Matter-parity constraints on particle spectrum in three-generation Calabi-Yau manifolds

R. Arnowitt
Center for Theoretical Physics, Department of Physics, Texas A&M University, College Station, Texas 77843

Pran Nath
Department of Physics, Northeastern University, Boston, Massachusetts 02115
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An analysis of the particle spectrum after intermediate scale breaking in the three-generation Calabi-Yau manifold $CP^3 \times CP^3 / Z_3$ is given. The spectrum is investigated within the framework of intermediate scale breaking which preserves matter parity and breaks the rank-six gauge group $SU(3)_C \times SU(3)_L \times SU(3)_R$ to the standard-model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. It is shown that a large number of particles and mirror particles associated with the extra families and mirror families on $CP^3 \times CP^3 / Z_3$ can all be made superheavy through mass generation induced by nonrenormalizable interactions that are present in the superpotential below the compactification scale. The final spectrum of the compactified superstring below the intermediate-mass scale breaking is shown to be the spectrum of the standard $N=1$ supersymmetry theory and a few additional exotic leptons. Some of these exotic leptons are hybrid structures containing equal mixture of leptons and mirror leptons.

I. INTRODUCTION

Since the advent of the $E_8 \times E_8$ heterotic string, its compactification on the manifold $M_4 \times K$, where $K$ is the compact six-dimensional Calabi-Yau manifold, has provided an attractive scheme for a unified four-dimensional theory. The Calabi-Yau compactification preserves $N=1$ supersymmetry and simultaneously one of the $E_8$ groups reduces to $E_6$ which may be viewed as the grand unification group of particle interactions at the compactification scale. However, the physically relevant manifold with three generations is nonsimply connected: it is the manifold $CP^3 \times CP^3 / Z_3$ (Ref. 3). On this manifold the group $E_6$ breaks to the group $SU(3)_C \times SU(3)_L \times SU(3)_R$ due to Wilson flux lines. The rank-six $[SU(3)]^3$ symmetry must be broken further to the rank-four standard-model gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_Y$. A standard scenario to achieve this is the so-called intermediate-mass scale breaking at $M_f \approx 10^{15}$ GeV. Above this mass scale but below the compactification scale, the massless spectrum of the $CP^3 \times CP^3 / Z_3$ compactification consists of nine families of leptons which belong to the $(1,3,\bar{3})$ representation of $SU(3)_C \times SU(3)_L \times SU(3)_R$, seven families of $(3,3,1)$ quarks, seven families of $(3,1,3)$ antiquarks, six families of $(1,3,3)$ mirror leptons, four families of $(3,3,1)$ mirror quarks, and four families of $(3,1,3)$ mirror antiquarks.

The phenomenology on three-generation Calabi-Yau manifolds has been investigated extensively recently and a number of difficulties with the model have been pointed out. Surprisingly, however, a detailed analysis of the intermediate-mass scale breaking as well as an analysis of the particle spectrum below the intermediate-mass scale has not been carried out. This analysis is rather crucial since the pattern of intermediate scale breaking determines the number of low-lying exotic particles. Too many of these low-lying exotic particles would be phenomenologically disastrous since they could generate Landau singularities as well as produce an unacceptably large value of $\sin^2 \theta_W$.

Recently the authors presented an analysis of intermediate scale breaking within the framework of matter-parity invariance imposed above the intermediate scale. This is needed to forbid rapid proton decay. The analysis used two different lepton multiplets and their mirror counterparts to generate vacuum expectation values (VEV's) for the SU(5)- and SO(10)-singlet fields $\nu^c$ and $\nu$. Remarkably the lowest-lying extrema maintain matter parity after symmetry breaking and prevent SU(2)$_L \times U(1)$ breaking at $M_f$.

In this paper we discuss the Goldstone and Higgs fields associated with intermediate-mass scale breaking. We discuss the absorption phenomenon which grows masses for the vector bosons in the reduction $SU(3)_C \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$. It is shown that the absorption involves linear combinations of the lepton multiplets and their mirror counterparts. The unabsorbed components are also linear combinations of the lepton multiplets and their mirrors. These hybrid structures are shown to possess electroweak masses and thus may be accessible at accelerator energies.

In addition to the lepton multiplets and their mirrors which enter in intermediate scale breaking and the three standard-model generations of quarks and leptons, there exist extra generations of lepton, quark, and antiquark multiplets and their mirrors which must be removed from the low-energy spectrum to avoid phenomenological disasters such as Landau singularities in coupling constants and an unacceptably large value of $\sin^2 \theta_W$. We ad-
dress this question in this paper and show that it is possible to find models where the unwanted spectrum of particles become superheavy. This is brought about by mass generation from the nonrenormalizable interactions in the superpotential that enter the theory below the compactification scale.

In Sec. II a brief review of symmetry-breaking analysis of Ref. 11 is given. In Sec. III Goldstone and Higgs analysis after intermediate scale symmetry breaking is carried out. Absorption of the lepton multiplets and their mirrors is analyzed. The mass growth of the vector bosons associated with the breaking of \( SU(3)_{\mathcal{C}} \times SU(3)_{\mathcal{L}} \times SU(3)_{\mathcal{R}} \) to the standard-model gauge group \( SU(3)_{\mathcal{C}} \times SU(2)_{\mathcal{L}} \times U(1)_{Y} \) is exhibited. The components of the lepton multiplets and their mirrors which are unabsorbed in the symmetry breaking are also discussed. In Sec. IV we show how mass generation from the nonrenormalizable interactions in the superpotential can make the unwanted modes superheavy.

