Tunable Ferrite-Based Negative Index Metamaterials for Microwave Device Applications

A Dissertation Presented

by

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DEDICATION

This dissertation is dedicated to my family and friends for all their support, encouragement, and inspiration.
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I would thank the staffs and students at CM3IC for their collaboration and help. I thank Dr. Yajie Chen for his instruction and sharing from his rich research experience. I thank Dr. Soack D. Yoon and Dr. Anton Geiler for training me many valuable experimental skills from day one. I thank Jinsheng (Jason) Gao for helping me with precise sample cutting and lithography.

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ABSTRACT

Metamaterials possessing simultaneous negative permittivity and permeability, and hence negative refractive index, have created intense interest since the beginning of this century in fundamental physics, material science, and microwave and optical engineering. The mainstream approach of realizing these properties is to combine metallic plasmonic wires and magnetic ring resonators. These metallic metamaterials can be adapted for different frequency ranges by design of device elements in proportion to targeted wavelengths. However, because the magnetic resonant properties is defined strictly by geometric parameters of the ring structures, these metamaterials suffer from narrow bandwidth and are not at all tunable. Alternatively, ferrite materials show a broad band of negative permeability near the ferromagnetic/ferrimagnetic resonance that can be tuned by a magnetic field in frequency. So there are great opportunities of realizing broad band and tunable negative index metamaterials (NIMs) using ferrites.

This research explores the negative permeability property of ferrite materials and the negative permittivity property of plasmonic metal wires concomitant in frequency to realize tunable negative index metamaterials (TNIMs). Further, these ferrite-based TNIMs were applied to demonstrate microwave devices. Different ferrite materials, including poly and single crystalline yttrium iron garnet (YIG) and scandium doped barium hexaferrite were utilized. Broadband, low loss and tunable NIMs were realized in X-, K-, and Q-band respectively. The minimum insertion loss is ~ 5.7 dB/cm and the maximum dynamic bandwidth is ~ 5 GHz for the K-band waveguide TNIM, ~ 5 dB/cm and ~ 3 GHz for the X-band microstrip TNIM, and -25 dB/cm and ~ 3 GHz for the Q-band waveguide TNIM. Continuous and rapid phase tunability of 160 degree/kOe was realized at 24 GHz for the K-band TNIM, and 70 degree/kOe at 9 GHz for the microstrip TNIM. Large phase tuning was also found in the Q-band TNIM using a hexaferrite. But the insertion loss needs to be reduced for it to be practical.

These demonstrations are the first to implement TNIMs in microwave device applications.
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1. Introduction

The development of RF and microwave devices has been progressing rapidly during the past two decades with the growth of wireless communication and personal electronics industries and the continuous demand from the defense industry. Correspondingly, novel electromagnetic (EM) materials having superior electrical and magnetic properties are in great need. For example, high $k$ materials are needed for the semiconductor industry to allow for reduction of gate oxide thickness of transistors; high permeability and low loss materials are desired at GHz range for the IC industry to improve the quality factor of inductors; and matched high permeability and permittivity materials will be ideal to shrink antennas’ size.

Electromagnetic materials can be categorized by the sign of their permittivity ($\varepsilon$) and permeability ($\mu$) to fall into four categories generally (in this chapter, both $\varepsilon$ and $\mu$ indicate the real values). The first category for the case when $\varepsilon > 0$ and $\mu > 0$. Most natural materials fall into this category, which allow electromagnetic waves to propagate through. The corresponding refractive indices ($n$) are positive. The second category is for the case when $\varepsilon < 0$ and $\mu > 0$. These materials include some noble metals such as silver and plasmonic metal structures working below their plasmonic frequency. The third category is for the case when $\varepsilon > 0$ and $\mu < 0$. Some metallic magnetic resonators such as split rings and some ferrite materials show this property
at the higher frequency side of resonance. In materials in these two categories, EM waves decay exponentially and cannot propagate through the media.

The last category is for the case when $\varepsilon < 0$ and $\mu < 0$ with corresponding $n < 0$ too. This type of materials was first proposed by Veselago in 1968 from a sense of symmetry of nature\(^1\. There are no natural materials having this property. So far, this class of materials can only be achieved by combining materials in the second or/and third category in proper geometrical structures. Normally, these constructions are small compared to the target wavelength so that they can be characterized by effective EM parameters like $\varepsilon$, $\mu$, and $n$. They are have been called negative index metamaterials (NIMs), left handed metamaterials (LHMs), or double negative metamaterials (DNM) in the community. A beam of light, or EM wave, incident upon the interface of positive and negative index media will be refracted "negatively", which stills obeys Snell's law. The phase velocity of the wave in the NIMs is negative. Interestingly, the earliest discussion of negative phase velocity was by Sir Arthur Schuster and H. Lamb in 1904\(^2\).

In general, NIMs are an emerging class of artificially designed and structured materials showing unprecedented EM properties in the microwave to terahertz regimes. In order to understand the methodology of NIM design, it is very helpful to examine the relation between the complex permittivity $\varepsilon = \varepsilon' + i\varepsilon'' = |\varepsilon|e^{i\phi}$, permeability $\mu = \mu' + i\mu'' = |\mu|e^{i\phi}$, and refractive index $n = n' + in'' = |n|e^{i\phi}$ in the
complex plane (here the physics notation is used wherein for engineering notation, "i" is replaced with "-j"). Because \( k = \sqrt{\varepsilon \mu}, \quad |k| = \sqrt{\varepsilon \mu} \) and \( \theta = (\phi + \varphi)/2 \).

Figure 1-1 shows four typical cases of \( \varepsilon \) and \( \mu \) interaction from a purely mathematical point of view. Note that both \( \varepsilon'' \) and \( \mu'' \) should be positive as a requirement of a passive medium. Figure 1-1(a) describes the materials of the first category discussed previously. When both \( \varepsilon' \) and \( \mu' \) are positive, \( \varepsilon' \) and \( \mu' \) are in the first quadrant, a positive \( n' \) results. \( n'' \) is small when both \( \varepsilon'' \) and \( \mu'' \) are small. So an EM wave can propagate through with little absorption. In Fig. 1-1(b) describes the materials in the third category, i.e. ferrites and magnetic resonators. In this case the resulting \( n'' \) is large. The materials appear to be very lossy so the EM wave cannot propagate through. Figure 1-1(c) describes the case of a NIM that is when both \( \varepsilon' \) and \( \mu' \) are negative and \( \varepsilon'' \) and \( \mu'' \) are small, which result in a negative \( n' \) and a small \( n'' \). The EM wave can propagate through the metamaterial with small absorption. At the same time, the phase velocity and group velocity are of opposite direction. The case in Fig. 1-1(d) is special where \( n' \) gets negative when \( \varepsilon' \) is negative and \( \mu' \) positive. However, the large \( \mu'' \) also results in large \( n'' \) which prevents wave propagation.
Fig. 1-1. Permittivity, permeability, and refractive index in the complex plane: (a) $\varepsilon' > 0$ and $\mu' > 0$, (b) $\varepsilon' > 0$ and $\mu' < 0$, (c) $\varepsilon' < 0$ and $\mu' < 0$, and (d) $\varepsilon' < 0$ and $\mu' > 0$ but with $\mu''$ much bigger than $\mu'$ and $\varepsilon''$ much smaller than $\varepsilon'$.

The NIMs exhibit many unusual EM properties such as backward wave propagation, negative refraction, near-field imaging, reverse Cherenkov radiation, etc. These unusual properties allow for novel applications such super lenses, leaky wave antennas, and miniature delay lines. Researches in NIMs have been carried out in many frequency domains spreading from microwave to infrared wave, and even visible light with the application of nanometer level fabrication techniques. The
mainstream design principle of NIMs is to combine one material or structure of negative $\varepsilon$ with another of negative $\mu$ concomitant in frequency to achieve negative $n$.

Notable NIMs are metallic resonant metamaterials$^{7,8}$, photonic crystals$^9$, and planar periodic arrays of passive lumped circuit elements$^{10}$. The basic principle of NIM is to induce an effective negative permittivity in an array of thin metallic wires and an effective negative permeability in an array of split-ring resonators (SRR). As shown in Fig. 1-2 (Ref. 7), the copper wire strips and square SRRs are on the two sides of fiber glass boards. In terms of wave polarization, the electrical field is along the wire strips and magnetic field is in the horizontal plane. The SRRs are placed in two orthogonal orientations in order to interact with the magnetic field in all directions in the horizontal plane. The negative permittivity and permeability are due to the plasmalike effect of the metallic mesostructures and the low-frequency magnetic resonance of the SRRs, respectively. In the equivalent circuit analysis, the wire strips and SRRs form many $LC$ resonators with tight coupling to each other. Various metal-based NIMs or LHMs have been proposed, including the widely studied rod-SRR structure, $\Omega$ ring$^{11}$, $S$ ring$^{12}$, short wire pairs$^{13}$, and some other variations. Generally the rings are like artificial magnetic atoms, the size of which is much smaller than the wavelength. So the metamaterials can be treated as homogeneous medias effectively.
In contrast, photonic crystals reveal negative refraction due to a different mechanism connected to the band structure ($f$ vs. $k$), which are normally demonstrated by showing the negative refraction of an incident beam of microwave or light. As an example for the metallic photonic crystals is that of a flat lens in Fig. 1-3 (Ref. 9) where the spacing between the metal cylinders is comparable to the wavelength. So the structure is not a true homogeneous media.

Fig. 1-2. Photograph of a NIM sample consisting of square copper SRRs and copper wire strips on fiber glass circuit boards. The rings and wires are on opposite sides of the boards. And the boards have been cut and assembled into an interlocking lattice (Ref.7).
In recent years excitement in NIMs has focused on the optical frequency range thanks to the advances in nanofabrication technology. Terahertz and magnetic response was obtained from a double SRR$^{14}$ and single ring$^{15}$ structures. Midinfrared magnetic response was demonstrated by using staple-shaped nanostructures$^{16}$. Visible light magnetic response was observed on paired silver strips$^{17}$ and EM coupled pairs of gold dots$^{18}$. Subsequently, near-infrared NIMs were demonstrated in a metal-dielectric-metal multilayer, and optical NIMs were developed using paired parallel gold nanorods$^{19}$, metal-dielectric stacks$^{20}$, fishnet structure$^{21}$, and some other variations. All these structures are based on structural EM resonant mechanisms. Figure 1- 4 shows a nano-scale NIM with fishnet structure of alternating layers of silver and magnesium fluoride. It shows negative index of refraction at an optical wavelength $\sim 1,600$ nm (Ref. 21). Similar to the metallic NIMs in the microwave range, there is strong magneto-inductive coupling between neighboring layers and
tight coupling between adjacent $LC$ resonators through mutual inductance results in a broadband negative index of refraction with relatively low loss.

Fig. 1-4. (a) Diagram of the 21-layer fishnet structure with a unit cell of 5860 x 5565 x 5265 nm$^3$. (b) SEM image of the 21-layer fishnet structure with the side etched, showing the cross-section. The structure consists of alternating layers of 30nm silver (Ag) and 50nm magnesium fluoride (MgF2), and the dimensions of the structure correspond to the diagram in (a). The fishnet structure shows negative index of refraction at optical wavelength $\sim$ 1,600 nm (Ref. 21).

Metamaterials relying on accurate geometric structures to match the negative $\varepsilon$ or $\mu$ over the same frequencies are generally limited by inherent narrow bandwidths and the lack of frequency tunability. Table 1-1 presents the working frequency and band width of major types of NIMs experimentally developed over the microwave range. In order to obtain negative $n$ at different frequencies, one has to alter the periodicity or size of the elements. Recently, ferroelectric materials have also been
loaded into split rings to enable frequency tunability by varying the bias voltage. However, the tunable bandwidth is still below 0.2 GHz mainly limited by the nature of SRRs used to generate negative $\mu$.

<table>
<thead>
<tr>
<th>Research Group</th>
<th>Structure Type</th>
<th>Dim</th>
<th>$f$ (GHz)</th>
<th>BW (GHz)</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shelby</td>
<td>Copper Wires &amp; SRRs</td>
<td>2D</td>
<td>10.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Eleftheriades</td>
<td>L-C Transmission Line</td>
<td>2D</td>
<td>1.2</td>
<td>0.2</td>
<td></td>
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<tr>
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<td>Metallic Photonic Crystal</td>
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<td>0.4</td>
<td></td>
</tr>
<tr>
<td>Pimenov</td>
<td>Ferromagnet-Superconductor</td>
<td>1D</td>
<td>90.0</td>
<td>Narrow</td>
<td>$T = 10$K, $H = 3$T</td>
</tr>
</tbody>
</table>

Table 1-1. Bandwidth and working frequency comparison of major types of NIMs demonstrated in microwave frequency range.

It has been long known that ferrite materials provide negative $\mu$ at the higher frequency side of their ferromagnetic resonance (FMR) in a transmission line in the microwave range. Since the frequency of the FMR shifts with the magnetic bias field $H$, the negative $\mu$ can be frequency tuned by $H$. Therefore, using ferrite materials instead of metallic resonators to generate negative $\mu$ can offer real-time frequency tunability of NIM and eliminates the inherent narrow bandwidth of resonant structures.

Wu has theoretically confirmed the feasibility of obtaining negative index for
metal-ferrite-metal superlattice structure with transfer function matrix calculations\textsuperscript{22}. Pimenov \textit{et al} have demonstrated negative index by forming alternative layers of ferromagnetic La\textsubscript{0.89}Sr\textsubscript{0.11}MnO\textsubscript{3} and superconductive YBa\textsubscript{2}Cu\textsubscript{3}O\textsubscript{7} to a superlattice\textsuperscript{23}. At 90 GHz, negative refractive index was demonstrated at a very low temperature of 10 K and a very high bias magnetic field of 3.1 Tesla. The superconductive layers supply negative permittivity when turning superconductive at sufficient low temperature. The required low temperature and high bias magnetic field severely prevent this approach from practical device applications.

In this dissertation, the work of designing, fabricating, and characterizing tunable negative index metamaterials (TNIMs) consisting of ferrite materials and metal wires will be presented in details. The works cover full or part of C- (4 - 8 GHz), X- (8 - 12 GHz), Ku- (12 - 18 GHz), K- (18 - 26.5 GHz) and Q- (30 - 50 GHz) bands. The design principles lie in the EM properties of ferrite materials and plasmonic metal structures. The properties of ferrite materials and plasmonic metal structures will be described in the following two chapters. The realization of TNIMs at different frequency bands by combining these two components will be presented.
2. Low Loss Ferrites

2.1. YIG Slab with In-plane Bias

Ferrite materials are insulating magnetic oxides generally ferromagnetic or ferrimagnetic. Because the collective spin magnetic momentum in ferrite is not zero under proper bias conditions, they possess a permeability tensor, which gives rise to anisotropic and nonreciprocal EM properties. Unlike most materials, they possess both high permeability and high permittivity at frequencies from dc to the microwave. Due to their low eddy current losses, there exist no other materials with such wide ranging value to electronic applications in terms of power generation, conditioning, and conversion. These properties also afford them unique value in microwave devices that require strong coupling to electromagnetic signals and nonreciprocal behavior. They are widely used in microwave devices like circulators, phase shifters, filters and isolators. These components are ideal for high power and high speed microwave applications and are especially useful for radar and communication systems.

For ferrite materials under zero or nonzero magnetic DC bias field, a ferromagnetic or ferrimagnetic resonance (FMR) can be achieve at a certain frequency of AC field excitation due to spin precession. The permeability of the ferrite tends to be dispersive near the FMR frequency. In order to understand this property and its link to TNIM applications, in the following we analyze an example of calculating the
permeability of a single crystal yttrium iron garnet ((Y₃Fe₂(FeO₄)₃, or YIG) slab. The YIG crystal structure is cubic. Comparable to spinel ferrites, its magneto-crystalline anisotropy energy is relatively small and the corresponding magnetic anisotropy fields \( H_A \) are typically 10's of Oe. Since the ferromagnetic resonant frequency is strongly dependent upon \( H_A \), the zero field FMR frequency of YIG falls near or below 1 GHz. This limits the frequency of devices employing YIG to C-, S-, and X-bands. In fact, for many of these applications the ferrite is biased by a field from a permanent magnet. The magnet serves to saturate the ferrite as well as shift the FMR to higher frequencies required for certain device applications.

As shown in Fig. 2-1, a single crystal YIG slab is positioned with the applied DC bias field \( \vec{H} \) along the z-axis, \( \vec{H} = H\hat{z} \). The dimension of the slab is \( a \times b \times c \). In the following, the complex permeability tensor is calculated step by step. The employed parameters are \( a = 0.2 \) mm, \( b = 5.0 \) mm, \( c = 10.0 \) mm, \( g = 2.0 \), \( H = 2500 \) Oe, \( 4\pi M_s = 1750 \) Gauss, \( H_a = 33 \) Oe and \( \Delta H = 10 \) Oe, where \( g \) is the gyromagnetic factor, \( 4\pi M_s \) the saturation magnetization, \( H_a \) the anisotropic field, and \( \Delta H \) the FMR linewidth associated with the magnetic loss or the damping of magnetic moment.
First, we assume that the magnetization is at an arbitrary direction denoted by the angles $\theta$ and $\phi$. And we write out the free energy as:

$$F = -M \cdot H + \frac{1}{2} \left( N_x M_x^2 + N_y M_y^2 + N_z M_z^2 \right) + \frac{K}{M^2} \left( M_x^2 M_y^2 + M_y^2 M_z^2 + M_z^2 M_x^2 \right). \quad (2-1)$$

Here $N_x, N_y$ and $N_z$ are demagnetizing factors associated with the dimensions of the sample under the conditions:

$$N_x + N_y + N_z = 4\pi \quad \text{and} \quad aN_x = bN_y = cN_z. \quad (2-2)$$

Given the previous defined dimensions, $N_x = 200\pi/53$, $N_y = 8\pi/53$, and $N_z = 4\pi/53$. So approximately, $N_x = 4\pi$ and $N_y = N_z = 0$ approximately.
\( K_1 < 0 \) is the first order cubic magnetic anisotropy constant and \( H_s = -\frac{2K_1}{M_s} \). Apply the constrain of minimum free energy by setting

\[
\frac{\partial F}{\partial \theta} = 0 \text{ and } \frac{\partial F}{\partial \phi} = 0. \tag{2-3}
\]

And then the equilibrium state of magnetization can be obtained as \( \theta = 0 \) and \( \phi = 0, \frac{\pi}{2} \) (the same for \( \theta = 0 \)).

From the free energy expression Eqn. 2-1, the internal field can be calculated as

\[
\hat{H}_m = -\nabla_M F . \tag{2-4}
\]

Therefore,

\[
\hat{H}_m = \hat{H} - 4\pi M \hat{x} = -\frac{2K_1}{M_s} \left[ \left( M_x^2 + M_z^2 \right) \hat{x} + \left( M_y^2 + M_z^2 \right) \hat{y} + \left( M_x^2 + M_y^2 \right) \hat{z} \right] + \frac{4K_1}{M_s} \left( M_x^2 + M_y^2 + M_z^2 \right) \left( M_x \hat{x} + M_y \hat{y} + M_z \hat{z} \right) . \tag{2-5}
\]

In Eqn. 2-1, the variable \( \hat{M} \) and \( \hat{H} \) should include both DC and time dependent AC components in a rigid consideration. So the rigid expression of Eqn. 2-1 and Eqn. 2-5 can be obtained by doing the substitutions:

\[
\hat{M} \rightarrow \hat{M} + \hat{m} \text{ and } \hat{H} \rightarrow \hat{H} + \hat{h} . \tag{2-6}
\]

The capital letters with subscript "o" denote the DC field and magnetic moment. And the small letters denote time dependent AC/RF field and magnetic moment. In magnitude, the AC components are very small compared to the DC components.

Under the equilibrium state of magnetization where \( \hat{M} \) is aligned with \( \hat{H} \) at \( z \) axis,
\( M_z = M_r \) and \( M_s = M_y = 0 \). Therefore by doing the substitution and ignoring the higher order terms in Eqn. 2-5, the effective internal field is obtained as:

\[
\frac{r}{H_{in}} = H_\hat{z} + h_\hat{x} + h_\hat{y} - 4\pi m_s \hat{x} - \frac{2K_r}{M_z^2} (m_s \hat{x} + m_y \hat{y}). \tag{2-7}
\]

This expression includes the AC demagnetizing field. Once the effective internal field is obtained, it can be applied to the equation of motion of the magnetic moment.

\[
\frac{d\hat{m}}{\gamma dt} = -m \times \hat{H}_a - M \times h_a, \tag{2-8}
\]

where \( \hat{H}_a \) is the DC internal field, \( h_a \) AC/RF internal field, and \( \gamma = 2\pi \times 2.8 \text{GHz} / \text{kOe} \) the gyromagnetic ratio. Eqn. 2-8 can be broken down in three directions and simplified by ignoring the higher order terms as follows. Because the DC magnetization is along \( z \) axis, so the \( z \) component of \( \hat{m} \) can be ignored.

\[
\frac{j\omega}{\gamma} (m_x \hat{x} + m_y \hat{y}) = -(m_x \hat{x} + m_y \hat{y}) \times (H_\hat{z}) - (M_s \hat{z}) \times \left[ (h_x \hat{x} + h_y \hat{y}) - 4\pi m_s \hat{x} + \frac{H_a}{M_y} (m_x \hat{x} + m_y \hat{y}) \right]
\]

\[
\Rightarrow \quad \begin{cases} 
\frac{j\omega}{\gamma} m_x = -m_x (H - H_a) + M_s h_y \\
\frac{j\omega}{\gamma} m_y = m_y (H - H_a) + 4\pi M_y m_s - M_y h_x 
\end{cases} \tag{2-9a}
\]

Eqn. 2-9a can be rewritten as

\[
\begin{cases} 
\frac{j\omega}{\gamma} m_x + (H - H_a) m_y = M_s h_y \\
-(H + 4\pi M_y - H_a) m_x + \frac{j\omega}{\gamma} m_y = -M_y h_x 
\end{cases} \tag{2-9b}
\]

So \( \hat{m} \) can be solved in terms of \( \hat{h} \) in the form of \( \hat{m} = [\chi] \hat{h} \). Solve Eqn. 2-9b and get
\[
m_x = \begin{bmatrix} M_x h_x & (H - H_a) \\ -M_x h_x & \frac{j \omega}{\gamma} \end{bmatrix},
\]
\[
m_y = \begin{bmatrix} \frac{j \omega}{\gamma} & M_y h_y \\ -(H + 4\pi M_x - H_a) & \frac{j \omega}{\gamma} \\ -M_y h_y & -(H + 4\pi M_x - H_a) \end{bmatrix},
\]

For simplification in notations, write
\[H_1 = H - H_a, \quad H_2 = H + 4\pi M_x - H_a, \quad \text{and} \quad \Omega^2 = H_1 H_2 - \frac{\omega^2}{\gamma^2},\]

and then the \(2 \times 2\) susceptibility is:

\[
[\chi] = \frac{M_s}{\Omega^2} \begin{bmatrix} H_1 & j \frac{\omega}{\gamma} \\ -j \frac{\omega}{\gamma} & H_2 \end{bmatrix}.
\] (2-10)

If considering the damping effect associated with the FMR linewidth, just do the replacement \(\omega \rightarrow \omega - j \frac{\Delta \omega}{2}\), where \(\Delta \omega = \gamma \Delta H\) in Eqn. 2-10. Consequently, the \(2 \times 2\) permeability tensor is:

\[
[\mu] = 1 + 4\pi [\chi] = \begin{bmatrix} 1 + \frac{4\pi M_x}{\Omega^2} H_1 & j \frac{4\pi M_x}{\Omega^2} \frac{\omega}{\gamma} \\ -j \frac{4\pi M_x}{\Omega^2} \frac{\omega}{\gamma} & 1 + \frac{4\pi M_x}{\Omega^2} H_2 \end{bmatrix}.
\]

Add in the third dimension components and get the \(3 \times 3\) permeability tensor as:

\[
[\mu] = \begin{bmatrix} 1 + \frac{4\pi M_x}{\Omega^2} H_1 & j \frac{4\pi M_x}{\Omega^2} \frac{\omega}{\gamma} & 0 \\ -j \frac{4\pi M_x}{\Omega^2} \frac{\omega}{\gamma} & 1 + \frac{4\pi M_x}{\Omega^2} H_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
\] (2-11)
Note that in the previous calculations, the CGS units system is used so the permeability tensor is relative. The next question is how the tensorial permeability affects the wave propagation. Since transmission, reflection, and absorption directly relate to the impedance $Z$ and propagation constant $\beta$ of a sample, Maxwell equations need to be employed to solve for them. Let's consider the slab in the center of a rectangular waveguide under the TE$_{10}$ mode as shown in Fig. 2-2, \( \vec{h} = (h_x, h_y, 0) \) and \( \vec{h} \propto e^{-j\beta z} \cdot e^{-jky} \) in the slab.

