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Dyadic Green’s function calculations on a layered dielectric/ferrite structure

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Dyadic Green’s function techniques have been applied to solve the transmission properties of a microstrip line fabricated on top of a single-crystal Y-type hexaferrite substrate. Current potentials are used to construct the Galerkin elements to facilitate solution accuracy even in the FMR region. Transmission coefficients of a microstrip line fabricated on a Y-type hexaferrite substrate were thus calculated, which compared reasonably well with measurements. © 2001 American Institute of Physics. [DOI: 10.1063/1.1360260]

I. INTRODUCTION

For a given geometry electromagnetic wave solutions arising from a point source satisfying the required (homogeneous) boundary conditions are termed Green’s functions.\(^1\) When the source distribution is known, the general solution can then be expressed as superposition of the Green’s functions. On the other hand, by using the Green’s function representation Maxwell equations are converted into integral forms from which the unknown source distribution can be solved numerically using the Galerkin’s method, for example.\(^2\)

Historically, the electromagnetic problem concerning a Hertzian dipole in the presence of a (lossy) semi-infinite dielectric substrate was analyzed by Sommerfeld.\(^1\) The importance of Sommerfeld’s work is that the mathematical solutions are endowed with physical meanings so that surface waves are generated from poles of the integrands and space waves from branch points.\(^1,2\) For a microstrip geometry consisting of an isotropic substrate backed by a ground plane the dipole source solutions, or the Green’s functions, can be written in closed forms. However, for a stratified structure containing multiple dielectric and magnetic layers it is generally not possible to find close-form solutions. In this work we construct the Green’s function solutions utilizing the spectral domain analysis, since, within each layer, Maxwell equations dictate plane-wave solutions whose spectral dispersion and polarization are already known.\(^3\)

In solving the integral equations containing the Green’s functions in the integrands current potentials have been conveniently used to construct the Galerkin elements, composing the unknown current distribution in the metal patch(es) appearing at the interface(s) of a layered structure.\(^4,5\) Three advantages result by using current potentials. Not only the symmetry of the patch reserves in the calculations, but also the vector Galerkin equations are converted into scalar ones. Most importantly, the condition for current continuity at metal–patch boundaries is automatically satisfied, requiring the normal component of the current to vanish at the bound-

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FIG. 1. The geometry of a stratified structure containing multiple layers. A planar circuit is located at the \(z=0\) plane.
The transfer–matrix method is applied in the spectral domain. A transfer matrix is a 4×4 matrix, and, for a given transverse spectral vector, \( k_p = (k_x, k_y)^T \), it correlates the tangential components of the rf \( e \) and \( h \) fields on both sides of a layer, Eq. (3) below. Here, the superscript \( T \) denotes transposition of a row vector into a column vector. Thus for the \( v \)th layer the transfer matrix can be written as

\[
\Gamma_v = \left[ \begin{array}{cccc}
\gamma_{11}(z) & \gamma_{12}(z) & \gamma_{13}(z) & \gamma_{14}(z) \\
\gamma_{21}(z) & \gamma_{22}(z) & \gamma_{23}(z) & \gamma_{24}(z) \\
\gamma_{31}(z) & \gamma_{32}(z) & \gamma_{33}(z) & \gamma_{34}(z) \\
\gamma_{41}(z) & \gamma_{42}(z) & \gamma_{43}(z) & \gamma_{44}(z)
\end{array} \right],
\]

(1)

so that

\[
\begin{pmatrix}
e_x(z_{v+1}) \\
e_y(z_{v+1}) \\
h_x(z_{v+1}) \\
h_y(z_{v+1})
\end{pmatrix} = \Gamma_v \begin{pmatrix}
e_x(z_v) \\
e_y(z_v) \\
h_x(z_v) \\
h_y(z_v)
\end{pmatrix},
\]

(2)

In Eq. (2) \( k_{vax}, M \gg \nu \gg -N, 4 \gg \alpha \gg 1 \), denotes the \( z \) component of the wave vector of the \( \alpha \)th eigenmode in the \( \nu \)th layer satisfying

\[
k_x^2 + k_y^2 = \epsilon_{\nu} \mu_{\nu} \omega(c)^2.
\]

(3)

