Dark Matter in SUGRA, Strings and Branes

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A brief review is given of dark matter in SUGRA, strings and branes. For SUGRA models the implications of Yukawa coupling unification on dark matter are discussed in the light of $g - 2$ and $b \rightarrow s\gamma$ constraints. A brief discussion is given of the dark matter in orbifold string compactifications under constraints of modular invariance and radiative breaking of the electroweak symmetry. Finally a new candidate for dark matter - an extra-weakly interacting massive particle or an XWIMP- is discussed. Such dark matter can arise in a wide class of models, including the Stueckelberg extensions of MSSM, in U(1) extensions of MSSM with off diagonal kinetic energy, and possibly in a wider class of models which may have a string/D-brane origin. Satisfaction of the relic density of XWIMPs consistent with WMAP is also discussed.

Keywords: Dark matter, sugra unification, strings, XWIMPS.

1. Introduction

In this talk we give a brief overview of the leading candidate for cold dark matter$^1$ in a broad class of models which includes the supergravity unified models (SUGRA), string models and brane models. In mSUGRA$^2$ the neutralino$^3$ arises as the least massive supersymmetric particle (LSP) over a broad region of the parameter space,$^4$ and with R parity it can be a candidate for cold dark matter. The nature of dark matter depends critically on the type of soft breaking and it is this aspect that differentiates the nature of dark matter in SUGRA models, vs dark matter in strings and branes. For instance, in orbifold compactifications of the heterotic string, the constraints of modular invariance play an important role in the nature of soft breaking, and hence on the nature of neutralino dark matter. Aside from the neutralino, even in supersymmetry there exist other possible candidates such as the gravitino, and the sneutrino. Specifically, the gravitino possibility has resurfaced recently.$^5$ In addition other dark matter candidates abound such as the Kaluza-Klein states$^6$ in extra dimensional models, Q balls,$^7$ as
well as a variety of other possibilities.\textsuperscript{8–10} To this list we will add a new candidate - an extra weakly interacting massive particle or an XWIMP.\textsuperscript{11} Such a particle can arise in a wide class of models including the Stueckelberg extensions of MSSM,\textsuperscript{12–16} the $U(1)_X$ extensions with off diagonal kinetic energy terms,\textsuperscript{11,17–19} and possibly in a broader class of models with string/D-brane origins.

The outline of the rest of the paper is as follows. In Sec.(2) we give an overview of dark matter in SUGRA models with focus on inclusion of Yukawa coupling unification. It is known that in this case the constraints of $b \to s + \gamma$, and the sign of the $\mu$ parameter play a central role in the analysis. Since the sign of $\mu$ is closely tied to the sign of the supersymmetric contribution to $g_\mu - 2$, the analysis in this case is highly constrained. In Sec.(3) we discuss dark matter in heterotic string models, and point out that here $\tan \beta$ in no longer a free parameter but is determined by the twin constraints of radiative breaking of the electroweak symmetry and by the constraints of modular invariance. In Sec.(4) we discuss the new candidate for dark matter- an extra-weakly interacting massive particle, and show that despite its extra weak interactions, it is possible to satisfy the WMAP relic density constraints.

2. Dark matter in SUGRA unification

Extensive investigations of the relic density in SUGRA models exist in the literature (for recent works see\textsuperscript{20–22}) and for the mSUGRA case this implies exploration of the parameter space spanned by the four conventional parameters: $m_0, m_{1/2}, A_0, \tan \beta$. Significant regions exist where WMAP constraints can be satisfied and these regions can be broadly labeled as the stau co-annihilation region, where coannihilation between the LSP and the stau produces relic densities consistent with WMAP, the resonance region where relic density constraint is satisfied due to the Higgs poles, and the hyperbolic branch (HB)\textsuperscript{23} where the relic density is satisfied due to a relatively large higgsino component of the LSP. These analyses are very sensitive to the nature of soft breaking and thus the inclusion of non-universalites in the soft breaking produce significant effects in the analysis. Non-universalites can appear in a variety of ways but these must be consistent with the flavor changing neutral current constraints. Such constraints can be respected by inclusion of non-universalites in the Higgs sector and in the gaugino sector, and several analyses exist where the Higgs sector\textsuperscript{24} and the gaugino sector\textsuperscript{25–30} non-universalites have been included in dark matter analyses. In addition to the above analyses of dark matter are also sensitive to CP
phases and we discuss this later.

