INVENTORY CONTROL ISSUES IN A DISASSEMBLY LINE

A Thesis Presented

By

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To
The Department of Mechanical and Industrial Engineering

In partial fulfillment of the requirements for the degree of

Doctor of Philosophy

In the field of
Industrial Engineering

Northeastern University
Boston, Massachusetts

August 2010
Dedicated to my family.
Abstract

Recent years have witnessed Reverse Supply Chain (RSC) become the center of attention for researchers and Original Equipment Manufacturers (OEMs). The fast depletion of virgin resources and the rapid increase of product returns from customers to original manufacturers for maintenance, repair or to be disposed of can be said to be one of the main reasons behind this interest. Electronics manufacturers have introduced state-of-the-art technologies in quick succession. As a result of this, the end-of-life (EOL) products returned by customers have grown significantly in volume. But often these EOL products are found to be in excellent working conditions (functional). Customers return them because of the various marketing programs or favorable incentives offered by service providers or manufacturers that create a “must have” sense in the minds of customers to acquire upgraded products. In the past two decades environmental concerns have influenced production processes, as environmental regulations have targeted pollution from industry. However, there is growing awareness that this may not be sufficient and it is increasingly recognized that the use and disposal phases, as well as the production phase of the product life cycle, are important. Environmentalists have always demanded that the manufacturing companies should take these products back and manage them in
an environmentally conscious manner. End-of-Life (EOL) products can be remanufactured, reused, recycled, or disposed of. However, manufacturers have not invested or engaged themselves in such initiatives because of the uncertainty associated with the process.

Many corporations have understood the economic and environmental benefits of minimizing the use of virgin resources. Also, due to evolving environmental legislation, they have started to comprehend the importance of the recovery process and are taking serious steps in restructuring their supply chain processes to meet the new regulations. The idea behind the change is to use materials and parts more than once before they are finally discarded. Thus, effective supply chain management is vital in gaining a competitive edge over other corporations. The take-back process is clearly more environment friendly than the traditional forward supply chain process as it “closes the loop” of the supply chain process and transforms the EOL products into new serviceable products. This new portion of the supply chain is known as Reverse Supply Chain. It has been found that the original supplier is in the best position to control the return process. The reverse supply chain logistics model operates independently of the forward supply chain that delivered the original product.

In comparison to regular supply chain management, the management of reverse supply chain is more challenging because it is much more reactive and less visible. A major challenge is inventory control and value management of EOL products. Due to the disparity between demand for parts and materials and their line yields, the decision maker
faces economical as well as physical constraints when trying to take a decision on how many products to take back and when to take them back, and when to keep them and for how long before they are finally disposed of.

The objective of this research is to develop inventory control policies of on hand inventory (OHI) of returned products and disassembled parts in such a way that total variable costs of the system is minimized and the profit is maximized, and also to provide management tools that can help improve the overall performance of the disassembly line. These inventory management and planning questions have not been fully addressed in the literature.

In this research, two types of approaches have been presented and tested to model the inventory control problem in context of disassembly. The first type of approaches is deterministic, while the second type of approaches is stochastic. The performances of the two approaches have been tested given the uncertainty of demand, return and product on the total solution cost. The results show that stochastic models outperform the deterministic model when uncertainty levels are high, and provide a better solution.

The various techniques presented in this dissertation provides knowledge that helps in understanding the challenges and opportunities associated with inventory control in a disassembly line context and form a body of knowledge that helps in addressing the inventory problems associated with a disassembly line.
Acknowledgements

First and foremost, I would like to express my sincere gratitude to my advisor and long time teacher during my years at Northeastern Professor Surendra M. Gupta. I knew Professor Gupta long before I started my graduate program in Northeastern University. He was a mentor and like a father to me. He was the first one to motivate me to join the PhD program. During the PhD process, he always encouraged me to express my ideas. He showed me how to research a problem and achieve goals. He spent endless time reviewing and proofreading my papers, asking me questions to let me think harder, and supported me during the difficult times in my research. He believed in me when I doubted myself. Without his encouragements, continuous guidance and insight, especially during my leave of absence from the University, I could not have finished this dissertation. Our weekly progress emails were our means of communication and way to keep in touch that gave me the motivation to work harder and make the best effort possible. His effort and patience will never be forgotten.

Besides professor Gupta, special thanks go to the members of my thesis committee, Dr. Sagar V. Kamarthi and Dr. Seamus M. McGovern for their support and encouragement.
Special thanks also go to my friends and colleagues at the Laboratory for Responsible Manufacturing (LRM) for their support and help during my PhD study. They made the lab like a home for us. I have enjoyed every minute of being on campus and in the lab. Special thanks go to Gun, Prasit, Satish, and Srikanth. A deep appreciation goes to my friends Amre, Onder, and Ali. You guys were great.

My deepest gratitude goes to my parents for their understanding and support during my academic years overseas for over a decade. Special thanks and my sincere appreciation goes to my wife, Jumana, for believing in me and pushing me to get back to work when I was frustrated, for her support during my long hours of work, and for understanding why I skipped breakfast with her on Saturdays.

Last but not least, special thanks go to the Graduate School of Engineering staff for their support and help in processing my paperwork and solving my problems. Special thanks go to the International Students and Scholars Institute (ISSI) and Mr. Sal Mazzone for his help and understanding in keeping track of my file, and his quick responses and solutions to my last minute problems. They made my experience in Boston and Northeastern very unique and enriching.
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LIST OF SYMBOLS

$\gamma_j$ The probability of part $i$ being disassembled

$1 - \gamma_j$ The probability of part $i$ being recycled or disposed of

$Q_{\text{dis}}^j$ Quantity disassembled of part type $j$ in a given period of time

$Q_{\text{rec}}^j$ Quantity recycled of part type $j$ in a given period of time

$Q_{\text{ret}}^i$ Quantity returned of core type $i$ in a given period of time

$W$ Space available at the workstation line expressed in cubic units designated

$w$ Space consumed by disassembled part at the workstation along the disassembly line

$INV^\text{max}_j$ Maximum inventory of part type $j$ that can be carried on the workstation

$\delta_j$ The total mass (or volume) of recyclable parts of type $j$

$I$ Set of core products that are eligible for disassembly

$J$ Set of parts and/or subassemblies disassembled

$K$ Set of quality levels of the end-of-life (EOL) product

$T$ Time period allowed for the disassembly operation

$t$ The time period of stage
$R$ Set of liquidations options available

$E(x)$ The expected value of $x$

$C^p_{i,t}$ Cost to collect, and transport one unit of core type $i$, in period $t$, from customer to origin

$C^{dc}_{i,k,t}$ Cost to disassemble one unit of core type $i$, with quality level $k$, in period $t$.

$C^{rec}_{i,k,t}$ Cost to recycle one unit of core type $i$, with quality level $k$, in period $t$.

$C^{holc}_{i,k,t}$ Cost to hold one unit of core type $i$, with quality level $k$, in period $t$. ($/\text{Unit/Time}$)

$C^{disc}_{i,k,t}$ Cost to dispose one unit of core type $i$, with quality level $k$, in period $t$.

$C^{diss}_{j,t}$ Cost to dispose one unit of subassembly $j$, in period $t$.

$C^{diss}_{j,k,t}$ Cost to dispose one unit of subassembly type $j$, with quality level $k$, in period $t$.

$C^{rec}_{j,t}$ Cost to recycle one unit of subassembly $j$, in period $t$.

$C^{hols}_{j,t}$ Cost to hold one unit of subassembly $j$, in period $t$. ($/\text{Unit/Time}$)

$C^{hols}_{j,k,t}$ Cost to hold one unit of subassembly $j$, with quality level $k$, in period $t$. ($/\text{Unit/Time}$)

$C^{pur}_{j,t}$ Cost of purchasing a new subassembly $j$ from an outside supplier in period $t$.

$C^{trac}_{i,k,t}$ Cost of transaction associated with selling one unit of core type $i$, with quality level $k$, in period $t$.
\[ C_{j,k,t}^{\text{tras}} \] Cost of transaction associated with selling one unit of subassembly/part type \( j \), with quality level \( k \), in period \( t \)

\[ Q_{i,k,t}^{\text{req}} \] Quantity returned of core type \( i \), with quality level \( k \), in period \( t \).

\[ T_{i,k,t} \] Tim to disassemble one unit of core type \( i \), with quality level \( k \), in period \( t \)

\[ AT_t \] Time allowed for disassembly operations in period \( t \)

\[ D_{j,t} \] Demand of subassembly type \( j \) in period \( t \)

\[ WC_{i,t} \] Inventory space availability of core type \( i \) in period \( t \) expressed in cubic units

\[ WS_{j,t} \] Bin space availability of subassembly type \( j \) in period \( t \) expressed in cubic units

\[ w_i \] Amount of space consumed by one unit of core type \( i \)

\[ w_j \] Amount of space consumed by one unit of subassembly type \( j \)

\[ CAP_{dc,i,t} \] Facility capacity along the workstation of returned cores in period \( t \)

\[ CAP_{d,i,t} \] Facility capacity along the workstation of disassembled parts in period \( t \)

\[ CAP_{i,sgr} \] Storage facility capacity of core products type \( i \)

\[ CAP_{j,sgr} \] Storage facility capacity of subassembly/part type \( j \)

\[ CAP_{d,dis,i,t} \] Disposal capacity in period \( t \)

\[ CAP_{i,rec} \] Recycling capacity in period \( t \)
\( \text{CAP}_n^{\text{dem}} \) Demand source \( n \) capacity in terms of cubic units

\( \psi_i \) Mass to unit conversion of core type \( i \)

\( \psi_j \) Mass to unit conversion of subassembly type \( j \)

\( \lambda_R \) Mean rate of the return per unit time of end-of-life (EOL) product Poisson distribution

\( \lambda_D \) Mean rate of the demand per unit time of the disassembled part Poisson distribution

\( \lambda_{r,k} \) Value of a recoverable inventory core product with quality level \( k \) ($/item)

\( \lambda_{s,k} \) Value of a serviceable inventory disassembled part with quality level \( k \) ($/item)

\( \lambda_n \) Value of a new part purchased from an outside supplier ($/item)

\( P_{i,k,t}^c \) Price of core product type \( i \), with quality level \( k \), in period \( t \)

\( P_{j,k,t}^s \) Price of part/subassembly type \( j \), with quality level \( k \), in period \( t \)

\( Q_{i,k,t}^{\text{retc}} \) Quantity of core type \( i \), with quality level \( k \), returned in period \( t \)

\( Q_{i,k,t}^{\text{dis}} \) Quantity of core type \( i \), with quality level \( k \), disassembled in period \( t \)

\( Q_{i,k,t}^{\text{disc}} \) Quantity of core type \( i \), with quality level \( k \), disposed of in period \( t \)

\( Q_{i,k,t}^{\text{rec}} \) Quantity of core type \( i \), with quality level \( k \), recycled in period \( t \)

\( Q_{i,k,t}^{\text{inv}} \) Quantity of core type \( i \), with quality level \( k \), remain at end of period \( t \)
$Q_{j,t}^{\text{diss}}$ Quantity of subassembly type $j$, with quality level $k$, disposed of in period $t$

$Q_{j,t}^{\text{recs}}$ Quantity of subassembly type $j$, with quality level $k$, recycled in period $t$

$Q_{j,t}^{\text{invs}}$ Quantity of subassembly type $j$, with quality level $k$, remain at end of period $t$

$Q_{i,k,t}^{\text{remc}}$ Quantity of core type $i$ beyond demand, with quality level $k$, remain at end of period $t$

$Q_{i,k,t}^{\text{remx}}$ Quantity of parts type $j$ beyond demand, with quality level $k$, remain at end of period $t$

$Q_{i,k,t}^{\text{liqc}}$ Quantity of core product type $i$, with quality level $k$, liquidated in period $t$

$Q_{j,k,t}^{\text{liqs}}$ Quantity of subassembly/part type $j$, with quality level $k$, liquidated in period $t$

$Q_{j,t}^{\text{pur}}$ Quantity of subassembly type $j$, purchased in period $t$

$R_{i,k,t}^c$ Set of liquidation options for core product type $i$, with quality level $k$, in period $t$

$R_{j,k,t}^s$ Set of liquidation options for subassembly type $j$, with quality level $k$, in period $t$

$V_{i,t}(x,r/k)$ Net profit generated from the sale of inventory of core type $i$, in period $t$, through hold or option $r$ liquidation given it is a quality level $k$

$V_{j,t}(x,r/k)$ Net profit generated from the sale of inventory of subassembly type $j$, in period $t$, through hold or option $r$ liquidation given it is a quality level $k$

$PR_{i,t}^c(ql)$ Most expected total attainable profit from the sale of core type $i$, with quality $ql$, in period $t$
\( PR^t_{j,t}(q2) \) Most expected total attainable profit from the sale of subassembly type \( j \), with quality \( q2 \), in period \( t \)

\( N \) Number of stages, or time periods in the planning horizon

\( n \) Label for current stage

\( h_c \) Out-of-pocket holding cost per core product per unit time

\( h_s \) Out-of-pocket holding cost per disassembled part/subassembly per unit time

\( \phi \) Inventory carrying charge per period

\( S_n \) Current state for stage \( n \), or the inventory available for allocation

\( S \) Current system state or the disassembled part inventory type

\( X \) End of life cycle decision, where it can be hold inventory (\( X = 0 \)), or liquidate inventory (\( X = 1 \))

\( X_n \) Decision variable for stage \( n \)

\( X^*_n \) Optimal value for \( X_n \) given \( S_n \)

\( f_n(S_n, X_n) \) The best total expected profit generated of the best overall policy for the remaining stages, given that inventory is at level \( S_n \), ready to start stage \( n \), and select \( X_n \) as immediate decision

\( F \) Threshold (safety stock) level in F-Policy problem
\( W \) \hspace{0.5cm} \text{Expected waiting time in the system in F-Policy Problem}

\( L \) \hspace{0.5cm} \text{Expected number of disassembled parts in the system in F-Policy Problem}

\( K \) \hspace{0.5cm} \text{Queue capacity in the F-Policy Problem}

\( p_{io} \) \hspace{0.5cm} \text{Probability that there is } i \text{ parts in the system and they are not allowed into the queue}

\( p_{ii} \) \hspace{0.5cm} \text{Probability that there is } i \text{ parts in the system and they are allowed into the queue}
Chapter 1

Introduction

This chapter provides a brief introduction to the dissertation and its contribution to the field of supply chain management. In Section 1.1, an overview of inventory management in disassembly context is presented. Section 1.2 presents the motivation behind this research. Section 1.3 provides the scope of this study and its contribution to the body of knowledge. And finally, in Section 1.4 the research outline followed in the dissertation is presented.

1.1 Overview

Recent years have witnessed a decrease in life cycle of products. Among others, electronic goods have witnessed a drastic reduction in their life span. In parallel, there has also been a discernible increase in the number of product returns. It is not uncommon for example, to locate take–back offers in print and visual media. How have companies responded to these trends? Significantly enough, they have sensed a business opportunity
having realized that there are economic benefits that can be reaped from these trends. Consequently, they have taken serious steps to modify their supply chain process in such a manner that they can integrate their current supply chain management (SCM) practices with these trends in order to be able to conduct business in professional and profitable manner. In Europe and Japan, government has set rules and regulation on proper handling of end-of-life (EOL) products at the end of its life cycle in order to reduce its harm on environment. Legislative examples include Japan’s law encompassing TV sets, refrigerators, air conditioners and washing machines and European Parliament’s end-of-life (EOL) vehicles directive and waste of electrical and electronic equipment (WEEE) directive. Strikingly, the stringent obligations outlined in the WEEE directive are binding not only for the European manufacturers but also for others that wish to sell their products within the European Union (EU) member states (Altekin et al, 2008). It has been observed that every year end-of-life vehicles generate between 8 and 9 million tons of waste in Europe which should be managed correctly. In 1997, the European Commission adopted a ‘Proposal for a Directive’ which aimed at making vehicle dismantling and recycling more environmentally friendly, set clear quantified targets for reuse, recycling and recovery of vehicles and their components and pushed producers to manufacture new vehicles also with a view to their recyclability (ec.europa.eu, 2009).

The end-of-life vehicle legislation enforced in Europe and Japan has certain aims in view (netregs.gov.uk, 2009). The principle aims may be stated as follows:
• Influence the design of vehicles so that they, and their components, are easier to recycle.

• Reduce the use of certain hazardous materials in car and van manufacturing, making it easier and safer to dispose of the materials in end-of-life (EOL) vehicles.

• Standardize treatment requirements and ensure that treatment facilities hold a permit and have equipment to prevent pollution.

• Increase the proportion of each vehicle that is recycled and reduce the quantity going to landfill.

Hence, there has been a collective effort between the government and corporations and this has been passed on to consumers to increase public awareness and to increase implementation of take-back programs (TBP). These programs prohibit consumers from disposing their products and allow Original Equipment Manufacturers (OEMs) to collect their products and use their parts and materials more than once before they are finally discarded.

Re-use of parts and materials from end-of-life (EOL) products have become an important alternative due to the fast depletion of virgin resources and the diminishing availability of landfill space. Usually, retrieved items are cheaper and environmentally friendly when processed to be used in a new or refurbished product. In view of the destruction of non-renewable natural resources, industries are becoming environmentally conscious and are restructuring their operations to achieve their goals while abiding by
the new rules and regulations. Reverse Supply Chain (RSC) is the concept evolved to implement such system.

By definition, reverse supply chain (RSC) involves all activities of retrieving a product at the end of its life cycle from customers all the way back to its origin. It includes collection, transportation, warehousing, disassembly operations, and redistribution to potential demand sources. In brief, there are five key components to the reverse supply chain (Guide and Wassenhove, 2009). These are as follows:

1. Product Acquisition: The used product must be retrieved.

2. Reverse Logistics: Once collected, used products are transported to some sort of facility for inspection.

3. Inspection and Disposition: The returned products are tested, sorted, and graded. Diagnostic tests may be performed to determine a proper action that recovers the most value from the returned product. If a product is new, it may be returned to the forward supply chain. Others may be eligible for some form or reconditioning while others may be sold for scrap or recycling.

4. Reconditioning: Some products may be reconditioned or completely remanufactured. Most people have seen products labeled *factory reconditioned* which implies it is used but like new and may have a warranty. Some products may have parts that can be extracted for reuse or as spare parts. Others go for salvage or recycling. Figure 1.1 is a representation of these options.
5. Distribution and sales: Reconditioned or remanufactured products may be sold in secondary markets where customers are unwilling to purchase a new product. In other instances the firm may need to create a new market if demand is not currently present. Of course, there are distribution needs in getting the product to the secondary market.

The appropriate handling of core products and disassembled parts is the main focus of this research. The right implementation of inventory control techniques could help minimizing the total system cost and increase the efficiency and productivity of the disassembly line.

![Figure 1.1 3 R’s option in product recovery](image)
1.2 Motivation

The ever increasing market competition and the successive introduction of new technologies in the market place, along with government regulations, have motivated and sometimes forced companies to be involved and take responsibility of handling end-of-life (EOL) products. One of the main concerns in handling an overwhelming number of returns and disassembled parts is appropriate storage and disposal of these items. Put differently, one of the most important challenges in reverse logistics is inventory management. Inventory management in reverse logistics is far more complex than traditional inventory control systems. Some of the reasons are uncertainty in demand and return, wide range of quality of products, and strict rules and regulations set by government.

A number of forces seem to be influencing the rapid increase in need for reverse logistics activities (enotes.com, 2009). These include:

- The green forces such as legislation and consumer awareness and concern. Frequently due to legislation, the original equipment manufacturer (OEM) is now responsible for final disposal of the product. The increasing value of return products increases the need for safe return from the field.

- Increased number of customer goods returned for credit as a result of increased demand for customer service and satisfaction. Large retail chains usually have an agreement with suppliers allowing them to return goods. While originally intended to cover failed products, it has expanded to cover perfect goods that simply have not
sold. From the consumer perspective, the buyer may return a good simply because they have decided not to keep it.

- Shortened product life cycles. As products become obsolete more quickly, the possibility of and potential for returns increases.
- The drive to reduce costs. Firms are striving to reuse potentially good items through reuse, recycling or secondary usage. For example, Ford Motor Company has a program for recycling plastic bumpers into tail light housings.
- Increase in e-commerce sales. The massive increase of sales made via the Internet is conducive to increased returns as consumers buy merchandise "sight unseen" only to be disillusioned or dissatisfied with their purchase.
- Increased demand for repairs, re-manufacturing, upgrades, or re-calibration.
- Potentially valuable products that are no longer viewed as such by the current user. Consumers may purchase a new TV or washer/dryer even though the one they own still has a useful life.
- Increased use of returnable or reusable containers.
- Warranty returns. For many items with warranties, the good is first returned and then its disposition determined.
- Rental returns. The proliferation of rental businesses ensures the return of used but still valuable furniture, appliances, and electronics.
- Product recalls. Products may be recalled by the manufacturer due to potential failure in the field or safety concerns.
In Europe and Japan, a highly developed infrastructure for take-back programs (TBP) already exists. These programs have been implemented by enforcing certain regulations. Some regulations prevalent in Europe (netregs.gov.uk, 2009) are outlined below.

- **End-of-Life Vehicles (Producer Responsibility) Regulations 2005 SI 263**
  
  Requires producers to register responsibility for vehicles placed on the market and apply for approval of their vehicle collection system. It introduces reuse, recovery and recycling targets for end-of-life (EOL) vehicles treated at authorized treatment facilities.

- **End-of-Life Vehicles Regulations 2003, SI 2635**
  
  It requires producers of vehicles to set up collection, treatment and disposal systems to make sure that components in vehicles can be recovered, reused and recycled at the end of their life.

  However the US still lacks that developed system since the regulations are not widely applied to all the states (epa.gov, 2009). Of products sold between 1980 and 2007, approximately 235 million units had accumulated in storage as of 2007. Also, out of the 2.25 million tons of TVs, cell phones and computer products ready for end-of-life (EOL) management, 18% (414,000 tons) was collected for recycling and 82% (1.84 million tons) was disposed of, primarily in landfills. From 1999 through 2005, recycling rate was relatively constant at about 15%. During these years, the amount of electronics recycled increased but the percentage did not because the amount of electronics sent for end of life management increased each year as well. For 2006-2007, the recycling rate increased to
18%, possibly because several states have started mandatory collection and recycling programs for electronics. But some states are still suffering from continuous disposal of hazardous materials and others are yet to face that problem in the long run. Table 1.1 and Table 1.2 show the classification of the end-of-life (EOL) products in storage as of year 2007, and it breaks down the percentages of products that has been recycled and disposed.

<table>
<thead>
<tr>
<th>Product Type</th>
<th>Number (million units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desktop computer</td>
<td>65.7</td>
</tr>
<tr>
<td>Computer monitors</td>
<td>42.4</td>
</tr>
<tr>
<td>Computers (notebooks)</td>
<td>2.1</td>
</tr>
<tr>
<td>Televisions</td>
<td>99.1</td>
</tr>
<tr>
<td>Hard copy peripherals</td>
<td>25.2</td>
</tr>
<tr>
<td>Total</td>
<td>234.6</td>
</tr>
</tbody>
</table>
Table 1.2 Recycling rates for different products

<table>
<thead>
<tr>
<th></th>
<th>Generated (million units)</th>
<th>Disposed (million units)</th>
<th>Recycled (million units)</th>
<th>Recycling Rate (by weight)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Televisions</td>
<td>26.9</td>
<td>20.6</td>
<td>6.3</td>
<td>18%</td>
</tr>
<tr>
<td>Computer Products*</td>
<td>205.5</td>
<td>157.3</td>
<td>48.2</td>
<td>18%</td>
</tr>
<tr>
<td>Cell Phones</td>
<td>140.3</td>
<td>126.3</td>
<td>14.0</td>
<td>10%</td>
</tr>
</tbody>
</table>

* include CPUs, monitors, notebooks, keyboards, mice, and hard copy peripherals.

In the US, product recovery process is profit driven, with emphasis on the material recovery (Gunter, 2004). However, some products are being remanufactured and parts are being reused as spare parts for existing products in the market or as refurbished parts in the manufacturing process of new products. The remanufacturing business is widely common in electronics and automobile industry. Companies involved in the remanufacturing process are rarely involved in the disassembly operations. Disassembly facilities usually play the role of "middle agent" as supplier of disassembled parts to manufacturing and remanufacturing companies.

This creates a business opportunity for standalone disassembly facilities to supply used parts to OEMs and remanufacturing companies as per their demand. However, methods and techniques should be established to ensure success and profitability of these companies. Thus, companies need to utilize the best processes in order to ensure that the efforts made do not cut too deeply into the bottom line. Yet, many reasons discourage OEMs from getting involved in this effort. Companies are becoming less vertically
integrated and focusing more on the core business, limited efficiency of the recovery process, and the uncertainty associated with its operations (Gunter, 2004).

An additional important issue in recent years has been the shift in recovery philosophy. Companies have realized the benefits of value recovery compared to material recovery. Due to that, companies have focused on recovering parts and modules that could be repaired and reconditioned to be used in new products. Using disassembled parts contribute to big saving in overall production costs. However, disassembling too many products usually results in excess inventory of parts that is beyond the demand. Thus, several questions have been raised when it comes to using inventory control methodologies in reverse logistics. Previous research in the literature has focused on hyper systems, where manufacturing and remanufacturing is considered as one system and where the optimal number of remanufactured products and parts is the challenge. However, in this case the main concern is when disassembly facility has no control over the items brought back. Most of the time, planners have to take decisions on the spot about its on hand inventory due to certain take-back program (TBP) standards. This gives rise to many questions of which mention may be made of the following.

a) How do we balance the inventory in all workstations?

b) What inventory control technique should be applied to control the system?

c) What performance measures should be considered?

d) How to incorporate the uncertainty in demand, returns, and quality to the model?

Such challenges motivate us to develop an inventory control policies, based on traditional inventory control models, which do not only help to improve the
1.3 Research Scope and Contribution

Reverse Logistics (RL) or Reverse Supply Chain (RSC) is a broad topic that covers many areas. These areas include handling of end-of-life (EOL) products to satisfaction of customers using refurbished parts and products. Owing to the complexity of the issue, the main focus of the research is inventory management of EOL products in a disassembly line context. In the first model, demand for disassembled parts and components is deterministic. Supply of end-of-life (EOL) products is known and given in advance. The quality of items is given after the product is sorted and tested and before it is sent to disassembly facility. As a result, the quality and quantity of each returned item type is known before disassembly operation begins. Prices of items are given which reflect the demand sources and the capacity levels of disassembly facilities have also be taken into account. The standard for disposal is also incorporated in the model due to government regulations. The model simulates the need of different demand sources and the on hand inventory that is being carried from period to period and also investigates different liquidation channels to sell it.

In the second model, stochastic techniques are applied. The demand and supply of end-of-life (EOL) products are probabilistic in terms of quality and quantity of products returned or disassembled. Also, prices of the items depend on the quality level. We
assume that the system has to keep inventory in terms of full totes. Partial tote is automatically liquidated. The remaining on hand inventory is carried to the next period.

In the third model, optimization through simulation modeling is used to investigate the inventory problem in a disassembly line. The supply of end-of-life (EOL) products and the demand of disassembled parts are given as levels. The rates of disassembly, inspection, and demand are considered and the relationships between the elements of the system are studied to simulate the long-term behavior of the system when sudden changes occur. A system dynamic approach is used to model the problem.

In the fourth model, the queueing and its application in inventory management is applied to the inventory of end-of-life (EOL) products in a disassembly line. The model assumes stochastic nature of the problem and the attempt is made to control the arrival of disassembled parts to the designated inventory area along the disassembly line. The model attempts to find the optimum queue size of certain disassembled parts and the optimum safety stock level that minimize the system total cost.

The main contribution of this research is in the area of Inventory control in a disassembly line context. Early research has considered the inventory control problem in a manufacturing/remanufacturing setting, also known as hybrid systems. However, this research focuses on an area that has got little attention in the literature, where standalone disassembly facilities have been considered, and face an overwhelming number of items being returned, disassembled, and possibly stored to satisfy future demand.
1.4 Research Outline

The interest of the research has been classified into four categories. These areas are
interrelated, but not necessarily dependent on each other. A result from one area has been
carried to the next in some cases in order to test the validity of the model.

In the first category, the inventory fluctuations and uncertainty in demand and
supply has been studied. The main purpose behind this is to observe the behavior of the
disassembled parts and subassemblies and its accumulation and also to observe the effect
on the overall performance of the disassembly line. Also, the challenges faced by the
planner which are different from those found in the traditional forward supply chain are
highlighted.

In the second category of the study, the financial constraints are considered. For
this, a break down the different costs and pricing techniques associated with the
disassembled parts and components are provided, such as holding and carrying cost,
transportation cost, and costs associated with acquiring end-of-life (EOL) products from
the market. Also, physical constraints are considered such as space and line capacity of
the disassembly facility and how it relates to the cost assigning process.

In the third category, deterministic and stochastic models are developed and the
findings of previous categories are incorporated to test the validity of the models.
Optimization modeling and spreadsheet analysis techniques are used to derive
conclusions.
Chapter 2

Literature Review

This chapter discusses the earlier researches done in the area of inventory control and reverse logistics. The most relevant research areas include: environmentally conscious manufacturing, closed-loop supply chain, disassembly and disassembly line, inventory management in reverse logistics, and inventory management in disassembly context. This information is crucial in order to understand and develop the work presented in this dissertation. Additional references are mentioned in each chapter when appropriate. The literature review a structure to first provide background information about the area of reverse logistics and disassembly, and then cover detailed review of the current related literature about the research interest, inventory and value management of end-of-life (EOL) products.
2.1 Environmentally Conscious Manufacturing (ECM)

Product recovery has been the center of attention of many original equipment manufacturers (OEMs), researchers, and government entities. This effort has been driven by increasing environmental deterioration, diminishing virgin resources, the overflow of waste sites, and increasing levels of pollution (Srivastava, 2007). National governments, international conferences, some industry associations, and a number of individual companies are increasingly emphasizing the need for a company to take responsibility both for the environmental consequences of its production and for the ultimate disposal of the products it produces. As a result, companies are increasingly challenged – by competitors, government regulations, public pressure, or good conscience – to improve and manage their processes and products from both an environmental and an economical standpoint. To be effective, these efforts, to the extent possible, need to address the energy uses and material flows related to the entire cycle of production, consumption and ultimate disposition (Deanna, 1994). Consequently, companies started to take serious steps to restructure their operations to develop and design more environmentally friendly products that generate less harmful impact on the environment. The increase in returns of end-of-life (EOL) products and their subsequent disposal have raised the issue of hazardous materials and their impact on the environment. Because of this, the area of Environmentally Conscious Manufacturing and Product Recovery (ECMPRO) has become a subject of interest in many levels (governments, OEMs, and consumers). Environmentally Conscious Manufacturing (ECM) is an emerging discipline that is
CHAPTER 2. LITERATURE REVIEW

concerned with developing methods for manufacturing new products from conceptual
design to final delivery and ultimately to product recovery and, as a last resort, end-of-life
(EOL) disposal such that all the environmental standards and requirements of managing
products are satisfied (Gupta, 2004).

Environmentally Conscious Manufacturing can be subdivided into two main
subtopics: green design, and green operations. Green design is an important part of
integrating design with manufacturing operations. Fiksel (Fiksel, 1996) has discussed the
design of the product by taking into account environmental considerations and by
combining green design, environmental safety, and health over the product’s life cycle
from production to disposal. An earlier paper (Moyer and Gupta, 1997) discussed the
effects of electronics assembly, disassembly and disposal on the environment and the
threats of continuing such improper trends and harmful practices of disposal. A later
paper (Gungor and Gupta, 1999) provided a detailed survey review of the issues
associated with disassembly and reverse logistics and the challenges associated with it. In
2009, Ilgin and Gupta provided a state-of-the-art comprehensive survey in the issues of
ECMPRO that covered all areas in the 1999 survey in addition to latest research on new
emerged areas. Green operations include the different processes and operations that are
designed in such a way so as to minimize the waste and hazardous materials remitted
during the assembly and disassembly process of products.

A study indicates the use of several “green indicators” to measure the implications
of the product design for the environment (Navin-Chandra, 1991). Some of these
indicators include percent of material recycled, and recycling potential. Other researchers
aimed to develop an understanding of how product design decisions could impact the environment compatibility (Glantsching, 1994). Georgiadis and Valchos (Georgiadis and Valchos, 2003) examined the impact of environmental issues on long-term behavior of a single product supply chain with product recovery. Issues examined are firm’s “green image” effect of customer demand, take back obligation, and state campaigns for proper disposal of used products.

Many researchers have considered product designs under certain legislation and regulations (Fleischmann et al., 2001), and (Das, 2002). There is sufficient literature available on design for remanufacturing and recycling (Henshaw, 1994), (Bellmann and Khare, 2000), and (Guide et al., 1999, 2000).

Green operations, also known as green manufacturing/remanufacturing is a key element in the study of current literature on environmentally conscious manufacturing. Such operations, in addition to others, are referred to in industry as product recovery. Product recovery refers to a broad set of activities designed to reclaim value from a product at the end of its useful life cycle (Srivastava, 2007). One of the key challenges that planners face in today’s industry is integrating remanufacturing operations with existing supply chain design (Savaskan et al., 2004). Two approaches are always considered when a decision is made to recover a product: recycling, or value recovery (Gunter, 2004). Recycling, mainly driven by economic and regulatory factors, is performed to retrieve material content of used and non-functioning products. Value recovery can be viewed as consisting of two distinct phases: disassembly and remanufacturing. Disassembly involves sorting, cleaning, testing, and parts separations.
Remanufacturing consists of repairing, refurbishing, reassembly, and repackaging. Not all products that are sent to disassembly facilities have to go through the two phases.

Ammons addressed the carpet recycling industry in the USA and the separation of nylon and disposal of the remainder (Ammons et al., 1999). Since recycling is economically driven, it finds application in automobile and electronics industry as well. Several researchers used goal programming techniques to examine the stream of materials and profitability of such system (Boon et al., 2000) and (Boon et al., 2003). Reimer et al. (Remier et al., 2000) modeled recycling of electronics from generators’, recyclers’, and materials processors’ points of view. Later, the model was extended by Sodhi and Remier (Sodhi and Remier, 2001) by presenting a mixed-integer linear programming model for recycling end of life electronics. Williams conducted intensive research in the area of electronics demanufacturing (Williams, 2006) which aimed at improving the process of disassembly and materials identification and separation.

The three major elements of value recovery are: repair, refurbish, and remanufacture. In addition, reuse is an option if the end-of-life (EOL) is in a working condition (Thierry et al., 1995). The purpose of repair is to bring the EOL products into a working condition, usually at a quality which is lower than the new products. The purpose of refurbishing is to bring the EOL product up to a certain “quality level”, while remanufacturing aims at returning a used product to “as new” condition. The following researches presented in the literature deals with repair and refurbishing of end-of-life (EOL) products: (Thierry et al., 1995), (Ferrer, 1997), (Dowlatshahi, 2000), (Guide, 2000), (Ferrer and Ayres, 2000), (King et al., 2004), (Brent and Steinhilper, 2005).
2.2 Reverse Supply Chain Management (RSCM)

2.2.1 Background

Many corporations have understood the economic and environmental benefits of minimizing the use of virgin resources. Also, due to strict environmental legislations, they have started to comprehend the importance of the recovery process and are taking serious steps in restructuring their supply chain processes to meet the new regulations. These new regulations include limitations on waste disposal and recycling requirements. The idea is to allow the use of materials and parts more than once before they are finally discarded. Remanufacturing has received increasing attention in the U.S. (Guide, 2000) because of its economic benefits, as well as regulatory and consumer demands for more environmentally friendly manufacturing and recycling. There are over 73,000 firms engaged in remanufacturing in the U.S., directly employing over 350,000 people (Lund, 1998). Remanufacturing provides the foundation for closed-loop supply chains and focuses on value-added recovery (Ketzenberg et al, 2003). One of the leading US recycling companies, Comprehensive Automotive Reclamation Services, has established its business based on a concept using a technology from the Netherlands (Gunter, 2004). In his dissertation, Gunter (Gunter, 2004) gives an intensive overview of some examples of the companies who are integrating operations to close the loop in the supply chain.
There are a number of important characteristics that needs to be managed, coordinated, and controlled if the reverse supply chain is to be economically viable (Blumberg, 2005). Some of these characteristics are as follows:

- **Uncertain flow of materials**—firms often do not know when a return item will arrive nor are they certain of its condition. The item may be *like new* or may require substantial repair or even disposal. Field service engineers often try a new part in a field failure, assuming the old part is bad. Subsequently, the old part is returned. When it turns out that the new part did not fix the problem, the old part is still returned as *bad*, thus creating a flow of mixed good and bad parts. Typically, 30 to 35 percent of high tech returns are perfectly good.

- **Customer diversity**—the return flow can be quite diverse and dependent upon the specific customer or end user. This may require considerable knowledge of specific customers and their use of the product.

- **Time**—from a cost or service perspective it may be desirable to return/repair/process an item as quickly as possible so that it may be quickly disposed of or reused.

- **Value improvement**—the firm will of course want to maximize the value of its return goods by transforming them into the state that will provide the most revenue or least cost.

- **Flexibility**—where demands fluctuate, the facility, transportation or other services may need to be flexible to support the firm's goals for the returned material.

- **Multiparty coordination**—since reverse logistics almost always involves multiple parties, an efficient and rapid real-time communication system or network is needed.
Effective supply chain management is vital in gaining a competitive edge over other corporations. As mentioned earlier, there are a number of important characteristics those need to be managed, coordinated, and controlled if the reverse supply chain is to be economically viable (Blumberg, 2005). By definition, supply chain management “involves functions such as production, purchasing, materials management, warehousing, inventory control, distribution, shipping, and transport logistics” (Jaber and Rosen, 2002). On the other hand, the definition of the Reverse Supply Chain (as adopted by the Council of Logistics Management) is “the process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in process-inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal”. In fact, the take-back process is more environment friendly than the traditional supply chain process as it “closes the loop” of the supply chain process and transforms the end-of-life (EOL) products into new serviceable products. These ideas have led to the evolution of the concept of Closed-Loop Supply Chain (CLSC) which brings together supply chain management with reverse supply chain management. For the purpose of this research, only the reverse supply chain is of interest. The topic is divided into three sub-categories: distribution planning, inventory management, and production planning.

2.2.2 Distribution Planning

Distribution planning in reverse logistics involves the design and implementation of a reverse network to physically transport end-of-life (EOL) products from end users back
to the point of origin via collection centers. The point of origin could represent the original equipment manufacturers (OEMs) or in some cases a third party. The reverse distribution can take place through a forward distribution channel, reverse distribution channel, or an integration of both (Fleischmann et al., 1997). Many researchers have treated the reverse network as an individual route. To date, very few researchers have considered an integrated forward-reverse distribution planning that operates simultaneously using simple approaches. In addition, no models in the literature study the combined routing problems. The current research is focused on the network design stage, but early research shows interest in designing a complete and integrated optimal concept model for a closed-loop supply chain for a manufacturing/remanufacturing facility that collects end-of-life (EOL) products in addition to manufacturing new products (Thierry et al., 1997). Thierry et al. (Thierry et al., 1997) used a linear programming model to determine the optimal cost of materials flow in the reverse network.

