END-WINDING LEAKAGE OF AEROSPACE HOMOPOLAR ALTERNATORS

Dr. Mulukutla S. Sarma
Dept. of Electrical Engineering
Northeastern University
360 Huntington Avenue
Boston, MA 02115, U.S.A.
TEL.: 617-437-

SYNOPSIS

An accurate knowledge of the magnetic field distribution is of great importance in finding the best designs for electrical machinery. It is the purpose of this paper to present a new numerical method for the determination of three-dimensional flux distribution in the end zone of a high-speed aerospace homopolar alternator, and for the calculation of end leakage reactance. The new analysis is applied to an experimental 95 kVA, 115/200 V, 3400 Hz, 40,800 r/min, three-phase, wye-connected, homopolar alternator.

The magnetic induction in the space around the endwinding region of a modern high-frequency, high-speed aerospace alternator is much larger than the corresponding one in the case of a standard 60-Hz energy converter. The losses caused by the alternating fields in structural parts, like end plates and end bells, are greatly increased. Therefore, it becomes essential to establish a method for finding the flux density distribution around the end windings of high-speed rotating machines. The flux linkages caused by this distribution is generally referred to as the end-winding leakage. The knowledge of the magnetic field in the end regions leads to the determination of losses, leakage reactances, and short-circuit forces. Many efforts have been made to find satisfactory methods for predicting the end leakage fields of ac machines in spite of the difficulties involved. The field distribution
is three-dimensional; the shapes of the windings and boundaries are complicated and of different permeabilities; and the reaction of the eddy currents induced in the boundary surfaces complicates the theory considerably.

Much research work has been done in the past on the end field problem. The available literature is mostly concerned with the turbogenerator end regions. The approaches suggested vary widely in type and in the magnitude of the approximations made. Use has been made of field-plotting methods including electrolytic tanks. Analytical approaches have applied the concepts of both scalar and vector magnetic potentials, and computers have been used to advantage. In general, either the representations of the winding shapes have been inadequate, or the boundary effects are not taken into account with enough accuracy. The end windings are represented as filaments or are replaced by approximate current sheets. These simplifications have not been made here. The analysis is specifically applied to the end-winding region of a homopolar inductor alternator.

Assumptions Made are given below:

1) The problem is treated as a magnetostatic one. Time derivatives, if present in the partial-differential equation, are replaced by spatial derivatives. The analysis is carried out for the instant at which the A phase is carrying the maximum current. 2) Because of periodicity, only one pole pitch is considered in the analysis. 3) The curvature of the machine is not considered, so that a Cartesian coordinate system can be used. It is well justified here because the homopolar alternator is a ten-pole machine and has a very short air gap. 4) The cross section of the slots and the teeth are assumed to be rectangular. 5) The stator core end surface from which the end windings
project is taken to be of infinite permeability. 6) The effects of air gap, slots, and short-pitched winding are adequately taken into account by fictitious conductor currents. The end shields, which are the boundary surfaces enclosing the end windings in the radial and axial directions, will be taken to have permeabilities of either zero, infinity, or a finite constant value. 8) The material regions of the end shields are considered approximately. The effect of any rings in the vicinity of the windings is not considered. All materials are taken to be isotropic and homogeneous. 9) The current density field is continuous and is distributed uniformly over the area of the cross section of a coil. The permeability of the current carrying regions is taken as that of air. 10) The winding is in two layers. It is conveniently represented with certain simplifications.

Although certain simplifying approximations are made here in representing the end winding, an attempt is being made for the first time to represent the dimensions of the conductor and the discrete current distribution as it exists in the actual machine.

