Enhancement of Network Monitoring
&
Security Analysis Using Phasor Measurement Units

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ABSTRACT

Among various facilitators of the smart grid are the phasor measurement units (PMU) which are rapidly populating substations in today’s interconnected transmission systems. This dissertation investigates the use of PMUs for monitoring the state and topology of transmission systems. The first part of the dissertation is concerned about optimal placement of these devices in order to achieve a measurement design that is robust against bad data, loss of measurements as well as network switching. Unlike previously considered PMUs that have unlimited channels, this study considers PMUs which are manufactured for monitoring branches and therefore have limited reach. Placement problem is formulated as an integer programming problem which is implemented and solved using IEEE standard test systems as well as three actual transmission systems.

The second part of the dissertation focuses on the use of PMUs for improving the external network model. The impact of changes in the external system model on internal state estimation and subsequent security analysis is analyzed first. It is shown that this impact can be minimized by strategically managing the real-time data exchange between the neighboring transmission systems. Both conventional as well as PMU measurements
are considered in the analysis. Developed strategy identifies the optimal conventional 
external measurements whose real-time exchange will bound errors in the static security 
analysis of the internal system within a desired threshold. The dissertation then extends 
this analysis to formulate the problem of tracking changes in external network topology 
using the internal network measurements along with PMU measurements from the external 
system. Given the importance of external topology errors in static security analysis of the 
internal system, solution of this problem is considered critical in operation of smart grids. 
Developed solution is tested using simulated scenarios on IEEE 118-bus test system 
involving both load and topology changes in the external system.
To My Parents Mahnaz and Ahmad Emami
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CHAPTER I

INTRODUCTION & BACKGROUND
\textit{1.1 Introduction}

Majority of the materials covered in this dissertation is about application of Phasor Measurement Units (PMUs) in power systems. Therefore, this chapter is mostly dedicated to review of the history of PMU and its development, as well as the related applications developed during past three decades.

After three decades from introduction of the first prototype of PMU, there are still many ongoing projects and researches to explore new benefits for using PMUs in power system and their advantages over the conventional measurements. Although there are numerous other applications for PMUs in power systems, most of the application described in this chapter, are those which are already available in form of commercial packages.

History of the PMU and its benefits for power systems in this chapter is followed by the contributions to this dissertation and finally a brief overview for the following chapters is represented.

\textit{1.2 Historical Overview of Phasor Measurement Units (PMUs)}:

Phasor measurement unit (PMU) is a device with capability of measuring the positive sequence of voltage and current phasors. Voltage phasors in power systems have been always of interest to the power system engineers. It is well known that the active power flow in a given line in power system is proportional to the difference between the phase
angles of terminals of that line [1]. Therefore, measuring the phase angles difference across the power system transmission lines is important to power system engineers.

The history of PMUs goes back to 1977 when a new symmetrical based algorithm was proposed in a paper [2] to protect the transmission line in power systems. Symmetrical components of three phase voltage and current are described in that paper. Calculation of the positive sequence voltages and currents in a power system was the first step in the procedure employed by the phasor measurement units. Positive sequence bus voltages of the network constitute the state vector of the power system [1]. A paper from Phadke [3], published in 1983 indicates the importance of the positive sequence voltage of the network.

Availability of the global positioning satellite system (GPS) in the same era, and the ability of the GPS [4] to represent the most accurate method to synchronize the power system measurements, enabled the researchers in Virginia Tech to build the first prototype model of the phasor measurement unit (PMU) in early 1980s. Figure 1, shows a picture of the first prototype PMU, built in Virginia Tech [1]. This prototype was followed by the first commercial PMU built in 1991 by Microdyne [5].
Figure 1.1. First prototype PMU built at Virginia Tech [1].

Power system disturbances have forced the power networks to be armed with wide area monitoring, protection, and control (WAMPAC). WAMPAC is used to improve the grid planning, efficiency, and energy training [6],[7]. In order for the WAMPAC system to operate properly, usage of latest advances in sensing, communication, computing, modeling, visualization, and algorithm techniques are required to study and model the power system [8].
Application of PMUs in power grids is required for real-time wide area monitoring of modern power systems, great grid condition visibility achievement, awareness about the system situation, stress evaluation, fast remedial action, and even analysis to support economical and reliable system operation under increased system and market operation complexity. Although, synchronization is not a new concept in power system, advances in science forced the time frame for synchronization to be diminished. Thanks to advances in synchronization and communication, the time frame for synchronized measurement, once used to be up to minutes is now reduced to microseconds. Although high precision time-tagged measurements are currently considered a luxury in power systems, as they can be only provided by PMUs, industry foresees the day when all measurements will be time synchronized with high precision and time tags will be a normal part of any measurements.

Currently, PMUs are the most sophisticated device used in power systems which utilizes the high accuracy computation and the availability of GPS signal. Although advanced techniques in measuring and synchronizing measurements are basics for PMU operation, advances in other areas are also required to achieve the benefits of PMUs. One such area is data communication where faster and more reliable communication channels have created the chance of streaming from remote PMU site to the power system center.

PMU technology can track the network dynamics in micro-second rate. This is a great advancement to the SCADA/EMS measurements where the refreshment rate is seconds to minutes. Collected data from different PMUs in the system will be sent to the Phasor Data Concentrator (PDC) to align data from different PMUs in the network. This is done based on the time tag. Next, the PMU data from PDC of each center is sent to the central facility
where all collected data from different PMUs in different utility systems will be synchronized. After processing the PMU data grid-wide, a snap-shot of the synchronized measurements from appropriate substations will be available at the sub-second rate.

As mentioned in [9], the main purpose of new technologies in wide area monitoring, control and protection is to cover the following three directions:

- Monitor, control, and protect the transmission lines from disturbances and their negative impacts such as blackouts.

- Increase transmission line capacity in specific transmission area, mainly between two different electricity markets to reduce the congestion between networks.

- Improving the transmission assets utilization by enhancement of planning, control, operation, and protection process and modeling.

Researches and development of phasor measurements and their applications in power systems during the past two decades, show that a system designed based on utilizing these devices are very effective to meet the above three objectives, and can satisfy the new wide area operational constraints in the following three major areas [8]:

- Real-time wide area monitoring

- Real-time wide area control

- Real-time wide area adaptive protection
Generic architecture for a wide area monitoring, protection, and control contains four different layers:

- Layer 1, phasor data acquisition - synchronized phasor devices such as PMUs and digital fault recorders are installed at the substation and are capable of measuring the phasor voltage and current phasor and frequency at the substation. Basic phasor measurement process derives positive sequence, fundamental frequency phasors from voltage and current of the substation.

- Layer 2, Phasor data management – PDC collects data from different PMUs as well as other PDCs and correlates them into a single data set. It can also stream the data set to the appropriate application, using the data buffer.

- Layer 3, Data service – this layer contains a set of necessary services to provide data to different applications. The major role of this layer is to provide data in proper format for different applications. Both appropriate format and fast execution are important in this layer as providing data in appropriate format in a desired time frame can leave enough time for running the application within the sampling time.

- Layer 4, Applications- this layer contains the following three sections: real-time wide area control, real-time wide area monitoring and real-time wide area protection.

Four layers mentioned above forms one possible architecture which enables monitoring, control and protection of the system based on utilization of PMUs.
I.3 Benefits of Using PMUs

Utilizing phasor measurements in power systems has been popular for the past several years. As vendors and power utility companies show interest in using PMUs on their systems, investment on PMU and their applications in power systems has increased, as a result application of PMUs in power systems has received more attention in research as well. Since the problem of using PMUs in power system for different new applications is an ongoing process, new methods are introduced to improve the power system monitoring and control, every day. Although benefits of PMUs are broader than what is listed below, the most visible ones can be listed as follows [8]:

- Monitoring and control of the system in real-time
- Improving the power system State Estimation (SE)
- Real-time Congestion management
- Benchmarking, validation and fine-tuning of system models
- Post disturbance analysis
- Power system restoration
- Overload monitoring and dynamic rating
- Adaptive protection
- Planned power system separation
Next, each benefit will be described briefly.

I.3.1 Real-time monitoring and control:

Real-time monitoring of the system provides the on-line information of the system to the power system operator. Accurate knowledge of current information of the system increase the power system efficiency under normal conditions, and helps the power system dispatcher to identify, anticipate, and modify disturbances in system abnormal conditions.

At present, energy management system (EMS) relies on the outcome of state estimator to monitor the security of the system. In order for state estimation to estimate the state of the system, topology of the system along with the system measurements provided with supervisory control and data acquisition (SCADA) are required. Even though providing the accurate measurements and topology of the system to the EMS is currently not possible, in ideal case data update and communication takes several seconds.

On the other hand, time synchronized devices are capable of directly measuring the positive sequence of the voltage for a given system. Therefore, given enough PMUs in a system, state of the system can be directly measured. While populating the entire system with PMUs is not economical, gradual implementation can demonstrate benefits of using PMUs.

One of the greatest advantages of PMUs is the fact that in case of unexpected incident, PMUs are capable of not only sending alerts to inform the internal power system operator,
but to have the capability of sending warnings to the neighboring network operator about the unexpected events as well. This can be accomplished by implementing the early warning system on operator’s desk that would trigger simple alarms. These alarms will be initiated in case of violation of power system operating condition. Base case limits in off-line operational condition and planning studies are set based on some assumptions which are usually restricted inorder to prevent the power system operation condition violation; one other benefit of real-time monitoring of phase angles of the system is to modify such limits for an operating power system. Capability of PMUs to directly measure the phase angles of the system enables the power system dispatcher to reduce the error margin and operates transmission system closer to its their real stability limits, while maintaining the safe security level [11]. One immediate impact of getting a system working closer to their security margins is to relax the need of investment on expensive upgrades to the existing transmission system [12]. Also, having the knowledge of actual condition of the system enhances the local and wide area protection system.

Monitoring and detecting the inter-area fluctuation modes can be used to improve the dynamic model of the system. Improving the dynamic model of the system can increase the reliability and accuracy of the outcome of the dynamic studies. These models, in turn, can be used to optimally chose the strategic location of system stabilizers. Real-time monitoring can also help the operator to prevent black outs. Although black outs are among low-probability events in power systems, they are considered among the costliest events that may happen to a network. In order for the benefits of real-time monitoring of a network to be achieved in preventing blackouts specific studies are required. These studies
include networks specific features as well as the anticipated accuracy and reliability of the method.

Voltage instability is also one of the most important threats to power systems. The symptom of voltage instability can be one of the followings:

- typically low system voltage profile
- heavy reactive line flows
- inadequate reactive support
- heavily loaded power system

Usually, one of the mentioned symptomatic events that may last in time frame of seconds to several minutes and sometimes even hours is followed by voltage collapse [13-15]. Voltage collapse usually is initiated by single or multiple contingencies which are considered low-probability contingencies. Voltage stability monitoring, requires frequent and accurate analysis of static and dynamic techniques [16-19]. Therefore, implementing the voltage stability is easier with PMUs as current monitoring devices are not capable of adequately monitoring this type of system dynamics.

While some of the power system applications require large number of PMUs for their implementation such as measuring the state of the system with PMUs, other applications can be initiated with low number of PMUs (applications such as monitoring of frequency
disturbance). While usage of large number of PMUs in a power system allow the power system operator to have simultaneous implementation of multiple functions, usually power utility companies would rather the gradual installation of PMUs in their system due to the costly process of PMU installation in power systems. It should be noted that even in case of gradual implementation, the initial design for PMU placement should be initiated based on the applications which requires lower number of PMUs while the final expansion in mind. In other words, installation of PMUs to achieve the short term investments of power system utilities should be consistent with the long term objective of these utility companies for PMU placement. As shown in chapter two, many utility companies have already installed several PMUs in their system; although the installed PMUs may satisfy some of the short term objectives for the company, unfortunately the installed PMUs have very limited impact (or in some cases no impact at all) on the optimal number of required PMUs for system observability or critical measurement, etc. In other word, long term investment is not considered in their investments. More practical installation of PMUs in such utility systems could have saved benefits for the utility company in their present and future projects.

\[ I.3.2 \text{ Power system state estimation} \]

To be able to explain the role of PMUs in improvement of power system state estimation, first we need to review the state estimation problem formulation. This section reviews the mathematical formulation that is used in current power system state estimator.
Theory and implementation of state estimation in power system is explained explicitly in [20].

State estimation solves an over determined system of nonlinear set of equation as an unconstrained weighted least square (WLS) problem.

The WLS problem minimizes the weighted sum of the squares of the residual errors as follows:

\[
\min J(x) = \frac{1}{2} (z - h(x))^T R^{-1} (z - h(x)) 
\]  

Where: \( x \) is the state vector of the system that includes the voltage magnitude and voltage phase angles of all buses in the system except for the slack bus. \( Z \) is the measurement vector, and \( h(x) \) is a non-linear vector function that correlates the measurements of the system to the states. \( \sigma_i \) being the standard deviation of each measurement, for a system with \( m \) measurements, \( R \) is a diagonal matrix as follows:

\[
R = diag\{\sigma_1^2, \sigma_2^2, ..., \sigma_m^2\} .
\]

To solve (I.1), the first order optimality conditions should be satisfied. This can be written as:

\[
g(x) = \frac{\delta J(x)}{\delta x} = -H^T(x)R^{-1}[z - h(x)] = 0, \quad H(x) = \frac{\delta h(x)}{\delta x} 
\]  

Using the Taylor series, and ignoring the higher order terms, \( g(x) \) can be written as follows:
\[ g(x^{k+1}) \approx g(x^k) + G(x)(x^{k+1} - x^k) = 0 \]  \hspace{1cm} (I.3)

Therefore:

\[ x^{k+1} = x^k - [G(x^k)]^{-1}g(x^k) \]  \hspace{1cm} (I.4)

Equation (I.4) is an iterative scheme known as Gauss-Newton method where \( k \) is the iteration index and \( x^k \) is the solution vector corresponding to iteration \( k \).

\[ G(x^k) = \frac{\partial g(x^k)}{\partial x} = H^T(x^k)R^{-1}H(x^k) \]  \hspace{1cm} (I.5)

\( G(x^k) \) is the gain or information matrix. Equation (I.4) is the so-called normal equations of the least squares method and iteration step \( \Delta x^{k+1} = x^{k+1} - x^k \) can be found only if the gain matrix is nonsingular.

WLS state estimation was introduced with Fred Schweppe in 1969 in his classic papers [21]-[23]. Power system state estimation was a very active area during the past four decades. Many researchers focused on the state estimation, and different methods have been tried besides WLS algorithm in hope of getting a better algorithm. Among the available state estimation methods WLS has been the most popular in practical implementations. In general, state estimation process consists of the following steps:

1- Data acquisition

2- Network topology

3- Observability analysis
4- Estimation of state vector

5- Detection of bad data

Coutto, Silva, and Falcao have a paper [24] that extensively investigates the power system state estimation bibliography in the first two decades (1968-1989). The comprehensive application and implementation of state estimation can be found in a book by Abur and Exposito [20], and Monticelli [25].

State estimation plays a key role in security frameworks as one of the major application used in energy management system (EMS).

Bose and Clements in [26] describe the role of state estimation in power systems control center including all abovementioned functions. This includes a survey about the numerical algorithms for state estimation, topology processing, bad data identification, and network observability.

I.3.2.i Practical Issues

Although it is very important to have an idea about how the state estimation is formulated and the way that numerical algorithm can improve the implementation of state estimation, it is equally important to have knowledge about how state estimation is used in practice.
The state estimation is a very large and complex process that includes both hardware and software acquisition. Presently, most of the state estimations are implemented in independent system operator (ISO) control centers. Real-time implementation of state estimation as well as practical experience has been reported in several papers. [27] shows the experiences with state estimation in EMS control center and covers deficiencies such as topology error and critical measurements. Panel discussion at 2005 IEEE PES general meeting was dedicated to addressing some of the problems faced by SE in practice and why SE had not achieved its expected role in power system industry. Among these papers was [28] where Allemong described three important categories required to have a successful state estimation implementation:

- Reliable set of measurements with redundancy
- Accurate knowledge of topology of the system based on the status of circuit breakers
- Accurate parameters of the network elements

This is consistent with the practitioners declaration about the issues mainly impede state estimation is practice [29]. These issues can be listed as follows:

1) Incorrect model parameters for the network elements

2) Incorrect topology of the system

3) Inadequate or faulty telemetry
4) Inconsistent phase metering

5) Inconsistency between meter placement in the field and in the computer model.

For example incorrect assignment of a flow measurement to equipment is very typical problem. Unfortunately the problem SE is facing today is more or less the same problems that it has been facing since its earliest industrial implementation. Although such issues relate to the infrastructure of state estimation rather than the state estimation algorithm, they deteriorate the outcome of state estimation. Abovementioned problems deserve serious attention to improve the state estimation outcome; however, researchers cannot do much in this matter besides recommendations.

I.3.2.ii. Role of PMUs in improving the State Estimation

State estimation is a tool that is widely used in power network control centers to improve the quality of directly telemetered data, to provide a way for direct monitoring of network conditions which are not directly available to power system dispatcher through telemetry, and to provide the best available estimate of network model that can be used as a starting point for further real-time power system application such as contingency analysis, congestion management, volt-VAR optimization, and constrained re-dispatch. State estimation and it’s subordinate applications such as bad data identification, parameter
estimation, breaker status estimation, and external model estimation are widely used in industry with different degrees of success.

As PMUs became the point of interest for researchers and industry, utilization of PMUs in power system applications became more popular. One such applications is the state estimation. The incorporation of PMUs in SE is numerically and algorithmically not difficult. One important change is the lack of reference bus. Among the advantage offered by PMUs for state estimation, the followings are probably the most commonly documented:

- Improved accuracy and robustness of bad data detection and identification
- Transformation of the non-linear SE equations into linear equations which in turn, lead themselves to a faster and more reliable solution algorithms.
- The synchronized measurements provided by PMUs in the external system can be directly used to maintain a more accurate external system model and merge it with the internal SE solution.

Another benefit of using PMUs in power system is their ability to directly measure the state of the power system. Therefore, given widespread availability of PMUs in a power system, the state of the system can be directly obtained at a higher rate without estimation. It is noted such an approach also carries the risk of bad data.

Currently, cost of the PMU as a device, cost of installation and related infra structure constitute the main obstacles for the industry for widespread PMU deployment. One
opportunity is to share the PMU infrastructure (such as communication and master system) to access both PMU and digital data available from digital relays. Application of PMU bases to access digital relay data can add redundancy to run the state estimation with higher frequency or to reduce the cost of PMU deployment.

I.3.2.iii. Issues addressed in this dissertation

This dissertation focuses on the following improvement of state estimation by PMUs:

- Network Observability

- Removal of critical measurements

- Robustness of the measurement system against contingencies

- Contingency analysis for internal system in an interconnected network

- External system topology error identification

I.3.3. Real-time Congestion Management

Congestion management is among top priority functions implemented by power schedulers in the power market; this application is also executed in real-time by grid operator. Congestion management is critical function has a major role in both generation dispatch which is a day-ahead market study and re-dispatch which is a real-time market application. The main task of congestion management is to satisfy demands of the network
optimally and economically without violating the power network limits (transmission line limits and voltage magnitude limits).

Real-time congestion management is trying to keep the real-time power flows across transmission lines within impeccable limits of transmission lines in case of dispatch adjustment with the constraint of minimizing the cost.

In traditional congestion management, the actual power flow on a transmission line is compared to the Nominal Transfer Capability (NTC) which is calculated in advance through offline techniques. The offline calculation is done based on the network constraints such as thermal limits, voltage limits, and stability limits. Based on [30] and [31] usually the constraints used in calculation of offline NTC is more restricted than what it should be; this will lead into unreasonable margins in congestion management which in turn will cause the unused transfer capacity. This lost opportunity is costly for dispatch process and should be avoided.

Since the PMU provide synchronized, more accurate system measurements, deployment of PMUs in power system networks enables the power system dispatch center to have a more accurate estimate and improved calculation of transmission line limits as well as the line flows. This in turn will cause the system work closer to its limits and as a result it will improve the system operation which leads into savings for utility companies, and ultimately can bring down the cost for power system costumers. High rate of sampling and accuracy of the PMUs, will lead to a faster and more accurate execution of real-time algorithm such as real-time transfer capability (RTC) in congestion management. In most
of transmission lines such RTC’s exceed the corresponding NTC’s; which in turn reduces the need for real-time congestion reduction [32], [33].

I.3.4. Validation and tuning of system models

Power system parameters are not always accurate. Since each part of the power system is modeled using a different set of system parameters (i.e. network, generators, loads, etc.), wrong parameters will cause error in system studies. The objective of validation of system model, and Parameter Estimation (PE) is to identify the doubtful parameters of the system and have an improved estimate of those parameters.

Generally, the power system parameters are entered to the modeling database manually, therefore errors occur inevitably in the power system models. Once an error enters the power system model database, it is difficult to identify the error, and it may remain undetected for several years.

In case of steady state parameter models estimation, model errors can be identified and be replaced with the corrected values, using an algorithm known as parameter estimation (PE). Having access to accurate synchronized measurements from PMUs can be used to improve the ability of PE to identify and correct steady-state modeling errors such as impedances and admittances.

While it is important to have an accurate model of the system for steady state, measuring of dynamic modeling parameter of the system is more difficult, and requires
accurate monitoring and knowledge of system feedback to expected and unexpected switching events. A quick review of available papers in this field such as [34], and [35] indicate that parameter estimation algorithm can be used in measuring of dynamic model characteristics as well.

\textbf{I.3.5. Post Disturbance Analysis}

Post disturbance analysis is the study and simulation of chronicle of events after a disturbance occurs in power system. To do so, an investigation should be done on data, recorded through data recorders available throughout the network. Data recorders are being used in industry for many years. However, working with traditional data recorders is very difficult and time consuming process due to the fact that the data available through such devices are not time synchronized and it is very difficult for the engineers to align the timeline of recorded data.

