Hardware and Software Implementation of a Passive Ultra-short Baseline Array Sonar

A Thesis Presented

by

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Abstract

Underwater mine detection and removal is of great importance for both military and commercial fleets. Currently, trained dolphins locate possible mines in the water column and attach a marker buoy. Divers then deploy satchel charges at the suspected sights at obvious personal risks. Autonomous underwater vehicles (AUVs) are currently being tested to replace divers in this effort, and they would track acoustic pingers to reach the possible mine location. Beacon tracking by autonomous underwater vehicles is clearly a vital component in this application. In this research we designed and constructed an ultra short baseline (USBL) passive sonar used to provide a method of bearing estimation. An acoustic pinger and a passive sonar were designed to operate at $10.5\,kHz$. The sonar employs a unique three-element, L-shaped array with a 1/4 wavelength hydrophone separation. The acoustic pinger and sonar also provide a noncoherent 4-FSK downlink to the AUV for command and control applications. The power consumption of the passive sonar including the DSP processor is less than one-half Watt and has a form factor of less than $50\,cm^3$, enabling power-efficient, compact deployments. A maximum-likelihood based estimator was developed to estimate the bearing angles of the acoustic pinger. The estimator is immune to multipath for delay spreads not exceeding $250\,ms$. Angle estimates are produced by a coarse initial grid search followed by several Newton-Raphson iterations.
Simulation results indicate that this system can estimate bearing angles to less than 1 degree of standard deviation with a received signal-to-noise ratio of 6 $dB$. 
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Chapter 1

Introduction

This thesis concerns the hardware and software development of a passive sonar for beacon tracking by an autonomous underwater vehicle (AUV). The motivation for this effort is mine neutralization. Trained dolphins are currently being used to find mines on the seabed and then attach a marker buoy. Navy divers then deploy satchel charges in the proximity of the markers, at great risk to their lives. The method of beacon tracking is the main focus of this thesis. We present an ultra short baseline (USBL) array used to provide a means of locating underwater mines. Here the beacon system is composed of a transducer, signalling circuitry, and power amplification. The passive sonar system is comprised of an L-shaped array of three hydrophones from Teledyne Benthos, filtering and amplification circuitry, and a DSP board which samples the three signals and estimates the relative angles of elevation and azimuth of the beacon. Due to size and drag constraints imposed by the AUV, a three-element, bilinear array was implemented in an L-shaped, 1/4-wavelength configuration.


1.1 Thesis Outline

This thesis is organized as follows. The remainder of this chapter provides background information on the transmitter board, receiver board, the sonar algorithm for bearing estimation, and related work. In this chapter, we also present the design constraints of the frequency and timing used for transmitting the sonar pinging signal and the impedance matching requirement for transmitting maximum power through the transducer. Chapter 2 describes the design circuitry of the transmitter board used to power the transducer/projector. Chapter 3 describes the receiver board onboard the AUV. Specifically, the main components of the receiver used to take the differential signal of the hydrophones, filter the incoming signal, gain adjustments, and power regulators are described in Chapter 3. Chapter 4 presents the algorithm for detecting the angle of elevation and azimuth from the received signal. The maximum log-likelihood estimate is used for the calculation of bearing using Newton-Raphson iterations.

This chapter will give a brief overview of the design constraints of the transmitter board and receiver board. It also presents the basic idea behind the algorithm for bearing estimation. Related work is also summarized in this chapter and then compared to the work presented in this thesis.

1.2 Transmitter Board Background

The transducer submerged in water will transmit a 10.5 kHz tone at a repetition rate of 4 Hz and a pulse duration of 4 ms. A Colpitts oscillator was used to create the sinusoidal signal and is buffered to prevent any load from affecting the oscillation frequency. A 555-timer is then used to create the necessary timing scheme to allow the oscillating signal to pass according to the timing constraint given above. The signal is then passed on to buffers that remove any
CHAPTER 1. INTRODUCTION

DC offset and then amplified by an 800 W car audio amplifier. The network line is impedance matched with a custom transformer and the signal finally powers a submerged transducer. As an extension to previous work, referenced at the end of section 1.5, the transmit board also permits the transmission of a 2-bit symbol using noncoherent frequency shift keying (FSK). As will be seen, this is achieved by superimposing one of four possible gated sinusoids with the sonar ping. The transmitter board and design is further discussed in detail in Chapter 2.

