INNOVATIVE USE OF PHASOR MEASUREMENTS IN STATE ESTIMATION AND PARAMETER ERROR IDENTIFICATION

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ABSTRACT

Innovative Use of Phasor Measurements in State Estimation and Parameter Error Identification (March 2014)

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State Estimation plays a significant role in power systems secure operation. It performs real time monitoring of the entire system and provides the system states to other security applications. Recently, the Phasor Measurement Units (PMUs) have been invented and deployed to power systems to provide both phasor magnitudes and phase angles. This research focuses on enhancing power system state estimation and parameter error identification through innovative use of phasor measurements.

The first part of the dissertation focuses on improving network parameter error identification through innovative use of phasor measurements. Previous work has shown that the parameter errors in certain topologies could not be detected or identified without incorporating phasor measurements. This dissertation firstly investigates a computationally efficient algorithm to identify all such topologies for a given system. Then a strategic phasor measurement placement is proposed to ensure detectability and identifiability of certain network parameter errors. In addition, this method is reformulated and extended to detect and identify isolated power islands after disturbances.

Another way to improve parameter error identification is to use multiple measurement scans instead of the normal single measurement scan. This dissertation investigates an alternative approach using multiple measurement scans. It addresses
limitations for parameter error in certain topologies without investing new measurements.

The second part of the dissertation concentrates on interoperability of PMUs in state estimation. Incorporating phasor measurements into existing Weighted Least Squares (WLS) state estimation brings up the interoperability issue about how to choose the right measurement weights for different types of PMUs. This part develops an auto tuning algorithm which requires no initial information about the phasor measurement accuracies. This algorithm is applied to tune the state estimator to update the weights of different types of PMUs in order to have a consistent numerically stable estimation solution. Furthermore, the impact of this tuning method on bad measurement detection is investigated.

All these methods have been tested in IEEE standard systems to show their performance.
To My Wife and Parents
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Table of Contents

1 Introduction ........................................................................................................................................... 1

1.1 Power systems secure operation and state estimation ................................................................. 1

1.2 State estimation functions .............................................................................................................. 4

1.3 Contribution of this dissertation ................................................................................................. 6

2 Identifying Single and Double Edge Cutsets in Power Networks ............................................... 9

2.1 Introduction ........................................................................................................................................ 9

2.2 Problem formulation ...................................................................................................................... 11

2.3 Review of existing method ........................................................................................................... 13

2.4 Proposed method ............................................................................................................................ 17

2.5 Simulation results ............................................................................................................................ 22

2.6 Conclusions .................................................................................................................................... 28

3 Strategic Placement of Phasor Measurements for Parameter Error Identification ......................................................... 30

3.1 Introduction ........................................................................................................................................ 30

3.2 Review of existing method ........................................................................................................... 33

3.3 Proposed method ............................................................................................................................ 35

3.4 The simulation results .................................................................................................................... 41

3.5 Conclusions .................................................................................................................................... 50

3.6 Post disturbance island identification via state estimation .......................................................... 51

3.6.1 Introduction ................................................................................................................................. 51

3.6.2 Proposed method ....................................................................................................................... 51

3.6.3 Simulation results ....................................................................................................................... 54

3.6.4 Conclusions ................................................................................................................................ 57

3.7 Conclusions of the chapter ............................................................................................................. 57
Identifying Parameter Error via Multiple Measurement Scans

4.1 Introduction

4.2 Limitations of the existing method

4.3 Proposed method

4.4 Additional observations

4.5 Simulation Results

4.6 Conclusions

Addressing Interoperability Issue of Phasor Measurements in State Estimation

5.1 Introduction

5.2 State estimator tuning for PMU measurements

5.2.1 Proposed Method

5.2.2 Simulation Results

5.2.3 Conclusions

5.3 An alternative way of determining the accuracy of a PMU measurement system

5.3.1 Proposed method

5.3.2 Simulation results

5.3.3 Conclusions

5.4 Impact of weights tuning in bad data detection in state estimation

5.4.1 Proposed method

5.4.2 Simulation results

5.4.3 Conclusions

5.5 Conclusions of the chapter
6 Conclusions and future work .................................................................96

6.1 Conclusions .........................................................................................96

6.2 Future work .........................................................................................97

References ...............................................................................................99

VITA ...........................................................................................................106

List of Publications ..................................................................................107
List of Figures

Fig. 1 On-line Static Security assessment: Functional Diagram ........................................4
Fig. 2 Example of a double edge cutset with multiple solutions .................................12
Fig. 3 Flowchart of the method to identify single and double edge cutsets ............16
Fig. 4 Flowchart of the algorithm to identify single and double edge cutsets ......18
Fig. 5 5-bus test system ................................................................................................21
Fig. 6 IEEE 14-bus system ..........................................................................................23
Fig. 7 IEEE 30-bus system .........................................................................................23
Fig. 8 IEEE 57-bus system .........................................................................................24
Fig. 9 Topology and placement of phasor measurements in Type 1 .....................36
Fig. 10 Topology and placement of phasor measurements in typical single edge
cutsets .........................................................................................................................37
Fig. 11 Topology and placement of phasor measurements in typical double edge
cutsets .........................................................................................................................37
Fig. 12 Topology and placement of phasor measurements in Type 2 ...................38
Fig. 13 Topology and placement of phasor measurements in critical k-tuples ....39
Fig. 14 Flowchart for strategic phasor measurement placement .........................40
Fig. 15 IEEE 14-bus system .........................................................................................42
Fig. 16 IEEE 30-bus system .........................................................................................43
Fig. 17 IEEE 57-bus system .........................................................................................43
Fig. 18 Flowchart of proposed method .....................................................................54
Fig. 19 IEEE 14-bus system .........................................................................................55
Fig. 20 IEEE 30-bus system .........................................................................................56
Fig. 21 Flowchart of the proposed method .................................................................67
Fig. 22 PMU measurements in power system .............................................................80
Fig. 23 Flow chart of the PMU tuning process ..................................................... 82
Fig. 24 Method through calibrating reference signal ....................................... 86
Fig. 25 Flowchart of robust bad data detection on PMU measurements .......... 91
List of Tables

TABLE 1 Single Edge and Double Edge Cutsets in 5-bus Test System........... 22
TABLE 2 A Tree of IEEE 14-bus System............................................................... 24
TABLE 3 Fundamental Cutsets of IEEE 14-bus System........................................ 24
TABLE 4 Combination Examples of Fundamental Cutsets of IEEE 14-bus System ......................................................................................................................... 25
TABLE 5 Combinations of Fundamental Cutsets of Large System .................. 25
TABLE 6 Simulation Results of IEEE 14-bus System ............................................... 26
TABLE 7 Single and Double Edge Cutsets in IEEE 14-bus System....................... 26
TABLE 8 Simulation Results of IEEE 30-bus System ............................................... 26
TABLE 9 Single and Double Edge Cutsets in IEEE 30-bus System....................... 27
TABLE 10 Simulation Results of IEEE 57-bus System ............................................... 27
TABLE 11 Single and Double Edge Cutsets in IEEE 57-bus System....................... 27
TABLE 12 Simulation Results of a Utility 4520-bus System ................................. 27
TABLE 13 Single and Double Edge Cutsets of IEEE 14-bus System in Different types........................................................................................................................ 44
TABLE 14 Single and Double Edge Cutsets of IEEE 30-bus System in Different types........................................................................................................................ 44
TABLE 15 Single and Double Edge Cutsets of IEEE 57-bus System in Different types........................................................................................................................ 44
TABLE 16 Results of Error Identification in IEEE 30-bus system ....................... 45
TABLE 17 Results of Error Identification in IEEE 14-bus system ............................ 46
TABLE 18 Results of Error Identification in IEEE 57-bus system ............................ 46
TABLE 19 Results of Error Identification in IEEE 30-bus system ....................... 47
TABLE 20 Test A Result of Multiple Error Identification in IEEE 14-bus system

TABLE 21 Test B Result of Multiple Error Identification in IEEE 14-bus system

TABLE 22 Strategic Placement of Phasor Measurements in IEEE test systems

TABLE 23 Islands identification in IEEE 14-bus system

TABLE 24 Islands identification in IEEE 30-bus system

TABLE 25 IEEE 14-bus system results of error identification

TABLE 26 IEEE 30-bus system results of error identification

TABLE 27 IEEE 118-bus system results of error identification

TABLE 28 Results of Multiple Error Identification in IEEE 14-bus system

TABLE 29 Results of Multiple Error Identification in IEEE 30-bus system

TABLE 30 IEEE 14-bus system results of error identification

TABLE 31 IEEE 14-bus system results of error identification

TABLE 32 IEEE 30-bus system results of error identification

TABLE 33 IEEE 30-bus system results of error identification

TABLE 34 measurement Configuration in IEEE 14-bus System

TABLE 35 Results of PMU Tuning Process in IEEE 14-bus System

TABLE 36 Normalized Errors of System States

TABLE 37 Results of PMU Accuracy Test (Errors shown in degree)

TABLE 38 Measurement Configuration in IEEE 14-bus System

TABLE 39 Results of PMU Tuning Process in IEEE 14-bus System

TABLE 40 Results of Bad Data Detection of Example 1

TABLE 41 Results of Bad Data Detection of Example 2
1 Introduction

Electric power systems are normally composed of generation, transmission and distribution systems and load. The generation part constitutes different power plants which supply the power to the entire systems. Transmission and distribution systems carry the power from generators and deliver it all the way to the power consumers which are called load in power systems. The transmission system is normally a meshed power grid with high voltage level. The high voltage is preferred due to different reasons such as reducing the power loss during power delivery. It contains a large number of substations which connects transmission lines, transformers and other systems device for system monitoring and control. The distribution system connects the transmission system and the load. It is typically configured in a radial manner with lower voltage level, where feeders root from the distribution substations and stretch their branches over the distribution area in a tree structure. Since the power energy cannot be stored on a large scale, the whole systems need to be monitored and controlled in a real time manner to facilitate their secure operation. This dissertation falls into this context which is to improve power system state estimation in order to enhance power system secure operation.

1.1 Power systems secure operation and state estimation

The operating condition of a power system in a time point can be determined if the network model and voltage phasors of all system buses are known, which are normally referred to the steady state of the power system. Traditionally, as referred in [1], the system may operate in one of three different states, which are namely normal, emergency and restorative states, with the change of operating conditions. A power system is in a normal state if all the loads in the system can be supplied power without
violating any operational constraints. A normal state is further said to be secure if the system can remain in a normal state following the occurrence of each contingency from a list of contingencies. Otherwise, the normal state is classified as unsecure where the system is normal but vulnerable with respect to some of the considered contingencies. Then some preventive actions must be taken to avoid its move into an emergency state. If the system condition already violates some constrains but can still supply power to all the loads in the system, the system is said to be in an emergency state. In this condition, immediate corrective action should be taken to bring the system back to a normal state. To bring the system to a normal state, corrective controls may be taken such as disconnecting various loads, lines, transformers or other equipment. Then, the load versus generation balance may have to be restored in order to start supplying power to all the loads. This operating state is called the restorative state, and the actions to be taken to make the system normal are restorative controls.

Therefore, the main goal of the system operator is to maintain the system in a normal and secure operation state. To achieve this goal, the first step is to provide real time monitoring of the system state. Since firstly introduced by Dr. Schweppe in [2]-[4], state estimation plays a significant role in power system secure operation. Every several minutes, it processes the measurements, which are obtained from different areas of the system by Supervisory Control and Data Acquisition (SCADA) systems, and provide the optimal estimate of the system state under this time point. Then the results are applied to other functions to provide further security analysis or preventive controls if needed. More details are shown in Fig. 1. Every several minutes, measurements from different areas of power systems, such as power injections and flows, are collected and sent to state estimator by SCADA. State estimator processes
these raw measurements and provides optimal system state with system models to other security functions. Based on its results, if the system is not normal, preventive controls will be applied to make the system move back to normal state. If the system is already in normal state, the contingency analysis will be conducted with certain load forecast results to further test whether the system is secure. If the system is normal but not secure, certain preventive control actions such as security constrained optimal power flow will be provided to control the system back to normal and secure state. All these functions together are called online steady state security assessment. This is operated by Energy Management Systems (EMS), which are computer software packages, in the control center in every several minutes. Therefore, state estimation is of great significance for power systems secure operation.
1.2 State estimation functions

Since first proposed by Fred Schweppes in 1970s [2]–[4], state estimator now is fully equipped in EMS in most of the control centers, which includes different functions to monitor system state, identify system vulnerabilities and provide data base to be used by other preventive controls. With the research and development of these decades, the state estimation can include several different functions [5]:

Topology processing: Provides the one-line diagram of the system using the gathered information of circuit breakers and switches.
Observability analysis: Determines whether the available measurements are enough to provide a unique system state solution. Identifies any existing unobservable branches and observable islands in the system.

State estimation solution: Provides the optimal estimate for the system state, which is the set of bus voltage phasors of the entire power system, based on the network model and the gathered measurements from the system. Based on these results, all the line flows, loads, transformer taps, and generator outputs are also estimated.

Bad data processing: Detects and identifies the gross errors in measurement set. Bad measurements will be eliminated or corrected if there is enough redundancy in the measurement configuration.

Parameter error processing: Detects and identifies network parameter errors, such as transmission line model parameters, tap changing transformer parameters, shunt capacitor or reactor parameters provided that there is enough measurement redundancy.

All these functions together facilitate the secure operation of the modern power systems. Recently, a new type of measurement namely synchronized phasors which are provided by phasor measurement units (PMUs) has been introduced by Dr. Arun G. Phadke [6]. These phasor measurements can provide not only the magnitude but also time synchronized phase angle information of the measured voltages and currents at buses. Note that direct measurement of phase angles of wide-area voltage and current phasors was not possible by SCADA measurements. With the installation of PMUs in power systems, these synchronized phasor measurements can be effectively applied by state estimators to improve estimation results. This dissertation is focused on new ways of using phasor measurements to enhance state estimation.
1.3 Contribution of this dissertation

This dissertation’s main objective is to improve state estimation via application of phasor measurements. Specifically, Chapter 2 to 4 are focused on improving network parameter error identification through innovative use of phasor measurements. Chapter 5 concentrates on addressing interoperability issues of phasor measurements in state estimation.

Previously, state estimators assumed perfect knowledge of network model parameters, which are all stored in the database and maintained by the state estimator. However, it is common knowledge that network parameters may have errors due to different reasons and may cause biased estimation results. Chapter 2 to 4 focus on how to detect and identify those parameter errors with the help of phasor measurements.

Chapter 2 presents a numerical algorithm to identify all single and double cutsets in power networks. The existing work on parameter error identification uncovers some limitations that, when it comes to single and double edge cutsets, the errors cannot be identified unless there are phasor measurements installed in appropriate places. In this chapter, an efficient algorithm is introduced to identify all those topologies from large scale power networks based on factorization of network incidence matrix. The method is simple to implement and computationally less demanding than other existing algorithms. Numerical results are provided to illustrate the utilization of the method as a pre-cursor to strategic phasor measurement placement to improve parameter error identification, which will be presented in Chapter 3.

In Chapter 3, a strategic placement of phasor measurements is proposed to improve network parameter error identification. It relies on the results obtained by the
algorithm presented in Chapter 2. The proposed strategy ensures identification of all those network parameter errors with limited number of phasor measurements. The method can be applied to the entire system or to a specified zone only where parameter errors are of particular importance. In addition, this method is reformulated and extended to identify power islands after disturbances with the help of phasor measurements.

Chapter 4 introduces an alternative method of parameter error identification via multiple measurement scans. The existing single scan method has some limitations that it fails to identify errors in certain types of parameters. The proposed method can make those errors identifiable by simply applying multiple measurement scans, which does not require any investments in new meters. In addition, a detailed analysis of the inherit limitations with the single scan method is provided. It also reveals the relationship between the number of measurement scans, the value of normalized Lagrange multipliers and normalized measurement residuals. This allows proper choice of the number of scans to be used. The method can be customized to individual systems and verifying the identification results.

Chapter 5 of this dissertation focuses on addressing interoperability issue of phasor measurements in state estimation. PMUs from different vendors may have different accuracies. Incorporating those PMUs into existing Weighted Least Squares (WLS) state estimation brings up the interoperability issue of PMUs. The challenge is how to choose the appropriate set of measurement weights for different PMUs to properly reflect their accuracies, since such measurements have never been previously used in state estimators. These pre-assigned weights directly impact successful detection and identification of bad PMU measurements. In this chapter, an auto tuning algorithm, which requires no initial information about the PMU accuracies, has been developed.
It tunes the state estimator to update the weights of different types of PMUs in order to have a statistically consistent and numerically stable estimation solution. Furthermore, the impact of this tuning method on PMU bad measurement detection has been investigated.

All the above outlined investigations lead to development of various methods to improve state estimation by innovative use of phasor measurements.
2 Identifying Single and Double Edge Cutsets in Power Networks

2.1 Introduction

EMS applications rely on the real-time network model which is maintained by the state estimator. State estimation is an important application of EMS. It processes measurements obtained from different areas of power systems and provides steady-state operating state for other advanced EMS application programs. Once receiving field measurement data, network parameter, network topology, and other information, state estimation filters incorrect data to ensure that the estimated state is correct. Successful execution of state estimation is dependent on the validity of the measurements, the topology data and the assumed network parameters. While the state estimator typically assumes perfect knowledge about the network parameters, it implicitly assumes that all parameters such as the reactance and resistances of transmission lines and transformers, the shunt capacitors or reactors, etc. are perfectly known and stored in the database located in the control center. However, this assumption may not be true.

The reasons for having erroneous parameters may be quite different. Transmission taps may be changed by operators without notifying the control center, transmission line parameters (resistances, inductances) may change due to aging or ambient temperature variations. These unreported changes in the parameters or human data handling errors will easily corrupt the database. Their detection and identification remains a challenging problem especially for large scale systems containing very large number of network elements. Furthermore, since state estimation is formulated assuming perfect knowledge of the network model and parameters, inconsistencies detected during state estimation will be blamed on measurement errors. This may lead to misidentification and elimination of good measurements as bad data. Identification
of parameter errors has therefore been an important and challenging problem which has attracted much attention in the literature [7]-[22]. State estimation can be a useful and powerful tool in detecting and identifying parameter errors. Using state estimator to avoid this and ensure an error-free database constitute the main motivation of this part of the research.

