BAD DATA DETECTION IN TWO STAGE ESTIMATION USING PHASOR MEASUREMENTS

A Thesis Presented

By

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Abstract

The ability of the Phasor Measurement Unit (PMU) to directly measure the system state, has led to steady increase in the use of PMU in the past decade. However, in spite of its high accuracy and the ability to measure the states directly, they cannot completely replace the conventional measurement units due to high cost. Hence it is necessary for the modern estimators to use both conventional and phasor measurements together.

This thesis presents an alternative method to incorporate the new PMU measurements into the existing state estimator in a systematic manner such that no major modification is necessary to the existing algorithm. It is also shown that if PMUs are placed appropriately, the phasor measurements can be used to detect and identify the bad data associated with critical measurements by using this model, which cannot be detected by conventional state estimation algorithm.

The developed model is tested on IEEE 14, IEEE 30 and IEEE 118 bus under various conditions.
Acknowledgement

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Table of Contents

Abstract .......................................................................................................................... i
Acknowledgement .......................................................................................................... ii
Table of Contents ........................................................................................................... iii
List of Figures ................................................................................................................ vii
List of Tables ................................................................................................................ ix

Chapter 1 - Introduction ............................................................................................... 1-1

1.1 History .................................................................................................................... 1-2
  1.1.1 Background Discussion of State Estimation ................................................. 1-3
  1.1.2 Background Discussion of PMUs ................................................................. 1-4
1.2 Motivation & Objective ......................................................................................... 1-5
1.3 Organization of Thesis ......................................................................................... 1-5

Chapter 2 - Conventional State Estimator ................................................................. 2-1

2.1 Construction of the System Model ....................................................................... 2-2
  2.1.1 Transmission line ......................................................................................... 2-2
  2.1.2 Shunt Capacitor or Reactor ......................................................................... 2-3
  2.1.3 Tap Changing and Phase Shifting Transformer ......................................... 2-3
2.2 Bus Admittance matrix ......................................................................................... 2-6
2.3 Maximum Likelihood Estimation ........................................................................ 2-8
2.4 WLS State Estimation ........................................................................................ 2-9
  2.4.1 WLS Algorithm ......................................................................................... 2-10
  2.4.2 The Measurement Function ...................................................................... 2-14
  2.4.3 The Measurement Jacobian ...................................................................... 2-15
2.5 Including PMU Data in State Estimator .............................................................. 2-20
  2.5.1 PMU Measurements Mixed with Conventional Measurements ....... 2-21
2.6 Conclusion ................................................................. 2-24

Chapter 3 - The Linear State Estimation ............................................. 3-1

3.1 Introduction to Linear State Estimation ................................. 3-1
3.2 Linear State Estimation with π-Equivalent ............................. 3-2
3.3 Matrix Formulation ......................................................... 3-4
   3.3.1 Current Measurement-Bus Incidence Matrix .................... 3-5
   3.3.2 Voltage Measurement Bus Incidence Matrix .................. 3-6
   3.3.3 Series Admittance Matrix ........................................... 3-8
   3.3.4 Shunt Admittance Matrix ............................................ 3-8
   3.3.5 System Matrix Formulation ......................................... 3-9
   3.3.6 An Alternative Formulation of the Linear State Estimator .... 3-11
3.4 Solution to the Linear State Estimation Problem ...................... 3-13
3.5 Adding Phasor Measurement through a Post-Processing Step ........ 3-14
3.6 Conclusion ..................................................................... 3-20

Chapter 4 - Bad Data Analysis ....................................................... 4-1

4.1 Introduction to Bad Data Analysis ............................................ 4-2
4.2 Measurement Residual and its Properties ............................... 4-4
4.3 Largest Normalized Residual $r_{N \text{max}}$ Test .......................... 4-7
   4.3.1 $r_{N \text{max}}$ Test and Conventional State Estimator ............. 4-7
   4.3.2 Strength and Weakness of the Test ............................... 4-14
4.4 Bad Data Associated with Critical Measurement ..................... 4-15
4.5 Bad Data Detection in Phasor Measurements .......................... 4-23
4.6 Conclusion ..................................................................... 4-24

Chapter 5 - Results and Conclusions .............................................. 5-1
5.1 Testing of the State Estimation ........................................5-1
  5.1.1 Creating Systems and Measurements ..........................5-1
  5.1.2 Results ................................................................5-2
5.2 Bad data.........................................................................5-8
  5.2.1 Single bad data (non-critical).................................5-8
  5.2.2 Single bad data (critical measurement)..................5-13
  5.2.3 Multiple Bad Data .................................................5-18
  5.2.4 Bad Data in Phasor Measurement .........................5-20
5.3 Conclusion...................................................................5-24
5.4 Future Work ..................................................................5-24

References .................................................................xii

Appendix A Topology and Measurement Data.....................A-1
  A.1 busdata.m ................................................................A-1
  A.2 linesdata.m ...........................................................A-6
  A.3 zconv.m .................................................................A-12
  A.4 current.m ............................................................A-23
  A.5 zpmu.m .................................................................A-24

Appendix B State Estimation algorithms ............................B-1
  B.1 Conventional state estimation (wlscart.m) ....................B-1
  B.2 Two stage state estimation (stage2.m) .......................B-5

Appendix C Miscellaneous Functions ..............................C-1
  C.1 Ybusfunc.m ..........................................................C-1
  C.2 line_mat_func.m ....................................................C-2
C.3 factoriseGby chol.m ................................................................. C-4
C.4 rotatemat.m ................................................................. C-5
C.5 displayout.m ................................................................. C-6
List of Figures

Figure 2.1: 2-port Π model of a transmission line ........................................2-3
Figure 2.2: Equivalent circuit of an off-nominal tap transformer ..................2-4
Figure 2.3: Equivalent 2-port π model of a Transformer .............................2-5
Figure 2.4: A simple 5 bus example network ..............................................2-7
Figure 2.5: Equivalent 2-port π model of a transmission line .......................2-14
Figure 3.1: Equivalent 2-port π model of a Transmission line ......................3-3
Figure 3.2: 5 bus system showing current measurements ............................3-6
Figure 4.1: Classification of bad data .........................................................4-3
Figure 4.2: Normalized residue for 5 bus system with true measurements ......4-13
Figure 4.3: Normalized residue for 5 bus system with single bad measurement ..4-13
Figure 4.4: Normalized residue after first stage with bad critical measurement ....4-22
Figure 4.5: Normalized residue after first stage with bad critical measurement ....4-22
Figure 5.1: IEEE 14 bus system ..................................................................5-3
Figure 5.2: IEEE 30 bus system ..................................................................5-4
Figure 5.3: IEEE 118 bus system .................................................................5-5
Figure 5.4: r^N of IEEE 14 bus single bad data(non-critical)using CSE ..........5-9
Figure 5.5: rN of IEEE 14 bus single bad data(non-critical)using TSSE ..........5-9
Figure 5.6: r^N of IEEE 30 bus single bad data (non-critical) using CSE ..........5-10
Figure 5.7: r^N of IEEE 30 bus single bad data (non-critical) using TSSE ........5-11
Figure 5.8: r^N of IEEE 118 bus single bad data (non-critical) using CSE ..........5-12
Figure 5.9: $r^N$ of IEEE 118 bus single bad data (non-critical) using TSSE........5-12
Figure 5.10: $r^N$ of IEEE 14 bus single bad data (critical) using CSE.................5-14
Figure 5.11: $r^N$ of IEEE 14 bus single bad data (critical) using TSSE................5-14
Figure 5.12: $r^N$ of IEEE 30 bus single bad data (critical) using CSE....................5-15
Figure 5.13: $r^N$ of IEEE 30 bus single bad data (critical) using CSE....................5-16
Figure 5.14: $r^N$ of IEEE 118 bus single bad data (critical) using CSE...............5-17
Figure 5.15: $r^N$ of IEEE 118 bus single bad data (critical) using CSE...............5-17
Figure 5.16: $r^N$ of phasor measurements for IEEE 14 bus system .................5-21
Figure 5.17: $r^N$ of phasor measurements for IEEE 30 bus system .................5-22
Figure 5.18: $r^N$ of phasor measurements for IEEE 118 bus system..............5-23
List of Tables

Table 2-1: Network parameter for the 5 bus system shown in figure 2.4 ...............2-7
Table 2-2: Measurement data for 5 bus system shown in figure 2.4 ..................2-19
Table 2-3: Voltage in polar form for 5 bus system shown in figure 2.4 ..............2-20
Table 3-1: Phasor measurements for 5 bus system shown in figure 2.4 ..............3-17
Table 3-2: Voltage in polar form for 5 bus system ........................................3-20
Table 4-1: Measurement vectors containing both true and false measurement ......4-9
Table 4-2: True states and states computed using single bad measurement ........4-10
Table 4-3: Residue for True measurement and 1 bad data .............................4-11
Table 4-4: Diagonal entries of covariance of residual matrix Ω .......................4-11
Table 4-5: Normalized residues for 5 bus system ..........................................4-12
Table 4-6: Performance of $r^N_{max}$ Test under different conditions ..............4-15
Table 4-7: States estimated after first and second stage with single bad data .......4-19
Table 4-8: Residue after first and second stage for single bad data .................4-19
Table 4-9: Diagonal entries of covariance of residual matrix Ω .......................4-20
Table 4-10: Normalized residues for 5 bus system ..........................................4-21
Table 5-1: True state of IEE 14 bus system .....................................................5-3
Table 5-2: True states of IEEE 30 bus system .................................................5-4
Table 5-3: True states of 118 bus system .........................................................5-7
Table 5-4: Largest normalized residue IEEE 14 bus ........................................5-8
Table 5-5: Largest normalized residue IEEE 30 bus........................................5-10
Table 5-6 Largest normalized residue IEEE 118 bus........................................5-11
Table 5-7: List of Critical measurements ...........................................................5-13
Table 5-8: Largest normalized residue IEEE 14 bus........................................5-14
Table 5-9: Largest normalized residue IEEE 30 bus........................................5-15
Table 5-10: Largest normalized residue IEEE 118 bus......................................5-16
Table 5-11: Multiple interacting non-conforming bad data ................................5-18
Table 5-12: Multiple interacting conforming bad data .....................................5-19
Table 5-13: Multiple non-interacting bad data.....................................................5-20
Table 5-14: Largest normalized residue IEEE 14 bus with Bad data in PMU.....5-21
Table 5-15: Largest normalized residue IEEE 30 bus with Bad data in PMU.....5-22
Table 5-16: Largest normalized residue IEEE 118 bus with Bad data in PMU....5-23
Chapter 1 - Introduction

Electric power systems account for a critical part of our society’s energy infrastructure. Over the years we have grown to depend on the near perfect reliability of these systems that have become a necessary part of our everyday lives. When we enter a room, we instinctively reach for the light switch without the slightest concern that it won’t turn on. All of our household appliances, communication devices, and almost all of our tools ranging from construction sites to our offices require electricity for operation. It is not as if we assume electricity will always be available, it is that we believe electricity will always be available.

This kind of reliability doesn’t happen without a great deal of effort from individuals such as electrical engineers and larger bodies such as electric utilities, universities, and government organizations. One of the key aspects to maintaining the reliability of a large system such as the electric power grid is finding a way to provide feedback to those that control it. Finding a way to accurately monitor the system has been the goal of engineers for the majority of the life of our electric grid. If system operators can be provided with appropriate information regarding the conditions of their systems, then they can use that information to make decisions that will improve not only the day-to-day reliability of the system but allow for engineers to plan more effectively for the future.

State estimation is facet of electric power engineering that has evolved out of these needs. Beginning in the 1960’s, engineers began developing ways to monitor their systems from a central control room. They developed communications systems to collect measurement information across the systems and developed
system models that could portray the structure of the network. A computer then took in all of this information and computed an optimized portrait of the systems operating conditions called the system state. With this set of information, an adequately trained professional could understand everything that is going on in the system and make operation and control decisions accordingly.

Many technologies have come along since the inception of state estimation that have improved its performance and drove it to become an integral part of control center operations. Today, Phasor Measurement Unit (PMU) technology will serve as the next step in improving the quality of the estimate of the system state, providing operators with better information to maintain a high level of system reliability. While PMUs are still noticeably more expensive that traditional measurement devices, the gains of monitoring the transmission level part of the system with these newer, time-synchronized, measurement devices balances the financial drawbacks.

The Main contribution of this thesis is to demonstrate the use of PMU in field of bad data analysis using a two stage state estimator, which is designed to use phasor measurements to improve results of conventional state estimator. A new bad data algorithm is developed based on largest normalized residual test that shows the Phasor measurements can be used to add redundancy to conventional measurements which are critical in nature, hence bad data in such measurement can be identified. The reason for using a two stage state estimator is the flexibility of adding PMU measurements separately to the existing state estimator without the need of reformulation, as the proposed algorithm uses results of existing estimator.

1.1 History

This section investigates the history and development of the two critical technologies that make up the heart of this thesis. First is the history of state
estimation in power systems. Second is the Phasor Measurement Unit (PMU) which forms an integral part of bad data analysis.

1.1.1 Background Discussion of State Estimation

The state of a system can be defined as the minimum set of parameters that must be known to fully perceive the operating conditions of the system. When applied to electric power systems, the system state is known as the set of complex voltages at every node in the network. In conjunction with the network topology and impedances (which are constant and known), every line flow or injection can then be calculated. In the past, operators collected a set of incomplete and unsynchronized measurements which they would subsequently insert into a load flow calculation in an attempt to find the operating point of the system [2]. This was a very inexact approach and most times led to divergence of the load flow equations.

As the size and complexity of the power system grows it becomes exceedingly more important for system operators to know how the system is behaving [1]. This becomes most evident when simple contingencies occur, cascade to neighboring systems and eventually cause large blackouts. In most historical examples, the scale of the blackout could have been reduced if the system operators had had better and more up-to-date information regarding their system’s operating condition. After the Northeast Blackout of 1965, engineers took large steps to overhaul the operator’s load flow and create a more reliable tool for operators to use [2]. The solution to this problem was static state estimation and was first proposed by Fred Schweppe in the 1960’s [4, 5, 6]. This involved collection of voltage magnitudes, current injections, and real and reactive power flows such that there were more measurements than state variables, thus forming an over-determined system. These measurements were assigned a weight based on their accuracy and inserted into “load flow-like” equations which were non-linear and required multiple iterations to arrive at a solution. The solution of this minimization was the
system state. Then operators could use this information to make decisions regarding
the control and operation of their power system.

Originally, measurement collection times (or scan times) could be very large
due to delays in the communication systems and the computation times of the non-
linear iterations could also be sizeable [2]. However, over several decades, state
estimation has evolved greatly from its infancy as a mathematical curiosity to its
now central role in the reliable operation and control of power systems [1]. Its
effectiveness and functionality have changed and grown as technology has
improved; as time has passed, power system operators have been provided with
more abundant and more accurate system measurements to keep them informed on
the operating conditions of the power grid.

1.1.2 Background Discussion of PMUs

Phasor Measurement Units (PMUs) are digital metering devices that use a
DFT algorithm in conjunction with a precisely timed GPS signal to provide
synchronized phasor measurements at different locations in the power system [7].
With GPS synchronization, this technology has the ability to synchronize
measurements despite the large distances which may separate metering points.
While PMUs were originally very costly, the cost associated with these devices has
fallen over the last several decades as computer & GPS technology has improved
[2]. While the cost of PMUs has decreased the cost of installation due to security
and communication has increased. Older problems that have plagued state
estimators such as scan time could be minimized with the use of synchronized
phasor measurements [2].
1.2 Motivation & Objective

PMU finds a wide variety of applications in modern power systems such as enhancement of state estimation, real-time stability assessment, detection of low frequency oscillations, and islanding, etc. Incorporation of these measurements into the state estimation formulation will require changes to be made in the existing state estimation code. As an alternative, a two-stage state estimation (TSSE) procedure is proposed in where the result of the conventional state estimator (CSE) was “improved” by a second stage estimation where PMU measurements along with state estimates of CSE are processed.

One of the main objectives of executing the state estimator is to detect and eliminate bad data. This will ensure an unbiased state estimate which will facilitate proper execution of all other application functions that rely on state estimator output at the control center. However, despite full observability, the system may have vulnerable areas so called “critical” measurements. Errors in these measurements will not be detectable. One benefit of introducing redundant PMU measurements will then be to let them transform these critical measurements into redundant ones, thus eliminating their vulnerability to bad data [15, 17]

In this Thesis the two stage State estimator is used to develop a new algorithm to detect bad data with minimal changes being made to existing state estimation module. The developed algorithm is tested on IEEE 14, 30 and 118 bus systems.

1.3 Organization of Thesis

This thesis is organized in the following way:
Chapter 2: Conventional State Estimation Techniques

This chapter presents the mathematical formulation of the algorithm employed by traditional state estimation techniques. It explores system component modeling, maximum likelihood estimation, weighted least squares estimation (including the WLS algorithm and matrix formulation), and a brief discussion concerning the state estimator consisting of phasor and conventional measurements combined together.

Chapter 3: Linear State Estimation

The chapter begins with the formulation of the positive sequence linear state estimation problem using phasor measurements exclusively. The formulation of the state equation is presented using first a simple two-port pi-model and then broken down into several independent types of matrices. Rules for the population of these matrices and the formation of the complete state equation are presented. An alternative formulation of state equation which uses only real values instead of complex values is described. The chapter concludes with the formulation of a linear post-processing step which forms the two stage state estimator.

Chapter 4: Bad Data Analysis

The chapter introduces various types of bad data and their causes. A mathematical model for bad data analysis is presented based on maximum normalized residual test. A comprehensive study of types of measurement and their effects on bad data is given followed by strengths and weaknesses if the test. The chapter then concludes with implementation of maximum normalized residual test on the two stage state estimator model and how phasor measurements can be used to detect bad data in critical measurements.
Chapter 5: Results and Conclusion

The chapter summarizes the results of true states obtained from TSSE for IEEE 14, 30 and 118 bus systems followed by the results of bad data analysis when various types of bad data are enforced on these bus systems. The chapter provides conclusions from results obtained, followed by scope for the future work.
Chapter 2 - Conventional State Estimator

Power system state estimation refers to the process of collection of measurements from around the system and computing a state vector of the voltage at each observed bus. While technology has improved state estimators and other control center applications over many decades, the fundamental concepts and algorithms behind these proven techniques remain much the same. Measurements which are non-linear functions of the system state are collected and load-flow-like calculations are performed iteratively to determine the most probable system state from the known information.

State estimation results are processed to identify errors in the meters and parameters given that there are enough redundant measurements. In addition, failure or loss of a meter can be detected. Topology of a network changes when a line is lost due to overloading or equipment failure. In such a case, the security of the system might be in danger since some other lines might get overloaded as well. State estimation results, when analyzed carefully, can give early warnings of such cases.

This chapter presents the background of power system such as modeling of various components and calculation of bus admittance matrix followed by mathematical basis for conventional state estimation techniques and investigates several reformulations of this algorithm to include phasor measurements in the estimator to improve the quality of the estimate.
2.1 Construction of the System Model

The state of the power system is dependent on several factors. These include system parameters such as resistance, reactance, and shunt susceptance of transmission lines and measurements such as real and reactive power flows, real and reactive power injections, and voltages which are measured, network topology, which is assumed to be known [1].

The measurements are periodically sent into the control center over a SCADA network. However, the transmission line parameters and physical system model are carefully constructed offline prior to implementation. The system model only changes dynamically due to line outages or other contingencies and is typically handled by a topology processor [1]. This section outlines the details associated with constructing the system model for use with conventional non-linear state estimation technique, from individual component modeling to the formulation of the large system matrices.

2.1.1 Transmission line

Transmission lines in power systems are three phase. It is assumed that transmission line fully transposed and all other series and shunt devices are symmetrical in all three phases. The generation and load at each phase is also assumed to be balanced [1]. Thus, single phase analysis can be used to simplify the transmission line model, which is represented by a two-port Π circuit [3, 8]. The equivalent circuit of transmission line is shown in figure 2.1 connecting bus k to m.

In the figure 2.1, \( V_k \) and \( V_m \) are voltages, \( I_k \) and \( I_m \) are current injection, at bus k and bus m respectively. \( R+jX \) is positive sequence line impedance and \( jB \) is half of total line charging susceptance.
By kirchhoff’s law the current injections at bus k and bus m can be written as

\[
\begin{bmatrix}
I_k \\
I_m
\end{bmatrix} = 
\begin{bmatrix}
y + jB & -y \\
-y & y + jB
\end{bmatrix}
\begin{bmatrix}
V_k \\
V_m
\end{bmatrix}
\]  \hspace{1cm} (2.1)

Where

\[
y = \frac{1}{R + jX}
\]  \hspace{1cm} (2.2)

### 2.1.2 Shunt Capacitor or Reactor

Shunt capacitors and reactors are devices which are installed in the network to serve as reactive power support and voltage control [1]. These are represented by their per phase susceptance at corresponding bus.

### 2.1.3 Tap Changing and Phase Shifting Transformer

A tap changing transformer is one of the most commonly used transformers in power system. It is used to step-up or step-down voltage by a scalar factor ‘a’, which is called tap ratio, without changing voltage phase angle. A tap changing transformer between bus k and bus m with off-nominal but in-phase tap, can be modeled as an ideal transformer with a series impedance as shown in Figure 2.2
The following equations hold true for the transformer shown in figure 2.2.

\[ v_l = \frac{1}{a} v_k \]  \hspace{1cm} (2.3)

\[ i_{lm} = a i_k \]  \hspace{1cm} (2.4)

\[ i_{lm} = y (v_l - v_m) \]  \hspace{1cm} (2.5)

Using equation (2.3), (2.4) and (2.5) and eliminating \( v_l \) and \( i_{lm} \) we get

\[ a i_k = y \left( \frac{1}{a} v_k - v_m \right) \]  \hspace{1cm} (2.6)

\[ i_k = \frac{y}{a^2} v_k - \frac{y}{a} v_m \]  \hspace{1cm} (2.7)

Also

\[ i_m = -i_{lm} = y (v_m - v_l) \]  \hspace{1cm} (2.8)

Again using (2.8) and (2.3) and eliminating \( v_l \) we get
Writing equation (2.7) and (2.9) in a matrix form, we get

\[
\begin{bmatrix}
I_k \\
I_m
\end{bmatrix} = \begin{bmatrix}
y/a^2 & -y/a \\
-y/a & y
\end{bmatrix} \begin{bmatrix}
V_k \\
V_m
\end{bmatrix}
\] (2.10)

Figure 2.3 shows the corresponding two port equivalent for a transformer represented by above set of equation.

Figure 2.3: Equivalent 2-port π model of a Transformer

The second type of transformer is the phase shifting transformer. Unlike tap changing transformer, phase shifters have complex tap ratio, denoted by ‘a’. A phase difference between primary and secondary side voltages can be created by this complex tap ratio. The main purpose of a phase shifter is to control the flow of power to prevent congestion. The equations change slightly change as

\[
v_l = \frac{1}{a} v_k
\] (2.11)

\[
i_{lm} = a^* i_k
\] (2.12)

Yielding the following set nodal equation

\[
\begin{bmatrix}
I_k \\
I_m
\end{bmatrix} = \begin{bmatrix}
y/|a|^2 & -y/a^* \\
-y/a & y
\end{bmatrix} \begin{bmatrix}
V_k \\
V_m
\end{bmatrix}
\] (2.13)
2.2 Bus Admittance matrix

After the network parameters such as transmission line series and shunt impedances, transformer impedances, and shunt capacitors and reactors have been defined they can be assembled together to construct the network model of the system. This is called the admittance matrix or the Y-Bus of the power system. The admittance matrix of a power system takes the following form.

\[
I = \begin{bmatrix}
  i_1 \\
  i_2 \\
  \vdots \\
  i_N
\end{bmatrix} = \begin{bmatrix}
  Y_{11} & Y_{12} & \cdots & Y_{1N} \\
  Y_{21} & Y_{22} & \cdots & Y_{2N} \\
  \vdots & \vdots & \ddots & \vdots \\
  Y_{N1} & Y_{N2} & \cdots & Y_{NN}
\end{bmatrix} \begin{bmatrix}
  v_1 \\
  v_2 \\
  \vdots \\
  v_N
\end{bmatrix} = Y \times V
\] (2.14)

Admittances are used instead of impedances because the admittance matrix can be populated by inspection while the corresponding impedance matrix is extremely difficult to populate by. The rules for populating the admittance matrix of a network are derived from Kirchhoff’s laws of current injections into a node. Sparing the derivation of the equations, there are two simple rules that can be used to construct the network admittance matrix by inspection.