One obtains for constant \( \mu \)

\[
\nabla \times (\nabla \times \vec{A}) = \mu \vec{J}
\]

which is the fundamental equation of the vector potential of the electromagnetic field. Defining arbitrarily that

\[
\nabla \cdot \vec{A} = 0,
\]

in can be shown that air as well as iron regions have to satisfy Laplace's equation of a vector quantity, and the current-carrying regions have to satisfy the well-known Poisson's equations of a vector quantity. The field equations for linear material regions of finite conductivity and permeability are also developed in the paper.
In order to overcome the difficulty of simultaneously satisfying the three scalar Poisson's equations and the condition of (2) by numerical methods, (1) itself will be solved for $A_x$, $A_y$, and $A_z$ in the interior of the three-dimensional region without enforcing the condition of (2). On the boundaries, however, it is convenient to impose the zero condition on the divergence of $A$ and then solve the set of scalar Poisson's equations. It should be observed that the component equations of (1) are coupled, and the corresponding difference expressions involve eighteen surrounding points instead of the six as in the uncoupled equations.

Other avenues of approach exist, one of which is to assume the condition of (2) and then numerically solve the uncoupled Poisson's equations. One may then argue from (1) that the gradient of divergence of $A$ is zero by viewing it as

$$\nabla(\nabla \cdot \vec{A}) = \nabla^2 \vec{A} + \nabla \mu. \quad (3)$$

However, this scheme is not employed here as it has not yet been fully investigated.

A successive-point iteration method incorporating the features of the extrapolated Liebmann method [39] and the over-and-under relaxation methods [40] is used in this work. The old value of the vector potential at a given lattice point is stored and a value is calculated by using the appropriate difference expressions developed earlier. The new value of the vector potential is computed as the sum of the old value and $\alpha$ times the residual, which is the difference between the calculated value and the old value. $\alpha$ is a relaxation factor. $\alpha$'s greater than unity denote overrelaxation, and $\alpha$'s less than one demote underrelaxation. Experience has shown that the gradual decreasing of $\alpha$ to considerably less than unity, as the iterative procedure progresses, aids the convergence. The algebraic sum of residuals as well as sum of the absolute values of the residuals are used to monitor convergence. To start with an
overrelaxation factor of about 1.3 has been found helpful for about the first one hundred of the iterations. Later when it is observed that the residues are not decreasing anymore, or when they show a tendency to increase, the reduction of the factor \( \alpha \) becomes useful. Although the problem under consideration is essentially linear, the governing equations of the three components of the vector potential are coupled. This complex coupling seems to be responsible for the fact that a relaxation factor much less than unity, which is effectively an underrelaxation factor, helps the convergence of the program as the iterations proceed. Sufficient convergence is assumed to have been attained when the average residual is of the order of 0.01 percent of the average vector potential. The number of iterations required for an acceptable convergence was found to be of the order of 300. The computer time per iteration was about 36 seconds on a CDC 3800 computer.

The difficulty of limited computer storage capacity is overcome by storing at one time in the core the information relevant to only three XZ planes, at the time when the calculations at the lattice points on the middle plane are performed. The adopted iterative scheme carries out the computations plane by plane (XZ) planes) along the Y direction, with the additional use of two magnetic tapes. All the computer programs have been written in FORTRAN IV. The listings of the programs for various cases that have been investigated are available on request.

The results show that the eddy currents in the end shields reduce the leakage reactance, and this reduction depends only very slightly on the permeability of the end-shield material. This is because the end shields are at a relatively large distance from the current-carrying regions and any effect based on the material properties of the end shields becomes
insignificant on the flux distribution in the end zone. The results depend very considerably on the design, the geometry, the proximity of the magnetic and nonmagnetic materials to the current carrying regions, and the nature of the materials of the supporting rings, end shields, so that each end-winding leakage reactance has to be calculated separately. The usual calculations cannot adequately take account of all design variations, while the method presented here can consider any different design and provide a greater understanding of the effects at the end windings than the other engineering formulas used currently in design offices.

As a byproduct of the method set forth here, an approximation of the eddy current losses in the end shields can be obtained based on the current flow obtained from the tangential magnetic field at the end shield surface, and by knowing the surface resistivity of the conducting walls. The method is advantageous as the eddy currents are also a result of the final solution and no additional assumptions or guesswork are necessary for their calculation.