Large tau and tau neutrino electric dipole moments in models with vectorlike multiplets

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It is shown that the electric dipole moment of the $\tau$ lepton several orders of magnitude larger than predicted by the standard model can be generated from mixings in models with vectorlike multiplets. The electric dipole moment (EDM) of the $\tau$ lepton arises from loops involving the exchange of the $W$, the charginos, the neutralinos, the sleptons, the mirror leptons, and the mirror sleptons. The EDM of the Dirac $\tau$ neutrino is also computed from loops involving the exchange of the $W$, the charginos, the mirror leptons, and the mirror sleptons. A numerical analysis is presented, and it is shown that the EDMs of the $\tau$ lepton and the $\tau$ neutrino which lie just a couple of orders of magnitude below the sensitivity of the current experiment can be achieved. Thus the predictions of the model are testable in an improved experiment on the EDM of the $\tau$ and the $\tau$ neutrino.

I. INTRODUCTION

In the standard model, the electric dipole moment (EDM) of the $\tau$ arises at the multiloop level and is extremely small [1], i.e., $d_\tau < 10^{-34}$ ecm. On the other hand, the current experimental limit on the EDM of the $\tau$ lepton [2] is

$$d_\tau < 1.1 \times 10^{-17} \text{ ecm.}$$

Thus an experimental test of the standard model prediction is beyond the realm of observability in any near future experiment since the theoretical values lies several orders of magnitude below the current experimental limits. A similar situation also holds for the EDM of the $\tau$ neutrino where the current experimental limit on the EDM of the $\tau$ neutrino is [2] (for related papers, see [6,7])

$$d_{\tau\nu} < 5.2 \times 10^{-17} \text{ ecm,}$$

while in the standard model extended by a singlet the EDM again arises only at the multiloop level and is many orders of magnitude below the experimental limit. In this paper, we investigate the possibility that the EDM of the $\tau$ and the $\tau$ neutrino may be much larger by several orders of magnitude in models where there is a small mixing of the third generation leptons with a mirror in a vectorlike generation. Such a mixing may put the $\tau$ lepton EDM and the $\tau$ neutrino EDM within the realm of observation with an improved experiment. Thus vectorlike combinations are predicted in many unified models of particle interactions [8,9] and their implications have been explored in many recent works [10–16]. Such vectorlike combinations could lie in the TeV region and would be consistent with the current precision electroweak data.

In this work, we allow for the possibility that there could be a tiny mixing of these vectorlike combinations with the sequential generations and these mixing very significantly affect the $\tau$ lepton moments and also the $\tau$ neutrino moments. The implications of such mixings on the magnetic moment of the $\tau$ neutrino and the anomalous magnetic moment of the $\tau$ were discussed in [17]. Here, we include the effects of $CP$ violation, see [18] and discuss the enhancement of the leptonic EDMs due to the mixings with mirrors in the vectorlike generations. For the analysis here, we will focus on the leptonic vectorlike multiplets. To simplify the analysis, we will assume that the mixings of the sequential generations with the ordinary heavy leptons in the vectorlike combinations are small and thus will ignore it. The inclusion of such mixings will affect our overall results only by a small factor of $\sim 2$. However, we are after much bigger effects, i.e., effects which are larger than the standard model results by as much as a factor of $10^{15}$. Thus in the following analysis we will focus on the mixings of the sequential generations with the mirrors in the vectorlike combinations and show that they have huge effects. The mixing of the mirrors with the sequential leptons will introduce $V + A$ interactions for the ordinary leptons. Now, there are very stringent constraints on such interactions for the first two generations and thus these mixings are effectively negligible and we suppress them in our analysis. On the other hand, for the third generation leptons, a small mixing is possible and consistent with the current experimental constraints [19]. We note in passing that a similar situation holds for the case of the third generation quarks [20].

The masses of the vector multiplets could lie in a large mass range, i.e., from the current lower limits given by the LEP experiment for color singlet states to the region in the several TeV mass range. If the mirror leptons are discovered at the LHC, the analysis here would be very relevant for the planning of experiments for the discovery of the EDMs of the $\tau$ and the $\tau$ neutrino. However, it is possible that the vectorlike multiplets have masses large enough...
that they might escape detection even at the LHC. This is specifically true for the leptonic vector multiplets since the discovery reach for them is typically much smaller at hadronic machines than for the color particles. However, even for this case, the contribution of the mirrors to the EDMs can be huge as shown at the end of Sec. IV. Specifically, it is shown there that with the mirror masses in the TeV range, the EDMs of the \( \tau \) neutrino and the \( \tau \) lepton can be \( O(10^{14}) \) larger than the standard model value and only a factor of \( 10^3 \) smaller than the current sensitivity. An improvement in sensitivity of this magnitude is not necessarily outside of future experiment. Further, it is possible that a large EDM for the \( \tau \) neutrino could have astrophysical implications.

The outline of the rest of the paper is as follows: In Sec. II, we give an analysis of the EDM of the \( \tau \) lepton allowing for mixing between the vectorlike combination and the third generation leptons. In this case, the contribution to the EDM of the \( \tau \) arises from the exchanges of mirror neutrino, sneutrino-mirror sneutrino, from the third generation leptons and slepton-mirror sleptons along with \( W \) boson, chargino, and neutralino exchanges. In Sec. III, a similar analysis is given for the EDM of the \( \tau \) neutrino with inclusion of the contributions that arise from exchanges of the leptons from the third generation and the mirrors, and also from the exchanges of the \( W \) bosons, charginos, sleptons, and mirror sleptons.