1. The \(ii^{th}\) element of the admittance matrix is the sum of all the admittance of all lines connected to bus \(i\), including shunt susceptance of the branch and susceptance of the capacitor or reactor connected to bus.

2. The \(ij^{th}\) element of the admittance matrix is the negative of the admittance connecting bus \(i\) and bus \(j\)

Once all the lines and shunts are populated, in order to include transformer between bus \(k\) and bus \(m\) use equation (2.13) and update the elements of Y-bus using following equations.
\[ Y_{kk}^{\text{new}} = Y_{kk} + y/|a|^2 \quad (2.15) \]
\[ Y_{km}^{\text{new}} = Y_{km} - y/a^* \quad (2.16) \]
\[ Y_{mk}^{\text{new}} = Y_{mk} - y/a \quad (2.17) \]
\[ Y_{mm}^{\text{new}} = Y_{mm} + y \quad (2.18) \]

The admittance matrix \( Y \) is generally complex in nature, structurally symmetric, very sparse for large networks, and non-singular provided that each island contains a connection to ground [3, 8].

**Example 2.1**

Consider the following simple power system network shown in figure 2.4 to demonstrate the construction of the network model.

The network data for the system shown in figure 2.4 is shown in table 2.1

<table>
<thead>
<tr>
<th>From Bus</th>
<th>To bus</th>
<th>R</th>
<th>X</th>
<th>Total line charging Susceptance</th>
<th>Tap a</th>
<th>Tap side bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.067</td>
<td>0.1710</td>
<td>0.0173</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.00</td>
<td>0.2091</td>
<td>0.00</td>
<td>0.978</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.00</td>
<td>0.5562</td>
<td>0.00</td>
<td>0.969</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0.00</td>
<td>0.1762</td>
<td>0.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.00</td>
<td>0.1100</td>
<td>0.00</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Susceptance of the shunt capacitor at bus 5 is 0.19

**Table 2-1:** Network parameter for the 5 bus system shown in figure 2.4

**Figure 2.4:** A simple 5 bus example network
The bus admittance matrix of transformer branch is given by equation (2.19) and equation (2.20)

\[
\begin{bmatrix}
i_2 \\
i_3
\end{bmatrix} = \begin{bmatrix}
-j5.00 & j4.89 \\
j4.89 & -4.78
\end{bmatrix} \begin{bmatrix}
v_2 \\
v_3
\end{bmatrix}
\] (2.19)

\[
\begin{bmatrix}
i_2 \\
i_5
\end{bmatrix} = \begin{bmatrix}
-j1.92 & j1.86 \\
j1.86 & -1.80
\end{bmatrix} \begin{bmatrix}
v_2 \\
v_5
\end{bmatrix}
\] (2.20)

The bus admittance matrix the entire 5 bus system is given below

\[
\begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
i_4 \\
i_5
\end{bmatrix} = \begin{bmatrix}
1.99 - j5.05 & -1.99 + j5.07 & 0 & 0 & 0 \\
-1.99 + j5.07 & 1.99 - j11.97 & j4.89 & 0 & j1.86 \\
0 & j4.89 & -j19.55 & j5.68 & j9.09 \\
0 & 0 & j5.68 & -j5.68 & 0 \\
0 & j1.86 & j9.09 & 0 & -j10.70
\end{bmatrix} \begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4 \\
v_5
\end{bmatrix}
\] (2.21)

### 2.3 Maximum Likelihood Estimation

The mathematical theory powering state estimation techniques is a statistical method called maximum likelihood estimation. It begins by creating the likelihood function of the measurement vector. The likelihood function is simply the product of each of the probability density functions of each measurement. Maximum likelihood estimation aims to estimate the unknown parameters of each of the measurements’ probability density functions through an optimization [1].

It is commonly assumed that the probability density function for power system measurement errors is the normal (or Gaussian) probability density function

\[
f(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}
\] (2.22)
where $z$ is the random variable of the probability density function, $\mu$ is the expected value, and $\sigma$ is the standard deviation. This function would yield the probability of a measurement being a particular value, $z$. Therefore, the probability of measuring a particular set of $m$ measurements each with the same probability density function is the product of each of the measurements probability density functions, or the likelihood function for that particular measurement vector [1].

$$f_m(z) = \prod_{i=1}^{m} f(z_i)$$  \hspace{1cm} (2.23)

where $z_i$ is the $i^{th}$ measurement and

$$[z] = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix}$$ \hspace{1cm} (2.24)

Maximum likelihood estimation aims to maximize this function to determine the unknown parameters of the probability density function of each of the measurements. This can be done by maximizing the logarithm of the likelihood function, $f_m(z)$, or minimizing the weighted sum of squares of the residuals [1]. This can be written as

$$\text{minimize} \sum_{i=1}^{m} W_i r_i^2$$ \hspace{1cm} (2.25)

subjected to $z_i = h_i(x) + r_i$

The solution to this problem is referred to as the weighted least squares (WLS) estimator for $x$.

### 2.4 WLS State Estimation

Power system state estimators use a set of measurements taken from the power system to determine the most likely system state from the given information.
The state estimator becomes a weighted least squares estimator with the inclusion of the measurement error covariance matrix which serves to weigh the accuracy of each of the measurements. The physical system model information and measurements are part of the equality constraints of the basic weighted least squares optimization and make this algorithm specific to power systems. This section presents the solution to the weighted least squares problem and the system matrix formulation including the measurement function matrix and measurement Jacobian matrix.

### 2.4.1 WLS Algorithm

Several texts present material developing the mathematical algorithms for the weighted least squares estimator [1, 2, 4]. This section and section 2.4.2 explains the algorithm in Cartesian coordinate system where states are expressed in terms of their real and imaginary parts. Consider a measurement vector denoted by $z$ containing $m$ number of measurements and a state vector denoted by $x$ containing $n$ number of state variables.

\[
[z] = \begin{bmatrix}
  z_1 \\
  z_2 \\
  z_3 \\
  \vdots \\
  z_m
\end{bmatrix} \quad [x] = \begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  \vdots \\
  x_n
\end{bmatrix} \tag{2.26}
\]

Conventional state estimation techniques employ measurement sets which are non-linear functions of the system state vector. These functions are denoted by $h_i(x)$ and can be assembled in vector form as well.

\[
[h(x)] = \begin{bmatrix}
  h_1(x_1 \ x_2 \ x_3 \ \cdots \ x_n) \\
  h_2(x_1 \ x_2 \ x_3 \ \cdots \ x_n) \\
  h_3(x_1 \ x_2 \ x_3 \ \cdots \ x_n) \\
  \vdots \\
  h_n(x_1 \ x_2 \ x_3 \ \cdots \ x_n)
\end{bmatrix} \tag{2.27}
\]
These functions, evaluated at the true system state would yield a measurement set containing the true measurement values. However, all of these measurements have their own unknown error associated with them denoted by $e$ and is shown in vector form in equation (2.28).

$$ [e] = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix} \quad (2.28) $$

The measurement errors are assumed to be independent of one another and have an expected value of zero. The state equation using non-linear functions to relate the system state vector to the set of measurements can now be written in its complete form.

$$ [z] = [h(x)] + [e] \quad (2.29) $$

From the previous section, the solution to the state estimation problem can be formulated as a minimization of following objective function.

$$ J(x) = \sum_{i=1}^{m} \frac{(z_i - h_i(x))^2}{R_{ii}} \quad (2.30) $$

This represents the summation of the squares of the measurement residuals weighted by their respective measurement error covariance. This can be rewritten as the following.

$$ J(x) = [z - h(x)]^T [R]^{-1} [z - h(x)] \quad (2.31) $$

Where $R$ is the covariance matrix of the measurement errors and is diagonal in structure. Each of the diagonal elements is the covariance of its respective measurement and all of the off-diagonal elements are zero because the measurements are assumed to be independent. To find the minimum of this objective function the
derivative should be zero. The derivative of the objective function is denoted by \( g(x) \).

\[
g(x) = \frac{\partial \mathbf{f}(x)}{\partial x} = -[\mathbf{H}]^T \mathbf{R}^{-1} [z - h(x)]
\]

(2.32)

Where \( [\mathbf{H}] = \frac{\partial \mathbf{h}(x)^T}{\partial x} \)

The matrix, \( \mathbf{H}(x) \), is called the measurement Jacobian matrix. Expanding the non-linear function \( g(x) \) into its Taylor series around the state vector \( x^k \) yields:

\[
g(x) = g(x^k) + G(x^k)(x - x^k) + \ldots = 0
\]

Where \( G(x^k) = \frac{\partial g(x)}{\partial x} = [\mathbf{H}]^T \mathbf{R}^{-1} [\mathbf{H}] \)

(2.33)

Ignoring the higher order terms of the Taylor series expansion the derivative of the objective function yields an iterative solution known as the Gauss-Newton method.

\[
x^{k+1} = x^k - [G(x^k)]^{-1} \cdot g(x)
\]

\[
= x^k + [G(x^k)]^{-1} [H]^T \mathbf{R}^{-1} [z - h(x)]
\]

(2.34)

\( \mathbf{H}(x) \) is measurement Jacobian, it is simply the derivative of the measurement function with respect to the state vector. The measurement function and measurement Jacobian can be constructed using the known system model including branch parameters, network topology, and measurement locations and type. \( \mathbf{R} \) is the error covariance matrix should also be constructed prior to the iterations with the accuracy information of the meters installed in the system.

\( \mathbf{G}(x) \) is called the gain matrix. It is sparse, positive definite and symmetric provided that the system is fully observable. The matrix \( \mathbf{G}(x) \) is typically not inverted (the inverse will in general be a full matrix, whereas \( \mathbf{G}(x) \) itself is quite
sparse), but instead it is decomposed into its triangular factors by Cholesky factorization and the following sparse linear set of equations are solved using forward/back substitutions at each iteration:

\[
G(x^k) \Delta x^{k+1} = -g(x^k) = \left[ H(x^k) \right]^T [R]^{-1} [z - h(x^k)]
\]

Where \( \Delta x^{k+1} = x^{k+1} - x^k \) \hspace{2cm} (2.35)

Equation (2.35) is called the Normal Equations. Cholesky factorization of \( G(x) \) gives a lower triangular matrix \( L \) (also known as Cholesky factor) such that:

\[
G(x^k) = [L] \cdot [L]^T \quad (2.36)
\]

Hence equation (2.35) becomes

\[
L \cdot L^T \cdot \Delta x^{k+1} = -g(x^k) \quad (2.37)
\]

By using forward and back substitution as explained in [1], the need for inverting the gain matrix is eliminated. Let us define a temporary vector \( u \) such that:

\[
L^T \cdot \Delta x^{k+1} = u \quad (2.38)
\]

\[
L \cdot u = -g(x^k) \quad (2.39)
\]

In equation (2.39), \( L \) is a lower triangular matrix. So, starting from the top, the first row of \( u \) can be found by scalar division, \( u_1 = -g_1(x^k)/L_{11} \). Then, replacing the value for \( u_1 \) in the second row \( u_2 \) can be found in a similar manner. This is the forward substitution.

Once vector \( u \) is obtained, equation (2.38) is used to find the difference \( \Delta x^{k+1} \). \( L^T \) is an upper triangular matrix. The scalar division starts from the last row this time, and the rows of \( \Delta x^{k+1} \) are calculated one by one. Since the last row is obtained first, this process is called back substitution.
For the first iteration the measurement function and measurement Jacobian should be evaluated at flat voltage profile, or flat start. A flat start refers to a state vector where all of the real part of voltages is 1.0 and imaginary part of voltages is 0. In conjunction with the measurements, the next iteration of the state vector can be calculated again and again until a desired tolerance is reached. Once the algorithm converges and states are obtained the error covariance of the estimated states are given as

\[
\text{cov}(x) = \left( [H]^T [R]^{-1} [H] \right)^{-1} \tag{2.40}
\]

Notice that in equation (2.40) the covariance of the estimated states is inverse of gain matrix.

### 2.4.2 The Measurement Function

There are many different types of measurements that exist in a scheme such as this. These include real and reactive power bus injections and flows, line current flow magnitudes and bus voltage magnitudes. In order to develop equations to relate the state vector to each of these types of measurements the two-port \( \pi \)-model is assumed for network branches.

![Figure 2.5: Equivalent 2-port \( \pi \) model of a transmission line](image)

Note the notation used in Figure 2.5, it is seen in the following set of equations describing measurement function. The first measurement type is the real
& reactive power injections. For an injection at bus m, the measurements can be expressed as functions of the state vector and elements of the bus-admittance matrix.

\[
P_m = \sum_{k=1}^{N} \left( G_{mk} (e_m e_k + f_m f_k) + B_{mk} (f_m e_k - e_m f_k) \right)
\] (2.41)

\[
Q_m = \sum_{k=1}^{N} \left( G_{mk} (f_m e_k - e_m f_k) - B_{mk} (e_m e_k + f_m f_k) \right)
\] (2.42)

And likewise with real and reactive power flows. The conductance and susceptance in these equations follows the notation of the above two-port \(\pi\)-model.

\[
P_{mk} = (e_m^2 + f_m^2) g_{mk} - g_{mk} (e_m e_k + f_m f_k) - b_{mk} (f_m e_k - e_m f_k)
\] (2.43)

\[
Q_{mk} = -(e_m^2 + f_m^2) (b_{mk} + b_{sm}) - g_{mk} (f_m e_k - e_m f_k) + b_{mk} (e_m e_k + f_m f_k)
\] (2.44)

Where, \(G_{mk} + jB_{mk}\) is \(mk^{th}\) element of the bus-admittance matrix, \(g_{mk} + jb_{mk}\) is the admittance of the series branch connecting buses m and k. \(jb_{sm}\) is the susceptance of the shunt branch at bus m. N is the total number of buses connected to bus m. \(e_m + jf_m\) is the voltage of bus m in Cartesian form. Hence state vector comprises of \(e\) and \(f\).

**2.4.3 The Measurement Jacobian**

While the measurement Jacobian is simply the derivative of the measurement function with respect to the state vector, for application purposes it is simpler to construct this matrix from a symbolic representation of the derivative of the measurement function. The measurement Jacobian has the following general structure.
The order of the measurement vector will correspond to the order of the rows in the measurement function, and therefore, the measurement Jacobian. While the above partitioning is not required, consistency between the measurement vector and these two matrices is important. Similarly, the columns will correspond to the order of the state vector. Once constructed, the Jacobian matrix elements are each non-linear functions of the state variable and are re-evaluated for each iteration of the estimation solution. There are generalized equations for each type of element that may appear inside of this matrix. These can be classified first by measurement type and second by variable with which the derivative has been taken with respect to. First are the partial derivatives of the Jacobian matrix elements corresponding to the real power injection measurements.

\[
H = \begin{bmatrix}
\frac{\partial P_m}{\partial f} & \frac{\partial P_m}{\partial e} \\
\frac{\partial Q_m}{\partial f} & \frac{\partial Q_m}{\partial e} \\
\frac{\partial P_{mk}}{\partial f} & \frac{\partial P_{mk}}{\partial e} \\
\frac{\partial Q_{mk}}{\partial f} & \frac{\partial Q_{mk}}{\partial e} \\
0 & \frac{\partial e_m}{\partial e} \\
\frac{\partial f_m}{\partial f} & 0
\end{bmatrix}
\] (2.45)

\[
\frac{\partial P_m}{\partial f_m} = \sum_{k=1}^{N} (G_{mk}f_k + B_{mk}e_k) + G_{mm}f_m - B_{mm}e_m 
\] (2.46)

\[
\frac{\partial P_m}{\partial f_k} = G_{mk}f_m - B_{mk}e_m 
\] (2.47)
Next are the partial derivatives of the Jacobian matrix elements corresponding to reactive power injection measurements.

\[
\frac{\partial P_m}{\partial e_m} = \sum_{k=1}^{N} (G_{mk}e_k - B_{mk}f_k) + G_{mm}e_m + B_{mm}f_m \tag{2.48}
\]

\[
\frac{\partial P_m}{\partial e_k} = G_{mk}e_m + B_{mk}f_m \tag{2.49}
\]

Next are the partial derivatives of the Jacobian matrix elements corresponding to reactive power flow measurements.

\[
\frac{\partial Q_m}{\partial f_m} = \sum_{k=1}^{N} (G_{mk}e_k - B_{mk}f_k) - G_{mm}e_m - B_{mm}f_m \tag{2.50}
\]

\[
\frac{\partial Q_m}{\partial f_k} = -G_{mk}e_m - B_{mk}f_m \tag{2.51}
\]

\[
\frac{\partial Q_m}{\partial e_m} = \sum_{k=1}^{n} (-G_{mk}e_k - B_{mk}f_k) + G_{mm}f_m - B_{mm}e_m \tag{2.52}
\]

\[
\frac{\partial Q_m}{\partial e_k} = G_{mk}f_m - B_{mk}e_m \tag{2.53}
\]

Next are the partial derivatives of the Jacobian matrix elements corresponding to real power flow measurements

\[
\frac{\partial P_{mk}}{\partial f_m} = 2g_{mk}f_m - g_{mk}f_k - b_{mk}e_k \tag{2.54}
\]

\[
\frac{\partial P_{mk}}{\partial f_k} = -g_{mk}f_m + b_{mk}e_m \tag{2.55}
\]

\[
\frac{\partial P_{mk}}{\partial e_m} = 2g_{mk}e_m - g_{mk}e_k + b_{mk}f_k \tag{2.56}
\]

\[
\frac{\partial P_{mk}}{\partial e_k} = -g_{mk}e_m - b_{mk}f_m \tag{2.57}
\]

Next are the partial derivatives of the Jacobian matrix elements corresponding to reactive power flow measurements.
\[
\frac{\partial Q_{mk}}{\partial f_m} = -2(b_{mk} + b_{sm})f_m - g_{mk}e_k + b_{mk}f_k
\]  
(2.58)

\[
\frac{\partial Q_{mk}}{\partial f_k} = g_{mk}e_m + b_{mk}f_m
\]  
(2.59)

\[
\frac{\partial Q_{mk}}{\partial e_m} = -2(b_{mk} + b_{sm})e_m + g_{mk}f_k + b_{mk}e_k
\]  
(2.60)

\[
\frac{\partial Q_{mk}}{\partial e_k} = -g_{mk}f_m + b_{mk}e_m
\]  
(2.61)

Next are the partial derivatives of the Jacobian matrix elements corresponding to voltage components measurements

\[
\frac{\partial e_m}{\partial e_m} = 1, \frac{\partial e_m}{\partial e_k} = 0 ; \frac{\partial e_m}{\partial f_m} = 0, \frac{\partial e_m}{\partial f_k} = 0
\]  
(2.62)

\[
\frac{\partial f_m}{\partial e_m} = 0, \frac{\partial f_m}{\partial e_k} = 0 ; \frac{\partial f_m}{\partial f_m} = 1, \frac{\partial f_m}{\partial f_k} = 0
\]  
(2.63)

Many times current measurements are also considered for state estimation as they are easily available. The problem with including these measurements is that the jacobian corresponding to these measurements are undefined at flat start. Hence they are neglected, but are included later as phasor measurement (described in chapter 3).

**Example 2.2**

Consider the 5 bus system shown in Figure 2-4 whose network data is shown in Table 2-1. A load flow was solved to develop the system state and measurement set and normally distributed errors were added to the measurements. Table 2-2 shows the measurement information for the system including type, location, value, and error covariance. A conventional weighted least squares state estimator is coded in Matlab following the derivation in this document.
<table>
<thead>
<tr>
<th>Number</th>
<th>Measurement type</th>
<th>Value</th>
<th>Covariance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>V_1</td>
<td>1.010</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>2</td>
<td>\theta_1</td>
<td>0.00</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>3</td>
<td>P_2</td>
<td>-0.4780</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>4</td>
<td>P_4</td>
<td>0.00</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>5</td>
<td>Q_2</td>
<td>0.0390</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>6</td>
<td>Q_4</td>
<td>0.2134</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>7</td>
<td>P_{1-2}</td>
<td>0.8187</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>8</td>
<td>P_{2-5}</td>
<td>0.1073</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>9</td>
<td>P_{3-5}</td>
<td>0.1877</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>10</td>
<td>Q_{1-2}</td>
<td>0.01845</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>11</td>
<td>Q_{2-5}</td>
<td>-0.0473</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>12</td>
<td>Q_{3-5}</td>
<td>0.0128</td>
<td>10^{-4}</td>
</tr>
</tbody>
</table>

**Table 2-2:** Measurement data for 5 bus system shown in figure 2.4

The initial jacobian matrix is as follows

\[
H = \begin{bmatrix}
-1.99 & 1.99 & 0 & 0 & 0 & -5.07 & 11.81 & -4.89 & 0 & -1.86 \\
-5.07 & 12.12 & -4.98 & 0 & -1.86 & 1.99 & -1.99 & 0 & 0 & 0 \\
1.99 & -1.99 & 0 & 0 & 0 & 5.07 & -5.07 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.86 & 0 & 0 & -1.86 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 9.09 & 0 & -9.09 & 0 \\
5.03 & -5.07 & 0 & 0 & 0 & -1.99 & 1.99 & 0 & 0 & 0 \\
0 & 1.97 & 0 & 0 & -1.86 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 9.09 & 0 & -9.09 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
The state estimator converges to the following set of values after 5 iterations.

\[
x = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix} = \begin{bmatrix} 1.010 \\ 0.984 \\ 1.037 \\ 1.071 \\ 1.032 \\ 0.00 \\ -0.150 \\ -0.197 \\ -0.203 \\ -0.216 \end{bmatrix}
\]

(2.64)

Voltage in polar form with magnitude and phase angle are given in table 2-3

<table>
<thead>
<tr>
<th>Bus</th>
<th>Voltage magnitude</th>
<th>Voltage phase angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.010</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.995</td>
<td>-8.65</td>
</tr>
<tr>
<td>3</td>
<td>1.055</td>
<td>-10.75</td>
</tr>
<tr>
<td>4</td>
<td>1.090</td>
<td>-10.74</td>
</tr>
<tr>
<td>5</td>
<td>1.054</td>
<td>-11.81</td>
</tr>
</tbody>
</table>

Table 2-3: Voltage in polar form for 5 bus system shown in figure 2.4

### 2.5 Including PMU Data in State Estimator

As stated previously, conventional state estimation techniques have improved over the last several decades, however, the fundamental concepts have remained the same. With the increased use of phasor measurement units in substations, engineers and operators have access to new types of measurements.
Power system state estimation is one of many applications that can foreseeably benefit from this technological development. Given the idea that PMUs actually measure the system state instead of indirectly estimating it, including these types of data in a state estimator could improve the quality of the state estimate [9]. There are two specific algorithms that include phasor measurements in conventional state estimators. The first method aims to mix the phasor measurements with the conventional measurements and solve in the same manner as before. The second includes the phasor measurements in a linear post-processing step with the output of the conventional state estimator. This method is explained in detail in chapter 3. The first method is described below.

