COMPUTER-AIDED ANALYSIS OF MAGNETIC FIELDS IN NONLINEAR MAGNETIC BEARINGS

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ABSTRACT
The high magnetic energy stored in rare earth-cobalt magnets allows the design of lightweight motors and magnetic bearings for high speed rotors. Not subject to wear and with the ability to operate under high vacuum conditions, magnetic bearings appear ideal for applications requiring high rotational speeds such as 100,000 rpm. Important applications are for turbo-molecular pumps, laser scanners, centrifuges, momentum rings for satellite stabilizations, and other uses in space technology. It is the purpose of this paper to present a two-dimensional nonlinear numerical analysis of the magnetic fields in a magnetic bearing, based on magnetostatic assumptions and finite-difference iterative techniques.

INTRODUCTION
Rare earth-cobalt magnets have opened the windows to new technologies for high-speed motors and magnetic bearings, thereby causing a breakthrough to a new generation of systems for various applications. New applications such as the energy storage devices, which could replace the chemical batteries of satellites, are being looked into. Lightweight high-speed motors with an output power of about 2 kW at speeds above 30,000 rpm are being developed for spinning machines, momentum wheels, and rings. Magnetic bearings are ideal when it is desired to rotate objects at high rotational speeds in excess of 100,000 rpm, and their use for operating up to several million rpm was demonstrated by Beams. Because of the absence of mechanical contact and lubricating fluids, magnetic bearings can be operated in high vacuum at higher speeds with extreme low friction, low noise, and longer operating life. With no risk of contamination of oil or gas, and with least heat dissipation, it is possible to ascertain clean, stable and accurate operating conditions with reliability and repeatability. In space technology the magnetic bearings are used successfully in reaction, momentum and energy wheels, helim pumps, and telescope pointing. Terrestrial applications include scanners, high vacuum pumps, beam choppers in high vacuum, energy storage wheels, and accurate smooth rotating machines.
The design of magnetic bearings involves the calculation of magnetic forces and stiffness as a part of designing electromechanical servo system. In earlier days, the magnetic force of a suspension block is approximately calculated with reasonable accuracy by assuming simple straight flux paths. However, higher magnetic flux densities are increasingly used for reducing the weight and size of magnetic bearings, particularly so in the case of a single axis servoed magnetic bearing which utilizes the fringing rings. In such cases, the nonlinear characteristics of the ferromagnetic materials become quite significant; analytical techniques fail to yield sufficiently accurate results. Hence, it becomes essential to take recourse to numerical analysis of the nonlinear magnetic fields with the aid of a high-speed digital computer in order to determine more accurately the flux distribution corresponding to various conditions of operation, compute leakage, and evaluate forces at the airgap, so as to optimize the design of nonlinear magnetic bearings.

The magnetic bearing in reaction wheel shown in Figure 1 has been chosen here for the computer-aided analysis of its nonlinear magnetic fields.
ASSUMPTIONS

The solution will be based on magnetostatic assumptions. Only a two-dimensional cross-section is considered and a Cartesian coordinate system is used. The ferromagnetic materials are considered to be homogeneous and isotropic, characterized by single-valued magnetization curves, thereby neglecting hysteresis effects. The permeability, \( \mu \), is then a single-valued function of the magnetic induction, \( B \), as given by the saturation curve. Piecewise linear representation of the magnetization curve for digital computer use has been done as in Reference [4]. Discrete excitation currents in the conductors will be replaced by uniform current-density field over the cross-section of the excitation coil. The samarium-cobalt permanent magnet is replaced by an equivalent electromagnet. Rationalized MKSA system of units has been used for the development and application of the theory. The magnetic induction outside the bounding rectangular contours chosen for the model is negligible.

MATHEMATICAL MODEL AND THE GRID SYSTEM

The cross-section of the magnetic bearing suspension block containing ferromagnetic material, samarium-cobalt permanent magnet, electromagnet, airgaps and fringing rings is covered by a 55 x 100 rectangular grid system as shown in Figure 2. Symmetry has been made use of in limiting the mathematical model that needs to be analyzed. In order to satisfy boundary conditions with sufficient accuracy and also follow the geometric contours as closely as possible, a very fine grating of the grid has been chosen at and near the boundaries. If such fine meshes were chosen everywhere for the sake of uniformity, the number of grid points would become prohibitively large. Thus it becomes advisable to settle for sets of nonuniformly spaced gridlines as shown in Figure 2. This arrangement makes it possible to choose a finer mesh where great accuracy is required to satisfy boundary conditions and follow the configuration closely, and a coarser mesh in the interior of the regions where the change of reluctivity is not too large. As seen in Figure 2, the grid system is such that each row is cut by the same columns and each column is, in turn, cut by the same rows. This procedure leads to a great simplification in the programming of the numerical solution on a digital computer.