![Fig. 2-2. YIG lab placed in the center of a rectangular waveguide with in-plane bias field perpendicular to the direction of wave propagation.](image)

From Maxwell equations (MKS units), we have

$$\nabla \times (\nabla \times \vec{h}) = \nabla (\nabla \cdot \vec{h}) - \nabla^2 \vec{h} = \omega^2 \varepsilon \mu \cdot \vec{h} = k\mu \cdot \vec{h}. \quad (2-12)$$

Let's break the expression down to the $x$ and $y$ components.
\[
\n\nabla \left( \nabla \cdot \hat{r} \right) = \nabla \left( \frac{\partial}{\partial x} h_x + \frac{\partial}{\partial y} h_y \right) \\
= \left( \frac{\partial^2}{\partial x^2} h_x + \frac{\partial^2}{\partial x \partial y} \right) \hat{x} + \left( \frac{\partial^2}{\partial x \partial y} + \frac{\partial^2}{\partial y^2} h_y \right) \hat{y}, \\
= (-k_i^2 h_x - \beta k_i h_y) \hat{x} + (-\beta k_i h_x - \beta^2 h_y) \hat{y} \\
- \nabla^2 \hat{r} = \left( \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) \hat{r}, \\
= (\beta^2 + k_i^2) \hat{r}, \\
= (\beta^2 + k_i^2) h_x \hat{x} + (\beta^2 + k_i^2) h_y \hat{y} \\
k_i^2 \hat{e} \cdot \hat{h} = \left( k_o^2 \mu_{sx} h_x + k_o^2 \mu_{sy} h_y \right) \hat{x} + \left( k_o^2 \mu_{sx} h_x + k_o^2 \mu_{sy} h_y \right) \hat{y}.
\]

So Eqn. 2-12 can be simplified to
\[
\left[ \begin{array}{cc}
\beta^2 - k_o^2 \mu_{sx} & -\beta k_i - k_o^2 \mu_{sy} \\
-\beta k_i - k_o^2 \mu_{sx} & k_i^2 - k_o^2 \mu_{sy} \\
\end{array} \right] \left[ \begin{array}{c}
h_x \\
h_y \\
\end{array} \right] = 0.
\]

Because the determinant of the coefficient matrix has to be zero to have non-zero solutions,
\[
\left| \begin{array}{cc}
\beta^2 - k_o^2 \mu_{sx} & -\beta k_i - k_o^2 \mu_{sy} \\
-\beta k_i - k_o^2 \mu_{sx} & k_i^2 - k_o^2 \mu_{sy} \\
\end{array} \right| = 0. \quad (2-13)
\]

Therefore (note that \( \mu_{sy} = -\mu_{yx} \)),
\[
\left( \beta^2 - k_o^2 \mu_{sx} \right) \left( k_i^2 - k_o^2 \mu_{yy} \right) - \left( \beta k_i + k_o^2 \mu_{yx} \right) \left( \beta k_i + k_o^2 \mu_{xy} \right) \\
= -k_o^2 \mu_{yx} \beta^2 - k_o^2 \mu_{sx} k_i^2 + k_o^4 \mu_{sx} \mu_{yx} + k_o^4 \mu_{sy}^2 = 0
\]

Solve for \( \beta^2 \), and then get
\[
\beta^2 = k_o^2 \frac{\mu_{sx} \mu_{sy} + \mu_{sy}^2}{\mu_{yx}^2} - k_i^2 \frac{\mu_{sx}^2}{\mu_{yy}^2}. \quad (2-14a)
\]

One thing very subtle here is that the procedure of solving for permeability tensor from the effective internal field and the equation of motion needs to be reconsidered.
Because when the ferrite slab is put in a transmission line, the actually propagation constant and AC field distribution will be solved by matching the boundary conditions. So in Eqn. 2-5, the AC demagnetizing field should not be considered. But the AC field still needs to be put into the anisotropy term. Therefore, when AC demagnetizing field is not considered in Eqn. 2-5, \( H_1 = H_2 = H - H_a \) in the permeability tensor and \( \mu_{xx} = \mu_{yy} \). Eqn. 2-14a turns into:

\[
\beta^2 = k_o^2 \frac{\mu_{xx} \mu_{yy} + \mu_{sy}^2}{\mu_{sy}} - k_i^2.
\]

(2-14b)

So an effective relative permeability can be defined as

\[
\mu_{eff} = \frac{\mu_{xx} \mu_{yy} + \mu_{sy}^2}{\mu_{sy}}.
\]

(2-15)

In Figure 2-3, the effective permeability of the single crystal YIG slab is plotted out with the material parameters given previously. \( \mu_{eff} = \mu' - j \mu'' \). With the 2500 Oe in-plane bias field, the FMR occurs near 9.0 GHz, which agrees with the theoretically calculations:

\[
f_{FMR} = \gamma' \sqrt{(H - H_a)(H + 4 \pi M_s - H_a)}
\]

\[
= 2.8 \sqrt{(2.5 - 0.033)(2.5 + 1.75 - 0.033)}.
\]

(2-16)

The inset shows the enlarged \( \mu' \) at the higher frequency side of FMR. The bandwidth of negative \( \mu' \) is approximately

\[
\Delta f_{\mu' < 0} = \gamma \cdot 2 \pi M_s = 2.45GHz.
\]

(2-17)

The frequency where \( \mu' \) goes from negative to zero is called antiferromagnetic resonance (AFMR).
The FMR frequency shifts roughly linearly with the magnetic bias field as does the negative region of $\mu'$. The tuning factor is approximately $\gamma' = 2.8\text{MHz/Oe}$ with the tunable negative region of effective permeability only limited by the bias field theoretically. Therefore, by combining this feature properly with a wide band negative permittivity material, a wide band frequency tunable negative index metamaterial (TNIM). However, there is a drawback of using the negative permeability near FMR. In Figure 2-3, the imaginary part of permeability, $\mu''$, peaks at the FMR frequency, which is associated with large magnetic loss. The span of the peak is decided by the FMR linewidth $\Delta H$. The magnetic loss can be very large near FMR. So the useful negative region of $\mu'$ is actually smaller. In Figure 2-4, the figure of merit ($|\mu'/\mu''|$)
is plotted out in the $\mu' < 0$ frequency region. $|\mu' / \mu''|$ reaches the peak $\sim 50$ near $10.4$ GHz, which is the optimal working frequency.

![Figure of merit of effective permeability of the YIG lab in the $\mu' < 0$ frequency region.](image)

**Fig. 2-4.** Figure of merit of effective permeability of the YIG lab in the $\mu' < 0$ frequency region.

In the TNIM case, the cause of insertion loss is complicated. Factors like dielectric loss, eddy current loss, ohm loss, and impedance mismatch are all involved. These will be discussed in later chapters. From the previous analysis, it can be concluded that an ideal ferrite material for TNIM application should possess small linewidth to lower the magnetic loss and large $4\pi M_s$ to have a wide static bandwidth. The requirement on the anisotropy field $H_a$ will depend on ferrite type, the way of biasing and the desired working frequency.
Fig. 2-5. YIG lab placed in the center of a rectangular waveguide with in-plane bias field parallel to the direction of wave propagation.

There are also other choices to bias the ferrite slab. Figure 2-5 shows another scenario that the bias field is parallel to the direction of wave propagation. In this case, the permeability tensor changes to

$$\begin{bmatrix}
1 + \frac{4\pi M_s}{\Omega^2} H_i & 0 & \frac{j4\pi M_s}{\Omega^2} \frac{\omega}{\gamma} \\
0 & 1 & 0 \\
\frac{-j4\pi M_s}{\Omega^2} \frac{\omega}{\gamma} & 0 & 1 + \frac{4\pi M_s}{\Omega^2} H_2
\end{bmatrix}.$$  \hspace{1cm} (2-18)

Because only consider the TE_{10} mode's propagation in the waveguide, which can be controlled by the cutoff frequency, the RF field still has only $h_x$ and $h_y$ components. Equation 2-15 is still valid except being simplified to

$$\mu_{\text{eff}} = \frac{\mu_{xx} \mu_{yy} + \mu_{xy}^2}{\mu_{yy}} = \mu_{xx}. \hspace{1cm} (2-19)$$
Fig. 2-6. Effective permeability ($\mu_{xx}$) of a YIG slab with in-plane bias field parallel to wave propagation.

In experiments, the AC/RF magnetic field will also have $z$ components due to the magnetic precession. Higher order modes, such like TM$_{01}$ mode, will be excited in the rectangular waveguide. However, because the higher order modes are small and detectors connected to the vector network analyzer only detect TE mode, it is practical to only consider TE$_{10}$ mode in the analysis.

Comparing the two bias conditions, one can conclude that the $\hat{H} \perp \beta$ case is more favorable than the $\hat{H} \parallel \beta$ case for the negative permeability applications. Because the precession motion of magnetic moment in the ferrite slab in the first case fits the TE$_{10}$ mode well. In addition, for the same bias field magnitude, perpendicular bias
achieves an FMR frequency about $\gamma' \cdot 2\pi M_s$ higher as shown in Fig. 2-6.

2.2. YIG Slab with Out-of-plane Bias

Another commonly used bias condition is when the DC field is applied out-of-plane as shown in Fig. 2-7. The permeability tensor can be similarly solved by following the steps shown before.

Fig. 2-7. YIG slab under out-of-plane magnetic bias field.

By doing the similar algebra of Eqn. 2-4, the internal field can be obtained as
\[
\frac{r}{H_{\text{eff}}} = \left( H - 4\pi M_s \right) \hat{x} + \frac{r}{h} + \frac{H}{M_s} \left( m_y \hat{y} + m_z \hat{z} \right). \tag{2-20}
\]

Similarly the permeability tensor can be solved by putting Eqn. 2-20 into the equation of motion (Eqn. 2-8).

\[
\frac{j\omega}{\gamma} r = \left[ H - 4\pi M_s \right] \hat{x} - \frac{r}{h} - \frac{H}{M_s} \left( m_y \hat{y} + m_z \hat{z} \right)
\]

\[
\Rightarrow \frac{j\omega}{\gamma} \left( m_y \hat{y} + m_z \hat{z} \right) = -\left( m_y \hat{y} + m_z \hat{z} \right) \times \left( H - 4\pi M_s \right) \hat{x} - \left( M_y \hat{x} \right) \times \left( h \hat{y} + h \hat{z} \right) + \frac{H}{M_s} \left( m_y \hat{y} + m_z \hat{z} \right)
\]

\[
\Rightarrow \begin{cases}
\frac{j\omega}{\gamma} m_y = -m_z \left( H - 4\pi M_s - H_a \right) + M_y h_z \\
\frac{j\omega}{\gamma} m_z = m_y \left( H - 4\pi M_s - H_a \right) - M_y h_y
\end{cases}
\]

\[
\Rightarrow \begin{cases}
\frac{j\omega}{\gamma} m_y + \left( H - 4\pi M_s - H_a \right) m_z = +M_y h_z \\
- \left( H - 4\pi M_s - H_a \right) m_y + \frac{j\omega}{\gamma} m_z = -M_y h_y
\end{cases}. \tag{2-21}
\]

Therefore, the \(2 \times 2\) susceptibility is

\[
\left[ \chi \right] = \frac{M_s}{\Omega^2} \begin{bmatrix}
H_1 & j\frac{\omega}{\gamma} \\
-j\frac{\omega}{\gamma} & H_2
\end{bmatrix}, \tag{2-22}
\]

where \(H_1 = H_2 = H - 4\pi M_s - H_a\), and \(\Omega^2 = H_1 H_2 - \frac{\omega^2}{\gamma^2}\).

So the permeability tensor is
\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 + \frac{4\pi M}{\Omega^2} H_1 & j\frac{4\pi M}{\Omega^2} \frac{\omega}{\gamma} \\
0 & j\frac{4\pi M}{\Omega^2} \frac{\omega}{\gamma} & 1 + \frac{4\pi M}{\Omega^2} H_2
\end{bmatrix}
\]  \hspace{1cm} (2-23)

In this case, the external bias field needs to overcome the demagnetizing field to orient the magnetic momentum along x-axis. Consequently, to obtain one FMR frequency, out-of-plane bias requires a larger (\(4\pi M\)) field than in-plane bias. So it is not favorable for our applications.

### 2.3. M- and Y- Type Hexaferrites

Considering proper low loss ferrite materials for constructing TNIMs, YIG is an ideal candidate for L- (1 - 2 GHz), S- (2 - 4 GHz), C- (4 - 8 GHz), and X- (8 -12 GHz) bands with a bias field less than 7000 Oe. The high quality single crystal YIG can have linewidth smaller than 1 Oe. And the commercial available polycrystalline YIG has linewidth around 25 Oe. For higher frequency bands applications, hexaferrites having the magnetoplumbite structure are capable due to their high anisotropy fields\(^{26}\).

In contrast to the garnets and spinel ferrites, the magnetoplumbite structure is hexagonal in symmetry. Because of the symmetry of the hexagonal crystal lattice, the
magneto-crystalline anisotropy energy is given by the equation:

$$w_K = K_1 \sin^2 \theta + K_2 \sin^4 \theta + K_3 \sin^6 \theta + K_3 \sin^6 \theta \cos 6(\phi + \varphi).$$

(2-24)

The angle \(\theta\) and \(\phi\) are polar coordinates and the constants \(K_i\) are the coefficients of the magnetocrystalline anisotropy. The phase angle \(\varphi\) is zero for a particular choice of the axis of the coordinate system.

When the term with \(K_1\) is dominant, the spontaneous magnetization is oriented parallel to the \(c\) axis for \(K_1 > 0\); the case of M-type hexaferrite. For \(K_1 < 0\), the spontaneous magnetization is oriented perpendicular to the \(c\) axis; the case of Y-type hexaferrite. In general the angle \(\theta_0\) between the direction of the spontaneous magnetization and the \(c\) axis is a function of \(K_1\) and \(K_2\) as

$$\sin \theta_0 = \sqrt{-K_1 / 2K_2}.$$

For \(\phi\) is constant, the anisotropy field \(H^A\) is defined as:

$$H^A_\theta = (1 / M_s) \left( \frac{\partial^2 w_K}{\partial \theta^2} \right)_{\varphi=\text{const}}.$$

(2-25)

So that for

$$\theta_0 = 0: H^A_\theta = H^A = 2K_1 / M_s,$$

(2-26a)

$$\theta_0 = 90: H^A_\theta = -(K_1 + 2K_2) / M_s,$$

(2-26b)

$$\sin \theta_0 = \sqrt{-K_1 / 2K_2}: H^A_\theta = 2(K_1 / K_2)(K_1 + 2K_2) / M_s.$$

(2-26c)

Among the most popular of the microwave hexaferrites are those derived from the barium M-type (BaM) hexaferrite, BaFe\(_{12}\)O\(_{19}\) (see Fig. 1b). Their utility stems in part
from the alignment of the easy magnetic direction along the crystallographic c-axis and the ability to process these materials with crystal texture. For example, the growth of BaM films with crystal texture leads to perpendicular magnetic anisotropy: a requirement for conventional circulator devices.

The M-type hexaferrite consists of spinel blocks (S) that are rotated 180 degrees with respect to one another and separated by an atomic plane containing the Ba atoms (R blocks). This plane of atoms breaks the crystal symmetry resulting in the hexagonal structure and large magnetocrystalline anisotropy energy. Remarkably, the magnetic anisotropy field in this ferrite is $\sim 17,500$ Oe, 1000 times greater than the cubic ferrites. For the same in-plane bias as Fig. 2-1 with the c axis also in-plane parallel to the bias field, the FMR frequency $f_{\text{FMR}}$ of M-type hexaferrite is given by:

$$f_{\text{FMR}} = \gamma' \sqrt{(H + H_A)(H + 4\pi M_s + H_A)}$$  \hspace{1cm} (2-27)

The large $H_A$ places the zero field FMR frequency near 36 GHz. As such, devices based upon this ferrite can operate at frequencies as high as Ka-band. For low frequency applications, the anisotropy field, $H^A$, of BaM can also be reduced by doping certain elements such like scandium (Sc). The $4\pi M_s$ of BaM is $\sim 3,300$ Oe and the linewidth $\sim 200$ Oe. Although the linewidth of BaM is much bigger than the one of YIG, which will cause large magnetic loss near FMR, the larger $4\pi M_s$ implies wider negative permeability range.

Another type of hexaferrite ideal for TNIM applications is the Y-type hexaferrite. Its
The chemical formula is $BaMe^{2+}Fe^{3+}_{6}O_{11}$, where $Me$ can be divalent ions of metals like Fe, Mn, Zn, Co, Mg, & Ni. Y-type hexaferrites can be self-biased and be oriented with the in-plane easy axis perpendicular to the $c$-axis and the effective saturation magnetization, $4\pi M_{\text{eff}} = 4\pi M_{s} + H^{A}$, is $\sim 12,000$ Oe$^{28}$. The anticipated frequency range of negative permeability is $\gamma' \cdot 2\pi M_{\text{eff}} = 16.8 \text{GHz}$ according to the previous calculations. Because the reported linewidth of Y-type hexaferrites is of the same magnitude as M-type, smaller magnetic losses can be achieved for the TNIM applications. The large negative $\mu'$ bandwidth gives the optimal working frequency farther away from FMR. In other words, a broad bandwidth and high figure of merit can be obtained. For the same in-plane bias as Fig. 2-1 with the $c$-axis perpendicular to the plane, the FMR frequency $f_{\text{FMR}}$ of Y-type hexaferrites is given by:

$$f_{\text{FMR}} = \gamma' \sqrt{H \left( H + 4\pi M_{s} + H^{A} \right)}$$  \hspace{1cm} (2-28)

The permeability tensor under the same bias condition as Fig. 2-1 of Y-type hexaferrite is given by:

$$\begin{bmatrix} 1 + \frac{4\pi M_{s}}{\Omega^2} H_{1} & j \frac{4\pi M_{s}}{\Omega^2} \frac{\omega}{\gamma} & 0 \\ -j \frac{4\pi M_{s}}{\Omega^2} \frac{\omega}{\gamma} & 1 + \frac{4\pi M_{s}}{\Omega^2} H_{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $H_{1} = H_{x} H_{y} = H + H^{A} + 4\pi M_{s}$, and $\Omega^2 = H_{1} H_{2} - \frac{\omega^2}{\gamma^2}$. 

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2.4. Ferromagnetic Metal - Permalloy

Ferromagnetic metals have both charge carrier and magnetism, among which permalloy (nickel iron alloy, 20% iron and 80% nickel) seems to be ideal for metamaterial applications with its good magnetic properties. The $4\pi M_s$ of permalloy is around 10,000 G and the FMR linewidth $\Delta H$ is 30 Oe. Due to its large $4\pi M_s$, permalloy requires relatively small magnetic bias field to shift the FMR frequency to the desired working frequencies. It also has a wide negative permeability region of around 14 GHz, which is estimated by $\gamma' \cdot 2\pi M_s$ as calculated before and its linewidth is small enough to avoid large magnetic losses.

However, the conductivity of permalloy (~7,000 S/m) has an exchanging coupling effect to the magnetism near the FMR. It will induce extra losses due to surface currents if combined with other plasmonic metal wires to construct a NIM. The surface currents will destroy the plasmonic effect in metal wires. If one uses permalloy to construct plasmonic wires trying to obtain negative permittivity, the magnetic effect will be too weak to generate negative permeability due to the small volume factor. Basically, by using permalloy itself, there are still difficulties and tradeoffs to overcome to tune its FMR and plasma frequencies independently to make simultaneous negative permeability and permittivity. The complex magnetoimpedance/exchange conductivity of permalloy near FMR also needs to be carefully examined to evaluate the feasibility for metamaterial applications. These
issues will be discussed rather independently in Chapter 7.

As a summary of this chapter, the permeability tensor of spinel structured single crystal YIG is calculated under different bias conditions. The effective permeability of a YIG slab placed in the center of a rectangular waveguide are also discussed under two in-plane bias conditions when the bias field perpendicular and parallel to the wave propagation. The properties of the negative permeability region, including the figure of merit and the bandwidth are carefully examined. The permeability properties of M- and Y- type hexaferrites are also discussed related to the YIG calculation. Their advantage for high frequency application up to millimeter wave range lies in the high anisotropy fields. Y-type hexaferrites can provide extra large bandwidth of negative permeability due to their large effective $4\pi M_{\text{eff}}$ contributed by the $H_A$ due to its in-plane (the hexagon plane) easy axis. The ferromagnetic metal permalloy is included in discussions as a possible NIM component although it is limited by its exchanging conductivity which induces extra electrical loss and also hurts the plasmonnic effect if only its magnetic property is needed to design NIMs. However, there may be opportunities in its exchange conductivity/magnetoimpedance, which is strongly coupled to the Lerentzian shaped permeability near the FMR.