![Graph](image)

**Fig. 1.** Analysis of the neutralino relic density with the $b - \tau$ Yukawa unification constraints in the $m_0 - M_{1/2}$ plane when the soft terms are universal and real with $\mu < 0$ ($\theta_\mu = \pi$), $A_0 = 0$, $m_t = 178$ GeV, and $\tan \beta = 45$. Areas contoured by the dashed line has a neutralino relic density which is inside the WMAP bounds. The area above the solid line predicts $m_b(M_Z) > 3 \times 10^2$ GeV, while the area inside the dashed (dot-dashed) line is excluded by the lower bound on $m_h$ (the upper bound on $BR(b \to s + \gamma)$). On the lower dark area $m_{\chi} > m_{\tilde{\tau}}$ while on the upper side EWSB is not achieved. The thin dashed line indicates $m_{\chi} + = 103$ GeV. Taken from Ref. 33

For the remainder of this section we will focus on the analysis of dark matter including the effects of Yukawa coupling unification. Thus in many unified models the $b$ and $\tau$ Yukawa couplings are related at high scales, e.g., $h_b \simeq h_\tau$. These are evolved down to the electroweak scale and constrained by experiment $m_\tau = 1.7463$ GeV, and $2.69\text{GeV} < m_b(M_Z) < 3.10\text{GeV}$. In some models one extends the above to a full Yukawa unification $h_\tau \simeq h_b \simeq h_t$. We note in passing that while $b - t - \tau$ unification in $SO(10)$ models with 10plet of Higgs for breaking of the electroweak symmetry requires a large $\tan \beta$, a large $\tan \beta$ is not a necessity when the symmetry breaking is achieved via alternative schemes (see, e.g., Ref. 34).

The sign of $\mu$ plays a central role in $b - \tau$ and $b - t - \tau$ unification. It is known that the supersymmetric contribution to $a_\mu = (g_\mu - 2)/2$ is directly correlated to the sign of $\mu$ and further that one can infer this sign experimentally. A positive $\mu$ is favored by the $b \to s\gamma$ since the parameter space of mSUGRA and of other models is less stringently constrained by it. On the other hand $b - \tau$ unification seems to favor a negative $\mu$. This is
so because $b - \tau$ unification requires a negative loop contribution to the $b$ quark mass, and the major contribution to this loop comes from the gluino exchange and its sign depends on $\mu \tilde{m}_g$. Many analyses exist which have worked to resolve this problem.\(^4\)–\(^43\) One such possibility is to use non-universalities.\(^43\) For example, in $SU(5)$ the gaugino masses transform like the symmetric product of $(24 \times 24)$ which can be expanded as $1 + 24 + 75 + 200$. Now for the singlet case one gets universality of gaugino masses at the GUT scale. However, if one considers the 24 plet case, then $M_3, M_2, M_1$ are in the ratio (-2, 3, 1), and one finds a relative minus sign for the gluino mass term. This gaugino mass pattern switches the sign of $(\mu \tilde{m}_g)$ from positive to negative, which allows one to achieve a $b - \tau$ unification with a positive $\mu$. Experimentally, the most recent analyses appear to favor a positive $\mu$.\(^44\) Still we discuss both $\mu$ signs for Yukawas unification and dark matter\(^a\). For positive $\mu$ the analysis of dark matter is given in Refs.,\(^43\) while for negative $\mu$ it is given in Ref.,\(^33\) and an exhibition of one case is given in Fig.(1) using the WMAP relic density of Ref.\(^31\),\(^32\) The analysis of Fig.(1) shows that Yukawa unification constraint allows for a satisfaction of the relic density constraint consistent with WMAP.\(^31\),\(^32\) CP phases also have a strong effect on dark matter\(^33\),\(^46\) but here one needs to pay attention to the satisfaction of the edm constraints which, however, can be satisfied even for large phases via the cancellation mechanism.\(^47\) In passing we draw attention to the recent improved analyses of $b \rightarrow s + \gamma$ which, as discussed above, has an important effect on dark matter. These improved analyses include the next to leading order (NLO) corrections enhanced by large $\tan \beta$.\(^48\),\(^51\) The most recent analysis of Ref.\(^51\) additionally includes the full array of CP violating effects and these results will be useful in future dark matter analyses. In unified models there is also a strong link between proton stability and dark matter\(^49\),\(^50\) a topic which is beyond the scope of this talk.

