NORTHEASTERN UNIVERSITY

Statistical Characterization of a Class of Underwater Acoustic Communication Channels

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy

in

Electrical Engineering (Communications, Controls and Signal Processing)

by

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The dissertation of Parastoo Qarabaqi is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Northeastern University

2014
DEDICATION

To my loving family.
EPIGRAPH

A transatlantic word with the Caspian Sea

—Siavash Kasraei
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ABSTRACT OF THE DISSERTATION

Statistical Characterization of a Class of Underwater Acoustic Communication Channels

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Professor Milica Stojanovic, Chair

Acoustic channel models provide a tool for predicting the performance of underwater communication systems prior to deployment, and are thus essential for system design. In this dissertation, we offer a statistical channel model, which incorporates physical laws of acoustic propagation (frequency-dependent attenuation, bottom-surface reflections) as well as the effects of inevitable random local displacements.

We focus on random displacements on two scales: small-scale effects, that involve distances on the order of a few wavelengths, and large-scale effects, that involve many wavelengths. Small-scale effects include scattering and motion-induced
Doppler shifting, and are responsible for fast variations of the instantaneous channel response; while large-scale effects describe the location uncertainty and changing environmental conditions, and affect the locally-averaged received power. We model each propagation path by a large-scale gain and micro-multipath components that cumulatively result in a complex Gaussian distortion. Random surface motion and transducer displacement introduce additional variation whose temporal correlation is described by Bessel-type functions. The total power, or the gain contained in the channel, averaged over small-scale, is modeled as log-normally distributed. The models are validated using real data obtained from four experiments. Specifically, experimental data are used to assess the distribution and the auto-correlation functions of the large-scale transmission loss and the short-term path gains. While the former indicates a log-normal distribution with an exponentially decaying auto-correlation, the latter indicates a conditional Ricean distribution with Bessel-type auto-correlation.

Based on the proposed model, we design a channel simulator which we employ to generate a time-varying channel whose statistical characteristics match with those of a real underwater channel. The simulated channel is applied to convey an OFDM signal to coherent and differentially coherent detectors, and the MSE performance of the experimental and simulated systems are shown to be similar.

Finally, we investigate the feasibility of adaptive power control using an experimental data set as well as theoretically. Based on the observed time-correlation properties of the large-scale channel gain, linear power prediction is employed and achievable power savings are obtained analytically (assuming a log-normal gain distribution) and experimentally. The results indicate that substantial power savings are possible over extended periods of time.
Chapter 1

Introduction

1.1 Motivation

Over the past few decades, motivated by the growing needs for wireless underwater communications, and in view of the emerging digital communication techniques, underwater acoustic (UWA) communications have shifted from military toward commercial [1]. Modern UWA communication systems are used in off-shore oil exploration, environmental monitoring, fish detection, sonar, and for communication between underwater vehicles, and it will not be long before wireless transmission of real-time underwater video from acoustically controlled vehicles will become possible. The availability of new applications depends on overcoming the harsh UWA channel. High frequency acoustic signals are attenuated by the underwater channel over long distances, hence extremely limiting the available bandwidth for communication. The acoustic signals are subject to multipath propagation with long delay spreading and extreme motion-induced Doppler effect, both due to the very low speed of sound underwater [2].

UWA communication systems have to be designed to operate in a variety of conditions that differ from the nominal ones due to the changes in system geometry and environmental conditions. Channel simulators enable system designers to allocate the appropriate resources (power, bandwidth) prior to system deployment, as well as to design appropriate signals and processing algorithms on both the physical link layer and the higher network layers, and are thus essential as a
precursor to costly deployments. The development of a channel simulator requires knowledge of a channel model.

One approach to channel modeling is by ray tracing and normal mode theory [3] which provide approximate and numerical solutions for the wave equations for a deterministic channel geometry and signal frequency. Such models are thus restricted to a single frequency and one realization of the time varying acoustic channel. A different approach to channel modeling is statistical analysis which is carried out in small and large time scales. The notions of small- and large-scale are justified by common sense, but an exact definition for them does not exist. We refer to variations that are caused by displacements spanning many wavelengths as large-scale variations, and to those that are caused by displacements on the order of one or a few wavelengths as small-scale variations. Large-scale variations are modeled as a consequence of system displacements that cannot be predicted using the nominal system geometry. Namely, while a nominal transmitter and receiver placement within a fixed system geometry and a fixed sound speed profile will yield a fixed nominal acoustic field, the actual field will vary due to uncertainty about the exact system geometry. Such an uncertainty is treated as random, leading to large-scale variation in the gains and delays of propagation paths. At the same time, once a particular large-scale displacement is given, additional small-scale variation will occur in the path gains and delays. This variation is modeled as a consequence of scattering and instantaneous motion. Considering motion on the order of 1 m/s, and frequencies on the order of 10 kHz, a wavelength is traversed during a sub-second interval. Such short intervals of time incidentally correspond to typical communication transactions (a packet or a frame of packets). Small-scale channel variations can thus be thought of as those variations that occur over a communication transaction.

Small-scale variations influence the instantaneous channel response, and consequently the instantaneous signal-to-noise (SNR) ratio. As such they are meaningful for the analysis of signal processing algorithms and network protocols, and the assessment of average bit error rate or packet error rate, conditioned on a particular large-scale realization. In contrast, large-scale variations influence
the locally-averaged SNR, causing it to vary over longer periods of time. As such they are meaningful for the analysis of top-level system functions such as power allocation and the assessment of outage probabilities and statistical coverage.

1.2 Previous Work

Conventional beam tracing tools, such as Bellhop [4], provide an accurate deterministic picture of an UWA channel for a given geometry and signal frequency, but they do not take into account random channel variation. In recent years, there has been a growing awareness of the need to develop statistical channel models that will lead to computationally efficient tools for numerical simulation. New tools have been developed that address this need to some extent. For example, the VirTEX algorithm [5] was developed to simulate the effect of channel variation in a manner that is computationally more efficient than repeated application of the Bellhop beam tracing. This algorithm operates by tracing multiple inter-related beams to assess the cumulative effect on the signal of a given frequency. However, the computational complexity may still be an issue. For example, simulating a channel with a Doppler distortion on the order of 15 Hz, requires at least 30 channel realizations per second which means a total of 5400 Bellhop runs for simulating a system with 2 transmitters and 6 receiver elements over a period of 15 seconds.

The World Ocean Simulation System [6] provides a platform for simulating UWA networks. This tool uses Bellhop to reproduce the propagation effects for a given channel geometry whose properties are extracted from various databases. Particularly, the user specifies the geographic coordinates of the desired channel location and the simulator queries the databases for measurements of sound speed and bathymetry, and samples of the bottom sediments. While the databases provide realistic values for the nominal channel properties, the effects of random channel variations are not taken into account. The wavefront model [7] offers a deterministic approach which provides approximations to the ray theory to efficiently model the effects of the curvature of the surface waves and the amplitude and arrival time fluctuations that they introduce [8].
Numerous studies have also been conducted to model the UWA channel stochastically, e.g. [9–21]. These studies are usually based on the analyses of experimental acoustic data collected in a particular location. Some authors find Ricean fading [10, 11] or Rayleigh fading [9, 13, 17] to provide a good match for their measurements, while others find log-normal distribution [12, 14] or the K-distribution [15, 16] to be a better fit. The variety of the proposed statistical models is due to experiment-specific properties, e.g. the deployment site and the type of signals used for probing, as well as the time intervals during which the channel is observed.

Statistical modeling of small-scale phenomena is a subject of on-going research, which points to both Rayleigh and Ricean fading, and no consensus exists yet on this topic. Modeling of large-scale phenomena has also been addressed only to a very limited extent (see e.g. [18] which shows some evidence of log-normal fading), while a few attempts have been made at unifying the small and large-scale models (see e.g. [10, 15, 19]). In Ref. [18], knowledge of the environmental conditions at an experiment site was used to repeatedly run the Bellhop model to generate an ensemble of channel responses which was then used to estimate the statistical properties of the channel. The changing environmental conditions which were taken into account included water temperature and salinity. The results obtained through the Bellhop channel simulator showed statistical properties similar to those obtained experimentally; however, an accurate agreement was not observed. This mismatch may be due to the fact that only partial knowledge of the environmental conditions was used, and that surface variation was not taken into account. In Ref. [22], the authors used the environmental data obtained from an acoustic experiment to generate varying surface shapes which were then imported into the Bellhop beam tracer to generate the corresponding acoustic field. The simulated data were compared with the experimental measurements and a good agreement was observed for calm ocean conditions. At high wind speeds, however, the simulation results deviated from the experimental measurements. This deviation may be caused by the inability of the surface shape generator to model breaking waves which occur at high wind speeds, causing formation of bubble
clouds, as well as acoustic focusing by wave curvature [23]. In [24], the formation of bubbles near the water surface was found to be the underlying cause for the changing behavior of the UWA channels at different wind speeds. The radius and the density of bubbles that are generated at different wind conditions, as well as their acoustical properties, were studied. Such knowledge can be used to further improve the existing UWA channel models.

Adaptive power and rate control algorithms have to be designed based on the statistics of the large-scale channel variations. While power control algorithms have been widely studied for wireless radio and satellite channels (see for example [25–27]), limited studies in this field have been carried out for UWA channels (see for example [28] and [29]).

1.3 Contributions

The goal of this study is to provide a mathematically rigorous analytical model that takes into account certain physical aspects of acoustic propagation, and to validate such a model using experimental data. In parallel, our goal is also to develop a numerically efficient simulation model that describes the channel response in a wide range of frequencies representative of an acoustic communication system. Based on the model developed for the large-scale channel gain, an effective prediction method is proposed which is aimed at implementing adaptive power control. The contributions of the thesis and a list of publications are described below.


- An approximate multipath propagation model is introduced in which the effects of path filtering and multipath are decoupled.

- Statistics of the path gains are estimated from experimental data gathered in the Narragansett Bay near the coast of Rhode Island, USA in March 2008.
The path gains are observed to hold a time-varying local average and an additional, more rapidly varying random component. The time-varying local average of the path gains will lead to time-varying average power which is shown to be possible to predict using a simple linear predictor. This fact motivates the use of adaptive power control in UWA systems.

- Experimental data recorded in Fall 2008 near the coast of Martha’s Vineyard in Massachusetts, USA are used to model the locally averaged power gain. The data shows that the gain expressed in decibels can be modeled as an autoregressive process of low order (AR-1 or AR-2) with Gaussian input and coherence time that is longer than the round trip delay and thus allows the implementation of feedback.

- A feedback-based power control algorithm is suggested that is aimed at keeping the SNR constant such that the outage probability does not exceed a predefined threshold. The achievable power savings are analytically obtained under log-normal fading for a trivial delayed estimate method (in which the future value of the gain is estimated as equal to the most recent feedback value) as well as linear predictions of order 1 and 2, which are suitable for channels with AR-1 and AR-2 fading, respectively.

- The performance of the prediction algorithm is obtained for the experimental gain, and its dependence on different algorithm parameters such as the averaging window and the prediction lag and order is investigated. Significant power saving is shown to be achievable by adaptive power control.

- For channels with varying distance, the mean value of the gain is modeled to maintain a log-distance relationship, while the additional variation obeys a log-normal distribution. The data obtained from an experiment conducted off the coast of Northern California, USA, in September 2010 is used to verify the proposed model. The variance of the log-normal process is observed to be independent of the distance for the range of distances considered, i.e. 100 m–1 km.

- The model is used to investigate the problem of utilizing an autonomous underwater vehicle (AUV) to collect data from an underwater sensor network. The sensors in the network are equipped with acoustic modems that provide noisy communication whose quality degrades logarithmically with distance. A block log-normal fading model is used for SNR in which instantaneous SNR is assumed to be constant over the duration of one block. AUV path planning methods are proposed that maximize the information collected while minimizing travel time or fuel expenditure.

- The large-scale model previously proposed for the gain is verified for data recorded south of Martha’s Vineyard in Summer 2010 over a distance range of 500 m–4 km. The model targets locally-averaged gain in which the small-scale effects are averaged out.

- The effective spreading factor, which describes the log-distance model, is observed to be frequency dependent, while the variance of the log-normal process depends on the system bandwidth.


- To represent small-scale scattering, each transmission path is modeled as consisting of micromultipath components that cumulatively result in a complex Gaussian multiplicative distortion, whose time- and frequency-correlation are assessed analytically.

- Motion of the surface and transmitter/receiver displacements introduce additional variation whose temporal correlation is modeled via Bessel-type functions.

- Statistics of the path gains estimated from data collected during four different experiments are presented that verify the proposed model. The experiments were carried out at various sites and during different times (including a relatively new experiment held off the coast of Kauai Island, HI, USA, in July 2011). Pre-processing of data consists of Doppler compensation carried out via resampling as well as phase tracking by using a combination of recursive least squares (RLS) and a second order phase-locked loop (PLL). The orthogonal matching pursuit (OMP) algorithm is then used to estimate the sparse channel impulse response.

- Based on the proposed model, a computationally efficient simulator is designed which incorporates physical laws of acoustic propagation (frequency-dependent attenuation, bottom/surface reflections), as well as the effects of inevitable random local displacements which are treated in small and large time scales. Small-scale effects include scattering and motion-induced Doppler shifting, and are responsible for fast variations of the instantaneous channel.
response, while large-scale effects describe the location uncertainty and changing environmental conditions, and affect the locally averaged received power.

- The log-normal model observed from experimental data for the gain is assessed analytically for wideband systems assuming that the path lengths are Gaussian distributed.


- The proposed acoustic channel simulator is used to generate an ensemble of impulse responses based on the statistical channel properties estimated from experimental data.

- The histograms and the time-correlation functions of the small-scale path coefficients and the large-scale channel gain are compared with those of the experiment and matching results are observed.

- The simulated channel is used to convey an OFDM signal to coherent and differentially coherent detectors, and the MSE performance of the experimental and the simulated channel are shown to be similar.
Chapter 2

Channel Modeling

An acoustic wave undergoes various effects in an underwater medium which are caused by different physical phenomena. In this chapter, we develop a model for the UWA channel by studying these effects. First, we describe acoustic attenuation caused by propagation, spreading and reflection losses in a time-invariant channel. We then study the effects of large-scale channel displacements. Small-scale scattering and the inevitable motion-induced Doppler effects are studied next.

2.1 Nominal Conditions, Large-Scale Uncertainty

2.1.1 Nominal conditions: decoupling path filtering from path gains

Nominal channel geometry, along with a specified sound speed profile, gives rise to the nominal response of an acoustic channel. This response characterizes a time-invariant system, and it can be assessed by beam tracing for typical acoustic communication frequencies.

The basic path loss experienced by a signal of frequency $f$ traveling over distance $l$ affects the received signal energy and is given by [30]

$$A(l, f) = A_0 l^k a(f)^l$$  \hspace{1cm} (2.1)
where $A_0$ is a scaling constant, $k$ is the spreading factor, and $a(f)$ is the absorption coefficient which can be obtained in dB/km using the Thorp’s empirical formula [31] as $0.11 \frac{f^2}{1+f^2} + 44 \frac{f^2}{1000+f^2} + 2.75 \times 10^{-4} f^2 + 0.003$, where $f$ is in kHz. Considering multiple propagation paths of nominal length $l_p, p = 0, \ldots, P - 1$, each path will act as a low-pass filter, whose transfer function, which affects the amplitude of the received signal, can be modeled as

$$\tilde{H}_p(f) = \frac{\Gamma_p}{\sqrt{A(l_p, f)}}$$  \hspace{1cm} (2.2)

where $\Gamma_p$ is the cumulative reflection coefficient encountered over $n_{sp}$ surface and $n_{bp}$ bottom reflections along the $p$-th path. For example, an ideal surface can be modeled by a reflection coefficient $\gamma_s = -1$, while each bottom reflection can be modeled by [30, Chapter 3]

$$\gamma_b(\theta_p) = \begin{cases} \frac{\rho_b \sin \theta_p - \rho \sqrt{(c/c_b)^2 - \cos^2 \theta_p}}{\rho_b \sin \theta_p + \rho \sqrt{(c/c_b)^2 - \cos^2 \theta_p}}, & \cos \theta_p \leq c/c_b \\ 1, & \text{otherwise} \end{cases}$$  \hspace{1cm} (2.3)

where $\theta_p$ is the grazing angle associated with the $p$-th propagation path, $\rho$ and $c$ are the density and the speed of sound in water ($\rho = 1000$ kg/m$^3$ and $c = 1500$ m/s, nominally), and $\rho_b$ and $c_b$ are the density and the speed of sound in bottom. For soft bottom, $c_b < c$, so that critical reflection never occurs.

Given the transfer function of each path, the overall transfer function of the multipath channel is obtained as

$$\tilde{H}(f) = \sum_p \tilde{H}_p(f)e^{-j2\pi f \tau_p}$$  \hspace{1cm} (2.4)

where $\tau_p$ is the propagation delay associated with the $p$-th path, i.e., $\tau_p = l_p/c - t_0$ for constant sound speed, measured in reference to some $t_0$, e.g., $t_0 = l_0/c$.

In general, each path is characterized by an impulse response of a different shape, and this fact prevents one from obtaining a tractable, simple channel model. To explore simplified versions, we examine an approximation to the function $\tilde{H}_p(f)$. In particular, let us express this function so as to include the dependence on the
Figure 2.1: Transfer function $\bar{H}_0(f)$ corresponding to a reference path of length 1 km and spherical spreading ($k=2$).