2.5. Fabrication Techniques of Ferrite Thick Films

As discussed in the previous section, in order to minimize the magnetic loss near
FMR and also to achieve negative permeability, small linewidth ferrite materials are needed. Generally single crystalline ferrite films show the best FMR linewidth properties. For example, single crystalline yttrium iron garnet (YIG) grown on gadolinium gallium garnet (GGG) can have a very small linewidth < 0.1 Oe, while the commercial available polycrystalline YIG has a linewidth ~ 25 Oe. The polycrystalline hexaferrites have FMR linewidth usually > 300 Oe, while single crystalline hexaferrites have linewidth < 100 Oe$^{29,30}$. Although ferrite thin films deposited on seed substrates have smaller linewidth than thick films or the bulk, they are not usable for most microwave devices due to the small volume. In order to realize negative permeability in a NIM structure using a ferrite, its volume factor needs to be large enough to generate a strong FMR effect. Therefore, usable ferrites have to be not only low loss, but also must exist as thick films or bulk forms.

One conventional way to grow high quality thick films of single crystalline ferrites is liquid phase epitaxy (LPE)$^{31}$. The LPE technique can grow high quality films up to 200µm on a substrate with a seed layer early deposited by thin film deposition technique such as laser pulse deposition (PLD). Conventional techniques to grow large bulk single crystalline ferrites include flux melt and Czochralski pulling methods$^{32,33}$. One experimental attempt to make single crystalline zinc doped Y-type barium ferrite is described in Appendix B.
3. Plasmonic Metal Structures

3.1. Plasma Frequency and Effective Permittivity

The plasmonic effect can be understood as collective oscillation of charge carriers. For metals, it happens in visible or ultraviolet frequency range. At lower frequencies dissipation destroys all trace of plasmon and typical Drude behavior sets in. It was proposed that periodic structures built of very thin wires dilute the average concentration of electrons and considerably enhance the effective electron mass through self-inductance\(^34\). The analogous plasma/cutoff frequency can be depressed into GHz or even THz bands.

The collective oscillation of charge carriers density can be described by a simple harmonic motion\(^35,36\). The plasma frequency, \(\omega_p\), is typically in the ultraviolet region of the spectrum. It can be described using the equation below:

\[
\omega_p^2 = \frac{ne^2}{\varepsilon_0 m_{\text{eff}}},
\]  

(3-1)

where \(m_{\text{eff}}\) is the effective mass of the charge and \(n\) the charge density. The plasmons have a profound impact on properties of metals, not least upon their interaction with electromagnetic radiation. Such an interaction produces a complex permittivity

\[
\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega - j\gamma)},
\]  

(3-2)
\( \gamma \) represents the damping, the energy dissipated into the system. Note that there is a notation difference, \( i \rightarrow -j \), between physics and engineering. Eqn. (3-2) shows that \( \varepsilon = \varepsilon' - j\varepsilon'' \) has negative real part below plasma frequency \( (\omega < \omega_p) \).

The plasma frequency can be lowered by reducing the effective change density \( n_{\text{eff}} \) or increase \( m_{\text{eff}} \) proposed by Pendry et al (Ref. 8). Considering a cubic lattice of intersecting thin straight metal wires with lattice constant \( a \) and wire radius \( r \),

\[
n_{\text{eff}} = n \frac{\pi r^2}{a^2}, \tag{3-3}
\]

\[
m_{\text{eff}} = \frac{1}{2} \mu \epsilon e^2 n \ln \left( \frac{a}{r} \right), \text{ and} \tag{3-4}
\]

\[
\omega_p^2 = \frac{n_{\text{eff}} e^2}{\varepsilon_m m_{\text{eff}}} = \frac{2\pi e^2}{\alpha^2 \ln \left( \frac{a}{r} \right)}. \tag{3-5}
\]

And the effective permittivity is:

\[
\varepsilon_{\text{eff}} = 1 - \frac{\omega_p^2}{\omega \left( \omega - j\varepsilon, \omega_p^2 / \pi r^2 \sigma \right)}, \tag{3-6}
\]

where \( \sigma \) is the conductivity of the metal. So \( \omega_p \) and consequently \( \varepsilon_{\text{eff}} \) can be tuned by the spacing and radius of the metal wires. Negative effective permittivity can be obtained with proper geometric diameters in the GHz band.

This 3D lattice of metal wires can be simplified to 2D or even 1D. Here 2D means periodicity on two directions and 1D on one direction keeping the same plasmonic effect when the electrical \( (e) \) field of propagating wave modes along the wires' longitudinal direction. In the transmission lines, it is not practical and either necessary to construct 3D lattice of metal wires. Because the wave propagates in one direction, a 1D lattice can obtain effective negative permittivity with the \( e \) field along the metal.
3.2. Effective Permittivity of 1D Metallic Wire Array

Consider a simple case as shown in Fig. 3-1. Five metal wires are placed in a transmission line with equal spacing \( a \). The radius of each wire is \( r \). The transverse electromagnetic (TEM) wave is propagating from the left such that \( E_r \) is parallel to the wire axis and \( H_\phi \) is perpendicular to the wire. The radius of the wires is sufficiently large compared to the skin depth, so that there is no wave propagation within the wire. This finite 1D periodic structure can also be analyzed qualitatively by Pendry's method. However, Eqn. 3-5 will not be accurate since it is deducted from infinite 3D lattice. A proper theoretic tool to analyze this scenario is transfer function matrix (TFM) theory\(^{37}\).
The metal wire element can be regarded as a lumped element so that its TFM is:

\[
A_w = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix},
\]

where

\[
Y = \left( \frac{l}{2\pi r} \sqrt{\frac{\omega \mu_0}{2\sigma}} (1 + j) + j \omega \mu_0 l \frac{\ln(a/r)}{2\pi} \right)^{-1}.
\]

\(r\) is the radius of the wire, \(l\) the length of the wire, \(a\) the distance between the wires, \(\omega\) the angular frequency, and \(\sigma\) the conductivity. The admittance of the wire comes from two parts, surface impedance and self-inductance. Surface impedance is related to the EM field within the wire and self-inductance is related to the EM field around wire. The region between the wires is treated as a continuous medium of air or dielectric and as such represented by:

\[
A_a = \begin{bmatrix} \cos(ka) & jZ \sin(ka) \\ j\sin(ka)/Z & \cos(ka) \end{bmatrix},
\]

where \(Z = \sqrt{\frac{\mu_0}{\varepsilon}}\), \(k = \omega \sqrt{\varepsilon \mu_0}\), and \(\varepsilon\) is the permittivity of the media. In the later chapters including ferrite in the TFM analysis, \(\mu_0\) is replaced by the effective permeability \(\mu_{\text{eff}}\) of the ferrite in the transmission line as calculated in Chapter II. There is no cross term between the TFM of wires and dielectric medium, because there is no direct EM interaction between them. Therefore, the TFM of the whole block in the dashed line box in Fig. 3-1, consisting five wires and four and two halves dielectric medium region is then:

\[
[A] = \left\{ [A_{a/2}] [A_w] [A_{a/2}] \right\}^5 \equiv \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}.
\]

Please note that in Eqn. 3-9, the unit cell is one wire and two halves \(a/2\) long.
dielectric region. The resultant matrix \([A]\) is still a \(2 \times 2\) matrix which allows us to express its elements, \(a_{ij}\), in terms of effective permittivity, \(\varepsilon_{\text{eff}}\), and effective permeability, \(\mu_{\text{eff}}\). Clearly, \(\varepsilon_{\text{eff}} \neq \varepsilon\) and \(\mu_{\text{eff}} \neq 1\) (Please note that both \(\varepsilon_{\text{eff}}\) and \(\mu_{\text{eff}}\) are relative values), since \(\varepsilon_{\text{eff}}\) and \(\mu_{\text{eff}}\) include the electromagnetic interaction between the wires and the dielectric medium.

The scattering S-parameters are related to the TFM as:

\[
S_{11} = \frac{a_{12} + (a_{11} - a_{22})Z_0 - a_{21}Z_0^2}{a_{12} + (a_{11} + a_{22})Z_0 + a_{21}Z_0^2} \quad \text{and} \quad (3-10)
\]

\[
S_{21} = \frac{2Z_0}{a_{12} + (a_{11} + a_{22})Z_0 + a_{21}Z_0^2}. \quad (3-11)
\]

where \(Z_o\) is the characteristic impedance of the medium at the input and output of lattice. In the case of Fig. 3-1, \(Z_o\) is the characteristic impedance of air. The effective refractive index \(n_{\text{eff}}\) and the effective impedance \(Z_{\text{eff}}\) (normalized to \(Z_o\)) may now be calculated in terms of \(S_{11}\) and \(S_{21}\) as\(^{39,40}\):

\[
n_{\text{eff}} = \pm \frac{c}{2\pi \cdot f \cdot L} \cdot \cos^{-1}\left(\frac{1 - S_{11}^2 + S_{21}^2}{2S_{21}}\right) \quad \text{and} \quad (3-12)
\]

\[
Z_{\text{eff}} = \pm \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}}, \quad (3-13)
\]

where \(c\) is the speed of light and \(L\) the total length of the device under test (DUT). \(L = 5a\) in this case. Then the effective relative permittivity \(\varepsilon_{\text{eff}}\) and permeability \(\mu_{\text{eff}}\) can be calculated by:

\[
\varepsilon_{\text{eff}} = \frac{n_{\text{eff}}}{Z_{\text{eff}}}, \quad (3-14)
\]

\[
\mu_{\text{eff}} = n_{\text{eff}} \cdot Z_{\text{eff}}. \quad (3-15)
\]
These two are often called the Nicolson-Ross-Weir equations. Here, consider a specific case of periodic copper wires and air medium. The geometric parameters are 
\[ r = 30\mu m \quad \text{and} \quad a = 1.0\,mm \quad \text{and} \quad l = 1.2\,mm \ . \quad \sigma = 5.8 \times 10^7 \, \text{siemens/m} \quad \text{and} \quad \varepsilon = \varepsilon_\infty . \]
S-parameters, \( n_{\text{eff}} \), \( Z_{\text{eff}} \), and \( \varepsilon_{\text{eff}} \) are calculated in Matlab\textsuperscript{TM} program and plotted out. As shown in Fig. 3-2 and Fig. 3-3, no transmission is allowed in the negative permittivity (\( \varepsilon' < 0 \)) region. The trends of \( \varepsilon' \) and S21 show that there is a cutoff frequency at a high frequency over 60 GHz which is illustrated in Fig. 3-4. The cutoff frequency is just the so-called "plasma frequency", \( f_p \).

![Fig. 3-2. Calculated S-parameters of the 1D array of copper wires in a transmission line.](image-url)

Fig. 3-3. Calculated effective permittivity of the 1D array of copper wires.

Fig. 3-5 plots out the complex refractive index $n$, impedance $Z$, permittivity $\varepsilon_{\text{eff}}$, and permeability $\mu_{\text{eff}}$ near the plasma/cutoff frequency $f_p \sim 60$ GHz. Both $n$ and $Z$ behave sharp changes at $f_p$, where $\varepsilon'$ crosses the zero line. However, $\mu_{\text{eff}}$ stays continuous and near flat with $\mu' \sim 1$ in the whole frequency range as expected by intuition. And $\mu_{\text{eff}}$ stays continuous and near flat with $\mu' \sim 1$ in the whole frequency range as expected by intuition.
Fig. 3-4. Calculated S-parameters of the 1D array of copper wires near its plasma frequency.

Fig. 3-5. Calculated complex (a) refractive index, (b) impedance, (c) permittivity, and
(c) permeability of the 1D array of copper wires near its plasma frequency.

Comparing Fig. 3-4 and Fig. 3-5, one can see that the first maximum transmission frequency is several GHz higher than $f_p$. So the first maximum transmission can be used to estimate $f_p$ in experiments.

The air medium has been assumed in this TFM calculations. But when combining the wires with ferrite, the wires will suffer high dielectric constant $>10$. This will significantly change $A_s$ in Eqn 3-8 and the following calculation results. The TFM calculations are conducted for a DUT consisting same copper wires but with dielectric medium with $\varepsilon_r=10$. The transmission and effective material prosperities are plotted in Fig. 3-6 and 3-7. Fig. 3-7(c) shows that $f_p$ is pushed down to ~18 GHz. The S21 and S11 show characteristic periodic behavior when $f > f_p$ due to resonance between the wires. In $f < f_p$ region, the negative $\varepsilon'$ is more frequency dispersive compared to the one in Fig. 5(c). And the values of $\mu_{\text{eff}}$ in Fig. 3-7(c) are very close to the same as the ones in Fig. 3-5(c).
Fig. 3-6. Calculated S-parameters of the 1D array of copper wires and dielectric medium near its plasma frequency.

The Matlab™ code is attached in Appendix A. One needs to be very careful when applying Eqn. 3-12 because the equation $a \cos(Z)$ in Matlab™ is complex and has many branches. One needs to choose the physical sign and the proper branch at different frequencies according to causality and continuity. This can be very subtle, especially near resonance where the phase is dispersive in frequency.
Fig. 3-7. Calculated complex (a) refractive index, (b) impedance, (c) permittivity, and (c) permeability of the 1D array of copper wires and dielectric medium near its plasma frequency.

The plasma frequency decreases as the dielectric constant of the medium in between the wires, $\varepsilon_r$, increases. And the relation is nonlinear as show in Fig. 3-8. In order to obtain negative permittivity, the higher $\varepsilon_r$ is, the denser the metal wires need to be. This can also be understand intuitively by the neutralization of the negative permittivity effect generated by the wires and the positive effect in the medium.
Fig. 3-8. Plasma frequency of the 1D array of copper wires and dielectric media versus dielectric constant of the dielectric medium.

The above TFM calculations assume 1D metallic wire array and ideal transmission lines with TEM mode wave propagation. However, in the real case, the chosen transmission lines to mount metamaterials may have wave modes other than TEM mode. For instance, the fundamental wave mode in a rectangular waveguide is $TE_{10}$ mode and the one in a microstrip line is quasi-TEM mode. Furthermore, the transmission line is not one dimensional. So the effective charge density is much diluted. The plasma frequency is expected to be much lower. Therefore, several rows of wires in parallel may be needed to achieve strong enough plasmonic effect, or, in another word, to ensure the plasma frequency high enough. In these cases, the estimation of the plasma frequency and the negative permittivity using TFM
calculations may go far off. Instead, high frequency 3D electromagnetic simulation tools such like Ansoft HFSS™ and CST Microwave Studio™ can model the S-parameters rather realistically and the effective material parameters can be determined using Eqn. 3-12 to 3-15.

### 3.3. HFSS™ Simulation of Plasmonic Wires

In this section, plasmonic wires are studied in detail using HFSS™ under different conditions in a microstrip line. First, this section describes HFSS™ simulations of one row of cylindrical copper wires in a microstrip line which is also intended as a component of NIM structures. The effective material parameters are calculated from the simulated S-parameters and they are also compared with the ones obtained from TFM calculations. Secondly, the effect of having a Teflon™ substrate for the microstrip line is also simulated. Thirdly, one row of rectangular copper wires on a 0.15 mm thick Kapton™ (dielectric constant \( \varepsilon' = 3.9 \)) and attached to one and two 0.8 mm thick gallium-arsenide (GaAs) (\( \varepsilon' = 12.9 \)) slabs are simulated in the microstrip line. Finally, two rows of wires attached to GaAs slabs, are simulated to show one way to shift the plasma frequency.

In Fig. 3-9, an HFSS™ model of a row of copper wires mounted in a microstrip line is illustrated. The geometric parameters are the same as in the TFM calculation \(( r = 30\mu m, \ a = 1.0mm, \ and \ l = 1.2mm \)). So the thickness of the substrate \( h \) is 1.2
mm, the same as the length of the wires. The width $w$ and the thickness $t$ of the copper strip are 5.6 mm and 0.4 mm respectively. The wires connect to the center strip and the ground to ensure the surface current flow, otherwise the plasmonic resonance cannot be generated and the TFM treatment of the wire will not be valid. This can be demonstrated in simulations and will be essential in experiments.

To determine the geometric parameters of the microstrip line, a small transmission line calculator named TX-Line is used. This tool can be found at the webpage of AWR Corporation. The TX-Line can also calculate stripline, coplanar waveguide, grounded coplanar waveguide, and slotlines. One can specify the substrate media and part of the geometric parameters in TX-Line and it calculates the unknown ones. Note that the characteristic impedance of the microstrip is frequency dependant so the impedance matching can not be broadband. For the only purpose of characterizing the material's EM properties, the impedance matching is not an issue. In experiments, the microstrip line is normally designed to match $50\Omega$ at the center of the working frequency range. Broadband impedance matching for microstrip line is not yet feasible.

The two rectangular sheets in Fig. 3-9 represent the waveports. There are specific requirement of wave ports setup which can be found on Ansoft's website under the subject of "Excitations and Boundary Conditions". Roughly, the height of the waveports need to be $6h$ to $10h$ and a proper width is $10w$ when $w > h$ or $5w$ when
In the waveport configuration, there is an option of "De-embed", which needs to be checked and the de-embedding length needs to be specified so that the S-parameters at the two surfaces at the front and the back of the DUT can be obtained. Without doing so, the two air lines at the two sides of the wire array will be included into the actual DUT because the reference planes are at the two waveports. The S-parameters cannot be directly inputted to Eqns. 3-12 and 3-13 for calculating effective material properties. In the configuration of Fig. 3-9, the total length of the transmission line is 30 mm. The length of the DUT (same as in Fig. 3-1) is 5 mm. The de-embedding lengths for the two waveports are both 17.5 mm. Boundary conditions in the simulation need to be specified carefully according to the HFSS™ manual.

Fig. 3-9. HFSS™ simulation model of one row of periodic copper wires in a microstrip line. The inset shows the enlarged view of the wires.

In Fig. 3-10, the simulated transmission and reflection properties are represented by
S21 and S11 from 5 to 25 GHz. The frequency sweep is rather large if under only one analysis setup, which generates meshes at one specified frequency and uses it for all the other frequencies in the sweep. In order to obtain accurate simulation results, the frequency range was divided into two as 5 to 15 GHz and 15 to 25 GHz to simulate. The two analysis setup frequencies were chosen at 10 GHz and 20 GHz. By examining the behavior of S21 and S11, one can tell the resonance frequency indicated by the dips near 15 GHz. This is where the permittivity changes from negative to positive. The wave transmission at its lower side is small and gradually rises with frequency.

There are differences between the simulation results and the one given by TFM calculations in Fig. 3-4. The plasma frequency given by the simulation is 40 GHz lower than the calculated one, which agrees with the qualitative estimation based on effective charge density. The simulated S21 at the lower frequency side is not as steep as the one given by the TFM calculation, which reveals that the negative permittivity does not increase in magnitude as rapidly as in Fig. 3-7 (c).
Fig. 3-10. Simulated magnitude of S21 and S11 of one row of five round copper wires in a microstrip line.

The simulated S-parameters in HFSS™ are exported into a txt file. The txt file was imported into Matlab™ program for calculating the effective material properties: $n_{\text{eff}}$, $Z_{\text{eff}}$, $\varepsilon_{\text{eff}}$ and $\mu_{\text{eff}}$. In the program, unwrapping the real part of the output of

$$\cos^{-1}\left(\frac{1-S_{11}^2+S_{21}^2}{2S_{21}}\right)$$

is necessary as in the TFM calculations. The sign also needs to be carefully chosen according to the condition $n'' > 0$ and $Z' > 0$. The results are shown in Fig. 3-11. The $\varepsilon'$ changes sign at the resonant frequency near 15 GHz, which confirm the qualitative analysis based on the S-parameters. All parameters show sharp peaks at this frequency too. The $n'$ stays positive while $n''$ is very large at the lower frequency side.
Fig. 3-11. Calculated complex (a) refractive index, (b) impedance, (c) permittivity, and (c) permeability from the simulated S-parameters of one row of five round copper wires in a microstrip line near its plasma frequency.

The simulation model in Fig. 3-9 uses air substrate for the microstrip line. In order to study the effect of dielectric substrate, Teflon™ substrate (dielectric constant $\varepsilon_r = 2.1$, dielectric loss tangent $\tan \delta_e = 0.001$) is used for the same simulation. Because it is also possible to embed copper wires in a Teflon™ layer or a PCB board in experiments.

Figure 3-12 shows the simulated S-parameters in this case. The curves are smoother compared with in Fig. 3-11 and the sharp peaks at the resonance disappear. The
retrieved parameters show that the plasma frequency shifts ~ 2 GHz to a lower frequency. The abnormal bend of the impedance and permeability in Fig. 3-13(d) is not physical. It comes from the numeric error of the simulation and the retrieving method.

![Graph of S21 and S11 magnitudes](image)

**Fig. 3-12.** Simulated magnitude of S21 and S11 of one row of copper wires in a microstrip line on a Teflon™ substrate.
Fig. 3-13. Complex (a) refractive index, (b) impedance, (c) permittivity, and (c) permeability calculated from simulated S-parameters of one row of five copper wires in a microstrip line on Teflon™ substrate.

In experiments, ordinary lithography is used to fabricate metal wires from copper laminate on Kapton™ ($\varepsilon_r = 3.9$, $\tan \delta_r = 0.001$) substrate and the cross-section of fabricated wires are rectangular instead of round. One dimension of the wire cross-section is decided by the thickness of the laminate. Figure 3-14 shows the simulated magnitude and phase of S21 and S11 of one row of rectangular copper wires in a microstrip line with no substrate. The cross-section of the wires is 0.025 x 0.3 mm$^2$ and Fig. 3-15 shows the retrieved parameters. One can tell the curves are close to Fig. 3-10 and 3-11. The plasma frequency is near 20 GHz.
Fig. 3-14. Simulated magnitude and phase of S21 and S11 of one row of five rectangular copper wires in a microstrip line.