II. SYMMETRY BREAKING ON CALABI-YAU MANIFOLDS AND GENERATION OF \( N \) AND \( \nu \) VEVS

The three-generation Calabi-Yau manifold is described by \( CP^{3} \times CP^{3} / Z_{3} \), where this manifold is defined through the intersection of the following polynomials involving eight complex coordinates \( x_{0}, x_{1}, x_{2}, x_{3} \) and \( y_{0}, y_{1}, y_{2}, y_{3} \):

\[
0 = P_1 = \sum x_i^3 + a_1 x_0 x_1 x_2 + a_2 x_0 x_1 x_3, \quad (2.1a)
\]

\[
0 = P_2 = x_0 y_0 + c_1 x_1 y_1 + c_2 x_2 y_2 + c_3 x_3 y_3 - c_4 x_2 y_3 + c_5 x_3 y_2, \quad (2.1b)
\]

\[
0 = P_3 = \sum y_i^3 + b_1 y_0 y_1 y_2 + b_2 y_0 y_1 y_3. \quad (2.1c)
\]

The above manifolds depend on nine complex parameters corresponding to the nine deformations of this manifold. This nine-parameter \( CP^{3} \times CP^{3} / Z_{3} \) manifold must be further constrained to obtain a phenomenologically viable manifold. One of the most important phenomenological constraints in superstring model building is the elimination of rapid proton decay. A remarkable invariance which is needed to forbid the existence of dangerously rapid proton decay is that of matter parity. Matter parity is defined by

\[
M_2 = CU_{Z}, \quad (2.2)
\]

where \( C \) acts on the Calabi-Yau coordinates \((x_0,x_1,x_2,x_3) \otimes (y_0,y_1,y_2,y_3)\),

\[
C = (1,1,\sigma) \otimes (1,1,\sigma), \quad \sigma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (2.3)
\]

and \( U_{Z} \) is an element of \( SU(3)_{\mathcal{C}} \times SU(3)_{\mathcal{L}} \times SU(3)_{\mathcal{R}} \) which is the group symmetry allowed on the manifold \( CP^{3} \times CP^{3} / Z_{3} \):

\[
U_{Z} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \otimes \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} \otimes \begin{pmatrix} -1 \\ 1 \end{pmatrix}. \quad (2.4)
\]

The \( C \)-invariant Calabi-Yau manifold is still a fairly large manifold depending on five complex parameters: i.e.,

\[
0 = P_1 = \sum x_i^3 + a_1 x_0 x_1 x_2 + x_0 x_1 x_3, \quad (2.5a)
\]

\[
0 = P_2 = c_0 y_0 + c_1 x_1 y_1 + c_2 x_2 y_2 + x_3 y_3 + c_4 x_2 y_3 + x_3 y_2, \quad (2.5b)
\]

\[
0 = P_3 = \sum y_i^3 + b_1 y_0 y_1 y_2 + y_0 y_1 y_3. \quad (2.5c)
\]

Above the intermediate-mass scale breaking where \( [SU(3)]^3 \) symmetry is exact, the particle interactions are separately \( C \) and \( M_2 \) invariant. However, below the intermediate-mass scale \( C \) invariance is broken through the VEV of the C-odd but \( M_2 \)-even SU(5)-singlet field \( \nu \). Consequently, below the intermediate mass \( C \) invariance no longer holds, though, as was seen in Ref. 11, \( M_2 \) invariance holds.

Below the compactification scale but above the intermediate scale breaking where the interaction structure of the theory is \([SU(3)]^3 \) invariant one may decompose the 27-plet into its \( SU(3)_{\mathcal{C}} \times SU(3)_{\mathcal{L}} \times SU(3)_{\mathcal{R}} \) parts:

\[
27 = L^i(1,3,\bar{3}) \oplus Q^a(3,\bar{3},1) \oplus Q^a(\bar{3},3,1). \quad (2.6)
\]

In Eq. (2.6), \( L \) are the leptons, \( Q \) the quarks, and \( C \) the antiquarks. The indices \( a, i, r = 1,2,3 \) are the \( SU(3)_c \), \( SU(3)_L \), and \( SU(3)_R \) indices. The \( SU(3)_R \) indices may be further decomposed into \( SU(2)_L \times SU(2)_R \) indices so that \( l = (\lambda, 3), \quad r = (\rho, 3) \) where \( \lambda, \rho = 1,2 \). The lepton, quark, and antiquark multiplets may now be explicitly written in familiar particle notation:

\[
L^\lambda = (H^\lambda, e^\lambda H'_\mu), \quad L^\lambda_3 = (\nu^\lambda, \nu^\lambda). \quad (2.7a)
\]

\[
Q^a_\rho = (u^a_\rho, d^a_\rho), \quad -Q^a_\rho = D^a, \quad (2.7b)
\]

\[
Q^a_0 = (u^a_0, -d^a_0), \quad -Q^a_0 = D^a_0. \quad (2.7c)
\]

In Eq. (2.7) \( q^{ak} = (u^{ak}, d^{ak}) \) and \( l^{ak} = (\nu^{kl}) \) are the quark and lepton \( SU(2)_L \) doublets while \( D^a \) and \( D^a_0 \) are the color Higgs triplets where the superscript \( c \) stands for the conjugate fields. A similar analysis holds for the 27 decomposition. Under \( U_{Z} \) the components of the 27-plet have even or odd transformations:

\[
U_Z \text{ even: } H^\lambda, H'^\lambda, D^a, D^a_0, N; \quad (2.8a)
\]

\[
U_Z \text{ odd: } q^{ak}, l^{ak}, u^a_0, d^a_0, l^c, v^c. \quad (2.8b)
\]

The \( C \) parity of the different families and mirror families are exhibited in Table I (Ref. 6). Matter parities of the families and mirror families can then be easily obtained using Eqs. (2.8) and Table I. These are listed in Table II. For convenience we introduce the notation:

\[
(r, s, t, \ldots; n, m, p, \ldots) = (C \text{ odd}; C \text{ even}). \quad (2.9)
\]

