A Stueckelberg Extension of the Standard Model

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Abstract

An extension of the standard model of electro-weak interactions by an extra abelian gauge boson is given, in which the extra gauge boson and the hypercharge gauge boson both couple to an axionic scalar in a form that leads to a Stueckelberg mass term. The theory leads to a massive $Z'$ whose couplings to fermions are uniquely determined and suppressed by small mixing angles. Such a $Z'$ could have a low mass and appear in $e^+e^-$ collisions as a sharp resonance. The branching ratios into $f \bar{f}$ species, and the forward-backward asymmetry are found to have distinctive features. The model also predicts a new unit of electric charge $e' = Q' e$, where $Q'$ is in general irrational, in the coupling of the photon with hidden matter that is neutral under $SU(2)_L \times U(1)_Y$.

The Stueckelberg mechanism $^{[1]}$ gives mass to abelian vector bosons without breaking gauge invariance on the Lagrangian, and thus provides an alternative to the Higgs mechanism $^{[2]}$ to achieve gauge symmetry breaking without spoiling renormalizability. The simplest case is that of one abelian vector boson $A_\mu$ coupling to one axionic scalar field $\phi$. Here the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{m^2}{2} A_\mu A^\mu$$
is made gauge invariant by splitting off the longitudinal degree of freedom $\phi$ through $A_\mu \to A_\mu + \frac{1}{m} \partial_\mu \phi$, and defining the gauge transformation $\delta A_\mu = \partial_\mu \epsilon$, $\delta \phi = -m \epsilon$. Thus, $\phi$ takes the role of the longitudinal component of the massive vector, and is subject to a Peccci-Quinn type shift symmetry, which arises for constant $\epsilon$. The interaction with fermions is described by the standard (renormalizable) coupling to a conserved current $L_{\text{int}} = g A_\mu J^\mu$, $\partial_\mu J^\mu = 0$. In the quantum theory, a gauge fixing term similar to the $R_\xi$ gauge is added to the Lagrangian

$$L_{\text{gf}} = -(2\xi)^{-1} (\partial_\mu A^\mu + \xi m \phi)^2,$$

such that the total Lagrangian reads

$$L + L_{\text{int}} + L_{\text{gf}} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{m^2}{2} A_\mu A^\mu + g A_\mu J^\mu - \frac{1}{2 \xi} (\partial_\mu A^\mu)^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{\xi m^2}{2} \phi^2$$

where the two fields have been decoupled, and renormalizability and unitarity are manifest. These ideas cannot be easily extended to the non-abelian case \[3\], because in the non-abelian extension of the Stueckelberg Lagrangian the longitudinal components of the vector fields cannot be decoupled from the physical Hilbert space, which spoils renormalizability and unitarity. For the mass of the non-abelian gauge fields in the standard model, a spontaneous symmetry breaking mechanism involving the Higgs phenomenon \[2\] is then required.\[1\]

We now want to explore the perspectives of a model that extends the standard model Lagrangian \[1\] by an abelian vector boson and Stueckelberg type couplings. To start with, we look at the relevant part of the standard model first. Let $A_\mu^a$ be the gauge bosons in the adjoint of $SU(2)_L$ with field strengths $F_{\mu \nu}^a$, $B_\mu$ the hypercharge $U(1)_Y$ vector with field strength $B_{\mu \nu}$, and $\Phi$ be the $SU(2)_L$ Higgs doublet. Then the relevant part of the standard model is given by

$$L_{\text{SM}} = -\frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu} + g_2 A_\mu^a J_{2 \mu}^a + g_Y B_\mu J_\mu^Y - D_\mu \Phi \mp D^\mu \Phi - V(\Phi^\dagger \Phi), \tag{2}$$

where $D_\mu \Phi$ is the gauge covariant derivative and $V(\Phi^\dagger \Phi)$ takes its minimum at $v^2/2$ as usual. For the minimal extension of \[2\] by a Stueckelberg Lagrangian, we add the degrees of freedom of one more abelian vector field $C_\mu$ for a $U(1)_X$ with field strength $C_{\mu \nu}$, and one axionic scalar $\sigma$.

