WIND-BORNE DEBRIS TRAJECTORY IN HIGH WINDS: APPLICATION TO THE PROTECTION OF TALL BUILDING FAÇADES

A Dissertation Presented

By

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Abstract

Non-structural damage to the façade of tall buildings, particularly those exposed to high wind speeds, is often caused by the impact of wind-borne debris. This dissertation describes the development of a probability-based framework for the analysis of debris trajectory in simulated boundary layer winds and for the prediction of the probability of impact against the vertical façade of these buildings. The work focuses on compact debris, i.e., objects of small dimensions and negligible mass moment of inertia (e.g., roof ballast elements, gravel, “bluff” shingles, etc.) The trajectories were computed for three different wind fields: (i) uniform wind field with constant horizontal velocity and no turbulence, (ii) “sudden” vertical gust superimposed to the uniform wind field and (iii) fully turbulent wind field.

Debris elevations at takeoff, drag coefficient and Tachikawa number were modeled as random parameters to estimate the trajectory and to derive “Iso-probability Impact Contours”. These contours describe the impact probability associated with “randomly flying” objects, as they hit against a cladding element located on the vertical façade of a building. Monte Carlo methods were used to estimate the probabilities. Object momentum and angle at impact were also calculated.
This study also proposed and investigated the use of a “Universal Probability Curve”, describing the probability of impact for objects against the façade of a benchmark tall building as a function of distance. It was proven that this curve is “universal” and can be used independently of wind velocity.

Moreover, turbulence effects on the trajectory of wind-borne debris were investigated. First, a “Küssner-like” sudden vertical gust model was proposed to account for the influence of local recirculation in the flow around a building. Second, trajectories were estimated in fully turbulent winds in 2D; it accounts for atmospheric turbulence and wind shear effect, by modeling a variable mean velocity with elevation. Synthetically-generated and partially-coherent turbulence time histories were digitally simulated by the wave superposition method.

Simulated trajectory results were compared to experimental data derived both from literature and wind tunnel experiments, conducted at Northeastern University.
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Chapter 1

Introduction

1.1 Statement of the problem

Wind-resistant design of tall buildings, based on performance of the structural and non-structural elements, has recently received significant attention by researchers in wind engineering (Smith and Caracoglia 2011). Non-structural damage to the façade of tall buildings, particularly those exposed to high winds, is often caused by the impact of wind-borne debris. “Wind-borne debris” (e.g., (Baker 2007)) are flying objects, transported by wind during extreme events such as hurricanes. For example, after wind-induced failure, building or building envelope components can become airborne as “missiles” and can cause significant damage to the surrounding buildings, especially, on cladding elements on the façade of a tall structure. It also has been observed as a principal cause of damage in extreme historical wind events. For example, in Figure 1.1 (a), the glass façade of the Hyatt hotel is shown after hurricane Katrina in New Orleans. The façade is severely damaged due to the debris impact, as confirmed by the forensic investigation described in a report by Bashor (Bashor et al. 2012). Also, after Hurricane
Ike (2006) in Houston, it was similarly observed that the glass openings in one building had been shattered as a result of wind-borne debris (Figure 1.1 (b)).

This dissertation describes the development of a probability-based simulation method for the analysis of debris trajectory in complex wind fields (e.g., influence of atmospheric boundary layer, wake effects generated by other structures or obstacles) and for the prediction of the probability of impact against the vertical building façade. Evaluation of the damage, induced by wind-borne debris, cannot be overlooked if the objective of the structural building design includes the assessment of “vulnerability” of vertical façades against wind hazards. While damage surveys following severe storms are frequently conducted, there is limited data on the flight distances and debris velocity during the flight and, most importantly, at impact. This is the reason why a proper model for trajectory in turbulent winds, followed by wind-tunnel validation, is necessary; this model would also enable the use of probability-based methods to investigate the vulnerability against debris impact.
1.2 Motivation

Traditionally, the study of wind-borne debris trajectory has been a concern for special structures such as nuclear power plants due to the fact that they are highly important facilities; the design of critical structures must include the analysis of damages and failures caused by extremely rare wind events, such as tornades (Simiu and Cordes 1976; Twisdale et al. 1979), which are not currently considered for other buildings. Consideration of wind-borne debris in the design of residential building has begun to attract the attention of researchers in recent years and, especially in the United States. The importance of wind-borne debris modeling became more obvious after damage was observed during the landfall of Hurricanes Katrina and Ike in urban areas due to wind-borne debris impact (Divel et al. 2010). Also, more recently, extensive non-structural
damages due to debris impact were observed after the passage of the Joplin tornado in 2011.

It was stated by Holmes (Holmes 2008) that four components should be considered for modeling wind-borne debris damage. These features are: (i) “wind-field model” (Holmes 2004; Vickery et al. 2000; Vickery and Twisdale 1995); (ii) “debris generation model”, which is needed to describe the conditions at which the debris become an air-borne (Kordi and Kopp 2011; Wills et al. 2002); (iii) “debris trajectory model”, addressing the questions regarding how far the debris can go and what velocity can be reached during the flight (Holmes 2004; Holmes et al. 2006; Lin et al. 2006; Moghim and Caracoglia 2012b; Tachikawa 1983; 1988); and, assuming the availability of a trajectory model, a “probability debris impact model” is needed to analyze the damage caused by wind-borne debris (Lin and Vanmarcke 2008; Moghim and Caracoglia 2012a; 2012b).

Wills et al. (2002) categorized wind-borne debris by their lift-off characteristics and aerodynamic properties into three groups: compact, plate-like, and rod-like. Most recent numerical and experimental work, investigated the prediction of the debris trajectory, predominantly in two dimensions (2D) (Lin et al. 2007; Tachikawa 1983). Holmes et al. (2006) used equations of motion to predict horizontal plate-type object trajectory in uniform 2D flow for applications to impact testing. Kordi et al. (2009) showed by wind tunnel experiments that the initial conditions of the debris, buoyancy parameter and initial rotation considerably affect the flight trajectory of flat plates in a uniform 2D flow.
Also, Lin et al. (2007) proposed empirical equations by using wind tunnel tests to predict compact debris trajectory and the terminal velocity. The terminal velocity is defined as the maximum velocity that is reached by an object, after which equilibrium between external forces is obtained and the object continues its steady motion without any changes in speed. More recently attention was devoted to three dimensions (3D) (e.g., (Noda and Nagao 2010; Richards et al. 2008)).

Despite these important advances, a realistic model for trajectory estimation and for prediction of the impact against the building envelope is still not available. Also, in spite of the fact that several studies have been carried out to estimate the debris trajectory, most of them only consider a uniform wind field. This assumption implies that the wind shear and turbulence effects are negligible (or simulated in an indirect and simplified way). Even though it has been suggested that the hypothesis of uniform wind field with constant velocity is acceptable due to the limited flight duration (Holmes 2004), the relevance of investigating the stochastic nature of debris motion in a fully-developed turbulent wind field has been recently pointed out (Karimpour and Kaye 2012).

In an attempt to investigate the stochastic nature of debris flight, this dissertation describes the implementation of a series of numerical models for the prediction of the risk of damage, caused by wind-borne debris. The objective of the available methods is to evaluate the damage risk at the “regional scale” (e.g., overall risk estimation for an entire residential area) for vulnerability of low-rise buildings (e.g., (Lin and Vanmarcke 2008;
(Barbato et al. 2013)). However, less attention has been paid to the “building scale” and the need for analyzing failure risk for the cladding of a large tall building. Tall buildings are important structures, which are particularly exposed to debris damage because of the large glass claddings on their façade (Divel et al. 2010); the repair costs and downtimes may be significant on a tall structure and must be avoided. For example, the results of a recent numerical study indicate that the projected repair costs on low-rise residential buildings of an idealized 100m by 100m area, impacted by the landfall of a hurricane, can vary between 6% and 16% of the initial value of the properties; most importantly the estimation of the repair costs can be underestimated by as much as 29% if debris is not considered (Chung Yau et al. 2011). Also, in urban areas loose particles on the top of a roof can present a potential hazard during hurricanes (Jain 2013). As the wind velocity increases during the storm, loose particles can become wind-borne and take off from the roof. These flying particles can “hit” other buildings, other objects and cause injury to humans. On occasion, they can cause serious life and/or property damage.

In order to clarify the risk associated with wind-borne debris flight, this study will investigate the behavior of the debris once it becomes an airborne, the rate at which it is removed from a “source” (e.g., another built-up roof), and the traveled flight distance.

Most importantly, the main motivation for this study is related to the need to propose a systematic method for the analysis of debris flight at the “building scale” (i.e. in the proximity of its façade) and evaluate, in a probabilistic setting, potential debris damage.
Currently, such a method does not exist; this dissertation is devoted to the advancement and verification of this research idea.

1.3 Dissertation outline

The material, included in this dissertation, is presented as eight self-contained pieces of research. As such each chapter will contain its own introduction, literature review, methodology, outcomes, summary and references. The chapters are as follows:

In Chapter 1, the introduction and research motivation were described. The importance and need for the study of wind-borne debris was described along with the overall research outline.

In Chapter 2, of this dissertation, the basic model for the trajectory of compact objects is analyzed and a numerical procedure is presented for solving the equations of motion. The trajectory result is validated by empirical equations proposed by Lin et al. (2007). In this chapter a uniform wind field, in which constant wind velocity is assumed during the debris flight, is used. Some important non-dimensional parameters that have an effect on the trajectory are also introduced.

The analysis of the results of a recent forensic study, conducted to analyze hurricane-induced damages caused by compact debris on the façade of a tall building in Florida, indicated that compact debris can have an unusual upward trajectory (Jain 2013) due to the effect of a nearby building on the flow. In order to investigate the effect of wake recirculation, induced by nearby buildings, on the trajectory a simplified “Küssner-like”
sudden vertical gust model is proposed in Chapter 3. This “Küssner-like” vertical upward gust is added to the equations of motion in the uniform wind field at the early stages of the flight. In order to further generalize the effect of a vertical gust a trajectory model in fully turbulent wind field is proposed, based on wave-superposition method proposed by (Di Paola 1998). The trajectory of compact debris is also investigated in fully turbulent wind field and compared with the uniform wind field results. The detail of the fully turbulent wind field model and the effect on compact object trajectory is described in Chapter 4.

The primary objective of this dissertation is to model the probability of impact on tall building façades. The probabilistic model, which is developed in this study, is discussed in Chapter 5 and 6 in 2D and 3D, respectively. The novel concepts of “Universal Impact Curves” and “Iso-Probability Contours” are presented and described. These curves and contours are estimated and compared for three different wind field models, described in previous chapters.

For validation of the model in Chapter 5 because of the stochastic nature of the problem and lack of the data from the field, wind tunnel experiments are carried out in the Northeastern University’s small-scale wind tunnel. The experimental setup and procedure are explained in Chapter 7. The comparison between numerical and experimental results is described and documented in this chapter.
The work discussed and explained in this dissertation has been published in (Moghim and Caracoglia 2012a; 2012b), where the analytical and numerical model was developed for three different wind fields and Moghim, Xia and Caracoglia (2013, submitted), where the experimental investigation and validation were presented.
1.4 References


Chapter 2

Basic model for trajectories in uniform wind and deterministic debris parameters (compact debris)

2.1 Introduction

In order to estimate the damage due to wind-borne debris, having a proper model for the trajectory of the objects is important. In this chapter, an existing model for the debris flight in uniform wind field was utilized to simulate “compact debris” trajectory. A compact debris is defined (Wills et al. 2002) as an object of small dimensions, negligible moment of inertia compared to its mass, and for which the aerodynamic effect of lift force can usually be neglected. Cubes and spheres can be used as a good approximation for the shape of compact objects. As was mentioned in Chapter 1, most available numerical and experimental studies have attempted to predict the trajectory of the compact objects in 2D uniform flow (Holmes 2004; Tachikawa 1983; Wills et al. 2002). Few studies only have analyzed in detail three-dimensional nature of the flight (e.g., (Noda and Nagao 2010; Richards et al. 2008)).

In the next sections trajectories of both “cubes” and “spheres” were assessed. The equations of motion were numerically solved in the time domain in 2D and 3D by reviewing a series of existing methods for trajectory analysis. Also, simulation results are
described and compared against the literature results. It should be noted that in this chapter the wind field is assumed to be uniform with a constant velocity $U$ acting on a horizontal plane and independent of the elevation.

2.2 Equations of motions

After debris starts to fly, it will accelerate until either it hits other objects or the ground. In the uniform wind field with nominal 2D flow, the aerodynamic forces on the debris can be calculated as (Holmes 2004):

\[
D = 0.5 \times \rho_{air} \times A \times C_D \times \left[ (u_z)^2 + (u_x - U)^2 \right] \quad (2.1 \text{ a})
\]

\[
L = 0.5 \times \rho_{air} \times A \times C_L \times \left[ (u_z)^2 + (u_x - U)^2 \right] \quad (2.1 \text{ b})
\]

\[
M = 0.5 \times \rho_{air} \times A \times l \times C_M \times \left[ (u_z)^2 + (u_x - U)^2 \right] \quad (2.1 \text{ c})
\]

In Equations (2.1) $D$, $L$ and $M$ are drag, lift, moment force, respectively; $\rho_{air}$ is the air density; $A$ is the reference “projected area” of the object; $l$ is a reference length; $u_x$ is debris horizontal velocity; $u_z$ is debris vertical velocity; and $C_D$, $C_L$ and $C_M$ are drag, lift and moment force coefficient respectively.

Following Newton’s second law and the quasi-steady theory Equations (2.2) describe the debris in 2D space (Lin et al. 2007):

\[
m \frac{d^2x}{dt^2} = m \frac{du_x}{dt} = \frac{1}{2} \rho_{air} A \left[ (U - u_x)^2 + u_z^2 \right] \left( C_D \cos \Phi - C_L \sin \Phi \right) \quad (2.2 \text{ a})
\]
In the previous equations Φ is the horizontal mean-wind direction angle (yaw); x and z are horizontal and vertical position of the debris; θ is the angular rotation; and Im is the mass moment of inertia.

However, for a compact object it is usually acceptable to neglect the lift force and rotational moment force. It has been shown (Holmes 2004) that the only important parameter, which mainly controls the flight of the compact objects, is drag force. Following this assumption Equations (2.3) describe the compact debris motion in 3D space (X, Y, Z) (Richards et al. 2008). If the generic position of the debris in 3D space is represented by x,y,z, components as a function of time t, the equations become:

\[
\frac{d^2x}{dt^2} = \frac{du_x}{dt} = \frac{\rho_{air}A}{2m} \cdot C_D \cdot V_{rel} \cdot \frac{U_x - u_x}{\sqrt{V_{rel}}},
\]

\[
\frac{d^2y}{dt^2} = \frac{du_y}{dt} = \frac{\rho_{air}A}{2m} \cdot C_D \cdot V_{rel} \cdot \frac{U_y - u_y}{\sqrt{V_{rel}}},
\]

\[
\frac{d^2z}{dt^2} = \frac{du_z}{dt} = \frac{\rho_{air}A}{2m} \cdot C_D \cdot V_{rel} \cdot \frac{-u_z}{\sqrt{V_{rel}}} - g.
\]
In the previous equations $C_D$ is the drag coefficient assumed as a constant parameter, independent of relative angle of attack between moving object and velocity, flow conditions (turbulent vs. laminar flow) as a first approximation for a compact object. No variation of mean-wind velocity $U$ with coordinate $z$ was assumed in this chapter because of the uniform wind field hypothesis; $u_j$ is the debris velocity component in each direction ($j=x,y,z$); $U_x$ and $U_y$ are the components of the wind speed vector on the horizontal plane ($U=[U_x, U_y]$).

The quantity $V_{rel}$ is the magnitude of the relative velocity between wind and debris, which can be determined by Equation 2.4 below:

$$V_{rel} = \left[ (U_x-u_x)^2 + (U_y-u_y)^2 + u_z^2 \right]$$ (2.4)

Following Tachikawa and Baker (Baker 2007; Tachikawa 1983), the equations of motion can be re-written in dimensionless form as:

$$\frac{d^2 \vec{x}}{dt^2} = KC_D \sqrt{\left[ (\cos \Phi - \vec{u}_x)^2 + (\sin \Phi - \vec{u}_y)^2 + \vec{u}_z^2 \right]} (\cos \Phi - \vec{u}_x), \quad (2.5 \text{ a})$$

$$\frac{d^2 \vec{y}}{dt^2} = KC_D \sqrt{\left[ (\cos \Phi - \vec{u}_x)^2 + (\sin \Phi - \vec{u}_y)^2 + \vec{u}_z^2 \right]} (\sin \Phi - \vec{u}_y), \quad (2.5 \text{ b})$$

$$\frac{d^2 \vec{z}}{dt^2} = KC_D \sqrt{\left[ (\cos \Phi - \vec{u}_x)^2 + (\sin \Phi - \vec{u}_y)^2 + \vec{u}_z^2 \right]} (-\vec{u}_z) - 1. \quad (2.5 \text{ c})$$
In Equations (2.5) the dimensionless object position variables are \( \bar{x} = xgU^{-2} \), \( \bar{y} = ygU^{-2} \), \( \bar{z} = zgU^{-2} \) with time \( \bar{t} = t g / U \); \( \bar{u}_r = u_r / U \) (with \( r = x, y, z \)) are velocity components. The quantity \( K = U^2 \rho_{air} A / (2mg) \) is the Tachikawa number (Tachikawa 1983; 1988), i.e., it is a dynamic similarity measure to define the relevance of aerodynamic forces on the object vs. gravity forces. For small \( K \), typically around 2 to 3, the aerodynamic forces become irrelevant; this value is common for large mass compact objects, which tend to “fall” instead of “flying horizontally”. In contrast, large \( K \) values are representative of small mass but larger area objects (e.g., small mass density material).

### 2.3 Early flight trajectory

In this section the non-dimensional equations of motion are simplified in 2D and the solution to obtain an approximate trajectory at early stages of the flight is explained. Based on Tachikawa (Tachikawa 1983) the dimensionless equations of motion for compact debris in the horizontal plane can be rewritten as:

\[
\frac{d^2 \bar{x}}{d\bar{t}^2} = K \left[ (1 - \bar{u}_x)^2 + \bar{u}_z^2 \right] (C_D \cos \Phi), \quad (2.6 \text{ a})
\]

\[
\frac{d^2 \bar{z}}{d\bar{t}^2} = K \left[ (1 - \bar{u}_x)^2 + \bar{u}_z^2 \right] (C_D \sin \Phi) - 1. \quad (2.6 \text{ b})
\]
In the previous equations $\cos \Phi = \frac{1-\bar{u}_x}{\sqrt{(1-\bar{u}_x)^2 + \bar{u}_z}}$ and $\sin \Phi = \frac{\bar{u}_z}{\sqrt{(1-\bar{u}_x)^2 + \bar{u}_z}}$ were employed. In early stages of the flight, the equations can be decoupled and solved for the horizontal and vertical component respectively. When $\bar{u}_x \to 0$ and $\bar{u}_z \to 0$, the following simplification can be used: $\cos \Phi \to 1$ and $\sin \Phi \to 0$. Therefore, the equations of motion will reduce to $\frac{d^2 \bar{x}}{dt^2} = KC_D$ and $\frac{d^2 \bar{z}}{dt^2} = -1$. Therefore, it can be deduced that $\bar{x} = 0.5KC_D\bar{t}^2$; $\bar{z} = -0.5\bar{t}^2$ and $\frac{\bar{z}}{\bar{x}} = -\frac{1}{KC_D}$. Based on this observation a linear trajectory is expected at the initial part of the flight, which mainly depends on the Tachikawa number $K$ and the aerodynamic drag coefficient $C_D$. It will be confirmed in the next sections that the horizontal trajectory of the compact debris is mainly controlled by the Tachikawa number.

2.4 Trajectory estimation of compact debris by using empirical equations

It has been observed through extensive wind tunnel experiments, conducted at Texas Tech University (Holmes 2008), that for majority of the flying debris horizontal trajectory is affected by $K$; also, in Section 2.3 it was shown that the horizontal non-dimensional debris velocity is a function of $K$ and horizontal distance traveled by the object. Lin et al. (Lin et al. 2007) proposed an empirical equation, based on a series of
observations in wind tunnel and verified by analytical studies, for the estimation the trajectory of compact objects. The same study (Lin et al. 2007) also showed that the horizontal dimensionless velocity may be approximated by an exponential function of the quantity $K\bar{x}$. Equation 2.7 below was proposed by Lin et al. to estimate the velocity of compact objects vs. horizontal displacement (traveled distance):

\[ \bar{u}_x = 1 - \exp(-1.6 K\bar{x}), \quad \text{(For cubes)} \]  
\[ \bar{u}_x = 1 - \exp(-1.0 K\bar{x}), \quad \text{(For spheres)} \]  

Moreover, it was also suggested during the same experiments (Lin et al. 2007) that the dimensionless quantity $K\bar{x}$ can be empirically related to $K\bar{T}$ and that a polynomial can be used to approximately express this relationship. Equations 2.8 show these polynomials for spheres and cubes:

\[ K\bar{x} \approx 0.405(K\bar{T})^2 - 0.036(K\bar{T})^3 - 0.052(K\bar{T})^4 + 0.008(K\bar{T})^5, \quad \text{(For cubes)} \]  
\[ K\bar{x} \approx 0.248(K\bar{T})^2 + 0.084(K\bar{T})^3 - 0.100(K\bar{T})^4 + 0.006(K\bar{T})^5, \quad \text{(For spheres)} \]

It should be noted that both Equations 2.7 and 2.8 are approximations for a constant velocity wind field. Finally, the same experiments indicated that the aerodynamic drag coefficient ($C_D$) for cubes and spheres can be taken to be equal to 0.8 and 0.5, respectively.
2.5 Deterministic trajectory model validation

In this section, the equations of motion are numerically solved by transforming the equations of motion to state-space formulation and by employing a 5th order Runge-Kutta method. Numerical results are compared to Lin’s equations, explained in Section 2.4, to identify a suitable value of the drag coefficient. Also, it should be noted that observations by Tachikawa (Tachikawa 1983), an acceptable value of the Tachikawa number in most full-scale applications can be taken as $2 < K < 7$. Therefore, this interval of $K$ values is assumed during this study to be consistent with previous investigators.

In order to compare simulated results with the empirical Equations (2.7) and (2.8), the 3D system of equations must be simplified to 2D: $\Phi=0$ was used to obtain a uniform wind field only in the $X$ direction; zero initial position and velocity for components other than $X$ were set to zero. As a first example, Figure 2.1 (a) and (b) shows the dimensionless quantity $Kx$ (related to the horizontal position or traveled distance) vs. $Kt$ for a sphere with $C_D=0.5$ and $K=3$ and a cube with $C_D=0.8$ and $K=3$. A very good agreement between simulated results and the empirical Equations 2.8 is observed in this figure.

Also, an additional study is carried out made to verify the accuracy of the numerical simulation. The numerical results are shown in Figure 2.2, Figure 2.3 and Figure 2.4. In Figure 2.2 and Figure 2.3 $\bar{r}_x$ versus $\bar{x}$ is shown for the trajectory of spheres and cubes.
respectively. As previously mentioned, since $K$ mostly controls compact debris trajectory, three values of $K$ are considered (low, intermediate and high).

![Graph](image1)

(a) Dimensionless $K\bar{x}$ (horizontal position) vs. time $K\bar{t}$ for a sphere with $C_D=0.5$

![Graph](image2)

(b) Dimensionless $K\bar{x}$ (horizontal position) vs. time $K\bar{t}$ for a cube with $C_D=0.8$

Figure 2.1 Comparison of compact debris dimensionless horizontal position as a function of dimensionless time group, derived from both empirical Equations (2.8) and numerical simulations.
Figure 2.2 Comparison dimensionless horizontal velocity vs. horizontal displacement for cubes ($C_D=0.8$) from empirical Equations (2.7) and simulated results: (a) $K=2$; (b) $K=3.5$; (c) $K=7$. 
Figure 2.3 Comparison of dimensionless horizontal velocity vs. horizontal displacement for spheres ($C_D=0.5$) from both empirical Equations (2.7) and simulated results: (a) $K=2$, (b) $K=3.5$, (c) $K=7$.

There is a good agreement between the numerical procedure used to simulate object velocity and the empirical equations in these figures. From interpretation of the graphs in the Figure 2.2, Figure 2.3 and especially in Figure 2.4, it is inferred that the higher the
Tachikawa number the sooner the debris reaches its terminal velocity \((\overline{u}_x = 1)\). Also, cubes can fly “faster” than spheres, as they reach the unit-valued dimensionless velocity much sooner (smaller \(K_T\) values) than spheres.

Figure 2.5 shows the importance of the Tachikawa number in trajectory model of compact object. Three different Tachikawa numbers are used to simulate the trajectory of both cubes (Figure 2.5 (a)) and spheres (Figure 2.5 (b)). From Figure 2.5 it can be observed that, since for larger Tachikawa number the aerodynamic to the effect of gravity is smaller (in relative terms), the debris tends to fly more and higher. Also, as evident in both figures and the solid black line case associated with the upper limit value \(K=7.0\), both cubes and spheres reach the terminal velocity \((\overline{u}_x = 1)\) almost three times faster than \(K=3.5\).

![Figure 2.4 Comparison of empirical and simulated compact debris velocity for a sphere and a cube having the same Tachikawa number \((K=2)\).](image-url)
Figure 2.5 Comparison of debris horizontal trajectory as $\bar{u}_x$ vs. $\bar{x}$ horizontal displacement at various $K$: (a) cube, (b) sphere.

The dimensionless trajectory ($\bar{\alpha}$ vs. $\bar{\alpha}$) of both a cube and a sphere is plotted in Figure 2.6 for $K=3$ and $K=5$. Further investigation of these figures confirms that the cube
fly not only faster but higher and longer than the sphere, mainly because of a larger drag force. Also, for larger $K$, the same object travels longer in the same period of time.

Cross-comparisons between the various cases, analyzed in this parametric study, are important since they provide indication on the traveled flight distance for various objects. The studies also confirm that there is an inherent variability in the trajectory, induced by the variations in the object properties.

If a compact gravel object of mass 3 kg is flying off a roof during a strong wind with mean speed 45 m/s (not an unrealistic value during a hurricane), the traveled distance in the horizontal direction will be approximately 20 meters after about 7 seconds; the same object would reach its terminal velocity after 15 seconds. On the contrary there is a compact, and non-streamlined and "heavy" shingle (about 3 kg), for which a limited contribution from the lift force can be postulated.
Figure 2.6 Typical dimensionless trajectory of compact object: (a) sphere, (b) cube.
2.6 References


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2.7 Appendix to Chapter 2

2.7.1 Runge Kutta Methods

A brief overview of Runge-Kutta methods is provided in this appendix. The information was derived from (Sewell 2005).