3. Dark matter in heretotic string models

As in SUGRA models, dark matter in heretotic string models is largely governed by the soft breaking. In orbifold compactifications one typically has a large radius- small radius symmetry, so that $R \rightarrow \alpha'/R$, and more generally an $SL(2, Z)$ modular invariance symmetry. There are many analyses which have looked at soft breaking with modular invariance\(^52\),\(^53\) and their implications (For a sample see, Refs.\(^54\)–\(^57\) and references therein). In

\(^a\)For an analysis of dark matter with quasi-Yukawa unification see Ref.\(^45\)
the analysis of Ref.\textsuperscript{55,56} the further constraint of radiative breaking of the electroweak symmetry was utilized. With the twin constraints of modular invariance and radiative breaking, $\tan \beta$ is no longer a free parameter but is determined in terms of $\alpha_{\text{string}}$ and the remaining soft parameters. Using this constraint an analysis of dark matter for $\mu > 0$ with WMAP constraint implies an upper limit on sparticle masses which lie within reach of the LHC, and further the neutralino-proton cross sections lie within reach of the dark matter detectors.\textsuperscript{56}

4. Extra weakly interacting dark matter

Recently a new candidate for dark matter has been proposed whose couplings with matter are weaker than weak or extra weak.\textsuperscript{11} The mechanism for its generation is exhibited in Fig.(2), and it depends on three sectors: a visible sector where the particles of MSSM reside, a hidden sector where fields do not have any direct interactions with the fields in the visible sector and a third sector\textsuperscript{58} which connects both to the fields of the visible sector and of the hidden sector. We label this third sector, the connector sector. A spontaneous breaking in the connector sector produces mixings between the neutralino states in the visible sector and the neutralino states in the hidden sector. If the LSP of the hidden sector (XLSP) lies lower than the LSP
of the visible sector, then the XLSP becomes the LSP of the entire system. This is the XWIMP. As an example we consider the $U(1)_X$ Stueckelberg extension\textsuperscript{12–16} where one has the $U(1)_X$ gauge fields $C_\mu, \lambda_C, D_C$. The connector sector is chosen to be a pair of chiral fields $\phi^{\pm}$\textsuperscript{59,60} which are charged under both $U(1)_X$ and $U(1)_Y$. We add to the mix a Fayet-Illiopoulos term\textsuperscript{61}

$$L_{FI} = \xi_X D_C + \xi_Y D_B$$

(1)

Elimination of the D terms then leads to the potential

$$V = \frac{g_X^2}{2} \left( Q_X |\phi^+|^2 - Q_X |\phi^-|^2 + \xi_X \right)^2 + \frac{g_Y^2}{2} \left( Y_0 |\phi^+|^2 - Y_0 |\phi^-|^2 + \xi_Y \right)^2$$

(2)

which on minimization gives $\langle \phi^+ \rangle = 0$, $\langle \phi^- \rangle \neq 0$. After spontaneous breaking two new mass parameters emerge. $M_1 = \sqrt{2}g_X Q_X < \phi^- >$, $M_2 = \sqrt{2}g_Y Y_0 < \phi^- >$, or alternately one can choose the new parameters to be $M_1$ and $\epsilon = M_2/M_1$. The hidden sector and the connector sector provide two additional neutralino fields, $\chi_S, \lambda_X$ which together with the four neutral fermionic states in the MSSM, $\lambda_Y, \lambda_3, \tilde{h}_1, \tilde{h}_2$ give a set of six Majorana spinors. In the basis $((\chi_S, \lambda_X); (\lambda_Y, \lambda_3, \tilde{h}_1, \tilde{h}_2))$ one finds a $6 \times 6$ Majorana mass matrix, whose eigen states we label $((\xi^0_1, \xi^0_2); (\chi^0_1, \chi^0_2, \chi^0_3, \chi^0_4))$, where $\chi^0_a$ $(a=1,2,3,4)$ are essentially the four neutralinos that appear in MSSM, and $\xi^0_\alpha$, $(\alpha = 1, 2)$ are the new states.