A numerical analysis of sizes of the EDM of the \( \tau \) lepton and the \( \tau \) neutrino are given in Sec. IV. In this section, we also give a display of the EMDs on the phases and mixings. Conclusions are given in Sec. V. Deductions of the mass matrices used in Secs. II and III are given in the Appendix.

II. EDM OF THE \( \tau \) LEPTON

Figure 1(a) produces EDM of the \( \tau \) (\( d_\tau \)) through the interaction of the \( \tau \) with the \( \tau \) and the neutrino and its mirror, and here we give the relevant part of the Lagrangian which is

\[
\mathcal{L}_{CC} = - \frac{g_2}{\sqrt{2}} W^-_{\mu} \sum_{j=1,2} \bar{\tau}_j \gamma^\mu [D^\nu \nu_j D^\nu_{L1j} D^*_{L1k} P_L] + D^\nu_{R2j} D^*_{R2k} P_R \nu_j + \text{H.c.},
\]

where \( D^\nu_{L1j} \) are the diagonalizing matrices defined in the Appendix. These matrices contain phases, and these phases generate the EDM of the \( \tau \) for the case of Fig. 1(a).

Figure 1(b) produces a contribution to \( d_{\tau} \) through neutralino exchange and the relevant interactions in this case are

\[
- \mathcal{L}_{\tau - \tau - \chi^0} = \sum_{j=1-2} \sum_{k=1-4} \bar{\tau}_j [C_{jk} P_L + F_{jk} P_R] \chi^0_{jk} \bar{\tau}_k + \text{H.c.},
\]

where

\[
C_{jk} = \sqrt{2} [\alpha_{\tau} D^\tau_{R11} \tilde{D}^\tau_{1k} - \gamma_{\tau} D^\tau_{R11} \tilde{D}^\tau_{3k} + \beta_{E_{\tau}} D^\tau_{R12} \tilde{D}^\tau_{4k} - \delta_{E_{\tau}} D^\tau_{R12} \tilde{D}^\tau_{2k}],
\]

\[
F_{jk} = \sqrt{2} [\beta_{\tau} D^\tau_{R11} \tilde{D}^\tau_{1k} - \delta_{\tau} D^\tau_{R11} \tilde{D}^\tau_{3k} + \alpha_{E_{\tau}} D^\tau_{R12} \tilde{D}^\tau_{4k} - \gamma_{E_{\tau}} D^\tau_{R12} \tilde{D}^\tau_{2k}].
\]

Figure 1(c) produces a contribution to \( d_{\tau} \) through chargino exchange and the relevant interactions in this case are

\[
- \mathcal{L}_{\tau - \tilde{\tau} - \chi^0} = \sum_{j=1-2} \sum_{k=1-4} \bar{\tau}_j [K_{jk} P_L + L_{jk} P_R] \bar{\tilde{\tau}}_k + \text{H.c.},
\]

where

\[
K_{jk} = - g_2 [D^\tau_{R11} \kappa_{\tau} U^\tau_{j2} \tilde{D}^\tau_{1k} - D^\tau_{R12} \tilde{U}^\tau_{j1} \tilde{D}^\tau_{4k} + D^\tau_{R12} \tilde{U}^\tau_{j2} \tilde{D}^\tau_{2k}],
\]

\[
L_{jk} = - g_2 [D^\tau_{R11} \kappa_{\tau} V_{j2} \tilde{D}^\tau_{1k} - D^\tau_{R11} \tilde{V}^\tau_{j1} \tilde{D}^\tau_{4k} + D^\tau_{R12} \tilde{V}^\tau_{j2} \tilde{D}^\tau_{2k}].
\]

Here, \( U \) and \( V \) are the matrices that diagonalize the chargino mass matrix \( M_C \) so that

\[
U^* M_C V^{-1} = \text{diag}(m_{\chi^+_1}, m_{\chi^+_2}),
\]

and \( \kappa_{\tau}, \kappa_{\tau}, \kappa_{e}, \kappa_{\nu}, \kappa_{\nu}, \kappa_{\nu} \), etc., that enter Eq. (7) are defined by

\[
\kappa_{\tau} = \frac{(m_{\tau}, m_{\nu})}{\sqrt{2}M_W \cos \beta}, \quad \kappa_{E_{\tau}} = \frac{(m_{E_{\tau}}, m_{\nu})}{\sqrt{2}M_W \sin \beta},
\]

Using these interactions, we have

\[
d_{\nu} = \frac{g_2}{32 \pi^2 M_W^2} \sum_{j=1,2} m_{\nu_j} \text{Im}(D^\nu_{L1j} D^*_{L1k} D^*_{R1k} D^*_{R1l} I_1) \left( \frac{m_{\nu_j}}{M_W} \right),
\]

\[
d_{\tilde{\tau}} = - \frac{g_2}{16 \pi^2} \sum_{j=1,2} \sum_{k=1,4} m_{\tilde{\tau}_j} \text{Im}(\eta_{jk}) A \left( \frac{m_{\tilde{\tau}_j}}{M_W} \right),
\]

\[
d_{\tilde{\chi}^0} = - \frac{1}{16 \pi^2} \sum_{j=1,2} \sum_{k=1,4} m_{\tilde{\chi}^0_j} \text{Im}(\phi_{jk}) B \left( \frac{m_{\tilde{\chi}^0_j}}{M_W} \right),
\]

