Remote Sensing of Marine Life and Submerged Target Motions with Ocean Waveguide Acoustics

A thesis presented

by

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to

The Department of Electrical and Computer Engineering

in partial fulfillment of the requirements
for the degree of

Doctor of Philosophy

in

Electrical Engineering

Northeastern University
Boston, Massachusetts

April, 2012
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Submitted to the Department of Electrical and Computer Engineering on April 17, 2012, in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical Engineering

Abstract

Many species of fish that inhabit the continental shelf waters can cause significant acoustic scattering at low- to mid-frequencies due to the large impedance contrast between their air-filled swimbladders and the surrounding water. In this thesis, we investigate the acoustic resonance scattering response from distributed fish groups both experimentally and theoretically including the effects of multiple scattering, attenuation, and dispersion in a random range-dependent ocean waveguide using an instantaneous wide-area imaging system. In navy sonar operations, the biological organisms can cause high false alarm rates or missed target detections since the biological scattering can be confused with or camouflage the returns from other discrete and distributed objects, such as underwater vehicles and geologic features. From an ecological perspective, the ability to instantaneously survey fish populations distributed over wide areas is important for fisheries management.

The low-frequency target strength of shoaling Atlantic herring (Clupea harengus) in the Gulf of Maine during their Autumn 2006 spawning season is estimated from experimental data acquired simultaneously at multiple frequencies in the 300 to 1200 Hz range using (1) a low-frequency ocean acoustic waveguide remote sensing (OAWRS) system, (2) areal population density calibration with several conventional fish finding sonar (CFFS) systems, and (3) low-frequency transmission loss measurements. The OAWRS system’s instantaneous imaging diameter of 100 km and regular updating enabled unaliased monitoring of fish populations over ecosystem scales including shoals of Atlantic herring containing as many as 200 million individuals, as estimated based on single scattering assumption and confirmed by concurrent trawl and CFFS sampling. The mean scattering cross-section of an individual shoaling herring is found to consistently exhibit a strong, roughly 20 dB/octave roll-off with decreasing frequency over all days of the roughly 2-week experiment, consistent with the steep roll-offs expected for sub-resonance...
scattering from fish with air-filled swimbladders.

A numerical Monte-Carlo model is developed to determine the statistical moments of the broadband matched filtered scattered returns from fish groups spanning over multiple range and cross-range resolution cells of a waveguide remote sensing system. It uses the parabolic equation to simulate acoustic field propagation in a random range-dependent ocean waveguide. The effects of (1) multiple scattering, (2) attenuation due to scattering, and (3) fish group 3D spatial configuration on fish population density imaging are examined. The model is applied to investigate (a) population density imaging of shoaling Atlantic herring during the 2006 Gulf of Maine Experiment (GOME06) and (b) examine the wide-area imaging of sparse aggregation of ground fish species, such as Atlantic Cod, in Ipswich Bay continental shelf environment using the waveguide remote sensing system. Incoherent intensities are shown to dominate the total scattered returns from distributed fish groups making single scattering assumption valid for inferring fish areal population densities from their matched filtered scattered intensities. Multiple scattering, attenuation, fish group 3D spatial configuration, and coherent effects, such as resonance shift, sub- and super-local-maxima are found to be negligible at the imaging frequencies employed and for the herring densities observed. Similar results are obtained for the sparsely aggregated cod, but coherent effects such as the multiple-peak in school resonance can be prominent at much lower fish densities. Attenuation due to scattering can be significant when the fish flesh viscosity is high, especially true for cod.

We also investigate approaches for instantaneous long-range passive source localization and tracking with a towed horizontal line-array in a random range-dependent ocean waveguide using passive waveguide acoustics. This is very important for many sonar applications, such as localizing and tracking underwater vehicles and vocalizing marine mammal populations. Instantaneous passive source localization applying the (1) synthetic aperture tracking, (2) array invariant, (3) bearings-only target motion analysis in modified polar coordinates via the extended Kalman filter, and (4) bearings-migration minimum mean-square error methods using measurements made on a single towed horizontal receiver array in a random range-dependent ocean waveguide are examined. These methods are employed to localize and track a vertical source array deployed in the far-field of a towed horizontal receiver array during the Gulf of Maine 2006 Experiment. The source transmitted intermittent broadband pulses in the 300-1200 Hz frequency range. All four methods are found to be comparable with average errors of between 9% to 13% in estimating the mean source positions in a wide variety of source-receiver geometries and range separations up to 20 km. In the case of a relatively stationary source, the synthetic aperture tracking outperformed the other three methods by a factor of two with only 4% error. For a moving source, the Kalman filter method yielded the best performance with 8% error. The array invariant was the best approach for localizing
sources within the endfire beam of the receiver array with less than 10% error.

Thesis Supervisor: Purnima Ratilal
Title: Associate Professor of Electrical and Computer Engineering
Acknowledgments

First and foremost, I would like to express my deepest gratitude to my advisor Prof. Purnima Ratilal for her supervision, support, inspiration and guidance in research and academic advising. Her enthusiasm and intelligence in scientific research and thorough understanding in the subject of ocean acoustics and remote sensing have greatly inspired me and sparked my keen interest in exploring the dark side of the ocean. She has been a great mentor and friend who has shared many of her life experience with me and always guided me with extreme patience through every step of my way to this point. She has also provided me plenty of opportunities to present at conferences, introduced me to other scientists in the field, and always encouraged me thinking independently as a scientist. It has been a great honor and pleasure to work with Prof. Purnima Ratilal over the past six years.

I would also like to thank my thesis committee members, Prof. Mark Niedre at Northeastern University, Prof. Zoi-Heleni Michalopoulou of New Jersey Institute of Technology, Dr. J. Michael Jech of Northeast Fisheries Center, and Dr. Redwood W. Nero of Southeast Fisheries Center for their keen interest in this thesis and for giving me numerous invaluable comments and sharing interesting discussion about various research topics during my Part II exam and thesis defense. My deep appreciation also goes to Prof. Nicholas C. Makris at MIT. His thorough knowledge and scientific insights in ocean waveguide acoustics and remote sensing, and many thoughtful discussions we had have inspired and helped me tremendously in gaining fundamental understanding of many research topics in acoustics. He is also one of the best teachers I have ever known who can explain the most sophisticated and obscure scientific problems in a way that is easy to understand.

I would like to thank Prof. Purnima Ratilal for supporting me as a Research Assistant during the first two and half years at Northeastern. I am also grateful to have been awarded the ONR Graduate Traineeship Award in Ocean Acoustics, which financially supported my research work in the subsequent years of my Ph.D study. My great appreciation also goes to the Office of Naval Research, the National Oceanographic Partnership Program, the Alfred P. Sloan Foundation, and the National Oceanic and Atmospheric Administration, who funded the Gulf of Maine 2006 experiment, and financially supported the research work in Census of Marine Life. I would like to thank Dr. Thomas Weber, Dr. Richard Menis, Dr. Hector Pena, Dr. Ruben Patel, and crew of the research vessel Hugh Sharp, who helped me survive from the very first scientific cruise of my life and made it full of memorable moments and happiness.

It has been a great pleasure to work with all of my colleagues in the Ocean Acoustics Group from both Northeastern University and MIT, and I have learnt a lot from
all of you. Without your company and support, it is hard to believe that I would be able to go through this 6-year tough journey on my own. I would like to thank Mark Andrews, Ninos Donabed, Ameya Galinde, Daniel Cocuzzo, Saumitro Dasgupta, Elizabeth Kusel, Duong Tran, David Reed, Fan Wu, Mohammad Mahdi Tajdini, Deanille Symonds, Tianrun Chen, Ioannis Bertatsos, Srinivasan Jagannathan, Ankita Jain, and Anamaria Ignisca. Getting to know all of you and the day-after-day long hours work in the lab we have been through together are the most precious experience of my life. I also would like to thank Brian Loughlin, Deanna Beirne, Anne Magrath, and Kristin Hicks from the Bernard M. Gordon Center for Subsurface Sensing and Imaging Systems for their endless administrative support. I also owe my deepest gratitude to Faith Crisley for her tremendous help and support when I was stuck in China.

I am deeply grateful for the encouragement and endless love from my family. My parents have been always supportive of my academic education since I was little, and my parents in law have been always motivating and supporting of me in the past 6 years. Lastly, but not least, my beautiful wife Li, who has made many sacrifices to accompany me to the US. I reminded myself everyday how fortune I am to have you in my life. None of these would have been possible without your endless support, motivation, caring and love, and I dedicate this thesis to you.
Dedicated to

my wife

Li Jia

my parents

Weijun Gong and Hua Liang

and my parents in law

Long Jia and Guangjie Hu
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Chapter 1

Introduction

Instantaneous remote sensing over vast areas of the continental shelf environment can be accomplished using low- to mid-frequency acoustic waves ranging from tens of Hertz to a few kiloHertz. At these frequencies, the acoustic wave can propagate efficiently over long ranges spanning tens to hundreds of kilometers with negligibly small attenuation in continental shelf environments of water depths typically less than 200 m as waveguide modes, making it the most suitable tool for underwater remote sensing [1, 2, 3, 4, 5]. Ocean waveguide acoustics has been utilized by large marine mammals for communication, navigation, and prey detection and localization [6, 7, 8]. Starting from the early 20th century, Navies around the world began to use ocean waveguide acoustics as a primary remote sensing tool for detection, localization, tracking, and classification of underwater vehicles [1, 2]. Besides military uses, the marine science community has also been extensively using waveguide acoustics in a wide variety of remote sensing applications, including active and/or passive sensing of marine life [3, 4, 9, 5, 10, 11, 12], oceanography [13, 14, 15], and geologic and geophysical properties of the ocean environment [16, 17, 18].
Many swimbladder-bearing fish species inhabiting the continental shelf environments can cause significant acoustic scattering at low- to mid-frequency due to the large impedance contrast between their air-filled swimbladders and the surrounding water. In navy sonar systems, these fish can cause high false alarm rates or missed detections since the biological scattering can be confused with or camouflage the returns from other discrete and distributed objects such as underwater vehicles and geologic features. To mitigate the false alarm rate and enhance the capability of navy sonar systems to detect, localize and discriminate man-made targets from biological clutter in continental shelf environments, it is essential to understand the mechanisms that cause strong acoustic scattering from fish groups. From an ecological perspective, the ability to instantaneously survey fish populations distributed over wide areas, remotely classify fish species based on their acoustic scattering, and accurately enumerate their abundance are critically important for fisheries management.

When imaging distributed groups of scatterers, such as fish groups, in a random range-dependent ocean waveguide using a waveguide remote sensing system that applies a matched filter, numerous issues must be considered when inferring the areal population densities, spatial distributions or mean target strengths of fish populations from their matched filtered scattered intensities. For instance, when large number of fish are present within each resolution cell of the waveguide imaging system, multiple scattering as well as attenuation through the fish group may affect the measured scattered levels for observed fish densities.

Besides its applications in active sensing, ocean waveguide acoustics is also widely used for passively localizing and tracking submerged acoustic sources or targets in waveguides. In naval operations, the ability to instantaneously and accurately detect, localize,
and continuously track submerged targets from long ranges using passive acoustic measurements made on a towed horizontal receiver line-array is vital for national defense. In marine mammal research, the ability to detect, locate, track and monitor the vocalizing marine mammal populations over wide areas is critically important for understanding their behavior and their relationship with the environment, and enhancing the capability of Navy to mitigate the effects of man-made sound on marine mammals.

The thesis is composed of three parts. First, together with *in-situ* trawl sampling, experimental data acquired simultaneously using (1) an ocean acoustic waveguide remote sensing (OAWRS) system and (2) two conventional fish finding sonar (CFFS) systems during an experiment conducted in the Gulf of Maine are used to study the acoustic resonance scattering characteristics and the abundance of shoaling Atlantic herring populations, and infer the plausible physical mechanisms that lead to prominent scattering from herring aggregations in active sonar imaging during their Autumn spawning season on Georges Bank. Second, a numerical Monte-Carlo model is developed to determine the statistical moments of broadband matched filtered fully scattered returns from distributed fish groups spanning multiple range and cross-range resolution cells of OAWRS system in a random range-dependent ocean waveguide. This model is applied to examine the effects of (i) multiple scattering, (ii) attenuation due to scattering, (iii) modal dispersion, and (iv) fish 3D spatial configuration on population density imaging of fish populations in continental shelf environments. Third, four distinct methods that can provide instantaneous or near-instantaneous localization of sources located in the *far-field* of a single towed horizontal receiver array are examined. The performance of these methods is experimentally demonstrated with data from Gulf of Maine 2006 Experiment.
In Chapter 2, to study the scattering characteristics, abundance and diurnal behavior of Atlantic herring (*Clupea harengus*), the most abundance species in and around Georges Bank during their Autumn spawning season [19], an experiment known as Gulf of Maine 2006 Experiment (GOME’06), was conducted in conjunction with the U.S. National Marine Fisheries Service annual Atlantic herring acoustic survey of the Gulf of Maine and Georges Bank, concentrating on areas where herring shoals were most likely to form. The OAWRS system’s instantaneous imaging diameter of 100 km and regular updating enabled unaliased monitoring of fish populations over ecosystem scales including shoals of Atlantic herring containing hundreds of millions of individuals [3, 4], as confirmed by concurrent trawl and CFFS sampling, which also showed high spatio-temporal coregistration with fish shoals imaged by OAWRS and provided local areal population densities, as well as depth and length distributions of the fish populations. Together with long-range transmission loss measurement, concurrent CFFS, and trawl data, the OAWRS imagery enabled (1) estimates of herring strength to be made at low frequencies (300-1200 Hz) from which physical scattering mechanisms may be inferred, (2) herring spatial distributions and abundance to be estimated over ecosystem scales, and (3) regular diurnal patterns in herring behavior to be determined during the Autumn spawning season on Georges Bank [4, 20]. The mean scattering cross-section of an individual shoaling herring is found to have a strong nonlinear dependency on frequencies in the range of OAWRS survey, consistent with the steep roll-off expected for sub-resonance scattering from fish with air-filled swimbladder [21]. These findings suggest that OAWRS can provide valuable remote species classification over wide areas since significant variations in the frequency dependence of target strength are expected across species due to differences in the resonance frequency of fish swimbladder.

In Chapter 3, it is shown theoretically that the areal population density of fish
groups can be estimated from their incoherently averaged broadband matched filtered scattered intensities measured using an OAWRS system with less than 10% error. When imaging distributed groups of scatterers using a waveguide remote sensing system that applies the matched filter in a random range-dependent ocean waveguide, numerous issues must be considered when inferring the areal population densities, spatial distributions, or mean target strengths of fish populations from their matched filtered scattered intensities. (1) Large numbers of individuals are often present within each resolution cell of the OAWRS system so that the scattered fields from the distributed group can either combined coherently or incoherently at the receiver [10, 22, 23, 24]; (2) Multiple scattering and attenuation due to scattering through the fish group may affect the measured scattered levels [23, 25, 26, 27, 28, 29, 30, 31, 10]; (3) Modal dispersion due to the slowly propagating high order modes that arrive later in time than the low order modes lead to delayed returns that affect the measured scattered levels over the range extent of the distributed group; (4) The broadband transmissions scintillate in both time and space due to dynamic ocean processes, such as internal waves and bathymetric variations [32, 33, 34]; (5) Scattered returns from fish and other targets are measured in time and then charted to range by multiplying the measurement time with a charting speed, modal dispersion and waveguide scintillation affect the charting speed and hence the localization accuracy of targets in a waveguide [35, 36]; (6) The matched filter operation is a coherent process that involves correlating the scattered field with the transmitted waveform [10, 37, 38]; however, the scattered returns from distributed groups are often assumed to be incoherent, and the spatial resolution from matched filtering in the waveguide [4, 3, 9, 5] is assumed to be the same as that for discrete targets in free space with coherently scattered returns.

To address these issues, a numerical Monte-Carlo model is developed to deter-
mine the statistical moments of the broadband matched filtered scattered field from a three-dimensional spatial distribution of fish with random sizes and species in a random range-dependent ocean waveguide. The model includes multiple scattering from the fish group and employs the complex scatter function to account for attenuation and dispersion due to scattering from an individual fish. It uses a range-dependent acoustic propagation model [39] based on the parabolic equation to simulate propagation in a range-dependent ocean waveguide. By incorporating randomness in the waveguide environment and the dynamic scatterer group, the model can account for statistical fluctuations typically present in OAWRS data. This is the only model to simultaneously analyze multiple scattering and attenuation from fish groups imaged at long ranges in an ocean waveguide with a broadband pulsed system employing the waveguide Green’s function for propagation and the matched filter to localize scatterers in range. Previous models for predicting and analyzing the scattered levels from objects in an ocean waveguide, either do not consider multiple scattering [40, 41] or are restricted to direct-path imaging systems [27, 29, 26, 28, 10], in which they only consider very small schools on the order of the acoustic wavelength with a limited number of individuals, where the coherent effects in multiple scattering, such as resonance shifts and sub- and super-resonance local maxima can be significant. Here, we focus on fish groups that extend tens to thousands of times the wavelength of the waveguide imaging system, where we show that the incoherent intensity dominates the scattered returns and resonance shift effects are negligible or absent. The model developed here can also be applied to analyze detection and imaging of other groups of discrete scatterers in a waveguide where multiple scattering may be significant, such as bubble clouds, swarms of AUVs, and pods of dolphins or whales.

In Chapter 4, we investigate instantaneous passive localization and tracking of
acoustic sources over long range with measurements made on a single towed horizontal receiver line-array in a random range-dependent ocean waveguide. An advantage of passive sensing with a towed horizontal array of hydrophones is that the bearing of a sound source can be directly obtained by beamforming the received signals so that only the range of the source to the receiver array has to be determined. When only a single horizontal moving receiver array is available, the passive techniques developed for localizing acoustic sources located in the far-field of the array can be categorized as either recursive type nonlinear filters in Bayesian framework using bearings-only measurements or far-field waveguide techniques. The nonlinear filters developed [42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56] have been focused on optimizing the efficiency, enhancing the stability and robustness of the localization algorithms by deriving the theoretical limits of the Cramer-Rao lower bound (CRLB) for the proposed nonlinear filter. The performance of these nonlinear filters have been primarily tested with simulated data and have limited applications to real or field data. The far-field waveguide techniques, such as matched field processing (MFP) [57, 58, 59] and waveguide invariant [60, 61, 62] are either computationally expensive or require a priori knowledge of the ocean environment, making them incapable of providing instantaneous passive source localization over long range in a random range-dependent ocean waveguide. Four distinct methods that can provide instantaneous or near-instantaneous localization of acoustic sources located in the far-field of a single towed horizontal receiver array are examined. They include (1) synthetic aperture array (SAT), which combines measurements made on adjacent or widely separated finite apertures of a single towed array and employs conventional triangulation ranging algorithm [63, 64] for localizing sources located in the far-field of the receiver array; (2) array invariant (AI) [65], a technique that exploits the dispersive modal arrival structure of the acoustic field in an ocean
waveguide to estimate the source range for sources located off the broadside beam of the receiver array; (3) the bearings-only target motion analysis in modified polar coordinates implemented using extended Kalman filter (MPC-EKF) where the bearing and range components of the source location and velocity state vector are decoupled [46], and (4) bearings-migration minimum mean square error (MMSE), also based on triangulation but combines sequential bearing measurements in a global inversion for the mean source position over the measurement time interval. These methods are applied to localize and track a vertical source array deployed in the far-field of a towed horizontal receiver array during GOME’06 [4, 9]. The source transmitted intermittent broadband pulses in the 300-1200 Hz frequency range. The performance of all four methods are evaluated for a wide variety of source-receiver geometries and range separations up to 20 km. The localization accuracy are determined by comparing the estimated source ranges with the true ranges obtained from GPS measurements of source and receiver locations. The effects of source range and bearing relative to the receiver array broadside on localization accuracy are also investigated.
Chapter 2

Low-frequency target strength and abundance of shoaling Atlantic herring (*Clupea harengus*) in the Gulf of Maine during the Ocean Acoustic Waveguide Remote Sensing 2006 Experiment

To study the scattering characteristics, abundance and diurnal behavior of Atlantic herring (*Clupea harengus*), the most abundant fish species in and around Georges Bank during their Autumn spawning season [19], an experiment using ocean acoustic waveguide remote sensing (OAWRS) was conducted in the Gulf of Maine from September 19 to October 6, 2006, concentrating on areas where herring shoals were most likely to form (Fig. 2.1). The experiment, known as OAWRS 2006, was conducted in conjunction with the U.S. National Marine Fisheries Service annual herring survey of the Gulf of Maine and Georges Bank. Fish populations were instantaneously imaged over a 100 km diameter area by a mobile OAWRS system [3] with minute-to-minute updates to form wide-area movies of fish activity over many diurnal cycles, demonstrating
the capacity of OAWRS to instantaneously image fish populations over wide areas in complex continental-shelf environments with highly variable bathymetry and oceanography. Shoals imaged by OAWRS typically comprised tens to hundreds of millions of individuals and stretched for many kilometers along the northern flank of Georges Bank. Concurrent conventional fish finding sonar (CFFS) surveys showed high spatio-temporal co-registration with fish shoals imaged by OAWRS, and provided local areal population densities, as well as depth and length distributions of the fish populations. Concurrent trawl sampling, which showed Atlantic herring to be the overwhelmingly predominant species comprising the large shoals [3, 4], enabled onsite species identification and direct biological measurements of such parameters as fish length, swimbladder geometry, stomach content, and sexual development.

Together with long-range transmission loss measurements, concurrent CFFS and trawl data, the OAWRS imagery enabled (1) estimates of herring target strength to be made at low frequencies (300-1200 Hz) from which physical scattering mechanisms may be inferred, (2) herring spatial distributions and abundance to be estimated over ecosystem scales, and (3) regular diurnal patterns in herring behavior to be determined during the Autumn spawning season on Georges Bank [4, 20].

The mean scattering cross-section of an individual shoaling herring is found to consistently exhibit a strong, roughly 20 dB/octave roll-off with decreasing frequency in the range of the OAWRS survey over many measurement days, consistent with the steep roll-off expected for sub-resonance scattering from fish with air-filled swim bladders. These findings suggest that OAWRS can provide valuable evidence for remote species classification over wide areas since significant variations in the frequency dependence of target strength are expected across species due to differences in resonance. This is
Figure 2.1: Location of OAWRS 2006 experiment on the northern flank of Georges Bank in the Gulf of Maine. Plus indicates location of moored OAWRS source array deployed on October 1 to 3 at 42.2089N, 67.6892W, the coordinate origin for all OAWRS images in this chapter. Circle shows typical area imaged by OAWRS, 100 km diameter and wider than Cape Cod, in 70 s. Geographic locations of trawls deployed by NOAA FRV Delaware II are overlain. Dots indicate trawls where herring were predominant species. In contrast, diamond indicates a trawl where silver hake and squids dominated. The gray dashed box bounds the area of OAWRS imaging during the OAWRS 2006 experiment.
because the dominant source of acoustic scattering at low and mid frequencies (less than 10 kHz) is the air-filled swimbladder for fish that have swimbladders [66]. Resonance frequencies depend on swimbladder volume, shape, ambient pressure, and the effect of surrounding tissues [67, 68]. For many fish species of economic importance in the size ranging from 10 to 50 cm, resonances are expected to range from several hundred Hertz to a few kiloHertz [67, 68, 69, 70]. Previous experimental investigation of resonance have been limited to small scale tank measurements with individual fish out of their natural environment or highly localized measurements made in situ with sources of rapid power roll-off below 1.7 kHz [68, 71, 72].

The population that spawns on the northern flank of Georges Bank (Fig. 2.1) is the largest herring stock in the Gulf of Maine, and has both ecological and economic importance [19]. It has been surveyed annually by the U.S. National Marine Fisheries Service for roughly one decade during the Autumn spawning season [73, 74, 75]. Current estimates of the Georges Bank herring stock varies from 500,000 to 1 million tons based on acoustic surveys and other assessment methods [75], respectively. The National Marine Fisheries Service acoustic survey employs highly localized CFFS measurements along widely spaced line transects, roughly 15 to 20 km apart [75], trawl sampling at selected locations, and takes roughly a week to cover the northern flank of Georges Bank from east to west. As a result, annual stock estimates may be highly aliased in both time and space. One of the primary goals for OAWRS 2006 is to provide images of fish populations over the vast areas they inhabit that are unaliased in both space and time [3] so that more reliable abundance estimates may be obtained.
2.1 Multi-Sensor Experiment Design and Resources

The OAWRS 2006 experiment was designed to coincide with the National Marine Fisheries Service annual herring survey on Georges Bank. It was conducted with four research vessels (RVs) that employed a suite of acoustic imaging sensors, several oceanographic monitoring systems, and trawls for species identification. The OAWRS vertical source array and towed horizontal receiving array were separately deployed from two medium sized UNOLS vessels, RV *Endeavor* and RV *Oceanus* respectively, for bistatic measurement of echo returns. The instantaneous areal coverage of the OAWRS system in a single transmission is shown in Fig. 2.1. The vertical source array transmitted a suite of individual Tukey-windowed linear frequency modulated (LFM) pulses of 1s duration and 50 Hz bandwidth centered at a suite of frequencies with a repetition interval of 150 s for each center frequency [76]. Broadband LFM pulses centered at 415 and 735 Hz were transmitted seconds apart, then after 75 s those centered at 950 and 1125 Hz were transmitted seconds apart and the process was repeated. Transmissions radiated with azimuthal symmetry about the OAWRS source array, for which more information is available in Ref. [33], with source level continuously monitored with two desensitized hydrophones deployed from RV *Endeavor*. Scattered returns were acquired with a horizontal receiving array, the ONR five-octave research array, towed by RV *Oceanus* along designated tracks. The multiple nested sub-apertures of the array span 50 to 3750 Hz frequency range. Returns measured within each linear section of the array are processed by beamforming and matched filtering with angular resolution shown in Table 2.1. The receiving array also contained one desensitized hydrophone which was used to measure transmitted signals from the source array for transmission loss and source level calibrations. Two calibrated acoustic targets made of air-filled rubber hose [77], approximately
30 m long and 7 cm in diameter with known scattering properties [78], were vertically deployed at selected locations to enable accurate charting of scattered returns in both range and azimuth. One of the targets was moored with lower end 5 m off the seafloor and the other was centered at 140 m in waters 200 m deep.

Table 2.1: OAWRS receiving array 3-dB angular resolution $\beta(\phi)$ at broadside $\phi = 0$ and endfire $\phi = \pi/2$, and aperture length $L$ as a function of imaging frequency $f_c$. A Hanning spatial window is applied in the beamforming.

<table>
<thead>
<tr>
<th>$f_c$ (Hz)</th>
<th>L (m)</th>
<th>$\beta(\phi = 0)/\beta(\phi = \pi/2)$ (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>415</td>
<td>94.5</td>
<td>3.15/31.4</td>
</tr>
<tr>
<td>735</td>
<td>47.25</td>
<td>3.56/33.3</td>
</tr>
<tr>
<td>950</td>
<td>47.25</td>
<td>2.75/29.3</td>
</tr>
<tr>
<td>1125</td>
<td>23.625</td>
<td>4.65/38.1</td>
</tr>
</tbody>
</table>

Over the course of the OAWRS 2006 experiment, more than 3,000 wide-area images of the ocean environment were acquired by the OAWRS system for each of the four LFM center frequencies leading to more than 12,000 images in total. Similarly, more than 12,000 transmission loss measurements were made over the survey area to calibrate our transmission loss model. The length of each RV Oceanus towed array track was typically 15 km. With a nominal tow speed of 2 m s$^{-1}$ for the receiver ship, a total of roughly 75 images of the ocean environment were generated per frequency along each track. Minute-to-minute updates of the OAWRS imagery made it possible to closely monitor herring activity over wide areas and observe patterns of spatial distribution evolve over the course of each day. The inherent left-right ambiguity about the horizontal line-array’s axis in the OAWRS images were resolved mainly by varying receiver ship heading, sometimes only slightly for several transmissions by what we call a “Crazy Ivan” for immediate results, as well as ship position. These approaches for ambiguity resolution are described in Refs. [79], [80], and [81].
Examples of the massive fish shoals instantaneously imaged by OAWRS near Georges Bank are shown in Fig. 2.2. The massive shoal imaged during midnight hours of October 4 within the 150 to 180 m bathymetric contour in Fig. 2.2(A), for example, extends 15 km by 5 km and comprises roughly 170 million fish distributed about several population centers. The area occupied by this shoal is approximately equal to that of Manhattan Island in New York. The fish population in the diffuse cloud region to the north is comprised of over 70 million individuals.

Concurrent localized imaging of fish aggregations at OAWRS-directed locations was conducted by two other research vessels, the RV *Hugh Sharp* and the NOAA FRV *Delaware II*, using two downward-directed CFFS systems, the SIMRAD EK60 and EK500 echosounders, respectively. Both the EK60 and EK500 echosounders insonify the water column directly beneath the survey vessel simultaneously at three frequencies to provide the local depth dependence of dominant fish layers within their instantaneous resolution footprints, of between 24 to 50 m diameter, and estimates of volumetric and areal fish population densities. Specifications of these two echosounders appear in Table 2.2. A Reson 7125 Seabat multi-beam sonar (400 kHz) system was also deployed from RV *Hugh Sharp* with an angular swath of 128°. It was useful in providing detailed 3D morphology of smaller fish groups located in the mid-water column [82]. A high-speed rope trawl[83] deployed by NOAA FRV *Delaware II* enabled species identification [75] at OAWRS-directed locations.

Physical oceanography was monitored by sampling water column temperature and salinity with expendable bathythermographs (XBT) and conductivity-temperature-depth (CTD) sensors at regular hourly intervals from all four research vessels. The water column sound speed profile was found to be relatively constant in space and time over
Figure 2.2: (A)-(C) OAWRS images of areal fish density zoomed-in around massive herring shoals, with densities exceeding 10 fish m$^{-2}$ in population centers. Measured during evening to midnight hours of October 4, 2 and 1, respectively. (A) The total population of herring in the large dense shoal is roughly 170 million, and that in the diffuse cloud outside the large shoal is roughly 70 million. Imaged shoal populations of herring are approximately 86 and 70 million respectively for (B) and (C). Uncertainty in the abundance estimate is 17-20%. Note that the figures are plotted on different scales, and the coordinate origin is the source location shown in Fig. 2.1.
Table 2.2: Conventional fish finding sonars, SIMRAD EK60 and EK500 specifications. The angular 3-dB beamwidth is denoted by $\beta$, the pulse duration by $PD$ and repetition rate by $RR$. The resolution diameter, $Res$, is calculated for 200 m water depth.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>$f$(kHz)</th>
<th>$\beta$(deg)</th>
<th>$Res$(m)</th>
<th>$PD$(ms)</th>
<th>$RR$(s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EK60</td>
<td>38</td>
<td>7</td>
<td>24</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>11</td>
<td>39</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>200</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EK500</td>
<td>38</td>
<td>12</td>
<td>42</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>7</td>
<td>24</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

the 2006 OAWRS survey as shown by the compilation of over roughly 200 samples taken during the experiment in Fig. 2.3. No mesoscale oceanographic features such as eddies were found or expected. The small fluctuations about the mean profile are due to mild internal wave activity that causes well-understood short-term Gaussian field fluctuations in acoustic transmission that have an intensity standard deviation that can be reduced to a small fraction of the mean by stationary averaging [84, 85, 32]. An instrumented tow cable was also deployed from the RV Hugh Sharp to provide continuous measurement of temperature. This oceanographic information was used to carefully update horizontal locations and depths of the OAWRS source (typically centered at 60-70 m) and receiving arrays (centered at 105 m) [76] to optimize OAWRS imaging of fish groups. Decisions were often based on the outputs of the acoustic propagation model RAM, based on the parabolic equation, for multi-modal waveguide transmission loss in the range-dependent Georges Bank environment with the hourly sound speed profile updates and known bathymetry [86].
Figure 2.3: Profiles of water-column sound speed from XBT and CTD measurements made from all four research vessels on the northern flank of Georges Bank and Georges Basin during OAWRS 2006.
2.2 Data Processing and Analysis

2.2.1 Generating instantaneous wide-area OAWRS images of the ocean environment

Wide-area images of instantaneous scattered intensity spanning 100 km in diameter were generated in near real-time for every broadband transmission centered at each of the four frequencies, $f_c=415, 735, 950, \text{ and } 1125 \text{ Hz}$. For each transmission, the pressure data on the receiving array were first beamformed to determine the azimuth of the arrivals, then matched filtered with the source signal, and charted in range using two-way travel time [81, 80, 79]. Each image was then mapped onto geographic space using the GPS latitude and longitude information of the source and receiving array. A nominal sound speed of 1475 m/s that minimizes charting errors was used to convert the travel-time of the signal to range [81, 35]. The range resolution $\Delta \rho$ of the OAWRS system is approximately 15 m after matched-filtering, and the azimuthal resolution $\beta(\phi, f_c)$ associated with each frequency band both at broadside and endfire is tabulated in Table 2.1. A Hanning spatial window was applied in the beamforming to reduce sidelobe levels by more than 30 dB from the main lobe. A detailed explanation of the image formation process is provided in Refs. [3], [80], and [81].

A standard procedure of averaging three consecutive instantaneous OAWRS images and two adjacent range cells is used for all OAWRS images presented here. This leads to an experimentally determined standard deviation in log-intensity of roughly 1.5 dB [4], consistent both with theory and previous experiments [81, 80, 3, 4, 33]. This standard deviation is negligible compared to the dynamic ranges of features in the OAWRS images and the variations in herring target strength measured across frequency in the OAWRS
2.2.2 Estimating areal fish population density from instantaneous OAWRS imagery

Here we describe how areal fish population density over wide areas may be estimated from OAWRS intensity images of the ocean environment. At typical OAWRS operating frequencies from hundreds of Hertz to a few kiloHertz, most fish are acoustically compact scatterers, with swimbladder sizes that are much smaller than the wavelength. The sonar equation approach is then valid for analyzing scattering from fish since the scattered field from each individual is omni-directional, making propagation and scattering factorable even in a waveguide [87]. The expected scattered intensity from fish aggregations after matched-filtering is dominated by the incoherent intensity or variance of the scattered field and multiple scattering effects are negligible for the densities found here as shown in Ref. [10] and [27]. As a result, given a source at $r_0$ transmitting a broadband signal with bandwidth centered at $f_c$, and a receiver at $r$, the expected scattered intensity, $\langle I_s(\rho_m, f_c)\rangle$, within the OAWRS resolution footprint of area $A(\rho_m | \Delta \rho, f_c)$ centered at horizontal location $\rho_m$ can be expressed as,

$$\langle I_s(\rho_m, f_c)\rangle = \sum_{i=1}^{M(\rho_m)} \left\langle |Q(f_c)|^2 \right\rangle (4\pi)^4 \left\langle |G(r_i | r_0, f_c)G(r | r_i, f_c)|^2 \right\rangle \times \left\langle \frac{|S(r_i, f_c)|^2}{k^2} \right\rangle \quad (2.1)$$

where $M(\rho_m)$ is the number of fish within the resolution cell, $r_i$ is the location of the $i$th fish, $|Q(f_c)|^2$ is the source intensity, $G(r_i | r_0, f_c)$ and $G(r | r_i, f_c)$ are the waveguide Green functions from the source to each scatterer and from each scatterer to the receiver respectively, $S(r_i, f_c)$ is the fish scatter function and $k$ is the wavenumber.
The expected intensity in a fluctuating waveguide from uniformly distributed targets within the resolution footprint can be approximated as

$$\left\langle I_s(\rho_m, f_c) \right\rangle \approx \left| \left\langle Q(f_c) \right\rangle \right|^2 \gamma(\rho_m, f_c) \sum_{i=1}^{M(\rho_m)} \left| \frac{S(r_i, f_c)}{k^2} \right|^2$$

where

$$\gamma(\rho_m, f_c) = \left| \left\langle \frac{4\pi}{2} G(r_m | r_0, f_c) G(r | r_i, f_c) \right\rangle \right|^2$$

for sufficiently narrow depth layers $H$ and areal footprints over which $\gamma(\rho_m, f_c)$ becomes effectively constant, as shown for the OAWRS 2006 fish shoal imaging in Ref. [20]. The last factor of Eq. (2.2) can be written as

$$\sum_{i=1}^{M(\rho_m)} \left| \frac{S(r_i, f_c)}{k^2} \right|^2 = M(\rho_m) \sigma(\rho_m, f_c)$$

where $P(r_i)$ is the probability density of finding the $i$th fish at location $r_i$, and $P(r_i) = \frac{1}{A(\rho_m | \Delta \rho, f_c) H}$ for uniformly distributed fish shoals, $\sigma(\rho_m, f_c)$ is the average scattering cross-section of an individual fish over the OAWRS resolution footprint and the depth layer, $n_{A,oawrs}(\rho_m) = M(\rho_m)/A(\rho_m | \Delta \rho, f_c)$ is the mean areal fish population density within the resolution footprint, and $A(\rho_m | \Delta \rho, f_c) \approx \rho_m \Delta \rho \beta(\phi, f_c)$ is the range and azimuth-dependent spatial resolution of the OAWRS imaging system [81].

Inserting Eq. (2.3) into Eq. (2.2) and taking $10 \log_{10}$ of both sides, we obtain the scattered intensity level in decibels,

$$L(\rho_m, f_c) \approx SL(f_c) + TTL(\rho_m, f_c) + SS_{oawrs}(\rho_m, f_c) + 10 \log_{10}(A(\rho_m | \Delta \rho, f_c))$$

where $L(\rho_m, f_c) = 10 \log_{10} \left\langle I_s(\rho_m, f_c) \right\rangle$, $TTL(\rho_m, f_c) = 10 \log_{10} \gamma(\rho_m, f_c)$ describes the expected second moment of depth averaged propagation to and from the fish layer aver-
aged over the resolution footprint of the OAWRS system, \( SL(f_c) = 10 \log_{10} \langle |Q(f_c)|^2 \rangle \) is the spectral source level, and \( SS_{oawrs}(\rho_m, f_c) \) is the scattering strength.

From Eqs. (2.2)-(2.4), OAWRS scattering strength can be expressed as,

\[
SS_{oawrs}(\rho_m, f_c) = TS_{oawrs}(f_c) + 10 \log_{10} \langle n_{A,oawrs}(\rho_m) \rangle
\]  

(2.5)

where \( TS_{oawrs}(f_c) = 10 \log_{10} \sigma(\rho_m, f_c) \) in units of dB re 1 m\(^2\) is the target strength corresponding to the average scattering cross-section of an individual fish over the OAWRS resolution footprint and depth layer within the bandwidth centered at \( f_c \).

The terms in Eq. (2.4) are evaluated separately for each of the four OAWRS LFM waveforms with different center frequencies \( f_c \). A calibrated stochastic transmission loss model based on the parabolic equation [86] for a range-dependent fluctuating ocean waveguide is used to estimate the random Green functions and determine \( TTL(\rho_m, f_c) \) following the approach described in the Appendix B and Ref. [33]. Expected source level is estimated from one-way propagated signals received by a desensitized hydrophone on the moving receiver array using the approach of Ref. [33]. The two monitoring hydrophones on the source ship were used to verify the source level estimates. Our analysis indicates the source transmitted a stable output over the course of each day.

The application of Eq. (2.4) to estimate scattering strength from OAWRS imagery is illustrated in Figs. 2 and 3 of Ref. [88] and in Ref. [89]. Scattering strength is a useful parameter for characterizing submerged objects, both distant and nearby, because it is independent of the spatially varying transmission loss and areal resolution footprint of the imaging system. Once the target strength expected of an individual fish is known, an areal fish population density image can be obtained from a scattering strength image [89] using Eq. (2.5). The target strength corresponding to the average scattering cross-section
of an individual fish at OAWRS operating frequencies is estimated by matching between OAWRS and CFFS areal fish population density measurements where simultaneous sampling through stationary fish populations is available.

