Strategic and Robust Deployment of Synchronized Phasor Measurement Units with Restricted Channel Capacity

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Abstract

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Synchronized phasor measurements are changing the way power systems are monitored and operated. Their efficient incorporation into various applications which are executed in energy management control centers requires strategic placement of these devices. Earlier studies which consider placement of synchronized phasor measurement units (PMUs) to be used for state estimation assume that these devices will have unlimited channel capacities to record as many phase voltages and currents as needed. What differentiates this study from those already reported in the literature is the fact that it accounts for the number of available channels for the chosen type of PMU since all existing PMUs come with a limited number of channels and their costs vary accordingly. This is shown to be a critical factor in strategic placement of these devices. In this study, a revised formulation of the placement problem and its associated solution algorithm will be presented. Examples will be used to illustrate the impact of having limited number of channels on the location and number of required PMUs to make the system observable. Developed methods will take into account existing injection measurements, in particular the virtual measurements such as zero-injections that are available at no cost at electrically passive buses.

Moreover, despite the advances in related technologies, it is almost impossible to guarantee occasional device or communication failure that will lead to loss of data to be received from a given PMU. This work is also aimed to illustrate how the measurement design can be made reliable against such events while maintaining the cost of PMU installations at a minimum by using strategically placed PMUs with the proper number of channels.

Furthermore, it is also demonstrated that depending upon the topology of the network, there will be an upper limit on the number of channels for the PMUs beyond which installation costs will not be reduced any further. Accordingly, numerical results of applying the developed optimization method to power systems with varying sizes and topologies will be presented to illustrate the typical numbers of PMUs and their channel capacities that are required for optimal performance. The results of this work will be more useful as the number of PMU installations increases to levels that will make the system fully observable based solely on PMUs with different number of channel capacities.
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To my beloved parents,
Selma and Hasan Korkalı,
with love and gratitude
“A good companion shortens the longest road.”

—Turkish Proverb
Chapter 1

Introduction

Power systems have long been monitored based on measurements provided by remote terminal units (RTU). These measurements typically include branch power flows, bus power injections, and magnitude of bus voltages. A critical quantity of interest is the phase difference between a given pair of bus voltage phasors in the system. Until recently, this quantity was not easily measurable. Direct measurement of phase angles of voltage and/or current phasors is now possible by phasor measurement units (PMUs) thanks to the availability of the Global Positioning System (GPS) that facilitates time synchronization of measured signals at geographically remote locations. However, even when there are no phase angle measurements, phase angle associated with each bus voltage phasor can be estimated along with its magnitude provided that there are sufficient number of power flow and bus injections measured with negligible time skew. This is accomplished by the help of a power system state estimator.

Having synchronized phasor measurements for bus voltages and branch currents in a given power system has a significant effect on the application functions in control centers. In particular, the state estimation application will be affected in a rather fundamental manner in that the problem formulation can transform from nonlinear to linear if sufficiently large number of such phasors can be measured. Hence, there is interest on the part of state estimator users with respect to the requirements of such a transformation, namely the cost associated with installing the right number...
and type of PMUs at strategic locations in order to drastically improve their state estimators.

1.1 Motivations for the Study

Synchronized phasor measurement units are rapidly populating power systems as their benefits become more and more evident for various power system applications. One such function is the state estimation [1]. In fact, state estimation provides the much needed real-time database for several application functions which facilitate power system control and efficient operation. Given the limited resources, it is practical to place these devices strategically in order to minimize the cost/benefit ratio.

Optimal placement methods to account for contingencies, loss of measurements as well as existing conventional measurements and zero-injections have already been presented in previous publications. A common assumption in all of these studies is that each PMU can measure unlimited number of voltages and currents. This assumption allows the PMU placement problem to be formulated in a straightforward manner since placement of a PMU at a given bus automatically ensures availability of phasors at all of its neighboring buses. This is true as long as the network connectivity and associated branch parameters are perfectly known, which is also commonly assumed by all state estimators. Available PMUs have limited number of channels which are used to sample voltage or current signals. These sampled signals are processed to generate a positive-sequence phasor voltage and current from the three-phase voltages and currents. Hence, channel capacities of PMUs may play an important role in their strategic placement for maximum coverage.

1.2 Contributions of the Thesis

The main contribution of this thesis is to recognize the effect of channel capacity of a given type of PMU on their optimal placement for network observability and
to develop an optimal solution to the PMU placement problem given a specified number of available channels for the candidate PMUs. As in earlier studies, problem formulation accounts for bus injections which may be known due to measurements or due to their zero values at passive buses.

In formulating the problem it is realized that for a given number of channel capacity there will be a finite combination of possible assignments of incident branches to a given PMU placed at a bus. Hence, the choices will increase with the number of incident branches for a given bus, but will remain bounded irrespective of the overall system size, thanks to the sparse interconnection of power system buses.

1.3 Thesis Outline

This thesis comprises five chapters. It is organized as follows. In the current chapter, the motivations for the research problem and our contributions to PMU placement problem are discussed.

In the succeeding chapter (Chapter 2), we first present the general background information about synchronized phasor measurement units and review the relevant literature to the existing PMU placement strategies flourished in the field of power systems state estimation.

Chapter 3 delineates the method of determining optimal number and locations of PMUs, so that the system state of an entire power system will be observable. In doing so, the technique to be introduced is a numerical procedure where the problem is formulated as an integer linear programming (ILP) problem. In addition, different cases are suggested for the modeling of zero-injection buses.

Chapter 4 is devoted to the simulation results of our optimization model including various conditions where zero-injections are considered and ignored, the sparsity of the studied networks is reduced fictitiously; and the reliability is maintained under a single PMU failure. Several case studies are conducted to evaluate the algorithm’s performance and effectiveness.
Finally, Chapter 5 concludes this thesis with the discussion on the benefits of the proposed formulation for optimal PMU placement along with its usability in the existing power systems. Also, we express our ideas about what can be done for further study.
Chapter 2

Optimal PMU Placement for State Estimation

This chapter is mainly devoted to the history of the evolution of synchronized phasor measurement units used for the purpose of state estimation and provides an overview of the applications and miscellaneous techniques that have been introduced so far in the power systems literature regarding PMU placement.

2.1 Historical Overview of Phasor Measurement Units

Synchronized phasor measurements are considered as a promising measurement tool for electric power systems. They supply positive-sequence voltage and current measurements synchronized to within a microsecond thanks to the availability of Global Positioning System (GPS) and the sampled data processing algorithms designed for computer relaying applications. Apart from the positive-sequence voltage and current measurements, these systems are able to quantify both local frequency and frequency rate-of-change. Moreover, they can be altered according to the several needs of users in order to extract the data relating to zero- and negative-sequence quantities, harmonics, as well as individual phase voltages and currents. Currently,
there are 24 commercial PMU manufacturers. In this respect, standards established by the IEEE Power System Relaying Committee have enabled the data sharing capability among distant units from different manufacturers. Considering that blackouts are occurring overwhelmingly on the existing power networks, widespread allocation of PMUs has gained tremendous interest. In particular, positive-sequence measurements provide the accessibility to the power system state at any instant. Various applications of synchronized phasor measurements have been presented in the literature, and more applications will certainly be developed in coming years [2].

The modern era of phasor measurement technology has its origin in research conducted on computer relaying of transmission lines. Early study on transmission line relaying with relays based on microprocessors unveiled that the available computer power in 1970s was not substantial enough to manage the computations required to execute all the line relaying operations.

Positive-sequence voltages of a network constitute the state vector of a power system, and it is of fundamental importance in all of power system analyses. The first paper to identify the importance of positive-sequence voltage and current phasor measurements, and some of the uses of these measurements, was published in 1983 [3], and this work can be viewed as the starting point of modern synchronized phasor measurement technology. The GPS was being entirely installed around these years. Later on, it became evident that this system provided the most effective way of synchronizing power system measurements over long distances. The first prototypes of the existing PMUs using GPS were developed at Virginia Tech in 1980s. These prototype PMUs built at Virginia Tech were placed at certain substations of the Bonneville Power Administration (BPA), the New York Power Authority (NYPA), and the American Electric Power Service Corporation (AEPSC) [2]. For the moment, a number of commercial manufacturers offer PMUs and placement of PMUs on the modern power systems is being carried out profoundly in many countries.

Along with the development of PMUs, there is substantial amount of continuing research on the applications of the measurements provided by the PMUs. In this respect, the recent advancement in synchronized phasor measurements is reaching
towards improvements in its maturity, and a majority of modern power systems across the globe are continually placing wide-area measurement systems made up of the phasor measurement units.