We assume the scalar field $\sigma$ to have Stueckelberg couplings to all the abelian gauge bosons, i.e. $B_\mu$ and $C_\mu$. However, we leave untouched the charged vector bosons and the charge assignment of the standard model fermions and assume they are neutral under $U(1)_X$.\[2\] Thus, the Lagrangian of Eq.\[2\] is extended by

$$L_{\text{St}} = -\frac{1}{4} C_{\mu \nu} C^{\mu \nu} + g_X C_\mu J_X^\mu - \frac{1}{2} (\partial_\mu \sigma + M_1 C_\mu + M_2 B_\mu)^2, \tag{3}$$

\[1\]The abelian Stueckelberg type of coupling among gauge bosons and axionic scalars appear frequently in models of (gauged) supergravity or string theory compactifications. The “anomalous” gauge transformation of the axions plays an important role in the (generalized) Green-Schwarz anomaly cancellation mechanism. In the framework of higher dimensional Kaluza-Klein models, as arise from string theory, the four-dimensional abelian vector field may be part of a larger non-abelian, and possibly simple gauge group, that is broken in the compactification.

\[2\]This important ingredient to our model is naturally realized if $C_\mu$ belongs to a hidden sector,
where $J_X^\mu$ is the (conserved) hidden matter current, not involving any standard model fields. The gauge invariance now reads $\delta_Y B_\mu = \partial_\mu \epsilon_Y$, $\delta_Y \sigma = -M_2 \epsilon_Y$ for the hypercharge, and $\delta_X C_\mu = \partial_\mu \epsilon_X$, $\delta_X \sigma = -M_1 \epsilon_X$ for $U(1)_X$. To decouple the various gauge bosons from the scalars, one has to add similar gauge fixing terms as discussed above (see e.g. [3]). With the above extension, and after the standard spontaneous electro-weak symmetry breaking the mass terms in the vector boson sector take the form $-\frac{1}{2} V_{ab} M^2_{ab} V^\mu_b \nu^\nu$, using $(V_\nu^T)_a = (C_\mu, B_\mu, A^3_\mu)_a$, with mass matrix

$$M^2_{ab} = \begin{pmatrix} M_1^2 & M_1 M_2 & 0 \\ M_1 M_2 & M_2^2 + \frac{1}{4} g_Y^2 v^2 & -\frac{1}{4} g_Y g_2 v^2 \\ 0 & -\frac{1}{4} g_Y g_2 v^2 & \frac{1}{4} g_2^2 v^2 \end{pmatrix},$$

where $v = 2M_W/g_2 = (\sqrt{2} G_F)^{-\frac{1}{2}}$, $g_2$ and $g_Y$ are the $SU(2)_L \times U(1)_Y$ gauge coupling constants, $M_W$ is the mass of the W boson, and $G_F$ is the Fermi constant. From $\det(M^2) = 0$ it is easily seen that one eigenvalue is zero, whose eigenvector we identify with the photon $\gamma$. Among the remaining two eigenvalues $M^2_\pm$, we identify the lighter mass eigenstate with mass $M_-$ with the Z boson, and the heavy eigenstate with mass $M_+$ as the $Z'$ boson. The limit $M_2 \to 0$ takes one to the standard model with $M^2 = \frac{1}{4} v^2 (g_2^2 + g_Y^2) + O(M_2^2)$ and $M^2_2 = M^2_1 + O(M_2^2)$. We pause here briefly to note that in our analysis we succeeded in obtaining a massless photon, whereas a previous attempt to obtain a Stueckelberg extension of the standard model failed for that reason [3]. It is evident from counting degrees of freedom, that only two out of the three gauge bosons $(C_\mu, B_\mu, A^3_\mu)$ can get massive by absorbing the two scalars that are available.