### 2.2.3 Estimates of areal fish population density from CFFS

The CFFS measurements at 38 kHz are used to provide local estimates of areal fish population density [90, 91, 75]. The $7^\circ$ 3-dB beamwidth yields an instantaneous circular survey area of 24 m diameter directly under the survey vessel at 200 m water depth. Volumetric scattering from all targets within the conical beam were measured. The localized areal fish population density in fish m$^{-2}$, denoted by $n_{A,cffs}$, can be estimated using

$$n_{A,cffs} = \frac{4\pi}{\tilde{\sigma}_{bs}} \int_{z_1}^{z_2} s_v dz$$  \hspace{1cm} (2.6)

where $s_v$ is the volume backscattering coefficient [92] in m$^{-1}$, $z_1$ and $z_2$ delimit the depth bounds for fish aggregations, $\tilde{\sigma}_{bs} = 4\pi10\langle TS_{cffs}\rangle/10$ is the mean backscattering cross-section of an individual at 38 kHz in units of m$^2$, where $\langle TS_{cffs}\rangle$ is the corresponding mean target strength at ultrasonic frequency.

The expected target strength for an individual fish at 38 kHz varies with species, depth and total fish length. Here, the expected $TS_{cffs}$ in dB of an individual herring of total length $L_{TL}$ in centimeters at depth $z$ in m is obtained from Eq. (5) of Ref. [93],

$$TS_{cffs} = 20\log_{10} L_{TL} - 2.3\log_{10} (1 + z/10) - 65.4,$$  \hspace{1cm} (2.7)

and then converted to $\sigma_{bs}$. The mean backscattering cross section $\tilde{\sigma}_{bs}$ is obtained as the weighted average over the total length and depth distribution of the fish aggregations.
From trawl surveys of the imaged fish populations in OAWRS, herring was the overwhelmingly dominant species comprising the large shoals, which had small fractions of redfish and silverhake. Estimates of the mean $TS_{eff}$ for individual herring and redfish based on our trawl measurements (Appendix A) of the length distribution are provided in Table 2.3. The expected target strength of herring and redfish over similar depth extent at 38 kHz are close, varying at most by 1 dB, albeit their different length distributions. In contrast, their low frequency target strength near resonance varies significantly, as discussed in Sec. 2.3.3. These modeled $TS_{eff}$ values are in good agreement with those obtained by experimentally analyzing the CFFS backscattered field from individual fish distinguishable in the periphery of various aggregations consistent with 0.1 dB mean squared errors reported in Ref. [93].

Figures 2.4(D) and (E) illustrate the application of Eq. (2.6) to estimate areal fish density for herring aggregations in the 120-180 m water depth range.

2.2.4 Estimating low frequency target strength by matching OAWRS and CFFS population densities

Here we describe our procedure for estimating the low-frequency target strength corresponding to the average scattering cross-section of an individual shoaling herring over the resolution footprint of the OAWRS system by correlating OAWRS data with simultaneous measurements made along CFFS transects. The target strength of herring at 950 and 1125 Hz is found to be significantly higher than at 415 and 735 Hz, making much lower herring densities observable at these higher frequencies. At the lower frequencies of 415 and 735 Hz, the herring target strength is weaker causing the scattered returns to be background saturated at moderate fish densities. Due to the receiving array’s
Figure 2.4: Herring target strength at 950 and 1125 Hz estimated by matching areal fish density in OAWRS and CFFS data acquired during midnight hours of October 2. (A)-(C) A sequence of instantaneous OAWRS scattering strength images zoomed into the region containing a massive herring shoal with overlain CFFS line transect (solid line) made at nominal tow-speed of 2.5 m s\(^{-1}\). (D) CFFS time-depth echogram provides local depth distributions of fish aggregations. Dashed lines at 23:30 EDT and 01:00 EDT correspond to transect start and end points \(\alpha\) and \(\Omega\), respectively. (E) The areal fish population densities inferred from CFFS measurements following Eq. (2.6) are plotted as a function of time in black, and the corresponding areal fish population densities in dB, \(10 \log_{10}(n_{A,cffs})\), are plotted in gray. (F) The OAWRS scattering strength measurements and (G) instantaneous target strength estimates along CFFS line-transects at 950 and 1125 Hz. Target strength estimates near the edge of shoals are not accurate because of non-stationarity. (H) Population of herring within the area shown in (A)-(C) determined with various OAWRS fish density \(n_A\) thresholds. Solid line gives population above the threshold, dotted line gives population below the threshold.
Table 2.3: Physical parameters of modeled fish species, and their measured target strength at 38 kHz with a CFFS.

<table>
<thead>
<tr>
<th>Species</th>
<th>Atlantic herring</th>
<th>Acadian redfish</th>
<th>Silver hake</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{FL}$ (cm)</td>
<td>19-30</td>
<td>15-39</td>
<td>2-35</td>
</tr>
<tr>
<td>$L_{TL}$ (cm)</td>
<td>20-34</td>
<td>16-41</td>
<td>2-35</td>
</tr>
<tr>
<td>Depth (m)</td>
<td>120-190</td>
<td>120-190</td>
<td>10-75</td>
</tr>
<tr>
<td>$\langle TS_{cdfs} \rangle$ (dB)</td>
<td>-39.7$^a$</td>
<td>-38.9$^b$</td>
<td>N/A</td>
</tr>
<tr>
<td>$\sigma_{cdfs}$ (dB)</td>
<td>1.3$^c$</td>
<td>2.4</td>
<td>N/A</td>
</tr>
<tr>
<td>$\rho_f^d$(kg m$^{-3}$)</td>
<td>1071</td>
<td>1080</td>
<td>1050</td>
</tr>
<tr>
<td>$\xi^e$(Pa sec)</td>
<td>50</td>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td>$\varepsilon^f$</td>
<td>5-10</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>$\kappa_{nb}^g$</td>
<td>0.05</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>$\kappa_z^h$</td>
<td>$x^i$</td>
<td>0.05</td>
<td>0.03</td>
</tr>
</tbody>
</table>

$^a$ Mean target strength of herring calculated using Eq. (2.7)
$^b$ Mean target strength of redfish calculated using equation in Ref. [94].
$^c$ Standard deviation of derived herring target strength at 38 kHz incorporating fish length and depth distribution from CFFS and trawl surveys.
$^d$ Fish flesh density.
$^e$ Viscosity of fish flesh.
$^f$ Major-to-minor axis ratio of fish swimbladder.
$^g$ Fish swimbladder volume to fish body volume ratio at neutral buoyancy depth.
$^h$ Fish swimbladder volume to fish body volume ratio at depth.

$x$ is a linear function of ambient pressure at depth given by $x = \kappa_{nb} P_{nb} z_{nb} / P_z$, where $P_{nb}$ is the ambient pressure at neutral buoyancy depth $z_{nb}$, and $P_z$ is the ambient pressure at any depth $z$.

Sub-aperture design, OAWRS images at 950 Hz have the best cross-range resolution, making this an optimal frequency for wide-area sensing. An alternative approach for target strength estimation, based on differencing pairs of OAWRS wide-area scattering strength images at two distinct frequencies, is applied in the next section to determine target strength at the lower frequencies. The target strength estimates are summarized in Table 2.4.

Calibrated acoustic targets were deployed on October 2-3 enabling independent and precise geographic charting of OAWRS images. By making small adjustments to
the charting speed and array orientation, scattered returns from calibrated targets were accurately charted to the correct range-azimuth resolution cell relative to the source and receiver. This ensures that scattered returns from all other targets, including the fish aggregations, have been accurately charted as well. In this section we focus on data acquired on October 2-3 when calibrated target data was available and present target strength estimates for other days in the Sec. 2.2.6.

Close to midnight on October 2, both OAWRS and CFFS systems simultaneously co-registered a massive herring shoal between the 150 to 180 m isobaths on the northern flank of Georges Bank, as shown in Figs. 2.4(A)-(C). The observations were made continuously over a 90-min period between 23:30 eastern daylight time (EDT) on October 2 and 01:00 EDT on October 3. Measurements from the two systems are highly correlated during the course of the observations because of the statistical stationarity of the fish populations even though their resolution footprints are significantly different. The OAWRS system monitored and sampled the temporal and spatial evolution of the shoal’s horizontal morphology at intervals of 75 s without aliasing. Concurrently, the CFFS system crossed the same shoal twice along a U-shaped transect with two parallel transects 1.5 km apart. The depth distribution of the fish population, within roughly 40 to 60 m of the seafloor, is relatively consistent across the two CFFS transects as shown in Fig. 2.4(D).

To accurately estimate low-frequency target strength, we confine our present analysis to contiguous space-time segments that consistently register significant, stationary scattering from fish aggregations in both OAWRS and CFFS systems. We derive threshold values for CFFS population density and OAWRS scattering strength. The segmented data above these thresholds are used for target strength estimation. The CFFS threshold
is set at 0.2 fish m$^{-2}$ as shown in Fig. 2.4(E). For OAWRS, two square areas of dimension 6.2×3 km$^2$ and 1.57×5.58 km$^2$ that continuously register significant fish scattering and diffuse background reverberation respectively, throughout the course of observation are first examined. The histogram of scattering strength values within these areas, averaged over multiple OAWRS images, are plotted in Fig. 2.5(D). The histograms are approximately Gaussian. The OAWRS threshold is then set at -50 dB for 950 Hz to distinguish fish scattering from the background. This threshold is roughly 2 standard deviations below the mean for the fish histogram and roughly 2 standard deviations above the background mean.

Employing Eq. (2.5), and assuming local stationarity of fish population, we set the areal fish density within the OAWRS resolution footprint to that simultaneously sampled by CFFS transect through the OAWRS footprint, $n_{A,oawrs} \approx n_{A,cffs}$. The resulting target strength estimates for fish in these contiguous shoaling regions at 950 Hz are shown in Fig. 2.4(G). The differences in target strength estimates along the transect are due to the fact that the OAWRS and CFFS systems have different resolution footprint sizes, and so the true mean fish areal densities within the OAWRS resolution cell may be overestimated or underestimated by the CFFS system given non-stationary spatial distributions, as occur at shoal boundaries. The combination of measurements from many space-time locations from both systems should yield mean target strength estimates with small variance by virtue of the law of large numbers as discussed in Sec. 2.2.7.

Similar statistical analyses have been conducted for OAWRS data at 1125 Hz, with estimated target strength appearing in Fig. 2.4(G). This approach is also applied to infer herring target strength at 950 and 1125 Hz using OAWRS and CFFS data on October 3,
Figure 2.5: The intensity of scattered returns from shoals is highly frequency-dependent. The histograms illustrate that it is easier to detect shoals over background regions at higher frequencies. Simultaneous trawls show shoals are overwhelmingly comprised of herring while background regions yield negligible herring (Table 2.4, Fig. A.1). (A), (C) and (E) OAWRS images of herring shoal acquired simultaneously at three distinct frequency bands centered at 415, 950 and 1125 Hz at 00:41:15 EDT on October 2. The color scale used in (A), (C), and (E) is the same as in Figs. 2.4(A-C). (B), (D) and (F) Histograms of scattering strength values at locations within the shoal (areas inside the dashed box) and in a background region (areas inside the solid box) plotted for comparison. The 735 Hz data is ambient noise limited in background areas due to weak source level and is not shown.
where two contiguous shoal segments are imaged. Estimated target strengths for these two segments are provided in Table 2.4. The target strength estimates at 950 Hz for the 3 data sets are consistent, with a standard deviation of roughly 1 dB.

The approach of this section is not used to estimate target strength at 415 and 735 Hz because the herring are much weaker scatterers at these frequencies, as seen in Fig. 2.5(A) where only the densest shoal population centers stand above background scattering levels. An alternative approach to estimate herring target strength at these lower frequencies is developed and applied in the next section.

### 2.2.5 Frequency dependence of target strength estimated by differencing OA WRS scattering strength images over wide areas

Here, we develop an alternative approach to estimate target strength expected of an individual shoaling herring by differencing pairs of OA WRS scattering strength images acquired at two distinct frequencies over the entire area of the shoal. We apply this to data at 415 and 735 Hz. The approach is illustrated by the conceptual diagram shown in Fig. 2.8. From Eq. (2.5), we observe that scattering strength in areas containing fish increases logarithmically with areal density $n_A$. Here, $f_1$ represents a low frequency, such as 415 Hz, where the target strength for fish is lower than at another frequency $f_2$, such as 950 Hz. The background scattering strength from sources other than fish is expected to be statistically stationary and can be identified by its mean level which is frequency dependent. The total scattering strength measured at any given pixel is a sum of the contribution from fish and other background effects. The difference between the total scattering strength across various pixels at the two frequencies then follows
Figure 2.6: Herring target strength at 950 and 1125 Hz estimated by matching areal fish density in OAWRS and CFFS data acquired after dusk hours of Oct 3. Similar to Fig. 2.4 but for the first contiguous shoal segment over a period of 55-min starting at 18:55 EDT on Oct 3. In (G), the target strength estimates near shoal edges are not accurate because of non-stationarity.
Figure 2.7: Herring target strength at 950 and 1125 Hz estimated by matching areal fish density in OAWRS and CFFS data acquired after dusk hours of Oct 3. Similar to Fig. 2.6 but for the second contiguous shoal segment over a period of 50-min approximately two hours after the first observation shown in Fig. 2.6.
the trend illustrated in Fig. 2.8(B), where at very low fish densities, the scattering strength is dominated by the background reverberation, and at very high fish densities by fish scattering. The difference in the scattering strength at low fish densities therefore provides a measure of the difference in background reverberation. The difference in the scattering strength at high fish densities is equal to the target strength difference for fish at these frequencies. If the target strength at one of the frequencies is known accurately, then the target strength at the other frequency can be obtained.

This approach is implemented for pairs of OAWRS images using 950 Hz as the base frequency. The difference in scattering strength is calculated and plotted for OAWRS data acquired between 22:00 and 22:45 EDT on October 3 in Figs. 2.9(A)-(C) for various frequency pairs. We observe the scattering strength difference in the background is roughly 1 dB between 1125 and 950 Hz, but the fish target strength difference is larger, roughly 7.5 dB. Between 950 and 415 Hz, the background scattering strength difference is roughly 1.5 dB, but the fish target strength difference is more than 17 dB. Between 950 and 735 Hz, no conclusion can be drawn about background scattering because the 735 Hz data was dominated by ambient and nearby shipping noise since the source level for this frequency was lower. The fish target strength difference between 950 and 735 Hz is roughly 7.5 dB. These results are tabulated in Table 2.4.

As the OAWRS imaging frequency increases from 415 Hz to 1125 Hz, the target strength of both fish populations and background levels also increase. From the histograms of Fig. 2.5, the scattering strength increase with frequency is greater for the fish shoals than background levels making it easier to detect fish aggregations at the higher frequency. At 415 Hz, only densely populated fish regions are distinguishable from the background. Fish densities at shoal peripheries are typically too low to be
Figure 2.8: (A) Schematic of scattering strength levels at two distinct frequencies $f_1$ and $f_2$ as a function of local fish density and (B) their difference at a given OAWRS pixel. The scattering strength difference equals the mean background level difference for low fish densities, while at high fish densities, the scattering strength differences equals the fish target strength difference.
Table 2.4: Mean low-frequency target strength estimates. The $T_s^{\text{cor}}$ estimates are obtained by correlating OAWRS with CFFS data along CFFS transect. This approach is only applied to OAWRS data at 950 and 1125 Hz. For the other frequencies, the $T_s^{\text{sc}}$ estimates are obtained by the approach of differencing OAWRS images. The Diff is the expected target strength difference between the given frequency and 950 Hz.

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<th>Diff (dB)</th>
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$^a$ Base target strength at 950 Hz used in the differencing approach
Figure 2.9: OAWRS scattering strength level differences for the indicated frequency pairs as a function of areal fish density in dB re 1m$^2$ for data acquired between 22:00 and 22:45 EDT on October 3. The scattering strength difference at high fish densities equals the target strength difference for the frequency pair shown.

detectable.

The background levels in Figs. 2.9(A)-(C) can be used to derive the minimum detectable fish densities in the OAWRS system at various frequencies. From Fig. 2.5(D), if we require that fish returns stand at least one standard deviation above the background mean to be detectable, then, scattering strength levels that are above roughly -52 dB at 950 Hz would be detectable. This corresponds to a minimum detectable areal density of roughly 0.1 fish m$^{-2}$. The minimum detectable densities at the other frequencies are tabulated in Table 2.5. These are based on scaling the fish densities up or down depending on the background mean scattering strength level at the other frequencies (Fig. 2.9) and also accounting for the target strength differences. These results are consistent with those obtained from analyzing the histograms in Fig. 2.5.
2.2.6 Estimating low-frequency target strength of shoaling herring without calibrated target deployment

Here we present the estimated low frequency target strength of a shoaling herring based on simultaneous OAWRS and CFFS sampling line-transects of large shoals on September 27 and 29, 2006 when no calibrated targets were available to aid in charting. Only minor adjustments of less than 1.5 degrees from the array heading sensors were needed to ensure optimal co-registration between OAWRS and CFFS locations of shoals, which were typically the only landmark available in the absence of the calibrated targets.
Figure 2.11: Herring target strength at 950 Hz estimated by matching the areal density of OAWRS and CFFS data acquired during the early morning hours of September 27. Similar to Fig. 2.4 but for a contiguous shoal 40-min segment starting at 06:10 EDT on September 27.
Figure 2.12: Another example of target strength estimation by differencing OAWRS scattering strength images over wide areas, for shoaling herring imaged by OAWRS on Sep 27. Results are plotted for three frequency pairs, each obtained with 20 consecutive OAWRS images acquired in same track on Sep 27.

The target strength estimates shown here are consistent with those presented for the other days when calibrated targets were available in the main text.

During the early morning hours of September 27, between 06:10 and 06:50 EDT, both OAWRS and CFFS co-registered a massive herring shoal 10-15 km southeast of the OAWRS source, spanning more than 2 km in range and 5 km in azimuth, as shown in Figs. 2.11(A)-(C). A dense layer of herring, shown in Fig. 2.11(D), was found consistently spanning 120-170 m in the water column by simultaneous CFFS line-transects. The resulting target strength estimates at 950 Hz obtained by means of the approach described in Sec. 2.2.4, are plotted in Fig. 2.11(G). The target strength estimates at 415, 735 and 1125 Hz are tabulated in Table 2.4 using the approach described in Sec. 2.2.5 and plotted in Fig. 2.12(A)-(C), where target strength at 950 Hz is used as the base frequency. No conclusion can be made for the target strength difference between 950 and 415 Hz, because fish density is not high enough to make scattering from herring at
415 Hz distinguishable from background levels. The same approaches are also applied to estimate the target strength expected for an individual shoaling herring imaged on September 29 between 07:25 and 07:50 EDT. The results are provided in Table 2.4.

2.2.7 Statistical analysis for low-frequency target strength of herring

Here we describe the statistical approach used to estimate the mean and standard deviation of the low frequency target strength of fish populations imaged concurrently by OAWRS and CFFS. We confine our analysis to regions where both systems image continuous segments of statistically stationary fish populations that span many resolution cells in both the CFFS and OAWRS systems.

Employing Eq. (2.5), let \( TS_i \) be independent estimates of low frequency target strength obtained for each resolution cell after sufficient spatial and temporal averaging in anti-log to eliminate the bias when converted to log units and to insure that the \( TS_i \) are Gaussian random variables [32]. Assuming stationarity, let their means be \( \overline{TS} \) and standard deviations be \( \sigma_{TS} \). Then, given \( N \) independent measurements of the target strength along the CFFS transect, the resulting linear estimator for mean target strength is [95]

\[
\hat{TS} = \frac{1}{N} \sum_{i=1}^{N} TS_i. \quad (2.8)
\]

and the corresponding estimator for the standard deviation of the target strength dis-
The variance of the mean target strength estimate is

\[
\text{Var}(\hat{TS}) = \frac{\hat{\sigma}_{TS}^2}{N} \tag{2.10}
\]

Applying Eq. (2.10), the standard deviation in the mean target strength estimate for the data in Fig. 2.4 is roughly on the order of 0.7-0.8 dB.

### 2.3 Results and Discussion

#### 2.3.1 Measured abundance

Herring areal population densities and abundances are estimated by subtracting the estimated target strength expected of an individual herring from OAWRS scattering strength images as explained in Sec. 2.2.2. Areal fish density is first calculated by applying Eq. (2.5) at each pixel in an OAWRS image. Integrating over the area of a shoal then provides an estimate of its abundance. We illustrate abundance estimation with OAWRS images generated at 950 Hz since these have the best spatial resolution.

Figures 2.4(A)-(C) show the areal fish density for a sequence of instantaneous OAWRS images close to the midnight hours of October 2 where average density within the shoal often exceeds 10 fish m\(^{-2}\). Figure 2.4(H) shows the population over time within the area shown in Figs. 2.4(A)-(C) for consecutive OAWRS images from 23:30 EDT on October 2 to 01:00 EDT on October 3. When no threshold is applied, we simply integrate the densities throughout the area. The total population of roughly 40
million fish includes fish in the shoaling region as well as diffuse fish clouds outside the shoal. Contributions from outside the fish shoal, which may include background from the seafloor, are estimated to account for less than 10% of the total population in the area shown.

To exclude background reverberation, a density threshold is selected to segment shoaling regions where fish scattering is dominant. Figure 2.4(H) shows the total population of fish above and below various density thresholds, 0.1, 0.2 and 0.5 fish m$^{-2}$. The optimal density threshold for segmenting the shoaling region is roughly 0.2 fish m$^{-2}$ since below this threshold populations stay fairly constant as expected for background levels. This threshold also corresponds to a scattering strength of roughly -50 dB which is roughly the value where the scattering strength histograms for background and shoaling regions intersect in Fig. 2.5. The population of fish in the shoaling region varies over time between 30 and 40 million.

The massive shoal shown in Fig. 2.2(A) is comprised of over 240 million fish, with roughly 170 million in the large consolidated shoal and 70 million in the diffuse fish aggregation region. The population of fish in the shoals of Figs. 2.2(B) and (C) are roughly 86 million and 70 million, respectively.

The error in the abundance estimates from OAWRS data presented in this paper is quantified here by the central limit theorem [96]. From Eqs. (2.4) and (2.5), error in $n_{A,oawrs}$ at any pixel is caused by a fixed OAWRS target strength estimate which is the same for all pixels and the OAWRS scattering strength which varies from pixel to pixel. Summing large numbers of pixels reduces the percentage error from scattering strength fluctuations in the population density estimate to negligible values by the law of large numbers, leaving the error to be dominated by that in the OAWRS target strength.
estimate, which for the 950 Hz imagery is consistently between 0.7-0.8 dB throughout the experiment, corresponding to an uncertainty in the abundance estimate of roughly 17 to 20%.

### 2.3.2 Measured low frequency target strength

Following the approach of Sec. 2.2.4 and 2.2.5, the target strength corresponding to the mean scattering cross-section of an individual shoaling herring is estimated as a function of frequency in the 300-1200 Hz range of the OAWRS system from measured scattered returns and measured and modeled transmission loss as shown in Figs. 2.15 and 2.16 and Table 2.4. The target strength data show a consistent dependence in both level and roll-off with decreasing frequency of roughly 20 dB/octave for all measurements, which spanned five shoals on four days. The invariance of the results from shoal to shoal and day to day is consistent with the low measured standard deviations obtained for each shoal by stationary averaging. The very strong roll-off in frequency is consistent with that found just below the resonance peak of a system undergoing damped harmonic oscillation [21].

### 2.3.3 Using measured low frequency target strength to infer swimbladder properties

Air-filled swimbladders typically comprise roughly 5% of fish body volume at neutral buoyancy depth [97]. To remain neutrally buoyant as hydrostatic pressure changes with depth, fish need to regulate the amount of gas in their swimbladders to maintain the 5% ratio [38]. Given this ratio and total fish volume, swimbladder volume can be
determined at any depth as can neutral buoyancy depth if the relationship between pressure and volume is known for the swimbladder. One highly plausible relationship is Boyle’s law [98, 99], where the product of pressure and volume remains constant, which has been demonstrated in the laboratory with a single dead herring [100], but has not been directly confirmed in the wild where it is difficult to make in situ measurements of the physiology of free-ranging fish at depth.

Herring are physostomes, fish with open swimbladders connected to the gut and colon [101]. There are three hypotheses by which herring inflate their swimbladders: (1) by gulping air at the surface [101, 102, 103, 104]; (2) by bacterial fermentation in the gut [101, 105]; and (3) by secretion of gas from the blood stream into the swimbladder [106, 107, 108, 109]. Nøttestad [109] found hypotheses (1) and (2) implausible for the Norwegian spring spawning herring in his study that remain in deep layers, are not observed near the surface, and are not feeding. This led him to suggest hypothesis (3). Nero et al. [68] conducted on site experiments on the northern flank of Georges Bank with spawning herring by adding weights to captured herring until they sank. They concluded “that these herring contained up to at least a three times greater volume of gas than a neutrally buoyant fish at the sea surface,” and arrived at neutral buoyancy depths as great as 60 m from these and low frequency acoustic target strength measurements. They found their results to be consistent with measurements of Pacific herring in Puget sound [105], and suggested hypotheses (2) and (3) as plausible explanations for their observations. Similarly, Fänge [108] supports hypothesis (3) by noting that “The herring lacks obvious gas depository structures (rete mirabile, gas gland), but has relatively high O₂ values in the swimbladder (up to 21.5%) [110], and observations of release of gas bubbles from vertically migrating herring [111] indicate that some gas secretion may occur.”
The conditions of the present experiment were not only similar to those of Nero et al. [68], but also to those of Nøttestad [109] in that the spawning herring were only observed in deep layers and not near the surface where it is unsafe due to predator attack as both Nøttestad [109] and Makris et al. (2009) [4] note, and a vast majority of the herring captured in trawl samples were observed to have no large prey (euphausiids and copepods) in their stomachs. The latter point, however, needs to be tempered because bacterial content was indeterminate in the samples, food resources are plentiful near the seafloor where the herring shoals of the present study were found, and gas production by bacteria in herring stomachs can last more than 90 hours [112] after ingestion. It is typically associated with a delay due to phase lag in bacterial growth [112]. Since the diffusion rate of gas out of the swimbladders of caged herring at fixed depth is also found to be small [38], corresponding to less than a 0.3 dB decrease in target strength per day, herring may maintain bacterial gas for long periods with minimal feeding on large prey. Such feeding is known to increase with gonadal development [113]. These facts suggest that in addition to Nøttestad’s [109] hypothesis (3), hypothesis (2) may also remain highly plausible for our experiment as suggested by Nero et al. [68] for a similar location and season.

Here we compare the estimated low-frequency target strength obtained from experimental data with that derived from Love’s widely used model for resonant scattering from a fish swimbladder [115, 67, 68]. This comparison enables estimates of swimbladder volume to be inferred for the shoaling herring observed in this study. Love models the fish swimbladder as an elongated-spheroidal, viscous, heat-conducting shell which encloses an air cavity with surface tension at the inner surface [115, 67]. This leads to well-understood damped resonance behavior. While the material in and around the swimbladder has more complex elastic composition and structural constraints than that
Figure 2.13: Fork length distributions of most frequently caught species, Atlantic herring, Acadian redfish, Haddock, and Silver hake, from trawls deployed on Georges Bank (Fig. 2.1). The mean fork length of herring is 24.2 cm with a standard deviation 6.8% of the 24.2 cm mean. The equation $L_{TL} = 1.103L_{FL} + 0.01$ [68] is used to convert herring’s fork length to the total length, where $L_{TL}$ and $L_{FL}$ are in cm. The mean fork length of redfish is 26.2 cm with a standard deviation 15% of the 26.2 cm mean. The equation $L_{TL} = 1.033L_{FL} - 0.038$ [114] is used to convert redfish’s fork length to total length. Silver hake’s fork length is the total length.
in the Love model that could lead to more complex scattering, the Love model has been successfully tested in experimental settings where strong resonances have been observed [115, 67, 116, 117, 118, 119, 68], and probably provides an accurate description of the first order physics near resonance. Following Refs. [68] and [120, 121, 119, 122], the swimbladder is modeled as a resonant, air-filled prolate-spheroid with a major-to-minor axis ratio of 5 to 10 and major-axis to fish length ratio of 0.13 to 0.17 based on our trawl samples and CFFS measurements for herring. Only changes in minor-axis are assumed to contribute to swimbladder volume change due to physiological constraints [102, 123, 124, 68]. We use the herring length distribution (Fig. 2.13(A)) measured from trawl samples, and depth distributions determined by CFFS line-transects. The fish weight (W) to length (L) relationship is approximated by a normal distribution with a mean given by an empirically determined length-weight regression (gray line) from length-weight measurement of 1219 herring samples and a standard deviation of ±20% of the mean calculated from the length-weight data (dots) as shown in Fig. 2.14. Properties of modeled fish, such as the flesh density, viscosity, and swimbladder volume at depth are tabulated in Table 2.3. Given these constraints, swimbladder volume or equivalently swimbladder semi-minor-axis is the only unknown variable in the Love model that can lead to a significant change in fish target strength at and below the resonance peak. Swimbladder volume is assumed to vary with pressure according to Boyle’s law [98, 99], from which neutral buoyancy depth can be uniquely determined. Neutral buoyancy depth is then assumed to be a Gaussian random variable with mean and standard deviation determined by least-squares fit between measured and modeled target strength.

The best least square fits between our measured target strength data and the Love model appear in Figs. 2.15 and 2.16(A-D) for five fish shoals imaged by OAWRS on four
Figure 2.14: Atlantic herring length-weight regression calibration. The dots are the length-weight data obtained from the trawl-survey conducted by U.S. National Marine Fisheries Service in conjunction with OAWRS 2006 experiment, and the gray solid line indicates the derived best-fit length-weight regression, which can be expressed as $W = aL^b$, where $W$ is the weight of herring in kg, $L$ is the fork length of herring in cm, $a$ and $b$ are empirical regression parameters. For this trawl dataset, $a = 3.35 \times 10^{-6}$ and $b = 3.35$. 
different days, and consistently show a broad resonance peak with maximum at roughly 1.5-1.7 kHz and swimbladder semi-minor axis of 3 to 5 mm. The model to data match is excellent, with negligible mean-squared error, which is significant because it would not be possible to obtain a good match between the measured frequency dependence and the Love model if the overall level of the measured target strength data had a significant scale factor error that was much larger than measured errors of roughly 1 dB. This consistency gives added confidence to the veracity of both the data and model. As expected from the roughly 20 dB/octave frequency roll-off of the data, the best fit of the model is for a resonance just above the highest frequency data point available in the current set of measurements. The model fits of Fig. 2.15 can be interpreted with the aid of Table 2.6 which shows the volume and corresponding swimbladder minor axes radii given the measured herring length distributions, as well as possible neutral buoyancy depths based on Boyle’s Law. Neutral buoyancy depths were found to correspond to roughly half the mean shoal depth given the measured herring length distribution, spanning 20-34 cm with a mean of 26.7 cm, and depth distribution in a layer between 120-190 m from trawl and CFFS sampling.

The localized measurements of Nero et al. [68] for herring target strength data in the 1.5 to 5.0 kHz range show target strength levels consistent with those found in our best fit curve for frequencies above roughly 2.2 kHz. This can be seen by noting their best-fit neutral buoyancy depth curve (50 m) falls within 1-2 dB of ours for all examples above roughly 2.2 kHz for the measured fish depth, length and density distributions of this study. The Love-model based neutral buoyancy depths and resonances found here are also consistent with those measured by OAWRS in 2003 south of Long Island, NY, in scattering from shoals that evidence suggest were also Atlantic herring [3, 125]. Arbitrarily constraining the neutral buoyancy depth to be near the surface, say at 4 m
Figure 2.15: Experimentally determined low-frequency target strength corresponding to the average scattering cross-sections of shoaling herring distributed between 120 to 185 m (Fig. 2.4), imaged with the OAWRS system from 23:30 EDT October 2 to 01:00 EDT on October 3 at 415, 735, 950, and 1125 (circles) with standard deviations (error bars). Comparison with Love-model mean target strength for shoaling herring, with physical parameters tabulated in Table 2.3, of different swimbladder semi-minor axes over the shoals depth distributions (lines). The best least-squares fits shown are obtained only using target strength estimates of each shoal (Table 2.4) with standard deviations less than 3 dB. Arrows indicate the target strength uncertainties due to potential masking from background scattering (Sec. 2.2.5) for given frequencies. The best fit means and standard deviations of inferred swimbladder volume, swimbladder semi-minor axes, and corresponding neutral buoyancy depths of each shoal were tabulated in Table 2.6.
Figure 2.16: Same as Fig. 2.15, but for (A) Shoaling herring, distributed between 135 to 175 m, imaged with the OAWRS system from 18:55 to 19:50 EDT on October 3; (B) Shoaling herring, distributed between 120 to 175 m, imaged with the OAWRS system from 22:00 to 22:45 EDT on October 3; (C) Shoaling herring, distributed between 120 to 185 m, imaged with the OAWRS system from 06:10 to 06:50 EDT on September 27 (Fig. 2.11); (D) Shoaling herring, distributed between 150 to 180 m, imaged with the OAWRS system from 07:25 to 07:50 EDT on September 29.
Table 2.6: Experimentally inferred means and standard deviations of swimbladder volume, $v_{sb}$ and $\sigma_{v_{sb}}$, semi-minor axis, $a_z$ and $\sigma_{a_z}$, over the depth distributions of the shoals, and corresponding means and standard deviations of neutral buoyancy depth, $d_{nb}$ and $\sigma_{d_{nb}}$, where neutral buoyancy depth is restricted to water column depths of 0-200 m in the least squares fit. All three parameters are assumed to be Gaussian random variables completely characterized by their respective means and standard deviations.

<table>
<thead>
<tr>
<th>Date</th>
<th>Time&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Layer Depth (m)</th>
<th>$v_{sb}$ (ml)</th>
<th>$\sigma_{v_{sb}}$ (ml)</th>
<th>$a_z$ (mm)</th>
<th>$\sigma_{a_z}$ (mm)</th>
<th>$d_{nb}$ (m)</th>
<th>$\sigma_{d_{nb}}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct 2</td>
<td>23:30-01:00</td>
<td>120-185</td>
<td>3.41</td>
<td>0.41</td>
<td>4.4</td>
<td>0.26</td>
<td>82</td>
<td>11</td>
</tr>
<tr>
<td>Oct 3</td>
<td>18:55-19:50</td>
<td>135-175</td>
<td>4.27</td>
<td>0.31</td>
<td>4.9</td>
<td>0.18</td>
<td>108</td>
<td>9</td>
</tr>
<tr>
<td>Oct 3</td>
<td>22:00-22:45</td>
<td>120-175</td>
<td>3.62</td>
<td>0.38</td>
<td>4.5</td>
<td>0.24</td>
<td>85</td>
<td>10</td>
</tr>
<tr>
<td>Sep 27</td>
<td>06:10-06:50</td>
<td>120-185</td>
<td>1.61</td>
<td>1.49</td>
<td>3.0</td>
<td>0.96</td>
<td>31</td>
<td>48</td>
</tr>
<tr>
<td>Sep 29</td>
<td>07:25-07:50</td>
<td>150-180</td>
<td>3.98</td>
<td>0.21</td>
<td>4.8</td>
<td>0.12</td>
<td>107</td>
<td>6</td>
</tr>
</tbody>
</table>

<sup>a</sup> Time periods during which both OAWRS and CFFS systems simultaneously co-registered contiguous shoal segments. The time is in EDT.

and using Boyle’s law for fish at 120 to 190 m depth yields a corresponding 3.7 kHz resonance and leads the Love model to a herring target strength 20-30 dB lower (Fig. 2.17) than that measured by the OAWRS system. Such low target strengths are not only inconsistent with the target strength measurements of Figs. 2.15 and 2.16, and Nero et al. [68], they also violate conservation of energy since the corresponding transmission losses required to match our measured sound pressure levels returned from the fish shoals would have to be less than those found in even a perfectly reflecting waveguide without any medium attenuation, i.e. the waveguide would have to somehow add energy to the source signal by two to three orders of magnitude. This can be seen in Fig. B.1 which shows the measured transmission loss to be within 2-3 dB of that found in a perfectly reflecting waveguide without attenuation.

A wide distribution of swimbladder volumes and corresponding neutral buoyancy depths within any shoal is likely and could potentially unify the various existing data sets by superposition, with larger swimbladder volumes dominating at the lower end of
Figure 2.17: (A) An example of Love-model target strength corresponding to the average scattering cross-section of an individual for mixed species content and swimbladder semi-minor axis shown. Comparison of Love-model target strength with experimentally determined mean target strength estimates of shoaling herring, distributed between 135 to 175 m, imaged with the OAWRS system from 18:55 to 19:50 EDT on October 3. Presence of redfish in trawl-determined percentage (dashed gray line) has negligible effect on best fit target strength compared to herring alone (solid black line), while including an unrealistic percentage of redfish yields far worse fits (dashed black line). The silver hake, found at shallower water depth (< 75 m) in trawls, have resonance peak above 3 kHz (dash-dot curve) making their contribution negligible. Herring target strength with a resonance frequency at 3.7 kHz (dashed gray line) based on Love’s model using length and depth distribution obtained from CFFS and trawl measurements is found to be neutrally buoyant at 4 m, and is 20-30 dB lower than those measured by the OAWRS system. (B) Same as (A) but for shallower herring neutral buoyancy depths. Arrows indicate potential target strength uncertainties for given frequencies.
the possible resonance spectrums and smaller volumes dominating the higher end. None of the systems used in the field so far, however, could test this since it would require acquisition of simultaneous data both well below and well above all contributing resonant frequencies. Since low frequency target strength measurements near resonance are far more sensitive to small changes in swimbladder minor-axis or volume than at much higher CFFS frequencies, such lower frequency measurements have the potential to resolve the in situ swimbladder volumes of wild herring at depth with much greater accuracy. The measurements of target strength for herring as a function of depth reported by Ona [93] at 38 kHz, for example, show standard deviations of 8-10 dB and minimal 3 to 5 dB reduction from the surface to 300 m [124] which falls well within these standard deviations. These standard deviations significantly exceed the 5.6 dB expected purely from stationary Gaussian field fluctuations by the Central Limit theorem [32], shown to apply to scattering from fish of random orientation by Dahl and Mathisen [126]. This may suggest a bias in fish orientations or a wide spread in swimbladder volumes in the Ona [93] data, which could easily include the span of volumes measured here and in Nero et al. [68]. Fässler et al. [100] have suggested that projected dorsal area variations exhibit greater depth variation than the mean target strength measurements of Ona [93]. We find the Ona [93] mean trend in depth to be consistent with that expected from Boyle’s Law since at their frequencies $ka$, the product of acoustic wavenumber to the semi-minor-axis of the swimbladder, is typically less than unity and target strength is no longer proportional to projected area as in the large $ka$ limit.