2.2 Applications of Synchronized Phasor Measurements in Power Systems

Advances and applications in research studies of synchronized phasor measurement units have been presented by several recent papers. Reference [4] introduces a possible approach towards formulating a standard that would facilitate interoperability of PMUs under transient conditions. In [5], the authors propose a wide-area network of phasor measurement units as a means for monitoring and control of voltage stability. A new technique for wide-area protection utilizing PMU is suggested in [6]. In particular, the proposed protection scheme is dependent upon the comparison of the positive-sequence voltage magnitudes for certain areas along with the differences of positive-sequence current phase angles for each line between two areas in the power system. The authors of [7] show that the realization of optimal measurement designs can be achieved in order to determine the types and locations of few extra measurements that will considerably improve the capability of topology error processing. To accomplish this task, emerging PMUs are suggested for use in addition to the conventional power flow and injection measurements. In order to help prevent a large-scale blackout, the authors of reference [8] present an online voltage security assessment scheme making use of synchronized phasor measurements as well as decision trees which are periodically updated.

With the latest progress in smart grid technology, the use of phasor measurement units has definitely drawn substantial interest in order to render power system reliability within transmission and distribution infrastructure. Hence, the utilization of wide-area monitoring systems (WAMS) using synchrophasor measurements has gained momentum achieving improved system monitoring, control, and protection.
In one of recent studies [9], the authors discuss the detailed architecture and the recent implementations and applications of a wide-area frequency monitoring network (FNET). In [10], it is shown that single line outages can be detected by using phasor angle measurements data provided by PMUs even if there is extremely limited coverage. In a two-paper set [11, 12], the authors present a PMU-based technique for fault detection/location as well as multifunction transmission line protection for both arcing and permanent faults by processing the synchronized voltage and current phasors. In order to avoid reclosure on a permanent fault, arcing fault discrimination technique is proposed via processing the synchronized harmonic voltage and current phasors.

In [13], energy function analysis has been adapted using phasor data in order to monitor the dynamic security of power transfer paths. Utilizing the phasor data provided by a system of well-located PMUs, the transfer paths and their associated parameters are identified and the transfer path reactances and equivalent inertias are estimated by using the power-angle curves and the oscillation frequencies. A wide-area identification of long-term voltage instability from the bus voltage phasors provided by synchronized phasor measurements is devised in [14, 15]. A generalized fault section selector, as well as fault locator, is proposed by Liu et al. [16] for multiterminal transmission lines based on synchronized phasor measurement units.

In [17], the authors investigate the feasibility of estimating the rotor angle of synchronous generators from the measurements of field voltage of the generator and terminal voltage measurements acquired from PMUs.

### 2.3 Related Work

In order to estimate the system state, power system state estimator makes use of the set of available measurements. Given a set of measurements and their corresponding locations, the network observability analysis will determine if a unique estimate can be found for the system state. This analysis are carried out offline during the initial phase of a state estimator installation in order to check the sufficiency of the existing measurement configuration. If the system is not found observable,
then additional meters may have to be installed at certain locations. Observability analysis is executed online prior to performing the state estimator. It ensures that a state estimate can be obtained using the set of measurements at the last measurement scan. Telecommunication errors, telemetry failures, or changes in topology may at times result in the cases where the state of the whole system cannot be estimated. Network observability test allows detection of such cases right before the execution of the state estimator. Observability of a given network is determined by the type and location of the available measurements as well as by the topology of the network. Therefore, the analysis of network observability exploits the graph theory since it has connection with networks, their respective equations and solutions. Also, the system is said to be topologically observable if the meters are placed such that there exists at least one spanning measurement tree of full rank [18]. On the other hand, installing a PMU at every bus in a wide-area interconnected network is neither reasonable nor prudent. For that reason, the optimal PMU placement problem deals with determining the minimum number of PMUs to achieve full network observability.

Intrinsically, the optimal PMU placement problem is shown to be NP-complete with a solution space having $2^N$ possible combinations for an $N$-bus electric network [19]. In this respect, it is regarded as a combinatorial optimization problem and a considerable amount of work has been done by several researchers, accordingly. These approaches are broadly classified into two main categories: the metaheuristic techniques and conventional deterministic optimization methods. As formulations based on metaheuristics (e.g., simulated annealing (SA), genetic algorithm (GA), Tabu search (TS), etc.) do not involve derivative of cost functions and the variables of the meter placement problem are discrete; they have been extensively used in dealing with discrete variables when solving the optimal PMU placement problem.

The utilization and development of PMUs are first introduced in [20] and [21]. An algorithm for finding the minimum number of PMUs required for power system state estimation is developed in [22] and [23] in which the simulated annealing (SA) method and the graph theory are utilized in formulating and solving the problem. Nuqui and Phadke [24], [25] utilize a simulated annealing (SA) technique in their graph-theoretic approach to determine the optimal PMU locations. In their work, a
Chapter 2. Optimal PMU Placement for State Estimation

A novel concept of “depth-of-unobservability” is presented and how this has an effect on the PMU placement is also shown. An optimal placement method founded on nondominated sorting genetic algorithm (NSGA) is proposed by Milošević and Begović [26]. The algorithm unites the graph theory and a simple GA to estimate each optimal solution of the objective function. The best tradeoff between the competing objectives is then searched by customized NSGA. This method is limited by the size of the problem as it requires more complex computations. Another GA-based procedure for the placement problem is presented by Marín et al. [19]. In this letter, the relationship between the number of current phasors that must be measured on each PMU and the required number of PMUs is also sought by the authors. Cho, Shin, and Hyun [27] propose three approaches aiming at alleviating computational burden of the optimal placement problem. First, SA method is modified in setting the initial temperature and cooling procedure. Second, direct combination (DC) method is suggested using a simple yet effective heuristic rule to identify the most effective sets in the observability sense. At the end, TS method is utilized to diminish the searching spaces effectively. A novel technique established upon TS and augmented incidence matrix is introduced by Peng, Sun, and Wang [28]. Aminifar et al. [29] investigate the applicability of immunity genetic algorithm (IGA) for minimal PMU placement problem. Chakrabarti et al. [30] propose a methodology based on binary particle swarm optimization (BPSO). In this study, the objectives of the optimization problem lie at the intersection of minimization of the required number of PMUs and maximization of the measurement redundancy. Analogously, Hajian et al. [31] use a modified BPSO algorithm as an optimization tool for obtaining the minimal number of PMUs for complete system observability. Sadu, Kumar, and Kavasseri [32] solve the placement problem by particle swarm optimization (PSO) algorithm, and the idea of introducing randomness in selecting the buses for the PMU placement is suggested by the authors. Chakrabarti and Kyriakides [33] propose binary search algorithm as a technique for solving the problem.

In addition to the metaheuristic methods, several conventional deterministic techniques are applied to the optimal PMU placement problem [34–46]. In [34] and [35], the algorithm for optimal placement of PMUs is developed using integer
programming (IP) established upon the network observability and installation costs of PMUs. Gou [36] makes a simplification in the placement algorithm by using ILP and considering both the presence and absence of the conventional flow and injection measurements. In his another simultaneously published work [37], the author extends the formerly developed model and generalizes the ILP formulation to satisfy various needs by integrating redundant PMU placement, full observability and incomplete observability cases. Dua et al. [42] propose another formulation using ILP. Integer quadratic programming (IQP) model is proposed as a solution method in [40] and [41].