In the realm of distribution planning other researchers introduced an integrated plan for forward logistics with the disassembly facility to determine the number, locations, and capacities of these facilities, given that forward chain stays constant. Ammons et al. (Ammons et al., 1997) proposed a similar idea to the previous model in a carpet recycling industry in the US using a multi-level capacitated facility location Mixed Integer Linear Programming (MILP) model to address the problem. Other early research considered the different activities that are part of a distribution planning effort: collection, testing, sorting, transportation, and processing. These end-of-life (EOL) products are usually generated from multiple sources and being brought back to the product recovery
facility in a “convergent” type process (Krikke et al., 1998). Limited research in the literature discusses the inspection and sorting activities and measures the benefits if performed at collection points or at a centralized collection facility. The idea of redesigning existing logistics networks is capturing attention both in business and research so that a network can accommodate the reverse flow of used products and allow for remanufacturing/reuse activities (Tibben-Lembke, 2002). Daugherty et al. (Daugherty et al., 2005) discussed the importance of information technology that will lead to superior performance in reverse logistics activities. Ravi et al. (Ravi et al., 2005) used analytical network process (ANP) and the balanced score card concept for analyzing reverse logistics alternatives. Pokharel and Mutha (Pokharel and Mutha, 2009) found that that research and practice in Reverse Logistics are focused on all aspects of Reverse Logistics —from collection of used products, their processing and finally to the outputs of processing, namely, recycled materials, spare parts, remanufactured products and waste material disposal. Review of literature done by them revealed that mathematical modeling in Reverse Logistics research is mainly focused on deterministic methods and there are limited research papers considering stochastic demand for the remanufactured products and supply of used products by the customer. Also, it was found that the pricing models for acquiring used products are still developing. Dekker (Dekker, 2005) accounted for uncertainty in product recovery network design using a stochastic-programming approach. Using a linear physical programming Pochampally et al. (Pochampally et al., 2003) where able to program it so that it allows the decision maker to express his preferences for values of criteria (to compare the alternatives) in terms of ranges of
different degrees of desirability. As an extension for his research, Pochampally et al. (Pochampally et al., 2004) introduced a multi-criteria three phase approach to select collection centers as well as recovery facilities. In their work, based on previous work done by Pochampally, Nukala and Gupta (Nukala and Gupta, 2005) employed a fuzzy approach in selecting potential recovery facilities using a multi-criteria analysis.

### 2.2.3 Inventory Management

Inventory management is a key element in the reverse logistics field. Inventory management involves the control of assets being produced for the purposes of sale in the normal course of the company's operations. The goal of effective inventory management is to minimize the total costs - direct and indirect - that are associated with holding inventories. However, the importance of inventory management to the company depends upon the extent of investment in inventory. Appropriate management of the return flow of end-of-life (EOL) products is important to the overall performance of the disassembly facility. Some of the third party businesses involved in reverse logistics activities can rely on traditional inventory control methods with a few modifications to suit their needs when they have full control over their ordering policies. However other original equipment manufacturers (OEMs) cannot apply the same techniques due to the uncertainty of the quantity, quality and timing of products returned or the fact that they have no control over the reverse flow of materials and the take back program. In some cases, this inventory is treated as an alternative source for spare parts. The main focus in the study of inventory management in reverse logistics is the control of external orders.
from outside suppliers and the internal (re)manufacturing operations such that fixed and variable costs are minimized. Buchanan and Abad (Buchanan and Abad, 1998) assume that returns are a stochastic fraction of the number of items in the market for each period and drive an optimal procurement policy depending on two state variables: on-hand inventory and number of items in the market.

Meanwhile Korugan and Gupta (Korugan and Gupta, 1998) consider a two echelon-inventory system using an open queueing network with finite buffer to model the system. Van der Laan et al. (Van der Laan et al., 1999) present a detailed analysis of different policies to control serviceable and recoverable stock, taking into account non-zero lead-times for both sources, and PUSH and PULL recovery policies are considered. The authors conclude that defining an inventory position as a basis for replenishment decisions is non-trivial in case of a large difference between the lead-times of the two sources. Minner (Minner, 2001) studied the effect of combining the problem of safety stock planning in a general supply chain with the integration of external and internal product return and reuse using an extension of a general strategic safety stock placement approach. Fleischmann et al. (Fleischmann et al., 2002) presented a basic model for controlling inventories taking into account stochastic item returns. This paper provides a step towards a systematic analysis of inventory control in the context of reuse. The model assumes Poisson demand and returns and derives an optimal control policy and optimal control parameters. Teunter (Teunter, 2004) studied the inventory systems with product recovery and derived simple formulae that determine the optimal lot sizes for the production/procurement of new items and for the recovery of returned items. These
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formulae are valid for finite and infinite production rates as well as finite and infinite recovery rates, and therefore more general than those in the literature. Vadde et al. considers the effect of pricing end-of-life (EOL) products on the inventory level taking into account inventory constraints such as disposal limits (Vadde et al., 2006).

2.2.4 Production Planning

The third aspect of reverse logistics is production planning. In some cases of reverse logistics, the use of traditional production planning methods could be adequate, for instance when considering reuse of returned products. When products are in good working condition they are directly sent to customers after cleaning. In this case, applying traditional techniques could work, and the focus is on distribution planning and inventory management. However, when products undergo remanufacturing, repair, or refurbishing activities, problems may arise and traditional methods considered earlier have to be redesigned based on the process considered. Early research by Gupta and Taleb (Gupta and Taleb, 1994) studied the scheduling of disassembly operations when demand occurs for different components. The models take into account the precedence relationship between components, and later they extended the model to consider parts commonality (Gupta and Taleb, 1996). Thierry (Thierry, 1997) compares different Materials Resources Planning (MRP) approaches for remanufacturing with respect to their behavior under uncertainty. A simulation study was conducted to compare different safety stock levels required to meet the system service level under stochastic demand and return. Guide (Guide, 2000) emphasized that production planning and control activities
are more complex for remanufacturing firms due to uncertainties from stochastic product returns, imbalances in return and demand rates, and the unknown condition of returned products. Seven complicating characteristics have been identified and discussed that require significant changes in production planning and control activities.

In reverse logistics, production planning process has no predetermined steps; rather it is highly dependable on the quality of the returned products. This exposes production planning to a higher level of uncertainty and requires a more sophisticated coordination when disassembly is required (Guide et al., 1997). In a later research, Guide et al. (Guide et al., 2000) studied the acquisition of end-of-life (EOL) products and the complications associated with this. Due to its complexity a careful coordination is required to avoid an uncontrolled level of accumulated core inventory, or a lower service level than expected. The authors proposed a formal framework for Product Acquisitions Management (PrAM) to coordinate, monitor, and provide an interface between reverse logistics and production planning. Guide et al. concluded that “one-size fits all” methodology does not work well in a closed-loop supply chain, and in recent research identified issues in production planning in closed-loop supply chain. Three scenarios were modeled: remanufacture-to-stock, reassemble-to-order, and remanufacture-to-order (Guide et al., 2003). Jayaraman (Jayaraman, 2005) introduced an analytical approach towards production planning and control for reverse logistics with product recovery and reuse using a mathematical programming model. Dowlatshahi’s (Dowlatshahi, 2005) case study provided an insight into the operations of two companies involved in remanufacturing/recycling activities within reverse logistics. Based on these studies, a
complete framework of remanufacturing/recycling activities in reverse logistics is provided. Valchos et al. (Valchos et al., 2007) used simulation modeling technique to address capacity planning issues in remanufacturing facilities in a reverse supply chain. The models help in assessing the long-term capacity planning policies for a facility using total generated profit as a measure of performance.

2.3 Disassembly and Disassembly Line

Disassembly plays an important role in product recovery by allowing selective separation of desired parts and materials. The disassembly line is the best choice for automated disassembly of returned products, a feature that will be essential in the future. It is, therefore, important that the disassembly line be designed and balanced so that it works as efficiently as possible. Gupta and McLean discussed issues related to disassembly and provided an overview of the research in the area of disassembly of products. The research issues associated with disassembly can be classified into four general areas (Gupta and McLean, 1996): design for disassembly, disassembly process planning, disassembly production planning, and operational planning issues. For the current research, the most relevant topics are the disassembly process and its related issues.

2.3.1 Design for Disassembly

Design for Disassembly (DfD) involves designing products in a way to facilitate recovery of parts at the end of their life cycle for the purpose of repair, reuse, remanufacturing or
recycling. Design for disassembly (DfD) is an important design concept to make products friendlier for maintenance and remanufacturing practices. The importance of Designing for Disassembly became apparent as recovering parts and materials from end-of-life (EOL) products increased in popularity. There are number of benefits of achieving efficient disassembly of products as opposed to recycling a product by shredding. Research in Design for Disassembly is taking place at many Universities and companies throughout Europe, Scandinavia and North America. Many of these institutions have produced work suggesting ways in which Design for Disassembly should take place. Knight (Knight, 1996) discusses the uses of Design for Assembly (DfA) analysis as a decision support tool for disassembly sequencing and to identify opportunities for redesign. Sodhi and Knight (Sodhi and Knight, 1998) describes the development of product analysis procedures for combined disassembly and bulk recycling such that design teams can evaluate the ease of disassembly and recycling of alternative design concepts at on early design stage and investigate the consequences of materials selection at the end-of-life stage. Veerakamolmal and Gupta (Veerakamolmal and Gupta, 1999) introduced a technique for analyzing the design efficiency of electronic products by using the Design for Disassembly Index (DfDI). Gungor (Gungor, 2006) presented a model that evaluates alternative connectors by including the three main concerns: (1) making product disassembly friendly; (2) making product assembly efficient; and (3) increasing the product performance when it is in-use. The results obtained from the model can benefit designers in making better decisions on selecting connectors to be used in the product. The Design for Disassembly index uses a Disassembly Tree (DT) which relies
on product structure to identify precedent relationships. Veerakamolmal and Gupta (Veerakamolmal and Gupta, 2000) provide an overview of state of the art techniques for Design for Disassembly (DfD), reuse, and recycling.

2.3.2 Disassembly Process Planning

In disassembly process planning, researchers focus on the required tasks to perform the disassembly, and the order that these tasks should be carried out. The subject is divided into two important issues: disassembly sequencing, and level of disassembly. Gungor and Gupta (Gungor and Gupta, 1997) proposed an evaluation method so that the best disassembly strategy could be chosen among alternatives. The authors also proposed a disassembly sequence generation heuristic that gives near optimal solution for a given product using time of disassembly of each component, disassembly direction, and joint types. Kuo (Kuo, 2000) used a component-fastener graph generated through a CAD model to determine optimal disassembly sequence. Moore et al. (Moore et al., 2001) used a Disassembly Petri Net (DPN) approach to generate a disassembly sequence plan from a geometrically-based precedence matrix, and then used this to generate all feasible Disassembly Process Plans (DPPs). The Petri Net formalism is a modeling tool for the study of general discrete event systems. Its mathematical and practical foundations are well developed, and it supports both the qualitative and quantitative aspects of a model. Lambert (Lambert, 2002) introduced another approach to generate disassembly sequencing plans using connection diagram and precedence relationship, thus the plan can easily be presented as an AND/OR graph. AND/OR graphs
are widely used to provide complete, concise representation of all feasible disassembly sequences for end-of-life (EOL) products. Gupta et al. (Gupta et al., 2004) studied a disassembly sequencing problem for a cell phone case in a disassembly line. The authors used a MATLAB program to develop a sequencing plan while addressing other disassembly related matters. Andres et al. (Andres et al., 2007) proposed a two-phase method to generate the optimal disassembly sequence in a cellular configuration disassembly system.

2.3.3 Disassembly Production Planning

In disassembly production planning, studying the issues related to system design is crucial to adapt to the unique requirement of disassembly operations. Wiendahl et al. (Wiendahl et al., 2001) described several flexible disassembly layouts for processing end-of-life (EOL) products, such as disassembly line, disassembly cell, and single workstation. Authors concluded that transferring practices and knowledge from one system to another is not easily performed due to lack of standardization. Ranky et al. (Ranky et al., 2003) introduces a simulation model to help in answering “what if” questions and validating different possible layout designs for disassembly facilities. Singh and Sharma (Singh and Sharma, 2006) provide a state of the art review on different layout problems. The authors also discuss the development of facility layout software using meta-heuristics such as simulated annealing (SA) and genetic algorithm.
2.3.4 Disassembly Operational Planning

A number of operational planning issues arise in disassembly systems. One of the important issues faced by disassembly facility planners is that of determining the number of products to take back for reuse, recycling, storage, and disposal. In addition, scheduling, routing and re-routing problems are common in the disassembly line due to uncertain nature of returned products. Kongar and Gupta (Kongar and Gupta, 2002) developed a multi-criteria decision making methodology for disassembly to order (DTO) systems to determine the optimal take back size of end-of-life (EOL) products. The goal is to determine the optimal combination of take back and disassembled items so that demand is met and physical, financial, and environmental constraints are satisfied. Later, the model was extended using many methodologies such as linear physical programming, and goal programming (Imtanavanich and Gupta, 2004) (Imtanavanich and Gupta, 2005). The authors also applied genetic algorithm and fuzzy goal programming techniques to the problem at a later stage (Imtanavanich and Gupta, 2006). Later, the authors presented a model to determine the optimal complete DTO plan which includes the total number of products to take back and details on disassembly level and sequence (Imtanavanich and Gupta, 2007).

Meanwhile, Kim et al. (Kim et al., 2006a) proposed an integer programming model to minimize the number of brought back products disassembled assuming single product type. Gungor and Gupta (Gungor and Gupta, 1999) address a new problem which is Disassembly Line Balancing Problem (DLBP) using a systematic approach to solve the
problem in order to utilize the line as efficiently as possible. Later, different approaches were used to tackle the problem. McGovern and Gupta (McGovern and Gupta, 2003) used a greedy algorithm with the goal to minimize number of workstations while addressing the hazardous and high demand components. Then, a two-optimal algorithm is used to: 1) balance the part removal sequence and 2) reduce the total number of workstations. The authors presented many valuable techniques to solve and compare the results while ensuring that all methods meet the objectives of: minimizing the number of workstations, obtaining a similar idle time across all workstation, and these methods are in fact feasible. The authors used Ant colony optimization, genetic algorithms, and H-K meta heuristics and compared with a greedy/hill-climbing heuristic hybrid. All techniques showed results consistent with authors’ conclusion (McGovern and Gupta, 2004, 2005). Udomsawat and Gupta (Udomsawat and Gupta, 2003) discussed the use of a Multi-Kanban mechanism in a disassembly line. The approach is based on a Kanban technique used in an assembly line, however it is adjusted to cope with the uncertainty nature of a disassembly line and to reduce the inventory level generated, allowing the system to run more effectively. The authors utilized the use of simulation modeling techniques (ARENA Software) to solve the problem. In later research the authors applied the technique to different products such as personal computers, automobiles and home appliances. Also, the model has been tested under different scenarios such as demand fluctuations, preventive disassembly line maintenance, and sudden line breakdown (Udomsawat and Gupta, 2005a, b, 2006).
CHAPTER 2. LITERATURE REVIEW

Other issues related to disassembly are appropriate pricing of end-of-life (EOL) products and inventory management and control of brought back products. A research by Vadde, Kamarthi, and Gupta (Vadde et al., 2007a, b) studied the optimal pricing strategy for reusable and recyclable components under given constraints such as quantities returned and yield rate. It also investigated the effect of competition between Product Recovery Facilities (PRF) to acquire and sell end-of-life (EOL) products. The authors also discussed appropriate pricing strategies that can enhance the performance of PRFs by clearing their inventory and increase their profit under different scenarios such as product obsolescence and inventory constraints (Vadde et al., 2006a, b).

In recent years there is one underlying factor which has become apparent, recycling of electronic and electrical goods is currently a very marginal (in terms of resulting profit) activity but one which needs to be encouraged more in the future. For this to happen it is important to ensure that products are designed in a way to make it as attractive as possible to recyclers. There is one way which will ensure that products will be recycled in greater quantities in the future, and that is to make products very quick and easy to disassemble (DfD). Of course, products should also be made from recyclable material. Products will be recycled if they can be disassembled and sorted very quickly. Because the majority of materials have a relatively small recycling value it is important that the disassembly cost is even smaller for the process to be financially attractive.
2.4 Inventory Management in Reverse Logistics

Inventory control models of product returns and recovered parts mainly differ in terms of the assumptions on demand and return and the recovery process. The complexity of these models stems from the uncertainty in the quality and quantity of end-of-life (EOL) products and the fact that disassembly facilities have little or no control over the returns behavior by end users and market demand of these products. Assuming that demand and returns are independent; the literature categorizes inventory control policies into deterministic models, and stochastic models. The latter are further classified into continuous review models and periodic review models.

2.4.1 Deterministic Models

In deterministic inventory models the inputs of the model are assumed to be known and constant. In particular, demand and returns are known over the planning horizon. Deterministic models use economic order quantity (EOQ) modeling as a basis to find an optimal tradeoff between fixed and variable cost. Researchers modified this formula to cope with product returns. Schrady (Schrady, 1967) proposed a first model of this type by considering a deterministic repair model with constant demand and return and fixed lead time of external orders and recovery processes. Nahmias and Rivera (Nahmias and Rivera, 1979) extended Schrady’s original model with finite repair rate and limited storage space. Mabini et al. (Mabini et al., 1992) extended the previous model by considering stock out service level and a multi-item system where items share the same
repair facility. Richter, (Richter, 1996) similar to Schrady’s model, introduced a different control policy for alternating procurement and recovery batches and studied the interdependency between cost parameters and return. Teunter (Teunter, 2001) introduced an EOQ policy for product return with zero remanufacturing lead time and product disposal. Koh et al. (Koh et al., 2002) derived a joint inventory model of EOQ for procured items from an outside supplier and EPQ for optimal inventory level of on hand recoverable inventory, to meet the deterministic demand. The assumption was made that recoverable products after going recovery process are considered “as new” products. Teunter (Teunter, 2004) derived lot size expression for the production (1, R) policy and procurement (P, 1) policy. Dobos and Richter (Dobos and Richter, 2004) studied an inventory system similar to the EOQ model, where a certain fraction of returns are recycled. Number of items remanufactured will reflect the demand, while the rest is disposed of. Oh and Hwang (Oh and Hwang, 2006) proposed an inventory policy for a recycling system for determining the optimal policy parameters, given that demand is deterministic and a fixed fraction of demands is returned and used as raw material for new products.

2.4.2 Stochastic Models

In general, researchers can classify the stochastic models in inventory control theory into two broad groups: dependent demand and return models, and independent demand and return models. In the former, there is a perfect correlation between demand and return of products, while in the latter there is imperfect correlation. Kiesmuller et al. (Kiesmuller et
al., 2001) presented a hybrid system with dependent demand and return that follows a Poisson process. Product returns depend on lead time products sold in previous periods. It is concluded that inventory level under the assumption of dependent demand and return is less than compared to independent demand and return. In reverse logistics, it is commonly assumed that demand and return are independent, and they are further classified into two groups: continuous review models and periodic review models. However, assumption of joint probability distribution of demand and return can be considered, as in the case of repair systems.

In stochastic periodic review inventory modeling, the focus is to find optimal policies over a finite planning horizon. The first model in this area modeled the problem with distinct inventories for recoverable and serviceable inventories (Simpson, 1978). The model considers three parameters \((L, M, U)\), policy to control ordering from outside supplier, remanufacturing, and disposal quantities when fixed cost and lead time are negligible. Kelle and Silver (Kelle and Silver, 1989) proposed a similar model to Simpson’s model, but taking into account a fixed cost and assuming that all returns are remanufactured (i.e., no disposal occurs). Cohen et al. (Cohen et al., 1980) considered a system where returns and manufactured products are stocked together, and returns can be directly used. It shows that under certain assumptions, ‘order up to’ level policy is optimal. Optimal inventory level cannot be achieved if returns exceed that target, given that disposal is not considered in this model. Inderforth extended Simpson’s model with nonzero lead times considered and proves that optimal policy exists when lead times of manufacturing and remanufacturing differ by at most one period (Inderforth, 1997).
identical lead times, his model becomes identical to Simpson’s model. Mahadevan et al. (Mahadevan et al., 2002) applied a PUSH-type policy model in a hybrid manufacturing and remanufacturing system and demand and return are independent and follow a Poisson process. The goal is to answer the questions of how and when to manufacture and remanufacture products. Later, the disposal option was investigated for a similar model (Teunter and Valchos, 2003). Kiesmuller (Kiesmuller, 2003) considers a recovery system for a single product with inventory for recoverable and serviceable inventory. The study determines the optimal cost of manufacturing, remanufacturing, and disposal rates for the system under two different policies of allowable and forbidden backorders. Eynan and Kropp (Eynan and Kropp, 2006) examines a periodic review system under stochastic demand with variable stock out costs. The authors use a Taylor series expansion to approximate the cost function and produce a simple cost function similar to deterministic models.

In stochastic continuous review inventory modeling, the focus is to find optimal policies over a time axis that is modeled continuously, thus minimizing the long term average cost per unit time. In this research, van der Laan et al. (van der Laan et al., 1996a) used an approximation procedure based on the net demand during procurement lead time. Also, van der Laan et al., (van der Laan et al., 1996b) compared several disposal strategies and their impact on the level of recoverable and serviceable inventory. Later, van der Laan and Salomon (van der Laan and Salomon, 1997) discussed a continuous a PUSH and PULL-control policy with disposal option. The authors conclude that average cost using a PULL-policy tends to be lower compared to a PUSH-policy.
Yuan and Cheung discuss a \((s, S)\) inventory policy modeling an end of-lease (rental) type problem, with product returns after a known period of time (Yuan and Cheung, 1998). Fleischmann et al., (Fleischmann et al., 2002) consider a standard \((s, Q)\) inventory model with Poisson demand and returns to compute optimal parameters and minimum expected average cost. Teunter (Teunter, 2002) studies a discounted cost inventory system with stochastic demand and return. The author proposes a simple EOQ model for manufacturing and remanufacturing quantities. Kim (Kim, 2005) uses a continuous time Markov decision problem to develop an inventory control policy expression that minimizes the discounted costs over an infinite horizon with finite storage capacity.

2.4.3 Inventory Management in Disassembly Facilities

Disassembly facilities are considered to be stand alone entities that operate separately from the Original Equipment Manufacturers (OEMs). Disassembly facilities do not perform manufacturing or remanufacturing activities, and they act as a supplier of raw materials, parts and subassemblies to manufacturing facilities. The management of inventory at these facilities is unique in the sense that process flow is divergent, thus returned products have the tendency to “explode” into parts and subassemblies. When analyzing such a system, multiple classes and multiple levels of products are considered which complicates the problem further. Very limited research exists in the area of disassembly, or demanufacturing, facilities and few have developed a framework for inventory management. Sutherland et al. (Sutherland et al., 2002) introduced a model to improve the overall performance of the disassembly system. The model considers the
accumulation of parts disassembled, associated holding cost, and comparison of different selling policies and their impact on system’s profitability. Later, the author developed analytical models to predict the optimal selling quantities and periods of disassembled components (Gunter and Sutherland, 2004). Gaudettee (Gaudettee, 2004) in his dissertation discussed the inventory planning for the purpose of remanufacturing in his research, he touched the uncertainty of the process and the effect of structural characteristics of the end-of-life (EOL) products in determining the take back quantities that minimizes cost for disassembly and how does that translate to inventory. Akcali and Bayindir (Akcali and Bayindir, 2006) investigated the effect of different rules to determine the opportunity cost of disassembled parts on the performance of the system, under an approximate inventory control policy. Ahiska (Ahiska, 2008) applied a Markov decision process (MDP) to the problem of inventory optimization of a single product recoverable manufacturing system where stochastic demand is either met by either newly manufactured products or remanufactured products. El Saadany (El Saadany, 2009) has discussed inventory management in reverse logistics with price and quality considerations. In his research he attempts to close the gaps that exists in the current research such as ignoring the switching costs between production and remanufacturing, the learning effects, the assumption that remanufactured products is with “as-new” quality as manufactured products, and the effect of ignoring disassembly of returned products and dealing with EOL products as a whole products, and that collection is independent of price and quality consideration. He concludes pure policy is not always optimal.
Chapter 3

Background

This chapter discusses the different issues associated with the disassembly line. The focus is to address some of the issues related to inventory control and its effect on the reverse supply chain in general and on the disassembly line in particular. It also addresses the techniques adapted to approach and model the inventory problem.

3.1 Issues in Disassembly Line

In the last decade, the attention toward establishing a "green" image by recycling end-of-life (EOL) products has increased rapidly. However, the attention was shifted from the old fashioned recycling methodology where materials are retrieved to value recovery instead. Value recovery avoids the destruction process of returned end-of-life (EOL) products and allows the reuse of disassembled modules, after refurbishing and repairs, more than once before it is discarded. This process is known as disassembly or demanufacturing.
In recent years, disassembly has gained a lot of attention due to its role in efficiently recovering valuable materials, parts, and subassemblies from end-of-life (EOL) products. This is due to the rigid environmental legislation, the economical and environmental benefits from reusing primary materials instead of virgin resources and the increase in environmental awareness and concerns of the consumers. However, the process of recovering parts and materials is challenging, as there are many associated complications that are unique to the disassembly process (Teunter, 2001). Most products are not designed for ease of disassembly and this has had a profound impact on efforts to design economically viable recovery systems (Guide et al, 2003). Although most products can be disassembled eventually, lengthy disassembly does not make for economic recycling as the cost of disassembly is likely to be much larger than the revenue gained through recycling the parts and materials from the product. It is for this reason that designing products for easy disassembly has increased in popularity enabling more of the product to be recycled economically.

The disassembly of an end-of-life (EOL) product is performed at a single workstation, in a disassembly cell, or on a disassembly line (Gungor and Gupta, 2002). Disassembly lines are proposed as one of the settings in which disassembly activities can be performed efficiently, because they can yield high productivity rates and are suitable for automation (Altekin et al, 2009). A disassembly line is the best choice for automated disassembly as it provides the highest productivity rate for end-of-life (EOL) products available in large quantities. For that reason, in recent years, a lot of researchers have started to concentrate on problems related to the disassembly lines. For instance, some of
the objectives of disassembly line balancing are to provide a feasible disassembly sequence, minimize the number of workstations, provide an equalized work content at each workstation (or minimize idle time at each workstation), and balance the line (ensure similar idle times at each workstation) as well as address other disassembly-specific concerns. Due to government regulations and increase of awareness among corporations and consumers the idea of reverse supply chain (RSC) became the center of attention for many private and public entities. The desire is to establish new channels to reuse returned items to minimize its environmental impact if it were disposed of.

In more recent time, additional challenges in disassembly theory started to emerge due to the growing need of end-of-life (EOL) products processing. This subject became increasingly important during the last decade of twentieth century. The systematic approach of the disassembly process was fueled by both the desire for precious and valuable materials recovery and the challenge for environmentally processing of post consumed products. This resulted in the discovery of new types of problems that had to be solved, such as those related to optimum disassembly level, the optimum clustering of waste streams (Lambert and Gupta, 2005), and other challenges associated with balancing the inventories generated at various workstations of a disassembly line. It is concluded that not only it is important to balance the work content at each workstation, it is also important to balance the inventory generated at each workstation by strategically siphoning off the parts and materials disassembled by providing a proper flow to the demand sources. These demand sources include storage facilities, remanufacturing facilities, raw materials suppliers, and other shops that provide used spare parts.
Other sources of challenges include the end-of-life (EOL) products and the parts themselves. The changing and sometime unpredictable characteristics of the products and its contents complicate the problem. These could lead to serious inventory problems because of the disparity between the demand and actual yield of the line. In addition, the speed of the line is important in determining the quantity disassembled and thus the quantity accumulated of each component type that could result in irregular work-in-process (WIP). Work-in-process needs to be handled appropriately to avoid running out of space or starving the demand sources. In addition, different parts may have to be handled differently and some materials (including hazardous materials) may require special treatment to minimize the harm to the environment. Other issues such as the types of material handling equipment requirements, the types of demand sources, and whether or not to fulfill the demand continuously or in batches could complicate the disassembly line operations.

### 3.2 Inventory Issues in Disassembly Line

There are many challenges and issues that are associated with inventory planning in disassembly line. The disassembly line setting consists of multiple workstations that are working in a sequence to disassemble end-of-life (EOL) products to subassemblies and parts. Inventory management is one of the major issues that arise in the disassembly line because of the disparity between the demand and the line yield. The fluctuation in inventory level is due to changing in returns behavior and the change in demand pattern.
This creates uncertainty in the acceptable level of on hand inventory (OHI) at the workstations. Most of the literature on inventory control related to reverse logistics focus on the order quantity so as to minimize the total average and/or discounted costs. Of course, most authors use the average cost models as they are easier to analyze and easier to solve for “near optimal” solutions than the discounted cost models (Teunter and van der Laan, 2002).

3.2.1 Challenges Associated with the Demand and Return

As opposed to an assembly line, where the demand arrives at the last workstation, demands in a disassembly line environment may arrive at any of the workstations. To fulfill the demand, parts are sent to the demand sources continuously or in batches of certain sizes. Some physical and functional defects discovered during disassembly or caused due to disassembly may lead to fluctuations in the supply of parts and subassemblies. The defects may also have an effect on the quality of the output further complicating the problem. Even though, we make an assumption that the demand sources accept the disassembled parts “as is”, it is still important to keep the quality of the parts recovered within certain limit in order to satisfy customers and avoid high rate of return.

Also, the return behavior of end-of-life (EOL) products is unpredictable and changes depending on many factors. For instance, different seasons play an important role of the type of products returned and when they are returned. For example, the number of disposable cameras returned after holiday season increases compared to other seasons. Also, the quick introduction of advanced technology always results in shortening
the life cycle products in the market. Consumers tend to return them to upgrade to newer models with better options, even though these products are still functional.

### 3.2.2 Challenges Associated with the Demand Source

Demand at a disassembly line may arrive from many possible demand sources. Examples of demand sources include remanufacturing facilities, raw materials suppliers, storage facilities and waste disposal limitation quota. Demands of parts tend to be in different quantities creating a mixture that makes forecasted demand and line yield somewhat unpredictable. Theoretically, it is perhaps easier to build a separate system and develop different mechanism for handling each demand source. However, that is obviously impractical due to its high cost and limited space availability at the designated workstations along disassembly line. A flexible disassembly line with a proper sorting/storing area and a specialized material handling system is a must to provide efficiency.

### 3.2.3 Challenges Associated with the Product

During the use phase of the product, consumers tend to upgrade or downgrade some of the products by adding, changing or removing some of the parts making the number of retrievable components unpredictable. There is also uncertainty associated with the condition and the reliability of these parts based on operating environment of the products during their use phase. Furthermore, the quantity of the products brought back to the disassembly facility is unknown. Such unpredictable characteristics tend to
complicate the inventory system associated with the disassembly line. Because of the line’s limitations and the continuous demand of disassembled parts, determination of availability of enough inventories is necessary in order to meet the demand while ensuring a proper flow.

3.2.4 Challenges Associated with the Logistical Process

Recovering products often create a lot of complications in reverse logistics (Teunter, 2001). Complications in scheduling, planning and handling are common. Work-in-process is a source of concern because the number of work pieces on a disassembly line tends to “explode” (one work piece splits into two or more work pieces after disassembly). A proper handling of these products and directing them to the right workstation is important to avoid blocking or starving of workstations and demand sources.

3.2.5 Other Challenges Associated with Reverse Logistics

There are other challenges and concerns that are associated with a disassembly line. For example, hazardous materials and parts may require special handling to avoid contaminating neighboring parts and other products. Special handling include skilled human workers and specialized material handling equipment for proper delivery of the parts to the various demand sources or disposing of them in an environmentally benign manner.

Materials handling is another concern. Type and scale of the automated materials handling system is an important component in the disassembly system design. Because of
the uncertainty associated with disassembly operations, the use of robots and automated systems are not as useful because they are often designed to perform pre-determined tasks (Lambert and Gupta, 2005). Any design to improve the inventory issues associated with a disassembly line should take certain priorities into consideration while performing disassembly tasks. For example, a higher priority should be given to highly demanded parts and, thus, should not stay in the system for a long time. It is also worth investigating how pricing of disassembled parts and/or materials would affect the operations.

### 3.3 Inventory Control Models in Disassembly Line

In this section, different inventory control models will be presented to approach the inventory management problem in the disassembly line. In disassembly facilities planners face many challenges when it comes to appropriately managing inventory. This challenge is due to many reasons, such as fluctuation in disassembled parts demand, supply of end-of-life (EOL) products, and returned products qualities and conditions. Advanced planning for inventory management is much more difficult when compared to a traditional manufacturing setup. This is mainly because of the uncertainty associated with the behavior of the system and its entities and to the number of take back products that are returned and number of certain parts and subassemblies that are demanded. This usually result in a carried inventory that is beyond the actual demand, resulting in higher recovery costs, increased inventory and carrying costs, and excessive charges of carrying low demand or low value parts.

The other factor is uncertain yield of different quality parts disassembled. Unlike traditional forward system, inventories in reverse logistics are independent of demand.
Disassembly facility has no control of the returned products and the time of returns. This return trend fluctuates depending on the season. Because of the take-back program requirements, these facilities are obligated to assume ownership of these products. This means a pile of many end-of-life (EOL) products with different quality levels that are beyond the facility’s capacity to handle. While in traditional system inventory is demand driven and managed using a Material Requirements Planning (MRP) techniques based on the end item requirements. Orders are placed taking in consideration the exact need to produce the demanded quantity. These parts are ordered such that it arrives just in time for assembly. Just in Time (JIT) ensure minimal levels of inventories of parts are carried thus reducing the system overall cost. When necessary, safety stock is considered to account for uncertainty. The basics of the traditional inventory system are applied to solve the complex inventory planning for reverse logistics with modifications. Generally, two approaches are considered: deterministic approach and probabilistic approach. The assumptions used are discussed in details in the following section.

### 3.3.1 Deterministic Approach: Model and Parameters

Inventory of disassembled parts in a reverse logistics environment is unique and different from traditional forward inventory control system. The uncertainty involved is a major challenge when planning for such system. A disassembly line with single product type is considered, with known configuration and parameters. To avoid complexity, the end-of-life (EOL) product configuration is known without any upgrades or down grades. Thus, products arrive at the disassembly line neither with no extra or missing parts. Usually products with modifications result in longer disassembly time, which in return increases the on hand inventory to be carried over. The demand planning is accounted for, since the
product yield is known in advance. Most planners assume that yield is deterministic to simplify their planning process. However, planning tends to get very difficult when combining the uncertainties of supply, demand, and line yield. Also, the demand for these disassembled parts may arise at any workstations along the disassembly line. These products have different quality levels depending on the environment they were operating in. For the purpose of this research, three quality levels will be introduced, which are good quality, average quality, and imperfect or underperforming quality level. Also, the model distinguishes between the different demands sources for the disassembled parts and subassemblies according to their operation type and all quality levels cannot be accepted. For example, third party demand sources that purchases used parts for the purpose of reuse in new or remanufactured product usually accept parts and subassemblies in a good quality level that is operating and still functions well. These parts might need cleaning and minimal repair before being used in refurbished products. Also, the demand for different quality parts and subassemblies vary, based on the demand source capacity and need. All associated costs, such as inventory and carrying costs, disposal costs, transfer to storage cost, are all based on the quality of these items. Parts that cannot be used to fulfill the demand is recycled and sold separately based on weight. Any additional inventory is carried over to the next period. Inventory and carrying cost is given and assumed to be deterministic.

The costs associated with the disassembly process and end-of-life (EOL) product varies based on weight, dimension, and space utilization of these parts. The effect of different costs assignments on the system profit will be studied in this research. Also, due to limited space availability along the workstations, the capacity constraint is taken into consideration when planning for such system. The amount of inventory to be stored along the line will be limited, and any extra inventory beyond the demand will be either sent to
remote storage facilities or will be disposed of. These effects combined will be incorporated when modeling the system behavior.

The first approach to apply is Linear Programming (LP) model technique to solve the problem in the deterministic approach. All modeling is conducted using Lingo 11.0 modeling package and spreadsheet analysis to analyze the data generated. Deterministic approach is widely used to simplify the advance planning, however, it does not provide an optimal result in terms of costs saving or profit generated. Other aspects of end-of-life (EOL) products could be discussed such as value remaining and how that affects the planner's decision of remanufacturing, reuse, or disposal.

The second approach utilizes the Dynamic Programming (DP) technique and spreadsheet analysis. The model ensures that a minimum amount of inventory is carried from one period to another, and all on hand inventory is disposed of at the end of the planning horizon. It assumes that inventory space along the workstation is limited by full totes, and any extra inventory or partial tote can be liquidated such that profit is maximized. The system will keep inventory in the size of full totes with future demand and profit maximization objective taken into account.

3.3.2 Stochastic Approach: Model and Parameters

Inventory control in reverse logistics can accurately be modeled in a probabilistic environment. The characteristic of uncertainty in end-of-life (EOL) products return is a major factor that distinguishes the reverse supply chain from the traditional forward supply chain. While modeling the behavior of the system, an important element that is taken into consideration is the probabilistic behavior. This adds another dimension to the
inventory planning of disassembled parts, which is core and disassembled parts yield. Planner of the inventory system usually has no control over the type of products return and the time at which they are returned. Also, the assumption is that demand sources always accept parts "as is" regardless of its condition. Thus the system does not distinguish between different quality levels. For simplicity, the models assumes demand occur for one parts of the products, where the balance of the end-of-life (EOL) product is negligible. The models ignore the product physical characteristics such as weight and/or dimension. To model the disassembly system, a probabilistic approach is introduced using system dynamics approach and queueing theory application. Some of the assumptions made when modeling the system are as follows:

- All end-of-life (EOL) products returned will be disassembled regardless of its quality level.
- It is a stand alone disassembly facility, meaning no other operation will take place except for inspection.
- Inventory carrying cost, waiting cost, disassembly cost, and fixed cost remain constant.
- Single part demand is modeled, and the balance disassembled is negligible.

The third approach to model the problem is system dynamics (SD) approach. It utilizes the stochastic nature of the problem with the goal of replacing uncertain parameters with the deterministic expected value, thus these parameters are modeled with all possible outcomes. System dynamics approach is generally applied to macro-systems such as production-inventory system, and the philosophy behind system dynamic simulation is founded on the concepts that industrial systems are like input-output systems, and thus the system state changes according to the changes in the rates of the inflow and outflow, and it reaches a steady state when the input rate equals to the output.
rate. Thus, based on these assumptions the system can be modeled using Euler’s first order differential equations using VenSim PLE 5.9 simulation software to model and forecast the system long term behavior to external changes in demand and how long it takes before the system stabilizes again.

The fourth approach is to apply the application of Markov chain theory to the inventory problem in disassembly context. The approach will allow the modeling of the problem under the F-policy technique to exercise control over the arrival of disassembled parts to the inspection queue for inspection service to be provided. The approach assumes that all queues have an optimal capacity and threshold levels, and parts should not be allowed to the system unless that threshold level is reached. All parts beyond the capacity are lost to the system.

It is expected that stochastic techniques will lead to results that are superior to deterministic methods results. The results of the techniques used will be presented and optimal inventory control policy under different scenarios will be provided. This will provide a valuable evaluation tool to the inventory planner when it comes to managing an agile inventory system in a disassembly line.
Chapter 4

Problem Statements and Research

Objectives

This chapter presents the problems studied in this research and its different scenarios, the research objectives, and provides a numerical example to illustrate the problem definition and the areas of concern.

4.1 Problem Statements

The prime focus of this research is to study the issues and problems related to inventory management and control in reverse logistics, specifically in the context of a disassembly line. Unlike the traditional forward logistics system, the reverse supply chain is much more complex due to the uncertainties involved. The main issues that need to be addressed in this research are as follow:
Accurately forecasting the demand and supply of the end-of-life (EOL) products and disassembled parts/subassemblies. The unexpected behavior of the products return or sudden changes in the demand levels could lead to excess or shortage in inventories.

Accurately estimating the inventory holding cost of end-of-life (EOL) product based on its quality level, size and shape, space utilization, remaining value, inventory disposal costs, and other costs associated with the process. These costs contribute significantly to the overall system performance, thus the near optimal estimation is essential when modeling the process.

Studying the behavior of inventory piling up at the different workstations along the disassembly line and identifying the bottle necks in the process and how the demand and supply control could affect the planner decision.