Evidence from the extensive OAWRS, CFFS and trawl surveys conducted during OAWRS 2006 offers no plausible alternative to herring as the primary constituent and source of scattering in the shoals imaged by OAWRS. Consideration of the effect of two other species, however, also present in many trawls but in far fewer numbers (Table A.1),
still provides useful perspective. These are silver hake and Acadian redfish. While silver hake were found at shallower depths than herring, their much shorter lengths (Fig. 2.13) lead to individual target strengths more than 20 dB lower than those measured at the location of the herring shoals at OAWRS frequencies (Fig. 2.17). Given that (1) both CFFS and trawl samples found silver hake in considerably lower areal densities than herring, (2) their individual target strengths are roughly 20 dB lower, and (3) OAWRS transmission loss was much greater in the shallower layers where silver hake resided as part of the experimental design, contributions from silver hake can be ruled out as a plausible explanation for shoals imaged by OAWRS. As shown in Fig. 2.17(A), the presence of redfish in the maximum percentages determined by trawl has a negligible effect on the best fit target strength compared to that of modeling herring alone (Figs. 2.15 and 2.16). This should be expected because the low frequency dependence of background returns when no shoals are present is effectively negligible compared to the dependence when shoals are present (Fig. 2.5) and simultaneous trawls showed the shoals to be overwhelmingly comprised of herring but the background to yield negligible amounts of herring (Table 2.4, Fig. A.1). So, even though redfish are physoclist species with trawl sample lengths (Fig. 2.13) typically greater than those of the herring, the measured areal densities of redfish are too low to have a significant impact on the average scattering cross-section measured in the observed shoals. Including greater percentages of redfish in the modeling, or including measured percentages and shallower neutral buoyancy depths for herring leads to far worse fits and high mean-square error when matched with the data as shown in Fig. 2.17B.
2.3.4 Space, Time and Frequency Dependencies

From another perspective, since the spatio-temporal population distributions of the large shoals versus background levels are consistent among OAWRS, CFFS, and trawls, as shown here and in Ref. [4], the spatio-temporal distributions of silver hake, redfish or any other contaminant species would have to consistently follow those of herring if they were a major contributor to OAWRS returns, which is both implausible and contrary to trawl and CFFS data as well as the frequency dependence of OAWRS returns in shoals versus background (Fig. 2.5). Also, neither our trawl nor CFFS data show any evidence of shallow fish layers that could account for the prominent OAWRS returns that co-registered in space and time with the deep shoals measured by simultaneous trawl and CFFS sampling. Indeed, considering experimental causality, it was consistently necessary for OAWRS to first find and then direct trawl and CFFS vessels to the locations of these deep shoals because they are so difficult to find with conventional methods given the fact they occupy areas many orders of magnitude smaller than the wide areas over which they may roam.

2.4 Summary

The low frequency target strength of Atlantic herring (*Clupea harengus*) is estimated from experimental data acquired from shoaling herring in the Gulf of Maine during the Autumn 2006 spawning season in the 300 to 1200 Hz range using simultaneous ocean acoustic waveguide remote sensing (OAWRS), conventional fish finding sonar (CFFS) and trawl surveys. The target strength expected of an individual is found
to have a strong nonlinear dependence on frequency consistent with resonant scattering from an air filled swimbladder given measured fish length, depth distributions, and experimentally inferred swimbladder volumes based on Love’s model, which indicate the herring remain negatively buoyant in layers near the seafloor for extended periods.

The OAWRS system used in this study employed an instantaneous imaging diameter of 100 km with regular minute-to-minute updates enabling unaliased monitoring of fish populations over ecosystem scales. This included detection and imaging of shoals of Atlantic herring containing hundreds of millions of individuals, as confirmed by concurrent trawl and CFFS surveys that were directed to the shoals’ locations by OAWRS. High spatial-temporal co-registration was found between herring shoals imaged by OAWRS and concurrent CFFS line transects.
Chapter 3

Effects of multiple scattering, attenuation and modal dispersion in waveguide sensing of fish

An ocean acoustic waveguide remote sensing system [9, 4, 5, 3, 127] can instantaneously image fish populations and continuously monitor their behaviors over continental shelf-scale regions. It utilizes the capacity of the ocean to act as a waveguide where sound can propagate long ranges via trapped modes. The imaging frequency typically employed by the system ranges from several hundred Hz to a few kHz, while its imaging diameter ranges from tens to hundreds of kilometers, determined by the source level, pulse repetition interval, and receiving array aperture [80, 2, 9, 5, 81].

The waveguide remote sensing system comprises of a vertical source array that transmits broadband pulses radiated with azimuthal symmetry in the horizontal to ensonify targets of interest, and a towed horizontal hydrophone array that receives the scattered returns, as shown in Fig. 3.1(a). The data measured by the receiver array are beamformed to determine the azimuths of the scattered returns, and then matched filtered and charted in range. The matched filter is applied to achieve high spatial
resolution in range localization and to optimize signal-to-noise ratios by exploiting the coherent gain of broadband signals with large time-bandwidth products much greater than one [128, 37, 129].

Numerous issues must be considered when inferring the areal population densities, spatial distributions, or mean target strengths of fish populations from their matched filtered scattered intensities measured with the waveguide remote sensing system. (1) Large numbers of individuals, ranging from tens to hundreds of thousands, are often present within each resolution cell of the imaging system so that the scattered fields from the distributed group can either combine coherently or incoherently at the receiver [10, 22, 23, 24]. (2) When large numbers of scatterers are present in a group, multiple scattering, as well as attenuation through the fish group may affect the measured scattered levels [23, 25, 26, 27, 28, 29, 30, 31, 10]. (3) The acoustic field propagates long ranges in an ocean waveguide through multiple paths as waveguide modes. Modal dispersion due to the slowly propagating high order modes that arrive later in time than the low order modes lead to delayed returns that affect the measured scattered levels over the range extent of the fish group. (4) The broadband transmissions undergo spreading and absorption losses over range and scintillate in both time and space due to dynamic ocean processes, such as internal and surface waves, as well as bathymetric variations [32, 33, 34]. (5) As with any active imaging system, scattered returns from fish and other targets are measured in time and then charted to range by multiplying the measurement time with a charting speed. Modal dispersion and waveguide scintillation affect the charting speed and hence the localization accuracy of targets in a waveguide [35, 36]. (6) The matched filter operation is a coherent process that involves correlating the scattered field with the transmitted waveform [10, 37, 38]; however, the scattered returns from distributed groups are often assumed to be incoherent, and the spatial
Figure 3.1: (a) Geometry of the bistatic acoustic imaging system in an ocean waveguide. (b) 3D spatial configuration of a large herring group containing 7831 individuals uniformly distributed within a volume similar to an ellipsoid that has axes dimensions of $L_x = 100$ m, $L_y = 125$ m, and $L_z = 33.33$ m, but with cross-range and depth extents cut at $\pm 50$ m and $\pm 10$ m respectively to the center of the herring group. The plotted volume has dimensions of $L_x = 100$ m, $L_y = 100$ m, and $L_z = 20$ m. (c) Modeled broadband two-way transmission loss $TTL_W$ calculated using Eq. (3.9) from a source array to potential fish locations and from the fish locations back to the receiver array center is shown as a function of fish range and depth in the Gulf of Maine environment for a source waveform of 50 Hz bandwidth centered at $f_c = 950$ Hz. (d) The $TTL_W$ obtained by first averaging the broadband two way propagated acoustic intensities over a 20 m thick fish layer centered at $z_s = 150$ m depth and then taking the log transform following Eq. (3.11). The error bars indicate one standard deviation in the broadband $TTL_W$ over the depth layer of the fish.
resolution from matched filtering in the waveguide [4, 3, 9, 5] is assumed to be the same as that for discrete targets in free space with coherently scattered returns.

To address these issues, a numerical Monte-Carlo model is developed to determine the statistical moments of the broadband matched filtered scattered field from a three-dimensional (3D) random spatial distribution of fish with random sizes and of random species in a random range-dependent ocean waveguide. The model includes multiple scattering from the fish group and employs the complex scatter function to account for attenuation and dispersion due to scattering from an individual fish. The model uses a range-dependent acoustic propagation model [39] based on the parabolic equation to simulate propagation in a range-dependent ocean waveguide. By incorporating randomness in the waveguide environment and the dynamic scatterer group, the model can account for statistical fluctuations typically present in the data. The model simultaneously analyzes the fully scattered field that includes multiple scattering from a fish group extended over multiple range resolution cells of the imaging system without segregating fish from individual cells, since the matched filter is applied to automatically localize the scatterers in range.

The model is applied to (1) examine population density imaging of shoaling Atlantic herring in the 2006 Gulf of Maine Experiment [4, 9] (GOME’06) during their fall spawning season near the northern flank of Georges Bank, and (2) investigate wide-area imaging of sparse aggregation of ground fish species, such as Atlantic cod, in Ipswich Bay continental shelf environment with an ocean acoustic waveguide remote sensing system. It is shown theoretically that high-resolution population density imaging of fish can be achieved in the random range-dependent ocean waveguide with less than 10% error.

The numerical model developed in this paper is the only model to simultaneously
analyze multiple scattering and attenuation from fish groups imaged at long ranges in an ocean waveguide with a broadband pulsed system employing the waveguide Green’s function for propagation and the matched filter to localize scatterers in range. There are several useful theoretical models for analyzing and predicting the scattered levels from objects in an ocean waveguide [40, 41], however, they typically do not consider multiple scattering within the group. Previous models [27, 29, 26, 28, 10] that include multiple scattering from fish groups are restricted to direct-path imaging systems since they employ the free-space Green’s function, and are implemented for time harmonic signals. These models consider very small schools on the order of the acoustic wavelength with a limited number of individuals. They focus on scenarios where coherent effects in multiple scattering, such as resonance shifts and sub- and super-resonance local maxima, can be significant. Here we focus on fish groups that extend tens to thousands of times the wavelength of the waveguide imaging system, where we show that the incoherent intensity dominates the scattered returns and resonance shift effects are negligible or absent. Scattering from fish groups in a waveguide is modeled in Refs. [130, 9, 5] for herring and in Ref. [5] for several other swimbladder bearing fish, such as Alaskan pollock, Peruvian anchovy, Argentine hake, Barents Sea capelin, and Southern blue whiting, applying the single scatter assumption and compared to seafloor reverberation.

The Monte-Carlo model developed here can also be applied to analyze detection and imaging of other groups of discrete scatterers in a waveguide where multiple scattering may be significant, such as bubble clouds, swarms of AUVs, and pods of dolphins or whales.
3.1 Theory

3.1.1 Numerical Monte-Carlo model for the statistical moments of the broadband matched filtered fully scattered field from a 3D random distribution of scatterers that includes multiple scattering in a random range-dependent ocean waveguide

Here, we describe the numerical Monte-Carlo model used to simulate the statistical moments of the broadband matched filtered fully scattered field that includes multiple scattering from a 3D random spatial distribution of random scatterers in a random range-dependent ocean waveguide. The model can be applied to calculate the total scattered field from a group of scatterers in any direction in a waveguide including the forward. The theoretical formulation in Appendix D follows the approach of Ref. [10] but is implemented here for a random range-dependent ocean waveguide by employing the waveguide Green’s function. Model verification is provided in Sec. 3.1.3.

Consider the general bistatic geometry for the problem illustrated in Fig. 3.1(a), where the origin of the coordinate system is placed at the air-water interface with the positive \( z \) axis pointing downward. A vertical source array centered at \( \mathbf{r}_0 = (x_0, y_0, z_0) \) and a horizontal receiver array centered at \( \mathbf{r} = (0, 0, z) \) are used to image a group of scatterers centered at \( \mathbf{r}_C = (x_C, y_C, z_C) \) in the far-field of both the source and receiver arrays. The group is distributed over a three dimensional volume where each \( p \)th scatterer is located at \( \mathbf{r}_p = (x_p, y_p, z_p) = (\rho_p, \phi_p, z_p) \), where \( \rho_p^2 = x_p^2 + y_p^2 \). The source transmits waveform \( q(t) \) with Fourier transform \( Q(f) \) and bandwidth \( B \).

For a specific realization of the ocean environment and scatterer group, the time-
dependent broadband matched filtered fully scattered field $\Psi_s(t_M)$ that includes multiple scattering is calculated using Eq. (D.6). By randomizing both the waveguide Green’s function and spatial distribution of scatterers according to known statistical properties, many independent realizations of the scattered field are generated. The statistically coherent and incoherent scattered intensities from the scatterer group are then obtained from the sample squared mean $|\langle \Psi_s(t_M) \rangle|^2$ and sample variance $\text{Var}(\Psi_s(t_M))$, respectively. The waveguide is randomized by incorporating linear internal waves along the range-dependent acoustic propagation paths from source to scatterer locations and from scatterer locations to the receiver as outlined in Sec. 3.2.1. The scatterer group is randomized through both the scatter functions and spatial locations of individuals in the group.

The acoustic field scattered from one scatterer to another, as formulated in Eq. (D.4), can be solved using either matrix inversion or an iterative approach [10]. The matrix inversion approach requires an $N \times N$-matrix to be inverted, which makes this approach computationally cumbersome for large groups [27, 10] of scatterers. For groups of scatterers with low densities and small target strength, the iterative approach, with each iteration simulating an increasingly higher order of scattering, converges fairly quickly therefore making it an efficient approach to calculate the fully scattered intensities including multiple scattering from groups of scatterers. In Sec. 3.4, all examples shown for herring aggregations with low densities, we apply the iterative approach. Computational constraints limit the number of scatterers that can be simulated in a reasonable amount of time. The multiply scattered returns from a maximum of 40000 individuals are simulated, producing 1.6 billion multiple scattering interactions for each iteration of scattering. The iterative approach will only converges under the condition that the spectral radius of the matrix $S^k \cdot G^k$ must be smaller than unity [27], where the matrix
$S^k$ and $G^k$ are defined as

$$S^k_{p,n} = \frac{S_p(\Omega_{qp}, \Omega_{pn}, k)}{k}$$

(3.1)

and

$$G^k_{p,n} = \begin{cases} G(r_n|r_p, f) & n \neq q \\ 0 & n = q \end{cases}$$

(3.2)

respectively, where $S_p(\Omega_{qp}, \Omega_{pn}, k)$ is the scatter function for the $p$th scatterer defined in Appendix D, and $G(r_n|r_p, f)$ is the inter-fish acoustic propagation in Eq. (D.4), whose definition is given by Eq. (3.12). For a volume of scatterers of spherical shape, using Gerschgorin’s circle theorem [131] to estimate the spectral radius, we obtain the convergence criterion for the iterative approach to be

$$\frac{3 |\mathcal{S}_p| N}{2 R} = 2\pi n_f R^2 |\overline{\mathcal{S}}_p| < 1$$

(3.3)

where $R$ is the radius of the spherical volume, $n_f$ is the volumetric density of the scatterer group, and $\overline{\mathcal{S}}_p$ is the ensemble averaged scattering magnitude of the group. This implies that for sufficiently high volumetric density or large scattering magnitude or both, the iterative approach may not converge. Since cod are bigger fish with higher target strength than herring, for all examples shown in Sec. 3.5, the matrix inversion approach is applied to calculate the fully scattered intensities including multiple scattering from groups of cod.

### 3.1.2 Modeling the complex scatter function distribution and 3D spatial distribution for a group of fish

At frequencies ranging from a few hundred Hz to several kHz, the air-filled swimbladders of most swimbladder-bearing fish are the primary source of scattering. A fish
swimbladder is modeled here as a damped air-filled prolate spheroid, as discussed in Appendix E. Fish swimbladders are acoustically compact at the operating frequencies of a typical waveguide remote sensing system because they are much smaller than the acoustic wavelength so that the scattered field from each individual fish is radially symmetric. Given a fish of known swimbladder volume, its complex scatter function is calculated using Eq. (E.1).

To model the complex scatter function distribution for fish in a group distributed over a given water depth, we first assume the fish fork length $L$ follows a Gaussian distribution with a mean and standard deviation obtained from available in-situ trawl sample measurements [9]. The distribution of fish body volume is next calculated from the fork or total length distribution by applying the empirically determined non-linear weight-length regression, Eq. (E.5). By assuming fish in the group are neutrally buoyant at the same depth, and fish swimbladder volume typically comprises roughly 4% to 5% of fish body volume when they are neutrally buoyant, the swimbladder volume for each individual fish at any depth $z$ can be calculated by assuming Boyle’s law, as discussed in Appendix E. The distribution of complex scatter functions for fish in the group is then obtained from their swimbladder volume distribution via Eq. (E.1).

To approximate the 3D spatial configurations fish groups adopt when they form either large shoals or small dense schools, we consider two models in the examples shown in Secs. 3.1.3, 3.4 and 3.5. In the first model, we assume a fully random 3D spatial configuration for fish in a group specified by its volumetric density $n_V$, which is a function of range, depth and cross-range. In the second model, [29, 26] the fish locations in a group are only partially random and their 3D spatial configuration is derived from a basic cubic cellular unit. In this model, each fish is first arranged to be in the center
of a cubic cellular unit with all closest neighbors spaced a mean distance $d$ determined by the volumetric density,

$$d = \left( \frac{1}{n_V} \right)^{1/3}.$$  \hspace{1cm} (3.4)

As fish swim, the distances between these closest neighbors will vary from the mean value $d$ over time. To account for this variation, individual fish locations are next randomly varied from their mean positions according to a Gaussian distribution with a standard deviation $\sigma_d$ assumed to be 30% of the mean inter-fish distance $d$. In both models, the fish areal density $n_A$ at a given horizontal range is obtained by integrating the volumetric densities $n_V$ over the depth distribution of the fish layer at that range.

3.1.3 Model verification

Here, the model developed in Sec. 3.1.1 and Appendix D is verified by comparison with examples provided in Refs. [27] and [29] where the time-harmonic scattered intensities that include multiple scattering are calculated for a small group of fish containing 13 individuals at frequencies ranging from tens of Hz to 5 kHz, imaged by a monostatic direct-path system in an iso-speed lossless non-random environment, where the free-space Green’s function is valid. All fish are identical with 40 cm total length and equivalent spherical swimbladder radius $a = 2$ cm. The school is centered at $z = 50$ m depth and all fish are assumed to be neutrally buoyant at 50 m depth.

The time-harmonic fully scattered field $\Phi_s(r, f)$ at frequency $f$ that includes multiple scattering for each realization of the fish group is calculated using Eqs. (D.1) to (D.4) where $B(\phi_p) = 1$ for all fish in the group. The statistically coherent and incoherent fully scattered intensities at frequency $f$ that include multiple scattering are obtained from
the sample squared mean $|\langle \Phi_s(r, f) \rangle|^2$ and sample variance $\text{Var}(\Phi_s(r, f))$, respectively, derived from 10 independent Monte-Carlo simulations. The total scattered intensity is the sum of the two [10], $\langle |\Phi_s(r, f)|^2 \rangle = |\langle \Phi_s(r, f) \rangle|^2 + \text{Var}(\Phi_s(r, f))$.

The coherent, incoherent and total school target strengths denoted by $TS_{coh,s}(f)$, $TS_{incoh,s}(f)$, and $TS_{tot,s}(f)$ respectively are calculated using

$$TS_{coh,s}(f) = 10 \log_{10} \left( |\langle \Phi_s(r, f) \rangle|^2 \right) - SL(f) + TTL_{FS}(r_s, f|r_0, r),$$

$$TS_{incoh,s}(f) = 10 \log_{10} \left( \text{Var}(\Phi_s(r, f)) \right) - SL(f) + TTL_{FS}(r_s, f|r_0, r),$$

and

$$TS_{tot,s}(f) = 10 \log_{10} \left( \langle |\Phi_s(r, f)|^2 \rangle \right) - SL(f) + TTL_{FS}(r_s, f|r_0, r),$$

where $SL(f)$ is the source level at frequency $f$, and $TTL_{FS}(r_s, f|r_0, r) = 10 \log_{10}(1/|r_s - r_0|^2) + 10 \log_{10}(1/|r - r_s|^2)$ is the free-space two-way transmission loss from the source to the school center $r_s$ and from the school center to the receiver, respectively.

The fish are distributed in a 3D volume following the partially random lattice configuration described in Sec. 3.1.2 of this article and also in Ref. [29]. Each fish position is randomized with a standard deviation of $\sigma = 4$ cm about the mean inter-fish spacing $d$ set to be (a) one fish body length, $d = 40$ cm, (b) quarter fish body length, $d = 10$ cm, and (c) four times the fish body length, $d = 1.6$ m, respectively in Fig. 3.2. The coherent, incoherent and total school target strength spectra are compared to the estimated $TS_{est,s}(f)$ spectrum obtained from

$$TS_{est,s}(f) = TS(f) + 10 \log_{10} N,$$

which assumes single scattering is valid, where $TS(f) = 10 \log_{10}(|S(f)/k|^2)$ is the target strength of a single fish. Here, the azimuthal angle of ensonification is kept constant.
and the school target strengths are estimated from backscattered intensities. This differs from the examples shown in Fig. 4 of Ref. [29] where the azimuthal angle of ensonification was randomly varied between 0 and 360 degree for each realization.

The school target strength spectra plotted in Fig. 3.2 are consistent with those shown in Ref. [29]. At the mean inter-fish spacing of one body length (Fig. 3.2(a)), the resonance frequency of the school is lower than the single fish resonance frequency. The scattering is dominated by the coherent intensity near resonance, while the incoherent intensity becomes important above 1 kHz. When the mean inter-fish spacing is reduced to a quarter body length (Fig. 3.2(b)), the school resonance frequency shifts to a frequency much lower than that of the single fish, consistent with the findings in Fig. 8 of Ref. [27]. The scattering is dominated by the coherent intensity for the most part below 5 kHz. When the mean inter-fish spacing is increased to four times the fish body length (Fig. 3.2(c)), there is negligible shift in the school resonance frequency compared with that of the single fish. A noticeable sub-resonance local maxima appears at about 100 Hz, and many more local maxima appear at frequencies above resonance. In general, the model developed here reproduces the key features and results reported in Refs. [26, 27, 29]. The slight differences in the school target strength local maxima at frequencies above resonance is mainly due to the fact that the azimuthal ensonification angle was not varied across the realizations here.

### 3.2 Modeling the 2006 Gulf of Maine Experiment

During GOME’06 [9, 4], an ocean acoustic waveguide remote sensing system was employed to image and study the abundance and diurnal behavior of shoaling Atlantic
Figure 3.2: Effect of varying inter-fish spacing on the school target strength spectra for a small fish group containing 13 individuals. The mean inter-fish spacings are (a) one fish body length, $d = 40$ cm, (b) quarter fish body length, $d = 10$ cm, and (c) four times the fish body length, $d = 1.6$ m. The coherent, incoherent, total and estimated school target strength spectra are calculated via Eqs. (3.5) to (3.8) respectively.
herring populations during their autumn spawning season along the northern flank of Georges Bank over a two week period. The 100 km imaging diameter of the waveguide remote sensing system for a measurement time period of roughly 75 s enabled both massive herring shoals extending tens of kilometers and estimated to contain hundreds of millions of individuals, as well as smaller groups extending roughly 50 to 200 m and comprising fewer than 1 million fish to be instantaneously imaged. The images were updated every 75 s to provide continuous monitoring of fish activity for many hours each day. Simultaneous measurements made using an ultrasonic conventional fish-finding sonar provided the local volumetric densities and depth dependence of the fish distributions [4, 9]. Detailed descriptions of the experimental set-up, data analysis, and results showing wide-area population density images of herring shoals are provided in Refs. [9, 4], along with supporting oceanographic, biological and environmental information.

Instantaneous wide-area images of the ocean environment were formed from broadband linear frequency modulated transmissions of $\tau = 1$ second duration and $B = 50$ Hz bandwidth with varying center frequencies in the 300 to 1200 Hz range. The time-bandwidth product of the signals transmitted was $\tau B = 50 \gg 1$ so that the matched filter or pulse compression [128, 37, 129] could be employed to reduce the horizontal range resolution from $\Delta \rho = c_{chart}\tau/2 = 750$ m to $\Delta \rho = c_{chart}/2B \approx 15$ m, a factor of 50 improvement. The resulting 15 m horizontal range resolution of the waveguide remote sensing system is comparable to the horizontal imaging diameter of the conventional ultrasonic fisheries echosounder of roughly 20 m at 200 m water depth. The waveguide system’s angular resolution $\beta(\phi_p)$ at azimuthal angle $\phi_p$ depend on the center frequency and array aperture used for beamforming (see Table I in Ref. [9]). It’s resolution footprint is dependent on both range and bearing from the receiving array, given by $A_{res}(\rho_p) = \rho_p\Delta \rho \beta(\phi_p)$. 79
Incoherent averaging over 3 to 5 consecutive instantaneous wide-area images at a given frequency band was used to reduce the standard deviations and minimize the effects of noise in the images [4, 9]. A single scattering assumption was employed to infer fish areal population density from the averaged scattered intensity level at a given resolution cell, after correcting for source level, a two-way transmission loss term obtained by averaging the expected waveguide Green’s function over the depth extent of the fish layer, the resolution footprint of the imaging system, and the low frequency target strength obtained from the mean scattering cross-section of a herring individual. The latter was estimated at each imaging frequency band by local calibration with simultaneous measurements from a conventional fish-finding sonar [9].

Here, we implement the rigorous theoretical model described in Sec. 3.1 to prove the assumptions and approaches employed in Refs. [9, 4, 5] for inferring herring areal population densities and abundances from wide-area scattered intensity images acquired with the waveguide remote sensing system. The assumptions include (1) the incoherent intensity dominated the scattered returns from the fish group, (2) the single scattering assumption is valid, (3) negligible attenuation and dispersion effects on the incident field from forward propagation through the fish group, (4) multi-modal waveguide dispersion is negligible in the scattered field from the fish group, (5) the minimum mean water column sound speed is used as an optimal sound speed to chart the scattered field measured as a function of time to range, and (6) negligible degradation in the range resolution and in the scattered intensity level after matched filtering with the source waveform in the ocean waveguide.
3.2.1 Modeling acoustic propagation in the random range dependent Gulf of Maine

Long-range broadband acoustic propagation through a random range-dependent ocean waveguide is calculated using the parabolic equation based range-dependent acoustic propagation model, RAM [39]. This waveguide propagation model takes into account the environmental parameters, such as the range-dependent bathymetry, seafloor geo-acoustic properties, the dynamic water-column sound speed profiles, and the locations and depths of the source and receiver arrays. It provides the complex waveguide Green’s function $G_W$, calculated at discrete frequencies over the signal bandwidth as a function of range and depth.

The acoustic fields propagated from the source array to the fish locations and from the fish locations to the receiver array center in Eqs. (D.2) and (D.4) are calculated separately. For the source array, the model coherently sums the acoustic fields from each element of the array for the field incident on the fish group. The source array of the waveguide remote sensing system has the effect of beaming the acoustic intensity preferentially into the low order modes of the ocean waveguide which can propagate long ranges with less attenuation than the high order modes.

Following the approach of Ref. [33], linear internal waves are simulated in the waveguide propagation model by updating the water-column sound speed profile every 500 m in range corresponding to the correlation length scale of internal waves in continental shelf environments [132, 34]. The profiles are randomly selected from over 200 experimentally measured sound speed profiles acquired during the course of GOME’06, shown in Fig. 3 of Ref. [9]. This approach simulates the effect of internal waves in
randomizing the acoustic field in the waveguide.

As an example, the broadband two-way transmission loss ($TTL$) in the range-dependent Gulf of Maine environment simulated stochastically with the waveguide propagation model is illustrated in Fig. 3.1(c). The $TTL_W$ at each potential fish location $r_p$ is calculated using

$$TTL_W(r_p, f_c | r_0, r) = 10 \log_{10} \left( \frac{(4\pi)^2}{E_0} \int_{f_c-B/2}^{f_c+B/2} |Q(f)|^2 |G_{W,s}(r_p|r_0, f)|^2 |G_{W,r}(r|r_p, f)|^2 df \right),$$

where $G_{W,s}(r_p|r_0, f)$ is the waveguide Green’s function describing the acoustic field propagation from the 10-element source array centered at $r_0$ to a potential fish location $r_p$, and $G_{W,r}(r|r_p, f)$ is the waveguide Green’s function from that fish location to the receiver array center $r$, and $E_0 = \int |Q(f)|^2 df$ is the source energy. In Fig. 3.1(c), the source array is centered at 65 m depth with 0.83 m spacing between the elements, transmitting a 50 Hz bandwidth Tukey windowed signal with center frequency of 950 Hz. The horizontal coordinate of the receiving array is co-located with the source, but its depth is located at $z_r = 105$ m. The propagation path is a transect from Fig. 2(a) of Ref. [9] with coordinates (0,0) to (0.5,-15) km in eastings and northings, beginning at the horizontal location of the source array and extending through a region containing a large fish shoal. The varying bathymetry along the path is plotted in black in Fig. 3.1(c), illustrating the gradual upslope from Georges basin to the edge of the bank. Note that the $TTL_W$ is determined by first averaging 100 independent Monte-Carlo realizations of the source spectrum-weighted square-magnitude Green’s function through the fluctuating ocean and then applying the log transform. Single realizations of the transmission loss exhibit spatial fluctuations due to modal interference. By averaging the scintillating acoustic intensities in both time and space in the dynamic ocean environment, the fluctuations
in transmission loss are reduced [33]. The seafloor at the GOM E’06 site is mostly sandy [133], modeled here with geo-acoustic parameters of sound speed 1700 m/s, density 1.9 g/cm$^3$, and attenuation 0.8 dB/wavelength. The examples in the following sections simulate the bistatic imaging of herring groups located within the main lobe of the receiver array beamed along the path indicated in Fig. 3.1 (c).

For fish distributed over a D=20 m depth layer centered at $z_s = 150$ m depth in the waveguide, Fig. 3.1(d) shows the $TTL_W$ obtained by first averaging the broadband two-way propagated acoustic intensities in Fig. 3.1(c) over the depth layer of the fish, and then taking the log transform, described by,

$$TTL_W(\rho_p, z_s, f_c | r_0, r) = 10 \log_{10} \left\{ \frac{(4\pi)^2}{E_0} \int_{z_s-D/2}^{z_s+D/2} \left\langle \int_{f_c-B/2}^{f_c+B/2} |Q(f)|^2 \times |G_{W,s}(\rho_p, z_p | r_0, f)|^2 |G_{W,r}(r | \rho_p, z_p, f)|^2 \times P(\rho_p, z_p) df \right\rangle dz_p \right\}. \quad (3.10)$$

where, $P(\rho_p, z_p)$ is the probability of finding a fish at location $(\rho_p, z_p)$. For a uniform distribution of fish, $P(\rho_p, z_p) = 1/D(\rho_p)$, where $D(\rho_p)$ is the layer depth of the fish group at horizontal location $\rho_p$, so that

$$TTL_W(\rho_p, z_s, f_c | r_0, r) = 10 \log_{10} \left\{ \frac{(4\pi)^2}{E_0 D(\rho_p)} \int_{z_s-D/2}^{z_s+D/2} \left\langle \int_{f_c-B/2}^{f_c+B/2} |Q(f)|^2 \times |G_{W,s}(\rho_p, z_p | r_0, f)|^2 |G_{W,r}(r | \rho_p, z_p, f)|^2 df \right\rangle dz_p \right\}. \quad (3.11)$$

The standard deviation in the broadband $TTL_W$ over the depth layer of the fish at ranges greater than 10 km is roughly 2 to 3 dB, as shown in Fig. 3.1(d), indicating the two-way propagated broadband acoustic field is only partially randomized.

The *inter-fish* acoustic propagation in Eq. (D.4) is approximated by the free space
Green’s function, $G_{FS}$,

$$G_{FS}(\mathbf{r}_n|\mathbf{r}_p,f) = \frac{e^{ik|\mathbf{r}_p - \mathbf{r}_n|}}{4\pi|\mathbf{r}_p - \mathbf{r}_n|}.$$  \hfill (3.12)

This is justified because (1) multiple scattering is significant primarily between adjacent fish where the scattered field is propagated directly from one fish to the other without interaction with the waveguide boundaries, (2) the waveguide transmission loss is well approximated by the free space transmission loss at short ranges on the order of the water depth, as can be seen in Figs. 2.29 and 2.30 of Ref. [134], where the transmission losses shown for two 100 m deep Pekeris waveguides can be approximated as spherically spreading at ranges smaller than 100 m. The inter-fish propagation depends on the 3D positions of fish in a group, as well as the sound speed in the medium between the fish. Here, we assume this to be a constant of $c=1482$ m/s, which is the mean sound speed at the fish group depth of 150 m (see Fig. 3 of Ref. [9]).

### 3.2.2 Scattering properties of Atlantic herring

Here, we apply the theory described in Appendix E and Sec. 3.1.2 to model the complex scatter function of an Atlantic herring individual and the corresponding result averaged over a group. Consider a group of $N = 10000$ fish centered at 150 m water depth, uniformly distributed over a 20 m thick depth layer with fork lengths derived from a Gaussian distribution of mean 24.2 cm and standard deviation 1.65 cm (Fig. 3.3(c)), consistent with the measured fork length distribution of trawl samples [9] collected during GOME’06. The corresponding distribution of inferred swimbladder volumes for fish in the group is plotted in Fig. 3.3(d) with mean $V_m = 3.68$ ml and standard deviation $\sigma_V = 0.84$ ml, consistent with those inferred from the herring groups imaged on Oct. 03, 2006 in Ref. [9] with estimated neutral buoyancy depth of 85 m. Note that even
though the herring fork length distribution is modeled as Gaussian, the swimbladder volume distribution is not Gaussian and is asymmetric about the mean because the swimbladder volume depends nonlinearly on the fork length. It also depends on the depth location and the neutral buoyancy depth of each fish in the group.

The complex scattering amplitude, defined as the ratio of complex scatter function to wavenumber, $S(k)/k$, is plotted as a function of frequency for a herring of mean swimbladder volume 3.68 ml in Fig. 3.3(a). This fish has a resonance frequency near 1.6 kHz. Figure 3.3(b) illustrates the target strength spectrum as a function of frequency for the fish with mean swimbladder volume and for fish with swimbladder volumes both one standard deviation larger and smaller than the mean, as well as the ensemble-averaged target strength obtained by averaging the fish scattering cross-sections over the group, $TS = 10 \log_{10} |S_p(k)/k|^2$. A larger fish has a lower resonance frequency and a higher target strength at resonance. In Fig. 3.3(a), the imaginary part, $\Im\{S(k)/k\}$, peaks near resonance which indicates that both the scattering response from the fish group and the attenuation through the group increases near resonance. The imaging frequencies employed by the waveguide remote sensing system during GOME’06 were in the sub-resonance range for the vast majority of herring imaged.

The roughly 1.6 kHz mean resonance frequency of the shoaling herring populations obtained in Ref. [9] from measurements made using the waveguide remote sensing system in the Gulf of Maine is comparable to published results of herring resonance frequency obtained with other local acoustic imaging systems [68, 127]. As discussed in the last paragraph of page 115 in Ref. [9], “A wide distribution of swimbladder volumes and corresponding neutral buoyancy depths within any shoal is likely and could potentially unify the various existing data sets by superposition, with larger swimbladder volumes
Figure 3.3: (a) The real and imaginary parts of the complex scattering amplitude $S(k)/k$ for a herring of mean swimbladder volume 3.68 ml. (b) The target strengths of fish with mean swimbladder volume (black) and of fish with swimbladder volumes both one standard deviation larger (light gray) and smaller (dark gray) than the mean. The ensemble-averaged target strength (dash-dotted black), $TS = 10 \log_{10} |S_p(k)/k|^2$, obtained by averaging over the scattering cross-sections of all fish in the group is also plotted. The normalized histograms (by total number of fish $N = 10000$) of (c) the fork length and (d) swimbladder volume for the herring group. The swimbladder volume shown in (d) is inferred from (c) and taking into account the depth of each individual fish in the group given a known neutral buoyancy depth at 85 m using Eq. (E.5) and (E.6), where $p = 3.35 \times 10^{-6}$ and $q = 3.35$ are empirically determined from length and weight measurements of trawl samples [9].
dominating at the lower end of the possible resonance spectrums and smaller volumes dominating the higher end. None of the systems used in the field so far, however, could test this since it would require acquisition of simultaneous data both well below and well above all contributing resonant frequencies.”

It should be noted that the data presented in Ref. [9] is the only data set existing for scattering from shoaling herring populations in the Gulf of Maine at frequencies below 1.5 kHz. The sources used in other studies [68, 127, 135] have sharp roll-offs in source level at frequencies below 2.5 kHz. The roughly 1.6 kHz mean resonance frequency measured with the waveguide remote sensing system in Ref. [9] was found to be consistent for the large herring shoals at roughly 150 m to 180 m water depth for all 10 days of observation in Fall 2006. This resonance was obtained by matching the low frequency roll-off in scattering from the large shoals which is the most robust method for estimating the resonance of a harmonic oscillator as discussed in Ref. [21] and shown in Sec. 3.4.8. The waveguide remote sensing system used to measure fish target strength in Ref. [9] was also calibrated against scattering from deeply submerged BBN cylindrical air-filled targets during GOME’06 [136].

3.3 Modeling wide-area population density imaging of sparse aggregation of Atlantic cod in Ipswich Bay using OAWRS system

Many ground fish species, such as Atlantic cod (Gadus morhua), are also known to congregate in dense groups during their spawning seasons. Ipswich Bay near Massachusetts and New Hampshire coastlines is one of the primary spawning grounds for
Atlantic cod during their spring spawning season. According to decades of annual acoustic and trawl surveys conducted by Massachusetts State and National Marine Fisheries Service, large number of cod are expected to migrate to Ipswich Bay to spawn. To explore the abundance, temporal spatial distributions and diurnal behavior of Atlantic cod populations, an OAWRS experiment is proposed to be conducted in Ipswich Bay, Massachusetts, concentrating on areas where dense cod aggregations are most likely to form. Fig. 3.4(a) shows the expected experimental sites in the Ipswich Bay environment off the coastline of Massachusetts.
Here, we implement the rigorous theoretical model described in Sec. 3.1 to investigate the effects of multiple scattering, attenuation due to scattering, and coherent effects, such as resonance shift and sub- and super-resonance local maxima, on wide-area population density imaging of cod aggregations from scattered intensity images to be acquired with an OAWRS system in potential future experiment conducted in Ipswich Bay continental shelf environment.

### 3.3.1 Modeling acoustic propagation in Ipswich Bay

The approach described in Sec. 3.2.1 are used here to simulate the long-range broadband acoustic propagation in Ipswich Bay continental shelf environment. Over 100 experimentally measured sound speed profiles, shown in Fig. 3.4(b), acquired during the Main Acoustic Experiment 2003 conducted in the New Jersey STRATAFORM, a similar continental shelf environment to Ipswich Bay, are used here to simulate the effect of linear internal waves in randomizing the acoustic field in the waveguide. As an example, the broadband \( TTL \) in the range-dependent Ipswich Bay continental shelf environment simulated stochastically with the waveguide propagation model is shown in Fig. 3.5. The vertical source array is centered at 30 m depth with 1.6256 m spacing between elements, transmitting a 50 Hz bandwidth Tukey-windowed signal with center frequency of 415 Hz. The horizontal coordinate of the receiver array is co-located with the source, but deployed at depth of 40 m. The propagation path is along the dashed line shown in Fig. 3.4(a). The bathymetry along the path is plotted in black in Fig. 3.5, indicating an extremely benign slope for the seafloor. The seafloor is assumed to be mostly sandy with geoacoustic parameters identical to those in Sec. 3.2.1. For fish distributed over a \( D=10 \) m depth layer centered at \( z_s = 75 \) m depth in the waveguide,
Fig. 3.5 shows the $TTL_W$ obtained from Eq. (3.10) by first averaging the broadband $TTL$ in Fig. 3.5 over the depth layer of the fish and then taking the log transform.

