Recently much interest has been generated by the possibility that a nontrivial scale invariant sector of an effective field theory [1] characterized by unparticles [2] could play a role in low energy physics. This has led to several further investigations of unparticle effects in collider physics and elsewhere [3]. In models of this type one assumes that the ultraviolet theory has hidden sector operators $O_{UV}$ of dimension $d_{UV}$ possessing an IR fixed point. These couple via a connector sector with the standard model operators $O_{SM}^\nu$ of dimension $n$, generating an effective interaction $O_{UV}O_{SM}^\nu/M_{UV}^{d_{UV}+n-4}$. It is then assumed that the fields of the hidden sector undergo dimensional transmutation at scale $\Lambda_U$ generating scale invariant unparticle fields $O$ with dimension $d_{UV}$ which give the interaction

$$\left(\frac{\Lambda_U}{M_U}\right)^{d_{UV}+n-4} \frac{O_{SM}}{\Lambda_U^{d_{UV}+n-4}}. \tag{1}$$

The operator $O$ could be a scalar, a vector, a tensor, or even a spinor. If $O$ is a tensor of rank two it could couple to the stress tensor and its exchange between physical particles could lead to a modification of Newtonian gravity. We discuss this issue in this Letter.

We work strictly within the framework where conformal invariance holds down to low energies, and thus we forbid scalar unparticle operators of dimension $d_{UV} < 2$ which could have couplings to the Higgs field of the type $H^2O$. The presence of a super-renormalizable operator destroys conformal invariance once $H$ develops a VEV [4]. In our analysis here we consider an effective operator of the type

$$\kappa_s \frac{1}{\Lambda_U^{d_{UV}^2}} \sqrt{g} T^{\mu\nu} O_{\mu\nu}^U \tag{2}$$

where $\kappa_s$ is defined by $\kappa_s = \Lambda_U^{d_{UV}} (\Lambda_U/M_U)^{d_{UV}}$. We assume that $O_{\mu\nu}^U$ transforms like a tensor under the general coordinate transformations, and thus the interaction of Eq. (2) gives an action which is invariant under the transformations. For convenience we assume that $O_{\mu\nu}^U$ is traceless.

The addition of Eq. (2) to the action changes the stress-energy tensor so that the new tensor is $T^{\mu\nu} = T^{\mu\nu} + (\kappa_s/\Lambda_U^{d_{UV}^2}) g^{\mu\nu} T_{\mu\nu}$. The conservation condition in this case is $T^{\mu\nu} = 0$. The interaction of Eq. (2) implies that the unparticles can be exchanged between massive particles, and this exchange creates a new force, a “fifth” force, which we call “ungravity” which adds to the force of gravity. We wish to compute the correction to the Newtonian gravitational potential arising from the exchange of the unparticles to the lowest order. In this case one may neglect all the gravitational effects and replace $g_{\mu\nu}$ by $\eta_{\mu\nu}$. The quantity that enters in the computation of the unparticle exchange contribution is the unparticle propagator

$$\Delta^{\mu\nu\rho\sigma}_U(p) = \int e^{iP_{\mu\rho}\cdot x} [O^U_{\nu\sigma}(x)O^U_{\rho\sigma}]^{(3)}(x) dx. \tag{3}$$

An analysis of this propagator using spectral decomposition [2,3] gives

$$\Delta^{\mu\nu\rho\sigma}_U(p) = \frac{A_{d_{UV}}}{\sin(\pi d_{UV})} p^{\mu\nu\rho\sigma}(\eta^{\mu\nu} + \alpha P^\mu P^\nu P_{\rho\sigma}) = (-\eta^{\mu\nu} + P^\mu P^\nu P_{\rho\sigma})$, \tag{4}$$

and

$$A_{d_{UV}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{UV}}} \Gamma(d_{UV} + 1/2) \Gamma(d_{UV} - 1) \Gamma(2d_{UV}). \tag{5}$$