Runge-Kutta methods are a family of numerical methods for solving ordinary differential equations. These methods include both explicit and implicit iterative schemes to achieve higher order accuracy in general. These are one-step methods that need multiple function evaluation per step.

An initial value problem of the form \( u' = f(t,u), \quad u(t_0) = u_0 \) is assumed (scalar form for simplicity). The simplest Runge-Kutta formula requires two function evaluations per step, which has the following form:

\[
\begin{align*}
    v_1 &= f(t_k, U(t_k)), \\
    v_2 &= f(t_k + \alpha h, U(t_k) + \alpha hv_1), \\
    U(t_{k+1}) &= U(t_k) + h [Av_1 + Bv_2].
\end{align*}
\]

The quantity \( U \) is used in the previous equation to designate approximate numerical solution, evaluated at the time steps \( t_k \) and \( t_{k+1} \). The quantities \( A, B \) and \( \alpha \) have to be determined in such a way that minimizes the truncation error. The truncation error \( (T) \) in this case is:
If the last term is expanded using bi-variate Taylor expansion and the following relation is used:

\begin{align*}
u' &= f(t, u), \\
u'' &= f_i(t, u) + f_r(t, u)u' \\
u''' &= f_u(t, u) + 2f_m(t, u)u' + 2f_{uu}(t, u)u'' + f_u[t, u]u''
\end{align*}

We have

\begin{align*}
T &= (1 - A)u' + \frac{1}{2}hu'' + \frac{1}{6}h^2u''' + O(h^3) \\
&\quad - B \left[ f + \alpha hu'f_u + \frac{1}{2} \alpha^2 h^2 f_{u} + \alpha^2 h^2 f_{uu} + \frac{1}{2} \alpha^2 h^2 (u')^2 f_{uu} + O(h^3) \right] \\
&\quad = (1 - A - B)u' + \left( \frac{1}{2} - B\alpha \right) hu'' + \left( \frac{1}{6} - \frac{1}{2} B\alpha^2 \right) h^2 (u'' - f_u u''') + \frac{1}{6} h^2 f_u u''' + O(h^3).
\end{align*}

In the previous expression \( u \) and its derivatives are evaluated at \( t_k \), and \( f \) and the derivatives are evaluated at \( (t_k, u(t_k)) \).

Because of the presence of the last term, the \( O(h^3) \) can not disappear independently of the values of parameters \( A, B \) and \( \alpha \). At most constant and \( O(h) \) terms will disappear if \( A, B \) and \( \alpha \) are chosen to be \( B=0.5/\alpha \) and \( A=1-0.5/\alpha \) and \( \alpha \) is arbitrary. In
this way second order accuracy will be obtained. However, the same procedure can be used to get any desired accuracy level.

The formulas of fifth order Runge-Kutta-Fehlberg, which was used in this dissertation to solve a system of first-order differential equations, are as follows (with \( u'=f(t,u) \)):

\[
\begin{align*}
    v_1 &= f(t_k, U(t_k)), \\
    v_2 &= f\left( t_k + \frac{1}{4} h, U(t_k) + \frac{1}{4} hv_1 \right), \\
    v_3 &= f\left( t_k + \frac{3}{8} h, U(t_k) + h \frac{3v_1 + 9v_2}{32} \right), \\
    v_4 &= f\left( t_k + \frac{12}{13} h, U(t_k) + h \frac{1932v_1 - 7200v_2 + 7296v_3}{2197} \right), \\
    v_5 &= f\left( t_k + h, U(t_k) + h \frac{8341v_1 - 32823v_2 + 39440v_3 - 28672v_4 - 9295v_5}{6048} \right), \\
    v_6 &= f\left( t_k + \frac{1}{2} h, U(t_k) + h \frac{-6080v_1 + 41040v_2 - 28352v_3 - 9295v_4 - 5643v_5}{20520} \right), \\
    U_5(t_{k+1}) &= U(t_k) + h \frac{33440v_1 + 0v_2 + 146432v_3 + 142985v_4 - 50787v_5 + 10260v_6}{282150}.
\end{align*}
\] (A5)

Besides accuracy, the stability of a method is crucial. Unfortunately the region of absolute stability of explicit Runge-Kutta methods is small for “stiff equations”. A stiff equation is a differential equation for which certain numerical methods for solving the equation become numerically unstable, unless the step size is taken to be extremely small. A differential equation of the form \( y' = f(t,y) \) is said to be stiff if its exact solution \( y(t) \) includes a term that decays exponentially to zero as time \( t \) increases, but whose
derivatives are much greater in magnitude than the term itself. An example of such a term is $e^{ct}$, where $c$ is a large, positive constant, because its $k$-th derivative is $c^k e^{ct}$. Because of the factor of $c^k$, this derivative decays to zero much more slowly than $e^{-ct}$ as $t$ increases. Because the error includes a term of this form, evaluated at a time less than $t$, the error can be quite large if the step size $h$ is not chosen sufficiently small to offset the large derivative (Lambers 2010). Fortunately, the differential problems analyzed in this investigation do not fall into this category; this additional requirement is not needed.
Chapter 3

Compact debris trajectories in uniform wind combined with a simplified vertical gust model

3.1 Introduction

In Chapter 2, an initial parametric study was carried out to analyze the trajectory of compact debris by analyzing sensitivity of the dynamic equations to various parameters influencing debris flight, especially the drag force coefficient and the Tachikawa number. The simulation procedure could be indirectly (in a simple way) utilized for analyzing the “vulnerability” of vertical building façades to debris impact due to high winds and, possibly, for structural building design. Nevertheless, the hypothesis of uniform wind velocity with constant flow speed with elevation and no turbulence was used as a simplifying assumption, as this has been traditionally done in the literature.

As discussed in the first chapter, modeling of debris trajectory for risk and performance analysis on residential buildings due to high winds has mainly focused on low-rise constructions (Divel et al. 2010). In most cases, fundamental work on trajectory estimation has investigated the debris trajectories in horizontal uniform wind fields both in 2D (Lin et al. 2007; Tachikawa 1983) and 3D (Richards et al. 2008). An attempt to incorporate boundary-layer wind profile information for the study of the random
trajectory of objects in turbulent winds has been recently done by combining the results of wind tunnel tests with Monte Carlo methods (Karimpour 2011). Few fundamental or experimental studies are available on the topic of turbulence and debris trajectory, since most researchers have accepted the fact that turbulence-induced deviations in the trajectory are of secondary importance (Holmes 2004). Nevertheless, since the objective of this work is the assessment of random trajectory, influenced by various sources of uncertainly, the question on the effects of turbulence cannot be overlooked.

In spite of the complex spatial and temporal flow features in the atmospheric boundary layer, a limited influence of turbulence, mainly the along-wind component, has been noted by Holmes (2004). As a consequence, the use of simpler constant-velocity field has been recommended (Holmes 2004), since the average debris flight duration is usually short in comparison with the time-varying effects of the surrounding turbulent flow. Nevertheless, in order to enable the analysis of the risk of impact against a typical-size cladding panel located on the vertical façade of a building turbulence effects should be included since the “average” trajectory due to horizontal uniform wind may possibly differ from the one incorporating local flow effects (Moghim and Caracoglia 2012). Moreover, recent forensic evidence on debris damage on tall building façades suggests the tendency of compact debris to possess an upward trajectory in extreme winds and events (Jain 2013), contradicting the simulation results reported in Chapter 2. The effect
of unusual upward trajectory for plate-like objects, dependent on surrounding flow features at takeoff, was recently observed in wind tunnel (Kordi and Kopp 2011).

In this chapter, a 2D theoretically-based and numerically-implemented model was proposed in an effort to calculate the trajectory of compact objects, perhaps more realistically, in a wind field, which includes simulated turbulence. This model will be later adapted in the following chapters.

The model is based on a simulated simplified vertical gust profile, fully coherent, superimposed to a wind field with horizontal uniform constant velocity. A constant “step-like” transverse gust front of finite time duration (vertical velocity component) was included in the modeling of trajectory. This flow feature was labeled as a “Küssner-like Vertical Gust” due to the similarity with the classical aerodynamics problem of an airfoil in steady horizontal flight encountering an unsteady vertical gust front (e.g., (Bisplinghoff et al. 1955)). In this chapter, appropriately modified equations of motion were used for numerically simulating the trajectory of both “cubes” and “spheres”. First, parametric analyses were conducted to evaluate the effects of vertical gust duration and magnitude on debris trajectory. Second, simulation results were compared to the trajectories numerically derived in the absence of wind turbulence; dependency on mass ratio, mean wind velocity and Tachikawa number (Tachikawa 1983) was investigated. Observations were also provided in an attempt to replicate the findings of the wind tunnel tests, described by (Kordi and Kopp 2011).
3.2 Formulation of the “Küssner-like” vertical gust model in horizontal wind

Experiments on the trajectory of “plate objects” were recently conducted at the University of Western Ontario (Kordi and Kopp 2011). It was suggested that the effect of turbulence during early steps of flight, either inherent in the free-stream flow or generated by obstacles and recirculation around buildings, could lead to considerable differences in observed trajectory of plate-like objects. The initial angle may also have a significant effect on the trajectory of plate-like debris (Tachikawa 1988).

Latter observations and former experimental findings confirmed the exploratory experiments in (Tachikawa 1988), which indicate that imposing a transverse velocity component, such as vertical turbulence during early flight, may have a significant effect on debris trajectory. For example, this effect was experimentally recorded in (Tachikawa 1988) when the horizontal distance traveled by the object prior to impact against a vertical target is of the order of $0 \leq \bar{x} \leq 5$ in dimensionless units, with $\bar{x} = xgU^{-2}$, $x$ being the dimensional horizontal coordinate (or position), $U$ the mean wind speed and $g$ the gravity acceleration. Also, a survey after hurricane carried out on a tall building in Florida indicated damage on the upper floors even though the debris most likely source was identified as the roof of a nearby shorter structure (Jain 2013).

It seemed therefore plausible to assume that, in an urban scenario and even for a compact object, “signature” turbulence in the vertical direction may be present as the
debris detaches from the edges or the roof of a nearby building. The modeling of an initial sudden gust was perceived of relevance in relation to the dynamic simulations discussed in previous sections. For this purpose the simplified “step-like” upward gust model was utilized to be compatible with observations in (Jain 2013; Tachikawa 1988). This fully-correlated vertical gust was superimposed at dimensionless time $\bar{t} = 0$ onto the horizontal wind field with constant and uniform $U$ (Figure 3.1). The definition of $\bar{t}$ may be found in Chapter 2.

The vertical gust was incorporated in the loading model used in conjunction with the equations of motion. The numerical tests were limited to 2D, expressed in dimensionless coordinates $\bar{x}$ and $\bar{z}$ (vertical), as in Equations (2.5) of Chapter 2, and for mean-wind yaw angle $\Phi=0$ (Figure 3.1). Also shown in Figure 3.1, the quantity $d^*$ (dimensional units) is used to represent the relative horizontal distance between the debris initial position and a hypothetical target building along the $x$ axis in the figure. Figure 3.1 shows a typical vertical gust time history in dimensional units. The duration and magnitude was expressed as a function of the total flight duration as $\gamma \times \bar{t}_{\text{flight}}$ with the total flight duration $\bar{t}_{\text{flight}}$ in dimensionless units being approximately estimated from the relative distance between the two buildings and horizontal mean wind speed $U$; the gust magnitude was expressed as $w = \delta \times U$.

Therefore, according to Figure 3.1 the 2D equations of debris motion in the XZ plane, described in Chapter 2, could be rewritten as:
\[
\begin{align*}
\frac{d^2 \tilde{x}}{dt^2} &= \frac{d\tilde{u}_x}{dt} = K \left[ (1-\tilde{u}_x)^2 + (\delta - \tilde{u}_x)^2 \right] \left( C_D \cos \Phi_s \right), \\
\frac{d^2 \tilde{z}}{dt^2} &= \frac{d\tilde{u}_z}{dt} = K \left[ (1-\tilde{u}_z)^2 + (\delta - \tilde{u}_z)^2 \right] \left( C_D \sin \Phi_s \right) - 1.
\end{align*}
\]  

(3.1 a) 

(3.1 b)

Figure 3.1 Turbulence effects on flight trajectory: (a) lateral view (schematic) of initial vertical velocity effects; (b) “Küssner-like” Vertical Gust of magnitude \( \delta U \) and duration \( \gamma_{flight} \) (dimensional units).

Equations (3.1) are valid for \( 0 \leq \tilde{t} < \gamma t_{flight} \). In these equations, \( C_D \) is the drag coefficient, \( \tilde{u}_r = u_r / U \) (with \( r=x, y, z \)) and \( u_r \) being the dimensional velocity components.
of the object. The angle $\Phi_s$ accounts for the relative velocity between the object and the flow by accounting for $w$ component, as shown in Figure 3.2 (a); it is determined as

$$\Phi_s = \tan^{-1} \left( \frac{\delta - \bar{u}_z}{1 - \bar{u}_s} \right). \quad (3.2)$$

In contrast, when $\bar{t} \geq \bar{t}_{flight}$ the “standard” flight equations below should be used:

$$\frac{d^2 \bar{x}}{dt^2} = \frac{d\bar{u}_x}{dt} = K \left[ (1-\bar{u}_s)^2 + \bar{u}_z^2 \right] (C_D \cos \Phi), \quad (3.3 \text{ a})$$

$$\frac{d^2 \bar{z}}{dt^2} = \frac{d\bar{u}_z}{dt} = K \left[ (1-\bar{u}_s)^2 + \bar{u}_z^2 \right] (C_D \sin \Phi) - 1. \quad (3.3 \text{ b})$$

In the latter equations the relative angle $\Phi$ is calculated from:

$$\Phi = \tan^{-1} \left( \frac{-\bar{u}_z}{1 - \bar{u}_s} \right). \quad (3.4)$$

For further clarification Figure 3.2 schematically shows the relative velocity “seen” by a flying object with and without effects of vertical gust.

Equations (3.1) and (3.3) were numerically solved, as discussed in the previous chapter. At any given point in time, the modulus of the dimensionless debris velocity and the instantaneous trajectory angle (slope) with respect to the horizontal plane were numerically calculated as

$$\bar{V} = |\bar{V}| = \sqrt{\bar{u}_s^2 + \bar{u}_z^2}, \quad (3.5)$$

$$\Psi_z = \tan^{-1} \left( \frac{\bar{u}_z}{\bar{u}_s} \right). \quad (3.6)$$
The quantities in Equations (3.5) and (3.6) were numerically estimated since the goal of this work also included investigation of the consequences following compact debris impact on a building façade.

Figure 3.2 Two-dimensional velocity vectors with respect to the moving compact debris: (a) by considering vertical gust for $0 \leq t < \gamma t_{flight}$, (b) in the absence of vertical gust for $t \geq \gamma t_{flight}$. The quantities $u_{x,m}(t)$ and $u_{z,m}(t)$ in the figure are dimensional velocity components of the object with respect to a fixed reference coordinate system.

One of the parameters influencing the severity of impact is the momentum of the object ($p$), which is directly proportional to the velocity modulus $|\vec{V}|$ (at impact point) and the mass of debris ($m$). The quantity $p$ is described in the equation below for 2D analysis and in dimensional units as

$$p = m \sqrt{u_x^2 + u_z^2}. \quad (3.7)$$

In Equation (3.7) the total momentum of the object ($p$) was calculated from the vector combination of horizontal ($u_x$) and vertical velocity ($u_z$). Equation (3.7) was rewritten in dimensionless form as
\[ p = \frac{p}{\rho_{\text{air}} (A \times l) U} = \frac{F_R^2}{2K} \bar{V}(K, \delta, \gamma). \] (3.8)

In Equation (3.8) the dimensionless modulus is \( \bar{V} = \bar{V}(K, \delta, \gamma) \) as defined by Equation (3.5). The normalization of the equation was performed with respect to an equivalent mass of air moving at the same velocity as the mean wind, with \( A \times l \) being the reference volume of the object. Equation (3.8) shows that the momentum is inversely proportional to Tachikawa number \( K \), proportional to \( \bar{V} \) but also to \( F_R = U / \sqrt{gl} \) (Froude number). The dependency on \( F_R \) emphasizes the relevance of flow-related gravity effects versus flow inertial forces. The effect of Froude number for a compact object was not evident from the inspection of the equations of motion Equation (3.3). This remark contradicts the observation that \( F_R \) is exclusively important for plate-like objects. Finally, the velocity modulus \( \bar{V} \) was written in Equation (3.8) as a function of various properties, i.e., \( \bar{V}(K, \delta, \gamma) \) as later discussed in chapter 3.4.1 on momentum.

### 3.3 Numerical simulations: trajectory results

A parametric study was conducted to investigate the effect of various combinations of \( \gamma \) and \( \delta \) on debris trajectory (deterministically). For comparison purposes with Section 2.5 in Chapter 2, the results for a cubic compact debris with \( C_D = 0.8 \) and \( K = 2 \) and for \( U=30 \) m/s are summarized in Table 3.1.
To quantify the wind condition for each case given in Table 3.1, three scenarios and thirteen cases were selected. It must be noted that the dimensionless magnitude $\delta$ was used in this study as an indirect measure of turbulence intensity, defined as the ratio between the standard deviation of the random turbulence component and the mean wind speed $U$.

In Table 3.1 case 0 coincides with the deterministic solution in the absence of vertical gust while cases 1 through 12 show the effects of different combinations of vertical gust.
duration ($\gamma$) and magnitude ($\delta$) on the debris velocity and instantaneous trajectory angle. The cases with $\delta=0.10$ can be used to simulate a moderate synoptic wind (Simiu and Scanlan 1996); the cases with $\delta=0.25$ approximately correspond to a hurricane wind, in which the turbulence intensity of the vertical turbulence component may be large (Caracoglia and Jones 2009); finally, $\delta=1.00$ is possibly compatible with very large vertical speed component, compatible with the updraft/downdraft effect typical of tornadic winds (Haan Jr et al. 2008). These values were used as reference in the context of this exploratory study; other wind conditions could be inferred and included for comparison among these categories.

In Table 3.1, $d^*$ is the horizontal distance “source/target” (along the $x$ axis in dimensional units), referenced to a horizontal wind speed $U=30$ m/s. Two values were analyzed: $d^*=10$ m and $d^*=30$ m, respectively equal to $d^*=d^*gU^2 = 0.11$ and $d^*=0.33$ in dimensionless units. The former distance represents an intermediate flight trajectory point, whereas the latter corresponds to the critical relative distance $d_{b,critical}$, derived as the boundary of Region III with $U=30$ m/s (“no impact”) by probabilistic analysis, which will be later explained in Chapter 5 (Figure 5.2). Moreover, in Table 3.1 the dimensionless modulus of the debris velocity vector ($|\vec{V}|=\bar{V}$, Equation (3.5)) and the instantaneous trajectory angle (slope) with respect to the horizontal plane ($\Psi_z$; Equation (3.6)) are shown.
Inspection of Table 3.1 revealed that the influence of initial vertical gust properties on the debris velocity and \( \Psi_z \) was limited for both \( d^* = 10\text{m} \) (\( \bar{d}^* = 0.11 \)) and \( d^* = 30\text{m} \) (\( \bar{d}^* = 0.33 \)). Some variation was observed in case 12, which was selected to maximize the effect of the vertical gust by assuming a gust duration equal to one half of the total debris flight duration and a the same gust intensity as the mean horizontal \( U \). Needless to say, case 12 was conceived as a limiting example; as a first approximation, this case is possibly less compatible with others and a realistic wind event.

In order to analyze in more detail the difference in trajectory between case 0 and case 12, time histories of vertical and horizontal dimensionless velocity components, \( \bar{u}_y \) and \( \bar{u}_x \), were plotted in Figure 3.3 for a “cube” with Tachikawa number equal to 2. In Figure 3.3 (a) the influence of the vertical gust on \( \bar{u}_y \) can be noticed during the early stages of the flight, dominated by an upward-direction loading effect; this trend is reversed after the vertical gust vanishes at about \( \bar{t} = 0.2 \). From the results in Figure 3.3 it can be inferred that, independently of the vertical gust magnitude and duration, cases 0 and 12 tend to have very similar velocity time histories (especially for \( \bar{u}_x \)) beyond a certain point in time; this remark confirms the predominant effect of gravity despite the initial propensity of the trajectory to the upward direction due to the vertical gust effect.
Figure 3.3 Comparison between (a) vertical, (b) horizontal dimensionless velocity of a debris with $C_D=0.8$ and $K=2$ (case 0 vs. case 12 in Table 3.1).

In Figure 3.4 the estimated debris trajectory of case 0 is depicted versus the curve corresponding to case 11 (vertical gust with $\gamma=0.5$ and $\delta=0.25$) and 12 (vertical gust with
\( \gamma = 0.5 \) and \( \delta = 1.0 \). Case 11 is more realistic, as normalized duration \( \gamma \) and magnitude \( \delta \) of vertical gust are compatible with a highly-turbulent boundary layer wind, e.g., with turbulence intensities (ratio between standard deviation of turbulence and mean speed) equal to 0.20. Dimensionless quantities \( \bar{x} \) and \( \bar{z} \) are employed in the plots. In Figure 3.4 (a), the difference between case 0 and 12 is considerable but, as mentioned before, case 12 is physically less significant. Nevertheless, in Figure 3.4 (b) the difference between the two curves is small. Therefore, it was concluded from Figure 3.4 (b) and Table 3.1 that the influence of vertical gust on the deterministic trajectory of compact debris is of marginal importance at low \( K \).

The combined effect of Tachikawa number and vertical gust was investigated in Figure 3.5, with \( K=2 \) and \( K=7 \) and vertical gust properties pertaining to case 11 of Table 3.1. Figure 3.5 (a) depicts the time histories of the horizontal debris velocity in dimensionless units (\( \bar{u}_x \) vs. \( \bar{t} \)); Figure 3.5(b) shows the debris trajectory (\( \bar{x} \) vs. \( \bar{z} \)). As anticipated, the larger the Tachikawa number the sooner the debris velocity reaches the terminal wind velocity due to a larger aerodynamic force in comparison with gravity effects, especially at initial stages (Figure 3.5 (a)). In Figure 3.5 (b) and for low Tachikawa number (\( K=2 \)) the debris flight is mainly controlled by gravity force and, once the effect of the initial gust disappears, the object rapidly falls downward. This was anticipated due to the limited relevance of aerodynamic loading at lower \( K \).
Figure 3.4 Dimensionless trajectory for various combinations of magnitude and duration of the vertical gust and debris with $C_D=0.8$ and $K=2$ (cases 0, 11 and 12 in Table 3.1).
Figure 3.5 Effect of the Tachikawa number $K$ on debris flight with vertical gust of duration $\gamma=0.50$ and magnitude $\delta=0.25$ for an object with $C_D=0.8$ (The thick dashed line corresponds to case 11 in Table 3.1).
Nevertheless, the case with $K=7$ (lightweight compact object) shows a clear variation in the anticipated trajectory. The dotted-line rectangular “box” in Figure 3.5 (b) corresponds, in dimensionless units, to a region of width approximately equal to $3\bar{d}^* \approx 0.99$ and height $\bar{H}_0 = 0.33$. The figure shows that, if a compact object with $K=7$ (solid thick line in Figure 3.5 (b)) is released from a nearby tall building, the deviation of the trajectory imparted by the vertical gust is significant. This object is likely to reach a target window on the lower floors of a downwind building located at a relative distance $\bar{d}' \approx 0.33$ on the horizontal plane (about 30 m at $U=30$ m/s).

### 3.4 Influence of compact debris properties on the relative distance “source/target” and for various gust properties

#### 3.4.1 Momentum

As described by Equation 3.5 and 3.6, the quantity $|\vec{V}| = \vec{V}$ can be used as a non-dimensional measure of debris momentum at impact and may be utilized in the evaluation of damage to cladding panels (glass windows).
Figure 3.6 Effect of Tachikawa number on $|\bar{V}|$ at $U=30$ m/s and for various relative distances “source/target”: (a) $d^*=5$ m ($\bar{d}^*=0.05$), (b) $d^*=10$ m ($\bar{d}^*=0.11$), (c) $d^*=20$ m ($\bar{d}^*=0.22$), (d) $d^*=30$ m ($\bar{d}^*=0.33$).

Figure 3.6 depicts the dimensionless velocity modulus $|\bar{V}|$ derived by numerically integrating Equations 3.1 and 3.3 at $U=30$ m/s for a debris with $C_D=0.8$ as a function of $K$ and for various vertical gust magnitudes ($\delta$), duration ($\gamma$) and $d^*$ (cases 0, 7, 11, and 12, in Table 3.1). This figure extends the results of the left column of Table 3.1 by analyzing...
relative distances $d^*=5$, 10, 20 and 30m, measured horizontally (Figure 3.1 (a)). These curves were also numerically evaluated for $K\rightarrow 0$, unrealistic condition, for completeness.

From the analysis of Figure 3.6 it can be observed that the presence of a vertical gust during early flight stages can have an influence on the momentum of the object as function of $d^*$. Observations corroborate the need for classifying the momentum equation (Equation 3.8) in terms of $\mathbf{V} = \mathbf{V}(K, \delta, \gamma)$.

For example, the velocity modulus at $d^*=5m$ ($d^* = d^* g U^{-2} = 0.055$) (Figure 3.6 (a)) at various $K$ for a highly turbulent vertical gust ($\gamma=0.5$ and $\delta=1.0$; solid line with “cruciform” marker) varies between 0.3 and 1.6, which confirms non-negligible vertical gust effects: at low $K$, $|\mathbf{V}|$ is smaller than the reference case without vertical gust effect (solid line with triangle marker), suggesting a beneficial effect of turbulence. In contrast, $|\mathbf{V}| \approx 0.70$ at large $K$ and with $\gamma=0.5$ and $\delta=1.0$ is 40% larger than the normalized momentum obtained for other combinations of $\gamma$ and $\delta$, for which variations are very small.

A crossing point between the case with $\gamma=0.5$ and $\delta=1.0$ and other curves can be seen at about $K=3$ in Figure 3.6 (a), approximately coincident with the boundary between the two regions of initial decrement and subsequent increment in the $|\mathbf{V}|$ curve with $\gamma=0.5$ and $\delta=1.0$. A similar crossing among the curves was observed in Figure 3.6 (b) at $d^*=10$ m ($d^* = 0.11$), occurring at larger $K$ in comparison with Figure 3.6 (a); crossing was
absent at larger separation distances $d^*$. Crossing point in Figure 3.6 (a) and Figure 3.6 (b) suggests that for this range of $d^*$ (or $\bar{d}^*$) the vertical gust effect could become important at initial stages of the flight, when $d^*$ is very small. The detrimental influence of a vertical gust could be an issue for lightweight compact debris. In contrast, as $d^*$ increases ($d^* > 25 \text{m}$, $\bar{d}^* > 0.27$), the effect of the vertical gust on momentum of a compact object is always beneficial, irrespective of $K$ (Figure 3.6 (c) and Figure 3.6 (d)), since a small reduction of $|\bar{V}|$ was consistently noticed, compared to the case with $\gamma=0$ and $\delta=0$.