The frequency-dependence that distinguishes the $p$-th path from the reference path is embodied in the term $a(f)^{l_p-l_0}$ in the expression (2.5). If this term could be approximated as constant, one could model all the paths by an impulse response of the same shape, and just a different gain. The fact that the absorption factor $a(f)$ has a value very close to 1 for a broad range of acoustic communication frequencies may justify an approximation of the form

$$a(f)^{-(l_p-l_0)/2} \approx a_0^{-(l_p-l_0)/2}$$  \hspace{1cm} (2.6)

where $a_0$ is the absorption factor corresponding to a frequency within the signal bandwidth $[f_0, f_0 + B]$. We examine the viability of such an approximation in Fig. 2.2. This figure shows the factor $a(f)^{-\Delta l_p/2}$ as a function of frequency, for several path length differences $\Delta l_p = l_p - l_0$. While the dependence on $\Delta l_p$ is exponential, the spacing between the curves appears to be linear because $a(f)$ is very close to 1, i.e. $a(f)^{-\Delta l_p/2} = [1+\epsilon(f)]^{-\Delta l_p/2} \approx 1 - \epsilon(f)\Delta l_p/2$. The values shown range only between about 0.9 and 1, indicating that the approximation (2.6) may indeed be valid, especially for small path length differences. The smallest path
Figure 2.2: Verifying the approximation (2.6). Note that one needs to focus only on the frequency range occupied by a given system. For example, if a system operates in the 10 kHz – 20 kHz acoustic band, the factor \( a(f)^{-\Delta \ell_p/2} \) varies only between 1 and 0.98 for \( \Delta \ell_p \) up to 40 m.

length difference shown, 15 m, corresponds to the relative path delay of 10 ms, a value that is within the multipath spread of the majority of shallow water channels. Note also that it suffices to judge the validity of our approximation only within the frequency range occupied by a given system. For example, if a system operates in the 10 kHz – 20 kHz acoustic band, the factor \( a(f)^{-\Delta \ell_p/2} \) varies only between 1 and 0.98 for \( \Delta \ell_p \) up to 40 m.

Using the approximation (2.6), we model the channel transfer function as

\[
\bar{H}_p(f) \approx \bar{h}_p \bar{H}_0(f)
\]

where the path gain is given by

\[
\bar{h}_p = \frac{\Gamma_p}{\sqrt{(\ell_p/\ell_0)^k a_0^{\ell_p-\ell_0}}}
\]

The constant \( a_0 \) may be taken as the absorption factor at any frequency within the operational bandwidth, e.g. the center frequency \( f_c \), or the lower/upper band-edge frequency, resulting in maximal/minimal path gain, respectively. Any choice should be fine, since the assumption is that \( a(f) \) does not change much over a typical bandwidth of an acoustic communication system.
From our discussion so far, it seems reasonable to adopt a channel model in which the effects of path filtering and multipath are decoupled such that each path contributes with a gain $\bar{h}_p$ and delay $\bar{\tau}_p$, while the filtering effect is the same for all the paths, and described by the function $\bar{H}_0(f)$. The overall channel transfer function is thus given by

$$\bar{H}(f) = \bar{H}_0(f) \sum_p \bar{h}_p e^{-j2\pi f \bar{\tau}_p}$$  \hfill (2.9)

### 2.1.2 Large-scale displacements (location uncertainty)

Transmitter/receiver displacements within a nominal channel geometry, as well as changes in the surface height (e.g. due to tides) or shape of the bottom, lead to an uncertainty about the exact system geometry. These displacements effectively cause the path length to deviate from the nominal as $l_p = \bar{l}_p + \Delta l_p$, where the variation $\Delta l_p$ is regarded as random.

The path delays $\tau_p$ are easily calculated for the lengths $l_p$, while the path gain $h_p$ is obtained by using $\bar{l}_p$ instead of $\bar{l}_p$ in the expression (2.8), which yields

$$h_p = \bar{h}_p \frac{1}{\sqrt{(1 + \Delta l_p/\bar{l}_p)^k a_0^{\Delta l_p}}}$$  \hfill (2.10)

Noting that for a typical system geometry we have that $\Delta l_p \ll \bar{l}_p$, and that $k \ll \bar{l}_p$, where $\bar{l}_p$ is expressed in meters, we proceed to make the following approximation

$$(1 + \frac{\Delta l_p}{\bar{l}_p})^k \approx 1 + k \frac{\Delta l_p}{\bar{l}_p} \approx (1 + \frac{k}{\bar{l}_p})^{\Delta l_p}$$  \hfill (2.11)

Fig. 2.3 illustrates the validity of this approximation for a given set of parameters.

With the approximation (2.11), the path gain can be expressed as

$$h_p \approx \bar{h}_p e^{-\xi_p \Delta l_p/2}$$  \hfill (2.12)

where

$$\xi_p = a_0 - 1 + k/\bar{l}_p$$  \hfill (2.13)
and the approximation follows from the fact that \( a_0 \approx 1 \). Note that the linear approximation in expression (2.11) could also be used; however, we prefer the exponential one since it guarantees that the gain will be positive. Note also that if the path length variation (location uncertainty) can be modeled as Gaussian, the path gain will be log-normally distributed. We will use this notion later in Chapter 3 when we discuss the overall channel gain. The overall transfer function is now given as

\[
H(f) = \bar{H}_0(f) \sum_p h_p e^{-j2\pi f \tau_p}
\]  

(2.14)

where location uncertainty is captured by the large-scale parameters \( h_p \) and \( \tau_p \).

### 2.2 Small-Scale Channel Characterization

The channel transfer function (2.14) captures only the large-scale effects, i.e. it does not provide any information about fine-scale phenomena such as scattering. Scattering is a major contributor to signal variations that typically appear as random. A signal of frequency \( f \) undergoes scattering on rough surfaces and objects whose dimension is on the order of a few signal wavelengths \( \lambda = c/f \). For example, the wavelength corresponding to an acoustic frequency component of 15 kHz is 0.1 m, hence the distances in question are called “small.”
To model scattering in an UWA channel, let us focus on a single propagation path, say path $p$. So far, we have modeled this path as having the gain $h_p$ and delay $\tau_p$. However, if scattering occurs along this path, it is split into a number of micro-paths

$$H(f) = H_0(f) \sum_p \sum_i h_{p,i} e^{-j2\pi f \tau_{p,i}}$$

(2.15)

where $h_{p,i}$ are the intra-path gains, and $\tau_{p,i} = \tau_p + \delta \tau_{p,i}$ are the intra-path delays. Both the gains $h_{p,i}$ and the delays $\delta \tau_{p,i}$ are treated as random to account for random placement of scattering points within a scattering field. With this fact in mind, we define the small-scale fading coefficient as

$$\gamma_p(f) = \frac{1}{h_p} \sum_{i \geq 0} h_{p,i} e^{-j2\pi f \delta \tau_{p,i}}$$

(2.16)

so that the overall channel transfer function is expressed as

$$H(f) = H_0(f) \sum_p h_p \gamma_p(f) e^{-j2\pi f \tau_p}$$

(2.17)

### 2.2.1 Probability density function of the small-scale coefficient $\gamma_p(f)$

Since the scattering points are separated by distances on the order of $\lambda$, the intra-path gains $h_{p,i}$ are likely to be similar, but the phases $2\pi f \delta \tau_{p,i}$, taken modulo $2\pi$, can differ substantially between the intra-paths of the same path. These phase variations will in turn cause a significant variation in $\gamma_p(f)$.

Assuming that the constituent terms of the small-scale coefficient $\gamma_p(f)$ in expression (2.16) are independent and identically distributed, the Central Limit Theorem implies a complex Gaussian distribution for $\gamma_p(f)$ when the number of micro-paths is sufficiently large. It is also possible to consider a situation in which there is a component whose delay is stable. In that case, the distortion is modeled by

$$\gamma_p(f) = \gamma_{p,0} + \sum_{i \geq 1} \gamma_{p,i} e^{-j2\pi f \delta \tau_{p,i}}$$

(2.18)
where $\gamma_{p,0}$ represents the relative coefficient of the stable path (whose $\delta\tau_{p,0} = 0$).

In general, the coefficient $\gamma_p(f)$ is complex Gaussian with mean $\bar{\gamma}_p(f)$ and variance $2\sigma^2_p(f)$. This distribution is conditioned on both the large-scale parameters $h_p, \tau_p$, and the small-scale path statistics $\bar{\gamma}_p(f), \sigma^2_p(f)$. While the former capture location uncertainty, the latter are influenced by the changing environmental conditions (e.g. surface roughness that changes with the wind/wave activity). Both sets of parameters can thus change over prolonged intervals of time.

The path statistics $\bar{\gamma}_p(f)$ and $\sigma^2_p(f)$ can be determined experimentally, or analytically if the distribution of the constituent terms in (2.16) is known. To illustrate an analytical approach, let us assume that the micro-path amplitudes have the mean value $\mu_p$ and variance $\nu^2_p \ll \mu^2_p$, and that the relative intra-path delays $\delta\tau_{p,i}$ are zero-mean Gaussian, with variance $\sigma^2_{\delta_p}$. Gaussian distributed delays can for instance result from a Gaussian distributed surface/bottom height. In that case, if we denote the surface and bottom variance by $\sigma^2_s$ and $\sigma^2_b$, respectively, we have that

$$\sigma^2_{\delta_p} = \frac{1}{c^2} (2 \sin \theta_p)^2 [n_{sp}\sigma^2_s + n_{bp}\sigma^2_b]$$  \hspace{1cm} (2.19)

The mean and variance of the scattering coefficients are then obtained as

$$\bar{\gamma}_p(f) = \mu_{p0} + \mu_p S_p \rho_p(f)$$  \hspace{1cm} (2.20)
$$2\sigma^2_p(f) = \mu^2_p S_p [1 - \rho^2_p(f)] + S_p \nu^2 \approx \mu^2_p S_p [1 - \rho^2_p(f)]$$  \hspace{1cm} (2.21)

where $S_p$ is the number of intra-paths and

$$\rho_p(f) = E\{e^{-j2\pi f \delta\tau_{p,i}}\} = e^{-(2\pi f)^2 \sigma^2_{\delta_p}/2}$$  \hspace{1cm} (2.22)

The last equality holds for Gaussian distributed zero-mean intra-path delays. We note that the path statistics are determined by the function $\rho_p(f)$, whose value depends on the parameter $f\sigma_{\delta_p}$, i.e. the standard deviation of the micro-path delay normalized by the signal wavelength. This function decays rapidly, falling below -10 dB for $f\sigma_{\delta_p} \geq 0.35$. In this regime, the independent real and imaginary com-
ponents of the scattering coefficient have (approximately) equal variances, leading to the Ricean-distributed magnitude.

It is also worth noting that regardless of the distribution of \( \delta \tau_{p,i} \), for high frequencies such that \( f \sigma_{\delta_p} \gg 1 \), the above model reduces to the one in which the phases \( \phi_{p,i} \) are uniformly distributed. Such a model is typically used to describe radio communication channels.

### 2.2.2 Requirement on the minimum number of micro-paths

In Section 2.2.1, a complex Gaussian distribution was obtained for the scattering coefficients provided that the number of micro-paths is sufficiently large. In this section, we find the minimum number of micro-paths that result in a cumulative complex Gaussian coefficient. To elaborate, let us use a shorthand notation and define a random variable

\[
Z = \frac{1}{S_p} \sum_{i=1}^{S_p} e^{-j2\pi f \delta \tau_{p,i}}
\]

(2.23)

where the \( S_p \) micro-path delays \( \delta \tau_{p,i} \) are i.i.d. \( \mathcal{N}(0, \sigma^2_{\delta_p}) \). The random variable \( Z \) represents the scattering coefficient \( \gamma_p \) normalized by \( S_p \). The normalization is made simply for convenience; it does not affect the analysis. The path index \( p \) is dropped from \( Z \), since the same analysis applies to different paths (they differ only in the micro-path delay variance \( \sigma^2_{\delta_p} \)). We now vary the number of micro-paths \( S_p \) and record \( N = 10^5 \) independent realizations of \( Z \). We denote by \( X \) and \( Y \) the real and imaginary parts of \( Z \), respectively, and measure (estimate) the probabilities \( P_{X_k}, P_{Y_k} \) that \( X, Y \) fall into bins of width \( \Delta \) centered around specific values \( x_k, y_k, k = 1, \ldots K \). Finally, we form the ratios \( P_{X_k}/\Delta \) and \( P_{Y_k}/\Delta \), which represent the estimated probability density functions (pdfs) of \( X \) and \( Y \), respectively.

Fig. 2.4 shows the examples of measured pdfs obtained with \( S_p=10 \) micro-paths (left), and \( S_p=30 \) micro-paths (right). The 95% confidence intervals for the measured pdfs are indicated on the histograms. Shown also are the theoretical normal pdfs (solid curves) with mean \( \rho_p \) and variance \( (1 - \rho^2_p)^2/2S_p \) for the real part, and mean 0 and variance \( (1 - \rho^4_p)/2S_p \) for the imaginary part. These pdfs
Figure 2.4: Measured (estimated) pdf and Gaussian approximation of the normalized scattering coefficient with 10 micro-paths (left) and 30 micro-paths (right). Measurements are based on $10^5$ independent realizations of micro-path delays. The 95% confidence intervals are also shown. Standard deviation of a micro-path delay is $\sigma_\delta = 0.07$ ms, and the signal frequency is $f=10$ kHz. Each figure shows the horizontal axis window of $\pm 3$ standard deviations around the mean value.

are almost identical whenever $\rho_p = e^{-(2\pi f)^2\sigma_\delta^2/2} \ll 1$ which is normally the case for high frequencies. The results of Fig. 2.4 are visually pleasing for both 10 and 30 micro-paths in the sense that they imply a good match between the measured and the normal pdf. Mathematically, we have quantified the goodness of the Gaussian fit by the Kullback-Leibler divergence (KLD) [32]. Denoting by $G_{Xk}$ and $G_{Yk}$ the Gaussian probabilities used to approximate the measured probabilities $P_{Xk}$ and $P_{Yk}$, respectively, the corresponding KLDs are defined as

$$D_X = \sum_{k=1}^{K} P_{Xk} \log \frac{P_{Xk}}{G_{Xk}}$$  (2.24)

$$D_Y = \sum_{k=1}^{K} P_{Yk} \log \frac{P_{Yk}}{G_{Yk}}$$  (2.25)

Since the underlying probability distributions change with the number of micro-
paths, so do the distances \( D_X \) and \( D_Y \).

**Figure 2.5**: Kullback-Leibler divergence between the measured and Gaussian distribution vs. the number of micro-paths. The two curves correspond to the \( X \) and \( Y \) components. Same parameters as in Fig. 2.4 are used.

Fig. 2.5 shows \( D_X \) and \( D_Y \) vs. the number of micro-paths \( S_p \). As expected, the difference between the measured pdf and the normal approximation diminishes with increasing \( S_p \). More importantly, we observe that a good agreement is reached with about 10 micro-paths, and does not change much thereafter. Hence, we can say that for all practical purposes CLT holds in this region. This observation also conforms to the ad-hoc notion that “eight sinusoids with uniformly distributed phases suffice to simulate a Gaussian process.” Finally, note that if \( X \) and \( Y \) are independent, then \( D_Z = D_X + D_Y \). The distance \( D_Z \) is shown in Fig. 2.6 for several values of the product \( f \sigma \delta_p \). We note an almost identical behavior whenever \( f \sigma \delta_p \geq 0.4 \). For lower values of this factor, the distance increases as the \( X \) component acquires a prominent mean and the variance of both \( X \) and \( Y \) pales in comparison. One may think of this situation as low frequency/smooth surface regime.

### 2.2.3 Correlation between paths

Assuming that the scattering process is independent between different paths, i.e. that reflection points of different paths are sufficiently far apart, we have that

\[
E\{\gamma_p(f)\gamma_q^*(f)\} = \bar{\gamma}_p(f)\bar{\gamma}_q(f) + \delta_{p,q}2\sigma^2_p(f)
\]  

(2.26)
Figure 2.6: Composite Kullback-Leibler divergence between the measured and Gaussian distribution vs. the number of micro-paths. Each curve represents the composite KL divergence (sum) of the X and Y terms. Different curves correspond to different $f\sigma_{\delta_p}$ factors.

Note that although the paths exhibit uncorrelated scattering, the above function is not zero in general, due to the non-zero mean values.

2.2.4 Correlation in the frequency domain

Frequency-correlation of the small-scale path coefficients is described by the function $E \{ \gamma_p(f + \Delta f, t) \gamma_q^*(f, t) \}$. In order to evaluate the frequency-correlation, the probability density function of the intra-path delays $\delta\tau_{p,i}$ has to be known. For the Gaussian distributed delays with zero mean and variance $\sigma_{\delta_p}^2$, frequency-correlation is obtained as

$$E \{ [\gamma_p(f + \Delta f) - \bar{\gamma}_p(f + \Delta f)] [\gamma_p(f) - \bar{\gamma}_p(f)]^* \}$$

$$= \mu_p^2 S_p \rho_p (\Delta f) [1 - \rho_p (\sqrt{2f(f + \Delta f)})] \approx \rho_p (\Delta f) 2\sigma_p(f)\sigma_p(f + \Delta f)$$

(2.27)

where the approximation holds well (error below 4%) for $\sigma_{\delta_p} \geq 0.4/f_0$. Fig. 2.7 illustrates the frequency-correlation function and its approximation for a typical range of communication frequencies and several values of the standard deviation $\sigma_{\delta_p}$. We note that depending upon the standard deviation of the path delays, there may be more or less correlation between the small-scale coefficients $\gamma_p$ within the signal bandwidth. While the particular function of Fig. 2.7 pertains to Gaussian
distributed delays, one can expect a similar trend for a different distribution as well, i.e. one can expect the correlation to vary across a wide bandwidth. In our analysis of experimental data in Section 2.4, we will assume that full correlation exists across the signal bandwidth, i.e. that a single value $\gamma_p$ suffices to describe small-scale effects. However, we note that this assumption may not always hold. Specifically, if $\sigma_{\delta p}$ is on the order of $1/f_0$, and the corresponding Ricean factor is low, there may be little correlation between $\gamma_p(f_0)$ and $\gamma_p(f_0 + B)$ in a wideband system. This fact may have serious implications on the performance of signal processing algorithms that rely on the assumption of frequency-invariant multipath coefficients.