### 3.3.2 Scattering properties of Atlantic cod

Similar to Sec. 3.2.2, here we consider a group of $N = 10,000$ cod centered at 75 m water depth, uniformly distributed over a 10 m thick depth layer with total lengths derived from a Gaussian distribution of mean 70 cm and standard deviation 10 cm (Fig. 3.6(c)), consistent with the measured total length distribution of trawl samples collected during 2011 synoptic acoustic and trawl surveys of spring spawning aggregations of Atlantic cod in Ipswich Bay by University of New Hampshire and NOAA in May and June 2011 [137]. In comparison to physostomes with open swimbladders, cod are physoclist, fish with closed swimbladders, who can freely adjust the amount of air in the swimbladder through their gas glands so that they can remain neutrally buoyant at any depth given sufficient amount of time to adjust their swimbladder volumes. Here, we assume that all cod have the same neutral buoyancy depth of 40 m. The corresponding distribution of inferred swimbladder volumes for fish in the group is plotted in Fig. 3.6(d) with mean $V_m=103.4$ ml and standard deviation $\sigma_V=31.4$ ml. The complex scattering amplitude for a cod of mean swimbladder volume 103.4 ml is plotted as a function of frequency in Fig. 3.6(a). This fish has a resonance frequency near 400 Hz, which corresponds to the peak of the imaginary part of complex scatter function $\Im\{S(k)/k\}$. 
Figure 3.5: (a) Modeled $TTL_W$ calculated using Eq. (3.9) from a source array to potential fish locations and from the fish locations back to the receiver array center is shown as a function of fish range and depth in Ipswich Bay continental shelf environment for a source waveform of 50 Hz bandwidth centered at $f_c = 415$ Hz. (b) The $TTL_W$ obtained by first averaging the broadband two way propagated acoustic intensities over a 10 m thick fish layer centered at $z_s = 75$ m depth and then taking the log transform following Eq. (3.11). The error bars indicate one standard deviation in the broadband $TTL_W$ over the depth layer of the fish.
Figure 3.6: Identical to Fig. 3.3, but for the Atlantic cod, whose total length distribution is a Gaussian with mean of 70 cm and standard deviation of 7 cm. (a) The real and imaginary parts of the complex scattering amplitude \( \frac{S(k)}{k} \) for a cod of mean swimbladder volume 103.4 ml. (b) The target strengths of fish with mean swimbladder volume (black) and of fish with swimbladder volumes both one standard deviation larger (light gray) and smaller (dark gray) than the mean. The ensemble-averaged target strength (dash-dotted black) is also plotted. The normalized histograms (by total number of fish \( N = 10,000 \)) of (c) the total length and (d) swimbladder volume for the cod group. The swimbladder volume shown in (d) is inferred from (c) and taking into account the depth of each individual fish in the group given an assumed neutral buoyancy depth at 40 m using Eq. (E.5) and (E.6), where \( p = 7.65 \times 10^{-6} \) and \( q = 3.0606 \) are empirically determined from historical length and weight measurements of Atlantic cod during trawl surveys [138].
3.4 Illustrative Examples A: Atlantic herring

Here, the theory described in Sec. 3.1 is applied to simulate the broadband matched filtered scattered field from groups of Atlantic herring imaged by the waveguide remote sensing system during GOME’06. The effects of multiple scattering, attenuation due to scattering, and modal dispersion on fish population density imaging are investigated. The charting speed required to accurately localize a fish group in range from its time-dependent scattered field and the range resolution of the imaging system after matched filtering in the ocean waveguide are examined.

The modeled source waveform is a Tukey windowed linear frequency modulated pulse of 1 s duration and 50 Hz bandwidth with a center frequency $f_c$ of either 415, 735, 950 or 1125 Hz, similar to those transmitted by the waveguide remote sensing system during GOME’06. The waveform centered at 950 Hz is considered in the majority of examples illustrated here since the images formed from this waveform had the best cross-range resolution with significantly high fish scattered intensity to background reverberant intensity ratios and were used extensively for population density estimation in Refs. [9, 4, 5]. The source and receiver arrays are centered at depths $z_0=65$ m and $z_r=105$ m respectively to match typical imaging geometries of the waveguide remote sensing system employed in GOME’06. The horizontal position of the source and receiver arrays are co-located in the examples presented here to simplify charting of scattered returns in range, although a bistatic geometry was employed in GOME’06. In all examples, the fish group is centered at 14.9 km horizontal range from the source and receiver arrays. The matched filtered returns in time are charted to horizontal range using $\rho = c_{chart} t/2$, where $c_{chart}$ is the optimal charting speed determined in Sec. 3.4.5. All results are
normalized for a 0 dB re 1 \( \mu \)Pa at 1 m source level. The bathymetric transect over which the signal propagates is provided in Sec. 3.2.1. The bottom is assumed to be sandy in all examples, unless otherwise specified. Roughly 100 independent broadband Monte-Carlo simulations are used to compute the scattered field statistics for each fish group discussed in this article, unless otherwise specified.

In all the examples to follow, we assume the cross-range resolution of the imaging system is equal to the cross-range extent of each fish group considered, and the main lobe is steered in the direction of the fish group so that we can approximate \( B(\phi_p) \approx 1 \) for all fish in the group. Scattered returns from the fish group are compared to background reverberation derived from GOME’06 data. The expected reverberation level \( RL \) at a given horizontal location \( \rho_p \) is estimated for each frequency band making use of the mean background scattering strength obtained from Fig. 5 of Ref. [9] and subtracting the broadband two-way transmission losses to horizontal location \( \rho_p \) from the source and receiver, and adding \( 10 \log_{10} A_{\text{res}}(\rho_p) \), where \( A_{\text{res}}(\rho_p) \) is the resolution footprint of the imaging system considered in the examples here.

For fish groups considered here, the maximum range and cross range extents are limited to 1 km and 100 m respectively, because of computational constraints in modeling the scattered field moments numerically by Monte-Carlo simulation. The fish group sizes considered here are large compared to the wavelength of the imaging system, and similar in dimension to the small schools imaged in GOME’06. For larger shoals imaged in GOME’06, the results obtained here are still applicable to each resolution cell of the imaging system. This is because (1) multiple scattering is negligible, as we will show, for the vast majority of fish groups imaged, (2) even at imaging frequencies and fish volumetric densities where multiple scattering is significant, multiple scattering is
important primarily between nearest neighbors spaced within several body lengths apart and negligible for fish with larger spatial separations. For the examples considered here, multiple scattering between neighbors located all $4\pi$ radian solid angle from an individual is included for the vast majority of fish, except for individuals near the cross-range periphery of each group where the multiple scattering is restricted to neighbors located within $2\pi$ radian solid angle. The multiply scattered fields from these latter individuals are expected to be negligibly small compared to the overall scattered level, given the large number of fish within each resolution cell of the imaging system.

3.4.1 Coherent and incoherent broadband matched filtered scattered intensities that including multiple scattering from a fish group compared to environmental reverberation

The coherent and incoherent broadband matched filtered fully scattered intensities from a group of Atlantic herring in the Gulf of Maine, which include multiple scattering, obtained from 100 independent broadband Monte-Carlo realizations are shown in Fig. 3.7(b) and (c). The imaging frequency band is centered at $f_c=950$ Hz. The 3D spatial configurations that fish group adopt are derived from random realizations of a uniform probability distribution function (PDF) in depth and cross-range, and a non-uniform PDF in range shown in Fig. 3.7(a). The PDF in range is uniform in the middle, but with the front and back edges tapered with a half-Gaussian function. The simulated fish group has a total of $N=10000$ individuals centered at $z_s=150$ m depth with group extending $L_r=100$ m in range, $L_{cr}=100$ m in cross-range, and $L_z=20$ m in depth, resulting in an average volumetric fish density of $N/(L_rL_{cr}L_z)=0.05$ fish m$^{-3}$ and an average areal fish density of $N/(L_rL_{cr})=1$ fish m$^{-2}$, matching typical densities found during GOME’06 shown in Figs. 4D and 4E of Ref. [9].
Figure 3.7: (a) The areal density plotted as a function of range for a fish group in the Gulf of Maine environment with sand bottom. The vertical lines indicate the range where the fish areal density is half its maximum value. (b) The incoherent $\text{Var}(\Psi_s(t_M))$ and coherent $|\langle \Psi_s(t_M) \rangle|^2$ broadband matched filtered fully scattered intensities that include multiple scattering from the fish group imaged using the waveform centered at $f_c=950$ Hz with 50 Hz bandwidth and 0 dB re 1 $\mu$Pa at 1 m source level. The fish scattered intensities are compared to the expected background reverberation estimated from GOME'06 data. (c) Identical to (b) but plotted in logarithmic scale. The error bar indicates the standard deviation of the broadband matched filtered fully scattered intensities from the fish group.
The variance or incoherent intensity exceeds the squared mean or coherent intensity by about 20 dB. The incoherent intensity stands roughly 12 dB above background reverberation implying the fish group with areal density of 1 fish \( m^{-2} \) is highly detectable. For higher fish densities typically found in the shoal centers exceeding 10 fish \( m^{-2} \), the fish scattered intensity to reverberation ratio will be proportionally higher, exceeding 20 dB.

For the given fish density, multiple scattering is found to be negligible for both the incoherent and coherent intensities. The first four orders of multiple scattering are included in the field, higher orders being insignificant. Differences between the singly scattered level and the fully scattered level that includes multiple scattering are less than 0.1 dB over the entire extent of the fish group as shown in Fig. 3.8 and discussed in the next section.

A fully randomized PDF was used in this section to model the 3D spatial configuration of the fish group. In Sec. 3.4.8 and Sec. 3.4.9, we show that for fish groups larger than the acoustic wavelength, the incoherent intensity dominating the fully scattered returns is independent of the specific 3D spatial configuration adopted by the group, but dependent on its areal and volumetric densities. For smaller fish groups on the order of the wavelength, we show the coherently scattered intensity dominates and is sensitive to the exact 3D spatial configuration adopted by the fish group.

3.4.2 Effects of multiple scattering and dependence on fish density and target strength

The single scattering assumption is often employed to estimate population density
from scattered intensity in both waveguide remote sensing systems and conventional ultrasonic fisheries echosounders, since the scattered intensity is linearly related to population density when this assumption is valid [10, 4, 3, 5, 139, 9, 38]. When multiple scattering contributes to the full scattered intensity, however, the population density estimation may become non-linear. Here we examine the dependence of multiple scattering on herring target strength, which is a function of the imaging frequency, and population density.

Herring scatter functions and corresponding target strengths are highly dependent on imaging frequency as shown in Fig. 3.3. The effect of varying herring target strength by varying the imaging frequency band is investigated in Fig. 3.8. We consider broadband waveforms with the same four center frequencies as those transmitted by the vertical source array in GOME’06. The parameters of the fish group and waveguide environment are identical to the example shown in Fig. 3.7. The scattered field levels from the fish group increase dramatically with target strength as the imaging frequency band increases towards resonance. Multiple scattering effects are only noticeable at the highest frequency band with $f_c = 1125$ Hz for the fish density considered. Even in this case, the differences between the singly scattered intensity levels and the fully scattered intensity levels that include multiple scattering are less than 0.5 dB.

The effects of multiple scattering are investigated with increasing fish densities in Fig. 3.9 for the $f_c = 950$ Hz band. The fish groups have identical dimensions $L_r=50$ m, $L_{cr}=50$ m, and $L_z=20$ m, but contain (1) $N=2500$, (2) $N=10000$, and (3) $N=40000$ individuals with fully randomized spatial configurations in each case. These examples correspond to fish volumetric densities of 0.05, 0.2, and 0.8 fish m$^{-3}$, and areal densities of 1, 4 and 16 fish m$^{-2}$, respectively, as shown in Fig. 3.9(a). All other properties of
Figure 3.8: Effect of varying the imaging frequency band on the incoherent matched filtered scattered returns from a fish group. (a) The areal density of the fish group. (b) The incoherent fully scattered intensity $\text{Var}(\Psi_s(t_M))$ that includes multiple scattering and singly scattered intensity $\text{Var}(\Psi_s^{(1)}(t_M))$ from the fish group are plotted as a function of the imaging frequency band. The expected background reverberant intensity estimated from GOME’06 data is also plotted for comparison. The background reverberation varies by roughly 2 to 3 dB from the lowest to the highest frequency band and the average across the 4 bands is plotted here. (c) Identical to (b) but plotted in logarithmic scale. The error bars show the standard deviation of the broadband matched filtered fully scattered intensities from the fish group at various imaging frequency bands.
Figure 3.9: Effect of varying the fish areal density on the broadband incoherent matched filtered scattered returns from a fish group imaged with frequency band centered at $f_c = 950$ Hz. (a) The areal fish densities of three distinct fish groups. (b) The incoherent fully $\text{Var}(\Psi_s(t_M))$ and singly $\text{Var}(\Psi_s^{(1)}(t_M))$ scattered intensities from the fish groups with imaging frequency band centered at 950 Hz are compared to background reverberation. (c) Identical to (b) but plotted in logarithmic scale. The error bars show the standard deviation of the broadband matched filtered fully scattered intensities from fish group with various fish areal densities.
the fish group and waveguide environment are identical to the example shown in Fig. 3.7. The variance or incoherent intensity increases roughly linearly with fish density, standing increasingly above background reverberation. The singly and fully scattered intensities are identical except when the fish areal density approaches and exceeds 16 fish m$^{-2}$. In this case, the fully scattered intensities differ from the singly scattered intensities by less than 0.5 dB. These differences also depend on attenuation as the field forward propagates through the fish group. The effect of attenuation in scattering is investigated in greater detail in the next section.

These examples validate the single scattering assumption employed in Refs. [9, 4, 5] to estimate herring areal population densities from their experimentally measured scattered intensities by the waveguide remote sensing system with frequency band centered at $f_c = 950$ Hz and below, since multiple scattering is negligibly small even at the highest areal/volumetric herring densities observed in GOME’06. For herring areal population density imaging with frequency band centered at $f_c = 1125$ Hz, multiple scattering effects may matter at the shoal centers with very high fish densities as we illustrate in the next section.

3.4.3 Attenuation from scattering through a fish group and its dependence on swimbladder resonance damping

The conditions under which multiple scattering becomes significant are often similar to that required for attenuation from forward scattering to become significant. When the volumetric density of the scatterers and their corresponding target strengths are high, multiple scattering leads to delayed returns that can enhance the overall scattered level over the range extent of the group. However, the close proximity of the scatterers to
each other also imply that the forward scattered field from one scatterer can interfere destructively with the incident field at the location of another scatterer, as described by the extinction or forward scatter theorem [23, 25, 140]. This leads to shadowing and a corresponding reduction in the scattered intensity from the second scatterer, since the total field incident on it is reduced. This attenuation is dependent on the imaginary part of the forward scatter function of the scatterers and their spatial separations. Both multiple scattering and attenuation are cumulative over the range extent of the group and the effects are opposing.

Determining the attenuation from forward propagation through a fish group requires an accurate characterization of swimbladder resonance damping. The damping coefficient is not easily measured particularly for live in-situ fish, since it depends on many factors and physical parameters of the swimbladder wall and surrounding fish flesh [141]. As discussed in Ref. [142], the Love model [115, 67] may oversimplify the resonant damping by modeling the fish flesh as a viscous fluid.

Here we investigate the effect of varying the damping coefficients [142] on the scattered intensity from a group of herring in Figs. 3.10 and 3.11 with imaging frequency bands centered at $f_c = 950$ Hz and $f_c = 1125$ Hz respectively. The fish group considered contains $N=8000$ individuals with volumetric density of 0.64 fish m$^{-3}$, areal density of 6.4 fish m$^{-2}$, and dimensions $L_r=100$ m, $L_{cr}=12.5$ m, $L_z=10$ m, as shown in Figs. 3.10(a) and 3.11(a). This group is elongated in range compared to its cross-range and depth dimensions to enhance the effects of multiple scattering and attenuation. All other properties of the fish group and waveguide environment are identical to the example shown in Fig. 3.7. Figures 3.10(b) and (c) and Figs. 3.11(b) and (c) include both viscous and radiation damping. These should be compared with the results in Figs. 3.10(d) and
3.11(d) where only radiation damping is included, and Figs. 3.10(e) and 3.11(e) with zero damping coefficient. When no damping is present [Figs. 3.10(e) and 3.11(e)] the scatter function is purely real and the fully scattered incoherent intensity that includes multiple scattering increases cumulatively above the singly scattered intensity over the range extent of the group. When damping is included, attenuation through the group negates the effects of multiple scattering and the overall backscattered intensity level compared to the singly backscattered intensity level depends on the amount of damping included in the model. The effects are more prominent when the imaging frequency band is centered at $f_c = 1125$ Hz as shown in Fig. 3.11.

The mean broadband incident intensity in the random waveguide, along with the mean total intensity forward propagated through the fish group are shown in Figs. 3.12 (a) and (b) for the imaging frequency bands centered at 950 and 1125 Hz respectively, where both intensities are averaged over the 10 m depth layer of the fish group. When damping is absent, the mean total forward propagated intensity is higher than the mean incident intensity because multiple scattering raises the overall level and the effect is cumulative over the range extent of the group. In the cases when damping is present, the total forward propagated intensities can fall below the incident intensity depending on the amount of damping included in the model. It should be noted that the incident intensity does not decay monotonically with range in Fig. 3.12 because of slight modal interference still present in the field even after averaging over 100 independent broadband Monte-Carlo realizations.

The analysis in this section indicates that both multiple scattering, and attenuation due to scattering are negligible in the long-range acoustic data at imaging frequency bands centered at 950 Hz and below and for the herring densities observed during
Figure 3.10: Effect of varying fish swimbladder damping on the incoherent matched filtered scattered returns with imaging frequency band centered at 950 Hz. (a) The areal fish density of the fish group. (b)-(e) The incoherent fully $\text{Var}(\Psi_s(t_M))$ and singly $\text{Var}(\Psi_s^{(1)}(t_M))$ scattered intensities from the fish group are plotted as a function of fish damping coefficient, and compared to background reverberation.
Figure 3.11: Identical to Fig. 3.10 but for imaging frequency band centered at 1125 Hz.
Figure 3.12: The mean total acoustic intensity forward propagated through the fish group in Figs. 3.10 and 3.11 are plotted as a function of fish swimbladder damping for imaging frequency bands centered at (a) $f_c = 950$ Hz and (b) $f_c = 1125$ Hz. The broadband incident and forward propagated (which include multiple scattering from other fish in the group) intensities are averaged over the fish layer depth and over the 100 independent Monte-Carlo realizations.
GOME’06. When the imaging frequency band is centered at 1125 Hz, the effects are non-negligible at the shoal centers and an accurate knowledge of swimbladder resonance damping is required to determine the amount of attenuation present in the data.

### 3.4.4 Effect of modal dispersion

The incoherent intensity scattered from the fish group at the tail end of the distribution in Fig. 3.7 decays gradually as a function of range compared to the front end of the distribution. This is caused by modal dispersion, slowly propagating high-order modes that arrive later in time than the low order modes. The level of the dispersed returns are highly dependent on the waveguide and seafloor type. Compare the results in Fig. 3.7 for the sand waveguide to Fig. 3.13 for the silt waveguide. The properties of the fish group, imaging geometry, and all other properties of the waveguide are identical in these two examples, with the exception of the seafloor type. The silt seafloor has sound speed 1520 m/s, density 1.4 g/cm³ and attenuation 0.3 dB/wavelength.

In the silt waveguide, the incoherent intensity scattered from fish at the tail end of the distribution decays more rapidly with range than in the sand waveguide and at a spatial rate approximately equal to that at the front end of the distribution. This is because the silt waveguide supports fewer number of propagating modes than the sand waveguide. The small contrast in sound speed between the water column and the seafloor allows only the very low order modes to propagate in a silt waveguide. For this reason, the effect of modal dispersion is minimized in the silt waveguide. The fewer number of propagating modes in the silt waveguide also implies that the total energy propagated through the waveguide is weaker. As a result, the scattered levels from the
Figure 3.13: Identical to Fig. 3.7 but for the silt waveguide.
fish group and the background reverberation are both lower in the silt waveguide than in the sand waveguide. The background reverberation for the silt waveguide is obtained by subtracting 15 dB from the experimentally determined reverberation in the sand waveguide. This is because bottom reverberation modeling in Refs. [40, 130] indicate that reverberation from the seafloor in the silt waveguide will be roughly 15 dB lower than in the sand waveguide.

3.4.5 Charting speed in the random range-dependent Gulf of Maine environment

Charting the time-dependent broadband matched filtered scattered intensity accurately in horizontal range for a target in a random ocean waveguide requires an optimal choice of the mean charting speed $c_{\text{chart}}$. In a waveguide imaging system, the scattered intensities from targets arriving at the same time instance at the receiver are charted onto an elliptical arc [80, 143] in the horizontal plane with their respective azimuths determined via beamforming. The source and receiver arrays are located at the foci of the ellipse since the total horizontal path length from source to target $|\rho_t - \rho_0|$ and target to receiver $|\rho - \rho_t|$ satisfy [80] $|\rho_t - \rho_0| + |\rho - \rho_t| = c_{\text{chart}}t$. For the monostatic imaging scenario, since $\rho = \rho_0$, the range-time relationship simplifies to $|\rho - \rho_t| = c_{\text{chart}}t/2$.

Determining the optimal charting speed in a random range-dependent ocean waveguide can be challenging because sound speed is a function of depth, range, and time. Furthermore, the acoustic modes of the waveguide each propagate with a different group speed. Here, we determine the mean charting speed to be that which maximizes the modeled mean time-dependent broadband matched filtered scattered intensity at the true horizontal range span of the fish group.
The optimal mean charting speed is obtained when the variance or incoherent intensity integrated over the range extent of the group, \( \int_{r_{s-L/2}}^{r_{s+L/2}} \text{Var}(\Psi_s(t))dr \), is maximized, since there are no other scatterers present in the waveguide and the incoherent intensity is dominant. The mean charting speed in the sand waveguide for the example of Fig. 3.7 is found to be 1475 m/s with a standard deviation of 1.9 m/s. For the silt waveguide, the mean charting speed is 1474.4 m/s with a standard deviation of 1.1 m/s. For both waveguides, their respective mean charting speeds correspond roughly to the group speed of mode 1 in that waveguide. The charting accuracy is limited by the range resolution of the imaging system of roughly 15 m for the examples considered here. The estimated mean charting speed for all other examples are provided in Table 3.1.

The charting speeds obtained here for actively localizing random scatterers in the random range-dependent ocean waveguide are approximately equal to the minimum of the mean water-column sound speed profile shown in Fig. 3 of Ref. [9]. This result is consistent with that obtained in Ref. [35] for passive localization of a deterministic source in a range-independent ocean waveguide.

### 3.4.6 Fish areal population density estimation

Here, we show that the areal population density of fish groups can be readily estimated from their incoherently averaged broadband matched filtered scattered intensities when the single scattering assumption is valid. The areal population density \( n_A \) is estimated by correcting for (1) the broadband source level \( SL \), (2) the broadband two-way transmission losses \( TTL_W \) obtained from waveguide Green’s functions averaged over multiple realizations and over the depth extent of the fish layer, (3) the broadband
Table 3.1: Standard deviations, $\sigma$, of the broadband matched filtered scattered intensities from fish groups and corresponding estimated charting speeds, $c_{chart}$. The standard deviation of the estimated charting speed is roughly 1 to 2 m s$^{-1}$.

<table>
<thead>
<tr>
<th>Figures</th>
<th>$f_c$ (Hz)</th>
<th>$B$ (Hz)</th>
<th>$n_V$ (fish m$^{-3}$)</th>
<th>SC$^a$</th>
<th>Seafloor type</th>
<th>$\sigma$ (dB)</th>
<th>$c_{chart}$ (m s$^{-1}$)</th>
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<td>lattice</td>
<td>sand</td>
<td>5.0</td>
<td>1474.5</td>
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$^a$ 3D spatial configuration of the simulated fish group.
target strength $TS$ obtained from the mean scattering cross-section of an individual fish, and (4) the resolution footprint $A_{res}$ of the waveguide remote sensing system via

$$10 \log_{10} \langle n_A(\rho_p) \rangle \approx L(\rho_p) - SL(f_c) - TTL_W(\rho_p, z_s, f_c) - TS(f_c) - 10 \log_{10}(A_{res}(\rho_p)), \quad (3.13)$$

where

$$L(\rho_p, f_c) = 10 \log_{10} \left\langle |\Psi(\rho_p, f_c)|^2 \right\rangle \quad (3.14)$$

is the level of the measured or modeled incoherently averaged broadband matched filtered scattered intensity charted to horizontal range $\rho_p$,

$$SL(f_c) = 10 \log_{10} \left\langle \int_{f_c-B/2}^{f_c+B/2} |Q(f)|^2 df \right\rangle, \quad (3.15)$$

the $TTL_W(\rho_p, z_s, f_c)$ is defined in Eq. (3.11),

$$TS(f_c) = 10 \log_{10} \left\langle \frac{1}{E_0} \int_{f_c-B/2}^{f_c+B/2} |Q(f)|^2 |S(k)/k|^2 df \right\rangle, \quad (3.16)$$

and $A_{res}(\rho_p) \approx \Delta \rho L_{cr}(\rho_p)$, where $L_{cr}(\rho_p)$ is the cross-range resolution, and $\Delta \rho = c_{chart}/2B$ is the range resolution after applying the matched filter. Reference [33] describes how the broadband transmission loss can be approximated using the single frequency transmission loss as a function of range in a fluctuating waveguide by first spatial averaging the center frequency transmitted intensity and then transforming to log,

$$TTL_W(\rho_p, z_s, f_c) \approx 10 \log_{10} \left\{ (4\pi)^2 \int_{z_s-D/2}^{z_s+D/2} \left\langle |G_W(\rho_p, z_p|\mathbf{r}_0, f_c)|^2 \right\rangle \times G_W(\mathbf{r}|\rho_p, z_p, f_c)|^2 P(\rho_p, z_p) \right\} dz_p \right\}. \quad (3.17)$$

This allows a rapid estimation of the fish areal population densities over wide areas for a single transmission and over multiple transmissions with varying source and receiver locations.
We consider two examples, the fish groups in Figs. 3.7 and 3.13 that extend only 6 resolution cells in range, and a much larger fish shoal that extends 60 resolution cells in range. The areal densities of the former fish groups in the sand and silt waveguides are estimated using Eq. (3.13) and are shown in Figs. 3.14(a) and (b) respectively. We consider two distinct frequency bandwidths for the source waveform centered at 950 Hz. When the imaging bandwidth is \( B = 50 \text{ Hz} \), the matched filter operation smooths over the edges of the distribution. This is because the corresponding matched filter resolution of 15 m is not sufficiently fine to resolve the edge of the fish distribution over range since the distribution decays rapidly from 1 to 0 fish m\(^{-2}\) over a 5 m range. By increasing the imaging bandwidth to \( B = 150 \text{ Hz} \), the edges of the distribution can be more accurately mapped [10] since the system has a finer range resolution of 5 m. For the fish groups in Fig. 3.14, the total population estimation error varies from 2\% to 18\% depending on the seafloor type and imaging bandwidth.

In contrast, a much larger fish group extending 1 km in range is modeled in Fig. 3.15 with imaging frequency centered at 950 Hz and \( B = 50 \text{ Hz} \). The fish group contains \( N=100000 \) fish, has dimensions \( L_r=1000 \text{ m} \), \( L_{cr}=100 \text{ m} \), and \( L_z=40 \text{ m} \), volumetric density of 0.025 fish m\(^{-3}\) and areal density of 1 fish m\(^{-2}\) as shown in Fig. 3.15(a). This fish group has a fully randomized 3D spatial configuration. All other properties of the fish group and waveguide environment are identical to the example shown in Fig. 3.7. Only the singly scattered field is calculated because multiple scattering is insignificant at this imaging frequency and volumetric density as shown in Sec. 3.4.2. In addition, neglecting multiple scattering allows the numerical model to simulate much larger groups of fish because multiple scattering computations require an \( N^2 \) matrix multiplication, whereas single scattering requires only vectors of dimension \( N \). The PDF in range for this group is uniform in the middle and tapered with half-Gaussian functions at the edges. The
Figure 3.14: (a) Estimating fish areal population densities from their incoherently averaged broadband matched filtered fully scattered intensities using Eq. (3.13) illustrated for the fish group in Fig. 3.7 in the sand waveguide as a function of imaging frequency bandwidth $B$ with $f_c = 950$ Hz. (b) Identical to (a) but for the fish group in the silt waveguide shown in Fig. 3.13.
Gaussian edge region contains 20% of the total fish population in this example.

The estimated areal densities match the true densities on average over most of the group’s extent except at the trailing edge of the distribution where the estimated densities are slightly higher due to modal dispersion. Figure 3.15(d) plots the cumulative true and estimated populations integrated over the range extent of the group. The integrated populations match fairly well throughout except at the trailing edge of the distribution. The total population estimation error is roughly 7.6%. Since the fish density decays gradually with range at the edges of this distribution, the 15 m range resolution after matched filtering is sufficiently fine to trace the edges, on average, for this example.

The examples in this section indicate that the range resolution of the imaging system after matched filtering the broadband scattered waveform in the random range-dependent ocean waveguide is approximately equal to that obtained in free-space, $\Delta \rho \approx c_{\text{chart}}/2B$. This is because the total fish population estimated from the scattered intensities using Eq. (3.13) are approximately equal to the true population when the range resolution is defined as in free-space. Dispersion in the multi-modal waveguide has negligible effect on the matched filter resolution for waveforms with bandwidths up to 150 Hz and center frequencies in the range of 300 to 1200 Hz investigated here.

### 3.4.7 Standard deviation of the broadband matched filtered scattered intensities from the fish group

The standard deviation of the scattered returns must be accurately known in order to determine error bounds in population density estimation and to determine sample
Figure 3.15: Estimating fish areal population densities from their incoherently averaged broadband matched filtered scattered intensities illustrated for a much larger fish group. (a) True and estimated areal fish densities are plotted as a function of range. (b) The incoherent $\text{Var}(\Psi_s^{(1)}(t_M))$ and coherent $|\langle \Psi_s^{(1)}(t_M) \rangle|^2$ broadband matched filtered singly scattered intensities from the fish group imaged using the waveform centered at $f_c=950$ Hz with 50 Hz bandwidth and 0 dB re 1 $\mu$Pa source level are compared to background reverberation. (c) Identical to (b) but plotted in logarithmic scale. The error bar indicates the standard deviation of the broadband matched filtered singly scattered intensities from the fish group. (d) The true and estimated fish populations integrated as a function of range.
sizes necessary for reducing uncertainty [144]. The standard deviations of the broadband matched filtered fully scattered intensities within each resolution cell of the imaging system, determined from 100 independent Monte-Carlo realizations for each of the fish groups considered in this section are tabulated in Table 3.1. The standard deviations are also indicated by error bars in the respective figures. They are approximately 5 dB for most of the fish groups considered here, implying that the received broadband matched filtered scattered fields can be approximated as statistically saturated with phases that are almost fully randomized [32]. This result is consistent with the measured intensity standard deviations determined from instantaneous images of the waveguide remote sensing system [4, 3, 9, 5]. There, a 6-sample averaging, from averaging adjacent pixels in range and three consecutive instantaneous wide-area images over time, was employed to reduce the standard deviations in the scattered intensity images to less than 1.5 dB [4, 3, 9, 5]. This standard deviation for each pixel is small compared to the dynamic range of scattered intensity levels spanned by the fish groups.

When the incoherent intensity dominates, the standard deviation of the scattered returns are independent of the spatial configuration adopted by the large fish group considered in Sec. 3.4.9 (see Table 3.1). The standard deviations of the scattered levels from the fish group considered here are larger than the standard deviations of the broadband two-way transmission losses in the ocean waveguide (see Sec. 3.2.1) since the former incorporates additional sources of randomness, such as individual fish spatial locations and scatter functions.
3.4.8 Coherent multiple scattering effects such as resonance shift and sub- and super-resonance local maxima

Multiple scattering from a dense fish group can lead to effects such as a shift in the resonance frequency for the group from that of a single fish, and sub- and super-resonance local maxima. These phenomena have been predicted and explained in Refs. [26, 27, 29]. Here, we show that these effects are only significant for fish groups that are very dense and small, on the order of the acoustic wavelength, where the coherently scattered intensity is significant. Fish groups observed in the Gulf of Maine were many orders of magnitude larger and had lower volumetric densities. The incoherent intensity dominated the scattered returns so that the coherent multiple scattering effects are negligible as shown here.

Two distinct fish groups are considered, one with dimensions much larger than the acoustic wavelength and the other with dimensions on the order of the wavelength. The large herring group contains 7831 individuals with volumetric density $n_V = 0.05$ fish m$^{-3}$ distributed over a volume, as shown in Fig. 3.1(b), with axes dimensions given by $L_x = 100$ m, $L_y = 100$ m and $L_z = 20$, while the smaller group contains 240 individuals distributed over a similar volume but with axes dimensions given by $L_x = 2$ m, $L_y = 2$ m and $L_z = 2$ m, with a volumetric density $n_V=37.5$ fish m$^{-3}$ that is many orders of magnitude denser than the herring groups typically observed during GOME’06. Both groups are centered at $z_s = 150$ m depth and are imaged by a monostatic direct-path imaging system. We consider two distinct 3D spatial configurations for both the large and small fish group, the fully randomized and the partially random lattice configurations discussed in Sec. 3.1.2.
Figure 3.16: Effect of the 3D spatial configuration on the time harmonic fully scattered field moments, including multiple scattering, as a function of frequency for a monostatic direct-path imaging system examined for (a) a large herring group containing 7831 individuals and (b) a small herring group containing 240 individuals. The coherent, incoherent, total and estimated school target strength spectra calculated via Eqs. (3.5) to (3.8) respectively are compared as a function of the fish group configuration.

The analysis in this section follow the approach outlined in Sec. 3.1.3, except that 100 independent Monte-Carlo realizations are used here to estimate the statistical moments of the scattered intensities, including multiple scattering, at each frequency. The coherent, incoherent, total and estimated school target strength spectra calculated using Eqs. (3.5), (3.6), (3.7), and (3.8) are plotted in Figs. 3.16(a) and (b) for the large and small fish groups respectively as a function of the fish group spatial configuration. For the large fish group, the incoherent intensity dominates the total scattered returns. The total school target strength spectrum is well approximated by the estimated school target strength spectrum assuming single scattering. For this case, the total target strength spectrum is independent of the specific 3D spatial configuration adopted by the fish group.

In contrast, for the small fish group, the coherently scattered intensity is signif-
icant, causing a shift in the resonance frequency of the group. It also leads to sub-
and super-resonance local maxima as shown in Fig. 3.16(b). In this case, the specific
spectrum of the coherent, incoherent and total school target strengths are dependent
on the 3D spatial configuration adopted by the fish group. The total school target
strength is roughly 2 to 3 dB larger when fish in the group follow the fully random
spatial configuration rather than the partially random lattice configuration.

The analysis in this section has important implications for the measurement of
resonance frequency of fish groups. For instance, a local direct path acoustic system
that has high spatial resolution in depth typically images significantly fewer number of
fish, ten to a few hundred individuals, within its resolution footprint. Such a system
may make errors in estimating the resonance frequency of a fish group especially if its
operating frequency does not span the mean resonance frequency range of the fish group.
This is because the system may measure the local super- or sub-resonance maxima
resulting from coherent scattering interactions between fish, as shown in Figs. 3.2 and
3.16(b). This is less of an issue for the waveguide remote sensing system since there are
significantly many more fish, typically several thousand or more, within each resolution
footprint so that scattering is dominated by the incoherent intensity that peaks at just
one frequency, the mean resonance frequency for the fish group.

3.4.9 Effect of fish group 3D spatial configuration

Here we examine the effect of the 3D spatial configuration of a fish group on the
statistical moments of its broadband matched filtered fully scattered intensities that
include multiple scattering in a random ocean waveguide. We consider a herring group
containing 7831 individuals with volumetric density $n_V = 0.05$ fish m$^{-3}$ distributed over a volume, as shown in Fig. 3.1(b), with axes dimensions $L_x = 100$ m, $L_y = 100$ m and $L_z = 20$ m. This herring group is located at range $r_s = 14.9$ km from the source and receiver arrays at depth $z_s = 150$ m and is imaged with a broadband waveform of 50 Hz bandwidth centered at $f_c = 950$ Hz in the ocean waveguide with sand seafloor.

In Fig. 3.17, we consider two distinct 3D spatial configurations for the fish group, the fully randomized and the partially random lattice configurations discussed in Sec. 3.1.2. The range extent of this fish group is much larger than the acoustic wavelength and the scattered returns are dominated by the incoherent intensity. The dominant incoherent broadband matched filtered fully scattered intensities that include multiple scattering are approximately equal for fish in the two spatial configurations and local differences are negligible small compared to the dynamic range of intensity fluctuations indicated by the error bars in Fig. 3.17(c). For this example, the specific 3D spatial configuration adopted by the fish group has negligible effect on the mean scattered intensity, which is consistent with the results shown in Fig. 3.16(a) for the same fish group, but imaged by a monostatic direct-path system in an iso-speed lossless non-random environment.

When the fish group dimension is on the order of the acoustic wavelength, the scattered intensities depend on the specific 3D spatial configuration adopted by the fish group as discussed in Sec. 3.4.8.
Figure 3.17: Effect of the 3D spatial configuration of a fish group on the statistical moments of its broadband matched filtered fully scattered intensities. (a) The areal fish densities in the two configurations. (b) The incoherent $\text{Var}(\Psi_s(t_M))$ and coherent $|\langle \Psi_s(t_M) \rangle|^2$ broadband matched filtered fully scattered intensities that include multiple scattering from the fish groups imaged using the waveform centered at $f_c=950$ Hz with 50 Hz bandwidth and 0 dB re 1 $\mu$Pa at 1 m source level. The fish scattered intensities are compared to the expected background reverberant intensities estimated from GOME’06 data. (c) Identical to (b) but plotted in logarithmic scale.
3.4.10 Determining a convergence radius to enable multiple scattering calculation from the large group of scatterers that span multiple beams and range resolution cells of the waveguide remote sensing system

Due to computational constraints in matrix size and speed, it is extremely challenging to implement multiple scattering for large shoaling fish populations containing hundreds of thousands of individuals that span multiple range and cross-range resolution cells of the waveguide remote sensing system in a range-dependent ocean waveguide. As discussed in Sec. 3.1.1, with current computational power, the multiply scattered returns can only be simulated for a maximum of 40,000 individuals within a reasonable amount of time.

In reality, only the multiply scattered field from \( N \) nearest neighbors spaced within several body lengths apart are primarily important and negligible for fish with larger spatial separations. Here, we determine a convergence radius, \( R_c \), where only the \( N \) nearest neighbors located within the spherical volume of radius \( R_c \) are important for the multiple scattering calculations. For the \( p \)th fish within the fish volume, the number of nearest neighbors \( N_p \) will vary as a function of its relative position with respect to the center of the fish volume. Given a constant convergence radius, for fish located near the center of the fish volume, multiple scattering between neighbors located all \( 4\pi \) radian solid angle from them are considered to be important, while for fish located near the periphery of fish volume, multiple scattering is only restricted to neighbors located within \( 2\pi \) radian solid angle. Therefore, \( N_p \) will decreases with the increasing relative distance from the \( p \)th fish to the center of the fish volume. We determine the convergence radius \( R_c \) for the \( m \)th herring individual that is located at the nearest vicinity of the fish.
school center, because this is the individual that is supposed to have the most number of nearest neighbors given a radius $R$.

Here, we determine the convergence radius $R_c$ for the $m$th herring individual centered in an ellipsoidal-shaped herring group that has a typically observed volumetric density of 0.01 fish $\text{m}^{-3}$ and axes dimensions of $L_r = 400$ m, $L_{cr} = 400$ m, and $L_z = 20$ m, using a direct-path imaging system. Equations (D.2) and (D.4) and the model described in Sec. 3.2.2 are used to simulate the time-harmonic fully scattered field from the $m$th herring but only including multiple scattering from the $N_m(R)$ nearest neighbors surrounding the $m$th herring, where $N_m(R)$ increases with increasing radius $R$. Herring are assumed to be distributed within the ellipsoidal-shaped volume following the partial random lattice spatial configuration described in Sec. 3.1.2. For each independent Monte-Carlo simulation, the spatial location of each herring individual is randomized from its mean location by a standard deviation that is 30% of the mean inter-fish spacing $d$ obtained from Eq. (3.4).

As radius $R$ increases and more fish are included for multiple scattering calculation, (1) both the time-harmonic singly scattered intensities and fully scattered intensities that include multiple scattering are expected to increase but at different rate, and (2) the differences between the singly and fully scattered intensities, $\epsilon$, defined as

$$\epsilon = \Phi_s(r, m, f) - \Phi^{(1)}_s(r, m, f)$$

(3.18)

are also expected to increase with increasing radius. Because the spherical spreading loss that acoustic field undertakes from one fish to another increases as a function of $d^2$, where $d$ is the inter-fish spacing, the differences, $\epsilon$, are expected to stay constant beyond a given radius $R_c$, where the multiply scattered field from fish with large spatial separations become negligible. To account for the multiple scattering and attenuation
effects from distributed fish group especially in the forward direction, both the real and imaginary part of the scattered field are considered when calculate the differences $\epsilon$. To account for the randomness in fish group and fluctuations typically present in the measured data, 100 independent Monte Carlo simulations are used to compute the scattered field statistics for the $m$th fish. The second moment of differences between the time-harmonic fully scattered intensities that including multiple scattering $\Phi_s(r, m, f)$ and the time-harmonic singly scattered intensities $\Phi_s^{(1)}(r, m, f)$ from the $m$th fish at frequency $f$, $\langle |\epsilon|^2 \rangle$, defined by

$$\langle |\epsilon|^2 \rangle = \left\langle \left| \Phi_s(r, m, f) - \Phi_s^{(1)}(r, m, f) \right|^2 \right\rangle$$ (3.19)

is used as a criteria to determine when the rate of increment in singly and fully scattered field becomes roughly the same as the radius $R$ increases. When the radius $R$ is greater than the expected $R_c$, the second moment of the differences, $\langle |\epsilon|^2 \rangle$, is expected to plateau and stay constant as a function of increasing $R$. In addition, the convergence radius $R_c$ is also expected to be a function of imaging frequency.