### 2.5.1 PMU Measurements Mixed with Conventional Measurements

This first method of including phasor measurements in a conventional state estimator does so by mixing the conventional measurements \([z_c]\) comprising of real and reactive power flows, injections, and voltage with a set of complex positive sequence voltage and current phasors measurements \([z_p]\). Note that phasor measurements in \(z_p\) are in Cartesian form. But phasors are actually measured in polar form, so the measurement error covariance of the phasor measurements \([R_p]\), is also in polar form. Hence, it must be transformed according to the transformation rule for converting from polar to Cartesian coordinates.

The relationship between incremental representations in polar and Cartesian coordinates is given below. Consider the real part of voltage expressed in terms Voltage magnitude and phase angle
\[ e = V \cos(\theta) \]  
\[ \therefore \Delta e = \cos(\theta) \Delta V - V \sin(\theta) \Delta \theta \]  

Similar equations for imaginary part can be written

\[ f = V \sin(\theta) \]  
\[ \therefore \Delta f = \sin(\theta) \Delta V + V \cos(\theta) \Delta \theta \]  

Extending this to all the buses and putting them in a matrix form would be like:

\[
\begin{bmatrix}
\Delta e_1 \\
\Delta e_2 \\
\vdots \\
\Delta f_1 \\
\Delta f_2 \\
\vdots
\end{bmatrix} =
\begin{bmatrix}
\cos(\theta_1) & 0 & 0 & -V_1 \sin(\theta_1) & 0 & 0 \\
0 & \cos(\theta_2) & 0 & 0 & -V_1 \sin(\theta_2) & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\sin(\theta_1) & 0 & 0 & V_1 \cos(\theta_1) & 0 & 0 \\
0 & \sin(\theta_2) & 0 & 0 & V_2 \cos(\theta_2) & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots 
\end{bmatrix}
\begin{bmatrix}
\Delta V_1 \\
\Delta V_1 \\
\vdots \\
\Delta \theta_1 \\
\Delta \theta_1 \\
\vdots
\end{bmatrix}
\]  

In equation (2.69) the matrix is called rotation matrix [9] and is represented as \( T_v \). A similar rotation matrix \( T_i \) can be defined for currents. The final transformation for all phasor measurements (both voltage and current phasor measurements) is given below by equations (2.70) and (2.71)

\[
\begin{bmatrix}
\Delta V \\
\Delta I
\end{bmatrix}_{\text{Cart}} = [T] \begin{bmatrix}
\Delta V \\
\Delta I
\end{bmatrix}_{\text{Polar}}
\]  

where \( [T] = \begin{bmatrix} T_v & 0 \\ 0 & T_i \end{bmatrix} \)  

By using the \([T]\), the error covariance of phasor measurements \([R_p]\) in polar coordinates can be transformed to its equivalent in Cartesian coordinates \([R'_p]\).

\[
[R'_p] = [T][R_p][T]^T
\]
The two measurement vectors $[z_c]$ and $[z_p]$ can be vertically concatenated to serve as the measurement vector for this augmented formulation of the conventional state estimator

$$
[z] = \begin{bmatrix}
[z_c] \\
[z_p]
\end{bmatrix} = \begin{bmatrix}
z_c \\
v_{\text{Real}} \\
v_{\text{Imag}} \\
I_{\text{Real}} \\
I_{\text{Imag}}
\end{bmatrix}
$$

(2.73)

Following the model from the previous section, the state estimation problem becomes the following.

$$
\begin{bmatrix}
z_c \\
z_p
\end{bmatrix} = \begin{bmatrix}
h_c(x) \\
h_p(x)
\end{bmatrix} + \begin{bmatrix}
e_c \\
e_p
\end{bmatrix}
$$

(2.74)

Where $h_p(x)$ is the non-linear function relating the system state vector to the phasor measurement vector $z_p$. The measurement error covariance matrix for the minimization takes the following form.

$$
[R] = \begin{bmatrix}
R_c & 0 \\
0 & R'_p
\end{bmatrix}
$$

(2.75)

The subscripts $c$ and $p$ are used for conventional and phasor measurements respectively. Also it should be noted that $R'_p$ is used instead of $R_p$ because our estimator is defined in Cartesian coordinates system. The measurement Jacobian matrix is similarly constructed.

$$
[H] = \begin{bmatrix}
h_c(x) \\
h_p(x)
\end{bmatrix} = \begin{bmatrix}
\frac{\partial h_c(x)}{\partial x} \\
\frac{\partial h_p(x)}{\partial x}
\end{bmatrix}
$$

(2.76)
As explained in previous section the states can be estimated iteratively like equation (2.34).

\[ \Delta x^{(2)} = [G(x)]^{-1}[H]^T[R]^{-1}[z - h(x)] \]  

(2.77)

Superscript is just used to differentiate it from results of the conventional state estimator. Using equation (2.72) to (2.75) and substituting in equation (2.76) we get

\[ \Delta x^{(2)} = [G(x)]^{-1}\left( [H_c(x)^T\ H_p(x)^T]\begin{bmatrix} R_c & 0 \\ 0 & R_p' \end{bmatrix}^{-1}\begin{bmatrix} z_c - h_c(x) \\ z_p - h_p(x) \end{bmatrix}\right) \]  

(2.78)

\[ \Delta x^{(2)} = [G(x)]^{-1}\cdot ([H_c^T\ R_c^{-1}]\ [z_c - h_c(x)] + [H_p^T\ R_p^{-1}]\ [z_p - h_p(x)]) \]  

(2.79)

Where \( G(x) \) can also be in expanded form as

\[ [G(x)] = [H_c(x)^T\ H_p(x)^T]\begin{bmatrix} R_c & 0 \\ 0 & R_p' \end{bmatrix}^{-1}\begin{bmatrix} H_c(x) \\ H_p(x) \end{bmatrix} \]  

(2.80)

\[ [G(x)] = \left( H_c^T(x)\ R_c^{-1}\ H_c(x) + H_p^T(x)\ R_p^{-1}\ H_p(x) \right) \]  

(2.81)

\[ [G(x)] = [G_c(x) + G_p(x)] \]  

(2.82)

2.6 Conclusion

This chapter discussed conventional state estimation techniques and presented the formulation of the weighted least squares solution of a non-linear state estimation algorithm. Despite its flaws, this type of implementation is the most prevalent in electric utilities and has proven itself over many decades. However,
PMU technology provides a more accurate and time-sensitive Avenue for measurement collection and therefore the inclusion of PMU data in state estimation is a natural evolution of the technology. Technique for using PMU data mixed with conventional measurement in state estimation was also presented and discussed. In the next chapter, the linear formulation of post processing step which processes PMU measurement separately. This forms the basis of Two Stage State Estimator (TSSE)
Chapter 3 - The Linear State Estimation

It was the prevailing idea throughout most of the development of conventional state estimators that the precise simultaneous collection of measurements across the system was something that could never be accomplished. A large assumption that held all conventional state estimation techniques together was that the static state of the power system changed very slowly and operators could afford to have significant scan times. Even though some estimators today have scan times of only a few seconds, this could still be an eternity for several desirable applications in protection and control. PMUs allow for the synchronized collection of phasor measurements and with this technology becoming so prevalent in utilities, it is inevitable that it will be used for state estimation applications [2].

3.1 Introduction to Linear State Estimation

As presented in the previous chapter, the inclusion of PMU technology in state estimation may come in several forms. Even a small number of these precise measurements can weigh heavily on the accuracy of the overall state of the system [2]. However, application of PMU technology to state estimation would be to have bus voltage phasors and line current phasors in addition to the conventional measurements of real and reactive power injections. Thereby adding redundancy and increasing robustness.

Advantages of having PMU measurements as inputs may help us resolve many issues associated with conventional state estimators. Because PMUs are synchronized with GPS, the problem of scan time becomes irrelevant. Once the problem of scan time has been erased, the only issue of time is the communication...
and computational delay between the collection of the measurements and the employment of useful information for decision making by the operation and control applications. Additionally, when using PMUs as metering devices, the state of the system is actually being directly measured. However, estimation is still necessary for including redundancy and bad data filtering. Because of this, the placement of the PMUs is critical even for a fully observable system with a sufficient amount of measurement redundancy [16, 17]. It will be seen in the next chapter that the redundancy can be gained not only from the line currents (which are linearly related to the system state) but also from bus voltage measurements.

This chapter outlines linear state estimation from a single phase perspective. First, the basic mathematical concept of linear state estimation will be presented on a $\pi$-model transmission line to demonstrate how voltage and current measurements are related to the system state. The next section outlines the formulation of the system matrices on a simple network and then, the complete linear state estimation equation is presented and solved. And finally, the second stage of the two stage state estimation is described, wherein PMU measurements are included by a slightly different formulation, by taking into consideration a preliminary system state has already been determined [9].

3.2 Linear State Estimation with $\pi$-Equivalent

To understand the fundamentals of linear state estimator it is best to begin with a simple two-port $\pi$-model equivalent of a transmission line as shown in [1]. The state of this simple system will be the voltage magnitude and angle at each end of the transmission line. If there is a PMU at each end of the transmission line then it can be assumed that the measurement set for this system will consists of the voltage phasors at each end of the line and the line flows leaving each end of the line. Recall that because of the shunt capacitance of transmission lines that the line
current on each side of a single line will not be the same. Consider the \( \pi \)-equivalent
of a transmission line shown in Figure 3-1.

![Figure 3.1: Equivalent 2-port \( \pi \) model of a Transmission line](image)

All values will be considered to be measured in polar form and hence must be converted to Cartesian. This is what gives the state equation its linear property. The system state is then the following complex vector

\[
[x] = \begin{bmatrix} V_m \\ V_k \end{bmatrix} \tag{3.1}
\]

The error-free measurement set is considered to be the vertical concatenation of the voltage phasors at each end of the transmission line and the line flows from each end of the transmission line.

\[
[z] = \begin{bmatrix} V_m \\ V_k \\ I_{mk} \\ I_{km} \end{bmatrix} \tag{3.2}
\]

The system state can clearly be related identically to the voltage measurements in this complex vector. However, the linear relationship between the system state and the line flows requires some effort. First, several quantities must be defined. The series admittance and shunt susceptance of the transmission line are the following.
\[ y_{mk} = (R_{mk} + jX_{mk})^{-1} \]  \hspace{1cm} (3.3)

\[ y_{m0} = jb_{sm} \]  \hspace{1cm} (3.4)

\[ y_{k0} = jb_{sm} \]  \hspace{1cm} (3.5)

Sparing the derivation using Kirchhoff’s laws, the relationship between the system state and the line current flows on this simple transmission line is as follows

\[
\begin{bmatrix}
I_{mk} \\
I_{km}
\end{bmatrix} =
\begin{bmatrix}
y_{mk} + y_{m0} & -y_{ij} \\
-y_{ij} & y_{ij} + y_{k0}
\end{bmatrix}
\begin{bmatrix}
V_m \\
V_k
\end{bmatrix} \hspace{1cm} (3.6)
\]

Then the complete state equation takes the following form

\[
\begin{bmatrix}
V_m \\
V_k
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
I_{mk} \\
I_{km}
\end{bmatrix} =
\begin{bmatrix}
y_{mk} + y_{m0} & -y_{ij} \\
-y_{ij} & y_{ij} + y_{k0}
\end{bmatrix}
\begin{bmatrix}
V_m \\
V_k
\end{bmatrix} \hspace{1cm} (3.7)
\]

In the following section, this equation will be broken down into a set of individual matrices that when combined will form the matrix that will relate a measurement set of a power system network to the system state. Sets of simple rules will be presented for the construction of each of these matrices and how to combine them.

### 3.3 Matrix Formulation

Discussed in this section are the rules for populating the matrices used in the state equation for single-phase linear state estimation. A simple fictitious 5 bus system has been used to demonstrate the construction of each of the matrices that are needed. These matrices are called the current measurement-bus incidence matrix, the voltage measurement-bus incidence matrix, the series admittance matrix,
and the shunt susceptance matrix. Then, the system matrix relating the system state to the set of voltage and current phasor measurements is calculated using these four matrices. Explicit rules for constructing each of these matrices are included in each section.

3.3.1 Current Measurement-Bus Incidence Matrix

The current measurement-bus incidence matrix is a matrix that shows the location of the current flow measurements in the network. It is a \( m \times n \) size matrix where \( m \) is the number of current measurements in the network and \( n \) is the number of busses which have a current measurement leaving the bus. It is populated using a few simple rules. These include:

1. Each row of the matrix corresponds to a current measurement in the network.
2. Each column of the matrix corresponds to a bus in the system which has a current measurement leaving the bus.
3. If measurement \( X \) (corresponding to row \( X \)) leaves bus \( Y \) (corresponding to column \( Y \)) then the matrix element \( (X, Y) \) will be a 1.
4. If measurement \( X \) (corresponding to row \( X \)) leaves bus \( Y \) heading towards bus \( Z \) (corresponding to column \( Z \)) then the matrix element \( (X, Z) \) will be a -1.
5. All remaining entries will be identically zero.

Consider the fictitious 5 bus transmission network shown below. The presence of a current measurement is represented by an arrow above a CT.
Figure 3.2: 5 bus system showing current measurements

The current measurement bus index matrix for this system in a single phase representation would look like the following

\[
[A] = \begin{pmatrix}
1 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & 0 \\
0 & -1 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 \\
0 & -1 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & -1 & 1
\end{pmatrix}
\] (3.8)

In figure 3.2, I₄ and I₆ are only used to show the 2 port π model as a part of bigger system. Using only one measurement instead of both will have no effect on result.

### 3.3.2 Voltage Measurement Bus Incidence Matrix

The voltage measurement-bus incidence matrix is very similar to the current measurement-bus incidence matrix. It shows the relationship between a voltage measurement and its respective location in the network. Its purpose can most easily be understood by inspection of the state equation introduced in the first section of this chapter.
Because PMUs are used to make measurements, the voltage measurements are actually a direct measurement of the system state and therefore only require a simple identity relationship between the measurement and the state. The voltage measurement-bus incidence matrix is a $m \times n$ matrix where $m$ is the number of voltage measurements in the network and $n$ is the number of busses which have a voltage measurement. This matrix is populated using the following rules.

1. Each row of the matrix corresponds to a voltage measurement in the network.
2. Each column of the matrix corresponds to a bus in the system which has a voltage measurement.
3. If measurement $X$ (corresponding to row $X$) is located at bus $Y$ (corresponding to column $Y$) then the matrix element $(X, Y)$ will be a 1.
4. All remaining entries will be identically 0.

Consider again the small fictitious system shown in the previous section. The location of the voltage measurements are shown by the circles on the busses. For a single phase representation, the voltage measurement-bus incidence matrix would look like the following.

$$
[I] = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
$$

(3.9)

This matrix takes the form of the identity matrix because each bus is home to a voltage measurement. It is true that if the voltage measurements or associated state variables were reordered then it would not resemble the identity matrix. This will be seen later in the chapter when we investigate the real world application of
this algorithm. What is important to take away from this relationship though is that
PMU measurements yield a direct measurement of the system state and are
therefore related identically to their respective state variable.

3.3.3 Series Admittance Matrix

The series admittance matrix is a diagonal matrix where the diagonal
elements are the admittances of the lines being measured. It is a \( m \times m \) matrix
where \( m \) is the number of current measurements in the network. It is populated
using a single rule:

1. For measurement \( X \), the (\( X, X \)) matrix element is the admittance of the
branch being measured.

Referring back again to the 5 bus system, the following matrix is the single
phase version of the series admittance matrix of the system where \( y_i \) is the
admittance of line \( i \).

\[
[Y] = \begin{bmatrix}
y_1 & 0 & 0 & 0 & 0 & 0 \\
0 & y_3 & 0 & 0 & 0 & 0 \\
0 & 0 & y_2 & 0 & 0 & 0 \\
0 & 0 & 0 & y_5 & 0 & 0 \\
0 & 0 & 0 & y_4 & 0 & 0 \\
0 & 0 & 0 & 0 & y_5 & 0 \\
0 & 0 & 0 & 0 & 0 & y_6
\end{bmatrix}
\]  

(3.10)

3.3.4 Shunt Admittance Matrix

The shunt admittance matrix is a matrix that relates the location of each
current measurement to the shunt admittance of the line that it is measuring. It is a
\( m \times n \) matrix where \( m \) is the number of current measurements in the network and \( n \)
is the number of the bus where the current being measured originates. It is populated using a single rule:

1. For measurement $X$ (corresponding to column $X$) leaving bus $Y$ (corresponding to row $Y$), the matrix element $(X, Y)$ is the shunt admittance of the side of the line where measurement $X$ was taken.

Referring back again to the 5 bus system, the following matrix is the single-phase version of the shunt admittance matrix of the system where $y_{i0}$ is the shunt admittance of line $i$.

$$
[Y_s] = \begin{bmatrix}
    y_{10} & 0 & 0 & 0 & 0 \\
    y_{30} & 0 & 0 & 0 & 0 \\
    0 & 0 & y_{20} & 0 & 0 \\
    0 & 0 & y_{50} & 0 & 0 \\
    0 & 0 & 0 & y_{40} & 0 \\
    0 & 0 & 0 & y_{50} & 0 \\
    0 & 0 & 0 & 0 & y_{60} \\
\end{bmatrix}
$$

(3.11)

### 3.3.5 System Matrix Formulation

Consider the following state equation

$$
[z] = [V] = [l] [x] + [e]
$$

(3.12)

It can be seen that the set of measurements, $[z]$ is a vertical concatenation of the set of voltage and current phasor measurements, respectively. The system state, $[x]$ is then related to the set of measurements by a vertical concatenation of the voltage measurement-bus incidence matrix and a system matrix composed of the series and shunt admittance matrices and the current measurement-bus incidence matrix.
\[ [M] = [y][A] + [y_s] \]  

(3.13)

This matrix, \( M \) relates the system state to the set of current flow phasor measurements. Then the state equation becomes the following.

\[ [z] = \begin{bmatrix} \mathbf{V} \\ \mathbf{1} \end{bmatrix} = \begin{bmatrix} \mathbf{yA} + y_s \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{e} \end{bmatrix} \]  

(3.14)

Consider again the fictitious 5 bus system present earlier. The system matrix, \( M \) will take the following form

\[
[M] = yA + y_s = \begin{bmatrix}
y_1 + y_{10} & -y_1 & 0 & 0 & 0 \\
y_3 + y_{30} & 0 & 0 & -y_3 & 0 \\
0 & -y_2 & y_2 + y_{20} & 0 & 0 \\
0 & 0 & y_5 + y_{50} & 0 & -y_5 \\
0 & -y_4 & 0 & 0 & y_4 + y_{40} \\
0 & 0 & -y_5 & 0 & y_5 + y_{50} \\
0 & 0 & 0 & -y_6 & y_6 + y_{60}
\end{bmatrix}
\]  

(3.15)

Then the two matrices are vertically concatenated. In the following equation, the full system state as well as each of the separate voltage and current measurements has been represented as they were in the diagram of the 5 bus system.
3.3.6 An Alternative Formulation of the Linear State Estimator

Recall the complex linear state equation where $[z]$ is the raw measurement vector of voltage and current phasors, $[x]$ is the system state, $[e]$ is a vector of the error in each of the measurements, and the system matrix is the vertical concatenation of the voltage measurement-bus incidence matrix and a product of several other system matrices.

\[
V_1 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
y_1 + y_{10} & -y_1 & 0 & 0 & 0 \\
y_3 + y_{30} & 0 & 0 & -y_3 & 0 \\
0 & -y_2 & y_2 + y_{20} & 0 & 0 \\
0 & 0 & y_5 + y_{50} & 0 & -y_5 \\
0 & -y_4 & 0 & 0 & y_4 + y_{40} \\
0 & 0 & -y_5 & 0 & y_5 + y_{50} \\
0 & 0 & 0 & -y_6 & y_6 + y_{60}
\end{bmatrix}
\]

\[
[y] = \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
x_9 \\
x_{10} \\
x_{11} \\
x_{12}
\end{bmatrix}
\]

\[
[z] = [V] [x] + [e]
\]
complex values it could be computationally beneficial if the linear state equation could be reformulated to use the real and imaginary parts of the phasor values separately. The following is a presentation of a non-complex formulation of the linear state equation.

First we define real and imaginary part of the system state

\[
\begin{align*}
[e] &= \text{real}\{x\} \\
[f] &= \text{imag}\{x\}
\end{align*}
\]

The linear state estimation equation shown in equation (3.17) can be rewritten in the following form

\[
\begin{bmatrix}
[e] \\
f_{PMU}^c \\
f_{PMU}^d
\end{bmatrix} =
\begin{bmatrix}
I & 0 \\
0 & I \\
gA + g_s & -bA - b_s \\
bA + b_s & gA + g_s
\end{bmatrix}\begin{bmatrix}
[e] \\
f
\end{bmatrix} + \begin{bmatrix}
\epsilon
\end{bmatrix}
\]

Where

\[
c \text{ and } d \text{ are real and imaginary part of current phasor}
\]

\[
gA + g_s = \text{real}\{yA + y_s\}
\]

\[
bA + b_s = \text{imag}\{yA + y_s\}
\]

And [II] is the exact same voltage measurement-bus incidence matrix formulated for the complex linear state equation.

It is true that the size of the matrices when using the complex equation is exactly one fourth of the size of the matrices with real and imaginary parts separated. At first glance, this might lead one to believe that the computation time for matrix inversion or multiplication would be substantially larger. However, remember that even though complex matrix is smaller in dimensions it has double the values and would require extra computation because the values are complex.
Although, when using the formulation with real and imaginary separate, one may be able to avoid quite a bit of unnecessary computation time in the application by avoiding combining and separating complex numbers.

### 3.4 Solution to the Linear State Estimation Problem

The state equation presented in last subsection is non-complex representation of the complex linear equation. The equation (3.20) can be simply put in a form that is analogous to the conventional state estimator described in chapter 2.

\[ [z] = [H][x] + [\epsilon] \]  \hspace{1cm} (3.23)

Since equation (3.23) is a linear equation, this allows for calculation of the system state with exactly one iteration, for every new set of measurements unlike conventional non-linear techniques which require an unknown number of iterations for each new state.

The measurement error covariance matrix \([R]\) is a diagonal matrix containing variance of measurements. The covariance matrix is defined in polar domain and must to converted to its equivalent in Cartesian domain \([R']\) using suitable transformation matrix as described in section 2.5.1 the solution to the states can be obtained as

\[ [x] = \left[ \left( H^T R'^{-1} H \right)^{-1} H^T R'^{-1} \right] [z] \]  \hspace{1cm} (3.24)
The solution to the line state estimation problem is analogous to the one conventional state estimation problem described in chapter and the term $H^TR^{-1}H$ is similar to gain matrix $G_m$. The gain matrix may be too sparse to find its inverse for a larger bus system. Hence the equation can be solved used Cholesky factorization and forward and back substitution.

### 3.5 Adding Phasor Measurement through a Post-Processing Step

One of the methods is already described at the end of chapter 2, where all phasor measurement and conventional measurement are mixed to form a single estimator. The problem with this kind of measurement is that the entire estimator code has to undergo reformulation again in order to include phasor measurement [9]. Also it is not easy to include current measurement in this estimator as corresponding entries in jacobian matrix are undefined [15].

In this section an alternative method for including phasor measurements in conventional state estimation techniques is described which does not require any major manipulation of the existing current state estimator. This method also addresses the problem associated with current measurements. It will be seen how this method mirrors the linear state estimation.

In order to include the phasor measurements in the estimation process without altering the current state estimator structure is to take the calculated system state from the conventional state estimator use the phasor measurements to enhance the estimate with a linear calculation. First the state estimator must either be defined in Cartesian coordinates or it must be converted from polar to Cartesian coordinates.
and the associated covariance matrix must be transformed accordingly. Since the estimator is already defined in Cartesian coordinates we can use the states and its error covariance directly. The error covariance matrix of the states, given by equation (2.40) is

\[
\text{cov}(x^{(1)}) = \left([H_c]^T [R_c]^{-1} [H_c]\right)^{-1} = [G_c]^{-1}
\] (3.25)

In the above \(x^{(1)}\) is estimates obtained at the end of conventional state estimation process. \(H_c\) and \(G_c\) are corresponding jacobian and gain matrix.