GPS has been used during the past decades as a universal time reference, allowing many devices provide time synchronize data, one such device is PMU. Deployment of PMUs have been recommended strongly by authorities after the US northeast, and Italy blackout in 2003 [6],[7].

Creating WAMPAC for post disturbance analysis purposes has not to meet rigid constraints as those required for real-time analysis due to the fact that reasonable amount of delays are acceptable in post disturbance analysis. Therefore, data can be stored in substations, and be transferred to the data center on a scheduled time frame. Next,
available software packages can assist the power system dispatcher to spot the desirable information through the tremendous amount of data.

GPS can be used for time synchronizing of the recorded data in recorders. Some utility companies in the US have already deployed such technology to reduce the time error problem in their recorded data. One of the benefits of such method is to improve the implementation of disturbance diagnosis. Experience shows that accurate time synchronized data available through GPS can cut the diagnostic time from several hours to a few seconds [10].

After Italian blackout of 2003, several PMUs were installed throughout the continent. Although the initial intention of installing PMUs in European network was to provide a time synchronized measurements, further unexpected advantages were realized after the reconnection of southeastern and western grids in 2004 [7].

I.3.6. Power System Restoration:

Power system dispatcher usually encounters a huge standing phase angle (SPA) difference across the breaker which connects two adjacent substations. In case of lack of knowledge of power system dispatcher about such huge difference in phase angle, closing of the circuit breaker in process of restoration the power system, may cause shock to the power system, which in turn may damage power equipments or even recurrence of system outage [36]. PMUs are devices that can be used to monitor the network in real-time. And
due to their ability of directly measuring the phase angles, operator can use the PMU collected data to avoid such subsequences.

**I.3.7. Overload Monitoring and Dynamic Rating**

Although many software and devices are already available to power system utilities that enable them to monitor the power system equipment [37], use of PMUs can make the monitoring more accurate. It also helps to monitor the network in a higher time resolution. Although PMU based monitoring of a overload and dynamic rating of a system have some degree of deficiency comparing to the system monitored by existing equipment, the advantages of using PMUs for such purpose is the fact that PMU can be used for many other applications as well, while the existing devices can only be installed and used for such individual task. In other words, multi-tasking aspect of PMU makes them more desirable than available conventional measurement devices.

Overload monitoring is known as an equipment monitoring application. Currently, the only available application for equipment monitoring using PMUs is thermal monitoring of overhead transmission lines. Since PMU can measure the voltage magnitude and the phase angle of the node, having PMU installed at both terminals of a given line, allow us to calculate the impedance of the line in real-time. Thus, the average temperature of the line can be estimated in real-time. The con of this application is the fact that PMUs cannot provide any information about the hot spot on transmission lines.
I.3.8. Adaptive Protection

Adaptive protection is initiated based on the idea that characteristics of relays and PMUs should be change to fit the existing power system operating condition. Introduction of phasor measurement units and certain relays has enabled such devices to adapt to the current condition of power system.

Conventional relays and protections, used in power system usually operate based on a inflexible and predetermined rules defined by the power system operator. In case of an event, these devices response based on the predetermined manner programmed in their characteristic. Creation of digital relays has initiated a new era in real-time response to the network changes. Digital relays are created based on two crucial characteristic that differentiate them from traditional relays and protectors: function of digital relays is determined with a software rather than predetermined manner. They also have real-time communication capability. This option is used to overrule the software decision.

Adaptive relaying were defined after the creation of digital relays in 1987 [38],[39]. The main reason behind the creation of digital relays was the change in power industry. In 80’s power system companies started to operate their system closer to the operating limits due to the economical and environmental constraints. Since then the emphasis was on the economic operation of the power system, which in turn, changed the application of traditional protections in power systems.

Application of PMUs as an adaptive relays has been investigated. Another amazing application of PMUs is the accurate measuring of the line impedance, which can be used in
fault location detection. The line impedance has a major role in fault location detection. Accurate line impedance can be calculated using two PMUs at terminals of the line. This benefit of using PMU in the system will be more highlighted knowing the fact that inaccurate fault location can prolong the process of fault identification and system restoration.

1.3.9. Planned Power System Separation

Planned separation of power system into islands is the last action taken when power system is facing a drastic unstable electromechanical oscillation, and separation is unavoidable. Under such circumstances, the only solution for the power system engineers is to create islands, and isolate them from the rest of the network, and merge them with the grid, later on as conditions are improved. In such a case where separation is inevitable, it is proffered to create islands on a planned basis rather than an unplanned basis. When creating an island, the ideal case is to have a balance in existing loads and generations in the island. On the other hand, this may not happen in real world, which means there might be unbalances between generation and loads in an island after separation from the grid. In such a case, generation outage or load shedding might be required to have equity between the loads and generations in the island.

Currently, two major methods are in use to accomplish the system separation: out of step relaying and remedial action schemes. Both these techniques are known as relaying applications. However, the remedial actions scheme is sometimes known as control
application as well [110]. Both of the abovementioned techniques rely on the predetermined estimated behavior of the system which relies on the state of the system, topology, outages, and load and generation levels. It is common in the power system for the predicted condition of the system to be quite different from the system condition in real world. Since the protections scheme settings are planned based on the calculated, and predetermined conditions of the system, in case of excessive error between the calculated conditions and actual system condition, the performance of protections would be inappropriate for the real system condition which in turn, may worsen the situation.

Planned system separation, can be hugely improved by using the PMU data rather than current available methods. Pre-calculated system behavior can be replaced with real-time system monitoring using phasor measurement available through PMUs. Application of such scheme improves the planned system separation due to the following reasons:

- Using the real-time measurements, the operator can recognize whether the system state is moving toward unstable state. Also, those generators that are most likely vulnerable of losing stability can be identified.

- Islanding boundaries can be identified in real-time based on the network conditions.
1.4. PMU, Contribution to This Dissertation

With more than twenty utility companies, already using PMUs in North America, and with the growing demand of PMU installation on power systems around the globe, different applications of phasor measurement units are of great value to researchers as well as industry.

Many PMU applications and their positive impact on power system applications are indicated in previous sections. However, mentioned applications are only those which are already used in industry, and most of them are available as commercial packages, meanwhile researchers are exploring new methods to substitute the conventional techniques with better ones using PMUs.

Majority of this dissertation is also related to Phasor Measurement Units and their application in power systems. Studied applications of PMUs in this dissertation can be listed as follows:

- Placement of PMUs to have a fully observable system without considering the conventional measurements
- Placement of PMUs to have a fully observable system, considering zero injections
- Removal of critical measurements using PMUs
- Create a more robust system against contingencies, using PMUs
- Robust placement of PMUs against PMU loss or bad data
• Improvement of internal state estimation and subsequent security analysis with PMUs

• Tracking external topology errors using PMU measurements from external system

1.5. Organization and contribution of the dissertation

This dissertation is organized in five chapters which are divided into two major parts:

The first part (chapter II) studies the application of PMUs in power system for achieving full observability, and in general, application of PMUs in an isolated power grid. The second part (chapter III, IV) studies the more practical application of PMUs in an interconnected system. This part of dissertation concentrates on the impact of having real-time access to the data available through external phasor measurement units to improve the internal power system applications such as state estimation and security analysis.

The chapters of this dissertation are:

I. Introduction & Background

II. Phasor Measurement Units (PMUs) & their applications in power system observability

IV. Analysis of Impact of external measurements

IV. Tracking of external system topology with Phasor Measurement Units
V. Conclusion and future work

- In chapter I, we present a historic background and a general overview of phasor measurement units, their developments and their growing applications in power systems, and contribution of PMU to this dissertation.

- In chapter II, the theory and implementation of observability of the power system as one of the major requisites for state estimation execution is addressed. A new method to identify the optimal number and placement of PMUs to achieve full system observability is presented. Issue of critical measurements and bad data identification and their removal, using phasor measurements is investigated; an algorithm to have a robust system against contingencies using PMUs is developed; and a new method to back up the PMUs against loss and bad data measurement is addressed. Optimization package (TOMLAB) is used to obtain the optimal placement of PMUs. The test results are shown for IEEE standard cases for verification, and then the methods are implemented on three different utility companies.

- In chapter III, first the problem of optimal real-time measurement exchange between neighboring systems in order to improve the internal state estimation and contingency analysis is investigated, then impact of real-time access to the external phasor measurements available through external PMUs, on internal state estimation and subsequent security analysis is discussed. Results on the standard IEEE cases are shown for validation. Optimization packages (TOMLAB and
LINGO) are used to identify the optimal real-time exchange of conventional measurement between internal and external systems.

- Chapter IV, investigates the problem of identification of line outage in the external systems. Currently, available methods (To the best of author’s knowledge) are not capable of identifying the external line outages based on the internal data and a few real-time measurements from external system. While available methods for external line outage identification using PMUs, record the impact of each and every external line outage on PMUs, and then try to match the real-case scenario with one of the pre-calculated cases, the proposed method is capable of identifying the external line outage using only internal data as well as external PMU measurements. The formulation and the test results on standard IEEE cases are shown; mixed integer non-linear programming (MINLP) is used to identify the line outage in the external system.

- Chapter V, presents a brief summary of this research and a discussion of possible future works.

The main contributions of this dissertation are:

a. Strategic placement of single channel PMUs for full observability, robust system against bad data or contingency

b. Identification of optimal set of external measurements to improve internal security analysis
c. Impact of real-time phasor measurement exchange between neighboring systems on security analysis

d. External topology error identification and subsequent improvement of internal security analysis
CHAPTER II

PHASOR MEASUREMENT UNITS (PMUs) & THEIR APPLICATIONS IN POWER SYSTEM OBSERVABILITY
II.1. Introduction

Phasor measurement unit (PMU) is a device with capability of measuring the positive sequence of phasor voltage and phasor current measurements. These measurements are synchronized through Global Positioning System (GPS) within a micro second. Phase angle of the voltage phasors of power system buses have been of interest due to the fact that active power in a transmission line in power system, is proportional to the sine of the angle difference of the phase angles of terminals of the line [1]. Since many of the planning and operational actions in power system rely on the flow of the real power on transmission lines, measuring the angle difference between the terminals has been concern for many years.

The main advantage of using PMUs in power systems is their capability of directly measuring the state of the system. Therefore, synchronized measurements provided by PMUs is great help to power system control systems especially in interconnected networks.

While PMUs are not yet found in every substation, their utilization in substations for protection and control functions is rapidly increasing. As they become available in large numbers they can provide valuable information for energy management system applications as well. One of these applications, which will be significantly affected by the introduction of PMUs, is the state estimator. One of the great advantages of PMU is the fact that PMU yields the synchronized measurements across the power system. Initial work on PMU placement is based on the assumption that PMUs will have infinite number of channels to monitor phasor currents of all branches that are incident to the bus where a PMU will be installed [40,41]. While there are manufacturers that produce PMUs with
several channels to measure phasor currents and voltages, the number of channels is typically limited. Also, note that PMUs capture samples of phase voltages and currents that are received from the instrument transformers connected to a bus or a breaker in the substation. The sampled three phase signals are converted into positive sequence phasors at regular intervals and then telemetered to the phasor data concentrators. Phasor measurements are used by various application functions at energy control centers. One such application is state estimation which not only provides the best estimate of the system state but also acts as filter for gross errors in analog and digital measurements. Moreover, results of state estimation are used by many applications as inputs and therefore have a significant impact on the overall performance of the energy management systems. Some of the applications that rely on state estimation results include real-time contingency analysis, voltage stability assessment, transient stability assessment, real-time power flow, security constrained optimal power flow, load forecasting as well as the market applications.

In addition to the types of PMUs discussed so far having several channels, there are also PMUs that are designed to monitor a single line or transformer, i.e. they provide positive sequence voltage at one terminal of a branch as well as the current phasors of the monitored branch. These types of PMUs will be referred as the “branch PMUs” in the sequel. Such PMUs are currently installed and in use at substations of large utility companies in the world [42] and preliminary studies are conducted to determine best locations for their placement [43],[44]. Certain utility companies are known to use branch PMUs and daisy-chain them as needed to increase capacity at a given bus. One section of
this study presents an optimal placement strategy for such companies to achieve a reliable monitoring system design at minimum installation cost.

II.2. Operating states of a power system

The operating condition of a power system at a given point can be determined if the topology of the system and complex voltage phasor of the system is known [20]. Since the system can be specified just using the complex phasor voltages corresponding to that system, the complex phasor voltage is called “static state” of the system. A given power system can move into one of the following three states [20],[55]: namely normal, emergency, or restorative.

1- Normal state: A system is said to work under normal condition if using the existing generation of the system, all loads can be fed with no violations in any operating constraints. Operating constrains includes the active and reactive powers which can be transferred through a transmission line, and lower limits of the bus voltage magnitudes. Topology of the system is always changing due to contingency which are transmission line or generator outage. Line outages may happen due to expected causes such as switching, maintenance, or unexpected reasons such as lightning. If the system remains in normal condition following the occurrence of a contingency, the system is called “secure-normal”. Otherwise the system is classified as “unsecure-normal” condition where the system is operating under the
normal condition without violating any operating condition, but the system is vulnerable with respect to some of the contingencies.

2- Emergency State: Unexpected events can have a significant impact on system operating condition; it may also cause violation of some of the operating constraints while the system is operating. In such case the system is said to be working under emergency condition. Emergency state needs immediate recovery action to bring the system back to the normal state and avoid the system collapse.

3- Restorative State: one way to overcome the emergency state is to discontinue various loads, lines, transformers or other equipments. Removing loads and transformers might be able to confine the operating condition into the constraints limits. This may lead to re-gaining the stability with reduced load and reconfigured topology. As a result, the load versus generation balance may have to be restored in order to start supplying power to all the loads including the one removed from the system. This operating state is known as restorative state.

The objective of power system operator is to maintain the system in the normal secure state as the operating condition of power system changes during the day. To be able to achieve this goal, power system dispatcher should continuously monitor the system condition, and operating state of the system. Necessary remedial actions should be taken in case the system found to be operating under insecure-normal condition. The sequence of actions which should be taken by power system operator is known as security analysis.
II.3. Static security assessment

The first step in security analysis is to monitor the current state of the system. To do so the operator should have access to both topology and measurements of the system. State of the system can be determined using the network model and current measurements through state estimation packages. Measurement received in at the control center might be of both traditional and/or phasor type. Traditional measurements includes active (real) and reactive (imaginary) power flowing through transmission lines, generator outputs, loads, bus voltage magnitudes, transformers tap status, and switchable capacitor bank values. Traditional measurements are usually collected with remote terminal units (RTUs). RTU is a device which interface in the physical world to a distribute control system or SCADA system by transmitting telemetry data to the system and altering the state of connected objects based on control messages received from the system. Phasor measurements are rather new measurements which can be provided to the control center through PMUs. These measurements include the phasor voltage of buses of the system as well as phasor current of transmission lines. Once the measurement set for the entire system is available to the control system, the collected measurements (traditional or phasor) can be used by state estimator to estimate the current state of the system and identify the gross measurement error. In general, state estimation processes the telemetered measurements to obtain the optimal estimate for the state of the system which are the voltage magnitude and phase angle of all buses of the system. State estimation works based on the redundancy of the telemetered analog quantities (MW, MVAR, and KV) or phasor measurements, and the most current topology of the system. Knowing the state of the power system, MW, MVAR, MVA flows through all transmission lines and transformers, and the MW and
MVAR injections of all loads/generators can be calculated. The calculated voltage and power values provide useful operational information in case of lack of telemetry. In case of existence of valid telemetry, the calculated quantities can be used to identify possible measurement errors.

Prior to executing the state estimation in power system, information about the topology of the system and measurements are required. Having the system model and measurement set, state estimation can be run for the system if and only if the system is “Observable”. A system is called observable if the state estimation can be solved for the system and a unique set of solution can be obtained. In general, state estimation includes the following functions [20]:

- Topology processors: which gathers circuit breaker status, and ON/OFF status of switches.

- Observability analysis: the observability of the system should be checked before executing the state estimation, as state estimation cannot be solved for unobservable systems.

- State estimation solution: determines the best estimate for the current system state. State of the system is the voltage magnitude and phase angle of system buses (except the reference bus).

- Bad data processing: can be executed if redundancy is available in measurement set. This function detects the existing errors in the measurement set.
Parameter and structure error processing: estimates network parameters such as shunt capacitors, transmission line parameters, tap changing transformer parameters, and detects structural errors in network configuration.

Based on above mentioned functions included in state estimation, power system state estimator is considered basic of real-time security analysis. As mentioned, prior to execution of the state estimator for a given system, the observability of the system should be checked. If the system is found unobservable then extra measurement may be required to be added to the system in order to achieve the system observability. One of the main contribution of this chapter is application of PMUs for full observability achievement.

II.4. Use of Phasor Measurement Units (PMUs) for network observability

Phasor Measurement Units are the most accurate and advanced instrument utilizing synchronous measurement technology available to power system engineers and system operators [45]. Many researchers have dedicated their attention to application of PMUs in power systems observability. When placed at the bus, a PMU can measure the phasor voltage of the bus, and different number of phasor currents of branches incident to that bus, depending on the type of PMU [46]. It is well known that having a PMU installed at each and every bus in the system, the state of the system can be directly measured through PMUs, and the entire system would be observed by PMUs. However, the cost of PMUs and their installation does not allow utility companies to install PMUs at every bus. Costly procedure of purchasing and installation of PMUs, motivated the optimal placement of the
PMUs in the system. In other words, utility companies are looking for the minimum number of PMUs that can observe the entire system. A power system is called observable if the state of the system can be uniquely identified [20,25].

The problem of strategic placement of PMUs in power system to have a fully observable system, has received a lot of attention from researchers. [41], and [47] investigate the problem of optimal placement of PMUs to observe the network using integer programming. In [48], authors propose an exhaustive search approach to determine the minimum number and optimal placement of PMUs for state estimation. Although the proposed method in this paper yields the global optimal solution, the method is yet computationally intensive for large systems. In [40], authors propose a method to identify the strategic location for PMU installation in the system based on the dual search method which uses both a modified bisecting search and a simulated annealing method. The modified bisecting search fixes the number of PMUs for which the simulated annealing-based method then attempts to find a placement set that makes the system topologically observable. A graph theoretic approach for placing PMUs based on incomplete observability is proposed in [49] where simulated annealing method is used to solve the pragmatic communication-constrained PMU placement problem. In [50] authors represent a method to identify the optimal placement of PMU for power system state estimation based on the minimum condition number of the normalized measurement matrix. [51] shows a technique of identifying the optimal PMU placement using the genetic algorithm (GA). Authors in [52], use the particle swarm optimization in power system, to obtain the optimal PMU placement for full observability of the system. In [53] a binary particle swarm optimization based method is used to minimize the number of required PMUs and
maximize the measurement redundancy. [57] presents a method to test the system observability, as well as identifying the observable islands in case of system unobservability.

Although, many researchers have investigated the problem of strategic placement of PMUs on power systems, to achieve full observability, all their studies is based on an unrealistic assumption that each PMU is capable of measuring the phasor voltage of a bus and the phasor current of all branches incident to that bus. Since number of branches incident to each bus is different, solving the optimization problem based on the assumption that each PMU can measure phasor current of all branches incident to the bus where PMU is installed is more of an idealistic assumption. In this work, a new type of PMU is considered. This type of PMU is capable of measuring the phasor voltage of the bus, as well as the phasor current of only one of the branches incident to the bus where PMU is installed, these types of PMUs will be referred to as “single-channel” PMUs or “branch-PMUs” in the sequel, and number of them are already installed in utility companies system. Different methods are used by researchers to test the observability of the system. List of the most commonly used method for observability check in power systems is as follows:

- Numerical method based on nodal variable formulation
- Numerical method based on branch variable formulation
- Topological observability check method
In this dissertation, the first method is used to check the observability of the system. This method is introduced by Monticelli and Wu in 1985 [54].

II.4.1. Numerical Approach for observability [54]

System observability is independent of branch parameters. Therefore, it can be assumed that all branch impedances are equal to 1 p.u., and all bus voltages are set equal to 1 p.u.. Thus, the linear decoupled power flow along the system branch can be written as:

\[ P = B \cdot \theta \]  \hspace{1cm} (II.1)

Therefore, in case of null estimated state \( \theta \), all branch flows would be equal to zero. Estimated state of the system can be calculated using the linear decoupled measurement models:

\[ H \cdot \theta = Z \quad \Rightarrow \quad \hat{\theta} = (H' \cdot H)^{-1}H' \cdot Z = G^{-1} \cdot t \]  \hspace{1cm} (II.2)

Where \( G = H' \cdot H \) is the gain matrix, and \( t = H' \cdot Z \).

Null estimate for state \( \hat{\theta} \) is expected for an observable system when all system measurements are equal to zero. But the system is called unobservable if a nonzero set of estimate for state \( \hat{\theta} \) exists which satisfy the following equations:

\[
\begin{cases}
H \cdot \hat{\theta} = 0 \\
P = B \cdot \hat{\theta} \neq 0
\end{cases}
\]  \hspace{1cm} (II.3)
If equation (II.3) is satisfied for a system, $\theta$, is called unobservable state, and those branches with non-zero flows in the second equation in (II.3) identify unobservable branches.

**II.4.2. Unobservable branches**

Any unobservable system is consisted of several observable islands. These observable islands are connected through unobservable branches. In other words, unobservable branches are those branches which separate observable islands in an unobservable system, following procedure can be followed to identify unobservable branches [54]:

Let us assume that the measurements are set equal to zero. Thus equation (II.2) can be written as follows:

$$(H' \cdot H) \cdot \theta = 0, \quad G = (H' \cdot H) \Rightarrow G \cdot \theta = 0$$

(II.4)

Assuming that the row and column, corresponding to the reference bus has been removed from measurement jacobian matrix (H):

- If $G$ is nonsingular, the system is Observable.