The second moments of the differences between the time-harmonic fully scattered intensities and the time-harmonic singly scattered intensities, $\langle |\epsilon|^2 \rangle$, from the $m$th herring within a group that contains 32,000 individuals are shown as a function of radius $R$ for increasing imaging frequencies centered at 415, 735, 950, 1125, and 1600 Hz. The second moments $\langle |\epsilon|^2 \rangle$ increases as the imaging frequency approaches the resonance frequency of herring’s swimbladder as expected. For the typical observed fish density, the second moments $\langle |\epsilon|^2 \rangle$ is shown to plateau and roughly stay constant when the radius $R$ is greater than 200 m for all imaging frequencies shown in Fig. 3.18. This numerically determined convergence radius of $R_c = 200$ m will be employed in Sec. 3.4.11 for simplification of multiple scattering calculation from a large herring shoal that
Figure 3.18: Second moments of differences between the time-harmonic fully scattered intensities that including multiple scattering and the time-harmonic singly scattered intensities from the $m$th fish, $\langle |\epsilon|^2 \rangle$, are plotted as a function of radius $R$ at multiple frequencies. As $R$ increases, more nearest neighbors surrounding the $m$th fish are included for multiple scattering calculation. $\langle |\epsilon|^2 \rangle$ plateaus and stay constant when $R$ is greater than 200 m for all imaging frequencies.
spans multiple range and cross-range resolution cells of the waveguide remote sensing systems in a random range-dependent ocean waveguide.

3.4.11 2D fully scattered field including multiple scattering from a 3D random fish group spanning multiple range and cross-range resolution cells of a waveguide remote sensing system

Here, the numerically determined convergence radius and the models described in Sec. 3.2 are used to simulate the broadband matched filtered fully scattered intensities that including multiple scattering from a large herring group spanning multiple range and cross-range resolution cells of the waveguide remote sensing system in a random range-dependent ocean waveguide. To simplify the numerical implementation, we only consider a monostatic waveguide remote sensing system where the horizontal positions of the source and receiver arrays are also co-located. The imaging frequency band is centered at $f_c = 950$ Hz with 50 Hz bandwidth. The simulated herring group contains a total of $N = 68,616$ individuals with volumetric density of $n_V = 0.05$ fish $m^{-3}$ uniformly distributed over a volume similar to an ellipsoid that is centered at $r_C = 3$ km horizontal range from the source and receiver arrays over the broadside direction of the receiver array and at $z_s = 150$ m depth with axes dimensions of $L_r = 100$ m in range, $L_{cr} = 800$ m in cross-range, and $L_z = 20$ m in depth. The cross-range extension of the modeled herring group spans more than 5 times of the width of broadside beam of the receiver array, and the range extension spans more than 6 times of the range resolution (15 m) of the waveguide remote sensing system. The 3D spatial configurations that fish group adopt are derived from the partial random lattice configuration. Similar to the examples shown in Sec. 3.4, the source and receiver arrays are centered at depth $z_0 = 65$ m and
$z_r = 105 \text{ m}$ respectively to match the typical imaging geometries of the OAWRS system employed in GOME’06. The results are normalized for $0 \text{ dB re } 1 \mu \text{Pa at } 1\text{m source level.}$

For this example, the array gain is fully exploited by beamforming the modeled array measurements. The mid-frequency linear aperture of the FORA array, which contains 64 linear-spaced hydrophones and has an aperture length of $L = 47.25 \text{ m}$ with a designed cut-off frequency of $f_{\text{cutoff}} = 1000 \text{ Hz}$ [9], is used to measure the scattered field from the herring group. The center of the modeled herring group coincides with the broadside direction of the horizontal receiver array. In azimuthal direction, the herring group is divided into several segments, each of which spans 1 degree beam of the receiver array. The waveguide Green’s function from source to fish locations within each of these segments are computed separately using the stochastic model described in Sec. 3.2.1. The correlation length scale of internal waves in cross-range direction is also assumed to be the same as in the range direction (500 m) to account for the linear internal wave effects in randomizing the acoustic field in the waveguide. Therefore, the sound speed profiles are updated every 500 m in both range and cross-range directions. The same scheme is applied for computing the waveguide Green’s function from fish locations to all 64 hydrophones of the receiver array. The time-harmonic fully scattered intensities $\Phi_s(r_i, f)$ that including multiple scattering from the herring group measured by the $i$th hydrophone of the receiver array are obtained from Eq. (D.3), where $r_i = (0, y_i, z_r)$ is the location of the $i$th hydrophone of the receiver array. Using the far-field approximation, the beamformed time-harmonic fully scattered field can be expressed as a function of array scan angle $\theta$,

$$\Phi_B(s, f) = \int_{-\infty}^{\infty} T(y_i) \Phi_s(y_i, z_r, f)e^{iksy_i}dy_i$$

(3.20)

where $s = \sin \theta$, $k = 2\pi f/c$, and $T(y_i)$ is the array taper function. Here, a Hanning
spatial window is applied to reduce sidelobe levels by more than 30 dB from the main lobe [9]. Substituting Eq. (D.3) into Eq. (3.20), the beamformed time-harmonic fully scattered field can be decomposed into,

$$\Phi_B(s, f) = \Phi^{(1)}_B(s, f) + \Phi^{(MS)}_B(s, f)$$

$$= \int_{-\infty}^{\infty} T(y_i) \Phi^{(1)}_s(y_i, z_r, f) e^{iksy_i} dy_i + \int_{-\infty}^{\infty} T(y_i) \Phi^{(MS)}_s(y_i, z_r, f) e^{iksy_i} dy_i$$  (3.21) a sum of the beamformed time-harmonic singly scattered field, $\Phi^{(1)}_B(s, f)$, and the beamformed time-harmonic multiply scattered field, $\Phi^{(MS)}_B(s, f)$.

After applying the matched filter (Eq. (D.5)) and Fourier synthesis, the beamformed and matched filtered time-dependent broadband fully scattered field $\Psi_M(s, t_M)$ from the large herring group, including multiple scattering, can be expressed as

$$\Psi_M(s, t_M) = \int_B \Phi_B(s, f) H(f|t_M) e^{-i2\pi ft} df$$  (3.22) an integral involving Eq. (3.20) over the signal bandwidth $B$. Substituting Eq. (3.21) into Eq. (3.22), the beamformed and matched filtered time-dependent fully scattered field can be decomposed into,

$$\Psi_M(s, t_M) = \Psi^{(1)}_M(s, t_M) + \Psi^{(MS)}_M(s, t_M)$$

$$= \int_B \Phi^{(1)}_B(s, f) H(f|t_M) e^{-i2\pi ft} df$$

$$+ \int_B \Phi^{(MS)}_B(s, f) H(f|t_M) e^{-i2\pi ft} df$$  (3.23) a sum of the beamformed and matched filtered time-dependent singly scattered field, $\Psi^{(1)}_M(s, t_M)$, and the beamformed and matched filtered time-dependent multiply scattered field, $\Psi^{(MS)}_M(s, t_M)$. 129
The coherent and incoherent beamformed and matched filtered broadband singly scattered intensities, $|\langle \Psi_M^{(1)}(s, t_M) \rangle|^2$ and $\text{Var}(\Psi_M^{(1)}(s, t_M))$, and the coherent and incoherent beamformed and matched filtered broadband fully scattered intensities, $|\langle \Psi_M(s, t_M) \rangle|^2$ and $\text{Var}(\Psi_M(s, t_M))$, from a large herring shoal in the Gulf of Maine, which include multiple scattering obtained from 30 independent broadband Monte-Carlo simulations are shown in Fig. 3.19(a)-(d), respectively. The variance or incoherent intensity exceeds the squared mean or coherent intensity by at least 20 dB over the entire 2D distribution of the herring group, which is consistent with the results shown in Sec. 3.4 for herring group with the same volumetric density and using the same imaging frequency band. In addition, by comparing Fig. 3.19(a) with 3.19(c), it is evident that the differences between the singly scattered field and fully scattered field that includes multiple scattering is negligible, which makes the single scattering assumption valid for inferring herring areal population densities directly from measured scattered field using the waveguide remote sensing system in a random range-dependent ocean waveguide.

### 3.5 Illustrative Examples B: Atlantic cod

Similar to Sec. 3.4, the theory described in Sec. 3.1 and models described in Sec. 3.3 are applied to simulate the broadband matched filtered scattered field from groups of Atlantic cod in the Ipswich Bay continental shelf environment, where the water depth varies between 40 and 80 m. The effects of (1) multiple scattering, (2) attenuation due to scattering, and (3) coherent effects, such as resonance shift and sub- and super-resonance local maxima, on fish population density imaging with waveguide remote sensing system are investigated.
Figure 3.19: (a) The incoherent $\text{Var}(\Psi_{M}^{(1)}(s,t_{M}))$ beamformed and matched filtered broadband singly scattered intensities from a herring group containing 68,616 individuals centered at 3 km from the co-located source and receiver arrays over the broadside direction of the horizontal receiver array, imaged using the waveform centered at $f_c = 950$ Hz with 50 Hz bandwidth and 0 dB re 1 $\mu$Pa at 1 m source level. The black dashed lines indicate the boundary of the herring group in range and cross-range direction with respect to the horizontal receiver array of the waveguide remote sensing system. (b) The coherent $|\langle \Psi_{M}^{(1)}(s,t_{M}) \rangle|^2$ beamformed and matched filtered broadband singly scattered intensities from the herring group. (c) The incoherent $\text{Var}(\Psi_{M}(s,t_{M}))$ beamformed and matched filtered broadband fully scattered intensities that including multiple scattering from the herring group. The 200 m convergence radius is applied to simplify the numerical multiple scattering implementation, in which only the multiple scattering between $N_p$ nearest neighbors of $p$th fish located within the spherical volume of radius 200 m are considered important and non-negligible. (d) The coherent $|\langle \Psi_{M}(s,t_{M}) \rangle|^2$ beamformed and matched filtered broadband fully scattered intensities that including multiple scattering from the herring group.
The modeled source waveform is the same as that used in Sec. 3.4 but with a center frequency $f_c$ of either 300, 415, 575, or 950 Hz, slightly different from those transmitted during GOME’06, because cod are bigger fish and their swimbladders are expected to resonate at lower frequencies than herring. The waveform centered at 415 Hz is considered in the majority of examples illustrated in this section since (1) the cod swimbladders are expected to resonate below 500 Hz, which will lead to a high fish scattered intensity to background reverberant intensity ratios and (2) it is expected that this waveform would provide scattered intensity images with good cross-range resolution [9, 4, 5]. The source and receiver arrays are centered at depths of $z_0=30$ m and $z_r=40$ m, respectively. Similar to the results shown in Sec. 3.4, the horizontal position of the source and receiver arrays are also co-located in the examples presented here to simplify charting of scattered returns in range, although a bistatic geometry might be employed in the future experiment. In all examples, the cod group is centered at 13.27 km horizontal range from the source and receiver arrays. All results are normalized for a 0 dB re 1 $\mu$Pa at 1 m source level. The bathymetric transect over which the acoustic wave propagates is provided in Sec. 3.3.1. The bottom is assumed to be sandy in all examples. 100 independent Monte-Carlo simulations are used to compute the scattered field statistics for each cod group discussed in this section. For cod groups considered here, the maximum range and cross range extents are limited to 500 m and 100 m respectively.

3.5.1 Coherent and incoherent broadband matched filtered scattered intensities that including multiple scattering from a cod group

The coherent and incoherent broadband matched filtered fully scattered intensities
from a group of cod in the Ipswich Bay continental shelf environment, which include multiple scattering, obtained from 100 independent broadband Monte-Carlo realizations are shown in Fig. 3.20(b) and (c). The imaging frequency band is centered at $f_c=415$ Hz. The 3D spatial configurations that cod group adopt are derived from the partially random lattice configurations discussed in Sec. 3.1.2. The areal density over the range extent of the cod group is shown in Fig. 3.20(a). The simulated fish group has a total of $N = 950$ individuals with volumetric density $n_V = 0.001$ fish m$^{-3}$ distributed within a volume centered at $z_s=75$ m depth with axes dimensions given by $L_r=500$ m in range, $L_{cr}=200$ m in cross-range, and $L_z=10$ m in depth. The $n_V = 0.001$ fish m$^{-3}$ fish volumetric density used here is the typical density of cod observed during the recent acoustic and trawl survey of spring spawning cod aggregations in Ipswich Bay [137].

Similar to what has been shown for herring in Fig. 3.7, the variance or incoherent intensity exceeds the squared mean or coherent intensity by about 20 dB, dominating the total scattered returns. For the given fish density, multiple scattering is found to be negligible for both the incoherent and coherent intensities. Differences between the singly scattered level and the fully scattered level that includes multiple scattering are less than 0.1 dB over the entire range extent of the fish group as shown in Fig. 3.21 and discussed in the next section.

### 3.5.2 Effects of multiple scattering and dependence on expected areal density and target strength of groups of cod

Here we examine the dependence of multiple scattering on cod target strength, which is a function of both the imaging frequency and population density.
Figure 3.20: (a) The areal density plotted as a function of range for a cod group in Ipswich Bay continental shelf environment with sandy bottom. The vertical lines indicate the range bounds of the cod group. (b) The incoherent $\text{Var}(\Psi_s(t_M))$ and coherent $|\langle \Psi_s(t_M) \rangle|^2$ broadband matched filtered fully scattered intensities that include multiple scattering, and the incoherent $\text{Var}(\Psi_s^{(1)}(t_M))$ and coherent $|\langle \Psi_s^{(1)}(t_M) \rangle|^2$ broadband matched filtered singly scattered intensities from the cod group imaged using the waveform centered at $f_c=415$ Hz with 50 Hz bandwidth and 0 dB re 1 $\mu$Pa at 1 m source level. (c) Identical to (b) but plotted in logarithmic scale. The error bar indicates the standard deviation of the broadband matched filtered fully scattered intensities from the cod group.
The effect of varying cod target strength by varying the imaging frequency band is investigated in Fig. 3.21. We consider broadband waveforms with the center frequencies, 300 Hz, 415 Hz, 575 Hz, and 950 Hz. The parameters of the cod group and waveguide environment are identical to the example shown in Fig. 3.20. The scattered field levels from the fish group increase dramatically with target strength as the imaging frequency bands approach the resonance frequency of cod swimbladder. Multiple scattering effects are only noticeable at the frequency band centered at \( f_c = 415 \) Hz for the fish density considered. Even in this case, the differences between the singly scattered intensity levels and the fully scattered intensity levels that include multiple scattering are less than 0.5 dB.

The effects of multiple scattering are investigated as a function of increasing fish density in Fig. 3.22 for the \( f_c = 415 \) Hz band. The fish groups have identical dimensions \( L_r = 100 \) m, \( L_{cr} = 100 \) m, and \( L_z = 10 \) m, but contain (1) \( N = 95 \), (2) \( N = 580 \), and (3) \( N = 2960 \) individuals with partially random lattice spatial configurations in each case. These examples correspond to fish volumetric densities of 0.001, 0.006, and 0.03 fish \( \text{m}^{-3} \), and areal densities of 0.01, 0.06 and 0.3 fish \( \text{m}^{-2} \), respectively, as shown in Fig. 3.22(a). The 0.3 fish \( \text{m}^{-2} \) is the densest cod aggregation observed from recently conducted acoustic and trawl surveys in Placentia Bay, Newfoundland [145] All other properties of the fish group and waveguide environment are identical to the example shown in Fig. 3.20. The singly and fully scattered intensities are roughly identical except when the fish areal density approaches the highest density considered. In this case, the fully scattered intensities differ from the singly scattered intensities significantly by more than 3 dB or a factor of 2. The attenuation through the range extent of a dense cod aggregation is prominent at frequencies near the swimbladder resonance of cod. This significant attenuation effect in scattering from cod will be investigated in greater detail in the next
Figure 3.21: Effect of varying the imaging frequency band on the incoherent matched filtered scattered returns from a fish group.  (a) The areal density of the fish group.  (b) The incoherent fully scattered intensity $\text{Var}(\Psi_s(t_M))$ that includes multiple scattering and singly scattered intensity $\text{Var}(\Psi_s^{(1)}(t_M))$ from the fish group are plotted as a function of the imaging frequency band.  (c) Identical to (b) but plotted in logarithmic scale. The error bars show the standard deviation of the broadband matched filtered fully scattered intensities from the fish group at various imaging frequency bands.
Figure 3.22: Effect of varying the fish areal density on the broadband incoherent matched filtered scattered returns from a fish group imaged with frequency band centered at $f_c = 415$ Hz. (a) The areal fish densities of three distinct fish groups. (b) The incoherent fully $\text{Var}(\Psi_s(t_M))$ and singly $\text{Var}(\Psi_s^{(1)}(t_M))$ scattered intensities from the fish groups with imaging frequency band centered at 415 Hz. The cumulative attenuation effects due to scattering are significant at the highest areal density considered. (c) Identical to (b) but plotted in logarithmic scale. The error bars show the standard deviation of the broadband matched filtered fully scattered intensities from fish group with various fish areal densities.
These examples show that the single scattering assumption is valid and should be employed to estimate cod areal population densities from their scattered intensities in the potential future experiment using the waveguide remote sensing system with frequency bands centered above or below the expected resonance frequency of cod aggregation with areal densities less than or equal to 0.06 fish m$^{-2}$, since multiple scattering is negligibly small. For areal population density imaging of cod at frequency bands near the swimbladder resonance of cod, attenuation due to scattering can be prominent at high fish densities, where the fully scattered returns can be significantly smaller than the singly scattered returns over the range extent of the cod aggregation, making the single scattering assumption invalid.

### 3.5.3 Attenuation from scattering through a fish group and its dependence on swimbladder resonance damping

Here we investigate the effect of varying the damping coefficients [142] on the scattered intensity from a group of cod in Figs. 3.23 and 3.25 with imaging frequency bands centered at $f_c = 415$ Hz and $f_c = 950$ Hz, respectively. The fish group considered contains $N = 2960$ individuals with volumetric density of 0.03 fish m$^{-3}$, areal density of 0.3 fish m$^{-2}$, and dimensions $L_r = 200$ m, $L_{cr} = 50$ m, $L_z = 10$ m, as shown in Figs. 3.23(a) and 3.25(a). All other properties of the fish group and waveguide environment are identical to the example shown in Fig. 3.20. Figures 3.23(b)-(d) and Figs. 3.25(b)-(d) include both viscous and radiation damping, but the viscosity of fish flesh is varied from 20 to 120 Pa s. These should be compared with the results in Figs. 3.23(e) and 3.25(e) where only radiation damping is included.
When viscous damping is included, attenuation through the group negates the effects of multiple scattering and the overall backscattered intensity level compared to the singly backscattered intensity level depends on the amount of viscous damping included in the model and the imaging frequency bands. For imaging frequency band centered at $f_c = 415$ Hz near the expected swimbladder resonance of modeled cod group, when large viscosity $\xi$ is used to model the complex scatter function of an individual cod, the attenuation effects are much more prominent than that of using a small $\xi$, as can be seen from Figs. 3.23(b)-(d). When small or no viscous damping is included, as shown in Figs. 3.23(d)-(e), the fully scattered incoherent intensity that includes multiple scattering increases cumulatively above the singly scattered intensity over the range extent of the group. As larger viscosity $\xi$ used in the model, the matched filtered singly scattered incoherent intensities also decreases due to the increasing energy loss from viscous damping, as shown in Fig. 3.24. In contrast, for imaging frequency band centered at $f_c = 950$ Hz, both multiple scattering and attenuation effects are found to be negligible and independent of varying damping coefficients, as can be seen from Fig. 3.25(b)-(e).

The mean broadband incident intensity in the random waveguide, along with the mean total intensity *forward* propagated through the cod group are shown in Figs. 3.26 (a) and (b) for the imaging frequency bands centered at 415 and 950 Hz respectively, where both intensities are averaged over the 10 m depth layer of the fish group. For imaging frequency band centered at 415 Hz, when viscous damping is absent, the mean total forward propagated intensity is higher than the mean incident intensity because multiple scattering raises the overall level and the effect is cumulative over the range extent of the group. In the cases when viscous damping is present, but with small viscosity, the total forward propagated intensities fall below the incident intensity 1/4
Figure 3.23: Effect of varying fish swimbladder damping on the incoherent matched filtered scattered returns with imaging frequency band centered at 415 Hz. (a) The areal fish density of the fish group. (b)-(e) The incoherent fully $\text{Var}(\Psi_s(t_M))$ and singly $\text{Var}(\Psi_s^{(1)}(t_M))$ scattered intensities from the fish group are plotted as a function of fish damping coefficient.
way through the fish group from the front end, where the attenuation effects dominate the cumulative multiple scattering effects. However, the cumulative multiple scattering effects built up slowly over the range extent of the fish group and overtake the attenuation effects gradually, so that the two effects roughly cancel each other at the back end of the fish group, as can be seen from Fig. 3.26(a). When large viscosity is used to model the complex scatter function of fish, the total forward propagated intensities can fall below the incident intensities because of the prominent attenuation effects. For imaging frequency band centered at 950 Hz, the total forward propagated intensities do not vary as a function of damping coefficients, implying that the cumulative multiple scattering and attenuation effects on forward propagated intensities are negligible.

The analysis in this section indicates that both multiple scattering, and attenuation due to scattering are expected to be significant for high cod densities and at the imaging frequency band near the expected swimbladder resonance frequency of modeled cod group. An accurate knowledge of cod’s swimbladder resonance damping is required to determine the amount of attenuation present in the data, which directly affects the accuracy in cod abundance estimation from measured scattered intensities.
Figure 3.25: Identical to Fig. 3.23 but for imaging frequency band centered at 950 Hz.
Figure 3.26: The mean total acoustic intensity *forward* propagated through the fish group in Figs. 3.23 and 3.25 are plotted as a function of fish swimbladder damping for imaging frequency bands centered at (a) $f_c=415$ Hz and (b) $f_c=950$ Hz. The broadband incident and forward propagated (which include multiple scattering from other fish in the group) intensities are averaged over the fish layer depth and over the 100 independent Monte-Carlo realizations.
with the OAWRS system. When the imaging frequency bands are off the swimbladder resonance frequency of modeled cod group, the effects of both multiple scattering and attenuation are shown to be negligible even with high cod densities, making the single scattering assumption valid for estimating cod areal population densities from their scattered returns measured in the potential future experiment by the OAWRS system.

Accurate characterization of swimbladder resonance damping is not an easy task, particularly for live in-situ fish, since it depends on many factors including physiological and physical parameters of fish swimbladder wall and surrounding tissues, which vary both temporally and across species. For instance, in Ref. [146] Sand and Hawkins have shown that not only is cod able to restore the volume of its swimbladder by the slow process of secretion or absorption of gas in order to maintain neutrally buoyant after a rapid depth change, but also it can quickly restore the volume of swimbladder via an active muscular response, which maintains a muscular tonus by increasing or relaxing the stiffness of the surrounding tissues. The maintenance of a tonus in the surrounding tissues implies that cod can potentially control the buoyancy actively, which may makes their resonance frequency exhibiting a temporal variation particularly during the night-time observations when they typically form sparse aggregations and exhibit regular vertical migrations.

3.5.4 Coherent multiple scattering effects such as resonance shift and sub- and super-resonance local maxima for cod population density imaging

Here, we examine the coherent effects, such as resonance shift and sub- and super-resonance local maxima, on population density imaging of cod aggregations.
Two distinct fish groups are considered, one with dimensions much larger than the acoustic wavelength and the other with dimensions on the order of a few acoustic wavelengths. The large cod group contains 950 individuals with volumetric density \( n_V = 0.001 \text{ fish m}^{-3} \) distributed within a volume of axes dimensions given by \( L_x = 500 \) m, \( L_y = 200 \) m and \( L_z = 10 \), while the smaller group contains 848 individuals distributed within a smaller volume of axes dimensions given by \( L_x = 10 \) m, \( L_y = 10 \) m and \( L_z = 10 \) m, with a volumetric density \( n_V = 1 \text{ fish m}^{-3} \) that is many orders of magnitude denser than the cod groups typically observed during recent acoustic and trawl surveys of spawning cod aggregations in Ipswich Bay continental shelf environment. Both groups are centered at \( z_s = 75 \) m depth and are imaged by a monostatic direct-path imaging system. We only consider the partially random lattice 3D configurations for both the large and small cod group.

The analysis in this section follow the approach outlined in Sec. 3.4.8. The coherent, incoherent, total and estimated school target strength spectra calculated using Eqs. (3.5), (3.6), (3.7), and (3.8) are plotted in Figs. 3.27(a) and (b) for the large and small fish groups respectively. In comparison to the results shown in Fig. 3.16(a) for herring, similar results are obtained for cod when the school dimensions are many orders of magnitude larger than the acoustic wavelength. In this case, the incoherent intensity dominates the total scattered returns, which is also well approximated by the estimated school target strength spectrum assuming single scattering.

In contrast, for the small dense cod group, resonance shifts to both sub- and super-resonance frequencies are evident, which result in multiple prominent peaks on the time harmonic fully scattered field moments, as shown in Fig. 3.27(b). The incoherent intensity dominates at sub-resonance frequencies which cause the school resonance shifting to
Figure 3.27: Coherent effects on the time harmonic fully scattered field moments, including multiple scattering, as a function of frequency for a monostatic direct-path imaging system examined for (a) a large cod group containing 950 individuals and (b) a small dense cod group containing 848 individuals. The coherent, incoherent, total and estimated school target strength spectra calculated via Eqs. (3.5) to (3.8) respectively are compared as a function of the fish group dimension.

lower frequencies than that of an individual fish, while the coherent intensity are significant at higher frequencies, which lead to a resonance shift towards the super-resonance frequencies of an individual fish. In this case, as the inter-fish spacing is approximately the order of fish length, the collective resonance behavior of dense fish group shown in Fig. 3.27(b) is consistent with the findings in Fig. 8 of Ref. [27].

3.6 Summary

A numerical model has been developed to determine the statistical moments of the broadband matched filtered scattered field including multiple scattering from a random 3D spatial distribution of random scatterers spanning multiple range and cross-range resolution cells of a waveguide remote sensing system in a random range-dependent
ocean waveguide by Monte-Carlo simulation. The model can be applied to analyze bistatic scattering from a scatterer group in any direction including the forward. When combined with the incident field, the model can accurately characterize attenuation in forward propagation through a group. The model is applied to (1) examine population density imaging of shoaling Atlantic herring in the Gulf of Maine during GOME’06, and (2) investigate wide-area imaging of sparse aggregations of Atlantic cod, an important ground fish species in the Ipswich Bay continental shelf environment using a waveguide remote sensing system in the 300 to 1200 Hz frequency range. The fish swimbladder is modeled as damped air-filled prolate spheroid surrounded by viscous flesh using a complex scatter function.

Analysis with the model indicate that (1) the incoherent intensity dominated the scattered returns from large herring groups imaged during GOME’06 and sparse aggregations of cod in the Ipswich Bay continental shelf environment, effects such as resonance frequency shift and sub- and super-resonance local maxima are negligible or absent, (2) the single scattering assumption is valid for inferring herring and cod areal population densities from their matched filtered scattered intensities over the imaging frequency range at densities typically observed during in-situ acoustic surveys, (3) waveguide dispersion had negligible effect on shoal population estimation, (4) the standard deviations of the instantaneous broadband matched filtered scattered intensities from the fish groups are roughly 5 dB indicating that the scattered fields are fully saturated and the standard deviations can be reduced by stationary averaging, (5) the charting speed for accurately localizing targets in range from their time-dependent scattered fields after matched filtering in a random waveguide is approximately equal to the average group speed of the lowest order mode, (6) the imaging system’s range resolution in the waveguide is approximately equal to its resolution in free space where it is inversely
proportional to the signal bandwidth, and (7) attenuation due to scattering in forward propagation through fish groups can negate the effects of multiple scattering and are highly dependent on the accurate characterization of swimbladder resonance damping.
Chapter 4

Accuracy of passive source localization approaches with a single towed horizontal line-array in an ocean waveguide

Here we investigate instantaneous passive localization and tracking of acoustic sources over long ranges with measurements made on a single towed horizontal line receiver array in a random range-dependent ocean waveguide. Towed horizontal receiver arrays are employed in a wide range of applications in the ocean, such as naval operations for detecting and tracking underwater vehicles [147]; active and/or passive sensing of marine life [3, 4, 9, 5, 10, 11, 12], oceanography [13, 14, 15], and ocean geology [16, 17, 18]; and for oil and natural gas exploration [148, 149, 150]. An advantage of sensing with a horizontal array of hydrophones is that the bearing of the sound source can be directly obtained by beamforming the received signals so that only the range of the source to the receiver has to be determined.

Four distinct methods that can provide instantaneous or near-instantaneous estimates of source range in the far-field of a single towed horizontal receiver array are
examined. They include (1) synthetic aperture tracking (SAT), which combines measurements made on adjacent or widely separated finite apertures of a single towed array and employs the conventional triangulation ranging algorithm for localizing sources located in the far-field of the receiver array; (2) array invariant (AI), a technique that exploits the dispersive modal arrival structure of the acoustic field in an ocean waveguide to estimate the source range for sources located off the broadside beam of the receiver array; (3) the bearings-only target motion analysis in modified polar coordinates implemented using the extended Kalman filter (MPC-EKF) where the bearing and range components of the source location and velocity state vector are decoupled [46], and (4) bearings-migration minimum mean square error (MMSE), also based on triangulation but combines sequential bearing measurements in a global inversion for the mean source position over the measurement time interval. These methods are applied to localize and track a vertical source array deployed in the far-field of a towed horizontal receiver array during the Gulf of Maine 2006 Experiment (GOME’06). The source transmitted intermittent broadband pulses in the 300-1200 Hz frequency range. The performance of all four methods are evaluated for a wide variety of source-receiver geometries and range separations up to 20 km. The localization errors are determined by comparing the estimated source ranges with the true ranges obtained from GPS measurements of source and receiver locations. The accuracy statistics obtained from localizing acoustic sources with known GPS positions can be used to gain insight into the performance of all four methods in various source tracking scenarios. This can aid in the selection of an optimal passive source localization methods and provide estimation bounds for a similar source tracking scenario when the source positions are unknown.

Other methods for passive source localization in an ocean waveguide include hyperbolic ranging [151, 152, 153, 154, 155] with measurement from two or more widely
separated single hydrophones or vertical receiver arrays, and triangulation [156, 157, 158] from measurements made on two or more horizontal receiver arrays. The source has to be located in the near-field of the overall sensor configuration in the hyperbolic ranging method so the detection range in practical applications are often limited to less than 10 km. When more than one receiver array is available, source localization can be accomplished by including adaptive state estimators in recursive algorithms using time-delay measurements [63, 159] or passive multi-path time-delay measurements of reflections from ocean boundaries [160, 161] to aid in the range estimates.

When only a single horizontal moving receiver array is available, the passive techniques developed for localizing acoustic sources located in the far-field of the array can be categorized as either recursive approaches using bearings-only measurements or far-field waveguide techniques that rely on waveguide effects such as modal dispersion or interference. A large number of nonlinear filters have been developed for bearings-only tracking of both nonmaneuvering [42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52] and maneuvering targets [53, 54, 55, 56]. These filters are designed to optimize the efficiency, enhance the stability and robustness of the localization algorithms by deriving the theoretical limits of the Cramer-Rao lower bound (CRLB) for the proposed nonlinear filters. These nonlinear filters have been primarily tested with simulated data and have limited application to real or field data. The SAT, MMSE and MPC-EKF approaches investigated here belong in the category of bearings-only measurement approaches. In addition to the array invariant (AI) method [65] investigated here, other waveguide techniques for passive source localization include matched field processing (MFP) [57, 58, 59] and the waveguide invariant [60, 61, 62]. Match field processing is computationally expensive and highly susceptible to waveguide modal interference mismatch when adequate knowledge of the ocean environment is not available. The waveguide invariant localizes a moving
broadband source from measured incoherent acoustic intensity striation patterns in frequency versus range plots, provided that the slope of the striation patterns are known accurately [162]. The waveguide invariant can be applied for near-instantaneous source localization only if the repetition rate of source transmission is high in order for the slope of incoherent intensity striations to be noted [163]. For the data analyzed here, this rate is 0.8 min$^{-1}$, which is far smaller than that required for a near-instantaneous source localization using the waveguide invariant.

The four distinct passive source localization approaches are described in Sec. 4.1. Data processing schemes applied for deriving and enhancing localization accuracy are presented in Sec. 4.2. In Sec. 4.3, the performance of source range estimates and source horizontal trajectory tracking by all four methods was experimentally demonstrated with data from the Gulf of Maine 2006 Experiment in four most representative source-receiver geometries. Localization accuracies are quantified by comparing the estimated source ranges and horizontal trajectories to those measured by the ship-board GPS modules for all four methods in five distinct tracking scenarios and the effect of source range and source bearing relative to the broadside direction of the receiver array on the localization accuracy are examined in Sec. 4.4.

4.1 Passive source localization methods with a single towed horizontal hydrophone line-array

4.1.1 Synthetic Aperture Tracking

Here, the theory of SAT is formulated for a moving receiver array in terms of pairs of bearing measurements of a source. The principle is to form a synthetic array
by combining spatially separated finite apertures of a single towed horizontal line-array and apply the conventional triangulation ranging algorithm [63, 64] to localize the source located in the far-field of the receiver array. At each time instance \( t \), the source range is determined as the third point of a triangle from the intersection of the straight lines of pairs of source bearing estimates. This process is repeated for every adjacent pair of source bearing estimates. Finally, the SAT maps the sequential source range estimates onto a cartesian grid to predict the source horizontal trajectory. The SAT assumes that the source is stationary over adjacent pairs of bearing measurements. It can be applied to localize and track a moving source if certain conditions are satisfied, such as the source is slowly drifting.

Consider the geometry shown in Fig. 4.1, where the origin of the coordinate system is placed at the receiver array center at the start of a tow track at time \( t = 0 \). The \( x \) coordinate points East and the \( y \) coordinate points North. The angular course of the array center \( \theta(t) \) is measured clockwise from true North, and the corresponding receiver array center positions \( r_r(t) = (x_r(t), y_r(t)) \) are determined from the ship-board GPS. With a high-resolution horizontal line-array [9], the bearing of the source can be estimated by conventional plane-wave beamforming [164, 9]. Let \( \hat{\beta}_0 = [\hat{\beta}_0(t_0), \hat{\beta}_0(t_1), \ldots, \hat{\beta}_0(t_N)] \) be the sequential source bearing estimates measured clockwise from true North. The projected source range \( \hat{R}_0 \), tangent to the receiver tow track as shown in Fig. 4.1, at each time \( t \) has the following relationship

\[
\int_{t-\Delta t}^{t} v_r(\tau) d\tau = \hat{R}_0(t)\{\cot[\hat{\beta}_0(t - \Delta t) - \theta(t)] - \cot[\hat{\beta}_0(t) - \theta(t)]\}
\]

(4.1)

with a pair of source bearing estimates \( \hat{\beta}_0(t) \) and \( \hat{\beta}_0(t-\Delta t) \), receiver array course \( \theta(t) \), and corresponding GPS-measured receiver array velocity \( v_r(\tau) \). Taking the time-derivative
on both sides of Eq. (4.1), we obtain

$$\widehat{R}_0(t) = \frac{\bar{v}_r(t) \sin^2 \gamma(t)}{\dot{\beta}_0(t)} \tag{4.2}$$

in terms of the time-derivative of source bearing estimates $\dot{\beta}_0(t) = d\dot{\beta}_0(t)/dt$ (rads s$^{-1}$) and mean receiver array velocity $\bar{v}_r(t) = \sqrt{(x_r(t) - x_r(t - \Delta t))^2 + (y_r(t) - y_r(t - \Delta t))^2}/dt$ (m s$^{-1}$), where $\gamma(t) = \dot{\beta}_0(t) - \theta(t)$ is in radian. The source range $\widehat{R}_{s SAT}^S(t)$ from the receiver array center at time $t$ is

$$\widehat{R}_{s SAT}^S(t) = \frac{\widehat{R}_0(t)}{\sin \gamma(t)} = \frac{\bar{v}_r(t) \sin \gamma(t)}{\dot{\beta}_0(t)} \tag{4.3}$$
and the source horizontal location \( \hat{\mathbf{r}}_{s}^{SAT}(t) = (\hat{x}_{s}^{SAT}(t), \hat{y}_{s}^{SAT}(t)) \) is obtained from

\[
\begin{align*}
\hat{x}_{s}^{SAT}(t) & = x_{r}(t) + \hat{R}_{s}^{SAT}(t) \cos \phi(t) \\
\hat{y}_{s}^{SAT}(t) & = y_{r}(t) + \hat{R}_{s}^{SAT}(t) \sin \phi(t)
\end{align*}
\]

(4.4)

where \( \phi(t) = \pi/2 - \hat{\beta}_{0}(t) \).

Since the SAT can provide an estimate of the source location from two adjacent source bearing measurements, it is nearly instantaneous. For a non-stationary source with velocity \( v_{s}(t) \), SAT can still provide accurate and robust source localization if the condition \( v_{r}(t) \gg v_{s}(t) \) is satisfied, as will be discussed in Sec. 4.3.

### 4.1.2 The array invariant method

The array invariant (AI) method [65] provides instantaneous source range estimation by exploiting the multi-modal dispersive behavior of guided wave propagation in a dispersive ocean waveguide. The AI is applied to passive beam-time intensity data obtained after conventional plane-wave beamforming and matched filtering of acoustic measurements received on a horizontal array of hydrophones. It has been shown in Ref. [65] that the migration angle of maximum beam-time intensity, defined as the array invariant \( \chi_{h} \), is invariant to environmental parameters but follows a known and unique dependence on source-receiver range. The AI method has been applied to field data for source-receiver separations of up to 10 km [65].

Here, we apply Eqs. (28) and (27) of Ref. [65] to estimate the array invariant \( \hat{\chi}_{h} \) and the source range \( \hat{R}_{s}^{AI} \) respectively from the beam-time intensity data \( L_{bt}(s,t) \). Similar to SAT, the AI-estimated source positions \( \hat{\mathbf{r}}_{s}^{AI}(t) = (\hat{x}_{s}^{AI}(t), \hat{y}_{s}^{AI}(t)) \) can be calculated
using Eq. (4.4), given the known receiver array position \( \mathbf{r}_r(t) = (x_r(t), y_r(t)) \) and \( \phi(t) \). The experimentally measured sound speed \( c(z) \) at receiver array depth \( z \) used in the calculations are obtained from expendable bathythermographs (XBT) and conductivity-temperature-depth (CTD) measurements shown in Fig. 3 of Ref. [9]. As explained in Ref. [65], because \( \hat{\chi}_h \) for a horizontal array is insensitive to range at near-broadside incidence, only data that have \( 10^\circ \leq |\hat{\beta}_0| \leq 90^\circ \) were used in range estimation.

