Nonmagnetic Particle Separation Using Ferrofluids Controlled by Magnetic Fields

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Abstract

The ability to manipulate and control small particles is important in many biomedical applications including cell sorting for diagnosis and cancer detection. Several commonly used cell isolation techniques include flow cytometry, magnetic labeling, and electrophoresis. Recently, research has focused on sized based methods such as deterministic lateral displacement, and ferrofluid methods utilizing time and spatially traveling magnetic fields. Here a new sized based method of nonmagnetic particle separation is presented and investigated. This new method utilizes magnetic separation in combination with ferrofluids. The separation is accomplished without magnetic labeling and with the application of a time uniform magnetic field which can be contrasted with the complicated field dynamics required by other methods.

In this technique, nonmagnetic particles are submerged in a ferrofluid which is subjected to a magnetic field. In this way the fluidic environment is controlled rather than the nonmagnetic particle directly. The resulting body forces on the fluid give rise to forces on the nonmagnetic particles. These resulting forces are similar to familiar buoyant forces. Like the buoyant force, this magnetic force is dependent on particle volume. Analysis shows that the nonmagnetic particles can be separated through a combination of magnetic forces in the direction of particle motion and drag forces that oppose this motion. This combination results in different position profiles in time for nonmagnetic particles of different sizes. Governing magnetic field and particle dynamics equations are developed and analytic solutions are obtained. Solution methods utilize numerical time integration tools to overcome difficulties associated with nonlinear governing dynamics. Experimentation is performed to validate the model developed.
This technology will have direct impacts in cell separation and sorting for use in biomedical applications. It opens possibilities for the development of a point-of-care device that is disposable and does not require complicated equipment. Such a device would not require extensively trained technicians or a laboratory setting. Separation time scales are very short compared to currently available methods.
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1.0 Introduction

Ongoing and ever increasing medical research often relies on the ability to interact with and manipulate molecules, cells, cell populations, viruses, bacteria, and proteins [1, 2]. There are several methods available for accomplishing these tasks and they vary in a number of ways. The approaches differ in the number of entities they are capable of manipulating, the magnitude of the forces and displacements they can impose, and method with which the manipulation is facilitated [2].

The goal of the manipulation can be varied. Often, the response of a cell to its environment and mechanical and chemical stimuli is investigated. For example, the mechanical behavior of cell membranes has been investigated through micropipette aspiration. The cell is drawn into a glass pipette, under a known pressure, whose inner diameter is smaller than the diameter of the cell. Its displacements and deformations are tracked with light microscopy [3].

The ability to control magnetic materials with the application of a magnetic field leads to various magnetic techniques for medical applications. The guidance of magnetic particles in body fluids and living organisms for drug delivery, cell navigation, and cancer treatment has been studied extensively in literature [4-6]. In an important magnetic technique, cells are navigated from one location to another or isolated via magnetic labeling or tagging [1, 7]. This process involves physically attaching a magnetic nanoparticle to the biological body of interest. Because binding sites on cells are targeted by various biological entities such as hormones or antibodies, the nanoparticles can be made to accurately target some specific type of cell. This is done by chemically modifying the surface of the nanoparticles by coating them with
biocompatible molecules and immunospecific agents. After the attachment has been made a magnetic field is applied to attract the magnetic nanoparticles and in turn, it’s attached entity of interest.

A manipulation technique related to tagging is magnetic twisting cytometry. With this method small magnetic beads are also used. A number of beads are attached to a cell for a different purpose. Here a magnetic field is applied to investigate the mechanical behavior of the cell under, for instance, cyclic loading [2].

Magnetic labeling although effective in many circumstances requires careful preparation of the nanoparticles. This requires a priori knowledge of cell markers. The engineering of the attachment is time consuming and case specific [1]. Also, this method requires modification of the target cell through the nature of the manipulation, specifically, the attachment of the bead. Because the function of living cells depends sensitively on their environment [2] the modification may change the cell in some way interfering with the desired operation.

Increasing health care demands call for affordable and rapid analysis and processing of biological materials. Attention has been paid to microfluidic devices for cell manipulation, DNA sequencing, drug discovery, and cancer research addressing these needs [8-13]. Of great importance are separation techniques for use in separating a number of one specific type of biological entity from a population of different types. These techniques are critical in the analysis, isolation, and processing of biological materials for virus or pathogen detection, diagnosis and early cancer detection [14-16].

Point-of-Care devices are those that can be used at the location of treatment such as a doctor’s office or an ambulance [17]. This is in contrast to large scale technologies
that require moving samples to a lab setting where complex equipment and trained personnel perform the analysis. The Point-of-care technologies offer rapid on site processing of samples allowing for medical professionals to quickly diagnose conditions and respond with treatment increasing a patient’s chances of recovery \[18, 19\]. Developing point-of-care separation techniques is therefore of great interest.

There are several separation techniques that are commonly used for the applications under consideration (see references above). One such technique is flow cytometry in combination with Florescence Activated Cell Sorting (FACS) \[20\]. Here the separation is accomplished by passing cells suspended in a flowing stream of fluid through a light or laser. Florescent and light scattering characteristics of the different types of cells are identified by a detector. After the cells pass the detector the stream is broken into droplets each containing a single cell. If a droplet contains a target cell an electric charge is placed on the droplet. The droplets then pass a deflector where the charged droplet containing the cell of interest is removed from the stream. Although variations and different systems exist these are the basic concepts of operation. While FACS is an industry standard it has some limitations. Complex equipment and a large number of cells are required and cell detection rate is limited by signal processing time \[18, 21\].

Another widely utilized separation method is Electrophoresis. Cells or particles suspended in a liquid can carry and electric surface charge which can be exploited for separation. A particle mixture is passed through a separator. An electric field is applied perpendicular to the stream where an electrostatic force is exerted on the particles. The particles then diverge into different paths based on their charge where they can be
collected at different locations. Electrophoresis requires relatively large sample quantities (tens of milliliters) and relatively long processing time (tens of minutes) [22].

Deterministic Lateral Displacement is another alternative offering possibilities in reliable size based cell separation [23]. A particle solution is passed through a separator containing comb like structures or post like obstacles. The flow splits up into different streams around the obstacles. If a particle’s size is larger than its stream it is displaced laterally while particles smaller than their streams follow that stream and are thereby separated. While this techniques offers promising directions in high resolution separation problems still exist with particle agglomeration around the structures. This disrupts the flow patterns and hinders separation [18].

There are many types of chromatography which involves passing the analyte through a stationary phase [24]. One size based scheme is exclusion chromatography [25, 26]. A solution containing differently sized particles is passed through a column of a packed media. The media consists of porous beads. The smaller particles are able to enter the pores of the beads and therefore follow longer migration paths than do larger particles. As a result the larger particles are eluted on average earlier than the smaller particles. However, migratory paths are not deterministic and therefore unexpected retention times can occur reducing the resolution of the separation.

A second sized based chromatographic separation technique is hydrodynamic chromatography [27, 28]. Hydrodynamic flow drives a sample of particle solution through a capillary. The flow pattern is the familiar parabolic profile where flow is fully developed. The particle radii limit the proximity of the particle center to the capillary wall. Because of this the smaller particles can get ‘closer’ to the walls and are slowed in
the boundary layer. The larger particles move with the bulk of the fluid closer to the center of the capillary with an average velocity greater than the average velocity of its carrier solvent [27]. So the smaller particles have smaller average velocities than larger particles. Again, the larger particles are eluted on average earlier than the smaller particles. This technique has the same low resolution issues associated with exclusion chromatography because of nondeterministic paths.

Recently, research has focused on the use of ferrofluids in combination with small scale particle manipulation. Ferrofluids are colloidal suspensions of ferromagnetic nanoparticles in some continuous medium such as water or oil. Besides these two components, ferrofluids are also composed of a long chain molecular species adsorbed onto the surface of the ferromagnetic particles. The small size (3-15 nm) of the particle component results in behavior that is largely influenced by Brownian motion. The surfactant in combination with Brownian motion renders these fluids stable against agglomeration and destabilizing van der Waals forces [29]. Ferrofluids have a wide range of applications from sealing technologies to biomedical applications. Although under strong magnetic fields concentration gradients of magnetic particles in the medium can exist, the vast majority of applications allows for the distribution of magnetic particles to be considered as homogeneous [30]. So, ferrofluids can be treated as homogeneous continuums under appropriate circumstances [31]. Therefore, they obey Navier-Stokes in combination with Maxwell equations and respond to magnetic body forces.