1.3 Receiver Board Design Constraints

The receiver built into the lamprey contains three hydrophones in an L-shape configuration. The L-shape configuration is unique in design. The hydrophone signals each pass through a differential amplifier for proper gain and buffering. The signal is then passed through a filter that allows a pass-band of approximately $8 - 11 \text{ kHz}$. Upon filtration, the signal passes through a programmable gain amplifier (PGA). As the lamprey approaches the signal source the PGA steps down the gain so as not to damage the front end of the receiver. The signal is then passed into a DSP chip which contains the bearing estimator algorithm. The power regulators that power the receiver are also discussed in Chapter 3.

1.4 Bearing Estimator Algorithm Overview

The estimator used is the maximum likelihood estimator (MLE). The L-shape configuration of the hydrophones is unique because two hydrophones can be used to estimate the elevation angle from one leg of the “L” and two hydrophones from the other leg are used to estimate the azimuth. Therefore there is a common hydrophone between the two when estimating. This was a major consideration
when preparing the algorithm. By using the truncated Taylor series for the log likelihood to approximate the zeros of the cost function, one was able to estimate the bearing angles. Specifically, a Newton-Raphson search was implemented. The starting point at which the algorithm begins is chosen by taking predefined points in elevation and azimuth and starting at the point which yields the minimum cost. The derivation of this algorithm is discussed in more detail in Chapter 4.

1.5 Related Work

Bearing estimation through USBL arrays is not only useful for mine detection with reference from a marked transducer. It is also used for guidance of autonomously operating vehicles (AOV’s), providing a means of guidance when docking large ships, mooring motion monitoring, or diver tracking. In [1], the Oceanographic Systems Laboratory (OSL) at the Woods Hole Oceanographic Institution (WHOI) have previously developed a USBL navigation system for a Remote Environmental Monitoring Unit (REMUS), where their new transceiver combines all the electronics and hydrophones in a compact, stand-alone package. The vehicle contains no other electronics except a power source and RS-232 interface cable. Their system is composed of a 4-channel planar hydrophone array. An embedded DSP processor performs the real-time signal to determine the bearing of the received signal. The 4-channel array is also acoustically baffled to prevent noise and multipath from the backside of the array and can be used as an external module. The vehicle can initiate interrogation pings as requests and replies for bearing estimation. If there was no detected signal a “no reply” message is sent back. Their bearing estimation is performed through conventional beam forming techniques. The four channels are fed into a matched filter. This is the equivalent of cross-correlating the received signal and a ref-
CHAPTER 1. INTRODUCTION

ereference signal. The way the signal is detected is by comparing the filter output with a programmed threshold \[2\]. The largest output of the four channels is then processed by the beamforming algorithm to determine the bearing. As one can see the components of hardware in their system is similar in concept to the hardware presented in this thesis. However, their system is composed of a 4-channel array, whereas we use an L-shaped 3-channel configuration. The reduction of hydrophones reduces both the vehicle drag and the hotel load significantly. Their algorithm processes the outputs of the four channels to provide relative power vs. the bearing angles. The bearing is estimated by calculating the peak of the beamformer surface. Whereas our algorithm uses maximum log-likelihood estimate from the received complex observation using a gradient decent search to find the bearing. Also, the operating frequency of their pinger is a range from \(20 - 30\, kHz\).

In \[3\], Sheridan presents an acoustic position measuring system (PMS) using a \(50\, kHz\) pulse with a phase reversal during the middle of the pulse. This pulse is then driven into a projector by a power amplifier. The received signal from the four hydrophone array setup is passed into two phase-lock reversal detectors during the middle of the pulse. The system uses a counter to determine the “number of cycles of a reference frequency between the two mid-pulse reversals.” This method creates a \(\Delta t\) between the two hydrophones. From this time difference, the bearing angles are calculated. T Also, the hardware used and operating frequency is different from the presented hardware designed later described in Chapter 3. Sheridan uses a four hydrophone array.

In \[4\], the concept of an inverted USBL is presented. The system is placed on an escorting ship and is used to trace an underwater vehicle (UV). The system measures direction and slant range to the pinger/target. With the addition of the ships attitude and heading reference (AHRS), the location of the UV can
be detected. Obviously, this means of detection is different than the algorithm presented here because of the additional information required to estimate the UV bearing. Larsen calculates the bearing angles from the range rate. The range rate is resolved by the Doppler frequency shift of the transducer reply signal used in [5, 6]. Doppler frequency shift concept is the main principle in Doppler Velocity Log (DVL) [6]. The range rate is proportional to the size of a known velocity vector projection onto the transducer and hydrophone and the unknown angle between this vector and the transducer can then be determined.