It is easily seen from Table I that \( r, s, t \) take on values \( 1,3, -3 \) for leptons, \( 3,4,5,6 \) for mirror leptons, \( -4, -6 \) for quarks and \( -1, -3 \) for mirror quarks, etc. Similarly \( m, n, p, \ldots \) take on values \( 1,3,5,7,8 \) for leptons and \( 1,2 \) for antileptons, etc.
As may be seen from Eqs. (2.2) and (2.8), the $N$ and $\bar{v}^c$ fields within a given multiplet which is an eigenstate of $C$ have opposite matter parities. Thus one needs two different multiplets, one $C$ even and the other $C$ odd, if only the matter-parity-even $N$ and $\bar{v}^c$ fields are to grow VEV’s so that rapid proton decay be forbidden. In the following we thus utilize two lepton multiplets $L_i$ ($i = 1, 2$) and their mirrors $\bar{L}_i$, where $i = 1$ is a $C$-even multiplet and $i = 2$ is a $C$-odd multiplet. If only the $N_9$, $N_{10}$, $\nu_9^c$, and $\bar{\nu}_9^c$ fields grow VEV’s, matter-parity invariance would be preserved. The effective potential which governs symmetry breaking below the compactification scale has the form

$$V = V_m + V_F + V_D,$$

where $V_m$ is a mass term arising from $N = 1$ supersymmetry breaking, $V_F$ is the $F$ term arising from the superpotential, and $V_D$ is the $D$ term generated by $SU(3)_L \times SU(3)_R$ gauge transformations. We discuss now each of these terms briefly. It is conventionally assumed that in this class of superstring models supersymmetry is broken so that soft supersymmetry-breaking mass terms are generated in the scalar-boson section. These soft (mass)$^2$ terms can turn negative through renormalization-group (RG) effects which signals spontaneous breaking which generates the intermediate-mass scale. An analysis of the renormalization-group equations below the compactification scale shows$^{15}$ that such a phenomenon does indeed occur in simple models, in fact, quite rapidly due to the large number of massless fields that enter below the compactification scale. Further, the evolution of the soft symmetry-breaking (mass)$^2$ need not necessarily be the same for the multiplets and their mirrors. Thus assuming that at a scale not far below the compactification scale, the squared masses of the scalar fields turn negative we may write for $V_m$ the form

$$V_m = -\sum_i m_i^2(x_i + y_i + z_i + \bar{u}_i),$$

$$-\sum_i \bar{m}_i^2(x_i + y_i + z_i + \bar{w}_i),$$

(2.11)

where

$$x_i = N_i N_i^\dagger, \quad y_i = v_i v_i^\dagger, \quad z_i = H_i^2 H_i^2, \quad u_i = H_i^2 H_i^2, \quad \bar{u}_i = \bar{H}_i^2 \bar{H}_i^2,$$

(2.12)

In Eq. (2.11) we have exhibited only the neutral fields which may grow VEV’s in the analysis of symmetry breaking at the intermediate scale. The term $V_F$ in Eq. (2.10) which is the $F$ part of the potential arises from a superpotential which in general consists of two parts: the first is a cubic interaction of type $(27)^3 + (27)^3$ while the second is a nonrenormalizable interaction which is an infinite-order expansion in the inverse of the compactification mass. This second part of the interaction is a reflection of the fact that the infinite number of heavy modes that arise in the superstring have been integrated out below the compactification scale. The nonrenormalizable terms in the superpotential are just the remnants of the coupling of these heavy fields to the light sector. Thus we write the superpotential as the sum of a renormalizable and a nonrenormalizable part:

$$W = W_r + W_{nr},$$

(2.13)

where

$$W_r = W_r(L, Q, Q^c) + \bar{W}_r(L, Q, Q^c).$$

(2.14)

We exhibit here explicitly for later use the form of $W_r(L, Q, Q^c)$:

$$W_r(L, Q, Q^c) = \lambda_{ABC}^1 e_{i} a_{i} u_{i}^A d_{i}^B D_{i}^C + \lambda_{ABC}^2 e_{i} a_{i} u_{i}^A d_{i}^B D_{i}^C + \lambda_{ABC}^3 e_{i} a_{i} u_{i}^A d_{i}^B D_{i}^C + \lambda_{ABC}^4 e_{i} a_{i} u_{i}^A d_{i}^B D_{i}^C + \lambda_{ABC}^5 e_{i} a_{i} u_{i}^A d_{i}^B D_{i}^C$$

$$- \lambda_{ABC}^1 N_{i} N_{i}^\dagger d_{i}^B d_{i}^C + \lambda_{ABC}^2 N_{i} N_{i}^\dagger d_{i}^B d_{i}^C + \lambda_{ABC}^3 N_{i} N_{i}^\dagger d_{i}^B d_{i}^C + \lambda_{ABC}^4 N_{i} N_{i}^\dagger d_{i}^B d_{i}^C + \lambda_{ABC}^5 N_{i} N_{i}^\dagger d_{i}^B d_{i}^C$$

(2.15)

The indices $A, B, C$ in Eq. (2.15) run over the various generations. For the Calabi-Yau manifold of Eq. (2.5), many of the couplings in Eq. (2.15) are automatically zero by $C$ invariance. Thus one has$^{11}$

$$\lambda_{rst}^{1,2,3,4} = 0 = \lambda_{mn}^{1,2,3,4},$$

(2.16a)

$$\lambda_{acr}^{+} = 0 = \lambda_{mn}^{+},$$

(2.16b)

An expression similar to Eq. (2.15) with restrictions simi-
TABLE II. List of the matter parities of particles and mirror particles in the full set of families and mirror families. \( r \) runs over \( C \)-odd values while \( n \) runs over \( C \)-even values. Matter parity of families and mirror families are the same.

<table>
<thead>
<tr>
<th>Matter parity</th>
<th>Particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_2 ) even</td>
<td>( l_r, e_r, v_r ) ( \rho_r, v_r, d_r ) ( D_r, \bar{D}_r, N_r ) ( H_r, H_r' )</td>
</tr>
<tr>
<td>( M_2 ) odd</td>
<td>( l_r, e_r, v_r ) ( \rho_r, v_r, d_r ) ( D_r, \bar{D}_r, N_r ) ( H_r, H_r' )</td>
</tr>
</tbody>
</table>

lar to Eq. (2.16) hold for the superpotential \( \mathcal{W}_I \left[ L, Q, \bar{Q}, \bar{Q}' \right] \). As already stated the nonrenormalizable part of the superpotential is in general an infinite-order expansion in the inverse of the compactification mass \( M_c \). We exhibit here only the part relevant for symmetry breaking. We assume for this purpose a dimension \( 2n (n \geq 2) \) term in the superpotential of the form

\[
W_{nr} = \frac{\lambda^2}{(M_c)^{2n-3}} \left( L' L \right)^n .
\]  

(2.17)

In Eq. (2.17) \( \lambda_i \) are the couplings of the heavy fields to the light fields. It is the superpotential of Eq. (2.17) rather than the cubic interactions of \( W_r \), which are \( F \) flat, that play the dominant role in symmetry breaking at the intermediate scale. The potential \( V_F \) arising from Eq. (2.17) is

\[
V_F = \frac{n^2}{M_c^{2n-6}} \sum \lambda^2_i (f_i f_i^\dagger)_{n-1} \times (x_i + \bar{x}_i + y_i + \bar{y}_i + z_i + \bar{z}_i + w_i + \bar{w}_i) .
\]

(2.18a)

where

\[
f_i = N_i \bar{N}_i + v_f \bar{v}_f + H_2^2 \bar{H}_2^2 + H_3^2 \bar{H}_3^2 \] .

