Hierarchies and Textures in Supergravity Unification

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It is proposed that supergravity unified models possess, in addition to the hidden and the visible sectors, a third sector which contains exotic matter with couplings to the fields of both the visible and the hidden sectors, and that its elimination leads to quark-lepton textures at the grand unified theory (GUT) scale with a hierarchy in powers of $M_G/M_P$. Simultaneously the GUT mass is computed in terms of the Planck/string scale. Textures in the Higgs triplet sector are computed using a complete set of texture sum rules, and it is shown that they modify predictions of proton decay in unified models.

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One of the puzzles of particle physics is the question of how the quark-lepton mass hierarchies arise and attempts have been made over the years to resolve this problem [1]. An interesting observation towards a solution by Georgi and Jarlskog [2] is that the quark-lepton mass pattern can be understood in terms of textures at the grand unified theory (GUT) scale, and there have been many further attempts to implement and explore this idea [3]. The simplest GUT theories, the ordinary or the supersymmetry (SUSY) ones, do not contain such textures. Thus while the minimal supersymmetric SU(5) model predicts $m_b/m_{\tau}$, which is in fair agreement with experiment, the ratios $m_{s}/m_{\mu}$ and $m_{d}/m_{e}$ are off by an order of magnitude. In this Letter we present a supergravity framework which incorporates the textures. Our starting point is the assumption that there exists a new kind of matter (exotic matter) beyond the spectrum of the minimal supergravity model and that this matter couples to both the visible sector fields and to the fields of the hidden sector [4,5]. The above assumption leads us to include a third sector in supergravity unified models where such matter resides [6]. After spontaneous breaking of supersymmetry the exotic matter becomes superheavy with masses of size $O(M_P)$, where $M_P$ is an effective superheavy scale of order the Planck scale. Integration over the spectrum of the exotic matter leads to a set of Planck slope terms scaled by powers of $M_G/M_P$ at the GUT scale. The ratio $M_G/M_P$ is determined dynamically in this model. We then show that Planck slope terms of this type are sufficient to generate the full textures. We also compute in this model the textures in the Higgs triplet sector and find that they are different from those in the quark-lepton mass sector. This result has important implications for proton decay lifetimes which we also discuss. We give now the details of the analysis. As discussed above, the superpotential we consider has three sectors:

$$W = W_V + W_H + W_{VH},$$

where $W_V$ is the superpotential in the visible sector which contains quarks, leptons, and Higgs fields; $W_H$ is the superpotential in the hidden sector which contains fields $z$ whose vacuum expectation value (VEV) growth is of $O(M_P)$ and leads to the breaking of supersymmetry; and $W_{VH}$ is the potential that contains exotic matter which couples to both the fields of the visible sector and of the hidden sector. For simplicity we assume that there are no operators with dimensionality greater than 3 in $W$. While the framework of the analysis we present is general and can be implemented for any GUT group, we focus here on the extension of the minimal SU(5) model. We assume that the superpotential in the visible sector is given by

$$W_V = H_{1x} f_{1ij} K_{ij} + H_{2} f_{2ij} J_{2xij} + [M_P s^2 + (\alpha_1/3) \Sigma^3 + (\alpha_2/2) s \Sigma^2] + W_R, \quad (2)$$

where $i, j (=1, 2, 3)$ are the generation indices, $(H_{1x})$ is the 5-plet of Higgs, $(H_{2})$ is the 5-plet of Higgs, $(\Sigma)$ is a 24-plet of Higgs whose VEV growth breaks SU(5) → SU(3) × SU(2) × U(1), and $s$ is a gauge singlet. $f_1$ and $f_2$ are the Yukawa couplings, and $J_{2ij}$ and $K_{ij}$ are the currents defined in Eqs. (3) and (4) in terms of the three generations of 5-plet of matter ($M_{i1}, i = 1, 2, 3$) and 10-plet of matter ($M_{i3}^H$). $W_R$ is the remaining part which gives masses to the Higgs triplets while keeping the Higgs doublet light. For exotic matter we make the simple assumption that it belongs to $\tilde{5} + 5$ representations of SU(5) (with equal numbers of $\tilde{5}$ and 5 for anomaly cancellation). The sector which contains exotic matter, i.e., $W_{VH}$, has two parts: the first part contains the coupling of exotic matter with the hidden sector fields $z_I$, while the second part couples exotic matter to the fields of the visible sector. In general, one needs three different types of exotic fields to preserve matter parity. We shall distinguish these by the subscript $a (a = 1, 2, 3)$. This is required by the fact that there are three different $\tilde{5}$- and 5-plets of currents that can be formed out of the fields of the visible sector, and thus one needs three different types of 5 and $\tilde{5}$ of exotic fields to couple to these. Thus the three
5-plots of currents of dimensionality 2 that can be formed out of the fields that enter the visible sector are