### 2.2.5 Correlation in the time domain

Time-correlation of the scattering coefficients is described by the function $E\{\gamma_p(f, t + \Delta t)\gamma_p^*(t)\}$. This function captures the effect of motion within the scattering field, which influences the coefficients $\gamma_p(f, t)$ through the time-varying micro-path delays. To assess the time-correlation function, power spectral density (psd) of $\delta\tau_{p,i}(t)$ has to be known.

Without loss of generality, let us assume that the Gaussian-distributed de-
lays $\delta \tau_{p,i}(t)$ obey a first-order auto-regressive process (AR-1)

$$
\delta \tau_{p,i}(t + \Delta t) = \alpha_{\delta p} \delta \tau_{p,i}(t) + w_{\delta p,i}(t)
$$

(2.28)

where $w_{\delta p,i}(t) \sim N(0, \sigma_{\delta p}^2 (1 - \alpha_{\delta p}^2))$, $\alpha_{\delta p} = e^{-\pi B_{\delta p} \Delta t}$, and $B_{\delta p}$ is the 3 dB width of the psd of $\delta \tau_{p,i}(t)$. The above relationship is not a binding one; it simply states that two values of delay are expected to look more alike if they are more closely spaced in time, i.e. that one does not expect the delays to vary completely erratically, but to show some coherence. The time-correlation function is then obtained as

$$
E\{|\gamma_p(f, t + \Delta t) - \bar{\gamma}_p(f)|[\gamma_p(f, t) - \bar{\gamma}_p(f)]^*\}
= \mu_p^2 S_p e^{-(1-\alpha_{\delta p})(2\pi f)^2 \sigma_{\delta p}^2} [1 - e^{-\alpha_{\delta p}(2\pi f)^2 \sigma_{\delta p}^2}] \approx 2\sigma_p^2(f) e^{-\pi B_p(f) \Delta t}
$$

(2.29)

where the approximation holds for $\Delta t \ll 1/B_{\delta p}$, and we have defined

$$
B_p(f) = (2\pi f \sigma_{\delta p})^2 B_{\delta p}
$$

(2.30)

as the effective Doppler bandwidth (Doppler spread) of the path coefficient $\gamma_p(f, t)$. Fig. 2.8 illustrates the effective Doppler spread, the time-correlation function and its approximation. As one can expect, stronger temporal correlation is observed for lower values of the standard deviation $\sigma_{\delta p}$.

**Figure 2.8**: The functions (a) effective Doppler spread, (b) time-correlation and its approximation, as defined in Eq. (2.29), for several values of the standard deviation $\sigma_{\delta p}$. The frequency $f$ is set to 10 kHz, and $B_{\delta p} = 0.1$ Hz.
The complete auto-correlation function of the process $\gamma_p(f, t)$ can also be expressed in closed form. Using the relationships (2.27) and (2.29), we obtain a generalization

$$E\{[\gamma_p(f_1, t + \Delta t) - \bar{\gamma}_p(f_1)][\gamma_p(f_2, t) - \bar{\gamma}_p(f_2)]^*\} = \mu_p^2 S_p e^{-(2\pi(f_1 - f_2))^2 \sigma_p^2} e^{-(1 - \alpha_p)(2\pi f_1)(2\pi f_2)\sigma_p^2 [1 - \rho_p(\sqrt{2\alpha_p f_1 f_2})]} \approx \rho_p(f_1 - f_2)e^{-\pi B_p(f_1, f_2)\Delta t} 2\sigma_p(f_1)\sigma_p(f_2)$$

(2.31)

where $f_{1,2}$ stands for either of the two frequencies $f_1$ or $f_2$ within the signal bandwidth. Setting $f_1 = f + \Delta f$ and $f_2 = f$, the above expression defines the function $C_{\gamma_p}(\Delta f, \Delta t)$, whose special forms obtained for $\Delta t = 0$ and for $\Delta f = 0$ in Eqs. (2.27) and (2.29) were illustrated in Figs. 2.7 and 2.8, respectively.\footnote{Note that this function depends on $f$ as well, although we explicitly denote only the dependence on $\Delta f$.}

### 2.2.6 Statistically equivalent model for $\gamma_p(f, t)$

Let us define an auxiliary AR-1 process

$$\Delta\gamma_p(f, t + \Delta t) = \alpha_p(f)\Delta\gamma_p(f, t) + w_p(f, t)$$

$$\gamma'_p(f, t + \Delta t) = \bar{\gamma}_p(f) + \Delta\gamma_p(f, t + \Delta t)$$

(2.32)

where $w_p(t) \sim \mathcal{CN}(0, 2\sigma_p^2(f)(1 - \alpha_p^2(f)))$, and $\alpha_p(f) = e^{-\pi B_p(f)\Delta t}$. This process is characterized by a Gaussian pdf, $\gamma'_p(f, t) \sim \mathcal{CN}(\bar{\gamma}_p(f), 2\sigma_p^2(f))$, and an auto-covariance function $E\{\Delta\gamma_p(f, t + \Delta t)\Delta\gamma_p^*(f, t)\} = 2\sigma_p^2(f)e^{-\pi B_p(f)\Delta t}$. Since its pdf and psd coincide with those of the process $\gamma_p(f, t)$, the two processes are statistically equivalent.

The significance of the auxiliary process $\Delta\gamma_p(f, t)$ is that it can easily be generated in computer simulation. To do so, a single recursive operation needs to be applied to a Gaussian input, as specified by Eq. (2.32).

To develop a full channel simulator, however, frequency-correlation needs to be embedded into the process $\Delta\gamma_p(f, t)$. To this end, let us discretize the frequency
axis in steps of $\Delta f_s$ and the time axis in steps of $\Delta t_s$, and let us define the vector 
$\Delta \gamma_p[n] = [\Delta \gamma_p(f_0, t_0 + n\Delta t_s), \Delta \gamma_p(f_1, t_0 + n\Delta t_s), \ldots]^T$. Let us also define the 
matrix $A_p = \text{diag}[\alpha_p(f_k)]$, where $\alpha_p(f_k) = e^{-\pi B_p(f_k)\Delta t_s}$.

We are now in a position to specify an AR-1 process as follows. Starting 
with some $\Delta \gamma_p[0]$, e.g. an all-zero vector, generate

$$\Delta \gamma_p[n + 1] = A_p \Delta \gamma_p[n] + w_p[n], \quad n = 0, 1, \ldots \quad (2.33)$$

where $w_p[n] \sim \mathcal{CN}(0, W_p)$, and $[W_p]_{k,l} = [1 - \alpha_p(f_k)\alpha_p(f_l)]\rho_p(f_k - f_l)2\sigma_p(f_k)\sigma_p(f_l)$. This particular choice of the process noise covariance matrix $W_p$ ensures the desired frequency-correlation, $E\{\Delta \gamma_p(f_k, t)\Delta \gamma_p^*(f_l, t)\} = \rho_p(f_k - f_l)2\sigma_p(f_k)\sigma_p(f_l)$, as well as the complete correlation given by the expression (2.31).

The above model retains computational simplicity, thus offering an appealing platform for channel simulation. After executing the recursion (2.33) over a time interval of interest and for frequencies corresponding to the desired signal bandwidth, all that remains to be done is to add the mean values $\bar{\gamma}_p(f_k)$ to the so-obtained coefficients $\Delta \gamma_p(f_k, n\Delta t_s)$. The result is a discrete-time discrete-frequency random process $\gamma'_p(f, t)$ whose statistical properties are equivalent to those of the sampled process $\gamma_p(f, t)$.

We emphasize that a channel simulator does not yield an exact replica of an actual channel because (i) it is only statistically equivalent to the process $\gamma_p(f, t)$ as we have defined it, and (ii) our definition of the process $\gamma_p(f, t)$ is itself incomplete. Namely, while it takes into account rough surface scattering, it does not take into account the particular shape of the surface or the effect of breaking waves which inject bubbles into the water column. Both of these effects can additionally alter the signal, or even completely occlude the surface [23, 24]. The above model should thus be regarded as a first step towards assessing the small-scale variations. Imperfect as it may be, this model offers a simulation platform that captures some of the acoustic channel effects in a computationally-efficient manner.
2.3 Motion-Induced Doppler Shifting

Motion of the transmitter/receiver or any reflection points in the channel leads to time-varying path delays $\tau_p$. Focusing on the small-scale phenomena, we are interested in variations that occur over short intervals of time (e.g. sub-second intervals). During such intervals, say $[t_0, t_0 + T]$, it is reasonable to assume that any motion occurs at a constant velocity, i.e. that it is only the velocity and not acceleration that matters. When that is the case, the path delays are modeled as $\tau_p(t) = \tau_p - a_p t$ where $t \in [0, T]$ and $a_p = v_p/c$ is the Doppler factor corresponding to velocity $v_p$.\(^2\) Note that the Doppler factors $a_p$ may (and likely will) vary with time and a more general quasi-stationary model can be introduced to address this fact. Specifically, one may wish to divide the time axis into consecutive intervals of constant velocity, and associate a Doppler scaling factor $a_p(n)$ with the $n$-th interval. We will keep this fact in mind, but drop time indexing for simplicity where appropriate.

At least three types of motion influence the Doppler factor: (i) unintentional transmitter/receiver motion, i.e. drifting, which gives rise to a Doppler scaling factor $a_{dp}$; (ii) intentional transmitter/receiver motion, i.e. vehicular motion, which gives rise to $a_{vp}$, and (iii) waves, i.e. surface motion, which gives rise to $a_{sp}$. If the Doppler factors corresponding to each of these types of motion are fixed, the corresponding Doppler shift is always the same. On the contrary, if the Doppler factors change randomly from one time interval to another, so do the shifts. The resulting effect is that of random Doppler shifting or Doppler spreading. The overall small-scale path coefficient can now be defined as

$$\tilde{\gamma}_p(f, t) = \gamma_p(f, t) e^{j2\pi a_p ft}$$

(2.34)

To characterize Doppler spreading, we focus on the auto-correlation function of the random process $e^{j2\pi a_p ft}$, i.e. $E\{e^{j2\pi a_p f \Delta t}\}$. Assuming independence

\(^2\)It is also possible to account for a path-dependent propagation speed.
between various factors contributing to motion, we have that

\[ R_p(\Delta t) = E\{e^{j2\pi f a_d \Delta t}\} E\{e^{j2\pi f a_v \Delta t}\} E\{e^{j2\pi f a_s \Delta t}\} \]  

(2.35)

Should any one of these components be regarded as deterministic, its expectation is dropped.

To characterize the drifting component, we assume that the transmitter and receiver drift at velocities \( v_{td}, v_{rd} \) in directions \( \theta_{td}, \theta_{rd} \) with respect to horizontal pointed toward each other. The relative speed, projected onto the \( p \)-th path, is

\[ v_{dp} = v_{td}\cos(\theta_p - \theta_{td}) - v_{rd}\cos(\theta_p + \theta_{rd}) \]  

(2.36)

and the corresponding Doppler factor is \( a_{dp} = v_{dp}/c \). If the transmitter/receiver drift in random directions, then \( E\{e^{j2\pi f a_d \Delta t}\} \) involves averaging over \( \theta_{td}, \theta_{rd} \).

Assuming that drifting is equally likely in any direction, and that it occurs independently for the transmitter and receiver, we have that

\[
E\{e^{j2\pi f a_d \Delta t}\} = \frac{1}{2\pi} \int_0^{2\pi} e^{j2\pi \frac{v_{td}}{c} f\cos(\theta_p - \theta_{td})} d\theta_{td} \times \frac{1}{2\pi} \int_0^{2\pi} e^{j2\pi \frac{v_{rd}}{c} f\cos(\pi + \theta_p + \theta_{rd})} d\theta_{rd} 
\]

\[ = J_0(2\pi \frac{v_{td}}{c} f \Delta t) J_0(2\pi \frac{v_{rd}}{c} f \Delta t) \]  

(2.37)

where \( \theta'_{td} = \theta_p - \theta_{td} \) and \( \theta'_{rd} = \pi + \theta_p + \theta_{rd} \) are uniformly distributed on the interval \([-\pi, \pi]\), and \( J_0(\cdot) \) is the Bessel function of the first kind and order zero. Assuming \( v_{td} = v_{rd} = v_d \), and \( a_d = v_d/c \), the above expression reduces to \( J_0^2(2\pi a_d f \Delta t) \).

Vehicular component of the Doppler effect is obtained similarly, except that those components of motion that can be estimated and compensated by synchronization are not to be regarded as part of the channel distortion. Hence, we regard \( a_{vp} = v_{vp}/c \) as the residual Doppler factor after initial synchronization. For example, if synchronization compensates for the pre-dominant Doppler factor corresponding to the projection of the transmitter/receiver intentional velocity onto
the reference path \( p=0 \), the effective Doppler factor is

\[
a_{vp} = \frac{1}{c} \left[ v_{tv} \cos(\theta_p - \theta_{tv}) - v_{rv} \cos(\theta_p + \theta_{rv}) \right] - \left[ v_{tv} \cos(\theta_0 - \theta_{tv}) - v_{rv} \cos(\theta_0 + \theta_{rv}) \right]
\]

\[
= \frac{1}{c} \left[ v_{tv} \left[ -2 \sin \left( \frac{\theta_p + \theta_0 - 2\theta_{tv}}{2} \right) \sin \left( \frac{\theta_p - \theta_0}{2} \right) \right]
+ v_{rv} \left[ 2 \sin \left( \frac{\theta_p + \theta_0 + 2\theta_{rv}}{2} \right) \sin \left( \frac{\theta_p - \theta_0}{2} \right) \right] \right]
\]

(2.38)

Assuming that the transmitter/receiver motion is equally likely in any direction \( \theta_{tv}/\theta_{rv} \), the auto-correlation function corresponding to vehicular motion is

\[
E \{ e^{j2\pi f_{a_{vp}} \Delta t} \} = J_0 \left( 2\pi \frac{v_{tv}}{c} \sin(\frac{(\theta_p - \theta_0)/2)}{f \Delta t} \right) \times J_0 \left( 2\pi \frac{v_{rv}}{c} \sin(\frac{(\theta_p - \theta_0)/2)}{f \Delta t} \right)
\]

(2.39)

Finally, to assess the surface component, let us focus on waves that cause a point on the surface to move up and down creating a displacement that varies sinusoidally in time, with amplitude \( A_w \) and frequency \( f_w \). Note that we are accounting only for the vertical surface motion, and not for the horizontal group velocity of the waves or the effect of an inclined surface (a study on scattering from inclined waves can be found in [8]). A signal impinging upon the \( j \)-th reflection point along the \( p \)-th path catches it in a random phase, i.e. at vertical velocity \( v_w \sin(\psi_{p,j} + 2\pi f_w t) \) where \( \psi_{p,j} \sim U[-\pi, \pi] \), and \( v_w = 2\pi f_w A_w \). Projections of this velocity onto the \( p \)-th path, summed over all surface reflection points, yield

\[
v_{sp} = 2v_w \sin \theta_p \sum_{j=1}^{n_{sp}} \sin(\psi_{p,j} + 2\pi f_w t)
\]

(2.40)

Assuming that reflection points are sufficiently far apart such that \( \psi_{p,j} \) are independent, time-correlation is obtained by taking the expectation over \( a_{sp} = v_{sp}/c \) with the angles \( \psi_{p,j} \) uniformly distributed over \( 2\pi \). The result is

\[
E \{ e^{j2\pi f_{a_{sp}} \Delta t} \} = \left[ J_0 (2\pi a_{wp} f \Delta t) \right]^{n_{sp}}
\]

(2.41)
where \( a_{wp} = 2v_w \sin \theta_p/c \) and \( n_{sp} \) is the number of surface encounters along path \( p \).

![Figure 2.9: Auto-correlation of the small-scale coefficient \( \tilde{\gamma}_p \) for (a) surface motion, (b) surface and drifting motions.](image)

Putting together Eqs. (2.29), (2.37)–(2.41), the complete auto-correlation function of the overall small-scale coefficient \( \tilde{\gamma}_p(f, t) \) is obtained as

\[
R_{\tilde{\gamma}_p}(\Delta t) = [\tilde{\gamma}_p^2(f) + 2\sigma_p^2(f)e^{-\pi B_p(f)\Delta t}] J_0^2(2\pi a_d f \Delta t) \\
\times J_0(2\pi \frac{2v_{t_v} \sin((\theta_p - \theta_0)/2)}{c} f \Delta t) \\
\times J_0(2\pi \frac{2v_{t_r} \sin((\theta_p - \theta_0)/2)}{c} f \Delta t) \\
\times \left[J_0(2\pi \frac{2v_w \sin \theta_p}{c} f \Delta t)\right]^{n_{sp}}
\]  

(2.42)

This function exhibits an overall Bessel-like behavior, dampened by the exponentially decaying correlation of the scattering coefficient \( \gamma_p \) (first term in the expression (2.42)). Fig. 2.9 shows \( R_{\tilde{\gamma}_p}(\Delta t) \) for two different scenarios: one in which surface motion alone is taken into account, and another in which surface and drifting motions are considered. In both scenarios, the first zero crossing of the overall Bessel-like correlation function, which indicates the coherence time, is determined by the maximum of the surface or drifting velocities and the number of surface reflections. It is also worth noting that the main lobe of the surface component, whose behavior is dictated by the Doppler factor \( a_{wp} \), narrows with each additional surface encounter. This fact implies that each surface encounter
adds to the overall time variation of the signal, which is intuitively satisfying. Most notably, as we shall see in Section 2.4, experimental results demonstrate similar behavior. The correlation function (2.42), although obtained analytically under a number of modeling assumptions, thus becomes helpful in explaining experimental observations.