In literature, ferrofluids are manipulated and pumped with some combination of spatially uniform, spatially varying, and time varying magnetic fields [32-34]. These
complicated field dynamics result in forces and torques on the magnetic fluid and can facilitate fluid flow, vorticity, and circulation [29]. With these combinations of non-uniform magnetic field, analysis is complex and paradoxical behavior is often observed. For example, under ac magnetic fields there is a critical magnetic field strength at which the pumping direction is reversed [32]. Much of the complex behavior in these varying fields is observed because the direction of the magnetization of the ferrofluid lags behind the applied magnetic field, owing to the presence of magnetic torques [35]. Further adverse effects can be observed such as heating.

A magnetic hole is a nonmagnetic particle submerged in a magnetic fluid or ferrofluid [36]. The concept of ferrofluids paired with the presence of magnetic holes leads to the investigation of ferrofluids for biomedical and cell and particle manipulation applications. A model for detecting magnetic holes such as viruses, cells, and bacteria in ferrofluid has been developed and is one such application [37]. Historically, tagging which has been discussed previously could be used in combination with magnetic sensors to detect the presence of a tagged cell. This method has been associated with difficulties because the field produced by the magnetic labels fluctuates significantly leading to false negative and false positive signals [37]. The new model developed in [37] overcomes these difficulties by detecting the cell of interest directly and eliminates the attachment processes required by tagging.

The concept of magnetic holes is also applied to new particle manipulation techniques [38, 39]. Latex beads in the size range of 100-300 nm are manipulated with a magnetic field without first labeling the beads as with tagging methods. The nonmagnetic beads are submerged in a magnetic fluid where magnetic cobalt islands are
patterned on a substrate. The latex beads assemble at energy minima. The locations of the energy minima can then be moved via super position of the magnetic field of the islands with an externally applied static or rotating magnetic field. When the minima are moved slowly enough the beads will follow their movements. In a related way, axial alignment of nanorods is controlled with the application of magnetic field gradients. The nanorods align perpendicular to an external magnetic field at low field strengths and parallel to the external magnetic field at high field strengths. The preference of a nanorod to align with one orientation verses another is a result of which factor dominates at a given field strength; orientational potential energy or positional potential energy [40, 41].

Another technique which utilizes ferrofluids for small scale nonmagnetic particle (NMP) navigation is seen in [42]. Here, a number of 300 µm wide electrodes are patterned on a printed circuit board. A ferrofluid channel containing microspheres is oriented perpendicular to the electrode array. The electrodes are used to create a traveling magnetic field [33, 34] within the fluid thus allowing manipulation of the NMPs.

Both the need for fast, effective separation technologies and the successfulness of ferrofluids applied to small scale particle manipulation technologies forms the motivation for this thesis. This thesis focuses on an entirely different method of NMP separation that does not require magnetic labeling but also utilizes ferrofluids. With the technique investigated here, a magnetic field is generated by a pair of magnetic coils with one located coaxially above the other. On the mid plane between these coils is a petri dish oriented horizontally and filled with ferrofluid. The petri dish contains the NMPs to be separated within a thin layer of ferrofluid forming a quasi two-dimensional system. The
vector components of the magnetic field from each coil are superposed and can be calculated and adjusted to desirable values at any point within the fluid layer. This configuration allows NMPs to be separated primarily based on size. The separation is accomplished with the application of a very simple time uniform magnetic field. This can be contrasted with the complicated field dynamics required by other ferrofluid methods [38, 39, 42]. Because of the simplicity of the equipment required, it is anticipated that a device with these capabilities has strong potential for transformation to a point-of-care device as it can selectively separate and does not require a controlled laboratory environment, but only a battery, a microcontroller, solenoids, ferrofluid, and a sample of NMPs to be analyzed.
2.0 Theoretical Development

2.1 Magnetic Holes

The method developed within this thesis is accomplished by submerging an NMP in a ferrofluid. Because the ferrofluid responds to magnetic fields it can be manipulated via an externally applied magnetic field gradient. The method is based on the concept that nonmagnetic bodies or magnetic holes in ferrofluid will experience repulsive forces from sources of magnetic field gradients [42].

2.1.1 Magnetostatic Force

The interrelationship of a magnetic moment in a medium with a magnetic field gradient results in a body force on the medium. This force is known as the magnetostatic force [43]. The void produced by the NMP possesses an effective magnetic moment equal in magnitude and opposite in direction to the displaced fluid [36]. Therefore, an NMP in a ferrofluid with magnetization \( \mathbf{M} \) is equivalent to a magnetic particle with magnetization \(-\mathbf{M}\) in a nonmagnetic fluid [44]. Consequently, the force on a nonmagnetic body submerged in a ferrofluid, provided the magnetic field is relatively constant across the body volume, can be written [29][32] as

\[
\vec{F}_M = -\mu_0 V (\mathbf{M} \cdot \nabla) \mathbf{H},
\]

where \( V \) is the volume, \( \mu_0 \) is the permeability of free space, \( \mathbf{M} \) is the magnetization of the ferrofluid and \( \mathbf{H} \) is the magnetizing field. The force expression in equation (1) has been found and utilized in different applications involving magnetic holes [29][43].

2.1.2 An Analogous Example

To illustrate the concept an analogy is presented. Consider the mechanics of a simple buoyancy experiment where an NMP of density \( \rho_{\text{NMP}} \), and volume \( V \), is submerge
in a container of water of density $\rho_W$, subject to gravity $g$, as shown in Figure 1. In this situation the NMP will experience a buoyant force $F_{\text{buoyant}} = gV\rho_W$. If $\rho_{\text{NMP}} < \rho_W$ the NMP will rise towards the surface in the direction opposite to gravity [45].

The buoyant force results from the gravitational body forces acting on the water which creates a pressure gradient around the NMP. If gravity were removed the NMP would not experience a buoyant force and will not rise to the surface regardless of any density difference. However, the buoyant force can be mimicked by a magnetic force which functions in a similar manner. The gravitational body forces acting on the fluid can be replaced with magnetic body forces by replacing the water with ferrofluid and applying a magnetic field gradient in the direction that gravity had previously acted as in Figure 2. In this way the NMP can still be moved towards the surface. Additionally, the
direction and magnitude of the magnetic body force is controllable because the magnetic field gradient can be applied in any direction and at any strength within operational limits.

2.1.3 Physical Demonstration

This concept can be demonstrated with simple physical experiments. In these experiments, a vial is suspended vertically and its centerline axis is in the direction of gravity as in Figure 3.

![Figure 3: NMP submerged in ferrofluid with no magnetic field present](image)

The vial contains ferrofluid and the test particle which can be viewed by the ring formed at the bottom of the vial where the particle contacts the glass (also shown by an arrow in Figure 3). Notice that the particle is denser than the ferrofluid and it sinks to the bottom of the vessel. The intention is to overcome the force on the NMP resulting from gravity by applying a magnetic field gradient to force the particle to the surface, despite
the fact that the Teflon® material is nonmagnetic. Here the buoyant force is not great enough to bring the NMP to the surface so a magnetic force will be used to assist buoyancy. A magnetic field gradient is applied in Figure 4 by holding a magnet pair near the vial. It can be observed that the particle rises to the surface of the fluid as predicted thus proving the concept [45].

Figure 4: A simple experiment where an NMP is submerged in a ferrofluid and has sunk to the bottom. A magnetic field gradient is applied assisting the buoyant force bringing the NMP to the surface.

2.1.4 Separation Geometry

To make use the magnetic hole concept for NMP separation some specifics of the technique must be established. NMPs of different sizes will be constrained to a thin layer of ferrofluid with a thickness just large enough to accommodate the particles in Figure 5. This forms a quasi two dimensional system with respect to the degrees of freedom of particle motion. Magnetic coils form the source of magnetic field gradient. The
ferrofluid layer is restricted to the plane that lies mid way between the coils. Equation (1) can be used to show that NMPs will be repelled from this source consistent with the magnetic hole concept. The force in (1) in the direction of NMP motion can be utilized in combination with drag forces opposing NMP motion to selectively separate particles based on their size.