Further, since we are interested in computing the effects of the unparticles to the lowest order, we can replace $T^{\mu\nu}$ by $T^{\mu\nu}_U$ and replace $T^{\mu\nu}_T = 0$ by $T^{\mu\nu}_U = 0$. This condition implies that momentum factors when acting on the source give a vanishing contribution, and the relevant part of $P^{\mu\nu\rho\sigma}$ can be written as $P^{\mu\nu\rho\sigma} = \frac{1}{2} (\eta^{\mu\nu} \eta^{\rho\sigma} + \eta^{\rho\sigma} \eta^{\mu\nu} - \alpha \eta^{\mu\nu} \eta^{\rho\sigma})$. For the case of the graviton exchange $\alpha = 1$, and retaining $\alpha$ in the analysis will provide a quick check to the graviton exchange limit.
We have carried out an analysis of the unparticle exchange along with the one graviton exchange and computed the effective potential in the nonrelativistic limit. We find

\[ V(r) = -m_1 m_2 G \left( \frac{1}{r} + \frac{\xi^2}{\Lambda_{dU}^4} \frac{(2 - \alpha)}{(2\pi)^2} \sqrt{\frac{2}{\pi}} \right) \]

\[ \times \frac{\Gamma(2 - d_{dU})}{\Gamma(2 d_{dU})} \frac{\Gamma(d_{dU} + \frac{2}{3})}{\Gamma(d_{dU} + 2)} f_{d_{dU}}(r), \]

where \( \xi = \kappa / \kappa \) and \( \kappa = M_{P1} \) where \( M_{P1} \) is the Planck mass \( M_{P1} = 2.4 \times 10^{18} \text{ GeV} \), and where \( f_{d_{dU}}(r) \) is given by

\[ f_{d_{dU}}(r) = 4\pi \int [d^3q/(2\pi)^3] e^{-iqr}/(q^2)^{2-d_{dU}}. \]

The first term in the parentheses in Eq. (6) is the Newtonian term, while the second term is the ungravity correction. One can easily check that the ungravity correction produces the correct Newtonian potential for the case \( d_{dU} = 1 \) and \( \alpha = 1 \) since \( f_{d_{dU}}(r) = 1/r \) in this case. However, for \( d_{dU} \) different from unity the ungravity effects produce an \( r \) dependence of the form \( 1/r^{2d_{dU}-1} \) which can be differentiated from the effects of ordinary gravitation. An explicit evaluation gives

\[ V(r) = -m_1 m_2 G \left[ 1 + \left( \frac{\Lambda_{dU}}{r} \right)^{2d_{dU}-2} \right], \]

\[ R_G = \frac{1}{\pi \Lambda_U} \left( \frac{M_{P1}}{M_*} \right)^{(1/d_{dU}-1)} \]

\[ \times \frac{\left(2(2 - \alpha) \frac{\Gamma(d_{dU} + \frac{2}{3})\Gamma(d_{dU} + \frac{1}{2})}{\Gamma(2 d_{dU})}\right)^{(1/2d_{dU}-2)}}{\sqrt{\pi}}, \]

where \( M_* = \kappa^{-1} \). The choice \( d_{dU} < 1 \) will produce corrections to the gravitational potential which fall off slower than \( 1/r \) and thus would modify the very large distance behavior of the gravitational potential, which appears undesirable. Thus a sensible constraint on \( d_{dU} \) is \( d_{dU} > 1 \) in which case the ungravity contribution to the potential falls off faster than \( 1/r \) and the short distance behavior will be affected. Constraints of conformal invariance in this case require \( [5,6] d_{dU} > 1 + s \), where \( s \) is the spin of the operator, and thus for a rank one tensor operator \( d_{dU} > 2 \) and for a rank two \( d_{dU} > 3 \). Let us now consider a spin zero unparticle operator with \( d_{dU} \geq 2 \) with coupling of the type \( \sqrt{g} T_{\mu}O^U / \Lambda_{dU}^{d_{dU}-1} \). In this case the modified Newtonian potential can be gotten from Eq. (7) by replacing \( (2 - \alpha) \) by \( 2 \). With this choice the corrections to the potential can begin with terms of \( O(1/r^{4+2\delta}) \), \( \delta > 0 \).