- If $G$ is singular, then we have to divide the $G$ matrix into four sub-matrices as following:
\[
\begin{bmatrix}
    G_{11} & G_{12} \\
    G_{21} & G_{22}
\end{bmatrix}
\begin{bmatrix}
    \hat{\theta}_a \\
    \hat{\theta}_b
\end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]  
(II.5)

Where \( G_{11} \) is a non singular matrix in matrix \( G \). now we have to assign any specific value to \( \hat{\theta}_b \). Let’s assume that this value is \( \bar{\theta}_b \) then we have:

\[
\hat{\theta}_a = -(G_{11})^{-1} G_{12} \bar{\theta}_b
\]  
(II.6)

Assuming that the corresponding solution for this specific value is shown by :

\[
\hat{\theta}^* = (\hat{\theta}_a, \bar{\theta}_b)
\]  
(II.7)

Then we can write:

\[
P_b^* = C \cdot \hat{\theta}^*
\]  
(II.8)

Those branches for which we have: \( P_b^*(i) \neq 0 \), will represent the unobservable branches. Unobservable branches can be transformed into observable branches by adding appropriate measurements to that branch.

**II.5. Achieving Observability in power system using Phasor Measurement Units**

While PMUs are not yet found in every substation, their utilization in substations for protection and control functions is rapidly increasing. As they become available in large number they can provide valuable information for energy management system application
as well. One of these applications, which will be significantly affected by the introduction of PMUs, is the state estimator. One of the great advantages of PMUs is the fact that PMU yields the synchronized measurements across the power system, as PMUs work in synchronization with GPS satellites. Also, these devise have the ability to calculate the synchronous component of the voltage and current in installation location. As mentioned earlier, the state of the power system consists of positive sequence of voltage phasors at all buses of the system. Therefore, PMUs have the ability of directly measuring the power state of the system.

Also, note that PMUs capture samples of phase voltages and currents that are received from the instrument transformers connected to a bus or a breaker in the substation. The sampled three phase signals are converted into positive sequence phasors at regular intervals and then telemetered to the phasor data concentrators. Phasor measurements are used by various application functions at energy control centers. One such application is state estimation which not only provides the best estimate of the system state but also acts as filter for gross errors in analog and digital measurements. Moreover, results of state estimation are used by many applications as inputs and therefore have a significant impact on the overall performance of the energy management systems. Some of the applications that rely on state estimation results include real-time contingency analysis, voltage stability assessment, transient stability assessment, real-time power flow, security constrained optimal power flow, load forecasting as well as the market applications.

Today, different types of PMUs with different number of output channels are manufactured by various companies. Each PMU is capable of measuring the phasor
voltage of the bus along with the phasor current of different number of branches incident to that bus. Each PMU is usually named based on the number of its available outputs (i.e. a PMU which is capable of measuring the phasor current of two branches incident to installation substation, is called two channel PMU). Although many researchers have dedicated their attention to installation of PMUs in power systems and their optimal locations, all of them (to the best of author’s knowledge) have worked with an idealistic PMU so called “multi-channel” PMUs, in other words all of previous research related to PMU placement in power system is done based on the assumption that PMUs will have infinite number of channels to monitor phasor currents of all branches that are incident to the bus where a PMU will be installed. While there are manufacturers which produce PMUs with several channels to measure currents and voltages, the number of channels is typically limited.

In addition to the types of PMUs discussed so far having several channels, there are also PMUs that are designed to monitor a single line or transformer. These types of PMUs will be referred to as the “branch PMUs” in the dissertation. One section of this study presents an optimal placement strategy for several utility companies to achieve a reliable monitoring system design at minimum installation cost. The problem of strategic placement of branch PMUs is done for three real power systems. This section of the dissertation first investigates the full observability of the system without taking any traditional measurements into consideration, next the problem of identification and transformation of bad data from power systems is investigated, and then zero injections are taken into account to reduce the number of required PMUs to observe the system. And at last, the problem of robustness of the system against probable contingencies is
investigated. For illustration, first problems are solved utilizing the multi-channel PMUs, which could be easier, and then formulations are adjusted in such a way to branch PMUs.

The problem of optimal PMU placement can be solved in different ways, with different assumptions. This part of study is dedicated to identify the optimal placement of PMUs for a given power system in order for the full observability to be achieved. In real world, the problem of optimal PMU placement is needed to be solved for an existing power utility system. Knowing the fact that any existing power system is an operating system, means that the existing topology with available set of measurements from the system constitutes an observable and executable system. Therefore, the first step to identify the strategic placement of PMUs in a given system, to have full observability is to remove all conventional measurements (including zero injections) from the system. Removal of all conventional measurements leaves a plane system with no measurements. The objective of this part of study is to populate the plane system with as minimum number of PMUs as possible so that state of all substations can be calculated through the data available from PMUs. The obtained result should be somehow validated. One way to validate the results is to evaluate the outcome with the expected answers. Although the expected number of PMUs for a system is an approximation, it is well documented that to have an $N$ bus system fully observed, roughly $(N/3)$ multi-channel PMUs are required [2].

In order to find the optimal placement of PMUs for a given system, following optimization program should be solved. The objective is to find the minimum number of PMUs in such a way that each and every bus in the system is reached by at least one PMU. One way to solve such problem is using integer programming package such as TOMLAB
[55,56]. The formulation for full observability using minimum number of multi-channel
PMUs can be written as:

\[
\text{Minimize } \sum_{i} \omega_i x_i \\
\text{Subject to } T . X \geq \hat{1}
\]  

Where \( \omega_i \) is the cost of the PMU installation at bus \( i \). \( X \) is a binary decision variable
to vector, whose entries are defined as:

\[
x_i = \begin{cases} 
1 & \text{if a PMU is installed at bus } i \\
0 & \text{otherwise}
\end{cases}
\]

\( T \) is the bus to bus connectivity matrix which can be defined as following:

\[
T_{i,j} = \begin{cases} 
1 & \text{if bus } i \text{ and bus } j \text{ are connected} \\
1 & \text{if } i = j \\
0 & \text{otherwise}
\end{cases}
\]

And \( \hat{1} = [1,1,\ldots,1] \)

The objective function minimizes the installation cost for the selected PMUs, and the
constraint function ensures that the entire system is observable.

To solve such equation “TOMLAB” software has been utilized. Let us explain the
method on a small system, such as IEEE 14 bus system, for illustration. The validity of the
proposed method in a small system such as IEEE 14 bus system can be easily verified by
comparing the network topology and PMU placement. To validate the method, IEEE 14
bus system, and IEEE 57 bus system, as well as IEEE 118 bus system, have been chosen as the tested.

![PMU placement for IEEE 14-Bus System](image)

**Figure.II.1.** PMU placement for IEEE 14-Bus System

<table>
<thead>
<tr>
<th>Number of Required PMU</th>
<th>Optimal PMU placement ignoring zero injections</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2,6,7,9</td>
</tr>
</tbody>
</table>

**Table.II.1.** Optimal PMU placement for IEEE 14 bus system

Table II.1 shows that 4 multi channel PMUs is required for full observability achievement in IEEE 14 bus system. Table II.2, shows the optimal PMU placement for IEEE 30 bus system, and Table II.3 represents the result for IEEE 57 bus system, and table
II.4 shows the result for IEEE 118 bus system. As we expected in all cases, the number of PMUs are roughly (1/3) of the bus numbers in the system.

**Table II.2.** Optimal PMU placement for IEEE 30 bus system

<table>
<thead>
<tr>
<th>Number of Required PMU</th>
<th>Optimal PMU placement ignoring zero injections</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2,4,6,9,10,12,15,18,25,27</td>
</tr>
</tbody>
</table>

**Table II.3.** Optimal PMU placement for IEEE 57 bus system

<table>
<thead>
<tr>
<th>Number of Required PMU</th>
<th>Optimal PMU placement ignoring zero injections</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>1,4,7,9,15,20,24,25,27,32,36,38,39,41,46,50,53</td>
</tr>
</tbody>
</table>

**Table II.4.** Optimal PMU placement for IEEE 118 bus system

<table>
<thead>
<tr>
<th>Number of Required PMU</th>
<th>Optimal PMU placement ignoring zero injections</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>2,5,9,11,12,17,21,24,25,28,34,37,40,45,49,52,56,62,63, 68, 73, 75,77,80,85,86,90,94,101,105,110,114</td>
</tr>
</tbody>
</table>
After showing that the method is working properly, the method is applied to three utility company systems. The method is applied to three utility company systems to obtain the strategic location of multi-channel PMUs on their systems. Utility “A” system contains 3404 buses including both “A” system substations and pseudo buses representing the external equivalent. First step was to eliminate the pseudo buses and corresponding measurements. It turns out that 1119 pseudo buses constitute 1071 external observable islands, and the remaining 2285 buses constitute a single large island. 1119 buses and their incident measurements are removed, and 2285 substations are designed as the “A” system in our works. The expected number of multi-channel PMUs to observe a given system is approximately \( \frac{N}{3} \). Having 2285 buses in the system, the expected number of PMUs is approximately 762. The study shows that 721 multi-channel PMUs are enough for system “A” to be fully observable, without usage of any conventional measurements.

Analysis of network observability for system “B” reviled several observable islands. Out of 8131 substations received as “B” system, 2508 buses constitute one large island, and the remaining buses constitute 4324 observable islands, representing the pseudo buses. These buses and their corresponding measurements are removed from the system and the single island of 2508 buses is referred to as system “B” at our works. The expected number of PMUs to fully observe such system is 837. The study shows that 1513 multi-channel PMUs are required to observe the main island in this utility system. This is because of the radial topology of the system. The study shows that 824 buses in system “B” have just one neighbor, and 797 buses in this system have only two neighbors. In other word, the “B” system is mostly a radial system. Note that in this part of study the plane network has been
used. In other word it is assumed that no conventional measurements or zero injections are available in the system.

Observability study for third utility company system (system “C”) was different as their system contains no pseudo buses at all. “C” has a 1459 bus system, and the study shows that 367 multi-channel PMUs are required to have this system fully observed. The expected number of required multi-channel PMUs for such system is 486. The only required data in this part of study is the topology of the system as no measurement is considered in this part of study.

The next task is to find the optimal placement of multi-channel PMUs considering zero injections. This can be accomplished by different methods. The method used here is the one explained in [41].

II.6. Review of incorporating zero injections in system observability [44]

A numerical method based on Integer Programming (IP) has been discussed previously. Given an n-bus system, PMU placement problem can be formulated as follows:

\[
\begin{align*}
\text{Minimize} & \quad \sum_i \omega_i \cdot x_i \\
\text{Subject to} & \quad f(X) \geq \hat{1}
\end{align*}
\]

(II.10) \[
\omega_i \quad \text{is the cost of installation of a PMU at bus } i.
\]
$X$ is a binary decision variable vector, whose elements are defined as:

$$
x_i = \begin{cases} 
1 & \text{if PMU is installed at bus } i \\
0 & \text{otherwise} 
\end{cases} 
$$

(I.11)

$f(X)$ is defined as:

$$
f(X) = A.X 
$$

(I.12)

The matrix ‘A’ is called the “connectivity” matrix and defined as:

$$
A_{i,j} = \begin{cases} 
1 & \text{if } i = j \text{ or } i \text{ and } j \text{ are connected} \\
0 & \text{otherwise} 
\end{cases} 
$$

(I.13)

The function $f(X)$ represents the constraints that ensure every bus is either assigned a PMU or has at least one PMU assigned to any one of its immediate neighbors. Note that in $f(X)$, “+” acts like a logical “OR”. For instance, in the IEEE 14 bus system, shown in figure 1, the constraint equation for bus 1 can be written as follows:

$$
f_1 = x_1 + x_2 + x_3 \geq 1 
$$

This implies that at least one PMU should be installed at bus 1 OR bus 2 OR bus 5 in order to make bus one observable. Details of the method can be found in [41] and [44]. Different methods have been proposed to modify the formulation for optimal placement of PMUs in case of considering zero injections. The topology method is used in this dissertation to take zero injection buses into account.
The main idea of topology transformation is described in [58]. The approach is to merge the zero injection bus with any of its neighbors. Note that however the choice of the neighbor bus to be merged is arbitrarily done; it will affect the optimal PMU placement. Different choices of neighbors will lead to different PMU placement strategies; however the number of PMUs will be always the same independent of the choice of the neighbor bus. As an illustrating example, a seven bus system will be used as shown in figure II.2.

![Figure II.2. A simple 7 bus system with zero injection at bus 3.](image)

As shown in figure II.2, bus 3 is the bus with zero injection. The connectivity matrix of such system can be written as:
\[
A = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 \\
\end{bmatrix}
\]

Forming the equations of \( f(X) \) yields:

\[
\begin{align*}
  f_1 &= x_1 + x_2 \\
  f_2 &= x_1 + x_2 + x_3 + x_6 + x_7 \\
  f_3 &= x_2 + x_3 + x_4 + x_6 \\
  f_4 &= x_3 + x_4 + x_2 + x_7 \\
  f_5 &= x_4 + x_5 \\
  f_6 &= x_2 + x_3 + x_6 \\
  f_7 &= x_2 + x_4 + x_7
\end{align*}
\]

Bus 3 will have to be merged with one of its neighbors, namely bus 4, 6 or 2. If bus 3 and 6 are merged, the system diagram will change as shown in Figure II.3, resulting in a new 6 bus system with a new node 6 which is actually a super-node obtained from marriage of bus 3 and 6. This changes both the topology as well as the connectivity matrix.
Figure II.3. System diagram after merging bus 3 and bus 6.

The connectivity matrix corresponding to the new system diagram shown in Figure II.3:

\[
A = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 1
\end{bmatrix}
\]

And the constraint vector function \( f(X) \) will be given by:

\[
f(X) = A_{\text{new}}X = \begin{cases}
    f_1 = x_1 + x_2 \\
    f_2 = x_1 + x_2 + x_6 + x_7 \\
    f_4 = x_4 + x_5 + x_6 + x_7 \\
    f_5 = x_4 + x_5 \\
    f_6 = x_2 + x_4 + x_6 \\
    f_7 = x_2 + x_4 + x_7
\end{cases}
\]
The study is done for both utility systems “A”, and “B”. System “A” has 649 zero injections, taking zero injections into account, the total number of 501 multi-channel PMUs are required to have a fully observable system. “B” has 1166 zero injections in their system, and incorporating zero injections into the calculations reduces the required PMUs to the total number of 584.

II.7. Review of Identification and transformation of “critical” measurements

It is well documented in the literature that bad data detection on analog measurements is only possible if those measurements are not critical [20]. Critical measurements are those measurements whose removal from the measurement set will cause unobservability. Hence, their measurement residuals are identically equal to zero irrespective of their accuracy and hence their errors cannot be detected by the commonly used largest normalized residual test.

In this part of the study, the objective is to determine the best locations for placing PMUs in order to create a measurement set that does not include any critical measurements. The utilized approach used in this dissertation is an extension of the method described in [59] which involves two stages. The first stage is determination of candidate PMU locations that will transform each identified critical measurement in the system into a redundant measurement. The second stage takes these candidates and makes an optimal selection out of these to find the optimal solution.
The method that is employed in order to determine the critical measurements and the candidate measurements to make these redundant can be implemented as summarized below:

1- Form the measurement jacobian matrix, $H$.

2- Augment $H$ by adding all the candidate PMUs at the bottom. Let the sub-matrix representing the rows corresponding to the candidate measurements be denoted by $H_c$.

3- Perform a rectangular LU factorization on $H$. This can be accomplished by using Peters-Wilkinson method described in [60]. Row pivoting during this factorization should be limited to the rows corresponding to the existing (not any of the candidate PMUs) measurements. If such pivoting proves inadequate, then it implies an unobservable system. In that case, row pivoting will be extended to the rows of candidate measurements so that a minimally observable set of measurements can be found to make the system observable first. The following steps assumes that the system is observable already (either initially or made observable via this extension) and will identify critical measurements within this set.

4- Note that after row pivoting, top n rows of $H$ will represent the minimally observable set of measurements ($H_0$) and is referred to as essential set of measurements. The next m-n rows of $H$ represent the redundant measurements and are referred to as non-essential measurements ($H_r$). Let the LU factors of $H$ be written as:
\[
H = \begin{bmatrix}
H_0 \\
H_r \\
H_c
\end{bmatrix} = \begin{bmatrix}
L_0 \\
M_r \\
M_c
\end{bmatrix} \cdot [U]
\] 

(II.14)

Where:

\(H_0\): whose rows correspond to minimally observable measurement set of “n” (number of columns in \(H\) measurements.

\(H_r\): whose rows correspond to redundant existing measurements.

\(H_c\): whose rows correspond to candidate PMU measurements.

Following equation can be written for entire system:

\[
\begin{bmatrix}
H_0 \\
H_r
\end{bmatrix} [x] = \begin{bmatrix}
Z_0 \\
Z_r
\end{bmatrix}
\] 

(II.15)

Where \(Z_0\) and \(Z_r\) are essential and non-essential measurements. Applying Peters-Wilkinson decomposition above equation can be written in following format:

\[
\begin{bmatrix}
H_0 \\
H_r
\end{bmatrix} = \begin{bmatrix}
L_0 \\
L_r
\end{bmatrix} [U]
\] 

(II.16)

Where \(L_0\) is an \(n \times n\) lower triangular matrix, \(L_r\) is a \((m - n) \times n\) rectangular matrix, and \(U\) is a \(n \times n\) upper triangular matrix. Therefore:
\[ z_0 = L_0 \cdot U \cdot x \]
\[ z_r = L_r \cdot U \cdot x \]  \hspace{1cm} (II.17)

Eliminating \( U \cdot x \) from equations (II.17) yields:

\[ z_r = L_r \cdot L_0^{-1} \cdot z_0 \]

\[ T = L_r \cdot L_0^{-1} \Rightarrow z_r = T \cdot z_0 \]  \hspace{1cm} (II.18)

This equation shows the linear dependency between the essential and redundant set of measurements. In other words, an essential measurement is critical if the column corresponding to that measurement is null.

Therefore to identify the critical measurement in a system, after following steps 1 through 5, one should calculate the test matrix \( T \) defined as:

\[
T = \begin{bmatrix} T_r \\ T_c \end{bmatrix} = \begin{bmatrix} L_r \\ L_c \end{bmatrix} \left[ \begin{bmatrix} L_0 \end{bmatrix} \right]^{-1}
\]  \hspace{1cm} (II.19)

If column \( k \) in \( T_r \) is null, it will correspond to a critical measurement. Then, the rows having nonzero entries in column \( k \) of \( T_c \) will correspond to the candidates that will transform this critical measurement into a redundant one.
II.8. Application to small test and large utility systems

A computer package is developed for identification of critical measurements. For validation, it is first tested on small systems. Once the validity of the method is verified, the method can be applied to the real world systems such as system “A” and “B”. As an example, consider the simple 6-bus system and its measurement configuration as shown in Figure II.4. Assuming candidate phase angle (PMU) measurements at every bus, the following results will be obtained by applying the procedure described above:

Figure II.4. 6-bus system measurement configuration.

\[
H_0 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 3 & -1 & 0 & -1 & 0 \\
0 & 0 & -1 & 0 & -1 & 2 \\
0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 1 & 0 & -1 & 0 \\
1 & -1 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
H_r = \begin{bmatrix}
0 & 1 & 0 & 0 & -1 & 0 \\
0 & -1 & 1 & 0 & 0 & 0
\end{bmatrix}
\]

\[
H_c = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]
The critical measurements can be identified by checking the null columns of $T_r$ given below:

$$T_r = \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0.5 & 0.5 \\ 0 & -0.5 & 0 & 0 & 0.5 & -0.5 \end{bmatrix}$$

Column 1 which corresponds to the phase angle (PMU) measurement at bus 1 will be ignored since it serves as the reference bus. That leaves columns 3 and 4 corresponding to the injection at bus 6 and the flow from 4 to 5. The candidates which can transform these two measurements into redundant ones are the rows with non-zero entry at columns corresponding to null column in $T_r$; this can be found by calculating $T_c$ which is given below:
Hence, phase angle measurements at bus 6 and 4 will be identified as the candidate measurements. Typically, this step will be followed by an optimization procedure, where an optimal subset of the identified candidate measurements will be determined. This is accomplished by solving another integer programming problem which is formulated as follows:

\[
\text{min} \sum_i C_i x_i \\
\text{s.t. } Q \cdot X \geq \hat{1}
\]  \hspace{1cm} \text{(II.20)}

Where: \( X \) is a binary array whose entries correspond to the candidate measurements (in this example, they are the phase angle measurements at buses 2 through 6) and they will be 1 or 0 depending on whether they are selected or not respectively.

\( C_i \) is the installation cost for PMU \( i \).

\( Q \) represents the critical measurement to candidate measurement incidence matrix, defined as follows:

\[
Q_{ij} = \begin{cases} 
1 & \text{if meas } j \text{ is a candidate for critical meas } i \\
0 & \text{otherwise} 
\end{cases}
\]  \hspace{1cm} \text{(II.21)}

\[
T_c = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & -1 \\
1 & -0.5 & 0 & 0 & 0.5 & -1.5 \\
1 & -0.5 & 0 & 1 & -0.5 & -1.5 \\
1 & -0.5 & 0 & 0 & -0.5 & -1.5 \\
1 & -0.5 & 0.5 & 0 & 0 & -1.5
\end{bmatrix}
\]
For this simple 6-bus example, since there are only two non-overlapping candidate measurements, Q matrix will be trivial, so will be the optimal solution, i.e. both of the candidate measurements will be selected.