Wong and Zoltowski in [7] find bearing by estimation of signal parameters via rotational invariance techniques, using ESPRIT [8]. Multiple Signal Classification (MUSIC) is also another method used to estimate the bearing [9]. MUSIC uses a different eigenstructure method and provides better accuracy with respect to bearing estimation when compared to ESPRIT [7]. When ESPRIT eigenvalues are nearly identical, MUSIC makes it unnecessary to use a normalization estimator to find any course reference estimates. Due to this, it is also unnecessary to have all three vector-field components going through the wavefront because the resulting array manifold still keeps its relationship with the source direction cosine.

In [10], the system is composed of two parts: an above water and underwater system. The underwater system is composed of a starboard sonar array and a transducer operating at a frequency of 150 kHz. [10] takes mutual coupling among array elements into consideration. The method of estimation used is based on multi-subarray subspace fitting, referred to as EDMSF. Based on eigen-subspace, the estimate to the number of sources is calculated. The number of sources is important when using EDMSF. For an \( L \)-element uniform array, when estimating signal subspace, the eigenvalue decomposition of the \( L \times L \) matrix can also be used to calculate how many signal sources are present. The results of
this decomposition help to reduce the computational load in determining the
direction of arrival (DOA) of the received signal. The method EDMSF presented
in [10] uses an improved iteration method for eigenvalue decomposition. This
improved method takes advantage of the fact that the covariance matrix is
positive definite on DOA signal processing. Upon every computation of the
maximum eigenvalue and eigenvector by the improved iteration, a new matrix
is constructed based off the original matrix where the maximum eigenvalue of
the new matrix is set to the second largest eigenvalue of the original matrix.

In [11], an super short baseline (SSBL) array is presented. The system
designed utilizes an equilateral triangle configuration for the hydrophones sepa-
rated by $\lambda$. However, in standard SSBL systems, the hydrophone elements are
separated by $\lambda/2$, which prevents angle aliasing and allows for bearing estima-
tion through inter-element phase differences. Separation by $\lambda$ makes bearing es-
timation more difficult and less accurate. The method described in does not use
cross-correlation techniques or additional hardware to estimate bearing. Many
pulses at various frequencies are transmitted where the phase for each pulse is
stored. The phase values that are stored lie along a straight line whose slope
is calculated by using a least squares fit [11]. The slope relates to the best-fit
line in relation to the phase difference as a function of frequency data. To de-
termine the estimation of the angles of arrival of the received signal, one must
know the relative time difference of the signal arrival for each hydrophone. The
estimate of the time delay depends on many factors such as signal bandwidth,
SNR, noise bandwidth, and multipath. Also, the hydrophone receiver electron-
ics convert the square wave signal into a sinusoidal pulse. Then the signal is
digitized and processed. Because the transmitter uses a square wave, the elec-
tronics design is simpler, the time of arrival computations are simpler and more
accurate compared to a traditional sinusoidal signal being transmitted.
A bearing calculation method in [12] uses the arrival times of a communications packet on each element in the equilateral triangle three hydrophone array. A modified USBL scheme is used where various signals are transmitted, where the chirp section of the signal determines the phase of the received signal at each element. The arrival times of the signal are found using the phase differences of the three signals. \( \omega \) is the center frequency of the phase affects the magnitude of the three vectors, however, this does not affect the direction. Then the dot product of each vector can provide a phase difference from the complex angle of the dot product. The phase difference then provides an equivalent time of arrival difference between each of the elements. Using a vector sum, a bearing estimate can be provided using the arrival times of the signal. Also, the estimated azimuthal angle is independent of the depression angle. This method utilizes phase differences to find arrival times and then compute the angles of arrival of the transmitted signal.

In [13], a circular array of eight hydrophones is used for bearing estimation. The frequency response of the receiver is in the range of \( 10 - 30 \text{kHz} \). A sinusoid with a pulse width of .75 ms is transmitted with variable frequency ranging from \( 11 - 25 \text{kHz} \). The system focuses on using super resolution beamforming techniques to circular arrays which involve taking snapshots of the covariance matrix to form an estimate described in [14]. Then by using the MUSIC spectrum the bearing estimates can be attained.

Bayesian tracking is discussed in [15]. The CPDF algorithm is three step algorithm that is presented. The steps consist of pre-processing, detection, and bearing estimation. The “Bearing-Time Record” is an array of intensity level measures on a 2-dimensional grid of bearing and time which is input into the system. At each scan, the input data is mapped by a Bayesian CPDF detector to form a posterior PDF, which undergoes a 2nd-order Markov diffusion process.
CHAPTER 1. INTRODUCTION

Bearing estimation through synthetic aperture using frequency measurements and mean-squared error is presented in [16]. A synthetic aperture is a vector addition of the received signals. The received signal from the array is amplitude and phase adjusted, where the vectors are added. The system does not rely on the delay of each observation from each element to determine bearing. To estimate the bearing, Fau and Wolcin, use the steered beam output between two time intervals to estimate the frequency. This provides two frequency measurements where Doppler frequency difference between the hydrophones can shape a relationship to bearing estimation. It is noted that the bearing is inversely proportional to the transducer operating frequency. Also, the bearing also has a strong dependence on the magnitude squared of the velocity difference between the hydrophones and without this information, bearing cannot be determined. As one can see, this is another case of Doppler shift frequency method to calculate bearing, different from using MLE and gradient decent search method.

Cadre and Tremois [17] model the input data from the hydrophones as a hidden Markov model (HMM) and use Viterbi dynamic programming algorithm to find the bearing estimate. HMM consists of two levels of discretization of the state-space. Therefore, the probability of positions are derived from the probability of velocities and given an observation it tries to maximize the probability density function (PDF).

In [18], a transducer pinging at 125 kHz is used and the receiver measures the time it takes for the signal to be received at each hydrophone. However, before communication takes place, the hydrophones transmit a ping to one another to acknowledge their separation distance. Again the system uses the timing difference between each hydrophone to determine a bearing. Besides operating frequency, the system differs from the system presented in this thesis by means
Koda and Shibata in [19] transmit an ultrawave FM wave whose frequency modulated by a sinusoidal reference signal. The transmitted carrier frequency is $95 \text{kHz}$ with a modulation frequency of $1 \text{kHz}$. The signal is received by an array of four hydrophones in a circular configuration and is fed into a direction detector. Angular frequency shifting method is used for bearing determination. Using the separation distance of the hydrophones, and making sure the array element spacing is less than $\lambda/2$, the transmitted frequency can be chosen arbitrarily. The phase difference between the demodulated signal is used to determine bearing angles. The geometrical relations between the phase and bearing angles are given in [19]. This method uses phase differences between the individual elements to find the bearing angles which differs from the method described in this thesis. The system also uses a different transmission scheme.

As one can see, from the several references and methods briefly summarized in this section, the L-shape configuration of the hydrophones that we use is unique. The hardware used in our system also uses a different operating frequency and sending scheme than the papers referred to in this section. Further, the addition of a 4-FSK downlink to the AUV is an extension to previous work from Randell and Collins [20], who have designed a prototype of an underwater narrow beam telemetry channel between two stations, where the direction of arrival (DOA) is required to adjust the orientation of the transducer to maintain a reliable communications link.

### 1.6 Summary

This chapter gave an introduction to the USBL array project. A brief overview of the sonar system was given, including the transmitter and receiver design constraints, and a description of the maximum likelihood estimator used to
estimate the elevation and azimuthal angles of the received signal. Also related work was briefly summarized to compare with the actual setup and methods used for bearing estimation presented in this thesis. The following chapters will describe in detail the main components of each subsystem, specifically the transmitter, receiver, and MLE algorithm.
Chapter 2

Transmitter Board

This chapter describes the fundamental design of the circuits used for the sonar transmitter. The transmitter board creates the sinusoidal signal and provides the proper gating scheme to be passed to the projector. The topics that will be discussed are the essential transmitter components such as the sonar and communication oscillators, the summing circuit used to combine both sinusoidal signals, the gating of the oscillating signal, and the filtration and amplification of certain frequency components of this signal. The impedance matching and high power amplification necessary to power the projector is also discussed. The design constraints pertaining to each section will also be acknowledged in detail.

2.1 Sonar Oscillator

Presented in Figure 2.1 is the drive circuit for the sonar projector. For this project, a frequency of 10.5 kHz was selected as the pinging frequency for the lamprey. A sinusoid with a frequency of 10.5 kHz is transmitted with a duration of 4 msec and repetition rate of 4 Hz. As shown in Figure 2.4, the 555-timer circuit is operated in the astable mode and creates a square wave with an appro-
appropriate pulse sequence. Due to the fact that the required square wave imposes a duty cycle that is not normally capable of the 555 timer, a diode is placed in parallel with $R_2$ such that during the charging cycle for a high output, $R_2$ is bypassed, resulting in a small RC time constant and allowing for a reduced duty cycle.

