CP Violation From Standard Model to Strings

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(Dated: February 1, 2008)

A review of CP violation from the Standard Model to strings is given which includes a broad landscape of particle physics models, encompassing the non-supersymmetric 4D extensions of the standard model, and models based on supersymmetry, on extra dimensions, on strings and on branes. The supersymmetric models discussed include complex mSUGRA and its extensions, while the models based on extra dimensions include 5D models including models based on warped geometry. CP violation beyond the standard model is central to achieving the desired amount of baryon asymmetry in the universe via baryogenesis and leptogenesis. They also affect a variety of particle physics phenomena: electric dipole moments, $g-2$, relic density and detection rates for neutralino dark matter in grand unified and standard based models, and sparticle production cross sections, and their decays patterns and signatures at hadron colliders. Additionally CP violations can generate CP even-CP odd Higgs mixings, affect the neutral Higgs spectrum and lead to phenomena detectable at colliders. Prominent among these are the CP violation effects in decays of neutral and charged Higgs bosons. Neutrino masses introduce new sources of CP violation which may be explored in neutrino factories in the future. Such phases can also enter in proton stability in unified models of particle interactions. The current experimental status of CP violation is discussed and possibilities for the future outlined.

PACS numbers: Valid PACS appear here

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The electric dipole moment of the neutron was less than showing experimentally in 1951 that the magnitude of James Smith then carried out the first such test by of an electric dipole moment for these needed to be for elementary particles, and thus the possible existence evidence for the parity symmetry for nuclear forces and (Wu et al., 1957), it was argued by many that the elementary electric dipole moments would vanish due to parity symmetry. However, in 1950 it was first observed by Purcell and Ramsey (Purcell, 1957). However, they were quoted in other publications (Lee and Yang, 1956; Ramsey, 1956; Smith, 1951).

The experimental results of Purcell, Ramsey and Smith while completed in 1951 were not published till much later (Smith et al., 1957). However, they were quoted in other publications (Lee and Yang, 1956; Ramsey, 1956; Smith, 1951).

The electroweak sector of the Standard Model contains one phase which appears in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The CKM matrix satisfies unitarity constraints including the well known unitarity triangle constraint where the three angles $\alpha, \beta, \gamma$ defined in terms of ratios involving the products of CKM matrix elements and their complex conjugates sum to $\pi$. In addition the quantum chromo dynamic (QCD) sector of the Standard Model brings in another source of CP violation - the strong CP phase $\theta_{QCD}$. The natural size of this phase is $O(1)$ which would produce a huge contribution to the electric dipole moment (EDM) of the neutron in violation of the existing experimental bounds. A brief discussion of these issues is given in Sec.(II). A review of the experimental evidence for CP violation and of the searches for evidence of other CP violation such as in the electric dipole moment of elementary particles and of atoms is given in Sec.(III). Here we discuss the current experimental situation in the K and B system. In the Kaon system two parameters $\epsilon$ (indirect CP violation) and $\epsilon'$ (direct CP violation) have played an important role in the discussion of CP violation in this system. Specifically the measurement

\[ \text{References} \]

I. INTRODUCTION

We begin with a brief history of the considerations that led to question the validity of CP symmetry as an exact symmetry for elementary particles. The history is tied to the issue of electric dipole moments and we need to retrace the steps back to 1950 when it was generally accepted that the particle electric dipole moments vanished due to parity symmetry. However, in 1950 it was first observed by Purcell and Ramsey (Purcell and Ramsey, 1951), that there was no experimental evidence for the parity symmetry for nuclear forces and for elementary particles, and thus the possible existence of an electric dipole moment for these needed to be tested experimentally. They and their graduate student James Smith then carried out the first such test by showing experimentally in 1951 that the magnitude of the electric dipole moment of the neutron was less than $3 \times 10^{-20}$ e.cm where $e$ is the charge of the proton. After the violation of parity symmetry proposed by T.D. Lee and C.N. Yang (Lee and Yang, 1956) was confirmed (Wu et al., 1957), it was argued by many that the elementary electric dipole moments would vanish due to the combined charge conjugation and parity symmetry, i.e., CP symmetry (or equivalently under a time reversal symmetry under the assumption of CPT invariance). However, it was then pointed out by Ramsey (Ramsey, 1958) and independently by Jackson and collaborators (Jackson et al., 1957) that T invariance was also an assumption and needed to be checked experimentally (A brief review of early history can be found in (Ramsey, 1998)). Since then the search for CP violations has been vigorously pursued. The CP violation was eventually discovered in the Kaon system by Val Fitch, James Cronin and collaborators in 1964 (Christenson et al., 1964). Shortly thereafter it was pointed out by Andre Sakharov (Sakharov, 1967) that CP violations play an important role in generating the baryon asymmetry in the universe. However, it has recently been realized that sources of CP violation beyond what exist in the Standard Model are needed for this purpose. In this context over the past decade a very significant body of work on CP violation beyond the Standard Model has appeared. It encompasses non-supersymmetric models, supersymmetric models, models based on extra dimensions and warped dimensions, and string models. There is currently no review which encompasses these developments. The purpose of this review is to bridge this gap. Thus in this review we present a broad overview of CP violation starting from the Standard Model and ending with strings. CP violation is central to understanding the phenomena in particle physics as well as in cosmology. Thus CP violation enters in K and B physics, and as mentioned above CP violation beyond the Standard Model is deemed necessary to explain the desired baryon asymmetry in the universe. Further, new sources of CP violation beyond the Standard Model could also show up in sparticle production at the LHC, and in the new generation of experiments underway on neutrino physics. In view of the importance of CP violation in particle physics and in cosmology it is also important to explore the possible origins of such violations. These topics are the focus of this review. We give now a brief outline of the contents of this review.

In Sec.(II) we give a discussion of CP violation in the Standard Model and of the strong CP problem. The electroweak sector of the Standard Model contains one phase which appears in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The CKM matrix satisfies unitarity constraints including the well known unitarity triangle constraint where the three angles $\alpha, \beta, \gamma$ defined in terms of ratios involving the products of CKM matrix elements and their complex conjugates sum to $\pi$. In addition the quantum chromo dynamic (QCD) sector of the Standard Model brings in another source of CP violation - the strong CP phase $\theta_{QCD}$. The natural size of this phase is $O(1)$ which would produce a huge contribution to the electric dipole moment (EDM) of the neutron in violation of the existing experimental bounds. A brief discussion of these issues is given in Sec.(II). A review of the experimental evidence for CP violation and of the searches for evidence of other CP violation such as in the electric dipole moment of elementary particles and of atoms is given in Sec.(III). Here we discuss the current experimental situation in the K and B system. In the Kaon system two parameters $\epsilon$ (indirect CP violation) and $\epsilon'$ (direct CP violation) have played an important role in the discussion of CP violation in this system. Specifically the measurement
of $\epsilon'/\epsilon$ rules out the so called superweak theory of CP violation while the measurement is consistent with the Standard Model prediction. In this section we also give an analysis of experimental constraints on the angles $\alpha, \beta, \gamma$ of the unitarity triangle discussed in Sec.(II). The current experimental limits of the EDMs of the electron, of the neutron and of $^{199}\text{Hg}$ are also discussed.

In Sec.(IV) we give a discussion of the CP violation in some non-supersymmetric extensions of the Standard Model. These include the Left-Right (LR) extensions, the two Higgs doublet model and extensions with more than two Higgs doublets. It is shown that such extensions contain more sources of CP violation. For example, the LR extensions with the gauge group $SU(2)_L \times SU(2)_R \times U(1)_Y$ and three generations contain seven CP phases instead of one phase that the Standard Model has. Similarly it is shown that the number of allowed CP phases increases with the number of Higgs doublets. Further, new sources of CP violation arise as one increases the number of allowed generations. CP violation in the context of supersymmetric extensions of the Standard Model are discussed in Sec.(V). Here one finds that the minimal supersymmetric standard model (MSSM) has a large number (i.e., 46) of phases which, however, is reduced to two phases in the minimal supergravity unified model (mSUGRA). However, more phases are allowed if one considers supergravity unified models with non-universal soft breaking at the grand unified (GUT) scale consistent with flavor changing neutral current (FCNC) constraints. A discussion of CP violation in extra dimension models is given in Sec.(VI). In this section we give an exhibition of the phenomena of spontaneous vs explicit CP violation. In this section we also discuss CP violation in the context of warped extra dimensions.

A discussion of CP violation in strings is given in Sec.(VII). It is shown that soft breaking in string models is parametrized by vacuum expectation values (VEVs) of the dilaton ($S$) and of the moduli fields ($T_i$) which carry CP violating phases. Additionally CP phases can occur in the Yukawa couplings. Thus CP violation is quite generic in string models. We give specific illustration of this in a Calabi-Yau compactification of an $E_8 \times E_8$ heterotic string and in orbifold compactifications. Here we also discuss CP violation in D brane models. Finally in this section we discuss the possible connection of SUSY CP phases with the CKM phase.

A discussion of the computation of the EDM of an elementary Dirac fermion is given in Sec.(VIII) while that of a charged lepton in supersymmetric models is given in Sec.(IX). In Sec.(X) we give an analysis of the EDM of quarks in supersymmetry. The supersymmetric contributions to the EDM of a quark involve three different pieces which include the electric dipole, the chromo electric dipole and the purely gluonic dimension six operators. The contributions of each of these are discussed in Sec.(X). Typically for low lying sparticle masses the supersymmetric contribution to the EDM of the electron and of the neutron is generally in excess of the current experimental bounds. This poses a serious difficulty for supersymmetric models. Some ways to overcome these are also discussed in Sec.(X). Two prominent ways to accomplish this include either a heavy sparticle spectrum with sparticle masses lying in the TeV region, or the cancelling mechanism where contributions arising from the electric dipole, the chromo electric dipole and the purely gluonic dimension six operators largely cancel.

If the large SUSY CP phases can be made consistent with the EDM constraints, then such large phases can affect a variety of supersymmetric phenomena. We discuss several such phenomena in Sec.(XI). These include analyses of the effect of CP phases on $g_\mu -2$, on CP even-CP odd Higgs mixing in the neutral Higgs sector, and on the b quark mass. Further, CP phases can affect significantly the neutral Higgs decays into $bb$ and $\tau\tau$ and the decays of the charged Higgs into $tb, \bar{\nu}_\tau\tau$ and the decays $H^\pm \to \chi^\pm \chi^0$. These phenomena are also discussed in Sec.(XI). Some of the other phenomena affected by CP phases include the relic density of neutralino dark matter, proton decay via dimension six operators, the decay $B^0 \to \mu^+\mu^-$, decays of the sfermions, and the decay $B \to \phi K$. These are all discussed in some detail in Sec.(XI). Finally in this section we discuss the $T$ and CP odd operators and their observability at colliders. An analysis of the interplay between CP violation and flavor is given in Sec.(XII). In this section we discuss the mechanisms which may allow the muon EDM to be much larger than the electron EDM, and accessible to a new proposed experiment on the muon EDM which may extend the sensitivity of this measurement by several orders of magnitude and thus make it potentially observable. In this section an analysis of the effect of CP phases on $B \to X_s\gamma$ is also given. This FCNC process is of importance as it constrains the parameter space of MSSM and also constrains the analyses of dark matter. Sec.(XIII) is devoted to a study of CP violation in neutrino physics. Here a discussion of CP violation and leptogenesis is given, as well as a discussion on the observability of Majorana phases.

Future prospects for improved measurement of CP violation in experiments are discussed in Sec.(XIV). These include improved experiments for the measurements of the EDMs, B physics experiments at the LHCb which is dedicated to the study of B physics, Super Belle proposal, as well as superbeams which include the study of possible CP violation in neutrino physics. Conclusions are given in Sec.(XV). Some further mathematical details are given in the Appendices in Sec.(XVI).
II. CP VIOLATION IN THE STANDARD MODEL AND THE STRONG CP PROBLEM

The electroweak sector of the Standard Model with three generations of quarks and leptons has one CP violating phase which enters via the Cabbibo-Kobayashi-Maskawa (CKM) matrix $V$. Thus the electroweak interactions contain the CKM matrix in the charged current sector

$$g_2 \bar{u}_i \gamma_\mu V_{ij} (1 - \gamma_5) d_j W^\mu + H.c.$$  \hspace{1cm} (1)

where $u_i = u, c, t$ and $d_j = d, s, b$ quarks. The CKM matrix obeys the unitarity constraint $(VV^\dagger)_{ij} = \delta_{ij}$ and can be parameterized in terms of three mixing angles and one CP violating phase. For the case $i \neq j$ the unitarity constraint can be displayed as a unitarity triangle, and there are six such unitarity triangles. Thus the unitarity of the CKM matrix for the first and the third column gives

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0.$$  \hspace{1cm} (2)

One can display this constraint as a unitarity triangle by defining the angles $\alpha, \beta, \gamma$ so that

$$\alpha = \arg(-V_{ud} V_{ub}^* / V_{cd} V_{cb}^*), \quad \beta = \arg(-V_{cd} V_{cb}^* / V_{td} V_{tb}^*),$$

$$\gamma = \arg(-V_{td} V_{tb}^* / V_{ud} V_{ub}^*)$$  \hspace{1cm} (3)

which satisfy the constraint $\alpha + \beta + \gamma = \pi$. One can parameterize CP violation in a way which is independent of the phase conventions. This is the so called Jarlskog invariant (Jarlskog, 1985) $J$ which can be defined in nine different ways, and one of which is given by

$$J = \text{Im}(V_{us} V_{ub}^* V_{cb} V_{cs}^*).$$  \hspace{1cm} (4)

An interesting observation is that the CKM is hierarchical and allows for expansion in $\lambda \simeq 0.226$ so one may write $V$ as a perturbative expansion in $\lambda$ which up to $O(\lambda^3)$ is given by

$$\left( \begin{array}{ccc} 1 - \frac{\lambda^2}{2} & \lambda A \lambda^3 (\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A \lambda^2 \\
A \lambda^3 (1 - \rho - i\eta) & -A \lambda^2 & 1 \end{array} \right)$$  \hspace{1cm} (5)

In this representation the Jarlskog invariant is given by $J \simeq A^2 \lambda^6 \eta$, and the CP violation enters via $\eta$.

The Standard Model has another source of CP violation in addition to the one that appears in the CKM matrix. This source of CP violation arises in the strong interaction sector of the theory from the term $\theta^L \overline{G} \overline{G}$, which is of topological origin. It gives a large contribution to the EDM of the neutron and consistency with current experiment requires $\theta = \theta + \text{ArgDet}(M_D M_L)$ to be small $\theta < O(10^{-10})$. One solution to the strong CP problem is the vanishing of the up quark mass. However, analyses based on chiral perturbation theory and on lattice gauge theory appear to indicate a non-vanishing mass for the up quark. Thus a resolution to the strong CP problem appears to require beyond the Standard Model physics. For example, one proposed solution is the Peccei-Quinn mechanism (Peccei and Quinn, 1977) and its refinements (Dine et al., 1981; Kim, 1979; Zhitnitskii, 1980) which leads to axions. But currently severe limits exist on the corridor in which axions can exist. There is much work in the literature regarding how one may suppress the strong CP violation effects (for a review see (Dine, 2000)). In addition to the use of axions or a massless up quark one also has the possibility of using a symmetry to suppress the strong CP effects (Barr, 1984; Nelson, 1984).

The solution to the strong CP in the framework of Left-Right symmetric models is discussed in (Babu et al., 2002; Mohapatra et al., 1997). Specifically in the analysis of (Babu et al., 2002) the strong CP parameter $\theta$ is zero at the tree level, due to parity (P), but is induced due to P-violating effects below the unification scale. In the analysis of (Hiller and Schmaltz, 2001) a solution to the strong CP problem using supersymmetry is proposed. Here one envisions a solution to the strong CP problem based on supersymmetric non-renormalization theorem. In this scenario CP is broken spontaneously and its breaking is communicated to the MSSM by radiative corrections. The strong CP phase is protected by a SUSY non-renormalization theorem and remains exactly zero while the loops can generate a large CKM phase from wave function renormalization. Another idea advocates promoting the $U(1)$ CP violating phases of the supersymmetric standard model to dynamical variables, and then allowing the vacuum to relax near a CP conserving point (Dimopoulos and Thomas, 1996). In the analysis of (Demir and Ma, 2000) an axionic solution of the strong CP problem with a Peccei-Quinn mechanism using the gluino rather than the quarks is given and the spontaneous breaking of the new $U(1)$ global symmetry is connected to the supersymmetry breaking with a solution to the $\mu$ problem (Demir and Ma, 2000). Finally, in the analysis of (Aldazabal et al., 2004) a solution based on gauging away the strong CP problem is proposed. Thus the work of (Aldazabal et al., 2004) proposes a solution that involves the existence of an unbroken gauged $U(1)_X$ symmetry whose gauge boson gets a Stueckelberg mass term by combining with a pseudoscalar field $\eta(x)$ which has an axion like coupling to $G \overline{G}$. Thus the $\theta$ parameter can be gauged away by a $U(1)_X$ transformation. The additional $U(1)_X$ generates mixed gauge anomalies which are canceled by the addition of an appropriate Wess-Zumino term. We will assume from here on that the strong CP problem is solved by one or the other of the techniques outlined above.
III. REVIEW OF EXPERIMENTAL EVIDENCE ON CP VIOLATION AND SEARCHES FOR OTHER EVIDENCE

There are currently four pieces of experimental evidence for CP violation. These consist of (i) the observation of indirect CP violation (\(\epsilon\)), and (ii) of direct CP violation (\(\epsilon'/\epsilon\)) in the Kaon system, (iii) the observation of CP violation in B physics, and (iv) an indirect evidence for CP violation due to the existence of baryon asymmetry in the universe. Thus far the experimental evidence indicates that the CP violation in the K and B physics can be understood within the framework of the standard model. However, an understanding of baryon asymmetry in the universe requires a new source of CP violation. We briefly review these below.

A. CP violations in the Kaon system

Historically the first indication for CP violation came from the observation of the decay \(K_L \to \pi^+\pi^-\). In order to understand this phenomenon we begin with the states \(K^0\) (with strangeness \(S = +1\)) and \(\bar{K}^0\) (with strangeness \(S = -1\)). From the above one can construct CP even and CP odd eigenstates,

\[
K_{1,2} = \frac{1}{\sqrt{2}}(K^0 \pm \bar{K}^0).
\]

One can arrange \(\bar{K}^0\) to be the CP conjugate of \(K^0\), i.e., \(CP|K^0| = |\bar{K}^0\rangle\), and in that case \(K_1\) is the CP even and \(K_2\) is the CP odd state. The decay of neutral K’s come in two varieties: \(K_0(K_L)\) with lifetimes \(\tau_S = 0.89 \times 10^{10}s\) and \(\tau_I = 5.2 \times 10^{-8}\) with dominant decays \(K_S \to \pi^+\pi^-, \pi^0\pi^0(K_L \to 3\pi, \pi\nu)\). If these were the only decays one would identify \(K_S\) with \(K_1\) and \(K_L\) with \(K_2\). However, the decay of the \(K_L \to \pi^+\pi^-\) provided the first experimental evidence for the existence of CP violation (Christenson et al., 1964). This experiment indicates that the \(K_S(K_L)\) mixtures of CP even and CP odd states and one may write

\[
K_S = \frac{K_1 + \bar{K}_2}{(1 + |\epsilon|^2)^{1/2}}, \quad K_L = \frac{K_2 + \bar{K}_1}{(1 + |\epsilon|^2)^{1/2}}.
\]

Experimentally one attempts to measure two independent CP violating parameters \(\epsilon\) and \(\epsilon'\) which are defined by

\[
\epsilon = \frac{\langle (\pi\pi)I=0|\mathcal{L}_W|K_L\rangle}{\langle (\pi\pi)I=0|\mathcal{L}_W|K_S\rangle},
\]

where \(\mathcal{L}_W\) is the Lagrangian for the weak \(\Delta S = 1\) interactions, and

\[
\epsilon' = \frac{\langle (\pi\pi)I=2|\mathcal{L}_W|K_L\rangle}{\langle (\pi\pi)I=0|\mathcal{L}_W|K_L\rangle} - \frac{\langle (\pi\pi)I=2|\mathcal{L}_W|K_S\rangle}{\langle (\pi\pi)I=0|\mathcal{L}_W|K_S\rangle}.
\]

The parameter \(\epsilon'\) is often referred to as a measure of direct CP violation while \(\epsilon\) is referred to as a measure of indirect CP violation in the Kaon system. An accurate determination of \(\epsilon\) has existed for many years so that

\[
|\epsilon| = (2.266 \pm 0.017) \times 10^{-3}.
\]

The determination of direct CP violation is more recent and here one has (Alavi-Harati et al., 1999; Burkhardt et al., 1988; Fanti et al., 1999)

\[
\epsilon'/\epsilon = (1.72 \pm 0.018) \times 10^{-3}.
\]

The above result rules out the so called superweak theory of CP violation (Wolfenstein, 1964) but is consistent with the predictions of the Standard Model. A detailed discussion of direct CP violation can be found in (Bertolini et al., 2000).

There are other Kaon processes where CP violation effects can, in principle, be discerned. The most prominent among these is the decay \(K_L \to \pi^0\pi^0\). This process is fairly clean in that it provides a direct determination of the quantity \(\epsilon_{\pi\pi}\). The Standard Model prediction for the branching ratio is (Buras et al., 2004)

\[
BR(K_L \to \pi^0\pi^0) = (3.0 \pm 0.6) \times 10^{-11}
\]

while the current experimental limit is (Anisimovsky et al., 2004)

\[
BR(K_L \to \pi^0\pi^0) < 1.7 \times 10^{-9}.
\]

This an improvement in experiment by a factor of around \(10^2\) is needed to test the Standard Model prediction. On the other hand significantly larger contribution to this branching ratio can arise in beyond the Standard Model physics (Buras et al., 2005, 2004; Colangelo and Isidori, 1998; Grossman and Nir, 1997). A new experiment, 391a, is underway at KEK which would have a significantly improved sensitivity for the measurement of this branching ratio and its results could provide a window to testing new physics in this channel.

We turn now to B physics. There is considerable literature in this area to which the reader is directed for details ((Bigi and Sandra, 1981, 1984; Carter and Sandra, 1980; Dunietz and Rosner, 1986). For reviews see (Barberio, 1998; Harrison and Quinn, 1996; Hiltlin and Stone, 1991; Nakada, 1994; Nardulli, 1993; Peruzzi, 2004; Quinn, 1998; Sanda, 2004; Stone, 2006)). CP violations can occur in charged B or neutral B decays such as \(B_d = \bar{b}d\) and \(B_s = bs\). In the \(B^0\) system the mass eigenstates can be labeled as \(B_H\) and \(B_L\) with

\[
|B_L\rangle = p|B^0\rangle + q|\bar{B^0}\rangle,
\]

\[
|B_H\rangle = p|B^0\rangle - q|\bar{B^0}\rangle
\]

where \(p(q)\) may be parameterized by

\[
p = \frac{1 + \epsilon_B}{\sqrt{2(1 + |\epsilon_B|^2)}},
\]

\[
q = \frac{1 - \epsilon_B}{\sqrt{2(1 + |\epsilon_B|^2)}}.
\]

A quantity of interest is the mass difference between these states, i.e., \(\Delta m_s = m_{B_H} - m_{B_L}\). Next let us consider a
state f which is accessible to both $B^0$ and $\bar{B}^0$. A quantity sensitive to CP violation is the asymmetry which is defined by

$$a_f(t) = \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)}$$  \hspace{1cm} (14)$$

where $B^0(t)$ ($\bar{B}^0(t)$) denote the states which were initially $B^0(\bar{B}^0)$. The analysis of the asymmetry becomes specially simple if the final state is an eigen state of CP. $A_f(t)$ may be written in the form

$$A_f(t) = A_f^0 \cos(\Delta m t) + A_f^1 \sin(\Delta m t)$$  \hspace{1cm} (15)$$

where

$$A_f^0 = 1 - |\lambda|^2, \ \ A_f^1 = \frac{-2Im|\lambda|}{1 + |\lambda|^2}.$$  \hspace{1cm} (16)$$

Here $\lambda \equiv q\bar{A}_f/pA_f$, where $A_f = \langle f|H|B^0 \rangle$, and $\bar{A}_f = \langle f|H|\bar{B}^0 \rangle$. An interesting aspect of $a_f$ is that it is free of hadronic uncertainties and for the Standard Model case it is determined fully in terms of the CKM parameters. This would be the case if only one amplitude contributes to the decay $B^0(\bar{B}^0) \to f$. More generally one has more than one diagram contributing with different CKM phase dependence which make the extraction of CKM phases less transparent. Specifically $B^0(\bar{B}^0)$ decays may in general involve penguin diagrams which tend to contaminate the simple analysis outlined above. Gronau and London have proposed an isospin analysis which can disentangle the effect of the tree and penguin contributions when the final states in $B^0(\bar{B}^0)$ are $\pi^+\pi^-$ and $\pi^0\pi^0$ which is useful in the analysis of all the CKM angles (Gronau and London, 1990; Gronau and London, 1991). The decay final states $J/\Psi K_S$ is interesting in that it is a CP eigen state and it has a large branching ratio and to leading order is dominated by a single CKM phase. Specifically, the relation $A_{J/\Psi K_S}/\bar{A}_{J/\Psi K_S} = 1$ holds to within a percent (Boos et al., 2004), $A_{J/\Psi K_S} = \sin(2\beta)$ and $\bar{A}_{J/\Psi K_S} = 0$. Thus $B^0(\bar{B}^0)$ decay into this mode gives a rather clean measurement of $\sin(2\beta)$. BaBar and Belle have both measured CP asymmetries utilizing the charm decays. Using the decays $B^0(\bar{B}^0) \to J/\Psi K_S$ and $B^0(\bar{B}^0) \to J/\Psi K_L$ BaBar and Belle have obtained a determination of the CP asymmetry $\sin(2\beta)$ and the world average for this is (Barberio et al., 2006)

$$\sin(2\beta) = 0.685 \pm 0.032$$  \hspace{1cm} (17)$$

While the analysis of CP asymmetries in the $J/\Psi K_S$ system is the cleanest way to determine $\sin(2\beta)$ there are additional constraints on $\beta$ that are indirect such as from $\Delta m_d$ and $\Delta m_s$. These lead to a constraint on $\beta$ with $\beta$ lying in the range (130, 310) at 95% C.L. (Charles et al., 2005; Long, 2005).

The determination of $\alpha$ comes from the measurement of processes of type $B^0 \to \pi^+\pi^-$, $\rho^+\rho^-$ since the combinations of phases that enter here are via $\sin(2(\beta + \gamma)) = -\sin(2\alpha)$. One problem arises due to the contribution of the penguin diagram Fig.(1) which does not contain any weak phase. The penguin diagram can thus contaminate the otherwise neat weak phase dependence of this process. A possible cure come from the fact that one can use the analysis of (Grossman and Quinn, 1998) to put an upper limit on the branching ratio for $B^0 \to \rho^0\rho^0$. The current determination of $\alpha$ gives $\alpha = (96 \pm 13 \pm 11)_{\gamma}$ (Stone, 2006). The determination of $\gamma$ comes from the charged decays $B^\pm \to D^0K^\pm$. The current experimental values from BaBar and Belle are $\gamma = (67 \pm 28 \pm 13)_{\gamma}$, and $\gamma = (67^{+14}_{-13} \pm 13)_{\gamma}$ (Asner and Sun, 2006; Stone, 2006). A detailed analysis of global fits to the CKM matrix can be found in (Charles, 2006; Charles et al., 2005).

We discuss now $D^0-\bar{D}^0$ system. In analogy with the neutral B system we introduce the two neutral mass eigen states $D_1, D_2$ defined by

$$|D_1\rangle = p|D^0\rangle + q|\bar{D}^0\rangle, \quad |D_2\rangle = p|D^0\rangle - q|\bar{D}^0\rangle.$$  \hspace{1cm} (18)$$

The D mesons are produced as flavor eigen states but they evolve as admixtures of the mass eigen states which govern their decays. The analysis of $D^0$ and $\bar{D}^0$ decays by BaBar(Aubert et al., 2007) and by Belle(Staric et al., 2007) finds no evidence of CP violation. For further details the reader is directed to (Nir, 2007b).

The fourth piece of experimental evidence for CP violation in nature is indirect. It arises from the existence of a baryon asymmetry in the universe which is generally expressed by the ratio

$$n_B/n_\gamma = (6.1^{+0.3}_{-0.2}) \times 10^{-10}$$  \hspace{1cm} (19)$$

An attractive picture for the understanding of the baryon asymmetry is that the asymmetry was generated in the very early history of the universe within the context of an inflationary universe starting with no
initial baryon asymmetry (for a recent review on matter-antimatter asymmetry see (Dine and Kusenko, 2004)). The basic mechanism how this can come about was already enunciated a long time ago by (Sakharov, 1967). According to Sakharov there are three basic ingredients that govern the generation of baryon asymmetry. (i) One needs a source of baryon number violating interactions if one starts out with a universe which initially has no net baryon number. Such interactions arise quite naturally in grand unified models and in string models. (ii) One needs CP violating interactions since otherwise would be a balance between processes producing particles vs processes producing anti-particle leading to a vanishing net baryon asymmetry. (iii) Finally, even with baryon number and CP violating interactions the production of a net baryon asymmetry would require a departure from thermal equilibrium. Thus one finds that one of the essential ingredients for the generation of the baryon asymmetry in the early universe is the existence of CP violation. However, the CP violation in the Standard Model is not sufficient to generate the desire amount of baryon asymmetry and one needs a source of CP violation above and beyond what is present in the Standard Model. Such sources of CP violation are abundant in supersymmetric theories.

In addition to the baryon asymmetry in the universe there are other avenues which may reveal the existence of new sources of CP violation beyond what exists in the Standard Model. The EDMs of elementary particles and of atoms are prime candidates for these. The largest values of EDMs in the framework of the Standard Model SM are very small. SM predicts for the EDM of the electron and for the neutron is given in (Sakharov, 1967). For the case of the neutron the value that ranges from $10^{-31}$ to $10^{-33}$ ecm (Bernreuther and Suzuki, 1991; Bigi and Uraltsev, 1991; Booth, 1993; Gavela et al., 1982; Khraplovich and Zhitnitsky, 1982; Shabalin, 1983).

So far no electric dipole moment for the electron or for the neutron has been detected, and thus strong bounds on these quantities exist. For the electron the current experimental limit is (Regan et al., 2002),

$$|d_e| < 1.6 \times 10^{-27} \text{ ecm} \quad (90\% \ CL). \quad (20)$$

For the neutron the Standard Model gives $d_n \sim 10^{-32 \pm 1}$ ecm while the current experimental limit is (Baker et al., 2006)

$$|d_n| < 2.9 \times 10^{-26} \text{ ecm} \quad (90\% \ CL). \quad (21)$$

In each case one finds that the Standard Model prediction for the EDM is several orders of magnitude smaller than the current experimental limit and thus far beyond the reach of experiment even with improvement in sensitivity by one to two orders of magnitude. On the other hand many models of new physics beyond the Standard Model generate much larger EDMs and such models are already being constrained by the EDM experiment. Indeed improved sensitivities in future experiment may lead to a detection of such effects or put even more stringent constraints on the new physics models. The EDM of the atoms also provides a sensitive test of CP violation. An example is Hg-199 for which the current limits are (Romalis et al., 2001),

$$|d_{He_s}| < 2 \times 10^{-28} \text{ ecm}. \quad (22)$$

IV. CP VIOLATION IN SOME NON-SUSY EXTENSIONS OF THE STANDARD MODEL

While the Standard Model contains just one CP phase more phases can appear in extensions of the Standard Model. In general the violations of CP can be either explicit or spontaneous. The CP violation is called explicit if redefinitions of fields cannot make all the couplings real in the interaction structure of the theory. The remaining phases provide an explicit source of CP violation. CP violation is called spontaneous if the model starts out with all the couplings being real but spontaneous breaking in the Higgs sector generates a non-removable phase in one of the vacuum expectation values in the Higgs fields at the minimum of the potential. Returning to CP violation in the extension of the Standard Model, such extensions could be based on an extended gauge group, on an extended Higgs sector, or on an extended fermionic content (see, for example, (Accomando et al., 2006)). An example of a model with an extended gauge sector is the left-right (LR) symmetric model based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ (Mohapatra and Pati, 1975). For $n_g$ number of generations the number of phases is given by $N_L + N_R$ where $N_L = (n_g - 1)(n_g - 2)/2$ is exactly what one has in $SU(2)_L \times U(1)_Y$ model and $N_R = n_g(n_g + 1)/2$ are additional set of phases that arise in the LR model. For the case of three generations this leads to 7 CP phases instead of just one CP phase that one has in the Standard Model. An analysis of EDM in LR models for the electron and for the neutron is given in (Frank, 1999a,b).