![Diagram of separator](image)

**Figure 5**: Geometry of the separator- A ferrofluid layer containing different size NMPs to be separated is constrained to the plane that lies mid way between a coil pair.

### 2.2 Magnetic Field

Equation (1) relies on a magnetic field and a magnetic field gradient. Therefore it is necessary to seek a formulation for the magnetic field. The method of field application is through the use of a coil pair as illustrated in Figure 6. Each coil consists of N turns with a current I, flowing through each in the same ‘sense’ or direction.
2.2.1 Field Derivation

Because the ferrofluid layer and the NMPs are assumed to be constrained to the mid plane between the coils (Figure 6) that plane is the region of interest when determining the field produced by the coils. The axis and mid plane symmetric geometry chosen simplifies the field calculations such that there is only a magnetic field component in the axial direction and the field strength varies only in the radial direction (i.e. the field gradient acts in the radial direction, $-\hat{e}_r$). This arrangement is very similar to a
Helmholtz coil [46, 47] except that the coil radii are allowed to differ from the separation distance between the two coils. With this arrangement the Biot-Savart law for a loop of wire can be used to calculate the magnetic field on the mid plane.

Although the coils used will have cross sectional height (h) and thickness (w), each coil can be modeled as a single curve. This is a very good approximation of the field if the cross sectional dimensions h and w are much less than the separation distance a, and the coil radius b, as in Figure 7.

Figure 7: Diagram for vector addition used to determine the magnetic field at any point P on the mid plane between the coils. The coils are represented by curves.
The Biot-Savart law for a segment of current carrying wire is

\[ dB = \frac{\mu_0 l d\vec{I} \times \vec{R}}{4\pi R^2}, \]  

(2)

where \( \vec{B} \) is the magnetic field, \( \mu_0 \) is the permeability of free space defined as \( 4\pi \times 10^{-7} \) H/m, \( d\vec{I} \) is a differential current element, and \( \vec{R} \) is the unit vector pointing from the differential current element to any point \( P \), on the mid plane, and \( H = \frac{B}{\mu_0} \).

To establish \( \vec{R} \), vector addition is performed as follows with the aid of Figure 7.

\[ \vec{v}_1 = b \cdot \cos \theta \, \hat{i} + b \cdot \sin \theta \, \hat{j} + 0 \hat{k}, \]  

(3)

\[ \vec{u}_1 = r \hat{i} + 0 \hat{j} + \frac{a}{2} \hat{k}, \]  

(4)

\[ \vec{r}_1 = \vec{u}_1 - \vec{v}_1 = (r - b \cdot \cos \theta) \hat{i} - b \cdot \sin \theta \hat{j} + \frac{a}{2} \hat{k}. \]  

(5)

Inserting these geometric relationships into (2) gives the magnetic field contribution of the lower coil.

\[ dB_1 = \frac{\mu_0 N l}{4\pi} \left[ \frac{d\vec{I}_1 \times \vec{r}_1}{r_1^2} \right], \]  

(6)

where \( N \) is the number of turns inserted to account for the total current flowing in the coil. The product \( NI \) represents the total theoretical current flowing in the curve representing the coil. Performing a similar procedure for the upper coil and combining the contributions yields

\[ \vec{r}_2 = \vec{u}_2 - \vec{v}_2 = (r - b \cdot \cos \theta) \hat{i} - b \cdot \sin \theta \hat{j} - \frac{a}{2} \hat{k}, \]  

(7)

\[ dB = dB_1 + dB_2 = \frac{\mu_0 NI}{4\pi} \left[ \frac{d\vec{I}_1 \times \vec{r}_1}{r_1^2} + \frac{d\vec{I}_2 \times \vec{r}_2}{r_2^2} \right]. \]  

(8)

The differential current element can be written as

\[ d\vec{I} = b \cdot d\theta \cdot (-\sin \theta \, \hat{i} + \cos \theta \, \hat{j}), \]  

(9)

Using (9) and expanding the numerators in the brackets in (8) gives
\[
d\vec{l} \times \vec{r}_1 = \frac{a-b \cdot d\theta}{2} \cos \theta \hat{i} + \frac{a-b \cdot d\theta}{2} \sin \theta \hat{j} + \left[ b^2 \cdot d\theta \cdot \sin^2 \theta - (r - b \cdot \cos \theta)(b \cdot d\theta \cdot 
\cos \theta) \right] \hat{k},
\]

(10)

\[
d\vec{l} \times \vec{r}_2 = -\frac{a-b \cdot d\theta}{2} \cos \theta \hat{i} - \frac{a-b \cdot d\theta}{2} \sin \theta \hat{j} + \left[ b^2 \cdot d\theta \cdot \sin^2 \theta - (r - b \cdot \cos \theta)(b \cdot d\theta \cdot 
\cos \theta) \right] \hat{k}.
\]

(11)

Because of the axis and mid plane symmetry, the magnitudes of \( r_1 \) and \( r_2 \) are the same allowing us to write

\[
r_0 = |r_1| = |r_2| = \left[ (r - b \cdot \cos \theta)^2 + (b \cdot \sin \theta)^2 + \left( \frac{a}{2} \right)^2 \right]^\frac{1}{2},
\]

(12)

Now, with (10)-(12) we can rewrite (8) as

\[
d\vec{B} = \frac{\mu_0 N I}{4\pi} \left[ \frac{d\vec{l}_1 \times \vec{r}_1 + d\vec{l}_2 \times \vec{r}_2}{r_0^3} \right].
\]

(13)

Writing out the vector products in (13) and simplifying shows

\[
d\vec{l}_1 \times \vec{r}_1 + d\vec{l}_2 \times \vec{r}_2 = 2 \cdot \left[ b^2 \cdot d\theta \cdot \sin^2 \theta - (r - b \cdot \cos \theta)(b \cdot d\theta \cdot \cos \theta) \right] \hat{k},
\]

(14)

\[
= 2 \cdot b \cdot (b - r \cdot \cos \theta) \cdot d\theta \hat{k},
\]

(15)

As expected there is only a component in the \( z \) direction due to symmetry. Inserting (12) and (15) into (13) gives the magnetic field at any point \( P \), on the mid plane between the coil pair

\[
dB_z = \frac{\mu_0 N I}{4\pi} \left[ \frac{2 \cdot b \cdot (b - r \cdot \cos \theta)}{(r - b \cdot \cos \theta)^2 + (b \cdot \sin \theta)^2 + \left( \frac{a}{2} \right)^2} \right] d\theta,
\]

(16)

Similar derivations were performed to represent the coils as both surfaces and volumes.

When the coils are assumed to be current carrying surfaces the field becomes
\[ dB_z = \frac{2\mu_0 NI}{4\pi h} \left[ \frac{b \cdot (b-r \cdot \cos \theta)}{[(r-b \cdot \cos \theta)^2 + (b \cdot \sin \theta)^2 + (z)^2]^\frac{3}{2}} \right] d\theta dz, \quad (17) \]

where \( h \) is the cross sectional height of the coils and \( z \) is the absolute value of the \( z \) component of the distance from the mid plane to a differential current element. When the coils are assumed to be current carrying volumes the field becomes

\[ dB_z = \frac{2\mu_0 NI}{4\pi hw} \left[ \frac{b \cdot (b-r \cdot \cos \theta)}{[(r-b \cdot \cos \theta)^2 + (b \cdot \sin \theta)^2 + (z)^2]^\frac{3}{2}} \right] d\theta dz db, \quad (18) \]

where \( w \) is the cross sectional width of the coils.

### 2.2.2 Model Verification

A similar procedure using the curve representation which produced (16) was performed by Calhoun [48] for the case of Helmholtz coils (\( a=b \)). In that paper, numerical hand calculations were performed for results in the range of \( 0 \leq r < b \) for the parameters \( N=72, I=3A, \ a=b=33cm, \ h=1.6cm, \ w=1.8cm \) and their validity was confirmed with experimentation. Both analytic and experimental results from that paper are shown in Figure 8.
To verify the results found in this thesis, field solutions were found computationally with Maple™ software using each of equations (16)-(18). They are plotted in Figure 9 for comparison with the results reported in [48]. Figure 9 displays each of the three curves found using different representations of the coils. The three curves are virtually indistinguishable from each other. This supports the assertion that the curve representation is valid when the parameters a and b are much larger than the cross sectional dimensions of the coils, h and w. Furthermore, they are practically identical to

Figure 8: Mid plane magnetic field with experimental and analytic results found by Calhoun [48]. Reprinted with permission from Calhoun, R. C., Am. J. Phys., 64(11), pp. 1399-1404, 1996, Copyright 1996, American Association of Physics Teachers.
the results from [48] as shown in Figure 8. In this thesis the curve representation was used primarily for computational efficiency.