Using an orthogonal transformation $O$ to diagonalize $M^2$, we go to the basis of eigenstates $E^T = (Z'_\mu, Z_\mu, A^3_\mu)$, where $V = O \cdot E$, $O^T \cdot M^2 \cdot O = M^2_D$, and $M^2_D = \text{diag}(M^2_{Z'}, M^2_Z, 0)$. For $O$ we use the parametrization

$$O = \begin{bmatrix} \cos \psi \cos \phi - \sin \theta \sin \phi \sin \psi & -\sin \psi \cos \phi - \sin \theta \sin \phi \cos \psi & -\cos \theta \sin \phi \\ \cos \psi \sin \phi + \sin \theta \cos \phi \sin \psi & -\sin \psi \sin \phi + \sin \theta \cos \phi \cos \psi & \cos \theta \cos \phi \\ -\cos \theta \sin \psi & -\cos \theta \cos \psi & \sin \theta \end{bmatrix}.$$  

The mixing angles $\theta$, $\phi$ and $\psi$ are given by

$$\tan(\theta) = \frac{g_Y}{g_2} \cos(\phi), \ \tan(\phi) = \frac{M_2}{M_1}, \ \tan(\psi) = \frac{\tan(\theta) \tan(\phi) M^2_W}{\cos(\theta)(M^2_{Z'} - (1 + \tan^2(\theta))M^2_W)}.$$  

In the limit $M_2 \to 0$ one has $\phi, \psi \to 0$ and $\theta \to \theta_W$, $\theta_W$ being the Weinberg angle.

Note that $\theta$ and $\phi$ only involve the ratio $M_2/M_1$, such that the overall scale drops out. This is tempting in the framework of string or supergravity models, since it may lead to observable effects even for much higher mass scales than we consider in this paper.
The couplings to the physical vector fields in $E$ are gotten by inserting the mass eigenstates into the interaction Lagrangian $\mathcal{L}_{\text{int}} = g_2 A_\mu J_2^\mu + g_Y B_\mu J_3^\mu + g_X C_\mu J_X^\mu$. We find the coupling of the photon given by $A_\mu^i (e Q J_{\text{em}}^\mu - e' J_X^\mu)$, where $e$ is the electric charge defined by

$$e = \frac{g_2 g_Y \cos(\phi)}{\sqrt{g_2^2 + g_Y^2 \cos^2(\phi)}}. \quad (5)$$

while $Q J_{\text{em}}^\mu = \left(T_3 + \frac{Y}{2}\right) J_{\text{em}}^\mu = J_1^\mu + J_2^\mu$, and $e' = e Q'$ where $Q' = \left(g_X / g_Y\right) \tan(\phi)$. Thus we see that the effect of the mixing of the Stueckelberg term effectively changes $g_Y$ to $g_Y' = g_Y \cos \phi$. Further, the photon also couples to the hidden sector fermions in $J_X^\mu$ with a basic unit of charge $e'$, which in general will be irrational and small for a small mixing angle $\phi$. Thus one has the interesting possibility of observing hidden sector particles which carry small irrational charges through the Stueckelberg phenomenon.

Next we discuss the couplings of the $Z$ and $Z'$ bosons.\(^4\) The couplings of the $Z$ and $Z'$ bosons with the visible sector fields read

$$\mathcal{L}_{\text{NC}} = \frac{g_2}{\cos(\theta)} \left[ Z_\mu (\cos(\psi) (\sin^2(\theta) Q J_{\text{em}}^\mu - J_2^\mu) - \tan(\phi) \sin(\psi) \sin(\theta) (Q J_{\text{em}}^\mu - J_2^\mu')) + Z_\mu' (\sin(\psi) (\sin^2(\theta) Q J_{\text{em}}^\mu - J_2^\mu) + \tan(\phi) \cos(\psi) \sin(\theta) (Q J_{\text{em}}^\mu - J_2^\mu')) \right].$$

To estimate the range of parameters allowed by current experimental limits, it is instructive to compute the correction $\Delta$ to the $Z$ boson mass relative to that of the standard model, i.e. $M_Z = \frac{g_Y}{2} \sqrt{g_2^2 + g_Y^2} + \Delta = M_0 + \Delta$, where $M_0$ is the formula for the $Z$ mass in the standard model at the tree level.