where \( I_1(r), A(r), \text{and } B(r) \) are defined as follows:
\[ I_t(r) = \frac{2}{(1-r)^2} \left[ 1 + \frac{1}{4} r + \frac{1}{4} r^2 + \frac{3 r \ln r}{2(1-r)} \right] \]
\[ A(r) = \frac{1}{2(1-r)^2} \left[ 3 - r + \frac{2 \ln r}{1-r} \right] \quad (11) \]
\[ B(r) = \frac{1}{2(1-r)^2} \left[ 1 + r + \frac{2 r \ln r}{1-r} \right] \]

and where
\[ \eta_{jk} = [-D_{R_{11}}^{r+} \nu_j \tilde{D}_{1k}^{r+} + D_{R_{21}}^{r+} \nu_j \tilde{D}_{3k}^{r+} - D_{R_{12}}^{r+} \nu_j \tilde{D}_{2k}^{r+} + D_{L_{11}}^{r+} \nu_j \tilde{D}_{1k}^{r+} - D_{R_{12}}^{r+} \nu_j \tilde{D}_{2k}^{r+}] \]
\[ \zeta_{jk} = 2(\alpha_{E_{1j}} D_{R_{11}}^{r+} D_{1k}^{r+} - \gamma_{\tau} D_{R_{11}}^{r+} D_{3k}^{r+} + \beta_{E_{1j}} D_{R_{12}}^{r+} D_{4k}^{r+} - \delta_{E_{1j}} D_{L_{11}}^{r+} D_{1k}^{r+} - \delta_{E_{1j}} D_{L_{21}}^{r+} D_{2k}^{r+}) \]

The matrix elements of $\tilde{D}_{\nu}^{r+\tau}$ are the diagonalizing matrices of the sneutrino and slepton 4 × 4 mass matrices (see the Appendix). The couplings that enter $\zeta_{jk}$ in Eq. (12) are given by
\[ \alpha_{E_{1j}} = \frac{g_2 m_\tau X_{4j}}{2 m_W \sin \beta}, \]
\[ \beta_{E_{1j}} = e X_{1j} + \frac{g_2}{\cos \theta_W} X_{2j} \left( \frac{1}{2} - \sin^2 \theta_W \right), \]
\[ \gamma_{E_{1j}} = e X_{1j} - \frac{g_3 \sin^2 \theta_W}{\cos \theta_W} X_{2j}, \]
\[ \delta_{E_{1j}} = - \frac{g_2 m_\tau X_{1j}}{2 m_W \sin \beta} \]
\[ \alpha_{\tau j} = \frac{g_2 m_\tau X_{3j}}{2 m_W \cos \beta}, \]
\[ \beta_{\tau j} = - e X_{1j} + \frac{g_2}{\cos \theta_W} X_{2j} \left( \frac{1}{2} + \sin^2 \theta_W \right), \]
\[ \gamma_{\tau j} = - e X_{1j} + \frac{g_3 \sin^2 \theta_W}{\cos \theta_W} X_{2j}, \]
\[ \delta_{\tau j} = - \frac{g_2 m_\tau X_{3j}}{2 m_W \cos \beta} \]

and where
\[ X'_{ij} = (X_{1j} \cos \theta_W + X_{2j} \sin \theta_W), \]
\[ X''_{ij} = (-X_{1j} \sin \theta_W + X_{2j} \cos \theta_W), \]

and where the matrix $X$ diagonalizes the neutralino mass matrix so that
\[ X^T M_{\phi} X = \text{diag}(m_{\phi_1}, m_{\phi_2}, m_{\phi_3}, m_{\phi_4}). \]

### III. EDM of the $\tau$ Neutrino

The EDM of the $\tau$ neutrino receives contributions from the diagrams of Fig. 2. Using the interactions of Eq. (3), the contributions from the loop diagrams of Figs. 2(a) and 2(b) are as follows:
\[ d_{\nu}^{(a)} = - \frac{g_2^2}{32 \pi^2 M_W} \sum_{j=1,2} m_\tau \text{Im}(D_{L_{11}}^{\nu^*} D_{L_{11}}^{\nu} D_{R_{11}}^{\nu^*} D_{R_{11}}^{\nu}) \times I_1 \left( \frac{m_\tau^2}{M_W} \right), \]
\[ d_{\nu}^{(b)} = - \frac{g_2^2}{32 \pi^2 M_W} \sum_{j=1,2} m_\tau \text{Im}(D_{L_{11}}^{\nu^*} D_{L_{11}}^{\nu} D_{R_{11}}^{\nu^*} D_{R_{11}}^{\nu}) \times I_2 \left( \frac{m_\tau^2}{M_W} \right), \]

where $I_1(r)$ is given in Eq. (11) and $I_2(r)$ by
\[ I_2(r) = \frac{2}{(1-r)^2} \left[ 1 + \frac{1}{4} r + \frac{1}{4} r^2 - \frac{3 r \ln r}{2(1-r)} \right]. \]