According to Figure 3.6, for $0<K<1$ the $|\bar{V}|$ modulus is predominantly controlled by gravity irrespective of $d^*$; the effect of vertical gust is almost imperceptible for $K<1$ since all curves are practically coincident. In contrast, for $K>1$ duration ($\gamma$) and magnitude ($\delta$) of the vertical gust have an influence on $|\bar{V}|$, as discernible for $d^*=20 \text{m}$ ($\bar{d}^* = 0.22$) in Figure 3.6(c).

All $K - |\bar{V}|$ curves in Figure 3.6 exhibit a horizontal asymptotic trend, showing independence on $|\bar{V}|$ as $K$ increases. The value of the asymptote appears to mainly depend on the relative distance $d^*$ (or $\bar{d}^*$), and to a much lesser extent of vertical gust properties. At larger $d^*$ (or $\bar{d}^*$, Figure 3.6(d)) the asymptote tends to $\bar{V} = |\bar{V}| \approx 1$, which
confirm literature observations (e.g., (Lin et al. 2007)) that compact objects with \(K>5\) are mainly “carried by the flow” as their velocity tends to the mean-wind speed \(U\).

Moreover, in all the curves in Figure 3.6 a minimum \(|\vec{V}|\) can be observed on the left-hand side of the graphs, the position of which seems to be mainly occurring in the “gravity driven region” with some minor influence of the vertical gust. The presence of such a minimum was interpreted as the “balance” between gravity and aerodynamics effects. The position of the minimum is not the same and the relative reduction of \(|\vec{V}|\) is also a function of \(d^*\). For example, this condition was recorded at about \(K=2\) in Figure 3.6 (c). Also, in Figure 3.6 (d) with \(d^*=30\)m \((\bar{d}^*=0.33)\) the reduction of \(|\vec{V}|\) at \(K=3\) is not negligible. This is an interesting remark for compact debris, from which it was deduced that at an intermediate \(d^*\) (or \(\bar{d}^*\)) the effect of the gravity is always dominant for \(K<2\) but could possibly be mitigated by high turbulence: for example, the curve relative to \(\gamma=0.5\) and \(\delta=1.0\) indicates \(\vec{V}=|\vec{V}| \approx 0.6\) at \(K=2\) in Fig. 6(c), which is smaller than the corresponding value at \(K=3\) in Figure 3.6 (d) as \(d^*\) is further increased.

### 3.4.2 Angle of impact

As described in Section 3.2, the instantaneous trajectory angle with respect to the horizontal plane could be calculated by using Equation 3.6. This quantity and the velocity at impact can be useful to estimate the potential damage on the (glass) building façade. In
this section the angle of impact for four different $d^*$ and combination of $\gamma$ and $\delta$ is investigated and shown in Figure 3.8.

As depicted in Figure 3.8, for the heavy objects with lower Tachikawa number the angle at impact is almost 90 degrees, indicating that the object “hits” the target vertically. This confirms that for heavier objects the flight is predominantly controlled by gravity. However, for lighter objects, with $K \geq 2$, the angle at impact depends on the relative distance between the target and initial object position and is influenced the vertical gust properties.

In Figure 3.8 (a), since the target is very close to the initial position of the debris ($d^*=5$) the angle at impact becomes positive for objects with $K \geq 2$ and for $\gamma=0.5$ and $\delta=1.0$, which shows that the impact occurs during the ascending trajectory in this specific case Figure 3.7 shows the definition of positive and negative $\psi_z$. On the other hand, for other combinations of $\gamma$ and $\delta$, $\psi_z$ remains negative for all Tachikawa numbers in Figure 3.8(a). This indicates that the object tends to follow a downward trajectory and suggests a limited influence of the vertical gust during initial stages of the flight. This trend is confirmed for larger $d^*$ since, for example in Figure 3.8(d) and $d^*=30$m, the gravity is always dominant and the angle at impact is typical of a downward trajectory in all cases.
Also, it can be seen from Figure 3.8 that, at smaller $d^*$, the effect of the vertical gust on $\psi_z$ is not negligible. Even in the case with $\gamma=0.50$ and $\delta=0.25$, which corresponds to a simulated vertical gust not incompatible with atmospheric winds, there is a difference in the angle at impact in comparison with the case $\gamma=0$ and $\delta=0$ at $d^*=30m$. Nevertheless, as the relative distance “source/target” $d^*$ increases (e.g., Figure 3.8 (d)) all the curves tend to overlap because of the limited effect of vertical gust on the trajectory at these relative distances.
Figure 3.8 Effect of Tachikawa number on angle of impact $\psi_z$ at $U = 30$ m/s and for various relative distances “source/target”: (a) $d^* = 5$ m ($\bar{d}^* = 0.05$), (b) $d^* = 10$ m ($\bar{d}^* = 0.11$), (c) $d^* = 20$ m ($\bar{d}^* = 0.22$), (d) $d^* = 30$ m ($\bar{d}^* = 0.33$).

3.5 Summary

This chapter extends the results and conclusions of a chapter 2, discussing the implementation of a numerical model for wind-borne compact debris trajectory
estimation (e.g., a particle-type object), controlled by aerodynamic drag with no influence of lift or moment.

An investigation was conducted to analyze potential effects of a “sudden” vertical gust during early flight stages on debris trajectory and the consequent risk of impact against the vertical façade of a tall building. A simulated fully-correlated upward gust of constant magnitude and short duration was used, simulating a highly turbulent wind scenario. A modification to the standard equations of motion for compact debris flight was introduced to account for the effect of this simulated gust and solved in both 2D.

A parametric study on trajectory was carried out in this chapter for various combinations of gust magnitude and duration. It was found that the effect of the gust model may on occasion be considerable for a compact object. This study also allowed for the estimation of the mass momentum for compact debris during impact. The momentum and impact angle of the object, in dimensionless form, showed dependency on the Tachikawa number and the distance travelled by the object from its initial position. For objects with large Tachikawa number the vertical gust influence on mass momentum cannot be neglected. Numerical studies indicate whether the momentum at impact can be 20% larger when the vertical turbulence is considered in the simulations for $d^* < 10m$. The most notable effect of the vertical gust is observed on the angle of impact, for which up to 40% difference can be recognized.
3.6 References


Chapter 4

Influence of a computer-generated turbulent wind field on the trajectory of compact debris

4.1 Introduction

As indicated in the previous chapter and in (Moghim and Caracoglia 2012b), most studies on debris trajectory adopt the hypothesis of a uniform wind field. This assumption implies that the wind shear and turbulence effects are usually neglected, acceptable due to a limited flight duration (Holmes 2004); stochastic debris motion in a fully developed turbulent flow field has been recently investigated, predominantly by wind tunnel experiments (Karimpour and Kaye 2012). Nevertheless, as shown in Chapter 3, debris momentum, impacting against a building façade for an origin from several distances, can be altered, if a simplified vertical gust model is employed for assessing flight trajectory (Moghim and Caracoglia 2012b). The remarks in Chapter 3 indicate that simulation of realistic atmospheric winds within the trajectory model is of relevance for an accurate stochastic analysis of the impacts.

In this chapter, a model and numerical algorithm was developed to simulate wind-borne debris trajectories in a fully-developed atmospheric boundary layer wind. The model makes use of synthetically-generated turbulence time histories; it accounts for
variable mean velocity field with elevation and turbulence. The simulation of a partially coherent wind field was based on the wave superposition method (Di Paola 1998). The digital simulation uses the principle of superposition of harmonic waves with random phase.

For the simulation of the turbulence field, a two-dimensional (2D) wind field model was adopted by following two major steps. First, the boundary layer and turbulence time histories were generated at the “inlet” of the wind field (from where the object originates its trajectory). Time histories of horizontal turbulence and vertical turbulence were digitally generated at equidistant points, equally spaced vertically and horizontally on a 2D grid of a reference continuum 2D region covering the potential debris flight range (Figure 4.1). The Kaimal Spectrum for horizontal turbulence (Kaimal et al. 1972) and the Lumley-Panofsky Spectrum (Simiu and Scanlan 1996) for vertical turbulence were utilized.

The grid geometry was obtained by discretizing the continuum in the figure. This grid of points was used to estimate the wind field velocity as the object moves into the field. For validation of these wind time histories, the horizontal and vertical turbulence spectra from the synthetic records were compared to the target spectral models (Kaimal et al. 1972).
Figure 4.1 Discretized two-dimensional grid for digital generation of wind velocity field.

Subsequently, the records of the time histories were “propagated”, internally to the wind field, without explicitly resolving for wind field at all locations but following two simplified numerical schemes. These utilize either Taylor’s “frozen turbulent” hypothesis (Panofsky and Dutton 1984) or a simplified “Eulerian-Lagrangian” formulation, later described. The latter designation was used to emphasize that an expression was employed to approximately replicate the features of the “Lagrangian turbulence wind spectrum” (for high-speed moving objects), even though turbulence was still synthetically generated on a large number of grid points in the field at each time instant, i.e., from the “Eulerian point of view”; this second method will be described in more detail in Section 4.2. The
reason for proposing a new scheme for turbulence generation is due to the fact that, in contrast with most applications in wind engineering (e.g., for line-like structures such as a tall building or a bridge), the stochastic turbulence field must be simultaneously estimated at several points on a large 2D region (Figure 4.1). This requirement increases the computational complexity and makes estimation of trajectory by Monte-Carlo methods time consuming. The two simplified approaches, outlined above, were therefore proposed to facilitate the computations; these are described in the next sub-sections. These were employed to overcome the computational burden induced by the need for repeating the generation of wind field scenarios many times in the framework of a stochastic trajectory estimation via Monte-Carlo sampling, which will be later described in Chapter 5 and 6 (e.g., (Moghim and Caracoglia 2012a)).

After generating the wind field, the trajectory of compact objects was estimated by means of a point mass dynamic model in state space and integrated by fifth-order Runge-Kutta method, as in previous chapters. Once the accurate simulation of the turbulence wind field was established, a series of trajectory examples, including the effects of horizontal and vertical turbulence, were generated in order to be compared to uniform wind field results. Trajectory results were discussed in relation to the derivation of the value of momentum and angle of debris at various certain distances from the object’s takeoff position (e.g. Section 3.4).
4.2 Wind field model

4.2.1 Synthetic generation of boundary layer and turbulence time histories at the inlet of the wind field

As outlined in the previous section, the wind field was built in two steps. First, assuming that the boundary layer mean wind field “flows” from left to right in Figure 4.1, the velocity and turbulence characteristics were generated at the grid points located at the “inlet boundary” of the field along the left side; these are indicated by solid dark (red) markers in Figure 4.1 along the vertical line at the horizontal coordinate x=0. The n discrete points, where the mean wind velocity and its fluctuations are generated, were positioned along the vertical inlet line; their position can be uniquely defined by their vertical coordinates $z_i$ with $i=1,2,\ldots,n$ in Figure 4.1.

The horizontal wind velocity at each grid point of the inlet ($x=0$) depends on elevation $z=z_i$; it can be represented as the summation of mean wind speed and a fluctuating zero-mean turbulence, as (dimensional units)

$$U(x = 0, z = z_i, t) = U_m(z) + u(x = 0, z, t)$$ (4.1)

In Equation 4.1 $U(x,z,t)$ is the total horizontal wind speed at a given point of coordinates $(x,z)$ in the 2D region of Figure 4.1 and time $t$; the vertical coordinate $z$ of the points varies but $x=0$. The term $U_m(z)$ is the mean value of the $U(x,z,t)$, and $u(x,z,t)$ is the
horizontal turbulence component. The mean value $U_m(z)$ of the mean field was derived from the logarithmic law (Simiu and Scanlan 1996):

$$U_m(z) = \frac{1}{\kappa} u_* \ln \left( \frac{z}{z_{0,r}} \right)$$

(4.2)

In Equation 4.2, $\kappa=0.4$ is the von Kármán constant, $z_{0,r}$ is the roughness length and $u_*$ is the friction velocity, which can be indirectly estimated using Equation 4.2, by assuming as an independent parameter the mean velocity at the reference elevation 10 m, $U_{m,10}=U_m(z=10)$, as: $u_* = \kappa U_m(z) / \ln \left( z / z_{0,r} \right)$. In the modeling of wind shear $U=U_m=0$ (identically) was used for $0<z<z_{0,r}$ as a first approximation.

The horizontal component of the wind turbulence (fluctuating part only) at the generic grid point “$i$” ($i=1,\ldots,n$) along the inlet was re-labeled as $u_i(t)=u(x=0,z_i,t)$ from Equation 4.1 for simplicity of notation. Each point becomes a (1V–1D) uni-variate one-dimensional stochastic process.

After combining each scalar process $u_i(t)$ into an $n$-dimensional vector $\mathbf{u}(t)$, the stochastic discretized field of the horizontal turbulence becomes an $n$-variate one-dimensional stochastic process. As mentioned before, the $\mathbf{u}(t)$ is a zero-mean stationary random vector process of dimension $n$, the features of which can be represented by its PSD (power spectral density) matrix $S_{uu}(\omega)$ of dimension $n \times n$ as a function of circular
frequency $\omega$. The cross-PSD scalar function between any two points along the inlet (Figure 4.1), i.e., the two stochastic processes $u_i$ and $u_j$, may be calculated using Equation 4.3 below:

$$S_{u_i u_j}(\omega) = \sqrt{S_{u_i}(\omega) \cdot S_{u_j}(\omega)} \times \exp(-\hat{f}_u)$$

(4.3)

In Equation 4.3 the imaginary part of the PSD function (quadrature spectrum) is neglected as it vanishes in homogeneous turbulence (Simiu and Scanlan 1996); $S_u(\omega)$ is the auto-power spectral density of turbulence at point $i$; the variable $\hat{f}_u$ depends on the distance between $i$ and $j$, as $\hat{f}_u = c_{zu} \cdot (\omega / \pi) |z_j - z_i| (U_u(z_j) + U_u(z_i))^{-1}$ with $\omega>0$ only (one-sided spectra and cross-spectra). In the latter expression $c_{zu}=10$ is a dimensionless decay parameter for horizontal turbulence (Simiu and Scanlan 1996), $z_i$ and $z_j$ are the elevations of the generic discrete points on the grid, measured from the ground.

The function $\text{Coh}(z_i, z_j; \omega) = \exp(-\hat{f}_u)$ is the square root of the coherence function (or, “co-coherence” (Bendat and Piersol 2000; Simiu and Scanlan 1996)). The use of co-coherence model replicates full-scale characteristics of stationary wind turbulence. The definition of the quantity $\hat{f}_u$ is well established in the wind engineering field (Simiu and Scanlan 1996); it is based on flow similarity, by noting the duality between turbulence frequency and geometric scales and the fact that large scale (low frequency) features
appear more coherent. The model is, however, approximate. For example at very low frequencies \( \hat{f}_u \to 0 \) suggests flow structures with very large coherence, irrespective of the relative distance between the two points (physically incompatible). Also, the parameter \( c_{z,u} \) is taken as a constant, which is calibrated by fitting measured full-scale wind turbulence cross-spectra at various locations by neglecting local variations, dependence on height and wind speed (Simiu and Scanlan 1996). Even though Equation 4.3 introduces simplifications in complex turbulence environments, it can be accepted for engineering purposes and was employed in this study.

For generating the horizontal velocity fluctuation equation the method of “wave superposition”, described in (Di Paola 1998) was used:

\[
u_i(t) \approx \sum_{k=1}^{N} \frac{\sqrt{\Delta \omega}}{\sqrt{\pi}} \cdot \left[ \Phi_u(\omega_k) \cdot (\Lambda_u(\omega_k))^{1/2} \right] \cdot [A_i \sin(\omega_k t) + B_i \cos(\omega_k t)]
\]

(4.4)

In Equation 4.4, \( \omega_k = k \Delta \omega \) are pulsations evaluated at discrete intervals with \( k=1, \ldots, N \) and \( \Delta \omega \) a circular frequency step; \( N \) is the total number of waves, employed by the wave superposition algorithm, which implies signal truncation beyond a sufficiently large cutoff frequency (Di Paola 1998). The real matrices \( \Phi_u \) and \( \Lambda_u \) of dimension \( n \) by \( n \) can be obtained from the diagonalization of the one-sided cross-PSD matrix at each frequency; this matrix can be decomposed as (Di Paola 1998):
\( S_{uu}(\omega_k) = \Phi_u(\omega_k) \Lambda_u(\omega_k) \Phi_u^T(\omega_k) \); \( \Lambda_u \) and \( \Phi_u \) are real since \( S_{uu}(\omega_k) \) is real symmetric by construction. The \( n \times 1 \) vector coefficients \( A_k \) and \( B_k \) in Equation (4.4) are employed to produce a random phase angle in each \( k \)-th synthetic wave, thereby conferring randomness to the wind field vector. The scalar terms of these vectors, designated as \((A_k)_j\) and \((B_k)_j\), must be chosen as two independent zero-mean Gaussian variables with variance equal to 0.5. For computational purposes, the generation of Gaussian variables used the “Box-Muller” algorithm. The Box-Muller algorithm is described in (Robert and Casella 2004); it was preferred to other synthetic generators because it can numerically produce two independent Gaussian variables from two independent uniform random variables with high accuracy. The requirement imposed on the variance of \((A_k)_j\) and \((B_k)_j\) was needed to satisfy the fact that the variance of the linear process \[ [A_k \sin(\omega_k t) + B_k \cos(\omega_k t)] \] for each \( k \)-th wave must be one (Di Paola 1998); this hypothesis is equivalent to a random process of unit amplitude and uniformly distributed random phase. The \( \Phi \) vectors, which depend on \( \omega_k \) (frequency by frequency), are often referred to as the “blowing mode shapes of the horizontal wind velocity field” (Di Paola 1998).

In order to simulate the cross-PSD function for the horizontal turbulence component the Kaimal spectrum was used; this spectrum model is valid for neutral atmosphere and is acceptable for engineering purposes. The expression of this spectrum as a function of \( \omega > 0 \) and for a point located at elevation \( z \) from the ground \( (S_u(\omega, z)) \) is derived from
(Kaimal et al. 1972; Simiu and Scanlan 1996): $\omega S_w(\omega; z) / u_z^2 = 2\pi \times 200 f_r (1 + 50 f_r)^{5/3}$ with dimensionless frequency $f_r = \omega z / (2\pi U(z))$.

An equation similar to Equation 4.4 was utilized for digitally generating the zero-mean partially correlated time histories of the vertical turbulence at the inlet boundary in Figure 4.1. The time histories of the vertical turbulence $w_i(t)=w(x=0,z_i,t)$ were synthetically produced at the same grid points ($i=1,\ldots,n$) along the inlet.

For vertical turbulence the Lumley-Panofsky spectrum was employed; this spectrum reads: $\omega S_w(\omega; z) / u_z^2 = 2\pi \times 3.36 f_r (1 + 10 f_r)^{5/3}$ for $\omega>0$ and $z>0$.

In the first study the random multi-variate processes $u$ and $w$ were assumed as non-correlated (refer to Section 6.4 for additional investigation). In this case, the digital generation of the random sample of $w_i(t)$ is independent of $u_i(t)$ and becomes:

$$w_i(t) \approx \sum_{k=0}^{K} \frac{\sqrt{\Delta \omega}}{\sqrt{\pi}} \cdot \left[ \Phi_w(\omega_k) \cdot \left( \Lambda_w(\omega_k) \right)^{1/2} \right] \cdot \left[ E_k \sin(\omega_k t) + F_k \cos(\omega_k t) \right]$$  \hspace{1cm} (4.5)

In the previous expression $(E_k)_i$ and $(F_k)_i$ are independent Gaussian random variables, defined as before to impart a random phase to each wave. The quantities $\Phi_w$ and $\Lambda_w$ can be found from the orthogonalization of the cross-power spectral density matrix of the vertical turbulence $S_{ww}(\omega_k) = \Phi_w(\omega_k) \Lambda_w(\omega_k) \Phi_w^T(\omega_k)$, estimated at the inlet boundary grid.
points as
\[ S_{w_i w_j}(\omega) = \sqrt{\left(\hat{S}_{w_i w_i}(\omega) \cdot \hat{S}_{w_j w_j}(\omega) \times \exp(-\hat{\beta}_u)\right)} \cdot \hat{f}_w = c_{z,\mu} \cdot (\omega / \pi) \left| z_j - z_i \right| (U_m(z_j) + U_m(z_i))^i \]
with \( c_{z,\mu} \approx c_{z,\mu} = 10 \) (Simiu and Scanlan 1996).

4.2.2 Propagation of turbulent-wind time histories, internally to the field, by “frozen turbulence” hypothesis

After determining the velocity profile and turbulence \( u_i(t) \) and \( w_i(t) \) at the inlet boundary of the field in 2-D, propagation of turbulence internally to the grid in Figure 4.1 was established by a simple rule. For example, if the debris object is located at the position \( (x, z) \) at time \( t \) during the flight, the wind velocity is needed locally at the same point. Therefore, the horizontal wind velocity (mean and turbulence) \( U(x, z, t) \) is needed:

\[
U(x, z, t) \approx U_m(z) + u(q\Delta x, i\Delta z, t) \\
\approx U_m(z) + u\left(0, i\Delta z, t - \frac{q\Delta x}{U_m(z)}\right) \\
= U_m(z) + u(t - q\Delta x / U_m)
\]

In Equation 4.6 \( U_m(z) \) is the mean wind speed at elevation \( z \). The total horizontal wind velocity at any given point in the field \( (x, z) \) was approximately estimated by using the turbulence synthetically generated at the point closest to the actual object location defined by coordinates \( x \) and \( z \) on the grid in Figure 4.1, i.e., by accepting that \( z \approx i\Delta z \) and \( x \approx q\Delta x \); the integer index \( q \) was employed to designate the grid point of horizontal coordinate \( x_q = q\Delta x \).
The Taylor’s hypothesis of frozen turbulence was used on the right-hand side of Equation 4.6 to propagate \( u \) from the inlet boundary at \((0,z)\) toward the center of the field at \((x,z)\); Equation 4.6 assumes that the turbulence propagates from the left boundary and “reaches” this point after time \( t=q\Delta x/U_m \). Also, a simplified notation is introduced in Equation 4.6 for \( x=q\Delta x \), which reads \( u(0,i\Delta z, t - q\Delta x / U_m) = u_i(q\Delta x / U_m) \) and uses Equation 4.4 with \( u_i(t) = u(x_0=0, z_i, t) \) to designate the grid points along the inlet boundary. This assumption reduces turbulence generation of the whole field of \( n \times m \) points to the synthetic generation to the \( n \) points on the left boundary. In a similar way the vertical turbulence component of the wind field was evaluated as \( w(x,z,t) = w_i(t-q\Delta x/U_m) \).

The numerically generated wind velocity field was therefore designated as a “Frozen Turbulence Field” (FTF).

4.2.3 Propagation of turbulent-wind time histories, internally to the field by simplified “Eulerian-Lagrangian” formulation

As discussed in (Holmes 2004), when the velocity of the debris approaches the mean flow speed, the relative velocity between debris and wind field becomes smaller. This observation leads to smaller frequencies “seen” by the moving debris (fluid-object interaction) and questions the validity of a quasi-steady hypothesis for flow and pressure loading. As suggested in (Holmes 2004), for high-velocity debris the Euler turbulence spectrum (Equation 4.3) can be inaccurate for simulating turbulence features around the
object; as a consequence, a Lagrangian turbulence spectrum, accounting for the relative velocity effect, should possibly be employed.

Previous investigators (Holmes 2004; Karimpour and Kaye 2012) have often neglected this remark because of the lack of experimental information on Lagrangian spectra. In contrast, several literature studies and models exist to describe the Euler spectrum (e.g., (Kaimal et al. 1972)). In this section the Lagrangian feature in the turbulence field, later applied to trajectory estimation, was investigated by modifying Equation 4.6 of the FTF. A simplified “Eulerian-Lagrangian” formulation was proposed to account for this effect.

It must be noted that the term “Eulerian-Lagrangian” must not be intended in the traditional sense of time-marching schemes for computational fluid dynamics; it is utilized to emphasize the fact that an equation is employed to approximately replicate the features of the Lagrangian turbulence wind spectrum for high-speed moving objects, even though turbulence generation was still carried out on a large portion of the field at all times (i.e., from the Eulerian point of view). According to this assumption, Equation 4.7 below describes the synthetic propagation of horizontal velocity and its fluctuations, internally to the field:

\[ U(x,z,t) \approx U_n(z) + u \left( t - \frac{q \Delta x}{U_n(z) - u_s(t)} \right) \] (4.7)
In the above equation the relative velocity between the moving object and external flow is employed; the quantity \( u_x(t) \) with \( 0<u_x(t)<U_m(z) \) is the horizontal debris velocity (point-mass object) with respect to the fixed reference in Figure 4.1, which corresponds to the position \((x,z)\) occupied by the debris at time \( t \). It must be noted that \((x,z)\) is used to designate both the generic point in the wind field but also the generic position of the debris along its trajectory.

This second case was labeled as “Simplified Eulerian-Lagrangian turbulence Field” (SELF).