Simulating the inventory accumulation and testing different liquidation vs. keep strategies to reduce inventory and generate profit. Disassembly line faces serious inventory problems because of the disparity between the demand and the line yields. Workstations along the disassembly line tend to experience different accumulation rates as well as different depletion rates depending on the demand these parts. Such fluctuation between inventory and demand creates uncertainties in inventory levels at each workstation. Because of the differences in the flow rates (in and out), parts start piling at the designated sorting areas at workstations, in some cases this behavior does not interfere with the work because of the parts sizes and quick turn over. In other cases, the accumulation of inventory could
CHAPTER 4. PROBLEM STATEMENTS AND RESEARCH OBJECTIVES

block the workstation. Figure 4.1 is a representation of the recovery process of the end-of-life (EOL) products at the disassembly facility.

Figure 4.1 The forward and reverse flow of parts in a recovery process

4.2 Research Objectives

The core objective of this research is to develop inventory policies to determine the appropriate inventory levels of end-of-life (EOL) core products, disassembled parts/subassemblies, quantity upper and lower bound, and an appropriate way to handle Work-in-Process (WIP) and maintain it at certain levels. The goal is to ensure continuous
profitability of the system while providing a proper flow to the demand sources and also
to balance the inventory generated by siphoning off parts and materials disassembled
while avoiding the stockpiling of parts at the workstations.

The first objective is to estimate the parameters and the costs involved in the
model. Near optimal results can be achieved by deriving near optimal expressions. Then,
based on the traditional inventory models, an inventory management system can be
designed. The system should effectively reduce the on hand inventory (OHI) while
ensuring the profitability of the system. The research should examine the effects of these
parameters on the overall performance of the system.

The second objective is to test the system under deterministic and stochastic
scenarios, and under certain constraints and limitations, while taking into consideration
the different attributes of the disassembled parts and subassemblies such as the sizes, the
quality level, uncertainty in the quantity and type of returned products (the supply
stream), and the fluctuation in demand and prices from time to time. The results are then
compared to determine the conditions under which the stochastic modeling outperforms
the deterministic techniques.

Third, a major objective of the research is to learn more about inventory control
techniques because of opportunities in this area for companies. Here, the investigation of
the two approaches will facilitate various organizations in their efforts to learn more
about the disassembly line and help in improving system performance. Organizations,
academics, and others can utilize the findings of the research to conduct additional
research on this vital topic.
As an example of the inventory control problem in a disassembly line, the disassembly of a personal computer (PC) is considered. This example provides a relevant and real application of a disassembly line problem and was taken from McGovern and Gupta (McGovern and Gupta, 2005). The purpose of this example is to show the impact of disassembly of personal computer (PC) on the total inventory generated at each workstation over a seven-day period. The PC consists of 8 components \( (n=8) \) with component removal times and associated demands shown in Table 4.1. The following precedence relationships hold between different tasks: 

\[ 1 \rightarrow 2, \, 1 \rightarrow 3, \, 1 \rightarrow 4, \, 1 \rightarrow 5, \, 1 \rightarrow 6, \]
\[ 1 \rightarrow 7, \, 1 \rightarrow 8, \, 2 \rightarrow 8, \, 3 \rightarrow 8, \, (2 \ OR \ 3) \rightarrow 6, \, 5 \rightarrow 4, \, 5 \rightarrow 8, \, 6 \rightarrow 8, \, 7 \rightarrow 4, \, 8 \rightarrow 7. \]

The arrow \( \rightarrow \) means “must precede”. The number of products to be disassembled is based on the highest demand of a component. The demand of RAM modules (RAM) is the highest (750). However, every product disassembled yields 2 modules, thus the actual needed products is 375. The next highest demand is for the Motherboards (MB) with 720 units needed. Thus quantity of PC to be disassembled is 720 units. The disassembly line speed allowed is 40 seconds for each workstation \( (CT=40) \) and is calculated by assuming an 8 hour shift per day or 28,800 seconds per day \( (L= 8 \times 60 \times 60 = 28,800 \text{ seconds}) \). Thus, time allowed for each workstation \( (CT) \) is equal to \( 28,800/720 = 40 \text{ S/WS} \).

The above example is an illustration of the disassembly technique of end-of-life (EOL) product (personal computer) and the demand for different parts of the PC. The purpose of the example is to show and illustrate mathematically the buildup of inventory when trying to satisfy demand for these parts from external demand sources. In the next chapters similar example will be given, but rather for a hypothetical product \( ABC \).
Table 4.1 Disassembly tasks of a personal computer (PC)

<table>
<thead>
<tr>
<th>Task</th>
<th>Removal of Component</th>
<th>Component Removal Time</th>
<th>Hazardous</th>
<th>Component Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PC top cover</td>
<td>14</td>
<td>No</td>
<td>360</td>
</tr>
<tr>
<td>2</td>
<td>Floppy drive</td>
<td>10</td>
<td>No</td>
<td>500</td>
</tr>
<tr>
<td>3</td>
<td>Hard drive</td>
<td>12</td>
<td>No</td>
<td>620</td>
</tr>
<tr>
<td>4</td>
<td>Back plane</td>
<td>18</td>
<td>No</td>
<td>480</td>
</tr>
<tr>
<td>5</td>
<td>PCI cards</td>
<td>23</td>
<td>No</td>
<td>540</td>
</tr>
<tr>
<td>6</td>
<td>RAM (2)</td>
<td>16</td>
<td>No</td>
<td>750</td>
</tr>
<tr>
<td>7</td>
<td>Power supply</td>
<td>20</td>
<td>Yes</td>
<td>295</td>
</tr>
<tr>
<td>8</td>
<td>Motherboard</td>
<td>36</td>
<td>No</td>
<td>720</td>
</tr>
</tbody>
</table>

For this example, we make the following assumptions:

- Component removal times are deterministic, constant, and integer.
- Each product undergoes complete disassembly.
- All products contain all components, with no additions, deletions, or modifications.
- Each task is assigned to one and only one workstation.
- The sum of the component removal times of all the components assigned to a workstation does not exceed $CT$.
- The precedence relationships among the components must not be violated.
- Components have no physical or functional defects.
Table 4.2 DLPB greedy algorithm and ACO solution

<table>
<thead>
<tr>
<th>Workstation</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allocated Task(s)</td>
<td>1,5</td>
<td>3,6,2</td>
<td>8</td>
<td>7,4</td>
</tr>
</tbody>
</table>

An optimal allocation of tasks for this example at various workstations is shown in Table 4.2. Thus, the inventory generated is spread over 4 workstations. Workstation 1 handles components 1 and 5. Workstation 2 handles components 3, 6 and 2. Workstation 8 handles component 8 and workstation 4 handles components 7 and 4. In addition to the disparity in quantities generated, the sizes and the mass of the components are also different.

Table 4.3 presents the cumulative inventory balance for each component recorded for seven days after disassembling 720 PCs every day and meeting the demand for each of the components as shown in Table 4.1. From Table 4.3, one can clearly observe the detriment created due to the disparity between the demand and the yield from disassembly. It is obvious that such rapidly increasing inventories could be devastating to the corporation. Carrying unnecessary inventory of disassembled parts that are beyond the demand will result in high carrying costs and will affect the profitability. Balancing the generated inventory is an important element when studying the inventory issues in a disassembly setting. In table 4.3, after satisfying demand, parts will accumulate from day one till day seven. This accumulation is because disassembly calculation was based on disassembled quantity of highest demanded parts. It is clear this approach will create future space problem and financial implication on the system profitability.
Table 4.3 Cumulative inventory balance for each component recorded for seven periods

<table>
<thead>
<tr>
<th>Component</th>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
<th>Day 5</th>
<th>Day 6</th>
<th>Day 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC</td>
<td>360</td>
<td>720</td>
<td>1080</td>
<td>1440</td>
<td>1800</td>
<td>2160</td>
<td>2520</td>
</tr>
<tr>
<td>FD</td>
<td>220</td>
<td>440</td>
<td>660</td>
<td>880</td>
<td>1100</td>
<td>1320</td>
<td>1540</td>
</tr>
<tr>
<td>HD</td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>400</td>
<td>500</td>
<td>600</td>
<td>700</td>
</tr>
<tr>
<td>BP</td>
<td>240</td>
<td>480</td>
<td>720</td>
<td>960</td>
<td>1200</td>
<td>1440</td>
<td>1680</td>
</tr>
<tr>
<td>PCI</td>
<td>180</td>
<td>360</td>
<td>540</td>
<td>720</td>
<td>900</td>
<td>1080</td>
<td>1260</td>
</tr>
<tr>
<td>RAM</td>
<td>690</td>
<td>1380</td>
<td>2070</td>
<td>2760</td>
<td>3450</td>
<td>4140</td>
<td>4830</td>
</tr>
<tr>
<td>PU</td>
<td>425</td>
<td>850</td>
<td>1275</td>
<td>1700</td>
<td>2125</td>
<td>2550</td>
<td>2975</td>
</tr>
<tr>
<td>MB</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

4.3 Conclusions

From the previous example, it is clear that the inventory of parts and subassembly tend to build up in a fast manner. Because of the disparity between the demand and the line yields, these parts should be appropriately handled and managed to reduce the costs of carrying such items. In this dissertation, management tools will be developed to simulate the system and attempt to reduce the inventory while ensuring demand for these parts are met during all periods, and minimum inventory is kept to satisfy any future demand.
Chapter 5

Inventory Management of End-of-Life (EOL) Products in a Disassembly Context

Today’s markets have become more competitive and complex to operate in for many original equipments manufacturers (OEMs), especially with recent government requirements for processing end-of-life (EOL) products. Thus, the collection and the disassembly operations of these returned products from the end users back to the origin for the purpose of reuse or final disposal has gained a lot of research attention from government agencies, companies and consumers, and thus led to numerous regulations and have made society more aware of the importance of green image. However, OEMs face many challenges when trying to adapt this philosophy. Some of the challenges revolve around inventory management and control parameters and assessing the values of these items. In this chapter, a short summary to present and explain the system behavior and introduce concepts used in literature based on traditional inventory control theory that take into account the unique characteristics of the disassembly line.
CHAPTER 5. INVENTORY MANAGEMENT OF END-OF-LIFE (EOL) PRODUCTS IN A DISASSEMBLY CONTEXT

5.1 Introduction

The area of reverse logistics has received increasing attention in recent years. The field of reverse logistics, also known as reverse supply chain, is the process of planning, implementing, and controlling effectively the inbound flow and storage of secondary goods and related information in the direction opposite to the traditional supply chain for the purpose of recovering value or final disposal of end-of-life (EOL) products. Due to governmental legislations, many organizations have started to restructure their processes to meet these rules such as limitations on waste disposal and recycling requirements. Also, the increased desire of the public in going green has helped to enlighten them of the advantages of using secondary materials. On the other hand, these corporations have started to realize the economical benefits of the recovery operations as a business opportunity to generate profit if implemented correctly. These corporations are now trying to “close the loop” of the supply chain process and transform the end-of-life (EOL) products into serviceable products with little or no harm to the environment. This process is known as reverse supply chain.

Reverse logistics is defined as “the role of logistics in product returns, source reduction, recycling, materials substitution, reuse of materials, waste disposal and refurbishing, and remanufacturing”. Similar to the functions of supply chain management, reverse supply chain handle the same issues but in a backward flow with many added uncertainties. The uncertainties in reverse supply chain as per Gaurang Patel et al. stem from the following:
• Uncertain flow of materials: most of the time disassembly facilities do not know when a product or a part will be returned or disposed of.

• Uncertain quality and wide variety of return products: disassembly facility deal with wide and random variety of products with uncertain quality levels.

• Customer dependent: the flow is highly diverse and depends on end user.

• Timing: stochastic and disassembly time is uncertain.

• Uncertain potential remaining value: disassembly facility aim is to maximize the value of assets returned at end-of-life cycle.

• Uncertain market demand: price and demand of secondary market is uncertain.

The lack of knowledge and know-how, and questions regarding the profitability of the disassembly process among the factors mentioned above discourage many original equipments manufacturers (OEMs) from participating and engraining in take-back programs (TBP). Stand alone disassembly facilities handle an overwhelming number of end-of-life (EOL) products from collection point at the end users end all the way to recovery or disposal. In this chapter, inventory and value management of end-of-life (EOL) products and disassembled parts are the main concern. This chapter summarizes important concepts used in the literature of reverse logistics and disassembly. A classification of inventory types is given, and an introduction to the traditional inventory concepts that has been applied in similar research. This chapter is a guideline tool that help planner investigate the best tool that apply to any particular situation. For further details, the reader is encouraged to refer to references section.
5.2 Inventory Classification of End-of-Life (EOL) Products

Inventory in reverse logistics in general, and in disassembly context in particular is more complex than traditional inventory in the forward chain. Unlike traditional inventory, where inventory represents materials and parts that are consumed, the focus is core products and disassembled parts that are being piled up instead. Despite the differences in the “flow of goods” in the chain, the analysis of inventory can be carried in similar ways.

At the disassembly facilities there are three types of inventory during the process:

i) Recoverable inventory of core products and disassembled parts

ii) Serviceable inventory of disassembled parts and subassemblies

iii) Recyclable inventory of both core products and disassembled parts

Recoverable inventory represent all the end-of-life (EOL) products that has been recovered from end users at the end of its life cycle with or without potential of value recovery. Most of the time, the collection of these products are required regardless of its condition. After collection, end-of-life (EOL) products go through intensive testing and quality check at the collection facility before it is finally dispatched to disassembly facility for further processing or final disposal. The second category of inventory is serviceable inventory which contains all core products and disassembled parts that have been inspected and found to be suitable for remanufacturing, refurbishing, or reuse. This category will cover a wide range of quality level and demand is usually met from this category. The third category type is the recyclable inventory that consists of all items with potential materials value for resale. Usually these materials are being used by raw materials suppliers to meet requirements of the original equipment manufacturers of
certain precious materials type. The flow of end-of-life (EOL) products into the disassembly facility and different inventory types is shown in Figure 5.1.

Figure 5.1 Products returns to disassembly facility
Most researchers have considered a hybrid manufacturing and remanufacturing systems or repair systems when studying the inventory management problem. However, in the scenarios considered here, and the rest of the thesis, disassembly facilities are considered to be stand alone facilities that do not involve manufacturing or remanufacturing activities, rather it acts as a supplier of parts and materials to remanufacturers and secondary markets after inspecting these products and parts for reusability.

End-of-life (EOL) products are collected and sorted at a centralized collection facility before they are sent to the disassembly facility where they are being inspected and either stored as a recoverable inventory and disassembled when needed, or disposed of. Based on the demand, and the capacity of the workstations along the line, end-of-life (EOL) products are worked on. Then, disassembled parts and subassemblies are grouped in batches and sent to satisfy the demand sources. Inventory of parts beyond the demand are either kept on hand, in remote storage facility, or disposed of if no future demands of these parts are expected in near future. The system can either have shortage of disassembled parts, thus procuring the balance from an outside third party supplier or excess inventory that can be carried on for extra carrying costs. This raises the following questions:

i) How to determine what items to be recycled or disassembled?

ii) When and what quantity to procure from an outside suppliers?

iii) What are the effects of products quality on its holding costs and market price?

iv) What is effect of different selling policies on the overall profit?
The answer to these questions contributes to the planner decision making process and can be expressed graphically in Figure 5.2.

![Figure 5.2 Effects of external factors on inventory in a disassembly environment](image)

With this in mind, this chapter will address the need for inventory management tool that take in consideration the unique characteristics of the disassembly line and how
the above questions have been addressed in the literature before. The answers to the above raised questions are very important in helping the planner implement an inventory control model of the end-of-life (EOL) products returned.

1. **How to determine what items to be recycled or disassembled?** As previously explained, the most unique and challenging aspect of inventory control in reverse logistics is the uncertainty of the line yields. The yield will depend heavily on the condition of item returned. Usually, good quality items will have higher remaining values; hence it is more desirable to disassemble these items and then utilize it for remanufacturing, refurbishing, or direct reuse. Average and low quality items can be used to distract the precious materials for the purpose of recycling and send the balance to be disposed of.

2. **When and what quantity to procure from an outside supplier?** Usually the shortage in inventory is secured from an outside third party supplier. However, backordering results costs more per item ordered, in addition to additional costs incurred when placing orders. Appropriate forecasting of demand and supply, and holding enough safety stock level to avoid stock-outs is a key element in reducing the cost and avoid excessive unnecessary ordering from other suppliers.

3. **What are the effects of products remaining value and quality on its holding costs and market price?** As the product is used, its value is consumed and the wear and tear of the parts causes the total value of the product to decrease rapidly, and catastrophic part failure leads to precipitous drop in the product’s value and eventually quality. Product that has been in the market for longer period tends to
have lower quality than newly introduced products. Yet this is highly stochastic matter and it varies from product to product and to gain better understanding it is important to know how each part performs in different scenarios. High value items cost more to keep and handle, and in return it is worth more in the market compared to low value items. However, holding items for long periods could be a burden on the system’s profit if their perceived marginal profit is less than what one would realize from liquidating these items sooner. Inventory planner has to balance this trade-off between cost and worth.

4. What is the effect of different selling policies on the overall profit?

Disassembled parts and subassemblies can be liquidated at the end of every day, or every other day, or when it reaches some critical level. It is evident that the size of the liquidation transaction and the holding costs influence the performance of the selling policy. The presence of high market prices and high demand of disassembled parts either by repair or in secondary markets discourage inventory from piling up at the disassembly facility. Usually, high inventory level lead to reduction in price and thus reduction in the overall profit. Appropriate examining of the selling policy to be used can maximize the system revenue and reduce cost.

As discussed above, and as shown in Figure 5.2 it is clear that not only demand and supply affect the current inventory levels at the disassembly facility, but other system constraints, environmental regulations, the economy, and the applied pricing strategy could all influence the profitability of such operation. In the next section, details discussion of the questions will be given.
5.3 Research Questions Associated with Disassembly Operations

Cost assignment is a key element for corporations to ensure profitability in a reverse supply chain business. Generally speaking, in order for any corporation to acquire a bigger market share for its product it should provide either (i) the same quality products as its competitors at a lower price or (ii) a better quality products than its competitors at the same price. In a disassembly environment, two distinguished inventory categories, beside recyclable inventory, are of concern: recoverable inventory and serviceable inventory. Although the same concept of assigning costs applies to both categories, yet handling both categories is somehow different, causing a variation in total costs associated with each inventory type.

The first question addressed a key factor in qualifying items for disassembly and recycling. The disassembly facility will face an overwhelming number of returns under different scenarios as discussed in previous chapters that needs to be managed properly. Some returns may fall under the warranty period, while others do not function anymore and have been in the market for long time and might be discontinued and thus returned for final disposal. It is the planner decision to determine which items are more desirable for disassembly and can be sent to disassembly facility. The disassembly facility has a limited capacity in terms of number of items it can disassemble, store, or dispose, and this will limit the planner ability to allocate all returns for the proper use.

It may be profitable for the disassembly facility to hold on to the relatively newer disassembled parts and subassemblies by paying the holding costs if their perceived marginal profits are higher than what one would realize by recycling or disposing these
parts. The effect of wear and tear is examined when parts are collected and thus quality of products is determined before it is sent to the disassembly facility. For on-hand recoverable inventory, the recovered item value can be expressed as follows

\[ C^{\text{disc}} \leq \lambda_{r,k} \leq C^{\text{purch}} \]  
\[ C^{\text{hold}} \leq C^{\text{purch}} - C^{\text{dc}} \]

Thus, for the system to be profitable the value of an on-hand item in the recoverable inventory should be greater than or equal to the cost of disposal of the item, and less than or equal to the value of a new procured item from an outside supplier. Also, the cost of holding inventory on hand should be less than or equal the difference between the cost of procuring from third party supplier and the cost of disassembling a returned item.

For on-hand serviceable inventory, the serviceable item value can be expressed as follows

\[ \text{material value} \leq \lambda_{s,k} \leq \text{functional value} \]
\[ C^{\text{hold}} \leq \lambda_n - C^{\text{recovery}} \]

At minimum, the value of an on-hand serviceable item should be greater than or equal to the value of its material contents, and less than or equal to its functional value. The minimum functional value, or cut-off value, represents the value of the materials in the part. Thus, if the functional value is any lower, it is more economical to recycle the materials of the disassembled parts or products than hold it in inventory or sell it to remanufacturing facilities. Also, the cost of holding inventory on hand should be less than or equal to the difference between the value of newly acquired item and cost of recovering a returned item. Correctly assigning the costs to end-of-life (EOL) products and disassembled parts will contribute making the system more economical and cost effective.
The second question, addressed the area of what to procure and when to procure items from an outside supplier. Thus, the following two cases may occur:

**Case 1**

\[ D \leq Q^{ret} \Rightarrow Q^{dc} = D, \quad Q^{disc} = Q^{ret} - D, \quad Q^{pur} = 0 \]  

(5.5)

In case of excess inventory, disassembly facility will disassemble only the demand quantity, after factoring in yields, and any balance quantity can be disposed of if there is no hike in demand in the future period and supply of returned end-of-life (EOL) items assumed to be steady. Thus, no procurement from an outside supplier is necessary.

**Case 2**

\[ Q^{ret} \leq D \Rightarrow Q^{dc} = Q^{ret}, \quad Q^{disc} = 0, \quad Q^{pur} = D - Q^{ret} \]  

(5.6)

In case of shortage inventory, disassembly facility will disassemble all the quantity returned, after factoring in yield, and no balance will be left for disposal. The difference will be secured from an outside supplier. An effective strategy in controlling the inventory level is the implementation of either a PULL or PUSH policy.

**PULL-Policies**

PULL-policies are when disassembly starts only if items are needed to satisfy demand. Using PULL-policy means that a decision has to be made either to send all returned end-of-life (EOL) products to disassembly line immediately or hold them at the recoverable inventory. PULL-policy is used to increase the serviceable inventory level, whenever it drops below a certain specified level.

**PUSH-Policies**

PUSH-policies mean that all returned end-of-life (EOL) products are sent to disassembly at once. Any excess inventory is kept in the recoverable inventory. PUSH-policy should not be used if the return rate is large because it leads to a large average recoverable inventory, which leads to increase in holding costs.
The third question, addressed the relation between the quality of the returned products and its holding cost and market price. As the product is used, its value is consumed and the wear and tear causes the total value of the product to decrease rapidly. The holding cost of disassembled parts and subassemblies is a fraction of its expected market price. In this research, the holding cost is assumed to be no more than 3% of the expected sale price of disassembled part.

As for the market price, as suggested by Gunter, if the price is fixed the calculation and other costs will be relatively simple to determine. However, market price is stochastic and varies as a function of time. Thus, Gunter et al. (Gunter et al., 2004) suggested that the price can be modeled as a random variable with given mean and standard deviation based on historical data, and thus the price data can be modeled using a time-series analysis to capture the inertia in the system using an autoregressive model which provide a reasonable approximation, and in the following form

\[ X_t = \phi X_{t-1} + a_t \]  

(5.7)

Where \( X_t \) the current value of market price, \( X_{t-1} \) is the price at the previous time period, and \( \phi \) is the autoregressive coefficient, and \( a_t \) is a normally independent distributed error with mean \( \mu \) and variance \( \sigma_a^2 \). Due to the absence of historical data of such end-of-life (EOL) products an autoregressive coefficient of 0.99 is assumed to be reasonable compared to other commodities with available data. The error variance \( \sigma_a^2 \) then can be determined using the following formula

\[ \sigma_a^2 = (1 - \phi^2)VAR(X_t) \]  

(5.8)

The coefficient value can be assumed to be equal to 1, then the model will reduce to what is called a “random walk” model where value does not change except for the error when moving from \( t - 1 \) to \( t \).
The fourth question addresses the importance of choosing what selling policy that will positively impact the system’s overall revenue. For example, liquidating inventory in a daily basis can help disassembly facility to clear inventory very quickly, thus allowing the generation of more space for higher value items or high demanded parts and reduction of average holding costs. However, daily liquidation will contribute to higher variable costs such as handling and transaction costs associated with these sales. On the other hand, carrying inventory for the purpose of taking advantage of future price hike or possible shortage in the market could be beneficial but with high risk. In case of market over supply, the prices will eventually go down and the business will suffer from unnecessary costs. For the purpose of this research, limited capacity along the disassembly line and at the different workstations is assumed, and thus the maximum inventory that can be carried on hand of each disassembled part type is limited and known. The planner has to balance between the line disassembly capacity, what is being used to satisfy the demand, and what is kept on hand for future use. Because of the nature of these used items, and also the possibility of containing hazardous materials in these parts, the time between liquidations and how often these items get liquidated is an important area of concern. The previous research questions have been raised early in the research to highlight some of the important areas of the inventory management tools to be used when handling end-of-life (EOL) products. However, details of deriving these conclusions are beyond the scope of this research, yet the models developed can be of a great aid of planning of a profitable business.
5.4 Costs and Cut-off Values Associated with Disassembly Operations

The concept of remaining value is an important determining factor in the disassembly operations decision making process. The widely used assumption is that any end-of-life (EOL) product will follow a pattern of average market price of similar or same products in its category and quality level. However, sometimes prior knowledge of price and expected future demand of items are not available and thus determining the accurate price is not an easy task. Thus, evaluating the functionality of the end-of-life (EOL) product could then be highly beneficial, and that will lead to generating the appropriate disassembly sequence and depth to distract parts that can generate profit. Theoretically, after product is collected from the end customer, it will go through sorting and inspection before it is sent to the disassembly facility. One can assume that inspection stage can determine the quality and functionality level of each and every part that will be disassembled. It is more understandable that inspection can provide a good idea of the product current condition. However, it is very difficult to predict the condition of all and every part that is included in the core product. This lack of knowledge can complicate the disassembly process and operations. Thus, for simplicity, this research assumes that all parts of a hypothetical product will be disassembled regardless of its condition and functionality. Later on, items that proven to be of a low value can be recycled or disposed.
of. Also, beside the planning to fully or partially disassemble a product, identifying the appropriate liquidation channels of these items is very important. Remanufacturing business for instance, will most likely have a minimum standard requirement of accepting parts of certain quality level. Any product that falls below that minimum requirement is disqualified and thus returned to the facility for non-conformity, while on the other hand recycling business will accept all quality levels as long as it consists of precious materials and the fact that capacity of the facility permits.

By looking into the life cycle of the product, it starts with suppliers that provide manufacturing companies with raw materials. After going through the manufacturing process, these raw materials are transformed to parts that after assembly become an important element of the final product. During this transformation, the part has gained added value after going through the process, and once combined with other parts, their total combined value as a final product is much higher than their individual values. In other words, parts are more valuable as a product than they are individually. Once the product is used, its value will decrease due to wear and tear factor. At the end of its life cycle, the value of the end-of-life (EOL) product does not necessarily be higher than the value of its parts. Thus, there exists two ways of determining the value of a product at the end of its life cycle: materials value or functional value.

In a previous study conducted by Akcali and Bayindir (Akcali and Bayindir, 2008) they have investigated the opportunity costs and inventory associated costs in a disassembly environment. The authors have investigated six rules to determine the opportunity cost. The decision of which rules will apply will purely depend on the
planner, type of product is being disassembled, and the disassembly facility in terms of operation. Typically, the inventory holding costs is assumed to be a fraction of the value of the carried inventory in order to reflect time value of money, and this concept will be assumed throughout this research. This cost is then used in the mathematical model with an objective of minimizing cost or maximizing profit (Akcali and Bayindir, 2008). This is known as Average Cost (AC) approach and is used widely among researchers as an approximation. The main trade-off in keeping a part in inventory is the relative revenue obtained from the liquidation of these parts, and the costs associated with satisfying the demand of these parts. However price determination is not identical even for the same part and that depends on the quality, quantity, and time of sale. Also, the inventory holding costs of the same item is not identical and it depends on the quality, space it occupies along the line, and the value remaining (either material value or functional value). These characteristics of a recovered part make it complicated to determine the inventory carrying cost, as it is not clear how to account for the costs incurred during the collection, inspection, and disassembly operation of end-of-life (EOL) product (Akcali and Bayindir, 2008).

In a typical disassembly facility setup, end-of-life (EOL) products arrive at the facility according to a Poisson process with mean rate $\lambda_r$, for simplicity this assumption might be relaxed and an equivalent deterministic value is used. Also, the demand for disassembled parts is driven by Poisson distribution with mean rate $\lambda_d$. Assuming a reverse network exists (collection, distribution center, and transportation to origin) the
disassembly facility will incur a unit cost of $C_{i,t}^p$ for each end-of-life (EOL) product that is collected and transported back to its origin, including any cost that might be paid to the owner to repossess the item. After that, a unit disassembly cost is incurred $C_{i,k,t}^{dc}$ to disassemble the end-of-life (EOL) product. This cost varies as per the quality of the product. After the disassembly operation is performed, demand is satisfied from disassembled parts generating revenue $P^s$ per unit part. The remaining parts are carried in inventory incurring a holding cost of $C_{f,t}^{hols}$ per part per unit time. The same concept applies to the core item as well. Other costs such as disposal cost, recycling cost, transaction cost are applied when appropriate.

As per Akcali and Bayindir (Akcali and Bayindir, 2008), the unit inventory holding cost is comprised of two components: out of pocket cost, and opportunity cost. These costs are defined as follow:

(i) Out-of-pocket cost: it is related to the physical storage and handling of end-of-life (EOL) products and disassembled parts/subassemblies; and

(ii) Opportunity cost: also known as cost of capital, which is a fraction of the amount tied up in inventory.

Thus, inventory carrying cost of core product is straightforward and is given by:

$$C_{i,k,t}^{hole} = h_c + \phi C_{i,t}^p$$  \hspace{1cm} (5.9)

However, setting the inventory holding cost of a specific disassembled part is more difficult since parts of the core product will share the different costs incurred during the operation including cost of acquisition, cost of disassembly, cost of sale transactions, and
any value added costs such as cleaning or minor repair. Thus three approaches are common in estimating the unit inventory holding cost of the disassembled part:

(i) Physical measure based, or amount of space consumed by one unit

(ii) Market value based, or most attainable value based on market condition

(iii) Recovered value based, or remaining value either material or functional

Thus, the shared costs incurred during the operation is split between all the parts and subassemblies according to a predefined criterion, with a fraction $f_j$ for each part disassembled. Thus, the unit inventory holding cost can be calculated as follow:

$$C_{j,k,t}^{\text{hols}} = h_s + \phi((C_{i,k,t}^p + C_{i,k,t}^{dc})f_j + C_{j,k,t}^{\text{tras}})$$

(5.10)

Other costs of disposal and recycling, as mentioned above, are assumed to be zero if parts are disposed of or recycled in the same period, hence no holding cost incurred. However, if parts are on hand more than one period thus equation 5.10 will apply with the extra incurred cost of $C_{j,t}^{\text{dis}}$ and $C_{j,t}^{\text{rec}}$ added to the equation, thus, in case of disposal:

$$C_{j,k,t}^{\text{hols}} = h_s + \phi((C_{i,k,t}^p + C_{i,k,t}^{dc})f_j + C_{j,k,t}^{\text{tras}} + C_{j,t}^{\text{dis}})$$

(5.11)

And in case of recycling, the equation will become:

$$C_{j,k,t}^{\text{hols}} = h_s + \phi((C_{i,k,t}^p + C_{i,k,t}^{dc})f_j + C_{j,k,t}^{\text{tras}} + C_{j,t}^{\text{rec}})$$

(5.12)

Under the physical measure based approach, the allocation of the fraction factor is related to the volume of the part and the space it consumes, the weight, or the number of parts in the original assembly. Under the market value based approach, it depended on the selling policy, market price, and demand-supply relationship; this will affect the revenue that can be attained. Under the recovered value, a more intensive testing is required to
determine the exact value remaining by testing the functionality of item. According to Akcali and Bayindir (Akcali and Bayindir, 2008) the following inventory rules can be applied when studying a disassembly system to determine the fraction \( f_j \). Using different rule will return different results. Choosing the correct rule will depend of the decision planner and the type of products that is being disassembled in the facility based on the three approaches above:

- Physical measure based approach

  1. *Volume or weight of the disassembled part*: This approach assumes that the out-of-pocket cost is related to the disassembled part volume or weight. Assuming that the parts of an end-of-life (EOL) product share the opportunity cost of holding the inventory similarly, i.e. according to their relative volume or weight. Thus, the allocation fraction for the opportunity cost can be based upon the ratio of the out-of-pocket inventory holding costs incurred for the product and the disassembled part, thus

\[
f = \frac{h_p}{(h_p + h_c)}
\]

(5.13)

  2. *Number of parts J*. Assuming that the parts are “uniform”, and thus the cost incurred can be allocated equally between them, thus

\[
f = \frac{1}{J}
\]

(5.14)

Physical measure approach is most common in the disassembly setup, mainly because of lack of market and price data of end-of-life (EOL) products. However, other approaches are also common.
Market based approach

1. *Sales value approach.* It is possible to set the inventory carrying cost proportional to the sales price of a disassembled part. In this case the allocation fraction for each part is proportional to its sales price, thus

$$ f = \frac{P^s}{P^s + P^{balance}} $$  

(5.15)

2. *Estimated net realized value.* Also, it is possible to set the inventory carrying cost proportional to the net revenue obtained for each part. In this case, the allocation fraction for each part is proportional to its net revenue, that is profit minus cost, thus

$$ f = \frac{(P^s - C^s)}{(P^s - C^s + P^{balance})} $$  

(5.16)

It is notable that market based approach is more suitable to use when the disassembly facility is specialized in the operation and sale of one major part rather than different parts.

Recovered value based approach

1. *Balance recovered value.* The sale of the balance of an end-of-life (EOL) product will possibly generate a revenue that can pay for the costs incurred during disassembly, and thus the fraction allocation can be assumed to be

$$ f = \max \left\{ C_{i,k,t} + C_{j,k,t}^{dc} + C_{j,k,t}^{mas} - P^{balance}, 0 \right\} $$  

(5.17)
2. *Net recovered value.* This approach will ignore any revenue collected from the balance, and assume that inventory value equal cost incurred, thus reducing equation 5.17 to

\[ f = C_{i,k,t}^p + C_{i,k,t}^{dc} + C_{j,k,t}^{trans} \quad (5.18) \]

Mainly in this research, the first approach, physical measure based will be assumed. Other approaches are suitable in different scenarios mainly when disassembly facility is involved in the disassembly and recovery of a high value item, such as car engines for example. The above approaches assume that disassembled part is valuable and should be kept in inventory, thus inventory carrying cost is incurred. To determine whether a disassembled part should or should not be carried on will depend on the facility capacity, available space, and if the part is worth keeping.

As the part is used, the value is consumed and its value will deteriorate rapidly. If the part failed at any stage and cannot be repaired, then the part is assumed to be zero value from functionality point of view. In such case, the part value is considered to be equal to the value of its material content. Although the part may hold some value, it is possible that the disassembly and remanufacturing process could be very expensive economically and does not generate any profit financially. Thus, according to Gunter (Gunter, 2004) it is reasonable to assume that the minimum acceptable value (lower limit), known as cut-off value, is the materials value, and if the value is below that then it is more economical to recycle the part rather than to remanufacture it or refurbish it. Also, even if that value is higher than the minimum value it might be still low to justify
any remanufacturing operation, thus a higher value (upper limit) should be determined, and any value above that limit will be considered for proper recovery.

5.5 Conclusions

In this chapter, classification of end-of-life (EOL) products inventory is given and discussed: recoverable, serviceable, and recyclable inventory. This helps the reader identify and distinguish between returns, and implement the appropriate inventory policy. The main research questions of what to disassemble, when to disassemble, and how much to disassemble and liquidate are highlighted to provide a clear road map of what are the important indicators to look for when studying a disassembly inventory policy. This chapter provides an overview of different authors’ approaches in determining costs associated with disassembly, and prices of end-of-life (EOL) products that goes through disassembly, thus ensuring the profitability of the facility. It is important to note that, although these approaches are valuable to the industry, not all approaches are applied in the coming chapters of this dissertation. Whenever appropriate, reference is given to any assumption and approach used in the methodology.
Chapter 6

Inventory Balancing in Disassembly Line: Linear Programming Approach

Disassembly line is crucial in recovering products in large quantities. It has recently gained a lot of attention from researchers, government agencies, and private entities due to its role in efficiently recovering valuable materials, parts, and subassemblies. In order to ensure continuous profitability of the stand alone disassembly facilities, inventory management tools have to be developed to maximize the system’s profit and minimize its cost. In this chapter, an inventory planning model is developed to predict the optimal selling strategy to minimize the inventory balance on hand.

6.1 Introduction

Due to the disparity between the demand of disassembled parts and subassemblies coming from end-of-life (EOL) products and the actual line yields, there are many
inventory problems that arise during the disassembly process. Although inventory management in disassembly context might share some operational characteristics with the traditional inventory management tools, it differs in several important factors. First, the end-of-life (EOL) products tend to “explode” when it goes through disassembly operations. This characteristic complicates the inventory planning further. Second, the uncertainty about the quality of returned products, quantity of items supplied and demanded, timing of returns, knowledge about previous modifications that have been done to the end-of-life (EOL) products or the working environment that product used to operate in could significantly affect the overall performance of the line, hence the management of the product flow. Therefore, decision such as the appropriate level of inventory to maintain in every period becomes very challenging. This chapter focuses on developing and testing a mathematical model in a multi-period environment that address the effect of product characteristics and government legislation on the inventory level of end-of-life (EOL) products. Although uncertainty is a major factor when addressing the inventory management problem in disassembly line, yet we assume deterministic input data for the demand of disassembled parts and the supply of end-of-life (EOL) products. Firms involved in the disassembly activities require production planning and control system. Such systems will allow decision makers to examine the system’s profitability before engaging in any recovery process. It will help providing a clear picture of the system’s behavior under certain line restrictions and capacity limitations.
6.2 Important Issues in Inventory Management of End-of-Life (EOL) Products

As stated in previous chapters, in order for standalone disassembly facilities to be economically profitable, an efficient and cost effective inventory management tools must be developed. These tools are necessary to minimize the inventory costs of the system. In disassembly environment the traditional management tools can be applied with modification to cope with the nature of the reverse supply chain. Original equipment manufacturers (OEMs) and disassembly facilities face similar challenging inventory problems. Disassembled parts demand uncertainty, supply of end-of-life (EOL) products uncertainty, and yield of parts uncertainty are the major problematic factors when studying the inventory problem in disassembly line.

Environmentalists have always demanded original equipment manufacturers (OEMs) to be more involved and be responsible for their products at the end of its life cycles. However, the uncertainty factor that is associated with the disassembly operations discourages OEMs from participating in such programs. Appropriately handling and accounting for uncertainty in disassembly operations contribute to the performance of the system and the profitability of the disassembly operations. The take-back program could possibly cause financial drain for the OEMs if not managed well and if the concerns raised above are not addressed effectively.
6.2.1 Handling Products Yield Uncertainty

In a disassembly line, inventory issues arise because of the fluctuation in the line yields. Workstations along the disassembly line tend to experience different accumulation rates of disassembled parts and subassemblies, and different depletion rates. The main reason behind that is the outside demand of the end-of-life (EOL) parts and subassemblies. Such differences create uncertainties in inventory levels of these parts and space requirements that should be allocated to store it at the workstations. For example, for an end-of-life (EOL) product, each part $i$ of the product will be disassembled and made available to demand sources with a probability $\gamma_j$, and will be recycled or disposed of with a probability of $1 - \gamma_j$. Thus the yield number of parts, or quantity disassembled of parts/subassemblies $Q^\text{diss}_j$ is a product of the quantity returned in any given period $Q_i^{\text{ret}}$ and the probability of the parts will pass for disassembly operations $\gamma_j$. For the purpose of simplicity we assume that the variables in the following equations, equation 1 and 2, are deterministic. It is more realistic to that assume these variables are probabilistic, thus covering all range of output. The number of parts $i$ disassembled is given by

$$Q^\text{diss}_j = Q_i^{\text{ret}} \cdot \gamma_j$$  \hspace{1cm} (6.1)

And the number of parts $i$ that is being recycled $Q^\text{recy}_j$ is given by

$$Q^\text{recy}_j = Q_i^{\text{ret}} \cdot (1 - \gamma_j)$$  \hspace{1cm} (6.2)

To illustrate, consider the example of a cell phone. For the purpose of this example, assume only two parts are to be disassembled from the phone, with part (1) is
the back cover and part (2) is the keypad. Furthermore, assume that the back cover has a probability $\gamma_1 = 0.65$ to pass inspection and the keypad has a probability of $\gamma_2 = 0.40$ to pass inspection. If 1000 ($Q^\text{ret}_1 = 1000$) cell phones were returned to be disassembled, we will expect a total of 650 back cover ($Q^\text{diss}_1 = 650$) and a 400 ($Q^\text{diss}_2 = 400$) keypads to be retrieved respectively from the end-of-life (EOL) products. Although many planners assume that the yield is deterministic, it is highly uncertain and changes based on the quality of the end-of-life (EOL) products and the environment it was operating in. For the purpose of simplicity of this model, deterministic data are used to illustrate the approach.