Traditionally, identification of parameter errors are based on residual sensitivity analysis [7]-[11] and state vector augmentation [12]-[20]. Recently, an alternative method proposed in [21] makes use of normalized Lagrange multipliers associated with network parameters that are modeled as simple constraints. The main advantage of this approach is that network parameter errors and analog measurement errors can be simultaneously handled without the need to select any pre-suspected set of parameters. However, it is shown that parameter error identification may have limitations under certain network topologies. These topologies are characterized by single and double edge cutsets. The parameter error on those topologies cannot be detected or identified without phasor measurements. Reference [22] shows some examples where this problem can be solved by introducing phasor measurements at appropriate locations. However, a general solution for this problem was not developed or presented there.

The research in this chapter is an extension of that work. This chapter presents a computational efficient method to identify all single and double edge cutsets of a given power network, where the phasor measurements are needed for parameter error identification. Relying on the capability of this method, next chapter will describe a general phasor measurement placement strategy to ensure error identification for the above network parameters using the limited number of phasor measurements.
2.2 Problem formulation

As mentioned, reference [21] proposed a parameter error identification algorithm by calculating the normalized Lagrange multipliers. It also showed that, using only conventional measurements, parameter errors on single edge cutsets could not be detected and errors in double edge cutsets could not be identified. The reasons for these limitations can be traced to the existence of multiple solutions for such topologies.

In a connected network, a single edge cutset is a branch without which (except its endpoints) the network will become disconnected. A double edge cutset is defined as a pair of branches without which (except their endpoints) the network will not be connected and adding either one of them will make the network connected again.

For a single edge cutset, removal of the branch will split the system into two disconnected islands. Therefore, without having synchronized measurements which will provide a redundant link between these potential islands, tie-line parameter error cannot be detected using only conventional measurements.

For a double edge cutset, assume that \( p_1 \) and \( p_2 \) are a set of values for two parameters in the system, while \( p_1' \) and \( p_2' \) are another set of values which are erroneous. If the same value of objective function can be obtained with two different solutions \( x_1 \) and \( x_2 \) as

\[
J(x_1, p_1', p_2) = J(x_2, p_1, p_2')
\]  \hspace{1cm} (2.1)

Weighted Least Squares (WLS) state estimation will converge to either one of the above solutions; therefore either set of parameter values can be identified as the valid set.
Consider a simple example shown as Fig. 2. If there are only conventional power flow and injection measurements $P_{1-2}$, $P_{1-3}$ and $P_i$, the relationship between the power flow and line parameters using dc model is that

\[ P_{1-2} = \frac{\theta_{1-2}}{x_{1-2}} \]  \hspace{1cm} (2.2)  
\[ P_{1-3} = \frac{\theta_{1-3}}{x_{1-3}} \]  \hspace{1cm} (2.3)

where $\theta$ is the voltage angle, and $x$ is the line reactance. Since voltage angles cannot be measured by conventional measurements, another set of parameter values $x_{1-2}'$ and $x_{1-3}'$ can yield the same solution of the power flow as

\[ P_{1-2} = \frac{\theta_{1-2}}{x_{1-2}} = \frac{\theta_{1-2}'}{x_{1-2}'} \]  \hspace{1cm} (2.4)  
\[ P_{1-3} = \frac{\theta_{1-3}}{x_{1-3}} = \frac{\theta_{1-3}'}{x_{1-3}'} \]  \hspace{1cm} (2.5)

where $\theta_2$ and $\theta_3$ in the observable island are still the same. Therefore, both parameter pairs $x_{1-2}'$ and $x_{1-3}'$ as well as $x_{1-2}$ and $x_{1-3}$ will satisfy all the measurements. Thus, parameter errors on double edge cutsets cannot be identified. Note that in (2.4) and (2.5), the only other variable which changes its value is the voltage phase angle at bus 1. Thus, this problem can be solved by adding a phasor measurement at the appropriate bus, in this case bus 1. More details can be found in [21]. Then the limitation of the measurement design noted in [21] is addressed by incorporating phasor measurements for some example cases in [22]. However, a systemic method to identify all these appropriate locations as well as provide a strategy of phasor
measurement placement based on parameter error identification is needed. Therefore, in this section, a computationally efficient algorithm to identify all single and double edge cutsets, where the phasor measurements are needed, is proposed.

2.3 Review of existing method

Review of literature on graph theory yields existing research on several related structures for graph connectivity. Some structures in the graph theory are defined based on the node connectivity. For example, a clique and quasi-clique are the structures where every node is connected to all other nodes or certain percentage of all other nodes [23]. In a k-core, each node is connected to at least k other nodes [24]. More related structures are the ones defined based on the branch connectivity. Reference [25] and [26] are focused on finding the minimum cut of a given graph, which provides the cutsets with the least cost for a weighted graph. Document [27] proposes a method to identify maximal k-edge-connected subgraphs from a large graph, whose definition is a connected graph that cannot be disconnected by removing less than k edges. Since the aim of this research is to find all the single and double edge cutsets in a graph, these methods cannot be directly used. There are also methods to identify all cutsets of a given graph [28]-[33]. Then, the single and double edge cutsets, which are cutsets with only one and two edges, can be obtained from the results. A basic method to identify all cutsets relies on defining fundamental cutsets from corresponding trees as a basis of the cutset space to enumerate all the cutsets. This method starts by searching a tree of the network using algorithms such as depth or breadth first search or transforming network matrices into Echelon form [28], [29]. Fundamental cutsets can then be obtained and defined as a basis of the whole cutset space. The whole set of cutsets can be generated by this basis through mod 2 addition.
Finally, among the results, the cutsets with only one and two branches can be obtained. Although these methods are technically sound, their computational load in searching a tree and enumerating all cutsets may become quite significant for large size systems.

The details of this method are documented is reviewed as below.

Given a directed network $G$ with $n$ buses and $b$ branches, a bus to branch incidence matrix of $G$ can be defined as

$$A_a = \begin{bmatrix} a_{ij} \end{bmatrix}$$  \hspace{1cm} (2.6)

where

$$a_{ij} = 1 \quad \text{if bus } i \text{ is one endpoint of branch } j, \text{ and the direction is pointing away from bus } i;$$

$$a_{ij} = -1 \quad \text{if bus } i \text{ is one endpoint of branch } j, \text{ and the direction is pointing toward bus } i;$$

$$a_{ij} = 0 \quad \text{if bus } i \text{ is not an endpoint of branch } j.$$

By deleting one row, a reduced bus to branch incidence matrix $A$ can be derived. Similarly, the reduced branch to bus incidence matrix $A^T$ can be obtained by transposing $A$. In a connected network, all the rows of a reduced bus to branch incidence matrix are linearly independent.

A tree of a connected network $G$ is a sub-network of $G$ which 1) covers all the buses of $G$; 2) is a connected sub-network without any loops. If a tree has been searched, the matrix $A$ can be partitioned into:

$$A = \begin{bmatrix} A_T & A_L \end{bmatrix}$$  \hspace{1cm} (2.7)

in which each column of $A_T$ represents a branch of the corresponding tree and each column of $A_L$ corresponds to a link. Matrix $A_T$ is nonsingular.
After obtaining a tree, a set of fundamental cutsets relating to the tree are defined as $n-1$ cutsets, each of which includes a different tree branch respectively. A cutset matrix representing $n-1$ fundamental cutsets is in the form of

$$D = [d_{ij}] = [1_{n-1} \quad D_L]$$

(2.8)

where

$$d_{ij} = 1 \quad \text{if branch } j \text{ is included in cutset } i \text{ with the same direction;}$$

$$d_{ij} = -1 \quad \text{if branch } j \text{ is included in cutset } i \text{ with the opposite direction;}$$

$$d_{ij} = 0 \quad \text{if branch } j \text{ is not included in cutset } i.$$

Hence, in matrix $D$, each row represents a cutset, each column of $1_{n-1}$ corresponds to a tree branch and each column of $D_L$ corresponds to a link.

The realization of this fundamental matrix is based on the relationship:

$$D = [1_{n-1} \quad D_L] = A^{-1}_T[A_T \quad A_L] = A^{-1}_T A$$

(2.9)

Based on these definitions, one possible method to identify all single and double edge cutsets by enumerating all the cutsets is shown in Fig. 3.
**Fig. 3** Flowchart of the method to identify single and double edge cutsets

*Step1:* Build a reduced bus to branch incidence matrix $A$ of a given network.

*Step2:* Search a tree of this network, and partition $A$ as in (2.7). Different algorithms to search a tree of a graph can be found in [28], [29].

*Step3:* Realize fundamental cutset matrix $D$ from (2.9), where the fundamental cutsets correspond to the tree defined in *Step2*. A single edge cutset, if it exists, will appear in the fundamental cutsets because it belongs to any tree of the network. As a result, the row containing only one nonzero element represents a single edge cutset. The branch corresponding to this nonzero element can be identified as a single edge cutset.

*Step4:* Define the fundamental cutsets found in *Step3* as a basis of the whole cutset space. Then all the cutsets, as well as edge disjoint unions of cutsets and empty set, can be enumerated by the basis using mod 2 addition. Among the results, the cutsets with exactly two branches will be the double edge cutsets. The reason to enumerate all cutsets is that a double edge cutset can be formed by two tree branches. They
cannot be identified by only one set of fundamental cutsets, because each fundamental cutset contains only one tree branch.

The process is shown in Fig. 3. It can be noted that, in this algorithm, the tree searching and enumeration of all the cutsets of a network are computationally demanding. This is true especially in the case of power systems which have large number of buses and also a large number of cutsets. In addition, it is neither necessary nor efficient to search trees and enumerate all the cutsets, because only the single and double edge cutsets are needed for our investigation. Hence, this section proposes a simpler and computationally less demanding alternative method which will be presented next.

2.4 Proposed method

In this section, a numerical method that enumerates all the single and double edge cutsets of a system based on factorization of the reduced incidence matrix will be presented. A flowchart describing the developed method, which avoids searching trees and generating all cutsets, is given in Fig. 4. The detailed steps are explained below.
Given a power network $G$ with $n$ buses and $b$ branches:

**Step1:** Compose the reduced branch to bus incidence matrix $A^T$ (transpose of the reduced bus to branch incidence matrix $A$).

**Step2:** Select a branch and remove the row corresponding to it. This yields a modified version of $A^T$ matrix, i.e. $A^T_{b-1}$. It is noted that, in this step, only one row is removed at once. The row number of the matrix $A^T_{b-1}$ is always $b-1$. 

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Fig. 4 Flowchart of the algorithm to identify single and double edge cutsets

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18
**Step3**: Use LU factorization to decompose $A_{b-1}^T$ into its lower-trapezoidal and upper-triangular factors, which is also used in Peter-Wilkinson algorithm for factorization in order to solve the weighted least squares problem [36]:

$$
\bar{A}_{b-1}^T = PA_{b-1}^T = LU
$$

(2.10)

where

$A_{b-1}^T$ derived from $A_{b-1}^T$ after row permutation;

$P$ permutation matrix in which nonzero values regard to the rows permutation;

$L$ unit lower-trapezoidal matrix;

$U$ upper-triangular matrix;

Steps of factorization process are presented below:

After removal of a branch, the reduced incidence matrix $A_{b-1}^T$ will have $b-L$ rows (one less branch) and $n-L$ columns (one reference bus removed). Before the $s$th factorization step, matrix $A_s^T$ will have the following form:

$$
A_s^T = \begin{bmatrix}
    u_{11} & u_{12} & \cdots & u_{1(s-1)} & u_{1s} & \cdots & u_{1(n-1)} \\
    l_{21} & u_{22} & \cdots & u_{2(s-1)} & u_{2s} & \cdots & u_{2(n-1)} \\
    \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
    l_{s1} & l_{s2} & \cdots & l_{s(s-1)} & a_{ss} & \cdots & a_{s(n-1)} \\
    \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
    l_{(b-1)1} & l_{(b-1)2} & \cdots & l_{(b-1)(s-1)} & a_{(b-1)s} & \cdots & a_{(b-1)(n-1)}
\end{bmatrix}
$$

(2.11)

where, $u_{ij}$ and $l_{ij}$ represent the elements of $U$ and $L$ respectively. In fact, each $a_{ij}$ ought to have a superscript $s$, to indicate the associated factorization step, but this is omitted to minimize notational clutter.

At the $s$th factorization step:

(i) Row pivoting: If $a_{ss}$ is zero, check all $a_{is}$ ($i$ from $s$ to $b-L$) in the $s$th column and find the first nonzero element $a_{ks}$ and exchange the $k$th and $s$th rows.
If no nonzero elements can be found, the branch being checked, which corresponds to the row removed in Step 2, is identified as a single edge cutset. Then directly go back to Step 2 and check the next branch.

(ii) For \( i > s \), replace \( a_{is} \) by \( l_{is} = a_{is} / a_{ss} \).

(iii) Calculate a new \( a_{ij} \) equal to \( a_{ij} - l_{is}a_{sj} \) for each \( j \) from \( s+1 \) to \( n-1 \), and store it in position \( (i, j) \).

After the above steps, the upper triangular and lower trapezoidal factors are obtained as:

\[
U = \begin{bmatrix}
  u_{11} & u_{12} & u_{13} & \cdots & u_{1(n-1)} \\
  0 & u_{22} & u_{23} & \cdots & u_{2(n-1)} \\
  0 & 0 & u_{33} & \cdots & u_{3(n-1)} \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & 0 & \cdots & u_{(n-1)(n-1)} 
\end{bmatrix}
\]

\[
(2.12)
\]

\[
L = \begin{bmatrix}
  1 & 0 & 0 & \cdots & 0 \\
  l_{21} & 1 & 0 & \cdots & 0 \\
  l_{31} & l_{32} & 1 & \cdots & 0 \\
  l_{41} & l_{42} & l_{43} & \cdots & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  l_{(b-1)1} & l_{(b-1)2} & l_{(b-1)3} & \cdots & l_{(b-1)(n-1)} 
\end{bmatrix}
\]

\[
(2.13)
\]

**Step 4:** Identify double edge cutsets from matrix \( L \). If a column of \( L \) has only one nonzero element whose diagonal entry is unity, the branch corresponding to the diagonal element, together with the branch being checked, are identified as a double edge cutset. In addition, if there are several columns satisfying this requirement, they all, including the removed branch, will be identified as a network critical k-tuple. In this k-tuple, each of the branch pairs will constitute a double edge cutset. Any single edge cutset identified at this step should be ignored since it already constitutes a cutset itself.

**Step 5:** Check every branch of the network. Repeat **Step 2** to **Step 4** for each branch at once and identify all single and double edge cutsets separately. The number of rows
of \( A^T_{b-1} \) in Step2 is always \( b-1 \). Then, the entire set of single and double edge cutsets can be enumerated.

A tutorial example is presented here to illustrate the details of the algorithm. The 5-bus system is shown in Fig. 5.

![5-bus test system](image)

As Step1, the reduced branch to bus incidence matrix \( A^T \) will be built, where the rows and columns correspond to branches and buses respectively. Note that the column corresponding to bus 5 is removed to obtain \( A^T \):

\[
A^T = \begin{bmatrix}
B_{1-2} & 1 & -1 & 0 & 0 \\
B_{2-3} & 0 & 1 & -1 & 0 \\
B_{2-5} & 0 & 1 & 0 & 0 \\
B_{3-4} & 0 & 0 & 1 & -1 \\
B_{3-5} & 0 & 0 & 1 & 0 \\
B_{4-5} & 0 & 0 & 0 & 1
\end{bmatrix}
\]  
\tag{2.14}

Removing the first row corresponding to branch 1-2 and applying factorization, the following matrix \( A^T_s \), will be obtained before the 1st step:

\[
A^T_s = \begin{bmatrix}
B_{2-3} & 0 & 1 & -1 & 0 \\
B_{2-5} & 0 & 1 & 0 & 0 \\
B_{3-4} & 0 & 0 & 1 & -1 \\
B_{3-5} & 0 & 0 & 1 & 0 \\
B_{4-5} & 0 & 0 & 0 & 1
\end{bmatrix}
\]  
\tag{2.15}

It is noted that, during the factorization, no nonzero elements can be selected in the first column. Therefore the factorization will be terminated and branch 1-2 will be identified as a single edge cutset.
Next, remove the second row corresponding to branch 2-3 and the following $L$ matrix will be obtained after factorization:

$$
\begin{pmatrix}
B_{1-2} & 1 & 0 & 0 & 0 \\
B_{2-5} & 0 & 1 & 0 & 0 \\
L = B_{3-4} & 0 & 0 & 1 & 0 \\
B_{3-5} & 0 & 0 & 1 & 1 \\
B_{4-5} & 0 & 0 & 0 & 1
\end{pmatrix}
$$

As shown in (2.16) both the first and second columns in matrix $L$ contain only zero entries except the unity diagonal entries. Branches corresponding to these two unity entries are branches 1-2 and 2-5. Since branch 1-2 is a single edge cutset, it will be ignored. Therefore branch 2-5 is identified as a double edge cutset together with branch 2-3.

Then after checking all the branches in Step5, the full set of single and double edge cutsets will be determined as below:

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>Single Edge and Double Edge Cutsets in 5-bus Test System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Edge Cutsets</td>
<td>$B_{1-2}$</td>
</tr>
<tr>
<td>Double Edge Cutsets</td>
<td>$B_{2-3}, B_{2-5}$</td>
</tr>
<tr>
<td></td>
<td>$B_{3-4}, B_{4-5}$</td>
</tr>
</tbody>
</table>

2.5 Simulation results

The proposed method is implemented and tested on the IEEE 14, 30 and 57-bus test systems shown in Fig. 6, 7 and 8 respectively. In addition, it’s also simulated on a 4520-bus (5495-branch) transmission system and a 6301-bus (20777-branch) internet network (Gnutella P2P network) from Stanford large network dataset collection (http://snap.stanford.edu/data/). The complete set of single and double edge cutsets for all systems are determined using both the proposed and existing methods. CPU times required by the proposed method for each system are provided. The complexity of the existing algorithm reviewed in Section 2.3 is also analyzed for comparison.
Fig. 6 IEEE 14-bus system

Fig. 7 IEEE 30-bus system
IEEE 14-bus system shown in Fig. 6 is analyzed using the existing algorithm reviewed in Section 2.3. It involves two main steps. First, a tree whose branches are listed in Table 2 is identified by reduced echelon form of reduced incidence matrix $A$. Then, all fundamental cutsets are obtained using (2.9) and listed in Table 3.