The new parameters in the model beyond those of the standard model are just $(M_1, M_2)$, but alternately, we can replace them by $(\Delta, M_Z)$. Once these are fixed the mixing angles $\theta$, $\phi$ and $\psi$ can be computed. We do not undertake a global fit of electro-weak data in this paper, although this should be a worthwhile effort to determine the best values for $(\Delta, M_Z)$. Rather here we demonstrate that one can pick specific parameters for the Stueckelberg sector, such that the fits of the standard model with the precision electro-weak data are left essentially undisturbed, basically by arranging that the couplings between the $Z'$ and visible fermions are small. Thus, as an example, suppose we choose $|\Delta| = 2$ MeV and $M_{Z'} = 150$ GeV, where the choice of $|\Delta|$ is consistent with the current error on the experimental determination of $M_Z$\(^4\). The above implies $\phi \sim 0.75^0$ and $\psi \sim 0.18^0$ (via $M_1 \sim 149$ GeV, $M_2 \sim 1.9$ GeV), and we use $\theta = \theta_W$.\(^5\) The smallness of these angles leaves the precision fits of the standard model with experiments intact. Typically, the couplings of extra gauge bosons are reduced relative to the couplings

\(^4\)As an obvious remark, our construction preserves the GIM mechanism\(^6\).

\(^5\)Later we shall often neglect terms that are suppressed by extra factors of $\sin(\psi)$ compared to factors only suppressed by $\sin(\phi)$. 

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of the Z by a factor $M_{Z'}/M_{Z'}$. There is, for instance, a vast literature on extra $U(1)$
gauge bosons in the context of grand unified models such as $SO(10)$ or $E_6$ [8],
string and D-brane models [9] and a variety of other schemes [10, 11]. For models
where the $Z'$ couples effectively with the same strength as the Z, except for a
Clebsch-Gordon coefficient, one needs a large $Z'$ mass to achieve consistency with
the precision electro-weak data [8, 11]. In the present example the couplings of
the $Z'$ to quarks and leptons are suppressed by $M_{Z'}/M_1 \sim 0.01$, which makes them
small already.\(^6\) It is interesting to look for properties of the current model that
lead to finite and potentially observable effects, such as quantities that depend
on ratios of couplings. The cleanest signatures for $Z'$ would show up in resonant
production in $e^+e^-$ collison at the $Z'$ mass peak. As we shall point out, one can
then use the branching ratios for the different decay channels into $f\bar{f}$ species,
and the forward-backward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$ as distinguishing features.

The partial decay widths of the $Z'$ vector boson into visible fermions can easily
be computed from rewriting the interactions of $Z'$ with fermions as

$$\mathcal{L}_{NC}^{(Z')} = Z' \sum_f \left[ g_L^f \bar{f}\gamma^\mu(1-\gamma_5)f + g_R^f \bar{f}\gamma^\mu(1+\gamma_5)f \right] \tag{6}$$

and using

$$\Gamma(Z' \rightarrow f\bar{f}) = \frac{M_{Z'}}{6\pi} \left( (g_L^f)^2 + (g_R^f)^2 \right) + \mathcal{O}\left( \frac{M_f}{M_{Z'}} \right). \tag{7}$$