\[ J_{1x} = H_{1y} \Sigma^{y}, \]

\[ J_{2xij} = -(1/8) \epsilon_{uvw} M^{uv} M_{ij}^{y}, \tag{3} \]

\[ J_{3xi} = M_{iy}^{y} \Sigma^{y}. \]

Similarly there are three different 5-plots of currents \( K_{a} = (K_{a_{1}}, K_{2}, K_{3}) \) which can be constructed out of the fields appearing in the visible sector. These are

\[ K_{1ij} = M_{x}^{y} H_{y}^{x}, \quad K_{2} = \Sigma^{y} H_{y}^{x}, \quad K_{3i} = M_{iy}^{y} H_{y}^{x}. \tag{4} \]

One introduces the 5-plot of exotic fields \( E_{ax} \) and the 5-plot of exotic fields \( F_{a} \) so that the interactions \( E_{ax} K_{a}^{y} \) and \( F_{a} J_{ax} \) are matter parity invariant. For generality we introduce \( n \) pairs of such fields and denote these by \( E_{ax}, F_{a}(x = 1,\ldots,n) \), and write \( W_{VH} \) as

\[ W_{VH} = c_{a_{1}a_{2}b} M_{x}^{y} E_{ax} F_{a_{2}} + \alpha_{x} F_{a} \Sigma^{y} E_{ax} \tag{5} \]

where \( \alpha_{x} \) and \( \Sigma^{y} \) are defined by

\[ \alpha_{x} = (E_{ax}, J_{ax}), \quad \Sigma^{y} = (F_{a}, K_{a}). \tag{6} \]

\[ W_{VH}^{e} = H_{1x} \left( U_{1ij} \frac{\Sigma}{M_{p}} + U_{2ij} \frac{\Sigma^{2}}{M_{p}} + U_{3ij} \frac{\Sigma^{3}}{M_{p}} + \cdots \right) \Sigma^{y} + M_{x}^{y} H_{3}^{x} \]

\[ + J_{2xij} \left( W_{1ij} \frac{\Sigma}{M_{p}} + W_{2ij} \frac{\Sigma^{2}}{M_{p}} + W_{3ij} \frac{\Sigma^{3}}{M_{p}} + \cdots \right) \Sigma^{y}. \tag{9} \]

We find that the effective interaction of quarks, leptons, and Higgs fields at the GUT scale contains a hierarchy of new mass scales via an expansion in \( \Sigma/M_{p} \) [7]. This expansion arises because of the couplings of the exotic fields with \( \Sigma \) as can be seen from Eqs. (5)–(7). After spontaneous breaking of the GUT symmetry \( \langle \Sigma \rangle = M(2, 2, 2, -3, -3) \) and the interactions of the Higgs doublets and Higgs triplets at the GUT scale are given by

\[ W = H_{i} \left( H_{i}^{a} + H_{i}^{a} \Sigma^{y} \right) + H_{i} \left( H_{i}^{a} + H_{i}^{a} \Sigma^{y} \right) + \epsilon_{abc} H_{i} \left( H_{i}^{a} \right) + \epsilon_{abc} H_{i} \left( H_{i}^{a} \right). \tag{10} \]