6.2.2 Handling Demand and Supply Uncertainty

Among many, demand and supply uncertainty is a major challenge for inventory management of end-of-life (EOL) products. Due to the unpredictable products failures demand and supply can fluctuate. When planning for inventory, planners can assume a dependent demand and supply of end-of-life (EOL) products or an independent demand and supply of end-of-life (EOL) products. In the former, there is a perfect correlation between both. In other words, supply of items is assumed to be a percentage of the market on hand inventory. While in the later, there is no correlation between the demand and supply of these items.

The management of inventory in this case becomes very challenging compared to forward supply chain scenario because it is much more reactive and less visible. The demand rates could change from period to period, and will fluctuate depending on the
items quality and as per the requirements of the demand sources. Also, planners usually deal with many different demand sources that apply different buying rules, and request a wide range of mixture of parts. Also, the lead time although it is not unique to the disassembly line, however it can complicate the planning further.

Traditional inventory management in the forward supply chain is managed using a Materials Resources Planning (MRP) tools. Parts are accounted for using this tool as per the master schedule, and then ordered from the suppliers at point in time using the lead time information available, thus parts can arrive just in time for assembly. If necessary, a safety lead time and a safety stock might be added to account for uncertainty. To illustrate, take the example of the 1000 cell phones that is demanded in the market. Thus a total 1000 keypads and a 1000 back covers will be required for the assembly operations. These items will be ordered to be arrived on time when assembly operations is about to start. However, in disassembly environment, demand for disassembled parts can occur at anytime with no pre-established rule such as in MRP technique. For the disassembly facilities, if no enough inventories is available at the time of order, the difference is secured by ordering from a third party suppliers of new items to satisfy the demand, which experience a much less uncertainty in products quality and lead time. Yet, this will result in extra costs of procuring new parts and transaction costs associated with the order, and sometimes the disassembly facility might have to offer a price discount as a compensation for the waiting time while placing a backorder on the required parts.
Similarly, when looking at the supply of end-of-life (EOL) products from customers back to the origin, this portion is affected by many factors. In lease and rental programs for vehicles, electronics, and furniture for example a better planning of when items will be sent back and in what quantity could be accounted for. The age of product that is being returned and how long it has been used in the market widely affect the quality of parts and subassemblies that can be recovered. Also, the nature of the working environment these products were used in and the amount of wear cause the parts quality to deteriorate rapidly even if the product has been introduced to the market recently. This task of correctly forecasting the supply is very difficult when associated with warranty returns, repair, and product up-gradation for more technologically advanced items especially with companies’ expansions in take-back programs.

For simplicity, demand and supply of end-of life (EOL) products and parts are assumed to be independent. Furthermore, it’s assumed to be known and constant over the planning horizon. Mainly, the planner faces economical constraints (incurring holding and transaction costs of excess or shortage of inventory) and physical constraints (capacity and space limitations along the workstations) when trying to make an informative decision of how many products to take back and when to take them back, and for how long to keep them before they are liquidated or disposed of.
6.2.3 Handling the Space Capacity of Workstations

In a disassembly line, workstations are where disassembly operations take place. Each end-of-life (EOL) product is routed through all the workstations, if necessary, for different disassembly operations. Each workstation is allowed a predefined cycle time before the product is sent to the downstream workstation. At a workstation, the disassembly operation is performed to extract parts and subassemblies from the core product. Each disassembled part that passes the inspection is sent to the inventory storage bin, from where they are grouped in batches and sent to the demand sources. All damaged parts that fail the inspection test are sent to the recycling bin to be recycled. The residual, which might include hazardous materials, is finally disposed of. The characteristics of the product to be disassembled play a role in planning for inventory. The size and shape of the parts, and the amount of space designated at the workstation along the disassembly line is scarce.

The available space at each workstation is a limited resource. The space is expressed in cubic unit designated as, $W$, and each part disassembled on the line consumes a certain amount of space, $w$. The space relationship is given by,

$$\sum_{i=1}^{Q^{\text{dis}}} w_j \leq W$$  \hspace{1cm} (6.3)

Where $Q^{\text{dis}}$ is the total quantity disassembled of part $j$ at any given time. By assuming each workstation has a limited storage space that can carry a maximum of $INV_j^{\text{max}}$ parts, then,
\[ INV_j^{\text{max}} \cdot w_j \leq W \]  \hspace{1cm} (6.4)

At any time during the disassembly operations, the above measures can be calculated: consumed space, available space, and the accumulation rates of the different parts.

All recyclables of part \( j \), are collected in a recyclable area, and sold at pre-determined price to raw materials suppliers. These are processed and sold to parts manufacturers, which will be eventually used in assembly of new products. The mass (or volume) of recyclables is given by \( \delta_j \),

\[ \delta_j = \sum \psi_j \cdot Q_j^{\text{recs}} \]  \hspace{1cm} (6.5)

Where \( \psi_j \) is the mass to unit (or volume to unit) mapping conversion. To illustrate, consider a disassembly line with three work stations, \( M = 3 \), and each workstations has a limited space. That means, each workstation can carry inventory up to a certain level before blockage occurs. For simplicity, we assume equal space for all workstations, thus using equation 6.4; the maximum inventory can be calculated per Table 6.1 below

<table>
<thead>
<tr>
<th>Size / Workstation</th>
<th>Workstation No. 1</th>
<th>Workstation No. 2</th>
<th>Workstation No. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W )</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>( w )</td>
<td>0.10</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>( INV_j^{\text{max}} )</td>
<td>1000</td>
<td>200</td>
<td>400</td>
</tr>
</tbody>
</table>
Thus workstation 1 can hold up to 1000 disassembled part type $j = 1$, workstation 2 can hold up to 200 disassembled parts type $j = 2$, and workstation 3 can hold up to 400 disassembled part type $j = 3$.

### 6.2.4 Handling Logistical Operations Uncertainty

Other areas in the disassembly line that need attention are the logistical operations of the line. Recovering end-of-life (EOL) products often creates a lot of complications in the reverse supply chain. Complications in planning, scheduling, and handling are very common. For example, hazardous materials require special handling to avoid contaminating neighboring products and parts, and might require skilled workers and specialized materials handling equipments for proper delivery of disposing it. Also, the transportation and collection centers planning is a major part of the chain that requires intensive planning and scheduling. As these areas are beyond this research interest, the model assumes a current transportation and collection network exists.

One unique characteristics of the disassembly line is that it is a divergent process, unlike the regular assembly line which is a convergent process. Work-in-process (WIP) is a source of planning concern because of the number of work pieces on a disassembly line tends to “explode” (one work piece splits into two or more work pieces after disassembly). A proper handling is required to avoid problems in scheduling and holding these items in inventory. For simplicity, it is assumed that one part is disassembled per workstation.
6.3 Inventory and Value Management of End-of-Life (EOL) Products

One of the most challenging issues in reverse logistics in general and in disassembly line in particular is the inventory and value management of end-of-life (EOL) products and disassembled parts and subassemblies. The uncertainty associated with the operations of disassembly line complicates the planning of inventory levels. This research assumes that disassembly facilities have no control of the return flow of end-of-life (EOL) products. The disassembly facility is obligated to handle its products that are currently in the market and is being returned for end-of-life disposal. The disassembly facility assumes the role of “middle agent” or supplier of disassembled parts and subassemblies, and raw materials provider, to the manufacturers and remanufacturers.

6.3.1 Inventory Management Problem in Disassembly Line: Overview

In this section, the disassembly line and disassembly operation whose general framework is shown in Figure 6.1 is considered. End-of-life (EOL) products are being collected from end customers at its end of life cycle and brought back through the reverse supply network to a disassembly facility. In many situations, there exists a centralized collection center where these products are usually being tested for functionality, graded
based on its quality levels and sorted into different groups before it is finally sent to a disassembly facility. In most cases, end-of-life (EOL) products are functional; however customers return them to upgrade for a state of the art technology thorough advertised trade-in programs. Also, there are the take-back programs (TBP) and warranty returns that account for a big share of return volume.

![Disassembly line operations overview](image)

Figure 6.1 Disassembly line operations overview

The returned end-of-life (EOL) products are disassembled into separate parts and materials. For large quantities of returned products, the best way to disassemble them is to do it on a disassembly line that consists of several disassembly workstations. The main purpose of disassembly operation is to satisfy the demand sources requirements of disassembled parts and subassemblies, and any precious materials that can be recovered. The balance is either kept in inventory to satisfy future demand of these parts or disposed of if it has no value or it is economically more feasible to incur some disposal cost or transaction costs rather than carrying it in inventory for potentially higher profit in the
future. The decisions involved in determining what type of parts and subassemblies should be kept in inventory against exercising the options of liquidating, disposing, or recycling these items become very important. Also, timing of these decisions is critical to the profitability of the system because all items kept in inventory incur holding costs and carry a risk of obsolescence. For further details, refer to Gunter (Gunter, 2004).

As discussed before, there are many significant differences between the forward supply chain and reverse supply chain. In a regular assembly line, the focus is on the newly procured parts that become part of the final assembly. The inventory of these items tends to reduce as the production process proceeds. In a disassembly line, the focus shift to the return stream of end-of-life (EOL) products, also known as core products. The inventory of these items, and its parts after going through disassembly operations, is complicated, since parts tend to accumulate and build up as the process continues and then get liquidated in batches when demand occur. The main cost contribution in disassembly line context is in carrying and holding costs of these parts because of the wide range of quality levels that can be carried and the actual worth of these items. Also, unlike the traditional assembly line there are disposal costs and transaction costs associated with disposing and liquidating the inventory. The objective of the system is to control the inventory levels of core products and disassembled parts in the system to satisfy demand sources and minimize the overall associated costs.

Many researchers have considered inventory models in repair or hybrid systems, where manufacturing and remanufacturing are considered together. This model assumes an independent disassembly facility that act as a middle agent to manufacturers. With this in consideration, this model addresses the need of an inventory system analysis using optimization techniques.
6.3.2 Inventory Management Problem in Disassembly Line: Problem Definition

In reverse logistics environment the control and monitoring of incoming and outgoing end-of-life (EOL) products and disassembled parts and subassemblies are more difficult than traditional methods in a forward supply network. The inventory problem in a disassembly context stems from the disparity between the demand for disassembled parts and subassemblies and the actual disassembly line yields. The variation in yields is mainly linked to the returned products quality levels. Disassembly facilities handle a large quantity of end-of-life (EOL) products with different quality levels. In this research, three quality levels are used to categorize the level of returned items. Items can be in a good quality level, average quality level, or imperfect quality level. For simplicity, this model assumes that products with any quality level most likely will yield disassembled parts with similar quality level. Handling the demand and supply uncertainty will result in workstations experiencing different accumulations rates of disassembled parts in its sub-store inventory level, and similarly different depletion and consumption rates of items when demand for these items occur. The difference in flow rates (flow-in and flow-out) will cause the piling of these parts at the designated areas at each workstation. Such differences create uncertainties in inventory and space requirements at the workstations. In some cases this behavior does not interfere with the work because of the parts size and quick turn over of these items. In other cases, the accumulation of inventory could block the workstation which will affect the disassembly operations and the facility’s ability to meet the demand.
Thus, when planning for disassembly operations some constraints are taken into account. In this model, the physical constraint is of interest to simulate the rapid buildup of disassembled parts and derive an optimal control policy. Physical constraints include the capacity of the disassembly facility in terms of units disassembled per 8 hours working day. Also, the government regulation and limit for disposal and recycling are also incorporated in the model. This model addresses the effect of these parameters on the system performance and overall costs.

Each part is disassembled in one separate workstation. At each workstation, there exists two storage bins: recoverable bin and recyclable bin. Parts that qualify for further conditioning operations such as remanufacturing or refurbishing are kept at the recoverable bin. While others that do not pass testing are kept at the recyclable bin. Demands for disassembled parts are met from the recoverable inventory, and any excess parts are either kept at the storage bin if space permits or sent to remote storage facility. Demands for raw materials are met from the recyclable bin and being liquidated after converting units to mass or volume.

Linear programming (LP) model is developed to simulate the inventory buildup and the liquidation of parts to the different demand sources based on quantity availability, capacity of demand sources, and the actual line yields. Recyclable inventory are converted to mass/volume and being sold out to raw materials suppliers. The problem is modeled using Lingo 11.0 optimization software.
6.3.3 Inventory Management Problem in Disassembly Line:

Assumptions

The inventory management problem in a reverse logistics settings involves many interrelated decisions. This model assumes a periodic review analysis over a defined planning horizon. A number of assumptions are made to simplify the analysis of the inventory system:

- The end-of-life (EOL) product is totally disassembled, or it maintains the same level of disassembly for all products.
- There is a single end-of-life (EOL) product with three parts, part A, part B, and part C.
- Core products have three quality levels: good, average and imperfect.
- Demand for disassembled parts and supply of end-of-life (EOL) products is known.
- All disassembled parts can be sold, or either recycled, or disposed of if it does not pass inspection.
- All end-of-life (EOL) products can be accepted by the disassembly facility and can be carried for future periods.
- Allowing the use of government limitations on disposal and recycling.
- The following notations are used: \( I \) for core products, \( J \) for parts and subassemblies, \( K \) for the different quality levels of core products, and \( T \) representing the time period allowed for disassembly.
- Unlimited supply of end-of-life (EOL) products exists.
In the following section, two-phase problem scenario using a Linear Programming (LP) technique is formulated to provide a management tool to the inventory problem in a disassembly line. For each of these phases, the problem objective is given; and problem constraints and known parameters are given. Also, the decision variables are given and discussed in details. The problem output is analyzed using Lingo 11.0 optimization package and spreadsheet analysis.

6.4 Inventory Management Problem Formulation

In this section, a modeling approach to the problem based on optimization technique is presented. The inventory problem in disassembly is modeled over a finite planning horizon. At the end of each period, the quantity disassembled $Q_{j}^{\text{dis}}$ of every part type $j$, and the quantity remaining on hand at the end of every period are determined.

What makes this problem unique is that (1) it considers the space allocation and availability along the disassembly line since space is a scarce source and thus it is an important factor when planning for handling disassembled parts and subassemblies, and (2) it takes into account the limits on disposal and recycling of end-of-life (EOL) products to comply with the government targets. This section discusses the mathematical structure of the model and the development of the objective function and problem constraints. For simplicity, the model assumes deterministic input data. The model is developed for a single hypothetical product over a multi-period planning horizon. The objective of the model is to minimize the system’s total cost and minimize the inventory carried from one period to another. Also, any shortages in the demanded disassembled parts/subassemblies are procured from an outside supplier for additional costs. The model
assumes that all recoverable parts and subassemblies are not distinguished against by the demand sources, thus they are treated equally, while returned core end-of-life (EOL) products will be categorized based on their quality levels.

The objective of the system is to control the inventory levels of core products and disassembled parts and subassemblies in the system to satisfy demand sources and minimize the sum of variable costs.

### 6.4.1 Multi-Period Problem: Phase One Numerical Results

The problem formulation is designed to solve for two important decision variables. It is designed to solve for:
- The end-of-life core products to be kept on hand and to be disassembled
- The disassembled parts and subassemblies to be carried in inventory, recycled, or disposed of

The problem formulation is given by equation (6.6) through equation (6.15).

\[
\begin{align*}
\text{MIN} & \sum_{i} \sum_{k} \sum_{t} \left( C_{i,k,t}^{dc} \cdot Q_{i,k,t}^{dc} + C_{i,k,t}^{disc} \cdot Q_{i,k,t}^{disc} + C_{i,k,t}^{recc} \cdot Q_{i,k,t}^{recc} + C_{i,k,t}^{holc} \cdot Q_{i,k,t}^{holc} \right) \\
\text{ST} & Q_{i,k,t}^{inv} = Q_{i,k,t}^{inv} + Q_{i,k,t}^{ret} - Q_{i,k,t}^{dc}, \forall i, k, t \\
& Q_{j,t}^{inv} = Q_{j,t}^{inv} + \sum_{i} Q_{i,k,t}^{dc} + Q_{j,t}^{pur} - Q_{j,t}^{disc} - Q_{j,t}^{recc} - D_{j,t}, \forall j, t \\
& \sum_{i} \sum_{k} T_{i,k,t} \cdot Q_{i,k,t}^{dc} \leq AT_{i}, \forall t
\end{align*}
\]
\[ \sum_k \sum_i Q_{i,k,t}^{dc} \leq CAP^{dc}, \forall t \]  
(6.10)

\[ \sum_k w_i \cdot Q_{i,k,t}^{inv} \leq WC_i, \forall i,t \]  
(6.11)

\[ w_j \cdot Q_{j,t}^{inv} \leq WS_j, \forall j,t \]  
(6.12)

\[ \sum_k \sum_i \psi_i \cdot Q_{i,k,t}^{dis} + \sum_j \psi_j \cdot Q_{j,t}^{dis} \leq CAP^{dis}, \forall t \]  
(6.13)

\[ \sum_k \sum_i \psi_i \cdot Q_{i,k,t}^{rec} + \sum_j \psi_j \cdot Q_{j,t}^{rec} \leq CAP^{rec}, \forall t \]  
(6.14)

\[ Q_{i,k,t}^{ret} \geq Q_{i,k,t}^{dc} \geq Q_{i,k,t}^{rec} + Q_{i,k,t}^{disc}, \forall i,k,t \]  
(6.15)

The non-negativity constraints apply for all variables, All variables \( \geq 0, \forall i,j,k,t \).

The objective function (6.6) minimizes the sum of disassembly, disposal, recycling, and holding costs of all core products and disassembled parts and subassemblies, where the decision variables are the associated quantities of core and disassembled parts. The following two terms are straightforward calculations of the on-hand inventory that is carried on from one period to another. In equation (6.7) it calculates the inventory of end-of-life (EOL) core products and keeps a separate count based on its quality levels. While in equation (6.8) the same concept applies, however all disassembled parts, assumed to be accepted by the demand source “as is”, thus treated equally when on hand inventory is calculated, and then any procured items from an outside source are added to the serviceable inventory. The holding of items is stochastic in nature, thus it is treated alike. The calculation of the expected excess inventory is also given by

\[ E(Q_{i,k,t}^{inv}) = \sum_{Q_{i,k,t}^{inv} = 1}^{INV^{inv}} (Q_{i,k,t}^{ret} + Q_{i,t}^{purc} - Q_{i,k,t}^{dc} - Q_{i,k,t}^{rec} - Q_{i,k,t}^{disc}) \cdot P(x = Q_{i,k,t}^{inv}) \]  
(6.16)
And thus the total holding costs is calculated by multiplying the number of items on hand by the holding cost for a single period. In some cases, holding costs could vary from one period to another, and thus total holding costs is calculated by summation of all periods costs. The calculation for the excess inventory of disassembled parts and subassemblies is the balance of quantity disassembled minus the quantity demanded after excluding recycling and disposal volume. Equation (6.9) ensures that time available for disassembly operation for all quantity to be disassembled is within the allowed time. In most cases, the assumption is that disassembly facilities work 8 hours shift per day, 5 days a week. Equation (6.10) ensures that the disassembly facility capacity is met during all periods, and the amount to be disassembled is less than or equal to the total facility capacity. The following two equations (6.11) and (6.12) ensure that the space allocation along the workstations for core products that were returned and for the disassembled parts and subassemblies are enough. Every workstation has a limited space availability to store end-of-life (EOL) products that yet to be sent to disassembly operations, and for already disassembled parts that are ready to be sent to demand sources. After satisfying the demand, the amount carried on should be within the available space limits. The last two equations of the problem formulation take into account the government regulations and the set target for disposal and recycling quota. The quantities to be disposed of and recycled are to be converted using a unit to mass or (volume) conversion factors, and then liquidated based on its volume. The conversion is necessary since recyclable materials are considered for the raw materials suppliers. In the next section, the problem
methodologies and numerical calculations is given to illustrate the approach. The problem complexity is also described briefly.

### 6.4.2 Problem Methodologies and Complexity

Because of the problem nature, and the uncertainty associated with the disassembly operation, usually the solution space is very large. Take for example the range of the quantity of core products to be disassembled in any period of time $Q_{i,k,t}^{dc}$ of all quality levels is bounded by an upper and lower limits that is given by

$$\text{Range of } Q_{i,k,t}^{dc}, \forall k = [1, \min(Q_{i,k,t}^{ret}, CAP_{i}^{dc})]$$

(6.17)

Multiplying equation (6.17) by the feasible quantity to be procured from an outside supplier will result in the space solution that tend to be very large even for smellers core products with 3 parts. To illustrate the approach, the following numerical example is given. The problem is based on a hypothetical product that consists of three parts $ABC$, the input data is assumed to be deterministic. The demand for the disassembled parts over the planning horizon of five periods is given by Table 6.2
Available space allocated in the disassembly line for core end-of-life (EOL) products is 250 cubic units ($WC_{1,2,3,4,5} = 250$), and the available space allocated along the three workstations is 100 cubic unit each ($WS_{j=1,2,3,5} = 100$). The amount of space consumed by one unit of core product is 0.70 ($w_{i=1} = 0.70$) and the amount of space that is consumed by one unit of disassembled part is given by the set $w_{j=1,2,3} = \{0.10,0.20,0.025\}$ for parts $A$, $B$, and $C$ respectively. The unit to mass (or volume) conversion for core products is $\psi_{1} = 0.625$, and the unit to mass (or volume) conversion for the parts is given by the set $\psi_{j=1,2,3} = \{0.225,0.35,0.05\}$. The available time
at each time period is 28,800 seconds (or 8 hours per day), and the time to disassemble a core product is given 7.50 s, 13 s, and 19 s for good, average, and imperfect quality level of returns respectively. The government regulations on disposal and recycling are set to 200, and 600 cubic units respectively. The associated costs of core and parts are given by the following two tables respectively.

<table>
<thead>
<tr>
<th>Table 6.4 Costs associated with returned EOL product</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EOL Product</strong></td>
</tr>
<tr>
<td>Quality level 1</td>
</tr>
<tr>
<td>Holding Cost $/unit</td>
</tr>
<tr>
<td>Disposal Cost $/unit</td>
</tr>
<tr>
<td>Recycling Cost $/unit</td>
</tr>
<tr>
<td>Disassembly Cost $/unit</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 6.5 Associated costs with the parts/subassembly level</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Part Level</strong></td>
</tr>
<tr>
<td>Part A</td>
</tr>
<tr>
<td>Holding Cost $/unit</td>
</tr>
<tr>
<td>Disposal Cost $/unit</td>
</tr>
<tr>
<td>Recycling Cost $/unit</td>
</tr>
<tr>
<td>Purchasing Cost $/unit</td>
</tr>
</tbody>
</table>

The approach used to solve the problem is based on a deterministic linear programming (LP) model that allows the backorder from an outside supplier in case of
shortage. The stochastic nature that stems from the uncertainty in the line yield has been reduced to its deterministic expected value in this model. This constraint is dealt with in more details in the next chapter. Because of that, usually using a deterministic model in an environment that is highly uncertain will results in a high cost solution when compared to the stochastic model. However, it is easier to use and implement in industry.

The disassembly process start with the end-of-life (EOL) core product being collected, sorted, and graded based on its quality and then sent to disassembly facilities to resume disassembly operation. Then it will be separated into three different bins for good, average, and imperfect quality. The assumption is that all disassembled products will add 1 unit to the on hand inventory. Then, demand for these disassembled parts will occur in every period and will be satisfied from the disassembled parts bins. Demand sources accept all quality levels. After satisfying the demand, parts remain in inventory is either carried on to the next period, recycled, or disposed of thus the overall cost of the system is minimized. For the example above, the near-optimal solution that balances the system inventory and minimize all variable costs are given in tables 6.6 thru 6.9.

<table>
<thead>
<tr>
<th>Period</th>
<th>Quality level 1</th>
<th>Quality level 2</th>
<th>Quality level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>680</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Period 2</td>
<td>589</td>
<td>211</td>
<td>0</td>
</tr>
<tr>
<td>Period 3</td>
<td>670</td>
<td>80</td>
<td>0</td>
</tr>
<tr>
<td>Period 4</td>
<td>0</td>
<td>800</td>
<td>0</td>
</tr>
<tr>
<td>Period 5</td>
<td>524</td>
<td>206</td>
<td>0</td>
</tr>
</tbody>
</table>

From the table above, it is clear that the system tends to disassembled parts with good and average quality level, and discards the imperfect quality products. The reason is that
the least quality products costs less to handle and hold in house compared to a higher quality items, thus it is more desirable to start to disassemble the higher costs items first, and subsequently ensuring demand is being met from a higher quality level items whenever possible, and thus disposing higher volume of least quality item, see Table 6.7.

Table 6.7 Disposed core products at every period $Q_{r,k,t}^{fisc}$

<table>
<thead>
<tr>
<th>Period</th>
<th>Quality level 1</th>
<th>Quality level 2</th>
<th>Quality level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>0</td>
<td>0</td>
<td>729</td>
</tr>
<tr>
<td>Period 2</td>
<td>0</td>
<td>0</td>
<td>494</td>
</tr>
<tr>
<td>Period 3</td>
<td>0</td>
<td>377</td>
<td>169</td>
</tr>
<tr>
<td>Period 4</td>
<td>0</td>
<td>0</td>
<td>328</td>
</tr>
<tr>
<td>Period 5</td>
<td>0</td>
<td>177</td>
<td>195</td>
</tr>
</tbody>
</table>

Because of the high cost of procuring items from outside suppliers, when necessary an order is placed to cover any shortages of disassembled parts. The lead time and order size requirements are negligible. The numbers of items procured from an outside supplier are shown in Table 6.8.

Table 6.8 Procured parts in every period $Q_{r,t}^{pur}$

<table>
<thead>
<tr>
<th>Period</th>
<th>Part A</th>
<th>Part B</th>
<th>Part C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Period 2</td>
<td>50</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Period 3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Period 4</td>
<td>100</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Period 5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Because of the limitations on the disposal process and high cost of carrying inventory core product with the least quality, imperfect quality level, are recycled, as per Table 6.9.

Table 6.9 Quantity of core products recycled in every period $Q_{rec}^{i,k,j}$

<table>
<thead>
<tr>
<th>Period</th>
<th>Quality level 1</th>
<th>Quality level 2</th>
<th>Quality level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>0</td>
<td>702</td>
<td>27</td>
</tr>
<tr>
<td>Period 2</td>
<td>0</td>
<td>0</td>
<td>495</td>
</tr>
<tr>
<td>Period 3</td>
<td>0</td>
<td>0</td>
<td>546</td>
</tr>
<tr>
<td>Period 4</td>
<td>0</td>
<td>60</td>
<td>268</td>
</tr>
<tr>
<td>Period 5</td>
<td>0</td>
<td>371</td>
<td>0</td>
</tr>
</tbody>
</table>

Although recycling costs are equal for all quality levels, to ensure a high level of service level when satisfying demand sources, it is preferable to use high quality level items, and keep imperfect quality items to satisfy the demand for raw materials. For the above example, this combination will result in the least total cost of $76,292.15$, which result in a near-optimal solution and will balance the inventory generated problem to the best extent possible. Any inventory problem will result in one of the following cases:

- Excess inventory that is beyond the demand for disassembled parts
- Shortage of inventory that requires procuring from an outside suppliers
- Balanced inventory where demand meets the supply of items

While shortages in inventory can further complicate the problem, when planner has to account for the opportunity costs from lost sales and backorders, the excess inventory scenario could be a burden on the inventory system and could result in a high
cost. In this example, the remaining on hand inventories (OHI) are given in Table 6.10 and Table 6.11. Step two of the solution methodology attempt to rectify that problem.

<table>
<thead>
<tr>
<th>Period</th>
<th>Quality level 1</th>
<th>Quality level 2</th>
<th>Quality level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>62</td>
<td>87</td>
<td>0</td>
</tr>
<tr>
<td>Period 2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Period 3</td>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Period 4</td>
<td>357</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Period 5</td>
<td>0</td>
<td>0</td>
<td>357</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period</th>
<th>Part A</th>
<th>Part B</th>
<th>Part C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>0</td>
<td>170</td>
<td>320</td>
</tr>
<tr>
<td>Period 2</td>
<td>0</td>
<td>340</td>
<td>300</td>
</tr>
<tr>
<td>Period 3</td>
<td>90</td>
<td>0</td>
<td>130</td>
</tr>
<tr>
<td>Period 4</td>
<td>0</td>
<td>80</td>
<td>320</td>
</tr>
<tr>
<td>Period 5</td>
<td>0</td>
<td>355</td>
<td>190</td>
</tr>
</tbody>
</table>

6.4.3 Problem Formulation in a Multi-Period Case: Phase-Two

As discussed in the previous section, the inventory of disassembled parts after satisfying the demand could start to pile and excess inventory will be carried on. Thus, the goal of phase-two formulation is to determine the possible liquidation vs. disposal management options that can maximize the profit of this sub-system of parts inventory, by offering
price incentives to demand sources. In this case, demand sources will distinguish between different quality levels. The objective is to maximize the profit generated which is the revenue (money collected from the sales of the disassembled parts and subassemblies), minus the associated costs (transaction cost, holding cost, transferring to storage cost).

The problem is formulated to solve for two decision variables, in anticipation of future demand:

- Quantity of cores and parts to be liquidated within demand sources capacity
- Quantity of cores and parts to be carried in inventory in anticipation of future demand

The problem formulation is given by equation (6.18) to (6.24).

\[
\text{MAX} \quad \sum_{i} \sum_{k} \sum_{t} P_{i,k,t}^c \cdot Q_{l,i,k,t}^{lqc} + \sum_{j} \sum_{k} \sum_{t} P_{j,k,t}^s \cdot Q_{l,j,k,t}^{lqc} - \sum_{i} \sum_{k} \sum_{t} C_{i,k,t}^{trac} \cdot Q_{l,i,k,t}^{lqc} - \\
\sum_{j} \sum_{k} \sum_{t} C_{j,k,t}^{trac} \cdot Q_{l,j,k,t}^{lqc} - \sum_{i} \sum_{k} \sum_{t} C_{i,k,t}^{holc} \cdot Q_{l,i,k,t}^{remc} - \sum_{j} \sum_{k} \sum_{t} C_{j,k,t}^{holc} \cdot Q_{l,j,k,t}^{remc} \tag{6.18}
\]

\[
\text{ST} \quad Q_{i,k,t}^{lqc} + Q_{i,k,t}^{remc} \leq Q_{i,k,t}^{invc}, \forall i,k,t \tag{6.19}
\]

\[
Q_{j,k,t}^{lqc} + Q_{j,k,t}^{remc} \leq Q_{j,k,t}^{invc}, \forall j,k,t \tag{6.20}
\]

\[
\sum_{i} \sum_{k=1} w_i \cdot Q_{i,k,t}^{lqc} + \sum_{j} \sum_{k=1} w_j \cdot Q_{j,k,t}^{lqc} \leq CAP_{n}^{dem}, \forall t \tag{6.21}
\]

\[
\sum_{i} \sum_{k=1} w_i \cdot Q_{i,k,t}^{lqc} + \sum_{j} \sum_{k=1} w_j \cdot Q_{j,k,t}^{lqc} \leq CAP_{n}^{dem}, \forall t, k \neq 1 \tag{6.22}
\]

\[
\sum_{i} \sum_{k} Q_{i,k,t}^{remc} \leq CAP_{n}^{tagec}, \forall t \tag{6.23}
\]

\[
\sum_{j} \sum_{k} Q_{j,k,t}^{remc} \leq CAP_{n}^{tagec}, \forall t \tag{6.24}
\]
And the non-negativity constraint apply to all variables, where all variables $t_{kji} \geq 0, \forall i, j, k, t$.

The objective function in equation (6.18) is to maximize the profit from the sale of remaining items after subtracting the costs associated with the sale transaction. In phase-one the assumption was disassembled parts can be used directly, recycled, or disposed of. In phase-two two main demand sources exists: remanufacturing and refurbishing. Remanufacturing will only accept products with good quality level, thus minimum cleaning and repair will be required. While refurbishing will accept average and imperfect quality levels that require more repair and up-gradation. The demand sources storage facility capacities and the in-house storage facility capacity of the disassembly facility are considered. These constraints are explained by equations (6.19) through (6.24). In the next section, the problem methodologies and numerical calculations is given to illustrate the approach.

### 6.4.4 Problem Methodologies and Complexity

To illustrate the approach of phase two, the following numerical example is given. There exist two types of demand sources: remanufacturing and refurbishing. Demand sources distinguish between the different qualities of disassembled parts where remanufacturing facilities will only accept parts that are categorized as good quality, while the rest can be accepted by refurbishing facilities. The problem is based on a hypothetical product that consists of three parts $ABC$. The input data is known and assumed to be deterministic,
and the on hand inventory is also to be assumed deterministic. In Table 6.11 the problem input parameters are given.

<table>
<thead>
<tr>
<th>Part/Subassembly Type</th>
<th>Quality Level</th>
<th>Space Req.</th>
<th>Profit/Unit</th>
<th>Holding Cost</th>
<th>Transaction Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>0.625</td>
<td>$37.00</td>
<td>$3.70</td>
<td>$1.50</td>
<td></td>
</tr>
<tr>
<td>Core Type 1</td>
<td>Average</td>
<td>0.625</td>
<td>$23.00</td>
<td>$2.30</td>
<td>$1.50</td>
</tr>
<tr>
<td></td>
<td>imperfect</td>
<td>0.625</td>
<td>$17.00</td>
<td>$1.70</td>
<td>$1.50</td>
</tr>
<tr>
<td></td>
<td>Good</td>
<td>0.1</td>
<td>$13.00</td>
<td>$1.00</td>
<td>$0.75</td>
</tr>
<tr>
<td>Part Type A</td>
<td>Average</td>
<td>0.1</td>
<td>$10.00</td>
<td>$1.30</td>
<td>$0.75</td>
</tr>
<tr>
<td></td>
<td>imperfect</td>
<td>0.1</td>
<td>$8.00</td>
<td>$0.70</td>
<td>$0.75</td>
</tr>
<tr>
<td></td>
<td>Good</td>
<td>0.2</td>
<td>$8.00</td>
<td>$0.50</td>
<td>$0.75</td>
</tr>
<tr>
<td>Part Type B</td>
<td>Average</td>
<td>0.2</td>
<td>$4.00</td>
<td>$0.55</td>
<td>$0.75</td>
</tr>
<tr>
<td></td>
<td>imperfect</td>
<td>0.2</td>
<td>$3.00</td>
<td>$0.20</td>
<td>$0.75</td>
</tr>
<tr>
<td></td>
<td>Good</td>
<td>0.25</td>
<td>$7.00</td>
<td>$0.33</td>
<td>$0.75</td>
</tr>
<tr>
<td>Part Type C</td>
<td>Average</td>
<td>0.25</td>
<td>$5.00</td>
<td>$0.40</td>
<td>$0.75</td>
</tr>
<tr>
<td></td>
<td>imperfect</td>
<td>0.25</td>
<td>$2.00</td>
<td>$0.13</td>
<td>$0.75</td>
</tr>
</tbody>
</table>

The phase two problem assumes that in order to liquidate the extra inventory on hand, the disassembly facility has to 1) distinguish between the quality levels of core products and disassembled parts (unlike phase one, where demand will accept part “as is”) and 2) sales prices and holding costs will fluctuate based on that quality type. Compared to phase one, holding costs of similar items are higher and that is due to the assumptions of storing items in storage facilities rather than along the workstation. These items are beyond the demand, thus offering discounted prices, and considering the
demand sources capacity are important. The model was run over a five period horizon and analyzed using Lingo 11.0 and spreadsheet analysis. Tables 6.12 thru 6.16 give the detailed results of periods 1 thru 5. The objective function value is calculated to be $178,770.80. The objective function value represents the total profit that can be generated. In the results tables it is clear that the system tendency of liquidating higher value items more often and keep lower valued items on hand, and by liquidating higher value parts faster it results in a reduction in holding costs and eventually a higher overall profit.

Table 6.13 Period 1 results of liquidated, remaining, and accumulated inventory

<table>
<thead>
<tr>
<th>Period</th>
<th>Type</th>
<th>Quality</th>
<th>OHI</th>
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Table 6.17 Period 5 results of liquidated, remaining, and accumulated inventory

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6.5 Conclusions

In this chapter, the disassembly line inventory management problem is discussed. A mathematical approach in a multi-period environment is presented using a two-phase linear programming (LP) model to balance the inventory build up at the workstations thus sum of variables costs are minimized and profit generated from the sales is maximized. The model assumes a deterministic input data.
Chapter 7

Inventory Balancing in Disassembly Line: Dynamic Programming Approach

Reverse Logistics (RL) is a critical topic that has captured the attention of government agencies, private entities, and researchers in recent years. This increase in the concern was driven by current set of government regulations, increase of public awareness, and the attractive economic opportunities for the private entities involved in this business. Also, environmentalists have always demanded Original Equipment Manufacturers (OEMs) to be more involved and responsible of their products at the end of its life cycle. However, take-back programs (TBP) could possibly bring financial burden to the Original Equipments Manufacturers (OEMs) if they are not managed properly. Thus, an efficient, cost effective management tools have to be developed to help planner makes decisions and manage the overwhelming number of returns. In this chapter, a dynamic programming tool is developed to help improving the system outcome and maximize its profit.
7.1 Introduction

In recent years, disassembly has gained a lot of attention due to its role in efficiently recovering valuable materials, parts, and subassemblies from end-of-life (EOL) products and allows it to be reused more than once before it is finally discarded. This is due to the rigid environmental legislations, the economical and environmental benefits from reusing primary materials instead of virgin resources and the increase in the environmental awareness and concern of the consumers about the “green image”. However, the practice of recovering parts and subassemblies is challenging, as there are many associated complications that are unique to the disassembly process. The disassembly of an end-of-life (EOL) product is performed at a single workstation, in a disassembly cell, or in a disassembly line. The disassembly line setting is the best choice for automated disassembly as it provides the highest productivity rate for end-of-life (EOL) products available in large quantities. For that reason, in recent years, a lot of researchers have started to concentrate on problems related to balancing the disassembly line. The objectives of balancing the disassembly line are to provide a feasible disassembly sequence, minimize the number of workstations, provide an equalized work content at each workstations (or minimize idle time at each workstation), and balance the line (ensure similar idle times at each workstation) as well as address other disassembly-specific concern. However, due to the disparity between demand for disassembled parts and their line yields, there are many inventory problems that arise during the disassembly process. Unlike the traditional assembly line, or forward logistics in general, the demand in reverse logistics setting for disassembled parts and subassemblies occur at all workstations and in different rate. If on hand inventory (OHI) at any given workstation is
less than the demand, the difference is secured either from current inventory level kept at the facility warehouses from previous periods or through an outside third party supplier. Because of the overwhelming number of returns in today’s market where manufacturers tend to introduce technological advancement in their products and customers tend to upgrade their old equipments with the best technology available. At their end of life cycle, many of these products that are still functional are traded for new ones under the umbrella of exchange program, warranty programs, or take back programs. These end-of-life (EOL) products, sent back in the reverse chain, go through collection centers for sorting, inspection, and testing before its finally grouped based on its quality and transported to it is final destination at the disassembly facilities. End-of-life (EOL) products will undergo disassembly operations as per market demand and its capacity. In some cases, disassembly facilities will have excess inventory that is beyond the actual period’s demand for disassembled parts and subassemblies. The demand planner will usually face economical and physical constraints when deciding how much inventory to liquidate, and when to liquidate and which liquidation channel will generate the highest possible revenue. Also, deciding to keep inventory on hand for longer periods while incurring some holding costs now for a potential higher profit in the future is very critical.