**TABLE 2 A Tree of IEEE 14-bus System**

| Tree Branches | $B_{1-2}$, $B_{1-5}$, $B_{2-4}$, $B_{2-3}$, $B_{4-9}$, $B_{4-7}$, $B_{7-9}$, $B_{9-10}$, $B_{9-14}$, $B_{10-11}$, $B_{6-12}$, $B_{6-13}$ |

**TABLE 3 Fundamental Cutsets of IEEE 14-bus System**

| $S_1$ | $B_{1-2}$, $B_{2-5}$, $B_{4-5}$, $B_{6-11}$, $B_{13-14}$ |
| $S_2$ | $B_{1-5}$, $B_{2-5}$, $B_{4-5}$, $B_{6-11}$, $B_{13-14}$ |
| $S_3$ | $B_{2-4}$, $B_{4-4}$, $B_{4-5}$, $B_{6-11}$, $B_{13-14}$ |
| $S_4$ | $B_{3-3}$, $B_{4-4}$ |
| $S_5$ | $B_{5-5}$, $B_{6-11}$, $B_{13-14}$ |
| $S_6$ | $B_{4-9}$, $B_{7-9}$, $B_{6-11}$, $B_{13-14}$ |
| $S_7$ | $B_{6-7}$, $B_{7-9}$ |
| $S_8$ | $B_{7-8}$ |
| $S_9$ | $B_{9-10}$, $B_{6-11}$ |
| $S_{10}$ | $B_{9-14}$, $B_{13-14}$ |
| $S_{11}$ | $B_{10-11}$, $B_{6-11}$ |
| $S_{12}$ | $B_{6-12}$, $B_{12-13}$ |
| $S_{13}$ | $B_{6-13}$, $B_{12-13}$, $B_{13-14}$ |
Table 3 shows that $S_8$ has only one branch and therefore is identified as a single edge cutset. All other cutsets can be enumerated by mod 2 addition of all combinations of these fundamental cutsets, which are listed in Table 4. Note that these combinations also include edge disjoint unions of cutsets and empty sets [28]. Since, for an $n$-bus system, the number of fundamental cutsets is $n-1$, all combinations including fundamental cutsets themselves, can be calculated by:

$$C^1_{n-1} + C^2_{n-1} + \ldots + C^{n-1}_{n-1} = 2^{n-1} - 1$$  \hspace{1cm} (2.17)

<table>
<thead>
<tr>
<th>Systems</th>
<th>Num. of Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 30-bus System</td>
<td>536870911</td>
</tr>
<tr>
<td>IEEE 57-bus System</td>
<td>7.2058×10^{16}</td>
</tr>
<tr>
<td>Utility 4520-bus System</td>
<td>Near Infinity</td>
</tr>
<tr>
<td>Internet 6301-bus System</td>
<td>Near Infinity</td>
</tr>
</tbody>
</table>

As shown in Table 4, other cutsets are obtained by combinations of fundamental cutsets. The total number of combinations for the IEEE 14-bus system is 8191. After enumerating all these 8191 combinations, the cutsets which have only two branches can be identified as double edge cutsets. The complexity of searching a tree and enumerating combinations of fundamental cutsets is evident even for this simple example and computational burden is expected to rapidly become prohibitively large even for medium size systems. Numbers of combinations to be enumerated for the
systems considered in this work are shown in Table 5 in order to illustrate this growth in computational burden.

b) Results obtained using the proposed method

Proposed algorithm is implemented in MATLAB [37] and applied to the above mentioned systems. Sparsity of the network matrices are exploited as much as possible using MATLAB’s Sparse Matrix library. The main hardware configuration of the desktop used for computations had a CPU with frequency of 1.8GHz and a 3 GB of RAM. The results and CPU times obtained for all the IEEE test systems are presented in Tables 6, 8 and 10. All single and double edge cutsets are listed in Tables 7, 9 and 11. Results obtained for the 4520-bus utility system and 6301-bus Internet system are presented in Table 12. Note that detailed lists for single and double edge cutsets cannot be listed due to their size, their numbers are given instead.

<table>
<thead>
<tr>
<th>TABLE 6 Simulation Results of IEEE 14-bus System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Single Edge Cutsets</td>
</tr>
<tr>
<td>Number of Double Edge Cutsets</td>
</tr>
<tr>
<td>Number of Critical k-tuples</td>
</tr>
<tr>
<td>CPU time</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 7 Single and Double Edge Cutsets in IEEE 14-bus System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Edge Cutsets</td>
</tr>
<tr>
<td>Double Edge Cutsets</td>
</tr>
<tr>
<td>Critical k-tuples</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 8 Simulation Results of IEEE 30-bus System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Single Edge Cutsets</td>
</tr>
<tr>
<td>Number of Double Edge Cutsets</td>
</tr>
<tr>
<td>Number of Critical k-tuples</td>
</tr>
<tr>
<td>CPU time</td>
</tr>
</tbody>
</table>
### TABLE 9 Single and Double Edge Cutsets in IEEE 30-bus System

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Single Edge Cutsets</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$B_{9,11}$</td>
</tr>
<tr>
<td></td>
<td>$B_{12,13}$</td>
</tr>
<tr>
<td></td>
<td>$B_{25,26}$</td>
</tr>
<tr>
<td><strong>Double Edge Cutsets</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$B_{6,8}$, $B_{8,28}$</td>
</tr>
<tr>
<td></td>
<td>$B_{6,8}$, $B_{9,10}$</td>
</tr>
<tr>
<td></td>
<td>$B_{12,14}$, $B_{14,15}$</td>
</tr>
<tr>
<td></td>
<td>$B_{10,21}$, $B_{21,22}$</td>
</tr>
<tr>
<td></td>
<td>$B_{15,23}$, $B_{23,24}$</td>
</tr>
<tr>
<td><strong>Critical k-tuples</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$B_{1,2}$, $B_{1,3}$, $B_{3,4}$</td>
</tr>
<tr>
<td></td>
<td>$B_{2,5}$, $B_{5,7}$</td>
</tr>
<tr>
<td></td>
<td>$B_{12,16}$, $B_{16,17}$, $B_{10,17}$</td>
</tr>
<tr>
<td></td>
<td>$B_{15,18}$, $B_{18,19}$, $B_{19,20}$, $B_{10,20}$</td>
</tr>
<tr>
<td></td>
<td>$B_{27,29}$, $B_{27,30}$, $B_{29,30}$</td>
</tr>
<tr>
<td></td>
<td>$B_{27,28}$, $B_{25,27}$, $B_{24,25}$</td>
</tr>
</tbody>
</table>

### TABLE 10 Simulation Results of IEEE 57-bus System

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Single Edge Cutsets</strong></td>
<td>1</td>
</tr>
<tr>
<td><strong>Number of Double Edge Cutsets</strong></td>
<td>8</td>
</tr>
<tr>
<td><strong>Number of Critical k-tuples</strong></td>
<td>8</td>
</tr>
<tr>
<td><strong>CPU time</strong></td>
<td>0.06 sec</td>
</tr>
</tbody>
</table>

### TABLE 11 Single and Double Edge Cutsets in IEEE 57-bus System

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Single Edge Cutsets</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$B_{32,33}$</td>
</tr>
<tr>
<td></td>
<td>$B_{1,2}$, $B_{2,3}$</td>
</tr>
<tr>
<td></td>
<td>$B_{4,5}$, $B_{5,6}$</td>
</tr>
<tr>
<td></td>
<td>$B_{1,16}$, $B_{12,16}$</td>
</tr>
<tr>
<td></td>
<td>$B_{1,17}$, $B_{12,17}$</td>
</tr>
<tr>
<td></td>
<td>$B_{22,23}$, $B_{23,24}$</td>
</tr>
<tr>
<td></td>
<td>$B_{36,40}$, $B_{40,46}$</td>
</tr>
<tr>
<td></td>
<td>$B_{41,42}$, $B_{46,42}$</td>
</tr>
<tr>
<td></td>
<td>$B_{31,45}$, $B_{45,43}$</td>
</tr>
<tr>
<td><strong>Double Edge Cutsets</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$B_{18,19}$, $B_{19,20}$, $B_{21,20}$, $B_{21,22}$, $B_{4,18}$</td>
</tr>
<tr>
<td></td>
<td>$B_{24,26}$, $B_{26,27}$, $B_{27,28}$, $B_{28,29}$</td>
</tr>
<tr>
<td></td>
<td>$B_{24,25}$, $B_{25,30}$, $B_{30,31}$, $B_{31,32}$, $B_{32,35}$, $B_{35,36}$</td>
</tr>
<tr>
<td></td>
<td>$B_{37,39}$, $B_{39,57}$, $B_{57,56}$</td>
</tr>
<tr>
<td></td>
<td>$B_{48,44}$, $B_{44,45}$, $B_{45,55}$</td>
</tr>
<tr>
<td></td>
<td>$B_{14,46}$, $B_{46,47}$, $B_{47,48}$, $B_{48,49}$</td>
</tr>
<tr>
<td></td>
<td>$B_{20,52}$, $B_{32,53}$, $B_{53,54}$, $B_{54,55}$, $B_{85}$</td>
</tr>
<tr>
<td></td>
<td>$B_{49,50}$, $B_{50,51}$, $B_{10,51}$</td>
</tr>
<tr>
<td><strong>Critical k-tuples</strong></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 12 Simulation Results of Large Systems

<table>
<thead>
<tr>
<th></th>
<th>Utility 4520-bus System</th>
<th>Internet 6301-bus System</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Single Edge Cutsets</strong></td>
<td>1493</td>
<td>1766</td>
</tr>
<tr>
<td><strong>Number of Double Edge Cutsets</strong></td>
<td>384</td>
<td>719</td>
</tr>
<tr>
<td><strong>Number of Critical k-tuples</strong></td>
<td>568</td>
<td>16</td>
</tr>
<tr>
<td><strong>CPU time</strong></td>
<td>83 sec</td>
<td>730 sec</td>
</tr>
</tbody>
</table>
Results provided in these tables show that all single and double edge cutsets can be identified by using the proposed method. As an off-line calculation program, the computation times even for a large scale power system are reasonable and computational burden is significantly less compared to existing graph based methods.

As stated, one of the motivations for this work was to facilitate strategic placement of phasor measurements to ensure identification of the parameter errors in a given system. Examples were presented in [22], where certain network topologies containing single or double edge cutsets, would have parameter errors that could only be identified by phasor measurements. Hence, the proposed method can be applied first to identify all such topologies in a given network, and then proper placement strategies will be developed to address parameter error identification problem. This step will be presented in the following chapter. Moreover, it is also anticipated that this cutest identification method may have other applications in power system planning, for instance in transmission expansion studies [38], where constraints can be placed to avoid such topologies being created.

2.6 Conclusions

Previous work on network parameter error detection and identification has shown that parameter errors on single and double edge cutsets cannot be detected or identified without phasor measurements. This chapter proposes a numerical method which identifies all the single and double edge cutsets for a given network, in order to address these vulnerabilities. The method relies on the factorization of reduced incidence matrix, which is simple to implement and computationally less demanding than other existing algorithms. Numerical results are provided to illustrate the utilization of the method as a pre-cursor to strategic phasor measurement placement to
improve parameter error identification, which will be presented in the following chapter.
3 Strategic Placement of Phasor Measurements for Parameter Error Identification

3.1 Introduction

Network parameter errors are known to exist in power system data bases and their detection and identification remains a challenging problem especially for large scale systems containing very large number of network elements. As mentioned before, state estimation can be a useful and powerful tool in detecting and identifying parameter errors and significant effort has been used to address this problem by various investigators in the past [7]-[22]. Network parameters such as transmission line resistances, reactances, line charging capacitances, transformer taps and susceptances of shunt capacitors/reactors are all stored in the network parameter database and are used by several network applications in building the network model. Unfortunately, transmission line resistances may change due to aging or ambient temperature variations. Transformer taps may be moved but inadvertently not recorded in the database. The database can be easily corrupted by these unreported changes in the parameters or human data handling errors. Avoiding this and ensuring an error-free database constitute the main motivation of this research.

Traditionally, identification of parameter errors are based on residual sensitivity analysis [7]-[11] and state vector augmentation [12]-[20]. The methods using residual sensitivity analysis are performed following a converged state estimation solution and require the user to identify a set of suspected parameters. In the methods using state vector augmentation, the suspected parameters are included in the state vector and both the state and parameters are simultaneously estimated. Two different related techniques have been proposed to deal with the augmented model: One is solution using normal equations. Except for some observability and numerical issues this
method is a straightforward extension of conventional SE model. The other is Solution based on Kalman filter theory. Under this approach several measurement samples are processed in sequence in order to recursively improve existing parameter values.

However, those traditional methods have some common limitations. The first limitation is a suspect parameter set is required to be selected before the parameter error identification. Considering the dimension of the power system, it is not mathematically practical to include all the network parameters in the system in the estimation matrix. Sometime the suspicious parameters are arbitrarily selected by experienced network operators. This suspect parameter set could also be generated by the algorithm based on measurement residuals. However, such algorithm may be biased because of the measurement error.

The second limitation is that the bad data in measurements have to be removed from the network before performing the parameter error identification or estimation. For the methods based on residual sensitivity analysis, the measurement error will bias the parameter error identification results. For the methods with augmenting state variables, the algorithm used to generate the suspect parameter set are severely influenced by the bad measurements. Thus the parameter error estimation based on such suspect parameter set may be incorrect.

The existing methods are not able to identify the network parameter error if a suspicious set of parameters are not provided. In the mean time, those methods are vulnerable to the analog measurement errors. Those are the two major disadvantages of the existing methods.

Recently, an alternative method proposed in [21] makes use of normalized Lagrange multipliers associated with network parameters that are modeled as simple
constraints. The main benefit of this approach is that it allows identification of measurement and parameter errors simultaneously without selecting any pre-suspected parameters. This is accomplished with very minor computational overhead due to a simple yet effective method of recovering Lagrange multipliers for all the network parameters in the system, from the measurement residuals of a standard state estimation solution [21].

While this method is very effective in most cases, it is shown to have a limitation when the system topology includes single and/or double edge cutsets. Such topologies do not lend themselves to identification of parameter errors if the available measurements do not contain phasors. So, this is not a shortcoming of the technique developed in [21] but rather an inherent limitation imposed by measurement design. Reference [22] shows some examples where this problem can be solved by introducing phasor measurements at appropriate locations. However, a general placement strategy was not developed or presented there. The research proposed in this chapter is an extension of that work. Chapter 2 presents a computational method to identify all single and double edge cutsets in a power network. After identifying all these cutsets, this chapter proposes a general phasor measurement placement strategy to ensure error identification for the above network parameters based on the method developed in [21].

In this chapter, Section 3.2 reviews the existing method. Section 3.3 presents the proposed strategy of phasor measurement placement. Simulation results are shown in Section 3.4. The Section 3.5 concludes this method. In addition, an extension of the proposed method, which identifies the island after disturbances, is presented in Section 3.6. At the end, Section 3.7 concludes the entire chapter.
3.2 Review of existing method

This existing method in [21] is briefly reviewed as below. The weighted least square (WLS) state estimation problem can be formulated as the following optimization problem when considering parameter errors:

\[
\begin{align*}
\text{Min:} & \quad J(x) = \frac{1}{2} r^T W r \\
\text{s.t.} & \quad r = z - h(x, p_e) \\
& \quad c(x, p_e) = 0 \\
& \quad p_e = 0
\end{align*}
\]  

where

- \( r \) measurement residual vector;
- \( W \) diagonal weight matrix;
- \( z \) measurement vector;
- \( h(x, p_e) \) nonlinear function of system state and parameters related to measurements;
- \( c(x, p_e) \) nonlinear function of system state and parameters related to zero injections;
- \( p_e \) vector containing parameter errors;
- \( x \) system state vector: voltage magnitudes and angles.

One method to solve this problem by applying Lagrange multipliers in [21] is reviewed here. The parameter errors can be identified by calculating normalized Lagrange multipliers corresponding to the parameter errors.

The Lagrangian is formed first as:

\[
L = \frac{1}{2} r^T W r - \mu^T c(x, p_e) - \lambda^T p_e - \gamma^T [r - z + h(x, p_e)]
\]  

(3.3)
where $\mu$, $\gamma$ and $\lambda$ are Lagrange multipliers of the three equality constraints in (3.2) respectively. Then after using first order optimality condition and Taylor approximations, a compact form can be obtained as:

$$
\begin{bmatrix}
0 & H_x^T W & C_x^T \\
H_x & I & 0 \\
C_x & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
r \\
\mu
\end{bmatrix} = 
\begin{bmatrix}
0 \\
\Delta z \\
c_0(x_0)
\end{bmatrix}
$$

(3.4)

where

$$H_x = \frac{\partial h(x,p_e)}{\partial x}$$

(3.5)

$$C_x = \frac{\partial c(x,p_e)}{\partial x}$$

(3.6)

$$\Delta x = x - x_0$$

(3.7)

$$\Delta z = z - h_0(x_0)$$

(3.8)

$x_0$ is the initial guess of the system state. $h_0(x)$ represents $h(x,0)$ and $c_0(x)$ denotes $c(x,0)$, where $p_e$ is replaced by 0 because of the equality constraint in (3.2). Equation (3.4) is just equivalent to the conventional WLS equation, which can be solved iteratively. Once the solution converges, the measurement residual vector $r$ and Lagrange multiplier vector $\mu$ can be calculated. Then the Lagrange multipliers $\lambda$ corresponding to parameter errors can be recovered by [21]:

$$\lambda = - \left[ WH_p \right]^T \left[ r \right] = -[H_p^T Wr + C_p^T \mu]$$

(3.9)

where

$$H_p = \frac{\partial h(x,p_e)}{\partial p_e}$$

(3.10)

$$C_p = \frac{\partial c(x,p_e)}{\partial p_e}$$

(3.11)

The normalized Lagrange multipliers $\lambda^N$ will be calculated through its covariance matrix $cov(\lambda)$:

$$\lambda_i^N = \frac{\lambda_i}{\sqrt{\lambda(i,i)}} \quad (i = 1, 2, \cdots, k)$$

(3.12)
where \( k \) is the total number of parameters, \( \Lambda(i,i) \) is the \( i\)th diagonal element in the covariance matrix \( \text{cov}(\lambda) \). Detailed calculation of \( \lambda_i^N \) is documented in [21].

Finally the normalized Lagrange multipliers \( \lambda^N \) and measurement residuals \( r^N \) will be computed to identify parameter errors and bad data. The entire identification process can be summarized as follows:

\textit{Step1}: Run WLS state estimation, calculate \( r, \mu \) and store results.

\textit{Step2}: Compute normalized Lagrange multipliers \( \lambda^N \) and normalized measurement residuals \( r^N \).

\textit{Step3}: Detect and identify parameter errors and bad data. The normalized Lagrange multipliers \( \lambda^N \) and measurement residuals \( r^N \) computed in \textit{Step2} are provided in sequence. The largest one is chosen and compared with the threshold, which is commonly chosen as 3.0 approximately corresponding to the 99.8% confidence level for standard normal distribution. If the value is larger than the threshold, the corresponding parameter or measurement will be identified as erroneous. Otherwise, no parameter error or bad measurement will be declared in the system.

