appealing to enhance the performance of such metasurface. Adding another layer to provide matching can be an interesting research topic.

Also, currently the metasurfaces introduced are working based on resonant nano-particles. The extreme phase and amplitude variations occur because of resonant behavior of the particle. Therefore, they are essentially narrow band. Investigating approaches to improve the bandwidth of the current structures is another interesting problem.
References


References


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References


References