Similarly, the loop contributions of Figs. 2(c) and 2(d) to $d_{\nu}$ are given by
\[ d_{\nu}^{(c)} = - \frac{1}{16 \pi^2} \sum_{j=1,2} \sum_{k=1}^4 \text{Im}(S_{jk} T_{jk}^{\nu^*}) B \left( \frac{m_\tau^2}{m_{\tilde{h}_k}^2} \right), \]
\[ d_{\nu}^{(d)} = \frac{1}{16 \pi^2} \sum_{j=1,2} \sum_{k=1}^4 \text{Im}(S_{jk} T_{jk}^{\nu^*}) A \left( \frac{m_\tau^2}{m_{\tilde{h}_k}^2} \right), \]

where $S_{jk}$ and $T_{jk}$ are given by
The mixing matrices between leptons and mirrors are diagonalized using bunitary matrices (see the Appendix). So we parametrize the mixing between $\tau$ and $E_{\nu}$ by the angles $\theta_{L}, \theta_{R}, \chi_{L}$, and $\chi_{R}$, and the mixing between $\nu$ and $N$ by the angle $\phi_{L}, \phi_{R}, \xi_{L}$, and $\xi_{R}$, where

$$D_{L}^{\tau} = \begin{pmatrix}
\cos \theta_{L} e^{i \chi_{L}} & -\sin \theta_{L} e^{-i \chi_{L}} \\
\sin \theta_{L} e^{i \chi_{L}} & \cos \theta_{L}
\end{pmatrix},$$

$$D_{L}^{\nu} = \begin{pmatrix}
\cos \phi_{L} e^{i \xi_{L}} & -\sin \phi_{L} e^{-i \xi_{L}} \\
\sin \phi_{L} e^{i \xi_{L}} & \cos \phi_{L}
\end{pmatrix},$$

and $D_{R}^{\tau}$ and $D_{R}^{\nu}$ can be gotten from $D_{L}^{\tau}$ and $D_{L}^{\nu}$ by the following substitution: $D_{L}^{\tau} \rightarrow D_{R}^{\tau}, \theta_{L} \rightarrow \theta_{R}, \chi_{L} \rightarrow \chi_{R}$ and $D_{L}^{\nu} \rightarrow D_{R}^{\nu}, \phi_{L} \rightarrow \phi_{R}, \xi_{L} \rightarrow \xi_{R}$. We note that the phases $\chi_{L,R}$ arise from the couplings $f_{3}$ and $f_{4}$ while the phases $\xi_{L,R}$ arise from the couplings $f_{3}$ and $f_{5}$ through the relations

$$\chi_{R} = \arg(m_{\nu} f_{3} + m_{E} f_{5}^*)$$

$$\chi_{L} = \arg(m_{\nu} f_{5} + m_{E} f_{3}^*)$$

$$\xi_{R} = \arg(-m_{\nu} f_{3} + m_{N} f_{5}^*)$$

$$\xi_{L} = \arg(m_{\nu} f_{5}^* - m_{N} f_{3}).$$

However, these four parameters are not independent since the input of three phases of $f_{3}, f_{4},$ and $f_{5}$ would produce these four parameters. For the case of lepton and neutrino masses arising from Hermitian matrices, i.e., when $f_{3} = f_{3}^*$ and $f_{5} = -f_{5}^*$, we have $\theta_{L} = \theta_{R}, \phi_{L} = \phi_{R}, \chi_{L} = \chi_{R} = \chi$, and $\xi_{L} = \xi_{R} = \xi$. Further, here we have the relation $\xi = \chi + \pi$, and thus the $W$-exchange terms of the EDMs for $\tau$ neutrinos and $\tau$ leptons vanish. However, more generally, the lepton and the neutrino mass matrices are not Hermitian and they generate nonvanishing contributions to the EDMs. Thus the input parameters for this sector of the parameter space are $m_{\nu}, m_{E}, f_{3}, f_{4}, m_{\nu}, m_{N}, f_{5}$ with $f_{3}, f_{4},$ and $f_{5}$ being complex masses with $CP$ violating phases $\chi_{3}, \chi_{4},$ and $\chi_{5}$, respectively. For the slepton mass 2 matrices, we need the extra input parameters of the supersymmetry (SUSY) breaking sector, $M_{L}, M_{R}, M_{\tau}, M_{\chi}, M_{\nu}, M_{N}, A_{\tau}, A_{E}, A_{\nu}, A_{N}, \mu, \tan \beta$. The chargino and neutralino sectors need the extra two parameters $m_{1}, m_{2}$. We will assume that the only parameters that have phases in the above set are $A_{E}, A_{N}, A_{\tau},$ and $A_{\nu}$. These phases are $\alpha_{E}, \alpha_{N}, \alpha_{\tau},$ and $\alpha_{\nu}$, respectively. To simplify the analysis, we set the phases $\alpha_{E} = \alpha_{N} = \alpha_{\tau} = 0$. With this in mind, the only contributions to the EDM of the $\tau$ lepton and $\tau$ neutrino arise from mixing terms between the scalar matter–scalar mirrors, fermion matter–fermion mirror, and finally between mirrors among themselves in the scalar sector. Thus in the absence of the mirror part of the Lagrangian, the EDMs of $\tau$’s and neutrinos vanish. We can thus isolate the role of the $CP$ violating phases in this sector and see the size of its contribution. The $4 \times 4$ mass$^{2}$ matrices of sleptons and sneutrinos are diagonalized numerically. Thus the $CP$ violating phases that would play a role in this analysis are