Many challenges are associated with balancing the inventories generated at various workstations of a disassembly line. Not only it is important to balance the work content at each workstation, it is also important to balance the inventory at each workstation by strategically siphoning off the parts and subassemblies disassembled by providing a proper flow to the demand sources. The remaining disassembled parts and subassemblies that are beyond the demand sources requirements can be either liquidated or carried on to the next period. Current on hand inventory (OHI) of disassembled parts
Chapter 7. Inventory Balancing in Disassembly Line: Dynamic Programming Approach

and subassemblies needs to be handled appropriately to avoid running out of storage space along the line, incurring unnecessary excessive costs beyond its actual remaining value, or starving the demand sources. In addition, some items should be handled differently and some parts that include hazardous materials may require special handling. In this chapter, the focus is on addressing the unique aspects to balancing the inventories generated at various workstations of a so called “balanced” disassembly line. Using a multi-period dynamic programming model the inventory control problem will be discussed and the methodology developed will provide a management tool for the decision maker to take corrective actions as needed based on current on hand inventory and expected future returns of end-of-life (EOL) products. The goal is to maximize the total profit generated from liquidating disassembled parts and subassemblies by thoroughly examining various liquidation channels options.

7.2 Inventory and Value Management of End-of-Life (EOL) Products

One of the most challenging issues in reverse logistics is inventory and value management of end-of-life (EOL) products and disassembled parts and subassemblies. Uncertainty associated with the process complicates the decision making process when it comes to appropriate inventory levels in order to meet current and future demand. As previous chapter, this research assumes that disassembly facility has no control over the return stream of end-of-life (EOL) products, and it is required to handle all returns in an environmentally friendly manner.
7.2.1 Inventory Management Problem in Disassembly Line: Overview

Inventory management and control is one of the important issues in Reverse Logistics (RL). This research focuses on disassembly line and the inventory of disassembled parts and subassemblies generated along the line. By focusing on the disassembly line operations, the appropriate management of returned end-of-life (EOL) products and disassembled parts and subassemblies becomes a very critical topic to discuss. The proper handling of these items along the line is crucial in improving the overall efficiency and productivity of the line while reducing the associated costs. However, ensuring the smoothness of the operations and satisfying the demand sources are complex problems that require the development of management tools that will facilitate the decision making process and allow planners to adapt and react more effectively. After performing the disassembly operations on returned end-of-life (EOL) products, the demand of disassembled parts are first met and the demand sources is satisfied. At any given time period, two case scenarios may occur: either a shortage of disassembled parts against the demand or an excess inventory that is beyond the demand. When shortages occur, a discount is provided to the customer as a compensation for the wait time until the next batch of returned items is brought back and disassembled, or the difference could be secured through a third party outside supplier and usually with a higher price tag. In case of excess inventory that is beyond the demand, the planner has to make the decision of how much inventory to keep and for how long versus what and when to liquidate. The current on hand inventory (OHI) can be liquidated immediately to avoid extra carrying
and holding costs if expected future returns will be enough to satisfy future demand, or can be carried to the next period if shortage is expected in the future or a hike in the prices, thus any incurred costs can be justified for higher revenue in the future. Demand for disassembled parts and subassemblies occur at different rates and at all workstations. Disassembly facilities have no control over the quality, quantity, and flow of these end-of-life (EOL) products from end customer back to its origin. These end-of-life (EOL) products have different quality levels corresponding to the different environments they were operating in. Uncertainty between the demand of disassembled parts and subassemblies and the disassembly line yields will create a fluctuation of the amount of inventory at the workstations. The main challenge is to balance the current on hand inventory (OHI) such that demand is met at all periods, while costs and inventory level is kept at minimum. Two sources of constraints limit the decision maker ability to manage the inventory of disassembled parts: 1) Physical constraints, and 2) Economical constraints. Physical constraints include demand sources capacities, available space along the workstations, warehouse storage space limitations. Economical constraints include holding cost, acquisition cost, recovery cost, disposal cost, product storage cost, and carrying cost (handling operations, potential obsolescence, spoilage cost, and insurance and taxes). In the next section, the problem of interest is defined. The decision maker has to find an accepted trade-off between the elements of the inventory that meets the planner inventory policy.
7.2.2 Inventory Management Problem in Disassembly Line: Problem Definition

Disassembly facilities are considered to be stand alone entities that operate independently from Original Equipment Manufacturers (OEMs). Unlike hybrid systems, where manufacturing and remanufacturing are considered as an integrated system, disassembly operations is managed and operated separately. One main difference is that disassembly facilities do not perform any manufacturing or remanufacturing activities in house, and it only acts as a supplier of parts and other raw materials to the manufacturers and other brokers. The role of disassembly facilities can be pictured as middle agents in the supply chain loop, and thus the management of the inventory at these facilities is unique in a sense that the flow of the end-of-life (EOL) products is divergent. This means that handling one end-of-life (EOL) product could result in inventories of many disassembled parts and subassemblies that need to be handled and stored separately. When analyzing the inventory system of a disassembly facility, multiple quality levels of end-of-life (EOL) products corresponding to its functionality and usability are considered. This assumption complicates the problem further especially in the presence of customers’ requirement of a minimum acceptable quality levels. In previous research, the assumption was made that demand sources will accept disassembled parts in “as is” condition. In this model, an attempt is made to address that uniqueness in the process by introducing multi-quality levels of end-of-life (EOL) products and parts. The focus is on the inventory and
value management portion of returned end-of-life (EOL) products that have been acquired from end customers through take-back programs back to its origin, either the original equipment manufacturer or a standalone disassembly facility. Assuming that a collection and transportation network exists, products received at the facility are being sorted, tested and grouped based on their quality levels and sent for disassembly operation. The returned products are received in the disassembly facilities in variety of conditions based on the environments they were operating in. After inspection, these products are categorized into three quality levels: good quality (usually products that have been in the market for less than one year), average quality (usually products that have been in the market for one to three years), and imperfect quality (usually products that have been in the market for more than three years). This categorization will help the planner forecast the yields of different parts quality levels expected from the disassembly operations. Demand sources are satisfied first from disassembled parts immediately, or from other supplier or remote storage facility if needed. These demand sources are repair workshops that directly reuse disassembled parts as spare parts for similar products that are still in the market, recycling facilities that extract high value materials before they discard the balance of the items, and some original equipment manufacturers (OEMs) that manufacture, remanufacture, and refurbish products and introduce them to other secondary markets. The objective of the Dynamic Programming (DP) model is to maximize the total profit generated by the system from the disassembly operations for all periods in the planning horizon. The profit is the expected revenues (money generated from the sales of the disassembled parts and subassemblies) minus the associated costs
(holding and transaction costs associated with the sales of these items). The model compares for every quality level of disassembled parts and subassemblies $j$ the estimated net profit generated if items type $j$ were processed with option $r$ against other liquidation options and against holding and liquidation in the future for a potentially higher profit. The Dynamic Programming (DP) model provides near optimal results that were obtained through extensive search based on spreadsheet analysis.

### 7.2.3 Inventory Management Problem in Disassembly Line: Problem Assumptions

The inventory management problem in a reverse logistics settings involves many interrelated decisions. This model assumes a periodic review analysis over a defined planning horizon. A number of assumptions are made to simplify the analysis of the inventory system:

- It is a single product model $i$ with three parts/subassemblies $j$: part A, B, and C.
- There are three different quality levels $k$ of the returned core products: good, average, and imperfect.
- The configuration of the products is three parts and/or subassemblies that have three possible quality conditions $k$: working condition, underperforming condition, and malfunctioning condition.
- Demand and returns are constant and known.
- On hand inventory (OHI), which is the balance remaining after demand is satisfied, is calculated based on a given transition probability.
• At the end of the planning horizon, or last period, no inventory is allowed to be carried on inventory. All inventories should be liquidated.

• Holding and transaction costs associated with different parts condition stay constant from period to period, but changing based on the liquidation option.

• Disassembled parts prices fluctuations from period to period are non-linear. Transition probability and its parts yields probability are given and known.

• There are six options for handling end-of-life (EOL) cores and its parts: hold, remanufacture, refurbish, repair, recycle, and direct reuse, and they are referred to as options \( r \): 0 to 5 respectively.

In the next section, the problem mathematical modeling approach and problem formulation is presented and discussed in details. For simplicity, the model assumes deterministic input data. The model is developed for a single hypothetical product over a multi-period planning horizon. A main distinguish from the previous approach is that the model assumes that the core and disassembled parts quality levels are interrelated based on a given transition probability.

### 7.3 Inventory Management Problem Formulation

In this section, a modeling approach to the problem based on optimization technique is presented. The inventory problem in disassembly is modeled over a finite planning horizon. At the end of each period, the quantity disassembled \( Q_j^{\text{diss}} \) of every part type \( J \), the quantity remaining on hand at the end of every period \( Q_j^{\text{rem}} \), and the quantity liquidated through any of the available \( r \) liquidation options are determined \( Q_j^{\text{liq}} \).
What makes this problem unique is that (1) it considers the space allocation and availability along the disassembly line since space is a scarce source and thus it is an important element when handling disassembled parts, hence any inventory that accounts for a partial tote is liquidated, and only full totes of parts are carried in inventory, and (2) it takes into account the future potential profit that can generated by the current inventory on hand if carried until next period. This section discusses the mathematical structure of the model and the development of the objective function and problem constraints. For simplicity, the model assumes deterministic input data. The model is developed for a single hypothetical product over a multi-period planning horizon. The objective of the model is to maximize the total system’s profit and minimize the inventory carried from one period to another. Also, any shortages in the demanded disassembled parts are procured from an outside supplier for additional costs. The model assumes that all end-of-life (EOL) products and recoverable parts will have different quality levels and operational conditions, thus they are not treated equally when calculating costs and price of sale. The objective of the system is to control the inventory levels of core products and disassembled parts and subassemblies in the system to satisfy demand sources and maximize the system profit.
7.3.1 Mathematical Model

One of the issues in inventory management in reverse logistics is uncertainty of the quality of items returned and their line yields. To account for it, uncertainty is modeled by conditional probabilities $P(q2/q1)$ such that

$$q1 \in K$$

(7.1)

Equation (7.1) ensures that all quality levels of returned end-of-life (EOL) products belong to the set of possible quality levels, and

$$q2 \in K$$

(7.2)

Equation (7.2) ensures that all quality levels of disassembled parts and subassemblies belong to the set of possible quality levels, and

$$\sum_{q2} p_{j,t}(q2/q1) = 1 \forall q2$$

(7.3)

Equation (7.3) ensures that the summation of probabilities of any given part/subassembly $j$, with quality levels $q2$, at any given period of time $t$, given that it comes from a known core product quality level $q1$, adds up to 1.00.

The symbol $q1$ represents the different quality levels of core products type $i$, and the term $q1 = \{1,2,3\}$, which reflects good, average, and imperfect condition. Also, the symbol $q2$ represents the different quality levels of disassembled parts and subassemblies type $j$, and the term $q2 = \{1,2,3\}$ which reflects working, underperforming, and malfunctioning condition.

In this model, the concept of probabilistic approach is considered. Different operational levels of every parts and subassembly disassembled from core products with a known quality level are considered. This is modeled using different conditional probabilities of distracting a certain quality part from a given quality of the core. In other words, a good quality core product for example could return a perfectly working part,
underperforming part, and/or malfunctioning part. Previously, the assumption made was
good quality products will return a working condition part, and all demand sources will
accept disassembled parts in “as is” condition. In the next section, the formulation of the
problem is presented using a dynamic programming (DP) modeling technique.

7.3.2 Problem Formulation

In this section, the inventory management problem is formulated. The goal is to develop a
model that systematically determines liquidation vs. hold strategy of the current on hand
inventory with the aim to maximize system’s profit. This requires making a sequence of
interrelated decisions, where each decision corresponds to one stage of the problem. The
optimal policy for the remaining stages is independent of the policy decision in the
previous stages, and this property is known as “Markovian Property”.

To calculate the total expected profit $PR$, the following mathematical expression is used

$$PR_{i,j}^c(q1) = \max_{x=0,1} V_{i,j}^c(x,r/k)$$

Equation (7.4) represents the most attainable profit generated through holding inventory
or liquidating core products, compared across all available liquidation options at period $t$.

$$PR_{i,j}^s(q2) = \max_{x=0,1} V_{i,j}^s(x,r/k)$$

Equation (7.5) represents the most attainable profit generated through holding inventory
or liquidating disassembled parts, compared across all available liquidation options at
period $t$, where $V_{i,j}^c(x,r/k)$, and $V_{i,j}^s(x,r/k)$, can be calculated as follows:

$$V_{i,j}^c(0,r/k) = -C_{i,j}^{inc} Q_{i,j}^{inc}$$

(7.6)
Equation (7.6) represents the net profit generated from holding on hand inventory OHI of core products type \(i\), with quality level \(k\), in period \(t\), for potential higher profit in the future.

\[
V^c_{i,t}(1, r/k) = -C^\text{holc}_{i,k,t} \ast (Q^\text{inv}_{i,k,t} - Q^\text{liqc}_{i,k,t}) + P^c_{i,k,t} \ast Q^\text{liqc}_{i,k,t} - C^\text{trac}_{i,k,t} \ast Q^\text{liqc}_{i,k,t}
\]  

(7.7)

Equation (7.7) represents the net profit generated from liquidating part of the on hand inventory of core products type \(i\), with quality level \(k\), in period \(t\).

\[
V^s_{j,t}(0, r/k) = -C^\text{holss}_{j,k,t} \ast Q^\text{inv}_{j,k,t}
\]  

(7.8)

Equation (7.8) represents the net profit generated from holding on hand inventory of part/subassembly type \(j\), with quality level \(k\), in period \(t\), for potential higher profit in the future.

\[
V^s_{j,t}(1, r/k) = -C^\text{holss}_{j,k,t} \ast (Q^\text{inv}_{j,k,t} - Q^\text{liqs}_{j,k,t}) + P^s_{j,k,t} \ast Q^\text{liqs}_{j,k,t} - C^\text{trass}_{j,k,t} \ast Q^\text{liqs}_{j,k,t}
\]  

(7.9)

Equation (7.9) represents the net profit generated from liquidating part of the current on hand inventory of parts/subassemblies type \(j\), with quality level \(k\), in period \(t\), and holds the balance for potential higher profit in the future.

Then the value of the function \(f_n(S_n, X_n)\) can be calculated by the summation of the future profit to the formulas (7.6), (7.7), (7.8), and (7.9), as follows:

\[
f_n(S_n, X_n) = V^c_{i,t}(x, r/k) + PR^c_{i,t+1}(q1)
\]  

(7.10)

Equation (7.10) represents the summation of the immediate profit generated in stage \(n\), through holding or liquidating core products and the maximum future expected profit of future stages (periods) \(n+1\) onward.

\[
f_n(S_n, X_n) = V^s_{j,t}(x, r/k) + PR^s_{j,t+1}(q2)
\]  

(7.11)

Equation (7.11) represents the summation of the immediate profit generated in stage \(n\), through holding or liquidating disassembled parts and subassemblies and the maximum future expected profit of future stages (periods) \(n+1\).

Thus, the best solution value is given by the following formula
In the next section, the algorithm is presented to illustrate how the Dynamic Programming (DP) model works to generate solutions, and the process of selecting the best solution among all.

### 7.3.3 Problem Algorithm

The Dynamic Programming (DP) model has several steps and carried out in the form of loops. First, it loops through a range of values of parts and subassemblies and different quality levels available. Then, it compares different hold and liquidation options available for every part type of certain quality level to find the optimal or near optimal solution before moving to the next stage (period). In doing so, it saves a set of solutions and then chooses the best answer from the saved set. The general steps from the algorithm are shown below for the disassembled parts and subassemblies:
1. For ∀j, and ∀q do the following
   (A) Calculate the net profit generated from the hold or sale of the current OHI, then
      (i) If \( V_{j,\delta}^{x}(1, r/k) \geq V_{j,\delta}^{x}(0, r/k) \), then proceed to the following two steps
          (a), and (b), else go to (ii)
          (a) Choose \( r_0 \in R_{j,\delta}^{x} \) such that \( V_{j,\delta}^{x}(1, r/k) = V_{j,\delta}^{x}(1, r_0 / k) \);
          (b) Set \( r^* = r_0 \) where \( r_0 = \{1,2,3,4,5\} \)
      (ii) Set \( r^* = 0 \)
   (B) Save solution
2. If \( t \geq 1 \), then assign \( t = t - 1 \), and go to step 1. Else go to step 3
3. Finish, save final solution

The same methodology and steps apply for on hand inventory of core products as well.

The general steps from the algorithm are shown below for the core products:

For ∀i, and ∀q do the following
   (A) Calculate the net profit generated from the hold or sale of the current OHI, then
      (i) If \( V_{i,\delta}^{\epsilon}(1, r/k) \geq V_{i,\delta}^{\epsilon}(0, r/k) \), then proceed to the following two steps
          (a), and (b), else go to (ii)
          (a) Choose \( r_0 \in R_{i,\delta}^{\epsilon} \) such that \( V_{i,\delta}^{\epsilon}(1, r/k) = V_{i,\delta}^{\epsilon}(1, r_0 / k) \)
          (b) Set \( r^* = r_0 \) where \( r_0 = \{1,2,3,4,5\} \)
      (ii) Set \( r^* = 0 \)
   (B) Save solution
2. If \( t \geq 1 \), then assign \( t = t - 1 \), and go to step 1. Else go to step 3
3. Finish, save final solution

In the next section, a numerical example is given to illustrate the approach of dynamic programming in inventory management of end-of-life (EOL) products and disassembled parts in a disassembly line context.
7.3.4 Numerical Results

To illustrate the mathematical modeling and algorithm presented in the previous sections, the disassembly line and the inventory model of an end-of-life (EOL) product that contains three parts $ABC$ is considered. The purpose is to demonstrate the impact in inventory when end-of-life (EOL) products are disassembled and to investigate the possible liquidation channels available. The hypothetical product will be disassembled into three separate parts, $A$, $B$, and $C$. The number of end-of-life (EOL) products to be disassembled in every period is based on returns and the facility capacity. The problem is modeled over five periods planning horizon ($t = 5$). Due to the different environments these products used to operate in, different quality levels of end-of-life (EOL) products as well as disassembled parts exist. Table 7.1 is presented to model this uncertainty of the quality levels.

<table>
<thead>
<tr>
<th>Quality Level</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>0.42</td>
</tr>
<tr>
<td>Average</td>
<td>0.35</td>
</tr>
<tr>
<td>Imperfect</td>
<td>0.23</td>
</tr>
<tr>
<td>TOTAL</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Given any quality level of an end-of-life (EOL) product, the disassembled parts and subassemblies could also vary in its quality levels. In other words, good quality end-of-life (EOL) product could yield a working condition part or an underperforming condition part depending on the part type. In some cases, it could result in a malfunctioning part that of no use or value. This will depend on many factors that will
determine what categories disassembled parts fall in. Thus, this uncertainty is modeled using conditional probability as presented by equation (7.3). The following tables represent the transition probability matrices used in the model of the different quality level core products and its line yields. Table 7.2 represents the probability matrix of parts disassembled from good core products.

Table 7.2  Transition probability matrix for $i=1$, $k=1$

<table>
<thead>
<tr>
<th>Subassembly/Part</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality Level</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Working</td>
<td>0.53</td>
<td>0.26</td>
<td>0.37</td>
</tr>
<tr>
<td>Underperforming</td>
<td>0.32</td>
<td>0.41</td>
<td>0.19</td>
</tr>
<tr>
<td>Malfunctioning</td>
<td>0.15</td>
<td>0.33</td>
<td>0.44</td>
</tr>
<tr>
<td>Total</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 7.3 represents the probability matrix of parts coming from average core products.

Table 7.3  Transition probability matrix for $i=1$, $k=2$

<table>
<thead>
<tr>
<th>Subassembly/Part</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality Level</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Working</td>
<td>0.62</td>
<td>0.29</td>
<td>0.39</td>
</tr>
<tr>
<td>Underperforming</td>
<td>0.22</td>
<td>0.34</td>
<td>0.44</td>
</tr>
<tr>
<td>Malfunctioning</td>
<td>0.16</td>
<td>0.37</td>
<td>0.17</td>
</tr>
<tr>
<td>Total</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Table 7.4 represents the probability matrix of parts coming from imperfect core products.

Table 7.4  Transition probability matrix for $i = 1, k = 3$

<table>
<thead>
<tr>
<th>Subassembly/Part</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality Level</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Working</td>
<td>0.32</td>
<td>0.50</td>
<td>0.30</td>
</tr>
<tr>
<td>Underperforming</td>
<td>0.48</td>
<td>0.19</td>
<td>0.30</td>
</tr>
<tr>
<td>Malfunctioning</td>
<td>0.20</td>
<td>0.31</td>
<td>0.40</td>
</tr>
<tr>
<td>Total</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Tables 7.2 thru 7.4 represent the probability matrices that any given quality core product that is returned at the end of its life cycle could yield, after disassembly, a wide range of quality parts ranging from working condition, to underperforming condition, to malfunctioning condition. In this model, five demand sources, also called liquidation channels are considered. The liquidation channels are presented in Table 7.5.

Table 7.5  Available Liquidation Channels $r$

<table>
<thead>
<tr>
<th>Hold/Liquidation Option</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hold</td>
<td>$r = 0$</td>
</tr>
<tr>
<td>Remanufacture</td>
<td>$r = 1$</td>
</tr>
<tr>
<td>Refurbish</td>
<td>$r = 2$</td>
</tr>
<tr>
<td>Repair</td>
<td>$r = 3$</td>
</tr>
<tr>
<td>Recycle</td>
<td>$r = 4$</td>
</tr>
<tr>
<td>Reuse</td>
<td>$r = 5$</td>
</tr>
</tbody>
</table>
Tables 7.6 thru 7.8 represent the costs of holding one part in inventory for one period. These costs vary based on the part type, quality level, and time period.

<table>
<thead>
<tr>
<th>Table 7.6  Holding costs of $i = 1, j = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part/Cost</td>
</tr>
<tr>
<td>Part A, Quality Level =1</td>
</tr>
<tr>
<td>Part A, Quality Level =2</td>
</tr>
<tr>
<td>Part A, Quality Level =3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 7.7  Holding costs of $i = 1, j = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part/Cost</td>
</tr>
<tr>
<td>Part B, Quality Level =1</td>
</tr>
<tr>
<td>Part B, Quality Level =2</td>
</tr>
<tr>
<td>Part B, Quality Level =3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 7.8  Holding costs of $i = 1, j = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part/Cost</td>
</tr>
<tr>
<td>Part C, Quality Level =1</td>
</tr>
<tr>
<td>Part C, Quality Level =2</td>
</tr>
<tr>
<td>Part C, Quality Level =3</td>
</tr>
</tbody>
</table>
Tables 7.9 thru 7.11 represent the transaction costs associated with sale of one disassembled part. It differs based on part, quality level, and time period.

<table>
<thead>
<tr>
<th>Part A, Quality Level</th>
<th>r = 1</th>
<th>r = 2</th>
<th>r = 3</th>
<th>r = 4</th>
<th>r = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality Level =1</td>
<td>0.98</td>
<td>0.9</td>
<td>0.87</td>
<td>0.99</td>
<td>1.3</td>
</tr>
<tr>
<td>Quality Level =2</td>
<td>0.83</td>
<td>0.24</td>
<td>0.48</td>
<td>0.75</td>
<td>0.98</td>
</tr>
<tr>
<td>Quality Level =3</td>
<td>0.99</td>
<td>0.65</td>
<td>0.87</td>
<td>0.87</td>
<td>0.55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part B, Quality Level</th>
<th>r = 1</th>
<th>r = 2</th>
<th>r = 3</th>
<th>r = 4</th>
<th>r = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality Level =1</td>
<td>1.68</td>
<td>1.58</td>
<td>3.41</td>
<td>1.38</td>
<td>1.33</td>
</tr>
<tr>
<td>Quality Level =2</td>
<td>2.89</td>
<td>2.44</td>
<td>2.36</td>
<td>2.33</td>
<td>2.55</td>
</tr>
<tr>
<td>Quality Level =3</td>
<td>2.36</td>
<td>2.2</td>
<td>1.33</td>
<td>2.25</td>
<td>2.33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part C, Quality Level</th>
<th>r = 1</th>
<th>r = 2</th>
<th>r = 3</th>
<th>r = 4</th>
<th>r = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality Level =1</td>
<td>1.35</td>
<td>4.36</td>
<td>4.31</td>
<td>4.31</td>
<td>2.39</td>
</tr>
<tr>
<td>Quality Level =2</td>
<td>3.25</td>
<td>3.32</td>
<td>2.39</td>
<td>3</td>
<td>2.99</td>
</tr>
<tr>
<td>Quality Level =3</td>
<td>3.22</td>
<td>3.22</td>
<td>3.36</td>
<td>3.58</td>
<td>3.77</td>
</tr>
</tbody>
</table>

The above probability transition matrices and the costs data, along with the supply of end-of-life (EOL) products and expected profit is compiled and results are introduced.
Given the assumption that demand is satisfied from disassembled parts inventory bins, and all remaining parts are considered excess inventory, thus an appropriate inventory management model to handle these parts is developed for all generated parts and subassemblies over five periods planning horizon. The expected sales prices from the sales and the liquidations of current inventory differ from period to period based on two factors: 1) liquidation option, and 2) quantity liquidated, see appendix B. The following Tables 7.12 thru 7.14 present the core returns and the parts yields.

### Table 7.12 Part A yields over the five period horizon

<table>
<thead>
<tr>
<th>Period/Yield</th>
<th>Core Returned</th>
<th>Part A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>K=1</td>
</tr>
<tr>
<td>Period 1 Yield</td>
<td>4140</td>
<td>2125</td>
</tr>
<tr>
<td>Period 2 Yield</td>
<td>2760</td>
<td>1416</td>
</tr>
<tr>
<td>Period 3 Yield</td>
<td>1380</td>
<td>708</td>
</tr>
<tr>
<td>Period 4 Yield</td>
<td>3450</td>
<td>1771</td>
</tr>
<tr>
<td>Period 5 Yield</td>
<td>4830</td>
<td>2479</td>
</tr>
</tbody>
</table>

### Table 7.13 Part B yields over the five period horizon

<table>
<thead>
<tr>
<th>Period/Yield</th>
<th>Core Returned</th>
<th>Part B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>K=1</td>
</tr>
<tr>
<td>Period 1 Yield</td>
<td>4140</td>
<td>1348</td>
</tr>
<tr>
<td>Period 2 Yield</td>
<td>2760</td>
<td>899</td>
</tr>
<tr>
<td>Period 3 Yield</td>
<td>1380</td>
<td>449</td>
</tr>
<tr>
<td>Period 4 Yield</td>
<td>3450</td>
<td>1124</td>
</tr>
<tr>
<td>Period 5 Yield</td>
<td>4830</td>
<td>1573</td>
</tr>
</tbody>
</table>
Table 7.14  Part C yields over the five period horizon

<table>
<thead>
<tr>
<th>Period/Yield</th>
<th>Core Returned</th>
<th>Part C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K=1</td>
<td>K=2</td>
</tr>
<tr>
<td>Period 1 Yield</td>
<td>4140</td>
<td>1494</td>
</tr>
<tr>
<td>Period 2 Yield</td>
<td>2760</td>
<td>996</td>
</tr>
<tr>
<td>Period 3 Yield</td>
<td>1380</td>
<td>498</td>
</tr>
<tr>
<td>Period 4 Yield</td>
<td>3450</td>
<td>1245</td>
</tr>
<tr>
<td>Period 5 Yield</td>
<td>4830</td>
<td>1743</td>
</tr>
</tbody>
</table>

The carried inventory from period to period is considered to be an excess inventory that should be liquidated immediately, or carried on if there is a potential future profit. Because of the different parts categorizations this will result in 9 hold vs. liquidate strategies for all parts and all quality levels ($3^2 = 9$). The strategies are explained in details below for the above numerical example, and complete solution is presented in Appendix C:

$i = 1, j = 1, k = 1$

- If OHI is 2125 units, then the parts are liquidated through liquidation option $r = 3$ (repair) down to the nearest acceptable level $s$, and hold the balance to the next period with potential profit of $26,210.77$

- Based on carried inventory, and expected yield on period 2, the current OHI is 2416 between 2K-3K, and the best option is to liquidate through option $r = 5$ (reuse) down to the nearest acceptable level $s$, and hold the balance to the next period with potential future profit of $14,563.18$
Based on carried inventory, and expected yield of period 3, the current OHI is 2208 between 2K-3K, and the best option is to liquidate through option \( r = 4 \) (recycle) down to the nearest acceptable level \( s \), and hold the balance to the next period with potential future profit of $13,253.70

Based on carried inventory, and the expected yield of period 4, the current OHI is 3271 between 3K-4K, and the best option is to liquidate through option \( r = 1 \) (remanufacture) down to the nearest acceptable level \( s \), and hold the balance to the next period with potential future profit of $11,999.22

Based on carried inventory, and the expected yield of period 5, the current OHI is 3749 between 3K-4K, and the best option is to liquidate through option \( r = 2 \) (refurbish) down to the zero inventory (end of planning horizon) with potential profit of $9,110.07

\( i = 1, j = 1, k = 2 \)

If OHI is 1332 units, then the parts are liquidated through liquidation option \( r = 2 \) (refurbish) down to the nearest acceptable level \( s \), and hold the balance to the next period with potential profit of $11,841.84

Based on carried inventory, and expected yield on period 2, the current OHI is 1888 between 1K-2K, and the best option is to liquidate through option \( r = 5 \) (reuse) down to the nearest acceptable level \( s \), and hold the balance to the next period with potential future profit of $14,282.78

Based on carried inventory, and expected yield of period 3, the current OHI is 1444 between 1K-2K, and the best option is to liquidate through option \( r = 2 \) (refurbish) down to the nearest acceptable level \( s \), and hold the balance to the next period with potential future profit of $9,528.00
Based on carried inventory, and the expected yield of period 4, the current OHI is 2110 between 2K-3K, and the best option is to liquidate through option $r = 0$ (hold) to the next period with potential future profit of $8,366.28

Based on carried inventory, and the expected yield of period 5, the current OHI is 3554 between 3K-4K, and the best option is to liquidate through option $r = 2$ (refurbish) down to the zero inventory (end of planning horizon) with potential profit of $8,600.68

Based on carried inventory, and the expected yield on period 2, the current OHI is 1455 between 1K-2K, and the best option is to liquidate through option $r = 1$ (remanufacture) down to the nearest acceptable level $s$, and hold the balance to the next period with potential future profit of $9,597.42

Based on carried inventory, and expected yield of period 3, the current OHI is 1228 between 1K-2K, and the best option is to liquidate through option $r = 4$ (recycle) down to the nearest acceptable level $s$, and hold the balance to the next period with potential future profit of $8,669.87

Based on carried inventory, and the expected yield of period 4, the current OHI is 1569 between 1K-2K, and the best option is to liquidate through option $r = 5$ (reuse) to the next period with potential future profit of $8,401.63

Based on carried inventory, and the expected yield of period 5, the current OHI is 1797 between 1K-2K, and the best option is to liquidate through option
CHAPETR 7. INVENTORY BALANCING IN DISASSEMBLY LINE: DYNAMIC PROGRAMMING APPROACH

$r = 5$ (reuse) down to the zero inventory (end of planning horizon) with potential profit of $1,307.01$

$i = 1, j = 2, k = 1$

- If OHI is 1248 units, then the parts are liquidated through liquidation option $r = 2$ (refurbish) down to the nearest acceptable level $s$, and hold the balance to the next period with potential profit of $47,254.70$
- Based on carried inventory, and expected yield on period 2, the current OHI is 1899 between 1K-2K, and the best option is to liquidate through option $r = 4$ (recycle) down to the nearest acceptable level $s$, and hold the balance to the next period with potential future profit of $48,323.77$
- Based on carried inventory, and expected yield of period 3, the current OHI is 1449 between 1K-2K, and the best option is to liquidate through option $r = 5$ (reuse) down to the nearest acceptable level $s$, and hold the balance to the next period with potential future profit of $37,871.92$
- Based on carried inventory, and the expected yield of period 4, the current OHI is 2124 between 2K-3K, and the best option is to liquidate through option $r = 2$ (refurbish) to the next period with potential future profit of $35,459.63$
- Based on carried inventory, and the expected yield of period 5, the current OHI is 2573 between 2K-3K, and the best option is to liquidate through option $r = 4$ (recycle) down to the zero inventory (end of planning horizon) with potential profit of $31,030.38$

$i = 1, j = 2, k = 2$

- If OHI is 1386 units, then the parts are liquidated through liquidation option $r = 3$ (repair) down to the nearest acceptable level $s$, and hold the balance to the next period with potential profit of $42,507.65$
• Based on carried inventory, and expected yield on period 2, the current OHI is 1924 between 1K-2K, and the best option is to liquidate through option \( r = 2 \) (refurbish) down to the nearest acceptable level \( s \), and hold the balance to the next period with potential future profit of $39,759.57

• Based on carried inventory, and expected yield of period 3, the current OHI is 1462 between 1K-2K, and the best option is to liquidate through option \( r = 3 \) (repair) down to the nearest acceptable level \( s \), and hold the balance to the next period with potential future profit of $32,825.37

• Based on carried inventory, and the expected yield of period 4, the current OHI is 2155 between 2K-3K, and the best option is to liquidate through option \( r = 5 \) (reuse) to the next period with potential future profit of $30,626.01

• Based on carried inventory, and the expected yield of period 5, the current OHI is 3618 between 3K-4K, and the best option is to liquidate through option \( r = 4 \) (recycle) down to the zero inventory (end of planning horizon) with potential profit of $35,709.66

\[ i = 1, j = 2, k = 3 \]

• If OHI is 1405 units, then the parts are liquidated through liquidation option \( r = 5 \) (reuse) down to the nearest acceptable level \( s \), and hold the balance to the next period with potential profit of $46,676.11

• Based on carried inventory, and expected yield on period 2, the current OHI is 1937 between 1K-2K, and the best option is to liquidate through option \( r = 5 \) (reuse) down to the nearest acceptable level \( s \), and hold the balance to the next period with potential future profit of $34,269.71

• Based on carried inventory, and expected yield of period 3, the current OHI is 1468 between 1K-2K, and the best option is to liquidate through option \( r = 3 \)
(repair) down to the nearest acceptable level $s$, and hold the balance to the next period with potential future profit of $27,159.15$

- Based on carried inventory, and the expected yield of period 4, the current OHI is 2171 between 2K-3K, and the best option is to liquidate through option $r = 4$ (recycle) to the next period with potential future profit of $24,532.43$

- Based on carried inventory, and the expected yield of period 5, the current OHI is 2639 between 2K-3K, and the best option is to liquidate through option $r = 3$ (repair) down to the zero inventory (end of planning horizon) with potential profit of $21,956.48$

$i = 1, j = 3, k = 1$

- If OHI is 1494 units, then the parts are liquidated through liquidation option $r = 1$ (remanufacture) down to the nearest acceptable level $s$, and hold the balance to the next period with potential profit of $115,322.80$

- Based on carried inventory, and expected yield on period 2, the current OHI is 1996 between 1K-2K, and the best option is to liquidate through option $r = 1$ (remanufacture) down to the nearest acceptable level $s$, and hold the balance to the next period with potential future profit of $107,228.40$

- Based on carried inventory, and expected yield of period 3, the current OHI is 1498 between 1K-2K, and the best option is to liquidate through option $r = 0$ (hold) and hold the balance to the next period with potential future profit of $87,995.49$

- Based on carried inventory, and the expected yield of period 4, the current OHI is 2245 between 2K-3K, and the best option is to liquidate through option $r = 2$ (refurbish) to the next period with potential future profit of $78,590.57$
Based on carried inventory, and the expected yield of period 5, the current OHI is 2743 between 2K-3K, and the best option is to liquidate through option $r = 1$ (remanufacture) down to the zero inventory (end of planning horizon) with potential profit of $59,358.52$

$i = 1, j = 3, k = 2$

- If OHI is 1254 units, then the parts are liquidated through liquidation option $r = 0$ (hold) and hold the balance to the next period with potential profit of $73,099.42$

- Based on carried inventory, and expected yield on period 2, the current OHI is 1836 between 1K-2K, and the best option is to liquidate through option $r = 1$ (remanufacture) down to the nearest acceptable level $s$, and hold the balance to the next period with potential future profit of $68,771.22$

- Based on carried inventory, and expected yield of period 3, the current OHI is 1418 between 1K-2K, and the best option is to liquidate through option $r = 3$ (repair) down to the nearest acceptable level $s$, and hold the balance to the next period with potential future profit of $57,156.58$

- Based on carried inventory, and the expected yield of period 4, the current OHI is 2045 between 2K-3K, and the best option is to liquidate through option $r = 4$ (recycle) to the next period with potential future profit of $52,948.30$

- Based on carried inventory, and the expected yield of period 5, the current OHI is 2463 between 2K-3K, and the best option is to liquidate through option $r = 3$ (repair) down to the zero inventory (end of planning horizon) with potential profit of $42,240.45$
If OHI is 1392 units, then the parts are liquidated through liquidation option $r = 3$ (repair) down to the nearest acceptable level $s$, and hold the balance to the next period with potential profit of $56,165.28$

Based on carried inventory, and expected yield on period 2, the current OHI is 1928 between 1K-2K, and the best option is to liquidate through option $r = 5$ (reuse) down to the nearest acceptable level $s$, and hold the balance to the next period with potential future profit of $53,638.80$

Based on carried inventory, and expected yield of period 3, the current OHI is 1464 between 1K-2K, and the best option is to liquidate through option $r = 1$ (remanufacture) down to the nearest acceptable level $s$, and hold the balance to the next period with potential future profit of $44,309.52$

Based on carried inventory, and the expected yield of period 4, the current OHI is 2160 between 2K-3K, and the best option is to liquidate through option $r = 1$ (remanufacture) to the next period with potential future profit of $40,989.92$

Based on carried inventory, and the expected yield of period 5, the current OHI is 2624 between 2K-3K, and the best option is to liquidate through option $r = 2$ (refurbish) down to the zero inventory (end of planning horizon) with potential profit of $32,878.72$

The acceptable level $s$, vary from period to period and according to the part type. The planner will be able to determine what would be acceptable for that profit such that profit can be maximized in the future periods. However, a full tote always contains 500 parts, and liquidation always occurs in batches of partial or full tote.
Based on end-of-life (EOL) returns and disassembled parts, planner can determine what level of inventory should be kept for the next period and the quantity to be liquidated such that profit is maximized. In the previous example, these levels were identified based on the quality levels and potential selling price in the current and future periods.