![Analytically Obtained Mid Plane Magnetic Field, B](image)

**Figure 9:** Analytic magnetic field B, obtained with Maple™ and found with the model developed here for comparison with results in [48].

### 2.2.3 Field Profile

When considering the problem of particle navigation and the geometry discussed previously it is apparent that the work within this thesis is concerned with not only the field but also the field gradient. Furthermore, knowledge of these quantities is necessary for radial distances greater than what is available from Figure 8 and Figure 9. With the
benefit of computational software we are able to plot solutions of the magnetic field over a greater range of radial distances. For the same case discussed above the magnetic field B, and magnetic field gradient dB/dr, are plotted in Figure 10 and Figure 11 for a broader overview of the field profile. As before, each of the three curves found with the three different representations of the coils are displayed in both Figure 10 and Figure 11 and are virtually indistinguishable. It should be noted that the helmholtz arrangement (i.e. a=b) is not ideal for our particle separation application. However, it is presented here to illustrate a fairly typical field and field gradient profile which can be compared with published results in [48] with Figure 8 and Figure 9 above.
Figure 10: Analytic magnetic field $B$, obtained with Maple™ and found with the model developed here for distances from $r = 0$ to $r > b$. 
2.3 Magnetic Force

2.3.1 Magnetic Forces on NMPs

The magnetic force on a magnetic hole in a ferrofluid was given in (1). Having dealt with the magnetic field we can now investigate the force further. The magnetization of the ferrofluid appearing in (1) is given as [43]
\[ \vec{M} = \chi \vec{H}, \]  
\( (19) \)

where \( \chi \) is the initial magnetic susceptibility of the ferrofluid. This is with the assumption that at low, uniform magnetic field strengths the magnetization can be assumed to be parallel to the magnetizing field. This is an advantage of the technique resulting from the application of a uniform magnetic field. This greatly reduces the complexity of the model and rules out the torque effects acting on the ferrofluid and determined by \( \vec{T} = \mu_0 (\vec{M} \times \vec{H}) \) [35]. This expression, however, becomes a crucial component of the analysis in the case of, for instance, spatially and temporally traveling magnetic fields [32-34]. Further simplifications can be made with consideration of the axis symmetric geometry. The magnetic force in (1) on the NMP reduces to [44-45]

\[ \vec{F}_M = -\mu_0 V M(H) \frac{dH}{dr} \hat{e}_r, \]  
\( (20) \)

where \( \hat{e}_r \) is the unit vector along the \( r \) axis.

2.3.2 Coil Selection

From this point further attention is focused on a different coil pair than those analyzed in sections 2.2.2 Model Verification and 2.2.3 Field Profile. To reiterate, those coils were used to aid in the development of a magnetic field formulation because published results are available for the validation of the model developed in this thesis. Moving forward, coils of more suitable parameters (b, N) were modeled and subsequently sought out and obtained. They are more suitable mainly because they are capable of producing greater magnetizing field strengths and magnetizing field gradients therefore producing greater forces on NMPs.

The parameters chosen dictate the scale of experiments that follow theoretical development. It will become clear that the success of a separation attempt depends on a
number of factors, including the coil parameters. These parameters along with separation distance a, and current I, must be carefully selected based on the scale of the separation and the size range of particle within that scale.

The coil parameters are not easily chosen as it is an iterative process. This is because the equation of motion to be developed in the sections that follow is a differential equation that likely has no closed form solution. This is primarily a result of the highly nonlinear nature of (16)-(18). Presently, parameters are chosen based on commercial availability and prudently considering limitations such as current throughput, spatial constraints, and relative ease and cost effectiveness of early experiments for proof-of-concept. The experimentation to follow will be scaled in accordance with the coils. This facilitates further development of the model to a point where the iterative process can be accomplished thus allowing for coil selection for any range of particle sizes. Although the development of the model here focuses on the millimeter scale, the principles are sound for navigating particles as small as 100nm [49].

2.3.3 Force Landscape

Equation (20) is the magnetic force acting on an NMP at some point P, within a ferrofluid. Having arrived at agreeable parameters for a new set of coils (b=25 mm, N=1000, I=1 Amp), the magnetic force per unit volume can be plotted along with the magnetizing field and the magnetizing field gradient. Although coil parameters b and N have been selected, the separation of the coils a, can be adjusted which in turn alters the profile of these quantities. To illustrate this Figure 12, Figure 13, and Figure 14 show how these quantities vary for increasing radial positions of the NMP, and also for varying values of separation distance, a. These plots are helpful when selecting a separation
distance at which the force on the NMP does not reverse sign. Such an occurrence hinders separations and can in fact cause all the particles to collect at the zero force locations. This type of case can be seen in Figure 14 at \( a \approx 30 \text{ mm} \) and approximately \( r = 31 \text{ mm} \) (see also Figure 11 where the gradient reverses sign). This is the reason why the Helmholtz configuration is not ideal as mentioned previously. The plots may also be used to assist in an optimization of separation which is beyond the scope of this thesis.

![Magnetizing Field vs a and r](image)

Figure 12: Magnetizing field \( H \), as a function of coil separation distance \( a \), and radial position \( r \), for the coil parameters \( b = 25 \text{ mm}, N = 1000, I = 1 \text{ Amp} \).
Figure 13: Magnetizing field gradient $dH/dr$, as a function of coil separation distance $a$, and radial position $r$, for the coil parameters $b=25$ mm, $N=1000$, $I=1$ Amp.
Figure 14: Force per unit volume on an NMP in the ferrofluid layer as a function of coil separation distance a, and radial position r, for the coil parameters b=25 mm, N=1000, I=1 Amp.

2.4 Particle Dynamics

To fully understand the dynamics of the NMPs in the ferrofluid subjected to an externally applied magnetic field we endeavor to develop an equation of motion. Up to this point discussions of force have been limited to the magnetic force only. When the
particles are in motion they are also subjected to drag forces. A heuristic discussion is presented to aid in the understanding of the more rigorous approach immediately to follow.

2.4.1 Conceptual Development

The intent is to separate particles based on their size. It is assumed that the NMPS are spherical. The population of NMPs will initially be located near the central axis of the coil pair \( r=0 \). The coils will be energized giving rise to particle motion as in Figure 1.

Different sized NMPs experience different forces even though they are subjected to the same magnetic field. The force they experience within that magnetic field is dependent on the volume of the NMPs as apparent in (20). Therefore, at any given location \( r \), a larger NMP will experience a greater magnetic force than one that is small. However, as the NMPs gain motion under such a force there are drag forces to contend with. A pertinent question is: Which force dominates? The magnetic force is dependent on volume and consequently the cube of NMP radius, \( F_M \propto V \propto r^3 \). While drag forces on the other hand are based on projected area and therefore depend only on the square of the radius, \( F_D \propto r^2 \). So, larger NMPs attain greater accelerations. As the NMPs travel sufficiently far from the coils to a region where the magnetic force is negligible, drag forces become dominant. As a result of the complicated relationship between the magnetic and drag forces, larger NMPs travel further.

This result may seem counter intuitive. However, for the reasons discussed above this is in fact the case. As discussed in 2.1.2 An Analogous Example, this phenomenon can be seen as analogous to Archimedes Principle which is now extended to the case of
NMPs in motion. The buoyant force, which is compared here to the magnetic force, is also dependent on volume. The nature of the drag forces for both cases remains the same. So, similar to the phenomenon described for the case of NMPs in ferrofluid, larger steam bubbles in boiling water rise faster than smaller bubbles. In this way the assertions here are consistent with examples in nature.