The simplest extension of the Standard Model with an extended Higgs sector is the so called two Higgs doublet model (Lee, 1973, 1974) (2HDM) which contains two $SU(2)$ doublets which have exactly the same quantum numbers $\Phi_i = (\phi_i^+, \phi_i^0)$, $i=1,2$. One problem with the model is that it leads to flavor changing neutral currents (FCNC) at the tree level if one allows couplings of both $\Phi_i$ to the up and down quarks. The FCNC can be suppressed by imposing a discrete $Z_2$ symmetry (Glashow and Weinberg, 1977) such that under $Z_2$ one has $\Phi_2 \rightarrow -\Phi_2$ and $u_R \rightarrow -u_R$ and the remaining fields are unaffected. Under the above symmetry the most general renormalizable scalar potential one can
write is
\[ V_0 = -\mu_1^2 \Phi_1^\dagger \Phi_1 - \mu_2^2 \Phi_2^\dagger \Phi_2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)^2 + (\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + H.c.) \] (23)

However, with an exact \( Z_2 \) discrete symmetry CP cannot be broken either explicitly or spontaneously in a 2HDM model (Branco, 1980a,b; Mendez and Pomarol, 1991). Thus to have CP in the 2HDM model one must allow for violations of the discrete symmetry, but arrange for suppression of FCNC. If the couplings allow for FCNC at the tree level, then they must be suppressed either by heavy Higgs masses (Branco et al., 1985; Lahanas and Vayonakis, 1979) or by adjustment of couplings or fine tunings so that FCNC are suppressed but CP violation is allowed (Liu and Wolfenstein, 1987).

However, the hard breaking of the \( Z_2 \) discrete symmetry is generally considered not acceptable. A more desirable possibility is violation of the discrete symmetry only via soft terms (Branco and Rebelo, 1985). Here the FCNC are not allowed at the tree level but the inclusion of the soft terms allows for CP violation. Such a term is of the form
\[ V_{soft} = -\mu_n^2 \Phi_n^\dagger \Phi_n + H.c. \] (24)

Soft breaking of the \( Z_2 \) symmetry can allow both explicit and spontaneous CP violation. Thus explicit CP violation can occur in \( V = V_0 + V_{soft} \) if one has (Grzadkowski et al., 1999) \( Im(\mu_n^4 \lambda_3) \neq 0 \). For the case when \( Im(\mu_3^4 \lambda_3) = 0 \) a spontaneous violation of CP can arise. Specifically, in this case one can choose phases so that \( \Phi_1^\dagger = v_1/\sqrt{2} \) (\( v_1 > 0 \)) and \( \Phi_2^\dagger = e^{i\phi} v_2/\sqrt{2} \) (\( v_2 > 0 \)) with the normalization
\[ \sqrt{v_1^2 + v_2^2} = 2m_W/g_2 = 246 \text{GeV}. \] (25)

The conditions for CP violation in a 2HDM model, both explicit and spontaneous, have more recently been studied using basis independent potentially complex invariants which are combinations of mass and coupling parameters. These invariants also are helpful in distinguishing between explicit and spontaneous CP violation in the Higgs sector. For further discussion, the reader is referred to the works of (Botella and Silva, 1995; Branco et al., 2005; Davidson and Haber, 2005; Ginzburg and Krawczyk, 2005; Gunion and Haber, 2005; Lavoura and Silva, 1994). While the spontaneous breaking of CP discussed above involves \( SU(2) \) Higgs doublets which may enter in the spontaneous breaking of the electro-weak symmetry, similar spontaneous violations of CP can occur in sectors not related to electro-weak symmetry breaking.

In the absence of CP violation, the Higgs sector of the theory after spontaneous breaking of the \( SU(2)_L \times U(1)_Y \) symmetry gives two CP even, and one CP odd Higgs in the neutral sector. In the presence of CP violation, either explicit or spontaneous, the CP eigenstates mix and the mass eigenstates are admixtures of CP even and CP odd states. The above leads to interesting phenomenology which is discussed in detail in (Grzadkowski et al., 1999; Mendez and Pomarol, 1991). The number of independent CP phases increases very rapidly with increasing number of Higgs doublets. Thus, suppose we consider an \( n_D \) number of Higgs doublets. In this case the number of independent CP phases that can appear in the unconstrained Higgs potential is \( (\lambda_5^3 \lambda_3) \neq 0. \) For the case when \( (\lambda_5^3 \lambda_3) = 0 \) a spontaneous violation of CP can occur. Such an analysis of the EDMs in the two Higgs model is given in (Barger et al., 1997; Hayashi et al., 1994). Finally, one may consider extending the fermionic sector of theory with inclusion of additional generations. Such an extension brings in more possible sources of CP violation. Thus, for example, with four generation of quarks the extended CKM matrix will be \( 4 \times 4 \). Such a matrix can be parameterized in terms of six angles and three phases (Barger et al., 1981; Oakes, 1982). Thus generically extensions of the Standard Model will in general have more sources of CP violation than the Standard Model. We discuss CP violation in supersymmetric theories next. While the spontaneous breaking of CP discussed above involves \( SU(2) \) Higgs doublets which may enter in the spontaneous breaking of the electro-weak symmetry, similar spontaneous violations of CP can occur in sectors not related to electro-weak symmetry breaking.

V. CP VIOLATION IN SUPERSYMMETRIC THEORIES

Supersymmetric models are one of the leading candidates for new physics (for review see (Haber and Kane, 1985; Martin, 1997; Nath et al.; Nilles, 1984)) since they allow for a technically natural solution to the gauge hierarchy problem. However, supersymmetry is not an exact symmetry of nature. Thus one must allow for breaking of supersymmetry in a way that does not violate the ultraviolet behavior of the theory and destabilize the hierarchy. This can be accomplished by the introduction of soft breaking. However, the soft breaking sector in the minimal supersymmetric standard model (MSSM) allows for a large number of arbitrary parameters (Dimopoulos and Georgi, 1981; Girardello and Grisaru, 1982). Indeed in softly broken supersymmetry with the particle content of MSSM additionally 21 masses, 36 mixing angles and 40 phases (Dimopoulos and Sutter, 1995), which makes the model rather unpredictable.

The number of parameters is significantly reduced in the minimal supergravity unified models under the
assumptions of a flat Kähler metric as explained below. The minimal supergravity model and supergravity model in general are constructed using techniques of applied N=1 supergravity, where one couples chiral matter multiplets and a vector multiplet belonging to the adjoint representation of a gauge group to each other and to supergravity. The supergravity couplings can then be described in terms of three arbitrary functions: the superpotential $W(z_i)$ which is a holomorphic function of the chiral fields $z_i$, the Kähler potential $K(z_i, z_i^†)$ and the gauge kinetic energy function $f_{\alpha\beta}(z_i, z_i^†)$ which transforms like the symmetric product of two adjoint representations. In supergravity models supersymmetry is broken in a so called hidden sector and is communicated to the physical sector where quarks and leptons live via gravitational interactions. The size of the soft breaking mass, typically the gravitino mass $m_3/2$, is $\sim \kappa^2 | W_h |$, where $W_h$ is the superpotential in the hidden sector where supersymmetry breaks and $\kappa = 1/M_{Pl}$, where $M_{Pl}$ is the Planck mass. The simplest model where supersymmetry breaks in the hidden sector via a super Higgs effect is given by $W_h = m_2 z$ where $z$ is the Standard Model singlet super Higgs field. The breaking of supersymmetry by supergravity interactions in the hidden sector gives $z$ a VEV of size $\sim \kappa^{-1}$, and thus with $m \sim 10^{10-11}$ GeV, the soft breaking mass is of size $\sim 10^3$ GeV.

In the minimal supergravity model one assumes that the Kähler potential has no generational dependence and is flat and further that the gauge kinetic energy function is diagonal and has no field dependence, i.e., one has effectively $f_{\alpha\beta} \sim \delta_{\alpha\beta}$. In this case one finds that the low energy theory obtained after integrating the GUT scale masses has the following soft breaking potential (Chamseddine et al., 1982; Hall et al., 1983; Nath et al., 1983)

$$V_{SB} = m_2 \tilde{\lambda}^\alpha \lambda^\alpha + m_0^2 z_0 z_0^† + (A_0 W^{(3)} + B_0 W^{(2)} + H.c.)$$ (26)

where $W^{(2)}$ is the quadratic and $W^{(3)}$ is cubic in the fields.

The physical sector of supergravity models consist of the MSSM fields, which include the three generations of quarks and leptons and their superpartners, and a pair of $SU(2)_L$ Higgs doublets $H_1$ and $H_2$ and their superpartners which are the corresponding Higgsino fields $\tilde{H}_1$ and $\tilde{H}_2$. For the case of MSSM one has

$$W^{(2)} = \mu_0 H_1 H_2,$$

$$W^{(3)} = \bar{Q}_Y U H_2 \tilde{u}^c + \bar{Q}_Y D H_1 \tilde{d}^c + \bar{L}_Y E H_2 \tilde{e}^c$$ (27)

Here $H_1$ is Higgs doublet that gives mass to the bottom quark and the lepton, and $H_2$ gives mass to the up quark. As is evident from Eqs(26) and (27) the minimal supergravity theory is characterized by the parameters $m_0, m_2, A_0, B_0$ and $\mu_0$. An interesting aspect of supergravity models is that they allow for spontaneous breaking of the $SU(2)_L \times U(1)_Y$ electroweak symmetry (Chamseddine et al., 1982). This can be accomplished in an efficient manner by radiative breaking using renormalization group effects (Alvarez-Gaume et al., 1983; Ellis et al., 1983; Ibanez and Lopez, 1984; Ibanez et al., 1985; Ibanez and Ross, 1982, 2007; Inoue et al., 1982).

To exhibit spontaneous breaking one considers the scalar potential of the Higgs fields by evolving the potential to low energies by renormalization group effects such that

$$V = V_0 + \Delta V$$ (28)

where $V_0$ is the tree level potential (Haber and Kane, 1985; Nath et al.; Nilles, 1984)

$$V_0 = m_1^2 | H_1 |^2 + m_2^2 | H_2 |^2 + (m_3^2 H_1 H_2 + H.c.) + \frac{g_2^2}{8} | H_1 |^4 + \frac{g_3^2}{8} + \frac{g_Y^2}{2} | H_1 |^2 - \frac{g_2^2}{8} | H_1 H_2 |^2$$ (29)

and $\Delta V$ is the one loop correction to the effective potential and is given by (Arnowitt and Nath, 1992; Carena et al., 2000; Coleman and Weinberg, 1973; Weinberg, 1973)

$$\Delta V = \frac{1}{64 \pi^2} Str(M^4 H_1 H_2) (\log \frac{M^2 (H_1 H_2)}{Q^2} - \frac{3}{2})$$. (30)

Here $Str = \sum_i C_i (2 J_i + 1) (-1)^{2 J_i}$, where the sum runs over all particles with spin $J_i$ and $C_i (2 J_i + 1)$ counts the degrees of freedom of the particle $i$ and $Q$ is the running scale which is to be in the electroweak region. The gauge coupling constants and the soft parameters are subject to the supergravity boundary conditions: $\alpha_2(0) = \alpha_G = \frac{1}{2} \alpha_Y (0); m_2^2(0) = m_3^2 + \mu_0^2, i = 1, 2$; and $m_3^2(0) = B_0 \mu_0$. As one evolves the potential downwards from the GUT scale using renormalization group equations (Jack et al., 1994; Machacek and Vaughn, 1983, 1984, 1985; Martin and Vaughn, 1994), a breaking of the electroweak symmetry occurs when the determinant of the Higgs mass matrix turns negative so that (i) $m_1^2 m_2^2 - 2 m_3^4 < 0$, and further for a stable minimum to exist one requires that the potential be bounded from below so that (ii) $m_1^2 + m_2^2 - 2 m_3^2 > 0$. Additionally one must impose the constraint that there be color and charge conservation. Defining $v_1 = < H_1 >$ as the VEV of the neutral component of the Higgs $H_i$, the necessary conditions for the minimization of the potential, i.e., $\partial V / \partial v_i = 0$, gives two constraints. One of these can be used to determine the magnitude $| \mu_0 |$ and the other can be used to replace $B_0$ by $\tan \beta z < H_2 > / < H_1 >$. In this case the low energy supergravity model or mSUGRA
can be parameterized by $m_0, m_\frac{1}{2}, A_0, \tan \beta$ and sign($\mu_0$). It should be noted that fixing the value $|\mu|$ using radiative breaking does entail fine tuning but a measure of this is model dependent (see, for example, (Chan:1997bi) and the references therein). The above discussion is for the case when there are no CP violating phases in the theory. In the presence of CP phases $m_\frac{1}{2}, A_0, \mu_0$ become complex and one may parameterize them so that

$$m_\frac{1}{2} = |m_\frac{1}{2}| e^{i \frac{\pi}{2}}, \quad A_0 = |A_0| e^{i \alpha_0}, \mu_0 = |\mu_0| e^{i \theta_\mu_0}. \quad (31)$$

Now not all the phases are independent. Indeed, in this case only two phase combinations are independent, and in the analysis of the EDMs one finds these to be $\xi_1 + \theta_\mu_0$ and $\alpha_0 + \theta_\mu_0$. Often one rotates away the phase of the gauginos which is equivalent to setting $\xi_2 = 0$, and thus one typical choice of parameters for the complex mSUGRA (cmSUGRA) case is

$$m_0, \; |m_\frac{1}{2}|, \tan \beta, \; |A_0|; \; \alpha_0, \; \theta_\mu_0 \quad (\text{cmSUGRA}). \quad (32)$$

However, other choices are equally valid: thus, for example, the independent soft breaking parameters can be chosen to be $m_0, \; |m_\frac{1}{2}|, \tan \beta, \; |A_0|, \; \alpha_0, \; \xi_3$. mSUGRA model was derived using a super Higgs effect which breaks supersymmetry in the hidden sector by VEV formation of a scalar super Higgs field. Alternately one can view breaking of supersymmetry as arising from gaugino condensation where in analogy with QCD where one forms the condensate $\bar{q} q$ one has that the strong dynamics of an asymptotically free gauge theory in the hidden sector produces a gaugino condensate with $< \lambda^0 \lambda > = \Lambda^3$. The above can lead typically to supersymmetry breaking and a gaugino mass of size $m_\frac{1}{2} \sim k^2 \Lambda^3$. With $|\Lambda| > \sim (10^{12-13})$ GeV one will have an $m_\frac{1}{2}$ again in the electroweak region (Dine et al., 1985; Ferrara et al., 1983; Nilles, 1982; Taylor, 1990).

The assumption of a flat Kähler potential and of a flat kinetic energy function in supergravity unified models is essentially a simplification, and in general the nature of the physics at the Planck scale can be quite different. For this reason one must also consider more general Kähler potentials (Kaplunovsky and Louis, 1993; Soni and Weldon, 1983) and also allow for the non-universality of the gauge kinetic energy function. In this case the number of soft parameters grows, as also do the number of CP phases. Thus, for example, the gaugino masses will be complex and non-universal, and the trilinear parameter $A_0$, which is in general a matrix in the generation space, will also be in general non-diagonal and complex. A simplicity assumption to maintain the appropriate constraints on flavor changing neutral currents is to assume a diagonal form for $A_0$ at the GUT scale. Additionally, the Higgs masses for $H_1$ and $H_2$ at the GUT scale could also be non-universal. Thus in general for the non-universal supergravity unification a canonical set of soft parameters at the GUT scale will consist of (Matalliotakis and Nilles, 1995; Nath and Arnowitt, 1997; Olechowski and Pokorski, 1995; Polonsky and Pomarol, 1995)

$$m_{H_i} = m_0 (1 + \delta_i), \; i = 1, 2$$

$$m_\alpha = |m_\alpha| e^{i \xi_\alpha}, \; \alpha = 1, 2, 3$$

$$A_\alpha = |A_\alpha| e^{i \theta_\alpha}, \; \alpha = 1, 2, 3 \quad (33)$$

which contain several additional CP phases beyond the two phases in complex mSUGRA. However, not all the phases are independent, as some phases can be eliminated by field redefinitions. Indeed in physical computations only a certain set of phases appear, as discussed in detail in (Ibrahim and Nath, 2000c) (also see Appendix XVI.E). It should be kept in mind that for the case of non-universalities the renormalization group evolution gives an additional correction term at low energies (Martin and Vaughn, 1994).

As is apparent from the preceding discussion radiative breaking of the electroweak symmetry plays a central role in the supergravity unified models. An interesting phenomena here is the existence of two branches of radiative breaking: one is the conventional branch known since the early eighties (we call this the ellipsoidal branch (EB)) and the other was more recently discovered, i.e., it is the so called hyperbolic branch (HB). The two branches can be understood simply by examining the condition of radiative breaking which is a constraint on the soft parameters $m_0, m_{1/2}, A_0$ of the form (Chan et al., 1998)

$$C_1 m_0^2 + C_3 m_{1/2}^2 + C_2 A_0^2 + \Delta \mu_{\text{loop}}^2 = \frac{M_2^2}{2} + \mu^2. \quad (34)$$

Here $\Delta \mu_{\text{loop}}^2$ is the loop correction (Arnowitt and Nath, 1992; Carena et al., 2000), and $m_{1/2} = m_{1/2} + \frac{1}{2} A_0 C_4/C_3$, where $C_i$ are determined purely in terms of the gauge and the Yukawa couplings but depend on the renormalization group scale $Q$. The behavior of radiative breaking is controlled in a significant way by the loop correction $\Delta \mu_{\text{loop}}^2$ especially for moderate to large values of $\tan \beta$. For small values of $\tan \beta$ the loop correction $\Delta \mu^2$ is small around $Q \approx M_2$, and the $C_i$ are positive and thus Eq.(34) is an ellipsoidal constraint on the soft parameters. For a given value of $\mu$, Eq.(34) then puts an upper limit on the sparticle masses. However, for moderate to large values of $Q \approx M_2$, $\Delta \mu^2$ becomes sizable. Additionally $C_i$ develop a significant $Q$ dependence. It is then possible to choose a point $Q = Q_0$ where $\Delta \mu^2$ vanishes and quite interestingly here one finds that one of the $C_i$ (specifically $C_1$) turns negative, drastically changing the nature of the symmetry breaking constraint Eq.(34) on the soft parameters. Thus in this case the soft parameters in Eq.(34) lie on the surface of a hyperboloid and thus for a fixed value of $\mu$ the soft parameters can get very large with $m_0$ getting as large as 10 TeV or larger. The direct observation of squarks and sleptons may be difficult on this branch, although charginos, neutralinos and even gluino may be accessible.
However, the HB does have other desirable features such as suppression of flavor changing neutral currents, and suppression of the SUSY EDM contributions. Further, HB still allows for satisfaction of relic density constraints with R parity conservation if the lightest neutralino is the lightest supersymmetric particle (LSP). We note in passing that the so called focus point region (Feng et al., 2000) is included in the hyperbolic branch (Baer et al., 2004; Chan et al., 1998; Lahanas et al., 2003).

There is a potential danger in supergravity theories in that the hierarchy could be destabilized by non-renormalizable couplings in supergravity models since they can lead to power law divergences. This issue has been investigated by several authors: at one loop by (Bagger and Poppitz, 1993; Gaillard, 1995) and at two loop by (Bagger et al., 1995). The analysis shows that at the one loop level the minimal supersymmetric standard model appears to be safe from divergences (Bagger and Poppitz, 1993). In addition to the breaking of supersymmetry by gravitational interactions, there are a variety of other scenarios for supersymmetry breaking. These include gauge mediated and anomaly mediated breaking for which reviews can be found in (Giudice and Rattazzi, 1999; Luty, 2005). Finally as is clear from the preceding discussion in supergravity models and in MSSM there is inclusion in the hyperbolic branch (Baer et al., 2004; Chan et al., 2003). HB still allows for satisfaction of relic density constraints further, as suppression of flavor changing neutral currents, and with R parity conservation if the lightest neutralino is the lightest supersymmetric particle (LSP). We note in passing that the so called focus point region (Feng et al., 2000) is included in the hyperbolic branch (Baer et al., 2004; Chan et al., 1998; Lahanas et al., 2003).

VI. CP VIOLATION IN EXTRA DIMENSION MODELS

Recently there has been significant activity in the physics of extra dimensions (Antoniadis, 1990; Antoniadis et al., 1998; Arkani-Hamed et al., 1998; Gogberashvili, 2002; Randall and Sundrum, 1999a,b). One might speculate on the possibility of generating CP violation in a natural way from models derived from extra dimensions (For an early work see (Thirring, 1972)). It turns out that it is indeed possible to do so (Branco et al., 2001; Burdman, 2004; Chaichian and Kobakhidze, 2001; Chang et al., 2001; Chang and Mohapatra, 2001; Dienes et al.; Grzadkowski and Wudka, 2004; Huang et al., 2002; Khlebnikov and Shaposhnikov, 1988; Sakamura, 1999). The idea is to utilize properties of the hidden compact dimensions in extra dimension models. Thus in extra dimension models after compactification the physical four dimensional space is a slice of the higher dimensional space and such a slice can be placed in different locations in extra dimensions. In the discussion below we will label such a slice as a brane. We consider now a simple argument which illustrates how CP violation in extra dimension models can arise (Chang and Mohapatra, 2001). Thus consider a $U(1)$ gauge theory with left-handed fermions $\Psi_i$ ($i=1-4$), where $i = 1, 2$ have charges +1 and $i = 3, 4$ have charges -1, and also consider a real scalar field $\Phi$ which is neutral. We assume that the fermion fields are in the bulk and the scalar field is confined to the $y = 0$ brane. The fields $\Psi_1, \Psi_2$ and $\Phi$ are assumed to be even and $\Psi_{3L}, \Psi_{4L}$ are assumed to be odd under $y \rightarrow -y$ transformation. Further, under CP symmetry define the fields to transform so that $\Psi_{1L} \rightarrow (\Psi_{3L})^c, \Psi_{2L} \rightarrow (\Psi_{4L})^c$, and $\Phi \rightarrow -\Phi$ where $(\Psi_{L})^c$ has the meaning of a 4D charge conjugate of $\Psi$.

One constructs a 5D Lagrangian invariant under $y \rightarrow -y$ transformation of the form

\[
M_5^{-1}\lambda_5 \delta(y)\Phi[\Psi_{1L}^T C^{-1} \Psi_{2L} - (\Psi_{3L})^T C^{-1}(\Psi_{4L})^c] \\
+ \mu[\Psi_{1L}^T C^{-1} \Psi_{2L} - (\Psi_{3L})^T C^{-1}(\Psi_{4L})^c] + H.c. (35)
\]

On integration over the $y$ co-ordinate the interaction terms in 4D arise from the couplings on the $y=0$ brane and thus the zero modes of the fields odd in $y$ are absent, which means that the effective interaction at low energy in $(\lambda \Phi + \mu)\Psi_{1L}^T \Psi_{2L}^{(0)}$ which violates CP provided $Im(\lambda^* \mu) \neq 0$. Next we discuss a more detailed illustration of this CP violation arising from extra dimensions. This illustration is an explicit exhibition of how violations of CP invariance can occur in the compactification of a 5D QED (Grzadkowski and Wudka, 2004). Thus consider the Lagrangian in 5D of the form

\[
\mathcal{L}_5 = -\frac{1}{4}V_M^2 + \bar{\Psi}(i\gamma^M \gamma^M - m_i)\Psi + \mathcal{L}_{gb}. (36)
\]

Here $V_M$ is the vector potential in 5d space with coordinates $z^M$, where $M = 0, 1, 2, 3, 5$ so that $z^M = (x^n, y)$, where $\mu = 0, 1, 2, 3$, and where $D_M = \partial_M + ig_5 q V_M$ is the gauge covariant derivative, with $g_5$ the
\[ V_M(z) \rightarrow V_M(z) + \partial_M \lambda(z), \]
and additionally under the CP transformations in 5D
\[ z^n \rightarrow \eta^M z^M, \quad V_M \rightarrow \eta^M V_M, \quad \psi \rightarrow P \eta^0 \gamma^2 \psi^* \]
where \( \eta^{1,2,3} = -1 = -\eta^{0,5} \) and \( P = 1 \). We compactify the theory in the fifth dimension on a circle with radius \( R \) assuming periodic boundary conditions for the gauge fields but assuming the twisted boundary condition for the fermion field
\[ \psi(x, y + R) = e^{i\alpha} \psi(x, y). \]

We turn now to another mechanism for the generation of CP violation in extra dimensional theories. This scenario is that of split fermions where the hierarchies of fermion masses and couplings are proposed to arise from a fermion location mechanism under a kink background wherein the quark and leptons of different generations being confined to different points in a fat brane (Arkani-Hamed and Schmaltz, 2000; Kaplan and Tait, 2000, 2001; Mirabelli and Schmaltz, 2000). To illustrate the fat brane paradigm consider the 4+1 dimensional action of two fermions
\[ S_5 = \int d^4x dy [\bar{Q} i \gamma_M \partial^M + \Phi_Q(y)] Q + \]
\[ + \bar{U} i \gamma_M \partial^M + \Phi_U(y)] U + \kappa HQ^* U. \]
The quantities \( \Phi_Q, \Phi_U \) are potentials which confine the quarks at different points in the extra dimension. As a model one may consider these as Gaussian functions centered around points \( l_q \) (i.e., functions of the form \( \exp(-\mu^2(y - l_q)^2) \)) and \( l_u \) where \( 1/2\sqrt{R} \) is the width of the Gaussian. After expanding the fields in their normal modes and integrating over the extra dimension the Yukawa interaction in 4D including the generation index will take the form
\[ \mathcal{L}_Y = \lambda_{ij}^u Q_i U_j H + \lambda_{ij}^d Q_i D_j H^*, \]
where \( \lambda_{ij}^u \) is defined by
\[ \lambda_{ij}^u = \kappa_{ij} e^{-\frac{1}{2} \mu_1^2 (l_{1i} - l_{1j})}, \]
and \( \lambda_{ij}^d \) is similarly defined. The above structure indicates that the Yukawa textures are governed by the location of the quarks in the extra dimension. Detailed analyses, however, indicate that this scenario leads to an
insufficient amount of CP violation to explain the value of $\epsilon_K$ in Kaon decay. Thus the scenario above gives a value of the Jarlskog invariant $J \lesssim 5 \times 10^{-9}$ while one needs $J \sim 10^{-5}$ to get the proper value of $\epsilon_K$. The above shortcoming can be corrected by extending the analysis to two extra dimensions (Branco et al., 2001). In this case one finds the Jarlskog invariant $J \sim 2.2 \times 10^{-5}$ which is of desired strength to explain CP violation in the Kaon decay. An extension to include masses for the charged leptons and neutrinos has been carried out in (Barenboim et al., 2001).

An analysis using the fermion localization mechanism for generating quark-lepton textures within a supersymmetric SU(5) GUT theory is carried out in the analysis of (Kakizaki and Yamaguchi, 2004) where the different SU(5) chiral multiplets are localized along different points in the extra dimension. The analysis allows one to generate a realistic pattern of quark masses and mixings and lepton masses. The CP violation is of sufficient strength here since $J \sim O(10^{-5})$. An additional feature of this model is that dimension 5 proton decay operators are also naturally suppressed due to the fact that these operators contain an overlap of wavefunctions of different chiral multiplets and are thus exponentially suppressed.

Similar analyses can be carried out in the framework of a non-factorizable geometry (Abe et al., 2001; Chang et al., 2000; Grossman and Neubert, 2000; Huber and Shafi, 2001) based on the metric

$$ds^2 = e^{-2\sigma(y)}(dx^2 - dy^2),$$

where $\sigma(y) = k|y|$. Under the $Z_2$ orbifold symmetry the 5D fermion transform as $\Psi(-y) = \pm \gamma_5 \Psi(y)$. The $\Psi_{\pm}$ have the mode expansion

$$\Psi(x, y)_{\pm} = \frac{1}{\sqrt{2\pi r}} \sum_{n=0}^{\infty} \psi_{n, \pm}(x) f_{\pm}^{(n)}(y).$$

The zero modes of $\Psi_{\pm}$ are the left-handed and the right handed Weyl spinors. Masses for these are generated by the 5D Higgs couplings which are of the form

$$\int d^4 x dy \sqrt{-g} \lambda_{ij} H \bar{\Psi}_i \Psi_j,$$

For the zero mode they give rise to a Dirac mass term of the form (Huber and Shafi, 2001)

$$m_{ij} = (2\pi r_c)^{-1} \int_{-\pi r_c}^{\pi r_c} dy \lambda_{ij} H(y) f_{i+}^{(0)}(y) f_{j-}^{(0)}(y)$$

where

$$f^{(0)} = \left( \frac{2\pi r_c (\frac{1}{2} - c) - 1}{2\pi kr_c (\frac{1}{2} - c)} \right)^{-\frac{1}{2}} e^{(2-c)\sigma}$$

where $c$ is a parameter that characterizes the location of the fermion in the extra dimension. For $c < 1/2$ the fermion is localized near the $y = 0$ brane while for $r = \pi r_c$ it is localized near $y = \pi r_c$. With the appropriate choice of the $c$'s one may generate a realistic pattern of quark masses and mixings and a realistic CKM matrix. However, an explicit determination of the Jarlskog invariant appears not to have been carried out. The texture models using extra dimensions do generally require a high level of fine tuning in the selection of locations where the fermions are placed. Thus models of this type do not appear very natural. For related works on CP violation and extra dimensions see (Dooling et al., 2002; Huang et al., 2002; Ichinose, 2002; Sakamura, 1999).

VII. CP VIOLATION IN STRINGS

We discuss now the possible origins of CP violation in SUSY, string and brane models (for review of string theory see (Green et al., 1987a,b; Polchinski, 1998a,b)). One possible origin is string compactification (Bailin et al., 1998a,b, 2000; Dent, 2001, 2002; Faraggi and Vives, 2002; Kobayashi and Lim, 1995; Witten, 1985; Wu et al., 1991). One may call this hard CP violation since this type of CP violations can exist even without soft terms. Now Yukawa couplings which are formed via string compactification will carry this type of CP violation and the CKM phase $\delta_{CKM}$ which arises from the Yukawas is therefore a probe of CP violation arising from string compactification (assuming there is no CP violation arising from the Higgs sector). A second source of CP violation is via soft breaking. If SUSY contributions to K and B physics turn out to be small, then one has a plausible bifurcation, i.e., the CP violations in K and B physics are probe of string compactification, and baryogenesis and other CP phenomena that may be seen in sparticle decays etc become a probe of soft breaking.

Regarding soft breaking in string theory, such an analysis would entail specifying the Kähler potential, the superpotential, and the gauge kinetic energy function on the one hand and the mechanism of breaking on the other. Each of these are model dependent. However, it is possible to parameterize the breaking as in gravity mediated breaking in supergravity. Thus one can write the general form of the soft terms in the form

$$V_{soft} = m^2_{\alpha} C_{\alpha \dot{\alpha}} + A_{\alpha \dot{\beta}} Y_{\alpha \beta \gamma} C_{\alpha} C_{\beta} C_{\gamma}$$

$$+ \frac{1}{2} (B_{\alpha \beta \mu \alpha \beta} C_{\alpha} C_{\beta} + H.c.) + \cdots ,$$

where the general expressions for the scalar masses $m_{\alpha}$, trilinear couplings $A_{\alpha \beta \gamma}$ and the bilinear term $B$ can be given. For the case when $K_{\alpha \dot{\beta}} = \delta_{\alpha \dot{\beta}} K_{\alpha}$, one has (Brignole et al., 1994; Kaplunovsky and Louis, 1993)

$$m_{\alpha}^2 = m_{\beta}^2/2 + V_0 - F^I \bar{F}^J \partial_\alpha \partial_\beta \ln(K_{\alpha}) ,$$

$$A_{\alpha \dot{\beta} \gamma} = c F^I (\partial_\gamma K + \partial_{\dot{\gamma}} \ln(K_{\alpha \dot{\beta}}) - \partial_\gamma \ln(K_{\alpha \dot{\beta} \gamma})) ,$$

$$B_{\alpha \beta} = c F^I (\partial_\gamma K + \partial_{\dot{\gamma}} \ln(\mu_{\alpha \beta}) - \partial_\gamma \ln(K_{\alpha} K_{\beta} \gamma)) + \cdots .$$
while the gaugino masses are given by
\[ m_a = \frac{1}{2R(f_a)} F^I \partial_I f_a. \]  
(51)

An efficient way to parameterize \( F^I \) is given by (Brignole et al., 1994)
\[
\begin{align*}
F^S &= \sqrt{3} m_{1/2} (S + S^*) \sin \theta e^{-i \gamma_S}, \\
F^I &= \sqrt{3} m_{1/2} (T + T^*) \cos \theta \Theta_i e^{-i \gamma_I},
\end{align*}
\]
(52)
where \( \theta, \Theta_i \) parameterize the Goldstino direction in the \( S, T_i \) field space and \( \gamma_S \) and \( \gamma_I \) are the \( F^S \) and \( F^I \) phases, and \( \Theta_1^2 + \Theta_2^2 + \Theta_3^2 = 1 \).