### 2.3.1 A note on motion-induced time-correlation functions

In Section 2.3, the time-correlation functions of various random processes $e^{j2\pi a_x t}$ were obtained where $a_x = \frac{v_x}{c}$ denotes the Doppler factor corresponding to the velocity $v_x = v_{xd} \cos(\psi)$ and we assumed that the phase $\psi$ is uniformly distributed on the interval $[-\pi, \pi]$. The index $x$ in the above model denotes any of the different types of motion, e.g. receiver drifting projected onto a certain path, or surface motion. When transducer movement is considered, $\psi$ models the random direction of movement, and when surface motion is considered, it models the random phase at which the wave catches the vertical surface motion on a reflection point on one path. The frequency shift corresponding to this motion equals $\nu = f \frac{v_{xd} \cos(\psi)}{c}$ and the maximum possible Doppler shift is $\nu_d = f \frac{v_{xd}}{c}$. The time-correlation function was shown to be of the Bessel form. The Doppler power spectrum, obtained as the Fourier transform of the auto-correlation function, thus equals

$$S_x(\nu) = \mathcal{F} \{J_0(2\pi \nu_d \Delta t)\} = \frac{1}{\pi \sqrt{\nu_d^2 - \nu^2}}, \quad |\nu| \leq \nu_d \quad (2.43)$$

Alternatively, one can follow the approach in [33, Chapter 5] to directly obtain the Doppler spectrum given the uniform distribution for the received signal power. Assuming a mean power of unity and considering the fact that waves from the directions $\psi$ and $-\psi$ lead to the same Doppler shift, the received power spectrum as a function of direction is

$$S_x(\psi) = \text{pdf}_\psi(\psi) + \text{pdf}_\psi(-\psi) \quad (2.44)$$
Using a variable transformation $\psi \to \nu$, the Doppler spectrum is obtained as

$$S_x(\nu) = \frac{1}{\pi} \left| \frac{d\psi}{d\nu} \right| = \frac{1}{\pi \sqrt{\nu_d^2 - \nu^2}}, \quad |\nu| \leq \nu_d$$

which is the same as the function derived in (2.43).

This analysis is based on the assumption of uniform distribution of the direction of motion. An alternative approach is to model the velocity to be uniformly distributed, i.e., $v_x \sim \mathcal{U}(-v_{xd}, v_{xd})$. It is easy to show that the time-correlation function under this assumption equals $\text{sinc}(2\nu_d \Delta t)$ where $\nu_d = f \frac{v_{xd}}{c}$ and the sinc function is defined as $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$. Since the Doppler shift is equally likely to take any value in the interval $[-\nu_d, \nu_d]$, the Doppler power spectrum is a rectangular function on this interval, which is readily available by Fourier analysis

$$S_x(\nu) = \mathcal{F}\{\text{sinc}(2\nu_d \Delta t)\} = \frac{1}{2\nu_d} \text{rect}\left(\frac{\nu}{2\nu_d}\right)$$

Fig. 2.10 shows the time-correlation functions and the corresponding Doppler spectra for the two models described above. Both models exhibit similar behavior, dictated by $\nu_d$. First zero-crossing is at $\frac{1}{2\nu_d}$ for the sinc model and somewhat earlier for the Bessel model. When more than one type of motion are present, as seen in Section 2.3, the overall motion-induced time-correlation function equals the prod-
uct of Bessel functions. The main lobe of the overall time-correlation function, which indicates the coherence time, narrows with each additional type of motion.

2.4 Experimental Results

2.4.1 Description of the experiments

We present experimental data collected during four experiments. Table 2.1 lists the operational frequency range used in each experiment ($B$), the distance between the transmitter and receiver ($d$), the water depth ($h_w$), the transmitter/receiver height above bottom ($h_{tx/rx}$), and the type of signals transmitted. Fig. 2.11 shows the setup maps of the experiments. In the first three experiments, the signals were custom-designed, transmitted over the channel, and recorded for subsequent off-line processing. The probing signals were pseudo-noise (PN) sequences, periodically repeated and binary phase-shift keying (BPSK) modulated onto the center frequency at full rate. In the fourth experiment, FM sweep signals were transmitted and received by the Woods Hole Oceanographic Institution (WHOI) micro-modem [34]. Only the received signal strength, and not the full channel responses, are available from this experiment.

Table 2.1: Nominal parameters of the experimental channels.

<table>
<thead>
<tr>
<th></th>
<th>$B$ [kHz]</th>
<th>$d$ [km]</th>
<th>$h_w$ [m]</th>
<th>$h_{tx}$ [m]</th>
<th>$h_{rx}$ [m]</th>
<th>signal type</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPACE</td>
<td>8–17</td>
<td>0.06, 0.2, 1</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>PN sequence</td>
</tr>
<tr>
<td>MACE</td>
<td>10.5–15.5</td>
<td>0.5–4</td>
<td>100</td>
<td>45</td>
<td>60</td>
<td>PN sequence</td>
</tr>
<tr>
<td>KAM</td>
<td>8.5–17.5</td>
<td>3</td>
<td>103</td>
<td>58</td>
<td>59</td>
<td>PN sequence</td>
</tr>
<tr>
<td>PS</td>
<td>8–12</td>
<td>0.2–1</td>
<td>130</td>
<td>3</td>
<td>130</td>
<td>FM sweep</td>
</tr>
</tbody>
</table>

The first experiment, called the Surface Processes and Acoustic Communications Experiment (SPACE) was conducted near the coast of Martha’s Vineyard in Massachusetts, in the fall of 2008. The carrier frequency was 13 kHz and the transmission rate was 6.5 kilobits per second (kbps). The signal was transmitted for three minutes every two hours. We refer to the active three-minute interval of each two-hour period as one epoch. The experiment lasted for 15 days. The
Figure 2.11: Experiment setup: (a) SPACE, (b) MACE, (c) KAM, (d) PS.

water depth was 10 m, and the transmitter and receiver were fixed at 4 m and 2 m above the sea-floor, respectively. Receivers located Southeast of the transmitter at distances of 60 m, 200 m and 1000 m, labeled in Fig. 2.11(a) as SE1, SE3, and SE5, respectively, recorded the signals.

The second experiment, Mobile Acoustic Communications Experiment (MACE), was conducted in the Atlantic Ocean about 100 miles south of Martha’s Vineyard in the summer of 2010. The receiver buoy, labeled as B1 in Fig. 2.11(b), was suspended at the depth of 40 m and the transmitter was towed at the depth of 50–60 m. The water depth was approximately 100 m and the distance varied between 500 m and 4 km. The carrier frequency was 13 kHz and the signals were transmitted continuously at 5 kbps.
The third experiment, known as the Kauai Acomms MURI (KAM), was conducted in July 2011 off the coast of Kauai Island, Hawaii. Two sets of data from this experiment are analyzed. Probing signals, recorded by the WHOI communication system were used for studying the channel characteristics throughout this chapter and in Sections 3.2 and 4.2. This system, which is labeled in Fig. 2.11(c) as Sta03 (transmitter) and Sta09 (receiver), operated in the 8.5 kHz to 17.5 kHz bandwidth. The transmitter and receiver were deployed approximately mid-way in 100 m water and were 3 km apart. The signals were modulated onto the carrier of 13 kHz and transmitted at the rate of 6.5 kbps during 9 minute epochs every two hours. For the discussion on system performance evaluation in Section 4.3, orthogonal frequency division multiplexing (OFDM) signals were used. These signals were obtained from Scripps Institute of Oceanography (SIO) transducers, which operated in the 20 to 32 kHz bandwidth and are labeled in Fig. 2.11(c) as Sta02 (transmitter) and Sta08 (receiver).

Finally, the fourth experiment, called the Pacific Storm (PS) experiment, was conducted on the submerged portion of San Andreas Fault off the coast of Northern California in September 2010. During this experiment, 3 second long data packets were repeatedly transmitted every 5 seconds from an autonomous underwater vehicle (AUV) to a surface ship. The signals occupied 4 kHz of bandwidth around a center frequency of 10 kHz. The AUV was moving at about 3 m above the bottom, at a depth of approximately 130 m. The transmission distance varied from 200 m to 1 km. In this experiment, the signals were automatically processed by the acoustic modems mounted on the two ends of the link. The received signal strength was recorded once a second.

2.4.2 Small-scale analysis

The signals from the SPACE, MACE and KAM experiments were used to assess the small-scale channel behavior. The signals were first resampled based on the received packet length to compensate for the motion-induced time scaling and frequency shifting which can be modeled by a rough Doppler factor. Resampling was notably necessary for the MACE data, where the transmitter moved at about
1 m/s creating a raw Doppler rate on the order of $10^{-3}$. Fine Doppler compensation was then carried out using a combined recursive least squares (RLS) estimator and a second order phase locked loop (PLL). Once these steps were completed, the orthogonal matching pursuit (OMP) algorithm [35] was used for channel estimation. Compressed sensing techniques, such as the OMP algorithm, are commonly used for estimation of UWA channels as they outperform conventional least-squares methods when the channel is sparse [36,37]. The resulting estimate of the baseband channel response was then used to extract the path gains.

Figs. 2.12–2.14 show the results obtained for the three experiments. Each figure shows an ensemble of channel responses (magnitude), histograms of several selected paths, and their auto-correlation functions. The channel responses are shown over the duration of 1 minute at a resolution of one bit. Several local maxima over the delay axis are visible in the figures, indicating channel taps over which the impulse response is the strongest. Physical path delays corresponding to the nominal channel geometry are marked by arrows and labeled as $s$, $b$, $sb$, $s2b$ etc., referring to surface, bottom, surface-bottom, surface-bottom-surface, etc. reflections.

The observed path delays deviate from the nominal ones because of location uncertainty, motion-induced Doppler effect, and intra-path delay spreading. In addition, time variability of the channel imparts a slowly varying mean onto each path. Also, if the geometry is such that two paths have similar lengths, path merging will occur, i.e. their arrival times will be too close to be distinguished given the delay resolution (finite bandwidth) of the system. Path spreading, which is a consequence of both the micro-path dispersion and bandwidth limitation, may occur uniformly at a tractable rate, as in the path labeled $p_0$ in the KAM experiment, or an intractable one, as in the path labeled $p_{bs}$. In order to take into account the effect of all contributing taps, whenever the phase information is not needed, several adjacent taps are combined in a root mean square (rms) fashion to form the absolute value of a given path gain. Slow variation of the mean, which is a consequence of the changing $h_p(t)$, is then removed. For example, such variation is evident in the path $p_s$ of the MACE experiment. In this case, in order to extract
Figure 2.12: SPACE experiment: (a) time-evolution of the magnitude baseband impulse response, (b)-(e) histograms of selected path magnitudes, and (f)-(i) time-correlation functions. The histograms are well approximated by Ricean fits whose K-factor is the largest for the direct (most stable) path. The time-correlation functions show Bessel-type fluctuations that are most evident on paths with a higher number of reflections (more sources of motion) and the longest coherence time is observed for the direct path.
Figure 2.13: MACE experiment: (a) time-evolution of the magnitude baseband impulse response, (b)-(e) histograms of selected path magnitudes, and (f)-(i) time-correlation functions. The system bandwidth is high enough to distinguish the different paths, but each path is spread over several adjacent taps. The effect of motion is evident even after Doppler compensation, resulting in short coherence times. In addition, time variability of the channel imparts a slowly varying mean onto each path.
Figure 2.14: KAM experiment: (a) time-evolution of the magnitude baseband impulse response, (b)-(e) histograms of selected path magnitudes, and (f)-(i) time-correlation functions. In order to take into account the effect of all contributing taps in the histogram of the absolute value of each path gain, several adjacent taps are combined in a root mean square fashion. Slow variation of the mean is then removed. The time-correlation function of each path gain is estimated over shorter time intervals during which the path mainly appears on a single tap.
the absolute value of the small-scale factor $\gamma_p(t)$, the slowly varying window average of the observed process $|h_p(t)\gamma_p(t)|$ is removed. When the phase information is not to be neglected, the signal statistics are estimated over shorter time intervals, i.e. several seconds.

The histograms shown in Figs. 2.12–2.14 are those of the estimated $|\gamma_p(t)|$ for several selected paths, along with a theoretical Ricean curve. The magnitude $|\gamma_p(t)|$ is estimated as the rms of the maximum tap and its significant neighbors, whose time-varying mean is removed. The stationarity of the data was tested using the Phillips-Perron test [38]. The null hypothesis that the estimated time series has a unit root, which gives rise to nonstationarity, was rejected for all data sets. Maximum likelihood method is used to estimate the parameters of the fitted Ricean curve. We have also measured the variance on individual segments of data (sliding window average, after removing the time-varying mean) and have found it to remain constant. This observation also confirms the stationarity of the data in the wide sense. The conditional Ricean distribution (conditioned on the slowly-varying mean) appears to provide a good fit in all three experiments. The 95% confidence intervals of the histogram bars are plotted. For almost every bar, the Ricean fit falls inside the confidence interval which can be interpreted as a qualitative measure of goodness of the fits. To further quantify the goodness of fit, the Jensen-Shannon (J-S) divergence [39] was used. The J-S divergence is a symmetric measure of the difference between two distributions $P$ and $Q$, and is defined as

$$D_{JS}(P||Q) = \frac{1}{2}D_{KL}(P||M) + \frac{1}{2}D_{KL}(Q||M)$$ (2.47)

where $M = \frac{1}{2}(P + Q)$ and $D_{KL}(P||Q) = \sum_i P_i \ln(P_i/Q_i)$ denotes the Kullback-Leibler divergence of the fit $Q$ from the distribution $P$. Table 2.2 lists the values of the J-S divergence between the estimated histograms and the fitted Ricean distributions (gray rows) for the different paths analyzed in Figs. 2.12–2.14. The J-S divergence values for a fitted Rayleigh distribution are also listed in the table (white rows). Ricean distribution provides a better fit for all path gain data sets. The Ricean $K$-factor is indicated in the figures along with its 95% confidence
Table 2.2: Jensen-Shannon divergence for Ricean fits (gray rows) vs. Rayleigh fits (white rows).

<table>
<thead>
<tr>
<th></th>
<th>direct</th>
<th>reflected 1</th>
<th>reflected 2</th>
<th>reflected 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPACE</td>
<td>5.1e-3</td>
<td>3.6e-3</td>
<td>6.6e-3</td>
<td>4.8e-3</td>
</tr>
<tr>
<td></td>
<td>0.185</td>
<td>0.166</td>
<td>0.091</td>
<td>0.064</td>
</tr>
<tr>
<td>MACE</td>
<td>5.3e-3</td>
<td>7.4e-3</td>
<td>6.4e-3</td>
<td>3.1e-3</td>
</tr>
<tr>
<td></td>
<td>0.135</td>
<td>0.142</td>
<td>0.151</td>
<td>0.090</td>
</tr>
<tr>
<td>KAM</td>
<td>5.1e-4</td>
<td>5.8e-3</td>
<td>5.8e-3</td>
<td>2.9e-3</td>
</tr>
<tr>
<td></td>
<td>0.194</td>
<td>0.162</td>
<td>0.130</td>
<td>0.079</td>
</tr>
</tbody>
</table>

interval. Its value for the direct path ($p=0$) is greater than that of the other paths, indicating a stable arrival as there is no surface dispersion.

Finally, the time domain auto-correlation functions of the path coefficients are plotted. We note a remarkable similarity with the theoretical results of Section 2.3 and the Bessel-like functions developed there. For all three experiments, the auto-correlations corresponding to the direct path show less fluctuation and a higher coherence time. All other paths show Bessel-type autocorrelation, and, as noted in Section 2.2, Doppler bandwidth that increases with the number of surface encounters.

### 2.4.3 The estimation algorithms

In this section, we briefly explain the estimation algorithms which were used for data analysis in Section 2.4.2. Initial resampling based on the received packet length is used for rough Doppler compensation and a combined RLS and second-order PLL algorithm compensates for fine or residual Doppler effect. Fig. 2.15(a) shows the rough Doppler factor estimated for a 2 h long recording during the MACE experiment. Two estimates are obtained, one based on the received packet length and the other one based on the speed of the transmitter ship, and a close agreement is observed. A negative Doppler rate (moving away of the transmitter ship) is observed during the first 40 minutes, then the ship appears to change direction and move toward the receiver during the second half of recording, causing a positive Doppler rate. This description matches with the actual tow pattern shown in Fig. 2.11(b). The residual Doppler factor, estimated for a 56 s long
Figure 2.15: (a) Rough and (b) residual Doppler factors estimated for data recorded during the MACE experiment. The received packet length well represents the rough Doppler caused by the movement of the transmitting ship. The residual Doppler factor has a much lower range and faster fluctuations.

portion of the transmission is shown in Fig. 2.15(b). The residual Doppler factor appears to be 10 times smaller than the rough Doppler, and bears faster variation.