Another case can be illustrated by considering the 14-bus system with its measurement configuration shown in Figure 5, where all measurements are critical. It can be shown that, by applying the above procedure, a single PMU measurement at bus 6 (e.g. monitoring branch 6-11) will eliminate all critical measurements. While this is an extreme case, it illustrates the power of placing few PMUs to make significant improvements in bad data detection capability.

The same method can be utilized to identify the critical measurements in system “A”, “B” and “C”. Implementation of the proposed method reviles that 167 critical measurements exist in system “A”, 113 of which are power flows and the rest are power

Figure.II.5. 14-bus system measurement configuration.
injections. It turns out that 101 multi-channel PMUs are needed to remove all critical measurements from this system. In case of system “B”, the study shows 1274 critical measurements, 997 of which are power flow measurements and the rest are power injection measurements. Utilizing the method shows that 489 multi-channel PMUs are required to transform all critical measurements of “B” system. Implementation of the method for system “C” reviles 340 critical measurements. Among those, 312 measurements are power injection and the remaining 28 measurements are power flows. The study shows that 47 multi-channel PMUs are required to remove all critical measurements from this system.

II.9. Robustness against contingencies and Effect of contingencies on network observability

Network observability depends on types and locations of measurements as well as the network topology. Anytime a change occurs in one of these factors, observability analysis has to be repeated to ensure that state estimation problem can still be solved.

While it is not possible to design the measurement system so that the system remains robust against any sort of switching that the system may experience, measurement design can take into account most likely contingencies that involve topology changes. This is quite valuable since it is during these contingencies that the assistance of the state estimator will be most needed. In this section of dissertation, the problem of strategic placement of PMUs against topology changes associated with expected contingencies will be investigated. The approach will be similar to the ones used for the solution of the above
two problems. The first stage involves determining whether the considered topology change actually causes any change in network observability. If the system remains observable after the topology change, then no action will be necessary. Else, a set of candidate PMUs will be identified such that placing any one of them will restore observability.

The Algorithm of identification of candidate PMUs is shown below.

1. Form the augmented jacobian $H$ which contains all existing measurements followed by the candidate PMU measurements. For each contingency, remove the line which is opened during the contingency and modify the relevant entries in $H$.

2. Factorize $H$ by limiting the row pivoting to the rows associated with the existing measurements only. If the contingency causes unobservability, a zero pivot will be encountered.

3. Trace the column below the zero pivot in the lower triangular factor and the candidate PMU measurements will be given by those with nonzero entries in this column. In order to illustrate this approach, consider the 6-bus system measurement configuration given in Figure II.6.

4. In addition to the given measurements, every bus will be assumed to have a candidate PMU. Line 2-3 outage will be used as a contingency example. Note that the system will become unobservable if line 2-3 is taken out of service.
Figure II.6: Contingency case: line 2-3 is out of service.

Modified $H$ after the removal of line 2-3 can be formed as follows:

$$
H = \begin{bmatrix}
2 & -1 & 0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & -1 & -1 \\
1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & -1 & 0 & -1 & 2 \\
1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

Its lower triangular factor $L$ will then be given by:
Tracing the column under the zero pivot (column 6), identifies PMU measurements at buses 3, 5 and 6 as candidates for recovering observability during this contingency.

This part of study is accomplished for the “A” utility company only. This company provided the list of their top 20 contingencies, and the study shows that only two of the contingencies have impact on system observability. In each case the observability of the system can be restored with a single channel PMU following the contingency occurrence.

II.10. Application of Single line monitoring PMU in power systems

II.10.1. Observability check

The assumption that each multi-channel PMU has the ability of measuring of as many phasor currents incident to a bus as possible is an unrealistic assumption. Clearly, the number of the channels available through each PMU has direct impact on the optimal
number and location of PMUs in the system for full observability achievement. As mentioned earlier, different type of PMUs is manufactured by different companies. Each PMU is capable of measuring different number of current phasors depending on the number of available output channels. This part of this dissertation investigates the strategic location of a PMUs which have only one output channel; this kind of PMUs are already installed in power utility systems (i.e. system “A”). Single channel PMU (branch PMU) is capable of measuring the phasor voltage and the phasor current of only one branch incident to the bus where PMU is installed.

Formulation in previous section was derived based on the assumption that each PMU has the ability of measuring a phasor voltage of the bus and phasor currents of all branches connected to that bus, and therefore a PMU is assigned to a bus and all branches incident to that bus. On the other hand each single channel PMU can measure the phasor voltage and only one current phasor measurement; therefore it is reasonable to assume that each single channel PMU is assigned to one branch instead of a bus. The formulation and procedure of finding strategic location of branch PMUs to have a fully observable system is similar to the one described for the case of multi-channel PMUs. The objective is to minimize the cost of installation subject to all buses to be reached through at least one branch PMU. Formulation can be written as follows:

$$\min \sum_{i} C_i x_i$$

$$s.t. \ A.X = f(X) \geq \hat{1}$$

(II.22)
Where $X$ is a binary decision variable vector, whose entries are defined as:

$$
x_i = \begin{cases} 
1 & \text{if a PMU is required for branch } i \\
0 & \text{otherwise} 
\end{cases}
$$

$C_i$ is the cost of installation of PMU at branch $i$.

A: is branch to bus connectivity matrix

$f(X)$ is a vector function, whose entries are non-zero if the corresponding bus voltage is solvable using the given measurement set and zero otherwise.

**II.10.2. PMU placement for full observability ignoring zero injections**

As shown above this method is different from the multi-channel case when forming the ‘A’ matrix. While the bus connectivity matrix is of interest in case of system observability using multi channel PMUs, in case of using single channel PMU to obtain the system observability, branch to bus connectivity matrix should be formed. As an example consider a five bus system shown in Figure II.7. The bus to branch connectivity matrix corresponding to the following five bus system will be:

![Network diagram for the 5-bus system](Image)

**Figure II.7.** Network diagram for the 5-bus system
\[
A_{\text{Branch-Bus}} = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}^T
\]

Which yields the constraint equations as follows:

\[
f(X) = A_{\text{Branch-Bus}} \cdot X = \begin{cases}
    f_1 = X_1 + X_2 \geq 1 \\
    f_2 = X_2 + X_3 \geq 1 \\
    f_3 = X_2 + X_4 \geq 1 \\
    f_4 = X_3 + X_4 \geq 1 \\
    f_5 = X_4 + X_5 \geq 1
\end{cases}
\]

Where \( X_i \) represents the branch ‘i’ in the system. Applying the this method to the IEEE 14 bus system yields the optimal location of the single channel PMU to have an observable system in case of ignoring zero injections. Results are shown in Table II.5. Similar results are obtained for the 30 bus system as shown in Table II.6.

Knowing the fact that for an N-bus system, minimum number of (N-1) measurements are required to have a fully observable system, and due to the fact that each branch PMU provides two measurements; The minimum expected number of single channel PMUs required to observe an n-bus system is the integer after \( \left\lceil \frac{N-1}{2} \right\rceil \).

After validation of the method on small systems, the method is applied to real utility company systems. Optimal placement of branch PMUs to have a fully observable system is obtained for system “A”, “B”, and “C”. Study shows that for system “A” which is a
2285 bus system, 1291 branch PMUs are required to have a fully observable system. Results for system “B” with 2508 buses indicate that 1541 PMUs are found to be installed on the system to observe the grid. In case of 1459 bus system of “C”, the study reviles that 820 single channel PMUs are required to make this system fully observable. Note that in this part of study no conventional measurements are considered to be involved in our calculations.

**Table II.5** PMU placement for 14-bus system

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<th>PMU</th>
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**II.10.3. Optimal PMU placement with considering of zero injections**

In this section, zero injection buses in the system will be used to effectively lower the number of required PMUs to make the entire system observable. Results indicate that there are a total of 649 zero injection buses in system “A” and 1166 zero injection buses in system “B”. Taking zero injections into account, using the merging bus method explained
for multi-channel PMUs, the total required number of single channel PMUs that will make the entire system observable is found to be 1005 for system “A” and 1144 for system “B”.

Table II.6  PMU placement for 30-bus system

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II.10.4. *Optimal PMU placement in order to transform all critical measurements*

The objective of this part of dissertation is to transform all the critical measurements into redundant ones by using single channel PMUs. Technical approach and problem formulation are similar to the ones described above for multi-channel PMUs with the only difference in forming of the Q matrix shown below:

\[
\min \sum_{i}^{L} C_i x_i
\]

\[s.t. \quad Q \cdot X \geq \hat{1}\]  \hspace{1cm} (II.23)

Where the elements of both X and Q matrices represent the branches rather than the buses of the system.

The procedure is first to find the critical measurements as discussed for the multi-channel PMU case, and then following the same procedure to obtain the candidate branches where PMUs will be installed. Hence, the matrix Q will be defined as:

\[
Q_{ij} = \begin{cases} 
1 & \text{if branch } j \text{ is incident to the candidate measurement for removing critical measurement } i \\
0 & \text{otherwise}
\end{cases}
\]

As in the above cases, the method is tested first on small test systems such as the IEEE 14 bus system before applying it to actual utility company systems. Figure II.8 shows the example system with all critical measurements.
Figure II.8. IEEE 14-Bus system with all critical measurements.

It is found that a single channel PMU placed at branch 6-11 will be sufficient to transform all critical measurements into redundant ones in this system. The result is shown in Figure II.9.

Figure II.9. Transforming critical measurements using single channel PMU at bus 6 (Red Box)
The method is then applied to both system “A” and system “B”. Note that 134 single channel PMUs are required to transform all critical measurements in “A”, and 849 single channel PMUs are required to transform all the critical measurements from system “B”.

II.10.5. Loss of PMUs and multiple contingencies

The problem of optimal placement of branch PMUs to have a fully observable system is discussed earlier in this dissertation. However, the possibility of a PMU failure is not considered in literature (to the best of author’s knowledge). The failure of any PMU may result in loss of network observability or it may decrease the capability to detect bad data in the system.

Another concern is the possibility of multiple contingencies occurring either simultaneously or in cascade within a short period of time. When such possibilities are taken into account, a robust PMU placement can be accomplished. The proposed method can increase the reliability of the system in such a way that failure of even several PMUs and multiple simultaneous contingencies can be handled. In general, the proposed method will work unless both original and backup PMUs that are incident to a given bus are simultaneously lost either due to the multiple contingencies or PMU failure.

The proposed method to find the backup set of PMUs for an already installed set of PMUs is as follows:
1- Find the optimal set of PMUs to have an observable system as discussed earlier in “observability check” section.

2- Remove all branches with PMU placements in step 1. Let all the columns of the matrix $A$ that correspond to PMU assigned branches be stored in a new matrix called $D$. Let the remainder of matrix $A$ be stored as matrix $P$. Each column of matrix $P$ corresponds to a branch with no PMU at either ends.

3- If there is no null row in $P$, no further action is required. $P$ matrix can be replaced as $A$ in (II.22) and the solution of integer programming will yield the optimal solution for backup PMUs of those found in step 2. Else, let row $k$ of $P$ be null. Find all columns of $D$ whose $k$ 'th row is nonzero. If only one column is found, simply augment $P$ by attaching this column to $P$. If more than one column with nonzero entries in row $k$ is found, choose the one whose receiving end bus is least repeated in $P$ matrix.

4- Replace $A$ matrix by the modified $P$. Solve the integer programming problem and obtain the new set of backup PMUs for the previous set.

For illustration, the method is applied to a simple 7 bus system shown in Figure II.10.
Figure II.10. a simple 7 bus system

The branch to bus incident matrix for this system is:

\[
A = \begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 \\
\end{bmatrix}
\]

The optimal original set of PMUs to have a fully observable system is \( X = [1 \ 1 \ 0 \ 0 \ 1 \ 0 \ 1] \) so \( D \) and modified \( P \) matrices can be written as:
$P$ matrix represents those branches of the system where no PMU is assigned. Null rows in $P$ matrix correspond to buses which cannot be reached by placing PMUs on remaining branches. This may happen if all of the branches incident to a given bus are already chosen in the original set of PMUs (Bus 2). Removing all those branches from the branch to bus incident matrix creates a null row for that bus in $P$. A special case is when there is a radial branch (4-6) which can only become observable by assigning a PMU to the branch. Since this branch will also belong to the original PMU set and there is no other connection to that bus, the row corresponding to the remote end bus (4) in $P$ will be null. In these two cases, columns corresponding to added branches are moved from $D$ to $P$ and hence integer programming solution will assign repeated PMUs to these branches, i.e. these branches will appear in both the primary and backup branch PMU sets.

In the above 7-bus system example, 2nd and 4th rows of $P$ are null. Therefore columns are to be added to $P$ from $D$. Both columns 1 and 2 of $D$ have nonzero entries in their 2nd row. Column 1 of $P$ represents branch 1-2, the receiving end of which is bus 1. Column 2 of $D$ corresponds to branch 2-3, receiving end of which is bus 3. Since bus 1 is repeated less in $P$, branch 2-1 is chosen to be added to $P$. Such a choice is not required for row 4.

\[
D = \begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
\quad
P = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
since there is only one candidate column with a nonzero row 4. Hence 1st and 3rd columns are added to P. The new P will then look like:

\[
P = \begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]

IP solution will be given as \(X=[0 \ 1 \ 0 \ 1 \ 1 \ 1]\). Hence, there are only two additional PMUs to be installed in the system as backups since PMUs in branches 1-2 and 4-6 are already installed as part of the primary set. Clearly for those cases when a PMU that is assigned to a radial branch fails, it will not be possible to avoid creation of observable islands. On the other hand, duplicate PMUs can be installed at opposite ends of such lines. While this will account for PMU failures, it can still not avoid unobservability if the line is disconnected due to a fault.

This method is tested on the IEEE 30 bus test system. Table II.7 shows the results of original and backup PMUs obtained for this case.
Table II.7 Original and backup PMU placements for the 30 bus system.

<table>
<thead>
<tr>
<th>Original PMU location</th>
<th>From Bus</th>
<th>To Bus</th>
<th>Backup PMU location</th>
<th>From Bus</th>
<th>To Bus</th>
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*: shows the repeated branches in both original and backup location.
II.11. **Strategic location of phasor measurements with consideration of existing PMUs**

With the increasing demand of PMU installation in power systems, most of the power utility companies in North America have already a few PMUs installed on their systems. Two of the utility companies which participated in this study have a few existing PMUs on their systems. This part of dissertation investigates the impact of existing PMUs on strategic placement of PMUs for different achievements including: full observability with/without considering zero injections, optimal PMU placement for transforming critical measurements. There is 16 and 11 single channel PMUs available in system “A” and “B” respectively. Clearly, number of required PMUs to have an observable system or to remove the critical measurements should be decrease if existing PMUs are installed on appropriate location in the system. Unfortunately, results indicate that most of the PMUs are installed on irrelevant locations with no positive impact on the number required PMUs for any application in power systems. As the results reviled only a few PMUs are installed on reasonable locations which can reduce the number of required PMUs for any power system application such as system observability and bad date removal, while most of the PMUs are installed on improper locations and cannot improve the required PMUs for any power system application. Although some of these PMUs may have been installed on the system to follow short-term objectives, it is always good to keep the long-term goal in mind. This test shows the necessity of costly procedure of reinstallation of PMUs in the system due to installation of PMUs on inappropriate locations, and highlights the importance of installation of PMUs based on the proposed method.
II.11.1. **PMU placement ignoring zero injections**

In this part of study all conventional measurements are ignored. Only existing PMUs in system “A” and “B” are considered as existing measurements. Incorporating 16 and 11 existing PMUs in system “A” and “B” respectively, does not change the optimal location of multi channel PMUs. It is recalled that 721 multi-channel PMUs are required for monitoring the system “A”, and 1513 multi channel PMUs are required to make the “B” system observable without considering any existing PMUs installed in the system. This is determined by solving the following Integer Programming problem:

\[
\min \sum_{i} C_i \cdot x_i \\
\text{s.t. } A \cdot X = f(X) \geq \hat{1}
\]  

(II.24)

Where \( C_i \) is the cost of installation of PMU at branch \( i \).

If the corresponding cost for installation of PMU at bus ‘\( i \)’ is set equal to zero, then the program will automatically choose the corresponding PMU for placement. Solving the above integer programming problem for the “B” system with 11 existing PMUs yields a total number of 1524 PMUs. This means that still 1513 new multi channel PMUs are required to make the system fully observable. Also, 1541 new single channel PMUs are required to have system “B” fully observable. Same method is applied to system “A”. It turns out that solving the integer programming, with considering the existing PMUs on system “A”, 1283 new single channel PMUs would be still required to have the system full observable. As mentioned in section (II.10.2.), 1291 single channel PMUs are required to
observe this system in case of ignoring the existing PMUs. In other words number of new PMUs required to be installed has dropped by 8.

II.11.2. PMU placement considering zero injections

Next, the problem of optimal single channel PMU placement is repeated by taking into account all zero injection buses. Recall from previous sections that 584 multi channel PMUs or 1144 single channel PMUs is required to observe system “B”. Taking the existing PMUs into account this number becomes 595 multi-channel or 1155 single channel PMUs. Out of these 595 PMUs, 584 of them will be new PMUs. In other words the existing PMUs have no effect on PMU placement. As mentioned in section (II.10.3), 1005 single channel PMUs are required to have the “A” system fully observable without consideration of existing PMUs. The study shows that considering the existing PMUs, 998 new single channel PMUs are still required for observing this system.

II.11.3. Optimal PMU placement in order to transform all critical measurements

The objective of this part of the study is to find the PMU placement for transforming all critical measurements into redundant ones while taking into account those PMUs already present in the system. Following the discussed steps in previous section, yields the optimal PMU placement to have a fully observable system. In this case taking the existing PMUs into account will improve the previously determined PMU placement results for
system “B” only. It turns out that one of the critical measurements can be removed by PMUs already installed in the system “B”. While no improvement is seen through the number of the PMUs required for transforming system “A” critical measurements.

Previous study shows that without considering existing PMUs, 488 multi channel PMU are required for removal of critical measurements. Using the existing 11 PMUs in the system, the required number of PMUs is found to be 498 (including the existing 11 PMUs). Hence, it is apparent that 487 new PMUs are still required. In other words, just one of the existing PMUs are located at strategically proper bus to help reducing the number of new PMUs. Given the large dimension of the system “B” (2508) and the very small number of existing PMUs (11) this result is not entirely unexpected.

This study highlights the importance of installing PMUs at recommended substations. As shown earlier, 16 PMUs are installed in system “A”, and 11 PMUs are installed in system “B” has no critical improvement on results of any of mentioned studies due to random choice of PMU locations. Since the installation of PMUs is pretty costly procedure, result of this study are expected to assist planning engineers who are planning to invest in this new technology as random installation of PMUs in power systems can cause nothing but waste of money.
II.12. Summary

Although the problem of strategic location of PMUs in power systems have received a significant attention in research, all available methods are developed based on the idealistic assumption that each PMU has ability of measuring current phasors of all branches incident to the bus where PMU is installed. As of today such PMUs are not manufactured yet. This motivates to look for a more realistic case where each PMU has a limited number of output channels. One of such PMUs with limited number of output channels is branch PMUs which is already installed in North American power system utilities.

This chapter of dissertation studies the problem of optimal location of single channel PMUs to have full observability, removal of critical measurements, and robustness against single or multiple contingency, and loss of PMUs.
CHAPTER III

ANALYSIS OF IMPACT OF EXTERNAL MEASUREMENTS
III.1. Introduction

This dissertation is mainly divided into two major sections: first section (chapter II) deals with the problem of maintaining a single observable island. Second part in this dissertation, studies the case where both internal and external systems are involved. As both operating condition and the topology of neighboring systems have direct impact on internal power system analysis, updating the internal system knowledge about external topology and operating condition can improve the accuracy of internal power system studies such as security assessment. It is well known that the real-time topology and measurements of external system are not always available to internal power system dispatcher. This chapter and chapter four of this dissertation study the impact of such external changes on internal state estimation and contingency analysis.

Chapter two investigates the power system applications and problems in case of dealing with a single observable island. Problem of observability check and state estimation, security analysis, and application of PMUs for such applications have been discussed in chapter two.

This chapter on the other hand investigates the impact of having real-time access to some of external measurement. This chapter contains two parts, first part (III.2) investigates the impact of having access to the real-time phasor measurements available through a few PMUs installed in external system; as most of the power system utility companies have a few PMUs already installed in their system; and second part of this chapter (III.3) focuses on the issue of optimally selection of external measurements whose
real-time update can improve the internal state estimation and subsequent security analysis the best.

III.2. Impact of synchronized phasor measurements on external network modeling

Monitoring of a power system involves measurements which must be sufficient to observe the entire system state. When operating large interconnected systems, each utility will have detailed information and data about its own system while having limited access to the measurements from its neighboring systems, which are collectively referred to as the “external” system. This lack of real-time information both on measurements as well as network model constitutes the single most important source of errors in subsequent contingency analysis that is run by each utility control center.

Changes in electric market push the transmission network to be operating closer to the system boundaries. In order to keep the system secure and reliable, precise security analysis should be done on the system. The knowledge of external system is highlighted here because security analysis of a system cannot be done regardless of having up-to-date data from adjacent networks. Traditionally, static equivalents are used to model the external system in an interconnected network. This is due to the fact that the unreduced power flow model of the external system is usually unknown to the internal system. Even if the internal system operator has real-time access to the unreduced model of external system, application of such model engages excessively heavy calculations.
Generally, there are two major types of external network equivalencing: REI and Ward equivalents. The REI equivalent [62] recommends replacing a group of external buses with a pseudo bus which is connected to the boundary buses. In the Ward equivalent [63], the external power system is represented with a fixed admittance matrix as well as an injection at the boundary buses. As explained in [64], and [65] the major problem with these traditional methods is their incapability in accurately modeling of external system behavior in case of change of operating condition. Many extensions of Wards and REI methods are available such as the one represented in [65], [66] whose objective is to deal with the linearity problem. The proposed method is to represent the external reactive power response to the shift in internal system. [65], [70],[71], [72] propose a method which represents the equivalent network with a reduced model of external system obtained from off-line studies. Next, the equivalent is updated in real-time to match the internal network solution at the boundaries.

