Accurate Cosmological Parameters and Supersymmetric Particle Properties

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Abstract

Future satellite, balloon and ground based experiments will give precision determinations of the basic cosmological parameters and hence determine the amount of cold dark matter in the universe accurately. We consider here two cosmological models, the $\nu$CDM model and the $\Lambda$CDM model, and examine within the framework of supergravity grand unification the effect this will have for these models on supersymmetry searches at accelerators. In the former example the gluino (neutralino) mass has an upper bound of about 720(100) GeV and gaps (forbidden regions) may develop at lower energies. In the latter case the upper bound occurs at gluino (neutralino) mass of about 520(70) GeV with the squarks and selectron becoming light when gluino (neutralino) masses are greater than 420(55) GeV. Both models are sensitive to non-universal soft breaking masses, and show a correlation between large (small) dark matter detector event rates and low (high) $b \rightarrow s + \gamma$ branching ratio.

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While particle physics and cosmology have always interacted in an important way, the development of supersymmetry (SUSY) has greatly deepened this relation. Thus SUSY models with R-parity invariance predict the existence of cold dark matter (CDM) which may make up between 40% and 80% of all the matter of the universe. SUSY allows one to construct theories that reach in energy up to the GUT scale \((M_G \cong 10^{16} \text{ GeV})\) and backwards in time to the very early universe and are consistent with all known data. In this letter we study the effects that accurate determinations of the cosmological parameters will have on the SUSY spectrum. The new satellites, MAP and Planck \([1]\) and the many ground based and balloon experiments, will measure the basic cosmological parameters, the Hubble constant \(H\), the cosmological constant \(\Lambda\), the amount of matter in the universe \(\Omega = \rho/\rho_c\) (where \(\rho\) is the density of matter and \(\rho_c = 3H^2/8\pi G_N\), \(G_N = \text{Newton’s constant}\)) etc. at the level of a few percent. While at present there are many possible cosmological models that fit the current data leading to the wide window \(0.1 \lesssim \Omega_{\text{CDM}} h^2 \lesssim 0.4\) (where \(h = (H/100) Mpc^{-1} s^{-1}\)), the new experiments will fix this quantity very accurately. This will then greatly restrict the allowed SUSY parameter space, which in turn will influence SUSY predictions at accelerators.

In this letter we consider two possibilities, \((i)\) the \(\nu\)CDM model (with CDM, neutrino hot dark matter (HDM) and baryonic (B) dark matter) and \((ii)\) the \(\Lambda\)CDM model (with CDM, a cosmological constant \(\Lambda\) and B dark matter). In order to analyse these, it is necessary to have a fixed SUSY dynamics, and we use here supergravity grand unification models with R-parity where gravity mediates the SUSY breaking in a hidden sector at a scale \(\gtrsim M_G\) and gravity as the messenger of this breaking to the physical sector \([2]\). We assume here universal gaugino masses at the GUT scale, and also universal scalar masses \(m_0\) for the first two generations of squarks and sleptons at \(M_G\) (to suppress flavor changing neutral currents). We parameterize the Higgs \(H_{1,2}\) and third generation scalar masses at \(M_G\) by \(m_{H_{1,2}}^2 = m_0^2(1 + \delta_{1,2}), m_{q_L}^2 = m_0^2(1 + \delta_3), m_{u_R}^2 = m_0^2(1 + \delta_4), m_{e_R}^2 = m_0^2(1 + \delta_5), m_{d_R}^2 = m_0^2(1 + \delta_6), \) and \(m_{L_L}^2 = m_0^2(1 + \delta_7)\), where \(q_L \equiv (\tilde{u}_L, \tilde{d}_L)\) is the \(L\) squark doublet, \(u_R\) the right up(top)-squark singlet etc. In addition, there are third generation cubic soft
breaking parameters $A_{ct}, A_{cb}, A_{or}$ for the $t, b$ and $\tau$ particles. In the following we restrict
the parameters to the range $m_0, m_\tilde{g} \leq 1 \text{ TeV}$, $|A_t/m_0| \leq 7$, $|\delta_i| \leq 1$, $\tan \beta \leq 25$, where $m_\tilde{g}$
is the gluino mass, $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$, $\langle H_{1,2} \rangle$ gives masses to the $(d, u)$ quarks and $A_t$ is the
t-quark parameter at the electroweak scale. The limit on $\tan \beta$ implies that $\delta_{5,6,7}$ and $A_{b,\tau}$
make only small contributions and so we set these to zero in the following.