\section{A. Complex Yukawa couplings in string compactifications}

The Yukawa couplings arise at the point of string compactification, and it is interesting to ask how the Yukawa couplings develop CP phases. It is also interesting to determine if such phases are small or large. Consider, for example, the compactification of the \( E_8 \times E_8 \) heterotic string on a six dimensional Calabi-Yau (CY) manifold. In this case the massless families are either (1,1) or (2,1) harmonic forms. For the case when hodge number \( h_{11} > h_{21} \), the massless mirror families are (1,1) forms while if \( h_{21} > h_{11} \) the massless families are (2,1) forms. For the case when the families are (1,1) the cubic couplings among the families have been discussed in (Strominger, 1985). The analysis for the case when \( h_{21} > h_{11} \) is more involved. One specific model of interest that can lead to complex Yukawa corresponds to compactification on the manifold \( \cal K_0 \) (Gepner, 1988; Schimmrigk, 1987)
\[
P^1 \equiv \sum_{i=0}^{3} z_i^3 + a_0 (z_1 z_2 z_3) = 0
\]
\[
P^2 \equiv \sum_{i=0}^{3} z_i x_i^3 = 0
\]
(53)
which is deformed from the manifold \( \cal K_0 \) (corresponding to the case \( a_0 = 0 \)) in the ambient space \( CP^3 \times CP^2 \) by a single (2,1) form \( (z_1 z_2 z_3) \). The \( \cal K_0 \) has 35 \( h_{21} \) forms and 8 \( h_{11} \) forms, giving an Euler characteristic \( \chi = 2 (h_{21} - h_{11}) \) and the number of net mass less families is \( |\chi|/2 \) (Sotkov and Stanishkov, 1988).

By modding out by two discrete groups \( Z_3 \) and \( Z_3' \) one gets a three generation model. The discrete symmetries are \( Z_3 \) and \( Z_3' \) where
\[
\begin{align*}
Z_3 : & \quad g : \quad (z_0, z_1, z_2, z_3 : x_1, x_2, x_3) \rightarrow \\
& \quad (z_0, z_2, z_3, z_1; x_2, x_3, x_1), \\
Z_3' : & \quad h : \quad (z_0, z_1, z_2, z_3 : x_1, x_2, x_3) \rightarrow \\
& \quad (z_0, z_1, z_2, z_3; x_1, \alpha x_2, \alpha^2 x_3).
\end{align*}
\]
(54)
where \( \alpha^3 = 1, \alpha \neq 1 \). The group \( Z_3' \) is not freely acting and leaves three tori invariant. These invariant tori have to be blown up in order to obtain a smooth CY manifold. Such a blowing up procedure produces six additional (2,1) and (1,1) forms which, however, leave the net number of generations unchanged. One considers now the flux breaking of \( E_6 \) on this manifold. If one embeds a single factor, \( Z_3 \) or \( Z_3' \) in the \( E_6 \), then \( E_6 \) can break to \( SU(3)^3 \) or \( SU(6) \times U(1) \) each of which leave the Standard Model gauge group unbroken. However, the case \( SU(6) \times U(1) \) cannot be easily broken further since an adjoint representation does not arise in the massless spectrum. Thus typically one considers the \( SU(3)^3 \) possibility.

In this case there are two possibilities : Case(A), where \( Z_3 \) is embedded trivially and \( Z_3' \) is embedded non-trivially, and case (B) where \( Z_3' \) is embedded trivially and \( Z_3 \) is embedded non-trivially. Now for case (A) one may choose \( U_g = (id)_C \times (id)_L \times (id)_R \), \( U_b = (id)_C \times \alpha (id)_L \times \alpha (id)_R \), where \( U_g \) is defined so that \( g \rightarrow U_g g \) is a homomorphism of \( Z_3 \) into \( E_6 \) \( \equiv U_g (Witten, 1985) \), and similarly for \( U_b \), where \( (id) \) stands for an identity matrix, and \( C_L, R \) stand for color, left and right -handed subgroups of \( SU(3)^3 \).

The analysis of Yukawa couplings in this case has been carried out and the couplings can be made all real. Thus in this case there is no CP violation arising in the Yukawa sector at the compactification scale.

We consider next case (B) where essentially one has an interchange in the definitions of \( U_g \) and \( U_f \) so that
\[
U_g = (id)_C \times \alpha (id)_L \times \alpha (id)_R, \\
U_b = (id)_C \times (id)_L \times (id)_R
\]
(55)
In this case the massless states that survive flux breaking of \( E_6 \) transform under \( Z_3 \) as follows
\[
Z_3 L = L, \ Z_3 Q = \alpha Q, \ Z_3 Q^c = \alpha^2 Q^c
\]
\[
Z_3 \bar{L} = \bar{L}, \ Z_3 \bar{Q} = \alpha^2 \bar{Q}, \ Z_3 \bar{Q}^c = \alpha \bar{Q}^c
\]
(56)
where the leptons transform as \( L(1, 3, 3) \), quarks as \( Q(3, 3, 1) \), and conjugate quarks as \( Q^c(3, 1, 3) \). The barred quantities represent the mirrors, so that \( \bar{L}(1, 3, 3) \), \( \bar{Q}(3, 3, 1) \), and \( \bar{Q}^c(3, 1, 3) \). In this model the number of generations and mirror generation are identical to that of the Tian-Yau model (Greene et al., 1986, 1987) so that there are 9 lepton generations and 6 mirror generations, 7 quark generations and 4 mirror quark generations, 7 conjugate quark generations and 4 mirror conjugate quark generations, providing us with three net families of quarks and leptons. The analysis of Yukawa couplings has been carried out on the manifold \( \cal K_0 \) by many author.

Our focus here is the \((27)^3 \) couplings which are unaffected by the instantons (Distler and Greene, 1988) and here one can use the techniques of (Candelas, 1988) to determine the couplings. An analysis for case (B) was carried out in (Wu et al., 1991). The Yukawa couplings determined in this fashion have unknown normalizations for the kinetic energy. However, symmetries can be used to obtain constraints on the normalizations. Including these normalization constraints into account it is found
that Yukawas depend on $\alpha$ in a non-trivial manner, and thus CP is violated in an intrinsic manner. Further, the CP phase entering in the coupling is large. The CP violation on the $K_0'$ manifold persists even when the modulus $a_0$ is real, so in this sense CP violation is intrinsic.

B. CP violation in orbifold models

Next we discuss the possibility of spontaneous CP violation in some heterotic string models. What we consider are field point limits of such models so we are essentially discussing supergravity models with the added constraint of modular invariance (T duality). The duality constraints have been utilized quite extensively in the analysis of gaugino condensation and SUSY breaking (Binetruy and Gaillard, 1991; Cvetic et al., 1991; Ferrara et al., 1990; Font et al., 1990; Gaillard and Nelson, 2007;Nilles and Olechowski, 1990) and have also been utilized recently in the analysis of spontaneous breaking of CP (Acharya et al., 1995; Bailin et al., 1997; Dent, 2001, 2002; Giedt, 2002).

The scalar potential in supergravity and string theory is given by (Chamseddine et al., 1982; Cremmer et al., 1982)

$$V = e^K [(K^{-1})^j_i D_i W D_j W^\dagger - 3W W^\dagger] + V_D, \quad (57)$$

where $K$ is the Kähler potential, $W$ is superpotential and $D_i W = W_i + K_i W$, with the subscripts denoting derivatives with respect to the corresponding fields. As noted above we now use the added constraint of $T$-duality symmetry. Specifically we assume that the scalar potential in the effective four dimensional theory depends on the dilaton field $S$ and on the (Kähler) moduli fields $T_i (i = 1,2,3)$, and it is invariant under the modular transformations (to keep matters simple, we do not include here the dependence on the so called complex structure $U$-moduli)

$$T_i \rightarrow T_i' = \frac{a_i T_i - ib_i}{ic_i T_i + d_i}, (a_i d_i - b_i c_i) = 1, \quad (58)$$

where $a_i, b_i, c_i, d_i \in Z$. Under the modular transformations, $K$ and $W$ undergo a Kähler transformation while the scalar potential $V$ is invariant. For the Kähler potential we assume essentially a no scale form (Lahanas and Nanopoulos, 1987)

$$K = D(z) - \sum_i \log(T_i + \bar{T}_i) + K_{IJ} Q_I Q_J + H_{IJ} Q_I Q_J,$$

where $D(z) = -\log(z)$, and for $z$ one may consider

$$z = (S + \bar{S} + \frac{1}{4\pi^2} \sum_i \delta_i^{GS} \log(T_i + \bar{T}_i)), \quad (59)$$

where $\delta_i^{GS}$ is the one loop correction to the Kähler potential from the Greene-Schwarz mechanism, and $Q$ are the matter fields consisting of the quarks, the leptons and the Higgs. For the superpotential in the visible sector one may consider

$$W_v = \tilde{\nu}_{I,J} Q_I Q_J + \lambda_{I,J,K} Q_I Q_J Q_K. \quad (60)$$

Under $T$-duality, $Q$’s transform as

$$Q_I \rightarrow Q_I \Pi_i(\sigma T_i + d_i s_i). \quad (61)$$

In general, $K_{IJ}, H_{IJ}, \mu_{IJ}$ and $\lambda_{I,J,K}$ are functions of the moduli. The constraints on $n_{Q_J}$ are such that $V$ is modular invariant. Analyses of soft SUSY breaking terms using modular invariance of the type above has been extensively discussed in the literature assuming moduli stabilization. In such analyses one generically finds that CP is indeed violated if one assumes that the moduli are in general complex.

However, minimization of the potential and stabilization of the dilaton VEV is a generic problem in such models and requires additional improvements. Often this is accomplished by non-perturbative corrections to the potential. Thus one might consider non-perturbative contributions to the superpotential so that

$$W_{np} = \Omega(\sigma) \eta(T)^{-6}. \quad (62)$$

Here $\eta(T)$ is the Dedekind function, and we have assumed a single overall modulus $T$, and $\sigma = S + 2\delta^{GS} \log \eta(T)$ and $\delta^{GS} = -(3/4\pi) \delta^{GS}$. Additionally one can assume non-perturbative corrections to the Kähler potential and treat $D(z)$ as a function to be determined by non-perturbative effects. The analysis shows that for a wide array of parameters minima typically occur at the self-dual points of the modular group, i.e., $T = 1$ and $T = e^{2\pi/6}$. However, for some choices of the parameters $T$ can take complex values away from the fixed point. Nonetheless CP phases arising from such points are very small since in the soft parameters they come multiplied by the function $G(T) = (T + \bar{T})^{-1} + 2dlog(\eta(T))/dT$ the imaginary part of which varies very rapidly as the real part changes. Thus large CP phases do not appear to arise using the moduli stabilization of the type above (Bailin et al., 1997).

The situation changes significantly if $W_{np}$ contains an additional factor $H(T)$ where

$$H(T) = \left(\frac{G_4(T)}{(\eta(T)^2)}\right)^m \left(\frac{G_6(T)}{(\eta(T)^3)}\right)^n P(j), \quad (63)$$

where $G_4(T)$ and $G_6(T)$ are Eisenstein functions of modular weight 4 and 6, $m, n$ are positive integers and $P(j)$ is a polynomial of $j(T)$ which is an absolute modular invariant. Alternatively $H$ can be expressed in the form

$$H(T) = (j - 1728)^{m/2} j^{n/2} P(j) \quad (64)$$

The form on $H(T)$ is dictated by the condition that no singularities appear in the fundamental domain. In this
case to achieve dilaton stabilization with $T$ modulus not only on the boundary of the fundamental domain but also inside the fundamental domain and thus $T$ has a substantial imaginary part. In this case it is possible to get CP phases for the soft parameters which can lie in the range $10^{-4} - 10^{-1}$ (Bailin et al., 1997). Thus with the absolute modular invariant in the superpotential large CP phases can appear in the soft breaking in orbifold compactifications of the type discussed above.

In the analysis of (Faraggi and Vives, 2002) the issue of CP violation and FCNC in string models with anomalous $U(1)_A$-dilaton supersymmetry breaking mechanism was investigated. Here scalar masses arise dominantly from the $U(1)_A$ contribution while the dilaton generates the main contribution to the gaugino masses. Further, the dilaton contributions to the trilinear terms and to the gaugino masses have the same phase. In this class of models the nonuniversal components of the trilinear soft SUSY breaking parameter are typically small and one has suppression of FCNC and of CP in this class of models.

C. CP violation on D brane models

Considerable progress has occurred over the recent past in the development of Type I and Type II string theory. Specifically D branes have provided a new and better understanding of Type I string theory and connection with Type IIB orientifolds. Further, the advent of D branes open up the possibility of a new class of model building (for recent reviews on D branes see (Blumenhagen et al., 2005, 2006; Polchinski, 1996)). Thus a stack of N D branes can produce generally an $SU(N)$ gauge group or a subgroup of it, and open strings with both ends terminating on the same stack give rise to a vector multiplet corresponding to the gauge group of the stack. Further, open strings beginning on one end and ending on another transform like the bifundamental representations and can be chiral. Thus these are possible candidates for massless quarks, leptons, and Higgs fields. A simple possibility for model building occurs with compactification on $T^6/Z_2 \times Z_2$. In addition to the axion-dilaton field $s$ the moduli space consists in this case of the Kähler ($s$) and the complex structure ($u_m$) moduli ($m=1,2,3$). For the moduli fields one has the Kähler potential of the form

$$K_0 = -\ln(s + \bar{s}) - \sum_{m=1}^{3} \ln(t_m + \bar{t}_m) - \sum_{m=1}^{3} \ln(u_m + \bar{u}_m).$$

(65)

Consider now complex scalars $C^{[99]}_i$ along the direction $i$ with ends of the open string ending in each case on a D9-brane. In this case one can obtain the Kähler potential including the complex scalar field by the translation $t_m + \bar{t}_m \to t_m + \bar{t}_m + |C^{[99]}_i|^2$. For the case of strings with both ending on the same $D5_1$ brane one can show using either T-duality (Ibanez et al., 1999) or by use of Born-Infeld action (Kors, 2006; Kors and Nath, 2004) that the Kähler potential is modified by making the replacement $s + \bar{s} \to s + \bar{s} - |C^{[5m_5m]}_m|^2$. For the case when one has both D9- branes and $D5_m$-branes the modified Kähler potential reads

$$K^{[99+55]} = -\ln(s + \bar{s} - \sum_{m=1}^{3} |C^{[5m_5m]}_m|^2) - \sum_{m=1}^{3} \ln(t_m + \bar{t}_m - |C^{[9m]}_m|^2 - \frac{1}{2} \sum_{n,p=1}^{3} \gamma_{mnp}|C^{[5p_5n]}_n|^2),$$

(66)

To construct the Kähler potential for the case when one has open strings with one end on D9-branes and the other end on $D5_m -$ branes, or for the case when open strings end on two different $D5$ branes, one can use the analogy to heterotic strings with $Z_2$-twisted matter fields (Ibanez et al., 1999; Kors and Nath, 2004). Alternatively one can use string perturbation theory (Bertolini et al., 2006; Lust et al., 2004, 2005). The result is

$$K^{[95]} = \frac{1}{2} \sum_{m,n,p=1}^{3} \gamma_{mnp} \frac{|C^{[95m]}_m|^2}{(t_n + \bar{t}_n)^{1/2}(t_p + \bar{t}_p)^{1/2}} + \frac{1}{2} \sum_{m,n,p=1}^{3} \gamma_{mnp} \frac{|C^{[5m_5n]}_m|^2}{(t_p + \bar{t}_p)^{1/2}(s + \bar{s})^{1/2}}.$$  

(67)

Explicit formulae for the soft parameters using these results are given in the literature. However, one needs to keep in mind the configurations of the type discussed above are the so called $1/2$BPS states, and in this case the spectrum of open states falls into $N = 2$ multiplets, which implies that the spectrum is not chiral. Similar considerations apply to open strings which start and end on $D_3$ and $D_7$ branes, and results for these can be obtained by using T dualities.

For realistic model building one needs to work with intersecting D branes. Thus in Calabi-Yau orientifolds of Type IIA one has D6-branes that intersect on the compactified 6 dimensional manifold. Sometimes it is convenient to work in the T-dual picture of Type IIB strings where the geometrical picture of branes intersecting is replaced by internal world volume gauge field backgrounds, called fluxes on the D9 and D5 branes. The fluxes $F_a$ where a labels the set of branes, are rational numbers, i.e., $F_a = n_a^m / n_m^m$, in order to satisfy charge quantization constrains. The fluxes determine the number of chiral families. Further, the condition that $N = 1$ supersymmetry be valid is a further constraint on the moduli and the fluxes and may be expressed in the form (Bachas, 1995; Berkooz et al., 1996; Kors and Nath, 2004)

$$\sum_{m=1}^{3} \frac{t_m + \bar{t}_m}{s + \bar{s}} F_a^m = \prod_{m=1}^{3} F_a^m$$

(68)
In the presence of fluxes the gauge kinetic energy function $f_a$ is given by

$$f_a = \frac{3}{m_a} \sum_{n=1}^{3} \gamma_{mnp} F_a^{(n)} F_a^{(p)} t_m . (69)$$

The computation of the Kähler metric for the case of an open string with both ending on some given stack $a$, $C^{[m_a]}_m$, can be computed by dimensional reduction(Kors and Nath, 2004) or string perturbation theory(Lust et al., 2004) and is given by

$$K^{[a]} = \frac{\left| C^{[a] m} \right|^2}{\prod_{m=1}^{3} (s + t_m + \bar{t}_m) (u_m + \bar{u}_m) \Gamma(1 - \theta^{[a]}_{m})^{1/2}} , \quad \Delta^{(m)}_a = \frac{3}{2} \sum_{n,p=1}^{3} \gamma_{mnp} (t_n + \bar{t}_n) (t_p + \bar{t}_p) \left( F_a^{(m)} \right)^2 . (70)$$

Now the technique above using the heterotic dual or Born-Infeld works for $\frac{1}{2}$BPS brane configurations. However, for the bifundamental fields $C^{[a b]}$ that connect the different stacks of branes with different world volume gauge flux one needs an actual string perturbation calculation and here the result for the Kähler potential is (Lust et al., 2004)

$$K^{[a b]} = \frac{\left| C^{[a b]} m \right|^2}{\prod_{m=1}^{3} (u_m + \bar{u}_m) \bar{\psi}_{[a] (m)} \theta_{[a] m}^{1/2}} \Gamma(\psi_{[a] (m)}^{(m)})^{1/2} \Gamma(1 - \theta^{[a]}_{m})^{1/2} ,$$

$$\theta^{[a]}_{m} = \arctan \left( \frac{F^{(m)}_a}{R(t_m)} \right) . (71)$$

Using the above one can obtain explicit expressions for the soft parameters. These have been worked out in detail in several papers. One can count the number of CP phases that enter in the analysis. They are the phases arising from $s, t_m, u_m$ ($n=1,2,3$). These can be reduced with extra restrictions such as, for example, dilation dominance which would imply only one CP phase $\gamma_k$.

D. SUSY CP phases and the CKM matrix

A natural question is if there is a connection between the soft SUSY CP phases and the CKM phase $\delta_{CKM}$. A priori it would appear that there is no connection between these two since they arise from two very different sources. Thus the $\delta_{CKM}$ arises from the Yukawa interactions (assuming there is no CP violation in the Higgs sector) which form the string view point originates at the point when the string compactifies from 10 dimensions to four dimensions. This is the point where we begin to identify various species of quarks and leptons and their couplings to the Higgs bosons. On the other hand soft SUSY phases arise from the spontaneous breaking of supersymmetry and enter only in the dimension less than 3 operators. Thus it would appear that they are disconnected. While this conclusion is largely true it is not entirely so.

The reason for this is that in SUGRA models the trilinear soft term $A_{i j \beta}$ contains a dependence on Yukawas so that(Kaplunovsky and Louis, 1993; Nath et al., 1983)

$$A_{i j \beta} = F^{ij} \partial Y^{i j \beta} + .. . (72)$$

Thus the phase of the Yukawa couplings enters in the phase of the trilinear coupling. However, the phase relationship between $A$ and $Y$ is not rigid, even for the case when there is no phase in the Yukawas one can generate a phase of $A$, and conversely even for the case when $\delta_{CKM}$ is maximal one may constrain $A$ to have zero phase. Further, it is entirely possible that the Yukawa couplings are all real and $\delta_{CKM}$ arises from CP violation in the Higgs sector as originally conjectured (Lee, 1973, 1974; Weinberg, 1976). A more recent analysis of this possibility is given in (Chen et al., 2007).

On a more theoretical level it was initially thought that CP violation could occur in string theory in either of the two ways: spontaneously or explicitly (Strominger and Witten, 1985). However, it was conjectured later that CP symmetry in string theory is a gauge theory and it is not violated explicitly(Choi et al., 1993; Dine et al., 1992). We do not address this issue further here.

VIII. THE EDM OF AN ELEMENTARY DIRAC FERMION

If the spin-1/2 particle has electric dipole moment EDM $d_f$, it would interact with the electromagnetic tensor $F_{\mu \nu}$ through

$$\mathcal{L} = - i d_f \bar{\psi} \sigma_{\mu \nu} \gamma_5 \psi F^{\mu \nu} . (73)$$

which in the non-relativistic limit reads

$$\mathcal{L} = d_f \bar{\psi} A \bar{\psi} \mathcal{E} \psi_A . (74)$$

where $\psi_A$ is the large component of the Dirac field. The above Lagrangian is not renormalizable, so it does not exist at the tree level of a renormalizable quantum field theory. However, it could be induced at the loop level if this theory contains sources of CP violation at the tree level. Thus suppose we wish to determine the EDM of a particle with the field $\psi_f$ due to the exchange of two other heavy fields: a spinor $\psi_i$ and a scalar $\phi_k$. The interaction that contains CP violation is given by

$$\mathcal{L} = L_{ik} \bar{\psi}_f P_L \psi_i \phi_k + R_{ik} \bar{\psi}_f P_H \psi_i \phi_k + H.c. . (75)$$

Here $\mathcal{L}$ violates CP invariance iff $Im(L_{ik} R_{ik}) \neq 0$. A direct analysis shows that the fermion $\psi_f$ acquires a one loop EDM $d_f$ which is given by

$$d_f = \frac{m_i}{16 \pi^2 m_k^2} Im(L_{ik} R_{ik}) (Q_i A (m_i^2/m_k^2) + Q_k B (m_i^2/m_k^2)) . (76)$$

where

$$A(r) = \frac{1}{2(1 - r)^2} (3 - r + \frac{2 nr}{1 - r})$$

$$B(r) = \frac{1}{2(1 - r)^2} (1 + r + \frac{2 nr}{1 - r}) . (77)$$
IX. EDM OF A CHARGED LEPTON IN SUSY

We discuss now the EDM of a charged lepton in MSSM using the results of the previous section. As mentioned in Sec.IV, in softly broken supersymmetric models as many as 40 additional phases can appear. However, only certain combinations of phases appear in a given process and the number of such combinations depends on the process. We discuss now the details.

In the computations here we use the Lagrangian of applied $N = 1$ supergravity for the case of MSSM fields with inclusion of soft breaking (Haber and Kane, 1985; Nath et al.; Nilles, 1984). The EDM of a charged lepton receives contributions from chargino, neutralino, and slepton exchanges. A discussion of the chargino and neutralino masses is given in Sec.(XVI.A) while a discussion of the slepton and squark masses is given in Sec.(XVI.B).

For the case of the charged lepton we find

$$d_{\text{e-chargino}}^E = \alpha_{\text{EM}} \frac{m_{\tilde{e}}^2}{4\pi \sin^2 \theta_W} \sum_{i=1}^{2} \tilde{m}_{\chi_i}^2 + \text{Im}(\Gamma_{\text{e})} A\left(\frac{m_{\tilde{e}}^2}{m_{\tilde{e}}^2}ight)$$

(78)

where $U$ and $V$ are as defined in Appendix XVI.A and where $\Gamma_{\text{e}} = (\kappa_{\text{e}} U_{i2}^* V_{ii}) = |\kappa_{\text{e}}| U_{i2}^* U_{Li1}$. A direct inspection of $\Gamma_{\text{e}}$ shows that it depends on only one combination, i.e., $\xi_2 + \theta_\mu + \theta_H$ where the phase $\theta_H$ comes from the Higgs sector and as discussed later is generated at the loop level.

The neutralino exchange contribution to the EDM of the fermion is as follows:

$$d_{\text{f-neutralino}}^E = \frac{\alpha_{\text{EM}}}{4\pi \sin^2 \theta_W} \sum_{k=1}^{4} \sum_{i=1}^{2} \text{Im}(\eta_{fi}) \frac{\tilde{m}_{\chi_i}^0}{M_{fj}}$$

$$\times Q_f B\left(\frac{M_{fj}^2}{\tilde{m}_{\chi_i}^0}\right)$$

(79)

where

$$\eta_{fi} = (a_0 X_{1i} D_{1f}^* + b_0 X_{3i} D_{3f}^* + \kappa_f X_i D_{f2}^*)$$

$$= (c_0 X_{1i} D_{2f}^* - \kappa_f X_i D_{f1})$$

(80)

where $a_0 = -\sqrt{2} \tan \theta_W (Q_f - T_{3f})$, $b_0 = -\sqrt{2} T_{3f}$, $c_0 = \sqrt{2} \tan \theta_W Q_f$, and in $X_{bi}$ $b=3(4)$ for $T_{3f} = -1(\frac{1}{2})$. The following three combinations of phases appear in $\eta_{fi}$: $\xi_1 + \theta_\mu + \theta_H$, $\xi_2 + \theta_\mu + \theta_H$ and $\alpha_f + \theta_\mu + \theta_H$. We note in passing that the contribution from the neutrino Yukawa couplings to the lepton electric dipole moment is computed in (Farzan and Peskin, 2004), and the charged Higgs contributions to the lepton EDM in a two-Higgs doublet model is discussed in (Kao and Xu, 1992).

X. EDM OF QUARKS IN SUSY

The quarks receive contribution from the electric dipole operator ($d_q^E$), from the chromoelectric dipole operator ($d_q^C$), and from the purely gluonic dimension six operator of Weinberg ($d_q^G$). Thus

$$d_q = d_q^E + d_q^C + d_q^G$$

(81)

We discuss these in further detail below.

A. The electric dipole moment operator contribution to EDM of quarks

The electric dipole moment operator receives contributions from the gluino, chargino and neutralino exchanges. The gluino exchange contributes to the EDM
of the quarks as follows
\[ d_{q-glino}/e = \frac{-2\alpha_s}{3\pi} m_0 q_i \text{Im}(\Gamma_{11}^{q}) \]
\[ \times \left( \frac{1}{M_{\tilde{q}_1}^2} B(m_0^2/M_{\tilde{q}_1}^2) - \frac{1}{M_{\tilde{q}_2}^2} B(m_0^2/M_{\tilde{q}_2}^2) \right), \quad (82) \]
where \( \tilde{q}_1 \) and \( \tilde{q}_2 \) are the mass eigenstates, and \( \Gamma_{1k}^{q} = e^{-i\xi_3} D_{qk} D_{q1k} \). The gluino mass is \( m_{\tilde{g}} \), and \( B(r) \) is as defined by Eq.(77). An explicit analysis gives \( \Gamma_{12}^{q} = -\Gamma_{11}^{q} \) where
\[ \text{Im}(\Gamma_{11}^{q}) = \frac{m_0}{M_{\tilde{q}_1}^2 - M_{\tilde{q}_2}^2} (m_0 [A_0] \sin(\alpha_q - \xi_3) + |\mu| \sin(\theta_\mu + \theta_H + \xi_3)|R_q|), \quad (83) \]
which holds for both signs of \( M_{\tilde{q}_1}^2 - M_{\tilde{q}_2}^2 \). It is easy to see that \( \text{Im}(\Gamma_{1k}^{q}) \) and \( \xi_3 + \theta_\mu + \theta_H \) or \( \alpha_q + \theta_\mu + \theta_H \) and \( \xi_3 + \theta_\mu + \theta_H \) are the same that enter in the above expression and \( \xi_3 + \theta_\mu + \theta_H \). The chromino contribution to the EDM for the up quark is as follows
\[ d_{u-chargino}/e = \frac{-\alpha_{EM}}{4\pi \sin^2 \theta_W} \sum_{k=1}^{2} \sum_{i=1}^{2} \text{Im}(\Gamma_{uik}) \frac{\tilde{m}_{\chi_u^+}}{M_{d_k}^2} \times \]
\[ (Q_q B(M_{\tilde{q}_k}^2) + (Q_u - Q_d) A(M_{\tilde{d}_k}^2)) \]
Here \( A(r) \) is as defined by Eq.(77) and
\[ \Gamma_{uik} = \kappa_u V_{u2}^{*} D_{d1k}(U_{11}^{*} D_{q1k}) - \kappa_d U_{d2k} D_{q1k} \]
and
\[ \kappa_u = \frac{m_u e^{-i\theta_H}}{\sqrt{2m_W} \sin \beta}, \quad \kappa_{d,e} = \frac{m_d,e}{\sqrt{2m_W} \cos \beta} \quad (86) \]
and explicitly
\[ \Gamma_{u11(2)} = |\kappa_u|(\cos^2 \theta d/2)[U_{L21} U_{R11}^{*}] \]
\[ - (+) \frac{1}{2} |\kappa_u| \kappa_{d,e} \sin \theta d[U_{L21} U_{R21}^{*}] e^{i(\xi_2 - \phi_d)} \]
(87)
The EDM here depends only on two combinations of phases: \( \alpha_d + \theta_\mu + \theta_H \) and \( \xi_2 + \theta_\mu + \theta_H \) with \( \xi_2 - \alpha_d \) being just a linear combination of the first two. A similar analysis hold for the chromino contributions to the down quark and one gets only two phase combinations which are identical to the case above with \( \alpha_d \) replaced by \( \alpha_q \). The neutralino exchange contribution to the EDM of quarks is given by Eq.(79). The sum of the gluino, the chromino and the neutralino exchanges discussed above gives the total contribution from the electric dipole operator to the quark EDM.

### B. The chromoelectric dipole moment contribution to the EDM of quarks

For the case of the quarks one has two more operators that contribute. These are the quark chromoelectric dipole moment \((d^C)^{\mu\nu} \) and the purely gluonic dimension six operator. For the operator \((d^C)^{\mu\nu} \) we have the effective dimension five operator
\[ \mathcal{L}_1 = -\frac{i}{2} d^C_{\mu\nu} q \sigma_{\mu\nu\rho} T^a q G^{\rho a}, \]
where \( T^a \) are the \( SU(3) \) generators. Contributions to \((d^C)^{\mu\nu} \) from the quarks of the gluino, the chromino, and from the neutralino exchange are given by
\[ d_{q-glino}^{C} = \frac{g_s \alpha_s}{4\pi} \sum_{k=1}^{2} \sum_{i=1}^{2} \text{Im}(\Gamma_{1k}^{q}) \frac{\tilde{m}_{\chi_u^+}}{M_{\tilde{q}_k}^2} B(M_{\tilde{q}_k}^2), \quad (89) \]
and
\[ d_{q-chargino}^{C} = -\frac{g_s g_0}{16\pi^2} \sum_{k=1}^{2} \sum_{i=1}^{2} \text{Im}(\Gamma_{qik}) \frac{\tilde{m}_{\chi_u^0}}{M_{\tilde{q}_k}^2} B(M_{\tilde{q}_k}^2), \]
where \( B(r) \) is defined by Eq.(77) and \( C(r) \) is given by
\[ C(r) = \frac{1}{6(r-1)^2}(10r-26 + 2rlnr - \frac{18lnr}{1-r}). \]
(91)
We note that all of the CP violating phases are contained in the factors \( \text{Im}(\Gamma_{1k}^{q}) \), \( \text{Im}(\Gamma_{qik}) \), and \( \text{Im}(\eta_{qik}) \). But these are precisely the same factors that appear in the gluino, the chromino and the neutralino contributions to the electric dipole operator.