In Eq. (4) \( \epsilon_{\nu} \) and \( \mu_{\nu} \) are the dielectric constant and the permittivity of the \( \alpha \)th eigenmode in the \( \nu \)th layer, \( \omega \) is the angular frequency, \( c \) the speed of light in vacuum, and Gaussian units have been used throughout this analysis. The tangential components of the \( e \) field and the \( h \) field associated with the \( \alpha \)th eigenmode in the \( \nu \)th layer are expressed as \( (e_{\nu x}, e_{\nu y})^T \) and \( (h_{\nu x}, h_{\nu y})^T \) in Eq. (2), respectively. As dictated by Maxwell equations, for an anisotropic medium the four eigenmodes in the layer, designated by the index \( \alpha \), are generally nondegenerate, resulting in different \( \epsilon \) and \( \mu \) values and propagation constants \( k \). The \( \Gamma \) matrices, and the transfer matrices, \( \Gamma_v \), for a dielectric layer and for a ferrite layer, Eq. (2), are given in Ref. 6.

The surface impedance matrix, \( Z(z) \), can be defined as follows:

\[
\begin{pmatrix}
e_x(z) \\
e_y(z) \\
h_x(z) \\
h_y(z)
\end{pmatrix} = Z(z) \begin{pmatrix}
e_x(z_v) \\
e_y(z_v) \\
h_x(z_v) \\
h_y(z_v)
\end{pmatrix},
\]

(5)

where the dependence of the quantities in Eq. (5), say, \( e_x, e_y, h_x, h_y \), and \( Z \), on \( k_x \) and \( k_y \) is understood. Thus when a transfer matrix is defined to translate the tangential components of the rf electromagnetic fields over one layer thickness, Eq. (3), the surface impedance will be transferred too, according to the following equation:

\[
Z(z_{v+1}) = [a_v Z(z_v) + b_v] [c_v Z(z_v) + d_v]^{-1},
\]

(6)

where \( a_v, b_v, c_v, \) and \( d_v \) are the 2×2 partition matrices of \( \Gamma_v \) given by

\[
\begin{pmatrix}
a_v & b_v \\
c_v & d_v
\end{pmatrix},
\]

(7)

Since, except at \( z = 0 \), the tangential components of the \( e \) and the \( h \) fields are continuous across layer interfaces, the transfer matrices can be multiplied together to provide an overall transformation relating the outermost boundaries of the structure to the \( z = 0 \) plane. Thus we define two overall transfer matrices, top and bottom, denoted as \( \Gamma_t \) and \( \Gamma_b \), respectively, as

\[
\begin{pmatrix}
e_x(0^+) \\
e_y(0^+) \\
h_x(0^+) \\
h_y(0^+)
\end{pmatrix} = \Gamma_t \begin{pmatrix}
e_x(M) \\
e_y(M) \\
h_x(M) \\
h_y(M)
\end{pmatrix},
\]

(8)

\[
\begin{pmatrix}
e_x(0^-) \\
e_y(0^-) \\
h_x(0^-) \\
h_y(0^-)
\end{pmatrix} = \Gamma_b \begin{pmatrix}
e_x(-N) \\
e_y(-N) \\
h_x(-N) \\
h_y(-N)
\end{pmatrix}.
\]

(9)

The relationship between the two column vectors \( (e_x(0^+)e_y(0^+)h_x(0^+)h_y(0^+))^T \) and \( (e_x(0^-)e_y(0^-)h_x(0^-)h_y(0^-))^T \) is determined from the boundary conditions imposed by the planar circuit at \( z = 0 \), as connected together by the use of the dyadic Green’s functions discussed below. Therefore we have

\[
Z(0^+) = [a_t Z(M) + b_t] [c_t Z(M) + d_t]^{-1},
\]

(10)

\[
Z(0^-) = [a_b Z(-N) + b_b] [c_b Z(-N) + d_b]^{-1}.
\]

(11)

where \( a_t, b_t, c_t, \) and \( d_t \) denote the partition matrices of the top transfer matrix, \( \Gamma_t \), for example. Surface impedance matrices corresponding to a/an short/open interface are given in Ref. 6.