2.4.2 Mathematic Development

To explain this mathematically, an equation of motion is developed by first applying Newton’s Law. In the analysis, ferrofluid is assumed to be a continuum and the Brownian motion of the magnetic nanoparticles of the ferrofluid, although crucial for fluid stability, has negligible effects on the NMPs and is neglected in the equation of motion. Furthermore, the van der Waals interactions are also considered to be much smaller than the magnetic forces. The NMP is subjected to both the magnetic force $F_M$ in (20) and a drag force $F_D$. Newton’s Law then becomes

$$m\ddot{r} = F_M - F_D. \quad (21)$$

The drag force $F_D$ is calculated using the fluid mechanics relationships [50],

$$C_D = \frac{F_D}{\frac{1}{2}\rho_f \dot{r}^2 A}, \quad (22)$$

$$Re = \frac{\rho_f \dot{r} D}{\mu}, \quad (23)$$

$$C_D = \frac{24}{Re} \left(1 + \frac{1}{6} Re^2\right) Re < 5000, \quad (24)$$

where $C_D$ is the drag coefficient that captures Reynolds numbers up to $5 \times 10^3$ very well, $\rho_f$ is the density of the fluid, $Re$ is the Reynolds number, $A$ is the projected area and $D$ is the diameter of the NMP. From Equations (22)-(24), the drag force $F_D$ is determined to be
\[ F_D = 12A \frac{\mu}{D} \hat{r} + 2A \frac{\rho^{2/3} \mu^{1/3}}{D^{1/3}} \hat{r}^{5/3}. \]  

(25)

Inserting (20) and (25) into (21) and decomposing area \( A \) and volume \( V \), the equation of motion becomes,

\[ \ddot{r} = -\frac{1}{\rho_{NMP}} \frac{d}{dr} \left( \frac{M(H)}{\rho_{NMP}} \right) - \frac{18 \rho_{fluid}^{2/3} \mu^{1/3}}{\rho_{NMP} D^{4/3}} \hat{r}^{5/3}, \]  

(26)

which carries the contributions of magnetic field strengths, fluid properties and characteristic size of the NMP considered. An analytical solution of the coupled equations (16) (where \( H = \frac{B}{\mu_0} \)) and (26) is extremely complex, if not impossible, even with simplifying assumptions. In order to circumvent these difficulties and to capture the fundamental understanding of the solutions of (26), we will visit numerical time integration tools in the following section.
3.0 Solution Method

Solutions can be obtained using the Simulink® toolbox accompanying Matlab®.

3.1 Simulations

The simulations made use of the new coil parameters discussed in 2.3.2 Coil Selection. The separation of the coils was chosen based on Figure 12, Figure 13, and Figure 14. A separation of 50 mm was chosen for two main reasons. First, as can be seen in Figure 14, the direction of the force does not change sign. As previously mentioned, this would interfere with and possible prevent separation. Second, at this value the magnetic force remains sufficiently strong providing enough force for expedient separation. The fluid viscosity $\mu$, and density $\rho_f$, are that of Ferrotec ferrofluid EFH1 ($6.64 \times 10^{-3}$ Ns/m$^2$ and 1210 kg/m$^3$ respectively). The NMP density $\rho_{NMP}$ is based on a typical density of red blood cells (1125 kg/m$^3$). The field and field gradient at these coil parameter values are shown in Figure 15 and Figure 16.
Figure 15: Magnetic field along the radial coordinate on the mid plane for the parameters $a = 0.05\, \text{m}$, $b = 0.025\, \text{m}$, $N = 1000$ turns, $I = 1\, \text{A}$.
Figure 16: Magnetic field gradient along the radial coordinate on the mid plane for the parameters $a = 0.05 \text{ m}$, $b = 0.025 \text{ m}$, $N = 1000 \text{ turns}$, $I = 1 \text{ A}$.

Numerical integration is performed with sufficiently small time steps ($10^{-6}$ seconds) solving for the motion of NMPs of different diameters $D = 2 \text{ mm} \ldots 11 \text{ mm}$ with 1 mm increments. It is assumed that there are no interactions among the particles (such as collisions) and all the particles start their motion at the same initial position at the central axis of the coils ($r = 0$) with significantly small velocity $10^{-4} \text{ mm/s}$ (visible in Figure 17). The small velocity is necessary because there is theoretically no force when a particle occupies the $r=0$ position. This is an unstable equilibrium position which is unlikely to be occupied in a real world setting. The small initial velocity can be seen as a small perturbation to the system.
3.2 Simulation Results

The position variations of different size NMPs as a function of time are obtained in Figure 17, where we reveal that NMPs with larger diameters travel a further distance along the radial direction. The acceleration and velocity profiles for the first two seconds of the simulation are visible in Figure 18 and Figure 19. Again, these profiles are a result of the complicated relationship between the magnetic and drag forces. Because all the NMPs in the simulation are different sizes and the magnetic force is based on particle volume, the NMPs experience different magnitudes of acceleration through a region where the magnetic force is nonzero (Figure 17). When they reach the boundary where the magnetic force becomes zero, they have nonzero velocities even though there is no longer a magnetic force in the direction of their motion. From this point further they are subjected only to drag forces which depend are the projected area of the different sized NMPs. This results in varying rates of deceleration.
Figure 17: Acceleration (magnetic force induced) and deceleration (drag induced) of different size NMPs results in different position profiles with respect to time. Positions of NMPs with different diameters, D = 2 mm ... 11 mm with 1 mm increments, are displayed.
Figure 18: Accelerations of different size NMPs with respect to time for diameters, D = 2 mm ... 11 mm with 1 mm increments. Larger NMPs attain greater accelerations.
Figure 19: Velocities of different size NMPs with respect to time for diameters, \( D = 2 \text{ mm} \ldots 11 \text{ mm} \) with 1 mm increments. Larger NMPs attain greater velocities.

3.3 Force Examination

It is helpful in the understanding of the particle dynamics to examine the forces on one NMP.

3.3.1 Temporal Examination

Both the magnetic and drag forces along with the resultant force for the 10 mm NMP are depicted in Figure 20 as a function of time for the first two seconds of simulation. These forces directly result in the position profile in time for the 10 mm NMP which is extracted from Figure 17 and clearly shown in Figure 21.
Figure 20: Magnetic and drag forces and the resultant force exerted on a 10 mm diameter NMP with respect to time for the first two seconds of simulation.
Figure 21: Position of a 10 mm diameter NMP with respect to time for the first two seconds of simulation.

While a temporal examination of the forces, accelerations, velocities, and positions show in Figures 17-21 is helpful in understanding how an NMP travels, a spatial examination is helpful in understanding why they travel in such a way.

3.3.2 Spatial Examination

While drag force and therefore total force are functions of not only position but also time, magnetic force is only a function of position. Figure 22 shows how these forces vary with the position of the 10 mm diameter NMP. This perspective allows for a comprehensible view of how the acceleration and velocity of an NMP are affected as it passes through different regions of the petri dish.
While this analysis is presented for one NMP, it differs quantitatively for NMPs of different sizes which is the central concept of this thesis and allows the separation to work. However, it remains the same qualitatively and therefore the following discussion applies to NMPs of all sizes.

While the NMP begins to travel from the central coil axis \((r=0)\) outward it experiences increasing magnetic and drag forces but the magnetic forces dominate at this stage (Figure 22). So the NMP experiences a rapid increase in the rate of acceleration and velocity in Figure 23 Figure 24. The NMP passes through a point of maximum magnetic force at a location of \(r\approx17\) mm at which point the magnetic force begins to

\[\text{Figure 22: Magnetic and drag forces and the resultant force exerted on a 10 mm diameter NMP with respect to radial position } r.\]
decay. It is nevertheless still the dominant force until it reaches a radial position approximately equal to the coil radius (r=25 mm). At this point the magnetic force still has a nonzero value but the drag forces begin to dominate and the NMP begins to decelerate and the velocity decreases (Figure 23 and Figure 24). The magnetic force continues to decay to a point past the coil radius where it becomes zero at r≈45 mm. Here the drag force takes over completely but the NMP has a nonzero velocity which is clearly visible in Figure 24. It should be noted that for NMPs of different sizes this velocity will be different. Not only will it be different but it will occur at a different time which can be interpolated from Figure 17, Figure 18, and Figure 19. The 10 mm NMP continues to travel while decelerating until it reaches a final position. For the 10 mm NMP, this is at a location of r≈84 mm. Again, this final value is different for NMPs of different sizes.
Figure 23: Acceleration of a 10 mm diameter NMP with respect to radial position $r$. 
Figure 24: Velocity of a 10 mm diameter NMP with respect to radial position r.