Here \( A^{E,U}, B^{E,U}, \) and \( C^{U} \) are matrices in the generation space and are determined in a power series expansion in \( \epsilon (\equiv M/M_{p}) \) using Eq. (9). In the analysis here we assume that Eq. (2) contributes only to the up-quark sector (even there only to the top quark mass) and all of the remaining quark and lepton masses arise from higher dimensioned operators that are generated when one integrates over the exotic fields, i.e., from Eq. (9). Thus we set \( f_{1} = 0 \) in Eq. (2). One can show purely on geometric grounds using only the general form of Eq. (9) that the textures in Eq. (10) satisfy three nontrivial sum rules. Two of these appear in the down-quark lepton sector while the third appears in the up-quark sector. Using Eq. (9) we find that the textures that enter in the down-quark lepton sector satisfy the relations

\[ A^{E} = P_{3} - Q_{3}, \quad A^{D} = P_{3} - Q_{2}, \tag{11} \]

\[ B^{E} = -P_{2} + Q_{3}, \quad B^{D} = P_{2} - Q_{2}, \]

Since \( E_{ax} \) and \( F_{a} \) have dimensionality 1 while \( J_{ax} \) and \( K_{a} \) have dimensionality 2, the assumption that \( W_{VH} \) has no operators of dimensionality greater than 3 then requires that

\[ C^{(2)}_{ab} = 0, \quad C^{(2)}_{a_{1}a_{2}b} = C^{(0)}_{a_{1}a_{2}b} \Sigma^{y}, \tag{7} \]

where \( C^{(a)a_{1}a_{2}b} \) is field independent.

After spontaneous breaking of supersymmetry one gets a mass generation in the heavy sector with a mass term of \( E_{ax} M_{a_{1}a_{2}b} F_{a_{2}} \), where \( M_{a_{1}a_{2}b} \equiv (z_{l}) C_{l a_{1}a_{2}b} \) and \( M_{a_{1}a_{2}b} \sim O(M_{p}) \). We proceed now to eliminate the superheavy fields \( E_{ax} \) and \( F_{a} \) where \( \mu = (aa) \) and at the same time carry out an expansion in \( \Sigma/M_{p} \). Thus, for example,

\[ E_{ax} = \frac{1}{\Sigma} \left( F_{a} \Sigma^{y} \right) M_{a_{1}a_{2}b} \]

\[ + \frac{1}{\Sigma} \left( F_{a} \Sigma^{y} \right) M_{a_{1}a_{2}b} \]

\[ \times \left( \Sigma^{y} \right) M_{a_{1}a_{2}b} \]

\[ + \cdots \]

and a similar equation holds for \( F_{a} \). Eliminating the heavy fields in Eq. (5) and carrying out a radi diagonalization, and assuming for simplicity that \( M_{a_{1}a_{2}b} = M_{p} \delta_{a_{1}b} \delta_{a_{2}b} \), then gives an effective superpotential at the GUT scale of the form

\[ \left( P_{n}, Q_{n} \right) \]
couplings can be computed in a power series expansion in $\epsilon$. We get

$$A^U = \sum_{k=0,1,2,\ldots} (-3\epsilon)^k W_k, \quad B^U = \sum_{k=0,1,2,\ldots} (2\epsilon)^k W_k, \quad C^U = B^U.$$

(16)

From the above we derive the sum rule

$$B^{U(k)} = (-2/3)^k A^{U(k)}, \quad k = 0, 1, 2, \ldots,$$

(17)

where $A^{U(k)}$ and $B^{U(k)}$ are defined analogously to $A^{E(k)}$ and $B^{E(k)}$. It is then seen that all the textures in the Higgs triplet sector can be computed in terms of the textures in the Higgs doublet sector via the three sum rules of Eqs. (13), (14), and (17) or alternately via Eqs. (14), (15), and (17). There are several possible solutions [3] to the textures $A^{E,D,U}$ at the GUT scale which can lead to acceptable low energy quark-lepton masses. We construct here one specific example.