$$\chi_{3}, \chi_{4}, \chi_{5}, \alpha_{E}, \alpha_{N}. \quad (23)$$

To reduce the number of input parameters, we assume $M_{a} = m_{0}, a = L, E, \tau, \chi, \nu, N,$ and $|A_{i}| = |A_{0}|, i = E, N, \tau, \nu$. In Fig. 3, we give a numerical analysis of the EDM of the $\tau$ lepton and discuss its variation with the parameter $\chi_{3}$ (left panel), with $|f_{3}|$ (middle panel), and with $\alpha_{N}$ (right panel). Regarding $\chi_{3}$, it enters $D^{\nu}, D^{\tau}, D^{\tau'},$ and $D^{\nu'},$ and as a consequence, all diagrams in Fig. 1 that contribute to the EDM of the $\tau$ are affected. The phase $\alpha_{N}$, however, enters only in the chargino exchange contribution since it enters $D^{\nu'}$ and thus only the part of the $\tau$ EDM arising from the chargino exchanged is affected by variations of $\alpha_{N}$. We note that the various diagrams in Figs. 1(a)–1(c) that contribute to the $\tau$ EDM can add constructively or destructively in the latter case, generating large cancellations reminiscent of the cancellation mechanism for the EDM of the electron and the neutron [18]. Of course, the desirable larger contributions for the $\tau$ EDM occur away from the cancellation regions. The analysis of Fig. 3 shows that a $\tau$ EDM as large $10^{-18}$–$10^{-19}$ ecm can be obtained which is only about 2 orders of magnitude below the current experimental limits of Eq. (1). A similar analysis for the $\tau$ neutrino EDM is given in Fig. 4. Here again, one finds that the $\tau$ neutrino EDM as large as $10^{-18}$–$10^{-19}$ ecm can be obtained and again it lies only a couple of orders of magnitude below the current experimental limit of Eq. (2).

As discussed in Sec. I, the scale of the vectorlike multiplets is unknown. They could lie in the sub-TeV region but, on the other hand, they could also be several TeV sizes and escape direct detection even at the LHC. This is especially true for leptonic vectorlike multiplets since the discovery reach for color singlet leptonic states at hadronic machines is typically much smaller than for the color particles. In this context, it is then interesting to investigate the contributions to the $\tau$ lepton EDM and the $\tau$ neutrino EDM from vectorlike leptonic multiplets in the TeV range. A comparison of these EDMs when the leptonic vectorlike multiplet lies in the sub-TeV region vs in the TeV region is given in Table I below. It turns out that the dependence of the EDMs on the masses of the mirror leptons is a rather complicated one. Thus there are supersymmetric and non-
supersymmetric contributions which have different dependencies on the mirrors and mirror slepton masses. Thus in certain regions of the parameter space as the mirror leptons masses grow, the chargino and neutralino contributions are suppressed much faster than the $W$-exchange terms. In the SUSY contributions, both the couplings and form factors that contain the mirror lepton masses explicitly decrease as the mirror spectrum increases. In the $W$-exchange term, there is a competition between the couplings and form factors. The first term decreases while the latter increases and, indeed, a suppression of the EDMs occurs but here it is much slower than in the SUSY case. These phenomena are illustrated in the analysis of Table I which shows that the suppression of the $W$-exchange terms in both $\tau$ and neutrino EDMs is at a much slower rate than the other components in this specific part of the parameter space.

The analysis given above shows that even if the vector-like particles lie in the TeV range, they could contribute a significant amount, i.e., $O(10^{-20})$ ecm which is $O(10^{14})$ larger than what the standard model predicts and only 3 orders of magnitude smaller than the current limits. The above results do not appear outside the realm of detection in a future experiment with improved sensitivity. Further,
the results above could have possible astrophysical implications.

V. CONCLUSION

In this paper, we have considered extensions of the minimal supersymmetric standard model with vectorlike multiplets. We have specifically focused on the leptonic minimal supersymmetric standard model with vectorlike superpotential of the model for the lepton part may be written in the form

\[ W = \epsilon_{ij}[f_1H_1^\dagger\tilde{\psi}_i\tilde{\tau}_L^c + f_1'H_2^\dagger\tilde{\psi}_i\tilde{\nu}_L^c + f_2'\tilde{\chi}^i\tilde{N}_L^c + f_3\tilde{\chi}^i\tilde{E}_{\tau L}] + f_3\tilde{\psi}_i\tilde{\tau}_L^c + f_4\tilde{\nu}_L^c\tilde{E}_{\tau L} + f_5\tilde{\nu}_L^c\tilde{N}_L^c. \]  

(A3)

Mixings of the above type can arise via nonrenormalizable interactions [9]. Consider, for example, a term such as \( 1/M_P v_j^2 N_i \Phi_1 \Phi_2 \). If \( \Phi_1 \) and \( \Phi_2 \) develop a vacuum expectation value of size \( 10^{9-10} \), a mixing term of the right size can be generated.

To get the mass matrices of the leptons and the mirror leptons, we replace the superfields in the superpotential by their component scalar fields. The relevant parts in the superpotential that produce the lepton and mirror lepton mass matrices are

\[ W = f_1H_1^\dagger\tilde{\psi}_i\tilde{\tau}_L^c + f_1'H_2^\dagger\tilde{\psi}_i\tilde{\nu}_L^c + f_2'\tilde{\chi}^i\tilde{N}_L^c + f_3\tilde{\chi}^i\tilde{E}_{\tau L} + f_4\tilde{\nu}_L^c\tilde{E}_{\tau L} + f_5\tilde{\nu}_L^c\tilde{N}_L^c. \]  

(A4)

The mass terms for the lepton and their mirrors arise from the part of the Lagrangian

\[ \mathcal{L} = -\frac{1}{2} \frac{\partial^2 W}{\partial A_{ij}\partial A_j} \psi_i \psi_j + H.c., \]  