3.4 Final Positions

Returning to an examination of all the NMPs, the final positions of the particles in Figure 17 are more neatly presented in Figure 25, where we can see the trends of the separation distances. The final positions of NMPs increase as size increases and it appears that separation becomes linear for sufficiently large NMPs under the assumptions that constructed the physical problem. We also investigate the relationship between separation distances and the sizes of the NMPs. This is presented in Figure 26. The particles clearly travel different distances, but in some instances there is some overlap. The NMPs on the lower end of the scale overlap significantly. This is because their characteristic length $D$ is small relative to the coil dimensions and therefore they do not
experience an adequate resultant force profile in time for sufficient separation. This is does not indicate a limitation of the method but rather a limitation of the specific design with the parameters used for the simulation. The model can be scaled to separate the particles on the lower end of the spectrum (2-4 mm) which are not separated when using the coil parameters in question.

The NMPs on the higher end of the scale again begin to overlap because the increasing distance NMPs travel with increasing size is not sufficient enough to compensate for the larger radii, mainly due to drag forces that become larger for increased velocities and ultimately increased Reynolds numbers. These details should carefully be taken into account at the design stage of the device; however, the feasibility of the particle separation technique is clear from Figure 26.

Figure 25: The final radial positions of NMPs of different sizes.
3.5 Effect of Current

We next investigate the effects of larger and smaller current values on particle separation. Specifically, we are interested in extracting similar results as in Figure 25, but only for particle diameters where separation becomes feasible, i.e. $D > 4$ mm. In Figure 27, we compare Figure 25 with two different cases where current is chosen as $I = 0.5$ A and $I = 1.5$ A. As expected, when the current increases (or decreases), nonmagnetic particles travel larger (or smaller) distances. One can also see that the slope of a linearly interpolated curve increases as current increases. This suggests that separation distances between nonmagnetic particles also increases for increased values of the current.
We finally present in Figure 28 how a decrease in the current, down to $I = 0.5$ A, affects the position progression of the nonmagnetic particles. At any snapshot of time, the order of the particles is exactly the same as seen in Figure 17, meaning that a smaller particle is always behind a larger one with respect to the origin. However, we observe a major difference between Figure 17 and Figure 28: an interesting selective particle separation phenomenon. This phenomenon suggests that at low current values, the acceleration of smaller particles is much smaller and therefore they fall very far behind the larger particles, which cover much longer travel distances in the transient regime, $t < 4$ sec in Figure 28. For instance, when $t = 2$ sec, the ratio of the distances travelled between the smallest particle and the largest particle is around 1:69 in Figure 28, while
the same ratio is around 1:2 in Figure 17. This can be exploited in a number of ways. The current can be ramped in increments so that larger particles can be separated and removed at lower current values and smaller particles can be separated at higher current values. A second variation is to shut the current off at some time before the particles arrive at their final location. For example, the final locations of the 2 mm and 3 mm NMPs in Figure 28 are very close and therefore there is not adequate separation. But if the current is shut off at $t=2$ s there is more than enough difference in position for separation. The latter is exploited later in experiments.

**Position as a Function of Time**

![Position variation of the nonmagnetic particles for current $I = 0.5$ A.](image)

**Figure 28:** Position variation of the nonmagnetic particles for current $I = 0.5$ A.
4.0 Experimental

An experimental device was designed which consisted of two main parts. A lead screw driven apparatus accurately fixed coil separation distance \( a \), and a custom built sealed container housed the NMPs and the ferrofluid.

4.1 Materials

Raw aluminum (6061-T6), Raw delrin (½” thick), Precision ACME threaded rod (3/8-8), bearing guide rail (1”), Plain bearing square guide blocks (for 1”rail), Precision bearing shaft with support rail (½”), pillow block fixed alignment PTFE lined bearing (for ½” shaft), machinable PET Precision ACME round nuts (3/8-8), ¼” id ABEC-1 steel ball bearings, steel and plastic socket head cap screw fasteners, ¼” polyurethane washers, Buna-N A65 durometer AS568A-049 O-rings, Borosilicate glass disks (5 ½” diameter), Brass single barbed tube fittings, and compact 1/8” PVC needle valves were purchased from McMaster-Carr. Ferrotec ferrofluid EFH1 was purchased from Amazing Magnets. An Aven digital microscope and accompanying software was purchased from Digi-Key Corporation. Magnetic coils were donated from Classic Coil of Bristol, CT. Plastic syringes (35 mL capacity) were donated by Northeastern University’s Mechatronics lab and Professor Constantinos Mavroidis. 1/8” id flexible PVC tubing was donated by Northeastern BajaSAE student organization.

4.2 Fabrication and Equipment

The majority of the fixture was fabricated by the PI using a Bridgeport knee mill, Kent USA KLS-1740 gap bed lathe, and a Miller Syncrowave 250DX Tungsten Inert Gas welder with AC/DC square wave power source. Machine shop time and tools were donated by Northeastern University’s BajaSAE student organization. The ferrofluid
container was fabricated in the Northeastern University mechanical lab by Jonathan 
Doughty with a Lagunmatic CNC milling machine. Multimeters and a DC power supply 
for energizing the magnetic coils were supplied by Northeastern Lab Director Kevin 
McCue.

4.3 Apparatus

The system includes the ferrofluid container which is constrained to the mid plane 
between the coils in Figure 29 and Figure 30. Its top is closed by the Borosilicate glass 
material so particle motion can be observed. A lead screw driven apparatus houses the 
magnetic coils. The separation distance of the coils can be varied from \( a=0-10 \) in. \( (0-254 \) mm). The ability to alter the ‘dish’ position allows the tests to be conducted with particle 
positions for \( r=0-6 \) in. \( (0-152.4 \) mm) from the centerline axis of the coil pair. The 
versatility of the design enables experiments to be conducted for many different magnet 
and coil configurations along with various different parameter selections such as 
separation distance, coil diameters and number of turns of the coils [45].

![Figure 29: The sealed container to house the ferrofluid and NMPs.](image)
4.4 Coil Modification

The coils were modified by inserting an iron core into their centers which is visible in Figure 29. The iron cores magnify the magnetic field strength [51] allowing for greater magnetic force to be applied to NMPs at lower current values. Every effort was made to modify the coils in such a way that the field was only amplified and the ‘shape’ of the field was not altered in the region of interest (i.e. the mid plane). Investigations later show that this attempt was largely successful.
The experiments used air bubbles as the NMPs primarily because they are visibly in ferrofluid which is generally opaque. Although this NMP selection permits visibility it also reduces the ease with which NMPs can be separated. This difficulty stems from the low density of the air bubbles (1.2041 kg/m³, air at 1 atm and room temperature). The increased field strength combats this difficulty.

Also, the lower current requirements were necessary because the maximum current was limited by the power supply that was available for the experiments. Also, the heating of the coils is reduced at lower current values. Heating adversely affects the repeatability of the experiments because as the coils heat up the resistance of the coils increases and the current fluctuates. With the addition of the iron core heating was eliminated for all practical purposes and a consistent current within 0.5 ± 0.002 Amps was achieved even after long periods (on the order of 1 hour) of coil energization.

4.5 Experimental Procedure

The coils were positioned with a separation distance of a = 75.4 mm. The entire fixture was leveled with leveling screws and a digital angle finder. The ferrofluid container was held on end with one valve vertically downward and one vertically upward. The lower valve was connected to a ferrofluid reservoir with the PVC tubing and the upper valve was connected to a syringe. Ferrofluid was drawn into the container through the bottom valve via the syringe at the top until the container was completely filled with no air pockets.

The container was then oriented in the experimental position in Figure 29. The container was free to slide clear of the coils during setup and later be positioned between the coil pair for the experiment to take place. Two air bubbles were injected into one
valve with a new syringe and the displaced fluid was collected as it exited the other valve. A large air bubble with characteristic length $D=17.9\,\text{mm}$ and a small bubble of characteristic length $D=2.3\,\text{mm}$ were added. Bubbles with significantly large size differences were added and their characteristic lengths were later measured with the aid of the digital microscope using pixels after calibration.