### C. The contribution of the purely gluonic operator to the EDM of quarks

The purely gluonic dimension six operator which contributes to the dipole moment is (Weinberg, 1989)
\[ \mathcal{L}_1 = -\frac{1}{6} d^G \epsilon_{\alpha\beta\gamma} G_{\alpha\mu \rho} G_{\beta\nu \sigma} G_{\gamma\lambda \sigma} e^{\mu\nu\lambda\sigma}, \]
(93)
where \( G_{\alpha\mu \rho} \) is the gluon field strength tensor, \( f_{\alpha\beta\gamma} \) are the Gell-Mann coefficients, and \( e^{\mu\nu\lambda\sigma} \) is the totally antisymmetric tensor with \( \epsilon^{0123} = +1 \). An analysis of \( d^G \) including the quark-squark-gluino exchange (see Fig.(4), where one of the loops contributing to this operator is shown) with gluino phase \( \xi_3 \) but with squark mass matrix treated real is given in (Dai et al., 1990). Including the phases from \( A_t \) and \( \mu \) in the squark mass matrix the
analysis of $\tilde{d}^G$ gives (Dai et al., 1990; Ibrahim and Nath, 1998a)
\[
\tilde{d}^G = -3\alpha_s(\frac{g_s}{4\pi m_\tilde{g}})^3(m_t(z_1^1 - z_2^2)Im(\Gamma^1_t^D))H(z_1^1, z_2^2) + m_b(z_1^1 - z_2^2)Im(\Gamma^D_b^G)H(z_1^3, z_2^4).
\]  
(94)

Here
\[
\Gamma^1_q = e^{-i\alpha_d}D_{q2k}D_{q1k}^*, \quad z_\alpha^q = (\frac{M_\tilde{q}_\alpha}{m_\tilde{g}})^2, \quad z_q = (\frac{m_q}{m_\tilde{g}})^2,
\]  
(95)

and $H(z_1, z_2, z_3)$ is defined by
\[
H(z_1, z_2, z_3) = \frac{1}{2} \int_0^1 \int_0^1 \int_0^1 dx \int_0^1 dy x(1-x)u \frac{N_1 N_2}{D^4},
\]  
(96)

where
\[
N_1 = u(1-x) + z_3 x(1-x)(1-u) - 2ux[z_1 y + z_2(1-y)],
\]
\[
N_2 = (1-x)^2(1-u)^2 + u^2 - \frac{1}{9} x^2(1-u)^2,
\]
\[
D = u(1-x) + z_3 x(1-x)(1-u) + ux[z_1 y + z_2(1-y)].
\]  
(97)

For the case when $m_\tilde{g}, m_\tilde{g} >> m_q$ one obtains for $H$ the following expression
\[
H \simeq -\frac{m_\tilde{g}^2}{m_q^2}I(z_3^2),
\]  
(98)

where $I(z)$ is defined by
\[
I(z) = \frac{1}{6(z-1)^2}[2(z-1)(11z-1) + (1 - 16z - 9z^2)lnz].
\]  
(99)

The contribution of the last two operators to the EDM of the quarks can be computed using the naive dimensional analysis (Manohar and Georgi, 1984). This technique can be expressed in terms of a rule using the ‘reduced’ coupling constants. Thus for example, for a coupling constant $g$ appearing in an interaction of dimensionality (mass)$^D$ and containing N field operators the reduced coupling is $(4\pi)^{2-N}M^{D-1}g$ where $M$ is the chiral-symmetry breaking scale and has the value $M = 1.19$ GeV. Thus the rule means that the reduced coupling of any term in the effective hadronic theory at energies below $M$ is given by a product of the reduced couplings of the operators appearing in the effective Lagrangian at energies below $M$, that produces this term. Using this rule for the chromoelectric and purely gluonic dimension six operators one finds there contribution to the EDM of the quarks are given as follows
\[
d_q^E = d_q^E, \quad d_q^C = \frac{e}{4\pi}d_q^C, \quad d_q^G = \frac{e}{4\pi}d_q^G,
\]  
(100)

where $\eta^E, \eta^C$ and $\eta^G$ are renormalization group evolution of $d_q, d_q^C$ and $d_q^G$ from the electroweak scale to the hadronic scale. A discussion of how these renormalization group factors are computed is discussed in Sec.(XVII.C). Their numerical value is estimated to be $\eta^E \approx 0.61$ (Degrassi et al., 2005), $\eta^C \approx \eta^G \sim 3.4$. The alternate technique to estimate the contributions of the chromoelectric operator is to use the QCD sum rules(Khriplovich and Zyablyuk, 1996). To obtain the neutron EDM, we use the non-relativistic $SU(6)$ quark model which gives $d_n = \frac{1}{2}d_d - \frac{1}{4}d_u$.

**D. The cancelation mechanism and other remedies for the CP problem in SUSY, in strings and in branes**

Thus MSSM contains new sources of CP violation and these phases would induce EDMs of the fermions in the theory. Taking the values of the parameters of the model at their phenomenologically favorable range ($m_{1/2} \sim m_0 \sim 100$ GeV, $\tan\beta \sim 10, \theta_\mu \sim \alpha_0 \sim 1$) one finds that the EDMs of the electron and neutron exceed the experimental bounds by several orders of magnitude. This problem is certainly a weakness of the low energy SUSY and needs to be corrected to make the theory viable. Various remedies have been suggested in the literature to overcome this problem. The first of these is the
suggestion that the first generation of sleptons and the first two generations of squarks are very heavy (Nath, 1991) (see also (Kizukuri and Oshimo, 1992)). This means the production and study of these particles at LHC will be difficult if not impossible. There is another reason that this possibility is not attractive is that, the annihilation rate of the lightest supersymmetric particle LSP may be too low in this range of masses and as a result the relic density of the LSP may be larger than the observed dark matter density. Another suggestion is that the phases are small $O(10^{-2})$ (Arnowitt et al., 1991, 1990; Braaten et al., 1990a,b; Dai et al., 1990; Dugan et al., 1985; Ellis et al., 1982; Franco and Mangano, 1984; Garisto and Wells, 1997; Gunion and Wyler, 1990; Polchinski and Wise, 1983; Weinberg, 1989). However, a small phase constitute a fine tuning and there will not be any interesting display of CP violation in colliders. Moreover, electroweak baryogenesis cannot take place in this case (Kuzmin et al., 1985). A third possibility first proposed in (Ibrahim and Nath, 1998a,b,c) is that there are internal cancelations among the various contributions to the neutron EDM and to the electron EDM, leading to compatibility with experiment with large phases and a SUSY spectrum that is still within the reach of the accelerators.

This is the most interesting solution because it leaves room for a host of non trivial CP violating as well as CP conserving phenomena to be discovered at colliders and elsewhere. By CP violating properties we mean those properties that vanish in the limit of CP conservation like the EDMs and the neutral Higgs bosons mixing. By CP conserving phenomena, we mean those properties that exist in the absence of CP violation but they differ if CP violation is included like $g_\mu - 2$. Following the work of (Ibrahim and Nath, 1998a,b,c) there is much further work on the cancelation mechanism in the literature (Abel et al., 2002; Accomando et al., 2000a,b; Barger et al., 2001; Bartl et al., 1999, 2001; Brhlik et al., 2000a, 1999a, 2000b, 1999b, 2001; Chattopadhyay et al., 2001; Falk and Olive, 1998; Falk et al., 1999b; Ibrahim and Nath, 1998c, 2000d; Pokorski et al., 2000).

As was shown above, the quark and the lepton EDMs in general depend on ten independent phases providing one with considerable freedom for the satisfaction of the EDM constraints. Numerical analyses show the existence of significant regions of the parameter space where the cancelation mechanism holds. We describe here a straightforward technique for accomplishing the satisfaction of the electron EDM and the neutron EDM constraints. For the case of the electron one finds that the chargino component of the electron is independent of $\xi_1$ and the electron EDM as a whole is independent of $\xi_3$. Thus the algorithm to discover a point of simultaneous cancelation for the electron EDM and for the neutron EDM is a straightforward one. For a given set of parameters we vary $\xi_1$ until we reach the cancelation for the electron EDM since only one of its components (the neutralino) is affected by that parameter. Once the electric dipole moment constraint on the electron is satisfied we vary $\xi_3$ which affects only the neutron EDM keeping all other parameters fixed. By using this simple algorithm one can generate any number of simultaneous cancelations. The EDM of the atoms also provide a sensitive test of CP violation. An example is the EDM of Hg-199 for which the current limits are given by Eq.(22). Among the phases that enter the EDM of Hg-199 is the phase $\alpha_\mu$. We note that $\alpha_\mu$ enters only in $d_H$ to one loop order, and thus it can be varied to achieve a simultaneous cancelation in $d_H$ and a consistency with the experimental limits. Illustrated examples of points in the parameter space where cancelations occur and all the EDM constraints are satisfied are given in Tables 1 and 2 in Sec.(XVI.D). It needs to be emphasized that while the cancelations among the various contributions to the EDMs are pretty generic the suppression of the EDMs for the electron and for the neutron do require fine tuning. On the positive side the above, of course, leads to a narrowing of the parameter space of the theory.

In theories where the Higgs mixing parameter $\mu$ obeys the simple scaling behavior as the rest of the SUSY masses the EDMs exhibit a simple scaling behavior under the simultaneous scaling on $m_0$ and $m_{1/2}$. In the scaling region the knowledge of a single point in the MSSM parameter space where the cancelation in the EDMs occurs allows one to generate a trajectory in the $m_0-m_{1/2}$ plane where the cancelation mechanism holds and the EDMs are small. Thus under the transformation $m_0 \rightarrow \lambda m_0$, $m_{1/2} \rightarrow \lambda m_{1/2}$ $\mu$ itself obeys the same scaling, i.e., $\mu \rightarrow \lambda \mu$ in the large $\mu$ region. In this case $d_e$ exhibits the scaling behavior

$$d_e \rightarrow \lambda^{-2} d_e$$

The same scaling relation holds for the electric and for the chromoelectric operators of the quarks

$$d_q^E \rightarrow \lambda^{-2} d_q^E, \quad d_q^C \rightarrow \lambda^{-2} d_q^C$$

![FIG. 5 Two loop Barr-Zee type diagrams that contribute to the EDMs in supersymmetry (Chang et al., 1999).]
For the gluonic dimension 6 operator we find the following scaling
\[ d^G_q \rightarrow \lambda^{-4} d'^G_q \]  
(103)
Thus the scaling property of \( d_q \) will be more complicated. However, as \( \lambda \) gets large the contribution of \( d^G_q \) will fall off faster than \( d'^G_q \) and in this case one will have the scaling \( d_q \rightarrow \lambda^{-2} d_q \) and so \( d_n \rightarrow \lambda^{-2} d_n \). Thus scaling property of EDMs allows one to promote a single point in the SUSY parameter space where cancelation occurs to a trajectory in the parameter space. With the scaling property one can arrange the cancellation mechanism to work for the EDMs over a much larger region of the parameter space (Ibrahim and Nath, 2000d) than would otherwise be possible (Pospelov and Ritz, 2005). The scaling phenomenon also has implications for the satisfaction of the EDM constraints in string and D-brane models (Ibrahim and Nath, 2000d). As stated already in general only certain phase combinations appear in the analysis of a given physical quantity. Some examples of such combinations are given in Table 3 in Sec. (XVI.E).

For other solutions to the SUSY CP problem see (Abel et al., 2001; Babu et al., 2000b; Dimopoulos and Thomas, 1996; Nir and Rattazzi, 1996).

E. Two loop contribution to EDMs

Two loop contributions to the EDMs can be quite significant. Thus the analysis of (Barr and Zee, 1990; Gunion and Wyler, 1990) showed that significant contributions to the EDM of the electron and of the neutron can result if the Higgs-boson exchange mediates CP violation. A more recent analysis in the same spirit is given by (Chang et al., 1999) for the MSSM case. Here the CP phases arising from the Higgs boson couplings to the stop and the sbottom enter and these are not stringently constrained by data. Thus CP phases in the third generation could be quite substantial consistent with the EDM constraints. We discuss now the two loop analysis in further detail. We assume that the large CP phases arise only in the third generation trilinear soft parameters \( A_{\tau,t,b} \) and the relevant two loop interactions arise via the CP-odd Higgs \( a(x) \) (see Fig.(5)) whose interactions with fermions and sfermions are given by

\[
\mathcal{L}_a = \frac{g m_f}{2 M_W} R_{fia} \tilde{f}_i \gamma_5 f + v x a (-\tilde{f}_1 \tilde{f}_1 + \tilde{f}_2 \tilde{f}_2),
\]  
(104)

where \( g \) is related to the W boson mass by \( M_W = g v / 2 \), \( R_f = \cot \beta (\tan \beta) \) for \( T_f^i = \frac{1}{\sqrt{2}} \). The diagrams of Fig.(5) give the following contribution to the EDM of a fermion at the electroweak scale

\[
d_f / e = \frac{3 \alpha_{em}}{64 \pi^3} \frac{R_{f}}{m_f^2} \sum_{q=t,b} \frac{\xi_q Q_f Q^2_q (F(x_{1q}) - F(x_{2q}))}{m_a^2} \]  
(105)

FIG. 6 Estimate of the size of two loop contribution to the EDMs in supersymmetry with phases only in the third generation (Chang et al., 1999).

where \( x_{1a} = (m_q / m_a)^2 \) (i=1,2), \( \xi_q \) (q=t,b) are defined by

\[
\xi_b = \frac{2 m_b \sin 2 \theta_b I_m (A_b e^{i \delta_b})}{v^2 \sin 2 \beta}, \\
\xi_t = \frac{m_t \sin 2 \theta_t I_m (m e^{i \delta_t})}{v^2 \sin^2 \beta},
\]  
(106)

and where \( \delta_q = \arg(A_q + R_q u^*) \). The function \( F(x) \) is given by the loop integral

\[
F(x) = \int_0^1 dy y (1-y) \ln \left( \frac{y(1-y)}{x} \right).
\]  
(107)

Similarly the contribution to CEDM at the electroweak scale is given by

\[
d_f^C / e = \frac{\alpha_{em}}{128 \pi^3} \frac{R_{f}}{m_f^2} \sum_{q=t,b} \xi_q [F(x_{1q}) - F(x_{2q})].
\]  
(108)

A numerical analysis of the EDM is given in Fig.(6) and indicates that one can satisfy the EDM constraints in
certain ranges of the parameter space. However, it remains to be seen how one can naturally suppress phases in the first two generations while allowing them only in the third generation. The reader is also directed to several other works on two loop analyses of EDMs: (Chang et al., 1990, 1991; Degrassi et al., 2005; Feng et al., 2006, 2005; Pilaftsis, 2002). Specifically, a complete account of all dominant 2-loop Barr-Zee type graphs in the CP violating MSSM is given in (Pilaftsis, 2002). The analyses of EDMs given in this section were based on the assumption of R parity conservation. For analyses of EDMs without R parity see (Fassler et al., 2006; Hall and Suzuki, 1984; Keum and Kong, 2001a,b).

XI. CP EFFECTS AND SUSY PHENOMENA

As noted earlier with the cancelation mechanism the phases can be large, and thus their effects could be visible in many supersymmetric phenomena (Aoki et al., 1999; Asatrian and Asatrian, 1999; Baek and Ko, 1999; Barr and Khalil, 2000; Choi and Drees, 1998; Choi et al., 2000b; Choi and Lee, 2000; Choi et al., 2000c; Dedes and Moretti, 2000a,b; Goto et al., 1999; Huang and Liao, 2000a,b, 2002; Kneur and Moultaka, 2000; Kribs, 2000; Ma et al., 1999; Mrenna et al., 2000; Okada et al., 2000). Below we discuss several of these phenomena and refer to the literature above for others.

A. SUSY phases and $g_\mu - 2$

The effects of CP violating phases on the supersymmetric electroweak contributions to $g_\mu - 2$ have been investigated (Ibrahim et al., 2001; Ibrahim and Nath, 2000a,c). The parameter $a_\mu = \frac{2\omega - 2}{2}$ is induced by loop corrections to the muon vertex with the photon field. In MSSM the muon interacts with other fermions $\psi_i$ and scalars $\phi_k$ through

$$\mathcal{L} = L_{ik} \bar{\mu} P_L \psi_i \phi_k + R_{ik} \bar{\mu} P_R \psi_i \phi_k + H.c.$$ (109)

where $\psi_i$ stands for the neutralino (chargino) and $\phi_k$ stands for the smuon (scalar neutrino). The one loop contribution to $a_\mu$ is given by

$$a_\mu = a_\mu^1 + a_\mu^2$$ (110)

Here $a_\mu^1$ comes from the neutralino exchange contribution and $a_\mu^2$ comes from the chargino exchange contribution so that

$$a_\mu^1 = \frac{m_\mu}{8\pi^2 m_i} Re(L_{ik} R_{ik}^* I_3) \frac{m_i^2}{m_i^2} \frac{m_k^2}{m_k^2}$$

$$+ \frac{m_\mu^2}{16\pi^2 m_i^2} (|L_{ik}|^2 + |R_{ik}|^2) I_2 \frac{m_i^2}{m_i^2} \frac{m_k^2}{m_k^2},$$ (111)

Here

$$I_1(\alpha, \beta) = - \int_0^1 dx \int_0^{1-x} \frac{dz}{\alpha z^2 + (1 - \alpha - \beta) z + \beta},$$

$$I_2(\alpha, \beta) = \int_0^1 dx \int_0^{1-x} \frac{dz}{\alpha z^2 + (1 - \alpha) z + \beta},$$

$$I_3(\alpha, \beta) = \int_0^1 dx \int_0^{1-x} \frac{1-z}{\alpha z^2 + (\beta - \alpha - 1) z + 1},$$

$$I_4(\alpha, \beta) = \int_0^1 dx \int_0^{1-x} \frac{z - z}{\alpha z^2 + (\beta - \alpha - 1) z + 1}$$ (113)

In the supersymmetric limit the soft breaking terms vanish and $a_\mu$ should vanish as well (Barbieri and Giudice, 1993; Ferrara and Remiddi, 1974). A careful limit of Eqs.(111) and (112) shows that in the supersymmetric limit the sum of the $W$ exchange contribution, in the
FIG. 8 An exhibition of the dependence of $a_\mu$ on a SUSY CP phase. The curves correspond to the four cases below (Ibrahim and Nath, 2000c): (1) $m_0=70$, $m_{1/2}=99$, $\tan\beta=3$, $|A_0|=5.6$, $\xi_1=-1$, $\xi_2=0.62$, $\theta_\mu=2.35$, $\alpha_{A_0}=4$; (2) $m_0=80$, $m_{1/2}=99$, $\tan\beta=5$, $|A_0|=5.5$, $\xi_1=-0.8$, $\xi_2=0.95$, $\theta_\mu=1.98$, $\alpha_{A_0}=0.4$; (3) $m_0=75$, $m_{1/2}=132$, $\tan\beta=4$, $|A_0|=6.6$, $\xi_1=-1$, $\xi_2=2.74$, $\theta_\mu=1.2$, $\alpha_{A_0}=-1.5$; (4) $m_0=70$, $m_{1/2}=99$, $\tan\beta=6$, $|A_0|=3.2$, $\xi_1=0.63$, $\xi_2=0.47$, $\theta_\mu=2.7$, $\alpha_{A_0}=-4$ where all masses are in GeV units and all phases are in rad.

standard model part, and of the chargino exchange contributions, in the supersymmetric counterpart, cancel. Thus

$$a_\mu^W + a_\mu^\chi^+ = 0$$ (114)

Similarly one can show that the Z boson exchange and the contribution of the massive modes of the neutralino sector in the supersymmetric limit cancel.

$$a_\mu^\sigma + a_\mu^\chi^0 \text{(massive)} = 0$$ (115)

One can show that the massless part of the neutralino spectrum in the supersymmetric limit gives the value of $-\alpha_{em}/2\pi$. Thus it gives the same magnitude but is opposite in sign to the famous photon exchange result.

The CP dependence of $a_\mu$ arises from the effect of the phases on the particle masses, and on their effects on $L_{ik}$ and $R_{ik}$ and significant variations can arise in $a_\mu$ as the phases are varied. An illustration of this phenomenon is given in Fig. (8). Because of the significant dependence of $a_\mu$ on the phases it is possible to constrain the CP phases using the current data on $a_\mu$. This is done in (Ibrahim et al., 2001). Further details on the analysis of this section are given in Sec.(XVI.F).

B. SUSY CP phases and CP even -CP odd mixing in the neutral Higgs boson sector

Another important effect of CP violating phases is their role in determining the spectrum and CP properties of the neutral Higgs fields arising due to mixings of the CP even-CP odd Higgs fields (Pilaftsis, 1998a,b; Pilaftsis and Wagner, 1999).

Such mixings between CP even and CP odd Higgs bosons cannot occur at the tree level, but are possible when loop corrections to the effective potential are included. To calculate such mixings we use the one loop effective potential as given by Eq.(28). We assume that the $SU(2)$ Higgs doublets $H_{1,2}$ have non-vanishing vacuum expectation $v_1$ and $v_2$ so that we can write

$$(H_1) = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 + \phi_1 + i\psi_1 \\ H_1^- \\ \end{pmatrix},$$

$$(H_2) = e^{i\theta_H} \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 + \phi_2 + i\psi_2 \\ H_2^+ \\ \end{pmatrix}. $$

(116)

For the present case with the inclusion of CP violating effects, the variations with respect to the fields $\phi_1, \phi_2, \psi_1, \psi_2$ give the following

$$-\frac{1}{v_1} \frac{\partial\Delta V}{\partial \phi_1} = m_1^2 + \frac{g_2^2 + g_1^2}{8} (v_1^2 - v_2^2) + m_3^2 \tan\beta \cos\theta_H,$$ (117)

$$-\frac{1}{v_2} \frac{\partial\Delta V}{\partial \phi_2} = m_2^2 - \frac{g_2^2 + g_1^2}{8} (v_1^2 - v_2^2) + m_3^2 \cot\beta \cos\theta_H,$$ (118)

$$\frac{1}{v_1} \frac{\partial\Delta V}{\partial \psi_1} = m_1^2 \sin\theta_H = \frac{1}{v_2} \frac{\partial\Delta V}{\partial \psi_2}, $$ (119)

where the subscript 0 means that the quantities are evaluated at the point $\phi_1 = \phi_2 = \psi_1 = \psi_2 = 0$. As noted in (Demir, 1999) only one of the two equations in Eq.(119) is independent.

One can have sizable contributions to the potential corrections from top-stop, bottom-sbottom (Choi et al., 2000a; Demir et al., 2000b; Ibrahim and Nath, 2001a; Pilaftsis and Wagner, 1999), $W - H^+ - \chi^0$ sector (Ibrahim and Nath, 2001a) and from the $\chi^0 - Z - h^0 - H^0$ sector (Ham et al., 2003; Ibrahim and Nath, 2002). The mass-squared matrix of the neutral Higgs bosons is defined by

$$M_{ab}^2 = \frac{1}{2} \langle \Phi_a \Phi_b \rangle,$$ (120)

where $\Phi_a = (a = 1 - 4)$ are defined by

$$\{\Phi_a\} = \{\phi_1, \phi_2, \psi_1, \psi_2\},$$ (121)

and the subscript 0 means that we set $\phi_1 = \phi_2 = \psi_1 = \psi_2 = 0$. The dominant contributions come from the stop, sbottom and chargino. With the inclusion of the stop, the sbottom, and of the chargino contributions one finds that $\theta_H$ is determined by

$$m_3^2 \sin\theta_H = \frac{1}{2} \beta_{h_1} |\mu| |A_{h_1}| \sin\gamma_1 f_1 (m_{h_1}^2, m_{\tilde{t}_1}^2) + \frac{1}{2} \beta_{h_2} |\mu| |A_{h_2}| \sin\gamma_2 f_1 (m_{h_2}^2, m_{\tilde{t}_2}^2) - \frac{g_2^2}{16\pi^2} |\mu| |\tilde{m}_2| \sin\gamma_2 f_1 (m_{\chi_1}^2, m_{\chi_2}^2),$$

(122)
where
\[ \beta_h = \frac{3h_i^2}{16\pi^2}, \quad \beta_h = \frac{3h_i^2}{16\pi^2}, \]
\[ \gamma_a = \alpha_{A_a} + \theta_\mu, \quad \gamma_b = \alpha_{A_b} + \theta_\mu, \quad \gamma_2 = \xi_2 + \theta_\mu, \] (123)
and \( f_1(x, y) \) is defined by
\[ f_1(x, y) = -2 + \log \frac{xy}{Q^2} + \frac{y + x}{x} \log \frac{y}{x}. \] (124)

The inclusion of the stau and the neutralino sectors in the analysis would contribute extra terms to Eq.(122) that are dependent on the phase \( \gamma_\tau = \alpha_{A_\tau} + \theta_\mu \) and where \( \gamma_1 = \xi_1 + \theta_\mu \). The tree and loop contributions to \( M_{ab}^2 \) are given by
\[ M_{ab}^2 = M_{ab}^{(2)} + \Delta M_{ab}^2, \] (125)
where \( M_{ab}^{(2)} \) are the contributions at the tree level and \( \Delta M_{ab}^2 \) are the loop contributions where
\[ \Delta M_{ab}^2 = \frac{1}{16\pi^2} \text{Str} \left\{ \frac{\partial M^2}{\partial \phi_a} \frac{\partial M^2}{\partial \phi_b} \log \frac{Q^2}{\mu^2} \right\} + \frac{M^2}{\partial \phi_a} \frac{\partial^2 M^2}{\partial \phi_a \partial \phi_b} \log \frac{M^2}{\mu^2} \delta_{ab}, \] (126)
and where \( e = 2.718 \). Computation of the \( 4 \times 4 \) Higgs mass matrix in the basis of Eq.(121) gives
\[
\begin{pmatrix}
M_{11} + \Delta_{11} & -M_{12} + \Delta_{12} & \Delta_{13}s_{\beta} & \Delta_{13}C_{\beta} \\
-M_{12} + \Delta_{12} & M_{22} + \Delta_{22} & \Delta_{23}s_{\beta} & \Delta_{23}C_{\beta} \\
\Delta_{13}s_{\beta} & \Delta_{23}s_{\beta} & M_{33}s_{\beta} + M_{33}c_{\beta} \\
\Delta_{13}c_{\beta} & \Delta_{23}c_{\beta} & M_{33}s_{\beta}C_{\beta} & M_{33}c_{\beta} \\
\end{pmatrix},
\] (127)
where \( M_{11} = M_{A_1}^2 + M_{A_2}^2, \ M_{12} = (M_{A_1}^2 + M_{A_2}^2)s_{\beta}c_{\beta}, \ M_{22} = M_{A_2}^2 + M_{A_3}^2, \ c_{\beta}, \ s_{\beta} = \cos \beta, \sin \beta, \ \text{and} \ M_{33} = M_{A_3}^2 + \Delta_{33} \), and where \( (c_{\beta}, s_{\beta}) = (\cos \beta, \sin \beta) \). Here the explicit Q dependence has been absorbed in \( m_\chi^2 \) which is given by
\[ m_\chi^2 = (\sin \beta \cos \beta)^{-1}(-m_1^2 \cos \theta + \frac{1}{2} \beta_h |A_\tau| |\mu| \cos \gamma_\tau f_1(m_{b_1}^2, m_{b_2}^2)) + \frac{g_2^2}{16\pi^2} |m_2|^2 |\mu| \cos \gamma_\tau f_1(m_{b_1}^2, m_{b_2}^2) + \frac{g_2^2}{16\pi^2} |m_1| |\mu| \cos \gamma_\tau f_2(m_{x_1}^2, m_{x_2}^2) + \Delta_\chi. \] (128)

Here \( \Delta_\chi \) is the contribution arising from the neutralino exchange and
\[ \Delta_\chi = -\frac{1}{16\pi^2} \sum_{j=1}^{4} \frac{M_{\chi_j}^2}{D_j} \left( \log \frac{M_{\chi_j}^2}{m_\chi^2} - 1 \right) \]
\[ (M_{\chi_j}^4 (-g_2^2 |\mu| |\tilde{m}_2|^2 \cos \gamma_\tau - g_1^2 |\mu| |\tilde{m}_1|^2 \cos \gamma_1) + M_{\chi_j} \rho_2 (-g_1^2 |\tilde{m}_1|^2 |\mu|^2 + g_1^2 |\tilde{m}_2|^2 |\mu|^2 |\tilde{m}_1|^2 \cos \gamma_2 + g_2^2 |\tilde{m}_1|^2 |\mu|^2 |\tilde{m}_2|^2 \cos \gamma_1) \]
\[ -g_2^2 |\tilde{m}_1|^2 |\mu|^2 |\tilde{m}_2|^2 \cos \gamma_2 - g_1^2 |\tilde{m}_2|^2 |\mu|^2 |\tilde{m}_1|^2 \cos \gamma_2) \]
where \( \Delta_\chi = \xi_1 - \xi_2 \). The first term in the second brace on the right hand side of Eqs.(128) is the tree term, while the second, the third and the fourth terms come from the stop, sbottom, stau and changino exchange contributions. The remaining contributions in Eq.(128) arise from the neutralino sector. For \( \Delta_\chi \) one has
\[ \Delta_{ij} = \Delta_{ij} + \Delta_{ij} + \Delta_{ij} + \Delta_{ij} + \Delta_{ij} \]
(130)
where \( \Delta_{ij} \) is the contribution from the stop exchange in the loops, \( \Delta_{ij} \) is the contribution from the sbottom exchange in the loops, \( \Delta_{ij} \) is the contribution from the stau loop, \( \Delta_{ij} \) is the contribution from the changino sector and \( \Delta_{ij} \) is the contribution from the neutralino sector. For illustration \( \Delta_{ij} \) are listed in Sec.(XVI.G).

We note that the phases come to play a role here through the squark, slepton, changino and neutralino eigen values of their mass matrices. We note that in the supersymmetric limit \( M_{\chi_i} = (0, 0, M_Z, M_Z), \ (M_{b_1}, M_{b_2}) = (M_Z, 0), \ M_{\chi_1} = M_{\chi_2} = M_W = M_B = \)
With this in mind one can see that all the radiative corrections to the potential vanish in the supersymmetric limit. By introducing a new basis $\phi_1, \phi_2, \psi_1, \psi_2$ where

$$\psi_{1D} = \sin \beta \psi_1 + \cos \beta \psi_2$$
$$\psi_{2D} = -\cos \beta \psi_1 + \sin \beta \psi_2,$$  \hspace{1cm} (131)

one finds that the field $\psi_{2D}$ decouples from the other three fields and is a massless state (a Goldstone field). The Higgs mass matrix $M^2_{Higgs}$ of the remaining three fields is given by

$$
\begin{pmatrix}
M_{11} + \Delta_{11} & -M_{12} + \Delta_{12} & \Delta_{13} \\
-M_{12} + \Delta_{12} & M_{22} + \Delta_{22} & \Delta_{23} \\
\Delta_{13} & \Delta_{23} & M^2_A + \Delta_{33}
\end{pmatrix}
\hspace{1cm} (132)
$$

We note that the basis fields $\{\phi_1, \phi_2, \psi_{1D}\}$ of the above matrix are the real parts of the neutral Higgs fields and a linear combination of their imaginary parts $\psi_i$. Thus these states are pure CP states where $\phi$ are CP even (scalars) and $\psi_{1D}$ is CP odd (a pseudoscalar). What we are interested here is the mixing between the CP even and the CP odd Higgs states in the eigen vectors of the above matrix and this mixing is governed by the off-diagonal elements $\Delta_{12}$ and $\Delta_{23}$. These are found to be linear combination of $\sin \gamma_t$, $\sin \beta t$, $\sin \gamma_1$, $\sin \gamma_2$, and $\sin \gamma_\tau$ where these phases are defined as in Eq. (123). In the limit of vanishing CP phases the matrix elements $\Delta_{12}$ and $\Delta_{23}$ vanish and thus the Higgs mass matrix factors into a $2 \times 2$ CP even Higgs matrix times a CP odd element. The effect of phases on CP even-CP odd Higgs boson mixings have been studied by (Choi et al., 2000a; Demir et al., 2000b; Ibrahim and Nath, 2001a; Pilafsis and Wagner, 1999) and found to be significant. It is shown that if a mixing effect among the CP even and the CP odd Higgs bosons is observed experimentally, then it is only the cancelation mechanism of EDMs that can survive(Ibrahim, 2001a). A more accurate determination of the VEV of the Higgs fields would require use of two loop effective potential. An improved accuracy and scale dependence should be obtained with the full two-loop effective potential(Martin, 2003).