It is seen that the choice $f_{1ij} = 0$, $f_{2ij} = \alpha_{ij3}$, and

$$\epsilon U_{1ij} = -\frac{1}{3} D \delta_{i3} \delta_{j3}, \quad \epsilon^2 U_{2ij} = \frac{7}{12} E \delta_{i3} \delta_{j3} + \frac{1}{3} \delta \delta_{i3} \delta_{j3}, \quad \epsilon^3 U_{3ij} = \epsilon^3 V_{3ij} - \frac{1}{27} (\delta_{i1} \delta_{j2} + \delta_{i2} \delta_{j1}),$$

(18)

$$-3\epsilon W_{1ij} = B(\delta_{i2} \delta_{j1} + \delta_{i1} \delta_{j2}), \quad 9\epsilon^2 W_{2ij} = C(\delta_{i1} \delta_{j2} + \delta_{i2} \delta_{j1}),$$

leads to

$$A^E = \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix}, \quad A^D = \begin{pmatrix} 0 & F e^{i\phi} & 0 \\ F e^{-i\phi} & E & 0 \\ 0 & D & \delta \end{pmatrix}, \quad A^U = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix},$$

(19)

which are the Georgi-Jarlskog textures when $\delta = 0$ [2,8].

From Eqs. (18) and (19) we find that in the down-quark and lepton sectors one has $D \sim O(\epsilon)$, $E \sim O(\epsilon^2)$, and $F \sim O(\epsilon^3)$, and in the up-quark sector one has $A \sim O(1)$, $B \sim O(\epsilon)$, and $C \sim O(\epsilon^2)$. These are the orders needed to correctly generate the mass hierarchies in the down-quark and lepton sectors and in the up-quark sector.

We turn now to a determination of the ratio $\epsilon$. Minimization of the potential with respect to the fields $s$ and $\Sigma$ determines the GUT scale in terms of the Planck scale, i.e., $\epsilon = -\lambda_1/(15\lambda_2^2)$ [9]. Thus, for example, with $\lambda_1 = -0.6$ and $\lambda_2 = 1.5$ one gets $\epsilon = 1/60$. With this value of $\epsilon$ one finds that the quark-lepton mass hierarchies are well reproduced with the (remaining) Yukawa couplings $O(1)$. Thus $M_P = 2.4 \times 10^{18}$ GeV implies an $M_G \sim 4 \times 10^{16}$ GeV which is about the correct GUT scale. We note that it is the $J_3 \cdots K_3$ interaction structure which gives the $\epsilon^2 V_{22}$ term in Eq. (18) and is responsible for breaking the down-quark and lepton mass degeneracy.

Remarkably, as is apparent from the sum rules discussed earlier the inputs of Eq. (18) which determine the quark-lepton textures of Eq. (19) can also be used to compute the textures in the Higgs triplet sector. We find

$$B^E = \begin{pmatrix} 0 & (-\frac{19}{27} + e^{-i\phi}) F & 0 \\ 0 & \frac{16}{3} E & 0 \\ 0 & 0 & (\frac{2}{3} D + \delta) \end{pmatrix},$$

(20)

$$B^D = \begin{pmatrix} 0 & -\frac{8}{27} F & 0 \\ -\frac{8}{27} F & -\frac{4}{3} E & 0 \\ 0 & 0 & -\frac{2}{3} D \end{pmatrix}, \quad B^U = \begin{pmatrix} 0 & \frac{4}{3} C & 0 \\ \frac{4}{3} C & 0 & -\frac{2}{3} B \\ 0 & -\frac{2}{3} B & A \end{pmatrix}.$$

(21)

Comparison of Eqs. (19)–(21) shows that the textures in the Higgs triplet sector are different than those in the Higgs doublet sector. We shall show elsewhere that the use of Planck corrections without the constraint of exotic sector gives ambiguous results for textures in the Higgs triplet sector. Textures affect the predictions of the proton decay modes. The analysis requires that one use renormalization group equations to scale down the textures in the quark-lepton sector from the GUT scale to the electroweak scale. One then goes to the basis where the quark-lepton mass matrices are diagonal by defining transformations

$$U_R A^U U_L = A^{U(d)}, \quad D_R A^D D_L = A^{D(d)}, \quad E_L A^E E_R = A^{E(d)}.$$

(22)

Next we turn to the baryon-number-violating interactions in this basis. After integrating over the heavy Higgs triplet fields the baryon-number-violating dim5 operator is given by

$$W_5 = \frac{1}{M_{H3}} (-e_L U^{(1)} B^{E(d)} V^{(1)} u_{Ld} + \nu U^{(1)} B^{E(d)} U^{(2)} e_{abc} u_{bl} P U^{(2)} B^{U(d)} V^{(2)} d_{el} + \frac{1}{M_{H3}} (u^a_0 A^{U(d)} V^{(3)} e_{abc} d^a_{cl} U^{(3)} B^{D(d)} V^{(4)} p^a u^c_l)).$$