### 7.4 Conclusions

In this chapter, Dynamic Programming (DP) model has been introduced to help planner formalize a hold versus liquidate strategy for inventory of all disassembled parts and subassemblies. The model assumes probabilistic input data for supply, that been reduced to its deterministic equivalent, to make it easier to analyze the problem. Different quality levels were introduced with the probability that any quality item can come from any quality parent (core) item. The research answers two important questions: 1) when to liquidate vs. when to hold and for how long, and 2) what option could generate a higher profit in the long run.
Chapter 8

System Dynamics Approach to the

Inventory Management Problem

In this chapter, a simulation approach is presented to model the inventory buildup and behavior in a disassembly setup in the long term. The approach is considered as a model for decision making process in a reverse-logistics environment. Given the descriptive nature of the approach, the problem is investigated under different scenarios of demand of disassembled parts to draw conclusions. This will allow the disassembly facility to examine different strategies of handling inventory of end-of-life (EOL) products and disassembled parts. System dynamics (SD) techniques are used to simulate the process and show its complexity.
8.1 Introduction

In recent years, Original equipments manufacturers (OEMs) are facing more challenges due to the shortening life cycle of products in the market and tighter than before government legislations. Electronics manufacturers for example are introducing new technologies in a fast manner causing the life span of products in the market to decrease significantly and high volume of products will enter the disposal stream. Countries like Japan and others in the European Union (EU) have set rules and regulations that require Original Equipment Manufacturers (OEMs) to take responsibilities of their end-of-life (EOL) products at the end of its products’ life cycle. The United States is still behind in that effort although some states and companies have taken some steps to adapt to these regulations. This in-bound stream of incoming and returned products results in many challenges to the Original Equipment Manufacturers (OEMs) that if handled properly can represent a profitable business, as some studies showed that companies that engage in "active value recovery" have an advantage over others with "no value recovery" policy over the long term (Lehr and Milling, 2009). Interest in the reverse logistics has emerged during the past two decades, shifting the thinking from traditional fast introduction of new products in the market to full accountability of the products manufactured, or in other words the interest has shift from "materials recovery" to "value recovery". Now the process does not stop at the end customers. The manufacturer is also obligated to bear all costs that may arise at the end of the product’s life cycle to fully acquire and recover any value remaining in these items. For instance, Dell Corporation started to charge a small
fee upfront for future collection of obsolete products once the customers decide that they do not need the product any more. Activities like return, inspection, disassembly, and recovery were hardly considered before and now became part of the important aspects of doing business. Companies now consider these end-of-life (EOL) products as an alternative source for secondary resources instead of virgin resources that are depleting very fast. This approach will allow the reuse of materials and parts more than once before they are finally discarded. This approach has environmental as well as economical benefits. As a result, companies are restricting their supply chain process and trying to “close the loop” creating what is known today as Closed-Loop Supply Chain (CLSC).

The management of a Closed-Loop Supply Chain (CLSC) is defined as “design, control, and operation of a system to maximize value creation over the entire life-cycle of a product with the dynamic recovery of value from different types of returns over time” (Guide and Wassenhove, 2002).

Disassembly facilities usually play the role of middle agents between end customers and manufacturers in providing the required materials and parts to be remanufactured and refurbished. Products then are sold in secondary market either locally or globally which proved to be a very profitable business channel. These parts also can be supplied as spare parts for old or discontinued products which are no longer carried in inventory. Others that do not qualify or pass inspection are either accepted by recycling facilities or might be disposed of. In order for these activities to be profitable, disassembly facility should be able to reduce the uncertainty involved with the quantity, quality, and timing of product returns. Eliminating uncertainty and carrying zero
inventory is a considered an ideal situation, which also is hardly achieved in reality. This uncertainty contributes to higher complexity of planning and controlling compared to the traditional forward supply chain. Thus the need for developing tools to overcome this complexity becomes very imminent.

Major challenge Original Equipment Manufacturers (OEMs) face now is how to implement a reverse supply chain network that is both cost effective and efficient. The rapid increase of return of products from end customers back to the origin is a main reason behind that interest. In this chapter, System Dynamics (SD) approach is introduced as a decision making tool to help the planners take corrective actions, and to simulate different scenarios in the reverse supply chain network with a disassembly facility to anticipate the behavior of the system under variety of situations. The system dynamics approach will allow examining the relationship between the different elements of inventory management in a disassembly context.

System dynamics approach is usually applied to macro-systems such as production-inventory system and is presented here to solve the inventory control problem in a disassembly setting. The philosophy behind it is founded on the concept that the system state changes as the inflow and outflow changes (i.e. the rate of return of end-of-life and the demand of disassembled parts and subassemblies). First, the model will address an important question that whether it is at all profitable for a disassembly facility to proceed with and engage in disassembly operations as a source of profit. Otherwise, if that was not the case the facility should accept that proper disposal of these end-of-life
(EOL) products and associated cost is part of its social obligation and environmental
duty. The different model elements and their connections are presented in Figure 8.1.

Figure 8.1 Model structure and connections between elements

In a reverse supply chain, the cycle starts from the end customer, or the user. After products reach its end of life cycle, the customer (demand source) is obligated to return the product back to its origin or to a middle agent for further processing. The products then go through collection and inspection stage, assuming that a collection and transportation network exists. During inspection, products are sorted based on their quality levels and categorized and batched in groups. Items that do not pass preliminary inspection are sent to disposal and are disposed of. However, due to government regulations disposal is limited and capacity constraints are enforced. In the disassembly stage end-of-life (EOL) products go through disassembly operations. The planner tries to
adjust the number of returns and quantity demanded per the demand sources such that minimum inventory is carried on and ensure that demand sources are satisfied. Thus, the goal is always to establish a low and fairly stable inventory to avoid value loses. The storage time is a main parameter for inventory planner as illustrated in Figure 8.2 adapted from Lehr and Milling. The longer product remain in recoverable inventory, the less desired it become for recovery operations as it loses an estimated value of 1% per week in case of electronics (Lehr and Milling, 2009).

![Figure 8.2 Effects of storage time of returned products on desired return rate](image)

Figure 8.2 Effects of storage time of returned products on desired return rate

(Lehr and Milling 2009)
Also, the age and remaining value of end-of-life (EOL) products affect its suitability for recovery operations. A product that is being lightly used and still in a good functional and physical quality than older products in the market are more attractive for disassembly facility as it requires minimal testing and repair and will generate high revenue in the secondary market. The relationship is presented in Figure 8.3 (Lehr and Milling, 2009).

![Graph](image)

**Figure 8.3 Relationship between remaining value and suitability for recovery**

After that, disassembled parts go through **clean up and minor repair** and then sent to the demand sources (reuse, remanufacture, refurbish, repair, or recycle). Refurbished items that are no longer needed in the primary market are technically assumed to be disposed of, however it is sometime exported and marketed in other
secondary market such as in the Japanese car industry where the automobile shredder residue (ASR) is being dumped into the sea to create artificial islands (Kumar and Yamaoka, 2006). Disassembled parts are collected in serviceable inventory and ready for distribution as per their quality levels. Higher quality level parts are more suitable for reuse, remanufacturing, and refurbishing, while average and lower quality level items are more suitable for repair or recycling.

In an ideal case, “there will be a win-win-win situation, in which customer profits from low prices of remanufactured products, the ecological environment is conserved from exploitation of resources, and the OEM is able to make economic profits from recovery activities and to establish an image as an environmentally friendly companies” (Seitz, 2004). Beside this fundamental question, the model has to take into account two main questions: process-oriented questions and product-oriented questions (Lehr and Milling, 2009). First, under the process-oriented questions some strategic options are examined, for example the different types of acquiring products from end customers and under which program (take-back, end of lease…etc) and selling policies to be used in the facility (daily, weekly, target price…etc). Also, the capacity of the facility, capacity of demand sources, and availability of collection facilities are considered. In the product-oriented arena other important factors concerning the type of product that should be included in the collection effort, whether to fully disassemble the end-of-life (EOL) product or partial disassembly is considered, the depth of disassembly to part level or subassembly level. The answer to these questions will determine the best disassembly sequence and disassembly plan that will be adapted. The model can be tested under the
effect of sudden change of demand or supply. Other critical external market factors could be modeled such as price hike; however it is beyond the scope of this research. This study provides knowledge that helps understanding the challenges and opportunities associated with handling inventory of end-of-life (EOL) products under demand fluctuations. In the next section, a methodology and background of system dynamics approach is given, then a detailed simulation model is presented and discussed and conclusion is given based on the results.

8.2 System Dynamics: Background

The inventory system of a disassembly line is analyzed in a closed-loop supply chain context through a dynamic simulation model based on the system dynamics theory (SD). The system dynamics was first introduced in the 1960's by Jay Forrester in his book Industrial Dynamics (Forrester, 1961) and mainly were applied in forward supply chain applications. It is a solution approach based on qualitative and quantitative logical thinking. The method is proven to be an effective tool to understand the complex problem within the supply chain structure. The methodology works as simulation tool to highlight and experiment the relationship between the inventory elements and the impact of external factors on a long-term basis. So far its application in a closed-loop supply chain has been very limited.

First models of system dynamics application in closed-loop supply chain was targeted at the automobile recycling and motivated by the recent government regulations
and environmental concerns (Zamudio-Ramirez, 1996). He studied the interactions between original equipment manufacturers (OEMs), consumers, and recycling companies and modeled the effect of legal and technological changes on the industry. Georgiadis and Vlachos (Georgiadis and Vlachos, 2004) examined the effect of environmental legislations in customer demand in a remanufacturing environment on long term decision making. The model is later extended to include the impact on capacity planning and collection when managing product returns at the end of its life cycle (Georgiadis et al., 2006). Verwater-Lukszo and Christina (Verwater-Lukszo and Christina, 2005) presents a novel modeling approach aimed at improving complex inventory management in a multi-product batch-wise industrial plant. Vlachos (Vlachos et al., 2006) presented a system dynamics model for strategic remanufacturing and collection capacity planning of a single product reverse supply chain for product recovery. The model presents the analysis of system operations such as product flows and stocks, considering capacity constraints, alternative environmental protection policies involving a take-back obligation. The authors validated the model by employing an indirect structural test and then proceeded with numerical investigation. The model analyzes various scenarios to identify efficient policies and further to answer questions about the long-term operations of reverse supply chain using total supply chain profit as the measure of performance. Schröter (Schröter, 2006) studied the spare parts management in a closed-loop context as part of OEM commitment to balance production of large volume of products and its capability to supply spare parts for it. Kumar and Yamaoka (Kumar and Yamaoka, 2007) they apply the system dynamics modeling approach into the Japanese automobile industry to
examine the relationship between reduce, reuse, recycle, and dispose of car parts. They performed qualitative and quantitative analyses on the basis of the stock flow diagram for the closed-loop supply chain. Lehr and Milling (Lehr and Milling, 2009) examined different strategies for collection and value recovery of end-of-life (EOL) products in electronics industry through dynamic simulation technique using system dynamics methodology. Also they have presented state of the art literature on system dynamics in closed-loop supply chain. The study shows “the economic potential of an engagement in value recovery activities and on the other hand highlight the high complexity and connectivity inherited with various value recovery processes”. Poles and Cheong (Poles and Cheong, 2009) uses system dynamics simulation for studying and managing complex feedback systems, particularly business and social system, to model an inventory control system in a remanufacturing process where production is integral with remanufacturing. They analyzed the total inventory cost influenced by returns rate affected by the external factors. In the next section, the details of the model build up and structure will be presented. The methodology of system dynamics applied will be given.

8.3 Simulation Model Structure

The model developed in this chapter represents a decision making tool for inventory planners in a reverse logistics environment. The model is developed using system dynamics simulation software VenSim PLE 5.9. In order to simulate the disassembly and inventory buildup process for the given hypothetical product, a model is developed using
System Dynamics (SD) methodology. The development of system dynamics model is based on the identification of resources and their states (Wolstenholme, 1990). Resources such as materials, people, cash, orders, etc are considered as levels. A state of the resource can be defined as an accumulation of the resource and the rate at which resources are depleted or enhanced between states is represented by rate variables. First order differential equations, such as Euler’s equations are used to build the model. The input and output rates determine the state levels over a time epoch and based on these stocks and flow diagram the model equations are derived. The model is capable of describing a number of elements in the disassembly process from queue size to the size of inventory buildup. Unlike previous models presented, this model will take the stochastic nature of the disassembly process in account. The supply of end-of-life (EOL) products, which is the return stream and the demand of disassembled parts, which is the demand stream, both are based on a pre-set level.

System Dynamics (SD) approach is generally applied to macro-systems such as production-inventory systems, macroeconomics systems, demographic transformation and national economies. Growth models for national economies have been successfully developed by many researchers. The philosophy behind system dynamics (SD) is founded on the concept that industrial systems are like input-output systems. The system state changes according to the change in the rate of inflow and outflow. Hence, any system can be viewed as an input-output system with input-output rates. The system dynamics (SD) model is characterized by the feedback loop that triggers the corrective, control, action. Generally speaking, if the inflow and outflow rates of the end-of-life (EOL) products supply and the demand for disassembled parts are known, then it is a straight forward problem scenario. However, due to the stochastic nature of the problem it is more complex than that. Similarly, another source of complexity that emerges in the
modeling is the life cycle of the product. Hence, a mathematical formulation of the problem can be derived and then verified using simulation software. Thus, the problem can be modeled as follows:

\[
EOL - INV - Level = L_{EOL} = L_A + L_B + L_C = L_{EOL(t-1)} + (R_{out} EOL - R_{in} EOL)\Delta t \quad (8.1)
\]

Similarly,

\[
L_A - a - t\Delta t = L_A(t + \Delta t) = L_A t + (R_{out} A - R_{in} A)\Delta t \quad (8.2)
\]

\[
L_B - a - t\Delta t = L_B(t + \Delta t) = L_B t + (R_{out} B - R_{in} B)\Delta t \quad (8.3)
\]

\[
L_C - a - t\Delta t = L_C(t + \Delta t) = L_C t + (R_{out} C - R_{in} C)\Delta t \quad (8.4)
\]

8.3.1 Inventory Simulation Model

The purpose of the model using VenSim 5.9 software is to simulate the buildup of the inventory of disassembled parts under a fluctuated demand setting. In previous chapters deterministic models were introduced to simplify the problem. However, in reality Disassembly facility usually has no control over the return stream of end-of-life (EOL) products and thus PUSH-system is usually assumed when modeling disassembly lines and the return will follow a statistical distribution rather being known in advance. However, planner can forecast in advance the expected return according to different criteria and products based on the incentive given to customers (80% can be collected) (Lehr and Milling, 2009) and convert it to deterministic equivalent. Under the assumption that a collection and transportation network exists, it is assumed that products arrive at the disassembly facility after being acquired from end users. Thus, the starting inventory level can be known. After disassembly operations, disassembled parts are added to the inventory to satisfy the different demand needs. Following each disassembly operation,
the disassembled part is inspected and added to the inventory of parts (for further processing by demand sources) disposed of. Disassembled parts that do not pass inspection or those with imperfect quality level and proven to be very expensive to recover its value go to the materials recovery stream, or recycling. For simplicity the product assumed in the simulation model are a hypothetical product that contains three parts $A$, $B$, and $C$. The product follows one sequence for disassembly thus parts are disassembled in the order of their placement $A$, $B$, and then $C$. Also, a full disassembly of the product will be adapted, however demand only occur for one major part, and the balance is negligible. Theoretically, because of the differences in demand of each part type, each disassembled part will be unique, and thus an inventory model will vary from one part to another. However, it is also possible that demand for parts coming from one product will be similar or the same, and thus share similar inventory model.

### 8.3.2 Overview of the Simulation Model

The model of the inventory buildup after the disassembly operation has been developed using VenSim PLE 5.9 simulation package. The model describes the different activities that take place in the disassembly facilities such as starting level of end-of-life (EOL) products, disassembly, inspection, addition to inventory (serviceable or recyclable), demand and ultimately disposal. The first step in the model is to inspect the end-of-life (EOL) products and items that pass inspection are sent to the end-of-life (EOL) inventory. After that, products are sent to be disassembled and others are disposed of or recycled. It is assumed that as product age increases its probability to qualify for disassembly and further processing decreases. Similarly, as parts become older it gets
expensive to recover its value by performing disassembly operations, and they become more and more suitable for recycling rather than value recovery, repair, or remanufacturing. After the decision is made to perform the disassembly operation, the end-of-life (EOL) product will be routed through the designated workstations, in this work the assumption is that each part is disassembled separately in a different workstation, meaning no coupling of tasks at any workstation. After the part is disassembled, it will be added to the inventory, and if it is rejected after disassembly it is either disposed of or recycled. Once the demand for this part occurs, withdrawal is made against the current inventory of parts and the balance is carried to the next period. The simulation model is capable of describing number of complex entities in the inventory system such as inventory size, work-in-process, inventory level, rates of disassembly and rates of inspection and sorting. The output file will provide a weekly inventory overview of the disassembly operations and the demand occurrences over a long planning horizon.

System dynamics (SD) model can be tested for confidence building in two ways: through structure tests or behavior tests as suggested by Forrester (Forrester et. al.). Structure tests (structure verification test, parameters verification test, extreme condition test, model border adequacy test and dimensional consistency test) compare the structure of the model with the structure of the real system so that every relationship between the elements of the real system is being compared with the relationship between corresponding elements of the model which is described by mathematical equations. Behavior tests (behavior reproduction test, border adequacy test, and behavior sensibility test) are conducted to determine whether the behavior of the model matches the behavior of the real system, and here the relationship between the structure and the model behavior is analyzed in particular care. The simulation model structure is presented in Figure 8.4 and describes the relationship between all the inventory elements of the system.
Figure 8.4 is known as stock and flow diagram. In the figure, the stocks are represented using rectangular boxes. The inflow and outflow rates are represented by double-lined arrows pointing in and out respectively and the rates of flow is represented by a valve over the line. Clouds are conventionally used to represent sources and sinks that are stocks outside of the model boundary. The arrows marked with a positive sign indicate a direct relationship between the factors, that is, an increase in a factor at the tail of the arrow will cause an increase of the factor at the head of the arrow, while the arrows marked with a negative sign indicate inverse relationship between the factors, that is an increase in a factor at the tail of the arrow will cause a decrease of the factor at the head of the arrow.
8.3.3 Building of the Simulation Model using VenSim PLE 5.9

The system dynamics (SD) model for the inventory management of end-of-life (EOL) products was developed using the components strategy approach in VenSim PLE5.9 simulation software. The first step is the model setting for time and units. Initial time and final time were set at 0 and 300 respectively and with a time step of 0.125. The units of time selected as weeks and type of integration used is Euler's method. The basis for developing the system dynamics model is the casual-loop diagrams (CLDs). It represents the interaction between the elements of the system and its effect on the system output. Similarly, the feedback loops are a characteristic feature of the system dynamic modeling. It is influenced by rate of inspection, inventory of disassembled parts, desired rate of disassembly and adjustment quantity of disassembled parts. The input to the model is the outside demand of the disassembled parts generated by the different demand sources which is an exogenous variable. Thus, the inventory planner has no control over this outside variable. To model the problem, different demand inputs are tested. The common demand input is when a sudden hike in demand of disassembled parts occurs. A pulse input happens when a hike in demand remains for a short period of time and then return to it is original level. A step input is a sharp hike in demand which remain steady afterward at the same height to which the demand was raised. A sine input is a cyclic variation of demand over a large length of period. The ups and downs in the demand oscillate the system violently for some time, and eventually the system accommodates these variations. A ramp input is a gradual linear hike in demand over a length of time.
8.4 Simulation Results and Outlook

To illustrate the interdependencies between the model elements and the effects of external forces, three demand scenarios are examined using the model. The first scenario is with a sudden hike in demand, known as \textit{step input} demand. The initial demand was set at 100 units/week, and a sudden demand increase by 20%. This sudden increase in demand caused a gradual increase of the current end-of-life (EOL) products inventory of 38.90% from 1,800 units to 2,500 units within 28 weeks span. The inventory of end-of-life (EOL) products drops after that for 15 consecutive weeks. The system reaches equilibrium state at week 60 with a steady level of end-of-life (EOL) products of 2,300 core products on hand, or 31.4% more than the initial level at week 0. As a result of a sudden increase in demand, the current inventory of “reusable” parts are depleted to cope with the demand before it is started to build up inventory as the disassembly operations starts. The inventory reaches its lowest level of 200 disassembled parts in 15 weeks span, before it stabilize again at 375 parts level in week 60.
In Figure 8.6 below, the rate of inspection and sorting, and rate of disassembly increases as reflect to the increase in demand. The rate of disposal and disposal quantity increase sharply before it decreases again to normal level after the system stabilizes. The inventory policy under sudden hike in demand can be built on the premises of the inventory system behavior. Thus, it is realistic to permit shortages in the early stages of the hike in demand. However, the equilibrium state maybe reached sooner if sufficient buffer stock is kept to take back the impact of such demands. The figures clearly show that the inventory system reacts slowly to the sudden positive change in demand, destabilizing the system before regains control.

Figure 8.6 Rates and levels associated with inventory system under step input
However, the sudden hike in demand can not only be satisfied by increasing the rate of inspection, sorting, and disassembly. It is important to realize that the increase in end-of-life (EOL) products inventory is because of the increase of the collection of products from the end customers back to recovery to satisfy the higher demand for the disassembled parts. Thus, a network of collection and transportation means from customers to origin has to exist to ensure smooth availability of end-of-life (EOL) products when needed.

In the second scenario, the demand input is sine input. When the demand rate is sinusoidal the demand rate varies accordingly. The inventory of end-of-life (EOL) products increases to nearly 15,000 core products by week 42. As a reflection, to meet the increase in demand, the inspection and sorting rate and the disassembly rate increases accordingly to the peak of 2,250 units/week in 36, and 42 weeks respectively before declining. The lag between the rate of disassembly and rate of inspection and sorting causes the inventory of end-of-life (EOL) core products to build up faster, and causing an increase in disposal rate to avoid carrying excess inventory on hand. At its peak, the disposal rate of end-of-life (EOL) products remains below the mark of 10% of total inventory of end-of-life (EOL) products. The sine input is a wave nature that is repeated over the length of the planning horizon. The accumulation of inventory of end-of-life (EOL) products and reusable parts reacts in a predictable manner to the change in demand rates compared to the previous scenario when the hike in demand is sudden. It is also observed that there is a steady exponential increase in the inventory of reusable parts. Under this scenario, the inventory policy is to keep buffer stocks of end-of-life (EOL) products if the holding cost is kept at minimum, or to adjust the sorting and inspection rate to cope with the change in demand. Adjusting the rates in a disassembly line, would require increasing the manpower at the workstation, or possibly minimizing
the sorting and inspection time. Figure 8.7 shows the variations in inspection and
disassembly rates against the change in demand rates. Also, the different inventory levels
of end-of-life (EOL) products and disassembled parts are also shown.

In the next chapter, the study of the disassembly line and the effect of changing
the input parameters such as arrival rate and service (inspection) rate on the accumulation
of the parts and the overall system cost are studied.

Figure 8.7 Rates and levels associated with inventory system under \textit{sine input}
However, the hike in demand can not only be satisfied by increasing the rate of inspection, sorting, and disassembly. Unlike the previous scenario, the hike repeats itself over the planning horizon, and thus it is important to realize that the increase in end-of-life (EOL) products inventory is because of the increase of the collection of products from the end customers back to recovery in a planned manner such that products arrive just in time for disassembly operation. Thus, a network of collection and transportation means from customers to origin has to exist to ensure smooth availability of end-of-life (EOL) products when needed.

In the third scenario, the demand input is a combination of pulse and ramp input merged together to create this scenario. The input function represents a sudden hike in demand added with a later steady, but slow, increase. The sudden increase in demand of disassembled parts creates a havoc in the inventory levels for a brief time. After that, the changes in inventory is due to the slow increase of demand over the planning horizon. The pulse input causes the rate of inspection and sorting to increase from 100 units/week to near 2,900 units/week, while the disassembly rate have increased to nearly 900 units/week from 100 units/week. The rate of inspection and sorting stays steady from week 45 onwards, when the demand is increasing but slowly, in a predictable manner, at a level of 400 units/week. Similarly, the rate of disassembly stays steady at the same level from week 60 onwards, with a 15 weeks lag. The inventory of end-of-life (EOL) products ready for disassembly stabilizes at week 60 onwards, and stays steady at 3,200 units level. The inventory of resubale parts increases to 5,250 units at week 23, and back to “normal” inventory level of 1,400 unit just before week 60. Again disposal rate remains within 10% limit of inventory when the system is stable with its peak at 1,350 units/week, and normal diposal level of around 400 units/week.
The inventory policy should account for the time delay between the inspection and sorting activities and the disassembly operations. In the previous scenario, 15 weeks is a time lag between both activities to stabilize the system. Inventory buffer of a minimum lead time demand is ideal in this situation to avoid high cost of unnecessary disassembly. The following Figure 8.8 shows the variations in inspection and disassembly rates against the change in demand rates. Also, the different inventory levels of end-of-life (EOL) products and disassembled parts are also shown. In the next section, sensitivity analysis is presented.

![Graphs showing rates and levels associated with inventory under pulse/ramp input](image-url)
The combination of both scenarios is rare in real life situation; however it provides a clear insight in how the system will behave. An agile inventory system and a collection and transportation network should exist in order to ensure proper delivery to the disassembly facility. In all the previous scenarios, delivery to demand sources are not considered as it is assumed that demand sources will arrange for delivery. It is realistic to assume that the consideration of delivery could complicate the problem further.

System Dynamics (SD) model approach helps inventory planner study the impact of the external forces, in this case change of demand behavior, on the inventory over a long period of time. It is important for the planner to ensure smooth delivery of parts to the demand sources when needed at a minimum cost. Delay in processing the demand because of lack of end-of-life (EOL) products inventory on hand, or shortage of disassembled parts or slow rates of inspection and disassembly can become a financial burden on the disassembly facility operations.

In this study, only change in demand was considered. Other factors can be considered such as variation of supply of end-of-life (EOL) products, price hike of the disassembled parts, and change of market conditions. Also, this model assumes the demand exists for one major part, multiple-parts demand also can be considered in the model.

In the next section, a sensitivity analysis is performed under two different cases: i) when the initial demand level increases, and ii) when the disassembly time of the end-of-life (EOL) product increases.
8.4.1 System Dynamics Original Model Sensitivity Analysis

The original model of the end-of-life (EOL) products inventory system assumes the demand is 100 units/week. To test the model behavior when some changes occur, the following two scenarios are presented and tested, when all other parameters remain unchanged: i) when the demand changes from 100 to 112 units/week, and ii) when the disassembly time changes from 8 to 16 minutes/product. The changes on the inventory of end-of-life (EOL) products and inventory of reusable parts inventory are shown in Figure 8.9 and Figure 8.10.

![EOL Inventory for Disassembly](image)

Figure 8.9 Changes in end-of-life (EOL) inventory level when: i) demand changes to 112 parts/week, ii) disassembly time changes to 16 min/product

An increase in demand of 12 units a week, equivalent to 12%, resulted in an increase of the inventory level of end-of-life (EOL) products to 1,200 units, equivalent to 50%. Similarly an increase of disassembly time triggers the initial on hand inventory of
end-of-life (EOL) core products to be increased to 2,200 units' level, equivalent to 25.7%, to cope with the demand of disassembled parts.

![Inventory of reusable Parts](image)

Figure 8.10 Changes in reusable parts inventory level when: i) demand changes to 112 parts/week, ii) disassembly time changes to 16 min/product

An increase in the demand or the increase of the disassembly time both have a negative effect on the inventory of reusable parts, causing a drop in the inventory level, before disassembly rate increases. It is clear that the system reacts better and twice as faster when the demand changes, compared to changes in the system parameter itself such as the increase of the disassembly time. In the first case, when the demand increases by 12% to 112 units/week, the inventory drops to nearly 200 parts inventory and recover by week 45. In the case of disassembly time increase, there is a sharp decrease in reusable parts inventory due to continuous demand and low line yield of parts, yet above the 200 parts level, and it takes longer time to stabilizes again.


8.5 Conclusions

The SD model highlights the complexity and relationship between the different inventory elements. When implementing an inventory policy, the SD model is a valuable tool that helps understanding the long term consequence of market changes on disassembly operations and inventory management of end-of-life (EOL) products, and in putting a forecasting plan of the supply and demand variations. Changes in demand of disassembled parts put more pressure on the facility to adjust its operations to meet this demand and thus, increasing the collection and recovery effort. This requires an agile take-back program and access to enough supply in the market. The disassembly facility should be ready and equipped to process an overwhelming number of products that are returned. It is assumed that by giving incentive to end customers to return their products at the end of its life cycle, the disassembly facility can gain control over the quantity and timing of the returns. Also, it is assumed that with the proper collection network in place, it is possible to account for 80% of the products in the market at their end of life cycle (Guide and Wassenhove, 2003).
Chapter 9

F-Policy Approach to the Inventory Management Problem

This chapter presents the application of F-Policy approach in inventory control in a disassembly line context. The Markovian analysis of end-of-life (EOL) products and disassembled parts is generally concerned with the stochastic processes where knowledge of previous “state” or “outcome” influences future predictions. The objective of this model is to study the queueing of disassembled parts along the workstations for inspection service to be provided and to investigate how to control the inventory levels of disassembled parts by controlling the queue length of different parts, thus cost is minimized and queues are not overwhelmed by excess number of parts.
9.1 Introduction

Controlling the arrival of end-of-life (EOL) products into a disassembly line is proven to be a very complex problem. These end-of-life (EOL) products go through a serious of processes starting by collection from end customer all the way to the disassembly facility before it is sent back again in the forward supply chain. This area has drawn a considerable attention in recent years by researchers, government agencies, and private entities. In a disassembly line, the assumption is that end-of-life (EOL) products arrive to the disassembly facility after “some time” in the market where it has been used. Thus, the return of these products back to its origin is stochastic in nature and follows a probability distribution. Unlike the traditional supply chain, it is very difficult for the planner to determine when the products will be returned. In some cases, this is possible when for example these products are under a lease program where the return date is set in advance. When products are returned, it starts to build up at the disassembly facility, or forming a queue, ready to be disassembled. Products then go through disassembly operations and different parts are disassembled and start to form different queues. The F-Policy approach in disassembly line studies the buildup of disassembled parts at the service (inspection) queue before parts are sent to the demand sources. The study attempts on finding the optimal combination of line capacity and threshold (safety stock) levels. Using queueing theory in the study of the behavior of parts movements and its application in inventory control is effective tool in the disassembly line operations. In the next section, background on the Markovian chain theory is presented.
CHAPTER 9. F-POLICY APPROACH TO THE INVENTORY MANAGEMENT PROBLEM

9.2 Markov Chain: Background and Description

Modern probability theory is generally concerned with the stochastic process where knowledge of previous “state” or “outcome” influences the future prediction. In 1907, Andrey Markov began the study of an important new type of chance process. In this process, the outcome of a given experiment can affect the outcome of the next experiment. This type of process is called a Markov chain.

Formally, a Markov chain is a discrete random process with the Markovian property. A discrete random process means a system which can be in different states, and which changes randomly in discrete steps. The Markov property states that the probability distribution for the system at the next step only depend on the current state of the system, and not additionally on the state of the system at previous steps. Since the system changes states randomly, it is difficult to predict the exact state of the system in the future. However, the statistical properties of the system in the future can often be described. The changes of the states of the system are called transitions, and the probabilities associated with various state changes are called transition probabilities. The system consists of a set of states $S$, where $S = \{s_1, s_2, \ldots, s_r\}$, and it starts at one of these states and move successively from one state to another, and each move is called a step. The Markov chain can be defined as follows:

$$\Pr(X_{n+1} = x | X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = \Pr(X_{n+1} = x | X_n = x_n)$$  \hfill (9.1)
Markov Chain analysis is a powerful modeling and analysis technique with strong application in systems where state transitions take place randomly but in a predictable manner. In general, the behavior of a system is represented by a state-transition diagram which consists of a set of discrete states that the system can be in, and define the frequency at which the transitions between these states take place, in such way that all chain of events within the system can be represented. Thus the Markovian model can be analyzed to provide the probability that a system can be at any given state at any point in time, the amount of time it spends in a given state, and the expected number of transitions between states. Stochastic inventory systems are typical areas of application of Markovian chain analysis. In the disassembly line, the movements of disassembled parts and end-of-life (EOL) products from one state to another can be modeled using Markov Chain theory. Also, the arrival of these disassembled parts to join a current inventory type can be modeled and analyzed using queueing theory. The two concepts are interrelated. As an example to illustrate the application of Markov chain in inventory control in disassembly line, the following states of the system is defined in a disassembly facility that distinguish between the different inventory types.

Assume that a hypothetical end-of-life (EOL) product arrives to a disassembly facility and it is added to the current inventory of “waiting-to-be-disassembled” end-of-life (EOL) products at the disassembly line. After end-of-life (EOL) product get disassembled, the disassembled part(s) are added to inventory of disassembled parts inventory ready to go for inspection, or directly at the serviceable parts inventory, recoverable parts inventory, used parts inventory, or for final disposal inventory. Each one of these inventory type is defined as a “state” of the
system where at any given point in time the end-of-life (EOL) product or disassembled part can be in. The disassembled parts can move from one inventory type to another, for example a part that is considered to be added to the serviceable part inventory needs no or minor cleaning and repair before it is been sent to the demand sources, however, after being used the same part can return through the reverse chain and after inspection it found that it needs major repair, thus this part after its 2nd recovery has been added to the recoverable inventory. Eventually, this part will be disposed of. These states are assumed to be continuously changing from one period to another. Probabilities associated with these states are called transition probabilities and are defined in Table 9.1.

<table>
<thead>
<tr>
<th>Inventory states and transition probabilities</th>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
<th>State 4</th>
<th>State 5</th>
<th>State 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1 EOLP</td>
<td>$P_{11}$</td>
<td>$P_{12}$</td>
<td>$P_{13}$</td>
<td>$P_{14}$</td>
<td>$P_{15}$</td>
<td>$P_{16}$</td>
</tr>
<tr>
<td>State 2 DAPI</td>
<td>$P_{21}$</td>
<td>$P_{22}$</td>
<td>$P_{23}$</td>
<td>$P_{24}$</td>
<td>$P_{25}$</td>
<td>$P_{26}$</td>
</tr>
<tr>
<td>State 3 SPI</td>
<td>$P_{31}$</td>
<td>$P_{32}$</td>
<td>$P_{33}$</td>
<td>$P_{34}$</td>
<td>$P_{35}$</td>
<td>$P_{36}$</td>
</tr>
<tr>
<td>State 4 RPI</td>
<td>$P_{41}$</td>
<td>$P_{42}$</td>
<td>$P_{43}$</td>
<td>$P_{44}$</td>
<td>$P_{45}$</td>
<td>$P_{46}$</td>
</tr>
<tr>
<td>State 5 UPI</td>
<td>$P_{51}$</td>
<td>$P_{52}$</td>
<td>$P_{53}$</td>
<td>$P_{54}$</td>
<td>$P_{55}$</td>
<td>$P_{56}$</td>
</tr>
<tr>
<td>State 6 DPI</td>
<td>$P_{61}$</td>
<td>$P_{62}$</td>
<td>$P_{63}$</td>
<td>$P_{64}$</td>
<td>$P_{65}$</td>
<td>$P_{66}$</td>
</tr>
</tbody>
</table>

Note that the probabilities highlighted in bold represent an absorbing state, where once the product or part is sent to disposal, that part can no longer be recovered through the reverse chain, hence the probability that a part stays at that state, or $P_{66}$ is equal to 1 and all probabilities from state 6 to other states are equal to 0. To illustrate we consider a
numerical example where state matrix is given by $S = \{0.2, 0.2, 0.2, 0.2, 0.2, 0\}$, and the transition probability matrix is given in Table 9.2.

<table>
<thead>
<tr>
<th>Inventory states and transition probabilities</th>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
<th>State 4</th>
<th>State 5</th>
<th>State 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>EOLP</td>
<td>0.05</td>
<td>0.25</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>DAPI</td>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>SPI</td>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>RPI</td>
<td>0</td>
<td>0.1</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>UPI</td>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>DPI</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

By solving the numerical example, steady state probability is reached after 9 steps and the ending probability state matrix are given by $S = \{0.0, 0.01, 0.05, 0.05, 0.04, 0.85\}$ and steady state probabilities is $P_{66} = 1$, when all items in the system are finally disposed of.

See APPENDIX D for detailed step by step transition matrix for the above example.

Queueing theory has application in inventory management and its application can be applied to the disassembly line. To control the inventory threshold and capacity, $F$-Policy approach is applied to the inventory problem of disassembled parts. In a disassembly line, it is reasonable to assume control over the arrival of parts to inspection service. A system with smaller queue sizes provides a pleasing environment, less waiting time, and quicker depletion of on hand inventory, and will result in reduction of cost. In the next section, $F$-Policy is applied to analyze the inventory along the disassembly line.
9.3 F-Policy Problem in Inventory Control of EOL Products

Management of a finite capacity queueing system M/M/1/K with the F-Policy technique is used to exercise control over the building of queues in a disassembly setup. Queues can be managed in one of two ways: either by controlling the arrivals of customers “goods” or by controlling the service being provided (Gupta, 1995). The first approach is usually concerned with controlling the number of customers being in the system, and the second approach focus is to minimize the time the server is being idle. Control the arrival to the system can be applied to controlling inventory of disassembled parts along the disassembly line, while the control service time could have a direct application in balancing the disassembly line itself. By controlling the number of entities in the queue one can achieve a reduced waiting time for the work in process (WIP) while carrying less inventory levels.

In some cases, customers in a queue may respond to the state of the system, i.e. busy queues may expect a lower arrival rate compared to other queues. However, in a disassembly setting arrival of disassembled parts to queues are state independent, thus an external policy might need to be applied to exercise control over the building of queues. The F-Policy used in this approach, the pioneering work in the issue of controlling arrival, was first investigated by Gupta (Gupta, 1995). Later Ke et al., have extended this model and test it under different scenarios (Ke et al., 2006, 2010).
9.3.1 Model Description

The F-Policy problem addresses the issue of controlling arrivals in a queueing system. Consider a single server finite capacity queueing system where parts arrive according to a Poisson distribution with mean $1/\lambda$ and the service time is exponentially distributed with mean $1/\mu$. If the system reaches its capacity $K$ (i.e. the system becomes full) no further parts are allowed to enter the system until enough parts that are already in the system have been served so that the number of parts in the system drops down to a threshold value of $F$ ($0 \leq F \leq K - 1$). At that point, a setup time is required to start allowing parts in the system which is exponentially distributed with mean $1/\beta$. From that point on, the system behaves normally until such time when it reaches its capacity at which time the above process is repeated all over again. Note that the parts not allowed in the queueing system are lost to the system. This system will be denoted by $F\text{-Policy}(\lambda, \mu, \beta, F, K)$.