Fig. 3-15. Complex (a) refractive index, (b) impedance, (c) permittivity, and (c) permeability calculated from simulated S-parameters of one row of five rectangular copper wires in a microstrip line.
Figure 3-16 shows the case that the rectangular copper wires are on a thin Kapton™ substrate. The Kapton™ substrate is 0.15 mm thick and 5.0mm long. Figure 3-17 and 3-17 show the simulation results, which are almost the same as in Fig. 3-14 and 3-15. So the Kapton™ substrate has small influence on the plasmonic effect. The plasma frequency is still near 20 GHz. The reason may be its small dielectric constant and thickness. However, the dielectric constants of ferrites are all bigger than 10. And in order to achieve strong enough FMR to realize negative permeability, the volume cannot be too small. This volume factor issue needs to be studied more in details to direct the experiments which is included in later chapters.

Fig. 3-16. HFSS™ simulation model of one row of rectangular copper wires on a 0.15 mm thick Kapton™ substrate in a microstrip line. The inset shows the enlarged view of the wires.
Fig. 3-17. Simulated magnitude and phase of S21 and S11 of one row of rectangular copper wires on a 0.15 mm thick Kapton™ substrate in a microstrip line.

Fig. 3-18. Complex (a) refractive index, (b) impedance, (c) permittivity, and (d) permeability calculated from simulated S-parameters of one row of rectangular copper
wires on a Kapton™ substrate in a microstrip line.

To avoid dealing with the magnetic properties in the simulation at this stage, wires on Kapton™ substrate attached laterally to a 0.8 mm thick GaAs slab is simulated because GaAs has a high dielectric constant \( \varepsilon_r = 12.9 \) close to ferrite. The S-parameters and effective material properties are similar to the previous two cases. The retrieved parameters show that the plasma frequency shifts \( \sim 2 \text{ GHz} \) lower to near 17 GHz; the reason lies in the lateral positioning of the wire array and the dielectric. In this case, only one GaAs slab stays at one side of the wire array. To construct a NIM composite, ferrite slabs may need to put at two sides of the wire array.

![Graph showing simulated magnitude and phase of S21 and S11](image)

Fig. 3-19. Simulated magnitude and phase of S21 and S11 of one row of rectangular copper wires on a 0.15 mm thick Kapton™ substrate laterally attached to a 0.8 mm thick GaAs slab in a microstrip line. The inset shows the composite structure.
Fig. 3-20. Complex (a) refractive index, (b) impedance, (c) permittivity, and (d) permeability calculated from simulated S-parameters of one row of rectangular copper wires on a 0.15 mm thick Kapton™ substrate laterally attached to a 0.8 mm thick GaAs slab in a microstrip line.

Therefore, another composite with two GaAs slabs laterally at each side is simulated. The results in Fig. 3-21 and 3-22. The results are still similar to the previous three cases. The plasma frequency shifts lower to near 15 GHz. This is quite different from the case of one-dimensional array predicted by TFM calculations, where the plasma frequency shifts ~ 40 GHz to a lower frequency when replacing the air media between wires with a \( \varepsilon_r' = 10 \) dielectric media.
Fig. 3-21. Simulated magnitude and phase of S21 and S11 of one row of rectangular copper wires on a Kapton™ substrate laterally attached to two 0.8 mm thick GaAs slabs as shown in the inset.

Fig. 3-22. Complex (a) refractive index, (b) impedance, (c) permittivity, and (c)
permeability calculated from the simulated S-parameters of one row of rectangular copper wires on a 0.15 mm thick Kapton\textsuperscript{TM} substrate laterally attached to two 0.8 mm thick GaAs slabs at each side in a microstrip line.

The above simulations confirm that attaching dielectric slabs to the lateral sides of the plasmonic wires weakens the plasmonic effect and lowers the plasma frequency. The frequency shift depends on the volume factor of dielectric slab. In order to design the plasmonic wires to have a high enough plasma frequency when specific volume of ferrites is necessary, one can increase the effective charge density by using smaller spacing, thicker wires, or attaching several rows of wires together.

![Graph showing simulated magnitude and phase of S21 and S11](image)

Fig. 3-23. Simulated magnitude and phase of S21 and S11 of two rows of rectangular copper wires on Kapton\textsuperscript{TM} substrates laterally attached to two 0.8 mm thick GaAs slabs as shown in the inset.

Figure 3-23 and 3-24 show the simulation results of the composite of two rows of
rectangular wires one Kapton\textsuperscript{TM} substrate attached to two 0.8 mm thick GaAs slabs, which is equivalent to adding one row of wires to the composite in Fig. 3-21. The plasmonic frequency is strengthened and the plasma frequency is shifted \( \sim 2 \text{ GHz} \) upward to near 17 GHz.

![Graphs showing refractive index, impedance, permittivity, and permeability](image)

Fig. 3-24. Complex (a) refractive index, (b) impedance, (c) permittivity, and (c) permeability calculated from simulated S-parameters of two rows of rectangular copper wires on Kapton\textsuperscript{TM} substrates laterally attached to two 0.8 mm thick GaAs slabs.

The above simulations on the plasmonic wires are all in a microstrip line. For a different type of transmission line, for instance, rectangular waveguide, the qualitative conclusion should also be valid. Because the wave modes are different, the sensitivity
of the effective permittivity to the geometric parameters varies. The key of designing the wire array for realizing negative index is to ensure the plasma frequency is above the desired working frequency range after it is combined with ferrites.
4. Measurement of Refractive Index

To demonstrate a NIM structure, the intuitive way is to show the negative refraction. This requires a large 3D structure, usually in a wedge shape in a free space measurement\textsuperscript{41}. By measuring the refraction angle of the wave front at the interface of the metamaterial wedge and air, positive or negative refraction can be decided and Snell's law can be used to calculate the effective refractive index (only the real part of the complex refractive index). Furthermore, in order to demonstrate the refraction, the metamaterial also has to be isotropic in the plane of wave propagation.

The TNIM demonstrations using high quality ferrites are in transmission line for the purpose of simplicity and also for their value towards microwave device applications. The experimental measurement data will mainly be S-parameters (transmission and reflection) collected using vector network analyzer (VNA). The proof of negative index is based on S-parameters. One direct way is to retrieve the permittivity \( \varepsilon = \varepsilon' - j\varepsilon'' \), permeability \( \mu = \mu' - j\mu'' \), refractive index \( n = n' - jn'' \) and impedance \( Z = Z' - jZ'' \). Ideally, from the complex \( S_{11} \) and \( S_{21} \) (reflection and transmission), four independent unknowns can be solved. As shown in Chapter 3, \( n \) and \( Z \) are firstly retrieved from \( S_{11} \) and \( S_{21} \) then \( \varepsilon = \frac{n}{Z} \) and \( \mu = n \cdot Z \).

This is not a trivial task even or a dielectric material\textsuperscript{42,43,44}. First, the measurement method needs to be broadband. Broadband complex permittivity, permeability, or both measurements are very sensitive, especially in the measurement of signal phase. Small errors in the measurement can lead to large errors in the final results after the
complicated retrieval procedure. The transmission line used here needs to be well calibrated to the two ends of the transmission line in broadband. For example, if the sample is in a rectangular waveguide, the calibration sets for the specific rectangular waveguide are needed to calibrate at the two ends of the waveguide, or in another word, at the two coaxial to waveguide adapters. The calibration sets that come with the VNA are for coaxial calibrations, which only calibrate the measurement system to the two coaxial connectors. For the Agilent E8364A PNA Series Network Analyzer used in this research, the two coaxial ports are of the SMA type. The standard transmission/reflection (TR) measurement is illustrated in Fig. 4-1.

Fig. 4-1. Illustration of transmission line measurement of material properties of a sample/DUT. Port 1 and port 2 denote calibration reference planes.

Secondly, the equations (Eqn. 3-12 to 3-15) used previously in Chapter 3, often called the Nicolson-Ross-Weir equations, becomes numerically unstable when the length of the sample is multiples of half wavelength as illustrated in Fig. 4-2. The reason is that at these frequencies, \(|S_{11}|\) becomes very small, which causes the instability. Furthermore, the error in the phase of \(|S_{11}|\) also increases as \(|S_{11}| \to 0\)

consequently\textsuperscript{46}.

Fig. 4-2. Determination of the relative permittivity of a PTFE sample as a function of frequency using the Nicolson and Ross equations (solid line) and the iteration procedure (dashed line) (Ref. 6).

To bypass this problem, many researchers resort to using short samples. However, use of short samples lowers the measurement sensitivity. In fact, to minimize the uncertainty in low-loss materials a relatively long sample is preferred.

Another way to solve this problem is to iteratively solve various combinations of Eqns. 4-1 to 4-5 producing a solution that is stable over the measurement spectrum (Ref. 6). The sample length $L$ and air line length $L_{air}$ can be treated as unknowns in the system of equations by solving combinations of Eqn. 4-1 to 4-5. The solution of these equations is then independent of reference plane position, air line length, and sample length. For example, Eqns. 4-4 and 4-5 constitute four real equations that are independent of reference plane. They can be solved as a system with both the sample length and the air line length treated as unknown quantities.
\[ |S_{21}| = \left| \frac{Z(1-\Gamma^2)}{1-Z^2\Gamma^2} \right|, \quad (4-1) \]
\[ |S_{11}| = \left| \frac{\Gamma(1-z^2)}{1-Z^2\Gamma^2} \right|, \quad (4-2) \]
\[ \frac{S_{11}S_{22}}{S_{12}S_{21}} = \frac{(1-\varepsilon/\mu)}{4\varepsilon/\mu} \sinh^2 \gamma L, \quad (4-3) \]
\[ \frac{S_{21}}{S_{21}^0} = \exp(\gamma_0 L) \frac{Z[1-\Gamma^2]}{1-Z^2\Gamma^2}, \quad (4-4) \]
\[ S_{21}S_{12} - S_{11}S_{22} = \exp\left[(-2\gamma_0)(L_{air} - L)\right] \frac{Z^2 - \Gamma^2}{1-Z^2\Gamma^2}, \quad (4-5) \]

where \( \gamma = j \sqrt{\frac{\omega^2 \varepsilon \mu}{c^2} - \left(\frac{2\pi}{\lambda_c}\right)^2} \), \( \gamma_0 = j \sqrt{\frac{\omega^2}{c^2} - \left(\frac{2\pi}{\lambda_c}\right)^2} \), and \( \Gamma = \frac{\gamma_0 \mu_0 - \gamma / \mu}{\gamma_0 \mu_0 + \gamma / \mu} \).

Finally, because the TNIMs consisting of ferrites work near the FMR, the broadband simultaneous determination of the complex permittivity and permeability has to extend across the resonant frequency, which makes all available methods very difficult or even impractical at all. Because near the FMR, there is large absorption and the complex permeability of ferrites is extremely frequency dispersive, which cause the phase measurements experience large errors and the iteration method to solve these equations become difficult to converge.

Fortunately, there is a rather simple way to qualitatively demonstrate negative refraction index in the transmission line. Because the EM wave does not propagate through a media with only negative permittivity or permeability, the existence of
simultaneous negative permittivity and permeability can be confirmed by showing a passband of the NIM sample at frequencies where no transmission is allowed with only the negative permittivity composite or the negative permeability composite. In our case, there is no transmission allowed with only the plasmonic wires below the plasma frequency as demonstrated in the theoretical calculations and HFSS™ simulations of Chapter 3. By combining such a wire array properly with ferrite slabs under magnetic bias, a passband that appears near the FMR frequency confirms simultaneous negative permittivity and permeability and consequently negative index.
5. K-band TNIM and Phase Shifter Using Single Crystalline YIG

5.1. K-band TNIM

In this chapter we present experimental results of a TNIM using single crystalline YIG and an array of copper wires in a K-band rectangular waveguide. The tunability is demonstrated from 18-23 GHz under an applied magnetic field with a figure of merit of 4.2 GHz/kOe. The tuning bandwidth is measured to be 5 GHz compared to 0.9 GHz for fixed field. We measure a minimum insertion loss of 4 dB (or 5.7 dB/cm) at 22.3 GHz. The measured negative refractive index bandwidth is 0.9GHz compared to 0.5 GHz calculated by the transfer function matrix theory and 1GHz calculated by finite element simulation. The control achieved on material parameters and tunability could pave the way for the development TNIM microstrip, stripline, and coplanar guided wave structures.

We demonstrate a scheme by which continuous frequency tuning of the negative index is possible by using a YIG film or slab. The effect of the YIG film is to provide a tunable negative permeability over a continuous range of frequencies on the high frequency side of the ferrimagnetic resonance. Complementary negative permittivity is achieved using a row of periodic copper wires. Fig. 5-1 shows a schematic diagram of this tunable NIM in a K-band waveguide. The composite structure consists of 8
copper wires spaced 1 mm apart and a multilayered YIG film with a total thickness of 400\( \mu m \). YIG films were deposited by liquid phase epitaxy on both sides of a GGG (gadolinium gallium garnet) substrate. The thickness of the GGG substrate was 500\( \mu m \). The condition for FMR was obtained by applying the external magnetic field, \( H \), along the x-axis (see Fig. 5-1).

![Fig. 5-1. Schematic diagram of the experimental setup showing the NIM composite inserted in a K-band waveguide. The composite structure consists of 8 copper wires spaced 1 mm apart and multilayered YIG films with a total thickness of 400\( \mu m \). The shaded regions are YIG films whereas the black lines represented copper wires. Notice that the ferrite is separated from the wires by nonmagnetic dielectric material.](image)

As can be seen in Fig. 5-1, an air gap is maintained between the periodic array of copper wires and the YIG slab in order to reduce the coupling between the wires and the ferrite. In the frequency regime where \( \mu' \) is negative the close proximity of the ferrite to the wires implies a reduction in net current flow in the wires and, therefore, toward positive \( \varepsilon' \). The separation also reduces the effective dielectric loss induced
by the interaction of the wires’ self field with $\mu''$. In the copper wires there are two sources of conduction: (1) the microwave electric field in a waveguide produces a current in the wire, and (2) any extraneous microwave magnetic field due to the ferrite excited in a precessional motion induces an electric field along the wires. Clearly, proximity of the ferrite plays an important role. At frequencies where $\mu'$ is negative, the induced microwave magnetic field is opposite to the field excited in a TE$_{10}$ mode of propagation in a waveguide. Hence, the induced current by mechanism (2) is opposite to the current resulting from the electric field in a waveguide, i.e., mechanism (1).

In fabricating the TNIM, the copper wires were prepared by traditional lithographic techniques resulting in an array of 8 wires, 25$\mu$m thick and 100$\mu$m in width, spaced 1 mm apart. The single crystalline YIG was cut to form two 7mm$\times$4mm pieces. The composite was placed in the center of a K-band (18-26 GHz) waveguide as depicted in Fig. 5-1. S-parameters were measured using an HP 8510 network analyzer (45 MHz - 40 GHz). An electromagnet was used to generate the external magnetic field.

The magnitude and phase of $S_{21}$ as measured using the network analyzer at K-band with a bias field of 6.9 kOe, are shown in Fig. 5-2. In order to improve the impedance mismatch at the input of the device, we used a pair of E-H tuners connected before and after the waveguide containing the NIM composite. The pass band where the
loss is smaller than 6 dB is 0.9 GHz centered at 22.3 GHz.

Fig. 5-2. Measured amplitude (solid line) and phase (dashed line) of $S_{21}$ of the TNIM composite inserted in the K-band waveguide.

The material parameters $n_{eff}$, $\varepsilon_{eff}$ and $\mu_{eff}$ can be determined from the measured scattering parameters by using Nicolson-Ross-Weir equations discussed in Chapter 4, which are rewritten in the following for readers' convenience.

\[ \varepsilon_{eff} = \frac{n_{eff}}{Z_{eff}}, \tag{5-1} \]

\[ \mu_{eff} = n_{eff} \cdot Z_{eff}, \tag{5-2} \]

\[ n_{eff} = \pm \frac{c}{2\pi \cdot f \cdot L} \cdot \cos^{-1} \left( \frac{1 - S_{11}^2 + S_{21}^2}{2S_{21}^2} \right), \tag{5-3} \]
The real and imaginary parts of index of refraction were calculated using Eqn. 5-3 and are shown in Fig. 5-3. When the composite structure is sufficiently narrow that the $E_f$, electric field, and the $H_f$, magnetic field, are approximately uniform over the center region of the waveguide. There is no longitudinal $E_f$ and a small (or negligible) longitudinal $H_f$ fields in the region. We define the refractive index as $n = n' + in''$, where $i = \sqrt{-1}$. Figure 5-3 shows the plot of $n_{eff}$, as a function of frequency. The ambiguity in Eqns.5-3 and 5-4 with respect to sign is eliminated by considering $n'' = \text{Im}(n) > 0$ and $Z' = \text{Re}(Z) > 0$. In Fig. 5-3, the major downward peak of $n'$ centered at 23 GHz represents the negative index region.

Fig. 5-3. Real (solid line) and imaginary (dashed line) parts of the index of refraction retrieved from experimental data.
To demonstrate the tunability of the NIM we have carried out measurements of
transmission coefficient in different external magnetic fields. Figure 5-4 shows peak
shift of the region of negative index with the applied field. While varying the field
from 5.8 to 7.0 kOe, we observed that the magnitude of $S_{21}$ shifted from 18 to 23 GHz.
(The E-H tuners were removed for this measurement since these are narrow band
devices.) The tuning bandwidth for which the insertion loss was varied from -10 dB to
-4 dB was 5 GHz, (the magnetic field was varied from 5.8-7 kOe). The fixed field
bandwidth was 0.9 GHz. The bandwidth was defined as the frequency points when
the insertion loss increase to -6 dB relative to the center frequency, 22.3 GHz. We
believe that the insertion loss can be further improved with the use of E-H tuners at
each magnetic field setting and refinement of the copper wire arrays. As a comparison,
SRR NIMs operating at the X-band have been reported to have bandwidths
$<0.7GHz$ GHz at a single frequency without the ability to actively tune $n^{48}$. 
Fig. 5-4. Demonstration of the frequency tuning of the TNIM using magnetic bias field. The large arrow denotes the direction of frequency shift with increasing magnetic field.

The experimental results presented above can be understood using the theoretical transfer function matrix (TFM) analysis. In analyzing the wave propagation in the ferrite and wire composite we assumed a TEM wave propagation within a simpler composite which has the wires and the ferrites pieces in series as shown in the inset of Fig. 5-5. We would expect that the TFM analysis to be reasonably accurate in predicting the frequency region where the index may be negative, since FMR condition is contained within the formulation of the TFM theory. We have also carried our finite element simulation (using Ansoft HFSS) to support the theory. In the simulation, a slab of the periodic copper wires and YIG pieces in series was set at
the middle of the waveguide, where the same material and geometrical parameters were applied as in the case of TFM analysis.

We assumed that the width of the wire is sufficiently large compared to the skin depth so that propagation effects within the wire may be ignored. Hence, the wire is treated as a lumped element rather than a continuous medium. We represent the wire as a lumped element having an admittance $Y$ as illustrated in Chapter 3. The TFM representing the wire is

$$[A_1] = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$  \hspace{1cm} (5-5)$$

where

$$Y = \left( \frac{l}{2\pi r} \sqrt{\frac{\omega \mu_0}{2\sigma}} \right)^{\frac{1}{2}} (1-i) - i\omega \mu_0 \ln \left( \frac{d}{r} \right)$$

$\sigma = \text{conductivity}, \, \mu_0 = \text{permeability of air}, \, l = \text{length of wire}, \, r = \text{radius of the wire}, \, d = \text{distance between wires and} \, \omega = \text{angular frequency}$. The admittance of the wire, $Y$, comes from two parts, surface impedance and self-inductance. Surface impedance is related to the electromagnetic field within the wire and self-inductance is related to the electromagnetic field around wire. Since the medium between two successive wires is continuous the TFM is simply

$$[A_2] = \begin{bmatrix} \cos(kd) & -iZ \sin(kd) \\ -i\sin(kd)/Z & \cos(kd) \end{bmatrix}$$  \hspace{1cm} (5-6)$$

where $Z = \sqrt{\frac{\mu}{\varepsilon}}, \, k = \omega \sqrt{\mu \varepsilon}, \, \varepsilon$ is the permittivity and $\mu = \frac{\mu_{xy}^2 + \mu_{xx}^2}{\mu_{xx}}$ of the YIG material. If the medium is not transversely uniform, average values of $\varepsilon$ and $\mu$ should be used.
The TFM representing the composite as a whole is simply
\[
[A] = \left[ [A_1] \cdot [A_2] \right]^N = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}.
\]
(5-7)

The fact that the resultant matrix \([A]\) is still a \(2 \times 2\) matrix allows us to express its elements, \(a_{ij}\), in terms of effective permittivity, \(\varepsilon_{\text{eff}}\), and effective permeability, \(\mu_{\text{eff}}\). Clearly, \(\varepsilon_{\text{eff}} \neq \varepsilon\) and \(\mu_{\text{eff}} \neq \mu\), since \(\varepsilon_{\text{eff}}\) and \(\mu_{\text{eff}}\) include the electromagnetic interaction between the wires and magneto-dielectric medium. And S-parameters are obtained using
\[
S_{21} = \frac{2Z_0}{a_{12} + (a_{11} + a_{22})Z_0 + a_{21}Z_0^2}
\]
and
\[
S_{11} = \frac{a_{12} + (a_{11} - a_{22})Z_0 - a_{21}Z_0^2}{a_{12} + (a_{11} + a_{22})Z_0 + a_{21}Z_0^2}.
\]

Fig. 5-5. Real and imaginary parts of the effective refractive index calculated from the transfer function matrix theory (black line) and simulated using finite element method (red line). The inset shows the simplified structure of copper wires and YIG films in
In order to test the reliability and accuracy of the TFM calculations, we have also calculated \( n'_\text{eff} \) and \( n''_\text{eff} \) using HFSS. In Fig. 5-5, the solid lines are the results calculated from the S-parameters obtained from the TFM calculation whereas the dash lines are the results calculated from the simulated S-parameters using HFSS\textsuperscript{TM}. Both results make use of Eq.(5-3). In Fig. 5-5, \( n'_\text{eff} \) is negative for both calculation methods. \( n'_\text{eff} \), calculated by HFSS, shows a wider bandwidth (~ 1 GHz), whereas TFM theory predicts a narrower bandwidth (~ 0.5 GHz). The large values in \( n'_\text{eff} \) for the TFM method is a reflection of overestimating the ferrite volume, noticing that the TFM analysis is one dimensional so that the thickness of the YIG films is not considered. While in the HFSS\textsuperscript{TM} simulation, we set the thickness of YIG films as 0.4mm.