The phasor measurements \([z_p]\) and estimated states from the conventional state estimator \([x^{(1)}]\) can be vertically concatenated to serve as the measurement vector for this augmented formulation of the conventional state estimator.

\[
[z_2] = \begin{bmatrix} x^{(1)} \\ z_p \end{bmatrix} = \begin{bmatrix} e^{(1)} \\ f^{(1)} \\ V_{\text{Real}} \\ V_{\text{Imag}} \\ I_{\text{Real}} \\ I_{\text{Imag}} \end{bmatrix}
\] (3.26)

Hence the above measurement set can be linearly related to the system states as:

\[
[z_2] = [H_2][x^{(2)}]
\] (3.27)

\[
\begin{bmatrix} x^{(1)} \\ z_p \end{bmatrix} = \begin{bmatrix} I \\ [H_p] \end{bmatrix} \begin{bmatrix} e^{(2)} \\ f^{(2)} \end{bmatrix}
\] (3.28)

where, \(x^{(2)}\) is state obtained after the post processing step. \(H_p\) is same as jacobian for the linear estimator given by equation (3.23) and \(I\) is identity matrix as system state \(x^{(2)}\) is identically related to the partition of the measurement vector which contains the calculated system states \(x^{(1)}\).
The solution to the above over determined linear state estimation problem is given as

\[
[\mathbf{x}^{(2)}] = [\mathbf{G}_2]^{-1}[\mathbf{R}_2^{-1}\mathbf{H}_2][\mathbf{z}_2]
\]  \hspace{1cm} (3.29)

Where \([\mathbf{G}_2] = [\mathbf{H}_2]^T[\mathbf{R}_2]^{-1}[\mathbf{H}_2]\) \hspace{1cm} (3.30)

\(\mathbf{R}_2\) is new covariance matrix, which includes both the error models for the calculated system state and the phasor measurement vector.

\[
[\mathbf{R}_2] = \begin{bmatrix}
\text{cov}(\mathbf{x}^{(1)}) & 0 \\
0 & \mathbf{R}_p'
\end{bmatrix} = \begin{bmatrix}
\mathbf{G}_c^{-1} & 0 \\
0 & \mathbf{R}_p'
\end{bmatrix}
\]  \hspace{1cm} (3.31)

It must be noted that \(\mathbf{R}_p\) is transformed to \(\mathbf{R}_p'\) to account for the use of Cartesian system.

It is obvious that as a linear post-processing step is far more convenient to use compared to the estimator in described in last subsection of chapter 2, which requires major revision of existing EMS software. It is very important to prove the equivalence of the two methods [9]. Let us consider \(\mathbf{G}_2\)

\[
[\mathbf{G}_2] = [\mathbf{H}_2]^T[\mathbf{R}_2]^{-1}[\mathbf{H}_2]
\]  \hspace{1cm} (3.32)

Where \([\mathbf{G}_2] = [\mathbf{H}_2]^T[\mathbf{R}_2]^{-1}[\mathbf{H}_2]\)

\[
[\mathbf{G}_2] = \begin{bmatrix}
\mathbf{I}^T & \mathbf{H}_p^T
\end{bmatrix} \begin{bmatrix}
\mathbf{G}_c^{-1} & 0 \\
0 & \mathbf{R}_p'
\end{bmatrix}^{-1} \begin{bmatrix}
\mathbf{I} \\
\mathbf{H}_p
\end{bmatrix}
\]  \hspace{1cm} (3.33)

\[
[\mathbf{G}_2] = \left(\mathbf{I}^T \mathbf{G}_c \mathbf{I} + \mathbf{H}_p^T \mathbf{R}_p'^{-1} \mathbf{H}_p\right)^{-1}
\]  \hspace{1cm} (3.34)

\[
[\mathbf{G}_2] = \mathbf{G}_c + \mathbf{G}_p
\]  \hspace{1cm} (3.35)
Upon careful observation equation (3.35) is similar to equation (2.82). Hence it explains the equivalence of two methods. The above mentioned process is used to process Phasor measurement. This becomes clearer with example described below.

**Example 3.1**

Let us again consider the 5 bus system from example 2.2 to understand this method. The two stages weighted least squares state estimator is coded in Matlab (see Appendix B.2) following the derivation in this document. We already have estimated states from conventional state estimator (See example 2.2). Let us include the following phasor measurement:

<table>
<thead>
<tr>
<th>Number</th>
<th>Measurement Type</th>
<th>Value</th>
<th>Rii</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$</td>
<td>V_3</td>
<td>$</td>
</tr>
<tr>
<td>2</td>
<td>$\angle V_3$</td>
<td>-10.75°</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>3</td>
<td>$</td>
<td>I_{3-4}</td>
<td>$</td>
</tr>
<tr>
<td>4</td>
<td>$\angle I_{3-4}$</td>
<td>79.26°</td>
<td>$10^{-4}$</td>
</tr>
</tbody>
</table>

*Table 3-1: Phasor measurements for 5 bus system shown in figure 2.4*

The state estimation equation is given by

$$\begin{equation} [z_2] = [H_2][x^{(2)}] \end{equation}$$

(3.36)

Let us begin by listing various matrices used in this linear state estimation technique starting with the measurement vector
\[ [z_2] = \begin{bmatrix}
1.010 \\
0.984 \\
1.037 \\
1.071 \\
1.032 \\
0.00 \\
-0.150 \\
-0.197 \\
-0.203 \\
-0.216 \\
1.036 \\
-0.197 \\
0.037 \\
0.1928
\end{bmatrix} \]

\[ \begin{aligned}
&\{e^{(1)}\} \\
&\{f^{(1)}\} \\
&\{e^{(PMU)}\} \\
&\{f^{(PMU)}\}
\end{aligned} \]  

(3.37)

And the \([H_2]\) matrix is

\[
[H_2] = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5.677 & -5.677 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(3.38)
The upper part of the $H_2$ matrix is relation between estimated states from conventional state estimator and states to be obtained after post processing step. And lower part is the linear relation between phasor measurements and states.

To solve the above linear equation, the error covariance matrix must be carefully defined. The inverse of error covariance matrix for given 5 bus is given below.

\[
\begin{bmatrix}
0.62 & -0.93 & 0.25 & 0 & 0.09 & 0.09 & -0.16 & 0.10 & 0 & 0.04 \\
-0.93 & 1.75 & -0.57 & 0 & -0.25 & 0.11 & -0.10 & -0.07 & 0 & -0.03 \\
0.25 & -0.57 & 1.56 & -0.39 & -0.84 & -0.10 & 0.12 & 0.03 & 0 & -0.02 \\
0 & 0 & -0.39 & 0.41 & 0 & 0 & 0 & 0 & -0.01 & 0 \\
0.09 & -0.25 & -0.84 & 0 & 0.99 & -0.04 & 0.04 & -0.02 & 0 & 0 \\
0.09 & 0.11 & -0.10 & 0 & -0.04 & 0.60 & -0.95 & 0.25 & 0 & 0.09 \\
-0.16 & -0.10 & 0.12 & 0 & 0.04 & -0.95 & 1.84 & -0.58 & 0 & -0.26 \\
0.10 & -0.07 & 0.03 & 0 & -0.02 & 0.25 & -0.58 & 1.53 & -0.37 & -0.82 \\
0 & 0 & 0.00 & -0.01 & 0 & 0 & 0 & 0 & -0.37 & 0.36 \\
0.04 & -0.03 & -0.02 & 0 & 0.09 & -0.26 & -0.82 & 0.99 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0.00 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.00 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.05 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.02
\end{bmatrix}
\]

The partition on the main diagonal in above matrix represents covariance of states and phasor measurements respectively. And off diagonal blocks is zero as phasor measurements are not related to estimated states.

Using all the above matrices and solving for states we get
Voltage in polar form with magnitude and phase angle are given in table 3-2

<table>
<thead>
<tr>
<th>Bus</th>
<th>Voltage magnitude</th>
<th>Voltage phase angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.010</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.995</td>
<td>-8.65</td>
</tr>
<tr>
<td>3</td>
<td>1.055</td>
<td>-10.75</td>
</tr>
<tr>
<td>4</td>
<td>1.090</td>
<td>-10.75</td>
</tr>
<tr>
<td>5</td>
<td>1.054</td>
<td>-11.81</td>
</tr>
</tbody>
</table>

Table 3-2: Voltage in polar form for 5 bus system

### 3.6 Conclusion

As a clear application of PMU technology to state estimation, this chapter presented the formulation of the linear state estimation problem using exclusively PMU measurements. A basic formulation using a two-port pi-model is first used, followed by discussions of the matrices that are used to develop the system matrix.
(similar to the measurement function of conventional state estimation techniques). This chapter has been structured to provide discrete rules for populating each of the matrix and using them develop a linear state estimator model. The discussion of linear state estimator is extended to develop a two stage hybrid estimation model where phasor measurements are systematically included in a linear post processing step. In the next chapter, the application of the two stage state estimator to bad data analysis is described.
Chapter 4 - Bad Data Analysis

State estimation algorithms fit measurements made on the system to a mathematical model in order to provide a reliable data base for other monitoring, security assessment and control functions. One of the essential functions of a state estimator is to detect measurement errors, and to identify and eliminate them if possible. These algorithms should be able to handle four types of errors:

1. *Measurement error*, these are random errors usually exist in measurements due to the finite accuracy of the meters and the telecommunication medium.
2. *Parameter error* these are the error caused due to uncertainty in model parameters such as line admittance.
3. *Bad data*, these are larger measurement error occur when meter have biases or wrongly connected.
4. *Topology error*, these are error that causes bad data in the state estimation which are caused by incorrect topological information.

In this chapter we limit our discussion to bad data analysis. Bad data are not easy to detect. Hence state estimators are added with more advanced features to identify and detect bad data. In this chapter we discuss bad data analysis techniques with WLS estimation technique and how phasor measurements help in improving the bad data analysis.
4.1 Introduction to Bad Data Analysis

Unlike some of the state estimator where measurements are analyzed for bad data along with state estimation procedure, while using the WLS estimation method, bad data analysis is done only after the estimation process by processing the measurement residuals. The analysis is essentially based on the properties of these residuals, including their expected probability distribution. These have been explained later in this chapter.

Bad data may appear in several different ways depending upon the type, location and number of measurements that are in error. They can be broadly classified as:

1. Single bad data: Only one of the measurements in the entire system will have a large error.
2. Multiple bad data: More than one measurement will be in error.

Multiple bad data may appear in measurements whose residuals are strongly or weakly correlated. Strongly correlated measurements are those whose errors affect the estimated value of each other significantly, causing the good one to also appear in error when the other contains a large error. Estimates of measurements with weakly correlated residuals are not significantly affected by the errors of each other. When measurement residuals are strongly correlated their errors may or may not be conforming. Conforming errors are those that appear consistent with each other. Multiple bad data can therefore be further classified into three groups [1]:

1. Multiple non-interacting bad data: Bad data in measurements with weakly correlated measurement residuals.
2. *Multiple interacting but non-conforming bad data:* Non-conforming bad data in measurements with strongly correlated residuals.

3. *Multiple interacting and conforming bad data:* Consistent bad data in measurements with strongly correlated residuals.

Figure 4.1 summarizes the classification of bad data discussed above.

![Figure 4.1: Classification of bad data](image)

A power system has different types of measurement spread through the system. These measurements have different properties and the outcome of estimator is not only dependent on their values but also their location. These measurements can be classified as following depending on their location.

- **Critical measurement:** A critical measurement is the one whose elimination from the measurement set will result in an unobservable system (A system is said to be observable if the set of given measurement is sufficient to find a unique estimate of all the states of the system).
• Redundant measurement: A redundant measurement is a measurement which is not critical. Hence removal of such measurement does not affect the observability of the system.

• Critical pair: Two redundant measurements whose simultaneous removal from the measurement set will make the system unobservable.

• Critical k-tuple: A critical k-tuple contains k redundant measurements, where removal of all of them will cause the system to become unobservable.

4.2 Measurement Residual and its Properties

Bad data analysis for a WLS state estimator is based on analysis of residual. In the following section the measurement residuals are analyzed for their mean, covariance, probability distribution curves, and their values for different types of measurements.

Consider the general state estimation equation

\[ z = h(x) + e \]  \hspace{1cm} (4.1)

After the states are estimated as described in chapter 2 and 3, the measurement residual is given as

\[ r = z - h(\hat{x}) \]  \hspace{1cm} (4.2)

where \( \hat{x} \) are the estimated states

It is recognized that the power system state estimation problem is not linear. However, as the iterations of the nonlinear estimator approach the converged solution, the problem becomes almost linear [9]. Hence the linearized model of the state estimation is given by
\[ \Delta z = H \cdot \Delta x + e \]  

(4.3)

Hence the solution to this equation is given as

\[ \Delta \hat{x} = (H^T R^{-1} H)^{-1} (H^T R^{-1}) \cdot \Delta z \]  

(4.4)

\[ \Delta \hat{x} = G^{-1} H^T R^{-1} \cdot \Delta z \]  

(4.5)

Hence estimated measurement \[ \Delta \tilde{z} = H \cdot \Delta \hat{x} = K \cdot \Delta z \]  

(4.6)

Where \( K = H G^{-1} H^T R^{-1} \) and is called Hat matrix. This hat matrix has properties similar to identity matrix. Hence following equations hold true.

\[ K \cdot K \cdot K \ldots K = K \]  

(4.7)

\[ K \cdot H = H \]  

(4.8)

\[ (I - K) \cdot H = 0 \]  

(4.9)

Residual matrix is difference of actual measured values and the estimated measured value

\[ r = \Delta z - \Delta \tilde{z} \]

\[ = (I - K) \Delta z \]

\[ = (I - K) \cdot (H \Delta x + e) \]

\[ = (I - K) e \]

\[ = Se \]  

(4.10)

Where, Matrix \( S \) is called the sensitivity matrix and has the following properties [1].

\[ S \cdot S \cdot S \cdots S = S \]  

(4.11)
\[ S \cdot R \cdot S^T = S \cdot R \] (4.12)

Since errors are assumed to be distributed according to Gaussian distribution with zero mean and variance defined by matrix R, the probability distribution of residual matrix can be computed as follows:

1. Mean:
\[ E(r) = E(S \cdot e) = S \cdot E(e) = 0 \] (4.13)

2. Covariance:
\[
\text{Cov}(r) = \Omega = \text{Cov}(S \cdot e) \\
= S \cdot \text{Cov}(e) \cdot S^T \\
= S \cdot R \cdot S^T \\
= S \cdot R \quad [\text{from}(4.12)] \\
= R - H \cdot G^{-1} \cdot H^T 
\] (4.14)

Hence measurement residues follow a normal distribution curve with zero mean and \( \Omega \) as covariance. The covariance matrix is real and symmetric the other properties are mentioned below [1].

\[
\Omega_{ij}^2 \leq \Omega_{ii} \cdot \Omega_{jj} \\
\Omega_{ij} \leq \frac{\Omega_{ii} + \Omega_{jj}}{2} 
\] (4.15) (4.16)

The off diagonal element of covariance matrix tells about the nature of interaction between two measurements. The measurement \( i \) and measurement \( j \) are said to be weakly interacting if \( \Omega_{ij} \leq \varepsilon \), else they are strongly interacting. Where, \( \varepsilon \) is threshold determined by the network topology and the desired level of selectivity [1].
The covariance of the measurement residual can also determine the nature of the measurement. If all the elements of a row or column of the matrix are zero then the measurement is a critical measurement. If two columns of the matrix are linearly dependent then the measurement pair forms a critical pair. Similarly if \( k \) columns are linearly dependent, they form a critical \( k \)-tuple measurement [1].

4.3 Largest Normalized Residual \( r_{N_{\text{max}}} \) Test

Largest Normalized residual test proves to be a very useful in detection and identification of bad data. Despite of its drawback it is one of the most commonly used technique in bad data analysis.

The measurement residual of \( i \)th measurement can be normalized by simply dividing its absolute value by the corresponding diagonal entry in the residual covariance matrix:

\[
r_i^N = \frac{|r_i|}{\sqrt{\Omega_{ii}}} = \frac{|r_i|}{\sqrt{R_{ii} \cdot S_{ii}}} \tag{4.17}
\]

The normalized residual vector follows a standard normal distribution with zero mean and unity covariance. Hence the largest entry of the vector can be compared to statistical threshold to detect bad data. This threshold can be chosen based on the desired level of detection sensitivity.

4.3.1 \( r_{N_{\text{max}}} \) Test and Conventional State Estimator

It can be shown that the largest normalized residual will correspond to the erroneous measurement (provided that it is neither a critical measurement nor a member of a critical pair). This property may hold true even for certain multiple bad data cases, where bad measurements are essentially non-interacting [1].
Consider the case where the only bad data occurs in measurement k and all the remaining measurements are free of errors, residue corresponding both bad good measurements are given as

\[
[r] = S \cdot e
\]  
(4.18)

\[
\begin{bmatrix}
  r_1 \\
  r_2 \\
  \vdots \\
  r_k \\
  r_m
\end{bmatrix}
= \begin{bmatrix}
  S_{11} & S_{12} & \cdots & S_{1k} & \cdots & S_{1m} \\
  S_{21} & S_{22} & \cdots & S_{2k} & \cdots & S_{2m} \\
  \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
  S_{k1} & S_{k2} & \cdots & S_{kk} & \cdots & S_{km} \\
  S_{m1} & S_{m2} & \cdots & S_{mk} & \cdots & S_{mm}
\end{bmatrix}
\begin{bmatrix}
  0 \\
  0 \\
  \vdots \\
  e_k \\
  0
\end{bmatrix}
\]  
(4.19)

\[
\therefore r_j = S_{jk} \cdot e_k \quad j = 1, 2, \cdots, m
\]  
(4.20)

Normalizing the residues corresponding to measurement

\[
r_j^N = \frac{S_{jk} \cdot e_k}{\sqrt{R_{jj} \cdot S_{jj}}} \quad j = 1, 2, \cdots, m \text{ and } j \neq k
\]  
(4.21)

\[
r_j^N = \frac{\Omega_{jk} \cdot e_k}{\sqrt{\Omega_{jj} \cdot R_{kk}}} \quad \text{Multiply and divide by } R_{kk}
\]  
(4.22)

\[
r_j^N \leq \frac{\sqrt{\Omega_{jj} \cdot \Omega_{kk}} \cdot e_k}{\sqrt{\Omega_{jj} \cdot R_{kk}}} \quad \text{From equation (4.15)}
\]  
(4.23)

\[
r_j^N \leq \frac{\sqrt{\Omega_{kk}} \cdot e_k}{R_{kk}}
\]  
(4.24)

\[
r_j^N \leq \frac{S_{kk} \cdot e_k}{\sqrt{R_{kk} \cdot S_{kk}}} = r_k^N
\]  
(4.25)
Equation (4.25) shows that normalized residual of an erroneous measurement is highest of the entire normalized measurement residual vector. The above inequality becomes a strict equality, if the measurements $j$ and $k$ form a critical pair, since the corresponding columns of $\Omega$ matrix will be linearly dependent.

**Example 4.1**

Let us consider the 5 bus example from chapter 2 we already have states estimated. Let us enforce bad data in one the measurement and try to identify the measurement using the above test.

<table>
<thead>
<tr>
<th>No.</th>
<th>Measurement type</th>
<th>Measurement value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>No bad data</td>
</tr>
<tr>
<td>1</td>
<td>$V_1$</td>
<td>1.010</td>
</tr>
<tr>
<td>2</td>
<td>$\theta_1$</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>$P_2$</td>
<td>-0.4780</td>
</tr>
<tr>
<td>4</td>
<td>$P_4$</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>$Q_2$</td>
<td>0.039</td>
</tr>
<tr>
<td>6</td>
<td>$Q_4$</td>
<td>0.2134</td>
</tr>
<tr>
<td>7</td>
<td>$P_{1-2}$</td>
<td>0.8187</td>
</tr>
<tr>
<td>8</td>
<td>$P_{2-5}$</td>
<td>0.1073</td>
</tr>
<tr>
<td>9</td>
<td>$P_{3-5}$</td>
<td>0.1877</td>
</tr>
<tr>
<td>10</td>
<td>$Q_{1-2}$</td>
<td>0.1845</td>
</tr>
<tr>
<td>11</td>
<td>$Q_{2-5}$</td>
<td>-0.0473</td>
</tr>
<tr>
<td>12</td>
<td>$Q_{3-5}$</td>
<td>0.0128</td>
</tr>
</tbody>
</table>

*Table 4-1: Measurement vectors containing both true and false measurement*
As shown in table 4-1, real power injection at bus 2 has error. A step by step method of bad data identification is presented below.

**STEP I:** Calculate the states and residues. Table 4-2 shows the comparison of states calculated using true states and set with 1 bad measurement.

<table>
<thead>
<tr>
<th>Bus</th>
<th>True States</th>
<th>States With 1 bad data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cartesian form</td>
<td>Polar form</td>
</tr>
<tr>
<td></td>
<td>( E )</td>
<td>( f )</td>
</tr>
<tr>
<td>1</td>
<td>1.010</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.984</td>
<td>-0.150</td>
</tr>
<tr>
<td>3</td>
<td>1.037</td>
<td>-0.197</td>
</tr>
<tr>
<td>4</td>
<td>1.071</td>
<td>-0.203</td>
</tr>
<tr>
<td>5</td>
<td>1.032</td>
<td>-0.216</td>
</tr>
</tbody>
</table>

**Table 4-2:** True states and states computed using single bad measurement.

Using the states calculate the residue. Table 4-3 shows the residues of true measurement set and single bad measurement set.

<table>
<thead>
<tr>
<th>No</th>
<th>Measurement</th>
<th>Residue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>True measurements</td>
</tr>
<tr>
<td>1</td>
<td>( V_1 )</td>
<td>(-1.353 \times 10^{-6})</td>
</tr>
<tr>
<td>2</td>
<td>( \theta_1 )</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>( P_2 )</td>
<td>(-1.001 \times 10^{-5})</td>
</tr>
<tr>
<td>4</td>
<td>( P_4 )</td>
<td>(-6.002 \times 10^{-7})</td>
</tr>
<tr>
<td>5</td>
<td>( Q_2 )</td>
<td>(-1.009 \times 10^{-6})</td>
</tr>
<tr>
<td>6</td>
<td>( Q_4 )</td>
<td>(-5.117 \times 10^{-7})</td>
</tr>
<tr>
<td>7</td>
<td>( P_{1-2} )</td>
<td>(-8.730 \times 10^{-6})</td>
</tr>
<tr>
<td>8</td>
<td>( P_{2-5} )</td>
<td>(3.669 \times 10^{-5})</td>
</tr>
</tbody>
</table>
STEP 2 Compute normalized residue. In order to compute normalized residue of the measurement we need only the diagonal values of covariance matrix. The diagonal entries of covariance matrix are given below.