Among the published techniques, a certain number of those take into account the power system contingencies broadly associated with the line outages and/or measurement losses [33, 38–44, 47–51]. The integration of such contingencies in the placement problem would certainly contribute to the reliable measurement designs. A sequential meter addition/elimination process based on the measurement sensitivities has been presented by Park et al. [47]. Abur and Magnago [38] propose an LP-based method in which a number of additional measurements are then systematically added to ensure full observability under the loss of any single network branch. The same authors propose a numerical algorithm based on the measurement Jacobian and sparse triangular factorization to optimally upgrade the measurements and yield a configuration which can remain robust against loss of single measurement and single branch outage without sacrificing network observability [48]. Xu, Yoon, and Abur [39] address a binary integer programming method taking into account the loss of a single PMU in order to lessen the vulnerability of state estimation to PMU breakdowns. The identical efforts to obtain a reliable measurement system based on numerical observability are made by Rakpenthai et al. [49]. The authors utilize the minimum condition number of the normalized measurement matrix as a criterion. Then, the sequential addition and elimination methods are employed to determine the essential measurements and to identify the redundancy measurements under the contingency, respectively. Later work by Chakrabarti and Kyriakides [33] propose a strategy utilizing a binary search algorithm to find the minimum number of PMUs for full topological observability under normal operating conditions, as well
as single branch outages. In the paper, the search process is said to be exhaustive; as a result, they aspire to overcome the restrictions of the conventional optimization methods such as the integer programming and the uncertainties of the evolutionary programming techniques such as the genetic algorithm. In their another collaborative works [40], [41], they propose an IQP approach to minimize the total number of PMUs required to maintain the complete observability of the system for normal operating conditions and under the outage of a single PMU or a transmission line. Also, they aim to provide the maximization of the measurement redundancy at all system buses. Dua et al. [42] devise a procedure for optimal multistage scheduling of PMU placement phased over multiple time horizons. Furthermore, they suggest zero-injection constraints be modeled as linear constraints in an ILP framework. The two indices, Bus Observability Index (BOI) and System Observability Redundancy Index (SORI), are utilized to rank the multiple solutions obtained via minimum PMU placement problem. In their generic PMU placement formulation, the authors offer some modifications to deal with the issues of PMU loss and communication line outage. Likewise, Abbasy and Ismail [43] study the impact of single PMU loss or multiple PMU losses on the decision strategy of the PMU placement problem. In [50] and [51], the authors come up with the so-called branch PMUs which are designed to monitor a single branch by measuring the associated current and terminal voltage phasors. Further, they also address the robustness of the measurement design by considering not only the cases of PMU loss or failure, but contingencies stemming from line or transformer outages. More recently, Aminifar et al. [44] offer a practical ILP-based model taking account of several contingency conditions involving communication constraints, loss of measurements, and line outages.

Chen and Abur [45], [46] propose an IP-based solution that leads to the smallest number of strategically located PMUs eliminating the measurement criticality in the system. In these papers, it is shown that the bad data detection and identification capability of a system can be enhanced greatly with few additional PMUs.

A fault location scheme for transmission networks using PMUs is developed and the idea of fault location observability is presented by Lien et al. [52]. A method for placing minimum number of PMUs to locate any fault in a power system is
proposed by Pokharel and Brahma [53]. The method is formulated on the basis of ILP structure which is introduced in [34]. Mahmoodianfard et al. [54] utilize a scheme based on decision trees to find an optimum PMU placement for voltage security assessment. In the work by Zhou et al. [55], a virtual data preprocessing technique and a matrix reduction algorithm are introduced to show the effectiveness in reducing the computational effort for determining the optimal placement set. The performance validation for the proposed algorithm is proven by applying the method of Lagrangian relaxation to calculate the lower bound of the minimal number of PMUs.
Chapter 3

Strategic Placement of Phasor Measurement Units with Optimal Number of Channels

3.1 Statement of the PMU Placement Problem

Power systems are assumed to operate in pseudo-steady-state due to the slow dynamics of system loads and generation. Hence, measurements of various quantities such as power flows, voltages, and currents at various substations are used to approximately determine the operating conditions of this pseudo-steady-state of the system. These measurements typically have time skew which may be in the order of minutes and therefore an estimate of the system state obtained based on these measurements will only be an approximation. Time skew among collected measurements is eliminated when the measurements are replaced by those provided by synchronized phasor measurement units (PMU). These devices are quickly becoming the preferred metering choice at bulk power transmission substations. They provide magnitude and phase angle measurements of bus voltages and branch currents for the positive-sequence components of three-phase signals. These measurements are time-stamped and phase angles are defined with respect to a common reference
determined by the global positioning satellite system. PMU measurements can improve the performance and capabilities of various network applications due to their unique features. One of the applications which will benefit most from availability of PMU measurements is the state estimator. In fact, if the entire network can be made fully observable by just using PMU measurements, then the state estimation problem will become linear and will be solved directly without requiring the use of iterative methods. If these devices can be strategically placed at proper substations in sufficient numbers, then such a transformation will be possible, making a very significant improvement in the performance of existing state estimators, essentially eliminating the issues of divergence due to numerical problems.

All of the aforementioned studies in Section 2.3 consider the PMU placement problem in such a way that these devices are assumed to have unlimited number of channels. Hence, placing them at a given bus ensures that in addition to the phasor voltage at that bus, phasor voltages of all its immediate neighbors will be available due to the monitored phasor currents along all the branches incident to that bus. In practice, every PMU comes with a channel limit and therefore a more realistic placement of the PMUs should take into account their varying channel capacities. The authors of [56] attempt to introduce the limit on the number of measurements each PMU can make and a modified PageRank placement algorithm is utilized in the importance modeling of the network nodes. However, the optimal numbers required to make the studied test systems observable are not provided explicitly. Rather, the effect of the number of measurements on the nodes to fully observe the network is given as a criterion for the usefulness of the method.

In this study, the PMU placement problem is revisited with the aim of relaxing the abovementioned assumption based on the fact that the bus voltage phasor and all current phasors along branches connected to that bus are available. The problem is reformulated where the number of channels for each PMU can be changed and the problem can be solved repeatedly to find the optimal locations. Furthermore, the formulation takes into account any existing injection measurements, in particular those virtual measurements provided by the zero-injections of passive buses. Afterwards, the formulation is extended to account for loss of PMUs so that the
final PMU measurement design remains robust against loss of a PMU due to device or communication link failures. Another contribution of this study is to recognize the effect of channel capacity of a given type of PMU on their optimal placement for network observability and to develop an optimal solution to the PMU placement problem given a specified number of available channels for the candidate PMUs.

The approach taken in this study is one of exhaustive search among all possible combinations at a given bus for a given limit on the number of available channels. In formulating the problem it is realized that for a given number of channel capacity there will be a finite combination of possible assignments of incident branches to a given PMU placed at a bus. Hence, the choices will increase with the number of incident branches for a given bus, but will remain bounded irrespective of the overall system size, thanks to the sparse interconnection of power system buses. The developed measurement placement procedure will be outlined and illustrated by examples in the succeeding sections. Precisely, the main goal is to allow optimal placement of PMUs which may have limited number of channels.

### 3.2 Proposed Formulation for the PMU Placement Problem

Formulation of optimal PMU placement problem for the case of varying channels will be briefly reviewed first. Subsequently, the revision of this formulation to account for a PMU loss will be described in Section 3.5.

Consider a PMU which has $L$ channels and installed at bus $k$ as shown in Figure 3.1. Also assume that bus $k$ is connected to $N_k$ number of buses. Note that the actual number of channels may be three times more since the phasor measurements are usually the positive-sequence components derived from sampled waveforms of three-phase signals. So, it is understood that the number of channels refers to the number of positive-sequence phasor measurements that can be produced by the considered PMU.
If the number of channels, \( L \), is larger than the number of neighbors \( N_k \), then a single PMU placed at the bus will provide phasor voltages at all its neighbor buses. Otherwise, there will be \( r_k \) combinations of possible channel assignments to branches incident at bus \( k \):

\[
r_k = \begin{cases} 
   N_k \binom{L}{N_k} & \text{if } L < N_k, \\
   1 & \text{if } N_k \leq L.
\end{cases}
\]

where the number of possible combinations of \( L \) out of \( N_k \) branches is defined as:

\[
\binom{N_k}{L} = \frac{N_k!}{(N_k - L)!L!}
\]  

(3.1)

Since a PMU is able to measure both the voltage phasor of the bus at which it is installed and the current phasors of all the lines in the neighborhood of this bus, the placement of PMUs becomes a problem with an objective of finding a minimal set of PMUs such that a bus must be observed at least once by the solution set of the PMUs. This leads us to define the binary connectivity matrix \( \mathbf{H} \) consisting of all possible combinations at a given bus for a given limit on the number of available
Chapter 3. Strategic Placement of Phasor Measurement Units with Optimal Number of Channels

channels such that each bus $k$ will have $r_k$ rows, each row containing $(L+1)$ nonzeros for the bus itself and its neighbor buses. However, when $N_k < L$, i.e., the number of branches incident to bus $k$ is less than the channel limit of the PMUs, the associated row needs to be kept unchanged. The channel limit constraints can thus be imposed so that a PMU placed at a bus will observe its neighboring buses by selecting the appropriate combination(s) of $L$.

