CP Violation via Electroweak Gauginos and the Electric Dipole Moment of the Electron

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The effective Lagrangian which exhibits explicitly the transmission of the CP-violating phases induced via the electroweak gaugino masses below the scale of the spontaneous breaking of SU(2) × U(1) electroweak gauge symmetry is deduced. The formalism is used to compute the one-loop electric dipole moment of the electron including the full set of neutralino states. Contributions of neutralinos other than the photino are found to be significant. It is shown that the current data exclude maximal CP violation in the electroweak sector except for selectron masses \( \gtrsim 200 \text{ GeV} \) and photino masses \( \gtrsim \) 750 GeV.

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Over the past year an improvement on the limits of the electric dipole moment (EDM) of the electron by more than 2 orders of magnitude\(^1\) over previous measurements\(^2\) has been reported. The recent results of the Amherst group (Murthy et al.\(^3\)) and of the Berkeley group (Abdullah et al.\(^4\)) for \( \delta_e \) are, respectively,

\[
(1a) \quad (-1.5 \pm 5.5 \pm 1.5) \times 10^{-26} \text{ cm}, \\
(1b) \quad (-2.7 \pm 8.3) \times 10^{-27} \text{ cm}.
\]

In view of this improvement in experiment and the possibility that the error in the most recent measurement Eq. (1b) may be further significantly reduced in the near future,\(^3\) a fresh look at quantitative evaluations of the electron’s EDM beyond the standard model becomes desirable. The existence of an electron EDM \( \delta_e \gg 10^{-38} \) e cm (where \( \sim 10^{-38} \) e cm is the value of \( \delta_e \) expected in the standard model) is a clean test of new physics beyond the standard model, since unlike the neutrino electric dipole moment which is plagued by large QCD correction factors\(^4\) and uncertainties,\(^5\) there are no corresponding QCD suppression factors or uncertainties in the evaluation of the electron’s EDM.\(^6\) Recently, several papers have reported a value of \( \delta_e \) in the vicinity of the results of Eqs. (1) in non-standard-model frameworks.\(^7\) In this Letter we shall be concerned with the electron’s EDM in supersymmetric theories.\(^8\) Although this problem has a long history,\(^9\) a full analysis of the electron’s EDM which exhibits its dependence on the several CP-violating phases that enter in the spontaneously broken supersymmetric theory has not been given. Thus in all of the previous analyses, quantitative discussions of contributions of the neutralino other than that of the photino exchange have not been given. In this Letter we present a framework which deduces the low-energy effective Lagrangian below the scale of SU(2) × U(1) spontaneous breaking with explicit exhibition of the effect of CP violation induced via weak gauginos. This effective Lagrangian is then used to compute the electron’s EDM at the one-loop level. One finds that for large gaugino masses the contribution of the neutralino states other than the photino are comparable to that of the photino. It is shown that the current data do not allow maximal CP violation in the electroweak sector except for selectron masses \( \gtrsim 200 \) GeV or photino masses \( \gtrsim \) 750 GeV.

As is well known, in softly broken supersymmetric theories the CP-violating phases may be isolated in a few soft-supersymmetry-breaking parameters by a suitable redefinition of fields, e.g., in the Higgs-field mixing parameter \( (\mu) \), in the Poloni constant \( (A) \), and in the gaugino masses \( (\tilde{m}) \).\(^8\) We assume the mass term in the gaugino sector to be SU(2) × U(1) invariant and to have the form

\[
L_m(\text{gaugino}) = - \frac{1}{2} \tilde{\lambda}^a \tilde{m}_1 e^{i \gamma^2 \phi'} \lambda^0 - \frac{1}{2} \tilde{\lambda}^a \tilde{m}_2 e^{i \gamma^2 \phi'^2} \lambda^a,  
\]

(2a)

where \( \bar{\lambda}^a (a = 1, 2, 3) \lambda^0 \) are weak gauginos for the SU(2) × U(1) gauge groups. One has in addition to Eq. (2a), the mass term for the Higgsinos and an off-diagonal mass term between gauginos and Higgsinos:\(^8\)

\[
L_m(\phi, - H) = - \tilde{\Gamma} e^{i \gamma^2 \phi} \tilde{H} \nonumber
\]

\[
- \frac{1}{2} m_1 \tilde{e}^{i \gamma^2 \phi} \tilde{H}^2 + \text{H.c.},  
\]