C. Effect of SUSY CP phases on the b quark mass

The running b quark mass is another object in MSSM where CP phases could have an impact. $m_b$ can be written in the form

$$m_b(M_Z) = h_b(M_Z) \frac{v}{\sqrt{2}} \cos \beta (1 + \Delta_b)$$ \hspace{1cm} (133)

where $h_b(M_Z)$ is the Yukawa coupling for the b quark at the scale $M_Z$ and $\Delta_b$ is the loop correction to $m_b$. The SUSY QCD and electroweak corrections are large in the large $\tan \beta$ region(Carena et al., 1994; Hall et al., 1994; Pierce et al., 1997). At the tree level the b quark couples to the neutral component of $H_1$ Higgs boson while the coupling to the $H_2$ Higgs boson is absent. Loop corrections produce a shift in the $H_2^0$ couplings and generate a non-vanishing effective coupling with $H_1^0$. Thus the effective Lagrangian would be written as (Babu et al., 1999; Carena and Haber, 2003)

$$-\mathcal{L}_{eff} = (h_b + \delta h_b) \bar{b} H_1 L + \Delta h_b \bar{b} b L H_2^{0*} + H.d.$$  \hspace{1cm} (134)

where the star on $H_2^0$ is necessary in order to have a gauge invariant Lagrangian. The quantities $\delta h_b$ and $\Delta h_b$ receive SUSY QCD and SUSY electroweak contributions. The QCD contribution arises from the corrections where gluinos and sbottoms are running in the loops. In the electroweak contributions, the sbottoms (stops) and the neutralinos (charginos) are running in the loops. The basic integral that appears in the expressions of $\delta h_b$ and $\Delta h_b$ involving heavy scalars $S_1, S_2$ and a heavy fermion $f$ is

$$I = \int \frac{d^4k}{(2\pi)^4} \frac{m_f + \gamma_k k^\mu}{(k^2 - m_f^2)(k^2 - m_{S_1}^2)(k^2 - m_{S_2}^2)}$$ \hspace{1cm} (135)

In the approximation of the zero external momentum this integral could be written in the closed form

$$I = \frac{m_f}{(4\pi)^2} f(m_{j_1}, m_{j_2}^2, m_{j_2}^2)$$ \hspace{1cm} (136)

where the function $f(m, m_i^2, m_j^2)$ is given by

$$f(m^2, m_i^2, m_j^2) = \left[ (m^2 - m_i^2)(m^2 - m_j^2)(m^2 - m_k^2) \right]^{-1} \times \left( m_i^2 m_j^2 \ln \frac{m_j^2}{m_i^2} + m_i^2 m_k^2 \ln \frac{m_k^2}{m_i^2} + m_j^2 m_k^2 \ln \frac{m_j^2}{m_k^2} \right)$$ \hspace{1cm} (137)

for the case $i \neq j$ and

$$f(m, m_i^2, m_j^2) = \frac{1}{(m_i^2 - m_k^2)^2} (m^2 \ln \frac{m_i^2}{m_k^2} + (m^2 - m_i^2))$$

for the case $i = j$. In the SUSY QCD the heavy fermion is the gluino and the heavy scalars are the sbottoms. In the chargino contribution, the chargino is the heavy fermion and the heavy scalars are the stops. In the neutralino part, the neutralino is the heavy fermion and the heavy scalars are the sbottoms.

The couplings $\delta h_b$ and $\Delta h_b$ are generally complex due to CP phases in the soft SUSY breaking terms. Electroweak symmetry is broken spontaneously by giving expectation values to $H_1^0$ and $H_2^0$. Thus one finds for the mass term

$$-\mathcal{L}_m = \frac{M_b}{h_b} \bar{b} \bar{b} L H_2^0 + H.c.,$$ \hspace{1cm} (138)

where

$$M_b = \frac{h_b \cos \beta}{\sqrt{2}} (1 + \frac{\delta h_b}{h_b} + \frac{\Delta h_b}{h_b} \tan \beta)$$ \hspace{1cm} (139)
Here $M_b$ is complex. By rotating the $b$ quark field
\[ b = e^{i/2\gamma_5\chi_b'} , \quad \tan \chi_b = \frac{\text{Im} M_b}{\text{Re} M_b} \quad (140) \]
one gets
\[ -\mathcal{L}_m = m_b \bar{b}_R b_L + H.c., \quad (141) \]
where $m_b$ is real and positive and $b'$ is the physical field.
\[ m_b = \frac{h_b v \cos \beta}{\sqrt{2}} \left( (1 + \delta_R)^2 + \delta_I^2 \right)^{1/2} \]
\[ \delta_R = \text{Re} \left( \frac{\delta h_b}{h_b} \right) + \text{Re} \left( \frac{\Delta h_b}{h_b} \right) \tan \beta \]
\[ \delta_I = \text{Im} \left( \frac{\delta h_b}{h_b} \right) + \text{Im} \left( \frac{\Delta h_b}{h_b} \right) \tan \beta \quad (142) \]
Thus one finds for the mass correction
\[ \Delta_b \approx \text{Re} \left( \frac{\Delta h_b}{h_b} \right) \tan \beta + \text{Re} \left( \frac{\delta h_b}{h_b} \right) \quad (143) \]
The SUSY CP violating phases in the SUSY QCD corrections are $\xi_3$, $\alpha_A$, and $\theta_\mu$. These come from the vertices of $\bar{b}b\bar{g}$ and $\bar{b}bH$. In the chargino part one finds the phases $\xi_2$, $\alpha_A$, and $\theta_\mu$. In the case of neutralino we have $\xi_2$, $\xi_1$, $\alpha_A$, and $\theta_\mu$. The corrections of the $b$ quark mass are found to be very sensitively dependent on $\theta_\mu$, $\xi_3$ and $\alpha_A$ as the values of these phases affect both the sign and the magnitude of the correction. Thus the correction can vary from zero to as much as 30% in some regions of the parameter space and can also change its sign depending on the value of these phases. The effect of $\xi_2$ is less important and $\xi_1$ is found to be the least important phase(Ibrahim and Nath, 2003c). Similar results hold for the $\tau$ lepton mass and for the top quark mass. For the $\tau$ lepton the numerical size of the correction is as much as 5% and for the top quark is typically less than a percent.

D. SUSY CP phases and the decays $h \to b\bar{b}$, $h \to \tau\bar{\tau}$

As was mentioned above, the spectrum of the neutral Higgs sector and its CP properties are sensitive to the CP violating phases through radiative corrections. The couplings of the quarks with the Higgs are also found to be dependent of these phases. Thus one can deduce the corrected effective interaction of the $b$ quark with the lightest Higgs boson $H_2$ as
\[ -\mathcal{L}_{int} = \bar{b}(C_b^- + i\gamma_5 C_b^P)bH_2 \quad (144) \]
where
\[ \begin{pmatrix} C_b^- \\ C_b^P \end{pmatrix} = \begin{pmatrix} \cos \chi_b & -\sin \chi_b \\ \sin \chi_b & \cos \chi_b \end{pmatrix} \begin{pmatrix} C_b^1 \\ C_b^2 \end{pmatrix} \quad (145) \]
\[ C_b^1 = \frac{1}{\sqrt{2}} (\text{Re} (h_b + \delta h_b) R_{21} + - \text{Im} (h_b + \delta h_b) \sin \beta) \]
\[ + \text{Im} (\Delta h_b) \cos \beta) R_{23} + \text{Re} (\Delta h_b) R_{22} \]
\[ C_b^2 = -\frac{1}{\sqrt{2}} (-\text{Im} (h_b + \delta h_b) R_{21} + - \text{Re} (h_b + \delta h_b) \sin \beta) \]
\[ + \text{Re} (\Delta h_b) \cos \beta) R_{23} - \text{Im} (\Delta h_b) R_{22} \]
The matrix $R$ is the diagonalizing matrix of the Higgs mass matrix
\[ R M^2_{Higgs} R^T = \text{diag}(m_{H_1}^2, m_{H_2}^2, m_{H_3}^2) \quad (146) \]
where we use the convention that in the limit of vanishing CP phases, one has $H_1 \to H$, $H_2 \to h$ and $H_3 \to A$. These elements $R_{ij}$ and the corrections $\delta h_b$ and $\Delta h_b$ are found to be sensitive functions of the CP violating phases and their values are all determined by SUSY radiative corrections of MSSM potential. The quantity $R_{b/\tau}$ defined as
\[ R_{b/\tau} = \frac{\text{BR}(h \to b\bar{b})}{\text{BR}(h \to \tau\bar{\tau})} \quad (147) \]
is found to be an important tool to discover supersymmetry. In Standard Model, it is given by
\[ R_{b/\tau}^{SM} = 3 \left( m_b^2 \over m_T^2 \right) \left( m_b^2 - 4m_T^2 \right)^{3/2} (1 + w) \quad (148) \]
where $(1 + w)$ is the QCD enhancement factor(Gorishnii et al., 1990).
\[ 1 + w = 1 + 5.67 \frac{\alpha_s}{\pi} + 29.14 \frac{\alpha_s^2}{\pi^2} \quad (149) \]
By identifying $m_b$ with $m_{H_2}$, the lightest Higgs boson in MSSM, we find a shift in $R_{b/\tau}$ value due to supersymmetric effect including the effects due to CP phases as follows
\[ \Delta R_{b/\tau} = \frac{R_{b/\tau} - R_{b/\tau}^{SM}}{R_{b/\tau}} \quad (150) \]
The quantity $R_{b/\tau}$ in MSSM depends on the CP phase via $C_b^S$ and $C_b^P$. Thus if a neutral Higgs is discovered and $R_{b/\tau}$ measured and found to be different from what one expects in the Standard Model, then it would point to a non-standard Higgs boson such as from MSSM (Babu and Kolda, 1999). The analysis of (Ibrahim and Nath, 2003b) that the supersymmetric effects with CP phases can change the branching ratios by as much as 100 % for the lightest Higgs boson decay into $b\bar{b}$ and $\tau\bar{\tau}$. Similar results are reported for the other heavier Higgs bosons. Thus the deviation from the Standard Model result for $R_{b/\tau}$ depends on the CP phases and it can be used as a possible signature for supersymmetry and CP effects. Similar analyses can also be given for the decay of the heavy Higgs, e.g., for $H^0 \to t\bar{t}$, $bb$ and to $\chi^+\chi^-$ (Eberl et al., 2004; Ibrahim, 2007) if allowed kinematically.
E. SUSY CP phases and charged Higgs decays $H^{-} \to \bar{b}b$, $H^{-} \to \bar{\nu}_\tau \tau$

In the neutral Higgs sector, the ratio $R^{h_0} = BR(h^0 \to bb)/BR(h^0 \to \tau \tau)$ is found to be sensitive to the supersymmetric loop corrections and to the CP phases. In an analogous fashion we may define the ratio $R^{H^{-}} = BR(H^{-} \to \bar{b}b)/BR(H^{-} \to \tau \bar{\nu}_\tau)$ and it is also affected by SUSY loop corrections, and is sensitive to CP phases. Thus the tree level couplings of the third generation quarks to the Higgs bosons

$$-\mathcal{L} = \epsilon_{ij} h_{b} \bar{b} R H^1_{i} Q^j_L - \epsilon_{ij} h_{t} \bar{t} R H^2_{i} Q^j_L + H.c.$$  \hspace{1cm} (151)

receive SUSY QCD and the SUSY electroweak loop corrections which produce shifts in couplings similar to the case for the neutral Higgs bosons. Thus the general effective interaction may be written as

$$-\mathcal{L}_{eff} = \epsilon_{ij} (h_{b} + \delta h_{b}^{1} \bar{b} R H^1_{i} Q^j_L + \Delta h_{b}^{1} \bar{b} R H^2_{i} Q^j_L - \epsilon_{ij} (h_{t} + \delta h_{t}^{1} \bar{t} R H^2_{i} Q^j_L + \Delta h_{t}^{1} \bar{t} R H^1_{i} Q^j_L + H.c.) \hspace{1cm} (152)$$

where

We note that in the approximation

$$\delta h_{b}^{1} = \delta h_{b}^{1}, \; \Delta h_{b}^{1} = \Delta h_{b}^{2}$$  \hspace{1cm} (153)

one finds that the above Lagrangian preserves weak isospin. This is the approximation that is often used in the literature (Carena and Haber, 2003). However, in general, the above approximation will not hold and there will be violations of weak isospin. In the neutral Higgs interaction with the quarks and leptons of third generation, we examined $\delta h_{b, \tau}^{1}$, $\Delta h_{b, \tau}^{1}$, $\delta h_{t}^{1}$ and $\Delta h_{t}^{1}$. In the charged Higgs interaction with these particles we should similarly examine $\delta h_{b, \tau}^{2}$, $\Delta h_{b, \tau}^{2}$, $\delta h_{t}^{2}$ and $\Delta h_{t}^{2}$. The latter corrections have SUSY QCD contributions when gluinos, stops and sbottoms are running in the loops and SUSY electroweak contributions when neutralinos and/or charginos, stops and/or sbottoms are running in the loops. The CP violating phases that enter $\delta h_{b, \tau}^{2}$ and $\Delta h_{b, \tau}^{2}$ are $\xi_{3}, \alpha_{A_{E}}, \alpha_{A_{B}}$, and $\theta_{\mu}$. The phases that appear in $\delta h_{t, \tau}^{1}$ and $\Delta h_{t, \tau}^{1}$ are $\xi_{1}$, $\xi_{2}$, $\alpha_{A_{E}}, \alpha_{A_{B}}$, and $\theta_{\mu}$. The phases that enter the corrections $\delta h_{t}^{1}$ and $\Delta h_{t}^{1}$ are the same as in $\delta h_{b}^{1}$ and $\Delta h_{b}^{1}$. One can measure the size of the violation of weak isospin by defining $r_{b}$

$$r_{b} = (|\Delta h_{b}^{1}|^2 + |\delta h_{b}^{2}|^2)^{\frac{1}{2}} (|\Delta h_{b}^{1}|^2 + |\delta h_{b}^{2}|^2)^{-\frac{1}{2}} \hspace{1cm} (154)$$

Similar ratios could be defined for the top and tau, $r_{t}$ and $r_{\tau}$. The deviation of these quantities from unity is an indication of the violation of weak isospin in the Higgs couplings. It is found that such deviations from unity can be as much as 50% or more depending on the region of the parameter space one is in. It is also seen that these measures are sensitive functions of CP violating phases (Ibrahim and Nath, 2004). The interactions of the charged Higgs are thus governed by the Lagrangian

$$-\mathcal{L} = \bar{b} (B_{bt}^{*} \bar{c} \gamma_{5} t) H^{-} + \bar{\tau} (B_{b\tau}^{*} \bar{\nu}_{\tau} \gamma_{5}) \nu H^{-} + H.c.$$  \hspace{1cm} (155)

F. SUSY CP phases and charged Higgs decays $H^{\pm} \to \chi^{\pm} \chi^{0}$

The decay $H^{\pm} \to \chi^{\pm} \chi^{0}$ is sensitive to CP violation phases even at the tree level. Inclusion of the loop corrections further enhance the effects of the CP phases. The tree level lagrangian for $H^{\pm} \chi^{\pm} \chi^{0}$ is

$$\mathcal{L} = \xi_{ij} H_{2}^{\pm} \bar{\chi}_{j} P_{L} \chi_{i} + \xi_{ij} H_{1}^{\pm} \bar{\chi}_{j} P_{R} \chi_{i} + H.c., \hspace{1cm} (157)$$
where ξ_{ij} and ξ'_{ij} are given by

\[
\begin{align*}
\xi_{ij} &= -g X_{ij} V_{i1}^* - \frac{g}{\sqrt{2}} X_{12} V_{12}^* - \frac{g}{\sqrt{2}} \tan \theta_W X_{1j} V_{12}^* \\
\xi'_{ij} &= -g X_{ij} U_{i1} + \frac{g}{\sqrt{2}} X_{12} U_{12} + \frac{g}{\sqrt{2}} \tan \theta_W X_{ij} U_{12}
\end{align*}
\]

(158)

The phases that enter the couplings ξ_{ij} and ξ'_{ij} are ξ_1, ξ_2 and θ_μ. The loop corrections produce shifts in the couplings and the effective Lagrangian with loop corrected couplings is given by

\[
L_{\text{eff}} = (\xi_{ij} + \delta \xi_{ij}) H_2^{*} \chi_j^{0} P_L \chi_i^{+} + \Delta \xi_{ij} H_2^{*} \chi_j^{0} P_L \chi_i^{+} + (\xi'_{ij} + \delta \xi'_{ij}) H_2^{*} \chi_j^{0} P_R \chi_i^{+} + + \Delta \xi'_{ij} H_2^{*} \chi_j^{0} P_R \chi_i^{+} + H.c.
\]

(159)

The phases that enter the corrections ∆ξ_{ij}, δξ_{ij} are ξ_1, ξ_2, ξ', \alpha_{ij}, \alpha'_{ij} and θ_μ. This dependence arises from the shifts in the vertices of the charginos with top and sbottoms, charginos with bottom and stops, neutralino with bottom and sbottoms, neutralino with tops and stops, W bosons with charginos and neutralinos, Z bosons with charginos and neutralinos, charged Higgs with neutralinos and charginos and charged Higgs with stops and sbottoms. All these vertices enter in the loop corrections. Thus \( L_{\text{eff}} \) may be written in terms of the mass eigenstates as follows

\[
L_{\text{eff}} = H^{+} \chi_j^{0} (\alpha_{ij}^{S} + \gamma_{ij} \alpha_{ij}^{P}) \chi_i^{+} + H.c.,
\]

(160)

where

\[
\begin{align*}
\alpha_{ij}^{S} &= \frac{1}{2} (\xi_{ij} + \xi'_{ij}) \sin \beta + \frac{1}{2} \Delta \xi_{ij} \cos \beta \\
&+ \frac{1}{2} (\xi_{ij} + \xi'_{ij}) \cos \beta + \frac{1}{2} \Delta \xi_{ij} \sin \beta, \\
\alpha_{ij}^{P} &= \frac{1}{2} (\xi_{ij} + \xi'_{ij}) \sin \beta + \frac{1}{2} \Delta \xi_{ij} \cos \beta \\
&- \frac{1}{2} (\xi_{ij} + \xi'_{ij}) \cos \beta - \frac{1}{2} \Delta \xi_{ij} \sin \beta.
\end{align*}
\]

(161)

From the above Lagrangian one can write down the decay rate of the charged Higgs into charginos and neutralinos.

\[
\Gamma_{ij}(H^{+} \to \chi_j^{0} \chi_i^{-}) = \frac{1}{4\pi M_{H^{+}}} \left( (m_{\chi_j^{0}}^2 + m_{\chi_i^{-}}^2 - M_{H^{+}}^2)^2 - 4m_{\chi_j^{0}}^2 m_{\chi_i^{-}}^2 \right)^{\frac{1}{2}} \\
\times (0.5(\alpha_{ij}^{S} + |\alpha_{ij}^{P}|^2)(M_{H^{+}} - m_{\chi_j^{0}} - m_{\chi_j^{0}})^2 \\
- 0.5(\alpha_{ij}^{S} + |\alpha_{ij}^{P}|^2)(2m_{\chi_j^{0}} + m_{\chi_j^{0}}))
\]

(162)

The charged Higgs decays are found to be more sensitive to the phases that enter both at the tree level as well as at the loop level such as θ_μ (Ibrahim et al., 2004) relative to the phases such as α_A which enter only at the loop level.

**G. Effect of CP phases on neutralino dark matter**

If the lightest neutralino is the LSP then with R parity invariance it is a possible candidate for cold dark matter, and in this case the relic density (Chattopadhyay et al., 1999; Falk and Olive, 1998; Falk et al., 1995; Gomez et al., 2005) as well as the rates in experiments to detect neutralinos will be affected by the presence of CP phases (Chattopadhyay et al., 1999; Falk et al., 1999a, 2000). We give a brief discussion of neutralino dark matter and highlight the effects of CP on neutralino dark matter analyses. A quantity of interest in experimental measurements is Ωdm/Ω where Ωdm/Ω is the dark matter density, and ρ_c is the critical matter density needed to close the universe where

\[
\rho_c = 3H_0^2/8\pi G N \sim 1.88 \times 10^{-27} \text{gm/cm}^3.
\]

Here H_0 is the Hubble constant, and h_0 is its value in units of 100km/sec.Mpc, and G_N is the Newtonian constant. The current limit from WMAP3 on cold dark matter is (Spergel et al., 2006)

\[
\Omega_{cdm}h^2 = 0.1045^{+0.0072}_{-0.0095}.
\]

In the Big Bang scenario the neutralinos will be produced at the time of the Big Bang and will be in thermal equilibrium with the background till the time of freeze out when they will go out of equilibrium. The procedure for computation of the density of the relic neutralinos is well known using the Boltzman equations. In general the analysis will involve co-annihilations and one will have processes of the type

\[
\chi_i^0 + \chi_j^0 \to f \bar{f}, WW, ZZ, WH, \cdots
\]

Additionally co-annihilations with staus, charginos, and other sparticle species can also contribute. Thus the relic density of neutralinos \( n = \sum_i n_i \) is governed by the Boltzman equation(Gondolo and Gelmini, 1991; Griest and Seckel, 1991; Lee and Weinberg, 1977)

\[
\frac{dn}{dt} = -3Hn - \sum_{ij} (\sigma_{ij} v)(n_i n_j - n_{i\text{eq}} n_{j\text{eq}})
\]

(166)

Here \( \sigma_{ij} \) is the cross-section for annihilation of particle types \( i, j \), and \( n_{i\text{eq}} \) the number density of \( \chi_i^0 \) in thermal equilibrium. Under the approximation \( n_i/n = n_{i\text{eq}}/n_{\text{eq}} \) one has the well known result

\[
\frac{dn}{dt} = -3Hn - (\sigma_{\text{eff}})(n^2 - (n_{\text{eq}})^2)
\]

(167)

where \( \sigma_{\text{eff}} = \sum_{ij} \sigma_{ij} \gamma_i \gamma_j \), and \( \gamma_i \) are the Boltzman suppression factors \( \gamma_i = \frac{n_{i\text{eq}}}{n_{\text{eq}}} \). Explicitly one finds that the freeze-out temperature is given by

\[
x_f = \ln \left[ x_f^{-\frac{1}{2}} (\sigma_{\text{eff}}) x_f m_1 \sqrt{\frac{45}{8\pi^6 N_f G_N}} \right]
\]

(168)
where $N_f$ is the number of degrees of freedom at freeze-out and $G_N$ is Newton’s constant. The relic abundance of neutralinos at current temperatures is then given by

$$\Omega_{\chi^0} h^2 = \frac{1.07 \times 10^9 \text{GeV}^{-1}}{N_f^2 M_{\text{Pl}}} \left[ \int_{x_f}^{\infty} (\sigma v u) \frac{dx}{x^2} \right]^{-1}. \quad (169)$$

Here $x_f = m_1/T_f$, $T_f$ is the freeze-out temperature, $M_{\text{Pl}} = 1.2 \times 10^{19}$ GeV, and $\langle \sigma v \rangle$ is the thermal average of $\sigma v$ so that

$$\langle \sigma v \rangle = \int_0^{\infty} dv_v v^2 \langle \sigma v \rangle e^{-v^2/4x}/\int_0^{\infty} dv_v v^2 e^{-v^2/4x}. \quad (170)$$

The diagrams that contribute to $\langle \sigma v \rangle$ include the s channel $Z$, $h$, and $A'$, $h'$, and the $t$ and $u$ channel squark and slepton exchanges. The Higgs boson, and sparticle masses are affected by the CP phases of the soft parameters. Further, the vertices are also affected. Inclusion of the loop corrections to the vertices further enhances the dependence on phases (Gomez et al., 2004b).

Specifically the Yukawa couplings of bottom quark and neutral Higgs bosons are found to be sensitive to $\xi_3$ if one includes SUSY QCD corrections in the analysis. A detailed analysis to study the sensitivity of dark matter to the b quark mass and to the neutral Higgs boson mixings is given in (Gomez et al., 2004b). It is found that the relic density is very sensitive to the mass of the b quark for large $\tan \beta$ and consequently also to the CP phases since the b quark mass is sensitive to the phases. In Fig.(11) we give an exhibition of the relic density and its sensitivity to phases. In the analysis presented in Fig.(11), the relic density was satisfied due to the annihilation through resonant Higgs poles, and one observes the sensitivity of the relic density to CP violating phases. The analysis of the relic density with inclusion of Yukawa unification constraint with inclusion of CP phases is given in (Gomez et al., 2005). An analysis of relic density in the presence of CP phases is also given in (Argyrou et al., 2004; Belanger et al., 2006a; Falk and Olive, 1998; Nihei and Sasagawa, 2004).

Typical dark matter experiments involve scattering of neutralinos of the Milky Way that reside in our vicinity with target nuclei. The basic lagrangian that governs such scattering is the neutralino-quark scattering with neutralino and quarks in the initial and final states. The relative velocity of the LSP hitting the target is small, and so, one can approximate the effective interaction governing the neutralino-quark scattering by an effective four-fermi interaction

$$L_{4f} = \bar{\chi} \gamma^\mu \gamma^5 \chi \bar{q} \gamma^\mu (AP_L + BP_R) q + C \bar{\chi} \chi m_q q \bar{q} + D \bar{\chi} \gamma^5 \chi m_q \gamma^5 \bar{q} + E \bar{\chi} \gamma^5 \chi m_q \gamma^\mu \bar{q} + F \bar{\chi} \chi m_q \gamma^\mu \gamma^5 \bar{q} \quad (171)$$

The deduction of Eq.(171) requires Fierz rearrangement which is discussed in Sec.(XVI.H) and further details are given in Sec.(XVI.I). The first two terms $A, B$ in Eq.(171) are spin-dependent interaction and arise from the $Z$ boson and the sfermion exchanges. The effect of CP violating phases enter via the neutralino eigen vector components and the matrix $D \tilde{q}$ that diagonalizes the squark mass matrix. Then the phases that play a role here are $\theta_\mu$, $\xi_1$, $\xi_2$ and $\alpha_{A_\gamma}$. The $C$ term represents the scalar interaction which gives rise to coherent scattering. It receives contributions from the sfermion exchange, and from the exchange of the neutral Higgs $H_1$ mass eigenstates. The term $D$ is non vanishing in the limit when CP phases vanish. However, this term is mostly ignored in the literature as its contribution is suppressed because of the small velocity of the relic neutralinos. In fact the contributions of $D, E$ and $F$ are expected to be relatively small and could be ignored. A significant body of work exists on the analysis of detection rates in the absence of CP phases (Arnowitt and Nath, 1996; Nath and Arnowitt, 1995, 1997), but much less so with inclusion of CP phases. Inclusion of the CP phases shows a very significant effect of CP phases on the detection rates (Chattopadhayay et al., 1999; Falk et al., 2000). The CP effects can be significant even with inclusion of the EDM constraints(Gomez et al.; Nihei and Sasagawa, 2004).

### H. Effect of CP phases on proton stability

CP violating phases can affect the nucleon stability in supersymmetric grand unified models with baryon and lepton number violating dimension five operators ((Sakai and Yanagida, 1982; Weinberg, 1982). For a recent review see (Nath and Perez, 2007)). Thus in a wide class of unified models including grand unified models, string and brane models baryon and lepton number violation

!![](image)
arises via dimension LLLL and RRRR chiral operators of the form

\[ \mathcal{L}_{5L} = \frac{1}{M} \epsilon_{abc}(P f_i^c V_i)_{ij}(f_j^d)_k l (\bar{u}_{Lk} d_{Lj}) (\epsilon^*_{Lk}(V u_L)_{al} - \bar{d}_{Lk} d_{Lj}) + \ldots + H.c. \]  

\[ \mathcal{L}_{5R} = -\frac{1}{M} \epsilon_{abc}(V f^u)_{ij}(P f^d)_k l (\epsilon^*_{Ri}(V u_R)_{aj} \bar{u}_{Rj} \bar{d}_{Rk}) + \ldots + H.c. \]  

(172)

(173)

Here \( \mathcal{L}_{5L} \) in the LLLL and \( \mathcal{L}_{5R} \) is the RRRR lepton and baryon number violating dimension 5 operators, \( V \) is the CKM matrix and \( f_i \) are related to quark masses, and \( P_i \) appearing in Eqs (172) and (173) are the generational phases given by \( P_i = (e^{i \gamma_i}) \) with the constraint \( \sum \gamma_i = 0 \) (i=1,2,3).

Using the above one generates the baryon and the lepton number violating dimension six operators by dressing the dimension five operators by the chargino, the gluino and the neutralino exchanges. The dressing loops contain the CP phases both via the sparticle spectrum as well as via the vertices. This can be explicitly seen by elimination the sfermion fields above via the relations

\[ \bar{u}_{iL} = 2 \int [\Delta^L_{ui} L_{ui} + \Delta^L_{IR} R_{ui}] \]  

\[ \bar{u}_{iR} = 2 \int [\Delta^R_{ui} R_{ui} + \Delta^R_{LL} L_{ui}] \]  

(174)

where \( L_{ui} = \delta L_i / \delta \bar{u}_i^L \), \( R_{ui} = \delta L_i / \delta u_i^R \). Here \( L_i \) is the sum of fermion-sfermion-gluino, fermion-sfermion-chargino and fermion-sfermion-neutralino and \( \Delta^s \) are the propagators. A detailed analysis of the specific mode \( p \rightarrow \bar{\nu} K^+ \) which is typically the dominant mode in supersymmetric decay modes of the proton is then given by the following with the inclusion of CP phases

\[ \Gamma(p \rightarrow \bar{\nu} K^+) = \frac{\beta_p^2 m_N}{M_h^2 32\pi f_{\pi}^2} (1 - m_K^2/m_N^2)^2 |A_{\nu, K}|^2 \mathcal{A}^2 (A_{L}^2) \]  

\[ |(1 + \frac{m_N(D + 3 F)}{3m_B})| (1 + \gamma^{kk}_1 + (e^{-i \xi_1} \gamma^{L}_1 + \gamma^{L}_2) \delta_{12} + \frac{A^R}{A^S} \gamma^{R}_{31} \delta_{13}) + \frac{2m_N}{3m_B} D (1 + \gamma^{kk}_1 + (e^{-i \xi_1} \gamma^{L}_1 - \gamma^{L}_2) \delta_{12} + \frac{A^R}{A^S} \gamma^{R}_{21} \delta_{13})^2 \]  

(175)

where

\[ A_{\nu, K} = (\sin 2\beta M_R^2)^{-1} \alpha_2^2 \beta m_c m_d^2 V_{11}^* V_{21} V_{22} \]  

(176)

In the above \( A_L(A_S) \) are the long (short) suppression factors, \( D,F \), \( f_\pi \) are the effective Lagrangian parameters, and \( \beta_p \) is defined by \( \beta_p U_L^2 = \epsilon_{abc} \epsilon_{\alpha \beta} \beta \left< \text{d} L_k \text{d} L_i \right> \) where \( U_L^2 \) is the proton wavefunction. Theoretical determinations of \( \beta_p \) lie in the range \( 0.003 - 0.03 \text{ GeV}^3 \). Perhaps the more reliable estimate is from lattice gauge calculations which gives(Tsutsui et al., 2004) \( |\beta_p| = 0.0096(09)(c_{+0}) \text{ GeV}^3 \).

CP violating phases of the soft SUSY breaking sector enter in the proton decay amplitude. The CP phases enter the dressings in two ways, via the mass matrices of the charginos, the neutralinos and the sfermions, and via the interaction vertices. Taking account of this additional complexity, the analysis for computing the proton decay amplitudes follows the usual procedure. This effect is exhibited by considering \( R_\tau \)

\[ R_\tau = \frac{\tau(p \rightarrow \bar{\nu} + K^+)}{\tau_0(p \rightarrow \bar{\nu} + K^+)} \]  

(177)

where \( \tau(p \rightarrow \bar{\nu} + K^+) \) is the proton lifetime with CP violating phases and \( \tau_0(p \rightarrow \bar{\nu} + K^+) \) is the lifetime without CP phases. This ratio is largely model independent. All the model dependent features would be contained mostly in the front factors which cancel out in the ratio. Since the dressing loop integrals enter in the proton decay lifetime in GUTs which contain the baryon and the lepton number violating dimension five operators, the phenomena of CP violating effects on the proton life time should hold for a wide range of models of GUTs. The baryon and lepton number violating operators must be dressed by the chargino, the gluino and the neutralino exchanges to generate effective baryon and lepton number violating dimension six operators at low energy. These dressing loops have vertices of quark-squark-chargino, quark-squark-neutralino and quark-squark-gluino. From this structure one can read the phases that might enter the analysis. The chargino one has the phases of \( \theta_{\mu}, \alpha_A, \) and \( \xi_2 \). The neutralino vertex has beside the above set, an extra phase \( \xi_1 \). The gluino vertex has the set of \( \theta_{\mu}, \alpha_A, \) and \( \xi_3 \). Following the standard procedure (Nath et al., 1985; Weinberg, 1982) one can obtain the effective dimension six operators for the baryon and lepton violating interaction arising from dressing of the dimension five operators. By doing so and estimating \( R_\tau \), one finds that this ratio is a sensitive function of CP phases (Ibrahim and Nath, 2000b). Modifications of the proton lifetime by as much as a factor of 2 due to the effects of the CP violating phases can occur. It is found also that the CP phase effects could increase or decrease the proton decay rates and that the size of their effect depend highly on the region of the parameter space one is in.