The short distance ungravity contribution is constrained by the recent precision submillimeter tests of the gravitational inverse square law \([7,8]\). The current experiment probes short distances up to around 0.05 mm, and no significant deviation from the inverse square law is seen. However, better precision experiments in the future will be able to explore the parameter space of the model more thoroughly. Returning now to Eq. (7), the quantity \( R_G \) may be constrained by experiment. The usual parametrization of the correction to Newtonian gravity in terms of a Yukawa term is not directly suitable for the present case. Instead, we extrapolate the power law limits in Table I of Ref. [8] to obtain an upper limit on \( R_G \) as a function of \( d_{dU} \).

The result of this exercise is shown in the left panel of Fig. 1, where the current data excludes the region above the curve. In the right panel in Fig. 1 we present an analysis of the allowed region of the \( \Lambda_{dU} - M_* \) parameter space which follows from Eq. (7) when combined with the constraint on \( R_G \). Here the regions below the curves are excluded by the current data.

It is of interest that for \( M_* \approx M_{P1} \) the value of \( \Lambda_{dU} \) required for proximity to the present bound is very low. In order to assess this possibility and explore the constraint of a conformal fixed point we examine an SU(3) gauge theory with \( N_f \) massless Dirac fermions. In this case an infrared fixed point occurs at a coupling \( [9] \alpha_s = -4\pi(11N - 2N_f)/(34N^2 - 10N_N - 3(N^2 - 1)N_f/N). \)

For values of \( N_f \) close to and below \( 11N/2 \) but above \( N_f = (100N^2 - 66)/(25N^2 - 15) \) where the chiral symmetry breaking occurs, one is in the region of a conformal fixed point. In this region the scale \( \Lambda_{dU} \) is roughly given by the scale \( \Lambda \) in Ref. [9]:

\[ \Lambda_{dU} \approx M_{dU} \exp \left[ \frac{-1}{b\alpha_3} \ln \left( \frac{\alpha_s}{\alpha(M_G)} \right) - \frac{1}{b\alpha(M_G)} \right], \]

where \( b = (11N - 2N_f)/6\pi \). Thus for \( N = 3 \), the region of the conformal fixed point is \( 16.5 > N_f > 11.9 \). To get an estimate we set \( M_{dU} = 1 \times 10^{16} \text{ GeV} \), \( \alpha(M_G) \approx 0.04, N = 3 \), and \( N_f = 12 \) and find an infrared fixed point at \( \alpha_s = 0.75 \) which gives \( \Lambda_{dU} \approx 10^{-11} \text{ GeV} \). This is an explicit demonstration that an IR fixed point can occur with \( \Lambda_{dU} \) very small, which is of interest in our analysis.

The modification of gravity discussed here differs from the modification induced by extra dimensions in several aspects \([10]\). First, in extra dimension Arkani-Hamed–Dimopoulos–Dvali models \([10]\) the corrections to the potential from extra dimensions falls off exponentially at large distances \( r/R > 1 \), where \( R \) is a compactification length scale, while at short \( r/R \ll 1 \), the \( r \) dependence has the form \( 1/r^{n+1} \) where \( n \) is an integer. This is to be contrasted with Eq. (7) where the correction from ungravity...
ity has the $r$ dependence of the form $1/r^{2d_U-1}$ both at short
as well as at large distances, and further $d_U$ can take on
noninteger values. Further, for the case of extra dimensions
the constraint that the physics of the solar system not be
modified eliminates $n = 1$ [10], and one has modifi-
cations of the Newtonian potential for $n = 2$ of the form $1/r^3$. For
the case of the warped extra dimension model [11–13] the
correction to the gravitational potential can interpolate
between $n = 1$ and $n = 2$ for the case with small warping
[12], i.e., between the form $1/r^2$ and $1/r^3$. However, both
for the warped and the unwarped dimension case the
analytic form of the correction to the potential is signifi-
cantly different from the one in ungravity. Thus it should
be feasible to distinguish between extra dimension models
including models with warped dimensions from the un-
gravity correction to the gravitational potential.