### 4.1.3 Recursive nonlinear filters for bearings-only TMA

There are many existing recursive type nonlinear filters in Bayesian framework available for bearings-only source localization and tracking with a single moving observer. A comprehensive review of these filters, their theoretical limitations and performance bounds can be found in Ref. [165]. Here the MPC-EKF is employed to estimate the source horizontal trajectory because (1) it is computationally efficient over other existing recursive Bayesian estimators such as the particle filters [50, 52], (2) it can provide asymptotically unbiased source states estimates [46], and (3) it automatically decouples the observable bearing from the unobservable range component of the estimated source states and prevents error covariance matrix ill-conditioning, which is the primary cause of filter divergence and instability [46]. This observability issue arises from the fact that the measured bearing history by a single array is ambiguous and not unique to the unknown target trajectories for a constant velocity receiver. Furthermore, receiver maneuver is a necessary prerequisite for all available recursive Bayesian estimators to obtain a stable and converged solution for bearing-only source localization and tracking. However, the majority of the receiver tracks conducted during GOME’06 were straight lines, making it unnecessary to apply the more computationally intensive approach, such
Table 4.1: Parameters used for initializing the target state vector and error covariance matrix for the MPC-EKF method.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_i ) (km)</td>
<td>mean = ( \bar{R}_i = (\hat{R}_i^{SAT} + \hat{R}_i^{AI})/2 )</td>
<td>Initial target range</td>
</tr>
<tr>
<td>( v_i ) (m/s)</td>
<td>( \bar{v}_i = 2 )</td>
<td>Initial target speed</td>
</tr>
<tr>
<td>( \beta_i ) (deg)</td>
<td>( \bar{\beta}_i = \hat{\beta}_0 )</td>
<td>Initial target bearing measured clockwise from the true north</td>
</tr>
<tr>
<td>( \alpha_i ) (deg)</td>
<td>( \bar{\alpha}_i = \hat{\beta}_0 + 180 )</td>
<td>Initial target course</td>
</tr>
<tr>
<td>( \sigma_{R_i} )</td>
<td>2</td>
<td>Standard deviation of range estimate</td>
</tr>
<tr>
<td>( \sigma_{v_i} )</td>
<td>1</td>
<td>Standard deviation of speed estimate</td>
</tr>
<tr>
<td>( \sigma_{\beta_i} )</td>
<td>1.5</td>
<td>Standard deviation of bearing estimate</td>
</tr>
<tr>
<td>( \sigma_{\alpha_i} )</td>
<td>( \frac{180}{\sqrt{12}} )</td>
<td>Standard deviation of course estimate, uniformly distributed between ([-90, 90])</td>
</tr>
</tbody>
</table>

\( ^a \) Initial target range  
\( ^b \) Initial target speed  
\( ^c \) Initial target bearing measured clockwise from the true north  
\( ^d \) Initial target course  
\( ^e \) \( \hat{R}_i^{SAT} \) and \( \hat{R}_i^{AI} \) are the first instantaneously estimated source range by the SAT and AI, respectively, and \( \bar{R}_i \) takes the mean of the two estimates  
\( ^f \) \( \hat{\beta}_0 \) is the first bearing measurements by the horizontal receiver array  
\( ^g \) The uncertainty in the target’s course is assumed to be uniformly distributed in the span of \([-90, 90]\). Therefore, it has a standard deviation of \( \sigma_{\alpha_i} = \frac{180}{\sqrt{12}} \)

Accurate initialization of the source state vector and error covariance matrix is essential for the MPC-EKF to obtain good tracking performance \([165, 46]\). To enhance the stability of the algorithm and minimize filter divergence, we use the averaged initial range estimates from the SAT and AI, \( \bar{R}_i = (\hat{R}_i^{SAT} + \hat{R}_i^{AI})/2 \), and the first bearing estimate \( \hat{\beta}_0 \) as inputs to initialize the MPC-EKF following the procedure described by Eqs. (G.1) to (G.26) in Appendix G. The other parameters used for initializing the state vector and error covariance matrix are tabulated in Table 4.1. As shown in Sec. 4.4, taking advantage of the SAT and AI approaches for initialization improves the efficiency of the MPC-EKF by a factor of more than two and significantly reduces the root mean-square (RMS) error of the estimated source states by more than 50% compared to the conventional strategy of arbitrarily selecting an initial source range as input (see Sec. (6.4.1.1) of Ref. [165]).
4.1.4 Bearing-migration Minimum Mean Square Error method

The MMSE method is derived for estimating the mean source position over a measurement time interval when multiple (> 2) source bearing measurements are available. For a towed receiver array and a spatially stationary source, the sequential source bearing measurements will follow a unique, unambiguous, and nonlinear migration route over time. Taking advantage of the one-to-one mapping between the source position and the bearing-migration path, the expected mean source position

\[
\tilde{\mathbf{r}}_{s}^{\text{MMSE}} = \arg \min_{\hat{\mathbf{r}}_{s} \in S} \text{MSE}(\hat{B}(\tilde{\mathbf{r}}_{s})) = \arg \min_{\hat{\mathbf{r}}_{s} \in S} E\left[\left(\hat{B}(\tilde{\mathbf{r}}_{s}) - \hat{B}_{0}\right)^{2}\right]
\]

(4.5)
is determined by minimizing the mean square error of sequential source bearing estimates, where \( \hat{B}(\tilde{\mathbf{r}}_{s}) \) is the theoretical source bearing for the source position \( \tilde{\mathbf{r}}_{s} \) within the search space \( S \) and \( \hat{B}_{0} \) is the measured source bearing from the receiver array obtained by conventional beamforming.

4.2 Data processing and analysis

4.2.1 Source bearing estimation

To determine the bearing \( \hat{\beta}_{0} \) of a passive source from acoustic measurements by the horizontal receiver array, the time-series data on each hydrophone of the array are tapered by a Hanning-window, converted to beam-time data by conventional plane-wave beamforming, and matched filtered with the source signal replica to form a two-dimensional matched-filtered beam-time intensity data \( L_{bt}(s, t) \), where \( s = \sin \beta' \) is the
array scan angle. Here, we implement the method previously used in Sec. IIIA of Ref. [65] to estimate the source bearing \( \hat{\beta}_0' \), and the expected signal reception time \( \hat{t} \) by the receiver array. The bearing estimate \( \hat{\beta}_0' \) by conventional beamforming is first measured with respect to the broadside direction of the receiver array and then converted to a clockwise measurement from the true North \( \hat{\beta}_0 \) by correcting for the corresponding array heading measurement \( \alpha \). To break the inherent left-right ambiguity about the horizontal line-array’s axis, the converted sequential bearing estimates as well as its ambiguity counterpart are plotted together as a function of time [79, 80] and the true bearing sequence is selected to be the one with minimum fluctuations. Using the sequential source bearing estimates \( \hat{\beta}_0 = [\hat{\beta}_0(\hat{t}_1), \hat{\beta}_0(\hat{t}_2), \cdots, \hat{\beta}_0(\hat{t}_N)] \), and corresponding expected signal reception time sequence \( \hat{T} = [\hat{t}_1, \hat{t}_2, \cdots, \hat{t}_N] \), the time-derivative of source bearing estimates \( \dot{\hat{\beta}}_0(\hat{t}_i) \) can be derived using

\[
\dot{\hat{\beta}}_0(\hat{t}_i) = \frac{\hat{\beta}_0(\hat{t}_i) - \hat{\beta}_0(\hat{t}_{i-1})}{\Delta t_i} \tag{4.6}
\]

where \( \Delta t_i = \hat{t}_i - \hat{t}_{i-1}, \ i = 2, \cdots, N \).

4.2.2 Minimizing the effect of bearing estimation error on source localization with the synthetic aperture tracking method

Due to the randomness and multi-modal dispersion in a typical range-dependent ocean waveguide and the physical limitation in the angular resolution of the horizontal line-array used in the GOME’06, the bearing estimates \( \hat{\beta}_0(\hat{t}) \) by conventional beamforming may deviate from the true bearing \( \beta_0(\hat{t}) \) by \( \psi(\hat{t}) \), which is a non-Gaussian and non-stationary random variable that is highly dependent on the source receiver geometry and spatial-temporal-varying ocean environment. This uncertainty in bearing estimates results in a diamond shaped uncertainty region in range estimates given a bearing
measurement standard deviation $\hat{\sigma}_{\theta}$. To improve the accuracy in range estimates, a nonlinear curve fitting technique is needed to reduce the variance in bearing estimates. There are many existing nonlinear regression models can be used, such as the nonlinear least-squares, higher-order polynomial equations, and piecewise linear functions. Here, we implement a running-window averaged linear least-squares (RWA-LLS) method to minimize the variance in bearing estimation. First, an ordinary linear least-squares (LLS) estimator is applied to the bearing estimates $\hat{B}_i = [\hat{\beta}_0(\hat{t}_i), \hat{\beta}_0(\hat{t}_{i+1}), \ldots, \hat{\beta}_0(\hat{t}_{i+p-1})]$ within a 20-min running window, where $\hat{\beta}_0(\hat{t}_i)$ is the $i$th bearing estimates and $i = 1, 2, \cdots, N - p + 1$, given $N$ total number of bearing estimates. After obtaining all LLS fits $\hat{B}_{li} = [\hat{\beta}_l(\hat{t}_i), \hat{\beta}_l(\hat{t}_{i+1}), \ldots, \hat{\beta}_l(\hat{t}_{i+p-1})]$ within all $N - p + 1$ running windows, the expected nonlinear fits of bearing estimates $\hat{B}_m = [\hat{\beta}_m(\hat{t}_1), \hat{\beta}_m(\hat{t}_2), \ldots, \hat{\beta}_m(\hat{t}_N)]$ can be calculated by

$$\hat{\beta}_m(\hat{t}_i) = \begin{cases} \frac{1}{i} \sum_{j=1}^{i} \hat{B}_{lj,q} & 1 \leq i \leq p \\ \frac{1}{p} \sum_{j=i-p+1}^{i} \hat{B}_{lj,q} & p + 1 \leq i \leq N \end{cases} \quad (4.7)$$

where $q = i - j + 1$, and $\hat{B}_{lj,q}$ is the $q$th element in $\hat{B}_{lj}$, for instance $\hat{B}_{3,2} = \hat{\beta}_3(t_4)$, where $\hat{B}_3 = [\hat{\beta}_3(t_3), \hat{\beta}_3(t_4), \ldots, \hat{\beta}_3(t_{p+2})]$. Instead of using $\hat{\beta}_0(\hat{t}_i)$ to estimate $\hat{R}_s(\hat{t})$ by Eq. (4.3), now $\hat{\beta}_m(\hat{t})$ will be used to calculate the time-derivative of source bearing estimates $\dot{\hat{\beta}}_m(\hat{t})$ by Eq. (4.6).

### 4.2.3 Inferring source horizontal trajectory and mean locations

In many long-range passive source localization applications, such as marine mammal localizations, in the majority of the scenarios, the submerged target does not travel along straight lines. However, the ability to provide a relatively accurate estimation of target’s moving direction and its mean position within a given period of time from long ranges is of greater interests and more essential than to infer its exact moving
path. To predict the kinematics of an acoustic source, a LLS estimator is used here to approximate the source horizontal trajectory. Given the instantaneously estimated source horizontal positions \( \hat{\mathbf{r}}_s = (\hat{\mathbf{X}}_s, \hat{\mathbf{Y}}_s) \), where \( \hat{\mathbf{X}}_s = [\hat{x}_s(\hat{t}_1), \hat{x}_s(\hat{t}_2), \ldots, \hat{x}_s(\hat{t}_N)]^T \) and \( \hat{\mathbf{Y}}_s = [\hat{y}_s(\hat{t}_1), \hat{y}_s(\hat{t}_2), \ldots, \hat{y}_s(\hat{t}_N)]^T \), the LLS fit \( \hat{\mathbf{X}}_m \) and \( \hat{\mathbf{Y}}_m \) can be obtained by the linear regression

\[
[\hat{\mathbf{X}}_m \hat{\mathbf{Y}}_m]^T = [\hat{\mathbf{a}} \hat{\mathbf{b}}]^T \mathbf{T}^T
\]

(4.8)

where \( \hat{\mathbf{X}}_m = [\hat{x}_m(\hat{t}_1), \hat{x}_m(\hat{t}_2), \ldots, \hat{x}_m(\hat{t}_N)]^T \), \( \hat{\mathbf{Y}}_m = [\hat{y}_m(\hat{t}_1), \hat{y}_m(\hat{t}_2), \ldots, \hat{y}_m(\hat{t}_N)]^T \), \( \mathbf{T} = [(\hat{t}_1, \hat{t}_2, \ldots, \hat{t}_N)^T 1]^T \), \( \hat{t}_i = \hat{t}_{i-1} + \Delta t_i \), \( \Delta t_i \) is the time interval between two consecutive source signal receptions on the receiver array, which varies with time because of a continuous range variation, \( \mathbf{1} \) is an \( 1 \times N \) matrix given by \( \mathbf{1} = [1, 1, \ldots, 1] \), and \( \hat{\mathbf{a}} = [\hat{a}_x, \hat{a}_y]^T \) and \( \hat{\mathbf{b}} = [\hat{b}_x, \hat{b}_y]^T \) are the linear regression coefficients and intercepts, respectively, which can be estimated from

\[
[\hat{\mathbf{a}} \hat{\mathbf{b}}]^T = (\mathbf{T}^T \mathbf{T})^{-1} \mathbf{T}^T \hat{\mathbf{D}}
\]

(4.9)

where \( \hat{\mathbf{D}} = [\hat{\mathbf{X}}_s, \hat{\mathbf{Y}}_s] \). The estimated mean source horizontal position \( \hat{\mathbf{r}}_s = (\hat{x}_s, \hat{y}_s) \) can be obtained by

\[
[\hat{x}_s \hat{y}_s] = \frac{1}{N} \sum_{i=1}^{N} \hat{\mathbf{D}}
\]

(4.10)

Using LLS fit \( \hat{\mathbf{X}}_m \) and \( \hat{\mathbf{Y}}_m \) of Eq. (4.8), we can infer the estimated source horizontal trajectory \( \hat{\mathbf{r}}_m = (\hat{\mathbf{X}}_m, \hat{\mathbf{Y}}_m) \) and calculate the smoothed instantaneous source range estimates \( \hat{\mathbf{R}}_m = [\hat{R}_m(\hat{t}_1), \hat{R}_m(\hat{t}_2), \ldots, \hat{R}_m(\hat{t}_N)]^T \) via

\[
\hat{\mathbf{R}}_m = \sqrt{(\hat{\mathbf{X}}_m - \mathbf{r}_r)^2 + (\hat{\mathbf{Y}}_m - \mathbf{Y}_r)^2}
\]

(4.11)

where \( \mathbf{r}_r = (\mathbf{X}_r, \mathbf{Y}_r) \) is the GPS-measured receiver array center positions.
Smoothed instantaneous source range estimates $\hat{R}_{m}$ of Eq. (4.11) and the mean source position estimate $\hat{r}_{s}$ of Eq. (4.10) will be compared to the GPS-measured “ground truth” values in Sec. 4.3 for evaluating the performance of each passive localization method in various source-receiver geometries.

4.3 Applications: Passive localization and motion tracking of source array in the Gulf of Maine 2006 Experiment

In this section, we apply the (1) SAT, (2) AI, (3) MPC-EKF, and (4) MMSE methods to localize and track the horizontal trajectory of a vertical source array using measurements on a single towed horizontal receiving line array during GOME’06. The source array was either moored or drifting with the current and located mostly in the far-field of the receiver array with source-receiver separations ranging between 1 and 20 km. It transmitted 1-s duration Tukey-windowed broadband linear frequency modulated signals with 50 Hz bandwidth in the 300 to 1200 Hz frequency range at 50 or 75 s intervals. The receiver array was towed along designated track lines ranging from 1.5 km to 40 km in length with an average track length of roughly 12 km. Detailed experimental setup are provided in Refs. [9] and [4].

Of the 42 distinct tracks of the towed receiver array during GOME’06, only results from 6 representative tracks are discussed in this section, while the results from the remaining tracks are summarized in Table 4.3. The 6 tracks considered here have either (1) the greatest source-receiver separations, (2) the longest source drifting path, (3) the source located within the endfire beam of the receiver array, or (4) a moored source.
The inferred source ranges and horizontal locations are compared to those obtained from GPS measurements.

For each track we quantify the normalized bias $\epsilon$ in the mean source horizontal position and the fractional root mean-squares (RMS) error $\varrho$ in the source range estimates obtained along the track. The normalized bias, which measures the accuracy of mean source position estimation, is calculated using

$$
\epsilon = \frac{E(\hat{r}_s - \bar{r}_s)}{R_s} = \frac{1}{R_s} \sqrt{\left(\hat{x}_s - \bar{x}_s\right)^2 + \left(\hat{y}_s - \bar{y}_s\right)^2}
$$

where $\hat{r}_s = (\hat{x}_s, \hat{y}_s)$ is the mean source position estimate obtained from Eq. (4.10), $\bar{r}_s = (\bar{x}_s, \bar{y}_s)$ is the GPS-measured mean source position, and $R_s$ is the mean source range for each receiver track. The fractional RMS error, which measures the sample variance of the range estimates along a track, is obtained from

$$
\varrho = \frac{\sqrt{E\left[\left(\hat{R}_m - R_s\right)^2\right]}}{R_s} = \frac{1}{R_s} \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left[\hat{R}_m(t_i) - R_s(t_i)\right]^2}
$$

where $\hat{R}_m$ are the linear-least-squares fitted instantaneous source range estimates obtained from Eq. (4.11), $R_s = [R_s(t_1), R_s(t_2), \ldots, R_s(t_N)]^T$ are the GPS-measured true source ranges, and $N$ is the total number of range estimates for an individual track. For the MPC-EKF, instantaneous source range estimates $\hat{R}_s^{EKF}$ are used to calculate the fractional RMS error instead of the linear-least-squares fitted instantaneous source range estimates. For the MMSE method, only the normalized bias of mean source position estimates are obtained by Eq. (4.12).

### 4.3.1 Greatest source receiver separation

Here, we consider track522_1 with the greatest source-receiver separations. The
source-receiver geometry is shown in Fig. 4.2(a). The black triangles indicate the start points of both the source and receiver arrays at the beginning of their respective tracks. The source ship drifted 0.905 km with the current, from northwest to southeast, over a measurement time of 0.5 hour. At the same time, the receiver array was towed approximately 2.239 km from southwest to northeast along an approximate straight track.

For this track, the source bearing measurements after beamforming had a constant bias of 0.4 degree compared to the true bearing derived from GPS measurements as shown in Fig. 4.2(b). This bias is within one angular resolution of the towed receiver array. In Fig. 4.2(b), we observe the time-derivative of the source bearing estimates after RWA-LLS fit of the bearing measurements do provide a good match to the corresponding GPS-derived bearing derivatives. This is expected to improve the accuracy of source localization by the SAT which depends on the bearing derivatives.
Figure 4.2: Source range estimation and source horizontal trajectory tracking results for track522_1 that has the most distant source-receiver separations. (a) Source-receiver geometry of track522_1, triangle indicates the start positions of the vertical source array and the center of the horizontal receiver array at the beginning of receiver track line; (b) Source bearing estimates $\hat{B}_0$ by a conventional beamformer and the corresponding RWA-LLS fitted source bearing estimates $\hat{B}_m$ obtained using Eq. (4.7) are compared to the true source bearings $B_0$ derived from GPS measurements. Time-derivative of source bearing estimates $\dot{\hat{B}}_0$ and the corresponding time-derivative of RWA-LLS fitted source bearing estimates $\dot{\hat{B}}_m$ are compared to the bearing derivatives $\dot{B}_0$ derived from the GPS-measured source bearings. (c) Estimated source horizontal trajectories and corresponding mean source positions estimated using the SAT (dashed black line and hexagon), AI (dashed-dotted black line and square), MPC-EKF (solid gray line and circle), and MMSE (black inverse triangle) methods are compared to the GPS-measured true source horizontal trajectory (solid black line) and its mean position (black diamond). (d) Instantaneous source range estimates $\hat{R}_s$ and the interpolated results after applying the LLS averaging $\hat{R}_m$ along the track using the SAT and AI methods are compared to the GPS-derived true source ranges $\hat{R}_s$. The instantaneous source range estimates made by the MPC-EKF method are also overlain.
The instantaneous source range estimates using the SAT and AI approaches and the interpolated results after applying linear-least-squares averaging along the track are plotted in Fig. 4.2(d). The estimated source ranges from the MPC-EKF method are also plotted there. The source range estimates along the track using all three methods are highly correlated with the GPS-derived ranges. The estimated source horizontal trajectories and corresponding mean source positions estimated using the SAT, AI, MPC-EKF and MMSE methods are shown in Fig. 4.2(c). The mean source position estimate using the MPC-EKF approach provided the best match to the true mean source position from GPS with the smallest bias, while the SAT provided the best interpolated source range estimates with the smallest RMS error after linear-least-squares averaging along the track. As expected from theory, the MPC-EKF estimates quickly converged to the true source location initially, but diverged again after a few iterations because of the absence of a receiver maneuver which is necessary for the MPC-EKF to remain convergent.

4.3.2 Longest source drifting path

Here, track 560.1 with the longest source drifting path is considered. The source-receiver geometry is shown in Fig. 4.3(a). Over a roughly 3.5-hour measurement time, the source ship drifted approximately 10.54 km in the presence of strong currents, while the horizontal receiver array was being towed approximately 23.406 km from southwest to northeast along the designated track line shown in Fig. 4.3(a).

In Fig. 4.3(b), the RWA-LLS fit of the bearing measurements and the time deriva-
Figure 4.3: Identical to Fig. 4.2, but for track560.1 that has the longest source drifting path. In (b), only the time-derivative of RWA-LLS fitted source bearing estimates are compared to the corresponding bearing derivatives derived from GPS-measurements. The derivative derived from the fit provide a good match to the true source bearing and its time-derivative derived from GPS measurements. For this case, the MPC-EKF estimated mean source position had the smallest bias, while the AI estimated source trajectory and ranges after linear-least-squares interpolation provided the smallest RMS error along the track. The SAT approach led to the largest RMS error for the source range estimates along the track because the source drifted with an average speed that was large, roughly half that of the receiver array tow speed, making the SAT invalid for this case. The MPC-EKF estimated source trajectory converged to the true source trajectory 2/3 of the way along the track from the maneuver of the receiver tow ship. However, it quickly
diverged from the true source trajectory toward the end of the track mainly because the MPC-EKF is particularly formulated for tracking the non-maneuvering target. In case of a gradually maneuvering target as shown in Fig. 4.3(d), the MPC-EKF estimated results deviate from the true source trajectory is not unexpected.

4.3.3 Source located within the endfire beam of the receiver array

![Diagram](https://via.placeholder.com/150)

Figure 4.4: Identical to Fig. 4.3, but for track532_1, a typical example with the vertical source array located predominantly within or close to the endfire beam of the receiver array.

Here, we consider tracks 532_1 and 564_2, that have the source array located predominantly within or close to the endfire beam of the receiving array. The source-receiver
geometries are shown in Figs. 4.4(a) and 4.5(a) respectively for the two tracks.

Figure 4.5: Identical to Fig. 4.3, but for track564,2, another typical example with the vertical source array located predominantly within or close to the endfire beam of the receiver array.

The angular resolution of the receiver array at endfire is approximately 29-38 degrees in the frequency range of source transmissions. Bearing estimates from beamforming can deviate from the true bearing by as much as 6-7 degrees at endfire, as shown in Figs. 4.4(b) and 4.5(b). Inaccuracies in bearing estimation can significantly degrade the localization performance of the SAT and MPC-EKF, because the bearing estimates are directly applied to range estimation in these approaches. In contrast, the array invariant source range estimates depend on source bearing via \( \frac{1}{\sin \beta_0} \) (Eq. (27) of Ref. [65]), which is approximately 1 near the endfire direction and is less sensitive to
bearing estimation error. As a result, the AI yields the best source range estimation results with the smallest mean biases and RMS errors along both tracks, while the SAT and MPC-EKF source range estimates diverge significantly from the true source ranges along both tracks until the error in bearing estimates become negligible, as can be seen from Figs. 4.4(b)-(d) and 4.5(b)-(d). Again, due to the maneuver of the source, as can be seen from Fig. 4.5(d), the MPC-EKF source range estimates diverge from the true values immediately following a convergence triggered by a small maneuver of the receiver array during the first half of track564_2.

### 4.3.4 Moored source

During tracks 570_6 and 571_6, the source was moored to the ground via a loose anchor that allowed the ship to drift within a horizontal radius of 400-500 m about the anchor. The source-receiver geometries are shown in Figs. 4.6(a) and 4.7(a), respectively. The SAT and MMSE methods led to the smallest biases for the mean source position and the SAT provided the smallest RMS error in the linear-least-squares fitted source horizontal trajectory by a factor of 5 and 3 over the AI and MPC-EKF methods, respectively. For track570_6, despite the receiver ship maneuver, which occurred roughly 20% of the way into the track and helped the MPC-EKF method quickly converge to the true source location, the estimated source trajectory with this method eventually diverged slightly from the true source location toward to the end of the track, as can be seen from Fig. 4.6(d), mainly because the MPC-EKF is primarily developed from tracking a moving source. In case of a relatively stationary source, the accuracy of this method is expected to degrade. For track571_6, the absence of a receiver ship maneuver directly leads to a diverged source range estimates by the MPC-EKF as expected by the theory. Again, we show that inaccurate bearing estimates can lead to the performance degradation for
Figure 4.6: Identical to Fig. 4.3, but for track570_6, during which the source ship was moored to the ground via a loose anchor allowing the ship to drift within a horizontal radius of 400-500 m about the anchor.

the SAT and MPC-EKF, while it has little effects on the performance of AI, as can be seen from Figs. 4.6(b)-(d) and Fig. 4.7(b)-(d).

4.4 Discussion

Here we quantify and statistically evaluate the performance of the four source localization approaches of Sec. 4.1 by considering all 42 tracks of the GOME’06 experiment. We also categorize the tracks into five distinct tracking scenarios; (1) moving source, (2) moored source, (3) source located within the endfire beam of the receiver array, (4)
Figure 4.7: Identical to Fig. 4.3, but for track571_1, another typical example with a moored source.
moving source but excluding cases when it was located within the endfire beam of the
receiver array, (5) moored source but excluding cases when it was located within the
endfire beam of the receiver array.
Table 4.2: Normalized bias ǫ of mean source position estimates and fractional
RMS error ̺ of instantaneous or LLS fitted instantaneous source range estimates along a given receiver track obtained by the SAT, AI, and MPC-EKF
methods, and the corresponding RMS error of instantaneous and RWA-LLS fitted source bearing estimates, µβ̂ and µβ̂m , for all 42 tracks of GOME’06. For
0
the MMSE method, only the normalized bias of mean source position estimates
are tabulated.

Track

a Status

bD

cR

s

dǫ

SAT

eǫ

AI

fǫ

EF K

gǫ

M M SE

h̺

SAT

i̺

AI

j̺

EF K

kµ

β̂

lµ

β̂m

(km)

(km)

(%)

(%)

(%)

(%)

(%)

(%)

(%)

(deg)

(deg)

521 1

DF

2.904

13.361

18.89

16.15

1.63

25.06

18.77

15.76

4.81

0.85

0.75

522 1

DF

2.239

18.679

1.34

4.67

0.70

2.14

1.55

5.09

1.03

0.51

0.43

530 1

DF

15.915

6.006

1.93

5.59

3.15

4.21

0.98

5.45

12.87

0.98

0.90

530 2

DF

12.276

5.112

21.06

8.15

8.14

9.92

23.28

14.22

15.81

3.82

3.13

530 3

DF

16.274

5.691

1.25

14.83

2.25

4.95

1.62

16.93

5.81

1.34

0.99

531 1

DF

14.693

4.516

19.77

9.68

2.65

9.25

20.14

8.72

10.31

3.03

2.12

531 2

DF

14.638

4.521

21.49

17.04

10.01

4.62

34.72

17.02

13.63

3.20

3.05

532 1

DF

4.278

9.217

3.03

3.71

24.87

54.25

12.22

2.57

28.87

2.15

2.10

532 5

DF

7.732

11.973

23.62

14.89

6.25

24.67

24.68

15.61

28.84

1.45

1.14

540 1

DF

22.895

8.219

5.29

2.46

2.36

13.46

12.53

3.86

9.29

3.58

3.39

540 2

DF

12.612

14.792

7.17

8.63

26.73

7.74

7.94

9.01

40.30

1.42

0.91

550 1

DF

10.133

9.913

26.48

23.91

15.82

558.85

38.98

23.99

21.21

6.89

6.18

551 1

DF

14.285

4.742

3.34

4.92

1.61

5.68

3.33

8.59

7.52

1.82

1.43

551 2

DF

9.954

9.247

47.25

7.85

1.62

49.09

46.67

9.25

11.87

0.62

0.52

551 3

DF

2.492

10.213

3.66

9.04

0.87

12.56

5.29

10.80

8.58

0.48

0.24

553 1

DF

16.304

7.018

7.30

10.95

9.94

11.06

8.89

14.40

16.91

2.15

1.23

560 1

DF

23.406

8.819

15.37

21.07

6.21

25.45

22.81

14.86

11.86

1.30

1.17

560 2

DF

9.787

14.412

3.76

8.09

1.73

4.24

7.27

8.35

9.08

1.22

1.18

564 1

DF

1.998

10.217

57.77

7.94

6.86

68.37

62.45

13.50

3.62

4.27

3.55

564 2

DF

4.624

4.439

54.81

7.96

12.40

28.83

66.26

9.01

53.45

3.06

1.66

564 3

DF

7.676

5.162

9.28

15.42

18.25

14.56

6.93

14.63

24.54

2.83

2.49

566 1

DF

8.477

7.637

12.32

6.36

1.72

36.60

16.33

8.30

14.72

1.85

1.25

570 1

MR

15.800

7.717

0.54

11.77

11.15

2.12

0.77

11.86

26.24

1.21

1.05

570 2

MR

21.650

8.833

1.32

7.98

0.52

1.77

0.87

10.67

7.94

1.42

1.29

570 3

MR

18.436

9.288

1.18

6.03

1.26

3.30

0.95

7.00

7.15

1.02

0.97

570 4

MR

18.530

9.037

2.02

13.20

5.51

2.21

2.12

12.87

9.29

1.61

1.18

570 5

MR

15.118

8.131

1.09

10.93

0.42

2.52

1.04

12.72

7.33

1.44

1.12

570 6

MR

8.810

11.137

2.75

6.40

2.66

3.34

4.27

7.91

6.27

1.86

0.82

570 7

MR

10.385

7.284

2.17

9.70

0.74

2.65

1.28

10.38

3.75

2.09

1.18

571 1

MR

25.146

8.970

2.28

9.23

3.45

2.41

2.65

15.27

13.16

1.18

0.94

Continued on next page

174


### Table 4.3

<table>
<thead>
<tr>
<th>Track</th>
<th>Status</th>
<th>( D ) (km)</th>
<th>( R_s ) (km)</th>
<th>( e_{\text{SAT}} ) (%)</th>
<th>( e_{\text{AI}} ) (%)</th>
<th>( e_{\text{EPK}} ) (%)</th>
<th>( e_{\text{MMSE}} ) (%)</th>
<th>( \mu_{\beta} ) (deg)</th>
<th>( \mu_{\beta_m} ) (deg)</th>
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*a* Status of vertical source array. DF indicates that the source array was drifting with current, while MR indicates that the source array was moored.

*b* Total distance traveled by the towed horizontal line-array per track.

*c* Mean source range of the track, where \( R_s = \frac{1}{N} \sum_{i=1}^{N} R_s(t_i) \).

*d* Normalized bias of mean source position estimates by the SAT obtained using Eq. (4.12).

*e* Normalized bias of mean source position estimates by the AI.

*f* Normalized bias of mean source position estimates by the MPC-EKF.

*g* Normalized bias of mean source position estimates by the MMSE.

*h* Fractional RMS error of LLS fitted instantaneous source range estimates by the SAT obtained using Eq. (4.13).

*i* Fractional RMS error of LLS fitted source range estimates by the AI.

*j* Fractional RMS error of instantaneous source range estimates by the MPC-EKF.

**k** RMS error of the bearing estimates \( \mu_{\beta} = E[(\hat{\beta} - \beta_0)^2] = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\hat{\beta}_i(t_i) - \beta_0(t_i))^2} \).

**l** RMS error of the bearing estimates after RWA-LLS fit, \( \mu_{\beta_m} = E[(\hat{\beta}_m - \beta_0)^2] = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\hat{\beta}_m(t_i) - \beta_0(t_i))^2} \).

The normalized bias \( e \) in the mean source position estimate and the fractional RMS error \( \rho \) in instantaneous source range estimates averaged over all tracks of GOME’06 and tracks for the 5 distinct tracking scenarios defined above are tabulated in Table 4.3. The normalized bias in the mean source position estimate averaged over all tracks of the experiment \( \bar{e} \) are comparable, ranging from 9% to 13%. However, the performance of the four source localization approaches vary significantly over the 5 tracking scenarios. (1) For the moving source, the MPC-EKF approach provided the best performance...
with an average of 8% normalized bias $\bar{\epsilon}_{ms}$ in mean source position estimates, while the AI yielded the least fractional RMS error $\bar{\eta}_{ms}$ of roughly 11% in instantaneous source range estimates over tracks. (2) For the moored source, the SAT and MMSE led to 4% and 6% normalized biases $\bar{\epsilon}_{ss}$ in mean source position estimates respectively, which are approximately two to three times more accurate than the AI and MPC-EKF approaches, and the SAT also provided the least fractional RMS error $\bar{\eta}_{ss}$ of roughly 5% in instantaneous source range estimates averaged over tracks. (3) When the source was located within the endfire beam of the receiver array, the AI provided the best mean source position estimates with a normalized bias $\bar{\epsilon}_{ef}$ of roughly 10% and a fractional RMS error $\bar{\eta}_{ef}$ of 12%, while the other approaches had normalized biases and fractional RMS errors at least two times larger. (4) By excluding scenarios when the source was located within the endfire beam of the receiver array for the moving source, the performance of the SAT improved significantly by roughly a factor of two, making it comparable to the MPC-EKF and AI approaches. (5) By excluding scenarios when the source was located within the endfire beam of the receiver array for the moored source, the range localization improved for all methods except the AI. For instance, the MPC-EKF normalized bias $\bar{\epsilon}_{nes}$ was reduced by a factor of two in comparison to $\bar{\epsilon}_{ss}$, while the SAT still gave the best performance with 2.7% normalized bias and 3.3% fractional RMS error. For the SAT and AI methods, in addition to the fractional RMS error obtained from Eq. (4.13) using LLS-fitted instantaneous source range estimates $\hat{R}_m$, the fractional RMS error calculated using instantaneous source range estimates $\hat{R}_s$ are also tabulated in Table 4.3 in parenthesis as comparison. It is shown that the fractional RMS errors are reduced by a factor of 2-3 after applying the LLS averaging.
Table 4.3: Ensemble averaged normalized bias in the mean source position estimates and the corresponding ensemble averaged fractional RMS error of instantaneous source range estimates over all 42 tracks of GOME’06 and for the five distinct source tracking scenarios defined in Sec. 4.3 for all four source localization methods.

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<tbody>
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<td>( \bar{\alpha} (%) )</td>
<td>42</td>
<td>10.65</td>
<td>9.24</td>
<td>8.98</td>
<td>13.06 b</td>
</tr>
<tr>
<td>( \bar{\alpha}_{ms} (%) )</td>
<td>22</td>
<td>16.64</td>
<td>10.42</td>
<td>7.82</td>
<td>19.84</td>
</tr>
<tr>
<td>( \bar{\alpha}_{ss} (%) )</td>
<td>20</td>
<td>4.05</td>
<td>7.93</td>
<td>10.26</td>
<td>5.93</td>
</tr>
<tr>
<td>( \bar{\alpha}_{ef} (%) )</td>
<td>9</td>
<td>34.59</td>
<td>9.73</td>
<td>21.37</td>
<td>37.15</td>
</tr>
<tr>
<td>( \bar{\alpha}_{nem} (%) )</td>
<td>15</td>
<td>9.19</td>
<td>9.81</td>
<td>7.59</td>
<td>14.45</td>
</tr>
<tr>
<td>( \bar{\alpha}_{nes} (%) )</td>
<td>18</td>
<td>2.72</td>
<td>8.50</td>
<td>5.48</td>
<td>3.70</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>( \bar{\gamma} (%) )</th>
<th>( \bar{\gamma}_{ms} (%) )</th>
<th>( \bar{\gamma}_{ss} (%) )</th>
<th>( \bar{\gamma}_{ef} (%) )</th>
<th>( \bar{\gamma}_{nem} (%) )</th>
<th>( \bar{\gamma}_{nes} (%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{\gamma} ) ( %)</td>
<td>42</td>
<td>12.85 (32.93)</td>
<td>10.66 (32.26)</td>
<td>15.76</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>( \bar{\gamma}_{ms} ) ( %)</td>
<td>22</td>
<td>20.17 (44.71)</td>
<td>11.36 (32.19)</td>
<td>16.59</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>( \bar{\gamma}_{ss} ) ( %)</td>
<td>20</td>
<td>4.81 (19.98)</td>
<td>9.89 (32.34)</td>
<td>14.84</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>( \bar{\gamma}_{ef} ) ( %)</td>
<td>9</td>
<td>39.37 (64.98)</td>
<td>12.16 (30.84)</td>
<td>32.98</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>( \bar{\gamma}_{nem} ) ( %)</td>
<td>15</td>
<td>12.03 (36.65)</td>
<td>10.50 (33.27)</td>
<td>14.47</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>( \bar{\gamma}_{nes} ) ( %)</td>
<td>18</td>
<td>3.31 (16.96)</td>
<td>10.22 (31.86)</td>
<td>10.27</td>
<td>N/A</td>
<td></td>
</tr>
</tbody>
</table>

The instantaneous source range estimates \( \hat{R}_s \) obtained from the SAT, AI and MPC-EKF approaches are plotted against the corresponding GPS-derived source ranges \( R_s \) for all 42 tracks of GOME’06 in Figs. 4.8(a)-(c) respectively. For the SAT and AI methods, the LLS-fitted instantaneous source range estimates \( \hat{R}_m \) obtained from Eq. (4.11) of all 42 tracks are also plotted in Figs. 4.8(a)-(b). The linear regressions

\[
\hat{R}_s = \hat{b}_l + \hat{a}_l R_s
\]

\[
\hat{R}_m = \hat{b}_l + \hat{a}_l R_s
\]

of instantaneous range estimates \( \hat{R}_s \) or LLS-fitted instantaneous range estimates \( \hat{R}_m \)
with respect to the the GPS measured values $R_s$ are shown in Figs. 4.8(a)-(c). The regression slopes $\hat{a}_t$ and the correlation coefficients $\rho$ between the instantaneous or LLS-fitted source range estimates and GPS-derived source ranges are tabulated in Table 4.4. A similar analysis is carried out for the mean position estimates in Fig. 4.8(d) and in Table 4.4 including the MMSE approach. The regression slopes $\hat{a}_{tm}$ and the correlation coefficients $\rho_m$ are high and close to 1 for the source position estimates compared to the GPS-measured source positions validating all four approaches for source localization with a towed horizontal receiver array.
Figure 4.8: (a) Instantaneous source range estimates (black cross) and the corresponding LLS averaged source range estimates (gray cross) along each individual track by the SAT method versus GPS measured source ranges for all 42 tracks of GOME’06. The solid black and gray lines are the linear regression defined by Eq. 4.14. The regression coefficients $\hat{a}_l$ are tabulated in Table 4.4; (b) Identical to (a) but for the range estimates made by the AI method; (c) Identical to (a) but only the instantaneous source range estimates made by the MPC-EKF method are plotted; (d) Mean source range estimated by the SAT (black cross), AI (gray triangle), MPC-EKF (black circle), and MMSE (black diamond) methods are plotted against the true mean source ranges for all 42 tracks of the GOME’06. The linear regression lines for the SAT (solid black), AI (dashed black), MPC-EKF (dashed-dotted black), and MMSE (solid gray) methods are overlain, and the corresponding regression coefficients $\hat{a}_{tm}$ for all four methods are tabulated in Table 4.4.
Table 4.4: Linear regression coefficients $\hat{a}_l$ and correlation coefficients $\rho$ of instantaneous or LLS fitted instantaneous source range estimates, $\hat{R}_s$ or $\hat{R}_m$, with respect to the GPS-measured source ranges $R_s$ for the SAT, AI, and MPC-EKF methods. Similar linear regression and correlation coefficients, $\hat{a}_{lm}$ and $\rho_m$, are calculated for the mean source position estimates with respect to the mean GPS-measured source positions by all four methods including the MMSE approach.