(23)

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In the above the first two sets of terms are the $LLLL$ type dim5 operators, and the last term is the $RRRR$ type dim5 operator, $P = U_L^1 U_R$, and $U^{(i)} (i = 1, 2, 3)$ are defined by
\[
U^{(1)} = E_L^T E_{1L}^*, \quad U^{(2)} = U_R^1 U_{1R}, \quad U^{(3)} = D_R^1 D_{1R},
\]
where $B^E$, $B^U$, and $B^D$ are diagonalized by the transformations
\[
E_{1L}^T B^E E_{1R}^* = B^{E(d)}, \quad U_R^1 B^U U_{1L} = B^{U(d)}, \quad D_R^1 B^D D_{1L} = B^{D(d)}.
\]
The matrices $V^{(i)}$ are defined so that
\[
V^{(1)} = U_L^1 E_{1R}^*, \quad V^{(2)} = U_{1L}^* D_L, \quad V^{(3)} = U_L^1 E_R^*, \quad V^{(4)} = U_L^* D_{1L}.
\]

There have been extensive previous analyses of proton stability [10] in the literature. All of these [10] have ignored the effect of the textures on $p$-decay lifetime. In the usual analyses one uses the assumption that $B^E = -A^E$, $B^D = A^D$, and $B^U = A^U$. In this limit $U^{(i)} (i = 1, 2, 3)$ are all unity, and $V^{(i)} (i = 1, \ldots, 4)$ equals the KM matrix $V_{KM}$. It has recently been realized that quark-lepton textures can affect low energy physics [3,11]. In this Letter we have given a full solution to the problem of how proton decay interactions depend on the textures in the context of a specific model. These textures are model dependent. Thus different assumptions about the $A^E,D,U$ textures at the GUT scale would in general lead to different $B^E,D,U$ textures in the Higgs triplet sector. One of the consequences of the above analysis is that one cannot diagonalize $A^E$ and $B^E$ by the same transformations. A similar situation holds for $A^D$ and $B^D$ and for $A^U$ and $B^U$. Because of this there are modifications of the predictions of $p$-decay lifetime. One can make a rough estimate of the size of the change that one expects. For the most dominant $p$-decay mode $p \rightarrow \bar{p} K$ we note that if we ignore the effect of the mismatch matrices $U^{(i)}$ and assume that $V^{(i)}$ are all the same, then the change in the lifetime of this decay mode can be estimated as follows. For the standard case discussed previously the $p$-decay width is proportional to $(m_e m_\mu)^2$. With the new textures the decay width will be proportional to $(4/9)m_e (16/9)m_\mu$ as can be seen by a comparison of the eigenvalues of $A$ and $B$. This would imply an enhancement in the $p$-decay lifetime for this mode by a factor of about 3. We also note that the appearance of the $CP$-violating phase in the GUT sector as given by Eq. (20) may have implications for cosmology, such as in the generation of baryonic asymmetry in the early universe.

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[7] There have been schemes previously discussed for generating fermion mass hierarchies which make use of heavy fields. [See H. Georgi, Nucl. Phys. B156, 126 (1979); S.M. Barr, Phys. Rev. D 21, 1424 (1980).] In these analyses the first and second generation masses have a radiative origin. In the mechanism we discuss here masses for all three generations arise at the tree level and the hierarchy generating parameter is the ratio of two mass scales rather than a loop effect.
[8] The textures of Eq. (19) with $\delta = 0$ lead to an exact $b-\tau$ unification at the GUT scale. One can generate a $b-\tau$ splitting by allowing a nonvanishing value for $\delta$. Since $\delta/D$ is $O(\epsilon)$, one can expect a few percent correction to the exact $b-\tau$ universality at the GUT scale.
[9] We note that the mass of the heavy $X$ vector boson associated with the breaking of the SU(5) symmetry is given by $M_v/M_P = 5\sqrt{2} g_s \epsilon$, and the Higgs triplet mass is given by $M_H^2/M_P = 5\Lambda_2 \epsilon$.