(2b)

where \( \phi = (H,H') \) are the SU(2) × U(1) Higgs doublets and \( T^a \) are the Gell-Mann matrices. We introduce now the notation\(^8\)

\[
\psi^0 = \cos \theta_w \lambda^0 + \sin \theta_w \lambda^3,  
\]

(3)

\[
\psi^1 = \sin \theta_w \lambda^0 - \cos \theta_w \lambda^3,  
\]

\[
\psi^2 = i \left[ - \sin \theta (H_1^3 - H_2^3) + \cos \theta (H_2^2 - H_2^2) \right],  
\]

\[
\psi^3 = - i \left[ \sin \theta (H_2^2 - H_2^2) + \cos \theta (H_1^3 - H_1^3) \right],  
\]

where \( \tan \theta = H_1^3 / H_2 \) and \( \theta_w \) is the Weinberg angle. We can write the mass matrix in the neutral gaugino-Higgsino sector as follows:

\[
L_m = - \frac{1}{2} \bar{\psi} \begin{pmatrix} \tilde{M} + i \gamma \tilde{m} \end{pmatrix} \psi,  
\]

where

\[
\begin{pmatrix} \tilde{M} + i \gamma \tilde{m} \end{pmatrix} = \begin{pmatrix} A & B \\ B & C \end{pmatrix},  
\]

\[
B = \begin{pmatrix} 0 & 0 \\ M & 0 \end{pmatrix},  
\]

\[
M = \begin{pmatrix} 0 & 0 \\ 0 & M \end{pmatrix}.  
\]
and $A$ and $C$ are given by
\[
A = \begin{pmatrix}
\bar{m}_e e^{i\gamma_5} & \bar{m}_\tau e^{i\gamma_5} \\
\bar{m}_\tau e^{i\gamma_5} & \bar{m}_e e^{i\gamma_5}
\end{pmatrix},
\]
\[
C = \begin{pmatrix}
\mu e^{i\gamma_5} \sin 2\alpha & \mu e^{i\gamma_5} \cos 2\alpha \\
\mu e^{i\gamma_5} \cos 2\alpha & -\mu e^{i\gamma_5} \sin 2\alpha
\end{pmatrix},
\]
and where $\bar{m}_e$, $\bar{m}_\tau$, etc., are defined by
\[
\bar{m}_e e^{i\gamma_5} = \bar{m}_e e^{i\theta} \cos^2 \theta_W + \bar{m}_\tau e^{i\theta} \sin^2 \theta_W,
\]
\[
\bar{m}_\tau e^{i\gamma_5} = \bar{m}_e e^{i\theta} \sin^2 \theta_W + \bar{m}_\tau e^{i\theta} \cos^2 \theta_W,
\]
\[
\bar{m}_e e^{i\gamma_5} = 2^{-1} (\bar{m}_e e^{i\theta} - \bar{m}_\tau e^{i\theta}) \sin 2\theta_W.
\]

The previous analyses proceed by diagonalizing directly, via a biunitary transformation, the complex symmetric matrix
\[
X = \begin{pmatrix}
A' & B \\
B & C
\end{pmatrix},
\]
where $(A', C)$ are $(A, C)$ with $\gamma_5$ replaced by unity in Eq. (4). This procedure does not make transparent the various CP-violating phases that enter in the formalism and to our knowledge has not been carried far enough to exhibit the transmission of these CP-violating phases below the scale of spontaneous breaking.