Description of the procedure can be illustrated using a 7-bus system example shown in Figure 3.2. Assuming a channel limit of 2 for the PMUs, the number of rows for each bus is found to be $r_1 = r_5 = r_6 = r_7 = 1$, $r_2 = 6$, and $r_3 = r_4 = 3$. In fact, consider the buses connected to bus 2, which are buses 1, 3, 6, and 7; therefore, the 2-combinations of this set will result in the pairs $1-3$, $1-6$, $1-7$, $3-6$, $3-7$, and $6-7$. In this sense, each row associated with bus 2 in the matrix $\mathcal{H}$ will include a 1 corresponding to bus 2 and its neighbor pairs. Accordingly, let $\mathcal{H}$ be defined as

\[
\mathcal{H} = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 \\
\end{bmatrix}
\]
Using this approach, the relevant PMU placement problem can be formulated as follows:

\[ \text{Minimize } \sum_{i=1}^{n} w_i x_i \]

Subject to \[ H^T X \geq U \]  
\[ X = [x_1 \ x_2 \ \cdots \ x_n]^T \]
\[ x_i \in \{0, 1\} \]

where

\[ n = \sum_{j=1}^{N} r_j, \quad U = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{N \times 1}, \quad \text{and} \quad w = [1 \ 1 \ \cdots \ 1]_{1 \times n}. \]

Here, \( w \) represents the vector of installation cost of the PMUs, elements of which are assumed to be uniform for simplicity, and \( x_i \) are binary variables for the PMU placement, and \( N \) denotes the number of buses in the system. In the above matrix inequality, \( X \) represents a binary vector of all possible PMU channel assignments.
Correspondingly, the nonzero entries in $\mathbf{X}$ will point to the rows of associated buses, voltage angles of which can be observed by these PMU measurements.

The solution of the aforecited PMU placement problem where the specified channel limit for PMUs is 2, that is,

$$\mathbf{X} = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

yields a total of 3 PMUs, two of which are to be located at bus 2, and one at bus 4, enabling the entire network observability. Indeed, one PMU installed at bus 2 measures the voltage phasor at bus 2 as well as the current phasors for branches $2-1$ and $2-6$; whereas, the other PMU at bus 2 measures branch current phasors $2-1$ and $2-7$. Furthermore, a PMU located at bus 4 will measure the voltage phasor at bus 4 and current phasor on two of the incident branches $4-3$ and $4-5$.

Similarly, the location of PMUs for the IEEE 14-bus system with the channel limit of 3 is illustrated in Figure 3.3. The optimal solution for this case points out that 4 PMUs are required to achieve full network observability. A PMU located at bus 2 will observe buses 1, 2, 3, and 5; the one at bus 6 will observe buses 6, 11, 12, and 13; and so on.

![Figure 3.3: PMU placement for IEEE 14-bus system when the channel limit is 3.](image-url)
3.3 Modeling of Zero-Injection Buses

The procedure of forming the constraint equations is straightforward for a system without conventional measurements and/or zero-injections. However, when the zero-injection pseudomeasurements are considered, the method of topology transformation introduced in [35] should be utilized in reforming the constraint equations.

Consider the IEEE 14-bus system shown in Figure 3.4, where the dot next to bus 7 indicates that bus 7 is a zero-injection bus. It is obvious that if the phasor voltages at any three out of the four buses 4, 7, 8, and 9 are known, then the fourth one can be calculated using the power balance equations at bus 7 where the net injection is known to be zero. Topology transformation method is then used to merge the bus having the zero-injection measurement, with the buses connected to that bus. In the selection of the candidate buses, three different strategies are first considered and comparatively evaluated:

3.3.1 Case 1—Arbitrary Selection of Neighbor Buses

In this case, the bus which has the injection measurement is merged with one of its arbitrarily chosen neighbors.

3.3.2 Case 2—Selection of Neighbor Buses which Have Minimum Number of Neighbors

In this case, the bus which has the injection measurement is merged with its neighbor having the least number of neighbors.
3.3.3 Case 3—Selection of Neighbor Buses which Have Maximum Number of Neighbors

In this case, the bus which has the injection measurement is merged with its neighbor having the most number of neighbors.

![Network diagram and measurement configuration for the IEEE 14-bus system.](image)

**Figure 3.4:** Network diagram and measurement configuration for the IEEE 14-bus system.

For Case 1, the appropriate algorithm is developed so that any of the neighbor buses connected to bus 7, say bus 9, is chosen randomly. Since bus 8 has only one neighbor, it is selected in the second case; whereas, bus 4 is to be selected in Case 3 since it has five neighbors connected to it. The network will be updated after the merger of zero-injection bus and the selected neighbor bus into a new single bus. In this context, it is realized that the number of required PMUs can be reduced with the growing system size, or almost equivalently, the increased availability of zero-injection buses.

Apart from these suggested cases involving intuition, we have developed a more systematic methodology to incorporate the zero-injection buses into the problem formulation by using linear constraints as shown in the following case.
3.3.4 Case 4—Modeling of Zero-Injection Buses as Linear Constraints

In this case, zero-injection buses, which provide “free” measurements to the system, are incorporated into the optimization formulation as done in [42]. Particularly, zero-injections can be used to reduce the number of required PMUs by selectively allowing some buses to be unreachable by the PMU measurements, as long as these buses belong to a certain set of buses. This set is defined as the union of all zero-injection buses and their immediate neighbors.

Let us define a set $\mathcal{N}_i$ as a set of buses including zero-injection bus $i$ and all its neighbors. Assuming “$\ell$” zero-injection buses to be present in the system, the following set can be defined:

$$ \Phi = \bigcup_{i \in \mathcal{I}} \mathcal{N}_i = \mathcal{N}_{i_1} \cup \mathcal{N}_{i_2} \cup \cdots \cup \mathcal{N}_{i_\ell}, $$

where $\mathcal{I} = \{i_1, i_2, \cdots, i_\ell\}$ designates the set of zero-injection buses.

Thus, the inequality constraints in (3.2) can be reestablished based on the above considerations as follows:

$$ \begin{bmatrix} \mathcal{H}^T & \mathcal{C} \\ 0 & \mathcal{D} \end{bmatrix} \begin{bmatrix} \mathcal{X} \\ b \end{bmatrix} \geq \begin{bmatrix} \mathcal{R} \\ c \end{bmatrix} \quad (3.3) $$

where

$$ \mathcal{C}_{jk} = \begin{cases} -1 & \text{if } j \in \mathcal{N}_i \text{ and } k \in \{\Phi_j\}, \\ 0 & \text{otherwise}. \end{cases} $$

$$ \mathcal{D}_{jk} = \begin{cases} 1 & \text{if } j \in \{\mathcal{I}_i\}_{i \in \mathcal{I}} \text{ and } k \in \{\Phi_m\}_{m \in \mathcal{N}_i}, \\ 0 & \text{otherwise}. \end{cases} $$
Chapter 3. Strategic Placement of Phasor Measurement Units with Optimal Number of Channels

\[ \mathbf{X} \text{, } \mathbf{R} \text{, } b \text{, and } c \text{ are vectors of dimension } n \text{, } N \text{, } |\Phi| \text{, and } |\mathcal{I}| \text{, respectively, with } |\{\bullet\}| \text{ denoting the cardinality of a set; and the matrices } \mathbf{H}, \mathbf{C}, \mathbf{0}, \text{ and } \mathbf{D} \text{ in (3.3) are of sizes } n \times N, N \times |\Phi|, |\mathcal{I}| \times n, \text{ and } |\mathcal{I}| \times |\Phi| \text{, respectively. Also, } c_i \text{ are equal to } (|N_i| - 1) \text{ such that } i \in \mathcal{I}. \]

In building the matrices \( \mathbf{C} \) and \( \mathbf{D} \), the set \( \{\Phi_i\}_{i \in \mathcal{E}} \) is defined such that the elements of set \( \Phi \) are indexed or labeled by means of set \( \mathcal{E} \). For the sake of convenience, the first row associated with the first partitioned matrix on the left-hand side of (3.3) splits up into two parts in such a way that the zero-injection buses and their neighbors are heaped together on the top of the new matrix. In this way, the elements of the matrix \( \mathbf{D} \) are clustered in the order of union set \( \Phi \). Moreover, the elements of vector \( \mathbf{R} \) on the right-hand side of (3.3) take on a “0” for the variables related to zero-injection buses, and a “1” for those of the remaining buses.