(2.18b)

Finally the \( D \) term of the potential in Eq. (2.10) arising from the \( SU(3)_L \times SU(3)_R \) gauge transformations is

\[
V_D = \frac{g^2}{24} \left[ \sum_i (x_i - \bar{x}_i + y_i - \bar{y}_i - \frac{1}{2} (z_i - \bar{z}_i + w_i - \bar{w}_i)) \right]^2 + \frac{g^2}{8} \left[ \sum_i (-z_i - \bar{z}_i + w_i - \bar{w}_i) \right]^2
\]

\[
+ \frac{g^2}{2} \left[ \sum_i (H_i^2 v_i^\dagger - v_i v_i^\dagger H_i) \right]^2 + \frac{g^2}{6} \left[ \sum_i \left[ x_i - \bar{x}_i - \frac{1}{2} (y_i - \bar{y}_i + z_i - \bar{z}_i + w_i - \bar{w}_i) \right] \right]^2
\]

\[
+ \frac{g^2}{8} \left[ \sum_i \left[ y_i - \bar{y}_i - z_i - \bar{z}_i + w_i - \bar{w}_i \right] \right]^2 + \frac{g^2}{2} \left[ \sum_i (N_i f_i f_i^\dagger N_i) \right]^2 .
\]

(2.19)

The terms proportional to \( \Delta^2 \) are in general quite important in that they often break the vacuum degeneracy and help locate the lowest minimum. It was also seen that the lowest minimum which automatically preserves matter parity also preserves \( SU(2)_L \times U(1)_Y \) invariance.

III. GOLDSTONE AND HIGGS PHENOMENA 

AND VECTOR-BOSON MASS GROWTH

We consider next the effect of the VEV growth of \( N \) and \( \nu^c \) on the spontaneous breaking of \( SU(3)_c \times SU(3)_L \times SU(3)_R \). The \( N_1 \) VEV growth yields

\[
SU(3)_c \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_Y \times SU(2)_L \times U(1)_Y .
\]

(3.1)

while the \( \nu^c \) VEV growth breaks the group further to the standard-model gauge group: i.e.,

\[
SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_Y \times SU(2)_L \times U(1)_Y .
\]

(3.2)
We can calculate the vector-boson masses and identify the Goldstone bosons that are absorbed. The vector-boson masses arise from an analysis of the kinetic energy of the lepton multiplet $L_i^0$ and the mirror lepton multiplet $L_i^-$ with $i = 1, 2$ corresponding to C-even and C-odd multiplets. The gauge couplings of these multiplets are

$$L_i^{\pm} = -\sum_i \langle D_{\mu} L_i^{\pm} \rangle (D_{\mu} L_i^{\pm} - \sum_i \langle D_{\mu} L_i^0 \rangle (D_{\mu} L_i^0),$$

(3.3)

where

$$\langle D_{\mu} L_i^{\pm} \rangle = \frac{ig_L}{2} A_{\mu L}^{\pm}(\lambda_R^2)^L L_i^{\pm} + \frac{ig_R}{2} A_{\mu R}^{\pm}(\lambda_R^2)^L L_i^{\pm}.$$

(3.4)

and a similar equation holds for $L_i^-$. In Eq. (3.4) $A_{\mu L,R}^{\pm}, \alpha = 1-8$ are the SU(3)$_{L,R}$ gauge fields. As a consequence of the VEV growth of Eq. (2.20) we find that 12 vector bosons grow masses. These are

$$A_{\mu L}^+, A_{\mu L}^-, A_{\mu R}^+, A_{\mu R}^-, A_{\mu L}^{11}, A_{\mu R}^{11}, \alpha = 4, 5, 6, 7,$$

(3.5)

where $A_{\mu L}^{11}$ and $A_{\mu R}^{11}$ are two linear combinations of $A_{\mu L}^+, A_{\mu R}^+$ orthogonal to the hypercharge boson $A_{\mu L}^- = g_L A_{\mu L}^L + g_R A_{\mu L}^R + v^2 g_R A_{\mu L}^{11}/(4g_R^2 + g_L^2)^{1/2}$. The vector-boson masses are

$$M_{L}^{\pm} = \frac{1}{\sqrt{2}} g_L e \sqrt{(N^2 + \nu^2, M_{L}^R, M_{L}^R} = \frac{g_R}{\sqrt{2}} e \sqrt{(N^2 + \nu^2, (M_{L}^{11})^2 = \frac{1}{2}(g_L^2 + g_R^2)(N^2 + \nu^2),$$

(3.6)

and $\sin\theta = g_R (\langle N_1^2 \rangle + 2g_R^2 (\langle N_2^2 \rangle)/2$.)