Radiative breaking of $SU(2) \times U(1)$ at the electroweak scale $\beta$ determines the Higgs
mixing parameter $\mu$ ($W_\mu = \mu H_1 H_2$) at the Z boson mass $M_Z$ to be $\beta$

$$\mu^2 = \frac{t^2}{t^2 - 1} \left[ \frac{1}{2} \left( 1 - 3D_0 \right) + \frac{1}{t^2} \right] + \frac{1}{2} \left( \frac{1 - D_0}{2} \left( \delta_3 + \delta_4 \right) - \frac{1 + D_0}{2} \delta_2 + \frac{1}{t^2} \delta_1 \right) m_0^2$$

$$+ \frac{t^2}{t^2 - 1} \left[ \frac{1}{2} \left( 1 - D_0 \right) \frac{A_R^2}{D_0} + C_\delta m_\tilde{g}^2 \right] - \frac{1}{2} M_Z^2 + \frac{1}{22} \frac{t^2 + 1}{t^2 - 1} \left( 1 + \frac{\alpha_1}{\alpha_G} \right) S_0 + \text{loop corrections} \ (1)$$

where $t \equiv \tan \beta$, $D_0 \simeq 1 - (m_t/200 \sin \beta)^2$, $A_R \simeq A_t - 0.613 m_\tilde{g}$, $S_0 = Tr Y m^2$ ($Y$
= hypercharge, $m^2 = $ masses at $M_G$), $\alpha_1 (M_Z)$ is the $U(1)$ gauge coupling constant, $\alpha_G \simeq 1/24$
is the GUT scale coupling constant and $C_\delta$ is given in $\beta$. In Eq. (1), $D_0$ vanishes at the
t-quark Landau pole and $A_R$ is the residue at the pole. Thus $D_0$ is generally small ($D_0 \leq 0.23$
for $m_t = 175 \text{ GeV}$) and hence the non-universal effects are governed approximately by the
combination $\delta_2 - (\delta_3 + \delta_4)$.

For much of the parameter space, one finds that $\mu^2 \gg M_Z^2$ and “scaling” relations hold
among the neutralinos ($\chi_i^0, i = 1, \ldots 4$) and the charginos ($\chi_i^\pm, i = 1, 2$) $\beta$:

$$2 m_{\chi_i^0} \simeq m_{\chi_i^\pm} \simeq m_{\chi_2^0} \simeq \left( \frac{1}{3} - \frac{1}{4} \right) m_\tilde{g}, \text{ and } m_{\chi_2^0} \simeq m_{\chi_3^0, 4} \gg m_{\chi_1^0}. \text{ Corrections to these relations are } O(M_Z^2/\mu).$$

One also finds that the light Higgs mass obeys $m_h \lesssim 120 \text{ GeV}$. The lightest neutralino $\chi_1^0$
is the CDM particle for almost all the parameter space.