### 4.3 Compact debris trajectory and momentum model

The equations of motion (Equations (2.2)) proposed by (Tachikawa 1983) and described in detail in Chapter 2 can be used to describe the 2D trajectory of compact debris, assimilated as a point-mass object moving in the horizontal and vertical directions in the region shown in Figure 4.1 Given the generic spatial position of the object in Figure 4.1 \((x,z)\), these equations can be written in dimensionless form as:

\[
\frac{d^2 \bar{x}}{dt^2} = \frac{d \bar{u}_x}{dt} = K \left[ \left( \bar{U}(\bar{x}, \bar{z}, t) - \bar{u}_x \right)^2 + \left( \bar{w}(\bar{x}, \bar{z}, t) - \bar{u}_z \right)^2 \right] \left( C_d \cos \Phi \right) \tag{4.8 a}
\]

\[
\frac{d^2 \bar{z}}{dt^2} = \frac{d \bar{u}_z}{dt} = K \left[ \left( \bar{U}(\bar{x}, \bar{z}, t) - \bar{u}_x \right)^2 + \left( \bar{w}(\bar{x}, \bar{z}, t) - \bar{u}_z \right)^2 \right] \left( C_d \sin \Phi \right) - 1 \tag{4.8 b}
\]
In Equations (4.8a) and (4.8b) the aerodynamic force is quasi-steady (with $C_D \neq 0$ constant drag coefficient); it is computed by considering the magnitude of the instantaneous relative velocity between the point-mass object at position $(x,z)$, time $t$ and the field. The generic position along the trajectory is normalized in accordance with Tachikawa’s dimensional analysis as 
$$
\bar{x} = x \cdot g \left( \frac{U_{m,10}}{2} \right)^{-2},
$$
and 
$$
\bar{z} = z \cdot g \left( \frac{U_{m,10}}{2} \right)^{-2},
$$
in which $U_{m,10}$ is taken as the mean wind speed at the reference height $z=10$ m; $K$ is the Tachikawa number, defined as 
$$
K = \frac{U_{m,10}^2 \rho_a A}{2mg},
$$
with $A$ being a reference projected area, $m$ the mass of the object and $\rho_a$ the air density. The absolute horizontal and vertical velocity components of the point-mass debris are $u_x$ and $u_z$, from which the dimensionless quantities are found as 
$$
\bar{u}_x = \frac{u_x}{U_{m,10}}, \\
\bar{u}_z = \frac{u_z}{U_{m,10}}.
$$

In Equations (4.8a) and (4.8b), $\bar{U}$ and $\bar{w}$ are horizontal (mean plus turbulence) and vertical components of the wind speed in dimensionless form, with $\bar{U} = \frac{U}{U_{m,10}}$ being evaluated at the grid point in Figure 4.1 in close proximity to the object either from Equation (4.6) or (4.7); $\bar{w} = \frac{w}{U_{m,10}}$ is determined in a similar way. In Equations (4.8a) and (4.8b) the relative angle $\Phi$ accounts for the relative velocity between the point-mass object and the flow; it was determined as 
$$
\Phi = \tan^{-1}\left[ \frac{(\bar{w} - \bar{u}_z)}{(\bar{U} - \bar{u}_z)} \right].
$$
The trajectory can be found by numerical integration of Equations (4.8); as in previous studies a 5th-
order Runge-Kutta method is used, after transforming the second-order system of differential equations to a first-order state space form.

It is worthy no note that it is usually difficult to define exactly the values of the $K$ due to the very large variability in sizes and mass of objects that may become wind-borne As a first approximation previous studies (e.g, (Tachikawa 1988)) have indicated $2<K<7$ as an acceptable interval for this quantity. The $K$ values, used for analyzing the numerical results, are based on this hypothesis.

Equations (4.8a) and (4.8b) must be integrated at any given point in time also to obtain the modulus of the velocity and the instantaneous trajectory angle (slope) with respect to the horizontal plane. These two quantities (velocity and angle at impact) can be calculated in dimensionless form as described in Section 3.2 by Equations (3.5) and to (3.6).

4.4 Numerical results

4.4.1 Turbulence time history for uncorrelated $u$ and $w$ – validation example

The method described in Section 4.2 was used to synthetically generate turbulence in horizontal and vertical directions on a simulated 2D field. Trajectory estimation in turbulent wind field for an urban or sub-urban exposure was considered for the purposes outlined in previous chapters. The mean wind profile was determined by assuming a roughness length $z_{0,r}=0.3$ m and by pre-selecting the value of the mean wind speed at the
reference height equal to 10 meters \((U_{m,10})\). The turbulence intensity of the horizontal turbulence varies with elevation according to the relationship \(I_u = 1/\log(z/z_{0,r})\) (Simiu and Scanlan 1996). This relationship can be used to estimate the typical standard deviation of turbulence in a stationary atmospheric wind field (“theoretical value”, \(\sigma_{u,\text{theoretical}} = \sqrt{6/u_*}\)); it is valid for built-up terrain with \(z_{0,r} > 0.3\) m (Simiu and Scanlan 1996) and can be employed in conjunction with the Kaimal spectrum. The vertical turbulence component was similarly obtained using the Lumley-Panofsky spectrum, as indicated in Section 4.2; the turbulence intensity was approximated as \(I_w = 0.75I_u\) (Dyrbye and Hansen 1997).

For turbulence generation, the resolution of the grid in Figure 4.1 was chosen to reflect a total depth of the grid equal to a reference dimension \(H_{0,u} = 20\) m; this dimension was defined as the upper limit of the initial object position for a generic object at \(t=0\), which was proposed in (Moghim and Caracoglia 2012a) as a suitable value for trajectory estimation in an urban (but open) exposure; the concept of “initial object position” (takeoff position) will be better described in the next chapters. The grid was found with \(\Delta z/H_{0,u} = 1/20\), and \(\Delta x = \Delta z\).

The first example of simulation was carried out to analyze a wind field with moderate mean wind velocity \(U_{m,10} = 30\) m. In order to confirm the validity of the numerical algorithm based on Equation 4.6 (“FTF scheme”), a synthetic realization of horizontal (stream-wise) wind fluctuations is shown in Figure 4.2. The 10-minute record of \(u\) is
based on Equation 4.4 with \( N=1000 \) waves; the circular frequency step \( \Delta \omega \) was derived by assuming an upper cut-off frequency equal to 10 Hz. The selection of \( N \) and the upper frequency was dictated by the ability of reproducing sufficiently small turbulence scales (of the same order as the debris reference dimension, usually between tenths of centimeters to less than one meter) and by preserving both sufficient resolution in the synthetic signal and broad-band turbulence features.

The turbulence intensity \( I_u \) of the horizontal turbulence was calculated from the data of the synthetic time-history record in Figure 4.2 (a) as 0.207 and was compared to the target value at the same elevation \( z=10 \) m, \( I_u=0.213 \). A study was conducted to analyze the effects of the selection of \( N \) on the resolution of the turbulence spectrum (not reported for brevity). The selection of \( N=1000 \) and upper frequency shows a good agreement between theoretical values and simulated record. Also, the theoretical value of the turbulence standard deviation \( \sigma_{u,\text{theoretical}} = \sqrt{6} / u_* = 6.38 \) m/s is very close to the value estimated from the record, as 6.34 m/s. The resolution in the equivalent spectrum of the synthetic record was investigated in Figure 4.2 (b), in which the power spectral density of the realization in Figure 4.2 (a) was plotted against the Kaimal model in a log-log scale. A very good match was observed between the two spectra below the cut-off frequency with maximum relative differences of the order of 5% only at high frequencies; this remark suggests that the procedure can be used for synthetic turbulence replication.
Finally, the comparison between synthetically generated turbulence $u$ and theoretical behavior was extended to the spatial features of the turbulence field by analyzing the cross-PSD functions (Equation 4.3) of the horizontal turbulence between two generic points of elevations $z_i$ and $z_j$, located on the inlet line of the field (Figure 4.1). In this second verification case the reference mean wind speed was taken as $U_{m,10}=30$ m/s.
Figure 4.3 shows the comparison between the normalized cross-spectra, i.e., the u-turbulence co-coherence function (square root of the coherence function), \( \text{Coh}(z_i,z_j;\omega) = \exp(-\hat{f}_u) \), for two sets of points with \( z_i=12\text{m} \) \( z_j=20\text{m} \) in Figure 4.3 (a) and \( z_i=12\text{m} \) \( z_j=28\text{m} \) in Figure 4.3 (b). The co-coherence \( \text{Coh}(z_i,z_j;\omega) = \exp(-\hat{f}_u) \) was calculated at various frequencies \( \omega/\pi>0 \) (in Hz) using both synthetic data and the cross-spectra in Equation (4.3), based on the Kaimal spectrum model (one-sided). The synthetic-data normalized spectrum, shown in the figures, was evaluated numerically by using 2048 FFT points; it employs 45000 data points over a 10-minute interval (50 samples/s). For compatibility with the definition of co-coherence the real part of this spectrum is depicted in the figures.

The correspondence between data and theoretical model (thick dashed line) was found to be acceptable with both functions following the same trend but with the synthetic data exhibiting some scatter at higher frequencies around the upper cut off (10 Hz). This behavior at higher frequencies is not unexpected; it has been noted in previous studies (Carassale and Solari 2006). It is believed to be influenced by the procedure used for numerical evaluation of the synthetic-data spectrum. Also, negative values displayed by the synthetic-data normalized cross-spectra, which are in contrast with the theoretical equation \( \text{Coh}(z_i,z_j;\omega) = \exp(-\hat{f}) \), can be noticed for frequencies above 1 Hz. These, variations are however of secondary importance, since they correspond to small
magnitude fluctuations in terms of the total absolute value of the turbulence cross-
spectrum (Equation 4.3).

Figure 4.3 Comparison between cross-PSD functions of the horizontal turbulence,
obtained from synthetically generated records, and theoretical expression in Equation
(4.3) in terms of “co-coherence” (Coh(z_i, z_j; \omega)) for U_{\text{m,10}}=30 m/s; (a) z_i=12 m, z_j=20 m;
(b) z_i=12 m, z_j=28 m.
4.4.2 Compact-object trajectory analysis in fully turbulent stationary wind

In this section the trajectory results are computed and discussed, using both the FTF Equation (4.6) and the proposed SELF in (4.7). For comparison with previous chapters the trajectory of a cube (\(C_D=0.8\)) was calculated in both uniform and turbulent wind fields. The result for one realization for a cube in fully turbulent wind field (\(U_{m,10}=30\text{ m/s}\)) under the assumption of frozen turbulence field (FTF) and simplified Eulerian-Lagrangian (SELF) field is shown in Figure 4.4.

Figure 4.4 shows a comparison of trajectories obtained with the two turbulence models and with a uniform wind field (no turbulence); in the latter case the wind speed was taken as constant with height and equal to \(U_{m,10}\), independently of \(z\). In Figure 4.4 (a) the comparison of the trajectories is depicted in dimensionless units (\(\bar{x}=0.109\) corresponds to 10 m at \(U_{m,10}=30\text{ m/s}\)), whereas in Figure 4.4 (b) the time history of the horizontal debris position is presented. The initial object position of the object was taken as \(\bar{z}=0.109\), which coincides with \(z=10\) m at \(U_{m,10}=30\text{ m/s}\) (i.e., at the center point of the inlet). In order to enable the comparison between FTF and SELF formulation, records are labeled as “Realization 1 (FTF)” and “Realization 1 (SELF)” in Figure 4.4 (a) and (b).
Figure 4.4 Dimensionless 2D trajectories (a) and horizontal velocity time histories (b) for a cube in fully turbulent wind field \((U_{m,10}=30\, \text{m/s})\) under the assumption of frozen turbulence field (FTF) and simplified Eulerian-Lagrangian (SELF) field; one sample realization of the trajectory is shown (Tachikawa number, \(K=3\)).
Since the wind field is random each trajectory is a random curve. Therefore, for the comparison between FTF and SELF results, trajectories must be obtained using the same realization of the stochastic field. For example, from the analysis of Equation 4.4 for \( u \) and Equation 4.5 for \( w \) it is observed that this can be accomplished by utilizing the same sequence of random phases, \( A_k, B_k, E_k, F_k \), in the wave superposition algorithm. Therefore, the same realization of the stochastic wind field was used in Figure 4.4 to analyze the differences between FTF and SELF. From the inspection of Figure 4.4 (a) it can be noted that the trajectory of the object is indeed affected by the combined wind shear and turbulence. There is limited difference in the numerical results between the FTF and SELF models during early flight. As the time progresses and the velocity of the object increases, the difference between the FTF and SELF results becomes more visible (Figure 4.4 (b)). This remark is compatible with the observation on the relevance of the Lagrangian spectrum approach as the absolute velocity of the object increases (Holmes 2004).

Finally, the variability in the trajectory of the same cube in a random field (Tachikawa number, \( K=3 \)) was evaluated by repeating 10 times the realization of the wind i.e., by using 10 different random phase-angle sets in the wave-superposition method. The distinct realizations of the FTF field were labeled as R1,…,R10. Results are shown in Figure 4.5. In most cases the object flies higher than the turbulence-free case (beyond \( \bar{x} > 0.33 \)).
Figure 4.5 Effect of FTF turbulence on 2D trajectory of a cube (with Tachikawa number $K=3$) – dimensionless trajectory for $I_u=0.21$ taking off at $z=10$ m

$$Z = zg \left( U_{m,10} \right)^2 = 0.11.$$  

Realizations R9 and R10 seem to result in a “backward trajectory”. This situation is perhaps not unusual; similar patterns and variations (as between trajectories R1 and R10) have been observed by (Holmes 2004) in a fully turbulent field for a sphere with $K=2.07$ by repeated runs of synthetic turbulence generation and trajectory estimation. The observation on unrealistic “backward trajectory” can possibly be explained by noting that the direction of the relative wind velocity vector at takeoff is not horizontal due to the random $w$ turbulence (simulated turbulence intensity of $w$ between 12% and 15% at reference height). As a result, at takeoff, the aerodynamic force magnitude and direction (in line with the wind velocity vector for a compact object) become random. In turn, the
variability of this force has an influence on the magnitude and direction of accelerations, imparted to the object at initial stages of the flight; this could result in trajectory variability. Even though the ensemble average of the trajectories in Figure 4.5 will not likely differ from the dashed curve (turbulence-free case), differences in the probability of reaching a downwind “target” of finite and small vertical dimension, conditional on the initial position at takeoff, are plausible. This variability is believed to have important implications on the probabilistic analysis of the trajectory, as documented in the next chapters.

4.4.3 Assessment of compact debris properties in the simulated wind field as a function of relative distance “source/target”

As described by Equation 3.5 in Chapter 3, the quantity \( |\mathbf{V}| = \bar{V} \) can be evaluated once the compact object reaches its target; this condition can be uniquely defined by setting the horizontal distance \( d^* \) (relative between source and target, Figure 3.1 (a)) traveled along its trajectory; in this section of this chapter the quantity \( d^* \) was relabeled as \( d_0 \).

The quantity \( |\mathbf{V}| = \bar{V} \) can be used as a non-dimensional measure of debris momentum at impact since it is proportional to the debris momentum in Equation 3.8; momentum can be indirectly utilized in the evaluation of damage to glass panels as a function of various debris properties. Similarly the angle \( \Psi_z \) in Equation (3.6), evaluated after the object has traveled and reached the same relative \( d_0 \), can be employed to fully characterize the main features of the debris motion in the proximity of a “target”.
Due to the observed variability in the debris trajectory (from Figure 4.5) using the synthetically-generated turbulent wind field, it is important to study the stochasticity of $|\vec{V}|=\bar{V}$ and $\Psi_z$. A parametric study was carried out to analyze the mean properties of random quantities.

As an example, the CDF (cumulative density function) of $|\vec{V}|=\bar{V}$ was plotted in Figure 4.6 by using the data generated from $N_r=1000$ repeated realizations of trajectory for a compact object with $K=5$, $C_D=0.65$, flying in a wind field with $U_{m,10}=30$ m/s and from its initial object position before takeoff (relative to the target) $d_0=10$ m. The latter quantity corresponds to $d_0 = d_0 g \left( U_{m,10} \right)^2 = 0.11$ in dimensionless units. The “plotting position” technique was employed to evaluate the empirical CDF from the data of $|\vec{V}|=\bar{V}$ (Mood et al. 1974). The empirical CDF was compared against three potential CDF models, selected to describe the behavior of a random variable $\bar{V} \geq 0$. The distributions are: log-normal, gamma and Weibull. The maximum likelihood method (Mood et al. 1974) was used to estimate the parameters of each CDF model from the data. Inspection of this figure by comparison of the distributions, revealed that the log-normal distribution appears to be more suitable, since this CDF model is qualitatively closer to the empirical function of $|\vec{V}|=\bar{V}$ in the upper tail; also it still preserves the main
information for smaller probability values below the lower quartile of the distribution in the interval \(0.45 < \bar{V} < 0.65\).

Figure 4.6 CDF of \(|V|\) for compact debris with \(K=5, C_D=0.65\), flying in a wind field with \(U_{m,10}=30\) m/s and with initial position before takeoff, relative to the target, \(d_0=10\) m (\(\bar{d}_0 = d_0 g \left(U_{m,10}\right)^2 = 0.11\)). The small figure on the upper right-hand corner is a “local zoom” of the main figure to highlight the variations in the upper tail.

A Kolmogorov-Smirnov statistical test (e.g., (Mood et al. 1974)) was also used to test the hypothesis that the empirical CDF can be suitably represented by each of the theoretical distributions, with the mean and standard deviation being estimated from the sample with \(N_P=1000\) realizations. The Kolmogorov–Smirnov test is a nonparametric test for the analysis of continuous, uni-variate probability distributions; it can be used to compare a sample of realizations of a random variable with a reference probability
distribution form. The test measures the possibility that a sample adheres to a given distributional form. From the analysis of the data sets, it was concluded that the hypothesis regarding the distributional form is rejected for the Weibull and Gamma distributions whereas the test is not negative for the log-normal distribution with a significance level of 0.15%, suggesting a more evident appropriateness of the log-normal form in comparison with the other distributions. The significance level defines the sensitivity of the test and the probability of inadvertently reject the hypothesis. Even though this analysis was limited to one test case, the transformation to dimensionless quantities renders the results applicable to a variety of wind velocity scenarios (by dynamic similarity), provided that the same non-dimensional groups are preserved (i.e., dimensionless velocity or momentum, Tachikawa number, turbulence intensity in the field); it also provided useful information to be used in later chapters of this work.

The PDF (probability density function) of the same data set was also analyzed and it is shown in Figure 4.7. A reasonable agreement of the empirical histogram with the log-normal curve was observed. To generate this figure the number of realizations \( N_P = 1000 \) was again utilized and the maximum likelihood method was used to estimate the parameters of the log-normal PDF.
Influence of the sample size $N_P$, selected for the numerical estimation of the statistical properties of $\left|\bar{V}\right|$, was also investigated in order to derive a suitable minimum value of $N_P$, beyond which limited differences are observed in the main features of the distribution. Selection of this value is also of relevance to limit the error in the probability analysis in this or subsequent chapters. In order to accomplish this objective, the PDF of the momentum was numerically estimated from several realizations of trajectories of a compact object with $C_D=0.65$, for a given $K=5$, relative target position $d_0=10$ m and for fully-turbulent wind field using the FTF scheme ($U_{m,10}=30$ m/s). Comparison of the PDF as a function of the total number of realizations is plotted in Figure 4.8.
Figure 4.8 PDF comparison for different sample size $N_p$ ($K=5$, $C_D=0.65$, $U_{m,10}=30$ m/s).

The analysis of Figure 4.8 suggests that the minimum $N_p$ that should be considered is 300 since beyond this value little variations in the PDF graphs can be observed (e.g., small deviations of the mode and unnoticeable differences in the tails). Therefore, in the remainder of this section, the reference values of each quantity ($\bar{V}$ and $\Psi_z$) were calculated using the sample median obtained with $N_p=500$, as an “optimal” reference quantity.

Figure 4.9 depicts the dimensionless velocity modulus $|\bar{V}|$ derived by numerically integrating Eqs. (4.8a and 4.8b) at $U_{m,10}=30$ m/s for a debris with $C_D=0.8$ as a function of $K$. The figure was produced for four different distances between source point and target, $d_0=5, 10, 20$ and 30m, measured horizontally. As indicated above 500 realizations of
turbulent wind field were generated (FTF and SELF schemes). After determining the trajectory for a given distance source/target $d_0$ and $K$, the median momentum of the sample was calculated and it is shown in the figures.

From Figure 4.9 it can be inferred that the turbulence has an effect on $|\vec{V}|$ of the object (implying, as shown in Equation 3.8, the indirect effect on normalized momentum). This difference depends on both the Tachikawa number and $d_0$; however, it can be seen that the relative variation of this quantity at impact is between 8% and 15%. Also, as $d_0$ increases the effect of the turbulence becomes more obvious, when the velocity of the object tends to reach the mean wind speed. At larger $d_0$ (Figure 4.9 (d)) the asymptote tends to $\vec{V} = |\vec{V}| \approx 1$; this remark confirms literature observations (e.g., (Moghim and Caracoglia 2012a)) that compact objects with $K>5$ are mainly “carried by the flow”, as their velocity tends to the mean wind speed $U$.

It can also be inferred from Figure 4.9 that in a turbulence-free wind the graphs seem to be linear; the relation between normalized momentum and $K$ appears to be approximately quadratic (Equation 3.8). However, as the turbulent wind field was introduced, a larger increment in $|\vec{V}|$ was in general noticed, with relative differences varying from about 5% for small $d_0$ (Figure 4.9 a) to about 30% for larger $d_0$ (Figure 4.9 d). Also the linear relationship between $|\vec{V}|$ and $K$ tends to disappear, suggesting a nonlinear dependency between the two quantities. Therefore, the figure confirms that
accurate turbulence simulation is important even for estimation of compact object momentum.

Figure 4.9 Dimensionless velocity modulus $|\vec{V}|$ at $U_{m,10}=30$ m/s vs. Tachikawa number $K$ of a sphere with $C_D=0.65$ for various “source/target” distances before takeoff: (a) $d_0=5$ m ($\tilde{d}_0 = 0.05$), (b) $d_0=10$ m ($\tilde{d}_0 = 0.11$), (c) $d_0=15$ m ($\tilde{d}_0 = 0.16$), (d) $d_0=20$ m ($\tilde{d}_0 = 0.22$).
Figure 4.10 Angle of impact vs. Tachikawa number $K$ of a sphere with $C_D = 0.65$ flying at $U_{m,10} = 30$ m/s and for various “source/target” distances before takeoff: (a) $d_0 = 5$ m ($\bar{d}_0 = 0.05$), (b) $d_0 = 10$ m ($\bar{d}_0 = 0.11$), (c) $d_0 = 15$ m ($\bar{d}_0 = 0.16$), (d) $d_0 = 20$ m ($\bar{d}_0 = 0.22$).

As described in Chapter 3.2, the instantaneous trajectory angle with respect to the horizontal plane can be calculated by using Equation 3.6. This quantity and the velocity at impact can be useful to estimate the potential damage on the (glass) building façade. In this section the angle of impact for four different $d_0$ and for three different wind field
conditions was investigated and it is shown in Figure 4.10. Also, in Figure 3.7 the
definition of the positive and negative angle is shown. As depicted in Figure 4.10, for
heavy objects with low $K$ the angle at impact is almost 90 degrees, suggesting that the
object tends to impact vertically and proves that the flight is governed by the gravity.
Also, from the same figure it is inferred that the effect of the turbulence is more notable
for the object with lower Tachikawa number. For the angle of impact the difference
between FTF and the turbulence-free case is noticeable and varies from 9% to 20%.

4.5 Summary

In this chapter, wind-borne debris trajectories in a fully-developed atmospheric boundary
layer were estimated using a partially coherent turbulent wind field. The wave
superposition method was used to synthetically generate the wind field. Numerical results
show that turbulence can indeed affect the trajectory of compact debris in comparison
with a uniform wind speed scenario. Also, the influence of turbulence on trajectory tends
to be limited during early flight; some minor dependency on the two hypotheses, utilized
for synthetic turbulence propagation, was however observed.

Debris velocity modulus $|\mathbf{V}|$ and angle of the impact $\Psi_z$ are the two important
properties that can cause damage to the façades of a tall building. In particular, the value
of $|\mathbf{V}|$ at impact can be related to the dimensionless momentum through Equation 3.8.
Due to the variability in the trajectory, induced by turbulence, these two quantities
become random variables. The median values of $|\vec{V}|$ was therefore employed to study the “stochasticity” of the phenomenon in relation to the distance $d_0$ between the building façade and the object position before takeoff in the next chapter.

It was shown that the turbulence has an effect on the median velocity modulus of the object, with differences depending both on $d_0$ and Tachikawa number ($K$); the relative variation of velocity modulus can be of the order of 15% at $d_0$, which corresponds to a 30% increment in the normalized object momentum (assuming the same Froude number). Also, turbulence effects become more noticeable for objects with low Tachikawa number ($2<K<4$) and for larger $d_0$ values as the velocity of the object tends to reach the mean wind speed. At $d_0 = 20$ m (about 0.22 in dimensionless units) an asymptotic trend to $\vec{V} = |\vec{V}| \approx 1$ was found; this remark confirms that compact objects, especially for $K>5$, are mainly transported by the flow since the absolute velocity of the point-mass object approaches the mean wind speed. Moreover, for objects with a higher $K$ the predominant influence of aerodynamic forces on gravity forces can lead to a larger $|\vec{V}|$ and therefore normalized momentum. For example, the momentum can increase by about 44% (increment of $|\vec{V}|$ equal to 20%) if $K$ is incremented from 2 to 7. The relationship between $K$ and $|\vec{V}|$ is linear when turbulence is neglected, suggesting a quadratic dependency of normalized momentum on $K$. However, the linear relationship $|\vec{V}|$ vs. $K$ is
replaced by a nonlinear curve when turbulence and wind shear are simulated. Moreover, for heavy objects with $2<K<3$ the impact angle often reaches ninety degrees; this observation suggests that the object tends to impact almost vertically and proves that the flight is still controlled by gravity effects, in spite of the presence of turbulence. The “Küssner-like vertical gust model”, proposed in the previous chapter to simplify the analysis of trajectory in turbulent fields, can still be used for preliminary investigations but could produce conservative results in terms of momentum and angle at impact; more investigation is needed to fully clarify this aspect for a wide range of wind and turbulence scenarios.
4.6 References


Chapter 5

Probabilistic analysis of trajectory in the proximity of tall buildings: uniform wind field

5.1 Introduction

This chapter is devoted to the derivation of a numerical model for the analysis of the random trajectory of a compact debris object. Randomness was introduced to reflect the fact that the physical properties of the flying objects are usually not known (e.g., mass, projected area used to estimate the aerodynamic loads, etc.) This hypothesis implies the need for a probabilistic analysis of trajectory even for uniform wind field conditions with constant mean wind velocity, independent of the elevation from the ground (wind shear and turbulence neglected). Statistical models for the prediction of wind-borne debris damage risk have been developed in recent years. However, the damage risk is usually evaluated at the “regional scale” and the models are often applied to study low-rise buildings (Lin and Vanmarcke 2008). No attention has been paid to the need for analyzing the same risk for the cladding of a large building, especially a tall structure, along its vertical façade. This analysis is of relevance since debris-induced damage in the proximity of the tall buildings has been reported (Divel et al. 2010; Jain 2013).
In this chapter, the model and numerical algorithm developed for the solution of the dynamic debris equations in uniform wind field and in the absence of uncertainty was utilized to derive an impact risk model to analyze the “damage risk” for the glass façade of a 183m tall building. Monte Carlo simulation (Robert and Casella 2004) was used to numerically evaluate this risk, under the assumption that the fundamental physical parameters and object properties, controlling the flight trajectory, are random variables. The probabilistic analysis enabled the estimation of the “probability of impact” against selected areas of the building envelope.