\(^6\)From the general formula for $\mathcal{L}_{NC}$ we then obtain

$$\Gamma(Z' \rightarrow l_i\bar{l}_i) = \frac{M_{Z'}}{96\pi} \left[ \left( -\frac{g^2_L + g^2_R \cos^2(\phi)}{\sqrt{g^2_L + g^2_R \cos^2(\phi)}} \sin(\psi) + g_Y \sin(\phi) \cos(\psi) \right)^2 
+ 4 \left( \frac{g^2_Y \cos^2(\phi)}{\sqrt{g^2_L + g^2_R \cos^2(\phi)}} \sin(\psi) + g_Y \sin(\phi) \cos(\psi) \right)^2 \right], \tag{8}$$

$$\Gamma(Z' \rightarrow \nu_i\bar{\nu}_i) = \frac{M_{Z'}}{4\pi} \left[ \frac{g^2_L + g^2_R \cos^2(\phi)}{\sqrt{g^2_L + g^2_R \cos^2(\phi)}} \sin(\psi) \right]^2,$$
Further, the couplings of $Z'$ to $W^+W^-$, $ZZ$, and Higgs fields are proportional to $\sin(\psi)$, since $A_\mu^3 = -\cos(\theta) \sin(\psi) Z' + \cdots$. In the limit of small $\psi$ we neglect these contributions, and replace $\cos(\psi) \sim 1$. With this simplification, the total decay width below or above the $t\bar{t}$ threshold is given by

$$\Gamma(Z' \to \sum_i f_i \bar{f}_i) = M_{Z'} g_{Z'}^2 \sin^2(\phi) \times \left\{ \begin{array}{ll} \frac{102}{288} \frac{2s}{5} \frac{1}{12\pi} & \text{for } M_{Z'} < 2m_t \\
15 & \text{for } M_{Z'} > 2m_t \end{array} \right. \right.$$

(9)

For $M_{Z'} = 150$ GeV, the total decay width lies in the range $0.6 - 80$ MeV for $\phi \sim 1^0 - 10^0$, while for $M_{Z'} = 1$ TeV it would lie in the range $4$ MeV $-$ $0.6$ GeV. This is to be contrasted with $Z'$ widths of up to $100$ GeV or above for a $Z'$ boson with mass of $1 - 2$ TeV, as in most other models [11]. Thus the observation of a sharp $Z'$ could be seen as potential signal for a Stueckelberg gauge boson.

An interesting phenomenon relates to the possibility of observing the hidden sector via the $Z'$ decay. In the analysis of Eq. (9) we have ignored the couplings of the $Z'$ to the hidden sector fields in the current $J_X^\mu$. If the masses of the hidden sector fields are smaller than $M_{Z'}$, there would be additional contributions to the decay width of $Z'$. Thus if kinematically allowed, the $Z'$ would decay into a pair of hidden sector fermions with small irrational electric charges $Q'e$. Because of the small charges of such particles, we expect that they would typically escape detection with the current sensitivity of detectors and would appear as electrically neutral. It should be challenging to find ways to detect such particles. We note that hidden matter which is neutral under $SU(2)_L \times U(1)_Y$ would still possess charges in units of $e'$, even if $C_\mu$ also coupled to the visible sector fields. However, in this case, the result of Eq. (9) would be modified, and the $Z'$ width will be broadened [12].

From the above, one can easily get the branching ratios of $Z'$ into different fermionic modes, which will have distinctive characteristics. For the decay of $Z'$ into up quarks and down quarks one finds $R_{u/d}^Z = \text{BR}(Z' \to u_i \bar{u}_i)/\text{BR}(Z' \to d_i \bar{d}_i) \sim 17/5$. This is to be contrasted with the result for the $Z$ decay in the standard model where $R_{u/d}^Z \sim 0.78$. Similarly, one has $R_{t/\nu}^Z = \text{BR}(Z' \to t_i \bar{t}_i)/\text{BR}(Z' \to \nu_i \bar{\nu}_i) \sim 5$, compared to $R_{t/\nu}^Z \sim 0.5$, and for the quarks consider e. g. $R_{b/\tau}^Z = \text{BR}(Z' \to b \bar{b})/\text{BR}(Z' \to \tau^+ \tau^-) \sim 1/3$, while $R_{b/\tau}^Z \sim 4.4$. Another ratio of interest is $R_{\ell/t}^Z = \text{BR}(Z' \to \text{had})/\text{BR}(Z' \to t^+ t^-) \sim 49/15$, below the $t\bar{t}$ threshold, and $22/5$, above, compared to $R_{t/\ell}^Z \sim 20.77$, and finally $\text{BR}(Z' \to \sum_i l^+_i l^-_i) \sim 44\%$ for below, and 38$\%$ above the $t\bar{t}$ threshold. For the $Z$ boson one has in the standard model $\text{BR}(Z \to \sum_i l^+_i l^-_i) \sim 10\%$. These branching ratios of the $Z'$ are in fact drastically different from the decay branching ratios of the $Z'$ in many other models [8, 11].