The block diagram of the Doppler compensation algorithm is shown in Fig. 2.16. The transmitted signal is denoted by $u$ and the received signal, down-converted and low pass filtered, is denoted by $v$. The actual channel is denoted by $w$ and its estimate is denoted by $\hat{w}$. The residual phase $\theta(n)$ is tracked by a greedy algorithm

$$\hat{\theta}(n) = \hat{\theta}(n - 1) + K_1 \xi(n) + K_2 \sum_{l=1}^{n-1} \xi(l)$$

(2.48)

where $\xi(n) = \mathfrak{R}\left\{ R(n - 1)e^{-j\hat{\theta}(n-1)} \right\}$, and $R$ is the pointwise cross-correlation of the resampled received signal and the estimated signal $u(n)\hat{w}(n - 1)$.

Since the actual experimental channels are not known, it is not possible to find the optimal algorithm parameters that achieve the minimum estimation error. Instead, the following guidelines are used for selecting the parameters.

The length of the channel vector is selected to be slightly larger than the multipath spread, which is obtained in practice by cross-correlation of the received signal with the replica of the PN signal in the baseband, and is observed to range
Figure 2.16: Block diagram of the Doppler compensation algorithm. The received signal is resampled based on the expected (in the absence of the Doppler effect) and the actual (observed from the experiment) packet lengths $T$ and $\hat{T}$, respectively. A combined RLS and PLL method is used to adaptively estimate the Channel $\hat{w}$ and the residual Doppler phase $\hat{\theta}$.

between 10–20 ms for the various experimental data presented in this study. The choice of the RLS forgetting factor $\lambda$ and regularization parameter $\delta$ as well as the PLL parameters $K_1$ and $K_2$ determine the algorithm performance. Fig. 2.17 shows the algorithm outputs for a 56 s long data recorded during the MACE experiment. The instantaneous gain $\tilde{g}$ (which is indicative of the total gain contained in the channel at each instance of time; see Chapter 3) and the residual frequency shifting $\Delta f$ are plotted for various values of $\lambda$ and $K_1$. On one hand, the RLS algorithm does not converge for very large values of $\lambda$, and on the other hand, the estimated instantaneous gain is highly affected by noise if $\lambda$ is selected to be too small. A reasonable behavior is observed around $\lambda = 0.999$. Similarly, the estimated residual frequency shift is extremely noisy at high values of $K_1$, and convergence problems arise at very low values of $K_1$. The algorithm performs best around $K_1 = 0.1$, and we set $K_2 = K_1/10$ [40]. The RLS regularization parameter $\delta$ is selected in accordance with the received SNR. Particularly, we use [41, Chapter 9]

$$\delta = \sigma_u^2 (1 - \lambda)^{\alpha} \quad (2.49)$$
where $\sigma_u^2$ is the variance of the input signal $u(n)$, and

$$
\alpha \in \begin{cases} 
\{1\}, & \text{high SNR (more than 30 dB)} \\
[-1, 0), & \text{medium SNR (on the order of 10 dB)} \\
(-\infty, -1], & \text{low SNR (less than -10 dB)} 
\end{cases}
$$

The SNR is dependent on multiple variables such as the system bandwidth, the transmission distance, and the environmental conditions, and is observed to range between 15 and 25 dB (in-band) for the presented experimental data.

Once the residual motion-induced phase is compensated for, the OMP algorithm is repeatedly applied to a sliding window of the received signal. The length of the window is selected to be on the order of the effective length of the exponential window of the RLS algorithm which equals $(1 - \lambda)^{-1}$. The stopping criterion of the OMP algorithm is set as the maximum number of iterations being at least equal to the number of dominant channel taps observed by the RLS algorithm$^3$, which is found to be fewer than 10.

The benefits of the OMP algorithm come at the price of its quadratic complexity. A Sparse Adaptive Orthogonal Matching Pursuit (SpAdOMP) algorithm was proposed in [42] whose performance was shown to be superior to that of the least squares estimation methods at the same (linear) computational complexity. In that study, autoregressive moving average (ARMA) and wireless Rayleigh fading channels were used as test cases for performance comparison. As an alternative to OMP, we used the SpAdOMP algorithm to estimate our experimental channels, and found their performance to be very similar, while the runtime of the latter was an order of magnitude lower than the former. Particularly, estimating a 56 s long channel at a time$^4$ resolution of 0.1 ms was completed in about 26 minutes using the OMP algorithm, while it only took 94 seconds using the SpAdOMP algorithm.

---

$^3$An alternative way to roughly estimate the number of dominant taps is to correlate the received signal with the replica of the PN signal in the baseband.

$^4$Not to be confused with delay.
Figure 2.17: The instantaneous gain $\tilde{g}$ and motion-induced frequency shift $\Delta f$ estimated by the Doppler compensation algorithm using various values of the RLS forgetting factor $\lambda$ and the PLL parameter $K_1$ for a 56 s long data recorded during the MACE experiment.
Chapter 3

The Channel Gain: Definition, Modeling and Applications to Power Control

3.1 Definition of the Channel Gain

So far, we have developed a model that represents the time-varying channel transfer function as

$$H(f, t) = H_0 \sum_p h_p \tilde{\gamma}_p(f, t) e^{-j2\pi f \tau_p}$$  \hspace{1cm} (3.1)

This function completely describes the channel for a given set of large-scale parameters $h_p$, $\tau_p$, the path statistics $\tilde{\gamma}_p(f)$, $\sigma_p^2(f)$, and the Doppler scaling factors $a_p$. We recall that these parameters may also vary with time, each at its own rate. The different time-scales at which these parameters vary give rise to the definition of different figures of merit for a communication system design.

An important figure of merit for communication systems is the overall channel gain, i.e. the overall energy, or power, contained in the channel response at a given time. We define the instantaneous channel gain for a system operating in
the frequency range \([f_0, f_0 + B]\) as

\[
\tilde{G}(t) = \frac{1}{B} \int_{f_0}^{f_0+B} |H(f, t)|^2 df
\]  

(3.2)

and the corresponding locally-averaged gain as

\[
G = E_\gamma \left\{ \tilde{G}(t) \right\}
\]  

(3.3)

If the bandwidth is large enough such that all multipath components are clearly resolvable, the gain will be dominated by individual multipath components \([12]\)

\[
G = \sum_p G_p h_p^2
\]  

(3.4)

where

\[
G_p = \frac{1}{B} \int_{f_0}^{f_0+B} \tilde{H}_0^2(f)[\bar{\gamma}_p(f)^2 + 2\sigma_p^2(f)] df
\]  

(3.5)

We refer to the gain \(G\) as the large-scale gain, and note that it depends on both the path coefficients \(h_p\) and the path statistics \(\bar{\gamma}_p(f), \sigma_p^2(f)\). The large-scale gain is thus a random variable, whose statistics are determined by those of the path gains \(h_p\). The path gains, and consequently the channel gain, can also be modeled as random processes, so as to explicitly take into account their time dependence \(h_p(t)\). The gain will then become a random process itself, which we denote as \(G(t)\).

We emphasize that the time scales involved in this type of modeling are larger than those used for small-scale effects, i.e. that \(h_p(t)\) changes more slowly than \(\gamma_p(f, t)\). Finally, we note that the path statistics can also change with time, as dictated by the environmental conditions. These changes are likely to occur on even longer time scales.

The distribution of the gain can be assessed analytically if the distribution of the path gains is known. Recalling the model (2.12), we have that

\[
G(t) = \sum_p G_p \bar{H}_p^2 e^{-\xi_p \Delta t_p(t)}
\]  

(3.6)
Assuming that the path lengths are Gaussian distributed with mean $\bar{l}_p$ and variance $\sigma^2_{l_p}$, the gain is a sum of log-normally distributed random processes, which can be modeled as a log-normal process itself (see [43] and the references therein). The mean and variance of the approximate log-normal sum can be calculated using the Fenton-Wilkinson method described in [43] as

$$
\bar{G} = \sum_p G_p \bar{l}_p e^{\xi_p^2 \sigma^2_{l_p}/2} \\
\sigma^2_G = \sum_p G_p^2 \bar{l}_p^4 e^{\xi_p^2 \sigma^2_{l_p}} (e^{\xi_p^2 \sigma^2_{l_p}} - 1)
$$

Thus, in the case of independent and small zero-mean Gaussian path length displacements, the gain behaves approximately as log-normally distributed. On the decibel scale, we have that

$$
g(t) = \bar{g} + \Delta g(t) \quad (3.7)
$$

where $\bar{g} = 10[2 \log_{10} \bar{G} - \frac{1}{2} \log_{10}(\bar{G}^2 + \sigma^2_G)]$, and $\Delta g(t)$ is zero-mean Gaussian with variance $\sigma^2_g = 10^2(\log_{10} e) [\log_{10}(\bar{G}^2 + \sigma^2_G) - 2 \log_{10} \bar{G}]$.

The model (3.7) holds for a fixed channel geometry, which defines the nominal attenuation for the transmission distance $d = \bar{l}_0$. If the nominal geometry changes, e.g. due to vehicular motion over a period of time, $\bar{g}$ will change accordingly. Specifically, $\bar{g}$ is expected to maintain a log-distance relationship

$$
\bar{g}(d) = g_0 - k_0 10 \log d / d_{ref} \quad (3.8)
$$

where $d_{ref}$ is a reference distance, e.g. $1$ m. The constant $g_0$ and the effective spreading factor or path loss exponent $k_0$, which depends on operational frequency, can be estimated from ensemble averages of the gains obtained at varying distances [12]. As will be discussed in Section 3.2, experimental data also show that the variance $\sigma_g^2$ is independent of the distance. To highlight the dependence of the mean gain on distance, we re-write Eq. (3.7) as

$$
g(d, t) = \bar{g}(d) + \Delta g(t) \quad (3.9)
$$
The model (3.8) can be used as a mathematical framework for designing Monte-Carlo simulations of the large-scale variations of the channel gain. For example, in [44], the model was used to investigate the problem of utilizing an autonomous underwater vehicle (AUV) to collect data from an underwater sensor network. The sensors in the network were equipped with acoustic modems that provided noisy communication whose quality degraded logarithmically with distance. A block log-normal fading model was used for SNR in which instantaneous SNR was assumed to be constant over the duration of one block. AUV path planning methods were proposed that maximize the information collected while minimizing travel time or fuel expenditure. It was shown that improved performance could be achieved if the AUV communicated with multiple nodes at once using a deterministic multiple access method. The study was extended in [45] by comparing the performance improvement for deterministic and random multiple access methods.

Time-correlation properties of the large-scale gain $g(d, t)$, described by the model (3.8), depend on the actual motion pattern, and are best left to specific examples. Experimental observations, which will be discussed in Section 3.2, seem to attest to an exponential auto-correlation of the gain $\Delta g$, which indicates the possibility to model the gain as an auto-regressive process. If the time-varying gain in an actual channel can indeed be modeled as a simple AR process (of order 1 or more), this fact has significant implications on the use of adaptive power control. Namely, it implies the possibility to predict the gain a few seconds ahead, thus allowing the transmitter to cater to the receiver’s needs at the time of signal arrival, rather than simply acting upon outdated information about the receiver’s state. A study on the prediction of the UWA channel impulse response was conducted in [28].

### 3.2 Statistics of Experimental Channel Gain

Data obtained from the four experiments described in Section 2.4.1 are used to study the large-scale variations of the window averaged channel gain. A
two-second long rectangular window is used. Fig. 3.1 summarizes the results. The figure shows four columns, each corresponding to one of the experiments. Each column contains four plots, showing the gain $g$ vs. time (for the duration of the experiment), the gain vs. distance, the histogram of the gain deviation $\Delta g$, and its autocorrelation.

In the PS experiment, the transmission distance was constantly changing as the AUV moved, hence the average gain varied noticeably (Fig. 3.1(a)). A continuum of distances was covered during this experiment, as shown in Fig. 3.1(e). The solid curve here represents the log-distance model (3.8) whose parameter $k_0$ is indicated in the figure, while the dots represent the actual gain. Since the AUV was frequently at the same distance from the receiver, there are many values of the actual gain for each distance. Least squares estimates of the log-distance model parameters were calculated from the data as

$$k_0 = -\frac{\sum_i \hat{g}(d_i) \cdot 10 \log d_i - \frac{1}{T} \sum_i \hat{g}(d_i) \cdot \sum_i 10 \log d_i}{\sum_i (10 \log d_i)^2 - \frac{1}{T} (\sum_i 10 \log d_i)^2}$$

$$g_0 = \frac{1}{T} \sum_i \hat{g}(d_i) + k_0 \frac{1}{T} \sum_i 10 \log d_i$$

(3.10)

where $\hat{g}(d_i)$ are the ensemble averages of gains measured at distances $d_i, i = 1, \ldots I$.

The MACE experiment similarly involved transmission from an AUV. In contrast, signals in the SPACE and KAM experiments were transmitted between fixed points. In the SPACE experiment, the signals were recorded at three different locations (three different transmission distances). Fig. 3.1(b) shows the time series of the gain for these three locations, while Fig. 3.1(f) shows the same data clustered around three points along the distance axis. Finally, in the KAM experiment, the signals were recorded at a single location. Hence, a log-distance model is not shown for this experiment. The following observations are also made about the various experiments:

- Each epoch of the SPACE experiment lasted 3 minutes, with approximately two idle hours between adjacent epochs. Similarly, 9 minute epochs were transmitted every two hours during the KAM experiment. In Figs 3.1(b) and 3.1(d), the idle
Figure 3.1: Results of large-scale data analysis: PS experiment (left), SPACE (mid-left), MACE (mid-right), and KAM (right). (a)–(d): Gain versus time (dotted) and the model (solid); (e)–(h): Gain versus distance (dotted) and the model (solid); (i)–(l): Histogram of $\Delta g$ and Gaussian approximation; (m)–(p): Auto-correlation of $\Delta g$ (solid) and AR-1 auto-correlation (dashed).

hours are shown as empty spaces between consecutive epochs, but they are not plotted to scale. The values of the gain for the PS experiment were quantized; hence, they appear at discrete levels in Fig. 3.1(e).

• The PS and MACE experiments lasted a few hours, while the SPACE and KAM experiments lasted several days, thus undergoing additional time varying phenomena on a larger time scale. While the gain variation at each distance does not exceed a few ($\sim 5$) dB for the PS and MACE experiments, the long lasting SPACE and KAM experiments experience gain variation of as much as 15 dB over the
course of a few days. This observation is notably important, as it indicates that very large savings are available from slow power control.

- In the SPACE experiment, the total (multi-hour) gain variation is more pronounced at longer distances (15 dB at 1 km, versus 7 dB at 60 m). There also appears to be some correlation between the distances, notably the longer ones (e.g. a decrease in the gain during the final epoch or hour 30). These multi-hour variations are likely caused by the changing environmental conditions. As the wind/waves/tides change, so do the statistical parameters of the large-scale phenomena. A study on the dependence of the gain on the environmental data is presented in Section 3.2.1.

- In Fig. 3.1(b), the values of the large-scale parameter $k_0$ are indicated for epochs at hours 12 and 90 (highlighted in the figure). In both of the long-lasting experiments, SPACE and KAM, the mean value of the gain is thus calculated separately for each epoch.

- In the MACE experiment, the variance $\sigma^2_{\Delta g}$ estimated during a calm day (evening of June 26th, 2010) was about three times lower than on a windy day (4.8 dB on the morning of June 25th). The corresponding coherence time was twice as long during calm conditions.

Figs. 3.1(i)-(l) show the histograms of the random component $\Delta g = g - \bar{g}$, along with a theoretical Gaussian curve with zero mean and variance that is estimated from the data and indicated in the figure. Taking into account the pseudo-stationary nature of the long-lasting experiments (SPACE and KAM), $\bar{g}$ is calculated separately for each epoch. For all the experiments, the variance $\sigma^2_{\Delta g}$ is calculated from the data at hand and indicated in the figure. The data suggest that $\sigma^2_{\Delta g}$ is invariant for the span of distances considered. If greater distance spans are of interest to a particular system, sectioning may be required. Overall, a good match is observed between the histograms and the Gaussian fit, which speaks in favor of the log-normal model for the large-scale channel gain.

Finally, the auto-correlation of $\Delta g$ is plotted in Figs. 3.1(m)-(p). Shown also in the plots is an exponentially decaying auto-correlation function $e^{-|\Delta t|/\Delta t_c}$
corresponding to an AR-1 process where \( \Delta t_{c1} = 1/f_{\Delta g} \). A good match is observed between the experimental data and the AR-1 model for small time differences, notably up to several seconds. This fact implies the possibility to predict the channel gain from past samples, the fact that bears an important implication for adaptive power control in acoustic communication systems. Namely, if the round-trip delay is within the coherence time of the channel gain, a feedback loop can be closed between the transmitter and receiver. Judging by the observations made during the SPACE and KAM experiments, power control can yield substantial savings (10 dB or more) over longer intervals of time.

### 3.2.1 Relationship with environmental data

We investigate the relationship between the channel gain measured during the SPACE experiment and the environmental data. Particularly, we focus on the measurements of the wave period and height collected at the Martha’s Vineyard coastal observatory (MVCO) which are available at [http://www.whoi.edu/mvco](http://www.whoi.edu/mvco). The wave characteristics are known to depend on the wind speed, the amount of time it has blown, and its fetch [46]. In general, stronger winds generate waves with higher periods. Waves whose period is between a few seconds to a few tens of seconds are known as swells.