(3.7)

For the purpose of knowing what low-energy spectrum of the theory is to determine what components of the lepton multiplets $L_i$ and the mirror multiplets $L_i$ are absorbed by the vector bosons in the process of spontaneous breaking. In general, an effective potential $V(\phi)$, where $\phi = \{\phi_a\}$ is a column symbol of all the scalar fields, the absorbed Goldstone bosons are given by the nonzero vectors $(t^a) \langle \phi_a \rangle$, where $t^a$ are the group generators for the representation of $\phi$. For our case $\phi_a = \{L_i^L, L_i^R, E_{\mu L}^L, E_{\mu R}^L\}$ and $t^a$ are SU(3) Gell-Mann matrices of SU(3)$_L \times$SU(3)$_R$. The following calculations neglect small corrections of $O(\Delta/M_c \Sigma/M_c)$. For the bosons absorbed by $A_{\mu L}^+, \alpha = 4, 5, 6, 7$ we find

$$\cos\theta = \frac{1}{\sqrt{2}} (v_1 + \bar{v}_1), \sin\theta = \frac{1}{\sqrt{2}} (H_{2}^0 + H_{2}^{0*}),$$

(3.8a)

$$\cos\theta = \frac{1}{\sqrt{2}} (e + \bar{e}_1), \sin\theta = \frac{1}{\sqrt{2}} (H_{2}^* - H_{2}^{0*}),$$

where

$$\cos\theta \equiv g_L (\langle N_1^1 \rangle + g_L^2 (\langle N_2^2 \rangle + g_R^2 (\langle N_2^2 \rangle)/2)^{1/2},$$

$$\sin\theta \equiv g_R (\langle \bar{v}_2 \rangle / (g_L^2 (\langle N_1^1 \rangle) + g_R^2 (\langle \bar{v}_2 \rangle)^2)^{1/2}.$$

The leading piece of the two nonvanishing VEV's is given in Eqs. (2.20a) and (2.20b). This implies that the contribution of $V_m$ and $V_{\phi}$ to $M_{ab}$ is of electroweak size, i.e., $O(\Delta^2/\Delta^2)$. Only $V_{\phi}$ can produce large mass contributions and only for those fields having $(\langle N_1^1 \rangle)^2$ or $(\langle \bar{v}_2 \rangle)^2$ as coefficients in $M_{ab}$. It is straightforward to pick these heavy fields out. The pieces in Eq. (3.9) containing $D_{\mu L}^a, \alpha = 4, 5, 6, 7$ yield

$$\phi_{L_{45}} = \cos\theta = \frac{1}{\sqrt{2}} (v_1 + \bar{v}_1),$$

(3.10a)

$$\phi_{L_{67}} = \cos\theta = \frac{1}{\sqrt{2}} (e + \bar{e}_1),$$

where

$$M_{L_{45}} = M_{L_{67}} = g_L (\langle N_1^1 \rangle^2 + (\langle \bar{v}_2 \rangle)^2)^{1/2}.$$
where
\[ M_{R(45)} = g_R \langle N_1 \rangle. \]  
(3.13b)
\[ M_{R(67)} = g_R \langle [N_7^2 + (v_f^2)]^{1/2} \rangle. \]  
(3.13c)

The heavy fields arising from the \( D_{2L} \) parts of \( V_D \) are more complicated to treat as the mass matrix links four fields: \( \text{Re} N_1, \text{Re} \tilde{N}_1, \text{Re} v_f, \text{Re} \tilde{v}_f \). A detailed discussion of this sector is given in the Appendix. There it is shown that the heavy fields are
\[ \phi = N_1 (\text{Re} N_1 - \text{Re} \tilde{N}_1) \pm N_7 (\text{Re} v_f - \text{Re} \tilde{v}_f) \]  
(3.14a)
with masses
\[ M_{\phi} = [a + b \pm (a - b)^2 + 4c^2]^{1/2}, \]  
(3.14b)
where
\[ N_1 = \frac{1}{2} \[ 1 - \frac{b - a}{\sqrt{(b - a)^2 + 4c^2}} \]^{1/2}, \]
\[ a = \frac{1}{2} (g_L^2 + g_R^2)(N_1)^2, \]
\[ b = \frac{1}{2} (g_L^2 + g_R^2)(v_f^2), \]
\[ c = \frac{1}{2} (g_L^2 - g_R^2)(N_7 v_f^2). \]  
(3.14c)
Note that \( M_{\phi}^2 > 0 \).

There are 72 real components in the multiplets of \( L_i, \bar{L}_i \) \((i = 1, 2)\) of which we have seen above 12 real components are the absorbed Goldstone modes and 12 components become superheavy. These 24 components are combinations of \( C \)-even and \( C \)-odd lepton states and their mirrors. Thus the remaining 48 components must also be linear combinations of leptons and their mirrors. If these fields have low mass, one might be able to sample remnants of the mirror world in accelerator experiments. These exotic leptons would thus carry the imprint of the Calabi-Yau manifold on which the superstring has compactified.

Of the 48 remaining components, two combinations are shown in the Appendix to be massless at \( M_1 \):
\[ \frac{1}{\sqrt{2}} (\text{Im} N_1 + \text{Im} \bar{N}_1), \quad \frac{1}{\sqrt{2}} (\text{Im} v_f + \text{Im} \bar{v}_f). \]  
(3.15)
They are expected to grow electroweak size masses at the \( M_W \) scale from RG corrections and \( SU(2) \times U(1) \) breaking. The above discussion has been model independent. To proceed further one must include additional detailed assumptions concerning the structure of the Calabi-Yau manifold. Thus \( N_1 \) and \( v_f \) couple to the lepton multiplets in the renormalizable cubic interactions producing a mass term of the type
\[ -\lambda_{1m1}^3 H_{m \lambda}^2 H_{\lambda}^* (N_1) - \lambda_{3m1}^3 H_{m \lambda}^2 H_{\lambda}^* (N_7). \]  
(3.16)
Thus any Higgs doublet for which \( \lambda_{1m1}^3 \) or \( \lambda_{3m1}^3 \) are nonzero will become superheavy. Phenomenologically, one requires at least one pair of Higgs doublets not to couple to \( N_1 \) and hence remain massless. This would be the doublet pair that at the \( M_W \) scale accomplishes the \( SU(2) \times U(1) \) breaking. Presumably this pair lies in the \( C \)-even sector so that matter parity is maintained. To have a Higgs doublet not coupling to \( N_1 \) requires a zero in the coupling constant matrix \( \lambda_{1m1}^3. \) One may check that such an "accident" does indeed occur for the symmetric Calabi-Yau manifold where the cubic Yukawa couplings are known. The simplest assumption is that there is only a single zero in \( \lambda_{1m1} \) (and no zero in \( \lambda_{3m1} \)). There are 32 real components associated with \( H_{\lambda}^1, H_{\lambda}^2, H_{\lambda}^3, H_{\lambda}^{1,2}, \lambda = 1, 2. \) Thus the above assumption implies that (aside from the usual fields associated with the three generations of the standard model) the fields in \( L_i, \bar{L}_i, \) \( i = 1, 2 \) that will remain light are \( 72 - (12 + 12 + 32) = 16 \) in number.