The first disadvantage of the previously proposed methods is the problem of updating all load flow information as the system condition changes. Second, is the problem of having real-time access to all measurements and breaker status from external system; which is not available to the internal power system dispatcher in most of the cases. [67], and [68] use neural networks to identify dynamic equivalent for the system. Larson and Germond applied neural networks in [69] to develop a steady state equivalent that stays accurate for a long period of time.

Extensive discussion on different methods for network equivalencing is represented in [70-74], [26]. Although [75] shows that more explicit representation of external system
yields more accurate security analysis, it is almost impossible to have an explicit model of external system in practice.

Incorrect modeling of the external system will lead to errors that might be unacceptably high under certain operating conditions. Hence, an improvement in the way external systems are monitored will have a significant impact on the operation of interconnected power systems. Recent increase in the number of synchronized phasor measurements installed in various substations in power systems provides an opportunity for making such improvement. Synchronized phasor measurements are typically available via the phasor data concentrators which provide time tags and therefore allow system-wide measurement data to be available at local control centers.

State estimators can take advantage of the availability of synchronized phasor measurements from the external system in order to improve the external network’s estimated state and network model. As a result, reliability and accuracy of the subsequent calculations involving system security such as the contingency analysis can be significantly improved.

While installation of phasor measurement units (PMUs) is uprising in power systems, their numbers are still low to allow observability of systems based exclusively on PMU measurements. Thus, it is worth investigating the incremental benefits to be gained by taking advantage of their limited presence in the external system. This part of dissertation studies this problem by simulating scenarios where the operating conditions, location and number of available PMU measurements as well as the network model change for a given test system and its neighbors, i.e. its external system. IEEE 118 bus system is used for this
purpose by designating subset of buses as the internal and the remaining buses as the external system buses. This designation leads to the definition of those buses in the external system having direct connections to the internal system, as the external boundary buses.

It is noted that the internal system plus the internal boundary buses typically constitute a single observable island based on the internal system real-time measurements. Hence, those measurements used for the external system are either pseudo-measurement provided by the load forecasting and/or generation scheduler functions or they may seldom contain actual real-time measurements received through inter-utility real-time measurement exchange. In either case, due to the lack of redundancy, their impact on the internal state estimation will be minimal or null. On the other hand, they will have a very significant impact on the contingency analysis associated with the internal system topology changes due to their effect in building the external network model. The objective of this study is to highlight the role which few available PMU measurements might play in improving the external network model.

Specifically, this part of dissertation investigates the role of the location, number and type of these measurements in an attempt to develop guidelines and strategies to optimally place them when a limited number of them are available for placement.

The first part of this chapter is organized as follows. Following two sections (III.2.1 and III.2.3) review the procedure of transforming external system base case loads into impedances and the well known Kron reduction [20] to obtain the external system model (III.3). Next, the comparative analysis of the estimated system state with few PMUs in the
external system versus the case of having no PMUs is represented (III.2.4). Note that in order to simulate changing operating conditions, bus loads and generation for the entire system are increased by 10%.

**III.2.1. Transforming Power Injections at External Buses**

The net current injection at a substation is zero if there is no load or generation at that location. Voltages of buses with zero current injections can be eliminated from the system of equations to obtain a reduced dimension equivalent network model.

Let us consider the utility system of interest as the internal system and the remaining part of the interconnected system as the external system. Those external system buses which are connected directly to an internal system bus will be referred to as external boundary buses. Figure III.1 shows the schematic of internal and external system.

![Schematic of an interconnected system](image_url)

**Figure.III.1.** Schematic of an interconnected system
In Figure III.1, those buses connected through the tie lines between the internal and external systems will be referred to as internal and external boundary buses respectively.

In order to represent the external system with an equivalent network model, all external system buses except for the boundary buses will be converted to zero injection buses. This is accomplished by transforming the base case injections into equivalent shunt admittances using the base case power flow solution as shown below:

\[ Y_k^* = \frac{(P_k + jQ_k)}{|V_k|^2} \] (III.1)

Where \( P_k \) is the net active power injection at bus \( k \)

\( Q_k \) is the net reactive power injection at bus \( k \)

\( V_k \) is the voltage magnitude at bus \( k \)

And \( Y_k \) is the equivalent shunt admittance at bus \( k \).

### III.2.2. Kron Reduction In Power Systems

Consider the following matrix equation:

\[
\begin{bmatrix}
  x \\
  0
\end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix}
  y \\
  z
\end{bmatrix} + \begin{bmatrix}
  e \\
  f
\end{bmatrix}
\] (III.2)

This can be written in form of separate equations as follows:
\[ x = ay + bz + e \]
\[ 0 = cy + dz + f \]  \hspace{1cm} \text{(III.3)}

Let’s eliminate the variable ‘z’ from the top equation, this yields:
\[ x = ay - bd^{-1}cy - bd^{-1}f + e \]  \hspace{1cm} \text{(III.4)}

This can be written in following form:
\[ x = (a - bd^{-1}c)y + e - bd^{-1}f \]  \hspace{1cm} \text{(III.5)}

The procedure can be applied to the part of equation with zero left-hand-side like the second equation in (III.3). Obviously, in this case the operation to accomplish our purpose will not change the top element in the left-hand-side (x).

Following the same procedure, the method can be applied to the power system buses with zero injections. The current-voltage equation for a given system can be partitioned into two parts: top part is non-zero injection buses, followed by zero injection buses:
\[
\begin{bmatrix} I \\ 0 \end{bmatrix} = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \]  \hspace{1cm} \text{(III.6)}

Where \( I \) is sub-matrix which contains the net injection at non-zero injection buses of the system. Equation III.6 is similar to (III.2); therefore, same pattern can be applied to power system equations. This will lead to:
\[ I = (Y_1 - Y_2Y_4^{-1}Y_3)V_1 \]  \hspace{1cm} \text{(III.7)}
Equation (III.7) yields the admittance to represent the external model equivalent. This can be written as follows:

\[ I = Y_{eq} V_1 \]

**III.2.3. Forming the Network Equivalent by Kron Reduction**

As shown in section III.2.2, the zero injection buses can be removed from the system with application of Kron reduction. It is shown in section (III.2.1) that injections of any buses in the system can be transformed into shunt admittances; the objective is to transform all external buses into the zero-injection buses using the abovementioned transformation. Once this transformation is applied, it is easy to eliminate the zero injection buses from the external system by Kron reduction as shown in (III.7). Note that the external boundary buses are retained during the transformation and elimination procedure. Hence, the resulting equivalent model will contain all the internal system buses as well as the original external system boundary buses.

Using the example of Figure III.1, Kron reduction will start with the following equation:

\[
\begin{bmatrix}
Y_{11} & Y_{12} \\
Y_{21} & Y_{22}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
= 
\begin{bmatrix}
I_1 = 0 \\
I_2
\end{bmatrix}
\]  

(III.8)
Where $Y_{11}$ is the admittance matrix corresponding to the external part of the system. $Y_{12}$ and $Y_{21}$ are the admittance matrices corresponding to the connections between the boundary buses, and $Y_{22}$ is the admittance corresponding to the internal part of the system.

Eliminating $V_1$ yields the following equivalent admittance:

$$Y_{22-eq} = (Y_{22} - Y_{21}Y_{11}^{-1}Y_{12})$$  \hspace{1cm} (III.9)

Which represents the internal system with the attached external equivalent network.

**III.2.4. Validation of the method**

**III.2.4.i. Impact of external synchronized measurements on internal state estimation**

It is well documented that the external network model is valid as long as the external system state is equal to the base case solution [65], [76], [77]. This is certainly not the case due to the load variations taking place continuously in the entire system. While it is possible to monitor these load changes by proper real-time measurements from the internal system, such measurements usually are not readily available for the external system.

It is known that external PMU measurements can be directly used by internal system operator due to time alignment. In an attempt to take advantage of the few available PMU measurements in the external system, this study investigates the effect of such measurements on the accuracy of the internal state estimation. In the extreme case when every external bus is equipped with a PMU, the best solution will be obtained. This study
will consider the more realistic case of having few sparsely distributed PMUs in the external system and investigate their effects.

This will be accomplished by using the IEEE 118- bus system as an example. The system is partitioned into two parts, an internal system and an external system and their corresponding boundaries as listed in Table III.1. Figure.III.2 shows the schematic of an IEEE 118 bus system.

![IEEE 118 bus system](image)

**Figure.III.2.** IEEE 118 bus system

Several external system equivalents are developed by considering different number of buses with PMU measurements in the external system. All the equivalents are developed using base case loading conditions. In developing these equivalents, in addition to the external boundary buses, those buses with assumed PMU measurements are retained as
well. In other words, external buses with PMUs are treated as internal boundary buses as they have both characteristics of internal boundary buses: they are connected to external buses, and their real-time measurements are available to internal power system operator.

In executing the state estimation solution, a different loading condition is considered. All loads and generation are increased by 10% and the corresponding operating condition is solved by using the entire system model. This solution will subsequently be used as the “perfect” reference in evaluating the accuracy of the internal system state estimation when using different external network equivalents.

**Table III.1.** Internal and external system for IEEE 118 bus system

<table>
<thead>
<tr>
<th>Internal system</th>
<th>Internal Boundary</th>
<th>External System</th>
<th>External Boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2,3,4,5,6,7,8,9,10,11,12, 13,14,15,16,17,18,19,20, 21,22,23,24,25,26,27,28, 29,30,31,32,33,34,35,36, 37,38,39,40,41,42,43,44, 45,113,114,115,117</td>
<td>24,38,42, 45</td>
<td>47,48,50,51,52,53,54,55,56,57, 58,59,60,61,62,63,64,66,67,68, 69,71,73,74,75,76,77,78,79,80, 81,82,83,84,85,86,87,88,89,90, 91,92,93,94,95,96,97,98,99,100, 101,102,103,104,105,106,107, 108,109,110,111,112,116,118.</td>
<td>46,49,65,70,72</td>
</tr>
</tbody>
</table>
When using the equivalents with PMU buses in external system, synchronized phasor measurements are taken from the perfect reference solution, representing the correct real-time measurements which will be available via the ICCP, and are time synchronized through GPS system. However, network equivalent branch parameters will have errors due to the fact that they correspond to the base case loading conditions.

Table III.2 shows the results of state estimation for the internal system and the equivalent for different cases where different number of PMU buses retained in the external system equivalent. The following error criterion will be used to compare the improvement in the accuracy of the estimated state for different cases considered:

\[
J = \frac{1}{N} \sum_{i}^{N} \left[ \left( \frac{\hat{V}_i - V^{BC}_i}{V^{BC}_i} \right)^2 + \left( \frac{\hat{\delta}_i - \delta^{BC}_i}{\delta^{BC}_i} \right)^2 \right]
\]  

(III.10)

Where N is the number of buses in the system

\( \hat{V}_i \) is the estimated voltage magnitude of bus i

\( V_i^{BC} \) is error free voltage magnitude of bus i.

\( \hat{\delta}_i \) is estimated phase angle at bus i.

And \( \delta_i^{BC} \) is error free phase angle at bus i.

It is evident from Table III.2 that the error criterion J is improved with increasing number of PMUs retained in the external system.
Table III.2. List of PMUs and their positive impact on accuracy of criterion J

<table>
<thead>
<tr>
<th>Number of PMUs installed in the external system</th>
<th>PMU list</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>N/A</td>
<td>0.0071</td>
</tr>
<tr>
<td>4</td>
<td>83, 94, 103, 111</td>
<td>0.0053</td>
</tr>
<tr>
<td>5</td>
<td>79, 88, 92, 104, 109</td>
<td>0.0043</td>
</tr>
<tr>
<td>20</td>
<td>48, 69, 73, 76, 78, 96, 93, 80, 116, 67, 60, 53, 51, 57, 86, 90, 102, 104, 108</td>
<td>0.0022</td>
</tr>
</tbody>
</table>

III.2.4.ii. Impact of external synchronized measurements on internal contingency analysis

While the effect of PMUs on state estimation appears significant, this can be minimized or even completely eliminated by properly choosing the measurement set corresponding to the external system. This set can be chosen as a strictly “critical” set, making their effect on the internal system null, or in other words making them dormant measurements [73]. However, this will not be of much use when the real-time network model is needed for subsequent contingency analysis as both external measurement and topology have their direct impact on internal contingency analysis.

Every control center identifies a list of important contingencies which are periodically analyzed in order to maintain system security. Obtaining an accurate contingency analysis
result greatly depends on the accuracy of the external network model. Hence, in addition to
the internal system state, the state estimator should provide a good approximation of the
external network model.

This can be accomplished by taking PMU measurements into account and estimating
the external system bus injections which will in turn be used in subsequent contingency
analysis. The location and number of PMU measurements used for this purpose will have
an impact on the accuracy of contingency analysis results. In this part of dissertation, only
branch outage type contingencies are considered for the internal system.

As an example, outage of line 26-30 is considered as the contingency in the internal
system part of IEEE 118 bus system. Different number of PMU installations is assumed
and for each case the results of contingency analysis is compared with the perfect reference
solution. Table III.3 shows the comparative results which use the following error criterion:

\[ J_c = \sum_i^N \left[ (\hat{V}_i + V_i^{BC})^2 + (\delta_i + \delta_i^{BC})^2 \right] \]  \hspace{1cm} (III.11)

Where \( \hat{V}_i \) is the estimated voltage magnitude of bus i, in case of contingency.

\( V_i^{BC} \) is error free voltage magnitude of bus i.

\( \delta_i \) is estimated phase angle at bus i, in case of a contingency.

And \( \delta_i^{BC} \) is error free phase angle at bus i.
Table III.3. List of external PMUs and their impact on internal contingency analysis

<table>
<thead>
<tr>
<th>Number of PMUs retained in the external system</th>
<th>List of external PMUs</th>
<th>$J_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>N/A</td>
<td>1.4906</td>
</tr>
<tr>
<td>3</td>
<td>89, 69, 80</td>
<td>1.10118</td>
</tr>
<tr>
<td>5</td>
<td>61, 86, 93, 102, 90</td>
<td>0.5436</td>
</tr>
<tr>
<td>12</td>
<td>69, 80, 89, 66, 100, 61, 107, 102, 86, 93, 84, 108</td>
<td>0.1694</td>
</tr>
</tbody>
</table>

It is evident from the results of Table III.3 that having higher number of PMUs in the external system improves the accuracy of the external system model which in turn improves the accuracy of contingency analysis. In the extreme case where every external bus contains a PMU, a perfect match between the results will be expected since the network model will not be approximated in any way. However, it may take a while before PMUs populate the systems with such redundancy and in the meantime it may be important to decide on the best locations for the few PMUs to make the largest impact on accuracy of internal system solution.

**III.3. Inter area real-time data exchange for improvement of static security analysis**

Power system operation relies on accurate and continuous monitoring of the operating conditions which include the network topology and state. In a multi-area interconnected
system, each area will have access to its own real-time data and topology information whereas it will have limited access to these quantities that are external to its area boundaries.

Section III.2 of this chapter studies the impact of having real-time measurements available through external PMUs. This part (section III.3) on the other hand, investigates the process of identification of optimal external conventional measurements to improve the internal state estimation and subsequent contingency analysis.

It is customary for the areas to use a static network equivalent to represent their external systems. Since this model is static, it will remain valid as long as the operating conditions in the external system also remain the same. Changes in the external system operating conditions and topology will affect the internal system state estimation and the subsequent security analysis results. This is observed and documented in previous section as well as [78], where the impact of having access to a limited set of PMUs in the external system on the accuracy of external network modeling is investigated. However, no recommendation is made about the optimal placement of the PMUs in the external system. This part of dissertation addresses the issue of selecting the optimal set of conventional external measurements so that the errors in internal system applications will remain below an acceptable threshold.

It is assumed that every area will have enough measurements to make the internal area fully observable. Then, the objective is to identify the set of real-time measurements that are needed from the external areas in order to ensure a desired accuracy level for internal system state estimation and security analysis.
The problem of external network modeling is well investigated by various researchers in the past several decades. The methods developed and presented in [65], [70], [73], and [79] address the issue of developing and maintaining external network equivalents based on real-time data available in the internal system. Measurements from the external system are assumed not to be readily available, at least not in large numbers. Sensitivity based selection of buffer areas to improve the performance of the equivalents is investigated in [80,81].

This part of dissertation formulates the external system measurement selection as a mixed integer programming problem whose objective is to minimize the number of such measurements while maintaining a pre-defined accuracy level for the internal system solution.

This section is organized as follows: section III.3.1 illustrates how the internal system state estimates can be related to the external system measurements using a first order approximation. Next, an optimization problem is formulated in section III.3.2. in order to identify the best set of external system measurements to exchange in real-time. Then the validation results of applying the method to the IEEE 118 bus system are presented in section III.3.3. In order to model the internal and external system, the IEEE 118 bus system is split into two sub-systems modeling the internal and external systems. And finally, the model which is obtained using the selected measurements is used to analyze contingencies.

The process of identification of external measurement set depends on the operating point of the system. Therefore, the method should be applied to a given system for different operating conditions of the network, based on the seasonal changes, and load and
generation schedules. The result yields different sets of external measurements; operator may need to get the accurate estimation for the internal states, as well as contingency analysis.

### III.3.1. Dependence of internal state estimation on external measurements

It is well documented that the solution of the state estimation problem depends as much on the internal system model and measurements as on the representation of the external system and its measurements. Consider an interconnected system as shown in Figure III.3, where the part designated as the internal system represents the area of interest and its neighboring systems are shown as external systems 1 through 3. Those buses that belong to external systems but have direct connections to the internal system will be referred as “external boundary” buses.

**Figure III.3.** Diagram of an interconnected system
Partitioning the measurements as well as the states into real, reactive and voltage phase and magnitude respectively, the first order approximation of the measurement equations will take the following form:

\[
\begin{bmatrix}
\Delta Z_P \\
\Delta Z_Q
\end{bmatrix} = [H] \cdot \begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix} + [e] \tag{III.12}
\]

Where \(\Delta Z_P, \Delta Z_Q\) are the incremental real and reactive measurements, \(H\) is the measurement Jacobian, \(\Delta \theta\) and \(\Delta V\) are the incremental changes in the voltage phase and magnitude, and \([e]\) is the measurement error.

Applying the weighted least squares method, the incremental state estimate will be given by:

\[
\begin{bmatrix}
\Delta \hat{\theta} \\
\Delta \hat{V}
\end{bmatrix} = (H' R^{-1} H)^{-1} \cdot H' \cdot R^{-1} \cdot \begin{bmatrix}
\Delta Z_P \\
\Delta Z_Q
\end{bmatrix} \tag{III.13}
\]

Rewriting (III.13) in compact form yields:

\[
\begin{bmatrix}
\Delta \hat{\theta} \\
\Delta \hat{V}
\end{bmatrix} = \mathcal{L} \cdot \begin{bmatrix}
\Delta Z_P \\
\Delta Z_Q
\end{bmatrix} \tag{III.14}
\]

Where \(\mathcal{L}\) quantifies the dependence of estimated state on the measurements.

The objective of this part of the study is to identify those measurements from external system which have the most significant impact on the internal state estimation and/or subsequent contingency analysis. The main idea is based on the supposition that since the
selected measurements will have the most significant effect on the internal system, updating those measurements will lead to a more accurate contingency analysis.

Assuming that the load flow solution for the entire interconnected system is available, $\Delta Z_P$ and $\Delta Z_Q$ can be calculated using a first order approximation around the base case operating conditions. The rows and columns of $\mathcal{L}$ are reordered so that internal buses / measurements are listed first followed by the external buses / measurements. After the reordering, the structure of the matrix $\mathcal{L}$ will take the form shown in Figure III.4 below.

![Figure III.4. Reordered $\mathcal{L}$ matrix](image)

Note that the same reordering applies to the rows of $\Delta Z_P$ and $\Delta Z_Q$ in (III.14). Denoting the state variables by $X$ and using subscripts ‘Int’ and ‘Ext’ to refer to the internal and external buses / measurements respectively, estimated states can be expressed in terms of the measurements as follows:
\[
\begin{bmatrix}
\Delta X_{\text{Int}} \\
\Delta X_{\text{Ext}}
\end{bmatrix}
= \begin{bmatrix}
\mathcal{L}_{11} & \mathcal{L}_{12} \\
\mathcal{L}_{21} & \mathcal{L}_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta Z_{\text{Int}} \\
\Delta Z_{\text{Ext}}
\end{bmatrix}
\]  

(III.15)

This equation shows the relation of the internal state with internal and external measurements, and can be used by IP packages to identify those external measurements with the most significant impact on internal state.

**III.3.2. Optimal selection of external measurements to improve internal state estimation**

Naturally, the best solution is reached when all measurements from the external system are monitored and telecommunicated in real-time. This corresponds to the case of having full access to the right hand side vector of measurements, both from the internal and external systems, in (III.15). Since this is usually not possible, the next best solution is to choose those columns of \(\mathcal{L}_{12}\) whose effects on the internal state estimation are less than an acceptable threshold, say \(\epsilon\). Hence, the following optimization problem can be set up in order to determine such set of columns:

\[
\begin{cases}
\min K^T \cdot U \\
\left| (\mathcal{L}_{12} \cdot Z_{\text{Ext}}) - \mathcal{L}_{12} \cdot (Z_{\text{Ext}} \otimes U) \right| < L
\end{cases}
\]  

(III.16)

Where: \(\otimes\) operation indicating element-by-element multiplication of two arrays. \(L\) is a vector whose entries are equal to \(\epsilon\), an acceptable error threshold, which is set by the user.
Using smaller values of $\varepsilon$ will lead to the selection of a higher number of external measurements to be updated in real-time.