The main contribution of this chapter is the generation of “Iso-probability Impact Contours”, based on the random trajectories, determined by numerical simulation. These contours describe the probability associated with a “randomly flying” object, as it impacts against the façade, conditional on the mean wind speed magnitude and the relative initial object position in the proximity of the structure.

Moreover, the numerical results will also show that it is possible to derive a “Universal Probability Curve”, independent of mean wind speed, to describe this probability of impact against the façade of a benchmark tall building, was developed in two dimensions. Moreover, the influence of the various wind field models, analyzed in the previous chapters, on the probability values is investigated.
5.2 Probability-based modeling of flight trajectory in uniform wind field

5.2.1 Probability model: description and assumptions
A damage risk model was developed in this section, which makes use of the concept of “Probability-of-Impact Curves” (PI Curves) for 2D analysis, and of “Iso-probability Impact Contours” for 3D analysis. These curves along with their derivation will be explained in the next sub-sections. These curves represent a simple “engineering instrument”, which could assist a more rational decision by designers about the need for protecting tall building façades against potential wind-borne debris damage. Predictions were carried out at the “local scale” (i.e., for cladding elements); a specific high-rise structure was employed as a pilot example.

For the generation of these curves the main physical quantities in the model of Equations 2.5, combined with the initial flight conditions, were assumed as random variables. These are: the (dimensional) initial object position on the vertical plane $z$ with respect to its “target” at $t=0$ and denoted as $H_0$ (Figure 5.1), drag coefficient $C_D$, Tachikawa number $K$ and wind direction angle $\Phi$. The initial position on the horizontal plane $(x_0, y_0)$ and the mean wind speed $U$ were assumed as pre-selected deterministic constants. More details about the effects of $U$ are discussed later in Section 5.2.2.2.

The statistical properties and probability distribution of the random parameters are summarized in Table 5.1. All random variables are independent, as a first approximation. The upper limit of the $H_0$ quantity ($H_{0,u}$) was selected to be compatible with debris flight
in an urban scenario, valid under the assumption that the debris may originate from nearby buildings, as discussed in previous chapters. The upper limit of the uniformly distributed random $H_0$, which describes the initial object position on a vertical line (i.e., its elevation on a vertical plane), was designated as $H_{0,u} = 20$ m (Figure 5.1 and Table 5.1) with $0 \leq H_0 \leq H_{0,u}$.

Figure 5.1 2D debris and target location - Elevation view for a window located on the first floor ($h_w$: height of the opening; $B$: building width; $z_w$: elevation from ground level).
The prediction of the probability of impact (PI) was derived by inspection of the flight trajectories, generated by numerical sampling; a total of $10^5$ sample points were employed to enable estimation of low-probability cases. The properties and strength of the glass panels were not considered in this study but could be readily incorporated in the probabilistic analysis. It must be noted that, in all 2D and 3D risk analyses, inelastic impact with the ground was anticipated; the interaction with the ground after initial impact was not modeled, even though “rebound effects” could still be plausible.

Table 5.1 Properties of the random variables.

<table>
<thead>
<tr>
<th>Random variable</th>
<th>Description</th>
<th>Probability distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>Initial object position ($Z$)</td>
<td>Uniform; $0 \leq H_0 \leq 20$ m</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Static drag coefficient</td>
<td>Gamma; Median($C_D$) = 0.65, STD($C_D$) = 0.50</td>
</tr>
<tr>
<td>$K$</td>
<td>Tachikawa number</td>
<td>Uniform; $2.0 \leq K \leq 7.0$</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Mean-wind yaw angle (XY plane)</td>
<td>Uniform; $\Phi_1 \leq \Phi \leq \Phi_2$ with grid-dependent $\Phi_1$, $\Phi_2$</td>
</tr>
</tbody>
</table>

5.2.2 Probabilistic model: numerical results in 2D

5.2.2.1 Trajectory analysis for $U = 30$ m/s

The CAARC building was used as the reference structure (Melbourne 1980); this building has a rectangular floor plan with dimensions $D=45.7$ m, $B=30.5$ m and height equal to 183 meters. It was selected due to its basic shape as a simple yet representative structure. The “target” is a large glass opening (window) of height $h_w=4$ m and width $b_w$, located on the vertical façade of the building. In a typical 2D simulation scenario, the width $b_w$ is irrelevant. The horizontal relative distance between the initial object position
and the building façade \((d_0 \text{ in Figure 5.1})\) is deterministic. Initial velocity of the object was set to zero. The probability of impact, conditional on \(d_0\), was determined by repeating the simulations at various discrete points with \(0 \leq d_0 \leq 60\text{m}\). Typical initial object positions (random) are shown in Figure 5.1; these are indicated by a “dot” marker. A window on the first floor is shown in Figure 5.1, with \(z_w=4\text{m}\) being the vertical elevation of the lower frame from the ground level.

The 2D trajectory results and estimation of impact probability are shown in Figure 5.2 for \(U = 30\text{ m/s}\) and for the first floor to the fourth floor. Each floor was assumed to have an inter-story height of 4 m. In Figure 5.2, as the horizontal coordinate of initial object position \((d_0)\) decreases, the probability of impact increases; this can be noticed in Figure 5.2 (a) and Figure 5.2 (b).

Three regions were identified in Figure 5.2: Region I, in which a limited decrement in the magnitude and slope of the probability-of-impact curve was observed, Region II, a “transition region” with a drastic diminution of probability of impact, and Region III with little or “no impact”. The \(d_{0,\text{critical}}\) value corresponds to the boundary between Region II and III; this distance was estimated from the curve as the \(d_0\) corresponding to probability of impact \(P_I=0.05\). This “critical relative distance” \(d_{0,\text{critical}}\), depends on \(b_w\) and \(z_w\). A moderate decrement of probability of impact can always be observed for \(0 < d_0 < d_{0,\text{critical}}\). The dominant effect of gravity was expected due simulated effect of the zero-lift aerodynamic force for a compact object (Lin et al. 2007). The value \(d_{0,\text{critical}}\) is also
proposed in this dissertation for the first time as an essential parameter for design purposes.

Figure 5.2 Probability of impact against the façade of the CAARC building. 2D random trajectory simulation for wind speed $U=30$ m/s, and a window located on the first four floors ($z_w = 4, 8, 12, 16$ m).
5.2.2.2 Influence of mean wind speed on 2D trajectory

The effect of the mean wind speed on the impact probability for compact debris was investigated in 2D. Simulations were restricted to a window located on the first floor of the CAARC building, evaluated at $U=30$ m/s in the previous section. The algorithm for probability analysis was re-run at $U=20$ m/s and $U=40$ m/s. The results are shown in Figure 5.3.

In Figure 5.3, the probability of impact was plotted versus dimensional horizontal relative distance ($d_0$) for $U=20$, 30 and 40 (m/s) for a window at elevation $z_w=4$ m with the upper limit of the uniform random variable $H_0$, describing the initial object position on a vertical line (plane), defined for $0 \leq H_0 \leq H_{0,u}$ with $H_{0,u}=20$ m (Figure 5.1 and Table 5.1). From this figure it can be observed that the transition from Region I to II (drastic reduction in probability of impact), described in the previous section, tends to appear at smaller $d_0$ for lower velocity; this fact suggests that the probability of impact would drastically reduce for moderate wind storms. In contrast, large-velocity winds (i.e., a hurricane) would make the building more vulnerable. In fact, the figure suggests that the critical relative distance $d_{0,\text{critical}}$ almost doubles when the speed increases from 30 m/s to 40 m/s, indicating a non-proportional dependency between $U$ and PI. Moreover, at low winds ($U=20$ m/s) it could be noted that the gravity effect would be dominant, causing the fall of the object to ground before the “target” is reached.
Figure 5.3 Probability of impact against the façade of the CAARC building. 2D random trajectory simulation for different mean wind speed $U$: Dimensional units for $z_w=4$ m; $z_w = \bar{h}_w = 0.044$ and random initial object position $0 \leq \bar{H}_0 \leq \bar{H}_{0,\alpha}$ with

$$\bar{H}_{0,\alpha} = H_{0,\alpha} g U^{-2} = 0.218.$$ 

In Figure 5.4, the probability of impact was re-plotted at various $U$ with the curves being normalized in dimensionless units by employing the following quantities: relative distance $\bar{d}_0 = d_0 g U^{-2}$, upper limit of initial (vertical) object position (Figure 5.1) $\bar{H}_{0,\alpha} = H_{0,\alpha} g U^{-2} = 0.218$; target cladding of dimensionless height $\bar{h}_w = h_w g U^{-2} = 0.044$ located at elevation $z_w = 0.044$ in Figure 5.1. This normalization was obtained after observing that, in the absence of uncertainty (Figure 5.1), Equations 2.5 are independent of $U$ if the geometric scaling follows the normalization proposed in (Tachikawa 1983). In these equations the parameters, needed to solve for the trajectory, are in fact $K$ and $C_D$.
only (Φ, if directionality is included). The relative coordinates of the initial object position and “target” must also be normalized to satisfy geometric scaling. It was therefore inferred that the same observations can also be applied to the probabilistic analysis, provided that the probability distributions of all the random variables in Table 5.1 are the same. At various $U$, this is obviously valid for $K$, $C_D$ (and Φ, if applicable) and it is true by enforcing scaling similarity to the limits of the uniform distribution of $H_0$; this can be accomplished by replacing $0 \leq H_0 \leq H_{0,u}$ with $0 \leq \bar{H}_0 \leq \bar{H}_{0,u}$.

It must be noted that in Figure 5.4 both $h_w$ and $z_w$ correspond to 4 m at $U=30$ m/s but vary according to non-dimensional similarity for other velocities. It can be seen that the dependency of impact probability on $U$ curves disappears, since all curves overlap on each other, suggesting that the relationship between probabilities and $\bar{a}_0$ is unique and independent of $U$; it is a function of: elevation $\bar{z}_w$ and dimension $\bar{h}_w$ of the window, $\bar{H}_{0,u}$ upper limit of initial object position (Figure 5.1). Therefore, it was concluded that repetition of the probability analysis at various $U$ could be avoided: by using the dimensionless “Universal Probability Curve” in Figure 5.4 the probability may be equivalently evaluated regardless of the wind speed. Also, the generation of the curves may be restricted to one value of $U$, since results can be converted to other $U$ after normalization. This observation is particularly useful since a family of universal curves
for compact debris (for example depending on $z_w$, $h_w$ and $H_{0,u}$) may be proposed as “engineering tools” for design.

Figure 5.4 Probability of impact against the façade of the CAARC building. 2D random trajectory simulation for different mean wind speed $U$: Dimensionless “Universal Probability Curve” as a function of $\bar{d}_{0,critical} = d_{0,critical} g U^{-2}$ for a window with $z_w = h_w = 0.044$ and random initial object position $0 \leq \bar{H}_0 \leq \bar{H}_{0,u}$ with $\bar{H}_{0,u} = H_{0,u} g U^{-2} = 0.218$.

In Figure 5.4 the “critical relative distance”, defined as the horizontal relative distance “source/target” beyond which probability of impact $P_I \leq 0.05$, can be uniquely derived in dimensionless units as $\bar{d}_{0,critical} = d_{0,critical} g U^{-2} \approx 0.41$. Since this quantity is still a function of $z_w$, $h_w$ and random $\bar{H}_0$, it was decided to analyze the variation of this quantity as a function of the elevation of the window $z_w$, i.e., for higher floors.
This last observation was investigated in Figure 5.5, which depicts the variation of \( \tilde{d}_{0, \text{critical}} \) as a function of \( \bar{z}_w \) for constant window height \( \bar{h}_w = 0.044 \) and a random initial (vertical) object position \( 0 \leq \bar{H}_0 \leq \bar{H}_{0,u} \) with \( \bar{H}_{0,u} = 0.218 \).

Figure 5.5 Dimensionless critical relative distance \( \tilde{d}_{0, \text{critical}} = d_{0, \text{critical}} g U^{-2} \) versus window elevation \( \bar{z}_w = z_w g U^{-2} \) (\( \bar{h}_w = 0.044 \), \( \bar{H}_{0,u} = 0.218 \)).

Figure 5.5 indicates that if the target window is located on a higher floor in normalized units (or, equivalently, when wind speed decreases) the critical relative distance decreases. The relationship between \( \tilde{d}_{0, \text{critical}} \) and \( \bar{z}_w \) appears to be linear in Figure 5.5. The reduction of relative distance should also approximately tend to
\[ \tilde{d}_{0, \text{critical}} = 0 \] as \( \tilde{z}_w \to \tilde{H}_{0, \text{w}} \), since no impact is possible if a compact debris is released from a point with \( \tilde{H}_0 < \tilde{z}_w \).

### 5.2.2.3 Influence of the simplified vertical gust model on 2D “Universal Probability Curve”

The effect of the simplified “Küssner-like” vertical gust model (described in Chapter 3) on the “Universal Probability Curve” was investigated in this section. In order to derive the curve the probability of impact was calculated by solving the equations of motion 3.1 and 3.3. In the Monte Carlo simulations \( U = 30 \text{ m/s} \) was used to derive the curve; the parameters of the gust model corresponding to moderate turbulence with \( \gamma = 0.50 \) and \( \delta = 0.25 \) were used, as in Table 3.1. The case with \( \gamma = 0.50 \) and \( \delta = 0.25 \) was selected since this scenario is conceivably representative of atmospheric turbulent winds. As shown in the previous section, the Universal Probability Curve is independent of the mean-wind horizontal velocity if an appropriate similarity scaling is employed (for geometry and velocity). Therefore, the numerical results were presented in dimensionless units. These can be found in Figure 5.6.

Figure 5.6 depicts the results in dimensionless units, calculated from \( U = 30 \text{ m/s} \) and a window of height 4 m (\( \tilde{h}_w = 0.044 \)), located on the first floor (\( \tilde{z}_w = 0.044 \)). The Universal Probability Curve for impact against the vertical façade, derived for a vertical gust of moderate duration and magnitude, was plotted against the results in the absence of turbulence (“no gust”), reproduced from Figure 5.4 in the previous section.
Figure 5.6 Influence of simulated vertical gust on dimensionless “Universal Probability Curve” as a function of $\tilde{d}_0 = d_0 g U^{-2}$ for a window with $\tilde{z}_w = \tilde{h}_w = 0.044$ and random initial object position $0 \leq \tilde{H}_0 \leq \tilde{H}_{0,\mu}$ with $\tilde{H}_{0,\mu} = H_{0,\mu} g U^{-2} = 0.218$.

According to Figure 5.6, the probability of impact for the points close to the CAARC benchmark building decreases when the simplified gust model is utilized. In contrast, for a relative distance from the building $\tilde{d}_0 \geq 0.4$ and beyond the effect of the turbulence is to increase the probability of impact, since the upward component of the simulated gust maintains a horizontal trajectory during flight instead of forcing the debris towards the ground because of gravity. Also, for $\tilde{d}_0 < 0.3$ and for flight originating near the CAARC building, a decrement in the probability of impact was observed; this decrement is approximately -1% at $\tilde{d}_0 = 0.2$ in comparison with the case with no gust. It is believed
that for \( 0 \leq \bar{d}_0 \leq 0.2 \) the simplified gust causes the debris to fly higher; therefore, the impact occurs at a position with \( \bar{z}_w > 0.044 + \bar{h}_w \), above the designated cladding (target). As one goes farther from the target the simplified gust prevents the debris from reaching the ground, as indicated by the increment in probability for \( \bar{d}_0 > 0.4 \). When the simulated turbulence is considered the transition to Region III of “no impact” (with PI<5%, as defined in previous section) is observed at a larger \( \bar{d}_0 \) in comparison with the reference case in Section 5.2.2.2. The “critical relative distance” can be estimated at about \( \bar{d}_0 = 0.55 \) from Figure 5.6, compared to Figure 5.4 of the previous section with \( \bar{d}_{0,\text{critical}} = d_{0,\text{critical}} U^{-2} \approx 0.41 \).

5.2.3 Probabilistic model: numerical results in 3D
5.2.3.1 Trajectory analysis for first floor, centerline window
It was shown in the previous section that a relationship could be established between relative horizontal distance from the initial object position \( (d_0) \) and probability of impact. In this section, a 3D trajectory analysis with assessment of the probability of impact was carried out, enabling the estimation of the “Iso-probability Impact Contours”.

The Iso-probability Impact Contours are lines connecting those initial object positions of equal impact probability and located in regions surrounding a specified building (the CAARC building in this study). Iso-probability Impact Contours are reproduced on the horizontal plane. The probability of impact is conditional on the horizontal-plane initial
object position, for example expressed in dimensional coordinates \((x_0, y_0)\), upwind if relative to façade; it depends on \(z_w\) and \(b_w\) of the window (or the equivalent dimensionless quantities).

In the study example, these lines were derived from the probabilities, evaluated for a “coarse” grid of points on the XY plane surrounding the CAARC building (5 m by 5 m horizontal-plane grid). The curves are reported in Figure 5.7 to Figure 5.10. All simulations were carried out for uniform wind with \(U = 30\) m/s, after recalling the results of Section 5.2.2.2, which provided a framework for generalization at other \(U\), based on dynamic similarity. In Figure 5.7 and Figure 5.8 the case of a window with \(z_w = 4\) m (first floor) and \(b_w = 4\) m, located in the center of the long façade of width \(D = 45\) m, is shown.

Initially, a “Quasi-2D flight” hypothesis was used in presenting the results in Figure 5.7. This hypothesis restricts the random wind direction on the horizontal plane to the interval of \(\Phi\), which can be “seen” by the target, depending on \((x_0, y_0)\). This choice maximizes the probability of impact and assumes the presence of a predominant wind direction. For example, if the object originates from the point denoted by “A” in Figure 5.7, the variable associated with angle \(\Phi\) is randomly generated for a restricted interval \(\Phi_1 \leq \Phi \leq \Phi_2\). Since the window is located in the middle of the “\(D\) face”, the results below and above the centerline are the same due to symmetry. In these figures, the values are indicated above and below the centerline to allow for the tracing of the Iso-probability
Impact Contours, later discussed. The percentage probability value is shown close to each “dot” marker, which coincides with the position \((x_0, y_0)\).

Figure 5.7 “Quasi-2D” percentage probability of impact (PI) against the façade of the CAARC building- 3D simulation for “random” compact debris and uniform wind with \(U=30\) m/s; the “target” window of width \(b_w=4\) m is on the 1st floor \((z_w=4\) m\); \(B=30\) m and \(D=45\) m are the CAARC’s depth and width (Dimensionless quantities: grid spacing \(\Delta x_g = \Delta y_g = 0.055\), \(z_w = b_w = 0.044\), \(H_{0,u} = 0.220\)).

In a second stage, dependency on wind direction was eliminated due to the observation that a unique value of \(\Phi\), for example coincident with undisturbed wind direction, can become elusive, for example because of potential interference effects.
causes by nearby buildings in an urban area or local topography on the mean flow field. Also, the mean approaching direction of an actual storm may not be known a priori. In the absence of a specific wind tunnel study, enabling a more accurate characterization of the local flow field for a specific structure, the interval \(0 \leq \Phi \leq 2\pi\) was used; this second scenario was labeled as “Fully 3D” and is depicted in Figure 5.8.

The results of “Quasi-2D” and “Fully 3D” are referred to \(U = 30\) m/s. In order to estimate the “Fully 3D” case, the “Quasi-2D” probabilities were multiplied by the ratio between the restricted-angle flight direction and the turn angle (in radians), i.e., by postulating that wind from any direction has equal likelihood of occurrence. For example the impact probability of point “A” becomes: \(P_{\text{I, Fully-3D}} = P_{\text{I, Quasi-2D}} \frac{\Phi_2 - \Phi_1}{2\pi}\) with \(\Phi_1\) and \(\Phi_2\) in radians.

From Figure 5.7, it can be seen that, as the initial object position \((x_0, y_0)\) is moved farther away from the opening along the centerline, the probability is compatible with the decrement observed in Figure 5.2(a) for a 2D analysis. The Iso-probability Impact Contours corresponding to “Quasi-2D flight” are shown for probability values equal to 15% and 20%.

In Figure 5.7, the probabilities associated with the “Quasi-2D” assumption are compatible with the 2D analysis; this is because the object, when the wind direction is \(\Phi_1 \leq \Phi \leq \Phi_2\), was assumed to “directly fly toward” the target. Also, as the points (dot markers) move farther away from the target, a sudden decrement in the probability values
can be observed (refer, for example, to those calculated for flight originating from points along the center line).

Figure 5.8 “Fully 3D” percentage probability of impact (PI) against the façade of the CAARC building for comparison with the results in Fig. 6: $U=30$ m/s; window of width $b_w=4$m at elevation $z_w=4$m ($z_w = \bar{z}_w = 0.044$). (Dimensionless quantities: grid spacing $\Delta \bar{x}_g = \Delta \bar{y}_g = 0.055$, $\bar{H}_{0,u} = 0.220$).

5.2.3.2 Trajectory analysis for upper floors, centerline window
The probability-based simulations were extended to a window located on the 2nd and 4th floors for $U=30$ m/s, above the median point of the façade, along the $D$ side. Results are
summarized in Figure 5.9 and Figure 5.10. For the 5th floor the probability of impact was found to be zero for each point and the figure was omitted; this is due to the fact that both $H_{0,u}$ and $z_w$ were set equal to 20m.

In Figure 5.9, both “Quasi-2D” and “Fully-3D” results are presented; by exploiting symmetry, the “Quasi-2D” results are depicted below the reference center line, whereas the “Fully 3D” case can be noted on the upper half of the plot.

The 10% Iso-probability Impact Contours for the second floor (Figure 5.9) can be observed in the close proximity of the building, considering the same grid point layout as for the first floor. There is a tendency to a reduction in probability as the window is moved to upper floors. These curves also suggest a more rapid diminution of probability with the point $(x_0, y_0)$ moving away from the façade in comparison with first floor. This aspect appears of relevance since it may potentially influence a decision by the designer on the need for protecting the façade. This decision might in fact account for the relative decrement of the actual risk on upper floors.
Figure 5.9 Iso-probability Impact Contours for a window along the median point of the façade: second floor; uniform wind speed with \( U = 30 \) m/s (Dimensionless quantities: grid spacing \( \Delta x_g = \Delta y_g = 0.055 \), \( b_w = 0.044 \), \( z_w = 0.088 \), \( H_{0,w} = 0.220 \); percentage probabilities are shown).

The results in Figure 5.9 confirm the 2D results and the considerable effect of the gravity on the flight conditions and trajectory. Due to the elevation of the second floor (8m) and the uniformly-distributed, random initial object position (with \( 0 \leq H_0 \leq 20 \) m), it
is believed that the object more likely hits the ground before reaching the target. This observation was corroborated by the results at higher floors.

Figure 5.10 Iso-probability Impact Contours for window along the median, fourth floor and uniform wind speed with $U=30$ m/s (Dimensionless quantities: grid spacing $\Delta \bar{x}_g = \Delta \bar{y}_g = 0.055$, $\bar{b}_w = 0.044$, $\bar{z}_w = 0.176$, $\bar{H}_{0,\mu} = 0.220$; percentage probabilities are shown).

From the analysis of Figure 5.9 it was noticed that, when the directionality effect is neglected ("Fully-3D" case), the probability values drastically decrease; as a
consequence, the Iso-probability Impact Contour corresponding to $\text{PI} \geq 10\%$ “disappears” from the plot. This observation confirms that it would be important to carefully consider the effects of $\Phi$ in a full-scale building with anticipated 3D flow effects.

Risk analysis also was performed for windows located on the 4th and 5th floors with $U = 30$ m/s. Figure 5.10 shows the Iso-probability Impact Contours for the 4th floor. According to these results the probability considerably decreases; the Iso-probability Impact Contours, for the same probability levels as in the floors below, can exclusively be formed for $\text{PI} \leq 10\%$.

5.2.3.3 Trajectory analysis for quarter-width window

The probability analysis was subsequently extended to a window located at one quarter of the building width ($D$ side) in Figure 5.11 (first floor) and Figure 5.12 (second floor). Since trajectories and probability values are no longer symmetric about the centerline, simulations were performed for each upwind point ($x_0, y_0$) of the grid.

As shown in Figure 5.11, as one moves farther away from the CAARC building, the distance between two consecutive Iso-probability Impact Contours reduces, compared to Figure 5.7. Interestingly, when the window is located on the second floor ($z_w = 8$ m; Figure 5.12) the extension of the region, delimited by the contours with $\text{PI} = 20\%$, is larger than the one found for a window on the first floor ($z_w = 4$ m in Figure 5.11). This observation holds true for other regions and other contours; it is opposite to the predictions for an opening along the centerline (Figure 5.7 to Figure 5.10).
Moreover, the probability tends to zero faster when \( z_w = 8 \text{m} \) (Figure 5.12), suggesting that for compact debris the effect of gravity force tends to enhance the descending trajectory slopes as the relative distance between initial object position and the façade increases.
Figure 5.12 Iso-probability Impact Contours for window at quarter width on the second floor and uniform wind speed with $U=30$ m/s (Dimensionless quantities: grid spacing $\Delta x_g = \Delta y_g = 0.055$, $b_w = 0.044$, $z_w = 0.088$, $H_{0,u} = 0.220$; percentage probabilities are shown).

5.2.3.4 Discussion on 3D simulation results

Further inspection of the Figure 5.9 to Figure 5.12 indicated that window panels located along the centerline of the building are more vulnerable to debris impact, since the overall surface area of the regions, bounded by same probability levels, is larger (for example, if one compares the contours on the second floor corresponding to PI $\geq 20\%$ in Figure 5.9 and Figure 5.12). These “vulnerability regions” become smaller not only on the first floors but also at higher levels. If decisions on protecting the building from
debris impact (e.g., by installing debris-resistant glass panels) were based on the actual risk, installation of debris-resistant glass might possibly account for these aspects. In addition, the risk analysis could possibly be extended to investigate the dynamic momentum of the random object at the impact.

As a second plausible countermeasure for reducing the risk of impact, the use of setbacks in the floor plan layout might be suggested: the layout of the setback may follow indications derived from the analysis of Iso-probability Impact Contours at each floor level for a vertical façade without modifications.

The simulations (Figure 5.7 to Figure 5.12) also suggested that for the first to fourth floors, irrespective of the exact window location, the risk of impact for compact objects is negligible when, in a 3D analysis, \(d_0 > 30\) m and \(U=30\) m/s for this specific scenario. This \(d_0\) value is also compatible with the \(d_{0,\text{critical}}\), estimated for the same wind speed from the 2D analysis in Section 5.2. The latter observation indicates that a 2D analysis could possibly be considered as acceptable for a preliminary investigation of a full-scale structure.