In this calculation, we have used the following parameters: \( \sigma = 5.8 \times 10^7 \text{ siemens/m} \), \( l = 0.43 \text{ cm} \), \( r = 12.5 \mu \text{m} \), \( d = 1.0 \text{ mm} \), \( 4\pi M_s = 1750 \text{ G} \) and \( H_{\text{ext}} = 6.9 \text{ kOe} \). The obvious conclusion is that the electromagnetic interaction and the dimensions of the waveguide affect the electrical parameters of the material constituents that make up the NIM. For example, if we calculate \( \varepsilon \) using HFSS\textsuperscript{TM} allowing for three dimensional variations and the \( TE_{10} \) mode of propagation in the waveguide, we find that
\[
\varepsilon \approx 1 - \left( \frac{\omega_p}{\omega} \right)^2, \quad \text{where} \quad \frac{\omega_p}{2\pi} = 27.2 \text{ GHz.}
\]
The above result compares well with
our one dimensional analysis which gives \( \frac{\omega_f}{2\pi} = 28 \text{ GHz} \). The behavior of \( \varepsilon \) as a function of frequency, as calculated from the one dimensional analysis, is very similar to that calculated using the HFSS\textsuperscript{TM} software. In essence, \( \varepsilon \) as calculated here, contains the correct "picture" of a negative permittivity which is important in our search for a negative index in waveguide structures.

It is well known that the FMR resonance can be tuned by the application of an external magnetic field, \( H \). Accordingly, the range of frequencies by which \( \mu \) is negative is given by

\[
\Delta f = f_0 - f_r \approx \frac{\gamma}{2\pi} (2\pi M_s),
\]

where \( f_r = \frac{\gamma}{2\pi} \sqrt{H(H+4\pi M_s)} \), \( \gamma/2 = 2.8 \text{ GHz} / \text{kOe} \), \( 4\pi M_s = 1750 \text{ G} \). \( f_r \) is the FMR frequency and \( f_0 \approx f_r + \frac{\gamma}{2\pi} (2\pi M_s) \). The subscript "0" is to denote the anti-resonance frequency where \( \mu' = 0 \). The object of tuning is to vary \( H \) such that \( f_r \) is varied according to Eqn. 5-8, and, therefore, the onset of the frequency range at which \( \mu' \) is negative, can be shifted with \( H \). However, the bandwidth of negative \( \mu' \) is approximately constant given by \( \frac{\gamma}{2\pi} (2\pi M_s) \). The factor of \( 2\pi M_s \) is a result of applying \( H \) in the film plane. If \( H \) is applied perpendicular to the film plane, \( \Delta f \) would scale as \( 4\pi M_s \). Clearly, if we are to realize a negative index we require both \( \mu' \) and \( \varepsilon' \) to be negative. This means that irrespective of \( H \), \( f_0 \) should not exceed \( f_p \), the electric plasma frequency of the wires below which \( \varepsilon' \) is negative.

If we choose \( f_0 = f_p \), the maximum applied field used for tuning is:
\[ H_{\text{max}} \approx \frac{\alpha_p}{\gamma} - 2\pi M_s. \] Note that it may not be possible to tune \( f_p \), if only conductive wires are used. However, since the region of negative \( \varepsilon' \) is broadband this does not pose a significant limitation to practical devices.

In summary, a broadband, low loss, and tunable NIM employing ferrite materials and copper wires is demonstrated. The tunability was demonstrated from 18 to 23 GHz under an applied magnetic field with tuning factor of 4.2 GHz/kOe. The \textit{tuning bandwidth} was measured to be 5 GHz compared to 0.9 GHz for fixed field, while the conventional SRR NIMs have bandwidths at X-band of <0.7GHz. We have measured a minimum transmission loss of 4 dB (or 5.7 dB/cm) at 22.3 GHz. Theoretical analysis using transfer matrix method (TFM) and finite element method simulations matches with the experimental results reasonably well. In future, we plan to use self-biased ferrites, which require smaller applied magnetic fields and transfer the tunable NIM technology to integrated microwave devices, microstrip, stripline, or coplanar guided wave structures. Furthermore, the utilization of semiconductor films instead of copper wires should provide the opportunity to tune permittivity by injection of carriers via light of field effect transistors.

### 5.2. TNIM Phase Shifter

This section describes a tunable compact phase shifter utilizing the K-band tunable NIM demonstrated. Near ferrimagnetic resonance, a phase shift tuning of 160
degree/kOe is achieved at 24 GHz. The insertion loss varies from -4.0 to -7.0 dB/cm.

Phase shifters are critical elements in several electronically tuned microwave systems in defense, space and commercial communications applications. Excessive cost and weight of the phase shifters has limited the deployment of electronically scanned antennas. While digital diode based phase shifters may withstand high power of the order of a few tens of watts, by virtue of their nature, the accuracy in phase shift is limited. Hence there is significant demand in the microwave industry for affordable, light weight, high power phase shifters. Microwave ferrite phase shifters can generally handle higher power than competing technologies. For example, commercial ferrite phase shifters can operate at an average power of up to one hundred watts and peak power two thousand watts. In ferrite phase shifters a change in permeability by the application of magnetic field causes a change in the phase velocity of the microwave signal traveling through the phase shifter. To be useful in microwave systems, a phase shifter should exhibit low insertion loss, minimal variation in the insertion loss with phase shift, and low return loss. Recent advances in metamaterials possessing NIM and strong dispersion characteristics with high value of \( \frac{dn}{d\omega} \) has opened the doors for novel microwave technologies, where \( n \) is the index of refraction, \( \theta \) is the phase change of the signal through the material, \( \Delta L \) is the length of the material and \( c \) is the velocity of light. The tunability and low loss observed in the TNIMs make them ideal materials for designing tunable, compact and light weight phase shifters.
Traditional ferrite phase shifters operate at frequencies away from the FMR in order to avoid absorption losses near the FMR frequency. As a result $\mu'$ is necessarily small. The passband realized in the negative index region can be tuned by the external magnetic field. The permeability of the TNIM was simultaneously tuned along with refractive index. The change in permeability or refractive index leads to a change in the phase velocity of the signal and, therefore, the phase of the transmission coefficient.

The advantage of using a ferrite NIM material for phase shifter application is that it allows the use of a ferrite in the negative $\mu'$ region near the FMR when $\mu'$ is relatively high and still maintains low losses. The low loss is a key property of the NIM composite. Near the FMR, the magnitude of $\mu'$ is larger than that at frequencies away from it. Assuming the loss factor to be about the same for the NIM and the conventional ferrite phase shifter, we would expect a much improved figure of merit using the NIM composite, since the phase shifts would be significantly higher due to higher differential $\mu'$ as illustrated in Fig. 5-6.
Fig. 5-6. Calculated complex permeability of high quality single crystal YIG films showing the different working frequency regions of traditional and TNIM phase shifters.

We have previously demonstrated a design scheme by which continuous frequency tuning of the negative index was possible using a YIG film or slab. Figure 5-7 shows schematic diagrams of the two cross sections of this tunable NIM in a K band waveguide. Two samples of lengths, 8.0 mm and 4.0 mm, were used on the direction of propagation. The sample was mounted in the K-band rectangular waveguide and transmission measurements were carried out using an HP 8510 network analyzer. In experiments, two narrow slots were cut on the top and bottom of the waveguide to mount the copper wires.
As shown in Fig. 5-8(a), a transmission peak centered at 22.5 GHz was observed in the magnitude of $S_{21}$. The dip near 22.4 GHz corresponds to the FMR. Another dip near 22.6 GHz is due to the effect of antiresonance. Note that near the FMR, the interaction between the wires and the YIG slabs is very strong causing modulations to $S_{21}$. 

Fig. 5-7. Schematic diagram of the TNIM composite mounted in a K-band waveguide from the back and side views.
Fig. 5-8. (a) Measured amplitude of $S_{21}$ for the NIM inserted in the K-band waveguide.

(b) Real part of the refractive index calculated from the phase change difference of the transmitted wave of the two samples with difference length.

In order to determine the real part of the index of refraction $n'$ unambiguously, we carefully measured the S-parameters of the two samples having different lengths. To measure the phase changes of the $S_{21}$ of the two samples, the reference planes of the scattering matrix were set to the two ends of the sample by de-embedding. By subtracting them, we got the absolute phase change, $\Delta \varphi$, of $S_{21}$ after propagating through the 4.0 mm NIM sample. $n'$ is calculated as described in the equations below. In the equations, $\Delta L$ is the length difference of the two samples, $k_0$ is the propagation constant of free space and $a$ is the transverse dimension of the K-band waveguide. In the square root, the second term is very small compared with the first one at high frequencies.

$$\Delta \varphi = \beta \cdot \Delta L = \Delta L \cdot \text{Re} \left[ \sqrt{(n' - jn'')^2 k_0^2 - (\pi/a)^2} \right] \approx \Delta L \cdot n' k_0$$  \hspace{1cm} (5-9)

$$n' \approx \Delta \varphi / \Delta L \cdot k_0$$  \hspace{1cm} (5-10)
We found that $\Delta \phi$ had a discontinuity at the FMR frequency and so is the index of refraction. As shown in Fig. 5-8(b), a negative refractive index region of 0.5 GHz width is determined from the measurements. Although theoretically YIG has a negative $\mu'$ region with a band width of up to 2.5 GHz, the small negative refractive index region is due to the small volume factor of the YIG slabs. Increasing the volume of the YIG will increase the absorption. Therefore there is a trade off between wide band width for negative index region and low loss. In addition the dielectric permittivity of the YIG slabs reduces the effective negative permittivity obtained from the plasmonic copper wires.

Fig. 5-9. Measured insertion phase and insertion loss versus the magnetic bias field at 24 GHz.
We studied the phase shift of a 10 mm long sample as well as the insertion loss performance of the 8 mm long sample as shown in Fig. 5-9. At 24 GHz, when the applied magnetic field was varied from 6.0 kOe to 7.0 kOe, the phase varied 160° with the insertion loss varying from -4.3 dB to -6.3 dB. At the lower field side, the phase change was smaller. The phase shifter was operated at the frequency above the ferrimagnetic antiresonance with a positive permeability with the material having a positive refractive index. At higher fields, the phase was more sensitive to field tuning, which corresponded to the negative refractive index region as illustrated in Fig. 5-6. Overall, the insertion loss had a variation of -2 dB, as a result of variation in wave impedance due to variation of the permeability.

In summary, a waveguide field tunable phase shifter is demonstrated using the K-band waveguide TNIM composite. It was the first application of its kind. Continuous and rapid phase tunability of 160 degree/kOe was realized with an insertion loss of -4 dB to -7 dB at 24 GHz. The phase change in the negative refractive index region is more sensitive to field tuning compared to the phase change of the positive refractive index region over the ferrimagnetic antiresonance. This introduces a new method of fabricating effective high power and compact phase shifters.
6. Microstrip TNIM and Phase Shifter Using Polycrystalline YIG

In Chapter 2 and 3, the independent behaviors of plasmonic metal wires and ferrite have been analyzed and demonstrated. The next step is to combine them together to demonstrate simultaneous negative permittivity and permeability. One can choose different transmission lines to construct TNIMs in, such as coaxial waveguides, rectangular waveguides, striplines, microstrip lines, and coplanar waveguides (CPWs). Generally, planar structures like microstrip lines and CPWs are favored because they are small in size and easy to accommodate to PCB circuits fabrications. Microstrip line has advantages in its planar structure and quasi TEM wave mode. Its open structure also makes it favorable for mounting a DUT.

In this chapter, we report the development of a planar microstrip TNIM that can form an integral part of TNIM-based RFIC devices such as a tunable phase shifter. The design, fabrication, and testing of a microstrip TNIM and phase shifter are described in detailed. Magnetic field tunable passbands resulted from negative refractive index were realized in X-band (7 - 12.5 GHz). Both simulated and experimental data show tunable passbands resulting from negative refractive index. Although the reported TNIM composite has a minimum 5 dB insertion loss mainly due to impedance mismatch and magnetic loss, this work would enable further developments of TNIM-based RFIC devices, including planar transmission lines, coplanar waveguides, compact delay lines, circulators, phase shifters, and antennas.
Fig. 6-1. (a) Schematic top and side views of the $10.0 \times 2.0 \times 1.2$ mm$^3$ TNIM composite. (b) Photo of the microstrip test fixture, a $5 \times 25$ mm$^2$ upper strip on the brass ground base relative to a U.S. quarter provided for a visual size comparison. The TNIM composite is mounted under the center of the upper strip.

As shown in the schematic diagram of Fig. 6-1(a), the TNIM is composed of two alternate layers of periodic copper wires etched on laminated Kapton™ substrates and polycrystalline yttrium iron garnet (YIG) slabs. Polycrystalline YIG slabs (real part of relative permittivity, $\varepsilon_r = 15.0$), cut and polished from high density bulk material, were used. The linewidth of the polycrystalline YIG, $\Delta H$, is ~25 Oe whereas the saturation magnetization, $4\pi M_s$, is 1750 G. Although polycrystalline YIG has higher magnetic losses near FMR than single crystalline one, it is commercially available in bulk forms. This feature helps achieve a broader NIM passband. Periodic copper wires on Kapton™ ($\varepsilon_r = 3.9$) were prepared by wet etching lithography using Dupont copper clad Kapton™ sheets. Mylar™ ($\varepsilon_r = 3.2$) spacers were cut from large sheets. The
dielectric loss tangents of YIG, Kapton™, and Mylar are rather small and negligible in this design. All the components were glued together using Krazy Glue™.

The two Mylar™ pieces act as spacers between the wires and YIG slabs. The thicknesses of YIG slabs, Kapton™ substrates, and Mylar™ spacers are 0.70, 0.15, and 0.25 mm respectively. The copper wires have a length of 1.2 mm, a cross section of $0.025 \times 0.3 \text{ mm}^2$, and a center to center spacing of 1.0 mm. The height of the composite TNIM is 1.2 mm, which is also the vertical distance between the upper strip and the ground plane of the microstrip. The 10.0 mm long TNIM composite is mounted under the center of the upper strip with the rf electrical field and dc magnetic bias field along the long axis of the copper wires. In this assembly, the TNIM composite and air at the two ends act as substrates for the microstrip. Figure 6-1(b) is the photo of the microstrip test fixture, the $5 \times 25 \text{ mm}^2$ brass upper strip on the ground base, positioned relative to a U.S. quarter to allow for size comparison.

![Simulated transmissions of the TNIM composite (blue circle), YIG slabs (red line), and Cu wires (green line). The magnetic bias field applied to the TNIM](image)
composite and YIG slabs is 3.5 kOe.

In order to elucidate and demonstrate the validity of the TNIM design, the transmissions in the TNIM composite, YIG slabs and copper wires are compared in finite element simulations using the Ansoft HFSS™ software suite as shown in Fig. 6-2. The applied magnetic bias field for the TNIM composite and YIG slabs is 3.5 kOe. Without the application of a magnetic bias field, the plasmonic effect of copper wires dominates, providing a negative permittivity so that there is no transmission below the plasma frequency (green line), which is designed to be much higher than 10 GHz. In the case of YIG slabs (red line), the dip in the region 7.7-8.4 GHz clearly indicates the FMR which results in negative permeability. Therefore, the broad transmission peak of the TNIM composite (blue line) is a result of negative refractive index. The dip near 9 GHz is attributed to the AFMR of the YIG material.
Fig. 6-3. (a) Simulated and (b) measured 1.0 GHz wide TNIM passbands of over -8dB transmission centered at 7.5, 8.0, and 8.8 GHz at magnetic bias fields of 3.0, 3.5, and 4.0 kOe respectively. (c) Center frequency of the TNIM passband increases linearly from 7.6 to 10.7 GHz with the bias field changing from 3.0 to 5.5 kOe.
A key advantage of the TNIM design is the magnetic field tunability of the passband over a wide range of frequency. In Fig. 6-3(a), the simulated transmissions of the TNIM composite under 3.0, 3.5, and 4.0 kOe show a clear frequency shift of the passband which is confirmed in measurements as shown in Fig. 6-3(b). As can be seen from Fig. 6-3(a) and 3(b), the simulated and measured results match very well validating the control achieved in the design on the TNIM and the dynamic bandwidth. The minimum insertion loss is around 5 dB. It is mainly due to the magnetic loss, eddy current loss, and impedance mismatch. The instantaneous bandwidth obtained is 1.0 GHz at a fixed bias field and the dynamic bandwidth is 2.3 GHz with a 1.0 kOe change in bias field. For comparison, the bandwidths of traditional SRR based NIM structures are generally smaller than 1.0 GHz. In Fig. 6-3(c), the curve of the center frequency of TNIM passband versus the bias field is plotted. The center of TNIM passband is tuned linearly from 7.6 to 10.7 GHz by varying the bias field from 3.0 to 5.5 kOe, resulting in a tuning factor of 1.2 MHz/Oe.
Fig. 6-4. Measured phase shift and corresponding transmission versus the magnetic bias field of the TNIM composite at 9.0 GHz. The insertion phase shifts 45° while the transmission varies from -6 to -10 dB with 0.7 kOe field change.

One significant feature of the TNIM is the realization of transmission on the high frequency side of the FMR, where the permeability of the YIG materials is negative and changes rapidly. At a fixed frequency in this region, by varying the bias field, the TNIM composite would experience a rapid change in permeability which can generate a change in the refractive index, propagation constant, and insertion phase. Therefore, a magnetic field tunable phase shifter based on the TNIM can be realized. In Fig. 6-4, the experimental performance of the TNIM composite as a phase shifter at 9.0 GHz is presented. The field tuning of the insertion phase at 9.0 GHz shows a shift of 45° as the applied bias field varies from 3.94 to 4.58 kOe. The corresponding transmission in the TNIM varies from -6 to -10 dB as the result of the change in wave impedance owing to the change in effective permeability. The tuning factor obtained is 70°/kOe.
In summary, a wide band microstrip TNIM using copper wires and polycrystalline YIG slabs is constructed to operate in X-band. The 1.0 GHz wide instantaneous TNIM passband, having a peak transmission of -5 dB, is tunable between 7.6 and 10.7 GHz by changing the bias field from 3.0 to 5.5 kOe. The TNIM composite can perform as a tunable phase shifter where the insertion phase shifts 45° for a field change of 0.64 kOe. The 10.0 × 2.0 × 1.2 mm³ TNIM composite is compact, mechanically robust, and compatible with planar microwave devices. Although the TNIM design still face the challenge of high insertion loss, this technology provides unique properties such as wide band and continuous frequency tunability of negative refractive index, which can enable designs and developments of innovative devices with enhanced functionality in small and light weight constructs. Furthermore, using this device configuration on a multiferroic substrate has the potential to allow for electric field tuning.

Furthermore, similar designs can be applied to lower frequencies to L-band (1 - 2 GHz) and higher frequencies up to K-band (18 - 26.5 GHz), which is mainly limited by the bias field. At lower frequencies, since smaller bias magnetic field is needed, the permanent magnet may be replaced by a coil. For example, for a TNIM working at 1 GHz, as a simple estimation, bias field around 360 Oe (1GHz / (2.8GHz/kOe) = 357Oe) will be needed. By replacing the polycrystalline YIG with single crystal one with ∆H < 10e, the insertion loss can be reduced significantly. However, high quality single crystal YIG does not have much market availability, either in thick films or bulk form, specially for small amount purchases.
7. Q-band TNIM and Phase Shifter Using Sc-doped BaM Hexaferrite

Extremely high frequency (EHF) is the highest radio frequency band which runs from 30 to 300 GHz. It is also referred to as millimeter wave. Compared to lower bands, terrestrial radio signals in this band are of very little use over long distance due to atmospheric attenuation. In the United States, the band 38.6 - 40.0 GHz is used for licensed high-speed microwave data links and therefore there is significant interest in developing novel microwave materials and devices near 40 GHz.

This chapter presents the work of constructing a TNIM performing near 40 GHz in a Q-band rectangular waveguide. The simple structured TNIM composite consists of one Sc-BaM slab and two rows of copper wires. The theoretically calculated µ of the Sc-BaM slab shows a negative region near its FMR, which is coincident with the negative ε generated by the copper wires. The tunable passband indicating the negative n is demonstrated in both experiments and simulations. The effect of ferrite volume factor (FVF) of the Sc-BaM slab is studied experimentally. The tradeoff between the desirable negative µ and the undesirable high dielectric constant of the ferrite is illustrated. As a direct device application, a nonreciprocal tunable phase shifter, based on this metamaterial, was demonstrated. At 42.3 GHz, with a 3.0 kOe field variation, a continuous and near linear phase tuning over 247° was obtained for forward propagation, whereas 75° phase tuning was obtained for the reverse wave propagation.
There are two major challenges to realize TNIMs in Q-band comparing with previous works at lower frequencies. First, the smaller wavelength results in higher sensitivity to geometric parameters of the assembly. Second, a suitable low loss ferrite material with high frequency FMR is needed. The small linewidth ($\Delta H \sim 500 \text{Oe}$ of the polycrystalline and $\Delta H < 10 \text{Oe}$ of the single crystalline) YIG material is no longer suitable because of its relatively small anisotropy field ($H_a \sim 200 \text{Oe}$) and subsequent low frequency FMR. Alternatively, the $H_a$ of barium hexaferrite (i.e. magnetoplumbite or Ba M-type ferrite) is too large, $\sim 17,000 \text{ Oe}$, resulting in a FMR frequency much higher than 40 GHz. The doping of Ba M-type ferrite with Sc has been shown to shift the FMR to frequencies as low as X-band$^{34}$. Appropriate doping will allow for the tuning of FMR near 40 GHz by varying the bias field.

Figure 7-1(a) shows an intuitive TNIM design consisting of two rows of copper wires and two Sc-BaM slabs. Because of the ferrite’s anisotropy, the composite performance is nonreciprocal in nature. The field distribution analysis in simulations shows that there is little field concentrating in the right ferrite slab sandwiched by the two rows of wires. Therefore, the design can be simplified by removing the right ferrite slab resulting in a design as in Fig. 7-1(b). The wave propagation behavior remains largely unchanged between the two designs.

In Fig. 7-1(b), the thicknesses of the Kapton™ substrate, copper wires and the Mylar™ spacer are 0.14 mm, 0.025 mm, and 0.72 mm, respectively. The width of individual wires is 0.3 mm and their longitudinal center to center spacing is 1.0 mm. The thickness of the Sc-BaM slab is 1.0 mm to achieve the optimal performance. Its easy axis is parallel to the magnetic bias field $H$. The height of all elements is 2.84 mm, the same as
the inner height of the Q-band rectangular waveguide. The composite length along the
direction of propagation is 6.0 mm. The two rows of copper wires ensure sufficient
plasmonic effect to generate negative $\varepsilon$ while maintaining simplicity for fabrication
and assembly. The Kapton™ substrates serve as spacers to reduce the coupling
between the ferrite and copper wires. The composite is mounted at the center of the
Q-band waveguide as shown in Fig. 7-1(c).