Fig. 3.2 shows the estimated gain and the measured wave periods and heights. The heights of swells and waves with shorter periods were recorded separately, and show some correlation. At certain epochs, the gain is observed to drop noticeably, indicating that the estimation algorithm failed to produce the channel. The estimated gain at those epochs is about 10 dB below the minimum value shown on the figure. These occurrences are mainly observed at high wave heights; i.e. around days 296 and 300–302. The figure also indicates an inverse relationship between the gain and the wave period. For example, around day 295, when the wave period is high, the gain is observed to reach a local minimum; and around day 297, when the wave period is low, the gain is relatively high. A thorough study on the environmental conditions, which takes into account a higher number of environmental factors, e.g. water salinity/temperature and wave di-
rection, and various experiment sites may lead to a mathematical model for the very slow (multi-hour) gain variation. Nevertheless, the observations presented in Fig. 3.2, together with the earlier conjecture on the correlation between the gain variation over different distances, provide some evidence on the dependence of the multi-hour gain variations on the environmental conditions.

### 3.3 Power Control: Theoretical Considerations

To establish a relationship between the received signal power and the channel gain, let us consider a practical UWA system which operates in a frequency bandwidth \([f_0, f_0 + B]\). Assuming that the transmitted signal has a flat power spectral density \(S_T\), its power is \(P_T = S_T B\). The received signal power is

\[
\tilde{P}_R(t) = \int_{-\infty}^{+\infty} \tilde{S}_R(f, t) df = \int_{f_0}^{f_0 + B} \tilde{S}_T |H(f, t)|^2 df = P_T \tilde{G}(t) \quad (3.11)
\]

where \(\tilde{G}(t)\) is the instantaneous channel gain as defined by Eq. (3.2).

Fig. 3.3 shows the variation of the instantaneous received power with time,
Figure 3.3: Channel gain over several hours and a zoomed-in illustration of the gain over three minutes. While the fast fluctuations of the instantaneous gain are impossible to track, it is feasible to set up an adaptive power control algorithm to predict the slowly varying local average of the gain.

as observed from the SPACE experimental data collected at a channel distance of 1000 m. The figure is a re-illustration with fine details of the data plotted in Fig. 3.1(b). The transmit power was kept constant during the experiment. The figure thus demonstrates a scaled version of the instantaneous received power. The process appears to have a slowly varying local average with superimposed faster variations which are due to the variation of the small scale factors $\tilde{\gamma}_p(f, t)$ as described in Chapter 2.

While a simple feedback mechanism cannot aspire to predict the fast variations of the instantaneous gain, it may be possible to predict the slower variations of the locally averaged gain estimated at averaging window of length $T_w$ as

$$g(t) = \frac{1}{T_w} \int_{t-T_w}^{t} \tilde{g}(s)ds$$  \hspace{1cm} (3.12)

Fig. 3.4 illustrates a feedback process where the receiver provides the transmitter with the gain $g(t)$ through a feedback channel. The averaging window $T_w$
is chosen to be 2 seconds in this case. The feedback signals are transmitted every $t_f$ seconds and used to predict the gain $t_l$ seconds ahead. In a practical power control system, the prediction lag $t_l$ is chosen to be at least the round-trip delay, which is the time by which the feedback signal will be outdated once the adjusted transmit signal arrives at the receiver. For the hypothetical 2 km long channel of Fig. 3.4, $t_l$ is chosen to be equal to the round-trip delay of 2.6 seconds. For the illustrated system, two feedback signals are transmitted during the prediction lag time ($t_f = t_l/2$).

![Figure 3.4](image.png)

**Figure 3.4:** Illustration of a receiver-to-transmitter feedback mechanism. Local averaging is performed over a window of length $T_w$. The feedback signals are sent every $t_f$ seconds, and used to predict the gain $T_l$ seconds ahead.

In the remainder of this section, we look at theoretical gains that are achievable by adaptive power control assuming that the fading model is known. We first define the power control algorithm and then analyze its performance under the AR-1 and AR-2 models assuming a log-normal distribution for $G(t)$, which, in what follows, we denote by $G$ for simplicity. A method based on an outdated gain estimate is also considered as a benchmark.

While the log-normal distribution assumption enables analytical assessment of the achievable power savings, it does not restrict the proposed power control method. Log-normal behavior of the gain has been observed in other underwater channels (e.g. [14]) as well as in the experimental channels studied in Section 3.2.
3.3.1 Description of the power control algorithm

In a conventional system where no power control mechanism is applied, a fixed margin is introduced to ensure that the SNR $\gamma = P_TG/P_N$ remains above a threshold $\gamma_0$ with probability $1 - P_{out}$, where $P_N$ is the noise power and $P_{out} = P\{\gamma < \gamma_0\}$ is the outage probability. When the transmit power is constant, the outage probability is given by $P_{out} = P\{G < P_N\gamma_0/P_T\} = P\{G < G_{out}\}$. Let $P_{T0}$ be the power needed to achieve $\gamma_0$ in the absence of fading ($G = \bar{G} = 10^{10}$); i.e., $P_{T0} = P_N\gamma_0/\bar{G}$. In the presence of fading, the power needed to achieve $\gamma_0$ with outage $P_{out}$ is $P_T = P_{T0}\bar{G}/G_{out}$. The factor $1/G_{out}$ represents the needed fixed margin.

Transmit power utilization can be improved if some knowledge of the gain is available at the transmitter. In an ideal adaptive power control scheme, the transmit power can be varied in accordance with the known fading gain $G$, so that the SNR is kept at the value $\gamma_0$; i.e., $\hat{P}_T = P_{T0}\bar{G}/G$. In this case, the channel never experiences outage. Extensions to the case of limited peak transmit power are possible as well, but we focus our attention on an unrestricted case for simplicity.

When the channel is not fully known, an estimate $\hat{G}$ is used in place of the true gain $G$ and a margin $K$ is introduced to ensure that the discrepancy between the estimated and the true channel gain does not lead to outage. The transmit power is now adjusted according to

$$\hat{P}_T = KP_{T0}\bar{G}/\hat{G}$$  \hspace{1cm} (3.13)

The optimal value of $K$ can be determined such that the requirement on $P_{out}$ is satisfied. With the above power adaptation, outage occurs whenever the transmit power, $KP_{T0}\bar{G}/\hat{G}$, is less than the threshold $P_{T0}\bar{G}/G$. Thus, the probability of outage can be calculated as

$$P_{out} = P\{KG/\hat{G} < 1\} = P\{g - \hat{g} < -10 \log K\}$$  \hspace{1cm} (3.14)

Finally, we calculate the difference between the fixed and the average ad-
justed transmit power levels as a measure of the achievable power savings
\[
S = 10 \log \frac{P_T}{\bar{P}_T} = p_T - \bar{p}_T \tag{3.15}
\]

The optimal value of the margin \(K\) can be determined from the expression (3.14) if the statistics of the error \(\varepsilon = g - \hat{g}\) are known. The error statistics depend on the specific prediction method used.

3.3.2 Prediction methods

A trivial choice for \(\hat{g}\) is to use directly the value provided by the receiver. This value, however, will be delayed by the round-trip time once it reaches the receiver. Let us denote by \(n\) the discrete instances of time separated by \(t_f\) and set the prediction lag to \(t_p = t_f\) for simplicity, and let us define \(g[n] = g(nt_f)\). The trivial estimate can thus be represented as \(\hat{g}_0[n] = g[n-1]\). Note that we assume ideal measurements at the receiver.

If the channel gain obeys an AR process with auxiliary noise whose parameters are known, a linear predictor can be used to estimate the gain. Here, we briefly describe two cases: an AR model of order 1 and 2.

An AR-1 model is described as
\[
\begin{align*}
g[n] &= \bar{g} + \Delta g[n] \\
\Delta g[n] &= a_1 \Delta g[n-1] + w[n]
\end{align*}
\tag{3.16}
\]

where \(w[n]\) is the discrete-time process noise and \(\bar{g}\) denotes the mean of \(g\). Let us denote the variance of \(g\), or equivalently, that of \(\Delta g\), with \(\sigma_{\Delta g}^2\). Based on this model, a linear predictor of order 1 yields
\[
\begin{align*}
\hat{g}_1[n] &= \bar{g} + \Delta \hat{g}_1[n] \\
\Delta \hat{g}_1[n] &= a_1 \Delta g[n-1]
\end{align*}
\tag{3.17}
\]

For an AR-1 process, the parameter \(a_1\) is related to the Doppler frequency \(f_{\Delta g}\) of
the channel as \( a_1 = e^{-2\pi f_{\Delta g}|t_f|} \). The variance of the estimate and the variance of the estimation error in this case are given by

\[
\begin{align*}
\sigma_{\hat{g}_n}^2 &= a_1^2 \sigma_{\Delta g}^2 \\
\sigma_{\epsilon_1}^2 &= (1 - a_1^2) \sigma_{\Delta g}^2
\end{align*}
\] (3.18)

For an AR-2 model, the gain variation is described as

\[
\Delta g[n] = a_1 \Delta g[n - 1] + a_2 \Delta g[n - 2] + w[n]
\] (3.19)

Based on this model, the gain is estimated as

\[
\begin{align*}
\hat{g}_2[n] &= \bar{g} + \Delta \hat{g}_2[n] \\
\Delta \hat{g}_2[n] &= a_1 \Delta g[n - 1] + a_2 \Delta g[n - 2]
\end{align*}
\] (3.20)

The parameters \( a_1 \) and \( a_2 \) are related to the parameters of the continuous-time model as

\[
\begin{align*}
a_1 &= 2e^{-\xi \omega_n t_f} \cos(\sqrt{1 - \xi^2} \omega_n t_f) \\
a_2 &= -e^{-2\xi \omega_n t_f}
\end{align*}
\] (3.21)

where \( \xi \) is the damping factor and \( f_n = \omega_n / 2\pi \) is the natural frequency. For \( \xi < 1/\sqrt{2} \), the second-order spectrum has two prominent peaks at \( f_p = \pm f_n \sqrt{1 - 2\xi^2} \).

The Doppler frequency, \( f_{\Delta g} \), is given by

\[
f_{\Delta g} = f_n \sqrt{(1 - 2\xi^2) + \sqrt{1 + (1 - 2\xi^2)^2}}
\] (3.22)

The variance of the estimate and the estimation error are given by

\[
\begin{align*}
\sigma_{\hat{g}_2}^2 &= A^2 \sigma_{\Delta g}^2 \\
\sigma_{\epsilon_2}^2 &= (1 - A^2) \sigma_{\Delta g}^2
\end{align*}
\] (3.23)

where \( A^2 = a_1^2 + a_2^2 + 2a_1^2a_2/(1 - a_2) \).
3.3.3 Power savings under log-normal fading

We calculate the power savings under log-normal assumption for three different estimation methods: the delayed estimate and linear prediction of order 1 and 2.

If $g$ is log-normally distributed, Eq. (3.14) can be expressed in closed form as

$$P_{out} = Q \left[ \frac{10 \log_{10} K}{\sigma_{\epsilon}} \right] \quad (3.24)$$

where $Q$ denotes the Q-function, $Q(x) = \frac{1}{2}(1-\text{erf}(x/\sqrt{2}))$. The outage probability can also be stated as

$$P_{out} = P\{G < G_{out}\} = Q \left[ \frac{\bar{g} - 10 \log G_{out}}{\sigma_{\Delta g}} \right] \quad (3.25)$$

Comparing Eqs. (3.24) and (3.25), we have that

$$K = \left[ \frac{\bar{G}}{G_{out}} \right]^{\sigma_{\epsilon}/\sigma_{\Delta g}} \quad (3.26)$$

where $G_{out}$ is specified in terms of $P_{out}$ as

$$g_{out} = \bar{g} - \sigma_{\Delta g} Q^{-1}[P_{out}] \quad (3.27)$$

The average transmit power needed with optimal $K$ is

$$\bar{P}_T = E[K P_{T0} \bar{G}/\hat{G}] = K P_{T0} 10^{\frac{\sigma_g^2}{2}} \quad (3.28)$$

In the fixed power scenario, the average transmit power is equal to the fixed power, $P_T = P_{T0} \bar{G}/G_{out}$. Substituting for $P_T$ and $\bar{P}_T$, the power savings $S$ are obtained as

$$S = \sigma_{\Delta g} Q^{-1}[P_{out}] (1 - \sigma_{\epsilon}/\sigma_{\Delta g}) - \frac{\sigma_{\bar{g}}^2}{2} \quad (3.29)$$

For linear prediction of order 1 and 2, $\sigma_{\bar{g}}$ and $\sigma_{\epsilon}$ are given by Eqs. (3.18) and (3.23), respectively, and the savings can thus be computed in closed form. These results can serve as an upper bound on the power savings of practical systems in
which the actual model parameters are not known, and adaptive prediction is used instead.

3.4 Power Control: Experimental Data Processing

Rather than relying on the assumption that the fading process obeys an AR model with known parameters, we apply a linear predictor with adaptively adjusted weights $a_m$ to the experimental data. A one-step prediction of order $M$ is made as

$$
\hat{g}_M[n] = \bar{g} + \Delta \hat{g}_M[n]
$$

$$
\Delta \hat{g}_M[n] = \sum_{m=1}^{M} a_m[n] (g[n - m] - \bar{g})
$$

(3.30)

where $\bar{g}$ is the mean estimated using an initial segment of the data. The $M$ prediction coefficients $\{a_m\}$ are updated using an RLS algorithm based on the error $g[n] - \hat{g}_M[n]$.

Before measuring the power savings that can be achieved by prediction, we investigate the effect of various parameters on the performance of the prediction algorithm.

3.4.1 The averaging window

First, let us examine the length of the window $T_w$ used for calculating the locally averaged values $g[n]$ that are then used to form the predictions $\hat{g}_M[n]$. If this window is too small, insufficient averaging of the instantaneous values will occur; if the window is too large, the time variation of the gain will not be well captured. To assess this trade-off, we define two types of errors. The first is the normalized averaging error, defined as

$$
e_1 = \frac{\sum_{n=1}^{N} |g[n] - \bar{g}|^2}{\sum_{n=1}^{N} |\bar{g}|^2}
$$

(3.31)
Figure 3.5: Mean squared averaging and prediction errors versus the window length $T_w$, obtained from SPACE experimental data. For large window lengths, the local average is smooth and hence well predictable, but it highly differs from the instantaneous gain, resulting in a high averaging error.

where $N$ is the number of the data points. The second is the normalized prediction error, which depends on the predictor order $M$ and is defined as

$$e_2(M) = \frac{\sum_{n=1}^{N} |\hat{g}_M[n] - g[n]|^2}{\sum_{n=1}^{N} |\tilde{g}[n]|^2} \quad (3.32)$$

One can also define the total error

$$e(M) = \frac{\sum_{n=1}^{N} |\hat{g}_M[n] - \tilde{g}[n]|^2}{\sum_{n=1}^{N} |\tilde{g}[n]|^2} \quad (3.33)$$

Using the Cauchy-Schwarz inequality, the total error is upper-bounded as

$$e(M) \leq e_u(M) = e_1 + e_2(M) + 2\sqrt{e_1e_2(M)} \quad (3.34)$$

Fig. 3.5 illustrates the averaging error $e_1$ and the prediction error $e_2(M)$ versus the averaging window $T_w$ for $t_f = 0.3$ s, 0.5 s and 1.3 s. The case $M = 0$ refers to the delayed estimate (no prediction). The figure shows that the prediction error and the averaging error exhibit opposite trends. For averaging windows longer than 2 seconds, $e_1$ is the dominant error. The total error in this case is mainly due to excessive smoothing of the gain variations and it cannot be noticeably reduced.
by increasing the order of the predictor. If the averaging window is very small, $e_2$ is dominant. In this case, insufficient averaging is performed, i.e. the local average tends to the instantaneous values which are changing too rapidly to be tractable via feedback. In light of these observations, $T_w = 2$ s is selected for our experimental channel.

Fig. 3.5 also shows that for a fixed $T_w$, a greater error is observed for lower values of the feedback interval $t_f$. Specifically, we note that a predictor of order $M$ achieves its best performance if the feedback interval $t_f$ is selected to be equal to the prediction lag $t_l$. Accordingly, we set $t_f = t_l$ for all further processing. A relatively low feedback rate has also been reported to boost the performance of prediction in mobile radio channels [26].

### 3.4.2 The prediction lag and order

The major factor that fundamentally limits the performance of prediction is the round trip delay which determines the prediction lag $t_l$. Fig. 3.6 illustrates the prediction error $e_2(M)$ as a function of $t_l$. The averaging window is fixed at 2 seconds. As expected, the prediction error increases with prediction lag. In a practical power control system, a prediction lag of at least the channel’s round-trip delay must be sustainable (1.3 s for our experimental channel).

![Figure 3.6: Mean squared prediction error versus the prediction lag $t_l$, obtained from SPACE experimental data. $T_w$ is set to 2 s. Linear predictors outperform the trivial delayed estimate method by more than one decibel at almost all $t_l$.](image-url)
Results of Fig. 3.6 can also be used to estimate the efficiency of prediction at greater transmitter-receiver distances. For example, for a 2 km channel undergoing the same fading as our 1 km experimental channel, there is an additional 1 dB of loss due to the longer propagation delay. The order 2 and 1 predictors provide similar performance; however, the outdated estimate starts to lag considerably as the delay increases.

Figure 3.7: Mean squared prediction error versus the predictor order for varying forgetting factor $\lambda$ of the RLS algorithm, obtained from SPACE experimental data. $T_w$ is set to 2 s and $t_l$ is set to the round-trip delay which is 1.33 s. One can significantly decrease the prediction error by selecting a low RLS forgetting factor (solid lines), however, at such situations, the total error is dominated by the averaging error which is insensitive to the prediction method (dashed lines).