Following the same procedure used to constrain extra dimensions\textsuperscript{62} one can put also a constraint on $\epsilon$, and one finds $\epsilon < 0.06$.\textsuperscript{11} The LEP and the Tevatron data put further constraints on the model,\textsuperscript{11} reducing further the size of $\epsilon$. Because of the smallness of $\epsilon$, the interactions of $\xi^0_\alpha$ with the visible sector fields will be suppressed by an additional factor of $\epsilon$ and thus the interactions of the XWIMP with the visible sector will be extra weak. Further, if the mass of either $\xi^0_1$ or $\xi^0_2$ is smaller than the mass of all of the MSSM particles, then the XLSP will be the LSP of the entire system and hence with R parity conservation, a candidate for dark matter. It was shown in\textsuperscript{11} that a similar situation arises for the $U(1)_X$ extension of MSSM with gauge kinetic energy mixing\textsuperscript{17–19} involving a mixing of the $U(1)_X$ and the $U(1)_B$ gauge field strengths. Supersymmetric version of this model leads to a form of the neutralino mass matrix which, although different in form, also produces an XWIMP. Further, one may conceive of other models where considerations of the type outlines above lead to an XWIMP and a candidate for cold dark matter. Models of the type discussed here may have a string/D-brane origin.\textsuperscript{64}

A priori it would appear that an XWIMP would not satisfy the relic density constraints as its extra weak interactions would not allow for an
Fig. 3. The allowed parameter space in the $m_0 - m_{1/2}$ plane, under the 1σ WMAP3 constraint in extended mSUGRA for the case $A_0 = 0$, $\tan \beta = (30, 50)$ (upper,lower), $\text{sign}(\mu) > 0$, $m_t = 171.4$ GeV, $m_{1/2} \in (0, 1.5)$TeV and $m_0 \in (0, 3.5)$TeV, and $\Delta$ in the range (0,0, 0.1). Regions eliminated by the light chargino mass constraint, by the light Higgs mass constraint, and by the $b \rightarrow s + \gamma$ constraint are also exhibited. Taken from Ref.11.

Efficient annihilation of excess CDM in the early universe. However, if any of the MSSM particles lie close to the XWIMP mass, then the XWIMPS can co-annihilate quite efficiently and the relic density constraints can be satisfied. Thus, for example, if the NLSP turns out to be $\chi^0 = \chi^0_1$, and if $\Delta = (m_{\chi^0} - m_{\xi^0})/m_{\chi^0} > 0$, then the XWIMP $\xi^0 = \xi^0_1$ relic density is given by

$$\sigma_{\text{eff}}(\text{XWIMP}) \simeq \sigma_{\chi^0\chi^0}^{\text{eff}} \left( \frac{Q}{1 + Q} \right)$$

(3)

where $Q \sim (1 + \Delta)^{3/2} e^{-x_f \Delta}$. If $\Delta << 1$ then $Q = O(1)$ and an efficient co-annihilation of XWIMPS can occur. A detailed analysis was carried out in Ref.11 using the packages of Ref.65,66 and of Ref.67,68. In the analysis we have also examined the effects of the $Z'$ pole on the relic density using the techniques of integration over the $Z'$ pole in thermal averaging.69,70

The result of the analysis11 is exhibited in Fig.(3) using the constraints of the three year WMAP data63 and using the parameter space of mSUGRA accessible at the LHC.71 Further, in the analysis of Ref.11 the sensitivity to the top mass72 was also investigated and the analysis found to have significant variations with a one σ variation in the top mass.

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References

1. For a recent work on the direct empirical evidence for the existence of dark matter in galaxies see, D. Clowe et al., astro-ph/0608407.