The Sc-BaM slab is specially positioned to the left hand side of copper wires because of
its anisotropic nature. The bias field, $H$, the wave propagation constant, $\beta$, and the
direction from the copper wires to their vicinal Sc-BaM slab, noted as $Y$, form a
right-handed chirality. For the high quality single crystal Sc-BaM, the anisotropy field,
$H_a$, is 9000 Oe, the linewidth, $\Delta H$, ~ 200 Oe, the saturation magnetization, 3300 G,
the dielectric constant, 12, and the dielectric loss tangent, $\tan \delta$, 0.0002. Its FMR
frequency can be estimated by $\gamma' \sqrt{(H + H_a)(H + H_a + 4\pi M)}$, where the
gyromagnetic ratio $\gamma' = 2.8 \text{GHz}/kOe$. But in real cases, the demagnetization field
determined by the form factor of the ferrite crystal also needs to be subtracted from $H$. 
Fig. 7-1. (a) Top views of the TNIM design consisting of two Sc-BaM slabs, two rows of copper wires on Kapton™ substrate, and a Mylar spacer and (b) the simplified TNIM consisting of only one Sc-BaM slab. The magnetic bias field $H$, the propagation constant $\beta$, and the directional vector from copper wires to their vicinal ferrite slab $Y$ form a right-handed triplet. (c) The schematic drawing of the TNIM composite mounted in a Q-band rectangular waveguide.

The effective $\mu$ of the ferrite slab under the extraordinary wave mode in the rectangular waveguide can be theoretically calculated as plotted out in Fig. 7-2(a). A
negative region of effective $\mu$ is obtained near 40 GHz with $H \approx 5000\text{Oe}$. The effective $\varepsilon$ retrieved from the simulated scattering parameters of the copper wires is also plotted in Fig. 7-2(b) where it is shown to be negative over the whole frequency range which itself allows for no wave propagation. In the frequency region where $\varepsilon$ and $\mu$ are concomitantly negative, the effective $n$ is negative. Wave propagation is allowed and a passband results. The ordinary wave mode propagating through the ferrite is neglected here because it does not propagate when the ferrite is combined with the copper wires.

Because $n = \pm \sqrt{\varepsilon \mu}$, where $\varepsilon = \varepsilon' - j\varepsilon''$, $\mu = \mu' - j\mu''$, $n = n' - jn''$, and the plus and minus signs are chosen by making $n'' > 0$, the frequency response of $n$ can be calculated as plotted out in Fig. 7-2(c). The $n'$ shows a prominent negative ($<-1$) region and a peak negative value of -3.6 between 39.5 and 43.3 GHz. However, the $n''$ is relatively large ($>0.5$) over the whole frequency range except between 42 GHz and 45 GHz. From the complex $n$, the figure of merit (FOM) can be calculated as $|n'/n''|$ in Fig. 7-2(d). The FOM has a peak value of 3.9 at 42.6 GHz. The bandwidth is 3.2 GHz of $FOM > 2$ and 2.1 GHz of $FOM \geq 3$. Because the peak of FOM is relatively low and narrow, we would expect large insertion loss only in the negative $n$ region. We can also tell that the drawback mainly comes from the contribution of $\mu''$ near FMR by reviewing Fig. 7-2(a). Therefore, in order to improve FOM, the main focus should be on reducing the $\Delta H$ of the ferrite.
Fig. 7-2. (a) Theoretically calculated permeability versus frequency of a Sc-BaM slab under the extraordinary wave mode in the Q-band rectangular waveguide. The bias field and easy axis are both in the slab plane. (b) The retrieved permittivity from simulated scattering parameters of the copper wires. (c) The calculated refractive index from the permeability and permittivity in (a) and (b). (d) The calculated figure of merit of the refractive index.

In experiments, the high quality single crystalline Sc-BaM slab was grown by liquid phase epitaxy. Thin slabs were cut out from the parent crystal to make the easy
magnetic axis in the slab plane. The copper wires were fabricated from copper clad on Kapton™ sheets using traditional lithographic procedures. The composite was assembled by fixing individual components to form the construct as Fig. 7-1(b), and was mounted in the center of a Q-band rectangular waveguide as Fig. 7-1(c). Silver paint was used to ensure low resistance contact between the wires' upper and bottom ends to the inner wall of waveguide, which was found essential to generate the plasmonic effect and subsequent negative $\varepsilon$. The Q-band waveguide is calibrated to the two ends by thru-reflect-line (TRL) method. The scatter parameters are measured using Agilent vector network analyzer.

In Figure 7-3, the measured scattering parameters, S21 and S12, of a 6 mm long TNIM composite at a bias field of 5.5 kOe are presented in comparison with the S21 of the copper wires. There is no transmission allowed by the copper wires as expected. Therefore, the passband allowed in S21 for the TNIM composite is the result of adding the ferrite slab. As analyzed in the theoretical calculation, the passband signals the occurring of negative $n$. The S12 near 40 GHz is weakened because of the nonreciprocal property of the Sc-BaM slab, where the copper wires and the ferrite interact destructively. The static bandwidth of the passband is approximately 5 GHz with $>-20\,dB$ transmission. The peak transmission is -15 dB near 40 GHz, where S12 shows a dip of -60 dB. The isolation is around 40 dB. Due to the existence of positive $\mu$ elements in the TNIM composite, the experimental $\mu$ is a collective effect and is different than the theoretically calculated one in Fig. 7-2(a). The bandwidth of the negative $\mu$ region depends upon the FVF of the Sc-BaM slab. And in all cases it should be smaller than the theoretic estimation of the one of the bulk Sc-BaM material as $\gamma' \cdot 2\pi M_s \approx 2.8 \times 1.65 \approx 4.6\text{GHz}$. 
Fig. 7-3. Measured S21 (blue line) and S12 (green line) of a 6.0 mm long TNIM composite containing a 1.0 mm thick Sc-BaM slab in comparison with the S21 of copper wires (black line) mounted in the center of a Q-band rectangular waveguide.

The major advantage of TNIMs is the frequency tunability of the passband. In Fig. 7-4(a), the tuning of the passband is demonstrated by varying the bias field $H$. The inset to Fig. 7-4(a) is a plot of the passband’s center frequency versus $H$. The center frequency shifts from 40.9 GHz to 43.9 GHz with $H$ changing from 4.0 kOe to 7.0 kOe. The tuning is close to linear with a tuning factor of 1.0 GHz/kOe. In response to a field variation of 3 kOe, the dynamic bandwidth of this TNIM design is around 8 GHz, adding the 5 GHz static bandwidth and 3 GHz of center frequency shift. Figure 7-4(b) shows the simulated results for comparison, which agree well with the experiments. The large insertion loss in the experiments is mainly because of the magnetic loss in the ferrite and the eddy current loss on the copper wires and silver paint. To overcome this drawback, the linewidth of the ferrite needs to be reduced. Finer fabricating and assembling technique needs to be explored in future.
In order to obtain negative $n$ with acceptable insertion loss, both $\mu$ and $\varepsilon$ need to be negative as illustrated in Fig. 7-2(c). For the TNIM composite, both of them are collective effects of all the elements including the ferrite slab, copper wires, Kapton™ substrates, Mylar™ spacer, and the surrounding air. Because the dielectric constant of
the ferrite is much higher comparing to the ones of Kapton™ and Mylar™, the FVF plays an important role in realizing not only negative $\mu$ but also negative $\varepsilon$.

Fig. 7-5. Measured S21s of 6.0 mm long TNIM composites under bias fields of 5.0, 5.5, and 6.0 kOe. The contained Sc-BaM slab is (a) 0.3 mm and (b) 1.3 mm thick respectively.

In order to study the FVF in the TNIM composite, albeit qualitatively, another two TNIM composites have been measured consisting of 0.3 mm and 1.3 mm thick Sc-BaM
slabs respectively. In the 0.3 mm thick Sc-BaM slab case, weaker tunable passbands are observed as in Fig. 7-5(a). The peak transmission is only -22 dB and the static bandwidth is much narrowed down. The reduction in bandwidth coincides with the decrease of the negative $\mu$ region as the result of a smaller FVF. Therefore, in order to obtain negative $\mu$, the FVF must be big enough. In the 1.3 mm thick Sc-BaM slab case, the passbands disappear completely. The whole composite behaves like a ferrite slab exhibiting FMR absorption peaks. The reason lies in the dielectric effect of the ferrite. It overwhelms the plasmonic wires. In another word, the effective $\varepsilon$ of the composite turned positive. Therefore, in order to achieve negative $\varepsilon$ and so negative $n$, the ferrite’s dielectric effect must be considered when determining the geometry of the periodic copper wires. The FVF cannot be too big, although it can ensure negative $\mu$. And the carrier density in the copper wires must be sufficient so that the plasmonic effect can be strong enough to withstand the destructive dielectric effect of the ferrite.

One significant feature of the TNIM design is the realization of transmission near 40 GHz on the high frequency side of the FMR of the Sc-BaM ferrite. In this region, the permeability is negative and changes rapidly with frequency. Therefore, by varying the magnetic bias field at a fixed frequency, the TNIM composite will experience a rapid change in its effective permeability, as well as in the negative refractive index, propagation constant, and insertion phase. Therefore, a magnetic field tunable phase shifter based on this TNIM design can be realized. Assuming the loss factor to be about the same for the TNIM and the conventional ferrite phase shifters, we would expect a much improved figure of merit using the TNIM composite, since the phase shifts would be significantly higher owing to higher differential permeability.
Fig. 7-6. Measured magnetic field tuned phase shift and corresponding insertion loss of (a) forward wave propagation and (b) backward wave propagation of the TNIM composite with a 1.0 mm thick Sc-BaM slab at 42.3 GHz.

In Fig. 7-6, the field tuning of the phase shift of the TNIM composite is presented together with the corresponding transmission coefficients. Nonreciprocal phase shifter characteristics are demonstrated by comparing the response to forward and backward wave propagation. In Fig. 5(a), a 247° continuous field tuning of the phase shift in the forward wave propagation is measured with a corresponding insertion loss varying
from 15 dB to 20 dB. The field tuning is nearly linear with a tuning factor of $82.4^\circ$/kOe. For the case of backward wave propagation, as in Fig. 5(b), the magnetic field is varied from 4.0 to 6.0 kOe, a $70^\circ$ field tuning of the phase shift with an insertion loss varying from 14 to 20 dB, is measured. However, when the field is increased from 6.0 to 7.0 kOe, the phase shift changes in the opposite direction and the insertion loss increases from 20 to 24 dB.

In summary, a simple structured TNIM composite consists of one Sc-BaM slab and two rows of periodic copper wires is demonstrated in Q-band waveguide. The theoretically calculated $\mu$, $\varepsilon$, $n$, and the FOM, together with the measured and simulated tunable passbands of the TNIM composite, are sufficient to explain the negative $n$ effect. The effect of FVF of the Sc-BaM slab is studied to confirm the tradeoff between the desirable negative $\mu$ and the detrimental dielectric property of the ferrite. It gives out a qualitative design guide of future metamaterial works utilizing ferrites. In order to reduce the insertion loss and improve the FOM of the TNIM for device applications such as tunable phase shifter, smaller linewidth ferrite material with high frequency FMR and finer assembling technique will be needed.

A nonreciprocal tunable phase shifter is also demonstrated as a direct device application of this tunable negative index metamaterial. A continuous and approximate linear phase shift, at 42.3 GHz, of $247^\circ$ is obtained for forward wave propagation. The $247^\circ$ corresponded to a 3 kOe variation in magnetic field. Consequently, a $75^\circ$ phase shift is measured in the backward wave propagation condition corresponding to a field shift of 2 kOe.
8. Exchange-conductivity of Permalloy

8.1. Intrinsic Wave Modes of Permalloy Film

In Chapter 2, it was pointed out that permalloy has the potential for novel metamaterial application due to its high conductivity and good magnetic properties. One would expect to use only permalloy to realize both plasmonic effect and ferromagnetic resonance so as to generate simultaneous negative permittivity and permeability. To clarify that, it is necessary to understand its behaviors, especially the coupling between its conductivity and magnetism near FMR. First, let's consider the following case shown in Fig. 8-1.

Fig. 8-1. Permalloy film biased out-of-plane. The wave vector $k$ is parallel to the bias field $H_{\text{ext}}$. 

\[ H_0 = H_{\text{ext}} - 4\pi M_s \]
$\mathbf{M}$ is the magnetic moment, $\mathbf{H}_o$ the internal field, and $\mathbf{k}$ the spinwave vector.

Apply the equation of motion of the magnetic moment as:

$$-j \frac{\omega}{\gamma} \mathbf{m} = \mathbf{M} \times \mathbf{h} + \mathbf{m} \times \mathbf{H}_o,$$  \hspace{1cm} (8-1)

The permeability tensor should be able to solve out. When no external AC field is considered, the AC field $\mathbf{h}$ generally consists three terms, the rf excitation field $\mathbf{h}_f = h_x \hat{\mathbf{x}} + h_y \hat{\mathbf{y}}$, the exchange field $\mathbf{h}_e$, the AC demagnetizing field $\mathbf{h}_d$. In this case when $\mathbf{k}$ is parallel to $\mathbf{H}_o$, $\mathbf{h}_d = 0$. The exchange field can be expressed as:

$$\mathbf{h}_e = -\frac{2A}{M^2} \mathbf{k} \cdot \mathbf{m}.$$  

where $A$ is the exchanging stiffness constant. Break the equation of motion into $x$ and $y$ components and then get:

$$-j \frac{\omega}{\gamma} m_x = \left( H_o + \frac{2A}{M} k^2 \right) m_y - Mh_y$$  \hspace{1cm} (8-2a)

$$-j \frac{\omega}{\gamma} m_y = \left( H_o + \frac{2A}{M} k^2 \right) m_x + Mh_x$$  \hspace{1cm} (8-2b)

Hence the magnetic susceptibility tensor can be solved out as:

$$[\chi] = \frac{M}{H_1H_2 - \left( \frac{\omega}{\gamma} \right)^2} \begin{bmatrix} H_1 & j \frac{\omega}{\gamma} \\ -j \frac{\omega}{\gamma} & H_2 \end{bmatrix},$$  \hspace{1cm} (8-3)

where $H_1 = H_2 = H_o + \frac{2A}{M} k^2$. And hence the permeability is obtained using $[\mu] = 1 + 4\pi [\chi]$. Thus the equation of motion can be summarized by the permeability tensor. To couple the equation of motion with Maxwell equations in order to solve out the dispersion relation of all the wave modes, the permeability tensor of the permalloy
film is introduced to Maxwell equations. After some tedious algebraic deductions, one obtains the dispersion relations for resonant modes:

$$\frac{\omega}{\gamma} = H_0 + \frac{2A}{M} k^2 - \frac{4\pi M_s}{k^2 - 1}, \quad (8-4)$$

Rewrite it as:

$$k^4 - k^2 \left[ k_o^2 + \frac{\omega / \gamma - H_o}{2A / M} \right] + \left( \omega / \gamma - H_o - 4\pi M \right) \frac{2A k_o^2}{M} = 0, \quad (8-5)$$

where \( k_o^2 = \omega_o^2 \varepsilon / \mu_o \) and \( \varepsilon = \frac{\sigma}{j\omega} \) for permalloy. This is a quadratic equation of \( k^2 \), from which two physical values of \( k \) can be solved. For the non-resonant mode, we substitute \( \frac{\omega}{\gamma} \rightarrow -\frac{\omega}{\gamma} \) in Eqn. 8-5. The FMR linewidth or Gilbert damping factor is included by substitution \( \frac{\omega}{\gamma} \rightarrow \frac{\omega}{\gamma} - \frac{\Delta H}{2} \) or \( \omega \rightarrow \omega (1 - j\alpha) \). For the case of \( H_o = 3500 Oe, \ 4\pi M_s = 10000, \ A = 1.14 \times 10^{-6} \text{erg/cm}, \ g = 2.10, \ \alpha = 4.33 \times 10^{-3}, \) and \( \sigma = 7 \times 10^5 S/cm \), the calculated dispersion relation for the resonant mode is plotted in Fig. 8-2(a) and 8-2(b).
Fig. 8-2 Dispersion relation of the resonant modes of a permalloy film biased out of plane and parallel to the wave propagation.

Fig. 8-3. Horizontally enlarged plot of the dispersion relation of the resonant modes of a permalloy film biased out of plane parallel to the wave propagation.
From Fig. 8-2 and 8-3, one can tell the wave mode $k_1$ is the major propagation mode with large figure of merit ($FOM = \left| \frac{\text{real}(k)}{\text{imag}(k)} \right|$). Although $\text{real}(k_2)$ goes to negative between the FMR near 3600 Oe and the ferromagnetic antiresonance (FAMR) near 13,800 Oe, $\text{imag}(k_2)$ is almost the same in magnitude in that region. So its FOM is too low for long wave propagation in this frequency region. Both non-resonant modes shown in Fig. 8-4 have the same character and contribute to the absorption.

Fig. 8-4. Dispersion relation of the non-resonant modes of a permalloy film biased out of plane parallel to the wave propagation. The inset shows the horizontally enlarged plot near zero.

In the following, we consider another case where the wave vector is perpendicular to the bias field.
Fig. 8-5. Permalloy film is biased by an external field out of plane. And the spinwave vector $k$ is perpendicular to the field.

By coupling the equation of motion and Maxwell equations, one obtains a cubed equation of $k^2$ corresponding to the "mixed" "resonant" magneto-dielectric modes.

$$k^6 + k^4 \left[ \frac{2H_o + 4\pi M}{2A/M} - k_0^2 \right] + k^2 \left[ \frac{H_o(4A/M)^2}{4A^2/M^2} - \frac{\omega^2}{\gamma^2} - \frac{2H_o + 4\pi M}{2A/M} k_0^2 \right] + \left[ \frac{\omega^2}{\gamma^2} - (\frac{H_o + 4\pi M}{4A^2/M^2}) \right]^2 k_0^2 = 0$$

(8-6)

There is also another equation $k^2 = k_0^2$ that is obtained representing the pure EM modes. Three spinwave modes can be solved from Eqn. 8-6 using numerical methods in Matlab™. Another three "non-resonant" modes can be solved by do the substitution $\frac{\omega}{\gamma} \rightarrow -\frac{\omega}{\gamma}$ as demonstrated before. Since Eqn. 8-7 only contains quadratic terms of $\frac{\omega}{\gamma}$, the "non-resonant" modes are the same as the "resonant" ones resulting in altogether only three spinwave modes. Please note that the words "resonant" and "non-resonant" follow the traditional terminology, which do not really mean there is resonant behavior or not. For $H_o = 1100 Oe$, the calculation results are plotted out in the following Figs. 8-6 and 8-7.
Fig. 8-6. Dispersion relation of the resonant modes of a permalloy film biased out of plane perpendicular to the wave propagation.

Fig. 8-7. Horizontally enlarged plot of the dispersion relation of the resonant modes of a permalloy film biased out of plane perpendicular to the wave propagation.

The composition of EM wave and spinwave modes in a sample is decided by the
frequency, the sample dimension, and boundary condition. Given a boundary condition, the composition of spinwave modes and hence the internal field distribution in the sample can be decided by matching the boundary condition. In the following section, a permalloy film with symmetric excitation at the two sides is studied at one fixed frequency and with changing bias field. The boundary condition is matched using the spinwave modes just solved. The internal field, effective permeability, exchange-conductivity, and surface impedance are obtained consequently.

8.2. Exchange-conductivity

Let's consider a permalloy film of thickness $d$ along $y$ direction and infinite along $x$ and $z$ direction. The magnetization is out-of-plane along $y$ direction.

![Diagram of Permalloy Film](image)

Fig. 8-8. Permalloy film with the magnetization out-of-plane, parallel to the wave propagation.
Assuming solutions of the form \( (h_i^+ e^{-jk_iy} + h_i^- e^{jk_iy}) = h_i \), where \( h_i \) is the internal microwave field corresponding to \( k_i \) and \( h_i^\pm = h_i \pm jh_i \) (the "±" sign indicates the direction of propagation and the subscript "i" the specific wave mode), boundary conditions that need to be considered are:

1. \( h \) being continuous at the two surfaces at \( y = 0,d \),

2. \( e \) being continuous at the two surfaces at \( y = 0,d \),

and (3) spin boundary conditions:

\[
A \frac{\partial m}{\partial y} - K_s m = 0, \quad (8-7)
\]

where \( K_s > 0 \) is the uniaxial anisotropy constant in the case of easy axis perpendicular to the film surface. As a result, a set of equations can be obtained as:

\[
\sum_{i=1}^{2} (h_i^+ + h_i^-) = h_0 \quad \text{at} \quad y = 0, \quad (8-8a)
\]

\[
\sum_{i=1}^{2} (h_i^+ e^{-jk_0d} + h_i^- e^{jk_0d}) = h_d \quad \text{at} \quad y = d, \quad (8-8b)
\]

\[
\sum_{i=1}^{2} Z_i(h_i^+ - h_i^-) = e_0 \quad \text{at} \quad y = 0, \quad (8-8c)
\]

\[
\sum_{i=1}^{2} Z_i(h_i^+ e^{-jk_0d} - h_i^- e^{jk_0d}) = e_d \quad \text{at} \quad y = d, \quad (8-8d)
\]

\[
\sum_{i=1}^{2} (P_i h_i^+ + R_i h_i^-) = 0 \quad \text{at} \quad y = 0, \quad (8-8f)
\]

\[
\sum_{i=1}^{2} (P_i h_i^+ e^{-jk_0d} + R_i h_i^- e^{jk_0d}) = 0 \quad \text{at} \quad y = d. \quad (8-8g)
\]

In the equations, \( Z_i = \frac{k_i}{\omega \varepsilon} \), \( e = \frac{\sigma}{j\omega} \), \( P_i = (K_s + jk_i A)\chi(k_i) \), \( R_i = (K_s - jk_i A)\chi(k_i) \).
\[ \chi(k_i) = \frac{4\pi M_s}{H_o + \frac{2A}{M_s}k_i^2} \]

where \( h_0, h_d, e_0, \) and \( e_d \) are given excitation condition at the two surfaces. As solved in the previous section, there are altogether two resonant and two non-resonant modes under this condition. For matching the boundary conditions we only consider the resonant modes for simplicity since the two non-resonant modes are both fast decaying. From Eqn. 8-8, the six unknowns, \( h_{i,0}, e_0 \) and \( e_d \) can be solved numerically given an excitation condition \( h_0 \) and \( h_d \). The results are presented in the following. And the Matlab\textsuperscript{TM} code is attached in Appendix C. Here the frequency is fixed at 10.29 GHz and the internal field \( (H_o = H_{ex} - 4\pi M_s) \) varies from 3000 to 4000 Oe and \( K_s \) is assumed to be zero at the two surfaces.