<table>
<thead>
<tr>
<th></th>
<th>SAT</th>
<th>AI</th>
<th>MPC-EKF</th>
<th>MMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^a$</td>
<td>0.915 (0.750)</td>
<td>0.970 (0.736)</td>
<td>(0.790)</td>
<td>N/A</td>
</tr>
<tr>
<td>$\hat{a}_{l}^b$</td>
<td>0.990 (0.971)</td>
<td>0.891 (0.866)</td>
<td>(0.925)</td>
<td>N/A</td>
</tr>
<tr>
<td>$\rho_m^c$</td>
<td>0.894</td>
<td>0.977</td>
<td>0.950</td>
<td>0.903</td>
</tr>
<tr>
<td>$\hat{a}_{lm}^d$</td>
<td>1.011</td>
<td>0.910</td>
<td>0.921</td>
<td>1.141</td>
</tr>
</tbody>
</table>

$a$ Correlation coefficients between the LLS fitted or instantaneous (in the parenthesis) source range estimates and GPS measured source ranges for the SAT, AI and MPC-EKF methods, as shown in Fig. 4.8(a)-(c).

$b$ Linear regression coefficients of the linear regression lines shown in Fig. 4.8(a)-(c). Those in the parenthesis are the results obtained using instantaneous source range estimates by the SAT, AI, and MPC-EKF methods.

$c$ Correlation coefficients between the mean source range estimates and the GPS measured mean source ranges as shown in Fig. 4.8(d).

$d$ Linear regression coefficients of the linear regression lines shown in Fig. 4.8(d).

Next we investigate the effect of source range and bearing relative to receiver array broadside on the localization accuracy. The fractional errors of the source range estimates are calculated using

$$\mu = \frac{|\hat{R}_m - R_s|}{R_s}. \quad (4.15)$$

For the MPC-EKF method, the instantaneous source range estimates $\hat{R}_s$ is used in Eq. (4.15) to calculate the fraction error instead of the LLS-fitted instantaneous source range estimates $\hat{R}_m$. The nonlinear curve fitted fractional errors using a first degree and third degree of polynomial equations for range and relative bearing respectively are plotted for the SAT, AI and MPC-EKF methods in Figs. 4.9(a) and (b), respectively. For the
SAT method, the polynomial fitted fractional errors of roughly 10% stay approximately constant as the true range increases implying that the localization errors will increase linearly with source range. This is expected because the range estimates by the SAT are sensitive to the time-derivative of source bearing estimates (Eq. (4.3)), which is derived from the output of a conventional beamformer, whose performance is expected to degrade as source range increases due to the advent of spurious effects unique to in a random dispersive ocean waveguide [166, 167, 40]. For both the AI and MPC-EKF methods, as shown in Fig. 4.9(a), the polynomial fitted fractional errors decrease linearly as a function of source range, which implies that the range estimation error will increase nonlinearly but more gradually as source range increases.

In Fig. 4.9(b), it is shown that the polynomial fitted fractional errors for the SAT slightly increase from less than 5% to 10% for relative source bearings between 0 and 60 degree with respect to the broadside of the receiver array, but followed by a substantial increase at a rate of 4 times faster than before as the bearings move towards the endfire. This third degree of polynomial curve fitting is justified because of a nonlinear degraded angular resolution of the horizontal line-array defined by Eqs. (1) and (2) of Ref. [81] as the array steers from the broadside to the endfire. In comparison to the SAT, the fractional errors increase more gradually for the MPC-EKF for bearings from broadside to endfire. This is mainly because the kinematics of target state estimates by the MPC-EKF are less sensitive to the bearing estimation errors than the SAT, and the nonlinear filter based approach employed in the modified polar coordinates can automatically decouples the observable bearing from the unobservable range component of the estimated source states [165], hence efficiently mitigating the effect of bearing measurement errors on the range estimates. According to the theory of the array invariant method, it is expected to provide the most accurate range estimates at endfire incidence.
Figure 4.9: Effect of source range and source bearing relative to the broadside direction of the receiver line array on the fractional errors of source range estimates, defined by Eq. (4.15) using the SAT, AI, and MPC-EKF methods. (a) The interpolated fractional errors after a nonlinear curve fitting are plotted against the GPS-derived true source ranges. (b) Identical to (a) but plotted against the relative source bearing derived from the GPS measurements.

with a horizontal line-array because $\hat{\chi}_h$ is more sensitive to $\hat{R}_s$ when the incidence migration of dispersive modes is fully exploited. This is justified by the results shown in Fig. 4.9(b), where the polynomial fitted fractional errors slightly decrease as a function of relative source bearing from broadside to endfire. Among the three approaches shown in Fig. 4.9, the AI method is shown to provide the most consistent performance with an averaged fractional error of 7 to 12% for all source ranges and bearings analyzed in this paper. These experimentally determined localization accuracy on range and relative source bearing are important for predicting the performance of all four localization approaches considered when it is applied for other passive localization applications, such as localizing acoustically vocal marine mammals, in a similar context when the true source locations are unknown.
4.5 Summary

Four methods have been employed to passively localize and track a vertical source array located in the far-field of a towed horizontal receiving line array using measurements from the Gulf of Maine 2006 experiment. The methods include the (1) synthetic aperture tracking, (2) array invariant, (3) bearings-only target motion analysis in modified polar coordinates via extended Kalman filter, and (4) bearings-migration minimum mean-squares error. The source array transmitted intermittent broadband pulses in the 300 to 1200 Hz frequency range and was located between 1 to 20 km from the receiver array. A total of 42 tracks of the towed receiver array with relative source bearing spanning from broadside to endfire of the towed array have been analyzed. For each track, a normalized bias of the mean source position estimates and the fractional root mean-squares error of the instantaneous source range estimates along a track were introduced to evaluate the performance of each method.

All four methods led to comparable 9% to 13% normalized biases in the mean source position estimates averaged over all 42 tracks, while the averaged fractional RMS error in instantaneous source horizontal trajectory tracking is between 10% and 16%. The AI was found to be the most consistent in its performance maintaining a roughly 9% normalized bias in mean source position estimate and 10% fractional RMS error over a wide range of tracking scenarios. The performance of the SAT, MMSE and MPC-EKF methods varied widely depending on the tracking scenario. The SAT led to as small as 3% normalized bias and 4-5% fractional RMS error when localizing moored sources, but was not reliable for localizing sources located within or near the endfire beam of the receiver array or fast moving sources. The MPC-EKF was shown to be the most
favorable approach for localizing moving sources not located within or near the endfire beam of the receiver array with 8% normalized biases and a 14% fractional RMS error. The only reliable technique for localizing sources within or near the endfire beam of the towed receiver array is the AI regardless of whether the source is moving or moored. The fractional errors in the instantaneous source range estimates were found to be a constant function of range for the SAT, but a linearly decreasing function of range for both the AI and MPC-EKF, implying that the instantaneous localization errors would therefore increase linearly with range for the SAT, but nonlinearly and more gradually for the AI and MPC-EKF. In terms of source bearing relative to the broadside of the receiver line-array, it is shown that the fractional errors slightly increase between 0 and 60 degree from broadside, but followed by a substantial increase at a rate that is 4 times greater than before as the bearing approaching endfire for the SAT. The fractional errors increase more gradually for the MPC-EKF than the SAT, but decrease with relative bearing from broadside to endfire for the AI.
Chapter 5

Conclusions

In this thesis, we investigated the acoustic resonance scattering characteristics, abundance, and population density imaging of shoaling fish populations with an ocean acoustic waveguide remote sensing system both experimentally and theoretically. We also examined the performance of four distinct passive source localization methods with experimental data made on a towed horizontal receiver line-array for a wide variety of source-receiver geometries.

The mean low-frequency target strength of shoaling Atlantic herring is estimated from experimental data acquired in the Gulf of Maine during the Autumn 2006 spawning season in the 300-1200 Hz range using simultaneous ocean acoustic waveguide remote sensing, conventional fish finding sonar, and trawl surveys. The mean target strength expected of a shoaling herring individual is found to have a strong roughly 20 dB/octave roll-off with decreasing frequency, consistent with the steep roll-off expected for sub-resonance scattering from fish with an air-filled swimbladder given measured fish length, depth distributions, and experimentally inferred swimbladder volumes based on Love’s model, which indicate the herring remain negatively buoyant in layers near the seafloor.
for extended periods. These findings suggest that OAWRS can provide valuable evidence for remote species classification over wide areas since significant variations in the frequency dependence of target strength are expected across species due to differences in resonance. The OAWRS system used in this study employed an instantaneous imaging diameter of 100 km with regular minute-to-minute updates enabling unaliased monitoring of fish populations over eco-system scales. This included detection and imaging of shoals of Atlantic herring containing hundreds of millions of individuals, as confirmed by concurrent trawl and CFFS surveys that were directed to the shoal’ locations by OAWRS. High spatial-temporal co-registration was found between herring shoals imaged by OAWRS and concurrent CFFS line-transects.

A numerical model has been developed to determine the statistical moments of the broadband matched filtered scattered field including multiple scattering from a random 3D spatial distribution of random scatterers spanning multiple range and cross-range resolution cells of a waveguide remote sensing system in a random range-dependent ocean waveguide by Monte-Carlo simulation. The model can be applied to analyze bistatic scattering from a scatterer group in any direction including the forward. When combined with the incident field, the model can accurately characterize attenuation in forward propagation through a group. The model is applied to (1) examine population density imaging of shoaling Atlantic herring in the Gulf of Maine during GOME’06 and (2) investigate wide-area imaging of sparsely aggregations of Atlantic cod, an important ground fish species in the Ipswich Bay continental shelf environments using a waveguide remote sensing system in the 300 to 1200 Hz frequency range. The fish swimbladder is modeled as a damped air-filled prolate spheroid surrounded by viscous flesh using a complex scatter function. Analysis with the model indicates that (1) the incoherent intensity dominated the scattered returns from large herring groups imaged during
GOME’06 and sparsely aggregated groups of cod expected to image in the Ipswich Bay continental shelf environment, effects such as resonant shift and sub- and super-resonance local-maxima are negligible or absent, (2) the single scattering assumption is valid for for inferring herring and cod areal population densities from their scattered intensities over the imaging frequency range and observed or expected areal densities, (3) waveguide dispersion had negligible effect on shoal population estimation, (4) the standard deviations of the instantaneous broadband matched filtered scattered intensities from the fish groups are roughly 5 dB indicating that the scattered fields are fully saturated and that the standard deviations can be reduced by stationary averaging, (5) the charting speed for accurately localizing targets in range from their time-dependent scattered fields after matched filtering in a random waveguide is approximately equal to the average group speed of the lowest order mode, (6) the imaging system’s range resolution in the waveguide is approximately equal to its resolution in free space where it is inversely proportional to the system bandwidth, and (7) attenuation due to scattering in forward propagation through fish groups can negate the effects of multiple scattering and are highly dependent on the accurate characterization of fish flesh viscosity.

The passive source localization methods namely (1) the synthetic aperture tracking, (2) the array invariant, (3) the bearings-only target motion analysis in modified polar coordinates via extended Kalman filter, and (4) the bearings-migration minimum mean-squares error methods have been employed to passively localize and track a vertical source array located in the far-field of a towed horizontal receiver line-array using data from the Gulf of Maine 2006 experiment. The source array transmitted intermittent broadband pulses in the 300 to 1200 Hz frequency range and was located between 1 to 20 km from the receiver array. A total of 42 tracks of the towed receiver array with relative source bearings spanning from broadside to endfire of the towed array have been
analyzed. All four methods led to comparable 9% to 13% normalized biases in the mean source position estimates averaged over all 42 tracks, while the averaged fractional RMS error in instantaneous source horizontal trajectory tracking is between 10% and 16%. The AI was found to be the most consistent in its performance maintaining a roughly 9% normalized bias in mean source position estimates and 10% fractional RMS error over a wide range of tracking scenarios. The performance of the SAT, MMSE and MPC-EKF methods varied widely depending on the tracking scenario. The SAT led to as small as 3% normalized bias and 4-5% fractional RMS error when localizing moored sources, but was not reliable for localizing sources located within or near the endfire beam of the receiver array or fast moving sources. The MPC-EKF was found to be the most favorable approach for localizing moving sources not located within or near the endfire beam of the receiver array with 8% normalized biases and a 14% fractional RMS error. The only reliable technique for localizing sources within or near the the endfire beam of the towed receiver array is the AI regardless of whether the source is moving or moored. The fractional errors in the instantaneous source range estimates were found to be a constant function of range for the SAT, but a linearly decreasing function of range for the AI and MPC-EKF methods, implying that the instantaneous localization errors would therefore increase linearly with range for the SAT, but nonlinearly and more gradually for the AI and MPC-EKF. In terms of source bearing relative to the broadside of the receiver line-array, the fractional errors were found to be gradually increasing between 0 and 60 degree from the broadside, followed by a substantial increasing as the bearing approaching the endfire for the SAT. The fractional errors increase more gradual for the MPC-EKF than the SAT, but decrease with relative bearing from broadside to endfire for the AI.
Appendix A

Concurrent National Marine Fisheries Service Annual Atlantic herring Acoustic Survey in the Gulf of Maine

In conjunction with OAWRS 2006 experiment, U.S. National Marine Fisheries Service conducted annual Atlantic herring survey of the Gulf of Maine and Georges Bank. Five trawls, 105, 106, 134, 137, and 139, targeted at 130-170 m depth (Table A.2) were deployed within or in the vicinity of the large fish shoals imaged by OAWRS system, which enabled onsite species identification and biological measurements. The locations of these trawls are shown in Fig. 2.1. The trawls 134, 137 and 139 were made with simultaneous OAWRS imagery, and trawls 137 and 139 were made directly through shoals as shown in Figs. A.1(B)-(C). As tabulated in Table A.1, Atlantic herring is found consistently to be the overwhelmingly predominant species in trawls 105, 106, 137, and 139. Trawl 134, shown in Fig. A.1(A), was deployed in a region with no shoal formed, but one would form in the vicinity 4 hours later, and like both OAWRS and CFFS does not show high concentrations of herring before the shoal formed. Trawl 105 was made
through shoals imaged by OAWRS 95 minutes before, and trawl 106 was made through shoals imaged by OAWRS 20 minutes later. In addition, a small percentage of juvenile silver hake, Acadian redfish, and haddock are also present in the trawls. Most of the juvenile silver hake were caught at shallower water depth (<75 m) as the deeper trawls were deployed. Histograms of the measured length distributions of the most frequently caught species are plotted in Fig. 2.13. The conversion of herring length to weight by regression analysis is given by \( W = 0.00335L^{3.35} \), based on length-weight measurements of 1219 herring caught in trawls throughout the course of the survey.

Table A.1: Concurrent high-speed rope trawl deployed by NOAA FRV Delaware II within or in the vicinity of large herring shoals imaged by OAWRS during the OAWRS 2006 experiment in the Gulf of Maine and Georges Bank at shoal depth. The number of most frequently caught species in each trawl deployment, including Atlantic herring, Acadian redfish, Silver hake, and Haddock are tabulated. Trawls 134, 137, and 139 were made with simultaneous OAWRS imagery, and trawls 137 and 139 were made directly through shoals as shown in Figs. A.1(B)-(C). In contrast, trawl 134 was made in a region with no shoal formed (Fig. A.1(A)), but one shoal would form in the vicinity 4 hours later. Trawl 105 was made through shoals imaged by OAWRS 95 minutes before, and trawl 106 was made through shoals imaged by OAWRS 20 minutes later.

<table>
<thead>
<tr>
<th>Trawl</th>
<th>Atlantic herring</th>
<th>Acadian redfish</th>
<th>Silver hake</th>
<th>Haddock</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>105</td>
<td>8030 (99.98%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8032</td>
</tr>
<tr>
<td>106</td>
<td>634 (96.79%)</td>
<td>0</td>
<td>14 (2.14%)</td>
<td>2 (0.31%)</td>
<td>655</td>
</tr>
<tr>
<td>134</td>
<td>3 (11.54%)</td>
<td>0</td>
<td>14 (53.85%)</td>
<td>0</td>
<td>26</td>
</tr>
<tr>
<td>137</td>
<td>333 (76.03%)</td>
<td>0</td>
<td>94 (21.46%)</td>
<td>0</td>
<td>438</td>
</tr>
<tr>
<td>139</td>
<td>796 (74.74%)</td>
<td>9 (0.85%)</td>
<td>208 (19.53%)</td>
<td>23 (2.16%)</td>
<td>1065</td>
</tr>
<tr>
<td>Total</td>
<td>9796 (96.54%)</td>
<td>9 (0.07%)</td>
<td>330 (2.67%)</td>
<td>25 (0.2%)</td>
<td>10216</td>
</tr>
</tbody>
</table>
Figure A.1: (A)-(C) Locations of trawls over simultaneous OAWRS images. Trawls 137 and 139 were made directly through shoals as shown in (B) and (C). In contrast, Trawl 134 was made in a region with no shoal (A), but one shoal would form in the vicinity 4 hours later. The OAWRS source locations are the coordinate origin in all OAWRS images. On October 3, the OAWRS source ship was moored at 42.2089N, 67.6892W. The trawls in (A)-(C) were deployed and towed along the solid lines starting at $\alpha$ and ending at $\Omega$. The dashed lines indicate the contours of 100, 150, 180, and 200 m water depth.

Table A.2: Concurrent high-speed rope trawl deployed by NOAA ship FRV Delaware II within or in the vicinity of large herring shoals imaged by OAWRS system during the OAWRS 2006 experiment in the Gulf of Maine and Georges Bank at shoal depth. The dates, times (Eastern Daylight Time), deploy depth, and geographic locations of the trawls are tabulated.

<table>
<thead>
<tr>
<th>Trawl</th>
<th>Date</th>
<th>Time</th>
<th>Depth (m)</th>
<th>Begin Lat</th>
<th>Begin Lon</th>
<th>End Lat</th>
<th>End Lon</th>
</tr>
</thead>
<tbody>
<tr>
<td>105</td>
<td>09/26</td>
<td>14:21:40-15:16:32</td>
<td>130-160</td>
<td>41°</td>
<td>68°</td>
<td>41°</td>
<td>68°</td>
</tr>
<tr>
<td>106</td>
<td>09/26</td>
<td>19:23:24-19:57:20</td>
<td>130-160</td>
<td>41°</td>
<td>68°</td>
<td>41°</td>
<td>68°</td>
</tr>
</tbody>
</table>
Appendix B

Calibration of broadband mean TL with one-way propagated experimental data in the northern flank of Georges Bank

In wide-area sonar applications, two-way TL must be efficiently and accurately estimated in order to invert for scattering strength or target strength of scatterers [3, 88]. Here, a range-dependent acoustic model (RAM), based on the parabolic equation, is used to estimate two-way TL expected over wide areas with measured bathymetry and oceanography of the northern flank of Georges Bank environment. For each OAWRS image, TL was computed along radials separated by roughly 1.5 degrees, half the receiver array broadside resolution, for forward transmission from the source, and again for return transmission to the receiver array, by multiple Monte-Carlo realizations per radial. Each Monte-Carlo realization employed a different measured sound speed profile every 500-m to incorporate the effects of the fluctuating ocean waveguide.

In the OAWRS 2006 experiment, a very complete set of more than 12,000 transmission loss measurements spanning ranges from hundreds of meters to tens of kilometers,
and more than roughly 200 sound speed profile measurements were made over the survey area. These were used to calibrate the parabolic equation model used to estimate wide-area transmission loss, population density and target strength. Measured sound speed was found to follow a roughly Gaussian distribution about the mean at each depth and to be relatively uniform in time and space over the entire OAWRS imaging area (Fig. 2.3) with no evidence of horizontal features besides the expected short term fluctuations from internal wave activity.

The best fit between measured and modeled transmission loss was obtained for expected sandy bottom conditions of sound speed $1.7 \text{ km s}^{-1}$, density $1.9 \text{ g cm}^{-3}$ and attenuation $0.8 \text{ dB } \lambda^{-1}$, and in-water column attenuation $6 \times 10^{-5} \text{ dB } \lambda^{-1}$, as shown in Fig. B.1 using the maximum likelihood method [33]. Figure B.1 shows the standard deviation of one way TL for instantaneous broadband measurements is roughly 4.8 dB, and after 6-sample averaging is on the order of 1.7 dB, following previous experimental results and theory [33, 32]. We find the the mean measured TL to be within 2-3 dB of that of a perfectly reflecting lossless waveguide of the same depth (dashed gray line in Fig. B.1).

The scattered field from large fish shoals after two-way propagation in a waveguide also obeys circular complex Gaussian field statistics by the central limit theorem and has experimentally measured standard deviation of roughly 1.5 dB, after our standard 6-sample averaging, for the current experiment [20]. An example of the measured pressure level of scattered returns after beamforming and matched filtering is shown in Fig. B.2(A). The lack of apparent speckle noise fluctuations in this image is a result of (1) our standard 6-sample (3-ping and 2-range-cell) intensity average, and (2) inherent variance reduction from application of the matched filter to fluctuating signals received in
Figure B.1: (A) Experimentally determined mean and standard deviation (4.78 dB) of 677 measured instantaneous broadband one-way TL data after matched filter with 950 Hz center frequency. Plotted as a function of range, with modeled one-way TL overlain. Transmission data acquired by a single desensitized hydrophone on OAWRS receiver array on October 1-3 2006. Modeled TL computed by Monte Carlo simulation with parabolic equation inputing measured oceanography and bathymetry of Georges Bank environment. Modeled TL (dashed gray line) in a perfectly reflective waveguide with no water-column attenuation by normal-mode model. (B) After 6-samples averaging, the RMS error is reduced to 1.7 dB, consistent with theory for stationary averaging of intensities of circular complex Gaussian random fields [32, 33].

The mean measured pressure level along the transect shown in Fig. B.2(A) is plotted in Fig. B.2(B) overlain with the experimentally determined standard deviations for our standard 6-sample intensity average. These measured standard deviations range from 1 to 1.6 dB as shown in Fig. B.2(B) or 26% to 45% of our standard 6-sample averaged OAWRS intensity data. These also follow that expected from theory and numerous past experiments of received circular complex Gaussian field (CCGF) data where internal waves, ocean turbulence, eddies and variations in scatterer orientation have been shown to cause short term intensity fluctuations with a standard deviation that can be reduced to a small fraction of the mean after matched filtering and stationary averaging [84, 168, 80, 32, 81, 125].
Figure B.2: (A) Measured pressure level of scattered returns after beamforming and match filtering in dB re 1 m, normalized to unit source power, with standard 6-sample (3-ping and 2-range-cell) intensity average. (B) Mean measured pressure level along the transect in (A) appears with experimentally determined standard deviations for our standard 6-sample (3-ping and 2-range-cell) intensity average.
Appendix C

Analytic model for statistical moments of matched filtered scattered field from a group of random scatterers

Here we formulate the theory for an active bistatic imaging system, composed of a source located at \( r_0 \), and a receiving array located at the origin of the coordinate system, imaging a group of scatterers centered at location \( r \) in the far-field of both the source and receiver. The formulation assumes that the receiver is not in the forward scatter direction, so that the scattered field at the receiver can be separated from the incident field. The position of any scatterer in the group is \( r_p = r + u_p \), where \( u_p \) is its displacement from the group center. The source transmits a broadband waveform \( q(t) \) with Fourier transform \( Q(f) \) and bandwidth \( B \). For imaging systems used when only single scattering from each object is significant, the time-harmonic scattered field received from the group can be expressed by summing the contribution from each scatterer [22, 25, 24, 169, 170],

\[
\Phi_s(r, f) = Q(f) \sum_p W(r_p)W_0(r_p)G(r_p|0, f)G(0|r_p, f)S_p(\Omega_i, \Omega, k)\frac{S_p(\Omega_i, \Omega, k)}{k}, \tag{C.1}
\]

where \( W(r_p) \) and \( W_0(r_p) \) are the beampatterns of the imaging system source and re-
receiver respectively, weighting the contribution from the \(p^{th}\) scatterer, \(G\) is the medium’s Green’s function, \(S_p(\Omega_i, \Omega, k)\) is the scatter function, which depends on the wavenumber \(k = \frac{2\pi f}{c}\), incident angle from the source \(\Omega_i\), and scattered angle in the direction of the receiver \(\Omega\). The individual scatterers in the group can be of arbitrary size compared to the wavelength. Here, we assume the group center coincides with the main response axis of the receiving array such that the beam-pattern may be approximated as \(W(r_p)W_0(r_p) \approx 1\) for scatterers within the main lobe of the array, and \(W(r_p)W_0(r_p) \approx 0\) for scatterers outside of this lobe. We therefore restrict the sum in Eq. (C.1) to \(N\), the total number of scatterers imaged within the main lobe of the imaging system, over the full range extent of the group’s distribution. Let \(k_i\) and \(k_s\) be the incident and scattered wave vectors for scatterers within the mainlobe in Eq. (C.1). Approximating the spherical spreading loss, Green’s function to each scatterer can be simplified as

\[
G(r_p | r_0, f) = \frac{e^{ik|r_p - r_0|}}{4\pi|r_p - r_0|} \approx G(r | r_0; f)e^{ik_i \cdot u_p},
\]

where Green’s function from the source to the group center is factored out since the scatterer distribution is in the far-field of both the source and receiver. Using similar approximations for the Green’s functions from scatterers to receiver, the scattered field from Eq. (C.1) now simplifies to,

\[
\Phi_s(r, f) = Q(f)G(r | r_0, f)G(0 | r, f) \sum_{p=1}^{N} e^{j(k_i - k_s) \cdot u_p} S_p(\Omega_i, \Omega, k) k.
\]

The matched filter is now applied [171, 37, 172, 129], which is typically a normalized replica of the original transmitted waveform expressed as

\[
H(f | t_M) = \frac{1}{\sqrt{E_0}} Q^*(f)e^{i2\pi f t_M},
\]

where \(t_M\) is the time delay of the matched filter and \(E_0 = \int |Q(f)|^2 df\) is the source energy. Using Fourier synthesis, the time-dependent matched filtered scattered signal
from Eq. C.3 is,

$$\Psi_s(t_M) = \int \frac{1}{k} \left( \sum_{p=1}^{N} e^{i(k_i - k_s) \cdot u_p} S_p(\Omega_i, \Omega, k) \right) \Xi(f) df$$

(C.5)

where

$$\Xi(f) = \frac{1}{\sqrt{E_0}} |Q(f)|^2 G(r_f | r_0, f) G(0 | r, f) e^{-i2\pi f(t-t_M)}. \quad \text{(C.6)}$$

The matched filtered signal in Eq. (C.5) depends on the phase contribution from each scatterer, and is randomized by \(N\), the total number of scatterers imaged within the sonar beam, \(u_p\) and \(S_p\), the location and scatter function for each scatterer respectively. In general, the position \(u_p\) can be a function of time.

The expected intensity of the matched filtered scattered returns is a sum of the mean field squared \(\langle |\Psi_s(t_M)|^2 \rangle\), or coherently scattered intensity, and the variance \(\text{Var}(\Psi_s(t_M))\), or incoherently scattered intensity. If we assume the random variables \(N\), \(u_p\) and \(S_p\) are mutually uncorrelated and the scatterers are identically distributed, then, the squared mean matched filtered scattered field becomes,

$$\left| \langle \Psi_s(t_M) \rangle \right|^2 = \left| \int \frac{1}{k} \left( \sum_{p=1}^{N} e^{i(k_i - k_s) \cdot u_p} S_p(\Omega_i, \Omega, k) \right) \Xi(f) df \right|^2$$

$$= \langle N \rangle^2 \left| \int \frac{1}{k} U(k_i - k_s) \langle S(\Omega_i, \Omega, k) \rangle \Xi(f) df \right|^2. \quad \text{(C.7)}$$

Here, \(U(\kappa)\) is the characteristic function \([173, 22]\),

$$U(\kappa) = \langle e^{i\kappa \cdot u} \rangle = \int e^{i\kappa \cdot u} p_u(u) du,$$ \quad \text{(C.8)}

which is a three-dimensional spatial Fourier transform of probability density function (PDF) \(p_u(u)\) of scatterer position. We assume that the expectation is taken over a measurement time where the distribution is statistically stationary. The coherent intensity in Eq. C.7 depends on the expected scatter function, \(\langle S(\Omega_i, \Omega, k) \rangle\), which in general depends on the incident and scattered angles, and the statistics of the shape, size, material
properties, and orientation of the scatterers in the group. The second moment of the matched filtered scattered field from Eq. (C.5),

\[
\langle |\Psi_s(t_M)|^2 \rangle = \int \int \frac{1}{k k'} \left( \sum_{p=1}^{N} \sum_{q=1}^{N} e^{i[(k_i - k_s) \cdot u_p - (k_i' - k_s') \cdot u_q]} S_p(\Omega_i, \Omega, k) S^*_q(\Omega_i, \Omega, k') \right) \Xi(f) \Xi^*(f') df df',
\]

depends on the correlation between two different particle’s phase contributions and scatter functions. To evaluate Eq. (C.10), we must in general specify the joint probability distribution function for the scatterers’ positions and scatter functions. Here, we assume that the scatter functions are statistically independent from each other and from their relative position among the group. We also assume that the scatterers’ relative positions and phase contributions are uncorrelated with the majority of the other scatterers within the imaging system resolution footprint. For many groups in nature, such as fish schools, bird flocks and insect swarms, each scatterer’s position depends only on adjacent and nearby scatterers within some small radius, \(r_c\), beyond which their positions can be considered independent [174, 175, 139], such that

\[
\lim_{|u_p - u_q| >> r_c} p(u_p, u_q) = p(u_p)p(u_q).
\]

The correlation between two particle’s positions is typically formulated in terms of the pair distribution function, \(g\) [24], defined by

\[
p(u_p, u_q) = \frac{g(u_p - u_q)}{V^2}
\]

where \(V\) is the group volume. For scatterers separated by distances greater than \(r_c\), their positions can be considered independent, as \(g\) approaches unity. When the imaging system beamwidth is much larger than \(r_c\), the contribution to the total scattered intensity from correlated scatterers is negligible. Under these conditions, the expectation over the
scatter functions and phase contributions simplify,

\[
\left\langle \sum_{p=1}^{N} \sum_{q=1}^{N} e^{i[(k_i - k_s) \cdot u_p - (k'_i - k'_s) \cdot u_q]} S_p(\Omega_i, \Omega, k) S^*_q(\Omega_i, \Omega, k') \right\rangle
\]

\[
= \int \sum_{p=1}^{N} \sum_{q=1}^{N} \left[ \left( e^{i[(k_i - k_s) \cdot u_p - (k'_i - k'_s) \cdot u_q]} \right) \langle S_p(\Omega_i, \Omega, k) S^*_q(\Omega_i, \Omega, k') \rangle \right] \delta_{pq}
\]

\[
+ \left( e^{i[(k_i - k_s) \cdot u_p]} \right) \left( e^{-j[(k'_i - k'_s) \cdot u_q]} \right) \langle S_p(\Omega_i, \Omega, k) S^*_q(\Omega_i, \Omega, k') \rangle p(N) dN
\]

\[
= \langle N \rangle \left[ U((k_i - k_s) - (k'_i - k'_s)) \langle S(\Omega_i, \Omega, k) S^*(\Omega_i, \Omega, k') \rangle
\]

\[
- U(k_i - k_s) U^*(k'_i - k'_s) \langle S(\Omega_i, \Omega, k) \rangle \langle S^*(\Omega_i, \Omega, k') \rangle \right]
\]

\[
+ \langle N^2 \rangle U(k_i - k_s) U^*(k'_i - k'_s) \langle S(\Omega_i, \Omega, k) \rangle \langle S^*(\Omega_i, \Omega, k') \rangle \rangle \Xi(f) \Xi^*(f') df df'.
\]

(C.12)

where \( \delta_{pq} \) is the Kronecker delta and \( p(N) \) is the probability of finding \( N \) scatterers in the group. Inserting Eq. (C.12) into Eq. (C.10) results in the following expression for the second moment,

\[
\langle |\Psi_s(t_M)|^2 \rangle = \int \int \frac{1}{kk'} \left[ \langle N \rangle \left[ U((k_i - k_s) - (k'_i - k'_s)) \langle S(\Omega_i, \Omega, k) S^*(\Omega_i, \Omega, k') \rangle
\]

\[
- U(k_i - k_s) U^*(k'_i - k'_s) \langle S(\Omega_i, \Omega, k) \rangle \langle S^*(\Omega_i, \Omega, k') \rangle \right]
\]

\[
+ \langle N^2 \rangle U(k_i - k_s) U^*(k'_i - k'_s) \langle S(\Omega_i, \Omega, k) \rangle \langle S^*(\Omega_i, \Omega, k') \rangle \rangle \Xi(f) \Xi^*(f') df df'.
\]

(C.13)

The second moment can be expressed as the sum of the coherent intensity in Eq. C.7 and the incoherent intensity given by the variance of the field,

\[
\text{Var}(\Psi_s(t_M)) = \langle |\Psi_s(t_M)|^2 \rangle - \langle |\Psi_s(t_M)|^2 \rangle
\]

\[
= \langle N \rangle \int \int \frac{1}{kk'} \left[ U((k_i - k_s) - (k'_i - k'_s)) \langle S(\Omega_i, \Omega, k) S^*(\Omega_i, \Omega, k') \rangle
\]

\[
- U(k_i - k_s) U^*(k'_i - k'_s) \langle S(\Omega_i, \Omega, k) \rangle \langle S^*(\Omega_i, \Omega, k') \rangle \right] \Xi(f) \Xi^*(f') df df'
\]

\[
+ \frac{\text{Var}(N)}{\langle N \rangle^2} \langle |\Psi_s(t_M)|^2 \rangle.
\]

(C.14)
The coherent intensity in Eq. (C.7) is proportional to $\langle N \rangle^2$. If the standard deviation in the number of scatterers is small compared to its mean, then the incoherent intensity in Eq. (C.14) is proportional to $\langle N \rangle$. This is important for population density imaging, since population estimation from the matched filtered intensity depends on whether scattering from a group is coherent or incoherent.

An expression is provided for the incoherent matched filtered intensity in the time domain in Refs. [129] and [176] as the output of a convolution between the magnitude square auto-correlation function of the source signal with the mean backscatter cross-section spatial distribution of the scatterer. This causes complications when there is dispersion in the medium or target since the pulse shape may be altered leading to changes in the range resolution. Furthermore, the incoherent intensity expression in Ref. [129] is only valid when the total number of scatterers in each resolution cell is a constant over time or when the coherent intensity is significantly smaller than the variance, as can be seen by comparison with Eq. (C.14) in this paper. The matched filtered intensity scattered from a continuum is derived in Eq. 13 of Ref. [177] for the ocean seafloor reverberation. However, the final expression for the expected matched filtered intensity, Eq. 14 of Ref. [177] does not retain the time-frequency exponential dependence. This term is essential, along with the wavenumber dependent term for describing how the variance retains high spatial resolution, as can be seen by comparison with Eq. 14 in this paper.

In practical imaging systems, the matched filtered scattered signal and its intensity as a function of time are charted to range by multiplying with the propagation speed of the signal and accounting for the distance travelled from source to scatterer and from scatterer to receiver.
Appendix D

Model for the broadband matched filtered fully scattered field from a given distribution of scatterers including multiple scattering

Here we describe the theoretical model used to calculate the broadband matched filtered fully scattered field from a given 3D spatial distribution of distinct scatterers that includes multiple scattering. The formulation follows the approach of Ref. [10]. The imaging geometry, source, receiver and fish coordinates are described in Sec. 3.1.1.

Given a group containing $N$ scatterers, the time-harmonic total scattered field received at frequency $f$, 

$$
\Phi_s(r, f) = \sum_{p=1}^{N} A(\phi_p) B(\phi_p) \Phi_s(r, p, f),
$$

(D.1)

is a sum of the fully scattered fields from each of the scatterers within the group, where $A(\phi_p)$ and $B(\phi_p)$ are the beampatterns of the imaging system source and receiver arrays respectively weighting the contribution from the $p$th scatterer. Here $A(\phi_p)$ can be approximated as 1 because the vertical source array of the waveguide remote sensing
system has an azimuthally symmetric beampattern. $\Phi_s(r, p, f)$ is the fully scattered field\cite{30, 31} from the $p$th scatterer, formulated here as a sum of the singly scattered field, $\Phi_s^{(1)}(r, p, f)$, and the multiply scattered field, $\Phi_s^{(MS)}(r, p, f)$,

$$\Phi_s(r, p, f) = \Phi_s^{(1)}(r, p, f) + \Phi_s^{(MS)}(r, p, f)$$

$$= Q(f)G(r_p | r_0, f)G(r | r_p, f) S_p(\Omega_i, \Omega_s, k) \frac{k}{k}$$

$$+ \sum_{q=1, q\neq p}^N \Phi_s(r_p, q, f) G(r | r_p, f) S_p(\Omega_{qp}, \Omega_s, k), \quad (D.2)$$

where $G$ is the medium’s Green’s function, $S_p(\Omega_i, \Omega_s, k)$ is the scatter function\cite{25, 178} for the $p$th scatterer which depends on wavenumber $k = \frac{2\pi f}{c}$ and the direction of the incident and scattered plane waves,\cite{25, 178, 24} $\Omega_i$ and $\Omega_s$, respectively. The scatter function can be used to model general scatterers of arbitrary size compared to the wavelength.

Upon substituting Eq. (D.2) into Eq. (D.1), the time harmonic fully scattered field becomes

$$\Phi_s(r, f) = \Phi_s^{(1)}(r, f) + \Phi_s^{(MS)}(r, f)$$

$$= \sum_{p=1}^N \mathcal{B}(\phi_p) \Phi_s^{(1)}(r, p, f) + \sum_{p=1}^N \mathcal{B}(\phi_p) \Phi_s^{(MS)}(r, p, f) \quad (D.3)$$

where $\Phi_s^{(1)}(r, f)$ is the time harmonic singly scattered field, and $\Phi_s^{(MS)}(r, f)$ is the time harmonic multiply scattered field from all $N$ scatterers received at frequency $f$.

In Eq. (D.2), the multiply scattered field $\Phi_s^{(MS)}(r, p, f)$ at $r$ from the $p$th scatterer depends on the sum of the acoustic fields first scattered from all other $N-1$ scatterers and incident on the $p$th scatterer, with each term given by $\Phi_s(r_p, q, f)$. In order to solve Eq. (D.2) for all $N$ scatterers, we must find the scattered field incident on each scatterer.
from all other $N-1$ scatterers. This is expressed as

$$
\Phi_s(r_n, p, f) = Q(f)G(r_p|f)G(r_n|r_p, f)\frac{S_p(\Omega_i, \Omega_{pn}, k)}{k} + \sum_{q=1, q\neq p}^{N} \Phi_s(r_p, q, f)G(r_n|r_p, f)\frac{S_p(\Omega_{qp}, \Omega_{pn}, k)}{k},
$$

where $n$ and $p$ are variables indicating that the field incident on $n$ is scattered from $p$, $\Omega_{qp}$ is the incident angle onto $p$ scattered from $q$ and $\Omega_{pn}$ is the scattered angle from $p$ scattered onto $n$. Equation (D.4) has a similar form as Eq. (D.2) in terms of both singly and multiply scattered fields. Equations (D.2) and (D.4) expressed in terms of the object’s plane wave scatter function are valid [87, 25] in the far-field of each scatterer, where $r_p$, $|r_0-r_p|, |r_n-r_p| > a_p^2/\lambda$, where $a_p$ is the largest dimension of the $p$th scatterer.