(A5)

where \( \psi \) and \( A \) stand for generic two-component fermion and scalar fields. After spontaneous breaking of the electroweak symmetry \( \langle H^0 \rangle = v_1/\sqrt{2} \) and \( \langle H^0_2 \rangle = v_2/\sqrt{2} \), we have the following set of mass terms written in 4-spinors for the fermionic sector:

\[ W = \epsilon_{ij}[f_1H_1^\dagger\tilde{\psi}_i\tilde{\tau}_L^c + f_1'H_2^\dagger\tilde{\psi}_i\tilde{\nu}_L^c + f_2'\tilde{\chi}^i\tilde{N}_L^c + f_3\tilde{\chi}^i\tilde{E}_{\tau L} + f_4\tilde{\nu}_L^c\tilde{E}_{\tau L} + f_5\tilde{\nu}_L^c\tilde{N}_L^c. \]  

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To get the mass matrices of the leptons and the mirror leptons, we replace the superfields in the superpotential by their component scalar fields. The relevant parts in the superpotential that produce the lepton and mirror lepton mass matrices are

\[ W = f_1H_1^\dagger\tilde{\psi}_i\tilde{\tau}_L^c + f_1'H_2^\dagger\tilde{\psi}_i\tilde{\nu}_L^c + f_2'\tilde{\chi}^i\tilde{N}_L^c + f_3\tilde{\chi}^i\tilde{E}_{\tau L} + f_4\tilde{\nu}_L^c\tilde{E}_{\tau L} + f_5\tilde{\nu}_L^c\tilde{N}_L^c. \]  

(A4)

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Here, the mass matrices are not Hermitian and one needs to use biunitary transformations to diagonalize them. Thus we write the linear transformations

\[
\begin{pmatrix}
\tau_R \\
E_{\tau_R}
\end{pmatrix} = D^\tau_R \begin{pmatrix}
\tau_L \\
E_{\tau_L}
\end{pmatrix},
\begin{pmatrix}
\tau_L \\
E_{\tau_L}
\end{pmatrix} = D^\tau_L \begin{pmatrix}
\tau_{1L} \\
E_{\tau_{2L}}
\end{pmatrix}.
\]

(A6)

such that

\[
D^\tau_R \begin{pmatrix}
f_1v_1/\sqrt{2}f_3 \\
f_3f_2v_2/\sqrt{2}
\end{pmatrix} D^\tau_L = \text{diag}(m_{\tau_1}, m_{\tau_2}),
\]

(7)

and the same holds for the neutrino mass matrix so that

\[
D^\nu_R \begin{pmatrix}
f_1v_2/\sqrt{2}f_3 \\
-f_3f_2v_1/\sqrt{2}
\end{pmatrix} D^\nu_L = \text{diag}(m_{\nu_1}, m_{\nu_2}).
\]

(A8)

Here, \(\tau_1, \tau_2\) are the mass eigenstates and we identify the \(\tau\) lepton with the eigenstate 1, i.e., \(\tau_1\), and identify \(\tau_2\) with a heavy mirror eigenstate with a mass in the hundreds of GeV. Similarly \(\nu_1, \nu_2\) are the mass eigenstates for the neutrinos, where we identify \(\nu_1\) with the light neutrino state and \(\nu_2\) with the heavier mass eigen state. By multiplying Eq. (A7) by \(D^\nu_R\) from the right-hand side and by \(D^\nu_L\) from the left-hand side and by multiplying Eq. (A8) by \(D^\tau_L\) from the right-hand side and by \(D^\tau_R\) from the left-hand side, one can equate the values of the parameter \(f_3\) in both equations and we can get the following relation between the diagonalizing matrices \(D^\tau\) and \(D^\nu\):

\[
m_{\tau_1}D^\tau_{2R1}D^\nu_{1L1} + m_{\tau_2}D^\tau_{2R2}D^\nu_{1L2} = -[m_{\nu_1}D^\nu_{2R1}D^\tau_{1L1} + m_{\nu_2}D^\nu_{2R2}D^\tau_{1L2}].
\]

(A9)

Next, we consider the mixings of the charged sleptons and the charged mirror sleptons. We write the superpotential in terms of the scalar fields of interest as follows:

\[
W = -\mu \epsilon_{ij} H^*_i H^*_j - \epsilon_{ij}[f_1 H^*_i \tilde{\psi}^j_L \bar{\nu}_L + f_1 H^*_i \tilde{\psi}^j_R \bar{\nu}_R + f_2 H^*_2 \chi^{+i} \bar{\psi}^j_L + f_2 H^*_2 \chi^{+i} \bar{\psi}^j_R + f_3 \epsilon_{ij} \tilde{\chi}^i \psi^j + f_4 \tilde{\chi}^i \tilde{\psi}^j + f_5 \psi^i \tilde{\psi}^j + f_6 \psi^i \tilde{\psi}^j + f_7 \tilde{\psi}^i \psi^j]
\]

(A10)