$U$: is a binary decision vector, whose entry is one if a column corresponding to an external measurement is selected, and zero otherwise.

$K$: is a vector representing the cost of monitoring an external measurement in real-time.

Note that the solution of (III.16) will be dependent upon the considered operating point or system loading since it will be a function of the measured values for the external system measurements. Hence, it is possible to identify different sets of external measurements for different loading and topological conditions.

Given an acceptable error tolerance $\varepsilon$, the optimization problem of (III.16) can be solved using an integer programming package. The solution will give a set of external measurements whose real-time updates will guarantee that the internal system state estimation solution error will remain within a certain boundary such as $\varepsilon$. In the extreme case of $\varepsilon=0$, all external measurements will be chosen as the optimal solution.

**III.3.3. Validation of method**

Once a solution is obtained, simulations are used to validate the chosen set of external measurements, both for internal state estimation as well as for subsequent contingency studies. The following procedure is carried out for this purpose note that it is assumed in
In this study, it was observed that the operating condition change between two consecutive scans by 20%, the procedure can be summarized as follows:

Obtain power flow solutions for the entire system both for base case as well as 20% above base case loading conditions.

1. For a given accuracy tolerance $L$, solve the integer programming problem of (III.16). Obtain the solution $U$, which identifies those external measurements to be updated in real-time.

2. Consider the power flow case corresponding to the loading conditions 20% above base case. Update all internal measurements and those from external which have been identified in step 1, keep the rest of external measurements at their base case power flow solution values.

3. Estimate the system state and all injections at external system buses.

4. Run contingency analysis for the internal system using the network model estimated in step 3. Compare the results with the exact solution which can be obtained using the entire system model with exact model.
III.3.3.i. Impact of real-time access to conventional external measurements on internal state estimation

IEEE 118 bus system will be used as the test bed in simulating the cases of validation. First, the system is divided into two non-overlapping areas representing the internal and external systems as described in previous section (table III.1 and figure III.2). It is assumed that the real-time measurements incident to the external boundary buses are available.

Simulations are carried out using four different values for $\varepsilon$. Chi square value is used to gauge the validity of the approximation. The Chi-square criterion is defined as:

$$J = \sum_{i=1}^{n} \frac{(Z'_i(i)-Z''_i(i))^2}{R_{ii}}$$  \hspace{1cm} (III.17)

Where $n$ is the number of internal measurements.

$Z'_i$, is the internal system measurement vector.

$Z''_i$, is the internal system estimated measurement.

$R_{ii}$ is the $i^{th}$ diagonal entry of error covariance matrix.

Table III.4, shows the results of the chi square test for different choices of $L$, accuracy tolerance vector.
Table III.4. Chi-Square test results for different external measurements update

<table>
<thead>
<tr>
<th>Error Bounds</th>
<th>Number of selected measurements</th>
<th>J(Chi-square results)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L = 0.01</td>
<td>24 external measurements out of 166</td>
<td>18637.14</td>
</tr>
<tr>
<td>L = 0.0001</td>
<td>61 external measurements out of 166</td>
<td>3462.20</td>
</tr>
<tr>
<td>L = 0.00001</td>
<td>69 external measurements out of 166</td>
<td>3.453</td>
</tr>
<tr>
<td>L = 1e-35</td>
<td>166 external measurements out of 166</td>
<td>0.521</td>
</tr>
</tbody>
</table>

Note the sudden drop in Chi Square value when the error threshold is changed from $10^{-4}$ to $10^{-5}$. The chi square value corresponding to the latter is no longer statistically significant, implying that the selection of 69 measurements will be sufficient to maintain an accurate internal solution.

Since the true state is known via the power flow result, the degree of approximation in the estimated state with respect to this true value can also be calculated as an alternative criterion given below:

$$\rho = \frac{1}{n} \sum_{i=1}^{n} \left( \left( \frac{v_i^{ref} - \hat{v}_i}{v_i^{ref}} \right)^2 + \left( \frac{\theta_i^{ref} - \hat{\theta}_i}{\theta_i^{ref}} \right)^2 \right)$$  \hspace{1cm} (III.18)

Where: $n$ is the number of internal buses.

$v_i^{ref}$: is the true solution for voltage of bus $i$.

$\theta_i^{ref}$: is the true angle of bus $i$.

$\hat{v}_i$ is the estimated voltage for bus $i$. 

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\( \hat{\theta}_i \) is the estimated angle for bus i.

Table III.5 shows the results of using the error metric defined in (III.18).

<table>
<thead>
<tr>
<th>Error Bounds</th>
<th>Number of external measurements selected to be update in real-time</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>L=10</td>
<td>0 external measurement are updated</td>
<td>0.1291</td>
</tr>
<tr>
<td>L=0.01</td>
<td>24 external measurement are updated</td>
<td>0.01358</td>
</tr>
<tr>
<td>L=0.0001</td>
<td>61 external measurement are updated</td>
<td>3.89*10^{-4}</td>
</tr>
<tr>
<td>L=0.00001</td>
<td>69 external measurement are updated</td>
<td>2.04*10^{-6}</td>
</tr>
</tbody>
</table>

**III.3.3.ii. Impact of real-time access to conventional external measurements on internal contingency analysis**

Performance of the estimated external system model based on the limited number of real-time updated measurements can also be tested by considering the contingency analysis. In this case, the external system model will be built using the state estimation results that are obtained based on the full internal and selected external measurement set. This model will then be used to solve the power flow problem where a contingency will be applied to the internal system. In this study, like the one in previous section (section III.2.4.ii), only line outage type contingencies are considered.
Consider the contingency where one of the main transmission lines of the internal system (line 26-30) is taken out of service. The true solution corresponding to this contingency can be obtained by solving the entire system with proper topology change. This solution can then be compared with the one obtained using the model based on limited external system measurements. An error metric similar to the one in (III.18) can be defined as follows:

\[
\beta = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{V_i^{\text{ref}} - V_i^{\text{up}}}{V_i^{\text{ref}}} \right)^2 + \left( \frac{\theta_i^{\text{ref}} - \theta_i^{\text{up}}}{\theta_i^{\text{ref}}} \right)^2
\]

(III.19)

Where:

- \( n \) is the number of internal buses.
- \( V_i^{\text{ref}} \): is the true solution for voltage of bus i. \( \theta_i^{\text{ref}} \) is the true angle of bus i.
- \( V_i^{\text{up}} \) is the power flow solution for voltage of bus i, using the updated measurements.
- \( \theta_i^{\text{up}} \) is the power flow solution for angle of bus i, using the updated measurements.

Table 6 shows the computed metric of (19) for different error tolerance limits corresponding to the contingency of line 26-30 being taken out.
Table III.6. Contingency case error metric of $\beta$

<table>
<thead>
<tr>
<th>Error Bound</th>
<th>Number of external measurements to be updated in real-time</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>L= 10</td>
<td>0 external measurement are updated</td>
<td>0.2698</td>
</tr>
<tr>
<td>L=0.01</td>
<td>24 external measurement are updated</td>
<td>0.00694</td>
</tr>
<tr>
<td>L=0.0001</td>
<td>61 external measurement are updated</td>
<td>0.000689</td>
</tr>
<tr>
<td>L= 0.00001</td>
<td>66 external measurement are updated</td>
<td>0.000579</td>
</tr>
</tbody>
</table>

The proposed method is applied to the same system when system is operating under different loading conditions. It is observed that the optimal set of external measurements that are to be updated is different for different loading conditions. This is expected since the sensitivity of estimation errors in the internal system will be dependent on the external system conditions. Therefore, in practice it is advisable to execute the optimization program on a periodic basis and let the operator request different subset of measurements from the external system to be exchanged in real-time for different loading conditions.

Application of the proposed method will yield different set of external measurements depending on the operating conditions. Hence, the operator can decide to exchange different sets of measurements from neighboring areas based on the seasonal load forecasting and anticipated changes in load and generation patterns.
III.4. Summary

In an inter-connected system, each system has real-time access to its own measurements and topology. On the other hand, knowledge of internal operator about external breaker status and measurements is very limited. In any interconnected system, the external measurements may have impact on internal state estimation. However, the internal system can design external measurements in their studies in such a way that all external measurements are critical measurements and as a result their impact on internal state estimation would be null.

On the other hand, internal security analysis is related to both external measurements and topology. This chapter investigates the impact of having real-time access to some of external measurements. Since many companies in North America have already several PMUs installed in their system, and due to availability of external PMU measurement to internal system in real-time, the first part of this chapter studies the impact of considering real-time measurements from external PMUs and their positive impact on internal state estimation and subsequent security analysis. Currently, number of available PMUs in power systems is low; therefore this study is followed by more general study where optimal set of conventional measurements in external system is identified to confine the state estimation error below a certain threshold.

Results on IEEE 118 bus system is used as a test bed, and the results are shown for verification of proposed methods.
CHAPTER IV

TRACKING OF EXTERNAL SYSTEM TOPOLOGY

WITH PHASOR MEASUREMENT UNITS
IV.1. Introduction

As indicated in the previous chapter as well, accurate state estimation and subsequent security analysis in any inter-connected system, not only relies on the precise measurement and topology of the system under study, it also depends on the accuracy of measurements and topology of the network used to model neighboring systems.

In most of the cases internal systems have access to the historical data of their neighbors which include both measurements and topology of the system. Available data from external system is valid as long as system operating condition and topology remains the same; available data from a network usually remains valid for a short period of time due to dynamic nature of power systems. Therefore, the problem of having real-time access to external measurements and topology remains a practical challenge.

Chapter III investigated the impact of having access to real-time external measurements through PMUs. it showed how having real-time access to a few external measurements could improve the internal state estimation and security analysis. Chapter III also studied the problem of optimal selection of traditional external measurements to optimally improve the internal contingency analysis.

Although the external measurements and operating condition have direct impact on internal security analysis as shown in chapter III, their impact on internal contingency analysis is not as significant. On the other hand, any unreported alteration in external topology may cause major changes in internal state estimation and subsequent contingency analysis. This will be the case for instance when there is a major line outage in the external system.
In an inter-connected power system, state estimator has real-time access to the measurements, and topology of the area under study, while its real-time knowledge about external measurements and topology is usually very limited. One way to have real-time access to external measurements is through phasor measurement units (PMUs). While PMU deployment has increased, during past decade, their availability still remains sparse for most power systems.

Most of the power system utility companies have done investments on installation of PMUs on their systems. While looking for long term objectives on their system, loaded with phasor measurements, most of utility companies in North America have a few PMUs already installed in their system; therefore, real-time external data for their neighbors are available through existing PMUs. These measurements can be used as a real time data, and can be a great help for identification of external topology changes and subsequent improvement of the internal security analysis.

Redundancy of measurements and accuracy of the topology are keys to error free estimate of operating state. It is noted that in any inter-connected system, the internal system plus boundary buses constitute an observable island based on the internal system measurements. Hence, those measurements used for the external system, are either pseudo-measurements provided by the load forecasting and/or generator scheduler functions or they may seldom contain actual real-time measurements received through inter-utility real-time measurement exchange. In either case, due to lack of redundancy, their impact on the internal state estimation will be minimal or null. Since external topology follows the same rules, its impact on internal state estimation is either null or minimal. On the other hand,
they may have a significant impact on contingency analysis associated with internal system
topology change, due to their effect on external network instruction; for example lack of
information about external system topology is identified as one of the major reasons in
previous black outs in North America [6]. Since the load and generation usually change
slowly, assuming that the period of collecting data from internal buses and those external
buses with PMUs is small enough that operating condition of the system does not change
drastically, shift in load or generation usually do not have a critical impact on internal
contingency analysis and therefore can be ignored. On the other hand, external topology
error may have crucial impact on internal security analysis.

The U.S electric transmission network is an interconnected network where an outage in
one state can cause cascading blackouts in other states or neighboring countries [81]. Most
of the utility companies in United States have a few PMUs available in their systems
whose phasor measurements are available to the internal power system dispatcher in real-
time. Therefore it is worth investigating the incremental benefits to be gained by taking
advantage of their limited presence in the system. One such advantage would be external
topology error detection with using real-time internal data along with a few real-time data
available through external PMUs which can in turn, help improving the internal security
analysis.

As mentioned, change in topology or operating condition will change the state of the
system. Although, the objective of this part of dissertation is application of a few phasor
measurement units in external system to identify the external line outage, it is important to
propose a method which can differentiate between the load change and topology error. The
The proposed method is capable of identifying whether the change in internal state is because of the external load change or it is due to an external line outage.

Due to significant impact of external load/generation and topology error on security analysis, there have been several studies done in the past on the external measurement error and topology error. While most of the studies have concentrated on the analog measurement error detection [21], [26], [27], [82-90] (i.e. [82-89], proposed methods that used weighted residual test to detect, identify, and suppress the bad data in power systems.) or error detection of the local system [91-103], there has been relatively few on external system line outage identification [104-105].

Topology errors usually fall into the following categories:

- The line is not included in the model when it is actually in service in the system.
- Line is included in the model when the line is out of service
- Line is open/not open ended in the model but not in the actual system

Authors in [91] propose a method for detecting internal topology error based on the assumption that terminals of the line have injections and a good state vector is produced through state estimation. The proposed method identifies the line outage if a gross error is detected at the terminals of a line. Otherwise the method is not capable of identifying the line outage. [92] uses the residual sensitivity matrix to identify the internal line outage. [93] proposes a new method for identification of line outage in internal system based the study shows that the line outage is not detectable in case that covariance of residuals are equal to zero, they show that covariance would be non-zero in presence of topological
error. [94], introduces a method for computing indices which qualify the degree of correlation between estimated quantities exhibiting the symptoms of an anomaly and data sensitive to the configuration errors most likely to occur under the operating condition. [95] uses a rule-base algorithm method to identify the topological error in the system under study. In this paper also, the implementation environment of a central European utility using the proposed method is presented. Clements and Costa in [96], use a practical algorithm for topology error identification and correction that is based on the use of normalized Lagrange multipliers for the identification of topology error. The method of normalized Lagrange multiplier is an extension of the normalized residuals method. Authors in [97] propose a new method to identify internal topology error on the basis of statistical test applied to the power flow estimates for both active and reactive power flows. These estimates are calculated by means of iteratively re-weighted least-square algorithm which exhibits good convergence rates regardless of ratio of X/R for a given line. [98] proposes another method based on the data projection technique for visualization purposes in order to show that normalized innovations provide excellent error discrimination capability when compared to other variables commonly used in bad data identification schemes. The normalized innovations are then fed as input to a constructive artificial neural network (ANN) based on the group method of data handling. [99] proposes a method which takes advantage of the idea of temporal consistency. Authors claim that the method does not restrict itself to a single quasi steady state snapshot. The main idea in [100] is to augment the state vector with the power flow through the circuit breakers and identify the status of breakers based on the estimated flow through them. The proposed method uses a two stage state estimation base method where in the first stage state
estimation is followed by another state estimation in which detailed substation model is used for those substations which are found suspicious in the first stage. Singh and Alvarado proposed a method in [101] for detecting the topology error in electric power networks by introducing status variables in non-linear $l_1$ norm problem which in turn, leads to introducing additional variables and constrains in conventional LAV state estimation. [102] builds a topology error identification method based on normalized Lagrange multiplier and hypothesis test using Bayes theorem. [103] proposes a method to develop an orthogonal state estimation capable of detecting and identifying topological error in the system under study. The proposed method extends rotational based state estimator in order to handle the network representation in substation level. The assumption about the status of branches can be tested by using hypothesis testing procedure which relies on probability values for each possible combination of breaker statuses. These probabilities are determined by using a particular form of Bayes theorem and do not require additional runs of the state estimator. The largest probability value in this method indicates the breaker status configuration which is best supported by the real-time measurements.

There have also been few studies reported on the problem of external topology error detection and identification [104, 105]. In [104] Tate and Overbye proposed a method to compare the observed changes in phase angles at PMU buses and used a recursive method to check which one of the line outages could minimize the difference between the observed phase angle change and the calculated one. This method compares the observed change in phase angles with the resulting phase angle shift due to each and every external line outage which may require significant computations for large systems. Earlier work by Alvarado
in [105] was based on partial factorization of the Jacobian matrix. Different set of boundary flows were calculated and compared with the measured boundary flows. Set of boundary flows with the minimum difference with the measured boundary flows were then picked as the current topology of the external system. This method also requires large storage as well as heavy computations when it is applied to a system with numerous external branches.

Available processors in power system are only capable of obtaining the topology of the local system, utilizing the local measurements [106], [107]. It is well documented that knowledge of internal system about the connectivity information of the external system in an inter-connected network is very important for the system operation [6].

Abovementioned literature review can be classified in following categories:

[91], and [93] proposed method based on the residual test. [92], and [106] use co-linearity test. [94], and [96] use normalized Lagrange multiplier. [95], [98], and [99] use rule-base or artificial neural network method. [97], [100], [101], and [107] proposed method based on least absolute value state estimation. [102], and [103] use geometrical based hypothesis testing for topological error identification. [104], and [105] used comparison the current situation with pre-recorded data for topology error identification.

This chapter of dissertation proposes a new method to identify the external line outage assuming that both topology and measurements of internal system are always known to the power system operator. For simplicity, and illustration, the linear decoupled model of the power system can be used to develop power system model, and extract the formulation.
The presented method in this chapter is one possible method which uses the local internal phase angles and those phase angles available from external PMUs for both base-case (pre-outage), and post-outage case, as well as the base-case topology of the entire system (internal and external system) to identify the external line outage.

The IEEE 30 bus system and 118 bus system is used as a test bed to verify the validity of the proposed method.

**IV.2. Statement of the problem**

The problem addressed in this part is identification of the external topology change, using the real-time internal state and internal measurements as well as the real-time phasor measurements which are available through external PMUs. The proposed method is developed using the decoupled linear power flow model. The formulation is then extended to the full AC model.

Accurate power system operation requires close monitoring of system topology and measurements. In a given inter-connected power system such as the one shown in figure IV.1, the internal system is defined as the part of the system which is under study, and the part external to the boundaries of internal system, is defined as the external system. Real-time information about internal measurements and internal topology is assumed to be available to internal power system dispatcher whereas, very limited information or no information about external measurements and external topology is available to the internal power system operator.
Inter-utility real-time data exchange became a very important application in interconnected systems during past a few decades. Real-time and historical information of any system in wide area networks will be sent to inter-control center, to be used by external systems. (Inter-control Center Communication Protocol) ICCP allows the real-time and historical exchange of information such as circuit breakers status, measurement data, scheduling data, energy accounting data, and operator messages [108]. Although, each system has a limited access to external system information through ICC, usually this information is not enough to accurately model the external system prior to executing the security analysis for a given system. Internal power system dispatcher can have real-time access to the external measurements collected through external PMUs. These time-stamped phasor measurements can be easily used by the power system operator due to their alignment and synchronization through GPS.

Assuming that each system, along with its measurements constitutes an observable island, the state of the system can be estimated independent of external measurements or
topology. On the other hand, to run the accurate internal security analysis in an inter-connected system, each system uses an equivalent to model attached external systems, therefore, inaccuracy in external topology or load/generation may have direct impact on the outcome of internal security analysis. The equivalent used to model the external system is built based on the topology and operating condition of external system in a snapshot. Used equivalent and the subsequent studies related to the equivalent model is accurate as long as the external topology and operating condition remains the same. Soon after a change occurs in external topology or load/generation, the new equivalent should be replaced to maintain the accuracy level of studies in the internal system. Operating condition as well as topology of external system changes frequently due to the dynamic nature of power system. To get the accurate security analysis in the internal system, the external equivalent should be updated every couple of minutes. Currently, such luxury of having access to frequent updated equivalent model for the external system is not possible due to lack of real-time communication and excessive amount of calculation. In such case, it is reasonable to track those changes from external system which has direct and critical impact on the internal security analysis.

Although having real-time knowledge of the external operating condition is important to have an accurate internal security analysis, their impact on internal contingency analysis is minimal knowing the fact that the load/generation of a system usually does not change drastically in a short period of time. On the other hand, when it comes to the topology error, external topology may have very significant impact on security analysis of the system and can occur unexpectedly, and in matter of seconds; especially if the line outage is an external major line. Therefore it is very critical for internal operator to be aware of
external topology changes. Since it is currently impossible to have real-time access to all circuit breakers status through ICCP, an approach to identify external line outages based on internal topology and measurements as well as a few real-time external phasor measurements available through ICCP can be a great help to utility companies.

IV.2.1. Decoupled power flow model

Consider the real and reactive power balance equations at bus $i$:

$$P_i = \sum_{k=1}^{n} |V_i||V_k|[G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)]$$  \hspace{1cm} (IV.1)

$$Q_i = \sum_{k=1}^{n} |V_i||V_k|[G_{ik}\sin(\theta_i - \theta_k) - B_{ik}\cos(\theta_i - \theta_k)] \hspace{1cm} i = 1,2,3,\ldots,n$$

Where $\theta_i$ is the phase angle of voltage at bus $i$.

$G_{ik}$ is the conductance of the line connecting bus $i$ to bus $k$.

$B_{ik}$ is the susceptance of the line connecting bus $i$ to bus $k$.

$|V_i|$ is the voltage magnitude at bus $i$.