In the disassembly line, end-of-life (EOL) products that arrive at the disassembly facility will be disassembled. Due to limited storage place along the line and manpower limitation, the control policy should be implemented to maintain the quality of inspection of these parts after disassembly. When the line capacity is full, the arriving disassembled parts will be restricted entrance until the capacity of line drops to a threshold safety stock level. When the safety limit is reduced to the threshold level, then the disassembled parts will be allowed entrance to the system ($F$-Policy) before it is sent to demand sources.
9.3.2 The Equilibrium Solutions

Let $p_{ij}$ where $0 \leq i \leq K$, and $(j = 0, 1)$ be the probability that there are $i$ parts in the system and the arrivals are either allowed $(j = 1)$ or not allowed $(j = 0)$ to enter the system. Thus, $(i, j) \in \{(i, j): i = 0, 1, 2, ..., K; j = 0\} \cup \{(i, j): i = 0, 1, 2, ..., K; j = 1\}$, and the equilibrium equations for $p_{ij}$ are given by:

\begin{align*}
\mu p_{i0} &= \beta p_{00} & (9.1) \\
\mu p_{i+1,0} &= (\mu + \beta) p_{i0} & 1 \leq i \leq F & (9.2) \\
p_{i+1,0} &= p_{i0} & F + 1 \leq i \leq K - 1 & (9.3) \\
\lambda p_{k-1,1} &= \mu p_{K0} & (9.4) \\
\mu p_{11} + \beta p_{00} &= \lambda p_{01} & (9.5) \\
\lambda p_{i-1,1} + \mu p_{i+1,1} + \beta p_{i0} &= (\lambda + \mu) p_{i1} & 1 \leq i \leq F & (9.6) \\
\lambda p_{i-1,1} + \mu p_{i+1,1} &= (\lambda + \mu) p_{i1} & F + 1 \leq i \leq K - 2 & (9.7) \\
\lambda p_{K-2,1} &= (\lambda + \mu) p_{K-1,1} & F \neq K - 1 & (9.8)
\end{align*}

Then a spreadsheet analysis is used to compute those probabilities by using the following boundary condition given by

$$
\sum_{i=0}^{K-1} \sum_{j=0}^{1} p_{ij} + p_{K0} = 1
$$

(9.9)

In the next sections, steady state solutions and measures of performance are introduced to calculate the effectiveness of the system.
9.3.3 Steady State Solutions

The equilibrium equations can be solved to derive the steady state solutions as follows:

from (9.1)-(9.3), solving will result in the following:

\[ p_{i0} = p_{00} \frac{1-\alpha}{F} \quad 1 \leq i \leq F \quad (9.10) \]

\[ p_{i0} = p_{00} \frac{1-\alpha}{\alpha_{F+1}} \quad F + 1 \leq i \leq K \quad (9.11) \]

Where,

\[ \alpha = \frac{\mu}{\mu + \beta} \quad (9.12) \]

From equation (9.4), (9.7), and (9.8) and by induction,

\[ p_{i1} = p_{00} \frac{\sigma(1-\alpha)(1-\sigma^{K-i})}{(1-\sigma)\alpha_{F+1}} \quad F + 1 \leq i \leq K - 1 \quad (9.13) \]

From equations (9.6) and (9.7) and by induction,

\[ p_{i1} = p_{00} \left\{ \frac{\sigma^{-i+F+2}(1-\alpha)^2}{(1-\sigma)(\sigma-\alpha)\alpha_{F+1}} - \frac{(1-\alpha)\sigma}{(\sigma-\alpha)\alpha_{i}} - \frac{\sigma^{K-i+1}(1-\alpha)}{(1-\sigma)\alpha_{F+1}} \right\} \quad 0 \leq i \leq F \quad (9.14) \]

Where,

\[ \sigma = \frac{\mu}{\lambda} \quad (9.15) \]

Substituting the values of \( p_{ij} \) from equations (9.10), (9.11), (9.13), and (9.15) in the boundary condition (9.9), simplifying and solving for \( p_{00} \), resulting in equation (9.16)

\[ p_{00} = \left\{ \frac{(K-F-\sigma)(1-\alpha)}{(1-\sigma)\alpha_{F+1}} - \frac{1-\alpha_F}{(\sigma-\alpha)\alpha_{F+1}} - \frac{\sigma^2(1-\sigma^{K})(1-\alpha)}{(1-\sigma)^2\alpha_{F+1}} + \frac{\sigma^2(1-\sigma^{F+1})(1-\alpha)^2}{(1-\sigma)^2(\sigma-\alpha)\alpha_{F+1}} \right\}^{-1} \]
9.3.4 Measures of Effectiveness

Some important measures of effectiveness for the problem are defined below:

\( p_e \): The probability that the workstation is providing inspection to the part;

\( p_s \): The probability that the server requires a startup time before starting again;

\( p_b \): The probability that the system is blocked (no entrance allowed);

\( E(S) \): Expected number of parts when server starts to allow parts entering the system;

\( E(B) \): Expected number of parts when the system is blocked;

\( W \): Expected waiting time in the system;

\( L \): Expected number of parts in the system;

The expressions for the above parameters are given by:

\[
L = \sum_{i=0}^{K} ip_{i0} + \sum_{i=0}^{K-1} ip_{i1}
\]  \hspace{1cm} (9.17)

\[
p_e = \sum_{i=0}^{K} P_{i0} + \sum_{i=0}^{K-1} P_{i1}
\]  \hspace{1cm} (9.18)

\[
p_s = \sum_{i=0}^{F} P_{i0}
\]  \hspace{1cm} (9.19)

\[
p_b = \sum_{i=0}^{K} P_{i0}
\]  \hspace{1cm} (9.20)

The Markov chain representing the queueing system is ergodic. Using Little’s formula, which is valid for ergodic systems, the expected time spent by a part waiting for inspection and getting service \( W \), can be calculated as follows:
\[ W = \frac{L}{\lambda'} \] (9.21)

The effective arrival rate is \( \lambda' \) can be calculated as follows:

\[ \lambda' = \lambda \sum_{i=0}^{K-1} p_{ni} \] (9.22)

\[ E(S) = \sum_{i=0}^{F} ip_{i0} \] (9.23)

\[ E(B) = \sum_{i=0}^{K} ip_{i0} \] (9.24)

### 9.3.5 Cost Analysis

Ke et al., (Ke et al., 2010) has constructed a total expected cost function for the \( F \)-Policy \( M/M/1/K \) queueing system with second optional service. The same approach can be applied here for one essential service (inspection) in which \( F \) and \( K \) are decision variables. The objective is to determine the optimum threshold level (safety stock) \( F^* \), and the optimum system capacity \( K^* \), simultaneously at minimum cost. The joint optimal values \( (F^*, K^*) \) and various measure of effectiveness are obtained based on assumed values assigned to the system. As the values changes, the combination of \( (F^*, K^*) \) will change accordingly. Thus, the total cost \( TC \) function can be obtained using

\[ TC(F, K) = C_{\text{hold}} \ast L + C_{\text{ins}} \ast p_c + C_{\text{setup}} \ast p_s + C_{\text{lost}} \ast \lambda \ast p_b + C_{\text{wait}} \ast W + C_{\text{fixed}} \ast K \] (9.25)
Where the costs are holding cost, inspection cost, setup cost, lost part to the system cost, waiting cost, and fixed cost per part per unit time respectively. Please note that setup cost only occurs when the level of inventory drops to the threshold level $F$. The costs parameters are assumed to be linear. As per Ke et al., an efficient and direct procedure is used to obtain $(F^*, K^*)$. The algorithm is presented below:

Step 1: Find the optimal threshold value $F$, for a given system capacity, $K$, i.e,

$$\min_{F} TC(F, K) = TC(F^*, K^*)$$

Step 2: Find the set of all minimum cost solutions for given $K$ values

Step 3: Find the minimum cost solution among the above set

$$\min_{F} \Theta = TC(F^*, K^*)$$

In the next section, a numerical example will be given to illustrate the approach

### 9.4 F-Policy Problem Numerical Example

End-of-life (EOL) products will arrive to the disassembly facility. After disassembly, the disassembled parts will queue at the designated workstation. Their arrival rate follows a Poisson process with rate $\lambda = 1.5$. The service (inspection) rate follows an exponential distribution with rate $\mu = 1.0$. The disassembly line setup time to start allowing disassembled parts in queue again is exponential random variable with rate $\beta = 0.1$. The costs associated with the inventory process of disassembled parts are:
Table 9.3 summarizes the results findings of the problem, where we can find that the expected minimum cost function can be achieved when the threshold level remain at minimum level 1, however when the capacity of the queue decreased to 7 the threshold level has increased to 2, and thus the minimum cost function attains at a minimum value of $62.26 at $F^* = 2$ and $K^* = 7$.

Table 9.3  The expected cost $TC(F^*, K)$ for given $K$

<table>
<thead>
<tr>
<th>$F$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>$TC(F,K^*)$</td>
<td>$75.71$</td>
<td>$69.42$</td>
<td>$63.16$</td>
<td>$69.00$</td>
<td>$77.99$</td>
<td>$72.54$</td>
<td>$67.27$</td>
<td>$62.26$</td>
<td>$79.31$</td>
<td>$73.86$</td>
</tr>
<tr>
<td></td>
<td>$77.99$</td>
<td>$72.54$</td>
<td>$67.27$</td>
<td>$69.00$</td>
<td>$77.99$</td>
<td>$72.54$</td>
<td>$67.27$</td>
<td>$62.26$</td>
<td>$79.31$</td>
<td>$73.86$</td>
</tr>
<tr>
<td></td>
<td>$78.56$</td>
<td>$73.11$</td>
<td>$67.87$</td>
<td>$62.89$</td>
<td>$79.31$</td>
<td>$73.86$</td>
<td>$68.64$</td>
<td>$63.68$</td>
<td>$79.31$</td>
<td>$73.86$</td>
</tr>
<tr>
<td></td>
<td>$80.20$</td>
<td>$74.75$</td>
<td>$69.52$</td>
<td>$64.57$</td>
<td>$81.20$</td>
<td>$75.72$</td>
<td>$70.46$</td>
<td>$65.48$</td>
<td>$81.20$</td>
<td>$75.72$</td>
</tr>
<tr>
<td></td>
<td>$82.24$</td>
<td>$76.70$</td>
<td>$71.36$</td>
<td>$66.25$</td>
<td>$83.26$</td>
<td>$77.61$</td>
<td>$72.10$</td>
<td>NA</td>
<td>$83.26$</td>
<td>$77.61$</td>
</tr>
<tr>
<td></td>
<td>$86.16$</td>
<td>$76.49$</td>
<td>NA</td>
<td>NA</td>
<td>$88.32$</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>$88.32$</td>
<td>NA</td>
</tr>
</tbody>
</table>
By testing the validity of the mathematical model over different capacity levels, it is reasonable to conclude from the above results that as the planner tends to decrease the queue capacity for any reason (work limitation, space utilization,… etc) the threshold (safety stock) level will increase in response. This increase can be explained by system attempt to adjust to meet the demand and reduce the risk of running out of inventory by increasing its threshold level by adding an extra “security layer”. When the system has enough space to accommodate large number of parts, providing a service to work-in-process becomes a priority over having a bigger inventory size. The numerical illustration provides an insight to the inventory planner on how to properly manage the disassembled parts from end-of-life (EOL) products and the above results in this case provides help to make the decision regarding the queue capacity, the threshold level, and optimal level of re-allowing parts to start accumulating for service at the designated workstation. Although the model will result in the minimum cost achievable by the system, the planner has the flexibility to adjust the system parameters (queue capacity and threshold level) in any way of preference. In other words, it is not necessary that minimum cost is achieved; the planner can decide that for an extra known cost, the stock level can be increased and thus risk of running out of stock is minimized. Take for example the combination \((F = 1, K = 8)\) with a total cost \(TC = $63.16\), if planner decides to increase the threshold level by 1 parts, to \(F = 2\), in other words by 100\%, this will result in an increase of $4.11, or an increase of 7\% in the total cost. However, in the case of \(F = 2, K = 7\) an increase of 1 part to the threshold level to \(F = 3\), or a 50\% increase,
will only result in an increase of $0.60, or 1% in the total cost. Another scenario is to increase the capacity of the queue and measure its impact on the total cost. For the current ideal situation for example \((F = 2, K = 7)\) if planner decides to increase the capacity of the queue up \(K = 10\), equivalent to 43% increase, this will result in a in increase in the total cost equals to $15.73, equivalent to 25% increase in the total cost. Although the planner is not required to adapt the inventory policy that minimizes cost at all times, it is likely that other non-cost metrics would be useful in situations similar to this case, when the most commonly used inventory systems objective is to minimize cost. The planner has the flexibility to assess every situation independently and determines the best inventory levels that provides highest service rate. In this example, demand of one disassembled part type is assumed. Demand for all disassembled parts can be investigated in the same manner. For the simplicity of the mathematical model illustration, the assumption is that demand only occurs for one part type, and the balance of end-of-life (EOL) product is sent for disposal. From the mathematical results, it can also be concluded that the function is truly convex and that the solution gives a minimum value. Further, based on the above setting of the problem, a sensitivity analysis is performed for changes in the optimal values of \((F^*, K^*)\) along with any changes in the system parameters. Under the optimal operating conditions, different values for \((\lambda, \mu)\) are considered and different system measures of effectiveness are displayed in Table 9.4 and compared and conclusions are provided based on the results.
9.4.1 Sensitivity Analysis

Table 9.4 shows the different scenarios under which the model was tested and evaluated for different value of $\lambda$ and $\mu$.

<table>
<thead>
<tr>
<th>$(\lambda, \mu)$</th>
<th>$(0.5, 1.0)$</th>
<th>$(0.7, 1.0)$</th>
<th>$(0.9, 1.0)$</th>
<th>$(0.4, 2.0)$</th>
<th>$(0.4, 3.0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(F^<em>, K^</em>)$</td>
<td>(3,7)</td>
<td>(3,7)</td>
<td>(4,7)</td>
<td>(2,7)</td>
<td>(2,7)</td>
</tr>
<tr>
<td>$TC(F^<em>, K^</em>)$</td>
<td>$33.15$</td>
<td>$38.40$</td>
<td>$44.31$</td>
<td>$22.25$</td>
<td>$20.23$</td>
</tr>
<tr>
<td>$L$</td>
<td>0.895</td>
<td>1.32</td>
<td>$1.42$</td>
<td>0.249</td>
<td>0.154</td>
</tr>
<tr>
<td>$E(B)$</td>
<td>0.1</td>
<td>0.479</td>
<td>1.066</td>
<td>0.0002</td>
<td>0.0000016</td>
</tr>
<tr>
<td>$E(S)$</td>
<td>0.01</td>
<td>0.045</td>
<td>0.166</td>
<td>0.0002</td>
<td>0.000015</td>
</tr>
</tbody>
</table>

The first three scenarios in the above table examine the effect of arrival rate changes and its effect in the system parameters and system total cost $TC$, when the service rate remains unchanged. The last two scenarios in the same table (Table 9.4) examine the effect of service rate changes when the arrival rate remains unchanged.

From Table 9.4, it is clear that as the value of $\lambda$ increases, the threshold level $(F^*)$ increases and the expected cost also $TC$ increases. However as $\mu$ increases, the optimal system parameters $(F^*, K^*)$ remain unchanged and at steady level $(2, 7)$ however the total cost $TC(F^*, K^*)$ decreases. It is reasonable to assume that the system total cost and inventory holding cost will increase in response to the increased level of arrival of disassembled parts to the queue waiting for inspection. Also it is reasonable to assume
that as the service rate increases, the system total cost slightly decreases because fewer parts will remain in inventory and the queue will clear up faster.

9.5 Conclusions

In this chapter, the F-Policy M/M/1/K queueing system was examined with one essential service for inspection of the disassembled parts in a disassembly line. The F-Policy is the pioneering work done by Gupta (Gupta, 1995). Later, the model was extended by Ke et al. (Ke et al., 2006). After the end-of-life (EOL) products arrive at the disassembly facility, and parts are disassembled then it starts to queue up at the designated workstation for essential service (inspection) to be provided. The model aims at determining the optimal system parameters of the queue capacity and the optimal threshold level, thus the total expected cost is minimized. Then, the system is tested under different input values to validate the model. The model in disassembly line will provide a tool to evaluate the performance of the workstation and the buildup of queues along the line. It will help the planner configure the queue size and inventory level when the rate of arrival of parts or service rate changes to cope with the demand.
Chapter 10

Conclusions

This chapter discusses the outline and the results of this research in the area of inventory management of end-of-life (EOL) products in the disassembly line. Disassembly facilities play an important role in facilitating the collection and disassembly of end-of-life (EOL) products for the purpose of recovering any value remaining or final disposal. Inventory management is one of the main concerns for original equipment manufacturers (OEMs) who are involved in these activities. The disparity between the demand for the disassembled parts and subassemblies and the disassembly line yields make inventory control of these parts a very challenging task. This disparity can lead to fluctuations in inventory of parts causing either a too low or too high inventory levels. Thus, to avoid these high costs associated with inventory extremes, efficient and cost effective inventory management policies become very critical. Adapting moderate inventory policies can allow the appropriate liquidation of inventory without risking the safety levels and will contribute to the system overall profit. This dissertation attempts to develop inventory control models for the end-of-life (EOL) products in a disassembly line taking into
consideration some important factors: disassembly line capacity, demand sources capacity, disassembly line space availability, deterministic and stochastic input for supply and demand, government environmental regulations for disposal and recycling quota, and variety of products quality levels.

In general, the area of inventory management of disassembled parts and subassemblies are divided into two main categories: deterministic and stochastic models. The nature of the approach is determined by the inputs of the system. In the deterministic approach two techniques are applied: linear programming (LP) modeling and dynamic programming (DP) modeling were information about costs, prices, demand and supply are known. In the stochastic approach two other techniques are applied: system dynamics (SD) modeling and queueing theory F-Policy approach in inventory management. The inputs of the model are expressed in levels, rates, and probability distributions. It can be concluded that the results of the stochastic model provide more accurate solution to the problem than the deterministic models. Supply and demand of end-of-life (EOL) products is often influenced by external forces such as seasonal effects, price of raw materials, availability of spare parts in the market, etc thus it is more realistic to assume stochastic values when studying the inventory models. However, for simplicity planners prefer to apply deterministic models that provide near optimal solutions to the problem, with good approximation to the optimal solution. In the next section, the research findings and future research are discussed.
10.1 Summary of Results and Findings

The inventory management models developed in this dissertation are useful tools for managing inventory of end-of-life (EOL) products. One model fits all type of inventory policy is hard to achieve in the disassembly setting. The end-of-life (EOL) products returns are characterized by its high uncertainty, thus the models attempt to offer an insight on how to control these inventories by handling uncertainties.

1. Chapter 5 is an introductory chapter that summarizes and evaluates the different tools that have been presented in the literature for similar models in reverse logistics and disassembly setup. In this chapter the following questions are answered: i) What to recycle and what to dispose of? ii) When and what to procure? iii) What are the effects of products quality levels on market price, and iv) what are the effects of selling policies on the system profitability? This chapter addresses these issues through the different techniques that have been applied by other researchers. It shows that that not only supply and demand affects inventory levels at the disassembly facilities, but other system constraints, environmental regulations, the economy, and the applied pricing strategy could all influence the bottom line. The determination of what to keep vs. what to liquidate and under what cases and how to assign holding costs are discussed.

2. Chapter 6 assumes deterministic model of the end-of-life (EOL) product returns. The demand and supply of end-of-life (EOL) products and disassembled parts are known. The model applies linear programming (LP) approach with the objective
to minimize the system total cost. The objective function and system constraints are developed and the following system criteria were considered: inventory capacity, demand source limitation, disassembled parts volume, recycling and disposal regulations. The model is developed over two phases: i) phase one objective is to minimize the cost of on hand inventory, and ii) phase two objective is to maximize revenue from inventory that is beyond demand.

3. Chapter 7 uses the quality levels of the disassembled parts as a basis for comparison. The model compares for every quality level of disassembled part the estimated net profit generated if that batch of parts are processed through certain liquidation option compared to other liquidation channels and against holding the parts and liquidating in the future for potentially higher profit. The DP model provides near optimal results through extensive search.

4. Chapter 8 uses the System Dynamics (SD) model approach to analyze the inventory of disassembled parts build up at the disassembly facility. The basic philosophy behind the model is that any system can be viewed as input-output system with flow rates. The change in flow rates changes the system state. The long term behavior of the system, the time it takes to re-stabilize and its reaction to sudden external forces such as demand changes are monitored. Policies under different demand scenarios are presented and discussed. Very limited research in the area of disassembly has applied the System Dynamics (SD) concept.

5. Chapter 9 investigates the application of queueing theory in inventory management of end-of-life (EOL) product. F-Policy approach is used to exercise
control of the arrival of disassembled parts along the disassembly line workstations. These parts accumulate for inspection services to be provided before it is sent to demand sources. The policy is sued to determine: i) the optimal line capacity $K^*$, and ii) the optimum threshold level $F^*$. If inventory drops to the threshold level, parts are allowed to queue up in the system to build up the inventory, and stop at the capacity. Minimum cost could be achieved by solving the possible range of $(F, K)$. It also provides a helpful tool for the planner to show the cost implication if to increase the confidence level by increasing the threshold level. To inventory planner, other non-cost metrics are of great importance beside financial metrics.

10.2 Future Research

The future research will include investigation of the multiple-product structure of the end-of-life (EOL) products inventory management system in the disassembly line. The models in this research mainly focused in a hypothetical single-product model. After disassembly operations, the product will always generate three different parts. Demand always occurs for each part separately, and independent from other parts demand. There is limited research in the literature that has investigated the inventory control problem in a disassembly context. Some have focused on a hybrid system with manufacturing and remanufacturing system considered, while others focused on total quantity of returns that will minimize the average cost of the system in either a standalone setup (only
remanufacturing) or in a disassembly facility. In this research a disassembly line setup is considered with product disassembly activities only and no recovery activities (remanufacturing, refurbishing, repair, etc).

In future research the following areas are of interest for further exploration and investigation:

- Demand for disassembled parts should not be limited to single part type from each demand source. Demand of combination of parts coming from one demand source is more realistic. The disassembly facility will try to balance all inventory of disassembled parts, thus these inventories become interrelated.
- Demand lead time should be considered as demand might occur between the demand and the actual delivery time.
- Demand sources constraint was limited to demand source capacity. However, other factors such as financial, physical, and environmental constraints should be included.
- It is realistic to assume that demand sources distinguish between the different quality levels of disassembled parts in the linear programming model.
- Supply lead time and disassembly manpower requirements should be accounted for in future research because its impact on the system solution.
- Multiple product scenarios with parts commonality should be investigated further since disassembly facility usually will handle variety of products of the same make and model. Other independent facilities will handle all products that fall in the same category such as cell phones, home appliances, etc.
• Products quality levels should take probability distributions to cover all possible outcome values. In this research deterministic values were used.

• The interrelationship between quality of parts and its effect on other parts quality might be addressed.

• To investigate the effect of using a time-series analysis model in pricing the end-of-life (EOL) products and the disassembled parts on the system's profitability.

• To compare the different setting rules for assigning the inventory holding costs for end-of-life (EOL) products, disassembled parts and the fraction value based on different assignment approaches (Akcali and Bayindir, 2008).

• Seasonal effects of the end-of-life (EOL) products supply can be studied and included in the future models.

• Application of forecasting techniques can be integrated to study the behavior of the product returns and demand of disassembled parts.

• System dynamics model is a powerful tool to be applied to the inventory control problem in a disassembly line. Further enhancement to the model is possible in terms of multiple parts demand, quality levels of the disassembled parts, parts and weigh and volume for space utilization, and capacities of line and demand sources.

• It is realistic to assume limited disassembly and storage capacity in the system dynamics model.
- Cost metrics analysis based on the system dynamics model will be useful in comparison of the system performance. Currently, non-cost metrics are considered.

- Further application of the queueing model using F-Policy problem can be extended to include a second optional service as per Ke et al. (Ke et al., 2010).

- Further investigation should be carried out to determine the value remaining in each and every disassembled part and further keep vs. liquidation decisions should be made in the light of these values.

- The economic demanufacturing quantity model (EDQ) developed by Gunter (Gunter, 2004) may be extended by adding the parts volume factor.

- The economic demanufacturing quantity model (EDQ) developed by Gunter can be integrated with the dynamic programming (DP) model instead of the full and partial tote liquidation constraint assumed in the model.
References


APPENDIX A

LINEAR PROGRAMMING MODEL APPROACH TO INVENTORY IN DISASSEMBLY LINE

The following codes are the Linear Programming (LP) models used to simulate the piling of disassembled parts and subassemblies along the disassembly line. The models presented here are used in chapter 6.

MODEL:
! SINGLE PRODUCT DISASSEMBLY MODEL;

SETS:
CORE /1/:WC, WI, YC;
SUBASSEMBLY /1 2 3/:WS, WJ, YS, CDISS, CRECS, CHOLS, CPUR;
QUALITY / 1 2 3/:CD, CDISC, CRECC, CHOLC, Time;
PERIOD / 1 2 3 4 5/:RTC, ATime, CAP;
QUANTITY_C(CORE, QUALITY, PERIOD):QD,QRET, QDISC, QRECC, QINVC;
QUANTITY_S(SUBASSEMBLY, PERIOD):D, QDISS, QRECS, QPUR, QINVS;
END SETS

!Objective Function;
MIN= 10*QD( 1, 1, 1)+ 4.25*QDISC( 1, 1, 1)+ 7.50*QRECC( 1, 1, 1)+ 1.30*QINVC( 1, 1, 1)+
       10*QD( 1, 1, 2)+ 4.25*QDISC( 1, 1, 2)+ 7.50*QRECC( 1, 1, 2)+ 1.30*QINVC( 1, 1, 2)+
       10*QD( 1, 1, 3)+ 4.25*QDISC( 1, 1, 3)+ 7.50*QRECC( 1, 1, 3)+ 1.30*QINVC( 1, 1, 3)+
       10*QD( 1, 1, 4)+ 4.25*QDISC( 1, 1, 4)+ 7.50*QRECC( 1, 1, 4)+ 1.30*QINVC( 1, 1, 4)+
       10*QD( 1, 1, 5)+ 4.25*QDISC( 1, 1, 5)+ 7.50*QRECC( 1, 1, 5)+ 1.30*QINVC( 1, 1, 5)+
       13*QD( 1, 2, 1)+ 4.25*QDISC( 1, 2, 1)+ 7.50*QRECC( 1, 2, 1)+ 0.90*QINVC( 1, 2, 1)+
       13*QD( 1, 2, 2)+ 4.25*QDISC( 1, 2, 2)+ 7.50*QRECC( 1, 2, 2)+ 0.90*QINVC( 1, 2, 2)+
       13*QD( 1, 2, 3)+ 4.25*QDISC( 1, 2, 3)+ 7.50*QRECC( 1, 2, 3)+ 0.90*QINVC( 1, 2, 3)+
       13*QD( 1, 2, 4)+ 4.25*QDISC( 1, 2, 4)+ 7.50*QRECC( 1, 2, 4)+ 0.90*QINVC( 1, 2, 4)+
       13*QD( 1, 2, 5)+ 4.25*QDISC( 1, 2, 5)+ 7.50*QRECC( 1, 2, 5)+ 0.90*QINVC( 1, 2, 5)+
       17*QD( 1, 3, 1)+ 4.25*QDISC( 1, 3, 1)+ 7.50*QRECC( 1, 3, 1)+ 0.70*QINVC( 1, 3, 1)+
       17*QD( 1, 3, 2)+ 4.25*QDISC( 1, 3, 2)+ 7.50*QRECC( 1, 3, 2)+ 0.70*QINVC( 1, 3, 2)+
       17*QD( 1, 3, 3)+ 4.25*QDISC( 1, 3, 3)+ 7.50*QRECC( 1, 3, 3)+ 0.70*QINVC( 1, 3, 3)+
       17*QD( 1, 3, 4)+ 4.25*QDISC( 1, 3, 4)+ 7.50*QRECC( 1, 3, 4)+ 0.70*QINVC( 1, 3, 4)+
\[17*QD(1, 3, 5) + 4.25*QDISC(1, 3, 5) + 7.50*QRECC(1, 3, 5) + 0.70*QINVC(1, 3, 5) +
\]
\[1.25*QDISS (1, 1) + 1.75*QRECS (1, 1) + 25*QPUR (1, 1) + 0.20*QINVS (1, 1) +
\]
\[1.25*QDISS (1, 2) + 1.75*QRECS (1, 2) + 25*QPUR (1, 2) + 0.20*QINVS (1, 2) +
\]
\[1.25*QDISS (1, 3) + 1.75*QRECS (1, 3) + 25*QPUR (1, 3) + 0.20*QINVS (1, 3) +
\]
\[1.25*QDISS (1, 4) + 1.75*QRECS (1, 4) + 25*QPUR (1, 4) + 0.20*QINVS (1, 4) +
\]
\[1.25*QDISS (1, 5) + 1.75*QRECS (1, 5) + 25*QPUR (1, 5) + 0.20*QINVS (1, 5) +
\]
\[1.25*QDISS (2, 1) + 1.75*QRECS (2, 1) + 17*QPUR (2, 1) + 0.40*QINVS (2, 1) +
\]
\[1.25*QDISS (2, 2) + 1.75*QRECS (2, 2) + 17*QPUR (2, 2) + 0.40*QINVS (2, 2) +
\]
\[1.25*QDISS (2, 3) + 1.75*QRECS (2, 3) + 17*QPUR (2, 3) + 0.40*QINVS (2, 3) +
\]
\[1.25*QDISS (2, 4) + 1.75*QRECS (2, 4) + 17*QPUR (2, 4) + 0.40*QINVS (2, 4) +
\]
\[1.25*QDISS (2, 5) + 1.75*QRECS (2, 5) + 17*QPUR (2, 5) + 0.40*QINVS (2, 5) +
\]
\[1.25*QDISS (3, 1) + 1.75*QRECS (3, 1) + 28*QPUR (3, 1) + 0.60*QINVS (3, 1) +
\]
\[1.25*QDISS (3, 2) + 1.75*QRECS (3, 2) + 28*QPUR (3, 2) + 0.60*QINVS (3, 2) +
\]
\[1.25*QDISS (3, 3) + 1.75*QRECS (3, 3) + 28*QPUR (3, 3) + 0.60*QINVS (3, 3) +
\]
\[1.25*QDISS (3, 4) + 1.75*QRECS (3, 4) + 28*QPUR (3, 4) + 0.60*QINVS (3, 4) +
\]
\[1.25*QDISS (3, 5) + 1.75*QRECS (3, 5) + 28*QPUR (3, 5) + 0.60*QINVS (3, 5) +
\]

!Inventory of core products;
\[QINVC (1, 1, 1) = QRET (1, 1, 1) - QD (1, 1, 1) - QRECC (1, 1, 1) - QDISC (1, 1, 1);
\]
\[QINVC (1, 1, 2) = QINVC (1, 1, 1) + QRET (1, 1, 2) - QD (1, 1, 2) - QRECC (1, 1, 2) - QDISC (1, 1, 2);
\]
\[QINVC (1, 1, 3) = QINVC (1, 1, 2) + QRET (1, 1, 3) - QD (1, 1, 3) - QRECC (1, 1, 3) - QDISC (1, 1, 3);
\]
\[QINVC (1, 1, 4) = QINVC (1, 1, 3) + QRET (1, 1, 4) - QD (1, 1, 4) - QRECC (1, 1, 4) - QDISC (1, 1, 4);
\]
\[QINVC (1, 1, 5) = QINVC (1, 1, 5) + QRET (1, 1, 5) - QD (1, 1, 5) - QRECC (1, 1, 5) - QDISC (1, 1, 5);
\]
\[QINVC (1, 2, 1) = QRET (1, 2, 1) - QD (1, 2, 1) - QRECC (1, 2, 1) - QDISC (1, 2, 1);
\]
\[QINVC (1, 2, 2) = QINVC (1, 2, 1) + QRET (1, 2, 2) - QD (1, 2, 2) - QRECC (1, 2, 2) - QDISC (1, 2, 2);
\]
\[QINVC (1, 2, 3) = QINVC (1, 2, 2) + QRET (1, 2, 3) - QD (1, 2, 3) - QRECC (1, 2, 3) - QDISC (1, 2, 3);
\]
\[QINVC (1, 2, 4) = QINVC (1, 2, 3) + QRET (1, 2, 4) - QD (1, 2, 4) - QRECC (1, 2, 4) - QDISC (1, 2, 4);
\]
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\[ Q\text{INVC}(1, 2, 5) = Q\text{INVC}(1, 2, 4) + Q\text{RET}(1, 2, 5) - Q\text{D}(1, 2, 5) - Q\text{RECC}(1, 2, 5); \]
\[ Q\text{INVC}(1, 3, 1) = Q\text{RET}(1, 3, 1) - Q\text{D}(1, 3, 1) - Q\text{RECC}(1, 3, 1) - Q\text{DISC}(1, 3, 1); \]
\[ Q\text{INVC}(1, 3, 2) = Q\text{INVC}(1, 3, 1) + Q\text{RET}(1, 3, 2) - Q\text{D}(1, 3, 2) - Q\text{RECC}(1, 3, 2) - Q\text{DISC}(1, 3, 2); \]
\[ Q\text{INVC}(1, 3, 3) = Q\text{INVC}(1, 3, 2) + Q\text{RET}(1, 3, 3) - Q\text{D}(1, 3, 3) - Q\text{RECC}(1, 3, 3) - Q\text{DISC}(1, 3, 3); \]
\[ Q\text{INVC}(1, 3, 4) = Q\text{INVC}(1, 3, 3) + Q\text{RET}(1, 3, 4) - Q\text{D}(1, 3, 4) - Q\text{RECC}(1, 3, 4) - Q\text{DISC}(1, 3, 4); \]
\[ Q\text{INVC}(1, 3, 5) = Q\text{INVC}(1, 3, 4) + Q\text{RET}(1, 3, 5) - Q\text{D}(1, 3, 5) - Q\text{RECC}(1, 3, 5) - Q\text{DISC}(1, 3, 5); \]

!Inventory of subassemblies;
\[ Q\text{INVS}(1, 1) = Q\text{D}(1, 1, 1) + Q\text{D}(1, 2, 1) + Q\text{D}(1, 3, 1) + Q\text{PUR}(1, 1) - D(1, 1); \]
\[ Q\text{INVS}(1, 2) = Q\text{D}(1, 1, 2) + Q\text{D}(1, 2, 2) + Q\text{D}(1, 3, 2) + Q\text{PUR}(1, 2) - D(1, 2); \]
\[ Q\text{INVS}(1, 3) = Q\text{D}(1, 1, 3) + Q\text{D}(1, 2, 3) + Q\text{D}(1, 3, 3) + Q\text{PUR}(1, 3) - D(1, 3); \]
\[ Q\text{INVS}(1, 4) = Q\text{D}(1, 1, 4) + Q\text{D}(1, 2, 4) + Q\text{D}(1, 3, 4) + Q\text{PUR}(1, 4) - D(1, 4); \]
\[ Q\text{INVS}(1, 5) = Q\text{D}(1, 1, 5) + Q\text{D}(1, 2, 5) + Q\text{D}(1, 3, 5) + Q\text{PUR}(1, 5) - D(1, 5); \]
\[ Q\text{INVS}(2, 1) = Q\text{D}(1, 1, 1) + Q\text{D}(1, 2, 1) + Q\text{D}(1, 3, 1) + Q\text{PUR}(2, 1) - D(2, 1); \]
\[ Q\text{INVS}(2, 2) = Q\text{D}(1, 1, 2) + Q\text{D}(1, 2, 2) + Q\text{D}(1, 3, 2) + Q\text{PUR}(2, 2) - D(2, 2); \]
\[ Q\text{INVS}(2, 3) = Q\text{D}(1, 1, 3) + Q\text{D}(1, 2, 3) + Q\text{D}(1, 3, 3) + Q\text{PUR}(2, 3) - D(2, 3); \]
\[ Q\text{INVS}(2, 4) = Q\text{D}(1, 1, 4) + Q\text{D}(1, 2, 4) + Q\text{D}(1, 3, 4) + Q\text{PUR}(2, 4) - D(2, 4); \]
\[ Q\text{INVS}(2, 5) = Q\text{D}(1, 1, 5) + Q\text{D}(1, 2, 5) + Q\text{D}(1, 3, 5) + Q\text{PUR}(2, 5) - D(2, 5); \]
\[ Q\text{INVS}(3, 1) = Q\text{D}(1, 1, 1) + Q\text{D}(1, 2, 1) + Q\text{D}(1, 3, 1) + Q\text{PUR}(3, 1) - D(3, 1); \]
\[ Q\text{INVS}(3, 2) = Q\text{D}(1, 1, 2) + Q\text{D}(1, 2, 2) + Q\text{D}(1, 3, 2) + Q\text{PUR}(3, 2) - D(3, 2); \]
\[ Q\text{INVS}(3, 3) = Q\text{D}(1, 1, 3) + Q\text{D}(1, 2, 3) + Q\text{D}(1, 3, 3) + Q\text{PUR}(3, 3) - D(3, 3); \]
\[ Q\text{INVS}(3, 4) = Q\text{D}(1, 1, 4) + Q\text{D}(1, 2, 4) + Q\text{D}(1, 3, 4) + Q\text{PUR}(3, 4) - D(3, 4); \]
\[ Q\text{INVS}(3, 5) = Q\text{D}(1, 1, 5) + Q\text{D}(1, 2, 5) + Q\text{D}(1, 3, 5) + Q\text{PUR}(3, 5) - D(3, 5); \]

!Time aval. constarints;
\[ 7.50*Q\text{D}(1, 1, 1) + 13*Q\text{D}(1, 2, 1) + 19*Q\text{D}(1, 3, 1) \leq 28800; \]
\[ 7.50*Q\text{D}(1, 1, 2) + 13*Q\text{D}(1, 2, 2) + 19*Q\text{D}(1, 3, 2) \leq 28800; \]
\[ 7.50*Q\text{D}(1, 1, 3) + 13*Q\text{D}(1, 2, 3) + 19*Q\text{D}(1, 3, 3) \leq 28800; \]
\[ 7.50*Q\text{D}(1, 1, 4) + 13*Q\text{D}(1, 2, 4) + 19*Q\text{D}(1, 3, 4) \leq 28800; \]
\[ 7.50*Q\text{D}(1, 1, 5) + 13*Q\text{D}(1, 2, 5) + 19*Q\text{D}(1, 3, 5) \leq 28800; \]

!Disassembly capacity constraints;
\[ Q\text{D}(1, 1, 1) + Q\text{D}(1, 2, 1) + Q\text{D}(1, 3, 1) \leq 800; \]
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\[ QD(1,1,2) + QD(1,2,2) + QD(1,3,2) \leq 800; \]
\[ QD(1,1,3) + QD(1,2,3) + QD(1,3,3) \leq 800; \]
\[ QD(1,1,4) + QD(1,2,4) + QD(1,3,4) \leq 800; \]
\[ QD(1,1,5) + QD(1,2,5) + QD(1,3,5) \leq 800; \]

!Space requirements for core products;
\[ 0.70 \times (QINVC(1,1,1) + QINVC(1,2,1) + QINVC(1,3,1)) \leq 250; \]
\[ 0.70 \times (QINVC(1,1,2) + QINVC(1,2,2) + QINVC(1,3,2)) \leq 250; \]
\[ 0.70 \times (QINVC(1,1,3) + QINVC(1,2,3) + QINVC(1,3,3)) \leq 250; \]
\[ 0.70 \times (QINVC(1,1,4) + QINVC(1,2,4) + QINVC(1,3,4)) \leq 250; \]
\[ 0.70 \times (QINVC(1,1,5) + QINVC(1,2,5) + QINVC(1,3,5)) \leq 250; \]

!Space requirements for subassemblies;
\[ 0.10 \times QINVS(1,1) \leq 100; \]
\[ 0.10 \times QINVS(1,2) \leq 100; \]
\[ 0.10 \times QINVS(1,3) \leq 100; \]
\[ 0.10 \times QINVS(1,4) \leq 100; \]
\[ 0.10 \times QINVS(1,5) \leq 100; \]
\[ 0.20 \times QINVS(2,1) \leq 100; \]
\[ 0.20 \times QINVS(2,2) \leq 100; \]
\[ 0.20 \times QINVS(2,3) \leq 100; \]
\[ 0.20 \times QINVS(2,4) \leq 100; \]
\[ 0.20 \times QINVS(2,5) \leq 100; \]
\[ 0.25 \times QINVS(3,1) \leq 100; \]
\[ 0.25 \times QINVS(3,2) \leq 100; \]
\[ 0.25 \times QINVS(3,3) \leq 100; \]
\[ 0.25 \times QINVS(3,4) \leq 100; \]
\[ 0.25 \times QINVS(3,5) \leq 100; \]

!Disposal Limitation of Core and Subassembly;
\[ 0.625 \times QDISC(1,1,1) + 0.625 \times QDISC(1,2,1) + 0.625 \times QDISC(1,3,1) +
0.50 \times QDISS(1,1) + 0.35 \times QDISS(2,1) + 0.225 \times QDISS(3,1) \leq 600; \]
\[ 0.625 \times QDISC(1,1,2) + 0.625 \times QDISC(1,2,2) + 0.625 \times QDISC(1,3,2) +
0.50 \times QDISS(1,2) + 0.35 \times QDISS(2,2) + 0.225 \times QDISS(3,2) \leq 600; \]
\[ 0.625 \times QDISC(1,1,3) + 0.625 \times QDISC(1,2,3) + 0.625 \times QDISC(1,3,3) +
0.50 \times QDISS(1,3) + 0.35 \times QDISS(2,3) + 0.225 \times QDISS(3,3) \leq 600; \]
\[ 0.625 \times QDISC(1,1,4) + 0.625 \times QDISC(1,2,4) + 0.625 \times QDISC(1,3,4) +
0.50 \times QDISS(1,4) + 0.35 \times QDISS(2,4) + 0.225 \times QDISS(3,4) \leq 600; \]
\[ 0.625 \times QDISC(1,1,5) + 0.625 \times QDISC(1,2,5) + 0.625 \times QDISC(1,3,5) +
0.50 \times QDISS(1,5) + 0.35 \times QDISS(2,5) + 0.225 \times QDISS(3,5) \leq 600; \]