Here we give an alternative formulation which renders the parametrization of the CP-violating phases in the spontaneously broken theory explicit. In this procedure to effect the diagonalization of $L_m$, we first make the transformation $\psi' = e^{i\gamma_5} \phi'$ on each of the $\psi'$, which transforms $L_m$ to the form
\[
L_m = \frac{1}{2} \chi (\phi')^2 + \bar{m}_e e^{i\gamma_5} \phi',
\]
where $\phi'$ and $\phi''$ are symmetric and parametrically dependent on $\beta_i$. Next, we make an orthogonal rotation to diagonalize $\phi''$. This yields $L_m = \frac{1}{2} \chi (P^2 M^\dagger P + i\gamma_5 \mu^0)' \chi$, where $P$ is the orthogonal matrix that diagonalizes $\mu^0$. We note that any further effort to diagonalize $P^2 M^\dagger P$ will undiagonalize $\mu^0$. At this point we utilize the freedom of the $\beta_i$ to set $\mu^0$ proportional to the unit matrix, followed by a final orthogonal rotation (Q) to diagonalize the $P^2 M^\dagger P$ term, which then renders the entire mass matrix diagonal while retaining the diagonality of the kinetic terms, i.e., one has
\[
L_m = \frac{1}{2} \bar{Z}_k [(\bar{m}_e e^{i\gamma_5})^2 + (-1)^{\theta_i} \bar{\gamma}_5 \mu^0] \bar{Z}_k,
\]
\[
\bar{Z}_k = (i\gamma_5)^{\theta_0} O_{ee}^T e^{-i\gamma_5 \beta_j} \phi',
\]
where $M^2 = (\lambda_k \lambda_k)$ and $\lambda_k$ are the elements of the diagonal matrix $M_D$ defined by $M_D = O^T M' O$, with $O = P Q$ an orthogonal matrix, and $\theta_0 = \theta_j = \lambda_k$.

The interaction Lagrangian which governs the couplings of the leptons with the neutralino states is given by
\[
L_{e\tilde{\nu}} = ie \sum_k (\bar{e}_R P_L \tilde{\nu}_R + \bar{e}_L P_R \tilde{\nu}_L) (-i\gamma_5)^{\theta_0} \bar{Z}_k + h.c.,
\]
\[
U_k = -O_{e0} e^{i\theta_0} - O_{1k} \tan \theta_w e^{i\theta_1},
\]
\[
V_k = -O_{e0} e^{-i\theta_0} + O_{1k} \cot \theta_w e^{-i\theta_1}.
\]
In Eqs. (6) CP violation arises from two sources. First, there is a common CP-violating mass insertion $(-1)^{\theta_0} \gamma_5$ for all neutralino states. Second, there are CP-violating contributions arising from CP-violating phases $\beta_0$, $\beta_1$ in the vertices given by Eqs. (6b) and (6c). Equation (6) is the desired supergravity effective Lagrangian in the neutralino sector below the SU(2)$_L \times$U(1)$\gamma_5$ electroweak-symmetry-breaking scale which exhibits explicitly the CP-violating phases for each of the neutralino mass eigenstates. Since we are in the mass-diagonal basis in Eqs. (6), we can compute directly the contribution of Eqs. (6) to the electric dipole moment of the electron at the one-loop level. The loop calculation proceeds by the exchange of neutralino and selectron fields and involves chirality flip. The mass matrix mixing the LR selectron states has the form
\[
\begin{pmatrix}
\tilde{m}^2_L & \tilde{m}^2_{LR} \\
\tilde{m}^2_{RL} & \tilde{m}^2_R
\end{pmatrix},
\]
where
\[
\tilde{m}^2_{LR} = \Delta e^{i\theta_0} = m_e [A e^{-i\theta_0} m_{3/2} + (v'/v) e^{i\theta_0}],
\]
and $m_{3/2}$ is the gravitino mass, $A e^{i\theta_0}$ is the (complex) Poloni constant, $\mu e^{i\theta_0}$ is the (complex) Higgs-field mixing parameter, and $v'/v = (H^2)/\langle H^2 \rangle$. The eigenvalues of the selectron mass matrix are $\tilde{M}^2_L \pm \frac{1}{2} [m_e^2 + m_{3/2} \pm (m_e^2 - m_{3/2})^2 + 4\Delta^2]^{1/2}$. We calculate the electric dipole moment of the electron at the one-loop level to be
\[
d_e = -\frac{e}{2\pi} \sum_k (-1)^{\theta_0} (O_{ek})^3 \sin (-1)^{\theta_0} \phi_k + \phi_0 - 2\beta_0 + 2^{-1} (O_{ik})^2 (-1 + \tan^2 \theta_W) \sin (-1)^{\theta_0} \phi_k + \phi_0 - 2\beta_1
\]
\[
+ 2^{-1} O_{ek} O_{ik} (-\cot \theta_w + 3 \tan \theta_w) \sin (-1)^{\theta_0} \phi_k + \phi_0 - \beta_0 - \beta_1) \frac{1}{2} \sin 2\Theta J(x_k - J(x_k +)] \tilde{M}^{-1},
\]
where $\tan \theta_0 = \tilde{\mu}_0 / \tilde{M}^0$, $\tan 2\Theta = 2\Delta / (\tilde{M}_e^2 - \tilde{M}_\tau^2)$, $x_k \pm \tilde{M}_k / \tilde{M}_k$, $\tilde{M}_k = (\tilde{M}_e^0 + \mu_0^2)^{1/2}$, and where $J(x)$ is the loop function
\[
J(x) = 2^{-1} (1 - x)^{-3} (x - x^3 + 2x^2 \ln x).
\]
For an explicit numerical analysis we consider the limit when \( m_{\tilde{\tau}} = 0 \) and \( \cos 2\alpha = \frac{m_{\tilde{\tau}}}{m_{\tilde{\tau}}^2} \). In this limit the modes \( \lambda = \psi^0 \) and \( \psi^2 \) decouple and only the \( 2 \times 2 \) submatrix which couples \( \psi^1 \) and \( \psi^2 \) need be diagonalized. After the \( e^{i\pi/6} \) transformation a direct inspection shows that conditions needed to diagonalize \( \tilde{\mu}^2 \) are \( \sin (\beta_1 + \beta_2) = 0 \) and