We note that purely kinematical corrections to the
Newtonian potential have been computed in general rela-
tivity [14]. The sign of this correction as well as its $r$
dependence differs from the one computed here. Further,
the effective $R_G$ in this correction is $R_{GR} = G(m_1 + m_2)/c^2$
and is of the size of the Planck length or smaller.
Thus in the context of submillimeter experiments these
corrections are negligible. Finally, it is interesting to note
that renormalization group analyses of quantum gravity in
4 and higher dimensions [15] show that the graviton propa-
gator near an ultraviolet fixed point scales as

$$\frac{1}{p^{2(1-\eta/2)}}$$

where $\eta = 4 - D$ near the fixed point with $D$ the number of space-time dimensions. This propagator
has a resemblance to the one that appears in Eq. (4). Of
course the typical length scale in quantum gravity is the
Planck length while the length scale in ungravity can lie in
the submillimeter region and be accessible to experiment.

The interaction operator $\kappa_\rho \sqrt{g} T_{\mu\nu} O^\mu_{\rho}/\Lambda_U^{2d_U-1}$ can also play a role in high energy scattering, and its domain of
validity is also constrained from that consideration.

Consider the process $f\bar{f} \rightarrow$ scalar unparticle ($f$ is a fermion), which would give a Feynman amplitude $M = m_i p_i v(p_2)/M_d \Lambda_U^{d_U-1}$, where $m$ is the mass of the fermion
and $p_1$ and $p_2$ are the incident momenta. Using the
notation and phase space calculation of [2], we find a cross
section

$$\sigma(f\bar{f} \rightarrow \mathcal{U}) = \frac{1}{4\pi} \left( \frac{m}{M_d} \right)^2 \frac{1}{\Lambda_U^{d_U-2}} A_d.$$  \hfill (9)

(Restriction to the inclusive reaction enables us to probe
dimensions $d_U > 2$ without encountering the pole term
$\sin(\pi d_U)$ [2].) Since the annihilation to the unparticle
proceeds through a single partial wave ($s$ wave), the cross
section is bounded by unitarity, $\sigma < 16\pi/s$. From Eqs. (5)
and (9) this gives an upper bound on the energy for the
compatibility of the effective unparticle Lagrangian with
unitarity [16]:

$$\sqrt{s} < \frac{1}{R_s} \left( \frac{64\pi M_{pl}}{A_d m} \right)^{1/d_U-1},$$  \hfill (10)

where we have expressed the unitarity constraint in terms of
the quantity $R_s = (1/\Lambda_U)(M_{pl}/M_d)^{1/d_U-1}$ propor-
tional to the quantity $R_G$ defined in Eq. (7). The present upper bound on $R_s$ (see Fig. 1) can be rewritten in terms of $R_s$:

for the region of interest $2 < d_U < 3$, a convenient pa-
rametrization $R_s^{\max} \approx 0.5 + 1.75(d_U - 2) \times 10^{12}$ GeV$^{-1}$ will suffice.

If the exchange of the scalar unparticle is to be consist-
et with the present Newton’s law experiments, yet have a
chance of showing up in future experiments, $R_s$ must lie
below $R_s^{\max}$ but above (say) 0.1. Inserting this in (10),
we obtain the following result: for the worst-case scenario,
with the fermion being the top quark, unitarity is not
violated up to 1.2 TeV for $2 < d_U < 2.3$. (Above this
energy, rescattering corrections are significant.) For the
other fermions, of course, the range of validity is larger.
Even if we require compatibility with perturbative QCD
for the light quarks (including the $b$-quark), which is a
tighter constraint, it allows $2 < d_U < 2.2$ for $\sqrt{s}^{\max} \approx 1.2$ TeV. There are similar bounds if $T_{\mu\nu}^\mu$ is saturated with the
 gluon trace anomaly. To sum up, we can maintain
compatibility of scale invariance with both high and low
energy constraints and simultaneously not rule out seeing
corrections to Newton’s law in the next generation of
gravitational experiments.