Finally, Fig. 3.7 shows the prediction error $e_2(M)$ as a function of the prediction order $M$ for several values of the RLS forgetting factor $\lambda$. Clearly, the performance improves with prediction order. Also, among predictors of the same order, the one with the lowest forgetting factor outperforms the others. This behavior is attributed to the fact that prediction delays are long.

Although the gain in this experimental channel varies slowly enough to be predicted very well using even the outdated estimate, several decibels of additional improvement is available from prediction. However, at high prediction orders, the total error is mainly due to the averaging error as the prediction error becomes negligibly small. To better clarify this fact, the upper bound on the total error (3.34) is also plotted in Fig. 3.7 (dashed line). As can be seen there, one may not
expect to considerably reduce the total error by selecting a better predictor.

### 3.4.3 Achievable power savings

Fig. 3.8 illustrates the power consumption corresponding to the SPACE’08 recordings in several situations. If we set the minimum necessary received power $p_{R0}$ so as to ensure that all transmissions in this observation interval are successful (bottom line), the total transmit power needed is $p_T$ (top line). This power corresponds to the fixed margin case. The received power in this case ($p_R = p_T + g$, on the decibel scale) is varying, exceeding the necessary minimum most of the time.

![Figure 3.8: Illustration of the transmitted and received power in cases of fixed margin ($p_T$ and $p_R$) and adaptive power control ($\hat{p}_T$ and $\hat{p}_R$). The difference $p_T - \hat{p}_T$ indicates the achievable power savings, which amounts to 8.7 dB for the SPACE experimental data.](image)

In contrast, if adaptive power control is employed, the transmit power can be adjusted to the minimum needed to ensure successful reception. In this case, the transmit power ($\hat{p}_T$) is varying, while the received power ($\hat{p}_R = \hat{p}_T + g$) stays fixed (bottom line). The average transmit power consumed in this case ($\overline{p_T}$) is shown in dashed line. The resulting savings amount to 8.7 dB.

In this example, we have assumed ideal power control to illustrate the possible savings. A practical method, based on adaptive prediction described in Section 3.4, will incur some penalty due to prediction lag. Note, however, that this penalty is negligible since the power varies slowly enough that prediction is almost ideal.
Using the analytical approach of Section 3.3, the theoretical bounds on power savings are obtained assuming an AR-1 model with parameter $a_1 = 0.77$ estimated from the data. Fig. 3.9 illustrates the savings $p_T - \hat{p}_T$ versus the outage probability. Note that the savings are obtained for the fixed $p_T$ of Fig. 3.8. The figure shows that more savings are possible if higher outage probability is allowed. The achievable savings corresponding to ideal prediction of the gain are also plotted as a benchmark (dashed line). We note that the difference between the savings corresponding to ideal prediction and the bound obtained for the AR-1 model is less than a decibel.
Chapter 4

Acoustic Channel Simulator

In this chapter, we describe an acoustic channel simulator which is designed based on the channel models proposed in Chapters 2 and 3. We emphasize that this simulator is designed to address multipath, rough surface scattering, motion-induced Doppler, and large-scale variation in the channel geometry; however, it does not take into account the surface curvature, or the effect of breaking waves which add extensive complexity to the model. In addition, it assumes Gaussian-distributed intra-path delays of the scattered paths. Hence, we regard this simulator as a first approximation for computationally-efficient modeling of a class of underwater acoustic channels.

4.1 Brief Description

The channel simulator takes into account physical aspects of acoustic propagation as well as the effects of inevitable random channel variations. Channel variations are classified into small-scale and large-scale, based on the notion of the underlying random displacement being on the order of a few or many wavelengths, respectively. While small-scale variations occur over short displacements, and correspondingly short intervals of time (e.g., sub-second) during which the system geometry and environmental conditions do not change, large-scale modeling treats variations caused by location uncertainty as well as varying environmental conditions.
The basic (time-invariant, deterministic) model of an acoustic channel is that of a multipath channel with additional low-pass filtering. Low-pass filtering accounts for energy absorption which is higher for higher acoustic frequencies. The signal also attenuates with distance, according to the energy spreading law (quadratic with distance for spherical geometry of spreading, linear for cylindrical). Fig. 4.1 illustrates this effect.

![Figure 4.1](image1)

**Figure 4.1**: Transfer function corresponding to a single reference path of length 1 km and spherical spreading ($k = 2$).

In a multipath channel, all the paths can be approximated as having the same reference transfer function, but a different gain and delay. A nominal channel geometry can be used to determine the nominal path gains and delays, $\bar{h}_p$ and $\bar{\tau}_p$, as shown in Fig. 4.2.

![Figure 4.2](image2)

**Figure 4.2**: (a) In shallow water with constant sound speed, geometry of the channel can be used to calculate nominal path lengths and angles of arrival. (b) In deep water, or in shallow water with depth-dependent sound speed, the Bellhop ray tracer can be used to determine the multipath profile.
Because of the location uncertainty, the actual path gains $h_p$ and delays $\tau_p$ deviate from the nominal ones. This type of random deviation is classified as a large-scale phenomenon. Given a particular realization of the large-scale parameters at some time $t_n$, additional variation is caused by small-scale phenomena such as random surface scattering and local motion (transmitter/receiver drifting, surface waves, etc.). Fig. 4.3 illustrates the two types of random phenomena.

**Figure 4.3**: (a) Deviations from the nominal channel geometry cause the path gains and delays to deviate from the nominal ones. (b) Surface scattering causes each path to split into a number of intra-paths, while unpredictable local motion causes random Doppler shifting.

The effect of time-varying multipath is modeled by the transfer function

$$H(f, t) = \sum_p h_p(t_n)\gamma_p(f, t)e^{-j2\pi f\tau_p(t)} \quad (4.1)$$

where $\gamma_p(f, t), p = 0, 1, \ldots$ represent the scattering coefficients, and $\tau_p(t) = \tau_{p0} - a_pt$ represent the path delays which vary in time according to the (random) Doppler scaling factors $a_p$. Scattering coefficients are modeled as complex-valued Gaussian processes, whose statistical properties (correlation in time and frequency) are determined from the variance $\sigma^2_{\delta_p}$ and the 3-dB bandwidth $B_{\delta_p}$ of the intra-path delays. Algorithm 1 describes the simulation method.

Matlab codes: set_channel_params.m which a user edits to set the channel parameters and executes to save them in data files, and channel_simulator.m which performs the simulation based on the given data files. The simulator can be set up to use Bellhop or a simplified multipath calculator for generating the large-scale realizations. The small-scale scattering coefficients can be generated directly from a set of intra-path delays, or based on a statistically equivalent model.

Algorithm 1: Channel simulator

<table>
<thead>
<tr>
<th>Line</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initialization: set $h_p$, $\tau_p$, $\sigma^2_{\delta p}$, $B_{\delta p}$</td>
</tr>
<tr>
<td>2</td>
<td>for each realization of the large-scale process do</td>
</tr>
<tr>
<td>3</td>
<td>set $h_p$, $\tau_p$</td>
</tr>
<tr>
<td>4</td>
<td>for each realization of the small-scale process on an observation interval $t \in [0, T_{obs}]$ do</td>
</tr>
<tr>
<td>5</td>
<td>for $p = 1, \ldots, P$ do</td>
</tr>
<tr>
<td>6</td>
<td>$\gamma_p(f, t) = \frac{1}{h_p} \sum_i h_{p,i} e^{-j2\pi f \tau_{p,i}(t)}$</td>
</tr>
<tr>
<td>7</td>
<td>$\tilde{\gamma}_p(f, t) = \gamma_p(f, t) e^{j2\pi a_p f t}$</td>
</tr>
<tr>
<td>8</td>
<td>$H(f, t) = \tilde{H}_0(f) \sum_p h_p \tilde{\gamma}_p(f, t) e^{-j2\pi f \tau_p}$</td>
</tr>
<tr>
<td>9</td>
<td>$\tilde{G}(t) = \frac{1}{B} \int_{f_0}^{f_0+B}</td>
</tr>
<tr>
<td>10</td>
<td>$G = E_{\gamma} { \tilde{G}(t) }$</td>
</tr>
</tbody>
</table>

To demonstrate some practical applications we use the simulator to re-generate the experimental channels which were discussed in Chapters 2 and 3 given the nominal geometries and the statistics of the experimental channels. In addition, the simulator is used to evaluate the performance of a system based on orthogonal frequency division multiplexing (OFDM). We compare the performance of the system operating on the simulated and the experimental channels. Finally, we use simulated data to study the impact of system properties, i.e. bandwidth and center frequency, on the model parameters. The fact that conveying such a study through experimental data would be impractical due to the limitations of actual UWA modems indicates the importance of the availability of a UWA channel simulator.
4.2 Channel Characteristics

For studying the channel characteristics, the PN sequence signals transmitted via the WHOI communication system, described in Section 2.4, are analyzed.

Assuming sinusoidally moving surface and transmitter/receiver drifting of several centimeters per second, an ensemble of small-scale coefficients \( \tilde{\gamma}_p(f, t) \) is generated. The statistically equivalent model for \( \Delta \gamma_p[n] \), given by Eq. (2.33), is used. The duration of the simulated data is set to one minute to match with the duration of the signals used in the KAM’11 experiment whose impulse response was studied in Section 2.4.2. The impulse response is re-plotted in Fig. 4.4 for convenience. The large-scale parameters are assumed to be fixed during this time interval. The simulated channel transfer function is formed using Eq. (2.17) and its inverse Fourier transform is used to obtain the time evolution of the path gains. The time-correlation properties of the simulated path gains are dictated by the variance \( \sigma^2_{\delta p} \) and the Doppler spread \( B_{\delta p} \) of the intra-path delays, as well as the motion-induced Doppler factors. Scattering occurs on spatial dimension on the order of the wavelength, which is 12 cm for the center frequency of our experimental system, and translates into a time variation of 0.07 ms. In the simulated channel, \( \sigma_{\delta p} \) corresponding to the surface reflected path is set to 0.05 ms, and its value...
increases with the number of reflections for higher order paths (e.g. \( \sigma_{\delta p} = 0.25 \) ms for the path with two surface and one bottom reflections).

From Eq. (2.31), the coherence time corresponding to the scattering process is inversely proportional to \( B_{\delta p} \). If \( B_{\delta p} \) is selected to be too high (i.e. more than 0.01 Hz), the coherence time of the path gains will become less than that observed in the experiment. On the contrary, if \( B_{\delta p} \) is selected to be too low (i.e. less than \( 1 \times 10^{-4} \) Hz), the time-correlation functions will be almost flat for the duration of the simulated signal. We have selected \( B_{\delta p} = 5 \times 10^{-4} \) Hz which is in between the two thresholds.

The periodic variations of the path delays about the nominal values in Fig. 4.4, are used to estimate the motion. We focus on paths \( p_0, p_s, p_{sb} \) and \( p_{bs} \), which are observed around the nominal delays and are hence considered as valid paths. Since all paths, including the direct path, exhibit such delay variations, the effect of transducer motion seems to be more dominant than surface fluctuation. Vehicular motion is not considered because no average delay shift is observed. Note that resampling to compensate for rough Doppler was not needed for this data set as no major Doppler shift was observed. In order to calculate the transducer drifting, we model the delay fluctuations as a sinusoid of amplitude 160 microseconds (on the delay axis) and period 15 seconds (on the time axis), as observed in the figure. The derivative of this function yields the drifting Doppler factor, which is related to the relative speed, projected onto the path direction, by a factor of \( 1/c \), where \( c \) denotes the speed of sound in water. Assuming that all paths have approximately same angles of arrival (horizontal channel with range much greater than depth), a sinusoidal drifting speed of amplitude 10 centimeters per second and a period of 15 seconds results in the observed Doppler factor.

The sound speed profile was set based on measurements made during the experiment. The effect of temperature seemed to be the dominant factor as the measurements showed constant sound speed of about 1537 meters per second for up to 40 m depth which decreased to about 1530 meters per second at higher depths.

We test the accuracy of the simulator by comparing the statistics of the
Table 4.1: Jensen-Shannon divergence and the K-factors of the Ricean fits.

<table>
<thead>
<tr>
<th></th>
<th>experiment</th>
<th>simulation</th>
<th>experiment</th>
<th>simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>J-S divergence</td>
<td>5.1e-4</td>
<td>5.8e-3</td>
<td>5.8e-3</td>
<td>2.9e-3</td>
</tr>
<tr>
<td>K-factor [dB]</td>
<td>22</td>
<td>15</td>
<td>12</td>
<td>10</td>
</tr>
</tbody>
</table>

simulated and the experimental channels. Table 4.1 and Fig. 4.5 summarize the properties of the histograms and the time-correlation functions of the small-scale path gains and the large-scale channel gain, estimated from experimental and simulated data.

Table 4.1 lists the properties of the histograms of the path gains. The J-S divergence between the estimated histograms and the fitted distributions is listed in Table 4.1 as a quantitative measure of the goodness of the fits. The negligible divergence indicates a good fit for all path gains, both for experimental and simulated channels. The Ricean K-factor is also listed in the table. Its value for the direct path ($p_0$) is greater than that of the other paths, indicating a stable arrival as there is no encounter with the surface.

Figs. 4.5(a) and (b) show the time domain auto-correlation functions of the direct and the bottom-surface reflected path coefficients, respectively. The correlation function corresponding to simulated data matches with that of the experimental data which approves the choice of the scattering and motion parameters used for simulation. Bessel-like auto-correlation functions caused by motion-induced Doppler are observed clearly for the bottom-surface-reflected path which undergoes higher Doppler distortion.

Figs. 4.5(c) and (d) show the histograms of the large-scale channel gain variation $\Delta g = g - \bar{g}$, where $\bar{g}$ denotes the average of $g$, estimated for experimental and simulated data, respectively. The data consisted of 585 minutes of transmission, over a duration of 130 hours (9 minutes every 2 hours). The figures also show the Gaussian fits. The 95% confidence intervals of the histogram bars are plotted. For almost every bar, the Gaussian fit falls inside the confidence interval which indicates the goodness of the fits. Finally, Fig. 4.5(e) shows the auto-correlation function of $\Delta g$ estimated for simulated and experimental data along with the auto-
Figure 4.5: Small-scale (top) and large-scale (bottom) statistics of the experimental and the simulated channels: time-correlation functions corresponding to (a) the direct path, and (b) the bottom-surface path; histogram of (c) experimental and (d) simulated gain and Gaussian approximation; (e) time-correlation of experimental and simulated gain and AR-1 auto-correlation.

correlation function of an AR-1 process. Both simulated and experimental gain seem to obey the AR-1 auto-correlation function especially for time differences of less than a minute.

4.3 System Performance

The data used for evaluation of the system performance were obtained from Scripps Institute of Oceanography (SIO) transducers, which were described in Section 2.4. The transmitted signal consisted of consecutive OFDM blocks with a varying number of carriers. We compare the performance of coherent and differentially coherent receiver with and without partial-FFT (P-FFT) demodulation [47, 48]. P-FFT demodulation is a technique in which the received OFDM block is divided into $P$ non-overlapping segments and FFT demodulation is applied on each segment separately. The outputs of the FFTs are weighted and combined.
The combiner weights are optimized for each carrier, so as to approximate the effect of optimal (channel-matched) pre-FFT filtering.

Figure 4.6: Average MSE performance for (a) KAM’11 experiment and (b) simulation. Averaging is performed over $5 \times 10^4$ QPSk symbols which were transmitted over the experimental channel in 3 frames separated by 3 hours. The simulated channel parameters are selected to match those of the experimental channel and similar receiver configurations are used. The MSE results demonstrate the same trend for the experimental and the simulated channels.

Fig. 4.6(a) shows the average MSE performance result for the KAM’11 experiment, utilizing 8 receiver elements (separated by 7.5 meters). The average SNR is observed to be 15 dB at each receiver element within the signal bandwidth. Averaging is performed over $5 \times 10^4$ QPSk symbols which were transmitted in 3 frames separated by 3 hours. The figure demonstrates that in this experiment, conventional differential detection is seen to be competitive to coherent detection for a large number of carriers (which coincides with high bandwidth efficiency). P-FFT further improves the performance of both techniques by reducing the inter-carrier interference which arises with a large number of carriers (long block duration makes the system more susceptible to the time-variation).

Fig. 4.6(b) provides MSE results for simulation, where the channel parameters are selected to match those of the experimental channel, observed for the experimental data as described in Sec. 4.2, and similar receiver configurations are used. The characteristics of the noise measured during the experiment are shown in Fig. 4.7. This noise is observed to have Gaussian distribution and a relatively
Figure 4.7: Noise statistics, estimated from 120 seconds of recording of ambient noise during the experiment: (a) psd, (b) pdf. The psd is obtained by the Welch's method, using an FFT of size 256 with 50% overlap.

flat psd in the bandwidth of interest (20 kHz–32 kHz). The noise used in simulation was thus modeled as white Gaussian, and its power was set to yield the same SNR as the one observed in the experimental data (15 dB).

Comparing the results of Figs. 4.6(a), 4.6(b), we see that the system performance obtained via simulation follows the same trend as that observed with real data. The discrepancies that occur affect the absolute values, but not the trend. For instance, the break-even point between the conventional coherent and differentially coherent detection occurs at 2048 carriers, after which differential detection outperforms coherent detection. This fact gives credibility to the proposed channel simulator, and indicates that it can indeed be used as a computationally-efficient tool for system optimization.