For the sake of illustration, consider 7-bus system shown in Figure 3.5 where single-channel PMUs are used and the dots designate the zero-injection buses present in the system. Then, we can build the sets \( \mathcal{N}_{\text{Bus} \ 4} = \{3, 4, 5, 7\}, \mathcal{N}_{\text{Bus} \ 6} = \{2, 3, 6\}, \) and \( \Phi = \mathcal{N}_{\text{Bus} \ 4} \cup \mathcal{N}_{\text{Bus} \ 6} = \{2, 3, 4, 5, 6, 7\} \) based on the definitions above. In this context, Ineq. (3.3) will take form of Ineq. (3.4) as shown in the following:

\[
\begin{bmatrix}
[\mathbf{H}_1^T]_{6 \times 16} & -1 & -1 & -1 & -1 & -1 & -1 \\
[\mathbf{H}_2^T]_{1 \times 16} & 0_{1 \times 6} & 1 & 1 & 1 & 1 \\
0_{2 \times 16} & 1 & 1 & 1 & 1
\end{bmatrix} \begin{bmatrix}
\mathbf{X} \\
\mathbf{u}_2 \\
\mathbf{u}_3 \\
\mathbf{u}_4 \\
\mathbf{u}_5 \\
\mathbf{u}_6 \\
\mathbf{u}_7
\end{bmatrix}_{9 \times 22} \geq \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
1 \\
3 \\
2
\end{bmatrix}_{9 \times 1} \quad (3.4)
\]
The inequalities $b_3 + b_4 + b_5 + b_7 \geq 3$ and $b_2 + b_3 + b_6 \geq 2$ with $b_i \in \{0, 1\}$ ensure that there is at most one unobservable bus in these sets provided that their observability is realized via the use of zero-injection buses in the corresponding sets. Indeed, $b_6 = b_7 = 0$, meaning that bus 6 is not reached by a PMU by taking advantage of its being zero-injection bus and bus 7 is observed via zero-injection bus 4. Figure 3.5 also illustrates the installed 3 PMUs along with the associated branches through which the corresponding buses are observed.

![Figure 3.5: Configuration of 3 one-channel PMUs in 7-bus system.](image)

3.4 Effect of Network Sparsity on PMU Placement

Buses in typical power networks are known to be sparsely connected. However, sparsity of systems may vary significantly depending on the geographic and operational requirements. In order to study the effect of sparsity on the PMU placement, systems with increasingly dense bus interconnections are defined. This is accomplished by systematically adding connections between second, third, etc. neighbors.

First, the binary adjacency matrix $A$ is defined to describe the topology of the network in which the $ij$-th entry is 1 if there is a connection between bus $i$ and bus $j$, and zero otherwise. All diagonal entries will also be 1 by default. The matrix $A$
for the simple network of Figure 3.2 will be given by:

\[
\mathbf{A} = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 
\end{bmatrix}.
\]

This matrix will be referred to as single-hop connectivity matrix since it represents
the connectivity of the buses with their immediate neighbors. Those buses that can
be reached from a given bus by a single hop over an existing branch will contain
nonzero entries. Similarly, matrices representing systems where buses have direct
connections to those buses that can be reached in two, three or more hops in the
original network can be easily generated by multiplying the single-hop connectivity
matrix by itself as many times as the number of hops [57]:

\[
\mathbf{A}^{(m+1)} = \mathbf{A}^{(m)} \times \mathbf{A}. 
\] (3.5)

Let \( \mathbf{A}^{(1)} = \mathbf{A} \). Then, \( \mathbf{A}^{(m+1)} \) is defined as follows:

\[
\mathbf{A}^{(m+1)}(i, j) = \begin{cases} 
m + 1 & \text{if } \mathbf{A}^{(m)}(i, j) = 0 \text{ and } \mathbf{A}^{(m+1)}(i, j) > 0; \\
\mathbf{A}^{(m)}(i, j) & \text{if } \mathbf{A}^{(m)}(i, j) > 0. 
\end{cases}
\]
Hence, the 2-hop connectivity matrix $\mathbf{A}^{(2)}$ will be given by:

$$
\mathbf{A}^{(2)} = \\
\begin{bmatrix}
1 & 1 & 2 & 0 & 0 & 2 & 2 \\
1 & 1 & 1 & 2 & 0 & 1 & 1 \\
2 & 1 & 1 & 1 & 2 & 1 & 2 \\
0 & 2 & 1 & 1 & 1 & 2 & 1 \\
0 & 0 & 2 & 1 & 1 & 0 & 2 \\
2 & 1 & 1 & 2 & 0 & 1 & 2 \\
2 & 1 & 2 & 1 & 2 & 2 & 1
\end{bmatrix}
$$

Bus 1 2 3 4 5 6 7.

which shows all the bus pairs that can reach each other within two hops. From the 1-hop connectivity matrix, we can obtain all $(m+1)$-hop connectivity matrices, thus all possible $(m+1)$-hop routes.

The binary version of the multi-hop connectivity matrix $\mathbf{B}^{(m+1)}$ can be defined by simply replacing nonzeros in the matrix $\mathbf{A}^{(m+1)}$ by 1’s, as given below:

$$
\mathbf{B}^{(2)} = \\
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
$$

Bus 1 2 3 4 5 6 7.

The effect of reduced sparsity on the number and location of required PMUs can then be studied by using this matrix instead of the original connectivity matrix. The corresponding network connectivity is shown in Figure 3.6.

Again, from the newly formed network topology, we can easily build the matrix $\mathbf{H}$ consisting of all potential set of combinations incident to each and every bus.
When the PMU channel limit is assumed to be 2, the number of rows related to each bus in the new matrix $\mathbf{H}$ becomes $r_1 = 6$, $r_2 = r_4 = r_6 = 10$, $r_3 = r_7 = 15$, and $r_5 = 3$. In this case, the solution of the PMU placement problem still yields 3 PMUs as the optimal number; however, the locations for these PMUs will now be at buses 1, 4, and 7.

### 3.5 Optimal Placement Accounting for Single PMU Loss

The initial studies consider a simplified model to represent the “reach” of individual PMUs. Each PMU is assumed to provide the voltage phasor at the bus it is connected and the current phasors at all of its neighbors as well. This assumption is relaxed in [58] where the effects of channel capacity of a given type of PMU on their optimal placement for network observability are taken into account.

In this section, the formulation is extended to account for loss of PMUs so that the final PMU measurement design remains robust against loss of a PMU due to device or communication link failures.
It is worth mentioning that PMUs are prone to failures like any other measuring
device even though they are highly reliable. Therefore, it is necessary to guard
against such unexpected failures of PMUs. In [34] and [39], the primary set of
PMUs is backed up by a secondary set which is determined based on the same
optimization formulation. In this study, the formulation of (3.2) is modified as done
in [41–43] to ensure that each bus will be observed by at least two PMUs. This
ascertains that a PMU loss will not lead to loss of observability. In the integer linear
programming framework, this can be easily achieved by multiplying $U$ by 2, viz.,
$U = [2 \ 2 \ \cdots \ 2]_{1\times N}^T$.

In this regard, the solution of the aforesaid PMU placement problem where the
specified channel limit for PMUs is 2, will be given as:

$$X = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T$$

which yields a total of 5 PMUs, two at buses 2 and 4 each and the remaining one
at bus 3, enabling the entire network to be observable even when the measurements
from any one of the PMUs are lost.

Similarly, the location of PMUs for the IEEE 14-bus system with the channel
limit of 3 is illustrated in Figure 3.7. The optimal solution for this case requires
placement of 9 PMUs to achieve full network observability under loss of a single
PMU.

Figure 3.8 illustrates the results of solving the above problem of PMU placement
for the IEEE 57-bus system assuming no channel limits for PMUs, accounting for
loss of a single PMU and making use of zero-injection measurements based on the
fourth case. The solution validates that each and every bus in the network is reached
at least twice either by PMUs or zero-injection measurements located at the bus or
its neighbors. Zero-injection buses are designated by dots next to the bus names in
Figure 3.8.
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Figure 3.7: Reliable placement against single PMU loss for the IEEE 14-bus system when the channel limit is 3.

Figure 3.8: Zero-injections and optimally placed 22 PMUs for the IEEE 57-bus system assuming no channel limits.
Chapter 4

Simulation Results

4.1 Conventional PMU Placement with Fixed Channel Capacity

Simulations of the proposed method are carried out on various power systems. The binary integer programming problem is solved using the TOMLAB /CPLEX Solver Package [59]. The simulation results for the optimum number of PMUs with respect to channel limits for the cases where zero-injections are ignored and considered, are presented in Tables 4.1 and 4.2, respectively. As shown in Table 4.1, simulations are carried out using five IEEE test systems as well as one larger-size system with 4520 buses. Also, Table 4.2 illustrates the results of simulations performed on the IEEE 14-, 30-, 57-, and 118-bus systems. For the case without zero-injection measurements, the upper channel limit of the PMUs is determined by the maximum number of branches incident to a certain bus in the corresponding test system. For those cases where zero-injection measurements are considered, these limits are the number of the branches incident to the fictitious bus, which is created by merging one or several actual buses. Four ways to account for zero-injections are considered and compared via simulations. The third case appears to have an advantage over the others in particular when using multichannel PMUs. Second case may, however, be a better choice when single-channel PMUs are to be used.
As evident from Table 4.2, zero-injections help reduce the number of required PMUs. In Table 4.1, $\nu_{\text{min}}/N$ is the ratio of the minimum number of required PMUs to the number of buses in the system.

<table>
<thead>
<tr>
<th>Channel Limit for the PMUs</th>
<th>System under study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IEEE 14-Bus</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td></td>
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<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
<tr>
<td>$\nu_{\text{min}}/N$</td>
<td>0.2857</td>
</tr>
</tbody>
</table>

4.2 Impact of Network Sparsity on Strategic Placement of PMUs

Considering the densely connected topologies, revised topologies with 2-hop connectivity are obtained for five IEEE test systems. PMU placement problem is then solved using these revised systems and the results are shown in Table 4.3. Among the five power systems studied, it is observed that this ratio ranges from 27% to 33%
in Table 4.1, and from 10% to 21% in Table 4.3. In a similar vein, one can clearly observe how the loss of sparsity leads to strategic placement of smaller number of PMUs having larger number of channels.

**Table 4.2: PMU Placement with Zero-Injections for IEEE Test Systems**

<table>
<thead>
<tr>
<th>IEEE Test System</th>
<th>Number of Zero-Inj.’s</th>
<th>Channel Limit for the PMUs</th>
<th>Number of PMUs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CASE 1</td>
<td>CASE 2</td>
</tr>
<tr>
<td>14-Bus</td>
<td>1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>30-Bus</td>
<td>6</td>
<td>1</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>12</td>
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<td>4</td>
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<tr>
<td></td>
<td></td>
<td>7</td>
<td>11</td>
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<tr>
<td>57-Bus</td>
<td>15</td>
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<td>57</td>
</tr>
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<tr>
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<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>28</td>
</tr>
<tr>
<td>118-Bus</td>
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<td>1</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>28</td>
</tr>
</tbody>
</table>
### Table 4.3: Conventional PMU Placement with 2-Hop Connectivity for Five IEEE Test Systems

<table>
<thead>
<tr>
<th>Channel Limit for the PMUs</th>
<th>IEEE Test System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14-Bus</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td>20</td>
<td>8</td>
</tr>
<tr>
<td>23</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td></td>
</tr>
<tr>
<td>$\nu_{\text{min}}/N$</td>
<td>0.2143</td>
</tr>
</tbody>
</table>

#### 4.3 Reliable Measurement Design Against Loss of PMUs

PMU placement problem as formulated in Section 3.5 is solved for power systems of different sizes. The solutions for the optimum number of PMUs for different channel limits for the cases where zero-injections are ignored and considered, are presented in Tables 4.4 and 4.5. Table 4.4 presents solutions that are obtained for five IEEE test systems as well as for a 4520-bus utility system when zero-injections are ignored. When the zero-injections are taken into account, results change significantly as shown in Table 4.5 for four IEEE test systems. Once again, for the cases with and without zero-injection measurements, the upper channel limit of the PMUs is determined by the maximum number of incident branches to a bus in the corresponding test system. Among the six power systems studied, it is observed that this ratio ranges from 58% to 69%. As evident from Table 4.5, zero-injections help reduce the number of PMUs required for complete network observability while maintaining robustness against single PMU failure.
Table 4.4: Reliable Placement Against Loss of PMUs without Zero-Injections for Miscellaneous Power Systems

<table>
<thead>
<tr>
<th>Channel Limit for the PMUs</th>
<th>System under study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IEEE 14-Bus</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
</tr>
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<td>6</td>
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<tr>
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<td>9</td>
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<tr>
<td>10</td>
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<tr>
<td>12</td>
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<tr>
<td>13</td>
<td></td>
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<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

$\nu_{\min}/N$: 0.6429 0.6667 0.5789 0.5763 0.6300 0.6914

4.4 Illustration of Unified PMU Placement Schemes

In order to provide a more comprehensive picture of our overall methodology and draw attention to the effect of channel limits on PMU placement accounting for various combinations of abovementioned criteria, we have illustrated a number of PMU placement strategies as shown in Figures 4.1–4.12. In cases where the zero-injection measurements are considered, we have bounded our simulations merely by the fourth case since it allows for a more systematic treatment of zero-injections.
Table 4.5: Reliable Placement Against Loss of PMUs with Zero-Injections for IEEE Test Systems

<table>
<thead>
<tr>
<th>IEEE Test System</th>
<th>Number of Zero-Inj.'s</th>
<th>Channel Limit for the PMUs</th>
<th>Number of PMUs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CASE 1</td>
<td>CASE 2</td>
</tr>
<tr>
<td>14-Bus</td>
<td>1</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>30-Bus</td>
<td>6</td>
<td>26</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>15</td>
<td>15</td>
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<tr>
<td></td>
<td>6</td>
<td>16</td>
<td>15</td>
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<td>10</td>
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<tr>
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<td>11</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>57-Bus</td>
<td>15</td>
<td>44</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>24</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>24</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>24</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>118-Bus</td>
<td>10</td>
<td>111</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>75</td>
<td>76</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>62</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>61</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>61</td>
<td>63</td>
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<td>62</td>
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<td>7</td>
<td>62</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>62</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>62</td>
<td>63</td>
</tr>
</tbody>
</table>
Figure 4.1: Optimally placed 7 PMUs for the IEEE 14-bus system when the channel limit is 1 (ignoring the zero-injections).
Figure 4.2: Optimally placed 15 PMUs for the IEEE 30-bus system when the channel limit is 1 (ignoring the zero-injections).
Figure 4.3: Optimally placed 19 PMUs for the IEEE 57-bus system when the channel limit is 2 (ignoring the zero-injections).
Figure 4.4: Optimally placed 41 PMUs for the IEEE 118-bus system when the channel limit is 2 (ignoring the zero-injections).
Figure 4.5: Zero-injection and optimally placed 3 PMUs for the IEEE 14-bus system when the channel limit is 4.
Figure 4.6: Zero-injections and optimally placed 7 PMUs for the IEEE 30-bus system when the channel limit is 4.
Figure 4.7: Zero-injections and optimally placed 14 PMUs for the IEEE 57-bus system when the channel limit is 3.
Figure 4.8: Zero-injections and optimally placed 29 PMUs for the IEEE 118-bus system when the channel limit is 5.
Figure 4.9: Reliable placement against single PMU loss for the IEEE 14-bus system when the channel limit is 3 (considering the zero-injection).
Figure 4.10: Reliable placement against single PMU loss for the IEEE 30-bus system when the channel limit is 3 (considering the zero-injections).
Figure 4.11: Reliable placement against single PMU loss for the IEEE 57-bus system when the channel limit is 3 (considering the zero-injections).
Figure 4.12: Reliable placement against single PMU loss for the IEEE 118-bus system when the channel limit is 3 (considering the zero-injections).
Chapter 5

Concluding Remarks and Further Study

5.1 Concluding Remarks

This thesis presents a new problem formulation and its associated solution based on mixed integer linear programming method for obtaining the best locations of synchronized phasor measurement units. The main contribution of the new formulation is the way it accounts for the available number of PMU channels. Furthermore, zero-injection measurements are incorporated into the problem formulation in order to further minimize the required number of PMUs. Applying the developed technique to different size systems, it is observed that PMUs having more than 4 channels (positive-sequence) may not reduce the overall installation cost for medium-size systems. Moreover, it is observed that the channel limits which reduce the overall installation cost will be larger for larger-size and/or more densely connected systems. In order to demonstrate the effect of sparsity on the required channel limits, certain test systems are artificially modified by increasing connectivity in a systematic manner. The results indicate that densely connected systems will allow efficient utilization of PMUs with large number of channels.
This study also extends the results of conventional PMU placement to the case where the solution is expected to be robust against failure of any single PMU. Any existing injection measurements in the system, in particular those virtual ones at passive buses with no generation or load, are also accounted for in the modified optimization formulation. In this case, results of simulations on different type and size test systems imply that using PMUs with large number of channels does not minimize the investment in the measurement system. In most cases, having more than 4 channels (positive sequence) does not reduce the required PMU count. Furthermore, by strategic placement of PMUs, a very reliable metering design can be achieved by placing PMUs at less than 70% of the buses in the system. This number may be reduced significantly by taking advantage of zero-injection buses.

Ultimately, these results may be useful for the system planners as well as PMU manufacturers when they make decisions on the next set of PMUs to be purchased and installed or to be designed and marketed, respectively.

5.2 Further Study

We have studied and solved the problem of using PMUs with limited input capabilities to achieve complete observability of the network. In other words, it is intended to monitor at most a fixed number of currents from a bus. As a further study, the PMU placement problem can be reinvestigated by taking into account the fact that each PMU may have variations in channel capacity for a particular placement strategy. Additionally, the costs for the proposed placement strategy and the prospective placement procedures may be comparatively evaluated in order to determine the best option. Undoubtedly, novel methodologies can also be implemented for modeling of zero-injection buses and reliable PMU placement to investigate the feasibility of further reducing the number of PMUs required for entire network observability.
Appendix A

Functions and Scripts Used in the PMU Placement Algorithm

A.1 Read Network Parameters and Build the Single-Hop Connectivity Matrix \( A \)

function \([y \ NoBran \ branch \ external\_bus \ internal\_bus]\) =
readAndBuildA(PowerFlowInputData,NoBus)

```matlab
% read parameters of the network
fnet = fopen(PowerFlowInputData,'r');
line = fgetl(fnet);
line = fgetl(fnet);

iter1 = 0;
iter2 = 0;
iter3 = 0;

while 1
```

51
line = fgetl(fnet);
iter1 = iter1 + 1;
if line(1:4) == 'BRAN'
    break;
end
bus(iter1,1) = str2num(line(1:4));
end
bus(end,:) = []; % this entry corresponds to -9999

for ii = 1 : NoBus
    % specify external bus numbers
    external_bus(ii,1) = bus(ii,1);
    % internal bus numbers
    internal_bus(external_bus(ii),1) = it;
end

while 1
    line = fgetl(fnet);
    if line(1:4) == '-999'
        break;
    end
    iter2 = iter2 + 1;
cir(iter2,1) = str2num(line(17));
    if cir(iter2,1) == 0
        iter3 = iter3 + 1;
        From_Bus(iter3,1) = str2num(line(1:4));
        To_Bus(iter3,1) = str2num(line(6:9));
    end
end
NoBran = iter3;
From_Bus = internal_bus(From_Bus);
To_Bus = internal_bus(To_Bus);

branch = [From_Bus To_Bus];

% create the bus admittance matrix, Y, with jX = j1.0.
ys = 1; [rowBran colBran] = size(From_Bus);

for iter = 1 : rowBran
    Yi(ys) = From_Bus(iter,1);
    Yj(ys) = To_Bus(iter,1);
    Yv(ys) = 1; ys = ys + 1;
    Yi(ys) = To_Bus(iter,1);
    Yj(ys) = From_Bus(iter,1);
    Yv(ys) = 1; ys = ys + 1;
    Yi(ys) = From_Bus(iter,1);
    Yj(ys) = From_Bus(iter,1);
    Yv(ys) = 1; ys = ys + 1;
    Yi(ys) = To_Bus(iter,1);
    Yj(ys) = To_Bus(iter,1);
    Yv(ys) = 1; ys = ys + 1;
end

y = sparse(Yi,Yj,Yv,NoBus,NoBus);

A = spones(y) + zeros(NoBus,NoBus);

disp('Matrix A is found to be as follows: ');

ST = fclose(fnet);
A.2 Find the Required Number of PMUs for Complete Network Observability (Ignoring Zero-Injection Measurements)

```matlab
clear;
clc;

A = readAndBuildA('pfinput14.dat',14);
% A = readAndBuildA('pfinput30.dat',30);
% A = readAndBuildA('pfinput57.dat',57);
% A = readAndBuildA('pfinput118.dat',118);

%% reduce the network sparsity by adding 2nd, 3rd, etc. neighbors
% B2 = spones(A ^ 2) + zeros(size(A,1))
% B3 = spones(A ^ 3) + zeros(size(A,1))

[m,n] = size(A);

L = [];

for j = 1 : n
    for i = 1 : j
        if A(i,j) == 1
            if j ~= i
                L(i,j) = 1;
                L(j,i) = 1;
            end
        end
    end
end
end L;
```
ChannelLimit = input('Choose a channel limit for the PMUs: ');

H = [];

for k = 1 : n
    if sum(L(:,k)) < ChannelLimit
        V = find(L(:,k));
        T = sparse(1,n);
        T(1,k) = 1;
        T(1,V) = 1;
        H = sparse([H;T]);
    else
        V = nchoosek(find(L(:,k)),ChannelLimit);
        [a,b] = size(V);
        T = sparse(a,n);
        for i = 1 : a
            for j = 1 : b
                T(:,k) = 1;
                T(i,V(i,j)) = 1;
            end
        end
        H = sparse([H;T]);
    end
    fprintf('%d
',k)
end
H;

tic

f = ones(size(H,1),1);
Hnew = -H';
b = -ones(n,1);
N = length(f);
x_L = zeros(N,1);
x_U = ones(N,1);

IntVars = ones(N,1);
PriLev = 1;
cpxControl.EPGAP = 0.1/100;
cpxControl.TILIM = 60*5;

[x, slack, v, rc, f_k, ninf, sinf, Inform, basis, lpiter, ...
  glnodes, confstat, iconfstat, sa, cpxControl, presolve] = ...
cplex(f, Hnew, x_L, x_U, -inf*ones(n,1), b, ... 
cpxControl, [], PriLev, [], IntVars);

disp('The optimum number of PMUs is: ')

fprintf('%d\n',sum(x))

toc
A.3 Find the Required Number of PMUs for Complete Network Observability (Considering Zero-Injection Measurements)

clear;
clc;

A = readAndBuildA('pfinput14.dat',14);
A = readAndBuildA('pfinput30.dat',30);
A = readAndBuildA('pfinput57.dat',57);
A = readAndBuildA('pfinput118.dat',118);

[m,n] = size(A);

L = [];

for j = 1 : n
    for i = 1 : j
        if A(i,j) == 1
            if j ~= i
                L(i,j) = 1;
                L(j,i) = 1;
            end
        end
    end
end

L;

ChannelLimit = input('Choose a channel limit for the PMUs: ');

H = [];
for k = 1 : n
    if sum(L(:,k)) < ChannelLimit
        V = find(L(:,k));
        T = zeros(1,n);
        T(1,k) = 1;
        T(1,V) = 1;
        H = [H; T];
    else
        V = nchoosek(find(L(:,k)),ChannelLimit);
        [a,b] = size(V);
        T = zeros(a,n);
        for i = 1 : a
            for j = 1 : b
                T(:,k) = 1;
                T(i,V(i,j)) = 1;
            end
        end
        H = [H; T];
    end
end
fprintf('%d
',k)
end
H;

tic

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% for the IEEE 14-bus test system
C = zeros(size(H,2),4);
C(4,1) = 1;    C(7,2) = 1;
\[ C(8,3) = 1; \quad C(9,4) = 1; \]

\[ D = \text{zeros}(1,4); \]

\[ D(1,1:4) = -1; \]

\[ R = -\text{ones}(n,1); \]
\% use \( R = -2 \times \text{ones}(n,1) \) for reliable PMU placement, instead

\[ R(4) = 0; \quad R(7) = 0; \quad R(8) = 0; \quad R(9) = 0; \]

\[ c = -3; \]

\[ f = [\text{ones(size}(H,1),1);\text{zeros(size}(D,2),1)]; \]

\[ H_{\text{new}} = [-H',C;[\text{zeros}(\text{size}(D,1),\text{size}(H,1)),D]]; \]

\[ \text{RHS} = [R; c]; \]

\[ X = \text{bintprog}(f,H_{\text{new}},\text{RHS}); \]

\[ X = X(1:\text{size}(H,1)); \]

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% for the IEEE 30-bus test system

% \text{C} = \text{zeros(size}(H,2),17);
%
% \%
% \%
% C(2,1) = 1; \quad C(4,2) = 1; \quad C(6,3) = 1; \quad C(7,4) = 1;
% C(8,5) = 1; \quad C(9,6) = 1; \quad C(10,7) = 1; \quad C(11,8) = 1;
% C(21,9) = 1; \quad C(22,10) = 1; \quad C(24,11) = 1; \quad C(25,12) = 1;
% C(26,13) = 1; \quad C(27,14) = 1; \quad C(28,15) = 1; \quad C(29,16) = 1;
% C(30,17) = 1;
%
% D = zeros(6,17);
% D(1,1:7) = -1; D(1,15) = -1; D(2,3) = -1;
% D(2,6:8) = -1; D(3,7) = -1; D(3,9:11) = -1;
% D(4,11:14) = -1; D(5,12) = -1; D(5,14:17) = -1;
% D(6,9) = -1; D(6,11) = -1; D(6,14:15) = -1;
%
% R = -ones(n,1);
% R(2) = 0; R(4) = 0; R(6) = 0; R(7) = 0;
% R(8) = 0; R(9) = 0; R(10) = 0; R(11) = 0;
% R(21) = 0; R(22) = 0; R(24) = 0; R(25) = 0;
% R(26) = 0; R(27) = 0; R(28) = 0; R(29) = 0;
% R(30) = 0;
%
% c = [-7 -3 -3 -3 -4 -3]';
%
% f = [ones(size(H,1),1);zeros(size(D,2),1)];
%
% Hnew = [-H',C;[zeros(size(D,1),size(H,1)),D]];  
%
% RHS = [R; c];
%
% X = bintprog(f,Hnew,RHS);
%
% X = X(1:size(H,1));

% %%%% for the IEEE 57-bus test system
% C = zeros(size(H,2),39);
% C(3,1) = 1; C(4,2) = 1; C(5,3) = 1;
\% C(6,4) = 1; C(7,5) = 1; C(8,6) = 1;
\% C(9,7) = 1; C(11,8) = 1; C(13,9) = 1;
\% C(14,10) = 1; C(15,11) = 1; C(18,12) = 1;
\% C(20,13) = 1; C(21,14) = 1; C(22,15) = 1;
\% C(23,16) = 1; C(24,17) = 1; C(25,18) = 1;
\% C(26,19) = 1; C(27,20) = 1; C(29,21) = 1;
\% C(32,22) = 1; C(34,23) = 1; C(35,24) = 1;
\% C(36,25) = 1; C(37,26) = 1; C(38,27) = 1;
\% C(39,28) = 1; C(40,29) = 1; C(41,30) = 1;
\% C(43,31) = 1; C(44,32) = 1; C(45,33) = 1;
\% C(46,34) = 1; C(47,35) = 1; C(48,36) = 1;
\% C(49,37) = 1; C(56,38) = 1; C(57,39) = 1;
\%
\% D = zeros(15,39);
\% D(1,1:4) = -1; D(1,12) = -1; D(2,4:6) = -1;
\% D(1,12) = -1; D(1,12) = -1; D(2,4:6) = -1;
\% D(2,21) = -1; D(3,7:9) = -1; D(3,30:31) = -1;
\% D(3,7:9) = -1; D(3,7:9) = -1; D(3,30:31) = -1;
\% D(4,13:15) = -1; D(5,14:16) = -1; D(5,27) = -1;
\% D(5,14:16) = -1; D(5,14:16) = -1; D(5,27) = -1;
\% D(6,16:19) = -1; D(7,17) = -1; D(7,19:20) = -1;
\% D(7,17) = -1; D(7,17) = -1; D(7,19:20) = -1;
\% D(8,22:24) = -1; D(9,24:26) = -1; D(9,29) = -1;
\% D(9,24:26) = -1; D(9,24:26) = -1; D(9,29) = -1;
\% D(10,25:28) = -1; D(11,26) = -1; D(11,28) = -1;
\% D(11,26) = -1; D(11,26) = -1; D(11,28) = -1;
\% D(11,39) = -1; D(12,25) = -1; D(12,29) = -1;
\% D(12,25) = -1; D(12,25) = -1; D(12,29) = -1;
\% D(12,38) = -1; D(13,11) = -1; D(13,32:33) = -1;
\% D(13,11) = -1; D(13,11) = -1; D(13,32:33) = -1;
\% D(14,10) = -1; D(14,34:35) = -1; D(15,27) = -1;
\% D(14,34:35) = -1; D(14,34:35) = -1; D(15,27) = -1;
\% D(15,35:37) = -1;
\%
\% R = -ones(n,1);
\% R(3:9) = 0; R(11) = 0; R(13:15) = 0;
\% R(11) = 0; R(13:15) = 0; R(13:15) = 0;
\% R(18) = 0; R(20:27) = 0; R(29) = 0;
\% R(20:27) = 0; R(29) = 0; R(29) = 0;
\% R(32) = 0; R(34:41) = 0; R(43:49) = 0;
\% R(34:41) = 0; R(43:49) = 0; R(43:49) = 0;
\% R(56:57) = 0;
\% c = [-4 -3 -4 -2 -3 -3 -2 -2 -3 -3 -2 -2 -2 -2 -3]';
\%
\% f = [ones(size(H,1),1);zeros(size(D,2),1)];
\%
\% Hnew = [-H',C;[zeros(size(D,1),size(H,1)),D]];  
\%
\% RHS = [R; c];
\%
\% X = bintprog(f,Hnew,RHS);
\%
\% X = X(1:size(H,1));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% for the IEEE 118-bus test system
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
\% C = zeros(size(H,2),32);
\% C(3,1) = 1; C(4,2) = 1;  C(5,3) = 1;
\% C(6,4) = 1; C(8,5) = 1;  C(9,6) = 1;
\% C(10,7) = 1; C(11,8) = 1;  C(17,9) = 1;
\% C(26,10) = 1; C(30,11) = 1;  C(33,12) = 1;
\% C(34,13) = 1; C(35,14) = 1;  C(37,15) = 1;
\% C(38,16) = 1; C(39,17) = 1;  C(40,18) = 1;
\% C(59,19) = 1; C(61,20) = 1;  C(63,21) = 1;
\% C(64,22) = 1; C(65,23) = 1;  C(68,24) = 1;
\% C(69,25) = 1; C(70,26) = 1;  C(71,27) = 1;
\% C(72,28) = 1; C(73,29) = 1;  C(80,30) = 1;
\% C(81,31) = 1; C(116,32) = 1;
\%
\% D = zeros(10,32);
\% D(1,1:5) = -1;  D(1,8) = -1 ;  D(2,5:7) = -1;
\% D(3,5) = -1;  D(3,9:11) = -1;  D(3,16) = -1;
\% D(4,12:18) = -1;  D(5,11) = -1;  D(5,15:16) = -1;
% D(5,23) = -1; D(6,19) = -1; D(6,21:22) = -1;
% D(7,20:23) = -1; D(8,23:25) = -1; D(8,31:32) = -1;
% D(9,26:29) = -1; D(10,24) = -1; D(10,30:31) = -1;

% R = -ones(n,1);
% R(3:6) = 0; R(8:11) = 0; R(17) = 0;
% R(26) = 0; R(30) = 0; R(33:35) = 0;
% R(37:40) = 0; R(59) = 0; R(61) = 0;
% R(63:65) = 0; R(68:73) = 0; R(80:81) = 0;
% R(116) = 0;

% c = [-5 -2 -4 -6 -3 -2 -3 -4 -3 -2]';

% f = [ones(size(H,1),1);zeros(size(D,2),1)];

% Hnew = [-H',C;[zeros(size(D,1),size(H,1)),D]];

% RHS = [R; c];

% X = bintprog(f,Hnew,RHS);

% X = X(1:size(H,1));

disp('The optimum number of PMUs is: ')
fprintf('%d
',sum(X))
toc
Appendix B

IEEE Test Systems Data Used in the PMU Placement Algorithm

This appendix section contains the data regarding the IEEE 14-, 30-, 57-, and 118-bus test systems [60], which are utilized in our simulations. The system information of these IEEE test systems is shown in Table B.1 given below:

<table>
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<tr>
<th>Test System</th>
<th>Number of Branches</th>
<th>Number of Zero-Injections</th>
<th>Zero-Injection Bus(es)</th>
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## B.1 IEEE 14-Bus Test System Data

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### Branch Data

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-999 LOSS ZONES FOLLOWS 1 ITEMS

1 IEEE 14 BUS

-99 INTERCHANGE DATA FOLLOWS 1 ITEMS
### B.2 IEEE 30-Bus Test System Data

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-999 BRANCH DATA FOLLOWS

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| 1 | 3 | 1 | 1 | 1 | 0 | 0.0452 | 0.1652 | 0.0408 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
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| 3 | 4 | 1 | 1 | 1 | 0 | 0.0132 | 0.0379 | 0.0084 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
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TIE LINES FOLLOWS 0 ITEMS -999

END OF DATA
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**08/25/93 UW ARCHIVE**

**IEEE 57 Bus Test Case**

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1 IEEE 57 BUS

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TIE LINES FOLLOWS 0 ITEMS -999

END OF DATA
### B.4 IEEE 118-Bus Test System Data

**08/25/93 UW ARCHIVE**

**100.0 1961 W IEEE 118 Bus Test Case**

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END OF DATA
Bibliography