First, the spinwave vectors of the two resonant modes are calculated as shown in Fig. 8-9.

**Fig. 8-9.** Spinwave vectors of the two resonant modes at 10.29 GHz versus the internal field.
Then the wave vectors are put into Eqn. 8-8 for solving $h_{i,2}$. Hence, the spatial distribution of internal magnetic field can be calculated by using the equation:

$$h(y) = \sum_{i=1}^{2} (h_i^+ e^{-ik_y y} + h_i^- e^{ik_y y}), \quad 0 \leq y \leq d.$$  

(8-9)

For the case of symmetric excitation where $h_o = h_d = 1$, the solved field distributions along y axis are shown in Fig. 8-8(a) and (b) with $H_o = 3400Oe$ and $d = 0.1\mu m$.

The film under symmetric excitation can also be regarded as a 2D simplification of a long cylindrical wire with uniform circumferential excitation magnetized along radial directions.

![Fig. 8-10. $h_x$ distribution along y axis of the permalloy film under symmetrical excitation at 10.29 GHz when $H_o = 3400$ Oe.](image)

In the given excitation, both $h_o$ and $h_d$ are both real, which means that the both only have $x$ components. So in Fig. 8-11, $h_z$ is zero at the two ends and grows to the center. In Fig. 8-10, $h_x$ attenuates lightly from the two ends to the center.
Fig. 8-11. $h_z$ distribution along $y$ axis in the permalloy film under symmetrical excitation at 10.29 GHz when $H_o = 3400$ Oe.

Fig. 8-12. Surface impedance of the permalloy film under symmetric excitation.

Using the solved $h_{t,2}$, $e_0$ and $e_d$ are obtained from Eqn. 8-8(c) and (d). So the surface impedance $Z_s = \frac{e_{0,d}}{h_{0,d}}$ is decided consequently as plotted in Fig. 8-12. It can also be called magnetoimpedance since it changes rapidly near the FMR with the bias magnetic field.
This surface impedance can be confirmed by using Poynting vector integration:

\[
\oint_s (E_x \times H_z') \, dl = -j \omega \left[ dV \left( \frac{E_x H_z^* - E_z H_x^*}{|E|^2} \right) \right]
\]  

(8-10)

The left hand side (LHS) equals to \( 2Z_x A_s \), where \( A_s \) is the area of the surface which can be cancelled at the two sides. The volume averaged permeability can also be calculated from the first term at the right hand side (RHS) of Eqn. 8-10 as

\[
\mu_v = \frac{\mu_0}{1} \int \frac{dy}{|H|^2 + M g H^* |H|^2},
\]  

(8-11)

which is compared with the effective permeability later in Fig. 8-14. For the details of the deduction, please refer to the corresponding lecture notes of Prof. Carmine Vittoria.55

Given \( h_0 \) and \( e_0 \), \( h_d \) and \( e_d \) can be solved out in Eqn. 8-8. Subsequently, the transfer function matrix is obtained. So the effective propagation constant, permeability, permittivity, characteristic impedance, and hence the exchange-conductivity can be obtained. They are plotted in the following figures in sequence.
Fig. 8-13. Effective propagation constant \( k_{\text{eff}} = \beta - j\alpha \) of the permalloy film near the FMR at 10.29 GHz.

In Fig. 8-13, the complex effective propagation constant \( k_{\text{eff}} = \beta - j\alpha \) shows \( L \)erentzian curves. \( \beta \) is negative at the lower field region below the FMR but with the almost same magnitude as \( \alpha \), which indicate only the negative effective permeability but not the permittivity because \( k_{\text{eff}} = \omega \sqrt{\varepsilon_{\text{eff}} \mu_{\text{eff}}} \). The black dash line of \( \frac{\pi}{2d} \) serves as a reference. Both \( \beta \) and \( \alpha \) show small ripples near 3200 Oe. It is not intrinsic but linked to the film thickness \( d \) since it was found to shift to the right as \( d \) increases.
In Fig. 8-14, the effective permeability calculated from transfer function matrix is compared with the volume average permeability. They matches with each other away from FMR while the former shows sharper FMR peaks. The characteristic impedance $Z_{ch}$ in Fig. 8-15 is different in magnitude from the surface impedance $Z_s$ in Fig.
For instance, at $H_o = 3000\, Oe$, $\text{Re}(Z_{ch}) \approx 0.33$ and $\text{Im}(Z_{ch}) \approx -0.33$, while $\text{Re}(Z_s) \approx 0$ and $\text{Im}(Z_s) \approx -0.08$. The characteristic impedance also shows ripples near 3200 Oe, which is related to the film thickness.

![Graph](image)

Fig. 8-16. Effective exchange-conductivity of the permalloy film near the FMR at 10.29 GHz.

In Fig. 8-16, the complex effective exchange-conductivity $\sigma_{eff}$ is obtained as well. The red line is the intrinsic conductivity $\sigma$ for comparison. In the off-resonance region, the effective conductivity is consistent with the intrinsic one and $\text{Im}(\sigma_{eff}) = 0$. However, near FMR and the ripples, $\text{Re}(\sigma_{eff})$ peaks off from $\sigma$. $\text{Im}(\sigma_{eff})$ turns to be slightly negative between the two peaks and positive above the FMR and below the ripple. Since $\varepsilon_{eff} = \frac{\sigma_{eff}}{j\omega}$, the positive $\text{Im}(\sigma_{eff})$ implies capacitive and the negative one plasmonic. In other words, $\text{Re}(\varepsilon_{eff}) < 0$ at the lower field side of FMR.
Unfortunately, \( \text{Im}(\varepsilon_{\text{eff}}) \) is much larger in amplitude that results in a very low figure of merit of \( \varepsilon_{\text{eff}} \) in this region. So the negative effective permittivity is not useful for metamaterials.

In conclusion, there is strong coupling between the conductivity and the magnetism of permalloy near FMR. The interaction makes the permalloy film slightly plasmonic near FMR where \( \mu'_{\text{eff}} \) is negative. There is still a small region at the lower field end near 3000 Oe where \( \mu'_{\text{eff}} \) is weakly negative and \( \sigma_{\text{eff}} = \sigma \). Nevertheless, if constructing a permalloy wire array hoping to obtain simultaneous negative permittivity and permeability, \( \mu'_{\text{eff}} \) will not still be negative considering the volume factor in a transmission, where the magnetism in permalloy wires is averaged out by surrounding air or dielectrics. In order to use permalloy to construct a plasmonic wire structure while still keeping \( \mu'_{\text{eff}} \) negative, one has to overcome the volume factor limit. There might be a small operation window in frequency. Furthermore, there will also be coupling effect between the electrical and magnetic properties near plasmonic resonance. But it can be avoided by designing the plasma frequency much higher than the FMR frequency. The major obstacle in using only permalloy to realize a TNIM lies in the tradeoff between keeping the volume factor big enough to obtain negative permeability and the effective charge carrier density low enough to obtain negative permittivity.
9. Summary and Discussion

The research of using ferrites to provide negative permeability at the high frequency side of FMR in NIMs has been presented in this dissertation. Contrary to conventional magnetic resonators made by metallic rings, whose resonant frequency strictly depends on geometric parameters, the FMR frequency can be shifted by external magnetic field. Hence, the negative permeability region can be shifted in a broad frequency range. Since broadband negative permittivity is provided by plasmonic wires below the plasma frequency, broadband and frequency tunable negative index can be realized by the simultaneous permittivity and permeability in frequency using ferrites in combination with plasmonic wires.

Difficulties in realizing TNIMs are based in the design and refinement of suitable ferrites largely because of their complicated near-resonance behaviors. Firstly, the FMR frequency depends not only on intrinsic material properties of ferrites, including saturation magnetization, anisotropy field, and FMR linewidth, but also on the biasing orientation and the demagnetizing field defined by shape factors. Secondly, the effective permeability of a ferrite sample in a transmission line is different from the diagonal components of permeability tensor. Neither is it the same as the theoretically calculated effective permeability for a specific wave mode considering the volume factor and field polarization. When a ferrite sample is assembled with plasmonic wires, the interactions change the local wave modes and field polarization, which
makes near-resonance behaviors even more unpredictable. Thirdly, the magnetic losses near FMR are significant. In order to minimize these, the working frequency for a TNIM design needs to be as far away as possible from FMR. However, the drawback is that the permeability gets less negative at the same time. Subsequently this tradeoff limits the magnitude of negative permeability beside the volume factor. Therefore, a small FMR linewidth is critical to realizing low insertion loss in TNIMs.

Plasmonic wires demonstrate effective negative permittivity below plasma/cutoff frequency. The 3D lattice of infinite long metal wires can be treated the same as plasmons with effective charge carrier density and mass. The simple case of 1D array of periodic metal wires in a transmission line can be treated as lumped elements using transfer function matrix theory. For accurate evaluation plasmonic behaviors of metal wires in a transmission line, finite element simulations closely match experimental data. It is found that dielectric media weakens plasmonic effect and lowers plasma frequency, while reducing the spacing between wires and adding rows in parallel have the opposite effects. In order to obtain negative permittivity when a wire array is attached to ferrite slabs, the detrimental effect of ferrites' high dielectric constant must be considered into designs.

The occurrence of negative refractive index for a composite in transmission can be identified by a passband near FMR and below plasma frequency, where in the plasmonic wire array itself, transmission is not allowed. Ferrites behave like
dielectrics without the bias field. Hence for a TNIM composite, there is no wave transmission without a proper bias field. Although ideally one would like to retrieve the refractive index, permittivity, and permeability for a TNIM composite, it is difficult at the resonant region where the phase jumps abruptly. The phase jump causes large error together with $2\pi$ uncertainty in phase measurements, especially for short samples. Unfortunately, the retrieved material parameters are sensitive to the phase of S-parameters.

In Chapter 5, a TNIM using single crystalline YIG and an array of copper wires was demonstrated in K-band rectangular waveguide. The tunability was demonstrated from 18-23 GHz under an applied magnetic field with a figure of merit of 4.2 GHz/kOe. The bandwidth was measured to be 5.0 GHz under tuning and 0.9 GHz at a fixed field. The measured minimum insertion loss was 4 dB (or $\sim$ 6 dB/cm) at 22.3 GHz. The experimental results were compared with approximate transfer function matrix analysis, which also gave out negative index band by representing wires and ferrite media with transfer function matrix in an ideal transmission line. A phase shift based on this K-band TNIM structure was demonstrated based on the large frequency dispersion of the tunable negative index. A phase shift tuning of 160 degree/kOe was achieved at 24 GHz. The insertion loss varies from -4 to -7 dB/cm.

In Chapter 6, a planar microstrip TNIM, that can form an integral part of TNIM-based RFIC devices, was fabricated. Polycrystalline YIG with $\sim$ 25 Oe FMR linewidth was
used in combination with copper wires fabricated from copper clad on KaptonTM substrate. Magnetic field tunable passbands resulted from negative index were realized at X-band (7 - 12.5 GHz). Both simulated and experimental data show tunable passbands resulting from negative refractive index. The 1.0 GHz wide instantaneous TNIM passband, having a peak transmission of -5 dB, was tunable between 7.6 and 10.7 GHz by changing the bias field from 3.0 to 5.5 kOe. The TNIM composite can perform as a tunable phase shifter where the insertion phase shifts 45° for a field change of 0.64 kOe with the insertion loss varying from 6 to 10 dB. The 10.0 × 2.0 × 1.2 mm³ TNIM composite is compact, mechanically robust, and compatible with planar microwave devices.

In Chapter 7, a TNIM working in the millimeter wave range was demonstrated in Q-band waveguide utilizing single crystalline Sc-BaM with FMR linewidth ~ 200 Oe. The doping of scandium lowers the $H_a$ of BaM, which lowers the zero field FMR frequency consequently. A TNIM composite consists of one Sc-BaM slab and two rows of periodic copper wires was demonstrated in Q-band waveguide. The theoretically calculated $\mu$, $\varepsilon$, $n$, and the FOM, together with the measured and simulated tunable passbands of the TNIM composite, are sufficient to explain the negative $n$ effect. The effect of ferrites' volume factor of the Sc-BaM slab was studied experimentally to confirm the tradeoff between the desirable negative $\mu$ and the detrimental dielectric effect of the ferrite. These studies provide a qualitative design guide of future metamaterial works utilizing ferrites. The measured insertion loss of the passbands was ~ -15 dB, which was yet too large for practical device applications. In order to reduce the insertion loss and improve the FOM of the TNIM
for device applications such as tunable phase shifter, smaller linewidth hexaferrites with high frequency FMR and finer assembling technique will be needed. Nonreciprocal tunable phase performance was also demonstrated as a direct device application of this TNIM. A continuous and approximate linear phase shift, at 42.3 GHz, of $247^\circ$ was obtained for forward wave propagation. The $247^\circ$ corresponded to a 3 kOe variation in magnetic field. Consequently, a $75^\circ$ phase shift was measured in the backward wave propagation condition corresponding to a field shift of 2 kOe.

In Chapter 8, a permalloy film was theoretically studied on its spinwave modes, surface permeability, and exchange-conductivity near the FMR. Strong coupling between the conductivity and magnetism was found in the permalloy film as it becomes slightly capacitive at the lower field side of FMR. The opportunities and obstacles of permalloy alone to realize negative permittivity and permeability simultaneously are discussed. The major difficulties lie in the ferrite volume factor requirement for realizing magnetic resonance.

Besides the mainstream focus of NIMs on their superior electromagnetic properties, the significance of TNIMs also lies in realizing transmission near FMR, which region once was not considered for any device applications. The interactions between ferrites and plasmonic wires at resonance are rich in physics and still far from being clear. A complete understanding of the mechanism will help to discover opportunities of realizing novel and enhanced EM materials. For instance, ideal 3D NIMs in forms of powder or liquid may lead to revolutionary applications such like invisible cloak and super lens.
Future work in the field of TNIMs, utilizing multifaceted properties of ferrites, the author believes holds great opportunities. The Y-type hexaferrites, because of their broadband negative permeability resulted from its in-plane easy axis and large effective magnetization, allows one to design TNIMs that operates at frequencies away from FMR. So chances are magnetic losses can be reduced. Furthermore, ferrite cylinders can possibly be constructed into arrays to form magnetic photonic crystals, properties of which can be manipulated by magnetic field. Nevertheless, the theoretical study proposed by S. Liu et al. in Physic Review Letters is far from experimental realization due to their inappropriate treatment of the EM wave modes for ferrite cylinders. Rigorous band structure calculations based on proper wave modes assumptions are suggested before any experimental attempts can be considered.
% Use transfer function matrix theory to calculate the permittivity
% of a 1D array of metallic wires.
clear all;
close all;

c = 3*10^8; % speed of light in 'm/s'
h = 0.0012; % length of wires in 'm'
r = 0.000030; % radius of wires in 'm'
Mu0 = 4*pi*10^(-7); % magnetic constant of air in SI units
Mur = 1; % relative permeability of the medium between wires
Mu = Mu0*Mur;
Eps0 = 8.854187*10^(-12); % permittivity of free space in 'F/m'
Epsr = 10; % relative permittivity of the medium between wires
Eps = Eps0*Epsr;
Sig = 5.8*10^7; % conductivity of copper in Siemens/m
a = 0.001; % wires' center to center spacing in 'm'
d = a/2;
Z0 = sqrt(Mu0/Eps0); % characteristic impedance of air
Z = sqrt(Mu/Eps); % characteristic impedance of the medium in between
L = 5*a; % total length of the DUT in 'm'
f = (10:0.2:40)*10^9; % in 'Hz'
LL = length(f);
S11 = []; S21 = [];
for ii = 1:LL
    Y = ( h/(2*pi*r) * sqrt(pi*f(ii)*Mu/Sig) * (1-i) -
\[ i \cdot f(ii) \cdot \text{Mu} \cdot h \cdot \log(d/r) \]  
\[ \wedge (-1); \]

\[ A1 = \begin{bmatrix} 1 & 0; \ Y & 1 \end{bmatrix}; \]

\[ k = 2 \cdot \pi \cdot f(ii) \cdot \sqrt{Eps \cdot Mu}; \]

\[ A2 = \begin{bmatrix} \cos(k \cdot d) & -i \cdot Z \cdot \sin(k \cdot d); \ -i \cdot \sin(k \cdot d)/Z \cdot \cos(k \cdot d) \end{bmatrix}; \]

\[ A = (A2 \cdot A1 \cdot A2)^5; \]

\[ S21i = 2 \cdot Z0/(A(1,2) + (A(1,1)+A(2,2)) \cdot Z0 + A(2,1) \cdot Z0^2); \]

\[ S11i = (A(1,2) + (A(1,1) - A(2,2)) \cdot Z0 - \]

\[ A(2,1) \cdot Z0^2)/(A(1,2) + (A(1,1) + A(2,2)) \cdot Z0 + A(2,1) \cdot Z0^2); \]

\[ S21 = [S21, S21i]; S11 = [S11, S11i]; \]

\[ end \]

\[ A = \arccos \left( \frac{1 - S11.\wedge 2 + S21.\wedge 2}{2 \cdot S21} \right); \]

\[ \phi = \text{real}(A); \]

\[ \phi_\_ = \phi; \]

% Wrapping the phase to get physical solutions.

\[ \text{for } ii = LL:-1:2 \]

\[ \text{if } \phi(ii) < \phi(ii-1) \]

\[ \phi_{\_}(ii) = 2 \cdot \pi - \phi(ii); \]

\[ \text{end} \]

\[ \text{end} \]

\[ \phi = \phi_{\_}; \]

\[ \phi = \text{unwrap}(\phi); \]

\[ \phi = \text{abs}(\phi); \]

\[ mS21 = 20 \cdot \log10(\text{abs}(S21)); \]

\[ mS11 = 20 \cdot \log10(\text{abs}(S11)); \]

\[ A = \phi + i \cdot \text{abs}(\text{imag}(A)); \]

\[ \text{Neff} = c./(2 \cdot \pi \cdot f \cdot L).\wedge A; \]

% Neff = Neff.*sign(imag(Neff)); % passive device constraint

\[ Zeff = \text{sqrt( } \{(1+S11).\wedge 2-S21.\wedge 2)/\{(1-S11).\wedge 2-S21.\wedge 2}\); \]

% relative impedance

\[ Zeff = \text{Zeff.*sign(real(Zeff)); % passive device constraint} \]
```matlab
Figure;
hold on;
plot(f/10^9, real(Zeff));
plot(f/10^9, imag(Zeff), 'r');
hold off;

Eps_eff = Neff./Zeff; % The effective permittivity calculated is relative.

n = [f'/10^9, real(Neff)', ',imag(Neff)'];
save n2.txt n -ASCII;

Zr = [f'/10^9, real(Zeff)', ',imag(Zeff)'];
save Zr2.txt Zr -ASCII;

Epsm = [f'/10^9, real(Eps_eff)', ',imag(Eps_eff)'];
save Epsr2.txt Epsm -ASCII;

Mu_eff = Neff.*Zeff;

Mum = [f'/10^9, real(Mu_eff)', ',imag(Mu_eff)'];
save Mur2.txt Mum -ASCII;

S_Para = [f'/10^9, mS21', ',mS11'];
save S_Para2.txt S_Para -ASCII;

Figure;
hold on;
plot(f/10^9, real(Mu_eff), 'Linewidth', 2); % relative permittivity
plot(f/10^9, imag(Mu_eff), '--', 'Linewidth', 2);
hold off;
title('Complex Relative Permeability', 'FontSize', 22);
xlabel('Frequency(GHz)', 'FontSize', 24);
ylabel('Relative Permeability', 'FontSize', 24);

h = legend('\mu''', '\mu''', 1, 'FontSize', 24);
```

Figure;
hold on;
plot(f/10^9, real(Zeff));
plot(f/10^9, imag(Zeff), 'r');
hold off;

Eps_eff = Neff./Zeff; % The effective permittivity calculated is relative.

n = [f'/10^9, real(Neff)', ',imag(Neff)'];
save n2.txt n -ASCII;

Zr = [f'/10^9, real(Zeff)', ',imag(Zeff)'];
save Zr2.txt Zr -ASCII;

Epsm = [f'/10^9, real(Eps_eff)', ',imag(Eps_eff)'];
save Epsr2.txt Epsm -ASCII;

Mu_eff = Neff.*Zeff;

Mum = [f'/10^9, real(Mu_eff)', ',imag(Mu_eff)'];
save Mur2.txt Mum -ASCII;

S_Para = [f'/10^9, mS21', ',mS11'];
save S_Para2.txt S_Para -ASCII;

Figure;
hold on;
plot(f/10^9, real(Mu_eff), 'Linewidth', 2); % relative permittivity
plot(f/10^9, imag(Mu_eff), '--', 'Linewidth', 2);
hold off;
title('Complex Relative Permeability', 'FontSize', 22);
xlabel('Frequency(GHz)', 'FontSize', 24);
ylabel('Relative Permeability', 'FontSize', 24);

h = legend('\mu''', '\mu''', 1, 'FontSize', 24);
h = gca;
set(h,'FontSize',24);

Figure;
hold on;
plot(f/10^9, phi);
plot(f/10^9, imag(A),'r');
title('\phi', 'FontSize',30);
hold off;

Figure;
hold on;
plot(f/10^9, real(Neff),'Linewidth',2); % relative permittivity
plot(f/10^9, imag(Neff),'--','Linewidth',2);
hold off;
title('Complex Refractive Index','FontSize',22);
xlabel('Frequency(GHz)','FontSize',24);
ylabel('n','FontSize',24);
h = legend('n''','n'',1,'FontSize',24);
h = gca;
set(h,'FontSize',24);

Figure;
hold on;
plot(f/10^9, real(Eps_eff),'Linewidth',2); % relative permittivity
plot(f/10^9, imag(Eps_eff),'--','Linewidth',2);
hold off;
title('Complex Permittivity','FontSize',22);
xlabel('Frequency(GHz)','FontSize',24);
ylabel('Permittivity','FontSize',24);
h = legend('\(\varepsilon''\)','\(\varepsilon''\)',1,'FontSize',24);

h = gca;
set(h,'FontSize',24);

Figure;
hold on;
plot(f/10^9,mS21,'LineWidth',2);
plot(f/10^9,mS11,'r','LineWidth',2);

h = gca;
set(h,'FontSize',24);
xlabel('Frequency (GHz)','FontSize',24);
ylabel('Magnitude (dB)','FontSize',24);
h = legend('S21','S11',1,'FontSize',24);
hold off;
B. Grow Y-type Hexaferrite Using Flux Melting Technique

Flux melt technique was a conventional way to grow high quality single crystal ferrite materials for microwave applications first introduced by Nielson in 1960. In this technique, the mixture of oxide powder is firstly dissolved in a solvent or flux at high temperature. The temperature is then cooled down very slowly causing the precipitation and growth of crystals. In the post process, crystals are separated from the flux by selective acid extraction normally at room temperature. The flux-melt technique can serve as a way to improve magnetic properties of Y-type hexaferrites particles and to reduce FMR linewidth of oriented disks. The flux melt technique is also relatively easy to adapt to prepare Y-type hexaferrite single crystal particles or even large bulk piece.