Equation (2.15) also shows renormalizable couplings of \( v_f^2 \) to \( H_{12}^* \lambda_1^1 \) and the matter-parity constraint (2.16) limits mass growth from these couplings to
\[ -\lambda_{2m1}^2 (H_{12}^* I_{\lambda m}^1 \langle v_f \rangle) + H_{1m}^1 I_{\lambda m}^1 \langle v_f \rangle). \]  
(3.17)
Thus if the coupling constant \( \lambda_{2m1} \) is nonzero, both \( I_{m1}^1, I_{m2}^2 \) will become superheavy, while if \( \lambda_{2m1} \) vanishes, they will remain light. Again which of these two possibilities occur depends on the details of the Calabi-Yau manifold.

We list in Table III the fields that remain light in \( L_i, \bar{L}_i, \) \( i = 1, 2 \) (over and above the three generations of the standard model) for the two possibilities.

**IV. SPECTRUM OF EXTRA GENERATIONS**

As discussed in the Introduction, the massless spectrum of the theory at the intermediate scale is rather large. It consists of nine families of \((1,3,3)\) leptons and six families of mirror \((1,3,3)\) leptons; seven families of \((3,3,1)\) quarks and four families of \((3,3,1)\) mirror quarks; and seven families of \((3,1,3)\) antiquarks and four families of \((3,1,3)\) mirror antiquarks. Below intermediate-scale breaking, one would like to have as few of the extra generations or mirror generations as possible. This is because in \( N = 1 \) supersymmetric theory there is little room beyond three generations if one wants to achieve consistency with the current experimental limit on \( \sin^2 \theta_W \).

In Sec. III we have seen that the breaking of \( SU(3)_c \times SU(3)_L \times SU(3)_R \) symmetry at the intermediate scale requires at least two lepton multiplets \( L_i (i = 1, 2) \), one \( C \) even and the other \( C \) odd, and their mirrors \( \bar{L}_i \). Certain linear combinations of these are absorbed while of the remaining, many become superheavy leaving only 8 real fields with electroweak masses in the low-energy

<table>
<thead>
<tr>
<th>( \lambda_{2m1} \neq 0 )</th>
<th>( \lambda_{2m1} = 0 )</th>
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<tbody>
<tr>
<td>( \frac{1}{\sqrt{2}} (\text{Im} v_f + \text{Im} \bar{v}_f) )</td>
<td>( \frac{1}{\sqrt{2}} (\text{Im} N_1 + \text{Im} \bar{N}_1) )</td>
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<td>( \frac{1}{\sqrt{2}} (\text{Im} N_1 + \text{Im} \bar{N}_1) )</td>
<td>( \frac{1}{\sqrt{2}} (\text{Im} v_f + \text{Im} \bar{v}_f) )</td>
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<tr>
<td>Re( N_2, \text{Im} N_2 )</td>
<td>Re( N_2, \text{Im} N_2 )</td>
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<td>( \frac{1}{\sqrt{2}} (\text{Re} N_1 + \text{Re} \bar{N}_1) )</td>
<td>( \frac{1}{\sqrt{2}} (\text{Re} N_1 + \text{Re} \bar{N}_1) )</td>
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<td>( \frac{1}{\sqrt{2}} (\text{Re} v_f + \text{Re} \bar{v}_f) )</td>
<td>( \frac{1}{\sqrt{2}} (\text{Re} v_f + \text{Re} \bar{v}_f) )</td>
</tr>
<tr>
<td>( I_{m2}^2 )</td>
<td>( I_{m1}^1 )</td>
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domain. As discussed in Ref. 11, one also has that below
the intermediate-mass scale breaking the mass matrix has
the general form

$$W_{\text{mass}} = (M_{m}^{T}I_{m} + M_{mn}^{T}l_{m} + \lambda_{n}^{T}q_{m}) + (M_{m}^{T}q_{m}) q_{m}.\quad (4.1)$$

An examination of the C-odd and C-even lepton and
quark states of Table I shows that three generations of
leptons and quarks remain massless below the
intermediate-mass scale. Also barring accidental zeros in
the mass matrix, the remaining families and mirror fam-
ilies would become massive. However, unless these extra
families and mirror families have superheavy masses, the
theory is not phenomenologically viable since it would
lead to Landau singularity in the color coupling constant
below the intermediate-mass scale and yield unacceptably
large values of $\sin^{2}\theta_{w}$. This is precisely the situation
that occurs if one straightforwardly extends the superpoten-
tial of Eq. (2.17) to run over all the families and mirror
families. Such an extension yields electroweak masses for
all the extra families and mirror families which would lead
to the phenomenological problem discussed above.

At present, theoretical evaluations from first principles
of the nonrenormalizable interactions of the superpoten-
tial on Calabi-Yau manifolds are not available. Here we
proceed phenomenologically to determine the desired
constraints on the nonrenormalizable part of the superpoten-
tial which can avoid the problem discussed above. These
represent the constraints that the Calabi-Yau mani-
fold of Eq. (2.5) must satisfy if it is to be phenomenologi-
cally viable. The desired form for the leading part of the
nonrenormalizable interactions of the superpotential for
extra generations $A \neq i$ is

$$\Delta W_{\text{nr}} = \sum_{A \neq i} \frac{\lambda_{AB}^{2}}{M_{c}} \left( L_{A} \bar{L}_{i} \right)^{\alpha \beta} (L_{A} \bar{L}_{i}) \quad (4.2)$$