Finally, it must be noted that the simulations were performed under the assumption of uniform wind velocity, with a constant \(U\) irrespective of actual height \(z\) and boundary-layer effects. The hypothesis of a constant \(U\) was accepted since simulations were restricted to elevations \(0 \leq z \leq 20\) m, with the latter upper limit corresponding to about 10\% of the CAARC building’s height (183 m). Even though the variation of wind shear
effect is possibly less relevant at lower floors, an influence of boundary layer profile cannot be excluded. A non-negligible variation of probability of impact would still be plausible if a more accurate description of the boundary layer were utilized. This observation led to the extension of the investigation on Iso-probability Impact Contours to the case which includes the simulation of boundary layer winds by means of the simplified vertical gust model, proposed in chapter 3. Results are discussed in the next sub-section.

5.2.3.5 Investigation on the effects of the simplified vertical gust model on “Iso-probability Impact Contours”

The influence of the simplified “Küssner-like” vertical gust model on the Iso-probability Impact Contours was investigated in this section. The CAARC benchmark building case was used for comparison of the results with those discussed in Section 5.2.3.1. As in Section 5.2.3.3 the case with $\gamma=0.50$ and $\delta=0.25$ in Table 3.1 was selected, since this scenario is possibly representative of atmospheric turbulence. The case of a window located at the centerline on the upwind façade was exclusively analyzed. The points $(x_0, y_0)$, denoted by a “dot marker” in Figure 5.7, were chosen in the proximity of the “target” building. It was shown in Section 5.2.3.3 that, if debris trajectory originates from these points, the probability of impacting against a window located on the vertical façade along the centerline is high, greater than or equal to 20% (based on a “Quasi-2D flight” hypothesis); these points are the most critical. Therefore, it was decided in this section to restrict the study needed to the region near the CAARC building enclosed within the 20%
“Iso-probability Impact Contour” curve. The simulations were restricted to a target window located at the first floor on the vertical façade, along the centerline of the face $D$, $h_w=4$ m, $b_w=4$ m. In order to compare the results with Figure 5.7, a mean horizontal wind with $U=30$ m/s was considered; the Tachikawa number, drag coefficient, initial vertical-plane position of the debris and wind yaw angle were assumed to be random variables with the same properties as those explained in Table 5.1. The equations of motion were readily adapted to 3D analysis; discussion is omitted for brevity.

In Figure 5.13 the percentage probabilities, which are shown in larger font size, not enclosed within parentheses, are those calculated by considering the effect of the vertical gust in the trajectory. The probability values shown in smaller font size, enclosed within parentheses, correspond to the case without turbulence; the latter were directly obtained from Figure 5.7.

It can be observed in Figure 5.13 that the probability of impact tends to decrease once the effect of the vertical gust is included. As a result, the 20% Iso-probability Contour is formed closer to the building. In this case, the effect of the turbulence appears to be beneficial since the probability of impact decreases more rapidly as the relative distance from the building façade increases.
Figure 5.13 Influence of simulated vertical gust on “Quasi-2D” percentage probability of impact (PI) against the façade of the CAARC building - 3D simulation for compact debris, uniform wind with \( U = 30 \text{ m/s} \) and Küssner-like vertical gust of duration \( \gamma = 0.50 \) and magnitude \( \delta = 0.25 \) – probability values in parenthesis correspond to Figure 5.7. The target window is located on the 1\(^{st}\) floor of the building (Quantities in dimensionless units are \( \Delta \bar{y}_g = 0.055 \), \( \bar{z}_w = 0.044 \), \( \bar{H}_0 = 0.220 \)).

### 5.3 Summary

In this chapter, using the estimated trajectory, a risk model was developed to investigate the probability of impact for compact debris missiles against the façade of a tall building. The CAARC building was used as a reference structure. The “target” was assumed to be a large cladding panel (window) of height 4m, located on the first four floors of the building.
façade. Monte Carlo methods were utilized for assessing the probabilities. Both 2D and 3D simulations confirmed that the random flight of compact debris is mainly gravity driven.

In 2D, the curves, which describe the conditional probability of impact against a cladding component on the façade for a compact object flying from an initial position at a given relative distance from the building, were initially estimated for a specific mean wind velocity. Three regions, based on the “PI curves”, were identified and interpreted in relation to pertinent features of the flight. A new measure (“critical relative distance”) was coined for characterizing the maximum relative distance, from which debris can reach its target. Subsequently, the effect of a variation in the wind speed was investigated. A “Universal Probability Curve” for compact object was determined in dimensionless form, independent of horizontal mean wind velocity, as a function of the dimension and elevation of the cladding panel.

The 3D simulations were compatible with the 2D results. Iso-probability Impact Contours were proposed to describe vulnerability of a specific cladding area to debris damage. These were numerically calculated for a hypothetical window, positioned at various floor elevations, either in the middle or at the quarter width of the exterior wall. In the 3D setting, Iso-probability Impact Contours corresponding to probability of impact greater than 5% to 20% were presented for “Quasi-2D” flight, with a prevailing wind direction being pre-set as the worst-case scenario for the cladding panel. The probability
values became almost negligible when “Fully 3D” simulations, irrespective of mean wind direction, were used; Iso-probability Impact Contours could not be calculated, suggesting that directionality must be carefully considered for impact risk assessment (locally).

Finally, the effect of a “Küssner-like” simplified vertical gust on the Iso-probability Impact Contours and the Universal Probability Curve for impact against a vertical façade curves was evaluated. The analysis of the former aspect indicated that the simulated gust tends to reduce the probability of impact on the façade of the building for a moderate-magnitude turbulence effect. On the other hand, the influence of moderate-magnitude simulated gust on the Universal Probability Curve showed that the critical distance can increase since the simplified vertical gust tends to maintain the trajectory of the object horizontal at initial stages, flying farther and longer instead of reaching the ground.
5.4 References


Chapter 6

Probabilistic analysis of trajectory in the proximity of tall buildings: Fully turbulent wind field

6.1 Introduction

In this Chapter, time histories of computer-generated wind records to generate fully turbulent wind field which described in detail in Chapter 4 are used in conjunction with the probabilistic methodology (Chapter 5) to analyze debris impact against tall building façades. The effect of the simulated wind field on the “Universal Probability Curve” (probability-of-impact curve) is investigated in this section.

As was described in Chapter 5, this curve describes probability of impact against a cladding element of prescribed dimension, located on the façade of a tall building, calculated for a compact object; this probability value is conditional on the initial distance of the object from the building before takeoff \(d_0\), measured on the horizontal plane. The three main features, controlling the trajectory, are: \(K\), \(C_D\) and initial elevation from the ground before takeoff \(H_0\). These variables are random; their random properties, summarized in Table 5.1, are used to estimate the Universal Probability Curve. A detailed description of this curve may also be found in Chapter 5.2 and (Moghim and Caracoglia 2012a; 2012b).
Moreover, the sensitivity analysis is carried to identify the relative importance of each of the input random variable ($K$, $C_D$, and $H_0$) on the “Universal Probability Curve” in both uniform and fully turbulent wind field. The results showed the Tachikawa number is more important than drag coefficient in trajectory and proposed probabilistic model.

Having understood that the “Universal Probability curve” is unique and independent of the wind velocity in uniform and turbulent wind field, the suggested application of such a curve (“Universal Probability curve” and “Iso-probability Impact Contours”) in practical engineering is explained. And as a final aspect the effect of the $u$-$w$ turbulence correlation on “Universal Probability curve” is investigated and the results are shown in this chapter.

6.2 Effect of turbulent wind field on “Universal Probability Curve”

In order to derive the Universal Probability Curve the probability of impact is calculated for a two-dimensional trajectory problem. Figure 5.1 depicts the geometry and the quantities employed to simulate debris trajectory and to numerically evaluate debris impact against a window panel (“target”) located on the façade of a reference tall building; the 2D problem is shown. Again, Monte-Carlo sampling methods are employed to construct this curve.

The influence of various wind field and boundary layer conditions is investigated by varying the speed $U_{m,10}$ and by considering both the FTF and SELF schemes for
turbulence propagation and generation of the curves. As in previous sections the roughness length $z_0, r=0.3$ m is selected to replicate sub-urban wind exposure.

In Figure 6.1 the Universal Probability curve is shown with and without considering the turbulence in the wind field. A total of 10000 realizations are utilized to evaluate the curve by Monte-Carlo sampling. The reference mean wind speed is $U_{m,10}=30$ m/s. The mapping of the probability employs Tachikawa’s dimensional analysis since it has been suggested (Moghim and Caracoglia 2012a) that the curve can be determined independently of mean wind speed in the case of turbulence-free uniform wind.

In order to satisfy the “universality” feature in the curve (its uniqueness), all the relevant quantities employed to trace the graph (Figure 5.1) must follow the same normalization; these are: elevation of the window panel from the ground, $\bar{z}_w = z_w g(U_{m,10})^2 = 0.044$; height of the window $\bar{h}_w = h_w g(U_{m,10})^2 = 0.044$; upper limit of the random distribution of the initial object position along the vertical axis before takeoff, $\bar{H}_{0,u} = H_{0,u} g(U_{m,10})^2 = 0.218$ (Table 5.1). As can be inferred from this figure the $\bar{a}_{0,critical}$ increases by 32% since in a turbulent wind the vertical component of the turbulence causes the debris to fly more before reaching the ground.
Figure 6.1 Dimensionless “Universal Probability Curve” as a function of 
\( \overline{d}_0 = d_0g(U_{m,10})^{-2} \) for a window with \( z_w = H_w = 0.044 \) and uniformly-distributed random initial object position with upper limit \( H_{0,u} = H_{0,u}g(U_{m,10})^{-2} = 0.218 \): comparison of the curves with and without turbulence at \( U_{m,10}=30 \text{ m/s} \).

The \( \overline{d}_{0,\text{critical}} \), or “relative critical distance”, is defined as the distance source/target beyond which the probability of impact is less than 5% (unlikely impact). In general, the probability that the object can “hit the target window” of standardized height \( H_w = 0.044 \) at an elevation \( z_w = 0.044 \) from the ground, will be larger in the fully turbulent wind field in Figure 6.1. Also, the rapid reduction in the probability curve, visible in the case of uniform wind field beyond \( \overline{d}_0 > 0.3 \), essentially disappears in a turbulent wind field because of the combination of the horizontal and vertical turbulence components, which
tend to reduce the effects of gravity in the control of the flight trajectory. The presence of turbulence therefore makes the building more vulnerable to impact (larger $d_{0,\text{critical}}$).

Figure 6.2 Dimensionless “Universal Probability Curve” as a function of $\bar{d}_{0} = d_{0} g \left( U_{m,10} \right)^{-2}$ for a window with $z_{w} = \bar{h}_{u} = 0.044$ and uniformly-distributed random initial object position with upper limit $\bar{H}_{0,u} = H_{0,u} g \left( U_{m,10} \right)^{-2} = 0.218$: Effect of mean wind speed $U_{m,10}$ in fully-developed turbulent field (“FTF scheme”).

In Figure 6.2 the effect of a variation in the mean wind speeds on the Universal Impact Curves are shown. As can be seen from this figure, the curves are very close to each other for most values of $\bar{d}_{0}$ with mean wind speeds $U_{m,10}$ equal to 20, 30 and 40 m/s. It can be concluded that the universality property of the probability-of-impact curve is still satisfactory since the curves, estimated through the various models by preserving the
same position and the same height of the target in dimensionless units, are almost all coincident.

The effect of the wind shear on the Universal Probability Curve is investigated and shown in Figure 6.3. It can be seen that, even though the probability is sensitive to the roughness length of the boundary layer, the probability of impact in a wind exposure with $z_{0,r}=0.3$ m (suburban areas with obstacles similar to Exposure B in ASCE 7-10) is less than the one observed in an open terrain with $z_{0,r}=0.03$ m (Exposure C).

Figure 6.3 Dimensionless “Universal Probability Curve” as a function of $\bar{d}_0 = d_0 g (U_{m,10})^2$ for a window with $\bar{u}_w = \bar{h}_w = 0.044$ and uniformly-distributed random initial object position with upper limit $\bar{H}_{0,u} = H_{0,u} g (U_{m,10})^2 = 0.218$. The curves were estimated at $U_{m,10}=30$ m/s for two different exposures (roughness length $z_{0,r}$).
It should also be noted that the former statement is true when the relative distance source/target is less than or equal to $d_{0,\text{critical}}$, which is the most important quantity for building façade protection. Also, in both curves a similar trend in the probability of impact is detected.

Figure 6.4 Effect of the “FTF” and “SELF” on “Universal Probability Curve” as a function of $\bar{d}_0 = d_0 g \left( U_{m,10} \right)^2$ for a window with $z_w = \bar{h}_w = 0.044$, uniformly-distributed random initial object position with upper limit $H_{0,u} = H_{0,u} g \left( U_{m,10} \right)^2 = 0.218$ ($z_{0,r}=0.30$ m and $U_{m,10}=30$ m/s).

The influence of turbulence propagation scheme (FTF and SELF, described in Section 4.2 of this dissertation) is investigated in Figure 6.4. This figure compares “Universal Probability Curves” for a window (target) with $z_w = \bar{h}_w = 0.044$, computed using both
methods. According to Figure 6.4 the probability of impact is usually smaller if the SELF numerical scheme is selected to create the turbulent wind field. The relative difference is of the order of 10% to 15% for most values of $d_0$. Nevertheless, the critical distance is almost the same, almost independent of the turbulence propagation scheme; moreover, the probability of impact consistently decreases as $d_0$ increases.

Figure 6.5 Dimensionless critical relative distance $\tilde{d}_{0,\text{critical}} = d_{0,\text{critical}} \left( \frac{U_{m,10}}{g} \right)^2$ versus window elevation $\tilde{z}_w = z_w g \left( \frac{U_{m,10}}{g} \right)^2$ ($\tilde{h}_w = 0.044, \tilde{H}_{0,wa} = 0.218$).

The last aspect of this section is analyzed in Figure 6.5, which shows the variation of $\tilde{d}_{0,\text{critical}}$ as a function of $\tilde{z}_w$ for a flying object with a random initial (vertical) object
position, uniformly distributed in the interval \(0 \leq H_0 \leq \bar{H}_{0,u}\). The figure confirms that the critical distance \(\bar{d}_{0,critical}\) increases when the wind field is turbulent. It can be also noticed from the figure that, if the target window is located on an upper floor in normalized units (or, equivalently, when the mean wind speed decreases), the relative critical distance decreases. The reduction of relative distance should also approximately tend to \(\bar{d}_{0,critical} \approx 0\) as \(\bar{z}_w \rightarrow \bar{H}_{0,u}\), since no impact is statistically possible if a compact object is released from a point with \(\bar{H}_0 < \bar{z}_w\). Also, as the height of the window \(\bar{h}_w\) increases a reduction of critical distance is possible in the case of turbulent wind.

6.3 Parametric study on the relevance of Tachikawa number, drag force coefficient and “release point” elevation on the “Universal Probability Curve”

In this section a sensitivity analysis is carried to identify the relative importance of each of the input random variable (\(K\), \(C_D\), and \(H_0\)) on the “Universal Probability Curve” in both uniform and fully turbulent wind field. In order to better characterize the relevance of each random variable this curve is regenerated, by eliminating the variability of one variable at a time, after substitution with the median value of the distribution. For example, in the first case \(C_D=0.65\) is used as a deterministic constant whereas \(K\), \(H_0\) are still random; in the second analyzed case a deterministic \(K=4.5\) is employed with \(C_D\) and \(H_0\) being random. The properties of the random parameters are same as before and are described in Table 5.1.
6.3.1 Uniform wind field

The “Universal Probability Curve” is recalculated for uniform wind field with a constant incident wind speed equal to 30 m/s and, for comparison purposes with previous results, for the window located on the first floor of the reference “target” building. The three cases, eliminating variability of one random variable at a time, are shown in Figure 6.6.

As can be seen from the analysis of Figure 6.6, sensitivity of this curve to the Tachikawa number is clearly more important than the effect of the drag force coefficient. In fact, there is little difference in the probabilities between the reference curve, which includes the complete probabilistic analysis (thick solid line; reproduced from Figure 6.1) and the case with a constant $C_D=0.65$. A first but minimal difference in the curve is perceived at about $d_0 \approx 0.35$ ($d_0=30$ m at 30 m/s), which is of the order of 13%, and later in the “no impact” region (for probability less than 0.05). This result also indicates that the importance of carefully selecting the probability density function of $C_D$ (and any possible limitations introduced by the assumption in Table 5.1) is secondary, since little modification in the curve for compact objects in uniform wind field is observed; assuming that $0.5 < C_D < 0.8$ appears as reasonable.
Figure 6.6 Parametric study on the relevance of random $K$, $C_D$, and $H_0$ of the “Universal Probability Curve” as a function of $\bar{d}_0 = d_0 g \left( U_{m,10} \right)^{-2}$ for a window with $\bar{z}_w = \bar{h}_w = 0.044$

and $\bar{H}_{0,u} = H_{0,u} g \left( U_{m,10} \right)^{-2} = 0.218$ - uniform wind field without turbulence.

On the other hand, the importance of the Tachikawa number is the dominant factor in the variability of the trajectory and for the derivation of these curves. In fact, once $K$ is set to be constant (dashed blue line in Figure 6.6), an increment of probability of impact is observable for $\bar{d}_0 > 0.23$ ($0 < d_0 < 20 \text{ m at } 30 \text{ m/s}$), which suggests that for $K=4.5$ the aerodynamic force tends to still direct the trajectory of the debris toward the “target” for objects originating from a source located farther away from the façade. However, for $\bar{d}_0 > 0.23$ ($d_0 > 20 \text{ m at } 30 \text{ m/s}$) the probability of impact drastically decreases indicating a reduction in the critical distance $\bar{d}_{0,\text{critical}}$ in comparison with the reference curve. This
evidence also proves that the Tachikawa number should be treated as a random variable to cover a possibly extensive variety of scenarios in calculating the probability of impact; the choice of a uniform probability distribution for $K$ (within the limits designated in (Tachikawa 1983; 1988) as plausible for full-scale analysis) is confirmed to be well suited for this purpose.

6.3.2 Fully turbulent wind field

In this section, the sensitivity analysis is extended to a fully turbulent wind field; numerical results are shown in Figure 6.7. The curve is evaluated for $U_{m,10}=30$ m/s and for the same turbulent wind field conditions as in Figure 6.1 ($z_{0,r}, I_u$, etc.); $u$ and $w$ turbulence components was simulated using the FTF scheme. The results are normalized as before. As can be inferred from this figure, the prevalent effect of the Tachikawa number in comparison with the drag coefficient is still evident. However, it is important to note that the influence of $K$ and $C_D$ on the curve is different when the wind shear and turbulence are introduced. As also observed in Figure 6.6, elimination of the variability in $C_D$, by assuming $C_D=0.65$ as a constant median value, does not have any perceivable effect on the “Universal Probability Curve” in uniform wind. Nevertheless the same conclusion does not apply to a fully turbulent field; a distinct increment in this curve can be seen in Figure 6.7 in the region of $d_0 > 0.23$ ($0<d_0<20$ m at $U_{m,10}=30$ m/s).
Figure 6.7 Parametric study Relevance of random $K$, $C_D$, and $H_0$ of the “Universal Probability Curve” as a function of $\bar{d}_0 = d_0 g \left( U_{m,10} \right)^2$ for a window with $\bar{z}_w = \bar{h}_w = 0.044$ and $\bar{H}_{0,u} = H_{0,u} g \left( U_{m,10} \right)^2 = 0.218$ - shear wind effects and turbulence (FTF) included.

It is conceivable that, during early flight stages, the vertical and horizontal turbulence components can influence the magnitude and direction of the aerodynamic force, which is not only controlled by the relative velocity between the object and the flow but also by the variation in $C_D$. A similar behavior is seen when $K$ is taken as deterministic for $0 < \bar{d}_0 < 0.23$. The effect of the Tachikawa number on the curve tends to disappear for $\bar{d}_0 > 0.23$ ($d_0>20$ m at $U_{m,10}=30$ m/s). From the figure it can be also inferred that the same distinction on the predominant importance of $K$ on $C_D$ cannot be established when wind shear and turbulence are included in the field, suggesting that both random variables
should be carefully analyzed. Also, more careful investigation should be considered in order to analyze and replicate, perhaps more accurately, atmospheric wind fields (Section 6.4).

6.4 Effect of u-w turbulence correlation on “Universal Probability Curve”

In the previous sections, the synthetic generation of wind shear and turbulence effects has been based on the hypothesis of independence between \( u \) and \( w \) turbulence components. This assumption is usually acceptable in wind engineering as a first approximation and for practical purposes. Theoretically, the independence between \( u \) and \( w \) components is valid for homogenous turbulence (Simiu and Scanlan 1996); a homogeneous wind turbulence field is acceptable at medium to large elevations from the ground and when the scale of the object is large compared to the reference turbulence length scales (tenths to one hundred meters, depending on elevation from the ground). Nevertheless, this may no longer be acceptable for flying objects, as noted in (Holmes 2004). In particular a negative correlation can be introduced in order to comply with the observation that an inverse correlation between \( u \) and \( w \) fluctuations is possible due to turbulent shear stresses near ground level. This correlation value has been quantified as much as -0.3 (Holmes 2004; Kaimal et al. 1972).

Therefore, the hypothesis of independence between \( u \) and \( w \) turbulence is relaxed in this section and a relative dependence between the \( u \) and \( w \) processes is introduced to
analyze the effects on compact object trajectory. An additional numerical study is carried out to estimate the relevance of this aspect on the estimation of the “Universal Probability Curve” in Figure 6.1, Figure 6.2 and Figure 6.3.

Since the random processes are no longer uncorrelated, the digital generation of the $u$ and $w$ time histories must be conducted concurrently. For turbulence propagation internally to the wind field the “FTF scheme” is exclusively utilized in this section. The digital signals are generated at the grid points of the inlet boundary as $u_i(t) = u(x=0,z_i,t)$ $w_i(t) = w(x=0,z_i,t)$ $(i=1,\ldots,n)$ by simultaneous spectral decomposition, also accounting for the $uw$ co-spectrum of the turbulence. For computational purposes, the points are re-organized in the column vector $\mathbf{v} = [u_1,\ldots,u_i,\ldots,u_n;w_1,\ldots,w_i,\ldots,w_n]^T$ of dimension $2n$; the generation of the generic element of this vector is similar to Equation 4.4 and 4.5:

$$v_i(t) \equiv \sum_{k=0}^{2N} \frac{\sqrt{\Delta \omega}}{\sqrt{\pi}} \cdot \left[ \Phi_v(\omega_k) \cdot \Lambda_v(\omega_k) \right]^{1/2} \cdot \left[ G_k \sin(\omega_k t) + H_k \cos(\omega_k t) \right]$$  \hspace{1cm} (6.1)

As before, $(G_k)_j$ and $(H_k)_j$ are independent Gaussian random variables. The wave-superposition method is evaluated by superimposing the effects of turbulence at the scales of $\omega_k$ with $k=1,\ldots,N$. The matrices $\Phi_v$ and $\Lambda_v$ can be found from the orthogonalization of the cross-power spectral density matrix of turbulence, frequency by frequency, $S_{uv}(\omega_k) = \Phi_v(\omega_k) \Lambda_v(\omega_k) \Phi_v^T(\omega_k)$. The matrix $S_{uv}(\omega_k)$ simultaneously combines
the effects of horizontal cross-spectra $S_{uu}(\omega_k)$, the vertical ones $S_{ww}(\omega_k)$ and the matrix of the cross “$uw$ co-spectra” $C_{uw}(\omega_k)$ as:

$$
S_{\nu \nu}(\omega_k) = \begin{bmatrix} S_{uu}(\omega_k) & C_{uw}(\omega_k) \\ C_{uw}(\omega_k) & S_{ww}(\omega_k) \end{bmatrix}
$$

(6.2)

The elements of the first two sub-matrices, $S_{uu}$ and $S_{ww}$, are estimated as in Section 4.2. The scalar terms of $C_{uw}(\omega_k)$ are evaluated as indicated in (Hemon and Santi 2003),

$$
C_{u,w} = \sqrt{C_{u,u}(\omega) \cdot C_{w,w}(\omega)} \cdot \sqrt{\exp(-\hat{f}_u) \cdot \exp(-\hat{f}_w)}, \text{ by taking the geometric average between co-coherence functions of the two independent processes and with } \hat{f}_u \text{ and } \hat{f}_w \text{ designated as in Section 4.2. The quadrature terms of the cross-spectra between } u \text{ and } w \text{ terms are neglected. The co-spectra between } u \text{ and } w \text{ are simulated by following the model proposed by (Kaimal et al. 1972). These are for } \omega > 0:
$$

$$
\omega C_{u,w}(\omega, z) / u_*^2 = 2\pi \times (-14 f_r (1 + 9.6 f_r)^{-2/3}) \text{ and dimensionless frequency } f_r \text{. The latter form ensures that a negative correlation between } u \text{ and } w, \text{ equal to -0.3, is established locally at the same point.}
$$

Figure 6.8 shows an example of simultaneous realization of $u$ and $w$ turbulence at 10 m height for a mean wind speed $U_{m,10}=30 \text{ m/s}$, by using the synthetic generation provided by Equation (6.1); a total of 1000 waves are used, as in Figure 4.2.
Figure 6.8 Simultaneous realizations of digitally-generated stream-wise and vertical velocity fluctuations at $z=10$ m for $U_{m,10}=30$ m/s.
Figure 6.9 shows the analysis of the $u$ spectrum (a), $w$ spectrum (b) and $uw$ Co-spectrum (c) for the signals taken from Figure 6.8; in Figure 6.9 (c) the $Co_{uw}$ cross-spectrum appears positive since it has been normalized with respect to the negative covariance $cov(u,w)$ between the digital signals.

Figure 6.9 Stream-wise $u$ (a), vertical turbulence (b) spectrum and $uw$ Co-spectrum (c) of the signal in Figure 6.8 - Comparison with theoretical models.
The comparison of the two spectra or the cross-spectrum with the theoretical models (Kaimal, Lumley, etc.) is very good; the spectral analysis employs 2048 Fourier points on a total of 45000 data over a ten-minute record. The correlation coefficient between the two signals is about -0.31, relatively close to the reference value (Holmes 2004; Kaimal et al. 1972).

![Graph](image)

Figure 6.10 Effect of uw correlation on the “Universal Probability Curve” as a function of $\overline{d}_0 = d_0 g \left( U_{m,10} \right)^2$ for a window with $\overline{z}_w = \overline{h}_w = 0.044$ and $H_{0,u} = H_{0,u} g \left( U_{m,10} \right)^2 = 0.218$.