As mentioned before, this method on parameter error identification can identify both parameter and measurement error respectively in the same time without pre-specifying the suspicious parameter set. But it has shown that the errors in parameters of single and double edge cutsets could not be detected or identified without incorporating phasor measurements. In order to address this deficiency, a strategic placement of phasor measurements is developed and will be presented in the next section.

\subsection{3.3 Proposed method}
Installing phasor measurements at all system buses is neither economical nor necessary. Therefore, a single and double edge cutsets identification method has been developed and presented in Chapter 2. This part of the work focuses on a systematic method of placing a minimum set of phasor measurements to ensure error detectability and identifiability for network parameter errors.

Two distinct topologies emerge after all single and double edge cutsets are identified. These will be briefly described first, followed by the proposed phasor measurement placement algorithm based on these topologies. Each single or double edge cutset will cut the network into two different sub-networks. Each critical k-tuple will cut the network into several sub-networks. The sub-networks can be either a single bus or several buses connected by branches. The sub-networks can have two possible configurations, where cutsets connect them radially or as a loop.

*Type 1: Cutsets radially connecting sub-networks*

Fig. 9 Topology and placement of phasor measurements in Type 1

In this configuration, as shown in Fig. 9, each sub-network is connected by single or double edge cutsets to each other in a radial manner. The number of sub-networks can be two or more. Each sub-network can be either a single bus or several buses and
branches. The strategic phasor measurement placement will be to place at least one phasor measurement in each sub-network. Two typical cases are shown below.

1) Typical single edge cutsets

Fig. 10 shows a typical topology of a single edge cutset, which connects two sub-networks, of which one sub-network is just a single bus. The parameter error on the single edge cutset cannot be detected with only conventional measurements. The strategy in this case is to place at least one phasor measurement at any bus in the sub-network and another one at the single bus.

![Fig. 10 Topology and placement of phasor measurements in typical single edge cutsets](image1)

2) Typical double edge cutsets

Fig. 11 Topology and placement of phasor measurements in typical double edge cutsets

Typical topology of a double edge cutset in a power network is shown in Fig. 11. The two branches connect two sub-networks, of which one sub-network is just a common bus. With only conventional measurements, the parameter error on either branch of the double edge cutset cannot be identified. The strategy in this case is to
place one phasor measurement at the common bus and at least another one at any bus in the sub-network.

**Type 2: Cutsets connecting sub-networks in loop**

Fig. 12 Topology and placement of phasor measurements in Type 2

Fig. 12 shows the loop configuration of sub-networks that are connected by single and double edge cutsets. Each sub-network can be either a single bus or several buses with branches. The strategic phasor measurement placement in this case will be to place at least one phasor measurement in each sub-network. A typical case which includes critical k-tuples, will be described below.

In a given power network, a set of branches constitutes a critical k-tuple when each branch pair forms a double edge cutset. A typical example is where a critical k-tuple connects several common buses and a sub-network in a loop. The parameter error on these branches cannot be identified by conventional measurements. The strategy to identify these errors will be to place one phasor measurement at each common bus and at least another one at any bus in the sub-network. Network topology and strategic placement of phasor measurements are shown in Fig. 13.
Usually there are more than one single or double edge cutsets in a power network, where they are configured as Type 1 and Type 2 topologies. Therefore, after going through the entire set of such cutsets of both types, all the sub-networks, where phasor measurements are needed, can be identified. Then, by placing one phasor measurement at each sub-network, the final strategic placement solution will be obtained. The steps of implementing these strategies are shown in Fig. 14.
After all such cutsets have been identified they are processed one by one to incorporate phasor measurements. Each cutset divides the system into two or more sub-networks. If there are no phasor measurements and no other identified cutsets for a sub-network, a phasor measurement will be assigned to any bus inside the sub-network. Else, no phasor measurement will be placed for this sub-network. This procedure ensures that a minimum set of phasor measurements which guarantee error detectability and identifiability is placed for each type of cutsets. The final strategic placement of phasor measurements will be obtained once all cutsets are processed.

An overview of the overall phasor measurement placement procedure will now be presented. It will be assumed that the system is observable with conventional
measurements, thus none of the phasor measurements are critical. Note that the objective is to identify parameter errors that cannot be identified without phasor measurements even when all possible conventional measurements are present. This is accomplished in three major steps:

1. **Step1:** For a given power network, identify all single and double edge cutsets. An efficient identification algorithm for this purpose has been developed and details were documented in [34] and [35].

2. **Step2:** Classify all these cutsets into two different types based on their topologies.

3. **Step3:** Incorporate phasor measurements based on the proposed strategies. Details of strategies and implementation process are presented as before.

After application of the proposed method, the strategic locations of phasor measurements for parameter error identification are obtained. Simulation results will be shown as follows.

### 3.4 The simulation results

The topologies of IEEE 14-, 30- and 57-bus systems are shown in Fig. 15, 16 and 17 with phasor measurement locations, where in the figures a small pointer has been introduced at each bus representing a phasor measurement. The proposed method described above has been implemented on these systems. Simulation results including the strategic locations of phasor measurements are presented below.
Fig. 15 IEEE 14-bus system
Step 1: Identify all single and double edge cutsets

The method is presented at Chapter 2. Results of the entire set of such cutsets in IEEE 14-, 30- and 57- bus systems have already been shown at Section 2.5.

Step 2: Classify all these cutsets into two types
In this step, all cutsets that are identified are further classified into two distinct types. Results of IEEE 14-, 30- and 57-bus systems are shown in Table 13, 14 and 15 respectively.

**TABLE 13 Single and Double Edge Cutsets of IEEE 14-bus System in Different types**

<table>
<thead>
<tr>
<th>Type 1</th>
<th>(B_{7,8})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(B_{1,2}, B_{1,5})</td>
</tr>
<tr>
<td></td>
<td>(B_{2,3}, B_{3,4})</td>
</tr>
<tr>
<td></td>
<td>(B_{4,7}, B_{7,9})</td>
</tr>
<tr>
<td></td>
<td>(B_{9,14}, B_{13,14})</td>
</tr>
<tr>
<td></td>
<td>(B_{6,12}, B_{12,13})</td>
</tr>
</tbody>
</table>

| Type 2   | \(B_{9,10}, B_{10,11}, B_{6,11}\)     |

**TABLE 14 Single and Double Edge Cutsets of IEEE 30-bus System in Different types**

<table>
<thead>
<tr>
<th>Type 1</th>
<th>(B_{9,11})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(B_{12,13})</td>
</tr>
<tr>
<td></td>
<td>(B_{25,26})</td>
</tr>
<tr>
<td></td>
<td>(B_{6,8}, B_{8,23})</td>
</tr>
<tr>
<td></td>
<td>(B_{6,6}, B_{9,10})</td>
</tr>
<tr>
<td></td>
<td>(B_{12,14}, B_{14,15})</td>
</tr>
<tr>
<td></td>
<td>(B_{10,21}, B_{21,22})</td>
</tr>
<tr>
<td></td>
<td>(B_{15,23}, B_{23,24})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type 2</th>
<th>(B_{1,2}, B_{1,4})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(B_{2,5}, B_{5,8}, B_{6,7})</td>
</tr>
<tr>
<td></td>
<td>(B_{12,16}, B_{16,17}, B_{10,17})</td>
</tr>
<tr>
<td></td>
<td>(B_{25,18}, B_{18,19}, B_{19,20})</td>
</tr>
<tr>
<td></td>
<td>(B_{27,29}, B_{29,30})</td>
</tr>
<tr>
<td></td>
<td>(B_{27,28}, B_{28,25}, B_{25,24})</td>
</tr>
</tbody>
</table>

**TABLE 15 Single and Double Edge Cutsets of IEEE 57-bus System in Different types**

<table>
<thead>
<tr>
<th>Type 1</th>
<th>(B_{32,33})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(B_{1,2}, B_{2,3})</td>
</tr>
<tr>
<td></td>
<td>(B_{4,5}, B_{5,6})</td>
</tr>
<tr>
<td></td>
<td>(B_{1,16}, B_{12,16})</td>
</tr>
<tr>
<td></td>
<td>(B_{1,17}, B_{12,17})</td>
</tr>
<tr>
<td></td>
<td>(B_{22,23}, B_{23,24})</td>
</tr>
<tr>
<td></td>
<td>(B_{55,40}, B_{40,56})</td>
</tr>
<tr>
<td></td>
<td>(B_{41,42}, B_{56,42})</td>
</tr>
<tr>
<td></td>
<td>(B_{41,43}, B_{11,41})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type 2</th>
<th>(B_{18,19}, B_{19,20}, B_{21,20}, B_{21,22}, B_{6,18})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(B_{24,26}, B_{26,27}, B_{27,28}, B_{28,29})</td>
</tr>
<tr>
<td></td>
<td>(B_{27,38}, B_{39,37}, B_{37,56})</td>
</tr>
<tr>
<td></td>
<td>(B_{26,48}, B_{44,45}, B_{45,15})</td>
</tr>
<tr>
<td></td>
<td>(B_{14,46}, B_{46,47}, B_{47,48})</td>
</tr>
<tr>
<td></td>
<td>(B_{29,52}, B_{32,55}, B_{53,54}, B_{34,55}, B_{9,55})</td>
</tr>
<tr>
<td></td>
<td>(B_{9,56}, B_{50,51}, B_{51,51})</td>
</tr>
<tr>
<td></td>
<td>(B_{24,25}, B_{25,26}, B_{30,31}, B_{31,32})</td>
</tr>
<tr>
<td></td>
<td>(B_{34,32}, B_{34,35}, B_{35,36})</td>
</tr>
</tbody>
</table>
Two illustrative examples for each type are presented to show the benefits of corresponding phasor measurement strategies. Then an additional example is provided to show the performance with multiple parameter errors.

i. Cutset Examples of Type 1

Table 16 shows results of parameter error identification for Type 1 cutsets. An error is introduced in the impedance of the line $B_{25-26}$ in IEEE 30-bus system, where this branch is identified as a single edge cutset of Type 1. Two tests are carried out as follows:

Test A: Using a full set of conventional measurements only, and no phasor measurements.

Test B: Same as test A except phasor measurements at bus 1 and 26 are added.

As evident from the results shown in Table 16, the parameter error in $B_{25-26}$ is undetectable when there are no phasor measurements, because the largest normalized Lagrange multiplier remains below the chosen threshold of 3. After integrating phasor measurements at bus 1 and 26, the error is easily detected. More examples can be found to prove that one phasor measurement at any bus of the sub-network (from bus 1 to bus 25) together with another at bus 26 will always be sufficient for this problem. Therefore, in this situation, at least one phasor measurement in the sub-network and one in the single bus are needed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test A (No PMs)</th>
<th>Test B (With PMs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{25-26}$</td>
<td>2.1625</td>
<td>5.7569</td>
</tr>
<tr>
<td>$r_{25-26}$</td>
<td>2.1469</td>
<td>2.5656</td>
</tr>
<tr>
<td>$r_{27-25}$</td>
<td>0.4136</td>
<td>1.7233</td>
</tr>
</tbody>
</table>
TABLE 17 Results of Error Identification in IEEE 14-bus system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test A (No PMs)</th>
<th>Test B (With PMs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{2,3}$</td>
<td>36.8862</td>
<td>$x_{2,3}$</td>
</tr>
<tr>
<td>$x_{2,4}$</td>
<td>35.4795</td>
<td>$x_{3,4}$</td>
</tr>
<tr>
<td>$x_{3,4}$</td>
<td>15.7297</td>
<td>$x_{4,5}$</td>
</tr>
</tbody>
</table>

Results of a second example of Type 1 are shown in Table 17. As identified earlier in Table 13, $B_{2.3}$ and $B_{3.4}$ in IEEE 14-bus system constitute a double edge cutset of Type 1. Bus 3 is the common bus and others form a sub-network. An error is introduced in the impedance of line $B_{2.3}$. Two tests are carried out as follows:

Test A: Using a full set of conventional measurements only, and no phasor measurements.

Test B: Same as test A except phasor measurements at bus 1 and 3 are added.

In test A where only conventional measurements are available, the parameter error in $B_{2.3}$ is misidentified as in $B_{3.4}$, which actually forms a double edge cutset with $B_{2.3}$. However, with one phasor measurement at bus 3 (the common bus) and another at bus 1, the error can be identified. Furthermore, placing a phasor measurement at any other bus in the sub-network will still enable identification of the error. On the other hand, if the phasor measurement at bus 3 is missing, the error cannot be identified.

\[ \text{ii. Cutset Examples of Type 2} \]

TABLE 18 Results of Error Identification in IEEE 57-bus system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test A (No PMs)</th>
<th>Test B (With PMs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{9,55}$</td>
<td>7.8165</td>
<td>$x_{52,55}$</td>
</tr>
<tr>
<td>$x_{52,53}$</td>
<td>7.5506</td>
<td>$r_{55,54}$</td>
</tr>
<tr>
<td>$r_{53,54}$</td>
<td>6.9657</td>
<td>$x_{9,55}$</td>
</tr>
</tbody>
</table>

A representative case of Type 2 in IEEE 57-bus system is created for illustration. $B_{29.52}$, $B_{52.53}$, $B_{53.54}$, $B_{54.55}$ and $B_{9.55}$ compose a critical k-tuple connecting corresponding sub-networks in a loop. A parameter error in impedance of the line $B_{52}$. 46
is introduced. Two tests as described below are simulated and results are shown in Table 18.

Test A: Using a full set of conventional measurements only, and no phasor measurements.

Test B: Same as test A except phasor measurements at buses 1, 52, 53, 54 and 55 are added.

In test A, the error of line impedance in $B_{52,53}$ is misidentified as in $B_{9,55}$. After applying phasor measurements at buses 1, 52, 53, 54 and 55, the error becomes identifiable. In addition, the phasor measurement at bus 1 can be substituted by any other bus in the sub-network. However, if the phasor measurements placed at common buses are removed, the error on the corresponding branches cannot be identified. Hence, implementing one phasor measurement on each sub-network is the right strategy in this case.

**TABLE 19 Results of Error Identification in IEEE 30-bus system**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test A (No PMs)</th>
<th>$\hat{\lambda}$</th>
<th>Test B (With PMs)</th>
<th>$\hat{\lambda}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{27,28}$</td>
<td>6.2086</td>
<td></td>
<td>$x_{25,27}$</td>
<td>6.9989</td>
</tr>
<tr>
<td>$x_{25,27}$</td>
<td>6.0216</td>
<td></td>
<td>$x_{27,28}$</td>
<td>5.4993</td>
</tr>
<tr>
<td>$x_{24,25}$</td>
<td>4.7338</td>
<td></td>
<td>$x_{24,25}$</td>
<td>4.3471</td>
</tr>
</tbody>
</table>

Another case of Type 2 in IEEE 30-bus system is shown here and the simulation results are presented at Table 19. In this system, $B_{24,25}$, $B_{25,27}$ and $B_{27,28}$ constitute a critical k-tuple which connects corresponding sub-networks in a loop. An error is introduced in line impedance of $B_{25,27}$. Again two tests are simulated:

Test A: Using a full set of conventional measurements only, and no phasor measurements.

Test B: Same as test A except phasor measurements at buses 1, 25 and 27 are added.
As shown in Table 19, the parameter error in $B_{25-27}$ is misidentified as in $B_{27-28}$, when no phasor measurements exist. After placement of phasor measurements at buses 1, 25 and 27 (one in each sub-network) in Test B, the error is successfully identified. Note that, each of the three phasor measurements can be substituted by one at another bus in the same sub-network to make all these errors identifiable. In addition, as shown in Table 16, $B_{25-26}$ is a single edge cutset. As a result of that example, one phasor measurement has to be located at bus 26. Thus, there will be no need to place another one at bus 25.

iii. Example with multiple parameter errors

This algorithm can also address the identification problem with multiple parameter errors. As documented in [21], in this case, the normalized Lagrange multiplier test will be applied repeatedly to identify the errors one by one. It means, in each time of the test, only one error will be identified and corrected. And the test need to be performed repeatedly until there is no error identified any more. A representative example in IEEE 14-bus system is shown as below.

In IEEE 14-bus system, three parameter errors have been introduced to the system simultaneously, which are the impedance errors in $B_{4-7}$, $B_{6-12}$ and $B_{9-14}$ respectively. Three involved cutsets are $B_{4-7}$ with $B_{7-9}$, $B_{6-12}$ with $B_{12-13}$, and $B_{9-14}$ with $B_{13-14}$. Two tests are performed as below

Test A: Using a full set of conventional measurements only, and no phasor measurements.

Test B: Same as test A except phasor measurements at bus 7, 12 and 14 are added.

| TABLE 20 Test A Result of Multiple Error Identification in IEEE 14-bus system |
|-------------------------------|-----------------
| Parameter | $\lambda$ |
| $x_{7,9}$ | 18.9308 |
| $x_{4,7}$ | 17.6541 |
| $x_{5,6}$ | 15.0906 |
TABLE 21 Test B Result of Multiple Error Identification in IEEE 14-bus system

<table>
<thead>
<tr>
<th>Step No.</th>
<th>Parameter</th>
<th>$\lambda^N$</th>
<th>Line with parameter error identified</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x_{4,7}$</td>
<td>19.0336</td>
<td>$B_{4,7}$</td>
</tr>
<tr>
<td></td>
<td>$x_{5,9}$</td>
<td>16.9090</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x_{5,6}$</td>
<td>14.2632</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$r_{6,12}$</td>
<td>8.0170</td>
<td>$B_{6,12}$</td>
</tr>
<tr>
<td></td>
<td>$r_{12,33}$</td>
<td>7.6319</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$r_{6,11}$</td>
<td>6.0619</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$x_{9,14}$</td>
<td>5.7494</td>
<td>$B_{9,14}$</td>
</tr>
<tr>
<td></td>
<td>$x_{13,14}$</td>
<td>4.4328</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x_{5,11}$</td>
<td>2.7240</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$r_{7,8}$</td>
<td>0.1527</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>$x_{8,7}$</td>
<td>0.0824</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x_{13,14}$</td>
<td>0.0752</td>
<td></td>
</tr>
</tbody>
</table>

The results of Test A and Test B are shown in Table 20 and Table 21 respectively. Table 20 shows that without phasor measurements, such parameter errors are misidentified as in $B_{7,9}$ at the first time the normalized test is implemented. By placing phasor measurements at bus 7, 12 and 14, the errors can be identified one by one. As shown in Table 21, in Step 1, the normalized test is performed and the error in $B_{4,7}$ is identified. After correcting this error by the method using augmented state vector [21], the normalized test is implemented again in Step 2 and the error in $B_{6,12}$ is identified. Because there is no pre-information assumed about the multiple errors, the normalized test will be performed until there is no parameter error detected, which, in this example, is Step 4. Then all the three parameter errors are identified successfully. More examples of multiple error identification can be found in [21], [22] and [39].

c) Step 3: Installing phasor measurements based on proposed strategies

Power networks commonly contain several cutsets of both types. Through systematic application of the proposed strategies to all of the considered test systems, as mentioned before, the strategic locations of phasor measurements for IEEE 14-, 30-
and 57-bus systems are obtained and listed in Table 22. The results are also shown in Fig. 15, 16 and 17 respectively.