Calculations of the detector event rates for the $\chi_1^0$ particles in the Milky Way incident
on a terrestrial nuclear target proceeds in two steps. One first calculates the relic density
$\Omega_{\chi_1^0} h^2$ remaining after annihilation in the early universe $\beta$. Two regions occur here: for
$m_{\chi_1^0} \lesssim 60 \text{ GeV}$ (or by scaling, $m_\tilde{g} \lesssim 450 \text{ GeV}$) the annihilation is dominated by $h$ and $Z$
s-channel poles $\beta$. For higher $m_{\chi_1^0}$, the $t$-channel squark/slepton poles become dominant.
(These two regimes will show up below in detector event rates.) One then restricts the
SUSY parameter space so that the allowed window of $\Omega_{\chi_1^0} h^2$ is satisfied, as well as current
accelerator SUSY bounds. (These latter limit $A_t/m_0 \gtrsim -0.5$ from the $t$-quark mass \cite{4} and eliminate most of the parameter space with $\mu < 0$ from the $b \to s + \gamma$ decay data \cite{11}.) One then calculates the expected terrestrial detector event rates \cite{11} $R [\text{events/kg d}]$ for this restricted parameter space.

The $\mu$ parameter and $\tan \beta$ play an important role in determining $R$. Thus large (small) $\tan \beta$ generally gives rise to large (small) $R$. Also as $\mu$ increases (decreases) one finds generally $R$ decreases (increases). $m_0$ and $m_{\tilde{g}}$ play a more indirect role: since the early universe annihilation cross section decreases with $m_{\chi_1^0}$ and hence with $m_{\tilde{g}}$, the upper (lower) bounds on $\Omega_{\chi_1^0} h^2$ result in upper (lower) bounds on $m_{\chi_1^0}$ and $m_{\tilde{g}}$. Similarly, annihilation decreases with increasing $m_0$, generally restricting $m_0 \lesssim 200 \text{ GeV}$ (so that the upper bound in $\Omega_{\chi_1^0} h^2$ is not violated) except in the domain $m_{\chi_1^0} \gtrsim 60 \text{ GeV}$ where $h$ and $Z$ poles produce such rapid annihilation that a large $m_0$ can be accommodated.

$\nu$CDM Model. We assume here that measurements of the cosmological parameters have yielded the central values of $\Omega_\nu = 0.20$, $\Omega_B = 0.05$ and $h = 0.62$, (which are in accord with current estimates) and that $\Omega_{\text{total}} = 1$. This implies $\Omega_{\chi_1^0} = 0.75$. The errors in which the Planck sattelite can measure the various quantities have been estimated in Refs. \cite{12,13} and we find from this

$$\Omega_{\chi_1^0} h^2 = 0.288 \pm 0.013 \quad (2)$$

This may be compared with the current estimate $0.1 \leq \Omega_{\chi_1^0} h^2 \leq 0.4$ and shows the remarkable accuracy future sattelite measurements may be expected to obtain. Fig. 1 shows the maximum and minimum event rates for a xenon detector for universal soft breaking with $0.275 \leq \Omega_{\chi_1^0} h^2 \leq 0.301$, the 1 std window of Eq. (2). One may compare this with Fig. 2 of Ref. \cite{5} which uses the current estimate $0.1 \leq \Omega_{\chi_1^0} h^2 \leq 0.4$, and shows the rise in event rates with increasing $m_{\chi_1^0}$ for $m_{\chi_1^0} \lesssim 60 \text{ GeV}$ ($m_{\tilde{g}} \simeq (7-8)m_{\chi_1^0} \simeq 450 \text{ GeV}$) the $h$ and $Z$ pole dominated region in the early universe annihilation cross section, and the fall off for higher $m_{\chi_1^0}$. Fig. 1 shows a narrowing of the predicted range of allowed event rates for $m_{\tilde{g}} \gtrsim 450 \text{ GeV}$ (relative to Fig. 2 of \cite{5}), and most striking the appearance of forbidden re-
regions, i.e. gaps, in the allowed values of $m_{\tilde{g}}$ in the region $m_{\tilde{g}} \cong 500 \text{ GeV}$ and $m_{\tilde{g}} \cong 600 \text{ GeV}$. Fig. 2, with $\delta_2 = -1 = -\delta_1$ shows the effects of non-universal soft breaking. The event rates are reduced, (as expected from Eq. (1) since $\mu^2$ is increased) and the gap at $m_{\tilde{g}} \cong 500 \text{ GeV}$ is significantly widened. For the case $\delta_2 = 1 = -\delta_1$, where $\mu^2$ is decreased, one finds the event rates are significantly increased i.e. $10 \gtrsim R \text{ (events/kg d)} \gtrsim 10^{-3}$, the upper bound being at the current sensitivity of NaI detectors [14]. However, for this case the gaps have disappeared, and there is now a lower bound of $m_{\tilde{g}} \gtrsim 420 \text{ GeV} \ (m_{\chi^0_1} \gtrsim 55 \text{ GeV})$. In all cases one finds $m_{\tilde{g}} < \sim 720 \text{ GeV} \ (m_{\chi^0_1} < \sim 100 \text{ GeV})$ in order that the upper bound, $\Omega_{\chi^0_1}h^2 \leq 0.301$, is obeyed.