Another important signature for the Stueckelberg $Z'$ is the forward-backward asymmetry $A_{\text{FB}}$ in $e^+e^- \to \mu^+\mu^-$ experiment. The forward-backward asymmetry parameter $A_{\text{FB}}$ is defined by $A_{\text{FB}} = (\int_0^1 dz \frac{dz}{dz} - \int_1^0 dz \frac{dz}{dz})/(\int_0^1 dz \frac{dz}{dz} + \int_1^0 dz \frac{dz}{dz})$. At the $Z'$ pole it can be approximated by $A_{\text{FB}}(s = M_{Z'}^2) \sim (3/4)(g_{l'}^2)^2/(g_{l'}^2)^2 + (g_{l'}^2)^2$, with universal couplings $g_{L,R}^e = g_{L,R}^\mu = g_{L,R}^\tau$, as defined through (6), and where we
have only taken contributions due to $Z'$ exchange into account. One finds

$$\frac{g_R^l}{g_L^l} = 2 \frac{(1 + \delta) \tan^2(\theta)}{(1 + \delta) \tan^2(\theta) - 1}, \quad \delta = \frac{\tan(\phi)}{\sin(\theta) \tan(\psi)} = \frac{M_Z^2 - M_Z'^2}{M_Z^2 - M_W^2} + \mathcal{O}(M_\beta^2).$$

We then have $2 \leq g_R^l/g_L^l < 3.3$ for $M_{Z'} > 140 \text{ GeV}$, and $g_R^l/g_L^l$ goes to 2 asymptotically, which is its value for small $\psi$. For $g_R^l/g_L^l \sim 2$ one has $A_{FB}(s = M_Z^2) \sim 0.27$. The asymmetry at the $Z$ pole on the contrary is $A_{FB}(s = M_Z^2) \sim 0.02$. Also of interest is the quantity $R_{\text{peak}}^{Z'}(\mu^+\mu^-) = \sigma(e^+e^- \rightarrow \mu^+\mu^-)/\sigma(e^+e^- \rightarrow \mu^+\mu^-)_{\text{QED}}$ at the $Z'$ pole and similarly the quantity $R_{\text{peak}}^{Z'}^{\text{charged lep + had}} = \sigma(e^+e^- \rightarrow \text{charged lep + had})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)_{\text{QED}}$ at the $Z'$ pole. Our analysis gives $R_{\text{peak}}^{Z'}(\mu^+\mu^-) = 9/(4\alpha^2) \sim 3.5 \times 10^4$, and $R_{\text{peak}}^{Z'}^{\text{charged lep + had}} = 333/(20\alpha^2) \sim 2.7 \times 10^5$. This is to be compared with $R_{\text{peak}}^{Z'}(\text{charged lep + had}) \sim 0.4 \times 10^4$. It would be interesting to see if experiment can reach sensitivity necessary to test the presence of sharp resonances in the region of the $Z'$ mass.

The phenomenology of the extended model at the hadron colliders and in cosmology should also be of interest and needs investigation. There are of course many avenues for further generalizations and modifications of our model, including for instance a Stueckelberg extension of the minimal supersymmetric standard model. This will be discussed elsewhere [12].

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