I. SUSY CP phases and the decay \( B_s^0 \rightarrow \mu^+ \mu^- \).

The branching ratio of the rare process \( B_s^0 \rightarrow \mu^+ \mu^- \) is another area where CP violating phase effects arise.

It is known that the standard model value is rather small while in supersymmetric models it can get three
orders of magnitude larger for large tan β (Arnowitt et al., 2002; Babu and Kolda, 2000; Baek et al., 2003; Bobeth et al., 2001; Buras et al., 2002; Chankowski and Slavianovska, 2001; Choudhury and Gaur, 1999; Dedes et al., 2002; Huang et al., 2001; Isidori and Retico, 2001; Mizukoshi et al., 2002; Xiong and Yang, 2002).

Detecting such large values of \( B^0_b \) would be a positive test for SUSY even before any nanoparticles are found. This decay is governed by the effective Hamiltonian

\[
H_{eff} = -\frac{G_F e^2}{4\sqrt{2}\pi^2} V_{tb} V_{ts}^\ast \times (C_S O_S + C_P O_P + C'_S O'_S + C'_P O'_P + C_{10} O_{10}) \Omega
\]

where \( C \)'s are the coefficients of the Wilson operators \( O \)'s defined by

\[
O_S = m_0 (\tilde{d}_i^a P_{Hb} \tilde{l} \tilde{l}), \quad O_P = m_0 (\tilde{d}_i^a P_{Rb} \tilde{l} \tilde{l}),
\]

\[
O'_S = m_0 (\tilde{d}_i^a P_{Lb} \tilde{l} \tilde{l}), \quad O'_P = m_0 (\tilde{d}_i^a P_{Lb} \tilde{l} \tilde{l}),
\]

\[
O_{10} = (\tilde{d}_i^a \gamma^\mu P_L b_i) \gamma_5 \gamma_5 \tilde{l}, \quad (179)
\]

and \( \Omega \) is the scale where the coefficients are evaluated. The branching ratio is a function of the coefficients \( C_S, P \) and \( C'_S, P \). In the counter term diagram (see Fig.(12)) which contributes to this ratio one can find vertices of \( \bar{b}bH_i, \bar{s}\tilde{\chi}^- \tilde{l} \) and \( \tilde{\mu}\tilde{H}_i \). The first two vertices are sensitive functions of the CP violating phases as was explained in the different applications above. The phases that play a major role here are \( \theta_{\mu}, \xi_2 \) and \( \alpha_{A_q} \). Gluino and neutralino exchange diagrams also contribute which brings a dependence on additional phases \( \xi_1 \) and \( \xi_3 \). Inclusion of these (Ibrahim and Nath, 2003a) shows that the branching ratio can vary in some parts of the parameter space by up to 1-2 orders of magnitude due to the effect of CP phases. A demonstration of the strong effect of the phases on B decay branching ratio is given in Fig.(13).

An analysis of this process using the so called resummed effective lagrangian approach for Higgs mediated interactions in the CP violating MSSM is given in (Dedes and Pilaftsis, 2003).

**J. CP effects on squark decays**

The interactions of \( \bar{q}_i \tilde{\chi}_i^+ \) and \( \bar{q}_i \tilde{\chi}_i^0 \) do have CP violating phases at the tree level. These interactions are important for squark decays into fermions and such decays are expected to show up in the Large Hadron Collider when squarks become visible. The Lagrangian that governs the squark decays is given by

\[
\mathcal{L} = g\bar{b}(R_{bij} P_R + L_{bij} P_L) \tilde{\chi}_i^+ \tilde{b} + g\bar{b}(R_{abj} P_R + L_{abj} P_L) \tilde{\chi}_j^0 \tilde{l} + g\bar{b}(K_{ij} P_R + M_{ij} P_L) \tilde{\chi}_i^0 \tilde{l} + H.c. \quad (180)
\]

where

\[
\kappa_{i(b)} = \frac{m_{i(b)}}{\sqrt{2} m_W \sin\beta \cos\beta} \quad (181)
\]

and where

\[
L_{bij} = \kappa_i V_{j3} D_{b11}
\]

\[
R_{bij} = -(U_{j1} D_{b11} - \kappa_i U_{j2} D_{b21})
\]

\[
K_{bi3} = -\sqrt{2} [\beta_{bi} D_{b11} + \alpha_{bi} D_{b21}]
\]

\[
M_{bi3} = -\sqrt{2} [\alpha_{bi} D_{b11} - \beta_{bi} D_{b21}] \quad (182)
\]

The corresponding quantities with subscript \( t \) can be obtained by the substitution \( b \to t, U \leftrightarrow V \). The couplings \( R \) and \( L \) are functions of the phases \( \theta_{\mu}, \xi_2 \) and \( \alpha_{A_q} \). The set of phases that enter the couplings \( K \) and
$corrections$ $produce$ $shifts$ $in$ $the$ $couplings$ $as$ $follows$

where $\tilde{\text{neutralinos, neutral Higgs, charged Higgs, squarks,}}$

and other tilde are similarly defined. The loops $B$

yses $of$ $stop$ $and$ $sbottom$ $decays$ $can$ $be$ $found$ $in$ $Bartl$
tances $that$ $already$ $appear$ $at$ $the$ $tree$ $level$. $Recent$ $anal-

cies$ $enhances$ $the$ $CP$ $dependence$ $of$ $the$ $masses$ $and$ $the$ $ver-

ces$ $where$ $they$ $enter$ $are$ $sensitive$ $to$ $the$ $CP$

$SM$, $the$ $CP$ $asymmetry$ $predicted$ $for$

An $interesting$ $phenomenon$ $concerns$ $the$ $fact$ $that$ $in$ $the$

physics $since$ $new$ $physics$ $also$ $enters$ $at$ $the$ $loop$ $level.$

Thus $the$ $process$ $presents$ $a$ $good$ $testing$ $ground$ $for$ $new$

level $contribution$ $and$ $proceeds$ $only$ $via$ $loop$ $corrections.$

$FIG.$ $14$ $A$ $sample$ $of$ $one$ $loop$ $diagrams$ $with$ $CP$ $phase$ $de-

pendent$ $vertices$ $that$ $contribute$ $to$ $the$ $decay$ $of$ $the$ $stops.$

$M$ $is$ $the$ $same$ $above$ $set$ $with$ $an$ $extra$ $phase$ $\xi_i.$ $The$ $loop$
corrections $produce$ $shifts$ $in$ $the$ $couplings$ $as$ $follows$

\[
L_{\text{eff}} = g\bar{t}_i (\bar{R}_{bij} P_R + \bar{L}_{bij} P_L) \tilde{\chi}_i^+ \tilde{b}_i \\
+ g\bar{b} (\bar{R}_{tij} P_R + \bar{L}_{tij} P_L) \tilde{\chi}_j^+ \bar{t}_i + g (\bar{K}_{tij} P_R + \bar{M}_{tij} P_L) \\
\tilde{\chi}_0 \tilde{b}_i + g (\bar{K}_{bij} P_R + \bar{M}_{bij} P_L) \tilde{\chi}_0 \bar{b}_i + H.c. \quad (183)
\]

where $\bar{R}_{bij} = R_{bij} + \Delta R_{bij}$ where $\Delta R_{bij}$ is the loop
correction and other tilde are similarly defined. The loops $that$ $enter$ $the$ $analysis$ $of$ $\Delta^s$ $have$ $gluinos,$ $charginos,$ $neutralin-

als,$ $neutral$ $Higgs,$ $charged$ $Higgs,$ $squarks,$ $W$ $and$ $Z$ $boson$
exchanges. The masses of sparticles as well $as$ the vertices $where$ $they$ $enter$ $are$ $sensitive$ $to$ $the$ $CP$
phases. The analysis using the loop corrected lagrangian enhances $the$ $CP$ $dependence$ $of$ $the$ $masses$ $and$ $the$
vertices $that$ $already$ $appear$ $at$ $the$ $tree$ $level.$ $Recent$ $anal-
yses$ $of$ $stop$ $and$ $sbottom$ $decays$ $can$ $be$ $found$ $in$ $(Bartl$
et $al.,$ $2003,$ $2004c;$ $Ibrahim$ $and$ $Nath,$ $2005)$

**K. $B \rightarrow \phi K$ and CP asymmetries**

Like $B \rightarrow X_s + \gamma,$ the decay $B \rightarrow \phi K_S$ has no tree
level contribution and proceeds only via loop corrections. $Thus$ $the$ $process$ $presents$ $a$ $good$ $testing$ $ground$ $for$ $new$
physics $since$ $new$ $physics$ $also$ $enters$ $at$ $the$ $loop$ $level.$

An $interesting$ $phenomenon$ $concerns$ $the$ $fact$ $that$ $in$ $the$
SM, $the$ $CP$ $asymmetry$ $predicted$ $for$ $B \phi K_S$ is $the$ $same$
as $B \rightarrow J/\Psi K_s$ to $O(\lambda^2)$(Grossman and Worah, $1997$).

The current value of the $B \rightarrow J/\Psi K_S$ experimentally is

\[
S_{J/\Psi K_s} = 0.734 \pm 0.055 \quad (184)
\]

which is in excellent agreement with SM prediction of

$\sin 2\beta = 0.715^{+0.055}_{-0.045} $. $Although$ $currently$ $the$ $experimen-
tal$ $value$ $for$ $S_{\phi K_S}$ $(Aubert$ $et$ $al.,$ $2004)$

\[
S_{\phi K} = 0.50 \pm 0.25 (\text{stat})^{0.07}_{-0.03} (\text{syst}) \quad (185)
\]
is consistent within $1\sigma$ of the SM prediction, its value $has$ $signi-

cantly$ $in$ $the$ $past$ $showing$ $a$ $2.7\sigma$ deviation $from$ $the$ $SM$ $prediction,$ $which$ $triggered$ $much$ $the-

oretical$ $activity$ $to$ $explain$ $the$ $large$ $deviation$ $(Agashe$ $and$ $Carone,$ $2003;$ $Arnowitt$ $et$ $al.,$ $2003;$ $Baek,$ $2003;$
$Chakraverty$ $et$ $al.,$ $2003;$ $Cheng$ $et$ $al.,$ $2004;$ $Chiang$ $and$
$Rosner,$ $2003;$ $Ciuchini$ $and$ $Silvestrini,$ $2002;$ $Datta,$
$2002;$ $Dutta$ $et$ $al.,$ $2003;$ $Hiller,$ $2002;$ $Kane$ $et$ $al.,$ $2003;$

Although the discrepancy has largely disappeared it is $still$ $in-

structive$ $to$ $review$ $briefly$ $the$ $possible$ $processes$ $that$ $could$ $make$ $a$ $large$ $contribution$ $to$ $the$ $B \rightarrow \phi K_S$
process. $It$ $should$ $be$ $noted$ $that$ $the$ $branching$ $ratio$

$BR(B^0 \rightarrow \phi K_S) = (8.0 \pm 1.3) \times 10^{-6}$ $is$ $quite$ $consis-
tent$ $with$ $the$ $SM$ $result.$

The time dependent asymmetries in $B \rightarrow \phi K_S$ are defined so that

\[
A_{\phi K}(t) = \frac{\Gamma(\bar{B}(t) \rightarrow \phi K_S) - \Gamma(B(t) \rightarrow \phi K_S)}{\Gamma(B(t) \rightarrow \phi K_S) + \Gamma(B(t) \rightarrow \phi K_S)} \\
= -C_{\phi K} \sin(\Delta m_B t) + S_{\phi K} \sin(\Delta m_B t) \quad (186)
\]

where $S_{\phi K_S}$ and $C_{\phi K_S}$ are given by

\[
C_{\phi K_S} = \frac{1 + |\lambda_{\phi K_S}|^2}{1 + |\phi_{\phi K_S}|^2}, \quad S_{\phi K_S} = \frac{2 Im \lambda_{\phi K_S}}{1 + |\lambda_{\phi K_S}|^2} \quad (187)
\]

where $\lambda_{\phi K_S}$ is defined by

\[
\lambda_{\phi K_S} = -e^{-2i(\beta + \delta \beta)} \frac{\bar{A}(B^0 \rightarrow \phi K_s)}{\bar{A}(B^0 \rightarrow \phi K_S)} \quad (188)
\]

where $\beta$ is as defined in the SM, and $\delta \beta$ is any possible
new physics contribution. Much of the work in trying to produce large effects within supersym-
metric models has focussed on generating corrections from flavor mixing in the quark sector using the mass
insertion method$(Gabbiani$ $et$ $al.,$ $1996;$ $Hall$ $et$ $al.,$ $1986).$

Thus, for example, an LL type mass insertion in the down quark sector will have the form

\[
(\delta^d_{L_L})_{ij} = (V^d_L(M_4)^2_{LL}V^d_L)_{ij} \quad (189)
\]

Here $(M^2_4)_{LL}$ is the LL down squark mass matrix, $V^d_L$ is the rotation matrix that diagonalizes the down squark
mass matrix, and $\bar{m}$ is the average squark mass. Similarly one defines the mass insertions $(\delta_{LR}^{ij})_{ij}$, $(\delta_{LR}^{ij})_{ij}$ and $(\delta_{RL}^{ij})_{ij}$. Among the supersymmetric contributions considered are the gluino-mediated $b \to sq\bar{q}$ with $q = u,d,s,c,b$ and Higgs mediated $b \to s\bar{s}s$. Typically it is found that the LL and RR insertions give too small an effect but chirality flipping $LR$ and $RL$ insertions can generate sizable corrections to $B\phi K_{S}$. Thus, for example, $|\langle \delta_{LR}^{ij} \rangle| \leq 10^{-2}$ can significantly affect $B \to \phi K_{S}$ while the constraints on $B \to X_{s}\gamma$ and $\Delta M_{s}$ are obeyed. The analysis in $B \to \phi K_{S}$ in supergravity grand unification with inclusion of CP phases was carried out by (Arnrott et al., 2003) and it was concluded that significant corrections to the asymmetries can arise with inclusion in the trilinear soft parameter $A$ with mixings in the second and third generations either in the up sector or in the down sector. A similar analysis of asymmetries in $B \to \eta'K_{S}$ have also been carried out by (Gabrielli et al., 2005).

L. T and CP odd operators and their observability at colliders

In the previous sections we have discussed the effects of CP violation on several phenomena. The list of CP odd or T odd (assuming CPT invariance) is rather long (For a sample, see, (Bernreuther and Nachtmann, 1991; De Rujula et al., 1991; Kane et al., 1992; Valencia, 1994)). We discuss briefly now the possibilities for the observation of CP in collider experiments. First we note that CP phases affect decays and scattering cross sections in two different ways. Thus in addition to generating a CP violating contribution to the amplitudes, they also affect the CP even part of the amplitudes which can affect the overall magnitude of decay widths and scattering cross sections. However, definite tests of CP violation can arise only via the observation of T odd or CP odd parts. As an example of the size of the effects induced by CP odd operators in supersymmetry on cross sections consider the process $e^{+}e^{-} \to t\bar{t}$. Here an analysis in MSSM including loop effects with CP phases gives (Christova and Fabbrichesi, 1993)

$$
\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega} \left( 1 + c \frac{\alpha_{s}}{\pi} \sin(\alpha_{A_{t}} - \phi_{\bar{q}})(\vec{J} \vec{p} \times \vec{k})\right)
$$

(190)

where $\vec{k}$ ($\vec{p}$) are the center of mass momentum of one of the initial (final) particles and $\vec{J}$ is the unit polarization vector of one of the produced $t$ quarks perpendicular to the production plane. $c$ depends on the details of the particle spectrum and can vary significantly depending on the particle spectrum. The choice $c \sim 1$ give the correction of the T-odd observable to be of size $(10^{-3} - 10^{-2})$ which is typically of the same size as the radiative corrections from the Standard Model. More generally in $e^{+}e^{-}$ colliders in the process $e^{+}e^{-} \to X$ with momenta $\vec{p}_{1}, \vec{p}_{2}, \vec{p}$ a product of the type $(\vec{\xi}_{i} \times \vec{\eta}_{j}) \vec{\xi}_{k}$ where $\vec{\xi}_{i}$ is either a momentum or a polarization will give a T-odd observable. For example one will have T-odd operators of the type (Gavela et al., 1989)

$$
T_{1} = (\bar{p}_{1} \times \vec{p}_{2}) \cdot \vec{S}_{e^{-}}
$$

$$
T_{2} = \vec{p} \cdot (\vec{S}_{e^{-}} \times \vec{S}_{e^{+}})
$$

(191)

More generally with several particles ($i = 1, .., n, n > 4$) one can form T-odd operator such as

$$
\epsilon_{\alpha\beta\gamma\delta} \vec{p}_{1}^{\alpha} \vec{p}_{2}^{\beta} \vec{p}_{3}^{\gamma} \vec{p}_{4}^{\delta}
$$

(192)

An example of such an operator is the squark decay $\tilde{t} \to t + l^{+}l^{-} + \chi_{l}^{0}$ which can also lead to an observable signal at the LHC (Langacker et al., 2007). A study of the effects of CP-violating phases of the MSSM on leptonic high-energy observables is given in (Choi et al., 2004a). An efficient way to observe CP violation is via use of polarized beams in $e^{+}e^{-}$ colliders which is of interest in view of the proposed International Linear Collider. A discussion on tests of supersymmetry at linear colliders can be found in (Baer et al., 2004) and a detailed discussion of tests of CP asymmetries is given in (Moortgat-Pick et al., 2005). A number of works related to the effects of CP on the Higgs and sparticle phenomena discussed in this section are (Accomando et al., 2006; Akeroyd and Arribi, 2001; Alan et al., 2007; Bartl et al., 2006, 2004a,b; Boz, 2002; Cheung et al., 2006; Choi et al., 2004b; Ghosh et al., 2005; Heinmeyer et al., 2004; Hollik et al., 1999, 1998). An interesting issue concerns the possibility of expressing CP odd quantities in terms of basis independent quantities for the supersymmetric case similar to the Jarlskog invariant for the case of the Standard Model. Recent works in this direction can be found in (Dreiner et al., 2007; Lebedev, 2003).

Finally we note that the computation of SUSY phenomena with CP phases is more difficult than computations without CP phases. In Sec. (XVIJ) we give a brief discussion of the tools necessary for the computation of SUSY phenomena with CP phases.

XII. FLAVOR AND CP PHASES

CP violation can influence flavor physics (for recent reviews see (Bigi, 2007; Fleischer, 2006; Schopper, 2006)) and thus such effects could be used as probes of the SUSY CP violation effects. This can happen in several ways. This could happen in CP violation effects in K and B physics, or if EDMs of leptons are measured and turn out to be in violation of scaling, and in possible future sparticle decays which may contain flavor dependent CP violating effects. Let us consider first CP violation in K and B physics. Essentially all of the phenomena seen here can be explained in terms of the CP violation with a Standard Model origin, i.e., arising from the phase $\delta_{CKM}$. This means that unless some deviations from the Standard Model predictions
are seen, the supersymmetric CP violation must be small. On the other hand if significant deviations occur from the Standard Model predictions then one would need in addition to the large CP phases a new flavor structure. An example of this is flavor changing terms arising from the off diagonal component in the LR mass matrix \((\delta_{ij})_{LR}(d) = (m^2_{ij}(d))_{ij}/\bar{m}_q^2\) (Demir et al., 2000a,b; Dine et al., 2001, 1993; Khalil and Kobayashi, 1999; Masiero and Murayama, 1999).

Further, if one adopts the view point that the entire CP phenomena in the K and B system arise from the supersymmetric CP phases (Brlhlik et al., 2000a; Frere and Belen Gavela, 1983) then one will need a new flavor structure. But such an assumption appears to be drastic since Yukawa couplings arising from string compactification will typically be complex. However, there are other ways in which CP violation can act as strong probes of flavor physics and vice versa. For instance, SUSY CP effects would be relevant in flavor changing neutral current processes such as \(b \rightarrow s + \gamma\) and in \(\mu \rightarrow e + \gamma\). Also if the EDM of the electron and the muon are eventually determined and a scaling violation is found, then such effects give us a connection between CP violation and flavor. Similarly the connection between CP and flavor can be obtained from collider data in the decays of sparticles.

In the following we discuss two specific phenomena where CP and flavor affects can be significant. Issue of flavor and CP violation is discussed in many papers (Ayazi and Farzan, 2007; Chang et al., 2003; Demir and Farzan, 2005; Demir et al., 2000b; Masiero and Murayama, 1999). Additional papers that discuss these issues are(Ellis et al., 2006; Farzan, 2007; Gronau, 2007; Pospelov et al., 2006a,b). CP and flavor violation in SO(10) is discussed in (Babu et al., 2005; Chen and Maniathappa, 2005; Dutta et al., 2005; Harvey et al., 1980; Nath and Syed, 2001).

A. \(d_\mu\) vs \(d_e\) and possible scaling violations

The EDM of the muon and the electron are essentially scaled by their masses, so that

\[ d_\mu/d_e \simeq m_\mu/m_e \quad (193) \]

The current experimental limits on the muon EDM are much less stringent than on the electron EDM, and thus it is reasonable to ask if the EDM of the muon could be much larger than the EDM of the electron. If so the improved experiments on the muon EDM may be able to detect it. Thus we explore the conditions under which significant violations of scaling may occur. Now we recall from our discussion of the EDM of the electron, that large EDM for the electron generated by the chargino exchange may be canceled by the contributions from the neutralino exchange. Thus one possibility in generating a large muon EDM is to upset this cancelation for the muon case. This appears possible by inclusion of flavor dependent nonuniversals in the soft parameters. To make this idea concrete we consider the chargino and neutralino exchange contributions to a lepton EDM are

\[ d_l = \frac{e_0}{4\pi m^2_{\mu_e}} \frac{\bar{m}_q}{m_{\nu}} \sum_{\nu} \sum_{\nu} m_{\nu_e} I_m(U^\nu_{il}V^\nu_{lj}) \frac{\bar{m}_q^2}{m_{\nu}} \]  

\[ + \frac{e_0}{4\pi m^2_{\mu_e}} \frac{\bar{m}_q}{m_{\nu}} \sum_{\nu} \sum_{\nu} m_{\nu_e} I_m(U^\nu_{il}V^\nu_{lj}) \frac{\bar{m}_q^2}{m_{\nu}} \quad (194) \]

where \(A, B\) and \(\kappa_i\) are defined earlier, and where \(\eta^l_{ik}\) is given by

\[ \eta^l_{ik} = [-\sqrt{2}\tan \theta_3_{l1} X_{l1} + \theta_3_{l2}] D_{l1k} - \]  

\[ -\kappa_i X_{l3} D^*_{l2k}] \sqrt{\tan \theta_3_{l1} X_{l1} D_{l1k} - \epsilon_{l2} X_{l3} D_{l1k}} \quad (195) \]

Here \(X\) diagonalizes the neutralino matrix \(M_{\chi_{l0}}\), and \(D_l\) diagonalizes the slepton (mass)\(^2\) matrix. The chargino exchange contribution depends on the single phase combination \(\xi_2 + \theta_3\), while the neutralino exchange contribution depends additionally on the phase combinations \(\theta_\mu + \xi_1\), and \(\theta_\mu + \alpha_{A_i}\). Non-universalities can be introduced in two ways: via sneutrino masses which enter in the chargino exchange, and via slepton masses that enter in the neutralino exchange diagram. One efficient way to introduce nonuniversalities in the slepton sector is via the trilinear coupling parameter \(A_1\) which can be chosen to be flavor dependent at the GUT scale. In this case the cancelation in the electron EDM sector would not imply the same exact cancelation in the muon sector and significant violations of the scaling relation can be obtained.

Since the violations of scaling arise from the neutralino sector we discuss this in further detail. Here the leading dependence of the lepton mass arises from \(n^l_{ik}\) while subleading dependence arises from the outside sneuon mass factors in Eq.(194). Thus to understand the scaling phenomenon and its breakdown we focus on \(n^l_{ik}\) which can be expanded as follows using Eq.(195).

\[ n^l_{ik} = a_0 c_0 X_{l1} D_{l1k} D_{l2k} + b_0 c_0 X_{l1} X_{l3} D^*_{l1k} D_{l2k} - \kappa_i a_0 X_{l1} X_{l3} D_{l1k}^2 - \kappa_i b_0 X_{l1} X_{l3} D_{l1k} D_{l2k}^2 + \kappa_i^2 X_{l1} X_{l3} D_{l1k} D_{l2k}^2 \quad (196) \]

where \(a_0, b_0\) and \(c_0\) are independent of the lepton mass. The first two terms on the right hand side of Eq.(196) are linear in lepton mass through the relation

\[ Im(D_{l11} D_{l12}) = - Im(D_{l21} D_{l22}) = \frac{m_l}{M^2_{l1} - M^2_{l2}} (m_0 |A_f| \sin \alpha_f + |m| \sin \theta_\mu \tan \beta) \quad (197) \]

The third, fourth and fifth terms on the right hand side of Eq.(196) have a leading linear dependence on the lepton mass through the parameter \(\kappa_i\) and have additional weaker dependence on the lepton mass through the diagonalizing matrix elements \(D_{ij}\). The last term in Eq.(196)
B. SUSY CP phases and the FCNC process $B \rightarrow X_s \gamma$

There are other effects of the CP violating phases on the phenomenological constraint arises from the measurement of the rare decay $B \rightarrow X_s \gamma$. This decay only occurs at the one loop level in the standard model (Altomari et al., 1988; Casalbuoni et al., 1993; Colangelo et al., 1993; Deshpande et al., 1987; Dominguez et al., 1988; Falk et al., 1994; Grinstein et al., 1988). The supersymmetric radiative corrections might be of the same order of magnitude as the standard model contribution (Baer et al., 1998; Barbieri and Giudice, 1993; Barger et al., 1993; Bertolini et al., 1991; Bertolini and Vissani, 1995; Diaz, 1993; Garisto and Ng, 1993; Goto and Okada, 1995; Hewett, 1993; Lopez et al., 1993; Nath and Arnowitt, 1994). It is recently been recognized that supersymmetric contributions can receive significant contributions from the next-to-leading order corrections (NLO) which are enhanced by large tan $\beta$. These are typically parameterized by $\epsilon$'s. In addition to the $\epsilon$'s there are other two loop (NLO) corrections which, however, are small and can be absorbed in a redefinition of the SUSY parameters (Carena et al., 2001; Degrassi et al., 2000). Currently the branching ratio of $B \rightarrow X_s \gamma$ is fairly accurately known experimentally (Abe et al., 2001; Aubert et al., 2002a,b, 2005; Barate et al., 1998; Chen et al., 2001) and imposes very significant constraints on model building. The current experimental value is

$$BR(B \rightarrow X_s \gamma) = (355 \pm 24^{+9}_{-10} \pm 3) \times 10^{-6}$$

as given by the Heavy Flavor Averaging group (Barberio et al., 2006). The Standard Model result with QCD corrections (Chetyrkin et al., 1997) including NLO gives (Gambino and Misiak, 2001) $BR(B \rightarrow X_s \gamma) = (3.73 \pm 0.30) \times 10^{-4}$. A similar robust prediction for supersymmetric models is needed. To analyze the NLO corrections for the supersymmetric case (for recent analyses see, one has to examine the effective Lagrangian describing the interaction of quarks with the charged Higgs fields $H^\pm$ and the charged Goldstones $G^\pm$ which we display below (see, e.g., (Belanger et al., 2002; Demir and Olive, 2002; Gomez et al., 2005, 2006))

$$\mathcal{L}_{\text{eff}} = \frac{g}{\sqrt{2}M_W}G^+\left\{ \sum_d m_d V_{td} \left[ 1 + \epsilon_d(d) \frac{\cot \beta}{1 + \epsilon_d \cot \beta} I_{RD_L} \right] + \sum_u m_u V_{u\mu} \left[ 1 + \epsilon_u'(u) \frac{\tan \beta}{1 + \epsilon_u \tan \beta} \bar{u}_L b_R \right] \right\}$$

$$+ \frac{g}{\sqrt{2}M_W}H^+\left\{ \sum_d m_d V_{td} \left[ 1 + \epsilon_d'(d) \frac{\tan \beta}{1 + \epsilon_d \cot \beta} \right] \bar{u}_L d_L \right\}$$

$$+ \sum_u m_u V_{u\mu} \left[ 1 + \epsilon_u'(u) \frac{\cos \beta}{1 + \epsilon_u \tan \beta} \right] \bar{u}_L b_R + H.c.$$  

(199)
Hamiltonian that governs the decay \( b \to s + \gamma \) where contributions to the NLO corrections to the epsilons in \( b \to s + \gamma \) decay. There are a total of 20 such diagrams.

where

\[
\epsilon_t(b) = \frac{\Delta h_t^2}{h_t} + \tan \beta \frac{\delta h_t^1}{h_t},
\]

\[
\epsilon'_t(b) = \frac{\Delta h_t^{1*}}{h_t} - \cot \beta \frac{\delta h_t^{1*}}{h_t},
\]

\[
\epsilon_b(t) = \frac{\Delta h_b^2}{h_b} + \tan \beta \frac{\delta h_b^1}{h_b},
\]

\[
\epsilon'_b(t) = \frac{\Delta h_b^{1*}}{h_b} + \cot \beta \frac{\delta h_b^{1*}}{h_b},
\]

and where \( \epsilon_{bb} \) and \( \epsilon_{tt} \) is given by

\[
\epsilon_{bb} = \frac{\Delta h_b^2}{h_b} + \cot \beta \frac{\delta h_b^1}{h_b},
\]

\[
\epsilon_{tt} = \frac{\Delta h_t^2}{h_t} + \tan \beta \frac{\delta h_t^1}{h_t}.
\]

Using the above Lagrangian along with the interaction of quarks and \( W \) bosons one can write down the contributions to Wilson coefficients \( C_7 \) and \( C_8 \) in the effective Hamiltonian that governs the decay \( b \to s + \gamma \) (for further details see (Belanger et al., 2002, 2006b; Kagan and Neu- bert, 1998, 1999))

\[
H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}^* V_{tb} \sum_{i=1}^8 C_i(Q) O_i(Q) \quad (202)
\]

where

\[
O_2 = (\bar{c}_L \gamma^\mu b_L)(\bar{s}_L \gamma^\mu c_L)
\]

\[
O_7 = \frac{e}{16\pi^2} m_b \bar{s}_L \sigma^{\mu \nu} b_R F_{\mu \nu}
\]

\[
O_8 = \frac{g_s}{16\pi^2} m_b \bar{s}_L \sigma^{\mu \nu} T^{a \nu} b_R G^{a \nu}_{\mu \nu} \quad (203)
\]

as

\[
C_{7,8}^W (Q W) = F_{7,8}^{(1)} (x_t) + \frac{(\epsilon_{bb}^* - \epsilon_{bb}^t(t)) \tan \beta}{1 + \epsilon_{bb}^* \tan \beta} F_{7,8}^{(2)} (x_t) \quad (204)
\]

\[
C_{7,8}^{H^\pm} (Q W) = \frac{F_{7,8}^{(1)} (y_t) + 1 + \epsilon_{bb}^t(s) \tan \beta}{1 + \epsilon_{bb}^t \tan \beta} F_{7,8}^{(2)} (y_t) \quad (205)
\]

where \( x_t \) and \( y_t \) are defined by

\[
x_t = \frac{m_t^2 (Q W)}{M_W^2}, \quad y_t = \frac{m_t^2 (Q W)}{M_H^2} \quad (206)
\]

and \( F_{7,8}^{(1)} \) and \( F_{7,8}^{(2)} \) are given by

\[
F_{7}^{(1)} (x) = x(7 - 5x - 8x^2) + \frac{x^2 (3x - 2)}{4(x - 1)^3} \ln x
\]

\[
F_{7}^{(2)} (x) = x(3 - 5x) + \frac{x(3x - 2)}{6(x - 1)^3} \ln x
\]

\[
F_{8}^{(1)} (x) = x(2 + 5x - x^2) - \frac{3x^2}{8(x - 1)^3} \ln x
\]

\[
F_{8}^{(2)} (x) = x(3 - x) - \frac{x}{2(x - 1)^3} \ln x \quad (207)
\]

The \( C_7 \) and \( C_6 \) terms receive dominant exchange contributions from the \( W \), charged Higgs and the charginos. The gluino and neutralino exchange terms can also contribute. The gluino exchange contributions have also been computed (Everett et al., 2002). However, it turn out that in the minimal flavor violation (MFV) scenario, the contributions from the gluino and neutralino exchanges are indeed relatively small. The analyses of \( b \to s^+ + \gamma \) in beyond the MFA scenario where generational mixings are taken into account have been carried out in the work of (Foster et al., 2005a,b; Hahn et al., 2005). The most complete analyses of \( B \to X_s \gamma \) in SUSY with the inclusion of NLO’ effects is given in (Buras et al., 2003; Degrassi et al., 2006; Gomez et al., 2006). Specifically in the analysis of (Gomez et al., 2005, 2006) it is shown that the \( \epsilon \)'s as well as the decay \( B \to X_s \gamma \) are sensitive to the CP phases.