The design of the fixture enabled accurate positioning of the dish in relationship to the coil axis with respect to a fixed reference point on the dish. The bubbles were carefully moved by hand with permanent magnets and the aid of the stationary microscope. They were located so that their centers were relatively equal distances from the dish reference which will be coincident with the coil axis when the dish is moved into position.

The dish was then moved into position between the coil pair. The current was switched on at $I=0.5\,\text{A}$. Because of the low bubble densities and reasons discussed at the end of section 3.5 Effect of Current, the current was shut off at $t=1\,\text{s}$. The dish was moved out from between the coil pair and the position of the bubbles with respect to the fixed reference was measured with the microscope. The procedure was repeated five times.

4.6 Experimental Results and Discussion

For each experiment the final position of the bubbles was significantly different from each other. As expected, the large bubble traveled farther than the small bubble each time. The final positions of both bubbles are tabulated in Table 1. Figure 31 provides a visual representation of the final positions of each bubble for each experiment.
**Table 1**: Final position results for bubble separation experiment.

<table>
<thead>
<tr>
<th>Measurement/Experiment # (N=5)</th>
<th>Distance Traveled [mm]</th>
<th>Distance Traveled [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubble 1, 17.9mm Diameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>35.6</td>
<td>14.7</td>
</tr>
<tr>
<td>2</td>
<td>34.6</td>
<td>17.6</td>
</tr>
<tr>
<td>3</td>
<td>37.7</td>
<td>17.4</td>
</tr>
<tr>
<td>4</td>
<td>35.3</td>
<td>16.8</td>
</tr>
<tr>
<td>5</td>
<td>34.5</td>
<td>17.8</td>
</tr>
<tr>
<td>Bubble 2, 2.3mm Diameter</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Confidence Interval (95%)

- \(34.05 \leq X_i \leq 37.03\)
- \(15.41 \leq X_i \leq 18.31\)

**Figure 31**: Final positions of each bubble for each instance of a bubble separation experiment.
A confidence interval is also calculated in Table 1. Using this information, if the experiment were conducted a sixth time the large bubble would occupy a final position located between 34.05 mm and 37.03 mm. The small bubble will occupy a location between 15.41 mm and 18.31 mm. Both claims are at a 95% confidence level. Based on this information the experiment is repeatable.

Snapshots of the bubbles were taken before and after each measurement. For one instance they are presented in Figure 32 and Figure 33. In Figure 32, the bubbles are at their initial location where they have been marked with a red outline for clarity. In Figure 33, they are at their final locations after the separation. Their initial locations are marked on the glass with paint as a visual reference. It is clear in these photographs that the bubbles significantly separate therefore we conclude that the attempt was successful.
Figure 33: Final positions of the different sized bubbles for one instance after the separation experiment. The bubbles are outlined in red for clarity.
5.0 Comparison of Analytic Model with Experiments

The Analytic model has been developed separately from the experiments. Even so, the evidence from both domains standing alone is strong. However, closing the loop by comparing the two will further strengthen the case of nonmagnetic particle separation by the method presented. The comparison is not straightforward. The experiments presented were a first round of experiments intended to prove and demonstrate the concept based on the principles established by the analytic model. They were never intended to match the conditions exactly. That is left to future experiments in the micro and nano scales. Nonetheless, attention is turned to a comparison of the analytic model with the experiments within the scope of this thesis with the understanding that discrepancies will exist. Despite these discrepancies, a comparison is developed with special attention paid to differences between the two and attempts to compensate for these differences.

5.1 Magnetic Field and Magnetic Force

The first difference that must be accounted for is the insertion of the iron core into the magnetic coils. The iron core magnified the magnetic field by an unknown amount and is not taken into account in the magnetic field derivations of section 2.2.1 Field Derivation. If the shape of the magnetic field did not change in the plane of interest and the cores were successful only in magnifying the field, which is the largely case as will be shown, then the analytic field can be multiplied by a constant factor to match actual values.
5.1.1 Field Measurements

The magnetic field was measured with a FW Bell Model 5080 Gauss meter. A simple holding fixture for the measuring probe was built out of delrin for use with the experimental apparatus in Figure 34. With the holding fixture in combination with existing features on the experimental apparatus the probe could be accurately positioned at different radial positions along the mid plane.

![Image of holding fixture and Gauss probe](image.png)

**Figure 34: Holding fixture and Gauss probe used in magnetic field measurements.**

1800 measurements were taken in several configurations. For separation distances of \(a=50\,\text{mm}, 62.7\,\text{mm},\) and \(75.4\,\text{mm},\) and for current values of \(I=0.5\,\text{A}, 1.0\,\text{A},\) and \(1.5\,\text{A},\) measurements were taken at radial distances beginning at \(r=0\) in \(1/16''\) (1.5875 mm) increments. This cycle was repeated three times. The three measurements at each location were averaged for each of the three separation distances and for each of the three current values. The field results for the \(0.5\,\text{A}\) current value are presented in Figure 35. The experimental values for the coils with an iron core are shown with the curves
produced by the analytic model which does not account for the iron core. As expected, the experimental curves indicate a stronger field than that produced by the analytic model. This is because of the presence of the iron core in the experimental measurements.

![Unscaled Analytic and Experimental Field](image)

**Figure 35:** Magnetic field on the mid plane for increasing values of $r$, beginning at $r = 0$ for coil parameters $b=25$ mm, $N=1000$, $I=0.5$ A. Both experimental curves with an iron core and analytic curves not accounting for an iron core are shown.

### 5.1.2 Analytic Field Scaling

The experimental curves are used to find a scaling factor for each of the three analytic curves. After scaling, the profiles of the new scaled analytic curves can be compared to the profile of the experimental curves to determine if the shape of the field was altered by insertion of the iron core. Once the scaled analytic curves are shown to be
in agreement with the experimental curves the continuous scaled analytic curves can be used to calculate the magnetic force of the experiments. Then a simulation can be run for comparison with the bubble separation experiments. The scaling factors for the 50 mm, 62.7 mm, and 75.4 mm separations become 3.31, 3.18, and 3.15 respectively. The new scaled analytic curves are shown along with the experimental curves in Figure 36. It is clear that the field shape is basically preserved and the scaled analytic curves match the experimental curve well.

Figure 36: Magnetic field on the mid plane for increasing values of r, beginning at r = 0 for coil parameters b=25 mm, N=1000, I=0.5 A. Both experimental curves with an iron core and scaled analytic curves are shown.
5.1.3 Scaled Analytic Force

The scaled analytic field shown in Figure 36 was used with equation (20) to find the scaled analytic force per unit volume as in section 2.3.3 Force Landscape. In Figure 37, this force is plotted for the three separation distances in question.

Figure 37: Scaled analytic force per unit volume on the mid plane for increasing values of r, beginning at r=0 for coil parameters b=25 mm, N=1000, I=0.5 A, and for three separation distances, a.

In Figure 38, the force curve produced by the separation distance a=75.4mm is extracted from Figure 37 since that is the separation distance used in the experiments. A sixth order polynomial is fitted to the curve for use in the simulations. This is not necessary but it allows the simulations to run faster by eliminating the need for
Simulink® to visit a Matlab® function at every time step. The polynomial fits the scaled analytic force curve very well with an $R^2$ value of 0.9998.

**Figure 38:** Scaled analytic force per unit volume on the mid plane for increasing values of $r$, beginning at $r=0$ for coil parameters $b=25$ mm, $N=1000$, $I=0.5$ A, and at the separation distance $a=75.4$ mm with a polynomial fitting.

5.2 Simulation of the Bubble Experiment

The force obtained in Figure 38 was used to run a simulation based on the conditions of the bubble experiment. The characteristics lengths of the bubbles found in the experiments were used ($D=17.9$ mm and $D=2.3$ mm). The initial small perturbation velocity used in the simulations of section 3.1 Simulations, was replaced by an initial
bubble position offset which was the radius of the large bubble (8.95 mm) and the current was shut off at t=1 s to match the conditions of the experiment. The simulations produced the results in Figure 39 and Figure 40.

Figure 39: Position of two NMPs with respect to time produced by simulation for the conditions of the bubble separation experiment. The scaled Analytic force is used.
Figure 40: The final positions of the NMPs and how they separate from each other produced by simulation for the NMPs of the bubble separation experiment.