This appendix presents our attempt to make large bulk piece of Zn$_2$Y (Ba$_2$Zn$_2$Fe$_{12}$O$_{22}$) using flux melting technique. Various microwave and material characterization results are included to confirm the composite and property of the produced crystals.

Crystals of Zn$_2$Y were prepared by crystallization from molten salts in platinum crucibles. The BaO-B$_2$O$_3$ serves as the melt which is less volatile, less viscous, and with lower liquidus temperatures compared with NaFeO$_2$ which was firstly used at an earlier time. A typical composition is 0.210 Fe$_2$O$_3$, 0.336 BaCO$_3$, 0.133 B$_2$O$_3$, and
0.071 ZnO in mole ratio. The four types of powder were mixed by hand first before being put into the ball milling with some alcohol for hours to assure the uniformity.

Then the platinum crucible holding the powder mixture around three quarter of its volume was heated in the furnace. If the crucible was too full, the flux would spill out when boiling at the peak temperature, which might damage the furnace. The heating temperature profile was set as shown in Table A-1. In order to have the mixture melt completely, the time at 1200°C has to be long enough. So it was set to be 6 hours for the trial. The larger amount of powder, the longer the staying time needs to be. The crystallization happens between 1200-1000°C. The cooling should be very slow down to 0.5°C to 2.0°C per hour. We used 25°C per hour (8 hours in total) for the trial. The furnace was subsequently cooled down to 800°C rapidly at which point the crucible could be taken out to the air to cool to room temperature.

<table>
<thead>
<tr>
<th>Temperature Range</th>
<th>Time (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25-800°C</td>
<td>3</td>
</tr>
<tr>
<td>800-1200°C</td>
<td>2</td>
</tr>
<tr>
<td>1200°C</td>
<td>6</td>
</tr>
<tr>
<td>1200-1000°C</td>
<td>8</td>
</tr>
<tr>
<td>1000-800°C</td>
<td>3</td>
</tr>
</tbody>
</table>

Table B-1. Temperature profile for growing single crystals of Zn₂Y using flux melt technique.

Hexagon crystals were seen on the surface of the flux. The diameter of the largest one was up to 1cm. The etchant used to remove the flux is 40% nitric acid, 40% acetic acid, and 20% water. Some crystals broke during this etching process due to the strain and stress change. The removed ones were tested by vibrating sample magnetometer
(VSM) and electron paramagnetic resonance (EPR). And the small crystal particles were ground into fine powder to measure X-ray diffraction (XRD).

From the VSM measurement as shown in Fig. A-1, one can tell that the easy axis is in the plane, a symbolic character of Y-type hexaferrite. Magnetic properties including the saturation magnetization, coercivity field, and anisotropy field were obtained as $4\pi M_s = 2,185 \text{G}$, $H_c = 3.4 \text{Oe}$, and $H_u : 10,000 \text{Oe}$.

Fig. B-1. VSM measurement of a flake of single crystal Zn$_2$Y.

A 0.2 x 3 x 3 mm$^3$ crystal piece was measured in EPR to obtain the linewidth information. The rough measurement got a linewidth around 50 Oe as shown in Fig. A-2. It can be reduced if the crystal was ground thinner and smoother to reduce the contribution of the rough surface. The periodic ripples on the main Lorentzian curve can be understood by the spinwave's quantization due to the geometric dimension.
Figure A-3 shows the XRD measurement results of the Zn$_2$Y single crystal powder. The major diffraction peaks match the standard profile at the bottom. The mismatched large peak at the right pointed out by the arrow was due to some nonmagnetic secondary phase since the $4\pi M_s$ is a little smaller than expected.

![Graph](image)

Fig. B-2. EPR measurement of Zn$_2$Y single crystal. The right photo shows the crystal size compared to a penny coin. The measured main line linewidth is around 50 Oe.
Fig. B-3. XRD measurement of the Zn$_2$Y single crystal powder.

Generally speaking, single crystal piece larger than 1 x 1 cm$^2$ can be useful for device applications such like phase shifters, band pass filters, or even negative index metamaterials. More trials with more powder in a larger platinum crucible, larger heating time, and much slower cooling can be done in order to get larger piece single crystals.
C. Matlab™ Code of Calculating Exchange-Conductivity

% Main program
% Permalloy film with H,M perpendicular to the film plane. k//M
% Calculating the surface impedance, characteristic impedance, 
% effective permeability, and exchange-conductivity of a permalloy film.
% CGS units.
clc;
clear all;
close all;

global d w mu0 eps Xi1s Xi2s k1s k2s Z1s Z2s h1r h1l h2r h2l
mu0 = 4*pi*10^(-9); % henry/cm
eps0 = 1/(36*pi)*10^(-11); % F/cm
Z0 = sqrt(mu0/eps0);
sigma = 0.7*10^5; % siemens/cm
A = 1.14e-6; % erg/cm
w = gamma*3500; % Hz
f = w/(2*pi); % Hz
eps = -j*sigma/w; % F/cm
k0_sq = -j*w*sigma*mu0;
\[ dw = dH*gamma/2; \]
\[ w = w - j*dw; \]

% Calculate the spinwave vectors of resonant modes.

\[
c1 = \text{ones}(1,L); \\
c2 = -( k0\_sq + (w/gamma-Ho)^{M/(2*A)} ); \\
c3 = M/(2*A) * k0\_sq * (w/gamma-Ho-4*pi*M); \\
k1\_sq = -c2/(2*c1) + \sqrt{c2^2 - 4*c1*c3}/(2*c1); \\
k2\_sq = -c2/(2*c1) - \sqrt{c2^2 - 4*c1*c3}/(2*c1); \\
k1 = \text{sqrt}(k1\_sq); \\
k1 = \text{sign}(-\text{imag}(k1)) \cdot k1; \% \text{Pick up the physical mode (non-growing).} \\
k2 = \text{sqrt}(k2\_sq); \\
k2 = \text{sign}(-\text{imag}(k2)) \cdot k2; \\
\]

figure(1); \% k vs H
hold on;
plot(real(k1),Ho,'b','Linewidth',2);
plot(imag(k1),Ho,'b--','Linewidth',2);
plot(real(k2),Ho,'r','Linewidth',2);
plot(imag(k2),Ho,'r--','Linewidth',2);
h = gca;
set(h,'Fontsize',26);
legend('Re(k_1)','Im(k_1)','Re(k_2)','Im(k_2)',2);
ylabel('H_o(Oe)','Fontsize',28);
xlabel('k(rad/cm)','Fontsize', 28);
hold off;

d = 0.2E-4; \% cm
Xi1 = 4*pi*M.//(Ho+2*A/M.*k1\_sq-w/gamma ); \%
Xi2 = 4*pi*M.//(Ho+2*A/M.*k2\_sq-w/gamma ); \%
Ks_0 = 0; Ks_d = 0; 
n_minus_2pi=0; 
n_plus_2pi=0; 
indicator=0; 

for i = 1:L 

% Solve the boundary matching equations.
S = [ 1 1 1 1; exp(-j*k1(i)*d) exp(j*k1(i)*d) exp(-j*k2(i)*d) exp(j*k2(i)*d); ... 
    Xi1(i)*(Ks_0+j*k1(i)*A) Xi1(i)*(Ks_0-j*k1(i)*A) 
    Xi2(i)*(Ks_0+j*k2(i)*A) Xi2(i)*(Ks_0-j*k2(i)*A); ... 
    Xi1(i)*(Ks_d+j*k1(i)*A)*exp(-j*k1(i)*d) 
    Xi1(i)*(Ks_d-j*k1(i)*A)*exp(j*k1(i)*d) 
    Xi2(i)*(Ks_d+j*k2(i)*A)*exp(-j*k2(i)*d) 
    Xi2(i)*(Ks_d-j*k2(i)*A)*exp(j*k2(i)*d)]; 

b = [1 1 0 0]'; % boundary condition 

h_vec(:,i) = inv(S)*b; % h_vec(:,i) = [h1+(i) h1-(i) h2+(i) h2-(i)]' 
% h_vec(:,i) = S\b; 

Z1(i) = k1(i)/(w*eps); 
Z2(i) = k2(i)/(w*eps); 

% Solve the transfer function matrix 
SP = [ j*Z1(i)*exp(-j*k1(i)*d) -j*Z1(i)*exp(j*k1(i)*d) 
      j*Z2(i)*exp(-j*k2(i)*d) -j*Z2(i)*exp(j*k2(i)*d); ... 
      exp(-j*k1(i)*d) exp(j*k1(i)*d) exp(-j*k2(i)*d) 
      exp(j*k2(i)*d); ... 
      Xi1(i)*(Ks_0+j*k1(i)*A) Xi1(i)*(Ks_0-j*k1(i)*A) 
      Xi2(i)*(Ks_0+j*k2(i)*A) Xi2(i)*(Ks_0-j*k2(i)*A); ... 
      Xi1(i)*(Ks_d+j*k1(i)*A)*exp(-j*k1(i)*d) 
      Xi1(i)*(Ks_d-j*k1(i)*A)*exp(j*k1(i)*d) 
      Xi2(i)*(Ks_d+j*k2(i)*A)*exp(-j*k2(i)*d) 
      Xi2(i)*(Ks_d-j*k2(i)*A)*exp(j*k2(i)*d)];
A_mat = A_matrix(SP, Z1(i), Z2(i));
all(i) = A_mat(1, 1);
D = A_mat(1, 1)*A_mat(2, 2)-A_mat(1, 2)*A_mat(2, 1);
% Solve the characteristic impedance.
Z_square = A_mat(1, 2)/A_mat(2, 1);
Z(i)= sqrt(Z_square); % For circular polarization, Z = j*sqrt(Mu_eff/Eps_eff).
Z(i)= sign(real(-j*Z(i)))*Z(i); % Pick up the physical solution.
% calculation of alpha and beta: keff = beta - j*alpha %%%
ZS(i) = -j*(A_mat(1,1)-1)/A_mat(2,1);
% Calculate the EM coupling factor between the surface and characteristic impedance
emcoupl(i) = ZS(i)/Z(i);
C(i) = A_mat(1,1)+(A_mat(1,2))/Z(i); % C = exp(k_eff*t)
% beta(i) = angle(C(i))/d;
% if beta(i) < -pi/(2*d)
% beta(i) = -beta(i) - pi/d;
% elseif beta(i) > pi/(2*d)
% beta(i) = -beta(i) + pi/d;
% end
% alpha(i) = log(abs(C(i)))/d;
phi = log(C(i));
alpha(i) = real(phi)/d;
beta(i) = imag(phi)/d;
k_eff(i) = beta(i) - j*alpha(i); %%%
mu_eff(i)=((k_eff(i))*Z(i))/(j*w*mu0);
eps_eff(i)=j*(k_eff(i))/(Z(i)*w);
sigma_eff = eps_eff.*j.*w;
% Z_eff(i) = j*sqrt((mu_eff(i)*mu0)./eps_eff(i));
Zs_0(i) = Z1(i)*(h_vec(1,i)-h_vec(2,i)) + ...
\[ Z_2(i) \cdot (h_{vec}(3,i) - h_{vec}(4,i)); \]
\[ Z_{s_d}(i) = Z_1(i) \cdot (h_{vec}(1,i) \cdot \exp(-j k_1(i) * d) - h_{vec}(2,i) \cdot \exp(j k_1(i) * d)) + \ldots \]
\[ Z_2(i) \cdot (h_{vec}(3,i) \cdot \exp(-j k_2(i) * d) - h_{vec}(4,i) \cdot \exp(j k_2(i) * d)); \]

\% Check the result by Poynting vector integration.
\[
X_{1s} = X_{11}(i); \ X_{12s} = X_{12}(i); \ k_{1s} = k_{11}(i); \ k_{2s} = k_{22}(i); \ Z_{1s} = Z_{11}(i);
\]
\[ Z_{2s} = Z_{22}(i); \]
\[ h_{1r} = h_{vec}(1,i); \ h_{1l} = h_{vec}(2,i); \ h_{2r} = h_{vec}(3,i); \ h_{2l} = h_{vec}(4,i); \]
\[ \text{Integ}(i) = \text{quadgk}(\text{@myfun0}, 0, d); \ \% 2Z_s(i) \text{ from Poynting vector integration} \]
\[ \text{Integ1}(i) = \text{quadgk}(\text{@myfun1}, 0, d) / d; \ \% \text{effective permeability by volume integration} \]

end

\% Plot out the \( h(y) \) at \( H_o = 3400 \) Oe.
\[ N = 2001; \]
\[ y = 0:d/200:d; \]
\[ h_{y} = h_{vec}(1,N) \cdot \exp(-j k_1(N) \cdot y) + h_{vec}(2,N) \cdot \exp(j k_1(N) \cdot y) + h_{vec}(3,N) \cdot \exp(-j k_2(N) \cdot y) + h_{vec}(4,N) \cdot \exp(j k_2(N) \cdot y); \]
\[ \text{figure}(1); \]
\[ \text{plot}(y, \text{real}(h_{y}), 'k', 'LineWidth', 2); \]
\[ h = \text{gca}; \]
\[ \text{set}(h, 'Fontsize', 28); \]
\[ \text{xlabel}('y(cm)', 'Fontsize', 28); \]
\[ \text{ylabel}('h_x', 'Fontsize', 28); \]
\[ \% \text{title}(['H_o = ', \text{num2str}(H_o(N)), 'Oe'], 'Fontsize', 28); \]
\[ \text{figure}(2); \]
\[ \text{plot}(y, \text{imag}(h_{y}), 'k', 'LineWidth', 2); \]
\[ h = \text{gca}; \]
\[ \text{set}(h, 'Fontsize', 28); \]
\[ \text{xlabel}('y(cm)', 'Fontsize', 28); \]

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ylabel('h_z','Fontsize',28);

% title(['H_o = ', num2str(Ho(N)), 'Oe'],'Fontsize',28);

% characteristic impedance
figure(3);
hold on;
plot(Ho,real(-j*Z),'Linewidth',2);
plot(Ho,imag(-j*Z),'--','Linewidth',2);
hold off;
xlabel('H_o(Oe)','Fontsize',28);
ylabel('Z_c_h(Ohm)','Fontsize', 28);
h = gca;
set(h,'Fontsize',28);
legend('Re(Z_c_h)','Im(Z_c_h)',2);
title('Characteristic Impedance','FontSize',28);

figure(4); % surface impedance, Zs vs H (from 4x4 matrix)
hold on;
plot(Ho, real(Zs_0),'r','Linewidth',2);
plot(Ho, imag(Zs_0),'r--','Linewidth',2);
hold off;
xlabel('H_o(Oe)','Fontsize',28);
ylabel('Z_s(Ohm)','Fontsize', 28);
legend('Re(Z_s)','Im(Z_s)',2);
h = gca;
set(h,'Fontsize',28);
title('Surface Impedance','FontSize',28);

% effective wave vector calculated from A matrix
figure(5);
hold on;
plot(Ho,beta,'LineWidth',2);
plot(Ho, alpha, '-','LineWidth',2);
plot([3000 4000], [pi/(2*d) pi/(2*d)], 'k--','LineWidth',1);
\% plot([3000 4000],-[pi/(2*d) pi/(2*d)], 'g--','LineWidth',1);
\% plot([3500 3500], [min(beta) max(alpha)], 'g--'); % reference line of the resonant field
hold off;
xlabel('H(Oe)','FontSize',28);
ylabel('k_{\text{eff}}(rad/cm)','FontSize', 28);
h = gca;
set(h,'FontSize',22);
\%title('Propagation Constant','FontSize',28);
legend('{\beta}','{\alpha}','{\pi/(2d)}',2);
\% effective permeability calculated from A matrix
\% and from Poynting vector integration
figure(6);
hold on;
plot(Ho, real(mu_eff), 'r','LineWidth',2);
plot(Ho, -imag(mu_eff), 'r--', 'LineWidth',2);
plot(Ho, real(Integ1), 'b','LineWidth',1.5);
plot(Ho, -imag(Integ1), 'b--', 'LineWidth',1.5);
\% plot([3500,3500], [min(real(mu_eff)) max(-imag(mu_eff))], 'g--');
xlabel('H_o(Oe)', 'FontSize', 28);
ylabel('{\mu_{\text{eff}}}', 'FontSize',28);
legend('{\mu_{\text{eff}}}', '{\mu_{\text{eff}}}', '{\mu_v}', '{\mu_v}',1);
h = gca;
set(h,'FontSize',28);
title('Permeability Comparison','FontSize',28);
hold off;
\% effective permittivity calculated from A matrix
figure(7);
hold on;
plot(Ho, real(eps_eff),'b','LineWidth',2);
plot(Ho, -imag(eps_eff),'b--','LineWidth',2);
plot([3500 3500], [min(real(eps_eff)) max(-imag(eps_eff))], 'g--');
xlabel('H(Oe)', 'FontSize', 24);
ylabel('
\epsilon_{eff}(f/cm)', 'FontSize',32);
legend('\epsilon_{eff}''', '\epsilon_{eff}'' Imag',4);
h = gca;
set(h,'FontSize',30);
title('Effective Average Permittivity','FontSize',24);
hold off;

% effective/exchange conductivity calculated from A matrix
figure(8);
hold on;
plot(Ho, real(sigma_eff),'b','LineWidth',2);
plot(Ho, -imag(sigma_eff),'b--','LineWidth',2);
plot(Ho, sigma,'r--','LineWidth',2);
h = gca;
set(h,'FontSize',28);
hold off;
xlabel('H_o(Oe)', 'FontSize', 28);
ylabel('\sigma_{eff}(S/cm)', 'FontSize',32);
legend('Re(\sigma_{eff})','Im(\sigma_{eff})','\sigma',4);
% title('Effective Conductivity','FontSize',28);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% function definition in an independent file
function f0 = myfun(y)

global w mu0 eps X1s X1s k1s k2s Z1s Z2s h1r h1l h2r h2l
f0 = -j*w*mu0*(X1s*(h1r*exp(-j*k1s*y) + h1l*exp(j*k1s*y)) + h2l)}
\[
Xi2s*(h2r*exp(-j*k2s*y) + h2l*exp(j*k2s*y))) .*
\]
\[
\text{conj( h1r*exp(-j*k1s*y) + h1l*exp(j*k1s*y) + h2r*exp(-j*k2s*y) + h2l*exp(j*k2s*y) ...}
\[
- j*w*mu0*(h1r*exp(-j*k1s*y) + h1l*exp(j*k1s*y) + ...}
\[
h2r*exp(-j*k2s*y) + h2l*exp(j*k2s*y)) .* conj( h1r*exp(-j*k1s*y) + h1l*exp(j*k1s*y) + ...}
\[
- j*w*eps*(Z1s*( h1r*exp(-j*k1s*y) - h1l*exp(j*k1s*y) ) + ...}
\[
Z2s*( h2r*exp(-j*k2s*y) - h2l*exp(j*k2s*y)) .*conj( Z1s*( h1r*exp(-j*k1s.*y) - h1l*exp(j*k1s*y) ) + ...}
\[
Z2s*(h2r*exp(-j*k2s*y) - h2l*exp(j*k2s*y)) );
\]

% function definition in an independent file

function f1 = myfun1(y)

global Xi1s Xi2s k1s k2s h1r h1l h2r h2l
f1 = ( Xi1s*(h1r*exp(-j*k1s*y)+h1l*exp(j*k1s*y)) +
Xi2s*(h2r*exp(-j*k2s*y)+h2l*exp(j*k2s*y)) ) .*
\[
\text{conj( h1r*exp(-j*k1s*y)+h1l*exp(j*k1s*y) +}
\[
h2r*exp(-j*k2s*y)+h2l*exp(j*k2s*y) ) ...}
\]
\[
+ (h1r*exp(-j*k1s*y)+h1l*exp(j*k1s*y) +
\[
h2r*exp(-j*k2s*y)+h2l*exp(j*k2s*y)) .*
\[
\text{conj( h1r*exp(-j*k1s*y)+h1l*exp(j*k1s*y) +}
\]
\[
h2r*exp(-j*k2s*y)+h2l*exp(j*k2s*y) );
\]

% function definition in an independent file

function f2 = myfun2(y)

global eps k1s k2s Z1s Z2s h1r h1l h2r h2l

f2 = eps*( Z1s*( h1r*exp(-j*k1s*y) - h1l*exp(j*k1s*y) ) +
\[
Z2s*( h2r*exp(-j*k2s*y) - h2l*exp(j*k2s*y)) ) ...}
\]
\[
.*conj( Z1s*( h1r*exp(-j*k1s.*y) - h1l*exp(j*k1s*y) ) +
\]

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Z2s*(h2r*exp(-j*k2s*y) - h2l*exp(j*k2s*y))

% function definition in an independent file

function A = A_matrix(SP, Z1, Z2)

alpha_1_plus = det(SP([2:4], [2:4]))/det(SP);
alpha_1_minus = -det(SP([2:4], [1, 3, 4]))/det(SP);
alpha_2_plus = det(SP([2:4], [1, 2, 4]))/det(SP);
alpha_2_minus = -det(SP([2:4], [1, 2, 3]))/det(SP);

beta_1_plus = -det(SP([1, 3, 4], [2:4]))/det(SP);
beta_1_minus = det(SP([1, 3, 4], [1, 3, 4]))/det(SP);
beta_2_plus = -det(SP([1, 3, 4], [1, 2, 4]))/det(SP);
beta_2_minus = det(SP([1, 3, 4], [1, 2, 3]))/det(SP);

a11 = j*Z1*(alpha_1_plus-alpha_1_minus) +
     j*Z2*(alpha_2_plus-alpha_2_minus);

a12 = j*Z1*(beta_1_plus-beta_1_minus) +
     j*Z2*(beta_2_plus-beta_2_minus);

a21 = (alpha_1_plus+alpha_1_minus) + (alpha_2_plus+alpha_2_minus);

a22 = (beta_1_plus+beta_1_minus) + (beta_2_plus+beta_2_minus);

A = [a11, a12; a21, a22];
REFERENCES


50 Agilent Application Note 1364-1, "De-embedding and Embedding S-Parameter Networks Using a Vector Network Analyzer."