As commented above one expects violations of the scaling relations of size $O(M_Z^2/\mu)$. For $\delta_2 = -1 = -\delta_1$, where $\mu^2$ is increased, the effects are reduced, but can become quite significant for $\delta_2 = 1 = -\delta_1$ in the region $m_{\chi^0_1} \lesssim 60 \text{ GeV}$. The effects of non-universal soft-breaking are suppressed, however, for $m_{\chi^0_1} \gtrsim 60 \text{ GeV} \ (m_{\tilde{g}} \gtrsim 450 \text{ GeV})$ since the universal contributions in Eq. (1) dominate.

$\Lambda CDM$ Model. We consider as a second example with $\Omega_{\text{total}} = 1$, the possibility that the new measurements show the existence of a large cosmological constant with $\Omega_{\Lambda} = 0.55$. In addition we assume here baryonic matter with $\Omega_B = 0.05$ and $h = 0.62$. This would imply $\Omega_{\chi^0_1}h^2 = 0.40$. (The above choice for $\Omega_{\Lambda}$ represents the current upper limit [15], while the value for $\Omega_{\chi^0_1}$ is approximately what is actually observed from large galactic clusters.) Using the estimated errors expected for the Planck satellite [12] one finds $\Omega_{\chi^0_1}h^2 = 0.154 \pm 0.017$. Fig. 3 shows the maximum and minimum event rates for a xenon detector for three examples of $\delta_1$. We again see a reduction of event rates for $\delta_2 = -1 = -\delta_1$ and the enhancement for $\delta_2 = 1 = -\delta_1$. This time one finds a low upper bound on $m_{\tilde{g}}$ of $m_{\tilde{g}} \lesssim 520 \text{ GeV} \ (m_{\chi^0_1} \lesssim 70 \text{ GeV})$ due to the relatively low value of $\Omega_{\chi^0_1}h^2$. For the case $\delta_2 = 1 = -\delta_1$ the event rates for $\mu > 0$ are relatively high, i.e. $(3 \times 10^{-3} - 1.0) \text{ events/kg d}$, and there is again a minimum gluino mass for this case: $m_{\tilde{g}} \geq 400 \text{ GeV} \ (m_{\chi^0_1} \geq 50 \text{ GeV})$. Thus the choice $\delta_2 = 1 = -\delta_1$ sharply restricts the allowed gluino and neutralino mass range.
As mentioned above, there exists a correlation between large (small) values of detector event rate $R$ and small (large) values of the branching ratio $B(b \to s+\gamma)$. This is exhibited in Fig. 4 which shows a scatter plot of values of $R$ and $B$. The current experimental value is $B = (2.32 \pm 0.67) \times 10^{-4}$ while the Standard Model (SM) with NLO corrections predicts $B = (3.51 \pm 0.31) \times 10^{-4}$. One sees that almost all points with $B < 2.8 \times 10^{-4}$ (i.e. > 2 std below the SM prediction) have $R > 0.1 \text{ events/kg d}$, while almost all points with $B > 3.0 \times 10^{-4}$ (i.e. > 1 std above the experimental value) have $R < 0.1 \text{ events/kg d}$. Future CLEO data should significantly reduce the experimental error in $B$, and hence may influence the expected size of $R$.