<table>
<thead>
<tr>
<th>IEEE test systems</th>
<th>Number of PMs</th>
<th>Strategic location of PMs (bus number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-bus</td>
<td>7</td>
<td>1, 3, 8, 10, 11, 12, 14</td>
</tr>
<tr>
<td>30-bus</td>
<td>18</td>
<td>1, 3, 5, 7, 8, 11, 13, 14, 16, 17, 18, 19, 20, 21, 23, 26, 29, 30</td>
</tr>
<tr>
<td>57-bus</td>
<td>33</td>
<td>2, 5, 16, 17, 18, 19, 20, 21, 23, 25, 26, 27, 28, 30, 31, 33, 34, 35, 39, 40, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 57</td>
</tr>
</tbody>
</table>

After placing phasor measurements at above locations, and using all available conventional measurements, all the parameter errors can be detected and identified successfully. The method can be applied either to the entire system or just to a specific zone where the parameter errors are crucial to be identified. The total number of required phasor measurements depends closely on the system topology. Larger numbers of single and double edge cutsets will necessitate placement of larger numbers of phasor measurements. Therefore, it is also anticipated that this method may be used in power system planning, for instance in transmission expansion studies [38], where constraints can be placed to avoid such topologies being created.

3.5 Conclusions

Previous work on network parameter error identification has uncovered some limitations when only conventional measurements are used. This chapter extends this work to address this limitation via strategic placement of phasor measurements. It relies on the previously developed capability to efficiently detect and identify single and double edge cutsets. Proposed placement strategy ensures identification of all those network parameter errors irrespective of their locations. It is tested on different size systems with different topologies. The method can be applied to the entire system
or to a specified zone only where parameter error identification may be of particular concern. Furthermore, it provides a simple tool for taking parameter error identification into account during system planning and expansion studies.

3.6 Post disturbance island identification via state estimation

3.6.1 Introduction

An extension of the above method is to detect and identify isolated power islands through state estimation. Interconnected power systems may split into different islands due to the line outages caused by the nature disasters like hurricanes. It is of great importance to detect and identify these islands in order to capture the actual network model for further security analysis, emergency control and restoration. Given the recent proliferation of distributed power sources in power systems, a large number of these islands may still operate in the so called restorative state avoiding a complete system-wide blackout. Furthermore, the state estimator may still be active due to the partial availability of SCADA system even after the islands are formed. Since phasor measurements in different islands can still be synchronized by GPS but not confirm each other as before islanding, the parameter error identification algorithm incorporating phasor measurements can be applied to identify outage lines and, further, all the islands of the system. This part of work considers such operating conditions and presents a systematic method to identify electrical islands through state estimation with the help of phasor measurements.

3.6.2 Proposed method

There have been published methods to detect and identify electrical islands via monitoring the transient response of the system [40]-[42], detecting abnormal power flow results [43]-[45] and checking network connectivity from graph theory [46]-[49].
In this part a different approach is presented where this problem is formulated as identification of multiple line outages using state estimation. Therefore, the method will use post disturbance steady state measurements following the system splitting. Since a line outage can be treated as a distinctive line parameter error, in which the line admittance becomes zero, the problem will be formulated and solved as a parameter error identification problem. It should be noted that this formulation implicitly assumes a network free of any major parameter errors at the time of the islanding event.

In this section, the parameter identification algorithm will be used and reformulated to identify line outages and further the isolated islands in a power system. Some assumptions have been made under this method. First, only the static state after system splitting will be considered. Second, although the measurements in the disconnected lines are considered to be lost, the electrical islands formed after splitting will constitute observable islands. Furthermore, each island will be assumed to have at least one phasor measurement (if no such measurement exists, then an arbitrary phase angle will be assigned to any one of the buses inside the electrical island). The line outages will be identified one at a time by repeated application of the parameter error identification algorithm. Finally, the network model of the system with different islands after splitting will be obtained.

Description of the steps involved in the implementation of the method is given below. The flowchart of the method is shown in Fig. 18.

*Step 1:* Run WLS state estimation. In this step, all the measurement values obtained are the steady state values after system splitting, but the network model is still the same one before splitting.
**Step2**: Calculate normalized Lagrange multipliers \( \lambda^N \). The computation method is reviewed as before.

**Step3**: Check if any line outage is identified. The largest normalized Lagrange multiplier calculated in Step2 is compared with the threshold, which is typically set at 3.0. If the value is higher than 3.0, the corresponding branch is identified as a disconnected line. Otherwise, no line outage will be suspected.

**Step4**: Change the network model. If a line outage has been identified in Step3, delete the corresponding line from the network model and go to Step5. Otherwise, stop.

**Step5**: Repeat Step1 to Step4 with the updated network model and same measurement values until there is no line outage identified in Step3.

Then the network model after system splitting will be obtained, by which all the isolated islands will be identified. The flowchart of this method is shown Fig. 18. Simulation results in IEEE 14- and 30- bus systems will be presented as below.
3.6.3 Simulation results

The proposed method is implemented using the IEEE 14-, and 30- bus test systems. Topologies of these systems including the assumed islands after system splitting are shown in Fig. 19 and 20. Assumptions mentioned above are made for all simulations. Each island is rendered observable after system splitting. Only the flow measurements corresponding to the disconnected lines are assumed to be lost. At least one phasor measurement in each island is installed.

The first example is shown in Fig. 19. The IEEE 14-bus system is randomly separated into two isolated islands due to a disaster. Corresponding disconnected branches are $B_{5-6}$, $B_{4-9}$, and $B_{7-9}$. The system is observable under conventional measurements as well as two phasor measurements at buses 1 and 6.
As described, disconnected lines are identified one at a time until there are no suspect lines left. Detailed results in all iterations are listed in Table 23.

![Fig. 19 IEEE 14-bus system](image)

**TABLE 23 Islands identification in IEEE 14-bus system**

<table>
<thead>
<tr>
<th>Iteration No.</th>
<th>Parameter</th>
<th>$\lambda^N$</th>
<th>Outage line identified</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x_{7,9}$</td>
<td>266.3374</td>
<td>$B_{7,9}$</td>
</tr>
<tr>
<td></td>
<td>$x_{4,7}$</td>
<td>249.8935</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$x_{5,6}$</td>
<td>195.3127</td>
<td>$B_{5,6}$</td>
</tr>
<tr>
<td></td>
<td>$l_{4,9}$</td>
<td>95.1562</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$l_{4,9}$</td>
<td>129.4057</td>
<td>$B_{4,9}$</td>
</tr>
<tr>
<td></td>
<td>$x_{4,9}$</td>
<td>124.1862</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$x_{12,13}$</td>
<td>0.0841</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>$x_{6,12}$</td>
<td>0.0828</td>
<td></td>
</tr>
</tbody>
</table>

Table 23 shows the top two normalized Lagrange multipliers for each identification cycle. The branch corresponding to the largest one is identified and removed from the existing network model before next identification cycle. In this example, in the first three cycles, all the disconnected lines are successfully identified. The results of the fourth cycle yield insignificant values hinting that no more suspect lines are left. Then the islands are easily identified. Note that having at least one phasor measurement per electrical island facilitates the process of solving the parameter error identification problem.
Second example involves the IEEE 30-bus test system as shown in Fig. 20. The system is split into two islands because of the outage of lines $B_{4,12}$, $B_{6,10}$, $B_{9,10}$ and $B_{24,25}$. The system is observable under conventional measurements. Besides, there are two PMUs installed in bus 1 and 24 respectively. The proposed method is implemented and simulation results are shown in Table 24.

**TABLE 24 Islands identification in IEEE 30-bus system**

<table>
<thead>
<tr>
<th>Iteration No.</th>
<th>Parameter</th>
<th>$\lambda^N$</th>
<th>Outage line identified</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x_{24,25}$</td>
<td>750.3891</td>
<td>$B_{24,25}$</td>
</tr>
<tr>
<td></td>
<td>$r_{25,27}$</td>
<td>657.8142</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$x_{9,10}$</td>
<td>515.5548</td>
<td>$B_{9,10}$</td>
</tr>
<tr>
<td></td>
<td>$l_{4,12}$</td>
<td>479.1360</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$l_{4,12}$</td>
<td>822.0933</td>
<td>$B_{4,12}$</td>
</tr>
<tr>
<td></td>
<td>$x_{4,12}$</td>
<td>758.7676</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$l_{6,10}$</td>
<td>522.9023</td>
<td>$B_{6,10}$</td>
</tr>
<tr>
<td></td>
<td>$x_{6,10}$</td>
<td>511.2897</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$s_{24}$</td>
<td>0.7121</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>$s_{28,6}$</td>
<td>0.4177</td>
<td></td>
</tr>
</tbody>
</table>

As shown in Table 24, the outage lines have been identified one at a time. Two islands are found after deleting these lines. This method is simple to integrate into
existing state estimators yet provides a useful feature in particular during unexpected natural disturbances causing random islanding in power systems.

3.6.4 Conclusions

Phasor measurements have already been incorporated into an existing algorithm to identify parameter errors in power system state estimation. Based on this algorithm, this section presents a systemic method to identify line outages and thereby the created electrical islands in a power system. Simulations that are performed using IEEE 14- and 30-bus test systems are presented to illustrate the performance of the method. This method is easy to integrate into existing state estimators in order to facilitate fast and reliable identification of islanding and thus help the subsequent restoration process.

3.7 Conclusions of the chapter

Existing work on network parameter error identification has uncovered some limitations that, when only conventional measurements are used, the parameter errors on single and double edge cutsets cannot be detected or identified. This chapter presents a strategic placement of phasor measurements in order to address those limitations. It relies on the previously developed method to efficiently detect and identify all single and double edge cutsets. Proposed placement strategy ensures identification of all network parameter errors on those topologies in power networks with the least number of phasor measurements. It can be applied to the entire system or to a specified zone only where parameter error identification may be of particular concern. Further, as an extension, a systemic method to identify power islands after disturbances is proposed. This method is easy to integrate into existing state estimators in order to identify islands and thus help the subsequent restoration process.
4 Identifying Parameter Error via Multiple Measurement Scans

4.1 Introduction

A parameter error identification method which is based on Lagrange multipliers corresponding to a single measurement scan has been reviewed in the above chapters. The main advantage of the method is its ability to successfully detect and identify parameter errors in large systems without the need to pre-specify a suspect parameter set. Despite this advantage, the method fails to identify errors in certain types of parameters even with high measurement redundancy.

In this chapter, an alternative approach to addressing these limitations will be described. The method has the advantage that it will not require any investments in new meters but will simply use multiple measurement scans instead. In this chapter, a detailed analysis of the inherit limitations of the single scan method will be provided. Then the detailed derivation of multiple scan method will be presented. In addition, a derivation will be given to reveal the relationship between the number of measurement scans, the value of normalized Lagrange multipliers and normalized measurement residuals. It allows proper choice of the number of scans to be used. This will be useful to customize the method to individual systems and verifying the identification results. Simulations of both single parameter error cases and multiple parameter error cases will be implemented in IEEE test systems. Simulations with different scan numbers will also be shown to verify the newly derived observations.

Use of multiple scans is not a new idea and was previously employed for the traditional augmented state estimation method in order to improve redundancy and allow observability of parameters [5]. In this chapter, the method is used to improve the performance of the Lagrange multiplier based method. Thus, the method’s
limitations for certain topologies as described in [21] are eliminated without adding any new meters.

4.2 Limitations of the existing method

The existing method in [21] is reviewed in Section 3.2. The effectiveness of this method is also related to measurement redundancy and network topology. The method may fail when it comes to the parameter errors existing in the area lacking of measurement redundancy. The detail analysis is provided as below.

The Lagrange multipliers, which are used to detect and identify parameter errors, are calculated by (3.9). Just ignoring the zero injections for simplicity of notation, the Lagrange multipliers and the covariance matrix can be computed by:

\[
\lambda = -[H_p^T W r] \tag{4.1}
\]

\[
cov(\lambda) = H_p^T W \text{cov}(r) W H_p \tag{4.2}
\]

In (4.1), if the measurement residuals \( r \) follow a normal distribution, the Lagrange multipliers \( \lambda \) will also follow a normal distribution with a different covariance. For a certain parameter \( i \), the corresponding Lagrange multiplier is

\[
\lambda_i = -\sum_{j=1}^{m} H_p^T (i, j) W(j, j) r(j) \tag{4.3}
\]

where \( m \) is the total number of measurements.

Regarding (4.3) the following cases should be considered:

(1) No measurement is related to the parameter \( i \). This is the case when the parameter error will not be detectable. In this case, the entire \( i \)th row of \( H_p^T \) will be equal to zero. It implies that:

\[
H_p^T (i, j) = 0 \tag{4.4}
\]

where \( j = 1, 2, ..., m \). Therefore, the Lagrange multiplier \( \lambda_i \) corresponding to this parameter will always be zero, which will prevent detection and identification of the
parameter error. New measurements need to be installed to make this parameter error detectable.

(2) Only one measurement is incident to parameter $i$. In this case, the parameter error will be detectable but not identifiable.

Assume that, among all $m$ measurements, only the $q$th measurement is incident to parameter $i$. Thus, there will be only one nonzero element in row $i$ of $H_p^T$, which is $H_p^T(i, q)$. Then the corresponding Lagrange multiplier will be:

$$
\lambda_i = -\sum_{j=1}^{m} H_p^T(i, j)W(j, j)r(j)
= -H_p^T(i, q)W(q, q)r(q) \tag{4.5}
$$

The normalized Lagrange multiplier is calculated using (3.12):

$$
\lambda_i^N = \frac{|\lambda_i|}{\sqrt{\lambda(i,i)}}
= \frac{|-H_p^T(i, q)W(q, q)r(q)|}{\sqrt{H_p^T(i, q)W(q, q)[\text{cov}(r)(q, q)]W(q, q)H_p^T(i, q)}}
= \frac{|r(q)|}{\sqrt{[\text{cov}(r)(q, q)]}}
= r(q)^N \tag{4.6}
$$

which is equal to the normalized residual of measurement $q$. It implies that it is impossible to distinguish between the parameter and the measurement errors. Identification of such parameter errors requires more redundancy. The contribution of the proposed method is to provide a no-cost alternative which overcomes this limitation via the use of multiple measurement scans. This is especially useful for certain types of parameter errors given in [21], which may not be identified even with a highly redundant measurement configuration. The detailed derivation of the proposed method will be presented and the technical benefits will be illustrated by simulation examples at the end of this chapter.
4.3 **Proposed method**

Incorporating multiple measurement scans into state estimation is not a new idea. It has been applied earlier to the augmented state estimation method to improve redundancy. In this chapter, the same approach is adopted to improve the method proposed in [21]. The method assumes that the erroneous parameter will remain unchanged while the operating point of the system will change significantly in comparison to the measurement noise between multiple measurement scans. Therefore, certain parameter errors which cannot be identified by single scan estimation will now be identified. Furthermore, since estimation for each scan is independent of the rest, the existing state estimation program can be repeatedly executed for different scans. There is no need to modify the existing state estimation code. The derivation of this algorithm is presented below.

Starting from (3.1) the WLS problem incorporating multiple measurement scans can be reformulated as follows:

\[
\text{Min: } J(x) = \frac{1}{2} \sum_{i=1}^{n} r_i^T W_i r_i \\
\text{s.t. } r_i = z_i - h_i(x_i, p_e) \quad (i = 1,2,\cdots,n)\\
\hspace{2cm} c_i(x_i, p_e) = 0 \quad (i = 1,2,\cdots,n)\\
\hspace{4cm} p_e = 0
\]

where

- \(W_i\) diagonal weight matrix of each scan;
- \(z_i\) measurement vector of each scan;
- \(r_i\) measurement residual vector of each scan;
- \(h_i(x_i, p_e)\) nonlinear function of system state and parameter errors related to measurements in each scan;
- \(c_i(x_i, p_e)\) nonlinear function of system state and parameter errors related to
zero injections in each scan;

\( p_e \) vector containing parameter errors;

\( x_i \) system state vector of each scan: voltage magnitudes and angles; 

\( n \) number of scans.

Forming the Lagrangian as:

\[
L = \sum_{i=1}^{n} \left( \frac{1}{2} r_i^T W_i r_i - \mu_i^T c_i(x_i, p_e) - \gamma_i^T [r_i - z_i + h_i(x_i, p_e)] \right) - \lambda^T p_e
\]  

(4.9)

where \( \mu_i, \gamma_i \) and \( \lambda \) are Lagrange multipliers of the equality constraints in (4.8).