<table>
<thead>
<tr>
<th>No.</th>
<th>Measurement type</th>
<th>True measurements</th>
<th>1 bad measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$V_1$</td>
<td>$1.04 \times 10^{-6}$</td>
<td>$8.47 \times 10^{-7}$</td>
</tr>
<tr>
<td>2</td>
<td>$\theta_1$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>$P_2$</td>
<td>$6.53 \times 10^{-6}$</td>
<td>$6.49 \times 10^{-6}$</td>
</tr>
<tr>
<td>4</td>
<td>$P_4$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>$Q_2$</td>
<td>$6.36 \times 10^{-6}$</td>
<td>$6.41 \times 10^{-6}$</td>
</tr>
<tr>
<td>6</td>
<td>$Q_4$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>$P_{1-2}$</td>
<td>$5.58 \times 10^{-6}$</td>
<td>$5.59 \times 10^{-6}$</td>
</tr>
<tr>
<td>8</td>
<td>$P_{2-5}$</td>
<td>$8.66 \times 10^{-5}$</td>
<td>$8.66 \times 10^{-5}$</td>
</tr>
<tr>
<td>9</td>
<td>$P_{3-5}$</td>
<td>$1.69 \times 10^{-6}$</td>
<td>$1.66 \times 10^{-6}$</td>
</tr>
<tr>
<td>10</td>
<td>$Q_{1-2}$</td>
<td>$6.83 \times 10^{-6}$</td>
<td>$6.93 \times 10^{-6}$</td>
</tr>
<tr>
<td>11</td>
<td>$Q_{2-5}$</td>
<td>$8.38 \times 10^{-5}$</td>
<td>$8.39 \times 10^{-5}$</td>
</tr>
<tr>
<td>12</td>
<td>$Q_{3-5}$</td>
<td>$1.63 \times 10^{-6}$</td>
<td>$1.59 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Table 4-4: Diagonal entries of covariance of residual matrix $\Omega$
Normalized residue calculated using equation (4.17) is given below.

<table>
<thead>
<tr>
<th>No.</th>
<th>Measurement type</th>
<th>True measurements</th>
<th>1 bad measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$V_1$</td>
<td>$1.04 \times 10^{-3}$</td>
<td>7.237</td>
</tr>
<tr>
<td>2</td>
<td>$\theta_1$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>$P_2$</td>
<td>$3.94 \times 10^{-3}$</td>
<td><strong>24.388</strong></td>
</tr>
<tr>
<td>4</td>
<td>$P_4$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>$Q_2$</td>
<td>$4.00 \times 10^{-4}$</td>
<td>0.786</td>
</tr>
<tr>
<td>6</td>
<td>$Q_4$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>$P_{1-2}$</td>
<td>$3.69 \times 10^{-3}$</td>
<td>23.37</td>
</tr>
<tr>
<td>8</td>
<td>$P_{2-5}$</td>
<td>$3.94 \times 10^{-3}$</td>
<td>24.386</td>
</tr>
<tr>
<td>9</td>
<td>$P_{3-5}$</td>
<td>$4.10 \times 10^{-3}$</td>
<td>24.159</td>
</tr>
<tr>
<td>10</td>
<td>$Q_{1-2}$</td>
<td>$4.59 \times 10^{-4}$</td>
<td>1.261</td>
</tr>
<tr>
<td>11</td>
<td>$Q_{2-5}$</td>
<td>$3.25 \times 10^{-4}$</td>
<td>0.536</td>
</tr>
<tr>
<td>12</td>
<td>$Q_{3-5}$</td>
<td>$6.58 \times 10^{-4}$</td>
<td>3.441</td>
</tr>
</tbody>
</table>

**Table 4-5:** Normalized residues for 5 bus system

**STEP 3:** Compare largest normalized residue to threshold. In this case the threshold is set to 1.0 as seen in table 4-5 all normalized residual corresponding to true measurements are far below the threshold. But in the case of single bad data the largest normalized residue is 24.39 and corresponds to real power injection at bus 2.

Hence bad is correctly detected and identified. All the calculation has been performed using matlab. The source code is given in appendix B. Figure 4-2 and figure 4-3 shows the plots of normalized residues for true measurements and the case of single bad data discussed above.
Figure 4.2: Normalized residue for 5 bus system with true measurements

Figure 4.3: Normalized residue for 5 bus system with single bad measurement
4.3.2 **Strength and Weakness of the Test**

The performance of largest normalized residual test depends on type of bad data and configuration. Its performance and limitations are discussed below for all possible types of bad data.

**Single Bad Data**

When there is single bad data, the largest normalized residual will correspond to the bad measurement, provided that it is not critical or its removal does not create any critical measurements among the remaining ones.

**Multiple Bad Data**

Multiple bad data may appear in 3 ways:

- **Non-interacting:**
  If $S_{ij} \approx 0$, then measurement $i$ and $j$ are non-interacting. In this case, even if bad data appear simultaneously in both measurements, the largest normalized residual test can identify them sequentially.

- **Interacting, non-conforming:**
  If $S_{ij}$ is significantly large, then measurements $i$ and $j$ are interacting. However, if the errors in measurement $i$ and $j$ are equal, then the largest normalized residual test may still indicate the bad data correctly.

- **Interacting, conforming:**
  If two interacting measurements have errors that are equal, then the largest normalized residual test may fail to identify either one.

Table 4-6 provides the summary of above discussion
<table>
<thead>
<tr>
<th>Bad data type</th>
<th>Detectable</th>
<th>Identifiable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single bad data</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Multiple bad data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-interacting</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Interacting, non-conforming</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Interacting, conforming</td>
<td>✓</td>
<td>✗</td>
</tr>
</tbody>
</table>

**Table 4-6:** Performance of $r_{\text{max}}$ Test under different conditions

The maximum normalized residual test gives desired result under various conditions, but there are fails to detect and identify error critical measurements. In the following section a new approach is present to detects and identify bad data in critical measurements.

### 4.4 Bad Data Associated with Critical Measurement.

From the previous discussions it is already established that it is not possible to identify the presence of bad data in critical measurements. As diagonal element in residual covariance matrix is zero. It is evident from the example 4.1 (see table 4-4) the covariance of injection at bus 4 is zero.

A new method is presented to remove vulnerability of the bad data in critical measurements. This can be achieved by placing PMU strategically such that it adds redundancy to these measurements [17]. Since these measurements are no more critical bad data in such measurement can be easily identified by proposed algorithm.
The two stage estimator adds PMU measurement to the existing state estimation with little changes. Using this model we extend the discussion of bad data analysis to identification of bad data in critical measurement. That is the system will be able to detect and identify the bad data in such measurement after second stage. As this idea can be implemented along with linear post processing step, the trouble of changing the bad data module is averted.

If all the measurements are to be mixed to form a single large estimator, the appended measurement matrix, jacobian matrix and the error covariance matrix are given by

\[ [z_3] = \begin{bmatrix} \mathbf{Z}_c \\ \mathbf{Z}_p \end{bmatrix} \] \hspace{1cm} (4.26)

\[ [\mathbf{H}_3] = \begin{bmatrix} \mathbf{H}_c \\ \mathbf{H}_p \end{bmatrix} \] \hspace{1cm} (4.27)

\[ [\mathbf{R}_3] = \begin{bmatrix} \mathbf{R}_c & 0 \\ 0 & \mathbf{R}_p' \end{bmatrix} \] \hspace{1cm} (4.28)

It should be noted that \( \mathbf{R}_p' \) is transformed to Cartesian from its polar form \( \mathbf{R}_p \). Gain matrix is given as

\[ [\mathbf{G}_3] = [\mathbf{H}_3^T \mathbf{R}_3^{-1} \mathbf{H}_3] \] \hspace{1cm} (4.29)

Using equations (4.27) and (4.28)

\[ [\mathbf{G}_3] = [\mathbf{H}_c^T \mathbf{H}_p'] \begin{bmatrix} \mathbf{R}_c & 0 \\ 0 & \mathbf{R}_p' \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{H}_c \\ \mathbf{H}_p \end{bmatrix} \] \hspace{1cm} (4.30)

\[ [\mathbf{G}_3] = \begin{bmatrix} [\mathbf{H}_c^T \mathbf{R}_c^{-1} \mathbf{H}_c] + [\mathbf{H}_p^T \mathbf{R}_p'^{-1} \mathbf{H}_p] \end{bmatrix} = [\mathbf{G}_c + \mathbf{G}_p] \] \hspace{1cm} (4.31)

Using (4.14), residual covariance matrix \( \Omega_3 \) is defined as
where, $\Omega_{cc}$, $\Omega_{cp}$, $\Omega_{cp}$ and $\Omega_{pp}$ are sub matrix of $\Omega_3$, $\Omega_{cc}$ and $\Omega_{pp}$ are residual covariance of conventional and phasor measurement respectively.

$$
\begin{bmatrix}
\Omega_{cc} & \Omega_{cp} \\
\Omega_{cp} & \Omega_{pp}
\end{bmatrix} = 
\begin{bmatrix}
R_c & 0 \\
0 & R_p
\end{bmatrix} - \begin{bmatrix}
H_c
\\
H_p
\end{bmatrix} \left[ G_c + G_p \right]^{-1} \begin{bmatrix}
H_c^T & H_p^T
\end{bmatrix}
$$

(4.34)

$$
\begin{bmatrix}
\Omega_{cc} & \Omega_{cp} \\
\Omega_{cp} & \Omega_{pp}
\end{bmatrix} = 
\begin{bmatrix}
R_c & 0 \\
0 & R_p'
\end{bmatrix} - \begin{bmatrix}
H_c \left[ G_c + G_p \right]^{-1} H_c^T \\
H_p \left[ G_c + G_p \right]^{-1} H_p^T
\end{bmatrix}
$$

(4.35)

Expression for sub matrix is $\Omega_{cc}$ corresponding to conventional measurement is

$$
\Omega_{cc} = \left[ R_c - H_c \left( G_c + G_p \right)^{-1} H_c^T \right]
$$

(4.36)

It is already shown in chapter 3 that the gain matrix of second stage of the two stage estimator is given as

$$
\left[ G_2 \right] = \left[ G_c + G_p \right]
$$

(4.37)

Hence $\Omega_{cc}$ is

$$
\Omega_{cc} = \left[ R_c - H_c \left( G_c + G_p \right)^{-1} H_c^T \right]
$$

(4.38)

Therefore we update the covariance matrix corresponding to conventional measurement after second stage using equation (4.38) and re-calculate normalized residues.

$$
\left[ \Omega_{c}^{new} \right] = \left[ R_c - H_c \left( G_c + G_p \right)^{-1} H_c^T \right]
$$

(4.38)
Note:

- $R_c$ and $H_c$ are already calculated in first stage (i.e. conventional state estimator).
- To get the desired result, PMU placement is the key. This is done to add redundancy to the critical measurement.
- In order to obtain correct normalized residue the residual vector should also be updated using the states calculated at the end of second stage $x^{(2)}$. The updated residual matrix can be obtained as follows.

$$r_c^{new} = z_c - h_c(x^{(2)})$$  \hspace{1cm} (4.39)

$$\text{updated } r_i^N = \frac{|r_i^{new}|}{\sqrt{\Omega_{ii}^{new}}}$$  \hspace{1cm} (4.40)

This concept is illustrated using example below.

**Example 4.2**

Let us consider the same 5 bus system used in example 4.1. It is seen that power injection at bus 4 is a critical measurement. It will be shown that if a bad data occurs in one of these measurements, it is undetected by the bad data module of first stage yet correctly picked at end of second stage.

PMU measurement used in example 3.1 is chosen such that it adds redundancy to critical measurement. Same measurement set from example 4.1 is used, except instead of changing $P_2$, $Q_4$ is changed from 0.2134 to -0.2134. Comparison between bad data analysis after both stages of two stage estimator is shown below to demonstrate the fact.

**STEP1:** Calculate the states and residues. Table 4-7 shows the comparison of states calculated after first and second stage.
Using the states, calculate the residue. Table 4-8 contains the residues calculated using states from table 4-7.

### Table 4-8: Residue after first and second stage for single bad data

<table>
<thead>
<tr>
<th>No</th>
<th>Measurement</th>
<th>Residue</th>
<th>First stage</th>
<th>Second stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$V_1$</td>
<td>$-1.353 \times 10^{-6}$</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\theta_1$</td>
<td>0</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$P_2$</td>
<td>$-1.001 \times 10^{-5}$</td>
<td>0.062</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$P_4$</td>
<td>$-6.684 \times 10^{-7}$</td>
<td>$2.901 \times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$Q_2$</td>
<td>$-1.009 \times 10^{-6}$</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$Q_4$</td>
<td>$-1.0372 \times 10^{-6}$</td>
<td>0.212</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$P_{1-2}$</td>
<td>$-8.730 \times 10^{-6}$</td>
<td>0.055</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$P_{2-5}$</td>
<td>$3.669 \times 10^{-5}$</td>
<td>-0.227</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>$P_{3-5}$</td>
<td>$-5.32 \times 10^{-6}$</td>
<td>0.031</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>$Q_{1-2}$</td>
<td>$-1.2 \times 10^{-6}$</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>$Q_{2-5}$</td>
<td>$2.977 \times 10^{-6}$</td>
<td>-0.005</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>$Q_{3-5}$</td>
<td>$-8.398 \times 10^{-6}$</td>
<td>-0.004</td>
<td></td>
</tr>
</tbody>
</table>
**STEP 2** Compute normalized residue. In order to compute normalized residue of the measurement we need only the diagonal values of covariance matrix. The diagonal entries of covariance matrix after first stage and after updating are:

<table>
<thead>
<tr>
<th>No.</th>
<th>Measurement type</th>
<th>Diagonal entries of $\Omega$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First stage</td>
<td>Second stage</td>
</tr>
<tr>
<td>1</td>
<td>$V_1$</td>
<td>$1.04 \times 10^{-6}$</td>
<td>$4.69 \times 10^{-5}$</td>
</tr>
<tr>
<td>2</td>
<td>$\theta_1$</td>
<td>0</td>
<td>$4.64 \times 10^{-5}$</td>
</tr>
<tr>
<td>3</td>
<td>$P_2$</td>
<td>$6.53 \times 10^{-6}$</td>
<td>$7.19 \times 10^{-6}$</td>
</tr>
<tr>
<td>4</td>
<td>$P_4$</td>
<td>0</td>
<td>$9.35 \times 10^{-5}$</td>
</tr>
<tr>
<td>5</td>
<td>$Q_2$</td>
<td>$6.36 \times 10^{-6}$</td>
<td>$7.38 \times 10^{-6}$</td>
</tr>
<tr>
<td>6</td>
<td>$Q_4$</td>
<td>0</td>
<td>$5.04 \times 10^{-5}$</td>
</tr>
<tr>
<td>7</td>
<td>$P_{1-2}$</td>
<td>$5.58 \times 10^{-6}$</td>
<td>$9.32 \times 10^{-6}$</td>
</tr>
<tr>
<td>8</td>
<td>$P_{2-5}$</td>
<td>$8.66 \times 10^{-5}$</td>
<td>$8.70 \times 10^{-5}$</td>
</tr>
<tr>
<td>9</td>
<td>$P_{3-5}$</td>
<td>$1.69 \times 10^{-6}$</td>
<td>$1.76 \times 10^{-6}$</td>
</tr>
<tr>
<td>10</td>
<td>$Q_{1-2}$</td>
<td>$6.83 \times 10^{-6}$</td>
<td>$1.23 \times 10^{-5}$</td>
</tr>
<tr>
<td>11</td>
<td>$Q_{2-5}$</td>
<td>$8.38 \times 10^{-5}$</td>
<td>$8.40 \times 10^{-5}$</td>
</tr>
<tr>
<td>12</td>
<td>$Q_{3-5}$</td>
<td>$1.63 \times 10^{-6}$</td>
<td>$1.69 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

*Table 4-9: Diagonal entries of covariance of residual matrix $\Omega*
Normalized residue calculated using equations (4.17) and (4.40) is given below

<table>
<thead>
<tr>
<th>No.</th>
<th>Measurement type</th>
<th>Normalized Residues</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First stage</td>
</tr>
<tr>
<td>1</td>
<td>$V_1$</td>
<td>$1.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>2</td>
<td>$\theta_1$</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>$P_2$</td>
<td>$3.94 \times 10^{-3}$</td>
</tr>
<tr>
<td>4</td>
<td>$P_4$</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>$Q_2$</td>
<td>$4.00 \times 10^{-4}$</td>
</tr>
<tr>
<td>6</td>
<td>$Q_4$</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>$P_{1-2}$</td>
<td>$3.69 \times 10^{-3}$</td>
</tr>
<tr>
<td>8</td>
<td>$P_{2-5}$</td>
<td>$3.94 \times 10^{-3}$</td>
</tr>
<tr>
<td>9</td>
<td>$P_{3-5}$</td>
<td>$4.10 \times 10^{-3}$</td>
</tr>
<tr>
<td>10</td>
<td>$Q_{1-2}$</td>
<td>$4.59 \times 10^{-4}$</td>
</tr>
<tr>
<td>11</td>
<td>$Q_{2-5}$</td>
<td>$3.25 \times 10^{-4}$</td>
</tr>
<tr>
<td>12</td>
<td>$Q_{3-5}$</td>
<td>$6.58 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

**Table 4-10:** Normalized residues for 5 bus system

**STEP 3:** Compare largest normalized residue to threshold. In this case the threshold is set to 1.0. All residuals calculated after first stage, are far below the threshold, indicating no bad data. But after second stage the largest residual is 29.95 can corresponds to reactive injection at bus 4.

Figure 4-4 and figure 4-5 shows the plots of normalized residues after each stage of the two stage estimator
Figure 4.4: Normalized residue after first stage with bad critical measurement

Figure 4.5: Normalized residue after first stage with bad critical measurement
4.5 Bad Data Detection in Phasor Measurements

Bad data analysis of PMU measurement is similar to the bad data analysis of conventional measurement. This done at the end of second stage where states are estimated again using phasor measurements mixed with estimates from first stage.

The residual matrix for the second stage is calculated as shown

\[ r_2 = z_2 - H_2 x^{(2)} \]

And the residual covariance matrix is given by

\[ \Omega_2 = R_2 - H_2 G_2^{-1} H_2^T \]

Hence normalized residual vector is

\[ r_{2(i)}^N = \frac{|r_{2(i)}|}{\sqrt{\Omega_{2(ii)}}} \]

Note:

- First 2n entries of the residual matrix correspond to states calculated from first stage i.e. conventional state estimator, where n is number of buses and remaining entries are phasor measurement.
- There are no critical phasor measurements, as the states from first stage add redundancy to phasor measurements.
- Since the model is formulated in Cartesian coordinate systems and measurements are in polar form. The technique can correctly identify the location but cannot identify the measurement. That is, it can identify which
bus has error in voltage or which line has error in current, but cannot tell whether bad data is the magnitude or the phase angle measurement.

Once the normalized residuals are obtained for phasor measurements, they must be appended to updated normalized residues of conventional measurements obtained by equation (4.40). And the entire list must me checked for largest normalized residual. It is equivalent to the single estimator where all measurements are mixed.

### 4.6 Conclusion

This chapter discussed the problem of bad data analysis when WLS method for state estimation is used. It begins with the role of measurement residual and its behavior under various types of measurements. A formulation of largest normalized test is also presented in this chapter followed by its advantages and weakness. The greatest weakness of this technique is the bad data identification in critical measurements. To overcome this shortcoming a new idea is presented where PMUs are used to add redundancy to critical measurement .This idea is implemented using two stage state estimator. This chapter concludes with the bad data detection in phasor measurements. In next chapter, all the concepts discussed so far have been tested on IEEE 14, 30 and 118 bus systems.
Chapter 5 - Results and Conclusions

The main objective of this thesis is to demonstrate the use of PMU in state estimation process and implementing them in bad data analysis such that bad data including critical measurements can be detected. In order to achieve this, the developed code must be tested against some test configurations in order to show its successful functionality. This chapter discusses the testing of the Matlab implementation of two stage state estimation and bad data analysis on IEEE 14, 30 and 118 bus systems.

5.1 Testing of the State Estimation

Testing of the two stage state estimator is systematically discussed in several subsections; beginning with a discussion on creating system matrices and measurements, followed by results of estimated states obtained by true measurement set.

5.1.1 Creating Systems and Measurements

IEEE 14, 30 and 118 are standard test bus system and the bus and line data can be found in reference [18]. These are listed in Matlab function bustdatas.m and linedatas.m respectively (see appendix A.1 and A.2). Using this data the each of the system is represented in software called Power Education Toolbox (P.E.T), an open source software designed for state estimation [19]. P.E.T is capable of giving power flow result which is used to create conventional power injection and power flow measurements. Each system designed is checked for observability using P.E.T. Once the system is completely observable, the marked measurements are listed in a
separate function \textit{zconv.m} (see appendix A.3) which is used in first stage i.e. conventional state estimator.

Similarly PMU measurements are generated as follows, the voltage measurements are obtained from Power flow result of P.E.T and current measurement are obtained using equation [3, 8].

\[ I_{mn} = \frac{S_{mn}^*}{V_m^*} = \frac{P_{mn} - jQ_{mn}}{V_m} \]  \hspace{1cm} (5.1)

Where \( I_{mn} \) is current flowing from m to n.

For a selected configuration Matlab function \textit{current.m} (see appendix A.4) generates all current measurements for given set of real and reactive power flow and sending end bus index. The current measurements and the voltage measurements are listed in Matlab function \textit{zpmu.m} (see appendix A.5) which is used in second stage i.e. linear post processing step.

The standard deviation of error is another important element as it is used to define the covariance matrix. Due to higher accuracy of PMUs, their standard deviation of error is smaller than that of conventional measurement. Standard deviation of error for conventional measurement is set to \( 10^{-2} \) and standard deviation of error for PMUs is set to \( 10^{-3} \).

\subsection*{5.1.2 Results}

\textbf{A IEEE 14 bus system}

The System is fully observable with 15 pairs of power measurements as shown in figure 5.1. It has 7 pairs of injection measurement and 8 pairs of flow measurement. Table 5-1 shows the true states of the system under unbiased conditions.
The System is fully observable with 38 pairs of power measurements as shown in figure 5.2. It has 15 pairs of injection measurement and 23 pairs of flow measurement. Table 5-2 shows the true states of 30 bus system under unbiased conditions.
Figure 5.2: IEEE 30 bus system

<table>
<thead>
<tr>
<th>Bus</th>
<th>Magnitude</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.060</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.043</td>
<td>-5.35</td>
</tr>
<tr>
<td>3</td>
<td>1.021</td>
<td>-7.53</td>
</tr>
<tr>
<td>4</td>
<td>1.012</td>
<td>-9.28</td>
</tr>
<tr>
<td>5</td>
<td>1.010</td>
<td>-14.16</td>
</tr>
<tr>
<td>6</td>
<td>1.010</td>
<td>-11.06</td>
</tr>
<tr>
<td>7</td>
<td>1.002</td>
<td>-12.86</td>
</tr>
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<td>8</td>
<td>1.010</td>
<td>-11.81</td>
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<td>1.082</td>
<td>-14.11</td>
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<td>12</td>
<td>1.057</td>
<td>-14.94</td>
</tr>
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<td>13</td>
<td>1.071</td>
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<tr>
<td>14</td>
<td>1.042</td>
<td>-15.83</td>
</tr>
<tr>
<td>15</td>
<td>1.038</td>
<td>-15.92</td>
</tr>
</tbody>
</table>

Table 5-2: True states of IEEE 30 bus system
C IEEE 118 bus system

It is fully observable with 150 pairs of power measurements as shown in figure 5.3. It has 39 pairs of injection measurement and 111 pairs of flow measurement. Table 5-3 shows the true states of the 118 bus system under unbiased conditions.