As for the boundary conditions, along the boundary rectangular contours chosen for the mathematical model, the magnetic vector potentials will be assumed as zero and fixed. Thus the enclosing contour represents an equipotential of the magnetic vector potential, \( A \), and, therefore, a line of flux.
GOVERNING FIELD EQUATIONS

The cross section consists of domains, the contours of which are described in rectangular coordinates. It becomes necessary, therefore, to develop the partial differential equations in Cartesian system of coordinates. The fundamental laws governing all electromagnetic fields can be expressed by Maxwell's equations. The solutions of the magnetostatic field problems that include discrete current-carrying regions need to satisfy

\[ \nabla \times [\mu (\nabla \times A)] = J \]  

(1)

where \( A \) is the magnetic vector potential, \( J \) the current density, and \( \mu \), the relucitivity, is the reciprocal of the permeability. For a two-dimensional case in \( XY \)-plane with \( Z \)-directed currents

\[ \frac{\delta}{\delta x} (\nu \frac{\delta A_x}{\delta x}) + \frac{\delta}{\delta y} (\nu \frac{\delta A_y}{\delta y}) = -J \]  

(2)

For numerical work, the partial differential equation (2) has been transformed into difference form. The difference equation can be obtained either by directly replacing the partial derivatives by partial difference quotients with the use of the first two terms of the Taylor's series expansion of the vector potential \( A \) around a grid point, or by applying Ampere's law around the closed contour. Details are omitted here because of space limitation.

ITERATIVE PROCEDURE

An iterative method is then developed for the numerical solution. It essentially consists of calculating the vector potentials by successive point relaxation method with the use of overrelaxation, and computing the local reluctivities from the existing vector potential values and then underrelaxing the reluctivities. The judicious use of over- and under-relaxation methods greatly accelerate the convergence of the iterative solution. Overrelaxation of the vector potentials with an overrelaxation factor of 1.3 and the underrelaxation of the reluctivities with an underrelaxation factor of 0.1 appear to yield faster convergence. Besides the use of these factors, a block relaxation method for accelerating the convergence of the iterative solution has also been employed, details of which are omitted here because of space limitation.

CONVERGENCE CRITERION

The convergence is judged by the fact that the residuals are well-behaved functions decreasing monotonically, and that the sum of the absolute values is of the order of the average value of the vector potentials.
Thereby, the average residual is of the order of about 0.02 percent of the average vector potential.

The number of iterations required for an acceptable convergence is very reasonable; it is only of the order of 100 for the first run, and less for further runs. The computer time required for each iteration is about one second on a Control Data Corporation CYBER 72 high-speed digital computer system, including the compilation time and time allowed for a reasonable print-output, calculated on the basis of a typical run for 100 iterations.

The computer program has been written in "FORTRAN IV" language. The boundary value problem under investigation requires computer storage of the order of 32K to 64K. These computer programs are run on the CDC CYBER 72 high-speed digital computing system located at Boston, Massachusetts, with Northeastern University.

CONCLUDING REMARKS

The computer program accomplishes the following objectives through the iterative procedure:

(a) Calculation of reluctivities from the vector potentials and underrelaxing them;
(b) Successive point relaxation of vector potentials and overrelaxing them;
(c) Block relaxation of vector potentials for acceleration of convergence through the application of additive correction method;
(d) Determination of the individual residuals, their algebraic sum, sum of the absolute values of the residuals, the absolute value of the maximum residual as well as its location;
(e) Computation of radial, tangential, and total flux densities from the converged vector potentials;

and

(f) Writing the output in a desired form and storing the results on the magnetic disk for further use, so that the program may be run for additional cases of interest.

An iterative procedure consisting of successive point relaxation of the vector potentials with an optimum over-relaxation factor, a block relaxation method for accelerating the convergence, relaxation of reluctivities with an optimum underrelaxation factor, and a constant monitoring of the convergence of the sum of the absolute values of the residuals as well as of the absolute value of the maximum residual has been described. Computer programs in Fortran language are developed and applied to find the flux distribution corresponding to various conditions of operation. The results are critically analyzed for optimized design of
nonlinear magnetic bearings.

The strength of the electromagnet and/or that of the permanent magnet can easily be changed to observe their effect on the leakage as well as the flux-density distribution, particularly at the air-gap level and fringing rings. The number of fringing rings and their location may also be easily changed in order to evaluate their effects on the forces at the air-gap level.

As for the actual results, for reasons of space limitations and others of proprietary nature, the flux-densities at the air-gap level in the vicinity of the fringing rings are only shown typically in Figure 3.

ACKNOWLEDGEMENTS

The authors would like to express their sincere thanks to Mr. Joe Lyman and the management of Cambion Corporation for their encouragement and support.
REFERENCES

* Work supported by Cambion Corporation