Let \( G(k_x, k_y) \) be the Green’s function dyad in the spectral domain. For a given current distribution in the interface, \( j(x', y') \), the generated tangential electric field at \( z = 0 \) is

\[
e_{x}(x, y) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp[i(k_x(x-x'))\exp[i(k_y(y-y'))] \\
\times \exp[i(k_x(x-x'))\exp[i(k_y(y-y'))] \\
\times G(k_x, k_y)j'(x', y').
\]

(12)
In Eq. ~\ref{eq:substrate}
substrate surfaces, the metal patch, and a cated microstrip line is characterized by the following pa-
s\[ E \] denotes the surface impedance, as defined by the ratio of voltage to current.\footnote{H. How, X. Zho, and C. Vittoria, in Encyclopedia of Electrical and Electronics Engineering, edited by J. G. Webster (Wiley, New York, 2000), Supplementary I, pp. 349–366.} For a finite length of the magnetic substrate. In order to compare with measurements, we calculate the impedance of the transmission line defined using the transmission line model if the impedance and magnetic losses, respectively. Other losses, for example, discontinuity resulted from the coax-microstrip adapters employed under transmission measurements, may also partially displace the current distribution due to the bias-field effect.

The outermost layers of the stratified structure can be air or metal, called open-circuited or short-circuited boundary conditions, respectively.\footnote{H. How and C. Vittoria, IEEE Trans. Microwave Theory Tech. \textbf{MTT-42}, 988 (1994).} However, due to radiation loss and conductor loss, as well as surface-wave loss, these boundaries present imperfect open and short circuits. The following calculations apply to a microstrip line fabricated on a magnetic substrate. In order to compare with measurements, we calculate the impedance of the transmission line defined by the ratio of voltage to current.\footnote{H. How, T.-M. Fang, and C. Vittoria, IEEE Trans. Microwave Theory Tech. \textbf{MTT-42}, 1939 (1994).} For a finite length of the microstrip line the transmission coefficients can thus be calculated using the transmission line model if the impedance and the wave propagation constant are known as a function of frequency.

\section*{III. RESULTS}

Experimentally, we have fabricated a microstrip transmission line using a single-crystal \textit{Y}-type hexaferrite as the substrate material. The composition of the substrate material is \textit{Ba}_2\textit{MgZnFe}_{12}\textit{O}_{22}, and the easy plane coincides with the substrate surfaces, the \textit{xy} plane. The hexaferrite substrate material was characterized using a vibrating sample magnetometer (VSM) to show a saturation magnetization \( 4\pi M_s = 2.86 \text{ kG} \) and an anisotropy field \( H_A = 7.94 \text{ kOe} \). The fabricated microstrip line is characterized by the following parameters: thickness \( d = 0.010 \text{ in.} \), width \( w = 0.0051 \text{ in.} \), length \( l = 4 \text{ mm} \), and dielectric constant \( \varepsilon_r = 18 \). The dielectric loss tangent \( \tan \delta_e \) was assumed to be 0.01 and FMR linewidth \( \Delta H = 100 \text{ Oe} \). Other properties of the fabricated microstrip line, as well as measurements, can be found in Ref. 6. The longitudinal current \( j_x(x) \) and the transverse current \( j_y(x) \) have been plotted in Ref. 6 to show the asymmetrical displacement of the current distribution due to the bias-field effect.

Figures 2 and 3 plot the calculated and measured transmission coefficient for the amplitude and the phase, respectively, assuming the frequency \( f = 20 \text{ GHz} \). In Fig. 2 the measured insertion loss is larger than calculated even outside the FMR region. Reasons for this may be that the dielectric loss tangent and FMR linewidth assumed by the calculations are smaller than their actual values, corresponding to electric and magnetic losses, respectively. Other losses, for example, discontinuity resulted from the coax-microstrip adapters employed under transmission measurements, may also partially explain the discrepancy. The discrepancy between theory and calculations in the FMR region is even bigger, due to the difficulty in obtaining good numerical accuracy in that region.

The calculated transmission phase basically confirms measurements, Fig. 3, showing a resonant structure when FMR is encountered. Of special notice, it is seen in Fig. 3 that phase shift occurs linearly in the low field region prior to FMR is encountered. Of special notice, it is seen in Fig. 3 that phase shift occurs linearly in the low field region prior to FMR, suggesting that a transmission line involving \textit{Y}-type hexaferrite material is a superior candidate for phase shifters, especially at high frequencies.

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