Figure 5.3: IEEE 118 bus system
<table>
<thead>
<tr>
<th>Bus</th>
<th>Magnitude</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.955</td>
<td>-19.02</td>
</tr>
<tr>
<td>2</td>
<td>0.971</td>
<td>-18.48</td>
</tr>
<tr>
<td>3</td>
<td>0.968</td>
<td>-18.14</td>
</tr>
<tr>
<td>4</td>
<td>0.998</td>
<td>-14.42</td>
</tr>
<tr>
<td>5</td>
<td>1.002</td>
<td>-13.97</td>
</tr>
<tr>
<td>6</td>
<td>0.990</td>
<td>-16.70</td>
</tr>
<tr>
<td>7</td>
<td>0.989</td>
<td>-17.14</td>
</tr>
<tr>
<td>8</td>
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<tr>
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<td>1.043</td>
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<tr>
<td>11</td>
<td>0.985</td>
<td>-16.99</td>
</tr>
<tr>
<td>12</td>
<td>0.990</td>
<td>-17.50</td>
</tr>
<tr>
<td>13</td>
<td>0.968</td>
<td>-18.36</td>
</tr>
<tr>
<td>14</td>
<td>0.984</td>
<td>-18.22</td>
</tr>
<tr>
<td>15</td>
<td>0.970</td>
<td>-18.51</td>
</tr>
<tr>
<td>16</td>
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</tr>
<tr>
<td>17</td>
<td>0.995</td>
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</tr>
<tr>
<td>18</td>
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</tr>
<tr>
<td>19</td>
<td>0.963</td>
<td>-18.69</td>
</tr>
<tr>
<td>20</td>
<td>0.958</td>
<td>-17.81</td>
</tr>
<tr>
<td>21</td>
<td>0.959</td>
<td>-16.23</td>
</tr>
<tr>
<td>22</td>
<td>0.970</td>
<td>-13.67</td>
</tr>
<tr>
<td>23</td>
<td>1.000</td>
<td>-8.75</td>
</tr>
<tr>
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<tr>
<td>60</td>
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<tr>
<td>Bus</td>
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<td>Angle</td>
</tr>
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<td>-----</td>
<td>-----------</td>
<td>--------</td>
</tr>
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<table>
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<th>Angle</th>
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</tr>
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<td>91</td>
<td>0.980</td>
<td>3.36</td>
</tr>
<tr>
<td>92</td>
<td>0.992</td>
<td>3.86</td>
</tr>
<tr>
<td>93</td>
<td>0.987</td>
<td>0.85</td>
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<tr>
<td>94</td>
<td>0.991</td>
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<td>-2.45</td>
</tr>
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<td>1.011</td>
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<td>0.962</td>
<td>-9.62</td>
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<tr>
<td>107</td>
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<td>109</td>
<td>0.968</td>
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<td>112</td>
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<td>113</td>
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<td>0.960</td>
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<tr>
<td>118</td>
<td>0.949</td>
<td>-8.05</td>
</tr>
</tbody>
</table>

**Table 5-3:** True states of 118 bus system
5.2 Bad data

The Bad data analysis module is designed based on the discussions in chapter 4, and it is tested on all three IEEE bus systems. The threshold set for the maximized normalized residual test is 1.0. Each system is simulated for different types of bad data. Most important of these are single bad data in non-critical measurement and single bad data in critical measurement. Other types of bad data simulated are the cases of multiple bad data and bad data in phasor measurements. Each of these is described in detail below.

5.2.1 Single bad data (non-critical)

This is the case where bad data occur in one the measurement which could be identified easily by both Conventional State Estimator and Two Stage State Estimator.

A IEEE 14 bus system

A single bad data is induced in real power flow from bus 2 to bus 3. Table 5-4 has the list of 5 largest normalized residues calculated using both CSE and TSSE.

<table>
<thead>
<tr>
<th>Number</th>
<th>Measurement</th>
<th>r^N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P_{2-3}</td>
<td>57.18</td>
</tr>
<tr>
<td>2</td>
<td>P_{4-2}</td>
<td>56.02</td>
</tr>
<tr>
<td>3</td>
<td>P_3</td>
<td>55.48</td>
</tr>
<tr>
<td>4</td>
<td>Q_3</td>
<td>10.13</td>
</tr>
<tr>
<td>5</td>
<td>Q_{2-3}</td>
<td>4.34</td>
</tr>
</tbody>
</table>

Table 5-4: Largest normalized residue IEEE 14 bus
The maximum normalized residue in both the cases correctly indicates bad data in real power flow from bus 2-3. Figure 5-4 shows all normalized residues of conventional measurements using CSE

Figure 5.4: $r^N$ of IEEE 14 bus single bad data (non-critical) using CSE

Figure 5.5 shows normalized residues of conventional and phasor measurements of IEEE 14 bus system using TSSE.

Figure 5.5: $r^N$ of IEEE 14 bus single bad data (non-critical) using TSSE
### IEEE 30 bus system

A single bad data is induced in real power injection at bus 2. Table 5-5 has the list of 5 largest normalized residues calculated using both CSE and TSSE.

<table>
<thead>
<tr>
<th>Number</th>
<th>Measurement</th>
<th>$r^N$</th>
<th>Number</th>
<th>Measurement</th>
<th>$r^N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P₂</td>
<td>8.26</td>
<td>1</td>
<td>P₂</td>
<td>8.58</td>
</tr>
<tr>
<td>2</td>
<td>P₂₋₅</td>
<td>5.32</td>
<td>2</td>
<td>P₁₋₃</td>
<td>5.47</td>
</tr>
<tr>
<td>3</td>
<td>P₂₋₄</td>
<td>5.08</td>
<td>3</td>
<td>P₂₋₅</td>
<td>4.94</td>
</tr>
<tr>
<td>4</td>
<td>P₁₋₃</td>
<td>5.04</td>
<td>4</td>
<td>P₂₋₆</td>
<td>4.11</td>
</tr>
<tr>
<td>5</td>
<td>P₂₋₆</td>
<td>4.89</td>
<td>5</td>
<td>P₂₋₄</td>
<td>4.10</td>
</tr>
</tbody>
</table>

**Table 5-5:** Largest normalized residue IEEE 30 bus

The maximum normalized residue in both the cases correctly indicates bad data in real power injection at bus 2. Figure 5-6 shows all normalized residues of conventional measurements using CSE.

![Figure 5-6](image-url)

**Figure 5.6:** $r^N$ of IEEE 30 bus single bad data (non-critical) using CSE
Figure 5.7 shows normalized residues of conventional and phasor measurements of IEEE 30 bus system using TSSE.

![Figure 5.7: $r^N$ of IEEE 30 bus single bad data (non-critical) using TSSE](image)

C IEEE 118 bus system

A single bad data is induced in reactive power flow from bus 23 to bus 24. Table 5-6 has the list of 5 largest normalized residues calculated using both CSE and TSSE.

<table>
<thead>
<tr>
<th>Number</th>
<th>Measurement</th>
<th>$r^N$</th>
<th>Number</th>
<th>Measurement</th>
<th>$r^N$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Q_{23-24}</td>
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<td>Q_{23-24}</td>
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<td>2</td>
<td>Q_{70-75}</td>
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<td>2</td>
<td>Q_{24-72}</td>
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</tr>
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<td>Q_{70-74}</td>
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<td>Q_{72-71}</td>
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<td>5</td>
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</table>

Table 5-6 Largest normalized residue IEEE 118 bus
The maximum normalized residue in both the cases correctly indicates bad data in reactive power flow from bus 23 to bus 24. Figure 5-8 shows all normalized residues of conventional measurements using CSE.

![Figure 5.8: $r^N$ of IEEE 118 bus single bad data (non-critical) using CSE](image)

Figure 5.8 shows normalized residues of conventional and phasor measurements of IEEE 118 bus system using TSSE.

![Figure 5.9: $r^N$ of IEEE 118 bus single bad data (non-critical) using TSSE](image)

5-12
It can be seen that the proposed bad data algorithm gives results in conjunction with results of established bad data algorithm for conventional state estimator.

5.2.2 Single bad data (critical measurement)

The systems are made observable such that there is certain number of critical measurements. The list of critical measurement is provided in table 5-7. Redundancy to these critical measurements is added by PMU measurements. The list of PMU measurements is given in appendix A.5

<table>
<thead>
<tr>
<th>Case study</th>
<th>Critical Measurements</th>
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</tr>
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<tr>
<td>IEEE 30 bus system</td>
<td>11, 12, 24, 27, 30</td>
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<tr>
<td>IEEE 118 bus system</td>
<td>34, 79, 113, 114, 117</td>
</tr>
</tbody>
</table>

| Table 5-7: List of Critical measurements |

The simulation of this case in each of the IEEE bus is provided below

A IEEE 14 bus system

The list Critical measurements are given in table 5-7. To add redundancy to these measurements 3 voltage and 2 current phasor measurements are used. A bad data is forced in reactive injection a bus 14. Five largest normalized residues are listed in table 5-8 comparing results of CSE and proposed TSSE. Figure 5.10 and figure 5.11 shows all normalized residues of conventional measurements using CSE and TSSE respectively.
<table>
<thead>
<tr>
<th>Number</th>
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</tr>
</thead>
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<tr>
<td>2</td>
<td>P_{4-2}</td>
<td>$2.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>3</td>
<td>P_{2-3}</td>
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</tr>
<tr>
<td>4</td>
<td>P_{1-2}</td>
<td>$1.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>5</td>
<td>Q_{1-2}</td>
<td>$1.3 \times 10^{-3}$</td>
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</tbody>
</table>

<table>
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<th>Number</th>
<th>Measurement</th>
<th>$r^N$</th>
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</thead>
<tbody>
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<td>Q_{11}</td>
<td>3.79</td>
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</tr>
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<td>Q_{4}</td>
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</table>

**Table 5-8:** Largest normalized residue IEEE 14 bus

**Figure 5.10:** $r^N$ of IEEE 14 bus single bad data (critical) using CSE

**Figure 5.11:** $r^N$ of IEEE 14 bus single bad data (critical) using TSSE
**IEEE 30 bus system**

The list Critical measurements are given in table 5-7. To add redundancy to these measurements 6 voltage and 5 current phasor measurements are used. A bad data is forced in real injection a bus 12. Five largest normalized residues are listed in table 5-9 comparing results of CSE and proposed TSSE. Figure 5.12 and figure 5.13 shows all normalized residues of conventional measurements using CSE and TSSE respectively.

<table>
<thead>
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<th>Number</th>
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<tbody>
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</tr>
</tbody>
</table>

Table 5-9: Largest normalized residue IEEE 30 bus

**Figure 5.12:** $r^N$ of IEEE 30 bus single bad data (critical) using CSE
The list Critical measurements are given in Table 5-7. To add redundancy to these measurements, 19 voltage and 15 current phasor measurements are used. A bad data is forced in reactive power flow from bus 63 to bus 59. Five largest normalized residues are listed in Table 5-10 comparing results of CSE and proposed TSSE. Figure 5.14 and figure 5.15 show all normalized residues of conventional measurements using CSE and TSSE respectively.

**Table 5-10:** Largest normalized residue IEEE 118 bus

<table>
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<th>Number</th>
<th>Measurement</th>
<th>$r^N$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$Q_{104}$</td>
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<td>$e_{63}$</td>
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<td>$Q_{103}$</td>
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<td>3</td>
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<td>$P_{104}$</td>
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<td>$e_{60}$</td>
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<td>$P_{104-105}$</td>
<td>$6.0 \times 10^{-3}$</td>
<td>5</td>
<td>$e_{54}$</td>
<td>11.85</td>
</tr>
</tbody>
</table>
Figure 5.14: \( r^N \) of IEEE 118 bus single bad data (critical) using CSE

Figure 5.15: \( r^N \) of IEEE 118 bus single bad data (critical) using CSE
From the above results it can be concluded that if one of the critical measurement is bad then it cannot be identified existing bad data algorithm, but using phasor measurements to add redundancy these errors can be easily be identified by the proposed algorithm.

5.2.3 Multiple Bad Data

The algorithm is tested for following cases of multiple bad data using TSSE and proposed bad data algorithm.

A Multiple interacting but non-conforming bad data in IEEE 14 bus system

A bad data is forced in reactive power injection at bus 12 and real power flow in bus 12 to bus 13. Table 5-11 is the list of 5 largest normalized residues for 3 iterations. First pass is result obtained when both bad data are present. Second pass is result when bad data from first pass is corrected and tested again for bad data. Final pass is the result when both bad data are corrected

<table>
<thead>
<tr>
<th>First pass</th>
<th>Second pass</th>
<th>Final pass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement</td>
<td>r^N</td>
<td>Measurement</td>
</tr>
<tr>
<td>P_{12-13}</td>
<td>9.62</td>
<td>Q_{12}</td>
</tr>
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<td>P_{12}</td>
<td>8.67</td>
<td>Q_{6-13}</td>
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<td>P_{6-13}</td>
<td>8.45</td>
<td>Q_{12-13}</td>
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<tr>
<td>Q_{12}</td>
<td>6.15</td>
<td>P_{12-13}</td>
</tr>
<tr>
<td>Q_{6-13}</td>
<td>5.68</td>
<td>P_{12}</td>
</tr>
</tbody>
</table>

Table 5-11: Multiple interacting non-conforming bad data

Table 5-11 shows that estimator correctly indicates bad data is the real power flow in line 12-13 in first pass. In the second pass reactive power injection at
bus 12 is identified. In the final pass no bad data is identified as all normalized residue are less than 1.0.

Hence Algorithm works correctly in case of multiple interacting bad data but non-conforming.

**B Multiple interacting and conforming bad data in IEEE 30 bus system**

A bad data is forced in real power injection at bus 30 and real power flow in line 30-27. Table 5-12 is the list of 5 largest normalized residues.

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<thead>
<tr>
<th>Number</th>
<th>Measurement</th>
<th>$r^N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>13.54</td>
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<tr>
<td>2</td>
<td>$c_{25-27}$</td>
<td>12.97</td>
</tr>
<tr>
<td>3</td>
<td>$d_{25-27}$</td>
<td>12.95</td>
</tr>
<tr>
<td>4</td>
<td>$P_{30-27}$</td>
<td>12.94</td>
</tr>
<tr>
<td>5</td>
<td>$P_{30}$</td>
<td>12.43</td>
</tr>
</tbody>
</table>

**Table 5-12**: Multiple interacting conforming bad data

Table 5-12 shows that TSSE indicates real power injection at bus 27 as bad data, but it is a true measurement. Hence Test fails to identify bad data.

**C Multiple non interacting in IEEE 118 bus system**

A bad data is forced in reactive power flow in line 118-76 and reactive power flow in bus 103-110. Table 5-13 is the list of 5 largest normalized residues for 3 iterations. First pass is result obtained when both bad data are present. Second pass is result when bad data from first pass is corrected and tested again for bad data. Final pass is the result when both bad data are corrected.
Table 5-13: Multiple non-interacting bad data

Table 5-13 shows that estimator correctly indicates bad data is the reactive power flow in line 118-76 in *first pass*. In the *second pass* reactive power flow in line 103-110 is identified. In the final pass no bad data is identified as all normalized residue are less than 1.0

Hence Algorithm works correctly in case of multiple non-interacting bad data.

**5.2.4 Bad Data in Phasor Measurement**

This is the last case of bad data. In this case we enforce bad data in one of the PMU measurements. It must be noted that PMU measurement values are in polar form, and the estimator is designed for Cartesian form. Hence the bad data module may not be able to point which precise measurement has error, but can detect and identify the bus or line having error. Each of the IEEE bus is simulated for this case and results are shown below.

**A IEEE 14 bus**

Bad data is enforced in phase angle of voltage at bus 7 the value is changed from -13.37 to -16.37 table 5-14 shows the list of 5 largest normalized residues. And figure 5.16 shows the plot of normalized residues in Cartesian form.
From table 5-14 it is clear that algorithm correctly detects that there is bad data in voltage phasor.

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<tr>
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Table 5-14: Largest normalized residue IEEE 14 bus with Bad data in PMU

**Figure 5.16:** $r^N$ of phasor measurements for IEEE 14 bus system

**B IEEE 30 bus system**

Bad data is enforced in magnitude of voltage at bus 12 the value is changed from 1.0571 to 1.2571. Table 5-15 shows the list of 5 largest normalized residues. And figure 5.17 shows the plot of normalized residues in Cartesian form.
<table>
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<tr>
<th>Number</th>
<th>Measurement</th>
<th>$r^N$</th>
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</thead>
<tbody>
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<tr>
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<td>$P_{17}$</td>
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</table>

Table 5-15: Largest normalized residue IEEE 30 bus with Bad data in PMU

From table 5-15 it is clear that algorithm can correctly detect that there is bad data in voltage phasor at bus 12.

![Figure 5.17: $r^N$ of phasor measurements for IEEE 30 bus system](image)

\( C\ \text{ IEEE 118 bus system} \)

Bad data is enforced in magnitude of current in line 52-53. Table 5-16 shows the list of 5 largest normalized residues. And figure 5.18 shows the plot of normalized residues in Cartesian form.
Table 5-16: Largest normalized residue IEEE 118 bus with Bad data in PMU

From table 5-16 it is clear that algorithm correctly detects that there is bad data in current phasor in line 52-53.

Figure 5.18: $r^N$ of phasor measurements for IEEE 118 bus system
5.3 Conclusion

This chapter deals with the simulation and testing of the designed two stage state estimator with IEEE 14 30 and 118 bus system. In the beginning it is described how measurements and other relevant data were obtained using P.E.T and other standard data, followed by simulation of state estimator to obtain estimated states assuming all the measurements are true. The results of state estimation show that designed two stage state estimator provides the result to the desired level of accuracy. In the next section model is tested for a variety of bad data. The system is put to test under different types of bad data occurring in different types of measurement, at different locations. Most important of these were bad data in critical measurements; as the objective of this thesis was to demonstrate that PMU can be used to add redundancy to critical measurements, without the need to redefine the state estimation algorithm. The estimator is also simulated for multiple bad data and it is seen that under most of the conditions the results were satisfactory but except when multiple bad data are interacting and conforming. Where the existence of bad data was shown, but it declared a good measurement as bad. Another condition where bad data algorithm is tested is the presence of bad data in phasor measurements; it successfully pointed out the location bad data. It can be concluded that algorithm is reliable in case of single bad data but one must be careful if there are multiple bad data in the measurement set.

5.4 Future Work

The greatest advantage of this model is the ease of design. The whole idea of two stage state estimator is to use the new phasor measurements to improve the existing conventional state estimator by addition of a simple linear post processing step. Hence it can be easily implemented on real life power systems. However this algorithm doesn’t take into consideration of memory usage. Hence it is very
important that sparse matrix operation must be considered before implementing them on larger systems.

To identify critical measurement it uses the fact that the element in residual covariance matrix corresponding to critical measurement is zero, which is not reliable always. There are many good observability algorithm those can used to check the observability and identify critical measurements. These algorithms also suggest optimum number of PMU required and their ideal locations. Implementing such ideas can increase usability of the model.
References


Appendix A  Topology and Measurement Data

A.1 busdata.m

```matlab
function busdt = busdatas(num)

% Type determines the nature of the bus
% Type = 1 Slack bus
% Type = 2 PV bus
% Type = 3 PQ bus

%%% Sample 5 Bus system used as example in Thesis document %%%

% |Bus| Type| B   | V   | theta |
busdata5 = [1  1  0  1.0100  0.00;
2  3  0  0.9950  -8.65;
3  2  0  1.0550  -10.75;
4  2  0  1.0900  -10.75;
5  3  0.19  1.0540  -11.81];
```

```
### IEEE 14 Bus System

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<th>theta</th>
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%  IEEE 118 Bus System

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    5  3  -0.40  1.0020  -13.97;
    6  2  0  0.9900  -16.70;
    7  3  0  0.9893  -17.14;
    8  2  0  1.0150  -8.95;
    9  3  0  1.0429  -1.70;
   10  2  0  1.0500  5.88;
   11  3  0  0.9851  -16.98;
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%Return the data for the selected bus system

switch num
    case 1
        busdt = busdata1;
    case 5
        busdt = busdata5;
    case 14
        busdt = busdata14;
    case 30
        busdt = busdata30;
    case 118
        busdt = busdata118;
end
### A.2 linesdata.m