The condition needed to avoid the phenomenological
difficulties is

$$n_{AB} < n_{i}, \quad A, B \neq i, \quad (4.3)$$

where $i = 1, 2$ refers to the C-even and C-odd multiplets
that enter in the intermediate scale breaking of Eq. (2.17)
and $A \neq i$ run over all other lepton and quark genera-
tions. Since the VEV’s of $L_{A}$ and $\bar{L}_{A}$ are zero, Eq. (4.2)
does not contribute to the extrema equations. Thus re-
results of Eq. (2.20) hold with Eq. (4.2) included. The
growth of $N_{i}$ and $V_{i}$ VEV’s then generate mass terms in
Eq. (4.2). For simplicity, we will assume from now on
that Eq. (4.2) is diagonal in $A$ and $B$ and write

$$\lambda_{AB} = \lambda_{AB} d_{AB}, \quad n_{A} = n_{AA}. \quad (4.4)$$

where

$$m_{A} = \frac{\lambda_{A}^{2}}{M_{c}} \left[ \frac{\Sigma_{1}^{2}}{4} \frac{\Sigma_{2}^{2}}{\lambda_{2}} \right] \times \left( \frac{n_{A} - 1}{2n_{i} - 2} \right) \quad (4.5)$$

In Table IV we present an estimate of $m_{A}$ setting $M_{c}$
equal to the Planck mass $M_{pl} = 2.4 \times 10^{18}$ GeV. One sees
that values of $n_{i} \geq 3$ are phenomenologically acceptable.
Thus with the superpotential of Eqs. (2.17) and (4.2) we can
break the $[SU(3)]^{3}$ symmetry spontaneously at the
intermediate scale and make all the extra families and mirror
families superheavy with masses $\geq 10^{14}$ GeV with $n_{i} \geq 3$.

The three massless generations that arise from the di-
agonalization of the mass matrix of Eq. (4.1) contain ex-
tra particles not found in the standard model. First they
contain six Higgs doublets which are probably too many
to give a satisfactory value for $\sin^{2}\theta_{w}$. However, as
discussed in Sec. III, the cubic interactions in Eq. (2.15) of
type $HH'N$ would be expected to give superheavy masses
to the extra Higgs doublets. In addition there are color
triplets $D^{a}$, $D_{a}^{c}$ which can mediate proton decay through
dimension five operators. From Eq. (2.15), however, one
finds that the VEV growth for the $N_{1}$ field generates a
mass term for $D^{a}$, $D_{a}^{c}$ of the form

$$-\lambda_{AB}^{2} D^{a}_{i} D_{i}^{c} \quad (4.6)$$

and thus gives superheavy masses to all the $D^{a}$, $D_{a}^{c}$
quarks. A recent analysis
demonstrates that the proton-
decay mediated by the $D$-quark exchange can be
make consistent with the existing proton lifetime lower
limits from Irvine-Michigan-Brookhaven (IMB) and
KAMIOKANDE experiments with a $D$-quark mass as
low as $10^{13}-10^{14}$ GeV. This is the mass range for the $D$
quarks expected from the mass growth of Eq. (4.6).

V. CONCLUSION

In this paper we have investigated the particle spec-
trum of the three-generation Calabi-Yau compactification
of the $E_{6} \times E_{6}$ superstring below the intermediate-mass
scale. The analysis was done under the assumption that
the symmetry breaking at the intermediate-mass scale
preserves matter parity which is needed to forbid the
dangerous proton-decay arising from dimension-four
operators. An analysis of the Goldstone and Higgs fields
and the mass growth of vector boson associated with the breakdown of the $\text{SU}(3)^3$ symmetry to the standard-model symmetry $\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$ was carried out. The unabsorbed components of two lepton families and two mirror lepton families which provide the Higgs fields in intermediate scale breaking were classified. It was shown that a number of unabsorbed components become superheavy through the cubic superpotential after the $N$ and $\nu^c$ VEV growth leaving a few fields with electroweak size masses. Further, it was is shown that all the remaining unwanted families and mirror families of quarks and leptons can be removed from the low-energy spectrum by generating superheavy masses for these particles through nonrenormalizable interactions that appear in the superpotential. The final low-energy spectrum of the theory was shown to be just the spectrum of the minimal $N=1$ supersymmetry theory except for the few exotic leptons listed in Table III. The phenomenology expected from these extra particles will be discussed elsewhere.

**APPENDIX**

We discuss here in detail the mass spectrum of the C-even field $N_1$ and the C-odd field $\nu_2$ which grow VEV's at the intermediate scale $M_f$ to break $\text{SU}(3)^3$ to the standard model. The effective potential for these fields are given by Eqs. (2.10)-(2.12), (2.18), (2.20d), and (2.21) with $V_D$ given by

$$ V_D = \frac{1}{8}(g_L^2 + g_R^2)(x_1 - \overline{x}_1 + \nu_2 - \overline{\nu}_2)^2 $$

$$ - \frac{1}{2}g_R^2(x_1 - \overline{x}_1)(y_2 - \overline{\nu}_2) \right. \) \hspace{1cm} (A1) $$

We see that $V_D$ produces coupling between the $N_1$ and $\nu_2$ fields. It is convenient to decompose $N_1, \overline{N}_1, \nu_2, \overline{\nu}_2$ into real and imaginary parts:

$$ N_1 = n_1 + i\overline{n}_1, \hspace{0.5cm} \overline{N}_1 = \overline{n}_1 + i\overline{\overline{n}}_1 $$

$$ \nu_2 = v_2 + iv_2, \hspace{0.5cm} \overline{\nu}_2 = \overline{v}_2 + i\overline{v}_2 \right. \) \hspace{1cm} (A2) $$

where the VEV's of $n_1, \overline{n}_1, v_2, \overline{\nu}_2$ all vanish. One may easily verify that the mass matrix elements of the imaginary parts all vanish, e.g.,

$$ \left( \frac{\partial^2 V}{\partial \nu_2^2} \right) = 4\langle n_1'-x_1 \rangle^2 \left( \frac{\partial^2 V}{\partial x_1' \partial x_1} \right) - 2\frac{\partial V}{\partial x_1} = 0 \right. \) \hspace{1cm} (A3) $$