Assuming that the voltage magnitude and phase angle at the reference bus (let’s assume bus one to be the reference bus), the state of the system can be defined as follows:
\[ \theta = \begin{bmatrix} \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}, \quad |V| = \begin{bmatrix} V_2 \\ \vdots \\ V_n \end{bmatrix}, \quad x = \begin{bmatrix} \theta \end{bmatrix} \]  

(IV.2)

Therefore, the right-hand side of IV.1 is function of \( x \), therefore:

\[ P_i(x) = \sum_{k=1}^{n} |V_i||V_k| [G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)] \]  

(IV.3)

\[ Q_i(x) = \sum_{k=1}^{n} |V_i||V_k| [G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k)] \quad i = 1, 2, 3, ..., n \]

Replacing the right hand side of (IV.1) with its equivalent, from (IV.3), equation (IV.1) can be writes as follows:

\[ P_i = P_i(x) \quad i = 1, 2, 3, ..., n \]

\[ Q_i = Q_i(x) \quad i = 1, 2, 3, ..., n \]  

(IV.4)

And a set of equation can be set up in the form of \( f(x) = 0 \), therefore:

\[ P_i(x) - P_i = 0 \quad i = 1, 2, 3, ..., n \]

\[ Q_i(x) - Q_i = 0 \quad i = 1, 2, 3, ..., n \]  

(IV.5)

This equation identifies \( 2n-2 \) entries of \( f(x) \). Thus:
\[ f_1(x) = P_2(x) - P_2 \]
\[ f_2(x) = P_3(x) - P_3 \]
\[ \ldots \]
\[ f_{2n-2}(x) = Q_n(x) - Q_n \]

\[ f(x) = \begin{bmatrix}
  P_2(x) - P_2 \\
  \vdots \\
  P_n(x) - P_n \\
  Q_2(x) - Q_2 \\
  \vdots \\
  Q_n(x) - Q_n 
\end{bmatrix} \] (IV.6)

Let Jacobian of \( f \) be called as \( J \), \( J \) can be partitioned as follows:

\[ J = \begin{bmatrix}
  J_{11} & J_{12} \\
  J_{21} & J_{22} 
\end{bmatrix} \] (IV.7)

Where each of the above mentioned sub-matrices is \((n - 1) \times (n - 1)\) dimension matrix. \( J_{11} \) contains entries made of \( \partial P_i(x)/\partial \theta_k \). \( J_{12} \) entries can be calculated with \( \partial P_i(x)/\partial |V_k| \). \( J_{21} \) has terms of \( \partial Q_i(x)/\partial \theta_k \), and \( J_{22} \) entries can be calculated with \( \partial Q_i(x)/\partial |V_k| \). \( J \) and \( x \), and therefore, \( \Delta x \) has been written in partitioned form. It only
remains to write the $f(x)$ in the partitioned form, therefore the so called mismatch vector can be written as follows:

$$
\Delta P(x) = \begin{bmatrix} P_2 - P_2(x) \\ \vdots \\ P_n - P_n(x) \end{bmatrix}, \quad \Delta Q(x) = \begin{bmatrix} Q_2 - Q_2(x) \\ \vdots \\ Q_n - Q_n(x) \end{bmatrix}
$$

$$
f(x) = -\begin{bmatrix} \Delta P(x) \\ \Delta Q(x) \end{bmatrix}
$$

Using (IV.2), (IV.7), and (IV.8) we will get:

$$
\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta |V| \end{bmatrix} = \begin{bmatrix} \Delta P(x) \\ \Delta Q(x) \end{bmatrix}
$$

Please note that in general case in calculations for power systems under usual operating conditions, the off-diagonal sub-matrices $J_{12}$ and $J_{21}$ in Jacobian matrix $J$, are quite small. The reason can be explained with the following example:

$$
\frac{\partial P_i}{\partial |V_k|} = |V_i| |G_{ik} \cos(\theta_i - \theta_k) + B_{23} \sin(\theta_i - \theta_k)|
$$

$$
\frac{\partial Q_i}{\partial \theta_k} = -|V_i||V_K||G_{ik} \cos(\theta_i - \theta_k) + B_{23} \sin(\theta_i - \theta_k)|
$$

Knowing the fact that transmission lines are mostly conductive the conclusion can be made that $G_{ik}$ is quite small. Also it is known that under normal operating condition, the
$(\theta_i - \theta_k)$ is quite small \[110\] (usually less than $10^\circ$). Therefore, the off-diagonal sub-matrices $J_{12}$ and $J_{21}$ can be ignored in equation (IV.9). The new equation is known as decoupled power flow equation of power systems.

\[
\begin{bmatrix}
J_{11} & 0 \\
0 & J_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta \theta \\
\Delta |V|
\end{bmatrix}
= 
\begin{bmatrix}
\Delta P(x) \\
\Delta Q(x)
\end{bmatrix}
\] (IV.10)

This can be written in the following form:

\[
J_{11} \Delta \theta = \Delta P
\]

\[
J_{22} \Delta \theta = \Delta Q
\] (IV.11)

Where $J_{11}$ and $J_{22}$ are block diagonal entries of jacobian matrix. $\Delta P$ and $\Delta Q$ are change in active and reactive power injections of the system respectively. Using the flat voltage profile, top equation in (IV.11) can be written as follows:

\[
B \Delta \theta = \Delta P
\] (IV.12)

Where $\Delta \theta$ is change in phase angle solution of the system due to change in $\Delta P$. $B$ matrix can be determined using the line status and network parameters. For a given system, $B$ matrix can be formed as shown in figure IV.2:
Figure IV.2. B matrix for a given system

\[ B = \begin{bmatrix} \text{a} & \text{b} \\ \text{b} & \text{c} \end{bmatrix} \]

Where \( b \) is the negative of susceptance of the line connecting bus \( k \) to bus \( m \). \( a \), and \( c \) are sum of susceptances of all branches incident to bus \( k \) and \( m \) respectively.

In linear decoupled power flow method, a given line (\( l \)) can be replaced by two power injections at terminals of line (\( l \)); the amount of power injection required to be injected at terminals of the removed line in order to emulate the line outage, is equal to the pre-outage flow on the removed line (\( l \)).

For illustration consider line (\( l \)) connecting bus \( k \) to bus \( m \) as shown in figure IV.3.a with the active power flow (\( P_{km} \)). Line (\( l \)) can be represented as two power injections at bus \( k \), and \( m \). since the linear decoupled method is used, change of active power flow due to transfer from one terminal of line (\( l \)) to another terminal is zero; therefore, power injections at both ends of the removed line have the same value which is equal to the pre-outage load flow of line (\( P_{km} \)), with positive sign at ‘to’ bus (\( m \)) and negative sign at ‘from’ bus (\( k \)).
Therefore, removal of any line in the system can be modeled by inserting two power injections at terminals of the removed line while keeping the line in service. It is clear from equation (IV.12) that a shift in power injections will result in change of phase angles of the system. Since the internal phase angles are always available to the internal power system dispatcher, the objective of the proposed method is to track the change in internal phase angles and find the corresponding external power injections which may cause such shift in internal phase angles. Location of these power injections in turn, can identify the removed line in the external system.

Let line $l$, connecting bus $k$ to bus $m$ in a given system with $B$ matrix (as shown in figure IV.2), be considered as a potential removal in the system. Note that in figure IV.2, only entries corresponding to bus $k$, and $m$ are represented and the rest of the entries are not shown as no change happens to them after the line outage in external system.
After the line outage, the topology of the system will change; new topology of the system can be defined based on the post outage configuration of the system. Thus, a new $B$ matrix can be defined and formed for post outage system ($B_1$). In this study, only single line outages are being considered as topology change. Therefore, post outage topology of the system ($B_1$) will be related to pre-outage topology of the system ($B$) through the following equation:

$$B_1 = B + \Delta B$$  \hspace{1cm} (IV-13)

Where $\Delta B$ represents the change in topology of the system. Assuming the single line outage happens between two consecutive measurement scans, $\Delta B$ would be a matrix with the same size as $B$, with only four non-zero entries which can be found in locations corresponding to terminals of the removed line (bus $k$, and $m$) as shown in figure IV.4 [109].

![Diagram of matrix $\Delta B$](image)

**Figure IV.4.** Change in $B$ due to removal of the line $l$. 
Using the active decoupled power flow equation for a system for pre-outage and post outage cases, yields:

\[ B\theta_0 = P_0 \]

\[ B\theta_1 = P_0 - \Delta P \]

\[ (B + \Delta B)\theta_1 = P_0 \]  \hspace{1cm} (IV.14)

Where \( \theta_1 \) is the new system phase angle after the line outage. Note that, even though bus injections \( P_0 \) remain the same (see the last equation in (IV.14)), the same angle solution \( \theta_1 \) can be obtained by assuming fixed topology but an equivalent change in injection vector \( \Delta P \) (middle equation in (IV.14)). Removal of \( P_0 \) from (IV.14) yields:

\[ \Rightarrow \Delta B\theta_1 = \Delta P \]  \hspace{1cm} (IV.15)

As mentioned earlier, \( \Delta B \) has only four non-zero entries as shown in figure IV.4. Therefore, the left hand side of equation (IV.15) is vector with two non-zero entries at locations corresponding to terminals of the removed line. This can be shown as:

\[ \Delta P = [0 \ldots p \ 0 \ldots -p \ 0 \ldots 0]^T \]  \hspace{1cm} (IV.16)

Equation (IV.16) shows that the post outage solution of the system can be obtained with replacement of post outage topology with:
• Pre-outage topology of the system plus

• A pair of power injections at terminals of the removed line. Power injections absolute value is equal to the pre-outage flow on the removed line, with positive and negative sign at “to bus” and “from bus” respectively.

The presented method in this chapter is developed based on the assumption that internal power system dispatcher has access to real-time and historical internal line status, internal MW, and MVAR, as well as base-case external measurements and line status. Having access to internal measurements and topology, state estimation can be run by power system operator for the internal system in real-time. Therefore, the phase angles of all internal buses are assumed to be available to internal system operator.

Having access to the base case external topology and measurements, any internal security analysis can be accurately implemented as long as the external topology and external operating condition remains in the base case condition. Any un-reported change in external topology, may lead to crucial errors in internal contingency analysis.

Internal system operator uses an equivalent to model attached external systems and equivalent of a given system relies on topology and operating condition of that system. In case of un-reported line outage, the internal power system operator assumes that un-reported line outage is still in place, and therefore the base case equivalent will be used to model the external system. Due to the change in external topology, the equivalent which is used to represent the external model is no longer valid, and therefore outcome of the subsequent security analysis will be in error. While the impact of an un-reported external line outage can be minimal if the outage happens to a minor line, unreported external line
outage can have significant impact on the outcome of internal contingency analysis if the removed line is one of the major lines in external system. Major lines are those which transfer large amount of active power.

The topology of the system may change due to many expected reasons such as maintenance, switching, and seasonal load change, or unexpected reasons such as storm, hurricane, and lightning. Also, real-time access to the status of all external circuit breaker is currently not possible; therefore, a method which can identify external line outages (especially major line outages) can be a great help to power system utilities to have more reliable security analysis on their system.

**IV.2.2. Problem formulation**

**IV.2.2.i. DC Formulation**

As mentioned, internal system operator has access to real-time and historical information about internal system (including branch status and measurements), as well as base case information about external line status and external measurements. Therefore, assuming that enough measurements are available to make the internal system fully observable (which is the case in most of utility power systems), real time internal state can be estimated. Hence, base case and real-time phase angles of internal system are always assumed to be known in this study. Also, it is assumed that internal system has access to the base case topology and power injections of external system, while they have no knowledge about real-time external system phase angles except for those buses with PMUs. Treating the external buses with PMU, as internal boundary buses, buses of the
system can be partitioned into two parts, the part that the internal operator has real-time access to, and the part that operator has no knowledge about. Therefore, equation (IV.12) can be partitioned into two parts as follows:

\[
\begin{pmatrix}
\Delta P_1 \\
\Delta P_2
\end{pmatrix} =
\begin{pmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{pmatrix}
\begin{pmatrix}
\Delta \theta_1 \\
\Delta \theta_2
\end{pmatrix} +
\begin{pmatrix}
e_1 \\
e_2
\end{pmatrix}
\]  \hspace{1cm} (IV.17)

Where top equation is corresponding to the internal system plus external buses equipped with PMUs and is identified by index 1, and the bottom part represents the external system in linear decoupled equation and is identified by index 2. \(e\), representing the error.

As shown in section IV.2.1, any line outage in the system can be represented with two power injections at terminals of the removed line, when the pre-outage topology of the system is in use. Therefore, it is assumed in this study, that the topology of the system always remains the same, while any line outage of the system is reflected in the equations as change in power injections. Constant topology assumption forces \(B\) matrix to remain constant. With fixed \(B\) matrix, right hand-side of equation (IV.17), and subsequently \(\Delta \theta_1\) and \(\Delta \theta_2\) would be always zero unless a change happens in load/generation of the system.

As mentioned earlier, real-time access to the external circuit breaker status is not currently available. Equation (IV.14) through (IV.16) indicate that post outage topology of a system is equivalent with pre-outage topology plus two injections at terminals of the removed line. For simplicity first consider the case that only topology of the system change while system operating condition remains constant between two data samples (this assumption will be relaxed later on). Topology change of the system can be modeled with
two power injections; since the change in operating condition is assumed to be zero, the line outage will be reflected as a vector with only two non-zero entries (due to the topology change) in left hand side of equation (IV.17). Identifying location of two non-zero entries in the left hand side of (IV.17) yields two ends of the removed line (based on calculations shown in (IV.14)-(IV.16)). $\Delta \theta_2$ is a vector of external phase angles. Since the internal system operator has no access to real-time external phase angles, $\Delta \theta_2$ should be removed from (IV.17). Elimination $\Delta \theta_2$ from (IV.17), yields:

$$\begin{bmatrix} B_{12}B_{22}^{-1}B_{21} - B_{11} & \end{bmatrix}\Delta \theta_1 + \Delta P_1 = B_{12}B_{22}^{-1}\Delta P_2 - B_{12}B_{22}^{-1}e_2 + e_1$$  \hspace{1cm} (IV.18)

External topology change will cause shift in internal state solution, which includes change in the internal phase angles. Since the $B$ matrix is assumed to be constant, equation (IV.18) blames $\Delta P_2$ for any change in $\theta_1$ between two data collection.

Let’s assume that external line (l) connecting bus $k$ to bus $m$ with pre-outage load flow ($p$), is considered as the external line outage. As discussed in section IV.2.1, the post outage system is equivalent with the system with pre-outage topology if appropriate injections ($p$ and $-p$) at bus $m$ and $k$, are imposed to the system. This can be written as:

$$\Delta P_2 = \begin{bmatrix} 0 & 0 & \cdots & p & 0 & \cdots & -p & 0 & \cdots & 0 \end{bmatrix}^T$$  \hspace{1cm} (IV.19)

Since the topology is assumed to be constant, and due to having access to both real-time and historical information of internal system, the left hand-side of equation (IV.18), as well
as the first two terms of right hand side of (IV.18), are always known to internal power system operator. Therefore (IV.18) can be written as:

\[
(B_{12}B_{22}^{-1}B_{21} - B_{11})\Delta \theta_1 + \Delta P_1 = J
\]

\[
B_{12}B_{22}^{-1} = \gamma
\]

\[
e = e_1 - B_{12}B_{22}^{-1}e_2
\]

\[
\Rightarrow J = \gamma\Delta P_2 + e
\]  

(IV.20)

For a system with \(N\) internal buses and \(M\) external buses, \(J\) is an \(N\) by 1 vector, and \(\gamma\) is an \(N\) by \(M\) matrix. \(\Delta P_2\) is an unknown vector with only two no-zero entries, representing terminals of the removed line. Substituting (IV.19) in (IV.20), formulation can be written as:

\[
J = \gamma p \begin{bmatrix}
0 \\
. \\
. \\
+1 \\
0 \\
. \\
. \\
-1 \\
0
\end{bmatrix} + e = \gamma p T + e
\]  

(IV.21)

Where \(T\) is an \(M\) by 1 vector and non-zero entries of \(T\) identify terminals of the removed line, and \(p\) is pre-outage load flow on the external line outage.

Equation (IV.21) can be solved as a non-linear mixed integer programming where both location and value of injections are unknown. This equation should be followed by an integer programming package to be solved.
One way to solve (IV.21) is to turn this equation into a mixed integer binary programming problem; this can be done by defining new matrices in such a way that $\Delta P_2$ can be represented as multiplication of a floating point by a binary vector. Appropriate matrices can be defined as follows:

$$J = \Lambda p (0 \ldots 1 \ 0 \ldots 1 \ 0 \ldots 0)^T = \Lambda p X$$  \hspace{1cm} \text{(IV.22)}$$

Where $\Lambda = [\gamma \ -\gamma]$, and $X$ is a binary vector of length $2M \times 1$. Equation (IV.22) can be written in form of a mixed integer programming problem as follows:

$$\begin{align*}
\rho &= \min ( \sum e_{2i} ) \\
\text{s. t.} & \quad J = \Lambda p X + e \\
& \quad \sum_i X_i = 2
\end{align*}$$  \hspace{1cm} \text{(IV.23)}$$

Where $x_i$ is the $i$th entry of vector $X$. Solutions of equation (IV.23), yields location and pre-outage load flow of the removed line in external system. Applying (IV.23) to any single line outage in external system with constant operating condition, yields two non-zero entries, the one in the first half of $X$ represents the ‘to-bus’ end of the line as it refers to the power injection with the positive sign, and non-zero entry at the second half of $X$, represents the ‘from-bus’ end of the removed line.
While equation (IV.23) can identify the line outage when real ac power flow solution is used or when both topology and operating condition change in case of using dc power flow solution, it cannot handle the case where just a line outage happens, or when the dc power flow solution is being used.

The proposed method needs to be validated by means of using power flow solution. The validation of the method is initiated by testing the model for the synthetic DC power flow solution. Since the DC power flow solution is an error free set of data, the validity of the method can be easily tested. Using the linear decoupled model to represent the power system is an approximation. Regardless of level of accuracy of the estimate, like any other approximation, this model introduces error in comparison with actual power flow solution. On the other hand, data which is produced to test the model, and is referred to as DC power flow solution is calculated to model the error free case. Therefore, the error term in equation (IV.18) should be dropped, and this equation should be slightly changed before it can be used for the error free case; otherwise equation (IV.23) will be always satisfied irrelevant of the choice of buses identified as terminals of the removed line.

In case of line outage with constant operating condition in error free generated data (dc case), the aggregated absolute value for error is always zero; therefore the IP problem equations should be modified. Dropping the term corresponding to error in formulations for the dc case changes equation (IV.23) as following for the case of line outage with constant operating condition:
IV.2.2.ii. \textit{Extension to the AC case}

Section IV.2.2.i, investigates the identification of external line outage, using the linear decoupled model, and calculated state of the system as well as power flow solution. Since the calculation is used, the obtained result is accurate enough that the IP package can identify ends of the removed line, based on available data from internal system, without need of any additional information from external system. Generally there are two approximations which are hidden in the proposed method. These approximations are the source of error in case of using actual power flow solution for a system (which is referred to as the ac case) can be listed as follows:

- The power mismatch in the transmission line. While in the DC case, any line outage can be modeled with adding and subtracting the pre-outage load flow of the line to the terminals of the removed line, in the ac case, the injection values are slightly different due to power loss on transmission line.

- Additionally, using the linear decoupled method to model the power system induces another term of error which is the model approximation error. This should be taken into account when an actual power flow solution is in use. Therefore, equation (IV.23) should be adjusted in case of using actual power flow solutions.
Based on discussion above, equation (IV.23) is ignoring two facts that can be represented as source of errors: first one is the error term due to usage of linear decoupled method to represent the system. This error is applied to entire system since it corresponds to the modeling of the system; the other is the error term representing the power loss due to power transfer from one end of the removed line to another. This error is only applied to terminals of the removed line. Therefore, equation (IV.23) should be changed in before it can be used for the AC case. The modified equation can be written as follows:

\[
\begin{align*}
\rho &= \min(\sum e_{2i}) \\
\sum_i A_i &= 2 \\
\sum_i B_i &= 1 \\
\forall i, \ A_i &\geq B_i \\
\forall i, \ |e_{2i}| &< \frac{p}{10}
\end{align*}
\] (IV.25)

Where \( \Lambda \), and \( J \) are matrices define in part a of this section, \( p \), and \( q \) are positive floating points, \( A \) is a binary vector with two non-zero entries, representing two ends of the removed line, \( C \) is a binary vector with one non-zero entry. The non-zero entry in \( C \) is corresponding to the power difference between the two terminals of the removed line in the actual ac case; in other words it represents the terminal of the removed line whose active power is less that the other one due to transmission power loss. \( e \) is a vector corresponding to the error. This vector is produced due to the approximation of the power system with the linear decoupled model. \( e_{2i} \) is the \( i^{th} \) entry of vector \( e_2 \).
The first constraint in equation (IV.25) validates equality of sides in equation (IV.28). The second constraint confines the number of non-zero entries of binary vector (A) to two; indicating terminals of the removed line. Third constraint about binary vector (C) allows C to only have one non-zero entry which is corresponding to the terminal of the removed line with less power injection. Next constraint forces the location of non-zero entry in C to be consistent with the location of one of the non-zero entries in A. This is necessary due to the miss-match power between two terminals of the line in actual AC case. The last constraint makes sure that the power injections at terminals of the removed line are larger that error entries. This constraint is especially helpful in case of change in operating condition with the constant topology. Removing of this constraint from the equation set will lead to wrong solution in case of load/generation change with constant topology in external system. In such condition, having the last constraint dropped in equation IV.25 will cause the IP package to detect a minor external line outage plus an error vector. This happens while no topology change is occurred in actual system.

As mentioned earlier, in order for (IV.23) to be solved for the ac case, additional constraints should be introduced to this equation. Although these changes seem to be necessary to solve the problem in case of using actual power flow solution, they may not be enough to find the accurate solution due to available errors. In other words, the error due to the model approximation and power loss on removed transmission line may cause multiple solutions for some line outages especially if pre-outage power flow of the remove line is not large enough to dominate both of the errors. Therefore, additional real-time data from external system may be required for the solution of the problem. Phasor
Measurement Units (PMUs) is will provide such real-time information about external system.