!Recycling Limitation on Core;
\[ QRECC(1,1,1) + QRECC(1,2,1) + QRECC(1,3,1) \geq QDISC(1,1,1) + QDISC(1,2,1) + QDISC(1,3,1); \]
\[ QRECC(1,1,2) + QRECC(1,2,2) + QRECC(1,3,2) \geq QDISC(1,1,2) + QDISC(1,2,2) + QDISC(1,3,2); \]
\[ QRECC(1,1,3) + QRECC(1,2,3) + QRECC(1,3,3) \geq QDISC(1,1,3) + QDISC(1,2,3) + QDISC(1,3,3); \]
\[ QRECC(1,1,4) + QRECC(1,2,4) + QRECC(1,3,4) \geq QDISC(1,1,4) + QDISC(1,2,4) + QDISC(1,3,4); \]
\[ QRECC(1,1,5) + QRECC(1,2,5) + QRECC(1,3,5) \geq QDISC(1,1,5) + QDISC(1,2,5) + QDISC(1,3,5); \]

!Recycling Limitation of Subassembly;
QRECS(1, 1)+QRECS(2, 1)+QRECS(3, 1)>=QDISS(1, 1)+QDISS(2, 1)+QDISS(3, 1);
QRECS(1, 2)+QRECS(2, 2)+QRECS(3, 2)>=QDISS(1, 2)+QDISS(2, 2)+QDISS(3, 2);
QRECS(1, 3)+QRECS(2, 3)+QRECS(3, 3)>=QDISS(1, 3)+QDISS(2, 3)+QDISS(3, 3);
QRECS(1, 4)+QRECS(2, 4)+QRECS(3, 4)>=QDISS(1, 4)+QDISS(2, 4)+QDISS(3, 4);
QRECS(1, 5)+QRECS(2, 5)+QRECS(3, 5)>=QDISS(1, 5)+QDISS(2, 5)+QDISS(3, 5);

DATA:
WC=250;
WI=0.70;
YC=0.625;
WS=100 100 100;
WJ=0.10 0.20 0.25;
YS=0.05 0.35 0.225;
CDISS=1.25 1.25 1.25;
CRECS=1.75 1.75 1.75;
CHOLS=0.20 0.40 0.60;
CPUR=25 17 28;
CD=10 13 17;
CDISC=4.25 4.25 4.25;
CRECC=7.50 7.50 7.50;
CHOLC=1.30 0.90 0.70;
Time=7.5 13 17 19;
RTC=925 1051 855 720 1091;
ATime=28800 28800 28800 28800 28800;
QRET= 742 527 678 349 524 789 124 457 860 754 756 989 716 596 552;
CAP=750;
D=680 850 660 900 730 510 460 750 720 375 360 500 620 480 540;
DISLIM=100;
RECCAP=200;

END DATA
END

MODEL:
! SINGLE PRODUCT DISASSEMBLY MODEL;
SETS:
CORE /1/: WC, WI, YC;
SUBASSEMBLY /1 2 3/: WS, WJ, YS, CDISS, CRECS, CHOLS, CPUR;
QUALITY / 1 2 3/: CD, CDISC, CRECC, CHOLC, Time;
PERIOD / 1 2 3 4 5/: RTC, ATime, CAP;
QUANTITY_C(CORE, QUALITY, PERIOD): QD, QRET, QDISC, QRECC, QINVC;
QUANTITY_S(SUBASSEMBLY, QUALITY, PERIOD): D, QDISS, QRECS, QINVS;
QUANTITY_P(SUBASSEMBLY, PERIOD):QPUR;
END SETS

!Objective Function;
MIN= 10*QD( 1, 1, 1)+ 4.25*QDISC( 1, 1, 1)+ 7.50*QRECC( 1, 1, 1)+
1.30*QINVC( 1, 1, 1)+
10*QD( 1, 1, 2)+ 4.25*QDISC( 1, 1, 2)+ 7.50*QRECC( 1, 1, 2)+
1.30*QINVC( 1, 1, 2)+
10*QD( 1, 1, 3)+ 4.25*QDISC( 1, 1, 3)+ 7.50*QRECC( 1, 1, 3)+
1.30*QINVC( 1, 1, 3)+
10*QD( 1, 1, 4)+ 4.25*QDISC( 1, 1, 4)+ 7.50*QRECC( 1, 1, 4)+
1.30*QINVC( 1, 1, 4)+
10*QD( 1, 1, 5)+ 4.25*QDISC( 1, 1, 5)+ 7.50*QRECC( 1, 1, 5)+
1.30*QINVC( 1, 1, 5)+
13*QD( 1, 2, 1)+ 4.25*QDISC( 1, 2, 1)+ 7.50*QRECC( 1, 2, 1)+
0.90*QINVC( 1, 2, 1)+
13*QD( 1, 2, 2)+ 4.25*QDISC( 1, 2, 2)+ 7.50*QRECC( 1, 2, 2)+
0.90*QINVC( 1, 2, 2)+
13*QD( 1, 2, 3)+ 4.25*QDISC( 1, 2, 3)+ 7.50*QRECC( 1, 2, 3)+
0.90*QINVC( 1, 2, 3)+
13*QD( 1, 2, 4)+ 4.25*QDISC( 1, 2, 4)+ 7.50*QRECC( 1, 2, 4)+
0.90*QINVC( 1, 2, 4)+
13*QD( 1, 2, 5)+ 4.25*QDISC( 1, 2, 5)+ 7.50*QRECC( 1, 2, 5)+
0.90*QINVC( 1, 2, 5)+
17*QD( 1, 3, 1)+ 4.25*QDISC( 1, 3, 1)+ 7.50*QRECC( 1, 3, 1)+
0.70*QINVC( 1, 3, 1)+
17*QD( 1, 3, 2)+ 4.25*QDISC( 1, 3, 2)+ 7.50*QRECC( 1, 3, 2)+
0.70*QINVC( 1, 3, 2)+
17*QD( 1, 3, 3)+ 4.25*QDISC( 1, 3, 3)+ 7.50*QRECC( 1, 3, 3)+
0.70*QINVC( 1, 3, 3)+
17*QD( 1, 3, 4)+ 4.25*QDISC( 1, 3, 4)+ 7.50*QRECC( 1, 3, 4)+
0.70*QINVC( 1, 3, 4)+
17*QD( 1, 3, 5)+ 4.25*QDISC( 1, 3, 5)+ 7.50*QRECC( 1, 3, 5)+
0.70*QINVC( 1, 3, 5)+
1.25*QDISS ( 1, 1, 1)+ 1.75*QRECS( 1, 1, 1)+ 0.20*QINVS( 1, 1, 1)+
1.25*QDISS ( 1, 2, 1)+ 1.75*QRECS( 1, 2, 1)+ 0.20*QINVS( 1, 2, 1)+
1.25*QDISS ( 1, 3, 1)+ 1.75*QRECS( 1, 3, 1)+ 0.20*QINVS( 1, 3, 1)+
25*QPUR( 1, 1)+

1.25*QDISS ( 1, 1, 2)+ 1.75*QRECS( 1, 1, 2)+ 0.20*QINVS( 1, 1, 2)+
1.25*QDISS ( 1, 2, 2)+ 1.75*QRECS( 1, 2, 2)+ 0.20*QINVS( 1, 2, 2)+
1.25*QDISS ( 1, 3, 2)+ 1.75*QRECS( 1, 3, 2)+ 0.20*QINVS( 1, 3, 2)+
25*QPUR( 1, 2)+

1.25*QDISS ( 1, 1, 3)+ 1.75*QRECS( 1, 1, 3)+ 0.20*QINVS( 1, 1, 3)+
1.25*QDISS ( 1, 2, 3)+ 1.75*QRECS( 1, 2, 3)+ 0.20*QINVS( 1, 2, 3)+
1.25*QDISS ( 1, 3, 3)+ 1.75*QRECS( 1, 3, 3)+ 0.20*QINVS( 1, 3, 3)+
25*QPUR( 1, 3)+

1.25*QDISS ( 1, 1, 4)+ 1.75*QRECS( 1, 1, 4)+ 0.20*QINVS( 1, 1, 4)+
1.25*QDISS ( 1, 2, 4)+ 1.75*QRECS( 1, 2, 4)+ 0.20*QINVS( 1, 2, 4)+
1.25*QDISS ( 1, 3, 4)+ 1.75*QRECS( 1, 3, 4)+ 0.20*QINVS( 1, 3, 4)+
25*QPUR( 1, 4)+
1.25*QDISS(1, 1, 5) + 1.75*QRECS(1, 1, 5) + 0.20*QINVS(1, 1, 5) +
1.25*QDISS(1, 2, 5) + 1.75*QRECS(1, 2, 5) + 0.20*QINVS(1, 2, 5) +
1.25*QDISS(1, 3, 5) + 1.75*QRECS(1, 3, 5) + 0.20*QINVS(1, 3, 5) +
25*QPUR(1, 5) +

1.25*QDISS(2, 1, 1) + 1.75*QRECS(2, 1, 1) + 0.40*QINVS(2, 1, 1) +
1.25*QDISS(2, 2, 1) + 1.75*QRECS(2, 2, 1) + 0.40*QINVS(2, 2, 1) +
1.25*QDISS(2, 3, 1) + 1.75*QRECS(2, 3, 1) + 0.40*QINVS(2, 3, 1) +
17*QPUR(2, 1) +

1.25*QDISS(2, 1, 2) + 1.75*QRECS(2, 1, 2) + 0.40*QINVS(2, 1, 2) +
1.25*QDISS(2, 2, 2) + 1.75*QRECS(2, 2, 2) + 0.40*QINVS(2, 2, 2) +
1.25*QDISS(2, 3, 2) + 1.75*QRECS(2, 3, 2) + 0.40*QINVS(2, 3, 2) +
17*QPUR(2, 2) +

1.25*QDISS(2, 1, 3) + 1.75*QRECS(2, 1, 3) + 0.40*QINVS(2, 1, 3) +
1.25*QDISS(2, 2, 3) + 1.75*QRECS(2, 2, 3) + 0.40*QINVS(2, 2, 3) +
1.25*QDISS(2, 3, 3) + 1.75*QRECS(2, 3, 3) + 0.40*QINVS(2, 3, 3) +
17*QPUR(2, 3) +

1.25*QDISS(2, 1, 4) + 1.75*QRECS(2, 1, 4) + 0.40*QINVS(2, 1, 4) +
1.25*QDISS(2, 2, 4) + 1.75*QRECS(2, 2, 4) + 0.40*QINVS(2, 2, 4) +
1.25*QDISS(2, 3, 4) + 1.75*QRECS(2, 3, 4) + 0.40*QINVS(2, 3, 4) +
17*QPUR(2, 4) +

1.25*QDISS(2, 1, 5) + 1.75*QRECS(2, 1, 5) + 0.40*QINVS(2, 1, 5) +
1.25*QDISS(2, 2, 5) + 1.75*QRECS(2, 2, 5) + 0.40*QINVS(2, 2, 5) +
1.25*QDISS(2, 3, 5) + 1.75*QRECS(2, 3, 5) + 0.40*QINVS(2, 3, 5) +
17*QPUR(2, 5) +

1.25*QDISS(2, 1, 1) + 1.75*QRECS(2, 1, 1) + 0.60*QINVS(2, 1, 1) +
1.25*QDISS(2, 2, 1) + 1.75*QRECS(2, 2, 1) + 0.60*QINVS(2, 2, 1) +
1.25*QDISS(2, 3, 1) + 1.75*QRECS(2, 3, 1) + 0.60*QINVS(2, 3, 1) +
28*QPUR(2, 1) +

1.25*QDISS(2, 1, 2) + 1.75*QRECS(2, 1, 2) + 0.60*QINVS(2, 1, 2) +
1.25*QDISS(2, 2, 2) + 1.75*QRECS(2, 2, 2) + 0.60*QINVS(2, 2, 2) +
1.25*QDISS(2, 3, 2) + 1.75*QRECS(2, 3, 2) + 0.60*QINVS(2, 3, 2) +
28*QPUR(2, 2) +

1.25*QDISS(2, 1, 3) + 1.75*QRECS(2, 1, 3) + 0.60*QINVS(2, 1, 3) +
1.25*QDISS(2, 2, 3) + 1.75*QRECS(2, 2, 3) + 0.60*QINVS(2, 2, 3) +
1.25*QDISS(2, 3, 3) + 1.75*QRECS(2, 3, 3) + 0.60*QINVS(2, 3, 3) +
28*QPUR(2, 3) +

1.25*QDISS(2, 1, 4) + 1.75*QRECS(2, 1, 4) + 0.60*QINVS(2, 1, 4) +
1.25*QDISS(2, 2, 4) + 1.75*QRECS(2, 2, 4) + 0.60*QINVS(2, 2, 4) +
1.25*QDISS(2, 3, 4) + 1.75*QRECS(2, 3, 4) + 0.60*QINVS(2, 3, 4) +
28*QPUR(2, 4) +

1.25*QDISS(2, 1, 5) + 1.75*QRECS(2, 1, 5) + 0.60*QINVS(2, 1, 5) +
1.25*QDISS(2, 2, 5) + 1.75*QRECS(2, 2, 5) + 0.60*QINVS(2, 2, 5) +
1.25*QDISS(2, 3, 5) + 1.75*QRECS(2, 3, 5) + 0.60*QINVS(2, 3, 5) + 28*QPUR(2, 5);

!Inventory of core products;
QINVC(1, 1, 1) = QRET(1, 1, 1) - QD(1, 1, 1) - QRECC(1, 1, 1) - QDISC(1, 1, 1);
QINVC(1, 1, 2) = QINVC(1, 1, 1) + QRET(1, 1, 2) - QD(1, 1, 2) - QRECC(1, 1, 2) - QDISC(1, 1, 2);
QINVC(1, 1, 3) = QINVC(1, 1, 2) + QRET(1, 1, 3) - QD(1, 1, 3) - QRECC(1, 1, 3) - QDISC(1, 1, 3);
QINVC(1, 1, 4) = QINVC(1, 1, 3) + QRET(1, 1, 4) - QD(1, 1, 4) - QRECC(1, 1, 4) - QDISC(1, 1, 4);
QINVC(1, 1, 5) = QINVC(1, 1, 4) + QRET(1, 1, 5) - QD(1, 1, 5) - QRECC(1, 1, 5) - QDISC(1, 1, 5);
QINVC(1, 2, 1) = QRET(1, 2, 1) - QD(1, 2, 1) - QRECC(1, 2, 1) - QDISC(1, 2, 1);
QINVC(1, 2, 2) = QINVC(1, 2, 1) + QRET(1, 2, 2) - QD(1, 2, 2) - QRECC(1, 2, 2) - QDISC(1, 2, 2);
QINVC(1, 2, 3) = QINVC(1, 2, 2) + QRET(1, 2, 3) - QD(1, 2, 3) - QRECC(1, 2, 3) - QDISC(1, 2, 3);
QINVC(1, 2, 4) = QINVC(1, 2, 3) + QRET(1, 2, 4) - QD(1, 2, 4) - QRECC(1, 2, 4) - QDISC(1, 2, 4);
QINVC(1, 2, 5) = QINVC(1, 2, 4) + QRET(1, 2, 5) - QD(1, 2, 5) - QRECC(1, 2, 5) - QDISC(1, 2, 5);
QINVC(1, 3, 1) = QRET(1, 3, 1) - QD(1, 3, 1) - QRECC(1, 3, 1) - QDISC(1, 3, 1);
QINVC(1, 3, 2) = QINVC(1, 3, 1) + QRET(1, 3, 2) - QD(1, 3, 2) - QRECC(1, 3, 2) - QDISC(1, 3, 2);
QINVC(1, 3, 3) = QINVC(1, 3, 2) + QRET(1, 3, 3) - QD(1, 3, 3) - QRECC(1, 3, 3) - QDISC(1, 3, 3);
QINVC(1, 3, 4) = QINVC(1, 3, 3) + QRET(1, 3, 4) - QD(1, 3, 4) - QRECC(1, 3, 4) - QDISC(1, 3, 4);
QINVC(1, 3, 5) = QINVC(1, 3, 4) + QRET(1, 3, 5) - QD(1, 3, 5) - QRECC(1, 3, 5) - QDISC(1, 3, 5);

!Inventory of subassemblies;
QINVS(1, 1, 1) = QD(1, 1, 1) - D(1, 1, 1);
QINVS(1, 2, 1) = QD(1, 2, 1) + QINVS(1, 1, 1) - D(1, 2, 1);
QINVS(1, 3, 1) = QD(1, 3, 1) + QPUR(1, 1, 1) + QINVS(1, 2, 1) - D(1, 3, 1);
QINVS(1, 1, 2) = QD(1, 1, 2) - D(1, 1, 2);
QINVS(1, 2, 2) = QD(1, 2, 2) + QINVS(1, 1, 2) - D(1, 2, 2);
QINVS(1, 3, 2) = QD(1, 3, 2) + QPUR(1, 2, 1) + QINVS(1, 2, 2) - D(1, 3, 2);
QINVS(1, 1, 3) = QD(1, 1, 3) - D(1, 1, 3);
QINVS(1, 2, 3) = QD(1, 2, 3) + QINVS(1, 1, 3) - D(1, 2, 3);
QINVS(1, 3, 3) = QD(1, 3, 3) + QPUR(1, 3, 1) + QINVS(1, 2, 3) - D(1, 3, 3);
QINVS(1, 1, 4) = QD(1, 1, 4) - D(1, 1, 4);
QINVS(1, 2, 4) = QD(1, 2, 4) + QINVS(1, 1, 4) - D(1, 2, 4);
\[
\begin{align*}
QINVS(1, 3, 4) &= QD(1, 3, 4) + QPUR(1, 4) + QINVS(1, 2, 4) - D(1, 3, 4); \\
QINVS(1, 1, 5) &= QD(1, 1, 5) - D(1, 1, 5); \\
QINVS(1, 2, 5) &= QD(1, 2, 5) + QINVS(1, 1, 5) - D(1, 2, 5); \\
QINVS(1, 3, 5) &= QD(1, 3, 5) + QPUR(1, 5) + QINVS(1, 2, 5) - D(1, 3, 5); \\
QINVS(2, 1, 1) &= QD(2, 1, 1) - D(2, 1, 1); \\
QINVS(2, 2, 1) &= QD(2, 2, 1) + QINVS(2, 1, 1) - D(2, 2, 1); \\
QINVS(2, 3, 1) &= QD(2, 3, 1) + QPUR(2, 1) + QINVS(2, 2, 1) - D(2, 3, 1); \\
QINVS(2, 1, 2) &= QD(2, 1, 2) - D(2, 1, 2); \\
QINVS(2, 2, 2) &= QD(2, 2, 2) + QINVS(2, 1, 2) - D(2, 2, 2); \\
QINVS(2, 3, 2) &= QD(2, 3, 2) + QPUR(2, 2) + QINVS(2, 2, 2) - D(2, 3, 2); \\
QINVS(2, 1, 3) &= QD(2, 1, 3) - D(2, 1, 3); \\
QINVS(2, 2, 3) &= QD(2, 2, 3) + QINVS(2, 1, 3) - D(2, 2, 3); \\
QINVS(2, 3, 3) &= QD(2, 3, 3) + QPUR(2, 3) + QINVS(2, 2, 3) - D(2, 3, 3); \\
QINVS(2, 1, 4) &= QD(2, 1, 4) - D(2, 1, 4); \\
QINVS(2, 2, 4) &= QD(2, 2, 4) + QINVS(2, 1, 4) - D(2, 2, 4); \\
QINVS(2, 3, 4) &= QD(2, 3, 4) + QPUR(2, 4) + QINVS(2, 2, 4) - D(2, 3, 4); \\
QINVS(2, 1, 5) &= QD(2, 1, 5) - D(2, 1, 5); \\
QINVS(2, 2, 5) &= QD(2, 2, 5) + QINVS(2, 1, 5) - D(2, 2, 5); \\
QINVS(2, 3, 5) &= QD(2, 3, 5) + QPUR(2, 5) + QINVS(2, 2, 5) - D(2, 3, 5); \\
QINVS(3, 1, 1) &= QD(3, 1, 1) - D(3, 1, 1); \\
QINVS(3, 2, 1) &= QD(3, 2, 1) + QINVS(3, 1, 1) - D(3, 2, 1); \\
QINVS(3, 3, 1) &= QD(3, 3, 1) + QPUR(3, 1) + QINVS(3, 2, 1) - D(3, 3, 1); \\
QINVS(3, 1, 2) &= QD(3, 1, 2) - D(3, 1, 2); \\
QINVS(3, 2, 2) &= QD(3, 2, 2) + QINVS(3, 1, 2) - D(3, 2, 2); \\
QINVS(3, 3, 2) &= QD(3, 3, 2) + QPUR(3, 2) + QINVS(3, 2, 2) - D(3, 3, 2); \\
QINVS(3, 1, 3) &= QD(3, 1, 3) - D(3, 1, 3); \\
QINVS(3, 2, 3) &= QD(3, 2, 3) + QINVS(3, 1, 3) - D(3, 2, 3); \\
QINVS(3, 3, 3) &= QD(3, 3, 3) + QPUR(3, 3) + QINVS(3, 2, 3) - D(3, 3, 3); \\
QINVS(3, 1, 4) &= QD(3, 1, 4) - D(3, 1, 4); \\
QINVS(3, 2, 4) &= QD(3, 2, 4) + QINVS(3, 1, 4) - D(3, 2, 4); \\
QINVS(3, 3, 4) &= QD(3, 3, 4) + QPUR(3, 4) + QINVS(3, 2, 4) - D(3, 3, 4); \\
QINVS(3, 1, 5) &= QD(3, 1, 5) - D(3, 1, 5); \\
QINVS(3, 2, 5) &= QD(3, 2, 5) + QINVS(3, 1, 5) - D(3, 2, 5); \\
QINVS(3, 3, 5) &= QD(3, 3, 5) + QPUR(3, 5) + QINVS(3, 2, 5) - D(3, 3, 5);
\end{align*}
\]
!Time available constraints;
7.50*QD(1, 1, 1)+ 13*QD(1, 2, 1)+ 19*QD(1, 3, 1)<=28800;
7.50*QD(1, 1, 2)+ 13*QD(1, 2, 2)+ 19*QD(1, 3, 2)<=28800;
7.50*QD(1, 1, 3)+ 13*QD(1, 2, 3)+ 19*QD(1, 3, 3)<=28800;
7.50*QD(1, 1, 4)+ 13*QD(1, 2, 4)+ 19*QD(1, 3, 4)<=28800;
7.50*QD(1, 1, 5)+ 13*QD(1, 2, 5)+ 19*QD(1, 3, 5)<=28800;

!Disassembly capacity constraints;
QD(1, 1, 1)+ QD(1, 2, 1)+ QD(1, 3, 1)<=800;
QD(1, 1, 2)+ QD(1, 2, 2)+ QD(1, 3, 2)<=800;
QD(1, 1, 3)+ QD(1, 2, 3)+ QD(1, 3, 3)<=800;
QD(1, 1, 4)+ QD(1, 2, 4)+ QD(1, 3, 4)<=800;
QD(1, 1, 5)+ QD(1, 2, 5)+ QD(1, 3, 5)<=800;

!Space requirements for core products;
0.70*(QINVC(1, 1, 1)+QINVC(1, 2, 1)+QINVC(1, 3, 1))<=250;
0.70*(QINVC(1, 1, 2)+QINVC(1, 2, 2)+QINVC(1, 3, 2))<=250;
0.70*(QINVC(1, 1, 3)+QINVC(1, 2, 3)+QINVC(1, 3, 3))<=250;
0.70*(QINVC(1, 1, 4)+QINVC(1, 2, 4)+QINVC(1, 3, 4))<=250;
0.70*(QINVC(1, 1, 5)+QINVC(1, 2, 5)+QINVC(1, 3, 5))<=250;

!Space requirements for subassemblies;
0.10*(QINVS(1, 1, 1)+QINVS(1, 2, 1)+QINVS(1, 3, 1))<=100;
0.10*(QINVS(1, 1, 2)+QINVS(1, 2, 2)+QINVS(1, 3, 2))<=100;
0.10*(QINVS(1, 1, 3)+QINVS(1, 2, 3)+QINVS(1, 3, 3))<=100;
0.10*(QINVS(1, 1, 4)+QINVS(1, 2, 4)+QINVS(1, 3, 4))<=100;
0.10*(QINVS(1, 1, 5)+QINVS(1, 2, 5)+QINVS(1, 3, 5))<=100;

0.20*(QINVS(2, 1, 1)+QINVS(2, 2, 1)+QINVS(2, 3, 1))<=100;
0.20*(QINVS(2, 1, 2)+QINVS(2, 2, 2)+QINVS(2, 3, 2))<=100;
0.20*(QINVS(2, 1, 3)+QINVS(2, 2, 3)+QINVS(2, 3, 3))<=100;
0.20*(QINVS(2, 1, 4)+QINVS(2, 2, 4)+QINVS(2, 3, 4))<=100;
0.20*(QINVS(2, 1, 5)+QINVS(2, 2, 5)+QINVS(2, 3, 5))<=100;

0.25*(QINVS(3, 1, 1)+QINVS(3, 2, 1)+QINVS(3, 3, 1))<=100;
0.25*(QINVS(3, 1, 2)+QINVS(3, 2, 2)+QINVS(3, 3, 2))<=100;
0.25*(QINVS(3, 1, 3)+QINVS(3, 2, 3)+QINVS(3, 3, 3))<=100;
0.25*(QINVS(3, 1, 4)+QINVS(3, 2, 4)+QINVS(3, 3, 4))<=100;
0.25*(QINVS(3, 1, 5)+QINVS(3, 2, 5)+QINVS(3, 3, 5))<=100;

!Disposal Limitation of Core and Subassembly;
0.625*QDISC(1, 1, 1)+0.625*QDISC(1, 2, 1)+0.625*QDISC(1, 3, 1)<=600;
0.625*QDISC(1, 1, 2)+0.625*QDISC(1, 2, 2)+0.625*QDISC(1, 3, 2)<=600;
0.625*QDISC(1, 1, 3)+0.625*QDISC(1, 2, 3)+0.625*QDISC(1, 3, 3)<=600;
0.625*QDISC(1, 1, 4)+0.625*QDISC(1, 2, 4)+0.625*QDISC(1, 3, 4)<=600;
0.625*QDISC(1, 1, 5)+0.625*QDISC(1, 2, 5)+0.625*QDISC(1, 3, 5)<=600;

0.50*(QDISS(1, 1, 1)+QDISS(1, 2, 1)+QDISS(1, 3, 1))<=600;
0.50*(QDISS(1, 1, 2)+QDISS(1, 2, 2)+QDISS(1, 3, 2))<=600;
0.50*(QDISS(1, 1, 3)+QDISS(1, 2, 3)+QDISS(1, 3, 3))<=600;
0.35*(QDISS(2, 1, 1)+QDISS(2, 2, 1)+QDISS(2, 3, 1))<=600;
0.35*(QDISS(2, 1, 2)+QDISS(2, 2, 2)+QDISS(2, 3, 2))<=600;
0.225*(QDISS(3, 1, 1)+QDISS(3, 2, 1)+QDISS(3, 3, 1))<=600;
0.225*(QDISS(3, 1, 2)+QDISS(3, 2, 2)+QDISS(3, 3, 2))<=600;
\[ 0.50 \times (Q_{DIS}(1, 1, 3) + Q_{DIS}(1, 2, 3) + Q_{DIS}(1, 3, 3)) + 0.35 \times (Q_{DIS}(2, 1, 3) + Q_{DIS}(2, 2, 3) + Q_{DIS}(2, 3, 3)) + 0.225 \times (Q_{DIS}(3, 1, 3) + Q_{DIS}(3, 2, 3) + Q_{DIS}(3, 3, 3)) \leq 600; \]
\[ 0.50 \times (Q_{DIS}(1, 1, 4) + Q_{DIS}(1, 2, 4) + Q_{DIS}(1, 3, 4)) + 0.35 \times (Q_{DIS}(2, 1, 4) + Q_{DIS}(2, 2, 4) + Q_{DIS}(2, 3, 4)) + 0.225 \times (Q_{DIS}(3, 1, 4) + Q_{DIS}(3, 2, 4) + Q_{DIS}(3, 3, 4)) \leq 600; \]
\[ 0.50 \times (Q_{DIS}(1, 1, 5) + Q_{DIS}(1, 2, 5) + Q_{DIS}(1, 3, 5)) + 0.35 \times (Q_{DIS}(2, 1, 5) + Q_{DIS}(2, 2, 5) + Q_{DIS}(2, 3, 5)) + 0.225 \times (Q_{DIS}(3, 1, 5) + Q_{DIS}(3, 2, 5) + Q_{DIS}(3, 3, 5)) \leq 600; \]

Recycling Limitation on Core:

\[ Q_{REC}(1, 1, 1) + Q_{REC}(1, 1, 2) + Q_{REC}(1, 1, 3) \geq Q_{DIS}(1, 1, 1) + Q_{DIS}(1, 1, 2) + Q_{DIS}(1, 1, 3); \]
\[ Q_{REC}(1, 1, 4) + Q_{REC}(1, 1, 5) \geq Q_{DIS}(1, 1, 4) + Q_{DIS}(1, 1, 5); \]
\[ Q_{REC}(1, 1, 2) + Q_{REC}(1, 1, 3) \geq Q_{DIS}(1, 1, 2) + Q_{DIS}(1, 1, 3); \]
\[ Q_{REC}(1, 1, 4) + Q_{REC}(1, 1, 5) \geq Q_{DIS}(1, 1, 4) + Q_{DIS}(1, 1, 5); \]

Recycling Limitation of Subassembly:

\[ Q_{RECS}(1, 1, 1) + Q_{RECS}(1, 1, 2) + Q_{RECS}(1, 1, 3) \geq Q_{DIS}(1, 1, 1) + Q_{DIS}(1, 1, 2) + Q_{DIS}(1, 1, 3); \]
\[ Q_{RECS}(1, 1, 4) + Q_{RECS}(1, 1, 5) \geq Q_{DIS}(1, 1, 4) + Q_{DIS}(1, 1, 5); \]
\[ Q_{RECS}(1, 1, 2) + Q_{RECS}(1, 1, 3) \geq Q_{DIS}(1, 1, 2) + Q_{DIS}(1, 1, 3); \]
\[ Q_{RECS}(1, 1, 4) + Q_{RECS}(1, 1, 5) \geq Q_{DIS}(1, 1, 4) + Q_{DIS}(1, 1, 5); \]
DATA:

WC=250;
WI=0.70;
YC=0.625;
WS=100 100 100;
WJ=0.10 0.20 0.25;
YS=0.05 0.35 0.225;
CDISS=1.25 1.25 1.25;
CRECS=1.75 1.75 1.75;
CHOLS=0.20 0.40 0.60;
CPUR=25 17 28;
CD=10 13 17;
CDISC=4.25 4.25 4.25;
CRECC=7.50 7.50 7.50;
CHOLC=1.30 0.90 0.70;
Time=7.5 13 19;
RTC=925 1051 855 720 1091;
ATime=28800 28800 28800 28800 28800;
QRET=942 827 878 749 924 789 657 860 754 756 989 716 696 752;
CAP=750;
D=680 850 660 900 730 510 460 750 720 375 360 500 620 480 540 756 943 486 159
456 357 951 789 456 123 978 645 312 951 784 984 651 324 659 751 359 759 351
456 852 741 963 511 445 654;
DISLIM=100;
RECCAP=200;

END DATA
END
APPENDIX B

EXPECTED SALES PRICES FOR DP MODEL

Input data for solving the Dynamic Programming (DP) model presented in chapter 7.

\( i = 1, j = 1, k = 1 \)

### Net Profit Matrix Period 5

<table>
<thead>
<tr>
<th>Parts Quantity Classification</th>
<th>( r = 0 )</th>
<th>( r = 1 )</th>
<th>( r = 2 )</th>
<th>( r = 3 )</th>
<th>( r = 4 )</th>
<th>( r = 5 )</th>
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<td>3.33</td>
<td>3.29</td>
<td>3.24</td>
<td>3.45</td>
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<tr>
<td>1.5K-2.5K</td>
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<td>3.33</td>
<td>3.29</td>
<td>3.24</td>
<td>3.45</td>
</tr>
<tr>
<td>2.5K-4.0K</td>
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<td>3.33</td>
<td>3.29</td>
<td>3.24</td>
<td>3.45</td>
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\[i = 1, \ j = 1, k = 2\]

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### Net Profit Matrix Period 1

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\[ i = 1, j = 2, k = 1 \]

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\[ i = 1, j = 3, k = 2 \]

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## APPENDIX C

### DYNAMIC PROGRAMMING (DP) MODEL RESULTS

\( i = 1, j = 1, k = 1 \)

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### APPENDIX C

#### Profit Generated From Option $r$ Optimal Decision

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### Profit Generated From Option \( r \) Optimal Decision

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### Profit Generated From Option $r$

#### Optimal Decision

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#### Optimal Decision

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### APPENDIX C

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### APPENDIX C

#### Profit Generated From Option r Optimal Decision

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## Appendix C

### Profit Generated From Option $r$ Optimal Decision

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\[ i = 1, j = 3, k = 1 \]

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\[ i = 1, j = 3, k = 3 \]

### Profit Generated From Option \( r \) Optimal Decision

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<th>( f^*(S_n, X_n) )</th>
<th>( X_n )</th>
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### Profit Generated From Option \(r\) Optimal Decision

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</table>
# APPENDIX D

## MATRIX OPERATIONS EXAMPLE 1, CHAPTER 9

### Data (one step transition matrix)

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<th>State 4</th>
<th>State 5</th>
<th>State 6</th>
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### 2 step transition matrix

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APPENDIX E

F-POLICY APPROACH TO THE INVENTORY MANAGEMENT PROBLEM

For $F = 1, K = 10$

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<td>5.27</td>
<td>5.34</td>
<td>5.62</td>
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<tr>
<td>$E(S)$</td>
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<tr>
<td>$E(B)$</td>
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Measures of effectiveness table for the case $K = 10$
**APPENDIX E**

\[ F = 1, K = 9 \]

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</thead>
<tbody>
<tr>
<td>( \pi_0 )</td>
<td>0.233048</td>
<td>0.025222</td>
<td>0.027744</td>
<td>0.0277443</td>
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<td>0.027744</td>
<td>0.027744</td>
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\[ F = 2, K = 9 \]

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</thead>
<tbody>
<tr>
<td>( \pi_0 )</td>
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<td>0.026757</td>
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<td>0.032376</td>
<td>0.032376</td>
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\[ F = 3, K = 9 \]

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<th>6</th>
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<tr>
<td>( \pi_0 )</td>
<td>0.2536728</td>
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<td>0.03069441</td>
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<td>0.033764</td>
<td>0.033764</td>
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$F = 4, K = 9$

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<tr>
<td>$\Pi_0$</td>
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<tr>
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<td>0.058531</td>
<td>0.061178</td>
<td>0.062488334</td>
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$F = 5, K = 9$

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$F = 6, K = 9$

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<td>$\Pi_0$</td>
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<td>0.031797</td>
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<td>0.038474</td>
<td>0.042322</td>
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<td>$Pi_1$</td>
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<td>0.054729</td>
<td>0.058204</td>
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<td>0.058253</td>
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<td>0.51</td>
<td>0.51</td>
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</tr>
<tr>
<td>$W$</td>
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<td>4.70</td>
<td>4.76</td>
<td>4.80</td>
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<tr>
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<td>1.58</td>
<td>1.61</td>
<td>1.64</td>
<td>1.66</td>
<td>1.66</td>
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<tr>
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Measures of effectiveness table for the case $K = 9$
**APPENDIX E**

**F = 1, K = 8**

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<th>7</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Pi0</td>
<td>0.233048</td>
<td>0.027469</td>
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<td>0.03021562</td>
<td>0.030216</td>
<td>0.030216</td>
<td>0.030216</td>
<td>0.030216</td>
<td>0.030216</td>
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<tr>
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<td>0.0562421</td>
<td>0.056894</td>
<td>0.055126</td>
<td>0.05247321</td>
<td>0.048494</td>
<td>0.042526</td>
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**F = 2, K = 8**

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<tbody>
<tr>
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<td>0.2958</td>
<td>0.02958</td>
<td>0.032538</td>
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<td>0.063203</td>
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<td>0.06529917</td>
<td>0.06215695</td>
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<td>0.050374</td>
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**F = 3, K = 8**

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<th>8</th>
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</thead>
<tbody>
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<td>0.2817131</td>
<td>0.028171</td>
<td>0.030988</td>
<td>0.03408729</td>
<td>0.037496</td>
<td>0.037496</td>
<td>0.037496</td>
<td>0.037496</td>
<td>0.037496</td>
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<tr>
<td>Pi1</td>
<td>0.0619472</td>
<td>0.06475</td>
<td>0.066136</td>
<td>0.06511654</td>
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<td>0.052772</td>
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<td>0.024997</td>
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\( F = 4, K = 8 \)

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<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Pi0 )</td>
<td>0.270198</td>
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<td>0.029722</td>
<td>0.032693944</td>
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<td>0.03956</td>
<td>0.03956</td>
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<tr>
<td>( Pi1 )</td>
<td>0.060555</td>
<td>0.063813</td>
<td>0.065998</td>
<td>0.06630278</td>
<td>0.063491</td>
<td>0.055677</td>
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</table>

\( F = 5, K = 8 \)

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<td>0.038254</td>
<td>0.042079</td>
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<tr>
<td>( Pi1 )</td>
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\( F = 6, K = 8 \)

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</tr>
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<tbody>
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<td>0.255049</td>
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\( F = 7, K = 8 \)

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<tr>
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\( F = 8, K = 8 \)

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<tr>
<td>(L)</td>
<td>2.16</td>
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<td>2.73</td>
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<td>0.42</td>
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<td>0.19</td>
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<td>(TC(F,K))</td>
<td>63.16</td>
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Measures of effectiveness table for the case \(K = 8\)
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<tbody>
<tr>
<td>$\Pi_0$</td>
<td>0.233048</td>
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<tr>
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**F = 4, K = 7**

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<tr>
<td>P0</td>
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<td>P1</td>
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**F = 5, K = 7**

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**F = 6, K = 7**

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<tr>
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<td>0.065138</td>
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$F = 7, K = 7$

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<tbody>
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<table>
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Measures of effectiveness for the case $K = 7$
Vita

Badr O. Johar was born in Jeddah, Saudi Arabia in 1979. In 1997, he graduated from Al-Anjal Private School in Jeddah, and then decided to pursue his higher education in the United States. He moved to Boston in January 1998. He received his Bachelor of Science degree in Industrial Engineering with honor from Northeastern University in Boston, Massachusetts in 2002 with a QPA of 3.73. In 2004, he received his Master of Science degree in Engineering Management from the same institution with a QPA of 3.85. He then started a graduate program in Mechanical and Industrial Engineering in Northeastern University in the fall of 2004, and became a PhD candidate in 2006, at the same time he has been a research associate at the Laboratory of Responsible Manufacturing (LRM). His research interests are in the areas of disassembly, product recovery, and production and inventory control. He has co-authored several technical papers presented at various national and international conferences and published in their respective proceedings. Due to his outstanding academic achievements, he has been elected to The National Society of Collegiate Scholars, The Golden Key Honor Society, and The National Honor Society for Industrial Engineering Students Alpha pi mu. Also, he is a member of Institute of Industrial Engineers (IIE) and the Saudi Council of Engineers (SCE). He also received many awards for his academic excellence over the course of his study at Northeastern University. In 2009, alongside with completing his thesis writing, he was appointed as Projects and Planning Manager at Yanbu Cement Company, a leader in cement manufacturing in the Middle East to oversee a new state-of-the-art expansion project (YC5) a 10,000 tons per day plant in collaboration with Europeans and Chinese firms with a total cost exceeding half a billion US dollars. He still hold that position to date.