XIII. CP PHASES IN \( \nu \) PHYSICS AND LEPTOGENESIS

Recent experiments discussed later in this section show that neutrinos are not massless. In general neutrinos
could have either a Dirac mass, a Majorana mass or perhaps a mixture of the two. For a neutrino to have a Dirac mass there must be a corresponding right handed neutrino to give a mass term of the type $m_D \nu_R^T \nu_R + H.c.$.

On the other hand, one can generate a Majorana mass term from just the left handed neutrinos, i.e., a mass term of the form $\nu_L^T C^{-1} m_L \nu_L + H.c.$, where $C$ is the charge conjugation matrix. For the case of three neutrino species the Majorana mass matrix is in general a symmetric mass matrix of the form (Mohapatra et al., 2004, 2005; Nunokawa et al., 2007)

$$\mathcal{M}_\nu = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & m_{\mu\mu} & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & m_{\tau\tau} \end{pmatrix}$$  

(208)

The Majorana neutrino mass matrix can be diagonalized by an orthogonal transformation so that

$$V^T \mathcal{M}_\nu V = \mathcal{M}_\nu^D$$  

(209)

where $V$ can be written as $V = UK$ and where the matrix $U$ is similar to the CKM matrix and $K$ is a diagonal matrix with two independent Majorana phases. For $U$ one can use the parametrization

$$U = \begin{pmatrix} c_1 c_3 & c_1 s_1 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ -s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & c_1 c_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix}$$  

(210)

where $c_1 = \cos \theta_{12}$, $c_2 = \cos \theta_{23}$, $s_3 = \cos \theta_{13}$ and similarly for $s_1$, $s_2$ and $s_3$, where $\theta_{ij}$ and $\delta$ are constrained so that $0 \leq \theta_{ij} \leq \pi/2$ and $0 \leq \delta \leq 2\pi$. The matrix $K$ is diagonal and can be taken to be

$$K = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_1} & 0 \\ 0 & 0 & e^{i\phi_2} \end{pmatrix}$$  

(211)

Thus we have three diagonal masses, three mixing angles and three phases which together exhaust the full 9 parameter set of the Majorana neutrino mass matrix. The Majorana CP phases do not enter in the neutrino oscillations, and only the Dirac phase $\delta$ does. Thus the oscillation probability from flavor $\nu_\alpha$ to $\nu_\beta$ is given by

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} U_{\alpha i} U_{\beta j}^* U_{\alpha' i'}^* U_{\beta' j'} \times \sin^2 (\Delta m_{ij}^2 L/4E_\nu)$$  

(212)

where $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$. From the solar neutrino and the atmospheric neutrino data (Abdurashitov et al., 1999; Ahmad et al., 2002a,b; Altmann et al., 2000; Ambrosio et al., 2001; Fukuda et al., 2000; Hampel et al., 1999) one finds that the neutrino mass differences are given by

$$\Delta m^2_{sol} = (5.4 - 9.5) \times 10^{-5} \text{ eV}^2,$$

$$\Delta m^2_{atm} = (1.4 - 3.7) \times 10^{-3} \text{ eV}^2.$$  

(213)

A fit to the solar and atmospheric data using the three neutrino-generations gives constraints only on the neutrino mass differences and on the mixing angles. One has

$$\Delta m^2_{sol} = |m_2^2 - m_1^2|,$$

$$\Delta m^2_{atm} = |m_3^2 - m_2^2|,$$

$$\sin^2 \theta_{12} = (0.23 - 0.39), \quad \sin^2 \theta_{23} = (0.31 - 0.72), \quad \sin^2 \theta_{13} < 0.054.$$  

(214)

An interesting aspect of Eq.(214) is that the mixing angles $\theta_{12}$ and $\theta_{23}$ are large with $\theta_{23}$ being close to maximal while $\theta_{13}$ is small. This feature was rather unexpected and quite in contrast to the case of the quarks where the mixings are small. An important point to note is that the neutrino oscillation experiments do not give us any information on the absolute value of the neutrino masses. Other experiments are necessary to provide information on the absolute values such as from cosmology and from the neutrinoless double beta decay. Thus from cosmology one has the following upper bound on each species of neutrino masses (Hannestad, 2003, 2004; Hannestad and Raffelt, 2004; Spergel et al., 2006)

$$\sum_i |m_{\nu_i}| < (0.7 - 1) \text{ eV}$$  

(215)

Similarly the neutrinoless double beta decay gives the following upper bound on the effective neutrino mass $|m_{ee}|$ (Bilenky, 2004; Klappdor-Kleingrothaus et al., 2001)

$$|m_{ee}| < (0.2 - 0.5) \text{ eV}$$  

(216)

where (Mohapatra et al., 2005)

$$|m_{ee}| = |\cos^2 \theta_{13} (m_1^2 \cos^2 \theta_{12} + m_2^2 \sin^2 \theta_{12} e^{2i\phi_1}) + \sin^2 \theta_{13} |m_3^2 e^{2i\phi_2}|$$  

(217)

Several scenarios for the neutrino mass patterns have been discussed in order to explain the data. One possibility considered is that the third generation mass is much larger than the neutrino masses for the first two. Among these are the following: (i) $|m_{\nu_3}| >> |m_{\nu_1,\nu_2}|$, (ii) $|m_{\nu_4} |\sim |m_{\nu_2}|$, $|m_{\nu_1,\nu_2}| >> |m_{\nu_3}|$, (iii) $|m_{\nu_1} |\sim |m_{\nu_2} |\sim |m_{\nu_3}|$, $|m_{\nu_1,\nu_2,\nu_3}| >> |m_{\nu_1}| - |m_{\nu_2}|$. Neutrino oscillations are sensitive to $\delta$ but not to the Majorana phases (Barger et al., 2002b, 1980). As is clear from Eq.(217), Majorana phases do enter in the neutrinoless double beta decay, but an actual determination of CP violation in $\nu_\beta/\beta$ appears difficult (Barger et al., 2002a).

We discuss now briefly the possible determination of $\delta$ in the next generation of neutrino experiments such as NOνA (Ayres et al., 2002, 2004) and T2KK (Hagiwara et al., 2006). We begin by noting that under the condition that CPT is conserved, the conservation of CP would require $P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = 0$. In the presence of CP violation this difference is non-vanishing.
Thus specifically one has (Barger et al., 2007; Nunokawa et al., 2007)

\[
P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) =
-16J \sin(\frac{\Delta m^2_{12}L}{4E}) \sin(\frac{\Delta m^2_{13}L}{4E}) \sin(\frac{\Delta m^2_{23}L}{4E}),
\]

(218)

where \(E\) is the neutrino beam energy, \(L\) is the oscillation length and \(J\) is the Jarlskog invariant for the neutrino mass matrix similar to the one for the quark mass matrix

\[
J = s_{12}c_{13}s_{23}c_{23}s_{13}c^2_{13} \sin \delta.
\]

(219)

We note that \(J\) depends on \(\theta_{13}\) and \(\delta\) both of which are currently unknown and thus one has only an upper limit for \(J\) so that \(J \leq 0.04\). Thus the observation of a CP violation via Eq. (218) depends on other factors. For example, \(J\) vanishes if \(\theta_{13}\) vanishes and thus the effect of CP violation via Eq. (218) would be unobservable. Similarly, if there was a degeneracy in the neutrino masses, for example if \(|m_{\nu_1}| \sim |m_{\nu_2}|\), then again the observation of CP violation via Eq. (218) would be difficult. However, aside from these extreme situations the process Eq. (218) holds the strong possibility that long baseline experiments should allow one to observe CP violation due to \(\delta\) in the neutrino sector. Two experiments are ideally suited for this observation. One of these is NO\(\nu\)A (Ayres et al., 2002, 2004) which will be 25ktot liquid scintillator detector placed 810 km away from the NuMI neutrino beam in Fermilab (see Sec.XIV). The configuration will allow runs in the neutrino as well as in anti-neutrino mode. The second possibility is the T2KK detector (Hagiwara et al., 2006) which is discussed in Sec.XIV.

### A. CP violation and leptogenesis

As already mentioned in Sec.1, achieving baryon asymmetry in the universe requires three conditions: violation of baryon number, violation of C and CP, and departure from thermal equilibrium. Quantitative analyses show that the Standard Model falls short of fulfilling these conditions. Specifically, the amount of CP violation is found not sufficient. Thus in the framework of the electroweak baryogenesis the effective CP suppression factor that enters is \(f_{CP}\) where (Farrar and Shaposhnikov, 1993; Shaposhnikov, 1986)

\[
f_{CP} = T^{-12}_C (m^2_1 - m^2_2)(m^2_1 - m^2_3)(m^2_2 - m^2_3)
(m^2_1 - m^2_3)(m^2_2 - m^2_3)(m^2_2 - m^2_3)s_{12}s_{23}s_{31} \sin \delta
\]

(220)

where \(s_{ij} = \sin \theta_{ij}\) and \(\theta_{ij}\) are the three mixing angles, and \(\delta\) is the CKM phase, and \(T_C\) is the temperature of the electroweak phase transition (EWPT). The EWPT is supposed to occur at values \(T_C \sim 100\) GeV, which leads to \(\delta_{CP} \sim (10^{-18} - 10^{-20})\). A rough estimate of baryon asymmetry in EWPT is \(B \sim 10^{-3}f_{CP}\) and the Standard Model in this case leads to \(B \sim (10^{-26} - 10^{-28})\)

which is far too small compared to the desired value of \(B \sim 10^{-10}\). Additionally there are stringent constraints on the Higgs mass which are already in violation of the current limits. Analysis of baryogenesis in MSSM relieves some of the tension both because there are new sources of CP violation, and also because the Higgs mass limits are significantly larger, e.g., \(m_h \leq 120\) GeV. However, the analysis requires a significant fine tuning of parameters.

An attractive alternative to conventional baryogenesis (for reviews see (Cohen et al., 1993; Riotto and Trodden, 1999)) is baryogenesis via leptogenesis ((Fukugita and Yanagida, 1986). For recent reviews see (Buchmuller et al., 2005; Chen, 2007; Nardi et al., 2006; Nir, 2007a)). The essential idea here is that if one can generate enough lepton asymmetry, then it can be converted into baryon asymmetry via sphleron interactions which violate \(B + L\) but preserve \(B - L\). Leptogenesis is a natural consequence of the See-Saw mechanism (Gell-Mann and SLansky, 1980; Glashow, 1979; Minkowski, 1977; Mohapatra and Senjanovic, 1980; Yanagida, 1979) which is a popular mechanism for the generation of small neutrino masses (see also (Schechter and Valle, 1980, 1982; Valle, 2006) for early work on the See-Saw phenomenology). To generate a See-Saw one needs heavy Majorana neutrinos and one can characterize the Lagrangian for the Majoranas by

\[
L_N = M_1 N_1 N_1 + \lambda_{ij} N_i L_\phi
\]

(221)

where \(N_i\) are the Majorana fields, and \(\lambda\) are in general complex and thus the \(\lambda\) terms violate CP. Further, \(L_N\) violates lepton number and \(B - L\). Thus the Lagrangian of the above type has the general characteristics that might lead to the generation of baryon asymmetry via leptogenesis. The CP violation occurs in the decay of the Majoranas because of the overlap of the tree and the loop.
One can define a CP asymmetry parameter so that
\[ \epsilon_1 = \sum_{a} \frac{\Gamma(N_i \to l_a\phi) - \Gamma(N_i \to \bar{l}_a\phi^\dagger)}{\sum_{a} \Gamma(N_i \to l_a\phi) + \Gamma(N_i \to \bar{l}_a\phi^\dagger)} \]  \hspace{1cm} (222)

For the case of just two Majorana neutrinos the analysis of \( \epsilon_1 \) gives

\[ \epsilon_1 = \frac{C(M_2^2 - M_1^2)}{M_1} \frac{Im(\lambda \lambda^\dagger)_{12}^2}{(\lambda \lambda^\dagger)_{11}^2} \]  \hspace{1cm} (223)

where \( C(z) = C_1(z) + C_2(z) \) where (Covi et al., 1996)

\[ C_1(z) = (8\pi)^{-1} \sqrt{z} [1 - (1 + z) \ln \frac{1 + z}{2} - \ln 2] \]

\[ C_2(z) = (8\pi)^{-1} \frac{\sqrt{z}}{1 - z} \]

For the case of two singlets and \( M_1 < M_2 \) one has

\[ \epsilon_1 = -\frac{3}{8\pi} \frac{M_1}{M_2} \frac{Im(\lambda \lambda^\dagger)_{12}^2}{(\lambda \lambda^\dagger)_{11}^2} \]  \hspace{1cm} (225)

Next consider the case when the initial temperature \( T_i \) is larger than the mass of the lightest singlet neutrino \( N_1 \). In this case neglecting the effect of the decays of the heavier neutrinos, one can write the Boltzmann equations that govern the number densities \( n_{N_1} \) and \( n_{B-L} \) so that (Buchmuller et al., 2002, 2005; Buchmuller and Plumacher, 2001)

\[ \frac{dn_{N_1}}{dx} = -(D + S)(n_{N_1} - n_{eq}^{eq}) \]

\[ \frac{dn_{B-L}}{dx} = -\epsilon_1 D(n_{N_1} - n_{eq}^{eq}) - W n_{B-L} \]  \hspace{1cm} (226)

Here \( x \equiv M_1/T \), and \( W = \Gamma_W/(Hx) \) is the washout term. The processes contributing to the Boltzmann equations are the decays, inverse decays, scattering processes with \( \Delta L = 1 \) and processes with \( \Delta L = 2 \), where \( D = \Gamma_D/(Hx) \) includes decays and inverse decays and \( S = \Gamma_S/(Hx) \) includes \( \Delta L = 1 \) scattering. Two parameters that enter prominently in the analysis are the effective mass \( \bar{m}_1 \) which is defined by

\[ \bar{m}_1 = \frac{(\lambda \lambda^\dagger)_{11} \langle \phi \rangle^2}{M_1} \]  \hspace{1cm} (227)

and the equilibrium neutrino mass \( m* \) defined by

\[ m* = \frac{16\pi^{5/2}}{3\sqrt{5}} \frac{g*}{M_{Pl}} \frac{\langle \phi \rangle^2}{\left| \langle \phi \rangle \right|^2} \]  \hspace{1cm} (228)

where \( g* \) is the total number of degrees of freedom (\( g* = 106.75 \) for SM). Numerically \( m* \sim 10^{-3} \text{eV} \).

The ratio \( \bar{m}_1/m* \) controls whether or not \( N_1 \) decays are out of equilibrium. When \( \bar{m}_1 < m* \) (the weak washout region), \( N_1 \) decay is slower than the Hubble expansion and leptogenesis can occur efficiently. For the case \( \bar{m}_1 > m* \) (the strong washout region) the back reactions that tend to washout are fast and leptogenesis is rather slow. However, even for \( \bar{m}_1/m* >> 1 \), a sufficient amount of lepton asymmetry can be generated. The solution to \( n_{B-L} \) can be obtained in the form

\[ n_{B-L}(x) = n_{B-L}^{eq} \exp(-\int_{x_1}^{x} dx' W(x')) - \frac{3}{4} \epsilon_1 \kappa(x) \]  \hspace{1cm} (229)

where \( n_{B-L}^{eq} = n_{B-L}(x = \infty) \), and where \( \kappa(x) \) is given by

\[ \kappa(x) = -\frac{4}{3} \int_{x_1}^{x} \frac{D}{D + S} \frac{dn_{N_1}}{dx} \exp(-\int_{x_1}^{x'} dx'' W(x'')) \]  \hspace{1cm} (230)

The \( (B - L) \) asymmetry is converted into baryon asymmetry by sphaleron processes so that

\[ \eta_B = \frac{a_{sph}}{f} n_{B-L}^{eq} - \frac{3}{4} \alpha_{sph} \epsilon_1 \kappa f \]  \hspace{1cm} (231)

where \( a_{sph} \) is the sphaleron conversion factor (\( a_{sph} \approx 28/79 \)), and \( f \) is a dilution factor \( f = n_{sph}^{eq}/n_e^{eq} \) which depends on the photon production from the beginning of leptogenesis till the point of recombination, and numerically \( f = 2387/86 \). One then has

\[ \eta_B \sim 10^{-2} \epsilon_1 \kappa f \]  \hspace{1cm} (232)

Now an upper limit on \( \epsilon_1 \) can be obtained assuming that \( N_1 \) decay dominates the asymmetry as assumed above with a hierarchical pattern of heavy neutrino masses, and assuming that the decay of \( N_1 \) occurs for \( T > 10^{12} \text{GeV} \). In this case one can deduce, under the assumption \( M_1/M_2 << 1 \), the result (Davidson and Ibarra, 2002)

\[ \left| \epsilon_1 \right| \leq \frac{3}{16\pi} \frac{M_1 (m_3 - m_2)}{|\langle \phi \rangle|^2} \]  \hspace{1cm} (233)

With \( |m_3 - m_2| \leq \sqrt{\Delta m_{52}^2} \approx .05 \text{eV} \), one finds a lower bound on \( M_1 \) so that

\[ M_1 \geq 2 \times 10^9 \text{ GeV} \]  \hspace{1cm} (234)

This result implies a lower bound on the reheating temperature, and this bound appears to be in conflict with the upper bound on the reheating temperature to control the gravitino overproduction for the supersymmetric case. Consequently several variants of leptogenesis have been studied such as resonant leptogenesis (Pilaftsis, 1997; Pilaftsis and Underwood, 2004, 2005), soft leptogenesis (Boubekeur et al., 2004; Grossman et al., 2003, 2004), and non-thermal leptogenesis (Fujii et al., 2002). The type of CP violation that occurs in leptogenesis involves neutrinos which are Standard Model singlets, and
hence have no direct gauge interactions with the normal particles, and in addition are very heavy. Thus a direct observation of CP violation that enters leptogenesis would be essentially impossible in laboratory experiments. However, in unified models CP phases could be inter-related across different sectors and thus indirect constraints on such phases could arise in such models.

B. Observability of Majorana phases

In the previous subsection we found that the leptogenesis does depend crucially on the Majorana phases (for a review of Majorana particles and their phases, see e.g., (Kayser, 1984, 1985)). However, these phases arise from heavy Majoranas and are not the same as the Majorana phases that arise in light neutrino mass sector. It was noted in our discussion of the neutrino masses that Majorana phases do not enter in neutrino oscillations which depend only on the Dirac phase. The Majorana phases do enter in the neutrinoless double beta decay. However, they do so only in a CP even fashion and further their observation in the $0\nu\beta\beta$ appears difficult. The question one might ask is in what processes the Majorana phases can enter in a manifestly CP odd fashion. It is known that one such process is neutrino-anti-neutrino ($\nu \to \bar{\nu}$) oscillations (Berneabeu and Pascual, 1983; de Gouvea et al., 2003; Schechter and Valle, 1981). The analysis of (de Gouvea et al., 2003) sets out some simple criteria for their appearance in scattering phenomena. Thus consider the amplitude for the process $X$ where

$$A_X = e^{i\xi_X} (A_1 + A_2 e^{i(\delta + \phi)})$$

where we have pulled out a common phase factor $e^{i\xi_X}$ so $A_1$ has no phase dependent factor multiplying it, $\delta$ is a CP even phase and $\phi$ is a CP odd phase. Then the mirror process $\bar{X}$ has the following amplitude

$$A_{\bar{X}} = e^{i\xi_X} (A_1 + A_2 e^{i(\delta - \phi)})$$

where $A_{1,2}$ are assumed not to contain any CP violating effects and are the same in processes $X$ and $\bar{X}$. The difference $\Delta \Gamma_{CP} = |A_X|^2 - |A_{\bar{X}}|^2$ is then given by

$$\Delta \Gamma_{CP} = 4A_1A_2 \sin(\delta) \sin(\phi)$$

(237)

The above simple analysis points to the following three conditions necessary for CP odd effects to arise in the process $X$ vs its mirror process $\bar{X}$. These are (a) the existence of two distinct contribution to the amplitude, (b) the two contributions must have a non-vanishing relative CP odd phase, and (c) they must also have a non-vanishing relative CP even phase. The analysis of (de Gouvea et al., 2003) considers application to the process

$$l^+_\alpha W^+ \to \nu \to l^+_\beta W^+$$

(238)

for which one has the amplitude

$$A_X = \sum_i (\lambda_i U_{\alpha i} U_{\beta i}) \frac{m_i}{E} e^{-i\frac{m_i^2 L}{2E}} S$$

(239)

where $E$ is the energy of the intermediate state which propagates a microscopic distance $L$, $U$ is the mixing matrix, and $S$ depends on the initial and the final states and on kinematical factors. For the CP-conjugate process $l^-_\alpha W^- \to l^-_\beta W^-$ one has

$$A_{\bar{X}} = \sum_i (\lambda_i U_{\alpha i} U_{\beta i}) \frac{m_i}{E} e^{-i\frac{m_i^2 L}{2E}} \bar{S}$$

(240)

where the combination $(\lambda_i U_{\alpha i} U_{\beta i})$ is free of the phase-convention (Bilenky et al., 1984; Kayser, 1984; Nieves and Pal, 1987, 2003). Limiting the analysis to the case of two generations we write

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

(241)

Under the approximation $\lambda_1 = 1 = \lambda_2$, $|S| = |\bar{S}|$, $\alpha = \epsilon$ and $\beta = \mu$ this leads to

$$\Delta \Gamma_{CP} = |A_X|^2 - |A_{\bar{X}}|^2 = \frac{m_1m_2}{4E^2} |S|^2 \sin^2 2\theta \sin \left( \frac{(m_2^2 - m_1^2)L}{2E} \right)$$

(242)

The above example satisfies all the criterion set forth earlier for a CP odd effect to appear. CP odd effects can also appear in lepton number violating meson processes such as $K^\pm \to \pi^\pm \mu^\pm \mu^\pm$. Thus, for example, if we write

$$A_{K^+} = e^{i\xi_{K^+}} (A_{1K} + A_{2K} e^{i(\delta_{K^+} + \phi_{K^+})}),$$
$$A_{K^-} = e^{i\xi_{K^-}} (A_{1K} + A_{2K} e^{i(\delta_{K^-} - \phi_{K^+})})$$

(243)

one will have $\Delta \Gamma_{CP} = |A_{K^-}|^2 - |A_{K^+}|^2$ given by

$$\Delta \Gamma_{CP} \propto 4A_{1K}A_{2K} \sin(\delta_{K^}) \sin(\phi_{K})$$

(244)

$\Delta L = 2$ contributions do arise with $R$ parity violation in supersymmetry and contribute to $\Delta \Gamma_{CP}$. However, the effect turns out to be extremely small. Some possible cosmological effects of CP violation are considered in (Khlopov and Petcov, 1981).

XIV. FUTURE PROSPECTS

A. Improved EDM experiments

There are good prospects of improving the EDM bounds significantly. Thus future experiments may improve the sensitivity of EDM experiments by an order of magnitude or more (Dzuba et al., 2002; Kawall et al., 2004; Kozlov and Derevianko, 2006) and in some cases
by a significantly larger factor (Lamoreaux, 2001; Semertzidis, 2004; Semertzidis et al., 2004). A recent review on the current experimental situation and future prospects regarding the electron electric dipole moment can be found in (Commins and DeMille, book chapter). Regarding the neutron EDM a sensitivity at the level of $1.7 \times 10^{-28}$ ecm could be achieved (Balashov et al., 2007) and even a sensitivity of $10^{-29}$ ecm is possible (Harris, 2007). Regarding the EDM of $^{199}$Hg improved measurements are in progress and a factor of 3-4 improvement over the next year or so is possible. Beyond that there are various projects aimed at improving the limit with diamagnetic atoms, using Xe-129, radioactive Ra or Rn. However, all of them are still in the development phase, so when one may expect better limits from these experiments is unclear. Regarding the muon EDM one proposed experiment (Semertzidis, 2004; Semertzidis et al., 2004) feasible at JPARC (Japan Proton Accelerator Reseach Complex) could extend the sensitivity to as much as $10^{-24}$ ecm. However, it appears that the earliest muons may become available at JPARC is 2016. However, recently another proposal for muon EDM has been made where the existing muon beam $\mu$EI at PSI could be used. It is claimed that the muon EDM with a sensitivity of better than $d_\mu \sim 5 \times 10^{-25}$ ecm within one year of data taking is feasible (Adelmann and Kirch, 2006). Currently there is also an exploration underway regarding the possible determination of the deuteron EDM using techniques similar to the ones used for muon EDM with the goal of reaching a sensitivity of $10^{-29}$ ecm (Semertzidis, 2007; Semertzidis et al., 2004).

**B. B physics at the LHCb**

LHCb is one of the four detectors at the LHC, the other three being ATLAS, CMS, and ELLIS. Of the these ATLAS and CMS are the main particle physics detectors dedicated to the search for new physics such as supersymmetry or extra dimensions. While the ATLAS and CMS can also study B physics their capabilities in this respect are rather limited. On the other hand LHCb is an experiment which is specifically dedicated to the study of B physics. Thus the B mesons produced in collisions at the LHC are likely to lie in angles close to the beam directions and a detector ideal for the study of B physics should be able to detect such particles. This is precisely what the LHCb is designed to do. Specifically the detection of charged particle will be accomplished by its Ring-Imaging Cherenkov (RICH) detector. The precise identification of the interaction region utilizes a vertex locator or VELO which can be used for B-tagging, and more generally for the separation of primary and secondary vertices. The number of B mesons produced at LHCb will be enormous. Even a luminosity of $10^{32}$ cm$^{-2}$s$^{-1}$ will lead to a number of $b\bar{b}$ events at the rate of $O(10^{11-12})$ per year. Thus the LHCb will have an unprecedented opportunity to study B physics in great depth.

C. Super Belle proposal

The B factories are an ideal instrument for the study of elements of the CKM matrix including the CP phase $\delta_{CKM}$. The analyses provided by the B factories at SLAC (BaBar) and at KEK (Belle) have given a wealth of data and have improved the measurements of the CKM elements. Specifically they have been able to measure time dependent CP asymmetries with good precision. Further improvements in the measurements of these elements will come only with significantly greater luminosity. The Super Belle proposal aims at achieving that by an upgrade of the KEKB collider to a luminosity of $10^{35-36}$ cm$^{-2}$s$^{-1}$. Such an improvement will also require an upgrade of the vertex detector for the Super Belle and specific proposals are under study (Kawasaki et al., 2006).

D. Superbeams, $\nu$ physics, and CP

The answer to the question whether or not CP phases appear in neutrino physics is of crucial relevance to our understanding of fundamental interactions. The observation of such phases in the light neutrino sector is possible using long baseline experiments and intense beams (Diwan et al., 2006; Marciano and Parsa, 2006; Marciano, 2001) and its observation will give greater credence to the hypothesis that such phases also appear in the heavy neutrino sector which enter in leptogenesis. Thus the AIP-2004 study recommends ”as a high priority, a comprehensive U.S. program to complete our understanding of neutrino mixing, to determine the characteristic of the neutrino mass spectrum, and to search for CP violation among neutrinos” (Freedman and Kayser, 2004). Such high priority efforts could include improved $\nu\beta\beta$ experiments, and super beams to study neutrino oscillations and detect CP phases. Specifically the study recommends ”a proton driver in the megawatt class or above and neutrino superbeam with an appropriate very large detector capable of observing CP violation and measuring the neutrino mass-squared differences and mixing parameters with high precision”. One such proposal is an upgraded Fermilab Proton Driver (FPD). Such an upgrade will improve the study of $\nu_\mu \rightarrow \nu_e$ oscillations by a significant factor (Geer, 2006). Thus the current Fermilab NuMI proton beam has $10^{13}$ protons at 120 GeV (a beam power of .2 megawatts). A secondary beam of charged pions is generated from the proton beam, and the pions then decay producing a beam of tertiary $\nu_\mu$ as they propagate along a long corridor to the target 735 km downstream. With .2 megawatt of proton beam power

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3 See, e.g., http://www-pnp.physics.ox.ac.uk/lhc/b/
one generates only $10^{-5}$ interactions in a 1kt detector at the far end. Thus an upgrade of the proton beam to deliver several megawatt of proton beam power coupled with an upgrade of the detector to 10 kt will significantly enhance the sensitivity of the detector to observe possible CP effects. A similar idea being discussed is T2KK where the far detector is put on the east coast of Korea along the Tokai to Kamioka (T2K) neutrino beam line (Hagiwara et al., 2006).

XV. CONCLUSIONS

We have attempted here to give a broad overview of CP violation and the effect of CP phases arising from physics beyond the Standard Model. We know that CP violation beyond what is allowed in the Standard Model must exist in order that one generate the desired amount of baryon asymmetry in the universe. We have examined the origin of such CP violation in some of the leading candidates for physics beyond the Standard Model. These include models based on extra dimensions, supersymmetric models with soft breaking, and string models. Specifically supersymmetric models and string models generate a plethora of new CP phases and one problem one encounters is that such phases lead to EDMs for the electron and for the neutron in excess of the current limits. One way to limit to these is to fine tune the phases to be small which, however, is not satisfactory from the point of generation of baryon asymmetry. What one needs is a mechanism which allows at least some of the phases to be large while suppressing their contribution to the EDMs. One possibility is suppression of the EDMs by having a heavy sparticle spectrum. However, this possibility puts the sparticle masses at least for the first two generations in the several TeV range and thus outside the reach of the LHC. An alternative possibility of controlling the EDMs is the cancelation mechanism which allows for large phases consistent with the stringent limits on the EDMs from experiment. If the cancelation mechanism is valid, then the effect of CP phases will show up at colliders in a variety of supersymmetric phenomena. We have discussed some of these phenomena in this report. One important such phenomenon is CP even -CP odd Higgs mixing which would lead to discernible signals at Hadron colliders and at a future International Linear Collider (ILC). Effects of CP could also be visible in $B^0 \rightarrow \mu^+ \mu^-$, in Higgs decays $h^0 \rightarrow b \bar{b}, \tau \bar{\tau}$ and in sparticle decays. Dark matter analyses are also affected, specifically the detection cross section for neutralino-nucleon scattering.

The future proposed experiments will investigate CP phenomena with vastly increased data. Chief among these is the LHCb experiment which is dedicated to the study of the B mesons. The proposed Super Belle will further add to these efforts. These will pin down the CKM matrix elements to a much greater precision than BaBar and Belle, and may shed light on the possibility whether or not new sources of CP violation are visible. However, if the sparticles are indeed observed, as one expects they will be, then a study of their branching ratios is likely to put significant limits on CP phases from sparticle decays.

Acknowledgments

We thank Norman F. Ramsey for communications regarding the early history of CP violation. Discussions and communications with a number of other colleagues are also acknowledged. These include Giuseppe Degrassi, Damien Easson, Yasaman Farzan, Mario Gomez, Maxim Kholopov, Olaf Kittel, Alberto Lerda, Apostolos Pilaftsis, and Jose Valle. Communications with Eugene Commins regarding the electron EDM experiment, with Philip Harris regarding the neutron EDM experiment, with Yannis K. Semertzidis regarding the muon EDM and the EDM of the deuteron, and with Michael Romalis on the EDM of $^{199}\text{Hg}$ are acknowledged. Assistance by Daniel Feldman and Zuowei Liu during preparation of the manuscript is acknowledged. The work was supported in part by the NSF grant PHY-0546568.