since $\left( \frac{\partial V}{\partial x_1} \right)$ vanishes at the extrema. Thus in this approximation all four imaginary parts are massless. Two combinations are the Goldstone bosons of Eq. (3.8e), and the two remaining combinations are entries in Table III. The four real parts, couple in the mass matrix $M^2$. Labeling rows and columns by $n_1, \overline{n}_1, v_2, \overline{\nu}_2$ we find

\[
M^2 = \begin{bmatrix}
    A + a & B - (aa)^{1/2} & c & -d \\
    B - (aa)^{1/2} & A + a & -\overline{d} & \overline{c} \\
    c & -d & C + b & D - (bb)^{1/2} \\
    -d & \overline{c} & D - (bb)^{1/2} & \overline{D} + \overline{d}
\end{bmatrix}
\]

where

\[
a = \frac{1}{2}(g_L^2 + g_R^2)x_1, \hspace{0.5cm} b = \frac{1}{2}(g_L^2 + g_R^2)y_2, \right. \) \hspace{1cm} (A5) $$

\[
c = \frac{1}{2}(2g_L^2 - g_R^2)\sqrt{x_1y_2}, \hspace{0.5cm} d = \frac{1}{2}(2g_L^2 - g_R^2)(x_1y_2)^{1/2}
\]

and

\[
A = \frac{4n^2(n-1)}{M_c^{4n-6}} \lambda_1^2(x_1 \overline{x}_1)^{n-2}[nx_1x_1 + (n-2)\overline{x}_1 \overline{x}_1], \right. \) \hspace{1cm} \right. (A6) $$

\[
B = \frac{4n^2(n-1)}{M_c^{4n-6}} \lambda_1^2(x_1 \overline{x}_1)^{n-2} \sqrt{x_1N(x_1 + \overline{x}_1)}. \right. \) \hspace{1cm}

In Eq. (A4), the overbar notation means replace $x_1, y_2$ by $\overline{x}_1, \overline{y}_2$ [e.g., $\overline{a} = (1/3q)(g_L^2 + g_R^2)x_1$, etc.] and $C, D$ can be obtained from $A, B$ by the replacement $x_1, \overline{x}_1, \lambda_1 \rightarrow \overline{x}_1, \overline{x}_1, \lambda_1$, $[x_1, y_2$, etc., are defined in Eq. (2.12.)] It is understood in the mass matrix that all fields are VEV's. The leading terms for these VEV's are given by

\[
(x_1)^{2n-2} \approx (x_1)^{2n-2} \approx (\Sigma)\frac{3M_c\lambda^2}{2n^2(2n-1)\lambda^2}, \right. \) \hspace{1cm} (A7a) $$

\[
(y_2)^{2n-2} \approx (y_2)^{2n-2} \approx (\Sigma)\frac{3M_c\lambda^2}{2n^2(2n-1)\lambda^2}, \right. \) \hspace{1cm} (A7b) $$

while $x_1 - x_1 = O(\Delta^2), \overline{y}_2 - y_1 = O(\Delta^2)$ are electroweak in size.11

It is convenient to introduce the combinations

\[
\overline{x}_1 = \frac{1}{2}(x_1 + \overline{x}_1), \hspace{0.5cm} \overline{y}_2 = \frac{1}{2}(y_1 + \overline{y}_2) \right. \) \hspace{1cm} (A8) $$

and rewrite (A4) as

\[
M^2 = M_0^2 + M_1^2 \right. \) \hspace{1cm} (A9) $$

where $M_0^2$ is the matrix $M^2$ with $A, B, C, D$ set to zero and $a$ and $\overline{a}$ set to $a$ and $\overline{a}$ set to $\overline{a}$, and $b$ set to $b$, and $c$ and $d$ set to $c$, where the tildes quantities are the original ones with $x_1, y_2$, etc., replaced by $\overline{x}_1, \overline{y}_2$ [e.g., $\overline{a} = (1/3)(g_L^2 + g_R^2)x_1$, etc.]. $M_1^2$ is then the remainder, i.e., $M_1^2 = M^2 - M_0^2$. From Eqs. (A7) and (A8) it is clear that $M_0^2$ is the leading
part of $M^2$ for large $M_e$, while $M^2_\phi$ is of (small) electroweak size. We may thus diagonalize $M^2$ perturbatively, by first solving for the eigenvalues and eigenvectors of $M^2_\phi$. One finds easily two superheavy eigenvalues and two massless values:

$$\lambda^{(0)}_\pm = \alpha + \beta \pm (\alpha - \beta)^2 + 4\varepsilon^2)^{1/2},$$

$$\lambda^{(0)}_{\lambda_4} = 0,$$

with corresponding eigenfunctions given in Eq. (3.14a) and

$$\phi_3 = \frac{1}{\sqrt{2}} (n_1 + \bar{n}_1), \quad \phi_4 = \frac{1}{\sqrt{2}} (n_2 + \bar{n}_2).$$

One may find the corrections to $\lambda^{(0)}_\pm$ by perturbation theory where $M^2_\phi$ is the perturbation. One easily sees that

$$\Delta \lambda_\pm = \lambda_\pm - \lambda^{(0)}_\pm \approx \frac{8}{2n-1} (N_+ \Sigma^2_1 + N_+ \Sigma^2_2),$$

where $N_\pm$ is given in Eq. (3.14c). A convenient procedure to calculate the nonzero correction to $\lambda_1$ and $\lambda_4$ is to take the determinant and trace of Eq. (A4). To leading order we have

$$\det M = \lambda_\phi \lambda_1 \lambda_+ \lambda_- \approx 16(n-1) \Sigma_1^2 \Sigma_2^2 \lambda^{(0)}_\pm \lambda^{(0)}_{\lambda_4},$$

and

$$\text{tr} M = \lambda_\phi + \lambda_1 + \lambda_+ + \lambda_- \approx \lambda^{(0)}_\phi + \frac{8}{2n-1} (\Sigma_1^2 + \Sigma_2^2)$$

showing that $\lambda_{3,4} > 0$ and hence nontachyonic.

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14. An extension of matter parity to include larger $Z_n$ ($n > 2$) groups has been investigated recently by G. Lazarides and Q. Shafi, Report No. UT-STPD-3/88 (unpublished).