Applying first order conditions yields:

\[
\frac{\partial L}{\partial x} = \sum_{i=1}^{n} (C_{ix}^T \mu_i + H_{ix}^T \gamma_i) = 0
\]  

(4.10)

\[
\frac{\partial L}{\partial r} = \sum_{i=1}^{n} (W_i r_i - \gamma_i) = 0
\]  

(4.11)

\[
\frac{\partial L}{\partial p} = \sum_{i=1}^{n} (C_{ip}^T \mu_i + H_{ip}^T \gamma_i) + \lambda = 0
\]  

(4.12)

\[
\frac{\partial L}{\partial \mu_i} = c_i(x_i, p_e) = 0 \quad (i = 1, 2, \cdots, n)
\]  

(4.13)

\[
\frac{\partial L}{\partial \gamma_i} = \gamma_i^T [r_i - z_i + h_i(x_i, p_e)] = 0 \quad (i = 1, 2, \cdots, n)
\]  

(4.14)

\[
\frac{\partial L}{\partial \lambda} = p_e = 0
\]  

(4.15)

where

\[
H_{ix} = \frac{\partial h_i(x_i, p_e)}{\partial x}
\]  

(4.16)

\[
C_{ix} = \frac{\partial c_i(x_i, p_e)}{\partial x}
\]  

(4.17)

\[
H_{ip} = \frac{\partial h_i(x_i, p_e)}{\partial p_e}
\]  

(4.18)

\[
C_{ip} = \frac{\partial c_i(x_i, p_e)}{\partial p_e}
\]  

(4.19)

\begin{align*}
(i = 1, 2, \cdots, n)
\end{align*}

Then Lagrange multipliers \( \lambda \) can be derived from (4.11) and (4.12) as:

\[
\lambda = -\sum_{i=1}^{n} (C_{ip}^T \mu_i + H_{ip}^T W_i r_i)
\]
\[- \left( H_p^T \bar{W} \bar{r} + C_p^{-T} \bar{\mu} \right) \]

\[= - \left[ \frac{W H_p}{C_p} \right]^T \cdot \left[ \bar{r} \right] \tag{4.20} \]

where

\[\bar{C}_p = \begin{bmatrix} C_{1p} \\ \vdots \\ C_{np} \end{bmatrix} \tag{4.21} \]

\[H_p = \begin{bmatrix} H_{1p} \\ \vdots \\ H_{np} \end{bmatrix} \tag{4.22} \]

\[\bar{\mu} = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_n \end{bmatrix} \tag{4.23} \]

\[\bar{r} = \begin{bmatrix} \bar{r}_1 \\ \vdots \\ \bar{r}_n \end{bmatrix} \tag{4.24} \]

\[\bar{W} = \begin{bmatrix} W_1 & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & W_n \end{bmatrix} \tag{4.25} \]

After the estimation solutions of all scans have been obtained, the Lagrange multipliers \(\lambda\) corresponding to the parameter errors will be retrieved by (4.20). It is noted that, comparing (4.20) to (3.9), the number of Lagrange multipliers maintain the same because it is assumed that all the parameters including erroneous ones do not change. However, the expression in (4.20) reflects that the value of each multiplier is different and more sensitive to the errors. It indicates that the redundancy is improved by the multiple measurement scans.

Since the state estimation of each scan is independent, \((3.4)\) can still be used to obtain the estimation result of each measurement scan separately. The system operators just need to implement the existing estimation program repeatedly at different time. Then all the solutions will be obtained and stored.
Equation (3.4) can also be expressed as an extended form as below to incorporate multiple measurement scans explicitly.

\[
\begin{pmatrix}
0 & H_x^T W & C_x^T \\
\bar{H}_x & I & 0 \\
\bar{C}_x & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\Delta x \\
\bar{r} \\
\bar{\mu}
\end{pmatrix}
= \begin{pmatrix}
0 \\
\Delta z \\
-c_0(x_0)
\end{pmatrix}
\]  \hspace{1cm} (4.26)

where

\[
\bar{H}_x = \begin{bmatrix}
H_{1x} \\
\vdots \\
H_{nx}
\end{bmatrix}
\]  \hspace{1cm} (4.27)

\[
\bar{C}_x = \begin{bmatrix}
c_{1x} \\
\vdots \\
c_{nx}
\end{bmatrix}
\]  \hspace{1cm} (4.28)

\[
\bar{\Delta}x = \begin{bmatrix}
\Delta x_1 \\
\vdots \\
\Delta x_n
\end{bmatrix}
\]  \hspace{1cm} (4.29)

\[
\bar{\Delta}z = \begin{bmatrix}
\Delta z_1 \\
\vdots \\
\Delta z_n
\end{bmatrix}
\]  \hspace{1cm} (4.30)

\[
c_0(x_0) = \begin{bmatrix}
c_{10}(x_{10}) \\
\vdots \\
c_{n0}(x_{n0})
\end{bmatrix}
\]  \hspace{1cm} (4.31)

\( \Delta x_i \), \( \Delta z_i \) and \( c_{i0}(x_{i0}) \) (\( i = 1, 2, \cdots, n \)) denote \( \Delta x \), \( \Delta z \) and \( c_0(x_0) \) of each measurement scan.

Note that there is no need to solve (4.26) iteratively to obtain estimation results of different scans simultaneously. It is instead used to calculate the normalized Lagrange multipliers. Let:

\[
\mathbf{S} = - \begin{pmatrix}
W H_p \\
C_p
\end{pmatrix}^T
\]  \hspace{1cm} (4.32)

\[
\bar{u} = \begin{bmatrix}
\bar{r} \\
\bar{\mu}
\end{bmatrix}
\]  \hspace{1cm} (4.33)

Then, (4.20) can be written in compact form as:
\[ \lambda = \bar{S} \cdot \bar{u} \]  

(4.34)

The covariance of \( \lambda \), \( \text{cov}(\lambda) \), can be derived as:

\[ \Lambda = \text{cov}(\lambda) = \bar{S} \cdot \text{cov}(\bar{u}) \cdot \bar{S}^T \]  

(4.35)

Now, let us define the inverse of the coefficient matrix in (4.26):

\[
\begin{bmatrix}
0 & H_x^T \bar{W} & C_x^T \\
\bar{H}_x & I & 0 \\
\bar{C}_x & 0 & 0
\end{bmatrix}^{-1} =
\begin{bmatrix}
E_1 & E_2 & E_3 \\
E_4 & E_5 & E_6 \\
E_7 & E_8 & E_9
\end{bmatrix}
\]  

(4.36)

Since \( \bar{c}_0(x) = 0 \) at the solution, \( \bar{u} \) can be calculated by:

\[ \bar{u} = \bar{\Phi} \cdot \bar{\Delta z} \]  

(4.37)

where

\[ \bar{\Phi} = \begin{bmatrix} E_5 \\ E_8 \end{bmatrix} \]  

(4.38)

Then the covariance of \( \bar{u} \) can be expressed as:

\[ \text{cov}(\bar{u}) = \bar{\Phi} \cdot \bar{W}^{-1} \cdot \bar{\Phi}^T \]  

(4.39)

Substituting (4.39) into (4.35), the covariance of \( \lambda \) can be calculated as:

\[ \Lambda = \text{cov}(\lambda) = \bar{S} \cdot \text{cov}(\bar{u}) \cdot \bar{S}^T \]

\[ = \bar{S} \cdot \bar{\Phi} \cdot \bar{W}^{-1} \cdot \bar{\Phi}^T \cdot \bar{S} \]  

(4.40)

Finally, the normalized Lagrange multipliers \( \lambda_i^N \) will be given by:

\[ \lambda_i^N = \frac{|\lambda_i|}{\sqrt{\Lambda_{ii}}} \quad (i = 1, 2, \cdots, k) \]  

(4.41)

where \( k \) is total number of the parameters. Meanwhile, the normalized measurement residuals of multiple scans will be calculated simultaneously to identify bad data.

The entire identification procedure incorporating multiple measurement scans is summarized below.

**Step1:** Run WLS state estimation of each measurement scan separately and store the results.
**Step 2**: Calculate the normalized Lagrange multipliers $\lambda^N$ and normalized measurement residuals $r^N$.

**Step 3**: Detect and identify parameter errors and bad data via the same method described before.

The flowchart of this proposed method is shown in Fig. 21. Compared to the single scan estimation, the proposed method has two main advantages. One is that the redundancy is significantly improved just by the algorithm itself without investing in any new meters. This enables identification of certain types of errors, which are not identifiable by the single scan estimator. The other is the fact that the existing estimators can be readily used for each scan. It can be implemented just by running an existing program at different times. Simulation results on IEEE standard test systems will be provided afterwards.
4.4 Additional observations

The limitation of the single scan estimators in Section 4.2 will be elaborated here to illustrate the technical improvement introduced by the proposed algorithm.

Using (4.20) and ignoring the zero injections for simplicity of notation, the Lagrange multipliers for multiple scans can be calculated by:

\[
\lambda = - \left( H_p^T \bar{W} \bar{r} \right)
\]
Assuming that the measurement configuration does not change from one scan to the next and neglecting changes in the operating point, (4.42) can be further approximated as:

\[
\lambda \equiv - [H_p^T W (r_1 + \cdots + r_n)]
\]  

(4.43)

where

\[
H_{1p}^T = \cdots = H_{np}^T = H_p^T
\]  

(4.44)

\[
W_1 = \cdots = W_n = W
\]  

(4.45)

Then its covariance matrix will be given by:

\[
cov(\lambda) = H_p^T W \text{cov}(r_1 + \cdots + r_n) W H_p
\]  

(4.46)

Also assuming that the measurement residuals between different scans are not correlated, (4.46) can be further simplified as:

\[
cov(\lambda) = H_p^T W [\text{cov}(r_1) + \cdots + \text{cov}(r_n)] W H_p
\]  

(4.47)

Consider the case where for each scan, there is only one measurement \( q \) which corresponds to the erroneous parameter \( i \). Then, the normalized Lagrange multiplier of parameter \( i \) can be calculated by (4.41), (4.43) and (4.47):

\[
\lambda_i^N = \frac{\lambda_i}{\sqrt{\lambda(\lambda)}}
\]

\[
= \frac{-H_p^T (i.q) W(q.q) [r_1(q) + \cdots + r_n(q)]}{\sqrt{H_p^T (i.q) W(q.q) \text{cov}(r_1(q)) + \cdots + \text{cov}(r_n(q)) W(q.q) H_p^T (i.q)}}
\]  

(4.48)

Assuming no change in the measurement configuration, no sustained gross error in measurements and a single sustained error in parameter \( i \) between scans, (4.48) can be further simplified using the following approximation:

\[
\lambda_i^N \equiv \frac{-H_p^T (i.q) W(q.q) [nr(q)]}{\sqrt{H_p^T (i.q) W(q.q) [n[\text{cov}(r(q))] W(q.q) H_p^T (i.q)}}
\]
where
\[ n r(q) \]
\[ = \frac{|nr(q)|}{\sqrt{n[\text{cov}(r)(q,q)]}} \]
\[ = \sqrt{nr(q)}^N \quad (4.49) \]

Comparing (4.49) to (4.6), it can be seen that the normalized Lagrange multiplier is \( \sqrt{n} \) times larger than the normalized residual, where \( n \) is the number of measurement scans used. Therefore, use of multiple scans allows differentiation of parameter errors from analog measurement errors for cases where this is impossible using a single scan estimation. The number of scans can be set by the user. Note that, two scans will be sufficient to make a parameter error identifiable. However, increasing the number of scans will improve the power of identification test by forcing the normalized quantities to assume relatively larger values. This observation of the relationship between normalized Lagrange multiplier and measurement residuals with multiple scans indicated by (4.49) will be verified by simulations in the next section.

### 4.5 Simulation Results

The proposed method is implemented and tested using the IEEE 14-, 30-, and 118-bus systems. All the cases provided below are comparatively evaluated using both the single snapshot and the multiple scan estimation methods. Some examples shown in [21] are re-used in order to illustrate the technical improvements of multiple scan method.

The limitation mentioned in Section 4.2 is that, when there is only one measurement incident to the erroneous parameter, the parameter error cannot be identified. This is due to the fact that the normalized Lagrange multiplier will always
be identical to the corresponding normalized residual. A typical example of this case is shown in [21] for a parameter error in a shunt capacitor/reactor where a reactive power injection is also measured. As illustrated in [21], using single scan estimation, the parameter error will be detected but not identified even with the highest possible local measurement redundancy.

Three examples including the ones shown in [21] are simulated and presented below. Errors are introduced to the shunt susceptances at bus 9 ($S_9$), bus 24 ($S_{24}$), and bus 34 ($S_{34}$) of the IEEE 14-bus, 30-bus and 118-bus systems respectively. A full set of conventional measurements (including all power flow, injection and voltage magnitude measurements) are used in these systems. The measurement weights are chosen based on the error variances used to create the synthetic measurement data and the weights remained the same during different measurement scans. Two tests are carried out: in test A, the single measurement scan estimation is used, whereas in test B the states are estimated using two measurement scans. The normalized Lagrange multipliers and normalized residuals are calculated for both tests and presented in Tables 25, 26 and 27 for the three systems.

The results in tables are organized in the following way. The first two columns show results of Test A with the single measurement scan method. The third and fourth columns present the results of Test B with double measurement scans. The largest three normalized Lagrange multipliers $\lambda^N$ and measurement residuals $r^N$ are listed in the tables. The first column shows the names of the parameters or measurements. The second column provides the value of their normalized Lagrange multipliers or measurement residuals. Note that only the largest one will be tested against the identification threshold to select the suspect bad data or parameter.
As evident in Table 25, for test A, the normalized measurement residual corresponding to the reactive power injection at bus 9 ($q_9$) is equal to the normalized Lagrange multiplier for the shunt susceptance ($S_9$). Therefore, the parameter or measurement errors can be detected but cannot be differentiated, i.e. identified. This limitation is lifted in test B which uses two measurement scans. Note that the normalized Lagrange multiplier for the erroneous parameter $S_9$ is now the largest among all normalized quantities. Furthermore, it is interesting to observe that the normalized residual for $q_9$ stays the same as in test A because the measurement residuals of different scans are independent. Tables 26 and 27 show similar results for the IEEE 30 and 118-bus systems. In addition to these tests, cases where errors are introduced to the reactive injection measurements have also been simulated and their errors were correctly identified by the proposed method.
The proposed method can also identify multiple parameter errors. In this case, as documented in [21], the errors will be identified one by one by repeating the normalized Lagrange multiplier test. In each identification cycle, only one error will be identified and corrected. The test needs to be performed repeatedly until no error is detected any more. Two examples using double measurement scans in IEEE 14-bus system and 30-bus system are shown below.

### TABLE 28 Results of Multiple Error Identification in IEEE 14-bus system

<table>
<thead>
<tr>
<th>Identification Cycle</th>
<th>Parameter/Measurement</th>
<th>$\lambda^N / r^N$</th>
<th>Parameter error identified</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$X_{1,5}$</td>
<td>58.9957</td>
<td>$X_{1,5}$</td>
</tr>
<tr>
<td></td>
<td>$X_{1,2}$</td>
<td>55.8358</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$X_{4,3}$</td>
<td>27.2402</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$S_{24}$</td>
<td>21.5902</td>
<td>$S_{24}$</td>
</tr>
<tr>
<td></td>
<td>$q_{24}(2)$</td>
<td>15.2682</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$q_{24}(1)$</td>
<td>15.2674</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$p_{1}(1)$</td>
<td>0.7423</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>$p_{2}(2)$</td>
<td>0.6937</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$p_{1,3}(1)$</td>
<td>0.3082</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 29 Results of Multiple Error Identification in IEEE 30-bus system

<table>
<thead>
<tr>
<th>Identification Cycle</th>
<th>Parameter/Measurement</th>
<th>$\lambda^N / r^N$</th>
<th>Parameter error identified</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$X_{2,6}$</td>
<td>53.1020</td>
<td>$X_{2,6}$</td>
</tr>
<tr>
<td></td>
<td>$X_{4,5}$</td>
<td>34.1718</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$X_{1,4}$</td>
<td>21.7401</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$S_{24}$</td>
<td>10.9721</td>
<td>$S_{24}$</td>
</tr>
<tr>
<td></td>
<td>$q_{24}(2)$</td>
<td>7.8153</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$q_{24}(1)$</td>
<td>7.7012</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$p_{1}(1)$</td>
<td>1.0215</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>$p_{2}(2)$</td>
<td>0.6985</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$p_{1,3}(1)$</td>
<td>0.4107</td>
<td></td>
</tr>
</tbody>
</table>

Table 28 shows the results obtained for the IEEE 14-bus system. Two parameter errors are introduced in reactance of line 1-5 ($X_{1,5}$) and shunt susceptance at bus 24 ($S_{24}$) simultaneously. Applying the two scan estimation, the error in $X_{1,5}$ is identified first. After correcting this error by the augmented state vector method [21], the normalized test is performed again and the error in $S_{24}$ is identified. Correcting that error and applying the normalized test again results in no more suspect bad data. Thus
both parameter errors are identified successfully. Similar simulation results are obtained for the IEEE 30-bus system and shown in Table 29.

In addition, another set of simulations are then undertaken in order to validate the observation presented in Section 4.4. It should be noted that all the simulation results are obtained without making any approximations, unlike the way (4.49) is derived.

<table>
<thead>
<tr>
<th>Test A (Three Scans)</th>
<th>Test B (Four Scans)</th>
<th>Test A (Five Scans)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter/Measurement</td>
<td>$\lambda^N/r^N$</td>
<td>Parameter/Measurement</td>
</tr>
<tr>
<td>$S_9$</td>
<td>25.7481</td>
<td>$S_9$</td>
</tr>
<tr>
<td>$q_9(1)$</td>
<td>15.2897</td>
<td>$q_9(1)$</td>
</tr>
<tr>
<td>$q_9(2)$</td>
<td>15.2746</td>
<td>$q_9(2)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test A (Three Scans)</th>
<th>Test B (Four Scans)</th>
<th>Test A (Five Scans)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter/Measurement</td>
<td>$\lambda^N/r^N$</td>
<td>Parameter/Measurement</td>
</tr>
<tr>
<td>$S_{24}$</td>
<td>19.9047</td>
<td>$S_{24}$</td>
</tr>
<tr>
<td>$q_{24}(2)$</td>
<td>11.7307</td>
<td>$q_{24}(2)$</td>
</tr>
<tr>
<td>$q_{24}(1)$</td>
<td>11.5988</td>
<td>$q_{24}(1)$</td>
</tr>
</tbody>
</table>

Simulations with three, four and five measurement scans using IEEE 14-bus system are carried out and the results are shown in Table 30. The measurement set and assumed parameter error are identical to the case shown in Table 25. Comparing the results shown in Table 30 and Table 25, it is observed that the Lagrange multiplier corresponding to the erroneous parameter increases with the number of measurement
scans (n). More specifically, as shown in Table 31, its value increases approximately by a factor of $\sqrt{n}$ with respect to the value obtained using a single scan. Similar results are obtained for IEEE 30-bus system as presented in Tables 32 and 33. These results strongly support the relationship derived in (4.49) despite the approximations used in its derivation. This observation can be useful in predicting the Lagrange multipliers when using multiple scans to further verify parameter identification results.

4.6 Conclusions

This chapter first provides a brief review of a recently developed method of parameter error identification and describes some inherent limitations of the method in detail. Then an extension of the method using multiple measurement scans is presented in order to address those limitations. The main advantage is that certain types of parameter errors that cannot be identified by the single scan method, become identifiable with this improvement. Furthermore, a simple expression that allows appropriate choice of the number of scans to be used is derived. The proposed algorithm can increase local redundancy without investing in new measurements. This is especially suitable for a large system with areas of low measurement redundancy but containing crucial parameters to monitor. This revision is easy to implement and provide an efficient tool to improve the accuracy and reliability of the parameter database at no additional cost.
5 Addressing Interoperability Issue of Phasor Measurements in State Estimation

5.1 Introduction

The use of Phasor Measurement Units (PMUs) has a history of over 20 years of research and development. Most recently several utilities and regional market operators have developed plans for large scale deployment of such a technology. Typically, these PMUs are manufactured and provided by different vendors. Therefore, many utilities will need to mix and match PMU solutions from multiple vendors due to various equipment purchasing practices and/or phased expansions of the system solution over an extended period of time. In addition, utilities may have to change PMUs from one vendor to another one due to aging or other problems. Since PMUs are still new comparing to the conventional measurements, there are few historical data can be obtained regarding their accuracies. All these factors bring a challenge into the state estimator that how these different PMUs will affect the state estimation results since they may have the different measurement accuracies. This chapter is focuses on addressing this interoperability issue in state estimator and providing systemic solutions in state estimation.