4.4 The Impact of System Properties on the Model Parameters

In this section, we study the dependence of the statistical channel properties on the system bandwidth and center frequency. As the channel statistics depend on the underlying distribution of the path length displacements, we will focus on specific examples. In particular, we consider a case in which the major source
of uncertainty is the transmitter/receiver and the surface height displacement. For each realization of the channel geometry, the effects of scattering and motion-induced Doppler are simulated using the model parameters specified in Section 4.2. For purposes of illustration, we will focus on an example channel whose parameters are given in Table 4.2.

| Table 4.2: Nominal parameters of the simulated channel. |
|----------------------------------|-----------------|
| distance                        | \( d = 1 \text{ km} \) (increasing up to 3 km) |
| water depth                     | \( h = 20 \text{ m} \) |
| transmitter height above bottom | \( h_T = 10 \text{ m} \) |
| receiver height above bottom    | \( h_R = 12 \text{ m} \) |

Let us assume that the transmitter and receiver are slightly displaced around their nominal locations by some heights \( \Delta h_T, \Delta h_R \) and a distance \( \Delta d \), and let us also assume that the surface height is displaced by some \( \Delta h \). Treating these displacements as random variables, we generate an ensemble of channel responses and evaluate the gain \( G \) for each realization. Specifically, let us assume that displacements are uniformly distributed, each on the interval \( \pm 2 \text{ meters} \).

Fig. 4.8 shows the results obtained for varying distance \( d \) and two values of the bandwidth, \( B = 10 \text{ kHz} \) and \( B = 2 \text{ kHz} \). Logarithmic scale is used, i.e. the figure shows the gain in decibels, \( g = 10 \log G \). The distance changes from 1 km to 3 km in steps of 50 m, and for each of the \( I = 40 \) distances considered, there are \( M = 200 \) realizations of the channel (transmitter/receiver/surface drift from the nominal). The reflection coefficients are calculated according to [30, Chapter 3] assuming a propagation speed of 1500 m/s in water and 1300 m/s in soft bottom.

As described by Eq. (3.7), two trends are indicated in the figure: (i) a general decrease in gain with distance, which is the natural consequence of energy spreading, and (ii) variation of the gain about some mean value, which is a consequence of varying propagation conditions. Shown in the figure in solid line is also the log-distance model suggested in Eq. (3.8). The constants \( g_0 \) and \( k_0 \) can be estimated using Eq. (3.10) from ensemble averages of the gains \( \tilde{g}(d_i) \) obtained at varying distances \( d_i, i = 1, \ldots I \). Specifically, if we have \( M \) values \( g_m[d_i] \) of the
Figure 4.8: Applying the model (3.7) to simulated data (channel example of Table 4.2, center frequency $f_c=15$ kHz). Plots on the left correspond to the bandwidth $B=10$ kHz; plots on the right correspond to $B=2$ kHz. (a), (b): Gain versus distance (dotted) and the model (solid); (c), (d): Histogram of $\Delta g = g - \bar{g}$ and the Gaussian approximation.

Gain for each of the $I$ distances $d_i$, we can calculate the ensemble averages as

$$\hat{\bar{g}}(d_i) = \frac{1}{M} \sum_{m=1}^{M} g_m[d_i], i = 1, \ldots I$$  \hspace{1cm} (4.2)

Assuming now that the model (3.8) holds, we proceed to form the ensemble of values

$$\Delta g_{m,i} = g_m[d_i] - \hat{\bar{g}}(d_i)$$  \hspace{1cm} (4.3)

for all realizations $m = 1, \ldots M$ and for all $i = 1, \ldots I$. The histogram of the so-obtained values is shown in Figs. 4.8(c) and (d). Shown also in these figures is
a normal pdf with mean zero and variance

\[ \sigma^2_{\Delta g} = \frac{1}{M I} \sum_i \sum_m \Delta g^2_{m,i} \]  

(4.4)

We note a good agreement between the histogram and the normal pdf for the two bandwidths considered, and especially for \( B=10 \) kHz. This observation supports our earlier conjecture that the gain \( G \) could be modeled as a log-normal random variable when the bandwidth is sufficiently large. We also note that the variance \( \sigma^2_{\Delta g} \) is greater for the 2 kHz bandwidth.

One may also wonder at how the gain would behave if calculated only for a very narrow band of frequencies, or in the extreme case, a single tone. For a tone of frequency \( f_c \), the gain simply represents the magnitude of the channel transfer function evaluated at \( f_c \), \( G(t) = |H(f_c, t)|^2 \). This function exhibits large variations as the path lengths vary about the nominal values causing multipath to combine constructively or destructively. The presence of frequent low values of the gain makes the actual pdf deviate considerably from the Gaussian. Hence, a log-normal approximation is not a good choice in this case. Note, however, that the gain calculated for a single frequency is not a representative figure of merit for a typical acoustic communication system which is inherently wideband. The appropriate figure of merit for such a system is the total received signal power, i.e. the total gain (average over all frequencies). This gain should influence the large-scale system design (transmit power allocation, coverage prediction, etc.).

### 4.4.1 The impact of bandwidth and the center frequency on the gain

System bandwidth and center frequency affect the values of the model parameters \( k_0 \) and \( \sigma^2_{\Delta g} \). Fig. 4.9 shows the effective spreading factor \( k_0 \) and the variance \( \sigma^2_{\Delta g} \) versus the center frequency for several values of the bandwidth. We observe that \( k_0 \) is heavily dependent on the center frequency, but almost insensitive to bandwidth. In contrast, the variance \( \sigma^2_{\Delta g} \) appears to be varying negligibly with the center frequency, but, as expected, decreases with bandwidth.
Figure 4.9: Model parameters (a) $k_0$ and (b) $\sigma_{\Delta g}^2$ versus center frequency $f_c$ for various values of bandwidth $B$, ranging from zero (light gray) to 10 kHz (dark).

These trends are illustrated in Fig. 4.9. Specifically, we model the parameters as

$$k_0 = \alpha_0 + \alpha_1 f_c + \alpha_2 f_c^2$$  \hspace{1cm} (4.5)

and

$$\sigma_{\Delta g}^2 = \beta_0 / B$$  \hspace{1cm} (4.6)

where the constants $\alpha$ and $\beta$ can be determined numerically for a given deployment scenario, or experimentally from the measured data. In the example of Fig. 4.10, which corresponds to the channel of Table 4.2, $k_0$ is modeled for $B=10$ kHz (the resulting values of $\alpha$ constants are indicated in the figure), and $\sigma_{\Delta g}^2$ is modeled for $f_c=15$ kHz (the resulting value of $\beta$ is indicated in the figure).

The fact that the variance $\sigma_{\Delta g}^2$ decreases with bandwidth should come at no surprise. In a wideband system, the gain is expected to vary the least because it is calculated as the average over a wide band of frequencies. As a result, its variation is influenced only by the path gains, and not by the faster varying path delays, as demonstrated in Eq. (3.4). In contrast, the gain observed over a narrower band of frequencies retains dependence on the path delays through the phase terms in the expression (3.1), and thus exhibits a greater overall variation. This fact is illustrated in Fig. 4.11, which shows the magnitude squared of a (normalized) channel transfer function at two different moments in time, $|H(f, t_1)|^2$ and $|H(f, t_2)|^2$. The
Figure 4.10: (a) Model (4.5) for the effective spreading factor $k_0$ as a function of center frequency $f_c$ and (b) model (4.6) for the variance $\sigma^2_{\Delta g}$ as a function of the bandwidth $B$.

two functions correspond to the nominal system parameters of Table 4.2 but two different receiver heights above the bottom, 11 m and 13 m. Shown in the figure are the values of the gain $G_{B_1}(t_1)$ and $G_{B_2}(t_2)$ corresponding to several bandwidths $B$. Clearly, the grains differ the least for the widest bandwidth.

Figure 4.11: Magnitude squared of the channel transfer function $|H(f,t)|^2$ is averaged over the system bandwidth $B$ to obtain the gain $G_B(t)$, as specified by the expression (3.2). This figure shows two cases (solid and dotted line) that occur as the receiver depth changes slightly from time $t_1$ to time $t_2$. Three different values are considered; $B=10$ kHz, $B=2$ kHz, and the extreme case of $B \rightarrow 0$. 

$B=10$ kHz

$\alpha_0=0.73, \alpha_1=-0.006, \alpha_2=0.004$

$\beta_0=9.7$
4.5 Co-Located versus Spatially-Distinct System Elements

Our model, so far, treats a single transmitter–single receiver communication system. Practical systems, however, use multiple co-located transducers to improve the spectral efficiency (e.g. by spatial multiplexing), the link quality (e.g. by exploiting the diversity gain) or the coverage (e.g. by beamforming). Co-located transducer elements experience the same or very similar channel geometry. When simulating such systems, one may exploit the dependence between the model parameters corresponding to the co-located channels. In contrast, nodes in a network or relays in a co-operative communication system are spatially-distinct and hence experience very different channels which have to be modeled independently. In this section, we study the dependence of the model parameters corresponding to different transducer elements in a system with multiple co-located transducers.

**Figure 4.12:** Geometry of a channel with reference receiving element $RX$, vertical array element $RX^v$, and horizontal elements $RX^h, RX^{h'}$ along with surface and bottom-surface reflections.

Fig. 4.12 shows a reference receiving element $RX$, a vertical array element
and horizontal array elements $RX^v$ and $RX^h$, which are equivalent to the vertical element, with respect to surface-reflected and bottom-surface-reflected paths, respectively. The surface-reflected mirror images $RX^v_s$ and $RX^h_s$, and bottom-surface-reflected images $RX^v_{bs}$ and $RX^h_{bs}$ are also shown in the figure. The path grazing angle is a fundamental physical property which appears in the derivation of many of the model parameters which were introduced in the previous chapters and are used in channel simulation. In what follows, we derive equations for calculating the grazing angle of receiving array elements given the grazing angle of the reference element. For the surface-reflected path of the horizontal element $RX^h_s$

$$\Delta \theta^h_s = \theta_s - \theta^h_s = \sin^{-1} \left( \frac{\Delta h_s}{l_s} \right) = \sin^{-1} \left( \frac{\Delta x \sin \theta^h_s}{R \cos \theta_s} \right) \approx \frac{\Delta x}{R} \sin(\theta_s - \Delta \theta^h_s) \cos \theta_s$$

(4.7)

where

$$\Delta h_s = \Delta x \sin \theta^h_s = \Delta y \cos \theta^v_s$$

(4.8)

$l_s$ denotes the length of the surface-reflected path for the reference receiver $RX$, and we have used the first order Taylor series approximation of the sine function around 0 and the fact that $\frac{\Delta x}{R} \cos^2 \theta_s \ll 1$. For a general path $p$

$$\theta^h_p = \theta_p - \frac{\Delta x}{R} \cos \theta_p \sin \theta_p$$

(4.9)

Similarly, for the vertical array element $RX^v_s$

$$\Delta \theta^v_s = \theta_s - \theta^v_s = \sin^{-1} \left( \frac{\Delta y \cos \theta^v_s}{R \cos \theta_s} \right) \approx \frac{\Delta y}{R} \cos^2 \theta_s$$

(4.10)

For a general path $p$

$$\theta^v_p = \theta_p - \frac{\Delta y}{R} \cos^2 \theta_p \text{sgn}(\sin \theta_p) = \theta_p - \frac{\Delta y}{R} \cos \theta_p \sin \theta_p | \cot \theta_p |$$

(4.11)
where the adjustment has been made to address negative grazing angles. Comparison of Eqs. (4.9), (4.11) indicates that the angular difference between vertically positioned array elements is larger than that of horizontally positioned elements by a factor of $|\cot \theta|$. Later in this section, we will see the implication of this fact in the study of spatial correlation between vertical and horizontal array elements.

First, let us revisit the dependence of some of the model parameters introduced in Chapter 2 on the ray grazing angle. Note that this discussion holds for co-located array elements and is not valid for spatially-distinct nodes which experience completely different channel geometries.

The large-scale path delay $\tau_p$ is dependent on the grazing angle through the path length $\bar{l}_p = d / \cos \theta_p$, where $d$ denotes the horizontal transmitter–receiver separation. The path gain $\bar{h}_p$ is also related to the grazing angle through the path length, as well as the reflection coefficient described by Eq. (2.3). The statistics of the scattering coefficient $\gamma_p$ are characterized by the variance $\sigma^2_{\gamma_p}$ of the intra-path delays, whose value is dependent on the path grazing angle, as shown in Eq. (2.19). The grazing angle also determines the Doppler factor affecting each path, as described by Eqs. (2.36), (2.38), and (2.40). Moreover, it determines whether the co-located elements experience uncorrelated scattering. Two reflected signals experience uncorrelated scattering if they are separated by at least half the signal wavelength. The exact horizontal separation between the $j$-th reflection points along the $p$-th path of two co-located elements is given by

$$
\Delta x_{p,j}^{h/v} = h_{p,r} \left[ \cot \theta_p^{h/v} - \cot \theta_p \right]
$$

where the upper index $h/v$ refers to either the horizontal or the vertical array placement and $h_{p,j}$ is the effective depth corresponding to the $j$-th reflection point along the $p$-th path, and is defined as

$$
h_{p,j} = \begin{cases} 
(h - h_T) + (j - 1)h & , \theta_p > 0 \\
h_T + (j - 1)h & , \theta_p < 0
\end{cases}
$$

As illustrated in Fig. 4.12, the element separation $\Delta x$, or equivalently $\Delta y|\cot \theta_p|$,
provides an upper bound on the reflection point separation. This fact indicates that if the equivalent horizontal array length is smaller than half the wavelength, i.e. if \( \Delta x < \lambda/2 \), or if \( \Delta y < |\tan \theta_p|\lambda/2 \), all reflections experience correlated scattering. Taking into account the fact that \( |\tan \theta_p| < 1 \) for almost all practical UWA channel geometries, the above bounds imply that horizontal antenna arrays are more prone to correlated scattering than vertical arrays, hence achieving lower spatial diversity. A study on spatial correlation in vertical arrays based on experimental data analysis was conducted in [49]. Historically, vertical arrays have been favored in the literature as the horizontal arrays experience longer multipath spread, and their element spacing, length, and deployment depth play important roles in their performance [1, 50]. However, in practice, vertical arrays are hard to deploy in specific arrangements, e.g. bottom-mounted or towed [51].

The model (2.40) for the Doppler effect induced by surface motion, assumes that the \( j \)-th reflection point of the \( p \)-th path catches the vertical surface motion in a random phase \( \psi_{p,j} \). This assumption is valid if the reflection points are sufficiently far apart\(^1\) which is the case in the context the model was proposed for, i.e. various reflections on one path. The separation \( \Delta x_{p,j} \) between the points of a single reflection \( p, j \) for two co-located elements results in a time of reflection difference

\[
\Delta t_{p,j} = \frac{\Delta x_{p,j}}{c \cos \theta_p}
\]

which translates into a phase difference

\[
\Delta\psi_{p,j} = 2\pi f_w \Delta t_{p,j} = 2\pi \frac{\Delta x_{p,j}}{\lambda_w \cos \theta_p}
\]

This relationship shows that the reflection phases corresponding to various array elements cannot be considered random, but are precisely defined. This implies that exploiting the dependence between the model parameters of co-located elements not only simplifies the simulation process, but it also is crucial for the correctness

\(^1\)The comparison is made with respect to the wavelength of the sinusoidal surface motion \( \lambda_w = c/f_w \), and not the signal wavelength \( \lambda \).
of calculations. Another such example is the direction of transducer motion which is assumed to be random in Eq. (2.36). Once the direction of motion is randomly generated for simulating the channel corresponding to the reference element, the same (or dependent) direction has to be considered for the other array elements. The physical structure of the array determines the exact dependence between the elements, e.g. suspended versus fixed array.

### 4.5.1 Nonlinear channel geometry

The study in Section 4.5 was developed under the assumption of linear channel geometry; i.e. constant sound speed across the water depth, and flat surface and bottom boundaries. The relationships obtained there are no longer valid if these assumptions are not satisfied. As a result of the inherent nonlinearities in such channels, each array element may see the channel as consisting of a different set of ray patterns, i.e. rays with a certain number of surface and bottom reflections and refracting through certain layers of the water column. The possible ray patterns for each array element in a channel with nonlinear geometry can be determined analytically. For example, the authors of [52] model these channels as multi-layer mediums with piece-wise linear sound speed, and establish a framework for calculating the number of eignrays that arrive at the receiver after passing through certain layers. Specifically, the problem is formulated as an $n$-th order polynomial root finding problem where $n$ is related to how the rays propagate into the different layers. This framework provides insight on why a certain number of ray patterns is observed in ray tracing of underwater channels. It also indicates that this number depends not only on the relative placement of the array elements, but also on the exact location of each element with respect to the layer boundaries.

Fig. 4.13 shows the transmission loss, calculated via Bellhop, for a channel with non-flat sound speed, bathymetry and surface shape. The transmission loss is observed to depend hugely on location, indicating that the multipath profiles seen by the different array elements may indeed be different, making our earlier analysis invalid for channels with nonlinear geometry. However, the exact channel
Figure 4.13: Transmission loss in a channel with depth-dependent sound speed and non-flat bottom and surface shape, calculated by Bellhop beam tracing. The acoustic field is hugely dependent on the location of the transducers.

behavior depends on the fine details of the geometry whose knowledge is rarely possible to obtain prior to deploying a real system. In the absence of such data, the discussion on spatial correlation presented in Section 4.5 provides a simple method for predicting the correlation properties of array elements before setting up an acoustic system.
Bibliography


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