```
function linedt = linedatas(num)
```

```
%%% BEGIN HELP %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% FUNCTION:    linedt = linedatas(num)
%%% DESCRIPTION: The function linedatas returns the branch data of
%%% the selected IEEE bus system
%%% Arguments:   nbus - Number of bus systems 14, 30 or 118 for IEEE
%%%                14 ,30 or 118 bus system.
%%% Outputs:     1) linedt - Line data consisting of sending end bus, %
%%%               receiving end bus,Resistance, Reactance, half of %
%%%               shunt susceptance and transformer tap, all in p.u. %
%%% Author:      Aditya Tarali
%%% END HELP %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
function linedt = linedatas(num)
```

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A-6
IEEE 30 Bus System

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%----------------------------------------------------------

%%%%      Sample 5 Bus system used as example in Thesis document      %%%%
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%            |  From |  To   |   R     |   X     |     B/2  |  X’mer  |
%            |  Bus  | Bus   |  pu     |  pu     |     pu   | TAP (a) |
linedata5   = 
%----------------------------------------------------------

% | From | To | R          | X          | B/2 | X’mer |
% | Bus | Bus | pu         | pu         | pu  | TAP (a) |
linedata5  = [ 1 2 0.06701 0.17103 0.0173 1;
   2 3 0.0 0.20912 0.0 0.978;
   2 5 0.0 0.55618 0.0 0.969;
   3 4 0.0 0.17615 0.0 1;
   3 5 0.0 0.11001 0.0 1];

%----------------------------------------------------------

A-11
% Return the data for the selected bus system

switch num
    case 1
        linedt = linedata1;
    case 5
        linedt = linedata5;
    case 14
        linedt = linedata14;
    case 30
        linedt = linedata30;
    case 118
        linedt = linedata118;
end

A.3  zconv.m

function zdt = zconv(num)

%%% BEGIN HELP %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% FUNCTION:  zdt = zconv(num)                                          %
%%% DESCRIPTION: The function zconv returns the conventional            %
%%%                measurement set of the selected IEEE bus system.        %
%%%                Arguments:  nbus - Number of bus systems 14, 30 or 118 for %
%%%                IEEE 14, 30 or 118 bus system.                         %
%%%                Outputs:  1) zdt - Measurement data consisting of        %
%%%                measurement number, type of measurement(1 Voltage        %
%%%                magnitude 2 Voltage angle 3 real power injection 4 reactive  %
%%%                power injection 5 Real power flow 6 reactive power       %
%%%                power flow), measurement value in p.u, Sending end       %
%%%                bus, receiving end bus and error covariance.            %
%%%                Author:  Aditya Tarali                                  %
%%% END HELP %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A-13

% Sample 5 Bus system used as example in Thesis document

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234  6  -0.0793  39  40  1e-4;
235  6  -0.0647  40  42  1e-4;
236  6  -0.0781  41  42  1e-4;
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255  6  0.6748  63  59  1e-4;
256  6  0.1927  66  67  1e-4;
257  6  0.1467  66  62  1e-4;
258  6  0.1215  67  62  1e-4;
259  6  -0.0460  68  81  1e-4;
260  6  -1.0365  69  68  1e-4;
261  6  0.1290  70  74  1e-4;
262  6  0.0994  70  75  1e-4;
263  6  -0.1074  71  73  1e-4;
264  6  -0.0315  72  71  1e-4;
265  6  0.2360  75  118  1e-4;
266  6  -0.0954  75  77  1e-4;
267  6  -0.2103  76  77  1e-4;
268  6  0.1733  77  82  1e-4;
269  6  0.0661  77  78  1e-4;
270  6  -0.5843  77  80  1e-4;
271  6  -0.2711  83  82  1e-4;
272  6  0.1207  83  85  1e-4;
273  6  0.1473  83  84  1e-4;
274  6  0.0758  85  88  1e-4;
275  6  0.0066  85  89  1e-4;
276  6  0.0509  86  85  1e-4;
277  6  0.1102  87  86  1e-4;
278  6  -0.0248  88  89  1e-4;
279  6  -0.1286  89  92  1e-4;
280  6  -0.0997  89  90  1e-4;
281  6  0.0446  90  91  1e-4;
%Return the data for the selected bus system

switch num
    case 1
        zdt = zdata1;
    case 5
        zdt = zdata5;
    case 14
        zdt = zdata14;
    case 30
        zdt = zdata30;
    case 118
        zdt = zdata118;
end
end
A.4 current.m

function [mag arg] = current( p,q,n)

s=(p+1i*q); % apparent power flow

bdt = busdatas(nbus); % nbus is number of bus in configuration
for l = 1:length(n)

% Retriving sending end voltage
m=n(l);
v = bdt(m,4).*(cosd(bdt(m,5))+1i*sind(bdt(m,5)));

I=(s(l)/v);
mag(l)=abs(I);
arg(l)=-1*round(angle((I))*18000/pi)/100;
end
A.5  zpmu.m

function zdt = zpmus(num)

%%% Sample 5 Bus system used as example in Thesis document %%%

% | Msnt | Type | Value   | From | To   | Rii   |
%---------------------------------|
% Voltage Magnitude              |
% 1   7  1.055      3  0  1e-6;  |
% Voltage Angle                  |
% 2   8  10.75      3  0  1e-6;  |
% Current Magnitude              |
% 3   9  0.1962     3  4  1e-6;  |
% Current Angle                  |
% 4   10 79.26      3  4  1e-6]; |

%%% IEEE 14 Bus System %%%

% | Msnt | Type | Value   | From | To   | Rii   |
%---------------------------------|
% Voltage Magnitude              |
% 1   7  1.01860     4  0  1e-6;  |
% 2   7  1.062       7  0  1e-6;  |
% 3   7  1.03580     14  0  1e-6;  |
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### Voltage Angle

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### Current Magnitude

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### Current Angle

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%------------------------------------ Voltage Magnitude ---------------------------------%  
1    7    0.9893    7    0    1e-6;  
2    7    1.0429    9    0    1e-6;  
3    7    0.9836   14    0    1e-6;  
4    7    0.9670   31    0    1e-6;  
5    7    0.9807   35    0    1e-6;  
6    7    0.98    36    0    1e-6;  
7    7    0.9568   52    0    1e-6;  
8    7    0.955   54    0    1e-6;  
9    7    0.985    59    0    1e-6;  
10   7    0.9932   60    0    1e-6;  
11   7    0.9687   63    0    1e-6;  
12   7    1.0034   78    0    1e-6;  
13   7    0.985    85    0    1e-6;  
14   7    1.015    87    0    1e-6;  
15   7    0.981    95    0    1e-6;  
16   7    0.9927   96    0    1e-6;  
17   7    0.993   113    0    1e-6;  
18   7    0.9604   114    0    1e-6;  
19   7    0.9738   117    0    1e-6;  
%------------------------------------ Voltage Angle -------------------------------------%  
20   8    -17.14    7    0    1e-6;  
21   8    -1.70     9    0    1e-6;  
22   8    -18.22   14    0    1e-6;  
23   8    -16.99   31    0    1e-6;  
24   8    -18.92   35    0    1e-6;  
25   8    -18.92   36    0    1e-6;  
26   8    -14.58   52    0    1e-6;  
27   8    -14.65   54    0    1e-6;  
28   8    -10.55   59    0    1e-6;  
29   8    -6.77    60    0    1e-6;  
30   8    -7.17    63    0    1e-6;  
31   8    -3.54    78    0    1e-6;  
32   8     2.56    85    0    1e-6;  
33   8     1.45    87    0    1e-6;  
34   8    -2.28    95    0    1e-6;  
35   8    -2.45    96    0    1e-6;  
36   8    -16.0   113    0    1e-6;  
37   8    -15.27   114    0    1e-6;  
38   8    -19.04   117    0    1e-6;  
%------------------------------------ Current Magnitude --------------------------------%  
39   9     0.3614    7    6    1e-6;  
40   9     4.2758    9   10    1e-6;  
41   9     0.1899   14   12    1e-6;  
42   9     0.1197   31   29    1e-6;
43  9  0.0641  35  36  1e-6;
44  9  0.3319  36  34  1e-6;
45  9  0.1104  52  53  1e-6;
46  9  0.0757  54  55  1e-6;
47  9  0.3166  59  54  1e-6;
48  9  0.4446  60  59  1e-6;
49  9  0.3362  61  64  1e-6;
50  9  0.3148  78  79  1e-6;
51  9  0.3846  85  84  1e-6;
52  9  0.1804  86  85  1e-6;
53  9  0.2298  96  97  1e-6;
54  10 170.09   7   6  1e-6;
55  10 175.16   9  10  1e-6;
56  10 148.95  14  12  1e-6;
57  10  60.12  31  29  1e-6;
58  10 102.78  35  36  1e-6;
59  10 139.9  36  34  1e-6;
60  10  25.46  52  53  1e-6;
61  10  26.27  54  55  1e-6;
62  10  18.4  59  54  1e-6;
63  10  1.05  60  59  1e-6;
64  10 149.98  61  64  1e-6;
65  10 140.89  78  79  1e-6;
66  10  16.75  85  84  1e-6;
67  10 162.19  86  85  1e-6;
68  10 116.51  96  97  1e-6;

%----------------- Current Angle -----------------%
54  10 170.09  7   6  1e-6;
55  10 175.16  9  10  1e-6;
56  10 148.95 14  12  1e-6;
57  10  60.12 31  29  1e-6;
58  10 102.78 35  36  1e-6;
59  10 139.9 36  34  1e-6;
60  10  25.46 52  53  1e-6;
61  10  26.27 54  55  1e-6;
62  10  18.4 59  54  1e-6;
63  10  1.05 60  59  1e-6;
64  10 149.98 61  64  1e-6;
65  10 140.89 78  79  1e-6;
66  10  16.75 85  84  1e-6;
67  10 162.19 86  85  1e-6;
68  10 116.51 96  97  1e-6;

%Return the data for the selected bus system

switch num
    case 5
        zdt = zdata5;
    case 14
        zdt = zdata14;
    case 30
        zdt = zdata30;
    case 118
        zdt = zdata118;
end
end
Appendix B  State Estimation algorithms

B.1  Conventional state estimation (wlscart.m)

% Power System State Estimation using Weighted Least Square Method..

% nbus represents the number of bus system
% nbus = 5 for 5 bus test system
% nbus = 14 for IEEE 14 bus system
% nbus = 30 for IEEE 30 bus system
% nbus = 118 for IEEE 118 bus system

ybus = ybusfunc(nbus); % Get YBus..
zdata = zconv(nbus); % Get  Conventional Measurement data..
[bsh g b] = line_mat_func(nbus); % Get conductance and susceptance matrix
type = zdata(:,2); % Type of measurement,
% type =1 voltage magnitude p.u
% type =2 Voltage phase angle in degree
% type =3 Real power injections
% type =4 Reactive power injection
% type =5 Real power flow
% type =6 Reactive power flow
z = zdata(:,3); % Measurement values
fbus = zdata(:,4); % From bus
tbus = zdata(:,5); % To bus
Ri = diag(zdata(:,6)); % Measurement Error Covariance matrix
e = ones(nbus,1); % Initialize the real part of bus voltages
f = zeros(nbus,1); % Initialize the imaginary part of bus voltages
E = [f;e]; % State Vector comprising of imaginary and real part of voltage
G = real(ybus);
B = imag(ybus);

ei = find(type == 1); % Index of voltage magnitude measurements.
fi = find(type == 2); % Index of voltage angle measurements.
ppi = find(type == 3); % Index of real power injection measurements.
qi = find(type == 4); % Index of reactive power injection measurements.
pf = find(type == 5); % Index of real power flow measurements.
qf = find(type == 6); % Index of reactive power flow measurements.
Vm=z(ei);
Thm=z(fi);
z(ei)=Vm.*cosd(Thm); % converting voltage from polar to Cartesian
z(fi)=Vm.*sind(Thm);

nei = length(ei); % Number of Voltage measurements(real)
nfi = length(fi); % Number of Voltage measurements( imaginary)
npi = length(ppi); % Number of Real Power Injection measurements.
nqi = length(qi); % Number of Reactive Power Injection measurements.
npf = length(pf); % Number of Real Power Flow measurements.
nqf = length(qf); % Number of Reactive Power Flow measurements.

iter = 0;
tol = 1;
while (tol > 1e-6)

%Measurement Function, h
h1 = e(fbus (ei),1); % voltage measurement
h2 = f(fbus (fi),1); % angle measurement
h3 = zeros(npi,1); % real power injection
h4 = zeros(nqi,1); % reactive power injection
h5 = zeros(npf,1); % real power flow
h6 = zeros(nqf,1); % reactive power flow

% Measurement function of power injection
for i = 1:npi
    m = fbus(ppi(i));
    for k = 1:nbus
        % Real injection
        h3(i)=h3(i)+(G(m,k)*(e(m)*e(k)+f(m)*f(k))... +B(m,k)*(-f(m)*e(k)-e(m)*f(k)));
    end
end

% Measurement function of power flow
for i = 1:npf

end

B-2
m = fbus(pf(i));
n = tbus(pf(i));

%        Real injection
h5(i) = (e(m)^2 + f(m)^2)*g(m,n) - (g(m,n)*(e(m)*e(n)+f(m)*f(n)) + b(m,n)*(f(m)*e(n)-e(m)*f(n)));

%       Reactive injection
h6(i) = -g(m,n)*(f(m)*e(n)-e(m)*f(n)) + b(m,n)*(e(m)*e(n)+f(m)*f(n)) - (e(m)^2 + f(m)^2)*(b(m,n)+bsh(m,n));
end

h = [h1; h2; h3; h4; h5; h6];

%Residual matrix difference of measurement and the non linear
r = z - h;

% Jacobian..

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%        Jacobian  Block 1: Derivative of voltage        %%%%%
%%%%%            with respect to states                      %%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
H11 = zeros(nei,nbus);  % Derivative of e wrt e
H12 = zeros(nei,nbus);  % Derivative of e wrt f
H21 = zeros(nfi,nbus);  % Derivative of f wrt e
H22 = zeros(nfi,nbus);  % Derivative of f wrt f
for k = 1:nei
    H11(k,fbus(k)) = 1;
    H22(k,fbus(n)) = 1;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%     Jacobian Block 2: Derivative of Power injection     %%%%%
%%%%
%%%%%                 with respect to states                  %%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
H31 = zeros(npi,nbus);  % Derivative of real power injection wrt e
H32 = zeros(npi,nbus);  % Derivative of real power injection wrt f
H41 = zeros(npi,nbus);  % Derivative of reactive power injection wrt e
H42 = zeros(npi,nbus);  % Derivative of reactive power injection wrt f
for i = 1:npi
    m = fbus(ppi(i));
    for k = 1:(nbus)
        if k == m
            for n = 1:nbus
                H31(i,k) = H31(i,k) + (G(m,n)*e(n) - B(m,n)*f(n));
                H32(i,k) = H32(i,k) + (G(m,n)*f(n) + B(m,n)*e(n));
                H41(i,k) = H41(i,k) -G(m,n)*f(n) - B(m,n)*e(n);
                H42(i,k) = H42(i,k) + (G(m,n)*e(n) - B(m,n)*f(n));
            end
        end
    end
end
\begin{verbatim}
end
H31(i,k) = H31(i,k) + f(m)*B(m,m) + G(m,m)*e(m);
H32(i,k) = H32(i,k) - e(m)*B(m,m) + f(m)*G(m,m);
H41(i,k) = H41(i,k) + f(m)*G(m,m) - e(m)*B(m,m);
H42(i,k) = H42(i,k) - e(m)*G(m,m) - f(m)*B(m,m);
else
H31(i,k) = G(m,k)*e(m) + B(m,k)*f(m);
H32(i,k) = G(m,k)*f(m) - B(m,k)*e(m);
H41(i,k) = (G(m,k)*f(m) - B(m,k)*e(m));
H42(i,k) = (-G(m,k)*e(m) - B(m,k)*f(m));
end
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%        Jacobian Block 3: Derivative of Power flow       %%%%%
%%%%%                 with respect to states                  %%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
H51 = zeros(npf,nbus);
H52 = zeros(npf,nbus);
H61 = zeros(nqf,nbus);
H62 = zeros(nqf,nbus);
for i = 1:npf
    m = fbus(pf(i));
    n = tbus(pf(i));
    H51(i,m) = 2*e(m)*g(m,n) - g(m,n)*e(n) + b(m,n)*f(n);
    H51(i,n) = -g(m,n)*e(m) - b(m,n)*f(m);
    H52(i,m) = 2*f(m)*g(m,n) - g(m,n)*f(n) - b(m,n)*e(n);
    H52(i,n) = -g(m,n)*f(m) + b(m,n)*e(m);
    H61(i,m) = -2*e(m)*(b(m,n)+bsh(m,n)) + g(m,n)*f(n) + b(m,n)*e(n);
    H61(i,n) = -g(m,n)*f(m) + b(m,n)*e(m);
    H62(i,m) = -2*f(m)*(b(m,n)+bsh(m,n)) - g(m,n)*e(n) + b(m,n)*f(n);
    H62(i,n) = g(m,n)*e(m) + b(m,n)*f(m);
end

% Measurement Jacobian, H..
H = [H11 H12; H21 H22; H31 H32; H41 H42; H51 H52; H61 H62];

% Gain Matrix, Gm..
Gm = H'*inv(Ri)*H;

%Objective Function..
J = sum(inv(Ri)*r.^2);

%Solving for states iteratively by cholesky factorization and forward
\end{verbatim}
B.2 Two stage state estimation (stage2.m)

% and back substitution.
gm = H' * inv(Ri) * r;
dE = factoriseGbychol(Gm, gm, length(E));
E = E + dE;

e = E(1:nbus);
f = E(nbus+1:end);
iter = iter + 1;
tol = max(abs(dE));
end
displayout(E,'a'); % Displaying output in tabular form

d% baddata
O = Ri - H * inv(Gm) * H';
od = diag(O);
cm = find(od <= 1e-12);
r(cm) = 0;
rN = abs(r) ./ sqrt(od);
displayout(rN,'b'); % Display rN in formatted form

%%% DESCRIPTION: This program Calculates states of test bus system in Cartesian coordinate systems Two stage sate estimation algorithm power which uses Phasor measurement. This program also performs bad data analysis using maximum normalized residual test.

Arguments: None

Outputs: Displays estimated states in polar form also displays the normalized residual corresponding to each measurement and indicates bad data if detected.

Author: Aditya Tarali

% nbus represents the number of bus system
% nbus = 5 for 5 bus test system
% nbus = 14 for IEEE 14 bus system
% nbus = 30 for IEEE 30 bus system
% nbus = 118 for IEEE 118 bus system

zpmu = zpmus(nbus); % Get phasor Measurement data..
type = zpmu(:,2); % Type of measurements,
\[ zp \text{ Rpmu}= \text{rotatemat}(\text{nbus}); \]  
\%Converts measurements and its error  
\%Covariance matrix from polar to Cartesian form

\[ fb \_pmu = zpmu(:,4); \]  
\% From bus..

\[ tb \_pmu = zpmu(:,5); \]  
\% To bus..

\[ vi = \text{find}(\text{type} == 7); \]  
\% Index of voltage phasor

\[ gi = \text{find}(\text{type} == 9); \]  
\% Index of current phasor

\[ nvi=\text{length}(vi); \]  
\[ ngi=\text{length}(gi); \]

\[ [\text{bsh g b}] = \text{line\_mat\_func}(\text{nbus}); \]  
\% Get conductance and susceptance matrix

\% Relation between measured voltage phasor and states

\[ \text{II} = \text{zeros}(\text{nvi, nbus}); \]
\begin{verbatim}
for \[k=1:nvi\]
    \[\text{II}(k,fb\_pmu(vi(k)))=1;\]
end
\end{verbatim}

\% Relation between measured current phasor and states

\[ \text{A}=\text{zeros}(\text{ngi, nbus}); \]  
\% Current incidence matrix

\[ \text{gseries}=\text{zeros}(\text{ngi, ngi}); \]  
\% Series conductance matrix of measured lines

\[ \text{bseries}=\text{zeros}(\text{ngi, ngi}); \]  
\% Series susceptance matrix of measured lines

\[ \text{bshunt}=\text{zeros}(\text{ngi, nbus}); \]  
\% Shunt admittance matrix

\begin{verbatim}
for \[k = 1:ngi\]
    \[\text{A}(k,fb\_pmu(gi(k)))=1;\]
    \[\text{A}(k,\text{tb}\_pmu(gi(k)))=-1;\]
    \[\text{gseries}(k,k)=g(fb\_pmu(gi(k)),tb\_pmu(gi(k)));\]
    \[\text{bseries}(k,k)=b(fb\_pmu(gi(k)),tb\_pmu(gi(k)));\]
    \[\text{bshunt}(k,fb\_pmu(gi(k))) = bsh(fb\_pmu(gi(k)),tb\_pmu(gi(k)));\]
end
\end{verbatim}

\% Relation between all phasors and states

\[ \text{H\_pmu1}=[\text{II};\text{zeros(size(II))}]; \]
\[ \text{H\_pmu2}=[\text{zeros(size(II))};\text{II}]; \]

\[ \text{H\_pmu21}=\{(\text{gseries}\*\text{A});(\text{bseries}\*\text{A}+\text{bshunt})\}; \]
\[ \text{H\_pmu22}=\{-(\text{bseries}\*\text{A}+\text{bshunt});(\text{gseries}\*\text{A})\}; \]

\[ \text{H\_pmu}=[\text{H\_pmu1} \text{ H\_pmu2}; \text{H\_pmu21} \text{ H\_pmu22}]; \]

\%invoke conventional state estimator

\[ [\text{E1} \text{ Ri\_cse} \text{ H1}]=\text{wlsconv}(\text{nbus}); \]
\[ \text{G0}= \text{H1}'/\text{Ri\_cse}\*\text{H1}; \]

\[ \text{z2}=[\text{E1};zp]; \]  
\% Second Stage measurement matrix

\[ \text{H2}=[\text{eye}(2\*\text{nbus});\text{H\_pmu}]; \]  
\% Second stage jacobain matrix

\%Define the inverse of measurement error covariance matrix using results of
\%first stage

\[ \text{offdiag}=\text{zeros}(2\*\text{nbus}, \text{length}(\text{Rmu})); \]
\[ \text{inv\_R2}=[ \text{G0} \text{ offdiag}; \]

B-6
offdiag' inv(Rpmu));

%%Solution of linear state estimation problem

% Gain Matrix, Gm..
Gm2 = H2'*inv_R2*H2;

%Solving for states by cholesky factorization
gm2 = H2'*inv_R2*z2;
E2=factoriseGbychol(Gm2,gm2,2*nbus);

displayout(E2,'a') % Displaying output in tabular form
e=E2(1:nbus);
f=E2(nbus+1:end);
CvE2= inv(H2'*inv_R2*H2); % Covariance of measurement residual for stage 2

%%
% residue conventional measurement
ybus = ybusfunc(nbus); % Get YBus..
Zdt = zconv(nbus); % Get Measurement data..
nbus = length(ybus); % Get number of buses..
type = zdt(:,2);

% Type of measurement,
% type =1 voltage magnitude p.u
% type =2 Voltage phase angle in degree
% type =3 Real power injections
% type =4 Reactive power injection
% type =5 Real power flow
% type =6 Reactive power flow
z_cse = zdt(:,3); % Measurement values..
fb_cse = zdt(:,4); % From bus..
tb_cse = zdt(:,5); % To bus..
G = real(ybus);
B = imag(ybus);

ei = find(type == 1); % Index of voltage magnitude measurements..
fi = find(type == 2); % Index of voltage angle measurements..
ppi = find(type == 3); % Index of real power injection measurements..
qi = find(type == 4); % Index of reactive power injection measurements..
PF = find(type == 5); % Index of real power flow measurements..
qf = find(type == 6); % Index of reactive power flow measurements..

Vm=z_cse(ei);
Thm=z_cse(fi);
z_cse(ei)=Vm.*cosd(Thm); % converting voltage from polar to Cartesian
z_cse(fi)=Vm.*sind(Thm);

nei = length(ei); % Number of Voltage measurements..
nfi = length(fi); % Number of Voltage measurements..
np = length(ppi); % Number of Real Power Injection measurements..
nqi = length(qi); % Number of Reactive Power Injection measurements..
\texttt{npf = length(pf);} % Number of Real Power Flow measurements.
\texttt{nqf = length(qf);} % Number of Reactive Power Flow measurements.

\%Measurement Function, \texttt{h}
\texttt{h1 = e(fb\_cse (ei),1);} %voltage measurement
\texttt{h2 = f(fb\_cse (fi),1);} %angle measurement
\texttt{h3 = zeros(npi,1);} %real power injection
\texttt{h4 = zeros(nqi,1);} %reactive power injection
\texttt{h5 = zeros(npf,1);} %real power flow
\texttt{h6 = zeros(nqf,1);} %reactive power flow

\%Measurement function of power injection
\texttt{for i = 1:npi}
\texttt{\quad m = fb\_cse(ppi(i));}
\texttt{\quad for k = 1:nbus}
\texttt{\% Real injection}
\texttt{\quad h3(i)=h3(i)+(G(m,k)*(e(m)*e(k)+f(m)*f(k)))...}
\texttt{\quad \quad +B(m,k)*((f(m)*e(k) - e(m)*f(k)))};
\texttt{\% Reactive injection}
\texttt{\quad h4(i)=h4(i)+(G(m,k)*(f(m)*e(k) - e(m)*f(k)))...}
\texttt{\quad \quad -B(m,k)*(e(m)*e(k)+f(m)*f(k)));
\texttt{\quad end}
\texttt{\end}

\%Measurement function of power flow
\texttt{for i = 1:npf}
\texttt{\quad m = fb\_cse(pf(i));}
\texttt{\quad n = tb\_cse(pf(i));}
\texttt{\% Real injection}
\texttt{\quad h5(i) = (e(m)^2 + f(m)^2)*g(m,n)...
\quad \quad -(g(m,n)*(e(m)*e(n)+f(m)*f(n))+b(m,n)*(f(m)*e(n)-e(m)*f(n))};
\texttt{\% Reactive injection}
\texttt{\quad h6(i) = -g(m,n)*(f(m)*e(n) - e(m)*f(n))+b(m,n)*(e(m)*e(n)+f(m)*f(n))...
\quad \quad -(e(m)^2 + f(m)^2)*(b(m,n)+bsh(m,n)));
\texttt{\end}

\texttt{h\_cse = [h1; h2; h3; h4; h5; h6];}
\texttt{r\_cse = z\_cse - h\_cse;}

\% bad data conventional measurement
\texttt{O\_cse=Ri\_cse - H1*CvE2*H1';}
\texttt{od\_cse=dTag(O\_cse);}
\texttt{cm = find(od\_cse<=1e-9);}
\texttt{r\_cse(cm)=0;}
\texttt{rN\_cse=abs(r\_cse)./sqrt(od\_cse);}
\% bad data PMU measurements
\texttt{r2=z2-H2*E2;}
O = inv(inv_R2) - H2*CvE2*H2';
od = diag(O);
    rN_stage2 = abs(r2)./sqrt(od);
    rN_pmu = rN_stage2(2*nbus+1:end);
%% all rNs combined
    rN = [rN_cse; rN_pmu];
displayout(rN,'c');
Appendix C   Miscellaneous Functions

C.1 Ybusfunc.m

function ybus = ybusfunc(nbus)    % Returns ybus
linedata = linedatas(nbus);    % Calling "linedatas.m" for Line Data
fb = linedata(:,1);    % From bus
tb = linedata(:,2);    % To bus
r = linedata(:,3);    % Resistance, R
x = linedata(:,4);    % Reactance, X
b = linedata(:,5);    % half Ground Admittance, B/2
a = linedata(:,6);    % Tap setting. Its value is 1 for transmission line
z = r + 1i*x;    % Z matrix
y = 1./z;    % To get inverse of each element
b = 1i*b;
busdata=busdatas(nbus);    % Calling "Busdatas.m" for shunt admittance
Bbus=li*diag(busdata(:,3));

nbranch = length(fb);    % no. of branches
ybus = zeros(nbus,nbus);    % Initialize YBus

% Adding Off Diagonal Elements
for k=1:nbranch
    ybus(fb(k),tb(k)) = ybus(fb(k),tb(k))-y(k)/a(k);
    ybus(tb(k),fb(k)) = ybus(fb(k),tb(k));
end
% Adding Diagonal Elements

for m =1:nbus
    ybus(m,m) = ybus(m,m)+Bbus(m,m);
    for n =1:nbranch
        if fb(n) == m
            ybus(m,m) = ybus(m,m) + y(n)/(a(n)^2) + b(n);
        elseif tb(n) == m
            ybus(m,m) = ybus(m,m) + y(n) + b(n);
        end
    end
end

C.2 line_mat_func.m

%%%%%%%%%%%%%%%%%%%%%%%%%%%%% BEGIN HELP %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%   FUNCTION:    [bbus,g,b] = line_mat_func(nbus)                     
%%%                                                                     
%%%   DESCRIPTION: The function line_mat_func returns shunt and series    
%%%                admittance of the IEEE bus system stance reactance       
%%%                and susceptance of the arrangement this is not same       
%%%                bus admittance matrix.                                   
%%%                                                                     
%%%   Arguments:   nbus - Number of bus systems 14, 30 or 118 for IEEE    
%%%                14, 30 or 118 bus system.                               
%%%                                                                     
%%%   Outputs:     1)  bbus - The shunt admittance matrix. That is      
%%%                bbus(m,k) is shunt admittance of the line              
%%%                connecting bus m to bus k lumped at bus m             
%%%                2)  g - Conductance of series admittance matrix.     
%%%                that is g(m,k) is series conductance of line          
%%%                connecting bus m and bus k                            
%%%                3)  b - susceptance of series admittance matrix.      
%%%                that is b(m,k) is series conductance of line          
%%%                connecting bus m and bus k                            
%%%                                                                     
%%%   Author:      Aditya Tarali                                       
%%%                                                                     
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% END HELP %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Line Data for B-Bus (Shunt Admittance)Formation.

function [bbus,g,b] = line_mat_func(nbus)

linedata = linedatas(nbus); % Calling "linedatas.m" for Line Data
fb = linedata(:,1);         % From bus
tb = linedata(:,2);         % To bus
r = linedata(:,3);          % Resistance, R
x = linedata(:,4);          % Reactance, X
\texttt{b\_sh = linedata(:,5);} \quad \% \text{half Ground Admittance, B/2}
\texttt{a = linedata(:,6);} \quad \% \text{Tap setting. Its value is 1 for transmission line}
\texttt{z = r + 1i*x;} \quad \% \text{Z matrix}
\texttt{y = 1./z;} \quad \% \text{To get inverse of each element}
\texttt{nbranch = length(fb);} \quad \% \text{no. of branches...}

\% \text{bbus is shunt admittance matrix of the lines bbus (i,j) and bbus (j,i) are}
\% \text{values of shunt admittance where i is sending end and j is receiving end.}
\% \text{for line both are same but for transformer they are different.}
\% \text{Y is series admittance matrix. g and b are series conductance and series}
\% \text{susceptance respectively.}

\texttt{bbus = zeros(nbus);} \quad \% \text{Initializing bbus}
\texttt{Y = zeros(nbus);} \quad \% \text{Initializing Y}

\texttt{for k=1:nbranch}
\quad \% \text{Bbus for a Transmission line}
\quad \texttt{if a(k)==1}
\quad \quad \texttt{bbus(fb(k),tb(k)) = b\_sh(k);} \\
\quad \quad \texttt{bbus(tb(k),fb(k)) = bbus(fb(k),tb(k));}
\quad \% \text{bbus for transformer with tap}
\quad \texttt{else}
\quad \quad \texttt{bbus(fb(k),tb(k)) = imag(y(k))*(1-a(k))/a(k)^2;} \\
\quad \quad \texttt{bbus(tb(k),fb(k)) = imag(y(k))*(a(k)-1)/a(k);} \\
\quad \texttt{end}
\quad \% \text{No condition is required for series admittance as tap ratio is 1 for}
\quad \% \text{transmission line}
\quad \texttt{Y(fb(k),tb(k)) = y(k)/a(k);} \\
\quad \texttt{Y(tb(k),fb(k)) = Y(fb(k),tb(k));}
\texttt{end}
\texttt{g=real(Y);} \\
\texttt{b=imag(Y);}
C.3  factoriseGbychol.m

function x=factoriseGbychol(A,b,n)
%obtain lower lower triangular matrix by cholesky factorization such that L*L'=A
L=chol(A,'lower');
%obtain lower lower triangular matrix by cholesky factorization
U=L';
x=zeros(n,1);
y=x;
%forward substitution...
for i=1:n
  y(i)=b(1)/L(1,1);
  ltemp=L(:,1);
  ltemp(1)=[];
  L(:,1)=[];
  b(1)=[];
  b=b-ltemp*y(i);
end
%back substitution
for i=n:-1:1
  x(i)=y(i)/U(i,i);
  utemp=U(:,i);
  utemp(i)=[];
  U(:,i)=[];
  U(i,:)=[];
  y(i)=[];
  y=y-utemp*x(i);
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%% BEGIN HELP %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% FUNCTION:  x=factoriseGbychol(A,b,n)                              %
%%% DESCRIPTION: The function factoriseGbychol solves equation Ax=b  %
%%% by factorizing A using cholesky factorization and               %
%%% solving by forward and backward substitution.                 %
%%% Arguments:  1) A - Matrix comprising of coefficient of set of  %
%%%   equation in this case is Gain matrix Gm                    %
%%%  2) b - Matrix containing right hand side of equations       %
%%%    in this case it is gm                                     %
%%%  3) n - is the no no of variable, here it is number of        %
%%%    states.                                                   %
%%% Outputs:     1) x is the solution set. In this it is dE at the  %
%%%                     end of each iteration                     %
%%% Author:      Aditya Tarali                                   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% END HELP %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
C.4 rotatemat.m

%%%%%%%%%%%%%%%%%%%%%%%%%%%%% BEGIN HELP %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% FUNCTION: [zcart Rpmu] = rotatemat(nbus)                        %
%%%                                                                      %
%%% DESCRIPTION: The function rotatemat transforms error covariance %
%%% and phasor measurements from polar to cartesian form %
%%% of the selected sample bus system %
%%%                                                                      %
%%% Arguments: nbus - Number of bus systems 14, 30 or 118 for IEEE %
%%% 14, 30 or 118 bus system.                             %
%%%                                                                      %
%%% Outputs: 1) zcart - phasor measurement in cartesian form. %
%%% 2) Rpmu - The resultant error covariance of %
%%% phasor measurement of the selected bus system after %
%%% transformation from polar domain to cartesian domain %
%%%                                                                      %
%%% Author: Aditya Tarali                                         %
%%%                                                                      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% END HELP %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [zcart Rpmu] = rotatemat(nbus)
zpmu = zpmus(nbus); % Get Measurement data..
type = zpmu(:,2); % Type of measurement, Vi- 7, Vth- 8, Ii - 9, Ith - 10
z = zpmu(:,3); % Measurement values..
v = find(type == 7); %index of voltage magnitude measurement..
vth = find(type == 8); %index of voltage angle measurement..
i = find(type == 9); %index of current magnitude measurement..
ith = find(type == 10); %index of current angle measurement..
z(vth)=z(vth)*pi/180;
z(ith)=z(ith)*pi/180;
% Error covariance matrix of phasor measurements in polar form
R1= diag(zpmu(:,6));
R11 = diag(cos(z(vth))); %V magnitude to e
R12 = -1*diag(z(v).*sin(z(vth))); %V angle to e
R21 = diag(sin(z(vth))); %V magnitude to f
R22 = diag(z(v).*cos(z(vth))); %V angle to f
%phi11 is sub matrix to transform voltage phasor from polar to Cartesian
phi11 = [R11 R12; R21 R22];

R33 = diag(cos(z(ith)));
R34 = diag(-z(i).*sin(z(ith)));
R43 = diag(sin(z(ith)));
R44 = diag(z(i).*cos(z(ith)));
%phi22 is sub matrix to transform voltage phasor from polar to Cartesian

\[
\begin{bmatrix}
R_{33} & R_{34} \\
R_{43} & R_{44}
\end{bmatrix}
\]

\[
\phi_22 = \text{zeros}(a, b);
\]

\[
\phi_11 = \text{zeros}(a, b);
\]

\[
\phi_21 = \text{zeros}(a, b);
\]

\[
\phi = \begin{bmatrix}
\phi_11 & \phi_12 \\
\phi_21 & \phi_22
\end{bmatrix}
\]

\[
R_{pmu} = \phi R_{i} \phi';
\]

\[
\begin{align*}
z_1 &= z(v) \cdot \cos(z(vth)); \\
z_2 &= z(v) \cdot \sin(z(vth)); \\
z_3 &= z(i) \cdot \cos(z(ith)); \\
z_4 &= z(i) \cdot \sin(z(ith));
\end{align*}
\]

\[
\text{zcart} = [z_1; z_2; z_3; z_4];
\]

C.5 displayout.m