The matched filter [128, 37, 129] is a normalized replica of the original transmitted waveform,

$$
H(f|t_M) = \frac{1}{\sqrt{E_0}}Q^*(f)e^{i2\pi ft_M},
$$

where $t_M$ is the time delay of the matched filter and $E_0 = \int |Q(f)|^2 df$ is the source energy. Applying Fourier synthesis, the time-dependent matched filtered fully scattered field from the scatterer group, including multiple scattering, can be expressed as,

$$
\Psi_s(t_M) = \int_B \Phi_s(r, f)H(f|t_M)e^{-i2\pi ft}df,
$$

an integral involving Eq. (D.1) over the signal bandwidth.

Substituting Eq. (D.3) into Eq. (D.6), the time dependent matched filtered fully scattered field can be decomposed into,

$$
\Psi_s(t_M) = \Psi_s^{(1)}(t_M) + \Psi_{s}^{(MS)}(t_M)
= \int_B \Phi_s^{(1)}(r, f)H(f|t_M)e^{-i2\pi ft}df + \int_B \Phi_s^{(MS)}(r, f)H(f|t_M)e^{-i2\pi ft}df,
$$

(D.7)
a sum of the time-dependent matched filtered singly scattered field, $\Psi_{s}^{(1)}(t_M)$, and the time-dependent matched filtered multiply scattered field, $\Psi_{s}^{(MS)}(t_M)$.
Appendix E

Modeling fish swimbladder as a damped air-filled prolate spheroid

There are several models [179, 120, 115] that describe the scattering properties of a swimbladder near resonance. Here we model a fish swimbladder near resonance as a damped air-filled oscillator [180, 27, 29, 26, 142] and take into account the prolate spheroidal shape of the swimbladder through a correction term to the resonance frequency developed by Weston [120], as well as the damping from viscosity of the surrounding fish flesh modeled by Love [115]. This model approximates the swimbladder as a viscous heat-conducting shell enclosing an air cavity with surface tension at the inner surface [115, 67] that leads to a damped resonance behavior. Given a fish of swimbladder volume \( V_z \) (m\(^3\)) at depth \( z \) (m), its complex scatter function is [180],

\[
S(k) = \frac{\left(\frac{f_0}{f} - 1\right)k\bar{a}}{\left(\frac{f_0}{f} - 1\right)^2 + \delta^2} + i\frac{\delta k\bar{a}}{\left(\frac{f_0}{f} - 1\right)^2 + \delta^2},
\]  
(E.1)

where \( f_0 \) is the resonance frequency (Hz) and \( \bar{a} = (\frac{3}{4\pi}V_z)^{1/3} \) is the equivalent radius (m) of the swimbladder. The dimensionless damping coefficient,

\[
\delta = \delta_{rad} + \delta_{vis} = k\bar{a} + \frac{\xi_f}{\pi\bar{a}^2 f \rho_f},
\]  
(E.2)
is a sum of the radiation damping $\delta_{\text{rad}}$ and the viscous damping $\delta_{\text{vis}}$. The contribution from thermal damping is assumed to be negligible [115]. In Eq. (E.2), the physical properties of fish, such as the fish flesh density $\rho_f$ (kg m$^{-3}$) and viscosity $\xi_f$ (Pa s) are usually empirically determined. For instance, the viscosity and flesh density for Atlantic herring have been experimentally found to be $\xi_f=50$ Pa s (Ref. [67]) and $\rho_f=1071$ kg m$^{-3}$ (Ref. [181]) respectively. The resonance frequency of a prolate spheroid shaped swimbladder is given by

$$f_0 = \frac{\zeta}{2\pi a} \sqrt{\frac{3\gamma P_z}{\rho_f}} ,$$

where $\gamma=1.40$ is the ratio of specific heats of air and water, and $P_z$ is the ambient pressure at depth $z$. The correction term for the spheroidal shaped swimbladder [120], $\zeta$, is given by

$$\zeta = \frac{\sqrt{2}(1 - \epsilon^2)^{1/4}}{\epsilon^{1/3}} \left\{ \ln \left[ 1 + \sqrt{1 - \epsilon^2} \right] - \frac{\sqrt{1 - \epsilon^2}}{1 - \sqrt{1 - \epsilon^2}} \right\}^{-1/2} ,$$

where eccentricity $\epsilon$ is the ratio of semi-minor to semi-major axis of the prolate spheroid. For the herring groups modeled in this paper, $0.1 < \epsilon < 0.2$ [9].

The swimbladder volume $V_z$ at depth $z$ of an individual from a given fish species may be estimated using length and weight measurements of trawl samples of the species, and the fact that for most swimbladder-bearing fish, the air-filled swimbladder typically comprises roughly 4 to 5% of fish body volume $V_b$ (m$^3$) at neutral buoyancy depth $z_{nb}$ (m) [97]. For a fish of fork length $L$ (cm), its body volume can be estimated from,

$$V_b = \frac{W}{\rho_f} = \frac{1}{\rho_f} pL^q$$

where $p$ and $q$ are regression parameters [138] empirically determined from length $L$ and weight $W$ measurements of trawl samples. The swimbladder volume at any depth $z$ can
then be determined using Boyle’s law,

\[ P_{nb} V_{nb} = P_z V_z, \quad (E.6) \]

where \((P_{nb}, V_{nb})\) are the ambient pressure and swimbladder volume at neutral buoyancy depth \(z_{nb}\), and \((P_z, V_z)\) are the ambient pressure and volume at any depth \(z\).

Applying the extinction theorem [25, 23] and substituting Eq. (E.2) into Eq. (E.1), the total extinction cross section \(\sigma_{ext}\) of fish swimbladder that accounts for energy removed from the forward direction can be expressed as a sum of the scattering \(\sigma_{sca}\) and absorption \(\sigma_{abs}\) cross sections [142],

\[ \sigma_{ext} = \frac{4\pi}{k^2} \Im \{S_f(k)\} = \sigma_{sca} + \sigma_{abs} \]

\[ = \frac{4\pi}{k^2} \frac{\delta k\tilde{a}}{\left(\frac{f_0^2}{f^2} - 1\right)^2 + \delta^2} \]

\[ = \frac{4\pi\tilde{a}^2}{\left(\frac{f_0^2}{f^2} - 1\right)^2 + \delta^2} + \frac{4\pi\tilde{a}^2(\delta_{vis}/\delta_{rad})}{\left(\frac{f_0^2}{f^2} - 1\right)^2 + \delta^2}, \quad (E.7) \]

where \(S_f(k)\) is the scatter function in forward direction. Note that the scattering cross section \(\sigma_{sca}\) is consistent with that of Love’s model [115, 67].

In summary, given the measured fish length and depth distributions, the empirically determined length-volume regression Eq. (E.5), and assuming the prolate spheroidal swimbladder has a major axis that is a constant percentage of the total fish length (usually 26 to 33% (Ref. [124, 68])) and only changes in the minor-axis of the swimbladder contributes to volume change because of physiological constraints in fish anatomy [68, 102, 123, 124], the complex scatter function of a given fish at frequency \(f\) in Eq. (E.1) then only depends on a single parameter, the swimbladder volume \(V_z\) at depth \(z\) or equivalently the neutral buoyancy depth.
Appendix F

Modified polar coordinate formulation of the bearings-only target motion analysis via extended Kalman filter

F.1 Single-sensor bearings-only tracking problem formulation

Consider a typical target-observer geometry shown in Fig. F.1, with target and single-sensor observer confined to the horizontal plane. The horizontal position of the target is given by coordinates \((x_t, y_t)\), and the target is assumed to move with a nearly constant velocity \((\dot{x}_t, \dot{y}_t)\), therefore its corresponding acceleration, defined as \((\ddot{x}_t, \ddot{y}_t)\), is approximated as 0. The target’s kinematic state vector is defined as

\[
x_t = [x_t \ y_t \ \dot{x}_t \ \dot{y}_t]^T
\]  \hspace{1cm} (F.1)

The single-sensor observer’s kinematic state vector is similarly defined as

\[
x^o = [x^o \ y^o \ \dot{x}^o \ \dot{y}^o]^T
\]  \hspace{1cm} (F.2)

and its corresponding acceleration is given by \((\ddot{x}^o, \ddot{y}^o)\). By introducing the relative target
state vector, defined as

\[ x \triangleq x^t - x^o = [x \ y \ \dot{x} \ \dot{y}]^T, \]  

the discrete-time target state equation at time \( t_{k+1} \) can be written as

\[ x_{k+1} = F_{k,k+1} x_k - w_{k,k+1}, \]  

where

\[ F_{k,k+1} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \]  

is the state transition matrix, \( T = t_{k+1} - t_k \) is the sampling interval, and

\[ w_{k,k+1} = \begin{bmatrix} w_{k,1} \\ w_{k,2} \\ w_{k,3} \\ w_{k,4} \end{bmatrix} = \begin{bmatrix} \int_{t_k}^{t_{k+1}} (t_{k+1} - u) \ddot{x}_u^o du \\ \int_{t_k}^{t_{k+1}} (t_{k+1} - u) \ddot{y}_u^o du \\ \int_{t_k}^{t_{k+1}} \dddot{x}_u^o du \\ \int_{t_k}^{t_{k+1}} \dddot{y}_u^o du \end{bmatrix}, \]  

is a vector that characterizes the single-sensor observer acceleration. We assume \( w_{k,k+1} \) is known at every instant of time since the single-sensor observer’s state vector \( x_k^o \) is usually provided by an onboard GPS module.

The bearing measurement \( z_k \) at time \( t_k \) is the angle from the single-sensor observer to the target measured with respect to the true north in clockwise direction, as shown in Fig. F.1, and is corrupted by additive measurement noise, which can be written as

\[ z_k = h(x_k) + \nu_k \]  

where \( \nu_k \sim \mathcal{N}(0, \sigma_\beta^2) \) is a zero-mean Gaussian white noise with variance \( \sigma_\beta^2 \), and

\[ h(x_k) = \arctan \left( \frac{x_k}{y_k} \right) \]  

is the true bearing angle.
Figure F.1: A typical two-dimensional target-observer geometry. \( \mathbf{v}_o = (\dot{x}^o, \dot{y}^o) \) is the velocity vector of the single-sensor observer. \( \mathbf{v}_t = (\dot{x}^t, \dot{y}^t) \) is the velocity vector of the submerged target. \((x^o, y^o)\) and \((x^t, y^t)\) are the initial position of the single-sensor observer and submerged target, respectively. \( z_1 \) and \( z_k \) are bearing measurements at time \( t_1 \) and \( t_k \).

Given a sequence of target bearing measurements \( \mathbf{z} = [z_1 \ z_2 \ \cdots \ z_k] \), and the discrete-time target state equation described in Eqs. (F.4) to (F.6), the objective of single-sensor bearings-only TMA is to estimate the relative target kinematic state vector \( \mathbf{x}_k \) at each time instant \( t_k \).
F.2 Modified polar coordinates formulation

The modified polar coordinates (MPC) state vector is defined by

\[ y_k \triangleq [y_{k1} \ y_{k2} \ y_{k3} \ y_{k4}]^T \]

\[ = \begin{bmatrix} \dot{\beta}_k & \dot{r}_k & \beta_k & 1/r_k \end{bmatrix}^T. \tag{F.9} \]

where

\[ r_k = \sqrt{x_k^2 + y_k^2} \tag{F.10} \]

\[ \beta_k = \arctan\left(\frac{x_k}{y_k}\right) \tag{F.11} \]

\[ \dot{r}_k = \frac{x_k \dot{x}_k + y_k \dot{y}_k}{r_k} \tag{F.12} \]

\[ \dot{\beta}_k = \frac{\dot{x}_k y_k - x_k \dot{y}_k}{r_k^2} \tag{F.13} \]

Using the approach demonstrated in Ref. [46], the equivalent discrete-time target state equation of in MPC is given by

\[ y_{k+1} = f^{\text{mpc}}(y_k) = \begin{bmatrix} f^{\text{mpc}}_1(y_k) \\ f^{\text{mpc}}_2(y_k) \\ f^{\text{mpc}}_3(y_k) \\ f^{\text{mpc}}_4(y_k) \end{bmatrix} = \begin{bmatrix} (\alpha_2 \alpha_3 - \alpha_1 \alpha_4)/(\alpha_1^2 + \alpha_2^2) \\ (\alpha_1 \alpha_3 + \alpha_2 \alpha_4)/(\alpha_1^2 + \alpha_2^2) \\ y_{k3} + \arctan(\alpha_1/\alpha_2) \\ y_{k4}/(\alpha_1^2 + \alpha_2^2)^{1/2} \end{bmatrix} \tag{F.14} \]

where \( \alpha_i, \ i = 1, 2, 3, 4 \) are defined as

\[ \alpha_1 = Ty_k - y_k (w_{k1} \cos y_{k3} - w_{k2} \sin y_{k3}) \tag{F.15} \]

\[ \alpha_2 = 1 + Ty_k - y_k (w_{k1} \sin y_{k3} + w_{k2} \cos y_{k3}) \tag{F.16} \]

\[ \alpha_3 = y_{k1} - y_k (w_{k3} \cos y_{k3} - w_{k4} \sin y_{k3}) \tag{F.17} \]

\[ \alpha_4 = y_{k2} - y_k (w_{k3} \sin y_{k3} + w_{k4} \cos y_{k3}) \tag{F.18} \]

Similarly, the equivalent measurement equation in MPC can be expressed as

\[ z_k = H_k y_k + \nu_k \tag{F.19} \]
where $H_k = [0 \ 0 \ 1 \ 0]$. Now, in the MPC, the discrete-time target state equation becomes nonlinear and the measurement equation becomes linear, as opposed to the reversed case in Cartesian coordinates (Eqs. (F.4) and (F.7)).

Here, we denote the Jacobian $F_{\text{mpc}}^k$ (linearized transition matrix) as

$$
\hat{F}_{\text{mpc}}^k = \left[ \nabla y_k \left[ f_{\text{mpc}}(y_k) \right] \right]^T \bigg|_{y=y_k|k},
$$

(F.20)

whose derivation is given in Sec. F.3. Straightforward application of the extended Kalman filter (EKF) to Eqs. (F.14) and (F.19), will yield the MPC estimation algorithm given by

$$
\hat{y}_{k+1|k} = f_{\text{mpc}}(\hat{y}_{k|k})
$$

$$
P_{k+1|k} = \hat{F}_{\text{mpc}}^k P_{k|k} (\hat{F}_{\text{mpc}}^k)^T
$$

$$
K_{k+1} = P_{k+1|k} H_y^T \left[ H_y P_{k+1|k} H_y^T + \sigma_{\beta}^2 \right]^{-1}
$$

$$
\hat{y}_{k+1|k+1} = \hat{y}_{k+1|k} + K_{k+1}[z_{k+1} - H_y \hat{y}_{k+1|k}]
$$

$$
P_{k+1|k+1} = [I - K_{k+1} H_y] P_{k+1|k}
$$

Lastly, the state vector estimated in the MPC need to be transformed back to the Cartesian coordinate for comparison with other methods. The MPC to Cartesian transformation is given by

$$
x_k = \frac{1}{y_{k4}} \begin{bmatrix}
\sin y_{k3} \\
\cos y_{k3} \\
y_{k2} \sin y_{k3} + y_{k1} \cos y_{k3} \\
y_{k2} \cos y_{k3} - y_{k1} \sin y_{k3}
\end{bmatrix}
$$

(F.21)

F.3 Linearized transition matrix for MPC-EKF

The Jacobian (linearized transition matrix) $\hat{F}_{\text{mpc}}^k$ for the MPC-EKF defined by Eq.
(F.20) can be rewritten as

\[ \hat{f}_{k}^{\text{mpc}} = \left[ \nabla_{y_k} \left[ f^{\text{mpc}}(y_k) \right] \right]|_{y=y_{k|k}}^{T} \]

\[= \begin{bmatrix}
\partial f_{1}^{\text{mpc}}(y_k)/\partial y_{k1} & \partial f_{1}^{\text{mpc}}(y_k)/\partial y_{k2} & \partial f_{1}^{\text{mpc}}(y_k)/\partial y_{k3} & \partial f_{1}^{\text{mpc}}(y_k)/\partial y_{k4} \\
\partial f_{2}^{\text{mpc}}(y_k)/\partial y_{k1} & \partial f_{2}^{\text{mpc}}(y_k)/\partial y_{k2} & \partial f_{2}^{\text{mpc}}(y_k)/\partial y_{k3} & \partial f_{2}^{\text{mpc}}(y_k)/\partial y_{k4} \\
\partial f_{3}^{\text{mpc}}(y_k)/\partial y_{k1} & \partial f_{3}^{\text{mpc}}(y_k)/\partial y_{k2} & \partial f_{3}^{\text{mpc}}(y_k)/\partial y_{k3} & \partial f_{3}^{\text{mpc}}(y_k)/\partial y_{k4} \\
\partial f_{4}^{\text{mpc}}(y_k)/\partial y_{k1} & \partial f_{4}^{\text{mpc}}(y_k)/\partial y_{k2} & \partial f_{4}^{\text{mpc}}(y_k)/\partial y_{k3} & \partial f_{4}^{\text{mpc}}(y_k)/\partial y_{k4} \\
\end{bmatrix}
\]

\[= \mathbf{B} + \mathbf{C} \mathbf{D} \]

where the element of matrices \( \mathbf{B} \), \( \mathbf{C} \), and \( \mathbf{D} \) are given by

\[ B_{i,j} = \frac{\partial f_{i}^{\text{mpc}}(y_k)}{\partial y_{k,j}} \]

\[ C_{i,j} = \frac{\partial f_{i}^{\text{mpc}}(y_k)}{\partial \alpha_j} \]

\[ D_{i,j} = \frac{\partial \alpha_i}{\partial y_{k,j}} \]

where \( \alpha_i, \ i = 1, 2, 3, 4 \) was defined by Eqs. (F.15) to (F.18). From Eqs. (F.14) to (F.18), we can evaluate the elements of matrices \( \mathbf{B} \), \( \mathbf{C} \), and \( \mathbf{D} \) with the result

\[ \mathbf{B} = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1/(\alpha_1^2 + \alpha_2^2)^{1/2} \\
\end{bmatrix} , \quad \text{(F.23)} \]

\[ \mathbf{C} = \begin{bmatrix}
c_{11} & -c_{21} & c_{13} & c_{32} \\
c_{21} & c_{11} & -c_{32} & c_{13} \\
c_{31} & c_{32} & 0 & 0 \\
c_{41} & c_{42} & 0 & 0 \\
\end{bmatrix} , \quad \text{(F.24)} \]
where

\[
c_{11} = \left[-\alpha_1(\alpha_2\alpha_3 - \alpha_1\alpha_4) - \alpha_2(\alpha_1\alpha_3 + \alpha_2\alpha_4)\right]/(\alpha_1^2 + \alpha_2^2)^2
\]

\[
c_{13} = \alpha_2/(\alpha_1^2 + \alpha_2^2)
\]

\[
c_{21} = \left[-\alpha_1(\alpha_1\alpha_3 + \alpha_2\alpha_4) + \alpha_2(\alpha_2\alpha_3 - \alpha_1\alpha_4)\right]/(\alpha_1^2 + \alpha_2^2)^2
\]

\[
c_{31} = \alpha_2/(\alpha_1^2 + \alpha_2^2)
\]

\[
c_{32} = -\alpha_1/(\alpha_1^2 + \alpha_2^2)
\]

\[
c_{41} = -\alpha_1\hat{y}_{k4}/(\alpha_1^2 + \alpha_2^2)^{3/2}
\]

\[
c_{42} = -\alpha_2\hat{y}_{k4}/(\alpha_1^2 + \alpha_2^2)^{3/2}
\]

and

\[
D = \begin{bmatrix}
T & 0 & d_{13} & d_{14} \\
0 & T & d_{23} & d_{24} \\
1 & 0 & d_{33} & d_{34} \\
0 & 1 & d_{43} & d_{44}
\end{bmatrix}
\]

where

\[
d_{14} = -[w_k \cos \hat{y}_{k3} - w_k \sin \hat{y}_{k3}]
\]

\[
d_{24} = -[w_k \sin \hat{y}_{k3} + w_k \cos \hat{y}_{k3}]
\]

\[
d_{34} = -[w_k \cos \hat{y}_{k3} - w_k \sin \hat{y}_{k3}]
\]

\[
d_{44} = -[w_k \sin \hat{y}_{k3} + w_k \sin \hat{y}_{k3}]
\]

\[
d_{13} = -\hat{y}_{k4}\hat{y}_{k3}
\]

\[
d_{23} = \hat{y}_{k4}\hat{y}_{k3}
\]

\[
d_{33} = -\hat{y}_{k4}\hat{y}_{k3}
\]

\[
d_{43} = \hat{y}_{k4}\hat{y}_{k3}
\]

Here, \(w_k\) are the elements of vector \(w_{k,k+1}\) defined by Eq. (F.6), and \(\hat{y}_{k3}\) and \(\hat{y}_{k4}\) are the 3rd and 4th element of the estimated target state vector \(\hat{y}_k\).
Appendix G

Initialization procedure implemented for bearings-only TMA using MPC-EKF

G.1 Initialization in Cartesian coordinates

Here, we first present the initialization procedure employed in the Cartesian coordinates. The initial position components of the relative target state vector $x_i = [x_i \ y_i]^T$ are derived using the first bearing measurement $\hat{\beta}_1$, and the prior knowledge of the initial target range. Suppose that the initial range prior is $r \sim \mathcal{N}(\bar{r}, \sigma_r^2)$, and the additive noise in bearing measurement is a zero-mean Gaussian white noise $(\beta_1 - \hat{\beta}_1) \sim \mathcal{N}(0, \sigma_\beta^2)$, the initial mean position components of the relative target state vector can be defined as

$$\bar{x}_i = \begin{bmatrix} \bar{x}_i \\ \bar{y}_i \end{bmatrix} = \begin{bmatrix} \bar{r} \sin \hat{\beta}_1 \\ \bar{r} \cos \hat{\beta}_1 \end{bmatrix} \quad \text{(G.1)}$$

and the corresponding error covariance matrix of $x_i$ is

$$P_{x_i} = \mathbb{E}[(x_i - \bar{x}_i)(x_i - \bar{x}_i)^T] = \begin{bmatrix} P_{xx} & P_{xy} \\ P_{yx} & P_{yy} \end{bmatrix} \quad \text{(G.2)}$$
where

\[
\begin{align*}
P_{xx} &= r\sigma^2 \beta^2 + \sigma^2 \sin^2 \beta_1 \\
P_{yy} &= r\sigma^2 \sin^2 \beta_1 + \sigma^2 \cos^2 \beta_1 \\
P_{xy} &= P_{yx} = (\sigma^2 - r^2\sigma^2) \sin \beta_1 \cos \beta_1 \\
\end{align*}
\] (G.3)

Similarly, suppose that the initial target speed prior is \( s \sim \mathcal{N}(\bar{s}, \sigma^2_s) \), and the initial target course prior is \( c \sim \mathcal{N}(\bar{c}, \sigma^2_c) \), the initial velocity component of the relative target state vector \( \mathbf{v}_i = [\dot{x}_i \dot{y}_i]^T \) can be defined as

\[
\mathbf{v}_i = \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = \begin{bmatrix} \dot{x}_i^* - \dot{x}_i^o \\ \dot{y}_i^* - \dot{y}_i^o \end{bmatrix} = \begin{bmatrix} s \sin c \ - \dot{x}_i^o \\ s \cos c \ - \dot{y}_i^o \end{bmatrix}
\] (G.6)

Therefore, the mean velocity component and the corresponding error covariance matrix of \( \mathbf{v}_i \) can be expressed as

\[
\begin{align*}
\overline{\mathbf{v}}_i &= \begin{bmatrix} \overline{x}_i \\ \overline{y}_i \end{bmatrix} = \begin{bmatrix} \bar{s} \sin \bar{c} - \dot{x}_i^o \\ \bar{s} \cos \bar{c} - \dot{y}_i^o \end{bmatrix}, \\
\mathbf{P}_{\mathbf{v}_i} &= E[(\mathbf{v}_i - \overline{\mathbf{v}}_i)(\mathbf{v}_i - \overline{\mathbf{v}}_i)^T] \\
&= \begin{bmatrix} P_{\overline{x}\overline{x}} & P_{\overline{x}\overline{y}} \\ P_{\overline{y}\overline{x}} & P_{\overline{y}\overline{y}} \end{bmatrix}
\end{align*}
\] (G.7)

where

\[
\begin{align*}
P_{\overline{x}\overline{x}} &= \bar{s}^2 \sigma^2_c \cos^2 \bar{c} + \sigma^2_s \sin^2 \bar{c} \\
P_{\overline{y}\overline{y}} &= \bar{s}^2 \sigma^2_c \sin^2 \bar{c} + \sigma^2_s \cos^2 \bar{c} \\
P_{\overline{x}\overline{y}} &= P_{\overline{y}\overline{x}} = (\sigma^2_s - \bar{s}^2 \sigma^2_c) \sin \bar{c} \cos \bar{c}
\end{align*}
\] (G.10)

Combining the results of \( \mathbf{x}_i \) and \( \overline{\mathbf{v}}_i \), the initial relative state vector and its corresponding error covariance matrix in Cartesian coordinates are

\[
\mathbf{x}_1 = \begin{bmatrix} \overline{x}_i \\ \overline{y}_i \end{bmatrix} = \begin{bmatrix} \bar{s} \sin \hat{\beta}_1 \\ \bar{s} \cos \hat{\beta}_1 \\ \bar{s} \sin \bar{c} - \dot{x}_i^o \\ \bar{s} \cos \bar{c} - \dot{y}_i^o \end{bmatrix}
\] (G.13)
and

\[ P_1 = \begin{bmatrix} P_{xx} & P_{xy} & 0 & 0 \\ P_{yx} & P_{yy} & 0 & 0 \\ 0 & 0 & P_{\dot{x}\dot{x}} & P_{\dot{x}\dot{y}} \\ 0 & 0 & P_{y\dot{x}} & P_{y\dot{y}} \end{bmatrix}, \quad (G.14) \]

respectively, where the elements of \( P_1 \) are given by Eqs. (G.3) to (G.5) and Eqs. (G.10) to (G.12).

### G.2 Initialization in the MPC

Here, we derive the equivalent initial target state vector and corresponding error covariance matrix of EKF in the MPC using the results derived for the Cartesian coordinates in Sec. G.1.

First, substituting Eqs. (F.10) to (F.13) into Eq. (F.9), we can rewrite the nonlinear transformation from the Cartesian to the MPC \( y = [\dot{\beta}, \dot{r}/r, \beta, 1/r]^T \) as

\[
y = g(x) = \begin{bmatrix} \frac{\dot{x}y - xy}{x^2 + y^2} \\ \frac{x\dot{x} + y\dot{y}}{x^2 + y^2} \\ \arctan(x/y) \\ 1/\sqrt{x^2 + y^2} \end{bmatrix}^{(G.15)}
\]

Then, the mean initial target state vector and corresponding error covariance matrix can be approximated as

\[
\overline{y} \approx g(E(x_1)) = g(\hat{x}_1) = \begin{bmatrix} g_1(\hat{x}_1) \\ g_2(\hat{x}_1) \\ g_3(\hat{x}_1) \\ g_4(\hat{x}_1) \end{bmatrix} = \begin{bmatrix} \frac{(\overline{x}_i\overline{y}_i - \overline{x}_i\overline{y}_i)}{(\overline{x}_i^2 + \overline{y}_i^2)} \\ \frac{(\overline{x}_i\overline{x}_i + \overline{y}_i\overline{y}_i)}{(\overline{x}_i^2 + \overline{y}_i^2)} \\ \arctan(\overline{x}_i/\overline{y}_i) \\ 1/\sqrt{\overline{x}_i^2 + \overline{y}_i^2} \end{bmatrix}^{(G.16)}
\]

and

\[
P_{1y} = E[(y - \overline{y})(y - \overline{y})^T] \approx GP_1G^T^{(G.17)}
\]
where

$$G = \nabla_x \left[ g^T(x) \right] \bigg|_{x=x_1}^T = \begin{bmatrix} \partial g_1(\hat{x}_1)/\partial x & \partial g_1(\hat{x}_1)/\partial y & \partial g_1(\hat{x}_1)/\partial \dot{x} & \partial g_1(\hat{x}_1)/\partial \dot{y} \\ \partial g_2(\hat{x}_1)/\partial x & \partial g_2(\hat{x}_1)/\partial y & \partial g_2(\hat{x}_1)/\partial \dot{x} & \partial g_2(\hat{x}_1)/\partial \dot{y} \\ \partial g_3(\hat{x}_1)/\partial x & \partial g_3(\hat{x}_1)/\partial y & \partial g_3(\hat{x}_1)/\partial \dot{x} & \partial g_3(\hat{x}_1)/\partial \dot{y} \\ \partial g_4(\hat{x}_1)/\partial x & \partial g_4(\hat{x}_1)/\partial y & \partial g_4(\hat{x}_1)/\partial \dot{x} & \partial g_4(\hat{x}_1)/\partial \dot{y} \end{bmatrix} \tag{G.18}$$

The individual element of $G$ is given by

$$\frac{\partial g_1(\hat{x}_1)}{\partial x} = -\bar{y}_i - 2\bar{x}_i\bar{\beta}_i, \quad \bar{\beta}_i = \bar{x}_i \bar{y}_i - \bar{y}_i \bar{x}_i - \bar{x}_i^2 + \bar{y}_i^2, \tag{G.19}$$

$$\frac{\partial g_1(\hat{x}_1)}{\partial y} = \frac{\bar{x}_i - 2\bar{y}_i\bar{\beta}_i}{\bar{r}_i^2}, \quad \bar{r}_i = \sqrt{\bar{x}_i^2 + \bar{y}_i^2}. \tag{G.20}$$

$$\frac{\partial g_2(\hat{x}_1)}{\partial x} = \frac{\bar{x}_i - 2\bar{y}_i\left(\frac{\bar{x}_i}{\bar{r}_i}\right)}{\bar{r}_i^2}, \tag{G.21}$$

$$\frac{\partial g_2(\hat{x}_1)}{\partial y} = \frac{\bar{y}_i - 2\bar{x}_i\left(\frac{\bar{y}_i}{\bar{r}_i}\right)}{\bar{r}_i^2}, \tag{G.22}$$

$$\frac{\partial g_3(\hat{x}_1)}{\partial \dot{x}} = \frac{\partial g_2(\hat{x}_1)}{\partial \dot{y}} = \frac{\partial g_3(\hat{x}_1)}{\partial \dot{x}} = -\frac{\partial g_4(\hat{x}_1)}{\partial \dot{y}} = \frac{\bar{y}_i}{\bar{r}_i^2}, \tag{G.23}$$

$$\frac{\partial g_3(\hat{x}_1)}{\partial \dot{y}} = -\frac{\partial g_2(\hat{x}_1)}{\partial \dot{x}} = \frac{\partial g_3(\hat{x}_1)}{\partial \dot{x}} = -\frac{\bar{x}_i}{\bar{r}_i^2}, \tag{G.24}$$

$$\frac{\partial g_3(\hat{x}_1)}{\partial \dot{x}} = \frac{\partial g_3(\hat{x}_1)}{\partial \dot{y}} = \frac{\partial g_4(\hat{x}_1)}{\partial \dot{x}} = \frac{\partial g_4(\hat{x}_1)}{\partial \dot{y}} = 0 \tag{G.25}$$

where

$$\bar{x}_i = \frac{\bar{x}_i\bar{y}_i - \bar{x}_i\bar{y}_i}{\bar{x}_i^2 + \bar{y}_i^2}, \quad \bar{r}_i = \frac{\bar{x}_i\bar{x}_i + \bar{y}_i\bar{y}_i}{\bar{x}_i^2 + \bar{y}_i^2}, \quad \bar{y}_i = \sqrt{\bar{x}_i^2 + \bar{y}_i^2}.$$
Bibliography


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List of Figures

2.1 Location of OAWRS 2006 experiment on the northern flank of Georges Bank in the Gulf of Maine. Plus indicates location of moored OAWRS source array deployed on October 1 to 3 at 42.2089N, 67.6892W, the coordinate origin for all OAWRS images in this chapter. Circle shows typical area imaged by OAWRS, 100 km diameter and wider than Cape Cod, in 70 s. Geographic locations of trawls deployed by NOAA FRV Delaware II are overlain. Dots indicate trawls where herring were predominant species. In contrast, diamond indicates a trawl where silver hake and squids dominated. The gray dashed box bounds the area of OAWRS imaging during the OAWRS 2006 experiment.
2.2 (A)-(C) OAWRS images of areal fish density zoomed-in around massive herring shoals, with densities exceeding 10 fish m$^{-2}$ in population centers. Measured during evening to midnight hours of October 4, 2 and 1, respectively. (A) The total population of herring in the large dense shoal is roughly 170 million, and that in the diffuse cloud outside the large shoal is roughly 70 million. Imaged shoal populations of herring are approximately 86 and 70 million respectively for (B) and (C). Uncertainty in the abundance estimate is 17-20%. Note that the figures are plotted on different scales, and the coordinate origin is the source location shown in Fig. 2.1.

2.3 Profiles of water-column sound speed from XBT and CTD measurements made from all four research vessels on the northern flank of Georges Bank and Georges Basin during OAWRS 2006.
2.4 Herring target strength at 950 and 1125 Hz estimated by matching areal fish density in OAWRS and CFFS data acquired during midnight hours of October 2. (A)-(C) A sequence of instantaneous OAWRS scattering strength images zoomed into the region containing a massive herring shoal with overlain CFFS line transect (solid line) made at nominal tow-speed of 2.5 m s$^{-1}$. (D) CFFS time-depth echogram provides local depth distributions of fish aggregations. Dashed lines at 23:30 EDT and 01:00 EDT correspond to transect start and end points $\alpha$ and $\Omega$, respectively. (E) The areal fish population densities inferred from CFFS measurements following Eq. (2.6) are plotted as a function of time in black, and the corresponding areal fish population densities in dB, $10 \log_{10}(n_{A,cffs})$, are plotted in gray. (F) The OAWRS scattering strength measurements and (G) instantaneous target strength estimates along CFFS line-transects at 950 and 1125 Hz. Target strength estimates near the edge of shoals are not accurate because of non-stationarity. (H) Population of herring within the area shown in (A)-(C) determined with various OAWRS fish density $n_A$ thresholds. Solid line gives population above the threshold, dotted line gives population below the threshold.
2.5 The intensity of scattered returns from shoals is highly frequency-dependent. The histograms illustrate that it is easier to detect shoals over background regions at higher frequencies. Simultaneous trawls show shoals are overwhelmingly comprised of herring while background regions yield negligible herring (Table 2.4, Fig. A.1). (A), (C) and (E) OAWRS images of herring shoal acquired simultaneously at three distinct frequency bands centered at 415, 950 and 1125 Hz at 00:41:15 EDT on October 2. The color scale used in (A), (C), and (E) is the same as in Figs. 2.4(A-C). (B), (D) and (F) Histograms of scattering strength values at locations within the shoal (areas inside the dashed box) and in a background region (areas inside the solid box) plotted for comparison. The 735 Hz data is ambient noise limited in background areas due to weak source level and is not shown.

2.6 Herring target strength at 950 and 1125 Hz estimated by matching areal fish density in OAWRS and CFFS data acquired after dusk hours of Oct 3. Similar to Fig. 2.4 but for the first contiguous shoal segment over a period of 55-min starting at 18:55 EDT on Oct 3. In (G), the target strength estimates near shoal edges are not accurate because of non-stationarity.

2.7 Herring target strength at 950 and 1125 Hz estimated by matching areal fish density in OAWRS and CFFS data acquired after dusk hours of Oct 3. Similar to Fig. 2.6 but for the second contiguous shoal segment over a period of 50-min approximately two hours after the first observation shown in Fig. 2.6.
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2.9 OAWRS scattering strength level differences for the indicated frequency pairs as a function of areal fish density in dB re $1\text{m}^2$ for data acquired between 22:00 and 22:45 EDT on October 3. The scattering strength difference at high fish densities equals the target strength difference for the frequency pair shown.

2.10 Similar to Fig. 2.5 but for 19:18:45 EDT on Oct 3.

2.11 Herring target strength at 950 Hz estimated by matching the areal density of OAWRS and CFFS data acquired during the early morning hours of September 27. Similar to Fig. 2.4 but for a contiguous shoal 40-min segment starting at 06:10 EDT on September 27.

2.12 Another example of target strength estimation by differencing OAWRS scattering strength images over wide areas, for shoaling herring imaged by OAWRS on Sep 27. Results are plotted for three frequency pairs, each obtained with 20 consecutive OAWRS images acquired in same track on Sep 27.
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\[ L_{TL} = 1.103L_{FL} + 0.01 \] [68] is used to convert herring’s fork length to the total length, where \( L_{TL} \) and \( L_{FL} \) are in cm. The mean fork length of redfish is 26.2 cm with a standard deviation 15% of the 26.2 cm mean. The equation 
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2.14 Atlantic herring length-weight regression calibration. The dots are the length-weight data obtained from the trawl-survey conducted by U.S. National Marine Fisheries Service in conjunction with OAWRS 2006 experiment, and the gray solid line indicates the derived best-fit length-weight regression, which can be expressed as 
\[ W = aL^b \], where \( W \) is the weight of herring in kg, \( L \) is the fork length of herring in cm, \( a \) and \( b \) are empirical regression parameters. For this trawl dataset, \( a = 3.35 \times 10^{-6} \) and \( b = 3.35 \).
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2.16 Same as Fig. 2.15, but for (A) Shoaling herring, distributed between 135 to 175 m, imaged with the OAWRS system from 18:55 to 19:50 EDT on October 3; (B) Shoaling herring, distributed between 120 to 175 m, imaged with the OAWRS system from 22:00 to 22:45 EDT on October 3; (C) Shoaling herring, distributed between 120 to 185 m, imaged with the OAWRS system from 06:10 to 06:50 EDT on September 27 (Fig. 2.11); (D) Shoaling herring, distributed between 150 to 180 m, imaged with the OAWRS system from 07:25 to 07:50 EDT on September 29.
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3.1 (a) Geometry of the bistatic acoustic imaging system in an ocean waveguide. (b) 3D spatial configuration of a large herring group containing 7831 individuals uniformly distributed within a volume similar to an ellipsoid that has axes dimensions of $L_x = 100$ m, $L_y = 125$ m, and $L_z = 33.33$ m, but with cross-range and depth extents cut at ±50 m and ±10 m respectively to the center of the herring group. The plotted volume has dimensions of $L_x = 100$ m, $L_y = 100$ m, and $L_z = 20$ m. (c) Modeled broadband two-way transmission loss $TTL_W$ calculated using Eq. (3.9) from a source array to potential fish locations and from the fish locations back to the receiver array center is shown as a function of fish range and depth in the Gulf of Maine environment for a source waveform of 50 Hz bandwidth centered at $f_c = 950$ Hz. (d) The $TTL_W$ obtained by first averaging the broadband two way propagated acoustic intensities over a 20 m thick fish layer centered at $z_s = 150$ m depth and then taking the log transform following Eq. (3.11). The error bars indicate one standard deviation in the broadband $TTL_W$ over the depth layer of the fish.

3.2 Effect of varying inter-fish spacing on the school target strength spectra for a small fish group containing 13 individuals. The mean inter-fish spacings are (a) one fish body length, $d = 40$ cm, (b) quarter fish body length, $d = 10$ cm, and (c) four times the fish body length, $d = 1.6$ m. The coherent, incoherent, total and estimated school target strength spectra are calculated via Eqs. (3.5) to (3.8) respectively.
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3.4 Expected experimental site of instantaneous Atlantic cod survey by OAWRS in Ipswich Bay near Massachusetts and New Hampshire coastlines. Plus indicates expected location of moored OAWRS source array at 42.8667N, 70.4583W. Circle shows typical area imaged by OAWRS, 50 km diameter in 35 s. Hexagon indicates the center location of cod groups employed in all examples shown in Sec. 3.5. Dashed line indicates the propagation path of acoustic transmissions from OAWRS source array to the fish group and from the fish group back to OAWRS receiver array.
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