PMUs are rapidly populating the power systems due to their ability of directly measure the state of the power system, as well as other advantages such as providing synchronized measurements, and high frequency and accuracy in their measurement data collection. Most of the power system utilities have a few PMUs already installed in their systems. Although, currently most of the utility systems do not use the PMU measurements available from the external system, utilizing such external measurements would be a great help in many power system studies. One of the benefits of using the real-time measurements available from external PMUs is to overcome the problem of getting multiple solutions for external line outages. For illustration, consider matrices $\gamma$ and/or $\Lambda$, where the number of rows represent the number of internal buses, and the number of columns is equal to the number of external buses (note that number of columns would be doubled in case of using $\Lambda$). Usually power networks in an interconnected system, are connected through a few buses (known as boundary buses) with limited number of connections. Therefore, matrix $B_{12}$ is a very sparse matrix. Figure IV.5, shows a schematic of a typical $B_{12}$ for a given system after re-ordering rows and columns in $B$ matrix in such a way that internal buses are on top followed by internal boundary buses.

Rows and columns of $B_{12}$ correspond to the internal and external buses respectively. Therefore the shaded part in figure IV.5 represents part of $B_{12}$ with non-zero entries, and the blank part is a null sub-matrix. Even the shaded part of $B_{12}$ is very sparse due to the fact that each one of the internal boundary buses usually has connections with a few
external buses only (usually less than five). Therefore, the sparsity of \( \gamma \) and subsequently, \( \Lambda \) is independent of structure of \( B_{22}^{-1} \).

In other words, \( \Lambda \) in equation (IV.25) is a sparse matrix with number of columns twice as many as number of those external buses without PMUs, and number of rows as many as number of internal boundary buses plus the number of external buses with PMUs. Due to the limited number of internal boundary buses in an interconnected system, and due to the error introduced to the model, solving equation (IV.25) is usually not feasible without assist of a few real-time external measurements. This problem can be resolved by using the available measurements from external PMUs.

External buses with PMUs have the every characteristic of internal boundary buses due to the fact their real-time data is available to internal power system dispatcher, and due to their direct connection with external buses. Therefore, those external buses with PMU can be treated as internal boundary buses, this will augment the number of rows and decreases the number of column in \( \Lambda \), which in turn will reduce the chance of getting multiple solutions through solving equation (IV.25) as higher number of equations should be satisfied with increment of the rows in \( \gamma \). In fact without application of external PMUs, multiple solutions may cause the IP to find incorrect solution.

Equation (IV.25) is a non-linear mixed binary problem, which can be solved using the proper IP package (i.e. GAMS [111]). Note that the IP problem in (IV.25) minimizes the aggregated absolute value of error.
The solution of (IV.25) yields two terminals of the external line outage. $p$ identifies the pre-outage flow of the removed external line, and $q$ represents the power loss between two ends of the external line outage due to being transferred from one end to another.

**IV.3. Simulation Results**

**IV.3.1. Simulation results using DC approximation**

Since the linear decoupled model, is an approximation to model the operation of power system, it is easier to check the validity of the proposed method for the error free system solution where power loss due to transfer from one end to another on a given line is zero. Once the method is validated using error free data, the method can be tested for actual ac power flow solution as well.

Note that in this part of the test, equation (IV.24) will be used as no error is introduced to data. In order to solve equation (IV.24), information about pre-outage topology of entire system, as well as pre-outage and post-outage information of internal phase angles is required. To simulate the post outage internal phase angle, linear decoupled model of power system will be used.
\[ B \Delta \theta = \Delta P \]

\[ \Rightarrow \Delta \theta = B^{-1} \Delta P \quad \text{(IV.26)} \]

Line outage in external system will clearly change the \( B \) matrix. Defining \( B' \) as the system post-outage susceptance matrix, new state of the system (\( \Delta \theta' \)) can be calculated as follows:

\[ \Delta \theta' = (B')^{-1} \Delta P \quad \text{(IV.27)} \]

The objective is to calculate a new set of power injection imposing of which to the pre-outage system topology (\( B \)) yields the post outage system solution. This can be accomplished as follows:

\[ \Delta P_1 = B \Delta \theta' \quad \text{(IV.28)} \]

Where \( \Delta P_1 \) represents a new set of injections required to obtain \( \Delta \theta' \) when pre-outage topology is in place. Since the operator has no knowledge about real-time external data, both \( \Delta \theta' \), and \( \Delta P_1 \) can be divided into two sub-matrices where the part corresponding to the internal system is followed by the part corresponding to the external system this can be formulated as follows:

\[ P_1 = \begin{bmatrix} P_{1-\text{internal}} \\ P_{1-\text{external}} \end{bmatrix}, \theta' = \begin{bmatrix} \theta'_{\text{internal}} \\ \theta'_{\text{external}} \end{bmatrix} \quad \text{(IV.29)} \]

Thus \( \Delta \theta_1 \), and \( \Delta P_1 \) in (IV.18) are defined as follows:
\[ \Delta \theta_1 = \theta'_\text{internal} - \theta_{\text{internal}-BC} \]

\[ \Delta P_1 = P_{1-\text{Internal}} - P_{1-\text{Internal}-BC} \]  

(IV.30)

Where \( \theta_{\text{internal}-BC} \) and \( P_{1-\text{Internal}-BC} \) are base case values for the internal phase angles and active power injections respectively.

To validate the method, IEEE 118 bus system [43] is used as a test bed. Please note that it is assumed in this study that the test is executed frequently enough to ensure that the change in operating condition remains below a certain threshold (2% of the base case loading).

To test the proposed method, system is first split into two sub-systems representing the internal and external systems in an interconnected network. Then, different external line outages are imposed to the external system as unreported external line outages to test the validity of the method.

For IEEE 118 bus system, it is assumed that internal system is constituted of buses 1 through 45, 113,114,115, and 117. And external system includes buses 46 through 112, 116, and 118. Schematic of the two sub-systems and their connections is shown in figure IV.6 and IV.7.
Figure IV.6. Internal and external system boundaries in IEEE 118 bus system

Figure IV.7. IEEE 118 bus system divided into two sub-systems
To test the accuracy of the proposed method on the IEEE 118 bus system as shown in figure IV.7, different line outages are imposed to the external system, and the outcome is calculated based on equations (IV.26)-(IV.30).

The updated data used as the post outage information in this section is error free since it is calculated using equations (IV.26)-(IV.30). Therefore, equation IV.24 which is an error free equation should be used to formulate the problem. This should be followed by usage of an IP package [112] to solve the problem.

Although it is critical for internal power system operator to be notified of major external line outages, due to their significant impact on internal security analysis, the method is capable of identifying both major and minor external line outages due to lack of error in this part of study. Results are shown in Table IV.1.

**Table IV.1.** Test result for 118 bus system

<table>
<thead>
<tr>
<th>Line outage</th>
<th>P (Pre-outage flow on removed line)</th>
<th>Solution found for X</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-1 at bus</td>
</tr>
<tr>
<td>59-63</td>
<td>7.608</td>
<td>59</td>
</tr>
<tr>
<td>77-80</td>
<td>3.003</td>
<td>77</td>
</tr>
<tr>
<td>60-61</td>
<td>6.810</td>
<td>60</td>
</tr>
<tr>
<td>68-116</td>
<td>0.948</td>
<td>116</td>
</tr>
<tr>
<td>96-82</td>
<td>0.332</td>
<td>82</td>
</tr>
<tr>
<td>100-101</td>
<td>0.532</td>
<td>100</td>
</tr>
<tr>
<td>103-104</td>
<td>0.562</td>
<td>104</td>
</tr>
<tr>
<td>80-98</td>
<td>1.631</td>
<td>98</td>
</tr>
</tbody>
</table>
The first four rows in table IV.1, correspond to major lines in external system, following four rows correspond to external minor lines. Although unreported minor external line outage does not have a significant impact on security analysis, being able to identify minor lines can increase the reliability of the proposed method.

Next, the proposed method will be tested for the case that both operating change and topology change happen between two samples. Contrary with the previous case, errors are introduced to the system due to the change in load/generation. In other words, for a system with $M$ external buses, $\Delta P_2$ is a full vector with $(M-2)$ small entries due to load change in the system and two large entries corresponding to terminals of the removed line. The small entries in $\Delta P_2$ can be treated as errors; therefore, (IV.23) should be used to solve the problem of this type.

Table IV.2, shows the solution of the IP problem for the outage of three major line in the external system, using the DC power flow solution in case of change of both topology and operating condition between two data selections. Note that the assumption is to run the test so frequently, that the change of operating condition is confined to a certain threshold ( 2% of the base case loading). The lower the change in operating condition, the higher the chance of identifying the terminals of the removed line correctly.
Table IV.2 Results for 118 bus system with 2% change in load/generation

<table>
<thead>
<tr>
<th>Line outage</th>
<th>P (power injection)</th>
<th>$\rho$</th>
<th>Solution found for X</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1 at bus</td>
<td>+1 at bus</td>
<td></td>
</tr>
<tr>
<td>77-80</td>
<td>3.742</td>
<td>0.084</td>
<td>77</td>
</tr>
<tr>
<td>59-63</td>
<td>6.285</td>
<td>0.068</td>
<td>59</td>
</tr>
<tr>
<td>60-61</td>
<td>7.285</td>
<td>0.073</td>
<td>60</td>
</tr>
</tbody>
</table>

**IV.3.2. Simulation results using full AC power flow solution**

Although the proposed method should have been tested for the dc power flow solution, the proposed method is not useful unless it is capable of identifying the line outages using the real-world power flow solution. The objective of this part of study is to validate the method when actual ac power flow solution is used. To validate the method, both IEEE 30 bus system and IEEE 118 bus system is used as testbed. The IEEE 30 bus system, as shown in figure IV.8 is divided into two sub-systems representing internal and external systems. It is assumed that the internal system is composed of buses 23 through 30, and the external system contains buses 1 through 22. Figure IV.8 shows the schematic description of the internal and external systems.
The proposed method is tested using two different scenarios. The first scenario is the case where the external operating condition remains approximately constant between two measurement scans before and after the line outage. In case the operating condition changes between two scans, these changes in load/generation will be treated as extra errors. It is critical to emphasize the importance of having external system PMU measurements in identifying the external line outages. While fault status of minor external lines does not have a notable impact on internal security analysis, wrong status of major external lines may cause serious consequences in security analysis, therefore identification of major external line outages is of interest of most of utility companies. This can be done with using the real-time data for all internal buses and data from those external buses whose real-time data available through PMUs.

Figure IV.8. IEEE 30 bus system split into two sub-systems
This part of study assumes that in case of IEEE 30 bus system, two PMUs are installed at buses 7 and 10 as external PMUs. The choice of PMU location is made based on the optimal placement of PMUs for observing the IEEE 30 bus system described in chapter two of the dissertation. Also, it is assumed as an additional constraint that no PMU should be installed at either terminals of the removed line. Obviously, it is easier to identify the line outage in case of having a PMU installed on one of the terminals of the external line outage. Table IV.3, shows the result for three different major line outages in external system, in case of having constant operating condition between two scans.

Table IV.3 Test result for IEEE 30 Bus system, line outage with constant operating condition

<table>
<thead>
<tr>
<th>Line Outage</th>
<th>p</th>
<th>Solution for A</th>
<th>q</th>
<th>ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>+1 @ bus</td>
<td>-1 @ bus</td>
<td></td>
</tr>
<tr>
<td>2-6</td>
<td>1.050</td>
<td>2</td>
<td>6</td>
<td>0.052@2</td>
</tr>
<tr>
<td>12-4</td>
<td>1.103</td>
<td>4</td>
<td>12</td>
<td>-0.055@12</td>
</tr>
<tr>
<td>3-4</td>
<td>7.382</td>
<td>3</td>
<td>4</td>
<td>0.426@3</td>
</tr>
</tbody>
</table>

Table IV.4 shows the results of solving equation (IV.25) using GAMS for four different line outages in IEEE 118 bus system. It is noted that in this part of study the calculation is done based on the assumption that operating condition of the system remains the same, and only topology of the system changes between two samples.
In this part, IEEE 118 bus system described in section IV.3.2.i, is used to validate the method for the ac case as well. Table IV.4 shows the test results for four different line outages in external system. Assist of PMUs are required to solve the problem in this part of study. Although high number of PMUs (14 external PMUs) are required to identify the minor external line outages, lower number of PMUs (8 PMUs) can handle accurate identification of major line outages in external system. Similar to the case for IEEE 30 bus system, location of external PMUs in IEEE 118 bus system is chosen based on the method described in chapter 2 of this dissertation.

**Table IV.4.** Test results for 118 bus system, line outage with constant operating condition

<table>
<thead>
<tr>
<th>Line outage</th>
<th>p</th>
<th>Solution found for A</th>
<th>q</th>
<th>ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>+1@ bus</td>
<td>-1@ bus</td>
<td></td>
</tr>
<tr>
<td>77-80</td>
<td>3.477</td>
<td>80</td>
<td>77</td>
<td>-0.056 @ 77</td>
</tr>
<tr>
<td>59-63</td>
<td>5.100</td>
<td>59</td>
<td>63</td>
<td>-0.2248 @ 59</td>
</tr>
<tr>
<td>60-61</td>
<td>7.216</td>
<td>61</td>
<td>60</td>
<td>0.012 @ 61</td>
</tr>
<tr>
<td>94-95</td>
<td>1.389</td>
<td>94</td>
<td>95</td>
<td>0.024 @ 94</td>
</tr>
</tbody>
</table>

Table IV.4, shows the results of external line outage identification, using (IV.25). Note that this part of study is done based on the assumption of constant operating condition of the external system. The first three rows in Table IV.4 represent the major line outage in external system, and the last one shows a minor line outage identified using the proposed method.
While this is important to be able of identifying the line outage when the operating condition remains constant between two samples, the more complicated, and realistic case is the case where both load, and topology change happens between two samples.

In this scenario, the assumption about approximately constant operating condition, will be relaxed. Therefore, this scenario involves a case where both topology and operating conditions vary between the two measurement scans. Please note that due to the fact that the change in operating condition is treated as an error in (IV.25), it is assumed that the test is run frequently enough so that the change in load/generation between two consecutive scans does not exceed 2% of the base case loading. The change of operating condition threshold can be achieved by running the test every couple of minutes. Please note that it is assumed that in case of IEEE 30 bus system, only two external PMUs are installed in the system whereas 8 PMUs are installed in the external system for IEEE 118 bus system.

In case that the internal system operator wishes to run the test less frequently, the probability of having changes in system operating conditions in excess of 2% will be higher and this may necessitate the use of a higher number of external PMUs in order for the method to identify the correct line outage. Table IV.5, shows the test results for line outages in external system in case of change in both topology and operating condition in external system in IEEE 30 bus case. Note that it is assumed in this study that the change in operating condition between each two consecutive scans is confined to 2% of the base case loading and only two external PMUs are installed at buses 7 and 10.
Table IV.5 Test result for IEEE 30 Bus system, line outage with 2% change in operating condition

<table>
<thead>
<tr>
<th>Line Outage</th>
<th>p</th>
<th>Solution for A</th>
<th>q</th>
<th>ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>+1 @ bus</td>
<td>-1 @ bus</td>
<td></td>
</tr>
<tr>
<td>2-6</td>
<td>0.979</td>
<td>2</td>
<td>6</td>
<td>-0.049@6</td>
</tr>
<tr>
<td>12-4</td>
<td>1.126</td>
<td>4</td>
<td>12</td>
<td>-0.056@12</td>
</tr>
<tr>
<td>3-4</td>
<td>8.732</td>
<td>3</td>
<td>4</td>
<td>0.362@3</td>
</tr>
</tbody>
</table>

Table IV.6, shows the test result for IEEE 118 bus system, when both topology and operating condition change between two samples. Results shown in table IV.6 are obtained for the case that loads/ generations in post outage system are increased by 2%.

Table IV.6. Test result for IEEE 118 Bus system, line outage with 2% change in operating condition

<table>
<thead>
<tr>
<th>Line outag</th>
<th>p</th>
<th>Solution found for A</th>
<th>q</th>
<th>ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>+1@ bus</td>
<td>-1@ bus</td>
<td></td>
</tr>
<tr>
<td>77-80</td>
<td>3.766</td>
<td>80</td>
<td>77</td>
<td>0.036 @ 80</td>
</tr>
<tr>
<td>59-63</td>
<td>5.653</td>
<td>63</td>
<td>59</td>
<td>-0.171 @ 59</td>
</tr>
<tr>
<td>60-61</td>
<td>7.195</td>
<td>61</td>
<td>60</td>
<td>0.009 @61</td>
</tr>
</tbody>
</table>

It is shown that the proposed method is capable of identifying the line outage for the following cases:
✓ Constant operating condition with topology change (tested for both DC and AC case)

✓ Both Operating condition and topology change (tested for both DC and AC case)

The only remaining condition which is not discussed so far in this section is the case of constant topology with dynamic operating condition between two consecutive samples. In the case of operating condition change with constant topology, no feasible solution can be found for equation (IV.25) due to the last constraint in (IV.25). As mentioned, presence of last constraint in (IV.25) forces the solver to find two non-zero entries (corresponding to the terminals of the removed line) whose absolute value is larger than other entries of $\Delta P_2$. Small entries in $\Delta P_2$ are reflected as entries in $e$ vector. In case of load/generation change with constant topology, $\Delta P_2$ is a full vector whose entries are all small due to load/generation change in entire external system. These small entries are created due to the approximation error and/or the change in load/generation. Integer programming solution as defined in equation (IV.25), will be looking for two large entries (line outage terminals) along with a vector of small entries (error). Therefore, in case of change in operating condition with no topology change, the optimal solution cannot be found with a pair of large entries in $\Delta P_2$, and thus, the optimization will return a no feasible solution warning for the problem (IV.25).
**IV.4. Summary**

Problem of the tracking of external topology, and their impact on the internal security analysis has been addressed and discussed in this chapter. Although the problem of topology error in the area under study is well addressed in the literature, problem of external line outage detection has received very little attention due to lack of real-time information about the external system.

Introduction of PMUs as a time synchronized devices which can provide phasor measurements to be used in different systems, in an inter-connected network is known as a prelude to many new studies in power systems. One of such studies is tracking of external topology which has a significant impact on internal security analysis.

A new method for line outage identification in external system is studies in this chapter. The method uses the real-time internal system solution as well as real-time phasor measurements form those external buses equipped with PMUs.

It is observed that the number of required PMUs in the external system is a function of the tolerated change in the operating condition between two consecutive scans.
CHAPTER V

Conclusions and Future Work
This dissertation is mainly concerned about utilization of phasor measurement units to improve monitoring of power transmission systems.

This dissertation can be summarized as follows:

- Full network observability using branch PMUs

Chapter II investigates the problem of network observability and robustness of the system against bad data, and contingencies using phasor measurement units (PMUs). The problem of observability is formed such that an optimal set of PMUs can be selected to achieve full observability. This chapter assumes that observability is derived for the internal system only and does not consider issue of external network modeling.

- Transforming critical measurements into redundant ones

Errors in critical measurements cannot be detected regardless of their accuracy. Chapter II investigates the problem of strategic location of branch PMUs to transform critical measurements into redundant ones so that their error can be detected.

- Robustness against loss of measurements and branches

Planned or unexpected line outages in the system may lead to network unobservability. Chapter II, investigates the problem of robustness against loss of measurements or loss of branches in detail.

- Impact of real-time exchange of conventional measurements between external and internal systems
In any interconnected system, internal system has limited access to real-time data from its neighboring areas through ICCP. Due to limited number of measurements which can be updated in real-time, it is important to identify set of measurements with the most significant impact on internal security assessment. Chapter III investigates the optimal choice of conventional external measurements with the most impact on internal state estimation and subsequent security analysis.

✓ Impact of external phasor measurements on internal security analysis

Chapter III investigates the impact of synchronized measurements available through external PMUs. As expected, available data from external system can improve both internal state estimation and contingency analysis.

✓ Detection of external network topology changes using PMUs in the external system

Based on limited communication between neighboring systems in an interconnected grid, real-time tracking of external network model cannot be easily accomplished by the internal operator. Chapter IV presents a method which enables the internal power system dispatcher to track changes in the external topology with the assistance of external phasor measurements.

Contributions of this dissertation can be listed as follows:

✓ Optimal placement of 2-channel PMUs (branch PMUs) which are commonly installed in several utility systems.
The developed method is implemented and tested on small test systems as well as large utility systems.

- Development and testing of a method to analyze the impact of different external network measurement exchanges on contingency assessment
- Development and testing of a method to determine the impact of external phasor measurements
- Development and testing of a method to track external system topology changes based on PMU measurements from external system

Recommended future work about the topics covered in this dissertation can be listed as below:

- study the case of system observability with PMUs with different output channels (Chapter II)
- find a method to identify the strategic location of external PMUs to improve the internal security analysis the best (Chapter III)
- The presented method in chapter IV utilizes mixed integer non-linear programming to track external topology changes. In an attempt to substitute the method presented in chapter IV with simpler formulation, Dantzing selector [119], and reweighted $l_1$ method [120] was used. Although these methods do not seem to work properly in our case, replacing the mixed integer non-linear problem with the linear one can improve efficiency of the method.
- As mentioned in chapter IV, in case of change in operating condition with constant topology, the package returns “infeasible solution” warning indicating that there is
no reason to suspect a topology change, but there is a possibly a change in operating condition only. A recommendation for future work in this regard can be changing the formulation in such a way that method can identify the change in load/generations instead of returning infeasible warning.

✓ Finally the computational efficiency of the method presented in chapter IV can be further improved to facilitate its online implementation.
REFERENCES:


