DYNAMIC ANALYSIS OF
ORTHOGONAL APPARENT POWER COMPONENTS
IN POLYPHASE UNBALANCED NETWORKS

By
Afsaneh Ghanavati

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE
AT
NORTHEASTERN UNIVERSITY
BOSTON, MASSACHUSETTS
JULY 2012

© Copyright by Afsaneh Ghanavati, 2012
# Table of Contents

Table of Contents iv

List of Tables vi

List of Figures vii

Acknowledgment ix

Abstract x

1 Introduction 1

1.1 The Steady State 7-Component Decomposition [4], [5] . . . . . . . . . . 4
1.2 Local Hilbert Space and Dynamic Phasors . . . . . . . . . . . . . . . . 10
1.3 Akagi-Nabae Instantaneous Power Components . . . . . . . . . . . 11
1.4 Summary of Contributions . . . . . . . . . . . . . . . . . . . . . . . . . 14

2 Dynamic Power Analysis 16

2.1 A 7/11-Component Dynamic Power Decomposition . . . . . . . . . . . 17
2.2 Steady State Analysis . . . . . . . . . . . . . . . . . . . . . . . . . . . 19
2.3 Dynamic Analysis of an Industrial Example . . . . . . . . . . . . . . . 25
2.4 Comparison with Akagi-Nabae Decomposition . . . . . . . . . . . . . 34

3 Concluding Remarks 40

3.1 Summary of Results . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 40
3.2 Future Research . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 41

Bibliography 42
Appendix A: Matlab code for computing the 7 components

Appendix B: Matlab function used for computing the 7 components

Appendix C: Matlab function for computing the sequence components of real and reactive power

Appendix D: Matlab function for computing the average Akagi reactive power

Appendix E: Matlab function for computing the instantaneous Akagi reactive power
List of Tables

1.1 Expressions for $\mu$ Variables ........................................ 7
1.2 Expressions for $\sigma$ Variables ....................................... 8
1.3 Power Components .......................................................... 8
1.4 Imbalance Indices ............................................................ 9

2.1 Power Components in Example 1 ....................................... 21
2.2 Sequence Components of $P$ and $Q_B$ in Example 1 ............. 21
2.3 Power Components (in kVA) in Example 2 ............................ 23
2.4 Sequence Components of $P$ and $Q_B$ in Example 2 ............. 24
# List of Figures

1.1 Circuit Schematic of a Polyphase System ........................................ 4  
1.2 Load Compensation ........................................................................... 6  
1.3 Seven-Component Decomposition ....................................................... 15  
1.4 Zero, Positive and Negative Sequences of $P$ and $Q_B$ .................... 15  

2.1 Balanced Supply Voltage and Unbalanced Y Connected Load ............. 20  
2.2 Circuit With Unbalanced Resistive Load .......................................... 22  
2.3 Portion of Single-Line Diagram of the MWPI Power System ............ 25  
2.4 Phase Voltages ................................................................................. 29  
2.5 Phase Currents .................................................................................. 29  
2.6 Dynamic Seven-Component decomposition ........................................ 30  
2.7 Dynamic 7-Component decomposition ............................................... 31  
2.8 Indices of Imbalance ......................................................................... 32  
2.9 Zero, Positive and Negative Sequences of $P$ and $Q_B$ .................... 32  
2.10 Zero, Positive and Negative Sequences of $P$ .................................... 33  
2.11 Zero, Positive and Negative Sequences of $Q_B$ ............................. 33  
2.12 $q_{a,\beta}(t) , q_{a,0}(t) , q_{\beta,0}(t)$ ....................................................... 36  
2.13 Instantaneous $s(t) , p(t) , q_{AN}(t)$ ................................................. 36  
2.14 Window–Averaged $q_{AN}(t)$ ............................................................. 37  
2.15 Average and Instantaneous Akagi Reactive Power .......................... 38
2.16 Akagi Result from $Q_+ - Q_-$
Acknowledgment

First, I would like to thank my advisor Professor Aleksandar Stanković for his guidance and help throughout this work. Prof. Stanković was always there to give advice and enlighten my path with his insightful comments. His friendliness toward others has taught me great life lessons as well.

I would also like to thank my co-advisor Professor Hanoch Lev-Ari for his careful proofreading and invaluable comments and for all his help and support that made the completion of this thesis possible.

I am also grateful to my parents Farzaneh and Mansoor Ghanavati whose unconditional love and support have made me become the person I am now.

Last but not least, I express my gratitude and love to my husband Hossein whose continuous encouragement has given me strength to go forward and to my children Ali and Aala who have brought tremendous amount of joy and hope into my life.
Abstract

A complete characterization of non-active power components is of increasing importance, given the abundance of nonlinear loads and distributed generators in modern power systems. Effective control of power quality can be achieved only when the contribution of imbalance and nonlinearities to non-active power is clearly understood. Starting with the work of Budeanu, many authors have aimed to characterize the concept of non-active power in the most general case of unbalanced, non-sinusoidal operation. The most detailed work to date appears to be that of Lev-Ari and Stankovic, who introduced a decomposition of apparent power consisting of seven components. This steady-state decomposition generalizes and refines the 5-component decomposition of Czarnecki, as well as those introduced earlier by Sharon and by Shepherd & Zakikhani.

In this thesis we apply the concept of dynamic phasors and a local Hilbert space to extend the steady-state 7-component decomposition of Lev-Ari and Stankovic to the transient regime, resulting in a novel dynamic (time-variant) 7-component power decomposition for polyphase systems with any number of phases. Our dynamic power decomposition captures transient behavior, while reducing to the standard (constant phasor) characterization in steady state. Thus, it directly relates to quantities that are part of utility regulations, such as harmonic distortion and power factor. We also propose a secondary decomposition of both the real power and of Budeanu’s “reactive” power into their respective symmetric sequence components.

We demonstrate the utility of our dynamic power decomposition by applying it
to actual data recorded in a paper plant during an outage. We also use the same example to illustrate the shortcomings of the “instantaneous” power decomposition of Akagi and Nabae.
Chapter 1

Introduction

Reduction of the negative effects of waveform distortion on the power properties of a circuit requires that these properties, in the presence of harmonics, be carefully defined and quantified. The instruments to provide suitable information about these properties have to be constructed. Therefore, for a long time there have been attempts aimed at formulating the power theory of nonsinusoidal systems and efforts to build adequate meters followed these attempts. One of the most widespread approaches was based on Budeanu’s concept of reactive power. Since then many theories have been reported in the literature trying to solve the problem using different approaches.

Starting with the work of Budeanu [1], many authors have aimed to characterize the concept of reactive power in the most general case, and to decompose the load current into physically meaningful mutually orthogonal components. The most detailed work to date appears to be that of Czarnecki [2], who introduced a decomposition consisting of five mutually orthogonal components. The analysis of these current components can be employed to identify the load characteristic (active/reactive, linear/nonlinear, balanced/unbalanced, etc.) and to attain the power factor improvement by means of a network of linear, passive, reactive elements. The major flaw of this decomposition, which is based on a phase-by-phase approach, is that it does not allow an easy and immediate handling of the interaction of harmonic and sequence
components [3].

A recently introduced new orthogonal 7-component decomposition [4], [5] generalizes and refines the one proposed by Czarnecki. This 7-component decomposition can be used to analyze the apparent power in both the steady-state and in the transient mode of operation. The dynamic power decomposition is based on the concept of dynamic phasors where it captures transient behavior and reduces to the constant phasors in steady-state.

A Hilbert space framework is used to formulate objectives and derive results. Voltage and current vectors are row vectors representing polyphase load voltage and current (nonlinear load, non-sinusoidal voltage source) which are viewed as elements in a Hilbert space of n-phase, square integrable, T-periodic waveforms, with a defined inner product. Such waveforms can be represented by their Fourier coefficients, also known as phasors. Moreover, the notion of phasor representation can be extended also to non-periodic waveforms by using a local Hilbert space defined by a sliding window of length T (= the steady state period length) [6], [7], [8]. The resulting dynamic phasors are constant in steady state operation, but become time-varying during transients. Thus, they can provide important information about the nature of the transient, as we shall demonstrate in chapter 2.

The main contribution of this thesis is to combine the notion of dynamic (Fourier) phasors with the static 7-component decomposition of [4], [5]. This results in a dynamic version of the 7-component decomposition, namely

\[ S^2(t) = P^2(t) + N_s^2(t) + N_u^2(t) + Q_B^2(t) + Q_s^2(t) + Q_u^2(t) + S^2(t) \]  

(1.1)

Each dynamic power component can be expressed in terms of dynamic phasors, and thus depends on the (polyphase) voltage and current waveforms in the interval \((t - T, t]\). As in the static case, the components \(N_u(t)\) and \(Q_u(t)\) are associated with unbalanced current flow, while the components \(N_s(t)\) and \(Q_s(t)\) are associated with spread over harmonics: these concepts are discussed in further detail in sections 1.1 and 1.2.
Quantities that are conserved in a network have several features that are important for engineering practice. Among them is the design of local compensation devices (e.g., shunt capacitors), which is greatly simplified by decoupling from other parts of the system which occurs if the regulated quantity is conserved in the network. In a system (such as public utility) with several owners and managers of energy processing components, the interpretation of rules and regulations is greatly simplified in the case of conservative quantities [9]. Of all the seven power components in (2.1) only two – $P(t)$ and $Q_B(t)$ – are network-conservative. This is apparently related to the fact that these two quantities can be expressed in terms of inner products [9]. In contrast, the remaining five components are all defined in terms of (unsigned) norms and explained in sections 1.2 and 1.3.

The two signed components of the decomposition (2.1) can be further decomposed into their symmetric sequence components, viz,

\[
P(t) = P_+(t) + P_-(t) + P_0(t) \tag{1.2a}
\]

\[
Q_B(t) = Q_{B,+}(t) + Q_{B,-}(t) + Q_{B,0}(t) \tag{1.2b}
\]

where $P_+$, $P_-$, and $P_0$ denote the positive, negative, and zero-sequence components of $P(t)$, and similarly for the components of $Q_B(t)$. We shall demonstrate in chapter 2 that these sequence components provide useful insight about system transients.

An alternative notion of instantaneous reactive power, known as the Akagi-Nabae reactive power, has been very successful in addressing a number of practical control problems in three phase, three conductor systems [10]. Its main application areas include active filtering and drive inverter control. However, this approach cannot be used with four conductor systems (which may involve a zero sequence current), or in systems with more phases. Moreover, because it relies on observed instantaneous voltage and current, the Akagi-Nabae reactive power can be extremely sensitive to noise and processing delays, so that some form of (ad-hoc) filtering is often used in practical implementations. We compare in chapter 2 our dynamic decomposition approach with both the Akagi-Nabae instantaneous reactive power and its sliding
1.1 The Steady State 7-Component Decomposition [4], [5]

We consider an $n$-phase system, i.e., a system with $n+1$ conductors (“wires”) in which the first $n$ are referenced either to a common ground or the $(n+1)$-st (“neutral”) conductor. Then we can define the $n$-dimensional voltage and current row vectors

$$v(t) \overset{\text{def}}{=} [v_1(t) \ v_2(t) \ \ldots \ v_n(t)]$$

$$i(t) \overset{\text{def}}{=} [i_1(t) \ i_2(t) \ \ldots \ i_n(t)]$$

where currents (as shown in Fig: 1.1) have reference directions “toward” the load.

Thus $v(t)$ and $i(t)$ are row vectors representing polyphase load voltage and current (nonlinear load, non-sinusoidal voltage source), which we view as elements in a Hilbert space of $n$-phase, square integrable, $T$-periodic waveforms, with the inner product

![Circuit Schematic of a Polyphase System.](image-url)
defined by

\[<x, y> \overset{\text{def}}{=} \frac{1}{T} \int x(s)y^\top(s) \, ds = \sum_{k=1}^{n} \sum_{l=-\infty}^{\infty} X_{k,l}Y_{k,l}^* \]  

(1.4)

where the superscript \(^\top\) denotes transposition, and the superscript * denotes a complex conjugate transpose. Here \(X_{k,l}\) is the phasor describing the \(k\)-th phase/\(l\)-th harmonic of a polyphase waveform \(x(t)\), namely

\[X_{k,l} = \frac{1}{T} \int_T \left[ x(s) \, e^{T_k} \right] e^{-j\omega s} \, ds \]  

(1.5)

\[e_k \overset{\text{def}}{=} \begin{bmatrix} 0 & 0 & \cdots & 1 & 0 & \cdots & 0 \end{bmatrix} \]  

(1.6)

with \(\omega = \frac{2\pi}{T}\), and similarly for \(Y_{k,l}\). The notation \(\int_T\) indicates integration over an interval of length \(T\), where \(T\) is the duration of a cycle (or the period in a steady state). We are interested in energy processing systems where this quantity is typically fixed in advance to 1/60 (or 1/50).

The various components of our power decomposition can be expressed entirely in terms of the phasors \(V_{k,l}\) and \(I_{k,l}\), associated with the polyphase voltage and current waveforms \(v(t)\) and \(i(t)\). In steady state these waveforms are periodic, so that the phasors \(V_{k,l}\) and \(I_{k,l}\) are independent of the location of the integration interval in (1.5) and (1.6).

In this terminology the rms value of the polyphase vector \(x\) is expressed as \(\|x\| = \sqrt{<x, x>}\), and the average real power delivered to the load is \(P = <i, v>\). Therefore from the Cauchy-Schwartz inequality

\[P \leq \|i\|\|v\| = S \]  

(1.7)

This shows that the apparent power \(S\) is the highest average power delivered to the load, among all loads that have the same rms current (we assume that the voltage across the load does not change). For a non-ideal load the mismatch between \(S\) and \(P \ (P < S)\) is the target of compensation devices(Fig: 1.2). The ideal (Fryze)
compensator achieves \( i_s(t) = i_F(t) \) where \( i_F(t) \) is defined as

\[
i_F(t) \overset{\text{def}}{=} \frac{\langle i, v \rangle}{\|v\|} v(t)
\]  

so that \( P = \|i_F\|\|v\| \). Based on the 7-component approach, the current difference of \( i(t) - i_F(t) \) can be decomposed into mutually orthogonal components with clear physical meanings and the decomposition is as follows ([4], [5]):

\[
i = i_F \oplus i_{gs} \oplus i_{gu} \oplus i_B \oplus i_{bs} \oplus i_{bu} \oplus i_{\perp}
\]  

where the symbol \( \oplus \) denotes the sum of mutually orthogonal components. Each component has a direct physical meaning, which can be conveniently described in terms of the equivalent linear load admittances [9]

\[
Y_{k,l} \overset{\text{def}}{=} \frac{I_{k,l}}{V_{k,l}} = g_{k,l} - jb_{k,l}
\]  

The orthogonality among various current components implies the possibility of defining powers by calculating squares of the norm of currents and multiplying with the square of the voltage norm throughout [4], resulting in a time-variant version of (2.1), namely,

\[
S^2 = P^2 + N_s^2 + N_a^2 + Q_B^2 + Q_s^2 + Q_a^2 + S_{\perp}^2
\]  

Figure 1.2: Load Compensation
The various power components in (1.11) can be expressed in terms of (weighted) means and variances of the two parameter sequences \( \{ g_{k,l} \} \) and \( \{ b_{k,l} \} \), as shown in Tables 1.1 - 1.3.

<table>
<thead>
<tr>
<th>( |V| )</th>
<th>( |I| )</th>
<th>( \mu_g )</th>
<th>( \mu_b )</th>
<th>( \mu_g(l) )</th>
<th>( \mu_b(l) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{\sum_{k,l}</td>
<td>V_{k,l}</td>
<td>^2} )</td>
<td>( \sqrt{\sum_{k,l}</td>
<td>I_{k,l}</td>
<td>^2} )</td>
</tr>
</tbody>
</table>

Table 1.1: Expressions for \( \mu \) Variables

Here \( \mu_g \) denotes the weighted mean of \( \{ g_{k,l} \} \) and \( \mu_b \) is the weighted mean of \( \{ b_{k,l} \} \). The various \( \sigma \) variables are standard deviations: \( \sigma_g \) corresponds to the entire double-indexed sequence \( \{ g_{k,l} \} \), and similarly for \( \sigma_b \). On the other hand \( \sigma_{gu} \) is the standard deviation of the subsequence \( \{ g_{k,l} : l \text{ fixed}, k = 1, 2, \ldots, n \} \), subsequently averaged over all harmonics.
Now we can express all the power components from equation (1.11) in terms of the parameters defined in the above tables.

Table 1.2: Expressions for $\sigma$ Variables

<table>
<thead>
<tr>
<th>$\sigma_g$</th>
<th>$\sigma_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{\frac{\sum_{k,l}(g_{k,l} - \mu_g)^2</td>
<td>V_{k,l}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\sigma_{gu}$</th>
<th>$\sigma_{bu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{\frac{\sum_{k,l}</td>
<td>g_{k,l} - \mu_g(l)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\sigma_{gs}$</th>
<th>$\sigma_{bs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{\sigma_g^2 - \sigma_{gu}^2}$</td>
<td>$\sqrt{\sigma_b^2 - \sigma_{bu}^2}$</td>
</tr>
</tbody>
</table>

Table 1.3: Power Components

<table>
<thead>
<tr>
<th>$S$</th>
<th>$P$</th>
<th>$N_s$</th>
<th>$N_u$</th>
<th>$Q_B$</th>
<th>$Q_s$</th>
<th>$Q_u$</th>
<th>$S_{\perp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$|v||i|$</td>
<td>$\mu_g |v|^2$</td>
<td>$\sigma_{gs} |v|^2$</td>
<td>$\sigma_{gu} |v|^2$</td>
<td>$\mu_b |v|^2$</td>
<td>$\sigma_{bs} |v|^2$</td>
<td>$\sigma_{bu} |v|^2$</td>
<td>$|i_{\perp}| |v|$</td>
</tr>
</tbody>
</table>

Here

$$\|i_{\perp}\| = \sqrt{\sum_{k,l \in B} \|I_{k,l}\|^2}$$

(1.12)

where $B = \{(k, l), V_{k,l} = 0\}$ is the set of all $(k, l)$-pairs with vanishing voltage phasors.
It is evident from these tables that the only signed power components are $P$, which depends on $\mu_g$, and $Q_B$, which depends on $\mu_b$. These two components can also be expressed directly in terms of the voltage and current phasors, namely

$$
P = \sum_{k,l} \text{Re}\{V_{k,l}I_{k,l}^*\}$$

$$
Q_B = \sum_{k,l} \text{Im}\{V_{k,l}I_{k,l}^*\}$$

(1.13)

The components $N_u$ and $Q_u$ are a result of phase imbalance. In order to obtain further insight into the cause of imbalance, we introduce here three distinct imbalance indices (Table 1.4).

<table>
<thead>
<tr>
<th>$\nu_V$</th>
<th>$\nu_I$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\text{Re}{\sum_l V_l V_l^T}}{\sum_l</td>
<td></td>
<td>V_l</td>
</tr>
</tbody>
</table>

Table 1.4: Imbalance Indices

Here $\nu_V$, $\nu_I$, and $\zeta$ are the imbalance indices for voltage, current and load, respectively. $V_l$ and $I_l$ are the polyphase (row vector) voltage and current phasors associated with the $l$-th harmonic. The superscript $T$ denotes transposition, while the superscript $^H$ denotes a complex conjugate (i.e., Hermitian) transpose.
1.2 Local Hilbert Space and Dynamic Phasors

We now turn to introduce an explicitly time-variant Hilbert space framework for real-valued polyphase signals with finite local power [9], viz.,

\[ \frac{1}{T} \int_{t-T}^{t} \|x(s)\|^2 ds < \infty, \text{ for all } t \] (1.14)

For a fixed “t” our space consists of finite signal segments, and we define the (time-variant) inner product as

\[ \langle x, y \rangle (t) \overset{\text{def}}{=} \frac{1}{T} \int_{t-T}^{t} x(s)y^\top(s) ds \] (1.15)

With this definition, \( P(t) = \langle i, v \rangle (t) \), and \( Q_B(t) = \langle i, Hv \rangle (t) \), for instance. The actual numerical calculation of \( P \) and \( Q_B \) or, more generally, of all the components in our 7/11-component decomposition of apparent power, is more convenient to carry out in terms of a time-variant Fourier series representation (see, e.g., [11]).

Our time-variant Fourier series representation is based on the concept of dynamic phasors. This means we associate a Fourier series representation with the finite polyphase signal segment \( \{x(s); \ t - T < s \leq t\} \) where we consider “t” as a parameter, viz.,

\[ x(s) = \sum_{k=1}^{n} \sum_{\ell=-\infty}^{\infty} \left[X_{k,\ell}(t)e^{j\ell\omega s}\right] e_k, \quad t - T < s \leq t \] (1.16a)

\[ X_{k,\ell}(t) = \frac{1}{T} \int_{t-T}^{t} \left[x(s)e_k^\top\right] e^{-j\ell\omega s} ds \] (1.16b)

where \( T \) is the duration of a cycle (or the period in a steady state). The harmonic coefficient \( X_{k,\ell}(t) \) is a function of the parameter \( t \) except when \( x(\cdot) \) is a periodic signal with period \( T \): in this case \( X_{k,\ell}(t) \) is independent of \( t \), and coincides with the standard Fourier coefficient. Notice that since \( x(\cdot) \) is real valued, its Fourier coefficients must be conjugate-symmetric, i.e., \( X_{k,-\ell}(t) = X_{k,\ell}^*(t) \).

The Parseval identity for the Fourier series (1.16a) and (1.16b) can be written in the form

\[ \langle x, y \rangle (t) = \sum_{k=1}^{n} \sum_{\ell=-\infty}^{\infty} X_{k,\ell}(t)Y_{k,\ell}^*(t) \] (1.17)
where we used our definition (1.15). This means that all inner products can be evaluated via appropriate dynamic phasors [9].

The 7-component decomposition (1.11) can now be evaluated on the window \((t - T, t]\) using the notion of a local Hilbert space and the associated dynamic phasors [9]. A detailed discussion is provided in chapter 2.

1.3 Akagi-Nabae Instantaneous Power Components

The Instantaneous Reactive Power (IRP) Theory, developed by Akagi, Kanazawa and Nabae [13], provides mathematical fundamentals for the control of switching compensators, known commonly as active power filters. When the IRP Theory is considered as a theoretical foundation for control algorithm design, it is irrelevant whether it interprets power properties of electrical circuits correctly or not. It is enough that it enables the achievement of the control objectives. However, when it is considered as a power theory one could expect that it does provide a credible interpretation of power phenomena in electrical systems. Having this expectation in mind, the following dilemma occurs. Power properties of three-phase, three-wire systems with only sinusoidal voltages and currents, i.e., even without any harmonic distortion, are determined by three independent features of the system,

- Permanent energy transmission and associated active power \(P\)
- Presence of reactive elements in the load and associated reactive power \(Q\)
- Load imbalance that causes supply current asymmetry and associated unbalanced power \(D\) (in the Czarnecki decomposition)

Thus, how can the IRP Theory, based on only two power quantities (\(p\) and \(q\)), identify and describe three independent power properties? Moreover, according to Akagi and Nabae who developed the Instantaneous Reactive Power \(p - q\) Theory, its development was a response to the demand to instantaneously compensate the reactive power. The shortcomings of the instantaneous compensation approach were
detailed in [12]. In this thesis we focus on the inadequacy of the IRP theory for monitoring and interpreting power quality (see section 2.4).

Akagi’s approach for characterizing power components [13] relies on the instantaneous polyphase voltage and current row vectors $v(t)$ and $i(t)$ as defined in (1.3), from which one can construct the instantaneous real power $p(t) = i(t)v(t)^\top$ and the instantaneous (unsigned) apparent power

$$s(t) \overset{\text{def}}{=} \sqrt{[v(t)v(t)^\top][i(t)i(t)^\top]} \quad (1.18)$$

The gap $s^2(t) - p^2(t)$ can be described in a general three-phase four-wire system, in terms of three power components [10], [14]. However in the special case when both the zero-sequence current and zero-sequence voltage vanish, i.e., when

$$[1 \ 1 \ 1]^T v(t)^\top = [1 \ 1 \ 1]^T i(t)^\top = 0 \quad (1.19)$$

the gap $s^2(t) - p^2(t)$ can be described in terms of a single power quantity known as the Akagi-Nabae instantaneous reactive power $q(t)$ [13]. This happens, for instance in a three-phase, three-conductor system (without a neutral wire) and using the virtual star point as a voltage reference.

The constraint (1.19) implies that the three-element vectors $v(t)$ and $i(t)$ occupy, in fact a two-dimensional subspace of $R^3$. One convenient way of showing this explicitly is via the Park transform

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \frac{\sqrt{2/3}}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3/2} & -\sqrt{3/2} \end{bmatrix} \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} \quad (1.20)$$

$$\begin{bmatrix} i_\alpha(t) \\ i_\beta(t) \end{bmatrix} = \frac{\sqrt{2/3}}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3/2} & -\sqrt{3/2} \end{bmatrix} \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} \quad (1.21)$$

where $v$ and $i$ denote voltages and currents, and $a$, $b$ and $c$ denote the three phases [15]. The Park transform preserves the Euclidean norm and the inner product,
so that
\[ v_\alpha^2(t) + v_\beta^2(t) = v_a^2(t) + v_b^2(t) + v_c^2(t) \] (1.22)
\[ i_\alpha^2(t) + i_\beta^2(t) = i_a^2(t) + i_b^2(t) + i_c^2(t) \]
and
\[ p(t) = v_\alpha(t)i_\alpha(t) + v_\beta(t)i_\beta(t) = v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t) \] (1.23)
Consequently
\[ s^2(t) = [v_\alpha^2(t) + v_\beta^2(t)][i_\alpha^2(t) + i_\beta^2(t)] \] (1.24)
and it follows that
\[ s^2(t) - p^2(t) = q^2(t) \] (1.25)
where
\[ q(t) = v_\alpha(t)i_\beta(t) - v_\beta(t)i_\alpha(t) \] (1.26)
Instantaneous compensation without energy storage, as proposed by Akagi and Nabae [10], aims to reduce \( q(t) \) to zero so that \( s(t) = p(t) \). The relation between this type of compensation and the better known Fryze compensation has been explored in detail in [12].

The original Akagi-Nabae components were restricted to the case of three-phase three-wire systems (the single-phase case cannot be derived from it as a special case). Indeed only in that case the instantaneous situation can be characterized by two quantities, the \( \alpha \) and \( \beta \) components, since the three-dimensional currents and voltages are actually two-dimensional. However in four-wire three-phase systems the four voltage and current components should be characterized as three-dimensional quantities. The Akagi-Nabae components are not sufficient to achieve this [16]. Extension of the Akagi approach to systems with more than three phases is not straightforward either.
1.4 Summary of Contributions

We have shown the static (steady state) $7/11$ – component decomposition of apparent power that was introduced in [4], [5] to the transient case. This was achieved by using the notion of dynamic Fourier phasors, defined on a sliding window [6], [7], and the associated local Hilbert space [11], [19]. The resulting dynamic $7/11$ – component decomposition was applied to an industrial (paper mill) example that exhibits significant transients during a voltage sag incident.

The time – evolution of the various dynamic power components provided a clear indication for the onset and duration of the transient and for the unbalanced nature of the fault. The epoch of fault onset was clearly visible in practically all components (except $P_0$, $Q_{B,0}$ and $S_\perp$). It was particularly noticeable in the plots for $N_u(t)$, $Q_u(t)$, $P_\perp(t)$ and $Q_{B,\perp}(t)$, which is consistent with the unbalanced nature of the fault in this incident.

The same example also demonstrated the weakness of the Akagi – Nabae “instantaneous reactive power” metric. Although $q_{AN}(t)$ does provide some indication of the transient onset (Fig 2.13), it is much “noisier” and harder to interpret as compared with our dynamic $7/11$ – decomposition. The following figures are examples of the $7/11$ – decomposition analysis on an industrial data.
Figure 1.3: Seven-Component Decomposition

Figure 1.4: Zero, Positive and Negative Sequences of $P$ and $Q_B$
Chapter 2

Dynamic Power Analysis

In this chapter, we combine the static (= steady-state) 7-component decomposition with the notion of dynamic phasors to obtain a dynamic (= time-variant) decomposition of apparent power viz.,

\[ S^2(t) = P^2(t) + N^2_s(t) + N^2_u(t) + Q^2_B(t) + Q^2_s(t) + Q^2_u(t) + S^2_{\perp}(t) \] (2.1)

Each component is evaluated using the same expressions (Tables 1.1 - 1.3) as on the static case, except that the voltage and current phasors we use are evaluated on a sliding window and thus may be time-varying in the presence of transients. As in the static case, the components \( N_u(t) \) and \( Q_u(t) \) are associated with unbalanced current flow, while the components \( N_s(t) \) and \( Q_s(t) \) are associated with spread over harmonics.

The key concept of a local Hilbert space (see section 1.2) makes it possible to extend the original derivation of the static 7-component decomposition [12] to the more general, non periodic case. Since \( V_{k,l}(t) \) and \( I_{k,l}(t) \) are now evaluated on the sliding window \( (t - T \ t] \), the resulting dynamic decomposition (2.1) can be used to indicate the onset of transients and to characterize their nature as we demonstrate in section 2.3.

In addition, we exploit the (dynamic) inner-product interpretations \( P(t) = \langle i, v \rangle (t) \)
and $Q_B(t) = \langle i, Hv \rangle(t)$ to further decompose these two components, viz.,

$$P(t) = P_+(t) + P_-(t) + P_0(t)$$

$$Q_B(t) = Q_{B,+}(t) + Q_{B,-}(t) + Q_{B,0}(t)$$

where $P_+(t)$ (resp. $Q_{B,+}(t)$) denotes the contribution of the positive sequence component, $P_-(t)$ (resp. $Q_{B,-}(t)$) is the contribution from the negative sequence component and $P_0(t)$ (resp. $Q_{B,0}(t)$) is associated with the zero-sequence component. Since the negative sequence components are negligible under normal operating conditions (balanced, steady-state) we can use these components – along with $N_u(t)$ and $Q_u(t)$ – as indications for imbalance. In particular $P_-(t)$ and $Q_{B,-}(t)$ provide valuable information about the onset and nature of unbalanced faults (see section 2.1 for details).

In this chapter, the useful tool of 7/11-element decomposition is used on several examples (where the systems are unbalanced and the waveforms are nonsinusoidal) including an industrial one, in order to obtain tangible insights about the systems’ behavior when the faults occur. The 11 components of $S(t)$ are plotted for different cases where they provide useful information about the transient behavior (balanced, unbalanced, linear, nonlinear) of the systems. The corresponding results are shown in sections 2.2 and 2.3.

## 2.1 A 7/11-Component Dynamic Power Decomposition

The notion of a local Hilbert space that we described in section 1.1 allows us to replace classical (steady state) Fourier phasors by dynamic phasors in the expressions (Tables 1.1-1.3) used to evaluate the various components of the static decomposition. This results in the dynamic decomposition (2.1) that can be applied both in steady state and during power transients, with any number of phases.

In three-phase systems we can rely on the inner-product interpretation of $P(t)$ and $Q_B(t)$ (see, e.g.,(1.14)) to split each one of these components into its respective
symmetric sequence sub-components [9]. This means we can use the decomposition of the polyphase current $i(t)$ into its sequence components, viz,

$$i(t) = i_+(t) + i_-(t) + i_0(t)$$  \hspace{1cm} (2.3)

so that

$$P(t) = \langle i_+, v \rangle (t) + \langle i_-, v \rangle (t) + \langle i_0, v \rangle (t)$$  \hspace{1cm} (2.4)

and similarly for $Q_B(t)$ . This results in the additive decompositions (2.2b). Symmetrical power components are a standard way to describe unbalanced operation and faults in three-phase systems. The theory of symmetrical coordinates or components was developed as the result of an analytical study of the performance of phase converters and polyphase machines operating on unbalanced systems. Practically all system faults are unbalanced. As a consequence, without this theory the modern system of relays could not be realized.

The most compact expressions for the symmetric sequence components of $P(t)$ and $Q_B(t)$ are

$$P_+(t) = \sum_l \text{Re}\{V_l^+(t)[I_l^+(t)]^*\}$$

$$P_-(t) = \sum_l \text{Re}\{V_l^-(t)[I_l^-(t)]^*\}$$  \hspace{1cm} (2.5)

$$P_0(t) = \sum_l \text{Re}\{V_l^0(t)[I_l^0(t)]^*\}$$

and

$$Q_{B,+}(t) = \sum_l \text{Im}\{V_l^+(t)[I_l^+(t)]^*\}$$

$$Q_{B,-}(t) = \sum_l \text{Im}\{V_l^-(t)[I_l^-(t)]^*\}$$  \hspace{1cm} (2.6)

$$Q_{B,0}(t) = \sum_l \text{Im}\{V_l^0(t)[I_l^0(t)]^*\}$$

Here \{\[V_l^+(t), V_l^-(t), V_l^0(t)\], \{I_l^+(t), I_l^-(t), I_l^0(t)\}\}, are the positive, negative and zero sequence components of the $l$-th harmonic dynamic phasor triplet of voltage and
current. The triplet sets are obtained via the transformation

\[
\begin{bmatrix}
I_0(t) \\
I_+^t(t) \\
I_-^t(t)
\end{bmatrix} = \frac{1}{\sqrt{3}}
\begin{bmatrix}
1 & 1 & 1 \\
1 & \alpha & \alpha^2 \\
1 & \alpha^2 & \alpha
\end{bmatrix}
\begin{bmatrix}
I_{a,l}(t) \\
I_{b,l}(t) \\
I_{c,l}(t)
\end{bmatrix}
\]

(2.7)

where \( \alpha = e^{j2\pi/3} = 1/120^\circ \). \( V_+^t(t), V_-^t(t), \) and \( V_0^t(t) \) are defined in a similar way.

The splitting of \( P(t) \) and \( Q_B(t) \) results in a “two-level” 7/11-component decomposition, viz.,

\[
S^2 = (P_+ + P_- + P_0)^2 + N_s^2 + N_u^2 + (Q_{B,+} + Q_{B,-} + Q_{B,0})^2 + Q_s^2 + Q_u^2 + S_\perp^2
\]

(2.8)

where all quantities are time-variant.

### 2.2 Steady State Analysis

In steady state operation all waveforms are periodic and can be represented by static Fourier coefficients. Since the location of the window \( t - Tt \) is immaterial for a periodic waveform, the Fourier dynamic phasors (1.16b) become time-invariant and coincide with the classical Fourier coefficients. Consequently, our dynamic power decomposition coincides in steady-state with the static 7-component decomposition of [4] and [5]. We provide here two simple examples to illustrate the utility of the (static) 7-component decomposition and the meaning of its individual apparent power components.

**Example 1 : Unbalanced RLC Load**

This example can also be found in [21]. In this example, we have a 3-phase system with an unbalanced load which consists of a resistor, a capacitor and an inductor (see Figure 2.1). The voltage and current phasors for each of the 3 phases are

\[
\begin{align*}
V_R &= 1/0^\circ \text{ V} & V_S &= 1/120^\circ \text{ V} & V_T &= 1/120^\circ \text{ V} \\
I_R &= 1/0^\circ \text{ A} & I_S &= 1/30^\circ \text{ A} & I_T &= 1/30^\circ \text{ A}
\end{align*}
\]
Figure 2.1: Balanced Supply Voltage and Unbalanced Y Connected Load

Notice that the supply voltage is balanced, but the load current is not. This is also evident from the (relative) values of the equivalent conductances \( \{g_k\} \) and susceptances \( \{b_k\} \). Since only the fundamental harmonic is present, these parameters have a single index, so that \( k = R, S \) or \( T \), viz.,

\[
\begin{align*}
g_R &= 1 & g_S &= 0 & g_T &= 0 \\
b_R &= 0 & b_S &= -1 & b_T &= 1
\end{align*}
\]

Notice that both \( \{g_k\} \) and \( \{b_k\} \) are spread unevenly among the three phases, so we should expect significant contributions to the apparent power \( S \) from the unbalanced components \( N_u \) and \( Q_u \). Also since there are no harmonics other than the fundamental, we should expect that \( N_s = Q_s = 0 \) as well as \( S_\perp = 0 \). This is indeed
Table 2.1: Power Components in Example 1

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>Ns</th>
<th>Nu</th>
<th>QB</th>
<th>Qs</th>
<th>Qu</th>
<th>S⊥</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.00</td>
<td>1.00</td>
<td>0</td>
<td>1.41</td>
<td>0</td>
<td>0</td>
<td>2.45</td>
<td>0</td>
</tr>
</tbody>
</table>

the case, as shown in Table 2.1. Thus in this example,

\[ S^2 = P^2 + N_u^2 + Q_u^2 \]

and we notice that \( P^2 \) contributes only \( 1/9 \) of \( S^2 \). Notice that \( Q_u^2/N_u^2 = 3 \). This is so because the statistical variance of the \( \{b_k\} \) sequence (\( \sigma_b^2 = 2/3 \)) is three times bigger than the variance of the \( \{g_k\} \) sequence (\( \sigma_g^2 = 2/9 \)). Also notice that the Budeanu reactive power \( Q_B \) vanishes, because the power flows into the capacitor and inductor branches cancel each other, viz,

\[ Q_B = Im \{ V_R I_R^* + V_S I_S^* + V_T I_T^* \} = Im \{ 1 + (-j) + j \} = 0 \]

In Table 2.2, the positive, negative, and zero sequence values of \( P \) and \( Q_B \) are shown. Notice that since the supply voltage is balanced, both \( V^- \) and \( V^0 \) vanish, so that

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
P_+ & P_- & P_0 & Q_{B,+} & Q_{B,-} & Q_{B,0} \\
\hline
1.00 & 0 & 0 & 0 & 0 & 0 \\
\hline
\end{array}
\]

Table 2.2: Sequence Components of \( P \) and \( Q_B \) in Example 1

\( P = P_+ \) and \( Q_B = Q_{B,+} (= 0) \). Thus non-zero \( P_- \) and/or \( Q_{B,-} \) occur only when both
the voltage and the current are unbalanced.

Example 2: Unbalanced Resistive Load

![Circuit Diagram]

Figure 2.2: Circuit With Unbalanced Resistive Load

This example, which is also adopted from [21], involves a balanced sinusoidal source with non-negligible source impedance feeding a resistive load connected between phases “R” and “S”. Our calculations address only the power components associated with the load (and not with the source impedance). Again, since only the fundamental harmonic is present, the equivalent conductances \( \{g_k\} \) and susceptances
\{b_k\} have a single index, and \( N_s = Q_s = 0 \), as well as \( S_\perp = 0 \). The values of all seven components are listed in Table 2.3.

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>( P )</td>
<td>( N_s )</td>
<td>( N_u )</td>
<td>( Q_B )</td>
<td>( Q_s )</td>
<td>( Q_u )</td>
</tr>
<tr>
<td>149.64</td>
<td>100.54</td>
<td>0</td>
<td>78.502</td>
<td>22.23</td>
<td>0</td>
<td>75.003</td>
</tr>
</tbody>
</table>

Table 2.3: Power Components (in kVA) in Example 2

Now,

\[
g_R = 1.3676 \quad g_s = 1.0579 \quad g_T = 0 \\
b_R = -0.4320 \quad b_S = 0.9683 \quad b_T = 0
\]

and we can see the large spread (= load imbalance) between phases. This translates into relatively high values for \( N_u \) and \( Q_u \). Notice that in this example the resistive load is connected between two phases \( R \) and \( S \) (instead of a phase and the ground). This can set the phase currents to be completely out of phase with the corresponding phase voltages, making the reactive part of the load capacitive. That is why we have a significant contribution to \( S \) from the \( Q_B \) term, viz.,

\[
Q_B = \text{Im}\{V_R I_R^* + V_S I_S^* + V_T I_T^*\} \\
= \text{Im}\{(5.6689 \times 10^4 - j1.7909 \times 10^4) + (4.3855 \times 10^4 + j4.0140 \times 10^4) + 0\} \\
= 22.231kVAR
\]

The positive, negative, and zero-sequence values of \( P \) and \( Q_B \) are listed in Table 2.4,
Table 2.4: Sequence Components of $P$ and $Q_B$ in Example 2

<table>
<thead>
<tr>
<th>$P_+$</th>
<th>$P_-$</th>
<th>$P_0$</th>
<th>$Q_{B,+}$</th>
<th>$Q_{B,-}$</th>
<th>$Q_{B,0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.032 \times 10^5$</td>
<td>$-2699.4209$</td>
<td>0</td>
<td>$2.28 \times 10^4$</td>
<td>$-597.12$</td>
<td>0</td>
</tr>
</tbody>
</table>

As in the previous example, the main indicators of imbalance are $N_u$ and $Q_u$. However, because the load voltage is slightly unbalanced, we obtain non-zero values (albeit relatively small) for the negative sequence components of $P$ and $Q_B$. 
2.3 Dynamic Analysis of an Industrial Example

Figure 2.3: Portion of Single-Line Diagram of the MWPI Power System
We now turn to analyze power flow during a fault (voltage sag) using data collected from a large scale paper mill. Load voltage and current information was collected from a one year power-quality study (2004 – 2005) in the Mazandaran Wood and Paper Industries (MWPI) which is the largest paper manufacturer in Iran with a production capacity of 175,000 tons of paper per year, including 90,000 tons of newsprint, printing, and writing paper and 85,000 tons of fluting paper [17]. The total power consumed by plant is 35 MW, consisting of the following sections:

- water treatment (3,000 kW) nominal
- wood-handling plant (6,821 kW)
- pulp plant (25,000 kW)
- chemical recovery plant (4,907 kW)
- steam production plant (3,887 kW)
- effluent treatment (2,500 kW)
- first paper machine or PM1 (22,000 kW)
- second paper machine or PM2 (12,200 kW)
- finishing (3,500 kW)

A portion of a single-line diagram is shown in Figure 2.3. The power distribution system of MWPI consists of 43 transformers as follows:

1. 36 at 2 MVA, 20 kV/400 V
2. 5 at 5 MVA, 20 kV/6.6 kV
3. 2 at 20 MVA, 20 kV/6.6 kV

The 20-kV incoming line is fed by a 90-MVA, 230-kV/20-kV network transformer. Some power-quality-related disturbances and problems have been reported in MWPI over the past few years. From the power-quality point of view, the paper machine section is the most important part because of its continuous process. Most of the variable-speed drives of the plant are used in the paper machine section and work continuously with a multi drive control strategy. AC and dc drives are very sensitive
to voltage sags because of the power electronic switches. There are 18 dc drives for PM2 (paper machine for producing 113- and 127-g fluting paper) and 22 dc drives and 16 vector control ac drives for PM1 (paper machine for producing 48-g newsprint paper). Nominal linear speed of PM1 is 1,000 m/min and PM2 is 600 m/min. Each dc drive has a microcontroller processor board, and its speed is regulated with static accuracy lower than 0.01 percent and dynamic accuracy lower than 0.1 percent with pulse transducer (1,024 pulse/cycle). The multidrive system control is performed by a central computer (Masterpiece). The cost of one minute of lost production is about 240 US dollars for MWPI. The cost increases if there is board or instrument damage. Therefore, every production stoppage due to the voltage sag will cost about 18,000 US dollars for the plant. The cost increases if there is board or instrument damage [17].

Two power analyzers were used to monitor power-quality parameters: one of them was installed on the 20 – kV incoming feeder of the plant (point A in Figure 2.3), and the other was portable and installed at point B in Figure 2.3. The sampling rate of the analyzer was about 7,000 samples/s (140 samples/cycle). It sampled all three voltages and three currents of the system. The voltages and currents of point A during the voltage sag are shown in Figures 2.4 and 2.5. These figures show ten cycles before and ten cycles after the fault that were saved in the analyzer for further analysis.

The voltage sag is evident in all three voltage waveforms, but especially in phase A. The current waveforms display significant harmonic distortion in steady state, as well as a noticeable transient during the voltage sag. The dynamic 7-component decomposition of this system is shown in Figure 2.6 and 2.7.

A close-up view of the seven components is shown in Figure 2.7. These two figures simply present Figure 2.6 in a two four-window format as shown in the previous plots. Significant values of $N_s$ and $Q_s$ (as can be seen in Figure 2.7) are due to the noticeable current distortion in steady state. Also although $N_s$ and $Q_s$ experience a transient, the variation is not as large as seen in the other components. This is due to the fact that the fault causes a significant increase in the load imbalance. Indeed, we observe
that both \( N_u \) and \( Q_u \) increase by (approximately) a factor of 4 during the transient.

Decomposing \( P \) and \( Q_B \) elements into zero-, positive- and negative- sequence components produces the plots shown in Figures 2.9 – 2.11. As can be seen in these figures, the positive-sequence elements of \( P \) and \( Q_B \) are the dominant ones (in steady state) among the three sequences which means that there is almost no voltage (or perhaps current) imbalance. The indices of imbalance for voltage, current, and load are calculated and presented in the Figure 2.8. The fact that the values of these indices are around zero in the steady state confirms that the voltage and current waveforms are almost balanced in the steady state.

The negative-sequence exists although it has a small value in both cases where it is a good indication of the unbalanced nature of the fault. Although \( P_- \) and \( Q_B_- \) are relatively small (even in transients), they experience a huge relative change with the onset of the transient. This change is similar, but even more noticeable, than the change in \( N_u \) and \( Q_u \) which are the main indicators of imbalance. However, because the load voltage is slightly unbalanced, we obtain small non-zero values (in comparison to the positive-sequence values) for the negative sequence components of \( P \) and \( Q_B_- \). Finally the zero-sequence components of \( P \) and \( Q_B \) are almost zero: this can be due to the configuration (in a 3-wire format) of the system.
Figure 2.4: Phase Voltages

Figure 2.5: Phase Currents
Figure 2.6: Dynamic Seven-Component decomposition
Figure 2.7: Dynamic 7-Component decomposition
Figure 2.8: Indices of Imbalance

Figure 2.9: Zero, Positive and Negative Sequences of $P$ and $Q_B$
Figure 2.10: Zero, Positive and Negative Sequences of $P$

Figure 2.11: Zero, Positive and Negative Sequences of $Q_B$
2.4 Comparison with Akagi-Nabae Decomposition

The Akagi-Nabae approach was developed for 3-phase systems in which the zero-sequence voltage and current components are guaranteed to vanish, as discussed in section 1.3. When this condition is not satisfied one needs three distinct “instantaneous reactive power” quantities fully characterize the gap $s^2(t) - p^2(t)$ [14]. Thus in every three-phase system

$$s^2(t) - p^2(t) = q_{\alpha,\beta}^2(t) + q_{\alpha,0}^2(t) + q_{\beta,0}^2(t) \quad (2.9a)$$

where

$$q_{\alpha,\beta}(t) \overset{\text{def}}{=} v_\alpha(t)i_\beta(t) - v_\beta(t)i_\alpha(t)$$
$$q_{\alpha,0}(t) \overset{\text{def}}{=} i_\alpha(t)v_0(t) - v_\alpha(t)i_0(t)$$
$$q_{\beta,0}(t) \overset{\text{def}}{=} i_\beta(t)v_0(t) - v_\beta(t)i_0(t) \quad (2.9b)$$

The voltage and current waveforms used to determine these power components are obtained via the Park transform, viz,

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \\ v_0(t) \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -\sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix}$$

$$\begin{bmatrix} i_\alpha(t) \\ i_\beta(t) \\ i_0(t) \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -\sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix}$$

We apply these definitions to our industrial example (see sec. 2.3) and plot the various instantaneous power quantities in Figures 2.12 and 2.13 (the corresponding phase voltages and currents were shown in Figures 2.4 and 2.5).

Figure 2.12 shows that although $q_{\alpha,0}(t)$ and $q_{\beta,0}(t)$ are not exactly zero, they are negligible with respect to $q_{\alpha,\beta}(t)$, which is the Akagi-Nabae instantaneous reactive
power. Thus in this example, the gap \( s^2(t) - p^2(t) \approx q_{\alpha\beta}^2(t) \), and we can ignore \( q_{\alpha,0}(t) \), \( q_{\beta,0}(t) \) for the purpose of our discussion. In addition, notice also that the transient has almost no effect on these two power quantities.

An examination of Figure 2.13 shows that the transient is noticeable in all three waveforms: \( s(t) \), \( p(t) \) and \( q_{AN}(t) \equiv q_{\alpha\beta}(t) \). However, the duration of the transient is not easily discernible from \( q_{AN} \), and we get no information about the nature of the fault that has caused the transient.

Figure 2.14 shows the average Akagi power and Figure 2.15 is a two-window plot of Akagi reactive power where in the top window the average Akagi power and in the bottom window, a zoomed part of instantaneous Akagi reactive power is shown. As it can be seen from the graphs, the average Akagi plot seems to be a more useful tool in observing the transient behavior in comparison to the plots of instantaneous Akagi power elements.

From the presented plots, it can be concluded that the 7/11 approach is a better indicator of the onset of the transient in comparison to the Akagi-Nabae approach. Another factor is that unlike Akagi-Nabae, the 7/11 approach is a well-refined one. It contains \( N_u \) and \( Q_u \) that are associated with the unbalanced current flow as well as \( N_s \) and \( Q_s \) that are the representations of nonlinearity. Therefore this can also be said that the 7/11 approach provides more information about the imbalance and nonlinearity of the fault that has caused the transient. In addition the 7/11 approach can be easily used for any number of phases where the Akagi-Nabae approach cannot be used for a single-phase system and cannot go further when the number of phases are greater than three.
Figure 2.12: \( q_{\alpha,\beta}(t), q_{\alpha,0}(t), q_{\beta,0}(t) \)

Figure 2.13: Instantaneous \( s(t), p(t), q_{AN}(t) \)
Figure 2.14: Window–Averaged $q_{AN}(t)$
Figure 2.15: Average and Instantaneous Akagi Reactive Power
Figure 2.16: Akagi Result from $Q_A = Q_p - Q_n$
Chapter 3

Concluding Remarks

3.1 Summary of Results

We have adopted the static (steady state) 7/11-component decomposition of apparent power that introduced in [4], [5] to the transient case. This was achieved by using the notion of dynamic Fourier phasors, defined on a sliding window [6], [7], and the associated local Hilbert space [11], [19]. The resulting dynamic 7/11-component decomposition was applied to an industrial (paper mill) example that exhibits significant transients during a voltage sag incident.

The time-evolution of the various dynamic power components provided a clear indication for:

• the onset and duration of the transient, and
• the unbalanced nature of the fault

The epoch of fault onset was clearly visible in practically all components (except $P_0$, $Q_{B,0}$ and $S_\perp$). It was particularly noticeable in the plots for $N_u(t)$, $Q_u(t)$, $P_-(t)$ and $Q_{B,-}(t)$, which is consistent with the unbalanced nature of the fault in this incident.

The same example also demonstrated the weakness of the Akagi–Nabae “instantaneous reactive power” metric. Although $q_{AN}(t)$ does provide some indication of
the transient onset (recall Fig 2.13), it is much “noisier” and harder to interpret as compared with our dynamic 7/11 – decomposition.

3.2 Future Research

Our analysis of the industrial example identified four power components – $N_u(t)$, $Q_u(t)$, $P_-$ and $Q_{B,-}(t)$ – are good indications of the onset of unbalanced faults. Future research should aim to distinguish between different types of unbalanced faults by classifying their dynamic power component (DPC) signatures.

More generally, the dynamic 7/11 – component decomposition should provide detailed information about the numerous types of power quality events – voltage magnitude and frequency variation, fluctuations, imbalance, distortion – allowing fast and reliable detection and identification of such events. examples of industrial records of different types of power quality events need to be analyzed with the objective of correlating event types with specific DPC signatures.
Bibliography


[21] M.E. Ragnarsdottir, “Java Program that Calculates the Decomposition of Load Current (and consequently the apparent power) into Seven Orthogonal components,” MS Project report, Department of Electrical and Computer Engineering, Northeastern University, 2008.

Appendix A: Matlab code for computing the 7 components

function Reactive_Power_Transient_Papersa0v1under

close all; clc;
tic;           \%shows the time of the simulation
Window_size=140;

\% 140 was our speculated number of points in one period. This number
\% matches with the sampling rate indicated in the "power of the paper"
\% industrial paper. Frequency is taken as 50 Hz

M=xlsread('sa0v1under.xls');
Voltage1=M(:,1); Voltage2=M(:,2); Voltage3=M(:,3);
Current1=M(:,4); Current2=M(:,5); Current3=M(:,6);

for i=1:(length(Voltage1)-Window_size+1)
    V_temp1=Voltage1(i:(i+Window_size-1)); I_temp1=Current1(i:(i+Window_size-1));
    V_temp2=Voltage2(i:(i+Window_size-1)); I_temp2=Current2(i:(i+Window_size-1));
    V_temp3=Voltage3(i:(i+Window_size-1)); I_temp3=Current3(i:(i+Window_size-1));
end
%Taking FFT of the voltage & current waveforms
V1=fft(V_temp1)/(Window_size);V1=V1(2:70);
V2=fft(V_temp2)/(Window_size);V2=V2(2:70);
V3=fft(V_temp3)/(Window_size);V3=V3(2:70);
I1=fft(I_temp1)/(Window_size);I1=I1(2:70);
I2=fft(I_temp2)/(Window_size);I2=I2(2:70);
I3=fft(I_temp3)/(Window_size);I3=I3(2:70);

% Making mat-files of the input data
fid=fopen('volt_mag_ph_1.mat','w');fprintf(fid,'%e 
',abs(V1));fclose(fid);
fid=fopen('volt_ang_ph_1.mat','w');fprintf(fid,'%e 
',angle(V1));fclose(fid);
fid=fopen('volt_mag_ph_2.mat','w');fprintf(fid,'%e 
',abs(V2));fclose(fid);
fid=fopen('volt_ang_ph_2.mat','w');fprintf(fid,'%e 
',angle(V2));fclose(fid);
fid=fopen('volt_mag_ph_3.mat','w');fprintf(fid,'%e 
',abs(V3));fclose(fid);
fid=fopen('volt_ang_ph_3.mat','w');fprintf(fid,'%e 
',angle(V3));fclose(fid);
fid=fopen('cur_mag_ph_1.mat','w');fprintf(fid,'%e 
',abs(I1));fclose(fid);
fid=fopen('cur_ang_ph_1.mat','w');fprintf(fid,'%e 
',angle(I1));fclose(fid);
fid=fopen('cur_mag_ph_2.mat','w');fprintf(fid,'%e 
',abs(I2));fclose(fid);
fid=fopen('cur_ang_ph_2.mat','w');fprintf(fid,'%e 
',angle(I2));fclose(fid);
fid=fopen('cur_mag_ph_3.mat','w');fprintf(fid,'%e 
',abs(I3));fclose(fid);
fid=fopen('cur_ang_ph_3.mat','w');fprintf(fid,'%e 
',angle(I3));fclose(fid);

%Calling out the main program
Ts=1/7000;
t=0:Ts:(max(size(S))-1)*Ts;
t=t(:);

subplot(2,1,1);plot(t,S,’k’,’linewidth’,2);
hold on;
plot(t,P,’k’);
plot(t,N_s,’:’);
plot(t,N_u,’k’);%’:’,’markersize’,20);
title(’Power Components’,’fontsize’,12,’fontweight’,’bold’);
set(gca,’YTick’,[0,0.5*10^-5,10^-5,1.5*10^-5,2*10^-5]);

subplot(2,1,2);plot(t,S,’k’);
hold on;
plot(t,Q_b,’k’);
plot(t,B_s,’k’);
plot(t,B_u,’k’);%’,’,’markersize’,50);
xlabel(’time in seconds’);
toc;
return;
Appendix B: Matlab function used for computing the 7 components

function [S1,P1,N_s1,N_u1,Q_b1,S_out_of_band1,S_in_band1,B_s1,B_u1,Q_B] =Harmonics_W_Output_3PH

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Here the main program starts %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%First we need to calculate the total magnitudes of V og I

%Lets find out how many harmonics in both V and I
load -ascii volt_mag_ph_1;
load -ascii volt_mag_ph_2;
load -ascii volt_mag_ph_3;

load -ascii volt_ang_ph_1;
load -ascii volt_ang_ph_2;
load -ascii volt_ang_ph_3;
load -ascii cur_mag_ph_1;
load -ascii cur_mag_ph_2;
load -ascii cur_mag_ph_3;
load -ascii cur_ang_ph_1;
load -ascii cur_ang_ph_2;
load -ascii cur_ang_ph_3;

total_harm_V = length(volt_mag_ph_1);
total_harm_I = length(cur_mag_ph_1);

% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% %So program works for 1 phase and 2 phase lets add zeros to
%increase vector length to max harmonics
if volt_mag_ph_1(1) == 0
    for i = 1:total_harm_V
        volt_mag_ph_1(i) = 0;
        volt_ang_ph_1(i) = 0;
    end
end
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if volt_ang_ph_1(1) == 0 && length(volt_ang_ph_1) == 1
    for i = 1:total_harm_V
        volt_ang_ph_1(i) = 0;
    end
end
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if volt_mag_ph_2(1) == 0
    for i = 1:total_harm_V
        volt_mag_ph_2(i) = 0;
        volt_ang_ph_2(i) = 0;
    end
end

% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if volt_ang_ph_2(1) == 0 && length(volt_ang_ph_2) == 1
    for i = 1:total_harm_V
        volt_ang_ph_2(i) = 0;
    end
end

% %%%%%%%%%%%%%%%%%%%%%%
if volt_mag_ph_3(1) == 0
    for i = 1:total_harm_V
        volt_mag_ph_3(i) = 0;
        volt_ang_ph_3(i) = 0;
    end
end

% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if volt_ang_ph_3(1) == 0 && length(volt_ang_ph_3) == 1
    for i = 1:total_harm_V
        volt_ang_ph_3(i) = 0;
    end
end
if cur_mag_ph_1(1) == 0
    for i = 1:total_harm_I
        cur_mag_ph_1(i) = 0;
        cur_ang_ph_1(i) = 0;
    end
end

if cur_ang_ph_1(1) == 0 && length(cur_ang_ph_1) == 1
    for i = 1:total_harm_I
        cur_ang_ph_1(i) = 0;
    end
end

if cur_mag_ph_2(1) == 0
    for i = 1:total_harm_I
        cur_mag_ph_2(i) = 0;
        cur_ang_ph_2(i) = 0;
    end
end

if cur_ang_ph_2(1) == 0 && length(cur_ang_ph_2) == 1
    for i = 1:total_harm_I
        cur_ang_ph_2(i) = 0;
    end
end

if cur_mag_ph_3(1) == 0
    for i = 1:total_harm_I
cur_mag_ph_3(i) = 0;
cur_ang_ph_3(i) = 0;
end
end
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if cur_ang_ph_3(1) == 0 && length(cur_ang_ph_3) == 1
    for i = 1:total_harm_I
        cur_ang_ph_3(i) = 0;
    end
end
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Now we need a for loop to calculate V_mag and I_mag
%(remember include every phase and all harmonics)
V_mag_temp = 0;
I_mag_temp = 0;
for i = 1:total_harm_V
    V_mag_temp = V_mag_temp + (volt_mag_ph_1(i))^2 + (volt_mag_ph_2(i))^2 +
                 (volt_mag_ph_3(i))^2;
end
for i = 1:total_harm_I
    I_mag_temp = I_mag_temp + (cur_mag_ph_1(i))^2 + (cur_mag_ph_2(i))^2 +
                 (cur_mag_ph_3(i))^2;
end
V_mag = sqrt(V_mag_temp);
I_mag = sqrt(I_mag_temp);
%Lets now calculate g_ik and b_ik and put them
%into a three vectors or one matrix.
%Lets start by creating three empty vectors for each phase

\[
\begin{align*}
g_{\text{ph}_1} &= []; \\
g_{\text{ph}_2} &= []; \\
g_{\text{ph}_3} &= []; \\
b_{\text{ph}_1} &= []; \\
b_{\text{ph}_2} &= []; \\
b_{\text{ph}_3} &= [];
\end{align*}
\]

%Lets now fill in the vectors

\[
\begin{align*}
&\text{if } \text{total}_\text{harm}_V \leq \text{total}_\text{harm}_I \\
&\quad \text{min}_\text{harm} = \text{total}_\text{harm}_V;
&\quad \text{max}_\text{harm} = \text{total}_\text{harm}_I; \\
&\text{else} \\
&\quad \text{min}_\text{harm} = \text{total}_\text{harm}_I;
&\quad \text{max}_\text{harm} = \text{total}_\text{harm}_V; \\
&\text{end}
\end{align*}
\]

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Adding even harmonics to the program.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

\[
\begin{align*}
&\text{for } i = 1:\text{min}_\text{harm} \\
&\quad \text{if } \text{volt}_\text{mag}_\text{ph}_1(i) == 0 \\
&\quad\quad g_{\text{ph}_1}(i) = 0; \\
&\quad\quad b_{\text{ph}_1}(i) = 0; \\
&\quad \text{else} \\
&\quad\quad g_{\text{ph}_1}(i) = (\text{volt}_\text{mag}_\text{ph}_1(i)\times\text{cur}_\text{mag}_\text{ph}_1(i)\times\cos(\text{cur}_\text{ang}_\text{ph}_1(i)-
\text{volt}_\text{ang}_\text{ph}_1(i)))/(\text{volt}_\text{mag}_\text{ph}_1(i))^2;
\end{align*}
\]
b_ph_1(i) = (volt_mag_ph_1(i)*cur_mag_ph_1(i)*sin(volt_ang_ph_1(i)-
cur_ang_ph_1(i)))/(volt_mag_ph_1(i))^2;
end
end

for i = 1:min_harm
    if volt_mag_ph_2(i) == 0
        g_ph_2(i) = 0;
        b_ph_2(i) = 0;
    else
        g_ph_2(i) = (volt_mag_ph_2(i)*cur_mag_ph_2(i)*cos(cur_ang_ph_2(i)-
volt_ang_ph_2(i)))/(volt_mag_ph_2(i))^2;
        b_ph_2(i) = (volt_mag_ph_2(i)*cur_mag_ph_2(i)*sin(volt_ang_ph_2(i)-
cur_ang_ph_2(i)))/(volt_mag_ph_2(i))^2;
    end
end

for i = 1:min_harm
    if volt_mag_ph_3(i) == 0
        g_ph_3(i) = 0;
        b_ph_3(i) = 0;
    else
        g_ph_3(i) = (volt_mag_ph_3(i)*cur_mag_ph_3(i)*cos(cur_ang_ph_3(i)-
volt_ang_ph_3(i)))/(volt_mag_ph_3(i))^2;
        b_ph_3(i) = (volt_mag_ph_3(i)*cur_mag_ph_3(i)*sin(volt_ang_ph_3(i)-
cur_ang_ph_3(i)))/(volt_mag_ph_3(i))^2;
    end
end
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Adding zeros at the end of g and b vectors so they will be as long as max_harm

for i = (min_harm+1):max_harm
    g_ph_1(i) = 0;
    g_ph_2(i) = 0;
    g_ph_3(i) = 0;

    b_ph_1(i) = 0;
    b_ph_2(i) = 0;
    b_ph_3(i) = 0;
end

%Lets now calculate sum_1 = sum(g_kl*norm(mag_v)^2)
%and the sum(norm(mag_v)^2)
sum_g = 0;
sum_b = 0;

for i = 1:min_harm

    sum_g = sum_g + g_ph_1(i)*(volt_mag_ph_1(i))^2 + g_ph_2(i)*(volt_mag_ph_2(i))^2 +
            g_ph_3(i)*(volt_mag_ph_3(i))^2;

    sum_b = sum_b + b_ph_1(i)*(volt_mag_ph_1(i))^2 + b_ph_2(i)*(volt_mag_ph_2(i))^2 +
            b_ph_3(i)*(volt_mag_ph_3(i))^2;
end
\[ \mu_g = \frac{\text{sum}_g}{(V_{mag})^2}; \]
\[ \mu_b = \frac{\text{sum}_b}{(V_{mag})^2}; \]

% Now let's calculate \((\sigma_g)^2\) and \((\sigma_b)^2\)
\[ \text{sum}_\sigma_g = 0; \]
\[ \text{sum}_\sigma_b = 0; \]

for \( i = 1: \text{total}_\text{harm}_V \) % Gtum liklegast haft her bara total_harm_V
\[ \text{sum}_\sigma_g = \text{sum}_\sigma_g + ((g_{ph_1(i)} - \mu_g)^2)*((\text{volt}_{mag}_{ph_1(i)})^2) + ((g_{ph_2(i)} - \mu_g)^2)*((\text{volt}_{mag}_{ph_2(i)})^2) + ((g_{ph_3(i)} - \mu_g)^2)*((\text{volt}_{mag}_{ph_3(i)})^2); \]
\[ \text{sum}_\sigma_b = \text{sum}_\sigma_b + ((b_{ph_1(i)} - \mu_b)^2)*((\text{volt}_{mag}_{ph_1(i)})^2) + ((b_{ph_2(i)} - \mu_b)^2)*((\text{volt}_{mag}_{ph_2(i)})^2) + ((b_{ph_3(i)} - \mu_b)^2)*((\text{volt}_{mag}_{ph_3(i)})^2); \]
end

\[ \sigma_g^2 = \frac{\text{sum}_\sigma_g}{(V_{mag})^2}; \]
\[ \sigma_b^2 = \frac{\text{sum}_\sigma_b}{(V_{mag})^2}; \]

% To calculate \((\sigma_gu)^2\) and \((\sigma_bu)^2\)
% we need to calculate \(\mu_g(l)\) and \(\mu_b(l)\) first.
% Let's start by creating an empty vector for \(\mu_g(l)\) and \(\mu_b(l)\)
\[ \mu_{g_1} = []; \]
\[ \mu_{b_1} = []; \]

for \( i = 1: \text{min}_\text{harm} \)
    if \( \text{volt}_{mag}_{ph_1(i)} == 0 \) \&\& \( \text{volt}_{mag}_{ph_2(i)} == 0 \) \&\& \( \text{volt}_{mag}_{ph_3(i)} == 0 \)

mu_g_1(i) = 0;
mu_b_1(i) = 0;
else
mu_g_1(i) = (g_ph_1(i)*(volt_mag_ph_1(i))^2 +
g_ph_2(i)*(volt_mag_ph_2(i))^2 +
g_ph_3(i)*(volt_mag_ph_3(i))^2)/((volt_mag_ph_1(i))^2 +
(volt_mag_ph_2(i))^2 + (volt_mag_ph_3(i))^2);
mu_b_1(i) = (b_ph_1(i)*(volt_mag_ph_1(i))^2 +
b_ph_2(i)*(volt_mag_ph_2(i))^2 +
b_ph_3(i)*(volt_mag_ph_3(i))^2)/((volt_mag_ph_1(i))^2 +
(volt_mag_ph_2(i))^2 + (volt_mag_ph_3(i))^2);
end
end
mu_g_1;
mu_b_1;

%Now lets calculate (sigma_gu)^2 and (sigma_bu)^2
sum_sigma_gu = 0;
for i = 1:min_harm %held etta se rett
sum_sigma_gu = sum_sigma_gu +
((abs(g_ph_1(i)-mu_g_1(i)))^2)*((volt_mag_ph_1(i))^2) +
((abs(g_ph_2(i)-mu_g_1(i)))^2)*((volt_mag_ph_2(i))^2) +
((abs(g_ph_3(i)-mu_g_1(i)))^2)*((volt_mag_ph_3(i))^2);
end

sum_sigma_bu = 0;
for i = 1:min_harm %held etta se rett
sum_sigma_bu = sum_sigma_bu +
((abs(b_ph_1(i)-mu_b_1(i)))^2)*((volt_mag_ph_1(i))^2) +
((abs(b_ph_2(i)-mu_b_1(i)))^2)*((volt_mag_ph_2(i))^2) +
((abs(b_ph_3(i)-mu_b_1(i)))^2)*((volt_mag_ph_3(i))^2);
end

sigma_gu_sq = sum_sigma_gu/(V_mag)^2;
sigma_bu_sq = sum_sigma_bu/(V_mag)^2;

% Now lets calculate (sigma_gs)^2 and (sigma_bs)^2
sigma_gs_sq = sigma_g_sq - sigma_gu_sq;
sigma_bs_sq = sigma_b_sq - sigma_bu_sq;

% Now lets calculate the norms for i_gs, i_bs, i_gu, i_bu
i_gs_norm = sqrt(sigma_gs_sq)*V_mag;
i_bs_norm = sqrt(sigma_bs_sq)*V_mag;

i_gu_norm = sqrt(sigma_gu_sq)*V_mag;
i_bu_norm = sqrt(sigma_bu_sq)*V_mag;

% Now lets calculate the final result for P, Q_b, N_s, N_u, Q_s and Q_u
S1 = 2*V_mag*I_mag;
S = format_str(S1);
P1 = 2*mu_g*(V_mag)^2;
P = format_str(P1);
Q_b1 = 2*mu_b*(V_mag)^2;
Q_b = format_str(Q_b1);
N_s1 = 2*i_gs_norm*V_mag;
N_s = format_str(N_s1);
N_u1 = 2*i_gu_norm*V_mag;
N_u = format_str(N_u1);
B_s1 = 2*i_bs_norm*V_mag;
B_s = format_str(B_s1);
B_u1 = 2*i_bu_norm*V_mag;
B_u = format_str(B_u1);
S_in_band1 = sqrt(P1^2+Q_b1^2+N_s1^2+N_u1^2+B_s1^2+B_u1^2);
S_in_band = format_str(S_in_band1);
S_out_of_band1 = sqrt(S1^2-S_in_band1^2);
S_out_of_band = format_str(S_out_of_band1);

function b = format_str(v)

    b = sprintf('%.2f', v);

    i = findstr('.', b);
    if length(i) == 0
        i = length(b) + 1; % works if format is '%1.0f', i.e. no dot
    end

    k = i - 3;

    % 999.0 => i = 4 k=1
    % 1000.0 => i = 5 k=2
while k > 1
    b = strcat(b(1:k-1), ',', b(k:end));
    k = k - 3;
end
return;
Appendix C: Matlab function for computing the sequence components of real and reactive power

function=real_and_reactive_power_decomposition

close all;clc;
Window_size=140;

% 140 was our speculated number of points in one period. This number matches with the sampling rate indicated in the "power of the paper" industrial paper. Frequency is taken as 50 Hz

%M for the sa0v1under file
M=xlsread('sa0v1under.xls');
Voltage1=M(:,1);Voltage2=M(:,2);Voltage3=M(:,3);
Current1=M(:,4);Current2=M(:,5);Current3=M(:,6);
alpha = exp(j*2*pi/3);

%symmetrical components transformation matrix
A = (1/sqrt(3)) *[1, 1, 1; 1, alpha, alpha^2; 1, alpha^2, alpha];

for i=1:(length(Voltage1)-Window_size+1)
    V_temp1=Voltage1(i:(i+Window_size-1)); I_temp1=Current1(i:(i+Window_size-1));
    V_temp2=Voltage2(i:(i+Window_size-1)); I_temp2=Current2(i:(i+Window_size-1));
    V_temp3=Voltage3(i:(i+Window_size-1)); I_temp3=Current3(i:(i+Window_size-1));

    %Taking FFT of the voltage & current waveforms
    V1=fft(V_temp1)/(Window_size);
    V2=fft(V_temp2)/(Window_size);
    V3=fft(V_temp3)/(Window_size);

    I1=fft(I_temp1)/(Window_size);
    I2=fft(I_temp2)/(Window_size);
    I3=fft(I_temp3)/(Window_size);

    V=transpose([V1, V2, V3]);
    I=transpose([I1, I2, I3]);

    Sz(i)=0; Sp(i)=0; Sn(i)=0;
    for k=2:Window_size/2
        Vpnzk=A*V(:,k);
        Ipnzk=A*I(:,k);
Sz(i)=Sz(i)+Vpnzk(1)*Ipnzk(1)';
Sp(i)=Sp(i)+Vpnzk(2)*Ipnzk(2)';
Sn(i)=Sn(i)+Vpnzk(3)*Ipnzk(3)';
end
end

Pz=2*real(Sz);Pp=2*real(Sp);Pn=2*real(Sn);
P=Pz+Pp+Pn;

Qz=2*imag(Sz);Qp=2*imag(Sp);Qn=2*imag(Sn);
Qb=Qz+Qp+Qn;

\[ t = \frac{1:\text{length(Voltage1)}-\text{Window}_\text{size}+1}{6.9452e+003}; \]

figure;
subplot(2,2,1);plot(t,Pz);title('Pz','fontsize',12,'fontweight','bold');
xlabel('Time (s)');ylabel('Pz (W)');
subplot(2,2,2);plot(t,Pp);title('Pp','fontsize',12,'fontweight','bold');
xlabel('Time (s)');ylabel('Pp (W)');
subplot(2,2,3);plot(t,Pn);title('Pn','fontsize',12,'fontweight','bold');
xlabel('Time (s)');ylabel('Pn (W)');
subplot(2,2,4);plot(t,P);title('P','fontsize',12,'fontweight','bold');
xlabel('Time (s)');ylabel('P (W)');

figure;
subplot(2,2,1);plot(t,Qz);title('Qz','fontsize',12,'fontweight','bold');
xlabel('Time (s)');ylabel('Qz (Var)');
subplot(2,2,2);plot(t,Qp);title('Qp','fontsize',12,'fontweight','bold');
xlabel('Time (s)'); ylabel('Qp ( Var )');
subplot(2,2,3); plot(t,Qn); title('Qn', 'fontsize', 12, 'fontweight', 'bold');
xlabel('Time (s)'); ylabel('Qn ( Var )');
subplot(2,2,4); plot(t,Qb); title('Qb', 'fontsize', 12, 'fontweight', 'bold');
xlabel('Time (s)'); ylabel('Qb ( Var )');
return;
Appendix D: Matlab function for computing the average Akagi reactive power

function Akagi_reactive_power_decomposition

close all;clc;
Window_size=140;

% 140 was our speculated number of points in one period. This number
% matches with the sampling rate indicated in the "power of the paper"
% industrial paper. Frequency is taken as 50 Hz

M=xlsread('sa0v1under.xls');
Voltage1=M(:,1);Voltage2=M(:,2);Voltage3=M(:,3);
Current1=M(:,4);Current2=M(:,5);Current3=M(:,6);

alpha = exp(j*2*pi/3);
% transformation matrix

A=(sqrt(2/3))*[1,-1/2,-1/2;0,sqrt(3)/2,-sqrt(3)/2];

for i=1:(length(Voltage1)-Window_size+1)
    V_temp1=Voltage1(i:(i+Window_size-1)); I_temp1=Current1(i:(i+Window_size-1));
    V_temp2=Voltage2(i:(i+Window_size-1)); I_temp2=Current2(i:(i+Window_size-1));
    V_temp3=Voltage3(i:(i+Window_size-1)); I_temp3=Current3(i:(i+Window_size-1));

    %Taking FFT of the voltage & current waveforms
    V1=fft(V_temp1)/(Window_size);
    V2=fft(V_temp2)/(Window_size);
    V3=fft(V_temp3)/(Window_size);

    I1=fft(I_temp1)/(Window_size);
    I2=fft(I_temp2)/(Window_size);
    I3=fft(I_temp3)/(Window_size);

    % Three phase voltages and currents
    V=transpose([V1,V2,V3]);
    I=transpose([I1,I2,I3]);

    V=A*V; I=A*I;

    QA(i)=0;
    for k=2:Window_size/2
        QA(i)=QA(i)+((V(2,:)*I(1,:)'-(V(1,:)*I(2,:)'));
    end
end
QA=2*QA/Window_size;

Ts=1/7000;
t=0:Ts:(max(size(QA))-1)*Ts;
t=t(:);

plot(t,QA);
title('Akagi Reactive Power','fontsize',12, 'fontweight','bold');
xlabel('Time in seconds');ylabel('QA in vars');
return;
Appendix E: Matlab function for computing the instantaneous Akagi reactive power

function Akagi_reactive_power_decomposition_noaverage
    close all; clc;
    tic; % shows the time of the simulation

    Window_size = 140;
    % 140 was our speculated number of points in one period. This number
    % matches with the sampling rate indicated in the "power of the paper"
    % industrial paper. Frequency is taken as 50 Hz

    for i = 1:length(Voltage1)
        M = xlsread('sa0v1under.xls);
        Voltage1 = M(:,1); Voltage2 = M(:,2); Voltage3 = M(:,3);
        Current1 = M(:,4); Current2 = M(:,5); Current3 = M(:,6);

        A = diag([sqrt(2/3) sqrt(2)/2 1/sqrt(3)]) *[1 -0.5 -0.5; 0 1 -1; 1 1 1];

        for i = 1:length(Voltage1)
Voltage1=M(i,1);Voltage2=M(i,2);Voltage3=M(i,3);
Current1=M(i,4);Current2=M(i,5);Current3=M(i,6);

V1=Voltage1;V2=Voltage2;V3=Voltage3;
I1=Current1;I2=Current2;I3=Current3;

V=transpose([V1,V2,V3]);
I=transpose([I1,I2,I3]);
V=A*V; I=A*I;

qab(i)=I(1,:).*V(2,:)-V(1,:).*I(2,:);
qab(i)=0.001*qab(i);
qao(i)=I(1,:).*V(3,:)-V(1,:).*I(3,:);
qao(i)=0.001*qao(i);
qbo(i)=I(2,:).*V(3,:)-V(2,:).*I(3,:);
qbo(i)=0.001*qbo(i);

end

Qab=filter(1/140*ones(1,140),1,qab);
Qao=filter(1/140*ones(1,140),1,qao);
Qbo=filter(1/140*ones(1,140),1,qbo);

Ts=1/7000;
t=0:Ts:(max(size(qab))-1)*Ts;
time=t(:);
figure(1)
orient tall
subplot(311), plot(time,qab,time,Qab,'r-');
xlabel('time (sec)')
ylabel('q_{\alpha,\beta} (kVAR)')
axis([0.0202 0.4 0 200])

subplot(312), plot(time,qao,time,Qao,'r-');
xlabel('time (sec)')
ylabel('q_{\alpha,o} (kVAR)')
axis([0.0202 0.4 -3 2])

subplot(313), plot(time,qbo,time,Qbo,'r-');
xlabel('time (sec)')
ylabel('q_{\beta,o} (kVAR)')
axis([0.0202 0.4 -3 2])

figure(2)
orient tall
subplot(211), plot(time,Qab,'r-');
xlabel('time (sec)')
ylabel('Q_{\alpha,\beta} (kVAR)')
axis([0.0202 0.4 60 110])

zoom=1400:1900;
subplot(212), plot(time(zoom),qab(zoom),time(zoom),Qab(zoom),'r-');
xlabel('time (sec)')
ylabel('q_{\alpha,\beta} (kVAR)')
axis([0.98*min(time(zoom)) 1.02*max(time(zoom)) 0 160])
return;