The mass\(^2\) matrix of the slepton–mirror slepton comes from three sources, the \(F\) term, the \(D\) term of the potential, and the soft SUSY breaking terms. Using the above superpotential and after the breaking of the electroweak symmetry, we get for the mass part of the Lagrangian \(L_F\) and \(L_D\) the following set of terms:

\[
-L_F = (m_\tau^2 + |f_3|^2)\bar{\tilde{\psi}}_L \tilde{\psi}_L + (m_\nu^2 + |f_3|^2)\bar{\tilde{\nu}}_R \tilde{\nu}_R + (m_\tau^2 + |f_4|^2)\bar{\tilde{\psi}}_L \tilde{\psi}_L + (m_\nu^2 + |f_4|^2)\bar{\tilde{\nu}}_R \tilde{\nu}_R + (m_\tau^2 + |f_4|^2)\bar{\tilde{\tau}}_R \tilde{\tau}_R
\]

\[
+ (m_\nu^2 + |f_4|^2)\bar{\tilde{\nu}}_R \tilde{\nu}_R + (m_\tau^2 + |f_3|^2)\bar{\tilde{\psi}}_L \tilde{\psi}_L + (m_\nu^2 + |f_3|^2)\bar{\tilde{\nu}}_R \tilde{\nu}_R + \{m_\tau \mu^* \tan \beta \bar{\tilde{\tau}}_R \tilde{\tau}_R - m_\mu^* \tan \beta \bar{\tilde{\nu}}_R \tilde{\nu}_R
\]

\[
- m_\nu \mu^* \cot \beta \bar{\tilde{\nu}}_R \tilde{\nu}_R - m_\nu \mu^* \cot \beta \bar{\tilde{\psi}}_L \tilde{\psi}_L + (m_{\tilde{\tau}} f_3 + m_{\tilde{\tau}} f_4) \bar{\tilde{\psi}}_L \tilde{\psi}_L + (m_{\tilde{\nu}} f_3 + m_{\tilde{\nu}} f_4) \bar{\tilde{\nu}}_R \tilde{\nu}_R
\]

\[
+ (m_\nu f_3 - m_{\tilde{\tau}} f_3) \bar{\tilde{\nu}}_R \tilde{\nu}_R + (m_{\tilde{\tau}} f_4 - m_{\tilde{\nu}} f_4) \bar{\tilde{\tau}}_R \tilde{\tau}_R + (m_{\tilde{\nu}} f_4 - m_{\tilde{\tau}} f_4) \bar{\tilde{\tau}}_R \tilde{\tau}_R + H.c.\}
\]

(A11)

and

\[
-L_D = \frac{1}{2} m_\nu^2 \cos^2 \theta_W \cos 2\beta (\bar{\tilde{\nu}}_L \tilde{\nu}_L - \bar{\tilde{\tau}}_L \tilde{\tau}_L + \bar{\tilde{\psi}}_L \tilde{\psi}_L - \bar{\tilde{\nu}}_R \tilde{\nu}_R) + \frac{1}{2} \frac{m_\nu^2 \sin^2 \theta_W \cos 2\beta (\bar{\tilde{\nu}}_L \tilde{\nu}_L + \bar{\tilde{\tau}}_L \tilde{\tau}_L - \bar{\tilde{\psi}}_L \tilde{\psi}_L - \bar{\tilde{\nu}}_R \tilde{\nu}_R + \bar{\tilde{\tau}}_R \tilde{\tau}_R) + 2 \bar{\tilde{\psi}}_L \tilde{\psi}_L - 2 \bar{\tilde{\tau}}_R \tilde{\tau}_R \}
\]

(A12)

Next, we add the general set of soft supersymmetry breaking terms to the scalar potential so that

\[
V_{soft} = M_{\tilde{\psi}}^2 \bar{\tilde{\psi}}^i \tilde{\psi}^i + M_{\tilde{\nu}}^2 \bar{\tilde{\nu}}^i \tilde{\nu}^i + M_{\tilde{\tau}}^2 \bar{\tilde{\tau}}^i \tilde{\tau}^i + M_{\tilde{\psi}}^2 \bar{\tilde{\psi}}^i \tilde{\psi}^i + M_{\tilde{\nu}}^2 \bar{\tilde{\nu}}^i \tilde{\nu}^i + M_{\tilde{\tau}}^2 \bar{\tilde{\tau}}^i \tilde{\tau}^i + M_{\tilde{\psi}}^2 \bar{\tilde{\psi}}^i \tilde{\psi}^i + M_{\tilde{\nu}}^2 \bar{\tilde{\nu}}^i \tilde{\nu}^i + M_{\tilde{\tau}}^2 \bar{\tilde{\tau}}^i \tilde{\tau}^i + f_{12} A_H \bar{\tilde{\psi}}^i \tilde{\psi}^i - f_{14} A_H \bar{\tilde{\psi}}^i \tilde{\psi}^i + f_{22} A_H \bar{\tilde{\psi}}^i \tilde{\psi}^i - f_{24} A_H \bar{\tilde{\psi}}^i \tilde{\psi}^i + H.c.
\]

(A13)

From \(L_{F,D}\), and by giving the neutral Higgs their vacuum expectation values in \(V_{soft}\), we can produce the mass\(^2\) matrix \(M_{\tilde{\psi}}^2\) in the basis (\(\tilde{\tau}_L, \tilde{\psi}_L, \tilde{\nu}_R, \tilde{\nu}_L\)). We label the matrix elements of these as \((M_{\tilde{\psi}}^2)_{ij} = M^2_{\tilde{\psi}}\), where
$M_{11} = M_{2L}^2 + m_2^2 + |f_3|^2 - m_2^2 \cos 2\beta \left( \frac{1}{2} - \sin^2 \theta_W \right),$

$M_{22} = M_{2E}^2 + m_2^2 + |f_4|^2 + m_2^2 \cos 2\beta \sin^2 \theta_W,$

$M_{33} = M_{3L}^2 + m_2^2 + |f_4|^2 - m_2^2 \cos 2\beta \sin^2 \theta_W,$

$M_{44} = M_{3\ell}^2 + m_4^2 + |f_3|^2 + m_2^2 \cos 2\beta \left( \frac{1}{2} - \sin^2 \theta_W \right).$  \hspace{1cm} (A14)