5.2 State estimator tuning for PMU measurements

State estimation problem is commonly formulated as an optimization problem which minimizes the weighted least squares of measurement residuals. Hence, an important parameter which is specified by the users of state estimators is the weight given to each measurement residual when solving this optimization problem. Historically, such weights were assigned based on the accuracy classes of measurements [5] and [49]. This may depend of the known or assumed accuracy of
the meters used in obtaining the measurement, as well as the errors involved in the chain of measurement devices and transformers between the point of actual measurement and the digital interface to the computational medium, such as the computer.

The recent trend is using PMU measurements in state estimators along with other conventional measurements [50]-[55]. It brings up the challenge of how to assign measurement weights that will be consistent despite the differences between the various types of measurements used by the estimator. This issue is significant in particular when one or more of the measurements carry errors. Most of the commonly used error detection and identification methods are based on processing of standardized measurement residuals, where the process of standardization involves user assigned measurement weights [56]-[58] and [21], [22]. Hence, inconsistently assigned weights may lead to incorrect decisions about the measurements. While some errors may be missed due to their low assigned weights, others may be flagged as bad data due to their very high assigned weights even if they carry statistically insignificant measurement errors.

This part presents a systemic method which can tune the weights associated with the PMU measurements, thus eliminating the need to pre-specify these weights. The approach does not depend on any manufacturer specified data, thus it can be used with different types of PMUs regarding the inter-operability issues.

5.2.1 Proposed Method

A. Problem formulation

In state estimation, the measurement equation is considered as:

\[
z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix} = \begin{bmatrix} h_1(x_1, x_2, \ldots, x_n) \\ h_2(x_1, x_2, \ldots, x_n) \\ \vdots \\ h_m(x_1, x_2, \ldots, x_n) \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix} = h(x) + e \tag{5.1}
\]
where

- \( z \) measurement vector with dimension of \( m \);
- \( x \) the system state vector with dimension \( n \);
- \( e \) measurement error vector with dimension of \( m \);
- \( h(x) \) nonlinear function of measurement with state vector.

The following basic assumptions are commonly made regarding the measurements:

1) The mean of each measurement error is zero:

\[
E(e_i) = 0, \quad i = 1, 2, \ldots, m
\] (5.2)

2) Measurement errors are independent so that

\[
cov(e) = E(e \cdot e^T) = R = \begin{bmatrix} \sigma_1^2 & \sigma_2^2 & \cdots & \sigma_m^2 \end{bmatrix}
\] (5.3)

where the \( \sigma_i \) is the standard deviation of the error of measurement \( i \), which indicates the accuracy of a measurement.

The weighted least squares estimation method aims to minimize the objective function given by:

\[
J(x) = \sum_{i=1}^{m} [z_i - h_i(x)]^2 \cdot W(i, i)
\]

\[
= [z - h(x)]^T \cdot W \cdot [z - h(x)]
\] (5.4)

where \( W = R^{-1} \) is called weight matrix, which is reciprocal of variance matrix. It is normally assumed that the weight matrix is given since the standard deviations of the measurements are known. However, this assumption may not be true, especially for PMUs. Therefore, the objective of the part is to design a self tuning algorithm to estimate the variances of PMU measurements, whose reciprocals are the weights applied to in state estimation.
Consider the first order approximation to the measurement equation around an initial state $x_0$:

$$\Delta z = H \Delta x + e$$  \hspace{1cm} (5.5)

where

$$\Delta z = z - h(x_0)$$  \hspace{1cm} (5.6)

$$H = \frac{\partial h}{\partial x}$$  \hspace{1cm} (5.7)

$$\Delta x = x - x_0$$  \hspace{1cm} (5.8)

Then the incremental change in state vector can be obtained by WLS estimator as follows [5]:

$$\Delta \hat{x} = G^{-1}H^TR^{-1}\Delta z$$  \hspace{1cm} (5.9)

where

$$G = H^TR^{-1}H$$  \hspace{1cm} (5.10)

The estimated measurement value is given by

$$\Delta \hat{z} = H \Delta \hat{x} = K \Delta z$$  \hspace{1cm} (5.11)

where

$$K = HG^{-1}H^TR^{-1}$$  \hspace{1cm} (5.12)

Note that $K$ has the property [5]:

$$(I - K)H = 0$$  \hspace{1cm} (5.13)

Then the measurement residuals can be calculated as below:

$$r = \Delta z - \Delta \hat{z}$$

$$= (I - K)(H \Delta x + e)$$

$$= (I - K)e$$

$$= Se$$  \hspace{1cm} (5.14)

where $S = I - K$ is called residual sensitivity matrix.

Using the special property below [5]:

78
\[ S \cdot R \cdot S^T = S \cdot R \] (5.15)

The covariance matrix of measurement residuals can be derived from (5.14):

\[
R_r = \text{cov}(r) \\
= S \cdot \text{cov}(e) \cdot S^T \\
= S \cdot R \cdot S^T \\
= S \cdot R 
\] (5.16)

As shown in (5.3), since \( R \) is diagonal, the diagonal elements in (5.16) will be related as follows:

\[
R(i,i) = \frac{R_r(i,i)}{S(i,i)} 
\] (5.17)

where \( R(i,i) \), \( R_r(i,i) \) and \( S(i,i) \) are the \( i \)th diagonal element of \( R \), \( R_r \) and \( S \) respectively. It is noted that the diagonal elements of \( R_r \) are just the variances of corresponding measurement residuals.

Equation (5.17) presents the relationship between variances of measurement residuals and variances of measurement errors, whose reciprocals are the weights. It allows the weights of measurements to be tuned through historical data by calculating variances of measurement residuals and sensitivity matrix. This theory has already been used to tune the weights of conventional measurements (including voltage magnitudes, power flows and injections) [59]. In this chapter, the algorithm will be revised and extended to the case of PMUs. An iterative self tuning method for PMUs will be presented with details.

One way to measure the state estimation accuracy is to use the metric based on the calculated normalized state errors given by [60]:

\[
NE = \frac{\| \hat{x} - x_{\text{perfect}} \|_2}{\| x_{\text{perfect}} \|_2} 
\] (5.18)

where
\( \hat{x} \) estimated state vector;

\( x_{\text{perfect}} \) perfect state vector before adding errors.

Normally it is not possible to calculate \( NE \) because of lack of information on \( x_{\text{perfect}} \). However, this will not be an issue when using the power flow solution to synthetically create system measurements for testing. This metric will be computed in order to illustrate the technical benefits of the proposed method on estimation accuracy.

### B. Proposed Iterative PMU Tuning Method

![PMU measurements in power system](image)

As shown in Fig. 22, PMUs provide significant benefits as they are deployed in power systems. They measure the phasor angles, which cannot be directly measured by any conventional measurements. As a novel measurement device, the statistical properties of its error may not always be well known. Also the errors of PMUs of different brands may vary. It is therefore important to design a tuning system, which can determine its error variance, whose reciprocal will be used as the weight in WLS estimator. In this section, an adaptive PMU tuning method will be presented in detail.

It is assumed that no preliminary information about PMUs’ error variances exists. Starting with large and equal variances for all PMUs, the tuning method will estimate
the variances of measurement errors iteratively according to the procedure outlined below.

**Step 1**: Take $k$ scans of the system measurements as historical data during a reasonable time period to ensure steady random error variances for all measurements. Note that the network topology and measurement set maintain the same during this period.

**Step 2**: Set up an initial guess of error variance for each PMU. The common value can be set as 1.0 for convenience if no other pre-information provided.

**Step 3**: Run WLS state estimation for all $k$ snapshots using the same variance as the initial guess in **Step 2**. Calculate all the PMU measurement residuals and their sample variances. Store the variances and corresponding sensitivity matrix.

**Step 4**: Compute the random error variance of each PMU using the variance of its measurement residuals and sensitivity matrix obtained in **Step 3** based on (5.17).

**Step 5**: Update the weight of the PMU by the reciprocal of variance calculated in **Step 4**, if the corresponding absolute deviation of this variance value compared to the previous one used in **Step 3** is larger than a threshold. Repeat **Step 3** to **Step 5** until all deviations are less than threshold or iteration limit has been reached. The weights of PMUs are the reciprocals of these corresponding variances.
Obtain k scans of system measurements
Set up the initial guess of PMU error variance as $R=I$. Weight matrix is $W=1/R$
Run state estimation for k scans respectively with weight matrix $W=1/R$
Calculate PMU measurement residuals $r_i$ for each scan and corresponding sensitivity matrix $S_i$
Store time series of residuals $r=[r_1, r_2, \ldots, r_k]$ and $S=S_i$
Calculate diagonal elements of $R_{\text{update}}$ by $R_{\text{update}}(i,i)=\text{cov}(r'(i,i))/S(i,i)$
Calculate maximum diagonal elements absolute deviation $\Delta = \max|R_{\text{update}}(i,i)-R(i,i)|$

Yes

Delta < Threshold

No

Update $R$ by $R=R_{\text{update}}$, only whose deviations are less than threshold
Obtain the PMUs weight matrix $W=1/R$

Fig. 23 Flow chart of the PMU tuning process

The entire process is shown in Fig. 23. The weights of PMUs are updated recursively at each iteration. As mentioned before, the system topology and measurement set do not change during different scans. Therefore, although the system states may change, during k scans within the same iteration where the weight matrix is the same, the approximation of sensitivity matrix $S$ will remain the same. $S$ will
change only if new weights of PMUs are updated for a new iteration. Also, when updating the weights, only those whose absolute deviations from their previously assigned values are larger than a certain threshold, will be updated. After this process, the weight matrix of corresponding PMUs will be obtained as results under convergence.

5.2.2 Simulation Results

In this section, the PMU tuning method will be implemented and tested on IEEE 14-bus system.

<table>
<thead>
<tr>
<th>Measurement Configuration in IEEE 14-bus System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Injection</td>
</tr>
<tr>
<td>Power Flow</td>
</tr>
<tr>
<td>Voltage Magnitude</td>
</tr>
<tr>
<td>PMU</td>
</tr>
</tbody>
</table>

In this example, eight measurement scans (k=8) are simulated for the system. The measurement configuration is maintained the same during all scans and is shown in Table 34. Notation “fully measured” means that each bus is assigned one power injection and one voltage magnitude measurement. Besides, each line is measured by two power flow measurements at its terminal buses. Measurement data of each scan are synthetically created by adding random errors with certain variances to a set perfect measurement value, which is obtained by solving the corresponding power flow problem. The bus loads are slowly varied between different scans and the power flow solutions will reflect these changes. The variances of errors added here will be used to verify the results of this tuning method. The original PMU weight is set as 1 and the threshold is chosen as $10^{-7}$.

<table>
<thead>
<tr>
<th>PMU</th>
<th>$R_{\text{perfect}}$</th>
<th>$R_{\text{origin}}$</th>
<th>$R_{\text{result}}$</th>
<th>$R_{\text{actual}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_2$</td>
<td>$10^{-6}$</td>
<td>1</td>
<td>$2.67 \times 10^{-7}$</td>
<td>$3 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\theta_6$</td>
<td>$10^{-6}$</td>
<td>1</td>
<td>$5.32 \times 10^{-7}$</td>
<td>$9 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\theta_7$</td>
<td>$10^{-4}$</td>
<td>1</td>
<td>0.0053</td>
<td>0.0052</td>
</tr>
</tbody>
</table>
It has to be clarified that all the conventional measurements and PMUs are tuned in this example. Only the results of PMUs are shown in Table 35 after the application of the proposed method. In Table 35, $R_{\text{perfect}}$ is the variance of the error added when creating synthetic measurement value. $R_{\text{origin}}$ is the initial guess of PMU error variance, whose reciprocal is used as weight. $R_{\text{result}}$ is the estimated variance of PMU errors after this tuning method. Since only eight scans are carried out, $R_{\text{actual}}$ is the actual sample variance of PMU errors, which can be closer to $R_{\text{perfect}}$ by increasing the scan number. Noted that the error at $\theta_7$ is set higher than others deliberately to test the performance of this method. It is evident from Table 35 that although only eight scans are used, and that $R_{\text{result}}$ is quite different from $R_{\text{origin}}$, it turns out to be very close to $R_{\text{perfect}}$ or $R_{\text{actual}}$ including the one corresponding to $\theta_7$. It can be concluded that the proposed PMU tuning method can successfully estimate the error variances of PMUs with no prior error information. Besides, it can identify different error classes between different PMUs. This will be further elaborated in future work, which will study the impact of these weights on subsequent bad data identification. In addition, normalized errors of state vectors are calculated and shown in Table 36 to illustrate the estimation accuracy. These normalized errors of system states have significantly dropped after the tuning.
procedure for every scan. Hence, the technique can be used to improve the system state estimation accuracy substantially. This method is easy to integrate into the existing state estimators. It can also be applied offline for the calibration of PMU measurements.

5.2.3 Conclusions

Section 5.2 presents a recursive PMU tuning algorithm to let the state estimation system estimate the variance of PMU errors and tune its corresponding weight using historical state estimation results. The method can be used both offline to calibrate PMU measurements and on-line as part of an existing state estimation program to estimate the PMU errors and tune the corresponding weights. The impact of using this method on bad data processing function will be investigated in the following sections.

5.3 An alternative way of determining the accuracy of a PMU measurement system

5.3.1 Proposed method

An alternative way of determining the accuracy of a PMU measurement system considers the case where users will have access to proper laboratory set-up that will allow calculation of the sample error variance of the PMU measurements directly from the finite sample of measured errors in the laboratory. It requires a laboratory test system set up including the entire chain of metering devices and communication systems modeling the actual PMU measurement system. Such a lab set-up was found at Texas A&M University and experiments were run at their facility. Collaboration and help provided by Texas A&M University’s research group, in particular Professor Kezunovic and his graduate students are gratefully acknowledged.
As mentioned before, since the measurement value is finally telemetered to the control center to be used by the state estimator, the measurement system errors will include the cumulative errors associated with PMUs and their instrument transformers, communication channels used by the data acquisition infrastructure and PDCs.

A test system of a typical PMU measurement system is shown in Fig. 24, which is composed of PMUs, their corresponding communication channels as well as a PDC. A reference signal is chosen for testing the PMU of interest. The output signal of the PMU system with respect to the reference signal is obtained from the output of PDC. For the same reference signal, the test is run as many times as possible to obtain high enough number of samples to make the sample variance converge to the true but unknown variance. Sample error will be defined as the absolute deviation between each measured output from the PDC and the reference signal. After obtaining all sample errors for the corresponding PMU system, with assumptions of (5.2) and (5.3), its sample error variance can be calculated as follows:

$$var(e) = \frac{1}{n-1} \sum_{i=1}^{n} [e_i - E(e)]^2$$  \hspace{1cm} (5.19)
where

\[ n \text{ sample number; } \]

\[ e_i \text{ sample error.} \]

This calculation will be repeated for each PMU under investigation.

These two methods provide tools to obtain the accuracies of various types of PMUs. The method presented in Section 5.2 takes advantage of the state estimator and estimates the accuracies iteratively, which requires access to the results of state estimation for several measurement scans. This method calculates the accuracies statistically through laboratory tests, in which the entire measurement chain including all devices and communication links should be taken into account. The results from two methods are expected not to differ provided that the actual sources of noise and errors are properly accounted for by the experimental set up.

### 5.3.2 Simulation results

The PMU measurement system including PMUs, communication channels as well as PDC that is used for this work is shown in Fig. 24. A reference voltage signal of known magnitude and angle is applied to each type of PMU and tested 100 times. For each type of PMU, 100 corresponding output signals are obtained and 100 absolute sample voltage phase angle errors of this PMU measurement system are calculated. The results of two different types of PMUs are shown in Table 36. As mentioned, the error shown here is defined as the absolute deviation between measured output voltage phase angle from the PDC and the reference voltage phase angle. The unit of the error is shown in degree.

After obtaining all these samples, the error variance of the phase angles measured by this PMU system can be calculated assuming zero expected errors. Computed error variances in “radians” for the two types of PMUs tested are $1.86 \times 10^{-6}$ and
6.64×10^{-6} as shown in Table 37. These values appear to be in the same order of magnitude as those shown in Table 35, which are used to simulate state estimation tuning method. If the same types of PMUs are used in the actual system, the results obtained by this method can be compared and validated using the ones obtained by state estimation results. However, it’s noted that, in this work, the PMU systems tested are not the ones incorporated into the state estimator. These methods provide two distinct ways to quantify accuracies of PMU measurement systems. They can be used to assign consistent weights to the measurements and thus improve the performance of the state estimation.