```
function displayout(A,ch)
switch ch
%block 1: A contains States
case 'a'
    nbus= length(A)/2;
```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%% BEGIN HELP %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%   FUNCTION:   displayout(A,ch)                                       %%%
%%%   DESCRIPTION: The function displayout presents A in the display     %%%
%%%                A can be states or normalised residues depending on    %%%
%%%                option of ch. If A consist of stages it changes        %%%
%%%                them to polar form to display. If A contains           %%%
%%%                normalised residues. It seperates them in its real     %%%
%%%                and reactive parts to display identifies bad data if    %%%
%%%                found depending on threshold. it is set to 1.            %%%
%%%                                                                      %%%
%%%   Arguments:   1) A - States or Normalised Residues                  %%%
%%%                   2) ch- Ch is the option that determines the A      %%%
%%%                               ch is 'a' then A contains states.       %%%
%%%                               ch is 'b' Then A contains normalised residues %%%
%%%                               after CSE that is first stage.        %%%
%%%                               ch is 'b' Then A contains normalised residues %%%
%%%                               after second stage. I contains normalized %%%
%%%                               resides corresponding both conventional and %%%
%%%                               phasor measurements                            %%%
%%%   Author:      Aditya Tarali                                         %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% END HELP %%%%%%%%%%```
\[ f = A(nbus+1:end); \]
\[ e = A(1:nbus); \]
\[ v = e + 1i * f ; \]
\[ V = \text{abs}(v); \]
\[ \text{Del} = \text{round}(\text{angle}(v) * 180 / \pi * 100) / 100; \]
\[ \text{fprintf('}n\text{t State Estimation} \n\n'); \]
\[ \text{fprintf('}t\text{----------------------------}n'); \]
\[ \text{fprintf('}t| \text{Bus} | \text{V} | \text{Angle} | \n\n'); \]
\[ \text{fprintf('}t| \text{No} | \text{pu} | \text{Degree} | \n\n'); \]
\[ \text{fprintf('}t\text{----------------------------}n'); \]
\[ \text{for} \ k = 1:nbus \]
\[ \text{fprintf('}t| %4g', k); \]
\[ \text{fprintf(', %8.3f', V(k)); \]
\[ \text{fprintf(', %8.2f |', Del(k)); \text{fprintf('}n'); \]
\[ \text{end} \]
\[ \text{fprintf('}t\text{----------------------------}n'); \]

%%% block 2 : A contains normalised residues after first stage %%%

\begin{verbatim}
if length(A)==12
    nbus=5;
elseif length(A)==32
    nbus=14;
elseif length(A)==78
    nbus=30;
else
    nbus=118;
end
\end{verbatim}

% normalised Residues corresponding to Convetional measurement
\begin{verbatim}
zdt_cse=zconv(nbus);
type=zdt_cse(:,2);
ppi=find(type==3);
qi=find(type==4);
pf=find(type==5);
qf=find(type==6);
ri(:,1)=A(ppi);
ri(:,2)=A(qi);
fbusi=zdt_cse(ppi,4);tbusi=zdt_cse(ppi,5);
rf(:,1)=A(pf);
rf(:,2)=A(qf);
fbusf=zdt_cse(pf,4);tbusf=zdt_cse(pf,5);
\end{verbatim}
\begin{verbatim}
rN=[ri;rf];
fbus=[fbusi;fbusf];tbus=[tbusi;tbusf];
\end{verbatim}
a = max(max(rN));  % Looking for max rN
[~, c] = max(rN);  % and its location
if rN(c(1), 1) == a
    b = c(1);
else
    b = c(2);
end
change = 0;
bad_data = 0;
% 0 - no bad data 1 - bad data in conventional data 2 - bad data in Pmu
fprintf('%t Normalised residue in conventional measurement\n');
fprintf('%t-------------------------------------------------\n');
fprintf('%t| S1 | From | To | rN | \n');
fprintf('%t| No | Bus | Bus | Real | Reactive | \n');
fprintf('%t-------------------------------------------------\n');
for m = 1:length(rN)
    if change == 0
        fprintf('%t---------------Power Injection\n');
        fprintf('%t-------------------\n');
        change = 1;
    end
    if change == 1 && m - 1 == length(rN)
        fprintf('%t-----------------power flow\n');
        fprintf('%t-------------------------------------------------\n');
        change = 2;
    end
    if rN(m, 1) ~= 0 && rN(m, 2) ~= 0
        fprintf('%t%4g %8.0f\n', m, fbus(m));
        if tbus(m) == 0
            fprintf('%8.0f', tbus(m));
        else
            fprintf('%8.0f\n', tbus(m));
        end
        fprintf('%5.3f', rN(m, 1));
        fprintf('%5.3f', rN(m, 2));
    else
        if tbus(m) == 0
            fprintf('%t%4g %8.0f\t%8.0f\t%8.0f%\t%8.0f\t%t\t%t\t%t\t%t\t%t\t\n', m, fbus(m), tbus(m), cm(m));
        else
            fprintf('%t%4g %8.0f\t%8.0f\t%8.0f%\t%8.0f\t%t\t%t\t%t\t%t\t%t\t\n', m, fbus(m), tbus(m), cm(m));
        end
    end
end
if a >= 2 && m == b
    fprintf('  <=-Bad Data');
    bad_data = 1;
end
fprintf('%n');
if bad_data==1
    if b<=length(ri)
        if rN(b,1)>rN(b,2)
            fprintf('Real power injection at bus %d is',fbus(b));
            fprintf(' bad with normalised residue = %5.2f',a);
        else
            fprintf('Reactive power injection at bus %d',fbus(b));
            fprintf(' is bad with rN = %5.2f',a);
        end
    elseif b>length(ri)
        if rN(b,1)>rN(b,2)
            fprintf(' Real power flow from bus %d to',fbus(b));
            fprintf(' bus %d with rN = %5.2f',tbus(b),a);
        else
            fprintf(' Reactive power flow from bus %d',fbus(b));
            fprintf(' to bus %d with rN = %5.2f',tbus(b),a);
        end
    end
else
    fprintf('no bad data detected');
end
fprintf('\n');

%Graphical representation of normalized residues

figure
subplot(2,1,1);
bar(1:length(ri),ri);
xmax=length(ri);
ymax= max(max(ri));
axis([0 xmax+1 0 ymax*1.2]);
if bad_data==1
    if b<=length(ri)
        text(b,a,...
            ['Bad data at bus ',num2str(fbus(b))],...
            'VerticalAlignment','bottom',...
            'HorizontalAlignment','center',...
            'FontSize',10);
    end
end
title('r^N of Power Injection wrt measurement number',...
    'fontsize',12);
legend('Real','Reactive')
```matlab
subplot(2,1,2);
xmin=length(ri)+1;
xmax=length(ri)+length(rf);
bar(xmin:xmax,rf);
ymax=max(max(rf));
axis([xmin-1 xmax+1 0 ymax*1.2]);
str='Bad data in line '
if bad_data==1
    if b>length(ri)
        text(b,a,
            [str,num2str(fbus(b)),
             '-',num2str(tbus(b))],
            'VerticalAlignment','bottom',
            'HorizontalAlignment','center',
            'FontSize',10);
    end
end
title('r^N of Power Flow wrt measurement number',
     'fontsize',12);
legend('Real','Reactive')

%%%% block 3 : A contains normalised residues after second stage %%%%%
case 'c'
    if length(A)==12+4
        nbus=5;
    elseif length(A)==32+10
        nbus=14;
    elseif length(A)==78+22
        nbus=30;
    else
        nbus=118;
    end
%normalised Residues corresponding to Convetional measurement
zdt_cse=zconv(nbus);
type=zdt_cse(:,2);

ppi=find(type==3);
qi=find(type==4);
pf=find(type==5);
qf=find(type==6);

ri(:,1)=A(ppi);
ri(:,2)=A(qi);
fbusi=zdt_cse(ppi,4);tbusi=zdt_cse(ppi,5);

rf(:,1)=A(pf);
rf(:,2)=A(qf);```
fbusf=zdt_cse(pf,4); tbusf=zdt_cse(pf,5);

rN_cse=[ri;rf];
fbus_cse=[fbusi;fbusf]; tbus_cse=[tbusi;tbusf];

%normalised Residues corresponding to Phasor measurement
offset = max(qf);
zdt_pmu=zpmus(nbus);
type=zdt_pmu(:,2);
% v is for voltage c is for current
vi=find(type==7);
vt=find(type==8);
vi=find(type==9);
ct=find(type==10);

rv(1,:)=A(offset+vi);
rv(2,:)=A(offset+vt);
fbusv=zdt_pmu(vi,4); tbusv=zdt_pmu(vi,5);

rc(:,1)=A(offset+ci);
rc(:,2)=A(offset+ct);
fbusc=zdt_pmu(ci,4); tbusc=zdt_pmu(ci,5);

rN_pmu=[rv;rc];
fbus_pmu=[fbusv;fbusc]; tbus_pmu=[tbusv;tbusc];

rN=[rN_cse;rN_pmu];
fbus=[fbus_cse;fbus_pmu]; tbus=[tbusi; tbus_pmu];
	n_cse=length(rN_cse);

a=max(max(rN)); % looking for max rN
[~,c]=max(rN); % and its location

if rN(c(1),1)==a
  b=c(1);
else
  b=c(2);
end
change=0;

bad_data=0;
% 0-no bad data 1-bad data in conventional data 2 -bad data in Pmu
fprintf('t |  Normalised residue in conventional measurement\n');
fprintf('t |  From  |  To   |         rN            |\n');
fprintf('t |  No  |   Bus  | Bus   |   Real  |   Reactive |\n');
fprintf('t ------------------------------------------------\n');
for m = 1:n_cse
  if change==0
fprintf('
-----------------Power Injection');
fprintf('-------------------
');
change=1;
end
if change==1 && m-1==length(ri)
    fprintf('
-----------------power flow');
    fprintf('---------------------
');
    change=2;
end
if rN(m,1)~=0 && rN(m,2)~=0
    fprintf('
%4g ', m);
    fprintf('%8.0f', fbus(m))
    fprintf('%8.0f', tbus(m));
    fprintf('%5.3f ', rN(m,1));
    fprintf('%5.3f ', rN(m,2));
else
    fprintf('
%4g', m);
    fprintf('%8.0f', fbus(m));
    fprintf('%8.0f', tbus(m));
    fprintf(' CM');
end
% Checking for bad data
if a>=2 && m==b
    fprintf('  <--Bad Data');
    bad_data=1;
end
fprintf('

--------------------------------------------------------------------------

|  Sl  |  From  |  To   |         rN            |
--------------------------------------------------------------------------
|  No  |   Bus  | Bus   |   Real  |   Reactive |
--------------------------------------------------------------------------
for m = n_cse+1:length(rN)
    if change==2
        fprintf('
|      |      |      |      | rN |
--------------------------------------------------------------------------
|      |      |      |      |  |
--------------------------------------------------------------------------
|      |      |      |      |  |
end
if change==3 && m-1==(length(vi)+n_cse)
    fprintf('
--------------------------------------------------------------------------

|      |      |      |      |      |
--------------------------------------------------------------------------
|      |      |      |      |      |
end
fprintf('
%4g', m);
fprintf(' %8.0f', fbus(m))
fprintf(' %8.0f', tbus(m));
fprintf('%5.3f ', rN(m,1));
fprintf('%5.3f ', rN(m,2));
if a>=2 && m==b
fprintf('   <--Bad Data');
bad_data=2;
end
fprintf('
');
end
fprintf('	 |------------------------------------------------|--|\n');

if bad_data==1
if b<=length(ri)
if rN(b,1)>rN(b,2)
fprintf('\nReal power injection at bus %d is',fbus(b));
fprintf(\' bad with normalised residue = %5.2f\',a);
else
fprintf(\'\nReactive power injection at bus %d\',fbus(b));
fprintf(\' is bad with rN = %5.2f\',a);
end
elseif b>length(ri)
if rN(b,1)>rN(b,2)
fprintf(\'\n Real power flow from bus %d to',fbus(b));
fprintf(\' bus %d with rN = %5.2f\',tbus(b),a);
else
fprintf(\'\n Reactive power flow from bus %d',fbus(b));
fprintf(\' to bus %d with rN = %5.2f\',tbus(b),a);
end
endif
elseif bad_data==2
if (b-n_cse)<=length(vi)
fprintf(\'\n Voltage phasor at bus is bad %d\',fbus(b));
else
fprintf(\'\n Current phasor from bus %d \',fbus(b));
fprintf(\' to bus %d is bad\',tbus(b));
end
else
fprintf(\'no bad data detected\');
endif
fprintf(\'\n\');

%Graphical representation of normalised residues
figure
subplot(2,1,1);
bar(1:length(ri),ri);
xmax=length(ri);
ymax= max(max(ri));
axis([0 xmax+1 0 ymax*1.2]);
if bad_data==1
if b<=length(ri)
text(b,a,[\'Bad data at bus ',num2str(fbus(b))],...\n\'VerticalAlignment\',\'bottom\',...
end
end
title('r^N of Power Injection wrt measurement number',
      'fontsize',12)
legend('Real','Reactive')

subplot(2,1,2);
xmin=length(ri)+1;
xmax=length(ri)+length(rf);
bar(xmin:xmax,rf);
ymax= max(max(rf));
axis([xmin-1 xmax+1 0 ymax*1.2]);
str='Bad data in line ';
if bad_data==1
  if b>length(ri)
    text(b,a,[str,num2str(fbus(b)),'-',num2str(tbus(b))],
        'VerticalAlignment','bottom',
        'HorizontalAlignment','center','FontSize',10);
  end
end
title('r^N of Power Flow wrt measurement number',
      'fontsize',12);
legend('Real','Reactive')

figure
subplot(2,1,1);
bar(1:(length(vi)+1),[rv;zeros(1,2)]);
xmax=length(vi);
ymax= max(max(rv));
axis([0 xmax+1 0 ymax*1.2]);
if bad_data==2
  if b-n_cse<=length(vi)
    text(b-n_cse,a,['Bad data at bus ',num2str(fbus(b))],
        'VerticalAlignment','bottom',
        'HorizontalAlignment','center','FontSize',10);
  end
end
title('r^N of Voltage phasor wrt measurement number',
      'fontsize',12)

subplot(2,1,2);
xmin=length(vi)+1;
xmax=length(vi)+length(ci);
bar(xmin:(xmax+1),[rc;zeros(1,2)]);
ymax= max(max(rc));
axis([xmin-1 xmax+1 0 ymax*1.2]);
str='Bad data in line ';
if bad_data==2
  if b-n_cse<=length(vi)
if b>length(ri)
    text(b-n_cse,a,...
        [str,num2str(fbus(b)),'-',num2str(tbus(b))],...
        'VerticalAlignment','bottom',...
        'HorizontalAlignment','center','FontSize',10);
    end
end
end
title('r^N of current phasor wrt measurement number',... 
    'fontsize',12);
end