$M_{12} = M_{21} = m_{E} f_3 + m_r f_4,$

$M_{13} = M_{31} = m_r (A_r^\nu - \mu \tan \beta),$  \hspace{1cm} (A15)

$M_{14} = M_{41} = 0,$

$M_{23} = M_{32} = 0,$

$M_{24} = M_{42} = m_{E} (A_r^\nu - \mu \cot \beta),$  \hspace{1cm} (A15)

$M_{34} = M_{43} = m_{E} f_4 + m_r f_3.$

Here, the terms $M_{11}, M_{13}, M_{31}, M_{33}$ arise from soft breaking in the sector $\tilde{\tau}_L, \tilde{\tau}_R.$ Similarly, the terms $M_{22}, M_{24}, M_{42}, M_{44}$ arise from soft breaking in the sector $\tilde{E}_L, \tilde{E}_R.$ The terms $M_{12}, M_{21}, M_{33}, M_{34}, M_{41}, M_{43, 44}$ arise from mixing between the staus and the mirror. We assume that all the masses are of the electroweak scale so all the terms enter in the mass$^2$ matrix. We diagonalize this Hermitian mass$^2$ matrix by the unitary transformation $\tilde{D}\nu^\dagger M_{\nu}^2 \tilde{D}\nu = \text{diag}(m_{\nu}^2, m_{\tilde{\tau}}^2, m_{\tilde{\tau}}^2, m_{\tilde{\tau}}^2).$ There is a similar mass$^2$ matrix in the sneutrino sector. In the basis $(\tilde{\nu}_L, \tilde{N}_L, \tilde{\nu}_R, \tilde{N}_R),$ we can write the sneutrino mass$^2$ matrix in the form $(M_{\tilde{\nu}}^2)_{ij} = m_{\tilde{\nu}}^2,$ where

$m_{\tilde{\nu}}^2 = M_{22}$

$m_{\tilde{\nu}}^2 = M_{33}$

$m_{\tilde{\nu}}^2 = M_{44}$

$m_{\tilde{\nu}}^2 = M_{11}$

$m_{\tilde{\nu}}^2 = M_{21}$

$m_{\tilde{\nu}}^2 = M_{31}$

$m_{\tilde{\nu}}^2 = M_{41}$

$m_{\tilde{\nu}}^2 = m_{E} f_3 + m_r f_4 - m_N f_3,$

$m_{\tilde{\nu}}^2 = m_{E} (A_r^\nu - \mu \tan \beta),$  \hspace{1cm} (A15)

$m_{\tilde{\nu}}^2 = 0,$

$m_{\tilde{\nu}}^2 = 0,$

$m_{\tilde{\nu}}^2 = m_N (A_r^\nu - \mu \tan \beta),$

$m_{\tilde{\nu}}^2 = m_N f_3 - m_r f_3.$

As in the charged slepton sector, here also, the terms $m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2$ arise from soft breaking in the sector $\tilde{\nu}_L, \tilde{\nu}_R.$ Similarly, the terms $m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2$ arise from soft breaking in the sector $\tilde{N}_L, \tilde{N}_R.$ The terms $m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2,$ arise from mixing between the physical sector and the mirror sector. Again, as in the charged lepton sector, we assume that all the masses are of the electroweak size so all the terms enter in the mass$^2$ matrix. This mass$^2$ matrix can be diagonalized by the unitary transformation $\tilde{D}\nu^\dagger M_{\tilde{\nu}}^2 \tilde{D}\nu = \text{diag}(m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2, m_{\tilde{\nu}}^2).$ The physical $\tau$ and neutrino states are $\tau = \tau_1, \nu = \nu_1,$ and the states $\tau_2, \nu_2$ are heavy states with mostly mirror particle content. The states $\tilde{\tau}, \tilde{\nu}; i = 1-4$ are the slepton and sneutrino states. For the case of no mixing, these limits are as follows: $\tilde{\tau}_1 \rightarrow \tilde{\tau}_L, \tilde{\tau}_2 \rightarrow \tilde{E}_L, \tilde{\tau}_3 \rightarrow \tilde{\tau}_R, \tilde{\tau}_4 \rightarrow \tilde{E}_R, \tilde{\nu}_1 \rightarrow \tilde{\nu}_L, \tilde{\nu}_2 \rightarrow \tilde{\nu}_L, \tilde{\nu}_3 \rightarrow \tilde{\nu}_R, \tilde{\nu}_4 \rightarrow \tilde{\nu}_R.$ The couplings $f_3, f_4,$ and $f_5$ can be complex, and thus the matrices $D_{L,R}$ and $D_{R,L}$ will have complex elements that would produce electric dipole moments through their arguments discussed in the text of the paper. Also, the trilinear couplings $A_{\nu,E,}\tilde{\tau}$ could be complex and produce electric dipole moment through the arguments of $\tilde{D}_{L,R}$ and $\tilde{D}_{R,L}.$ We will assume for simplicity that this is the only part in the theory that has $CP$ violating phases (for a recent review of $CP$ violation, see [18]). Thus the $\mu$ parameter is considered real along with the other trilinear couplings in the theory. In this way we can automatically satisfy the constraints on the EDMs of the electron, the neutron and of Hg and Thallium.