\[
\tan 2\beta_1 = \frac{(\tilde{\mu}^2 \sin \phi_2 - \mu \sin \phi_1) / (\mu \cos \phi_1 + \tilde{\mu} \cos \phi_2)}{\mu \cos \phi_1 + \tilde{\mu} \cos \phi_2}.
\]

The mass eigenvalues \( \lambda \) of \( \tilde{\mu} \) are

\[
\mu^2 = \frac{1}{2} (\tilde{\mu}^2 + m_{\tilde{\tau}}^2 \pm \sqrt{(\tilde{\mu}^2 - m_{\tilde{\tau}}^2)^2 + 4\tilde{\mu}^2 m_{\tilde{\tau}}^2 \cos (\phi_1 + \phi_2) \cos (\phi_1 - \phi_2))}.
\]

Further the results of Table I show that the experimental bound on \( d_e \) of Eqs. (1) already constrains \( CP \) violation in the electroweak sector of the supersymmetric theory. Thus, for example, Table I shows that maximal \( CP \) violation in the electroweak sector is ruled out except for selectron masses \( \geq 200 \text{ GeV} \) or photino masses \( \geq 750 \text{ GeV} \). Further from Table I, one finds that for a normal size spectrum of the photino and of the selectron, the \( CP \)-violating phase is required by the experimental limit on \( d_e \) to be significantly smaller, i.e., \( \sim 10^{-2} \) of the maximum value. This limit will become even more stringent as the experimental limits on \( d_e \) improve.

In conclusion, we note that the method discussed here to deduce the effective Lagrangian Eq. (6) can be extended directly to the chargino sector and the resulting effective supergravity Lagrangian can be used for the investigation of a full array of \( CP \)-violating phenomena induced via the supersymmetric electroweak sector below the scale of \( SU(2)_L \times U(1)_Y \) breaking.

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3E. D. Cummins (private communication).


6For a recent review, see W. Bernreuther and M. Suzuki, Rev. Mod. Phys. (to be published); S. M. Barr and W. J. Mar-


11The result of Eq. (6) reduces to the standard result (Refs. 8 and 10) when $CP$-violating phases are set to zero, i.e., $\mu_0 = 0$ and $\beta = 0$.

12Our conclusion on the allowed region of maximal $CP$ violation is not sensitive to the approximation used here. Further, the approximation is not unreasonable in view of the fact that $\tilde{m}_{12} \approx -0.36 \tilde{m}$, and the current data from the CERN $e^+e^-$ collider LEP is consistent with $\tan \alpha \leq 0.77$. [See J. Lopez and D. V. Nanopoulos, Mod. Phys. Lett. A 5, 1259–1264 (1990).]