XVI. APPENDICES

A. Chargino and neutralino mass matrices with phases

Here we give some details on the diagonalization of the chargino and neutralino mass matrices which are in general complex. These appear in the analysis of Secs. (IX) and (X). We consider the chargino mass matrix first. Here we have

$$M_C = \begin{pmatrix} \tilde{m}_2 & \sqrt{2m_W} \sin \beta \\ \sqrt{2m_W} \cos \beta & \mu \end{pmatrix}$$

The chargino matrix $M_C$ is not hermitian, is not symmetric and is not real since $\mu$ and $\tilde{m}_2$ are complex. For simplicity we analyze its diagonalization for real $\tilde{m}_2$ and complex $\mu$. Generalization for complex $\tilde{m}_2$ and $\mu$ is straightforward. $M_C$ can be diagonalized by using the following biunitary transformation

$$U'^* M_C V^{-1} = M_D$$

Here $U'$ and $V$ are hermitian matrices and $M_D$ is a diagonal matrix which, however, is not yet real. $U'$ and $V$ satisfy the relation

$$V(M_C^* M_C) V^{-1} = \text{diag}(|\tilde{m}_{\chi_2}|^2, |\tilde{m}_{\chi_1}|^2)$$

$$= U'^* (M_C^* M_C) (U'^*)^{-1}$$

We may parameterize $U'$ so that

$$U' = \begin{pmatrix} \cos \frac{\theta_1}{2} & e^{i\phi_1} \sin \frac{\theta_1}{2} \\ -e^{-i\phi_1} \sin \frac{\theta_1}{2} & \cos \frac{\theta_1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \cos \frac{\theta_1}{2} & \sin \frac{\theta_1}{2} e^{i\phi_1} \\ -\sin \frac{\theta_1}{2} e^{-i\phi_1} & \cos \frac{\theta_1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \cos \phi_1 & \sin \phi_1 \\ -\sin \phi_1 & \cos \phi_1 \end{pmatrix}$$
\( \tan \theta_1 = 2 \sqrt{2} m_W (\tilde{m}_2^2 - |\mu|^2 - 2m_W^2 \cos 2\beta)^{-1} \)

\[
(\tilde{m}_2^2 \cos^2 \beta + |\mu|^2 \sin^2 \beta + |\mu| \tilde{m}_2 \sin 2\beta \cos \theta_\mu)^{\frac{1}{2}}
\]

and

\( \tan \phi_1 = |\mu| \sin \theta_\mu \sin (\tilde{m}_2 \cos \beta + |\mu| \cos \theta_\mu \sin \beta)^{-1} \)

Similarly we parameterize \( V \) so that

\[
V = \begin{pmatrix}
\cos \frac{\theta_2}{2} & \sin \frac{\theta_2}{2} e^{-i\phi_2} \\
-sin \frac{\theta_2}{2} e^{i\phi_2} & \cos \frac{\theta_2}{2}
\end{pmatrix}
\]

where

\( \tan \theta_2 = 2 \sqrt{2} m_W (\tilde{m}_2^2 - |\mu|^2 - 2m_W^2 \cos 2\beta)^{-1} \)

\[
(\tilde{m}_2^2 \sin^2 \beta + |\mu|^2 \cos^2 \beta + |\mu| \tilde{m}_2 \sin 2\beta \cos \theta_\mu)^{\frac{1}{2}}
\]

and

\( \tan \phi_2 = -|\mu| \sin \theta_\mu \cos \beta (\tilde{m}_2 \sin \beta + |\mu| \cos \theta_\mu \cos \beta)^{-1} \)

We wish to choose the phases of \( U' \) and \( V \) so that the elements of \( M_D \) will be positive. Thus we define \( U = H \times U' \) where

\[
H = (e^{i\gamma_1}, e^{i\gamma_2})
\]

where \( \gamma_1, \gamma_2 \) are the phases of the diagonal elements of \( M_D \) in Eq.(246). With the above choice of phases one has

\[
U^* M_C V^{-1} = \text{diag}(\tilde{m}_{x_1}^+, \tilde{m}_{x_2}^+) \]

Our choice of the signs and the roots is such that

\[
M_{\tilde{m}_{x_1}^+}(\tilde{m}_{x_2}^+) = \frac{1}{2} [\tilde{m}_2^2 + |\mu|^2 + 2m_W^2] (+) (-)
\]

\[
\frac{1}{2} [2(m_2^2 - |\mu|^2)^2 + 4m_W^2 \cos 2\beta + 4m_W^2] (\tilde{m}_2^2 + |\mu|^2 + 2m_W^2 |\mu| \cos \theta_\mu \sin 2\beta)^{\frac{1}{2}}
\]

where the sign chosen is such that \( \tilde{m}_{x_1}^+ < \tilde{m}_{x_2}^+ \) if

\( \tilde{m}_2^2 < |\mu|^2 + 2m_W^2 \cos 2\beta \).

For the neutralino mass matrix \( M_{\tilde{\chi}_0} \) one has

\[
\begin{pmatrix}
\tilde{m}_1 & 0 & -M_{Zs_Wc_\beta} & M_{Zc_Wb_\beta} \\
0 & \tilde{m}_2 & -M_{Zs_Wc_\beta} & M_{Zc_Wb_\beta} \\
-M_{Zs_Wc_\beta} & M_{Zc_Wb_\beta} & 0 & -\mu \\
M_{Zs_Wc_\beta} & -M_{Zc_Wb_\beta} & \mu & 0
\end{pmatrix}
\]

In the above \( s_W = \sin \theta_W, c_\beta = \cos \beta \) and \( s_\beta = \sin \beta \). The matrix \( M_{\tilde{\chi}_0} \) is a complex non hermitian and symmetric matrix, which can be diagonalized by a unitary transformation such that

\[
X^T M_{\tilde{\chi}_0} X = \text{diag}(m_{\chi_1}, m_{\chi_2}, m_{\chi_3}, m_{\chi_4})
\]

\[
B. \text{Squark and slepton mass}^2 \text{ matrices with phases}
\]

In this appendix we give details on the diagonalization of the squark and slepton mass matrices that appear in Secs.(IX) and (X). We consider the squark \( (mass)^2 \) matrix

\[
M_{\tilde{q}}^2 = \begin{pmatrix}
M_{\tilde{q}_{11}} & M_{\tilde{q}_{12}} \\
M_{\tilde{q}_{21}} & M_{\tilde{q}_{22}}
\end{pmatrix}
\]

For the up squark case one has

\[
M_{\tilde{u}_{11}} = M_Q^2 + m_u^2 + M_d^2 (\frac{1}{2} - Q_u s_W^2) \cos 2\beta
\]

\[
M_{\tilde{u}_{12}} = m_u (A_u - \mu \cot \beta)
\]

\[
M_{\tilde{u}_{21}} = m_u (A_u - \mu^* \cot \beta)
\]

\[
M_{\tilde{u}_{22}} = m_u^2 + m_d^2 + M_d^2 Q_u s_W^2 \cos 2\beta
\]

Thus the squark mass\(^2\) matrix is hermitian and can be diagonalized by the unitary transformation

\[
D_u^T M_{\tilde{u}}^2 D_u = \text{diag}(M_{\tilde{u}_{11}}, M_{\tilde{u}_{22}})
\]

where one parameterizes \( D_u \) so that

\[
D_u = \begin{pmatrix}
\cos \frac{\theta_u}{2} & \sin \frac{\theta_u}{2} e^{-i\phi_u} \\
-sin \frac{\theta_u}{2} e^{i\phi_u} & \cos \frac{\theta_u}{2}
\end{pmatrix}
\]

Here \( M_{\tilde{u}_{21}} = |M_{\tilde{u}_{21}}| e^{i\phi_u} \) and we choose the range of \( \theta_u \) so that \( -\frac{\pi}{2} \leq \theta_u \leq \frac{\pi}{2} \) where \( \tan \theta_u = \frac{|M_{\tilde{u}_{21}}|}{M_{\tilde{u}_{21}} + M_{\tilde{u}_{22}}} \). The eigenvalues \( M_{\tilde{u}_{11}}^2 \) and \( M_{\tilde{u}_{22}}^2 \) can be determined directly from Eq.(260) so that

\[
M_{\tilde{u}_{1(1)}(2)} = \frac{1}{2} (M_{\tilde{u}_{11}} + M_{\tilde{u}_{22}})(+)(-)
\]

\[
\frac{1}{2} [(M_{\tilde{u}_{11}} - M_{\tilde{u}_{22}})^2 + 4|M_{\tilde{u}_{21}}|^2]^{\frac{1}{2}}
\]

The \((+)\) in Eq.(264) corresponds to the case so that for \( M_{\tilde{u}_{11}} > M_{\tilde{u}_{22}} \) one has \( M_{\tilde{u}_{11}} > M_{\tilde{u}_{22}} \) and vice versa. For our choice of the \( \theta_u \) range one has

\[
\tan \theta_u = \frac{2m_u |A_u m_0 - \mu^* \cot \beta|}{M_{\tilde{u}_{11}}^2 - M_{\tilde{u}_{22}}^2}
\]

and

\[
\sin \phi_u = \frac{m_0 |A_u| \sin \alpha_u + |\mu| \sin \theta_u R_u}{|m_0 A_u - \mu^* \cot \beta|}
\]

where \( R_u = \cot \beta \). The analysis for the down squark case proceeds in a similar fashion with the following changes

\[
M_{\tilde{d}_{11}}^2 = M_Q^2 + m_d^2 - M_d^2 (\frac{1}{2} + Q_d s_W^2) \cos 2\beta
\]

\[
M_{\tilde{d}_{12}} = m_d (A_d - \mu \tan \beta)
\]

\[
M_{\tilde{d}_{21}} = m_d (A_d - \mu^* \tan \beta)
\]

\[
M_{\tilde{d}_{22}} = m_d^2 + m_d^2 + M_d^2 Q_d s_W^2 \cos 2\beta
\]
The other changes are the modification of expressions for \( \theta_4 \) and \( \phi_d \). They read

\[
\tan \theta_4 = \frac{2m_d |A_d m_0 - \mu^* \tan \beta|}{M_{d11}^2 - M_{d22}^2} \quad (268)
\]

and

\[
\sin \phi_d = \frac{m_0 |A_d| \sin \alpha_d + |\mu| \sin \theta_4 R_d}{m_0 A_d - \mu^* \tan \beta}. \quad (269)
\]

where \( R_d = \tan \beta \). Finally for the case of the electrons

\[
M_{e11}^2 = M_E^2 + m_e^2 - M_Z^2 \left( \frac{1}{2} - s_W^2 \right) \cos 2\beta
\]

\[
M_{e12}^2 = m_e (A_e^* - \mu \tan \beta)
\]

\[
M_{e22}^2 = m_E^2 + m_e^2 - M_Z^2 \sin 2\beta \cos 2\beta \quad (270)
\]

Expressions for \( \theta_4 \) and \( \phi_e \) are identical to the case of the down quark with the replacement of \( d \) by \( e \).

C. RG evolution of electric dipole, color dipole and purely gluonic operators

In this Appendix we discuss the renormalization group (RG) evolution of the EDMs discussed in Sec. (X). As discussed in the text, there are three competing operators that contribute to the EDM of the neutron. These are

\[
O_E = -i \frac{\bar{q} \sigma_{\mu\nu} \gamma_5 q F^{\mu\nu}}{2}
\]

\[
O_{qC} = -i \frac{\bar{q} \sigma_{\mu\nu} \gamma_5 T^a q G^{\mu\nu}}{2}
\]

\[
O_G = -\frac{1}{6} f^{abc} G_a G_b \tilde{G}_c. \quad (271)
\]

The one loop RG evolution of the electric dipole and of the color dipole operators can be easily obtained using their anomalous dimensions since these operators are eigenstates under the renormalization group. Evolving these operators from a high scale \( Q = M_Z \) to a low scale \( \mu \) one finds

\[
O_i(\mu) = \Gamma^{-\gamma_i / 3} O_i(Q) \quad (272)
\]

where

\[
\Gamma = \frac{G_s(\mu)}{G_s(Q)}, \quad \gamma_C = (29 - 2N_f)/3,
\]

\[
\gamma_E = 8/3, \quad \beta = (33 - 2N_f)/3, \quad (273)
\]

where \( N_f \) is the number of light quarks at the scale \( \mu \). Regarding the purely gluonic dimension six operator it obeys the following renormalization group equation (Boyd et al., 1990; Braaten et al., 1990a,b; Dai et al., 1990; Weinberg, 1989)

\[
\frac{\partial}{\partial \mu} O_G = \frac{\alpha_s(\mu)}{4\pi} \left( \gamma_G O_G - 6 \sum q m_q(\mu) O_{qC} \right), \quad (274)
\]

where \( \gamma_G = -3 - 2N_f \). The gauge coupling \( \alpha_s \) and the running quark mass satisfy the RG equations

\[
\frac{\partial}{\partial \mu} g_s(\mu) = -\beta \frac{\alpha_s(\mu)}{4\pi} g_s(\mu), \quad (275)
\]

and

\[
\frac{\partial}{\partial \mu} m_q(\mu) = \gamma_m \frac{\alpha_s(\mu)}{4\pi} m_q(\mu). \quad (276)
\]

where \( \gamma_m = -8 \). The above operators contribute to the CP violating Lagrangian multiplied by coefficients which must cancel their \( \mu \) dependence. This allows one to obtain for the coefficients the following relations

\[
d^{(E,C,G)}(\mu) \simeq \Gamma^{(E,C,G)}(\beta \mu) d^{(E,C,G)}(Q), \quad (277)
\]

where \( Q \) is the high scale. In implementing the RG evolution one uses the matching conditions due to crossing the heavy thresholds for \( q=b,c \). Thus, for example,

\[
d^G(m_q) = d^G(m_+^q) + d^G(m_-^q) \frac{1}{8\pi} \frac{\alpha_s(m_q)}{m_q}. \quad (278)
\]

Using this technique one can evolve the EDMs from the electroweak scale \( Q = M_Z \) down to the hadronic scale \( \mu \). A more up-to-date discussion of the RG evolution of operators including the mixings between the electric and the chromoelectric operators is given in (Degrassi et al., 2005).

D. Satisfaction of the EDM constraints in the cancelation mechanism

Here we give some examples of the parameter points where the cancelation mechanism discussed in Sec. (X.D) works to produce \( d_e, d_n \) and \( d_{Hg} \) consistent with the current limits. Table 1 gives three sets of points (a), (b) and (c) for which the corresponding EDMs \( d_e, d_n \) and \( d_{Hg} \) are listed in Table 2 where \( C_{Hg} \) is related to the \( \tilde{d}_E, \tilde{d}_C, \tilde{d}_G \) by \( C_{Hg} = |\tilde{d}_E - \tilde{d}_C - \tilde{d}_G| \). Using the experimental constraints on \( d_{Hg} \) one obtains the following constraint on \( C_{Hg} \)

\[
C_{Hg} < 3.0 \times 10^{-26} \text{cm}. \quad (279)
\]

The values of \( C_{Hg} \) listed in Table 2 are consistent with the above experimental constraint.

| case | \( m_0, m_{1/2}, |A_0| \) | \( \alpha_4, \xi_1, \xi_2, \xi_3 \) |
|------|-----------------|-----------------|
| (a)  | 200, 200, 4     | 1.5, 659.633   |
| (b)  | 370, 370, 4     | 2.6, 653.672   |
| (c)  | 320, 320, 3     | .8, 4, 668.6   |

<table>
<thead>
<tr>
<th>case</th>
<th>( d_e ) (ecm unit)</th>
<th>( d_n ) (ecm unit)</th>
<th>( C_{Hg} ) (cm unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>1.45 \times 10^{-21}</td>
<td>9.2 \times 10^{-21}</td>
<td>7.2 \times 10^{-27}</td>
</tr>
<tr>
<td>(b)</td>
<td>-1.14 \times 10^{-21}</td>
<td>-7.9 \times 10^{-21}</td>
<td>2.87 \times 10^{-26}</td>
</tr>
<tr>
<td>(c)</td>
<td>-3.5 \times 10^{-21}</td>
<td>7.1 \times 10^{-21}</td>
<td>2.9 \times 10^{-26}</td>
</tr>
</tbody>
</table>
E. Combination of CP phases in SUSY processes

The various phenomena discussed in Secs.(IX) and (X) involve several specific combinations of CP phases. Below we exhibit these combinations.

Table 3: Examples of CP phases in SUSY phenomena

<table>
<thead>
<tr>
<th>SUSY Quantity</th>
<th>Combinations of CP phases</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \rightarrow \nu_i K^+$</td>
<td>$\xi_1, \xi_2, \xi_3 + \theta_1, \alpha_{A_{i,h}} + \theta_1$</td>
</tr>
<tr>
<td>$b \rightarrow s + \gamma$</td>
<td>$\alpha_{A_{i,h}} + \theta_1, \xi_1, \xi_2, \xi_3 + \theta_1$</td>
</tr>
<tr>
<td>$H^0$ mixing and spectrum</td>
<td>$\alpha_{A_{i,h}} + \theta_1, \xi_1, \xi_2, \xi_3 + \theta_1$</td>
</tr>
<tr>
<td>$H^+ \rightarrow \chi^\mu \chi^\nu$</td>
<td>$\alpha_{A_{i,h}} + \theta_1, \xi_1, \xi_2, \xi_3 + \theta_1$</td>
</tr>
<tr>
<td>$y_{\mu} - 2$</td>
<td>$\alpha_{A_{i,h}} + \theta_1, \xi_1, \xi_2, \xi_3 + \theta_1$</td>
</tr>
<tr>
<td>$q \rightarrow q\chi$</td>
<td>$\alpha_{A_{i,h}} + \theta_1, \xi_1, \xi_2, \xi_3 + \theta_1$</td>
</tr>
<tr>
<td>Dark matter</td>
<td>$\alpha_{A_{i,h}} + \theta_1, \xi_1, \xi_2, \xi_3 + \theta_1$</td>
</tr>
<tr>
<td>$H^0 \rightarrow \chi^\mu \chi^\nu$</td>
<td>$\alpha_{A_{i,h}} + \theta_1, \xi_1, \xi_2, \xi_3 + \theta_1$</td>
</tr>
<tr>
<td>$d_e (d_\nu)$</td>
<td>$\alpha_{A_{i,h}} + \theta_1, \xi_1, \xi_2, \xi_3 + \theta_1$</td>
</tr>
<tr>
<td>$d_\nu$</td>
<td>$\alpha_{A_{i,h}} + \theta_1, \xi_1, \xi_2, \xi_3 + \theta_1$</td>
</tr>
</tbody>
</table>

In the Table (3) $\theta_1$ is defined so that $\theta_1 = \theta_h + \theta_H$ and the rest of phases are defined as in Eqs.(31) and (33).

F. Details of $g_\mu - 2$ analysis in SUSY with CP Phases

Here we give further details of the analysis of $a_\mu$ discussed in Sec.(XI.A) but limiting ourselves to the case when the muon mass can be neglected relative to other masses. The chargino exchange contribution is given by

$$a_\mu^{\chi^-} = a_\mu^{21} + a_\mu^{22},$$

where for $a_\mu^{21}$ and $a_\mu^{22}$ we consider now the limit where $I_3(\alpha, \beta)$ and $I_4(\alpha, \beta)$ that appear in Eq.(113) have their first arguments set to zero. In this case one has

$$I_3(0, x) = -\frac{1}{2} F_3(x), \quad I_4(0, x) = -\frac{1}{6} F_4(x)$$

where

$$F_3(x) = \frac{1}{(x-1)^3} (3x^2 - 4x + 1 - 2x^2 \ln x)$$

$$F_4(x) = \frac{1}{(x-1)^4} (2x^3 + 3x^2 - 6x + 1 - 6x^2 \ln x).$$

In the limit considered above one has the following explicit expressions for the chargino contributions

$$a_\mu^{21} = \frac{m_\mu^2 \alpha_{EM}}{2\pi \sin^2 \theta_W} \sum_{i=1}^{2} \frac{1}{M_{\chi_i^0}} \text{Re}(\kappa_{\mu} U_{i2}^* V_{i1}) F_3(M_{\chi_i^0}^2 M_{\chi_i^0}^2)$$

and

$$a_\mu^{22} = \frac{m_\mu^2 \alpha_{EM}}{24\pi \sin^2 \theta_W} \sum_{i=1}^{2} \frac{1}{M_{\chi_i^0}^2} (|\kappa_{\mu} U_{i2}|^2 + |V_{i1}|^2) F_4(M_{\chi_i^0}^2 M_{\chi_i^0}^2).$$

where

$$\kappa_{\mu} = \frac{m_\mu}{\sqrt{2} M_W \cos \beta}$$

Next we discuss the neutralino exchange contributions to $a_\mu$. These are given by

$$a_\mu^{\chi^0} = a_{11}^{\mu} + a_{12}^{\mu},$$

where

$$a_{11}^{\mu} = \frac{m_\mu^2 \alpha_{EM}}{2\pi \sin^2 \theta_W} \sum_{j=1}^{4} \sum_{k=1}^{4} \frac{1}{M_{\chi_j^0}^2} \text{Re}(\eta_{\mu j}^k) I_1(M_{\chi_j^0}^2 M_{\chi_j^0}^2),$$

and

$$a_{12}^{\mu} = \frac{m_\mu^2 \alpha_{EM}}{4\pi \sin^2 \theta_W} \sum_{j=1}^{4} \sum_{k=1}^{4} \frac{1}{M_{\chi_j^0}^2} X_{\mu j}^k I_2(M_{\chi_j^0}^2 M_{\chi_j^0}^2),$$

Here $\eta_{\mu j}^k$ is defined by

$$\eta_{\mu j}^k = -\left(\frac{1}{\sqrt{2}} [\tan \theta_W X_{1j} + X_{2j}] X_{1k}^* - \kappa_{\mu} X_{3j} D_k^* \right)$$

$$\left(\sqrt{2} \tan \theta_W X_{1j} D_k + \kappa_{\mu} X_{3j} D_k^* \right)$$

and $X_{\mu j}^k$ is defined by

$$X_{\mu j}^k = \frac{m_\mu^2}{2 M_W \cos \beta} \left| X_{1j} \right|^2$$

$$+ \frac{1}{2} \tan^2 \theta_W |X_{1j}|^2 (|D_k|^2 + 4|D_{k2}|^2)$$

$$+ \frac{1}{2} |X_{2j}|^2 |D_{k1}|^2 \cos \theta_W |D_{1k}|^2 \text{Re}(X_{1j} X_{j2})$$

$$+ \frac{m_\mu \tan \theta_W}{M_W \cos \beta} \text{Re}(X_{3j} X_{1j} D_k D_{k2}^*)$$

$$- \frac{m_\mu}{M_W \cos \beta} \text{Re}(X_{3j} X_{2j} D_k D_{k2}^*).$$

In the limit when the muon mass is neglected relative to other masses and the first argument in the double integral is taken to be zero one finds a simplification of the form factors so that

$$I_1(0, x) = \frac{1}{2} F_1(x), \quad I_2(0, x) = \frac{1}{6} F_2(x),$$

where

$$F_1(x) = \frac{1}{(x-1)^3} (1 - x^2 + 2x \ln x),$$

and

$$F_2(x) = \frac{1}{(x-1)^4} (-x^3 + 6x^2 - 3x - 2 - 6x \ln x).$$
G. Stop exchange contributions to Higgs mass\(^2\) matrix.

For completeness we give here an analysis of the one loop contributions from the stop sector with inclusion of CP violating effects in the analysis of CP even-CP odd Higgs mixings discussed in Sec. (XI.B). The contribution to the one loop effective potential from the stop and top exchanges is given by

\[
\Delta V (\tilde{t}, t) = \frac{1}{64\pi^2} \sum_{a=1,2} 6M^2_t (\log \frac{M_t^2}{Q^2} - \frac{3}{2}) f_2(m^2_{t_1}, m^2_{t_2})
\]

Using the above potential our analysis for \(\Delta_{ij}t\) gives

\[
\Delta_{11t} = -2\beta_h m_t^2 |\mu|^2 \left( \frac{(A_t | \cos \gamma_t - |\mu|\cot \beta)^2}{(m^2_{t_1} - m^2_{t_2})^2} f_2(m^2_{t_1}, m^2_{t_2}) \right)
\]

\[
\Delta_{22t} = -2\beta_h m_t^2 \frac{|\mu| |A_t| |\cos \gamma_t - |\mu|\cot \beta \cos \gamma_t|^2}{(m^2_{t_1} - m^2_{t_2})^2} f_2(m^2_{t_1}, m^2_{t_2}) + 2\beta_h m_t^2 \ln \left( \frac{m^2_{t_1}}{m^2_{t_2}} \right)
\]

\[
4\beta_h m_t^2 \frac{|A_t| |\cos \gamma_t - |\mu|\cot \beta \cos \gamma_t|^2}{(m^2_{t_1} - m^2_{t_2})^2} \ln \left( \frac{m^2_{t_1}}{m^2_{t_2}} \right)
\]

\[
\Delta_{12t} = -2\beta_h m_t^2 \frac{|\mu| |A_t| \sin \gamma_t - |\mu|\cot \beta |A_t| |\cos \gamma_t|^2}{(m^2_{t_1} - m^2_{t_2})^2} f_2(m^2_{t_1}, m^2_{t_2})
\]

\[
\Delta_{23t} = -2\beta_h m_t^2 \frac{|\mu| |A_t| \sin \gamma_t - |\mu|\cot \beta |A_t| \sin \gamma_t}{\sin \beta (m^2_{t_1} - m^2_{t_2})} f_2(m^2_{t_1}, m^2_{t_2}) + 2\beta_h m_t^2 |A_t| \sin \gamma_t \ln \left( \frac{m^2_{t_1}}{m^2_{t_2}} \right)
\]

\[
\Delta_{31t} = -2\beta_h m_t^2 |\mu|^2 |A_t|^2 |\sin \gamma_t|^2 \frac{\sin \gamma_t |\mu|\cot \beta - |A_t| |\cos \gamma_t|^2}{\sin \beta (m^2_{t_1} - m^2_{t_2})^2} f_2(m^2_{t_1}, m^2_{t_2}).
\]

In the analysis above the D terms of the squark (mass)\(^2\) matrices are ignored to obtain approximate independence of the renormalization scale Q similar to the analysis of (Carena et al., 2000; Denir, 1999).

H. Fierz rearrangement relations involving Majoranas

Fierz rearrangements are known to be very useful when manipulating interactions involving four fermions. Specifically such Fierz rearrangements are needed in the analysis of Sec. (XI.G). Here we give these relations for the case when two of the fermions are Majoranas (such as neutralinos) and the other two are quarks. Thus any four fermi interaction with two Majoranas and two quarks can be written as involving the following combinations

\[
\bar{x} \chi_i q \bar{q}, \bar{x} \gamma^\mu \gamma^5 q \bar{q}, \bar{x} \gamma^\mu \gamma^5 \chi_i q \bar{q}, \bar{x} \gamma^\mu \gamma^5 \chi_i q \bar{q}
\]

For convenience define the 16 gamma matrices as follows

\[
\Gamma^A = \{1, \gamma^0, i\gamma^i, \gamma^0 \gamma^5, \gamma^i \gamma^5, \gamma_5, i\sigma^{0i}, \sigma^{ij} \} : \ i, j = 1 \sim 3
\]

with the normalization

\[
tr(\Gamma^A \Gamma^B) = 4\delta^{AB}
\]

The Fierz rearrangement formula with the above definitions and normalizations is

\[
(u_1 \Gamma^A u_2)(u_3 \Gamma^B u_4) = \sum_{C,D} F_{CD}^{AB} (u_1 \Gamma^C u_4)(u_3 \Gamma^D u_2)
\]

where \(u_j\) are Dirac or Majorana spinors and

\[
F_{CD}^{AB} = \frac{-1}{16} tr(\Gamma^C \Gamma^A \Gamma^D \Gamma^B)
\]

and where the +ve sign is for commuting u spinors and the -ve sign is for the anticommuting u fields. In our case we have to use the -ve sign since we are dealing with quantum Majorana and Dirac fields in the Lagrangian. We give below the Fierz rearrangement for four combinations that appear commonly in neutralino-quark scattering. These are

\[
\bar{x} q \bar{q} \chi = \frac{1}{4} \bar{x} \gamma^\mu q \bar{q} + \frac{1}{4} \bar{x} \gamma^\mu \gamma^5 q \bar{q} + \frac{1}{4} \bar{x} \gamma^\mu \gamma^5 \chi_i q \bar{q},
\]

\[
\bar{x} \gamma^\mu q \bar{q} \chi = \frac{1}{4} \bar{x} \gamma^\mu \gamma^5 q \bar{q} \gamma^\mu q \bar{q} - \frac{1}{4} \bar{x} \gamma^\mu \chi_i q \bar{q},
\]

\[
\bar{x} \gamma^\mu \gamma^5 q \bar{q} \chi = \frac{1}{4} \bar{x} \gamma^\mu \gamma^5 \chi_i q \bar{q} - \frac{1}{4} \bar{x} \gamma^\mu \chi_i q \bar{q},
\]

\[
\bar{x} \gamma^\mu \gamma^5 \chi_i q \bar{q} \chi = \frac{1}{4} \bar{x} \gamma^\mu \gamma^5 q \bar{q} - \frac{1}{4} \bar{x} \gamma^\mu \gamma^5 \chi_i q \bar{q}
\]

The metric used above is \(\eta_{\mu\nu} = (1, -1, -1, -1)\), and since \(\chi\)'s are Majoranas we have used the properties \(\bar{x} \gamma_i \chi = 0\) and \(\bar{x} \sigma_{\mu\nu} \chi = 0\).

I. Effective four-Fermi interaction for dark matter detection with inclusion of CP phases

In this appendix we give a derivation of the four fermi neutralino-quark effective Lagrangian with CP violating
phases given in Sec.(XI.G). We begin by discussing the squark exchange contribution. From the fundamental supergravity Lagrangian of quark-squark-neutralino interactions

\[ -\mathcal{L} = \bar{q} [C_{qL} P_L + C_{qR} P_R] \chi \bar{q} i + \bar{q} [C'_{qL} P_L + C'_{qR} P_R] \chi \bar{q} q + H.c. \]  

(307)

the effective lagrangian for \( q - \chi \) scattering via the exchange of squarks is given by (Chattopadhyay et al., 1999; Falk et al., 1999a).

\[
\mathcal{L}_{eff} = \frac{1}{M^2_{q1} - M^2_\chi} \bar{q} [C_{qL} P_L + C_{qR} P_R] \chi \bar{q} [C_{qL} P_L + C_{qR} P_R] \chi \\
+ \frac{1}{M^2_{q2} - M^2_\chi} \bar{q} [C'_{qL} P_L + C'_{qR} P_R] \chi \bar{q} [C'_{qL} P_L + C'_{qR} P_R] \chi
\]

(308)

where

\[
C_{qL} = \sqrt{2}(\alpha_{d0} D_{q11} - \gamma_{q0} D_{q21}), \\
C_{qR} = \sqrt{2}(\beta_{d0} D_{q11} - \delta_{q0} D_{q21}), \\
C_{qL}' = \sqrt{2}(\alpha_{d0} D_{q12} - \gamma_{q0} D_{q22}), \\
C_{qR}' = \sqrt{2}(\beta_{d0} D_{q12} - \delta_{q0} D_{q22}),
\]

(309)

and where \( \alpha, \beta, \gamma, \) and \( \delta \) are given by

\[
\alpha(j) = \frac{g m_{u(d)} X_{4(3)} j}{2 m_W \sin \beta (\cos \beta)}, \\
\beta(j) = \frac{\beta_{u(d)} X_{3j}^*}{\cos \theta_W}, \\
\gamma(j) = \frac{\gamma_{u(d)} X_{3j}}{\cos \theta_W}, \\
\delta(j) = \frac{\delta_{u(d)} X_{3j}^*}{2 m_W \sin \beta (\cos \beta)}.
\]

(310)

Here \( g \) is the \( SU(2)_L \) gauge coupling and

\[
X_{ij} = X_{ij} \cos \theta_W + X_{ij} \sin \theta_W, \\
X_{ij}^* = -X_{ij} \sin \theta_W + X_{ij} \cos \theta_W.
\]

(311)

The effect of the CP violating phases enter via the neutralino eigenvector components \( X_{ij} \) and via the matrix \( D_{ij} \) that diagonalizes the squark mass matrix.

Using the Fierz rearrangement one can obtain now the coefficients A, B, C, D, E and F that appear in Eq.(171) in a straightforward fashion (Chattopadhyay et al., 1999; Falk et al., 1999a). The first two terms \( \langle A, B \rangle \) are spin-dependent interactions and arise from the \( Z \) boson and the sfermion exchanges. For these one has

\[
A = \frac{g^2}{4 M^2_W} \left[ |X_{30}|^2 - |X_{40}|^2 \right] \left[ T_{3q} - e_q \sin^2 \theta_W \right] \\
- \frac{|C_{qR}|^2}{4(M^2_{q1} - M^2_\chi)} - \frac{|C_{qR}'|^2}{4(M^2_{q2} - M^2_\chi)}
\]

(312)

\[
B = - \frac{g^2}{4 M^2_W} \left[ |X_{30}|^2 - |X_{40}|^2 \right] e_q \sin^2 \theta_W + \\
\frac{|C_{qL}|^2}{4(M^2_{q1} - M^2_\chi)} + \frac{|C_{qL}'|^2}{4(M^2_{q2} - M^2_\chi)}
\]

(313)

The terms \( C, D, E \) and \( F \) receive contributions from sfermions and from neutral Higgs and can be calculated using similar techniques.

J. Computational tools for SUSY phenomena with CP phases

The numerical analysis of supersymmetric phenomena with CP phases is significantly more difficult than for the case when the phases are absent. First most numerical integration codes for the renormalization group evolution, sparticle spectra and for the analysis of sparticle decays and cross sections are not equipped to handle phases. Second any physically meaningful set of parameters which include phases must necessarily satisfy the stringent EDM constraints which also require care. A significant progress has been in this direction by the so called CPsuperH (Lee et al., 2004), which is a Fortran code that calculates the mass spectrum and decay widths of the neutral and charged Higgs bosons in MSSM with CP phases. Obviously there is significant room for further progress in this area.

References

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