The effect of uw correlation on the “Universal Probability Curve” is investigated in Figure 6.10. The figure shows the probability curves as a function of the dimensionless horizontal distance $\overline{d}_0 = d_0 g \left( U_{m,10} \right)^2$ between “source” and “target” for a window with
\( \bar{z}_w = \bar{h}_w = 0.044 \) and uniformly-distributed random initial vertical object position with upper limit \( \bar{H}_{0,\sigma} = H_{0,\sigma} g \left( U_{m,10} \right)^2 = 0.218 \).

Figure 6.10 compares the results in Figure 6.1 (“FTF scheme” only) obtained for uncorrelated turbulence wind field by repeating the Monte-Carlo sampling at a series of representative points along the \( \bar{d}_0 \) axis. These simulations are preliminary but suggest a non-negligible reduction in the probability when the correlation between \( u \) and \( w \) is replicated. The relative reduction is of the order of 10% to 15% and tends to increase for larger \( \bar{d}_0 \). This phenomenon has been tentatively interpreted as related to the fact that a negative correlation between horizontal \( u \) and vertical \( w \) may possibly exacerbate a downward trajectory of the object and a diminution of the “successful” impacts. It is also suggested that the overestimation of probability for uncorrelated turbulence is favorable from the point of view of safety margins; the results, described in the previous sections, can therefore be accepted from a practical standpoint.

### 6.5 Prospective application of the “Universal Probability Curve” to engineering design

The “Universal Probability Curve” is intended to be utilized by a structural engineer who is in charge of assessing the vulnerability of the vertical façade of a given tall building to the impact of compact objects. For example, if a decision has to be made on the need for protecting a certain area of the façade of a tall building to minimize the risk of impact, the
critical distance $\bar{d}_{0,\text{critical}}$ can be used. If nearby obstacles, other structures or debris-generating sources can be located in a region with $\bar{d}_0 < \bar{d}_{0,\text{critical}}$, countermeasures should be considered by the designer for façade protection. Since the $\bar{d}_{0,\text{critical}}$ is dimensionless, conversion to dimensional values should account for the mean wind speed derived from the wind maps, provided by a structural design standard.

Also, these curves could possibly be coupled with damage (“fragility”) curves of the glass cladding, based on its dimensions and material properties (e.g., (Gavanski and Kopp 2011)), to ascertain damage and collapse risk. In a similar way, information derived from this curve may be used by forensic engineers to possibly analyze damage patterns on tall buildings after severe wind events (e.g., (Jain 2013)).

6.6 Summary

The effect of turbulence on the 2D Universal Probability Curve (probability-of-impact curve) was examined. The rapid decrement in the probability curve as the dimensionless distance source/target increases beyond 0.3, which was observed in uniform winds, almost disappears because of the combination of the horizontal and vertical components of the wind velocity. In spite of the turbulence variations, this curve is still similar qualitatively and quantitatively at various mean wind speeds (approximately unique).

The effect of terrain roughness on the probability curves appears secondary since the overall trend probability vs. random initial position is approximately the same. It is also
shown that the “critical distance” source/target is larger when turbulence is considered in
the simulations. Two numerical schemes (FTF and SELF) were proposed for the synthetic
generation of fully turbulent wind field; their influence on the impact probability was also
investigated. The probability values are usually smaller when the SELF method is
employed.

In the last part of the study, investigations also analyzed the effects of dependency vs.
independency of turbulence components on the probability of impact; a diminution of
probability was noted when the negative correlation between vertical and horizontal
turbulence is digitally replicated. The overestimation of probability for uncorrelated
turbulence is, however, favorable from the point of view of safety margins. Therefore, the
results, obtained for uncorrelated turbulence components, are acceptable from a practical
standpoint.
6.7 References


Chapter 7

Experimental validation of a numerical model for estimating wind-borne compact debris

7.1 Introduction

In this chapter wind tunnel tests were designed and executed at Northeastern University to validate the fully-turbulent wind and trajectory model proposed in Chapters 2 and 4 for smooth and turbulent wind field respectively. Tests were conducted in both uniform flow and grid-generated turbulent flow to simulate the main features of atmospheric boundary layer winds (at a wind tunnel scale) and to study the effects of turbulence on trajectories. A rectangular grid, placed upstream of the takeoff position, was used to generate the turbulence features.

The trajectory of compact objects of various sizes and shapes (cubes and spheres) was captured and analyzed using high speed digital videos. The object motion was investigated on a vertical plane at different mean flow speeds.

In the case of smooth flow, wind tunnel trajectories, which were compared to simulation results, showed a very good agreement. Comparisons were subsequently extended to investigate the effect of turbulence on trajectory and for validating the trajectory model, by using a series of tests in turbulent flow field. The influence of the
Tachikawa number on the trajectory of compact debris was also experimentally investigated; the interpretation of the experimental results indicates that cubes tend to fly farther and faster than spheres due to a larger drag forces.

7.2 Description of the wind tunnel experiments

7.2.1 Wind-tunnel setup

Model experiments were carried out in the closed circuit tunnel, which is operated by 15-HP 4-pole AC electric motor/fan at a maximum output of 1740 RPMs. The schematic view of the wind tunnel is shown in Figure 7.1. The cross section of the test chamber is 559 mm by 559 mm (22” by 22”). The longitudinal length of the test chamber is approximately 3000 mm (118”). The maximum flow velocity is about 18 m/s in smooth flow, prior to any modifications to the flow or the motor/fan.

The experiments consisted in observing and recording trajectories of small compact debris objects, which were released from a pre-defined position in the test chamber. A “release mechanism” (Figure 7.2) was designed to hold the objects before takeoff. Two small vertically-oriented plexi-glass panels were used, one of which was fixed to the support in Figure 7.2 whereas the second one was connected to a solenoid with a spring-loaded plunger to “clamp” the objects before each test (Figure 7.2). One of the plexi-glasses was fixed but the other one was allowed to move horizontally by the solenoid mechanism. The motion of the plunger was computer controlled to enable the automatic
release of the objects. When the solenoid engaged, the mobile plexi-glass started to move and release the object into the flow.

Figure 7.1. Configuration of the NEU’s closed-circuit wind tunnel: (a) schematic (b) longitudinal view of the test chamber.

The design of the “release mechanism” was performed in accordance with two criteria: (i) it must be sufficiently stiff to hold the object in a pre-defined position until airflow velocity in the chamber has reached a desired speed; (ii) the release must be
instantaneous to replicate an initial takeoff condition with zero velocity, by reducing uncontrollable impulsive forces on the object which could tamper the repeatability and accuracy of initial takeoff conditions; (iii) the apparatus must be compact to minimize interference with surrounding flow.

Figure 7.2 Debris “release mechanism” (launch support) installed in the wind tunnel.

The vertical position of the release system, i.e., the initial takeoff position, was kept constant during all the tests (400 mm from the bottom of the test chamber). Tests were carried out in both smooth flow and grid-generated flow field.

A high-speed monochrome digital camera was used to record each experiment; approximately 250 frames per second were captured; the resolution of the camera was 640 pixels by 480 pixels. The object trajectories were determined by post-processing the frames via image recognition software; the software MaxTRAQ by “Innovision System Inc.” was employed for this purpose. In each frame the horizontal and vertical position of the center of the point-mass object (in pixels) was identified through the software. The video data were subsequently exported; the exported data consisted of a video frame
identifier and object coordinates (x and z) in pixels. A “reference grid” with 2” by 2” (50.8 mm by 50.8 mm) square sections was included in the test setup and placed in the near background of the picture. Utilizing the grid and the output of the image recognition software, the pixels were converted to Cartesian coordinates. The camera setup and orientation of reference grid were designed to capture trajectories, which primarily evolved in a vertical plane corresponding to the longitudinal mid-section of the test chamber.

Parabolic-like trajectories in this vertical plane were anticipated for both smooth flow and turbulent flow due to the combined effects of gravity forces and horizontal airflow effects. Initial observations suggested however that a “more pronounced” parabolic shape should be observed for the uniform flow condition.

In order to reproduce turbulent flow a screen formed with a square mesh, which was placed upstream of the object’s release position, was utilized. Figure 7.3 shows the location of the grid in the wind tunnel. The grid mesh spacing $M=40$ mm, defined as the distance between the centerlines of the square mesh elements, was used. The flow measurement instrumentation was placed downstream of the grid. The horizontal flow velocity (mean value $U$ and fluctuations $u$) was measured by a hot wire probe and flow sensor, located adjacent to the release mechanism and in front of a Pitot tube sensor (Figure 7.3).
Measurements from the hot wire probe (Dantec, type 55P16) and Constant Temperature Anemometer (Model “Dantec - MiniCTA”, type 54T36) were employed in conjunction with the readings from a Setra differential pressure transducer connected to the Pitot tube. Figure 7.3 also shows the location of the Pitot tube and hot-wire in the wind tunnel. The outputs of the hotwire (in volts) were used to detect instantaneous flow.
features; the mean flow velocity recordings derived from a pressure sensor connected to a Pitot tube were used to calibrate/correlate the voltage of the anemometer to mean flow velocity. The calibration of the voltage readings by the hot wire CTA was based on the King’s Law; calibration was repeated at the beginning of each experiment in order to alleviate instrumental drift. More details on the generation of turbulent flow are described in Section 7.2.3.

Figure 7.4 presents an example of video data acquisition and processing procedure for a cube of side length \( l=6.3 \) mm flying in turbulent flow field (test No. 8 in Table 7.1). In Section 7.2.2 the tests will be described in more detail. Figure 7.4 (a) to (d) reproduce four representative video frames (close-up view), captured by the high-speed camera, in which the instantaneous position of the object is highlighted by a circle marker. The release point is set as the origin of the Cartesian axes and four frames are selected to show the trajectory. Rotation of the cube is also discernible in the frames. The reference grid (2” by 2” squares), employed for conversion from pixels to physical coordinates, can be seen in the background. Figure 7.4 (e) shows a typical plot of the \( \bar{z} \) (vertical) vs. \( \bar{x} \) (horizontal) position of the object over time in dimensionless physical coordinates, obtained after post-processing the video data. The experimental data relative to the four frames in Figure 7.4 (a) to (d) are displayed by circle markers in the figure.
Figure 7.4. Video data acquisition and processing procedure for a cube of $l=6.3$ mm in turbulent flow (test No. 8 in Table 7.1): (a) to (d) reproduce four illustrative video frames (the reference grid, used for conversion from pixels to physical coordinates, can be seen in the background); (e) estimation of experimental trajectory (thick solid line) in physical coordinates by post-processing the data of these and other frames.

7.2.2 Description of experiments

Five different sizes of cubes and spheres were used as the representatives of the compact debris. In Table 7.1 the characteristics of the debris were described. Also, in Figure 7.5 two examples of the cube and sphere were used in the experiments are shown. Mean wind speeds ($U$) ranged from 7 to 17 m/s. Model dimensions and wind speeds were designed to provide a range of debris side ratio and support condition to replicate, at a wind-tunnel scale, the typical features of full-scale compact debris objects. Flow similarity based on the Tachikawa number was used to interpret and relate the results of the experiments to full scale. The Tachikawa number was defined as
\[ K = U^2 \rho_a A / (2mg) \], with \( A \) being a reference projected area and \( m \) the mass of the object; \( \rho_a \) is the airflow density, \( g \) the gravity acceleration and \( U \) the mean flow speed. For small \( K \) the aerodynamic forces become irrelevant; small \( K \) values are typical of large-mass compact objects, which tend to “fall” rather than “fly horizontally”. In contrast, large \( K \) values are representative of small mass but larger area objects (e.g., objects having small density).

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Object type</th>
<th>Diameter or ref. dimension (mm, inch)</th>
<th>Mass (gr)</th>
<th>( U ) (m/s)</th>
<th>( K )</th>
<th>( Re )</th>
<th>Turbulence Intensity (%)</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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<td>6.3 (1/4”)</td>
<td>0.13</td>
<td>11.9</td>
<td>2.8</td>
<td>5084</td>
<td>21.8</td>
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<tr>
<td>2</td>
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<td>6.3 (1/4”)</td>
<td>0.13</td>
<td>14.6</td>
<td>4.2</td>
<td>6194</td>
<td>20.5</td>
</tr>
<tr>
<td>3</td>
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<td>0.13</td>
<td>17.9</td>
<td>6.3</td>
<td>7618</td>
<td>19.9</td>
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<tr>
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<td>7.9</td>
<td>2.6</td>
<td>1594</td>
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<tr>
<td>5</td>
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<td>12.2</td>
<td>6.2</td>
<td>2450</td>
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</tr>
<tr>
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<td>0.25</td>
<td>13.4</td>
<td>2.8</td>
<td>7144</td>
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<td>4.4</td>
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<td>3.9</td>
<td>10971</td>
<td>21.4</td>
</tr>
</tbody>
</table>

Notes: \( U \), mean flow speed; \( K \), Tachikawa number; \( Re \), Reynolds number.

In this study the range \( 2 \leq K \leq 7 \) was used in the experiments as a representative interval of full-scale object items (Tachikawa 1983; 1988); this number is a measure of the relevance of aerodynamic forces vs. gravity forces in relation to the object motion. The
debris material for was selected to satisfy the dynamic similarity constraints; as a result, cubes and spheres made of polyethylene and cast acrylic were employed.

![Image of polyethylene and cast acrylic cubes and spheres](image)

**Figure 7.5** Two examples of a cube and a sphere, tested in the wind tunnel.

Detailed properties of the compact spheres and cubes for tests conducted in turbulent flow are shown in Table 7.1; in this table the test cases corresponding to turbulent flow are exclusively reported. In the table the Reynolds number $Re$, based on mean $U$ and reference dimension of the objects, is also reported. The turbulence intensity varied between 19% and 23% in the turbulent flow field tests. The models were released at the initial height of 400 mm from the bottom of the chamber (Figure 7.3) and were placed at the edge of the small plexi-glass panels (Figure 7.2) to reduce local flow effects of the release mechanism on initial conditions of the trajectory. Four trials were repeated and the “best one” was used to minimize local flow effects and interference of the release mechanism at takeoff.
7.2.3 Grid-generated turbulence analysis and turbulence spectrum model

As described in the previous sections, tests were conducted both in smooth flow and in a homogeneously turbulent flow field by placing a grid formed with a square mesh in the uniform flow, upstream of the release position for the object. Even though the passive generation of homogeneous and isotropic turbulence was possibly limited by the size of the wind tunnel and the distance between centerlines of the grid mesh spacing, \( M=40 \) mm, \((\text{Comte-Bellot and Corrsin 1966}; \ (\text{Lavoie et al. 2007})\)), this turbulence generation mechanism was considered appropriate for the validation of the debris trajectory model. Active Turbulence generation devices (Bienkiewicz et al. 1983) could have been employed to increase turbulence length scales and to better satisfy the geometric ratio between turbulence scales and compact debris size in atmospheric wind fields (approximately 100 to 1); nevertheless these were not considered for simplicity and also because the primary objective of the tests was to primarily validate the numerical model with possibly less emphasis on the exact replication of full-scale conditions.

The characteristics of the flow field were analyzed by measuring the horizontal component of the flow speed (mean \( U \) and fluctuating component \( u \)), measured by the hot-wire probe and flow sensor after calibration against the readings of the differential pressure transducer connected to the Pitot tube (Figure 7.3). The flow instrumentation was placed downstream with respect to the location of the grid mesh at a distance equal to about ten times the \( M \) value and at the same elevation as the release system. The mean
flow speed $U$ was varied in order to satisfy the Tachikawa number similarity of the tested spheres and cubes with full-scale debris objects (Section 7.2.2).

Figure 7.6 shows a typical record of the horizontal flow fluctuations and compares the readings obtained from the two instruments at mean speed $U = 11.4$ m/s.

![Figure 7.6 Time history of along-wind horizontal flow velocity – sample record at a mean flow speed $U = 11.4$ m/s and turbulence intensity of the zero-mean fluctuations ($u$) $I_u=0.22$.](image)

The sampling rate was set to 4000 Hz during the experiments in order to derive basic characteristics of the horizontal fluctuating component $u$; these included the standard deviation $\sigma_u$, the turbulence intensity $I_u$, the length scale in the longitudinal flow direction $L_u^x$ and, later, the power spectral density (PSD) $S_u$ of the fluctuations. As expected, limited instrumental fidelity of the pressure transducer in detecting high-frequency time-dependent fluctuations can be noticed in Figure 7.6. Subsequent analysis of the records in the range of $U$ between 10 m/s and 15 m/s indicated that turbulence had $I_u=22\%$ (Table
7.1), almost independent of the $U$ in the chamber. This $I_u$ value was considered as satisfactory and compatible with actual atmospheric turbulence in high winds. The mean flow profile on a typical vertical plane of the test chamber was also analyzed; little variation of $U$ with vertical position $z$ was observed in the $z$-$x$ plane region where the trajectory was expected, indicating the acceptability of the hypothesis of a fully-developed homogeneous turbulent flow field with $U$ independent of height $z$.

The turbulence length scales were found from $L_u^x = U \sigma_u^{-2} \int_0^\infty R_u(\tau) d\tau$, i.e., by numerical integration of the auto-covariance function $R_u$ of the $u$ fluctuations (Simiu and Scanlan 1996). Experimental results suggested a limited influence of the $U$ on the length scales with $L_u^x=10$ mm; this remark confirmed that the length scales were basically controlled by the thickness of square mesh elements on the grid. These $L_u^x$ values were of the same order as the reference dimension of the spheres and cubes (variable between 3 and 8 mm). Even though the geometric ratio $L_u^x$ to object dimension was possibly incompatible with full-scale flows, the $L_u^x$ obtained experimentally were considered as acceptable for the purpose of validation of the debris trajectory model.

The debris trajectory model requires the synthetic generation of turbulence time histories of $w$ and $u$ turbulence fluctuations on a $z$-$x$ vertical plane, based on spectral decomposition. Therefore, the PSD of both $u$ ($S_u$) and $w$ ($S_w$) was necessary. The normalized spectrum model of $S_u$ was determined from the following generalized spectrum model (Tieleman 1995):
\[
\frac{S_u}{\sigma_u^2} = \frac{A}{1 + Bf^\alpha} \times \left( \frac{L_u^c}{U} \right)
\] 

(7.1)

In the above equation, \(A\), \(B\) and \(\alpha\) are constant parameters, which were found by analysis of the experimental data; \(f = nL_u^c/U\) is a dimensionless frequency (with \(n\) frequency in Hz). The similarity of the proposed spectrum with the von Kármán spectrum must be noted (Simiu and Scanlan 1996). The spectrum model in Equation 7.1 was also normalized by \(\left( \frac{L_u^c}{U} \right)\) to eliminate dependency of the spectral amplitude \(A\) on flow conditions and to enable the simultaneous use of records at various \(U\). The spectral ordinates of \(S_u\) were derived by digital spectral analysis of 40-second \(u\) digital records at mean flow speeds \(11<U<16\) m/s by employing 65536 Fourier points and by splitting each record into 512-point overlapping segments to produce the spectral estimates.

The regression of the \(S_u\) data in conformity with Equation (7.1) was conducted in two steps. Initially, the normalized \(S_u\) data at various mean flow speeds were analyzed in the inertial sub-range of the turbulence \((f > 0.1)\). The coefficient \(\alpha = 1.3426\) was found by comparing the trend of the “energy cascade” along the \(f\) axis at various \(U\). Even though a small discrepancy with respect to the theoretical value \((5/3)\) was observed, this value of \(\alpha\) was considered as acceptable in the context of this experimental analysis. At a second stage, the normalized \(S_u\) data at various \(U\) were treated as part of a same population by “stacking” the signals as a function of \(f\); quantities \(A\) and \(B\) were determined by least squares, i.e., by minimizing the error between the data and the model as a function of \(f\).
The following values were found: $A=4.655$ and $B=18$. This operation is graphically summarized in Figure 7.7. In this figure the normalized PSD of the along-wind fluctuations $u$ with direction parallel to the longitudinal axis of the wind tunnel, recorded at $U$ between 11.4 m/s and 15.3 m/s, are compared to the spectrum model. The figure confirms the similarity among the PSD data sets and the agreement of the data with the turbulence spectrum model (thick solid line) with $A=4.655$, $B=18$ and $\alpha=1.3426$.

Finally, under the hypothesis of isotropic turbulence within the experimental chamber, the properties and spectrum of the vertical turbulence $w$ were assumed the same as those derived for the $u$ component.

![Figure 7.7 Normalized power spectral density (PSD) of the along-wind $u$ velocity fluctuations, recorded at various mean flow speeds, vs. turbulence spectrum model (length scale $L_u^x$, mean flow speed $U$, frequency $n$).](image)
7.3 Wind tunnel results and model validation (smooth flow)

7.3.1 Model description for smooth flow
In order to validate the compact debris trajectory results, computed by numerical integration of the equations of motion in smooth flow (uniform field), a first series of wind tunnel experiments were carried out. In these tests the square-grid mesh was temporarily removed to simulate smooth flow. In this section a series of trajectory test results are presented for spherical objects, the characteristics of which are summarized in Table 7.2; these tests were numbered sequentially as reported in this table; they were conducted at a mean flow with mean speed $U$ varying between 7.7 m/s and 14.1 m/s. After removal of the mesh the average value of the turbulence intensity of the horizontal flow fluctuations, measured during the tests, was equal to 0.9%. This value is very small in comparison with the ones reported in Section 7.2.2; the hypothesis of smooth flow was therefore considered satisfactory.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Object type</th>
<th>Diameter, $d$ (mm, inch)</th>
<th>Mass (gr)</th>
<th>$U$ (m/s)</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>Sphere</td>
<td>6.3 (1/4”)</td>
<td>0.13</td>
<td>12.4</td>
<td>2.8</td>
</tr>
<tr>
<td>13</td>
<td>Sphere</td>
<td>6.3 (1/4”)</td>
<td>0.13</td>
<td>14.1</td>
<td>4.0</td>
</tr>
<tr>
<td>14</td>
<td>Sphere</td>
<td>3.2 (1/8”)</td>
<td>0.01</td>
<td>7.7</td>
<td>2.5</td>
</tr>
<tr>
<td>15</td>
<td>Sphere</td>
<td>7.9 (5/16”)</td>
<td>0.25</td>
<td>13.1</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Notes: $U$, mean flow speed; $K$, Tachikawa number.
A short review of the trajectory model (Tachikawa 1983) is reported below. The
trajectory of a point-mass object can be uniquely defined by its horizontal position \(x\) and vertical position \(z\) with respect to a frame of reference, and time \(t\). The equations of motion for compact debris, traveling in 2D smooth flow field of constant speed \(U\), can be expressed in the following dimensionless form:

\[
\frac{d^2 \bar{x}}{d\bar{t}^2} = K \left[ (1 - \bar{u}_x)^2 + \bar{u}_z^2 \right] (C_D \cos \beta),
\] (7.2a)

\[
\frac{d^2 \bar{z}}{d\bar{t}^2} = K \left[ (1 - \bar{u}_x)^2 + \bar{u}_z^2 \right] (C_D \sin \beta) - 1.
\] (7.2b)

In the previous equations the dimensionless position variables are \(\bar{x} = xgU^{-2}\) and \(\bar{z} = zgU^{-2}\) with dimensionless time \(\bar{t} = tg / U\); the dimensionless velocities of the object become \(\bar{u}_x = u_x / U\) and \(\bar{u}_z = u_z / U\) with \(u_x = dx / dt\) and \(u_z = dz / dt\); the direction sine and cosine for the projection of the aerodynamic force in the horizontal and vertical direction are \(\cos \beta = (1 - \bar{u}_x) / \sqrt{(1 - \bar{u}_x)^2 + (\bar{u}_z)^2}\) and \(\sin \beta = \bar{u}_z / \sqrt{(1 - \bar{u}_x)^2 + (\bar{u}_z)^2}\); the Tachikawa number, \(K = U^2 \rho_{air} A / (2mg)\) was defined in the previous section. The parameter \(C_D\) is the static drag coefficient, which is taken as constant independent of \(Re\) and of the relative angle of attack between the moving object and flow velocity.

The equations of motion were numerically solved using the 5th-order Runge-Kutta Method with the time step \(10^{-4}\) seconds in dimensional time units in order to enable resolution of motion variations induced by turbulent flow, as later discussed in Section
7.4.2. A drag coefficient $C_D=0.5$ was used in the simulations. Spheres were exclusively tested in smooth flow.

7.3.2 Analysis of wind tunnel experimental results
The comparison of wind tunnel experimental trajectories vs. simulated ones is shown in Figure 7.8 to Figure 7.10 (2D). There is a very good agreement between the simulated trajectory and wind tunnel results. Figure 7.8 shows the trajectory of a sphere with diameter $d=6.3$ mm, determined for two values of $U$ and $K$ (tests. No. 12 and 13 in Table 7.2). From this figure it can be inferred that the higher the Tachikawa number for the same object the more the sphere tends to fly in the flow field. Similar experimental results for spherical object of other dimensions can be seen in Figure 7.9 (tests No. 14 in Table 7.2) and Figure 7.10 (test No. 15). Even though the value of $U$ and trajectory in physical coordinates are distinct, the normalization based on $K$ leads to very similar dimensionless trajectory results.

In all figures some (but negligible) deviation of the experimental results from the numerical simulations can be observed; this difference tends to increase later in the flight path. One explanation for this small difference can possibly be related to the choice of $C_D=0.5$, which was derived from literature recommendations for full-scale trajectory analysis at large Reynolds numbers, $Re$ (Lin et al. 2007).
Figure 7.8 Dimensionless trajectory of a sphere in smooth flow with $d=6.3$mm: (a) $K=2.8$ (test No. 12), (b) $K=4.0$ (test No. 13).

In the smooth-flow wind tunnel tests the $Re$ was variable between 5000 and 10000; these values are representative of sub-critical flow regimes; for spherical objects the use of a more accurate value of $C_D$ would therefore be needed, which could explain
differences obtained experimentally in comparison with the simulations, especially in the range of horizontal object positions with $\bar{x} > 0.2$ (e.g., Figure 7.8a).

Figure 7.9 Dimensionless trajectory of a sphere in smooth flow with $d=3.2\text{mm}$, $K=2.5$ (test No. 14).

Figure 7.10 Dimensionless trajectory of a sphere in smooth flow with $d=7.9\text{mm}$, $K=2.7$ (test No. 15).
7.4 Wind tunnel results and model validation (turbulent flow)

In this section the simulated trajectory of both cubes and spheres in turbulent flow, with characteristics stated in Table 7.1, was validated by wind tunnel tests. As described in Sections 7.2.2 and 7.2.3, the square grid mesh (e.g., Figure 7.3) was used to generate homogeneous turbulence in an attempt to replicate conditions close to full scale.