There are discrepancies between the simulations and the bubble separation experiment despite having scaled the magnetic field which is expected. Recall the bubble experiment where the large bubble traveled 35.5 mm on average and the small bubble traveled 16.9 mm on average. Here the large bubble travels nearly 60 mm and the small bubble travels about 45 mm. There are two main factors which still have not been taken into account. First, the analytic model assumes that the NMPs are spheres. At low Reynolds numbers, a bubble moving through a fluid at its terminal velocity maintains a spherical shape because the hydrostatic pressure across the interface is balanced by the normal stress [52]. So it is common in practice to model a bubble in an infinite fluid as a sphere under many conditions. However, those conditions are not the same as those observed in the bubble separation experiment. In the experiment, relatively high Reynolds numbers are considered and the bubbles are not at their terminal velocity. Additionally, the presence of surfactants can play an important role increasing the drag in the presence of contaminants [53].
Second, the bubbles in the experiment are pushed against the glass surface by a buoyant force. This buoyant force is not in the direction of motion (i.e. it acts perpendicular to the two dimensional plane in which the bubbles are free to travel) so it was ignored in the equation of motion, equation (26). Additionally, its presence and the fact that the bubbles are pushed against the glass surface is the reason the NMPs, bubbles in this case, are visible at all. But this comes at a price. There are additional drag forces and interfacial forces that were unanticipated by the analytic model. Discrepancies between the simulation and the experiment are attributed the bubbles’ shapes and to these forces which were left unaccounted for in the model. It is believed that the latter has more of an impact on the dynamics of the separation than does the former [53].
6.0 Modification of the Analytic Model

First steps have been made toward accounting for the originally ignored interaction between the bubbles and the glass. It should first be noted that the following discussion and analysis concerns work that is at a very early stage and requires a great deal of further development. It is presented here as a foundation on which further development can be built.

To incorporate the additional interfacial force an additional term is added to the equation of motion as

\[
\ddot{r} = - \frac{1}{\rho_{NMP}} \frac{\mu M(H)}{dH} \frac{dH}{dr} - \frac{18\mu}{\rho_{NMP} D^2} \cdot \ddot{r} - 3 \frac{\rho_{\text{fluid}}^{2/3} \mu^{1/3}}{\rho_{NMP} D^{5/3}} \cdot \dot{r}^{5/3} - kD^\alpha. \tag{27}
\]

The goal is to seek constants \(k\) and \(\alpha\) that are the same for both NMPs for which the simulation will predict the same final locations observed in the experiments. Notice that the other terms are left in the equation of motion to preserve the other fluid mechanics contributions to the dynamics. But in reality these will also change because the flow pattern is altered by the presence of the boundary so the formulation is incomplete from the onset. But at this point (27) can be considered an empirical fitting based on observed experiments.

Since we assume that the additional drag term is closely related to the bubble surface area in contact with the glass surface it is expected that \(\alpha\) is approximately equal to 2. Simulations were run with all other values remaining the same as in section 5.2 Simulation of the Bubble Experiment, and the current is again shut off at \(t=1\) s. The constants \(k\) and \(\alpha\) were adjusted for the small bubble by trial and error until simulations predicted its final location to a point within the confidence interval in Table 1. Then the final location of the large bubble was checked. What is found is that \(k=600\) and \(\alpha=1.95\)
predict final locations for both bubbles to within their confidence intervals. The results from the simulation producing this are shown in Figure 41 and Figure 42.

In Figure 41 the added term appears to dominate the dynamics of the 2.3 mm NMP because the position profile in time is linear. The 2.3 mm NMP never reaches velocities as great as the 17.9 mm NMP which results in the linear behavior for the smaller particle. Longer simulations show that when the 2.3 mm NMP attains greater velocities the nonlinear position profile is observed like that exhibited by the 17.9 mm NMP for the simulation shown. If the position profiles in time produced by this empirical fitting are accurate remains to be seen. Additional experimentation with equipment capable of generating time resolved location data for the NMPs could be used to ascertain this information.
Figure 41: Position of two NMPs with respect to time produced by simulation making use of a modified version of the equation of motion for an empirical fitting to the bubble experiment. The scaled Analytic force is used.

Figure 42: The final positions of the NMPs and how they separate from each other produced by simulation making use of a modified version of the equation of motion for an empirical fitting to the bubble experiment. The scaled Analytic force was used.
7.0 Conclusions

An analytic model for the dynamics of magnetic holes in a quasi two dimensional ferrofluid layer subjected to an externally applied, time uniform magnetic field has been developed. Field formulations were derived for the magnetic field on the mid plane between a coil pair and are consistent with published results. The model shows that, based on simulations employing numerical time integration, sized based separation of nonmagnetic particles, including cells is achievable. The separation is accomplished by a unique relationship between magnetic forces on nonmagnetic particles proportional to their volume and drag forces on the nonmagnetic particles proportional to their projected area. The magnetic forces arise as a result of magnetic body forces on the ferrofluid in the presence of a magnetic field gradient. These body forces give rise to a force similar to buoyancy on the nonmagnetic particle even though it is insensitive to the magnetic field. The drag forces result from the particles motion through the ferrofluid.

Experimentation in the millimeter scale confirms these findings. The experiments reveal that a larger bubble always travels further than a smaller bubble under the constraints that constructs the physical problem. The model and the experiment can be scaled down to the micro and nano scale for separating particles as small as 100nm based on the construction of the model and as per the literature.

Sized based particle separation using ferrofluids controlled by magnetic fields offers several advantages over conventional techniques. The time uniformity of the magnetic field reduces the complexity of the model which is in stark contrast to other ferrofluid manipulation methods. Because of the simplicity of the equipment and the speed of the separations, which are on the order of seconds, the technique presented has
strong potential to be transformed to a point-of-care diagnostic device in the foreseeable future. Such a device would respond to increasing healthcare demands and facilitate expedient biological processing allowing medical professionals to respond to patients needs quickly increasing their chances of recovery.
8.0 Recommendations

For cases where particle interaction with container boundaries is negligible the analytic model takes all significant forces contributing to NMP dynamics into account. Furthermore, the physical construction of a device or experiments that fall into this category is in the realm of possibility. However, model modification to account for the additional interfacial forces observed in the bubble separation experiments is in its infancy. The empirical fitting should be developed further and based off a large number of experiments using a range of bubble sizes and a number of different configurations. Alternatively, and perhaps a superior approach, a step back can be taken and the analytic model describing the bubble experiments can be constructed solely on theory as was the model for the case of no boundary interactions.

The theory established in this thesis lays the ground work for down scaling experiments for particle sizes comparable to that of cells and other biological entities for which the technique is applicable. Micro scale simulations show that scaling down the model also scales down the time scale of the separation to the order of milliseconds. Additionally, separation becomes easier in one respect with a small scale model. As particle size decreases, Reynolds number decreases along with velocity. This means that the second term in (24) becomes negligible resulting in greater separation. The analytic model needs little or no modification for this scaling. However, particle-particle interactions, such as collisions, have been ignored. This should be taken into account in future studies. Experiments in the micrometer scale with a large number of heterogeneous different sized particles supporting the findings here are the next logical step towards realizing a point-of-care diagnostic device. Care should be taken to avoid
the difficulties associated with the extra forces arising in the bubble separation experiments so a direct verification of the analytic model can be made. Small scale fabrication is the only obstacle in taking this step which is a hurdle that is easily overcome.

The incremental current ramping should also be exploited as discussed in section 3.5 Effect of Current. This has the potential to separate a large range of particle sizes without the need for several designs of different sizes. Also, temporally varying current control should be investigated. Switching the current on and off repeatedly may excite the larger particles while leaving the smaller particles relatively undisturbed. If this is successful, the switching frequency’s effect on particle motion should also be investigated.

Another promising opportunity to pursue would be the implementation of closed loop feedback control. The progression of the separation could be controlled based on position information fed to a controller. Incorporation of a closed loop feedback control system would reduce the reliance on a complicated analytic model and all forces acting on the particles during separation would not have to be anticipated before hand and calculated into the model for separation to work. Disturbances and uncertainties can be accounted for by the controller making way for a reliable separation device even with the inevitable unexpected factors that will contribute to particle dynamics.
9.0 References