Corrections to Coulomb’s law can also be similarly
computed if one assumes couplings of a vector unparticle
operator $O_{\mu}^\rho$ to the conserved em current $J^\mu$ with
an interaction of type $(e_\rho/\Lambda_U)^2 J^\mu O_{\mu}^\rho$, where $d_U > 2$. An
analysis similar to the above gives the following modified
Coulomb potential:

$$V_c(r) = \frac{K e_1 e_2}{r} \left[ 1 + \frac{R_c}{r} \frac{2d_U-2}{2d_U-1} \right].$$

$$R_c = \frac{1}{\pi\Lambda_U} \left( \frac{e_1}{e_2} \right)^{1/d_U-1} \frac{2}{\Gamma(2d_U)} \frac{\Gamma(d_U + \frac{1}{2})}{\Gamma(2d_U)} \left( \frac{d_U}{2d_U-2} \right)^{1/2d_U-2}. $$

Coulomb’s law is not tested beyond the Fermi scale. Setting
$R_c < 10^{-13}$ cm, $d_U = 2$, and keeping $\Lambda_U = 10^{-11}$ GeV, one finds the constraint $e^\rho/e < 10^{-11}$. Thus
a sensitive probe could unravel the effects of unparticle
exchange to Coulomb’s law below such scales.

In summary, we have investigated the implications of a
scenario where conformal invariance of the hidden sector
strictly holds down to very low energies. This requires
constraining the dimensionality of the scalar unparticle
operators which might couple to the Higgs field so that
$d_U > 2$ in order not to spoil the conformal invariance of
the hidden sector. Under the assumption that a traceless
rank two unparticle operator can couple to the stress tensor,
we have computed corrections to the inverse square law
and find scale invariant power law corrections which can be
discriminated from similar corrections from extra dimen-
sion models. We also find the corrections from the
exchange of a scalar operator (with $d_U > 2$) which couples to the trace of the stress tensor. These corrections are testable in future experiments on the submillimeter probes of gravity. We note in passing that the analysis of spin 2 operators in the context of collider phenomenology is discussed in [17]. Corrections to Coulomb’s law from the exchange of vector unparticle operators were also computed.

Finally, we remark that the fractional modifications of the inverse square law was studied by Dvali [18] and was seen to lead to strong coupling effects. Dvali’s discussion was premised on infrared modifications of gravity which dominate the Einsteinian term at a scale $r \gg r_c$ which leads to the strong coupling referred to above. However, in our case, the modification of gravity at large scales does not dominate the Einsteinian term. In momentum space, the conformal propagator goes like $P^{(2d-4)}$, which for $d > 1$ is suppressed relative to the Einstein case, $P^{(-2)}$, while the propagator considered in the Dvali analysis behaves as $P^{(-2\alpha)}$ ($\alpha < 1$), which indeed dominates the Einsteinian term. Thus our setup escapes the strong coupling effect encountered in [18].

We end with a note of caution, in that a fully consistent formulation of unparticles does not exist and this feature carries over also to ungravity. Nonetheless, if unparticle stuff exists, and one assumes strict conformal invariance of the hidden sector, a new gravitational force, ungravity, could generate power law modification of gravity, and the new effects fall within the range of future submillimeter tests of gravity. Further, it is possible to distinguish between modifications of corrections due to extra dimensions and corrections from ungravity effects. It should be interesting to build explicit models of the hidden sector where strict conformal invariance is realized while also realizing couplings via a connector sector to the standard model fields of the type discussed here. The strict conformal invariance of the hidden sector required by our model is also suggestive of an AdS5 connection. However, such issues lie outside the scope of this work.

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Note added.—Recently, another work [19] in a similar spirit examined the correction to the long range forces from couplings to the baryon and lepton number currents and found that such corrections are significantly constrained by data.

References:

[16] See N. Greiner, in Ref. [20], for another application of unitarity.