7.4.1 Model description for turbulent flow

In order to numerically simulate the trajectory in the turbulent wind field the numerical model, described in (Moghim and Caracoglia 2013) and in Chapter 4, was used. A summary of the model for flow field generation is provided in this section. The 2D model of the wind field makes use of synthetically-generated turbulence time histories. It accounts for variable mean flow speed with elevation (wind shear effect) and stationary horizontal and vertical fluctuations, partially correlated in space (time-dependent turbulence). The digital simulation employs the principle of superposition of harmonic waves with a random phase (Di Paola 1998). The model, discussed in this chapter, is adapted from the model presented in Chapter 4 in order to simulate the turbulence features replicated in the wind tunnel experiments. Even though these features were possibly not fully compatible with those of atmospheric turbulence, the simplifications were accepted in the context of validation of the numerical model and procedure, proposed earlier.
In a full-scale boundary-layer field the fluctuations are digitally generated at a series of discrete points on a vertical plane, located on an idealized grid (Figure 7.11 (a)). In order to replicate as accurately as possible the experimental conditions in the wind tunnel (Figure 7.11 (b)), the wind shear effect, which was part of the original model (Figure 7.11 (a)), was eliminated to reproduce the fully-developed homogeneous turbulent flow, measured in the chamber. In fact, the depth of the boundary sub-layer with predominant flow shear effects and mean velocity deficit in the wind tunnel was in fact negligible, approximately estimated to be less than 5 mm in all the tests.

Time histories of horizontal ($u$) and vertical ($w$) fluctuations, using the same spectrum model described in Section 7.2.3 for both components, were digitally generated at equidistant points, equally spaced vertically and horizontally on the 2D grid approximately corresponding to the mid-span longitudinal section in the chamber (Figure 7.11 (b)). Discrete points were chosen to simulate the continuous 2D flow region, describing the flight range (Figure 7.11 (b)); this grid of points was used to estimate the flow velocity as the object moves within this field.
Figure 7.11 Discretized two-dimensional grid for digital generation of turbulent flow field: (a) full-scale field with shear effects, (b) wind tunnel flow field.
The model for flow field generation operates in two steps. First, it assumes that the flow field “flows” from left to right in Figure 7.11 (b); the mean flow $U$ constant along the height of the chamber was taken from the experimental measurements; $u$ and $w$ fluctuations were computed at the discrete points located on the left side of Figure 7.11 (b) at the “inlet boundary” of the field; these are indicated by solid dark (red) markers along the vertical line at the horizontal coordinate $x=0$ (dimensional space). These $n'$ discrete points, which are positioned along the vertical line at the inlet, can be uniquely defined by their vertical coordinates $z_i$ with $i=1,2,...,n'$ in Figure 7.11 (b) ($x=0$). The generation of the horizontal flow component at each inlet point depends on elevation $z=z_i$; the total horizontal velocity can be expressed as the summation of mean $U$, independent of $z$, and a fluctuating zero-mean term ($u$):

$$U_{tot}(x=0,z=z_i,t) = U + u(x=0,z_i,t).$$

(7.3)

In Equation 7.3 $U_{tot}$ is the total flow speed at a given point in the 2D region of Figure 7.11b and time $t$ with $x=0$. The horizontal component of $u$ at the generic grid point “$i$” ($i=1,...,n'$) along the inlet in Figure 7.11 (b) is re-labeled as $u_i(t)=u(x=0,z_i,t)$ from Equation 7.3. The collection of discrete $u_i$ at the inlet becomes a uni-variate one-dimensional stochastic process. This process can be combined into a stochastic vector $\mathbf{u}(t)$, the features of which can be represented by its PSD matrix $\mathbf{S}_{uu}(n)$ of dimension $n' \times n'$ as a function of frequency $n$. The cross-PSD scalar function between any two points along the inlet (Figure 7.11 (b)), i.e., the two stochastic processes $u_i$ and $u_j$, can be
calculated from $S_{uu}(n) \equiv S_u(n) \times \exp(-\hat{f}_u)$, in which the variable $\hat{f}_u = c_{z,u} \cdot n / U |z_j - z_i|$ depends on the distance between $i$ and $j$. This equation assumes that turbulence is homogenous and isotropic in the chamber. The cross-PSD of the fluctuations, $S_{uu}(n)$, is a real function with the quadrature spectrum being neglected (Simiu and Scanlan 1996). The PSD $S_u(n)$ is the same at all points. The coefficient $c_{z,u}=10$ (Simiu and Scanlan 1996) is a dimensionless decay parameter for horizontal turbulence; $z_i$ and $z_j$ are the elevations of the generic points on the grid. For generating the horizontal velocity fluctuation Equation 7.4 below is used (Di Paola 1998):

$$u_i(t) \cong \sum_{k=0}^{N} \sqrt{\Delta n} / \sqrt{2} \cdot \left[ \Phi_u(n_k) \cdot (\Lambda_u(n_k))^{1/2} \right]_i \cdot \left[ E_k \sin(2\pi n_i t) + F_k \cos(2\pi n_i t) \right]$$  (7.4)

In Equation 7.4, which is derived from Chapter 4 and is shown in dimensional units, $n_k=k \Delta n$ are frequencies evaluated at intervals with $\Delta n$ a frequency step and $k=1,...,N$; $N$ is the total number of waves, truncated at a sufficiently large cutoff value (Di Paola 1998). As indicated in chapter 4, the real matrices $\Phi_u$ and $\Lambda_u$ of dimension $n' \times n'$ can be obtained from the diagonalization of the one-sided cross-PSD matrix at each frequency; this becomes $S_{uu}(n_k)=\Phi_u(n_k)\Lambda_u(n_k)\Phi_u^T(n_k)$; $\Lambda_u$ and $\Phi_u$ are real since $S_{uu}(n_k)$ is real symmetric. The $n' \times 1$ vector coefficients $E_k$ and $F_k$ are employed to produce a random phase angle in each $k$-th synthetic wave; the scalar terms of these vectors must be chosen as two independent zero-mean Gaussian variables with variance equal to 0.5 (Di Paola 1998).
In order to simulate the $S_u(n)$ function for Equation 7.4 the wind spectrum model in Equation 7.1 with $A=4.655$, $B=18$ and $\alpha=1.3426$ (Section 7.2.3) must be used, in which the variance of $u$ ($\sigma_u^2$) was directly estimated from the measurements in each test. The upper cutoff frequency, needed for truncation of Equation 7.4, was chosen to be compatible with the turbulence scales resolved by the spectrum model in the wind tunnel, approximately corresponding to dimensionless frequency $f = 1$ in Section 7.2.3.

The selection of the total number of waves $N=500$ in Equation 7.4 was dictated by the need for preserving sufficient resolution in the digitally simulated time histories at various turbulence scales; the number $N=500$ was also determined from the results of a Section 4.4.3, in which negligible variation of the empirical probability density function of the debris momentum (velocity modulus), calculated at various time instants, was noted beyond this value of $N$.

An equation similar to Equation 7.4 was utilized for digitally generating the zero-mean partially correlated time histories of the vertical fluctuations ($w$) at the inlet in Figure 7.11 (b). For the $w$ component a similar set of equations was developed. The hypothesis of independency between the random samples of $w_i(t)$ and $u_i(t)$ was used.

After determining the velocity profile and turbulence $u_i(t)$ and $w_i(t)$ at the inlet of the 2D field (Figure 7.11 (b)), propagation of turbulence internally to the grid was established by a simple rule. For example, if the object position inside the chamber is $(x,z)$ at time $t$,
the wind velocity is needed locally at the same point. For the horizontal component the mean $U$ and turbulence $u(x,z,t)$ can be evaluated by the following equation:

$$
U_{x\nu}(x,z,t) \approx U + u(q\Delta x, i\Delta z, t)
= U + u(0, i\Delta z, t - \frac{q\Delta x}{U})
= U + u(t - q\Delta x / U).
$$

Equation 7.5 implies that the total horizontal wind velocity at any given point in the field $(x,z)$ is approximately estimated by using the turbulence synthetically generated at the point closest to the actual object location defined by coordinates $x\approx q\Delta x$ and $z\approx i\Delta z$ on the grid in Figure 7.11 (b); the integer index $q$ was employed to designate a generic grid point of horizontal coordinate $x_q=q\Delta x$. The Taylor’s hypothesis of frozen turbulence was used on the right-hand side of Equation 7.5 to propagate $u$ from the inlet boundary at $(0, z)$ with $z\approx i\Delta z$ toward the center of the field; Equation 7.5 assumes that the turbulence, observed at $t=0$ along the inlet, propagates from the left boundary from and “reaches” this point at time $t=q\Delta x/U$. Also, a simplified notation was used in Equation 7.5 for $x=q\Delta x$, with $u(0, i\Delta z, t - q\Delta x / U) = u_i(q\Delta x / U)$ by using Equation 7.4 with $u_i(t)=u(x_0=0, z_i, t)$. In a similar way the $w$ turbulence component was evaluated as $w(x,z,t)=w_i(t-q\Delta x/U)$.

The numerically generated flow field was designated as a “Frozen Turbulence Field” (FTF) and is based on the model, developed for replication of full-scale stationary wind turbulence (Figure 7.11 (a)). The main difference is the representation of $S_{u\nu}(n)$ at full scale; more details may be found in (Moghim and Caracoglia 2013a; 2013b).
For the trajectory estimation, the original model, proposed by Tachikawa (Tachikawa 1983) for uniform flow field in Equation 7.2, was modified to describe the 2D trajectory of compact debris inside the chamber and to account for the homogeneous turbulence.

The equations of motion of the point-mass object in dimensionless form become:

\[
\frac{d^2\bar{x}}{dt^2} = K \left[ (\bar{U}_{\text{tot}} (\bar{x}, \bar{z}, t) - \bar{u}_x)^2 + (\bar{w}(\bar{x}, \bar{z}, t) - \bar{u}_z)^2 \right] (C_D \cos \beta), \tag{7.6a}
\]

\[
\frac{d^2\bar{z}}{dt^2} = K \left[ (\bar{U}_{\text{tot}} (\bar{x}, \bar{z}, t) - \bar{u}_x)^2 + (\bar{w}(\bar{x}, \bar{z}, t) - \bar{u}_z)^2 \right] (C_D \sin \beta) - 1. \tag{7.6b}
\]

In Equation 7.6a and 7.6b, \( \bar{U}_{\text{tot}} \) and \( \bar{w} \) are horizontal (mean plus turbulence, Equation 7.5) and vertical components of the flow velocity speed rewritten in dimensionless form as \( \bar{U}_{\text{tot}} = U_{\text{tot}} / U = 1 + u / U \) (Equation 7.5) and \( \bar{w} = w / U \). In Eqs. (7.6a) and (7.6b) the angle \( \beta \) accounts for the relative velocity between the point-mass object and the flow; it can be determined as \( \beta = \tan^{-1} \left[ (\bar{w} - \bar{u}_z) / (\bar{U}_{\text{tot}} - \bar{u}_x) \right] \). The trajectory was found by numerical integration of Eqs. (7.6); as in the previous section a 5th-order Runge-Kutta method was used with time step \( 10^{-4} \) seconds in dimensional units. This choice was dictated by the need to numerically resolve motion variations of the same order of magnitude as the smallest fluctuation in the flow velocity for turbulent flow. Both spheres and cubes were considered in the experiments (Table 7.1); the drag coefficient for spheres and cubes are, respectively \( C_D = 0.5 \) (Section 7.3.1) and \( C_D = 0.8 \). It must be noted that, since the flow field is turbulent, the trajectory in Equation 7.6 becomes a random curve.
7.4.2 **Analysis of wind tunnel experimental results**

In this section the trajectories, which were computed at various $U$ and for various object types and dimensions as explained in Section 7.4.1, were systematically compared to the experimental trajectories in wind tunnel. Due to the turbulent flow direct comparison between numerical trajectory and a single experiment was not possible. Each experimental trajectory is in fact the realization of a stochastic process; therefore the comparison should be carried out by considering a suitable ensemble of experimental trajectories and an equivalent quantity, numerically computed (average trajectory, deviations from the mean trajectory, etc.). Since repetition of experiments with similar conditions several times was not possible, analyses were conducted by evaluating the ensemble average of the numerical trajectory at each time instant and by evaluating the deviation of a single wind tunnel realization from this average curve.

The average numerical trajectory was evaluated by repeating 15 times the trajectory simulations in a turbulent flow field for each test case in Table 7.1. In order to faithfully replicate the wind tunnel conditions, the mean flow speed $U$ and turbulence intensities were reproduced from the experimental measurements (Table 7.1). Figure 7.12 shows as an example, the computation of the average numerical trajectory based on the ensemble of 15 realizations (R1 to R15) for a sphere with diameter $d=3.2$ mm and $K=2.6$, simulating the results of test No. 4 in Table 7.1 at $U=7.9$ m/s. In this figure the average numerical trajectory is shown with a thick dark continuous line.
Figure 7.12 also shows the 95% confidence intervals of the average trajectory estimator, computed for the furthest point in both directions where maximum variability was noticed among the curves; the confidence region for this point is enclosed within a rectangular boundary (dashed line) at the bottom of the graph. The variation along both axes is about one order of magnitude smaller than the mean estimator; this confirms that the deviation of the ensemble average from the true value is small and the number of numerical repetitions (15) was acceptable.

Figure 7.12 Evaluation of average numerical trajectory by ensemble averaging of 15 realizations in turbulent flow (R1,…,R15) - 2D dimensionless trajectory of a sphere with diameter $d=3.2$ mm, $U = 7.9$ m/s and $K=2.6$ (replicating test No. 4 in Table 7.1).
The analysis of experimental results vs. average numerical trajectories included both spheres and cubes of various dimensions (diameter \(d\) or reference side length \(l\)). The results are shown in Figure 7.13 for spheres and Figure 7.14 for cubes in dimensionless units. As described in previous sections, the Tachikawa number, \(K\), was selected in a suitable range between 2 and 7. From the interpretation of the results in Figure 7.13, very good agreement was observed between average numerical trajectory and the experimental result at the various \(K\). Also, analysis of Figure 7.13 suggested that for all tests the average numerical trajectory is above wind tunnel tests for most cases; deviations of the single experimental realization from the average numerical trajectory is almost negligible at the initial flight stages and tends to increase to about 10% later in time (for \(\bar{x} > 0.1\)).

Also, it can be seen that for larger \(K\) the object tends to fly longer and higher before reaching the bottom of the chamber as a result of a larger horizontal aerodynamic force in comparison with the vertical gravity effect. These observations are the same for the cubes in Figure 7.14, even though differences between numerical simulations and experimental results are more pronounced with cubes than with spheres (around 15 to 20% for \(\bar{x} > 0.1\)).

Larger deviations may possibly be explained by observing that a rotation of the cubes, altering the kinetic energy of the object, was noted in the experiments; this effect is not simulated at all in Eqs. (7.6a) and (7.6b) and should perhaps be carefully considered in the modeling of compact object trajectories. From Figure 7.14 it can also be concluded
that a cube with the same $K$ as a sphere would fly farther due to the influence of a larger drag coefficient on the debris motion.

Figure 7.13 Dimensionless trajectory of a sphere in turbulent flow with: (a) $d=6.3$mm, $K=2.8$ (test No. 1); (b) $d=6.3$mm, $K=6.3$ (test No. 3); (c) $d=3.2$mm, $K=2.6$ (test No. 4); (d) $d=3.2$mm, $K=6.2$ (test No. 5)
Figure 7.14 Dimensionless trajectory of a cube in turbulent flow with: (a) $l=6.3\text{mm}$, $K=2.1$ (test No. 7); (b) $l=6.3\text{mm}$, $K=4.4$ (test No. 8); (c) $l=9.5\text{mm}$, $K=2.0$ (test No. 10); (d) $l=9.5\text{mm}$, $K=3.9$ (test No. 11).

In all the cases shown in Figure 7.13 and Figure 7.14 some deviations of a single experimental realization from the average numerical trajectory are expected; these are,
however, within the inherent variability of the stochastic process, i.e., of the same order of magnitude as the various realizations in Figure 7.12.

Another important observation can be seen from Figure 7.15 to Figure 7.17. In these figures experimental trajectories of compact objects with the same $K$ but with variable size (and $U$) were compared. Even though the variance of the turbulence in the experiments was different, the turbulence intensity was approximately the same (Table 7.1), thereby preserving dynamic flow similarity according to the dimensional analysis in Equation 7.6 (a) and 7.6 (b).

![Figure 7.15 Effect of the size of the object on dimensionless trajectory of a sphere in turbulent flow with: $d=6.3\text{mm}, K=2.8$ (test No. 1), $d=3.2\text{mm}, K=2.6$ (test No. 4) and $d=7.9\text{mm}, K=2.8$ (test No. 6).]
Figure 7.16 Effect of the size of the object on dimensionless trajectory of a cube in turbulent flow with: \( l=6.3\text{mm}, K=2.1 \) (test No. 7) and \( l=9.5\text{mm}, K=2.0 \) (test No. 10).

Figure 7.17 Effect of the size of the object on dimensionless trajectory of a cube in turbulent flow with: \( l=6.3\text{mm}, K=4.4 \) (test No. 8) and \( l=9.5\text{mm}, K=3.9 \) (test No. 11).
It can be noticed in the figures that most trajectories are similar at about the same $K$ in spite of the variability introduced by turbulence; this fact confirms that $K$ is still the most important parameter in the trajectory of compact objects in turbulent flow as in uniform flow. Also, at the same $K$ the flight of a sphere and a cube is qualitatively the same (i.e., dynamic similarity is still valid); this remark was expected since aerodynamic forces, acting on the object, would be the same.

### 7.5 Summary

In this chapter compact object trajectories in a homogeneous-turbulence flow field and a uniform flow field were experimentally determined in a small-scale wind tunnel at Northeastern University. The motivation for the tests was the validation of a recent model for simulating wind-borne debris trajectory in boundary layer winds. The trajectory of the objects was captured using a high speed camera and obtained by post-processing the video frames. Tests were performed on spheres and cubes at different mean flow speed and turbulence conditions. The experimental trajectories were compared to numerical trajectories, computed by numerically solving the 2D equations of motion with and without simulated turbulence.

Preliminary tests were carried out in uniform flow at various flow speeds and on objects of different size in the range of Tachikawa numbers between 2 and 7. Good agreement was seen between numerical and experimental results. Tests under homogeneous turbulence conditions were carried out by installing a square mesh grid in
the wind tunnel. The spectrum of the flow fluctuations was estimated by regression analysis of the turbulence data on a general model. This spectrum was used in conjunction with the wave superposition method to numerically calculate the trajectory of the objects by using a 2D model, based on Taylor’s “frozen turbulence” hypothesis. Numerical and experimental results are in a very good agreement and confirm the suitability of this simple model for turbulence propagation. Experimental results, corroborated by numerical simulations, suggested that turbulence can indeed affect the flight of compact objects in comparison with smooth flow, even though influence of turbulence on trajectory tends to be small during early flight. Three-dimensional effects were on occasion detected in the wind tunnel, by noticing the deviation of the objects from the vertical plane. These should possibly be analyzed in the future.

Finally, the effect of Tachikawa number on the flight trajectory was investigated. The experimental results suggest that for the same type of object (sphere or cube) the trajectory remains approximately the same if the Tachikawa number similarity is preserved. Also, it was concluded that cubes tend to fly farther and faster than spheres due to a larger drag coefficient.
7.6 References


Chapter 8

Summary and Conclusions

8.1 Summary

The primary intent of the research, described in this dissertation, was to develop a “framework” (i.e., a numerical procedure) for the analysis of debris flight at the “building scale” (i.e. in the proximity of its façade) and to evaluate, in a probabilistic setting, potential for damage. Currently, this framework does not exist; this dissertation was devoted to the advancement and verification of this research idea.

A general discussion on the state of the art regarding the wind-borne debris modeling was carried out in Chapter 1. Also, the motivation of this research was presented and discussed in detail. In particular, it was emphasized that a fast and efficient numerical procedure was absent and was needed for the investigation of structural performance.

Chapter 2 of this dissertation describes the basic model for the trajectory of the compact objects. A state-space model combined with numerical integration was used to efficiently solve the equations of motion. Validation of the model and procedure was carried out by comparing a series of trajectory simulations with literature results. In particular, the validation employed empirical equations proposed by Lin (Lin et al. 2007),
which had been used to interpret the results of a series of wind tunnel debris flight tests, conducted at Texas tech University at a reduced scale and at full-scale. In this chapter, the hypothesis of a uniform wind field was used; a constant horizontal wind velocity was assumed during the debris flight. The chapter closes by analyzing the significance of some fundamental non-dimensional parameters that have an effect on the trajectory.

Chapter 3 describes the results of the investigation on the effect of wake interference, for example caused by another building in the proximity of the analyzed structure, on the trajectory. This chapter was inspired by empirical evidence that upward trajectories of compact objects are possible in some cases. A simplified (sudden) vertical gust model was proposed in order to simulate these effects; a “Küssner-like” upward gust was included in the equations of motion of the model in a uniform wind field at the early stages of the object flight.

Chapter 4 summarizes the investigation on the trajectory of compact objects in a fully turbulent wind field. A new model for the numerical simulation of debris trajectory, based on the wave-superposition method (Di Paola 1998), was proposed and developed. The model includes both the influence of wind shear and turbulence. A parametric study of debris trajectory in a random wind field was presented; numerical simulations were compared with the uniform wind field results. The details of the fully turbulent wind field and the effect on compact object trajectory are described in this chapter.
Chapter 5 and 6 synthesize the information presented in the previous chapters to investigate the main goal of this dissertation, which is the estimation of the probability of debris impact on tall building façades. The probabilistic model, which was developed in this study, is presented in detail in Chapter 5 and Chapter 6 in 2D and 3D. The novel concepts of “Universal Impact Curves” and “Iso-Probability Contours” are described and discussed. These curves and contours were developed for the three different wind field models, described in the previous chapters, and compared.

Finally, experimental tests were necessary to validate the trajectory model in fully turbulent wind field because of the lack of the data from the field on stochastic trajectory of debris. Wind tunnel experiments were carried out in the small-scale wind tunnel at Northeastern University. The set up and procedure were explained in Chapter 7. A comparison between experimental results and the numerical model was carried out and shown in this chapter.

8.2 Conclusions

The main conclusions of the work are summarized below.

Regarding Trajectory Modeling: (Chapter 3 and Chapter 4)

- The “Küssner-like” upward gust model (simulated fully-correlated upward gust of constant magnitude and short duration) was used to approximately simulate the effects of a large vertical gust near a building structure at object flight takeoff. A modification to the equations of motion for compact debris flight was used to
account for the effect of this simulated gust. It was concluded in both 2D and 3D that uniform wind field assumption may not be completely valid for estimation of compact objects trajectory.

- The new model for debris trajectory in a fully-developed atmospheric boundary layer wind was used to analyze the influence of a realistic partially coherent turbulent wind field on trajectory. Numerical results show that turbulence can significantly influence the trajectory of compact debris in comparison with the uniform wind speed model, which is currently employed in the literature; turbulence affects the trajectory of compact objects during early flight stages.

- Both the “Küssner-like” upward gust model and the model in fully-developed turbulent wind field, employed in conjunction with the point-mass dynamic model, indicate that upward trajectory during early flight is possible for compact object.

- Dimensionless debris velocity modulus $|\vec{V}|$ and angle of impact $\Psi_z$ are two important parameters that can be used to estimate damage to the cladding elements of a tall building. The dissertation shows that these two quantities can be used to quantify the debris momentum at the time of impact in a dimensionless form. Also, it is demonstrated that $|\vec{V}|$ and $\Psi_z$ are approximately independent of the mean wind speed. Although approximate, the “Küssner-like“ upward gust
model” can still be used for preliminary investigations in fully turbulent winds since it produces conservative results in terms of dimensionless momentum ($\|\mathbf{V}\|$) and $\Psi_z$.

**On Risk Assessment Modeling: (Chapter 5 and Chapter 6)**

- A risk model was developed to investigate the probability of impact for compact debris missiles against the façade of a tall building. A new probability-based measure (“critical relative distance”) was coined for characterizing the maximum relative distance, from which a debris object can successfully reach its target with at least the chance of 5%.

- A “Universal Probability Curve” for compact object was generated for the first time to analyze the risk of impact for 2D debris motion. The work shows that this curve can be determined in dimensionless form. Most importantly, the curve is independent of the magnitude of the mean wind speed even in the case of turbulent wind flow.

- The effect of turbulence and wind shear (boundary layer wind) on “Universal Probability Curve” is investigated; the rapid diminution of impact probability, which was observed in uniform winds, almost disappears in the case of boundary layer winds.
• For a complete 3D risk analysis, “Iso-probability Impact Contours” were proposed in this work for the first time to describe in a simple but efficient way the vulnerability of a cladding component to the impacts. Numerical results of the 3D probability-based simulations were compatible with the 2D results and the Universal Probability Curve.

Experimental Validation of the Models: (Chapter 7)

• The proposed trajectory models, especially the new model for turbulent-flow trajectory, were validated by comparing numerical simulations with a series of wind tunnel tests (2D trajectory only).

• Experimental results confirm that turbulence can influence the flight of compact objects in comparison with smooth flow.

• The experimental tests suggest that, for the same type of object (sphere or cube), the trajectory remains approximately the same if the Tachikawa number similarity is preserved.

• It was observed in the experiments that cubes tend to fly farther and faster than spheres due to a larger drag coefficient.

8.3 Recommendations for Future Research

• More detailed investigations are needed to better capture the three-dimensional effects, which were observed in the wind tunnel in the turbulent-flow trajectory experiments.
• The simulation model and results in turbulent wind field should be expanded to analyze trajectories of plate-type debris objects. Wind tunnel experiments, using two high-speed cameras at the same time, would be needed to capture the 3D motion of plates. Analysis of 3D motion features may also be needed for compact spheres and cubes; deviation of the object trajectory from a vertical plane was in fact detected, on occasion, in the wind tunnel experiments.

• Additional studies are necessary to extend the theory of “Iso-Probability Impact Contours” and “Universal Impact Contours” for plate-type debris both in uniform and turbulent wind field.

• Further investigation of the problem and more simulations are required to investigate the sensitivity of trajectories to initial conditions (i.e., object velocity) at takeoff.

8.4 Outcome of the research: List of Deliverables

As of October 2013, the work, described in this dissertation, has been published or is in the process of being published in a series of journal articles, conference proceedings and poster presentations. A description of these items is provided below for completeness.

8.4.1 Journal Publications (Published or Under Review)


8.4.2 Full Papers in Conference Proceedings


8. Moghim, F., and Caracoglia, L., “Effects of an initial vertical gust on the trajectory of wind-borne compact debris in horizontal winds”, *Proceedings of the International Conference of the Engineering Mechanics Institute (EMI2011)*, American Society of Civil Engineers, Northeastern University, Boston, Massachusetts, USA, June 02-04,
2011, (extended abstract and presentation only).


8.4.3 Poster Presentations


8.5 References