**TABLE 37 Results of PMU Accuracy Test (Errors shown in degree)**

<table>
<thead>
<tr>
<th>Test #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>0.078</td>
<td>0.077</td>
<td>0.078</td>
<td>0.078</td>
<td>0.077</td>
<td>0.078</td>
<td>0.078</td>
<td>0.077</td>
<td>0.078</td>
<td>0.078</td>
</tr>
<tr>
<td>on PMU1</td>
<td>0.143</td>
<td>0.135</td>
<td>0.15</td>
<td>0.157</td>
<td>0.145</td>
<td>0.148</td>
<td>0.144</td>
<td>0.144</td>
<td>0.154</td>
<td>0.13</td>
</tr>
<tr>
<td>Error</td>
<td>0.078</td>
<td>0.078</td>
<td>0.077</td>
<td>0.078</td>
<td>0.078</td>
<td>0.077</td>
<td>0.078</td>
<td>0.078</td>
<td>0.078</td>
<td>0.078</td>
</tr>
<tr>
<td>on PMU2</td>
<td>0.147</td>
<td>0.139</td>
<td>0.158</td>
<td>0.153</td>
<td>0.151</td>
<td>0.161</td>
<td>0.163</td>
<td>0.147</td>
<td>0.156</td>
<td>0.147</td>
</tr>
<tr>
<td>Test #</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td>Error</td>
<td>0.078</td>
<td>0.077</td>
<td>0.078</td>
<td>0.078</td>
<td>0.078</td>
<td>0.078</td>
<td>0.078</td>
<td>0.078</td>
<td>0.078</td>
<td>0.078</td>
</tr>
<tr>
<td>on PMU1</td>
<td>0.138</td>
<td>0.15</td>
<td>0.143</td>
<td>0.137</td>
<td>0.159</td>
<td>0.148</td>
<td>0.153</td>
<td>0.143</td>
<td>0.147</td>
<td>0.135</td>
</tr>
<tr>
<td>Error</td>
<td>0.077</td>
<td>0.078</td>
<td>0.078</td>
<td>0.077</td>
<td>0.078</td>
<td>0.078</td>
<td>0.078</td>
<td>0.078</td>
<td>0.078</td>
<td>0.077</td>
</tr>
<tr>
<td>on PMU2</td>
<td>0.133</td>
<td>0.142</td>
<td>0.138</td>
<td>0.151</td>
<td>0.149</td>
<td>0.149</td>
<td>0.157</td>
<td>0.146</td>
<td>0.141</td>
<td>0.142</td>
</tr>
<tr>
<td>Test #</td>
<td>41</td>
<td>42</td>
<td>43</td>
<td>44</td>
<td>45</td>
<td>46</td>
<td>47</td>
<td>48</td>
<td>49</td>
<td>50</td>
</tr>
<tr>
<td>Error</td>
<td>0.078</td>
<td>0.077</td>
<td>0.078</td>
<td>0.078</td>
<td>0.077</td>
<td>0.078</td>
<td>0.078</td>
<td>0.077</td>
<td>0.078</td>
<td>0.078</td>
</tr>
<tr>
<td>on PMU1</td>
<td>0.129</td>
<td>0.143</td>
<td>0.142</td>
<td>0.151</td>
<td>0.141</td>
<td>0.153</td>
<td>0.156</td>
<td>0.146</td>
<td>0.145</td>
<td>0.15</td>
</tr>
<tr>
<td>Error</td>
<td>0.077</td>
<td>0.077</td>
<td>0.077</td>
<td>0.077</td>
<td>0.078</td>
<td>0.077</td>
<td>0.077</td>
<td>0.077</td>
<td>0.078</td>
<td>0.077</td>
</tr>
<tr>
<td>on PMU2</td>
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<td>0.153</td>
<td>0.133</td>
<td>0.154</td>
<td>0.163</td>
<td>0.158</td>
<td>0.15</td>
<td>0.162</td>
<td>0.136</td>
<td>0.142</td>
</tr>
<tr>
<td>Test #</td>
<td>PMU1</td>
<td>PMU2</td>
<td>PMU1</td>
<td>PMU2</td>
<td>PMU1</td>
<td>PMU2</td>
<td>PMU1</td>
<td>PMU2</td>
<td>PMU1</td>
<td>PMU2</td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
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</tr>
<tr>
<td>61</td>
<td>0.078</td>
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<td>0.078</td>
<td>0.129</td>
<td>0.078</td>
<td>0.151</td>
<td>0.078</td>
<td>0.157</td>
<td>0.078</td>
<td>0.135</td>
</tr>
<tr>
<td>62</td>
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<td>0.077</td>
<td>0.147</td>
<td>0.078</td>
<td>0.151</td>
<td>0.078</td>
<td>0.157</td>
<td>0.078</td>
<td>0.135</td>
</tr>
<tr>
<td>63</td>
<td>0.078</td>
<td>0.142</td>
<td>0.077</td>
<td>0.143</td>
<td>0.078</td>
<td>0.146</td>
<td>0.078</td>
<td>0.159</td>
<td>0.078</td>
<td>0.145</td>
</tr>
<tr>
<td>64</td>
<td>0.078</td>
<td>0.143</td>
<td>0.078</td>
<td>0.147</td>
<td>0.077</td>
<td>0.158</td>
<td>0.077</td>
<td>0.169</td>
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<td>0.154</td>
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<td>0.078</td>
<td>0.143</td>
<td>0.078</td>
<td>0.147</td>
<td>0.078</td>
<td>0.158</td>
<td>0.077</td>
<td>0.169</td>
<td>0.078</td>
<td>0.154</td>
</tr>
<tr>
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<td>0.143</td>
<td>0.078</td>
<td>0.147</td>
<td>0.078</td>
<td>0.158</td>
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<td>0.169</td>
<td>0.078</td>
<td>0.154</td>
</tr>
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<td>0.078</td>
<td>0.147</td>
<td>0.078</td>
<td>0.158</td>
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<td>0.169</td>
<td>0.078</td>
<td>0.154</td>
</tr>
<tr>
<td>68</td>
<td>0.078</td>
<td>0.143</td>
<td>0.078</td>
<td>0.147</td>
<td>0.078</td>
<td>0.158</td>
<td>0.077</td>
<td>0.169</td>
<td>0.078</td>
<td>0.154</td>
</tr>
<tr>
<td>69</td>
<td>0.078</td>
<td>0.143</td>
<td>0.078</td>
<td>0.147</td>
<td>0.078</td>
<td>0.158</td>
<td>0.077</td>
<td>0.169</td>
<td>0.078</td>
<td>0.154</td>
</tr>
<tr>
<td>70</td>
<td>0.078</td>
<td>0.143</td>
<td>0.078</td>
<td>0.147</td>
<td>0.078</td>
<td>0.158</td>
<td>0.077</td>
<td>0.169</td>
<td>0.078</td>
<td>0.154</td>
</tr>
</tbody>
</table>

5.3.3 Conclusions

This section provides an alternative method to obtain PMU accuracy information, which is based on laboratory tests modeling the chain of error sources influencing the overall errors of PMU measurements. Both this method and the one via state estimator tuning can be used to improve the performance of state estimators when PMU measurements are incorporated in their measurement sets.

5.4 Impact of weights tuning in bad data detection in state estimation

An on-line tuning method for determining weights for PMU measurements has been presented in Section 5.2. The method does not rely on any preliminary information such as manufacturer’s data about the measurement devices and determines the weights according to historical data on the measurements over a certain period of time, thus eliminating the need to pre-specify these weights by the
user. It can be used with different types of PMUs, addressing the issue of interoperability as well. Using accurate weights for measurements, in particular for new PMU measurements is important for successful bad data detection. The objective of this section is to illustrate the benefits of applying the derived tuning process to improve error detection and identification of PMU measurements.

5.4.1 Proposed method

The proposed method requires complete access to the results of state estimation for a sequence of measurement scans. Accurate measurement weights of PMUs can be obtained without pre-specifying them before estimation. This is utilized in the proposed method to improve the robustness of bad data detection. The flowchart of proposed method is shown in Fig. 25. The detailed explanations are introduced as follows.

*Step1*: Take an initial guess of measurement weights for each PMUs instead of pre-specifying this data based on historical accuracy data.

*Step2*: Apply the measurement tuning algorithm to PMU measurements and obtain their weights. The algorithm details are reviewed as before.

*Step3*: Run state estimation using the weights obtained in *Step2* for PMUs and bad data detection process. The normalized measurement residual test is performed to detect and identify bad data using the measurement weights obtained in *Step2* for PMUs.

*Step4*: Obtain the robust bad data detection and identification result.
This method can improve the performance of bad data detection of PMU measurements by using the accurate weights obtained by measurement tuning. It is easy to integrate to existing state estimator and enhances the robustness of bad data processing.

5.4.2 Simulation results

The proposed method has been implemented in IEEE 14-bus system. The simulation results are presented as below.

**TABLE 38 Measurement Configuration in IEEE 14-bus System**

<table>
<thead>
<tr>
<th>Power Injection</th>
<th>At bus 2, 5, 9, 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Flow</td>
<td>At branch 1-5, 2-3, 2-4, 3-4, 4-7, 4-9, 5-6, 6-12, 6-11, 7-8, 9-14, 9-10, 10-11, 12-13, 13-14</td>
</tr>
<tr>
<td>Voltage Magnitude</td>
<td>At bus 2, 3, 4, 5, 6, 8, 9</td>
</tr>
<tr>
<td>PMU</td>
<td>At bus 2, 3, 4, 5, 6, 8, 9</td>
</tr>
</tbody>
</table>

In this example, eight measurement scans are obtained for different loading conditions. The measurement configuration is shown in Table 38, which corresponds to an observable system. Measurement configuration maintains the same for all scans and measurement values are computed synthetically as done in [61]. It is noted that
the weights for the conventional measurements are assumed to be already assigned. Only the weights of PMUs are tuned in this example. The initial guess for the error variances of PMUs is randomly set as 0.0001 radian².

**TABLE 39 Results of PMU Tuning Process in IEEE 14-bus System**

<table>
<thead>
<tr>
<th>PMU</th>
<th>R_perfect</th>
<th>R_origin</th>
<th>R_results</th>
<th>R_actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ₂</td>
<td>10⁻⁶</td>
<td>10⁻⁴</td>
<td>2.71×10⁻⁷</td>
<td>2.3×10⁻⁷</td>
</tr>
<tr>
<td>θ₃</td>
<td>10⁻⁶</td>
<td>10⁻⁴</td>
<td>9.26×10⁻⁸</td>
<td>4×10⁻⁷</td>
</tr>
<tr>
<td>θ₄</td>
<td>10⁻⁶</td>
<td>10⁻⁴</td>
<td>7.41×10⁻⁷</td>
<td>2.4×10⁻⁴</td>
</tr>
<tr>
<td>θ₅</td>
<td>10⁻⁶</td>
<td>10⁻⁴</td>
<td>8.42×10⁻⁷</td>
<td>1.8×10⁻⁷</td>
</tr>
<tr>
<td>θ₆</td>
<td>10⁻⁶</td>
<td>10⁻⁴</td>
<td>2.63×10⁻⁷</td>
<td>6×10⁻⁷</td>
</tr>
<tr>
<td>θ₇</td>
<td>10⁻⁶</td>
<td>10⁻⁴</td>
<td>3.78×10⁻⁸</td>
<td>1.5×10⁻⁷</td>
</tr>
<tr>
<td>θ₈</td>
<td>10⁻⁶</td>
<td>10⁻⁴</td>
<td>8.69×10⁻⁷</td>
<td>4×10⁻⁷</td>
</tr>
</tbody>
</table>

The results of simulations are shown in Table 39 after the application of the tuning process. There are seven PMUs are tuned in this example. \(R\_\text{perfect}\) is error variance used when adding the random error to perfect phase angle to create synthetic PMU measurement value. \(R\_\text{origin}\) is the initial guess of error variance made for PMU. \(R\_\text{results}\) is result obtain from tuning process, which is estimated error variance of PMU. \(R\_\text{actual}\) is the actual sample variance of PMU errors of these eight scans. With the scan number increased, \(R\_\text{actual}\) is anticipated to converge to \(R\_\text{perfect}\). As shown in Table 39, the error variance of each PMU can be successfully estimated. It can be expected that the results will be more accurate by increasing the number of scans used as well as the measurement redundancy.

After this tuning procedure, the proposed method utilizes the weights of PMUs in bad data detection. Two simulation examples are provided to show its benefits.

**TABLE 40 Results of Bad Data Detection of Example 1**

<table>
<thead>
<tr>
<th>Test A (Without tuning)</th>
<th>Test B (With tuning)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement</td>
<td>(r^b)</td>
</tr>
<tr>
<td>(</td>
<td>V_6</td>
</tr>
<tr>
<td>(θ_2)</td>
<td>1.7526</td>
</tr>
<tr>
<td>(</td>
<td>V_4</td>
</tr>
<tr>
<td>(Q_{2,8})</td>
<td>1.4206</td>
</tr>
<tr>
<td>(</td>
<td>V_8</td>
</tr>
</tbody>
</table>
The first example is to investigate the impact of tuning on bad data detection when PMUs are under weighted. Two cases are presented for comparison with the results shown in Table 40 respectively.

Test A: Without tuning process, the initial guess of PMU measurement weight made before is directly used for bad data detection, where the PMUs are with less measurement weights.

Test B: Through proposed method, the weights of PMU measurements obtained by tuning process are applied for bad data detection.

In this example, an error has been introduced to $\theta_2$, whose value has been changed from -5.06 to -6.06 degrees. In Test A, the weights of PMUs are set as reciprocals of $R_{\text{origin}}$ in Table 39. In Test B, the reciprocal of $R_{\text{result}}$ are assigned as weights of PMUs after tuning. As shown in Table 40, without tuning for PMUs, the largest normalized measurement residual is less than 3.0, which is commonly chosen as the threshold as mentioned. It indicates that, without tuning, the measurement error at $\theta_2$ cannot be detected or identified. By contrast, after implementing the proposed method, such error has been successfully detected and identified in Test B, where the normalized residual corresponding to $\theta_2$ is in the top and beyond the threshold. It can be concluded that, under weighting a PMU measurement may cause its bad data undetectable. By tuning the weights of PMUs, the proposed method detects and identifies this error successfully.

The second example is to simulate the scenario when PMU measurements are over weighted and investigate the impact of proposed method. In this example, no measurement error is introduced. All the measurements only have their random noises. Two tests are performed to compare the results with or without measurement tuning.
Test A: Without tuning process, the PMU measurement weight is made as reciprocal of $10^{-8}$, which is considered as over weighted.

Test B: The measurement weights of PMUs obtained by tuning, which are $R_{result}$ in Table 39, are used for bad data detection.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>$r^A$</th>
<th>Measurement</th>
<th>$r^B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_2$</td>
<td>5.8195</td>
<td>$P_{4,9}$</td>
<td>2.4227</td>
</tr>
<tr>
<td>$\theta_5$</td>
<td>5.2493</td>
<td>$P_{13,14}$</td>
<td>1.9625</td>
</tr>
<tr>
<td>$P_5$</td>
<td>4.7343</td>
<td>$\theta_9$</td>
<td>1.7754</td>
</tr>
<tr>
<td>$P_2$</td>
<td>4.6599</td>
<td>$P_{6,12}$</td>
<td>1.5099</td>
</tr>
<tr>
<td>$\theta_4$</td>
<td>3.3441</td>
<td>$Q_{2,4}$</td>
<td>1.4209</td>
</tr>
</tbody>
</table>

As shown in Table 41, in Test A, the largest normalized residual corresponding to $\theta_2$ is beyond than the threshold. It indicates that, although there is no measurement error, $\theta_2$ is detected as erroneous incorrectly. However, by assigning accurate weights of PMUs through proposed method, no measurement error is detected in Test B. Therefore, over weighting PMUs may bring up false error detection at certain measurements. This problem can be solved by measurement tuning through proposed method. It can be concluded that the proposed method improves the robustness of bad date detection and identification of PMUs in state estimation.

5.4.3 Conclusions

Previous work on assigning measurement weights of PMU measurements develops a recursive tuning algorithm to let the state estimation system estimate the variances of PMU errors and tune their weights respectively. This section investigates its impact on bad data detection of PMU measurements and presents a systemic method to improve bad data detection of PMU measurements. The proposed method is convenient to integrate to existing WLS state estimators and enhances the robustness of error detection and identification for PMU measurements.
5.5 Conclusions of the chapter

In this chapter, a recursive tuning algorithm has been developed to estimate the variances of PMU errors and assign corresponding measurement weights in the state estimation without any pre-information about these data. Then an alternative method has been presented by accessing proper laboratory set-up that will allow calculation of the sample error variance of the PMU measurements directly from the finite sample of measured errors. In addition, the impact on bad data detection of PMU measurements has been investigated. An improved method has been proposed to integrate existing WLS state estimators and enhances the robustness of error detection and identification for PMU measurements.
6 Conclusions and future work

6.1 Conclusions

This dissertation focuses on improving power system state estimation by innovative use of phasor measurements. Chapter 2 to 4 focus on parameter error identification by incorporating phasor measurements strategically. Chapter 5 addresses the interoperability issues of phasor measurement units in state estimation. The main contributions of this dissertation are listed as below:

(1) A numerical algorithm is developed to identify all single and double edge cutsets in large scale power networks. This algorithm is based on factorization of graph incidence matrix, which is computationally efficient and easy to implement. It provides the capability of identifying all topologies for strategic placement of phasor measurements based on parameter error identification.

(2) A strategic phasor measurement placement is proposed to improve parameter error identification. The existing method encounters a limitation that the parameter errors on single and double edge cutsets cannot be identified without phasor measurements. This systematic method provides a solution to ensure identification of all those network parameter errors with the least number of phasor measurements. It can be applied to the entire system or certain areas requiring particular attention on parameter errors.

(3) A post disturbance island identification method via state estimation is introduced. It can identify power islands based on the state estimation results with the help of phasor measurements.

(4) An alternative parameter error identification method using multiple measurement scans is presented. This method successfully identifies certain parameter errors, which cannot be identified by existing single scan method, without requiring
new measurements. It also derives the relationship between the number of measurement scans, the value of normalized Lagrange multipliers and normalized measurement residuals. This allows proper choice of the number of scans to be used to customize the proposed method and verify the estimation results.

(5) A PMU weights tuning method is proposed to assign appropriate measurement weights for different PMUs incorporated in state estimator. It takes advantages of the state estimation results to estimate the PMU accuracies iteratively, which eliminates the need of pre-specifying the weights for PMUs. An alternative method is then proposed to calculate the accuracies statistically through laboratory tests, in which the entire measurement chain including all devices and communication links should be taken into account.

(6) The impact on bad data detection and identification are investigated to illustrate the benefits brought by proposed tuning method. Then a systemic method is presented to improve bad data detection of PMU measurements. This method is convenient to integrate to existing WLS state estimators and enhances the robustness of error detection and identification for PMU measurements.

All the above outlined investigations have been simulated on different systems, whose results are shown in different chapters respectively. The entire dissertation lies on introducing different methods to improve power system state estimation by innovative ways of applying phasor measurements.

6.2 Future work

The system model used in this dissertation is a single phase positive sequence power network. It assumes that the system is always balanced, which is the most
common case in transmission network. Today’s utilities are also moving toward smart power distribution systems with typical characteristics including higher penetration of distributed generation (DG) from renewable energy, incorporation of smart sensors, and demand response (DR). All of these applications turn to an acute need for a smart state estimator in the distribution level to provide real time systems monitoring and control inside. Different from transmission network, the distribution system is not always 3-phase balanced. So a 3-phase distribution system state estimation is needed. Previously, lack of sensors in distribution systems limited the performance of state estimator. However, with the recent evolution of smart sensors such that PMUs have been deployed into distribution level, it brings a great foundation of developing a distribution level state estimator. Therefore, future research may be focused on distribution system 3-phase state estimation. Based on that, the measurement and parameter error identification are still challenges which require future research.
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VITA

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List of Publications