The domain $m_{\tilde{g}} \lesssim 400 \text{ GeV}$ corresponds to the $Z$ and $h$ pole dominated region in the early universe annihilation cross section and hence $m_0$ can be large so that squark and slepton masses can be large. For $m_{\tilde{g}} \gtrsim 420 \text{ GeV}$ $m_0$ is generally small ($m_0 \lesssim 100 \text{ GeV}$) so that the upper bound on $\Omega_{\chi_1^0} h^2$ not be violated, and so in this domain the lightest slepton, $\tilde{e}_R$, could have a mass as low as $(85-90) \text{ GeV}$, at the edge of detectability of LEP190. The squarks are also relatively light in this domain i.e. $\lesssim (400-500) \text{ GeV}$.

**Conclusions.** We have considered in this letter what the future precision measurements of the basic cosmological parameters will imply for accelerator searches for SUSY. In the $\nu$CDM model the gluino mass obeys $m_{\tilde{g}} \lesssim 720 \text{ GeV}$, and more significantly there can be forbidden regions at $m_{\tilde{g}} \approx 500 \text{ GeV}$ and $m_{\tilde{g}} \approx 600 \text{ GeV}$. Violations of the gaugino scaling laws can become significant for $m_{\chi_1^0} \gtrsim 65 \text{ GeV}$. For the $\Lambda$CDM model one finds that $m_{\tilde{g}} \lesssim 520 \text{ GeV}$ ($m_{\chi_1^0} \lesssim 70 \text{ GeV}$), and for $m_{\tilde{g}} \gtrsim 420 \text{ GeV}$ ($m_{\chi_1^0} \gtrsim 55 \text{ GeV}$) one has relatively light squarks and can have a light $\tilde{e}_R$ selectron. In both models there is a general correlation between large dark matter detector event rates and small $b \to s+\gamma$ branching ratio (e.g. $B(b \to s+\gamma)$ lying significantly below the SM predictions). In general both models are sensitive to non-universal SUSY soft breaking. The analysis given here shows that accurate determinations of the cosmological parameters will significantly constrain many aspects of what would be expected at accelerators. While detection of SUSY dark matter is only one supersymmetric...
phenomenon, one might expect to learn from this two items: the magnitude of the event rate and the mass of the $\chi_1^0$. These could greatly further constrain what would be expected at accelerators.

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REFERENCES


[12] A. Kosowsky, M. Kamionkowski, G. Jungman and D. Spergel, Nucl. Phys. Proc. Suppl. 51B, 49 (1996). We assume here an angular resolution of $0.17^\circ$ and a noise level of $\omega^{-1} = 1.3 \times 10^{-17}$.


FIGURES

FIG. 1. Maximum and minimum event rates for $\mu > 0$ for a xenon detector as a function of $m_{\tilde{g}}$ for the 1 std range of Eq. (2) of the $\nu$CDM model with $\delta_i = 0$.

FIG. 2. Same as Fig. 1 with $\delta_2 = -1 = -\delta_1$, $\delta_3 = 0 = \delta_4$.

FIG. 3. Maximum and minimum event rates for a xenon detector for the 1 std band of the $\Lambda$CDM model with $\delta_1 = 0 = \delta_2$ (solid), $\delta_2 = -1 = -\delta_1$ (dotted), $\delta_2 = 1 = -\delta_1$ (dashed) for $\delta_3 = 0 = \delta_4$. The high mass discrete points are for $\delta_2 = 1 = -\delta_1$.

FIG. 4. Scatter plot of $R$ vs $B(b \rightarrow s + \gamma)$ for the $\Lambda$CDM model, $\mu > 0$ and $\delta_i = 0$ for the 1 std range.
The figure shows a scatter plot with the $b \gamma$ branching ratio on the x-axis and $R_{\gamma\epsilon}/\text{kg.da}$ on the y-axis. The data points range from $10^{-4}$ to $10^0$ on the y-axis and from $0.00010$ to $0.00050$ on the x-axis.