For the signal to propagate a sinusoidal wave must be present. This signal is created in the subcircuit labeled *Sonar Oscillator* in Figure 2.1 and is implemented using a Colpitts oscillator with an oscillation frequency of $10.5\, \text{kHz}$. The frequency of the oscillator is determined by two capacitors and one inductor, governed by the following equation

$$f_o = \frac{1}{2\pi \sqrt{L \left( \frac{C_1 C_2}{C_1 + C_2} \right)}}. \quad (2.1)$$

where the capacitor connecting the collector and emitter of the transistor, $C_3$, in our case, was chosen to be $0.1\, \mu\text{F}$. A 2N222 transistor was chosen due to its fast switching characteristic. To have a stable oscillation there are a few requirements. The instability criteria must be met; the input impedance of the oscillator must have a negative value neglecting any reactive components. Also the magnitude of the negative resistance must also be larger than the magnitude of the inductor. Resonant devices $C_2$ and $L$ were chosen to be $80\, \text{nF}$ and $5.1\, \text{mH}$ respectively to produce our desired frequency of $10.5\, \text{kHz}$.

The oscillator will be operating continuously to avoid any transient effects and to maintain stable operation and consistency throughout experimentation. The Colpitts oscillator is known to have a DC offset in its output signal approximately equivalent to the voltage supply used to bias the circuit, in this case $5\, \text{V}$. The DC offset is not a desired feature of the circuit and its removal will be discussed later. The output signal of the Colpitts oscillator signal is buffered
using a voltage follower to prevent loading of the oscillator that could alter the
frequency or amplitude of the sinusoidal signal. The unit-gain voltage follower is
constructed using an operational amplifier with direct feedback to the negative
terminal from the output, where the input signal is fed through the positive
input terminal of the op-amp. Analog Devices’ OP-27 was utilized in this case,
for its wide bandwidth and extremely low bias current (±10 nA), voltage off-
set (80 nVpp) and low-noise feature (3.5 nV/Hz). The op-amp is powered with
±12 V rails to ensure that the op-amp voltage rails pk-pk is large than the signal
amplitude pk-pk, thereby eliminating clipping.

2.2 Communication Oscillators

There are four other oscillators used for communication purposes to send spec-
ified commands to the lamprey other than the pinging operation. The frequen-
cies for these oscillators 8.25, 8.75, 9.25, and 9.75 kHz, will be summed with
the sonar signal of 10.5 kHz and transmitted through the projector, shown in
Figure 2.2.

The frequencies of the communication oscillators are spaced 500 Hz apart
and the reason will be discussed in a later section. Again, a .1 μF capacitor
was used to couple the collector and emitter, and using (2.1), \( C_2 \) and \( L \)
were calculated according to the frequencies listed above. The biasing circuit to power
each of the Colpitts oscillators are identical.

Only one of the four communication oscillators will be summed with the
sonar signal. The selection of the communication frequency is accomplished by
means of four separate transistor switches operating in the saturation region,
one for each oscillator. The output of the oscillators are connected to the col-
lector of each transistor. When a 5 V bias is applied to the base, the signal
is passed through from collector to emitter, which is grounded. Therefore in
Figure 2.1: Transmitter Circuit to drive the projector with the sonar and communication signal.
Figure 2.2: Sonar and communication Colpitts oscillators.
theory, when a certain frequency is undesired from the oscillators, that signal is shorted to ground, and the remaining one signal is passed through to be further summed with the 10.5 kHz sonar signal. Since all the communication signals are merged together after the switches, a 100 kΩ resistor is placed in series with all the oscillators’ signals after the switches to isolate the signal from the ground connections. Therefore, when three of the four signals are grounded, the desired signal still passes through. Nevertheless, there is some voltage drop across the 100 kΩ resistors, therefore reducing the amplitude of the selected communication signal. Due to this reduction from the isolating resistors, each communication oscillator will also be buffered similar to the sonar oscillator using an OP-27 operational amplifier to prevent any load on the oscillator with a voltage gain of approximately 4.

2.3 Summer Circuit

The sonar signal of 10.5 kHz and the selected communication signal will be summed together before being gated through the 555-timer. As shown in Figure 2.1, the output of the two buffers, which are the sonar and communication signal, are summed. However, these oscillating signals contain a large DC-offset, as this is a factor mentioned previously concerning the Colpitts oscillator characteristics. Therefore, summing the two signals each containing a large DC-offset could result in a combined signal whose DC-offset is as high as 12 V. There is an additional requirement for some of the remaining voltage rails of the op-amp of the summer circuit to be used for the actual oscillating signal, whose amplitude is approximately 5 V. This leads to the use of a DC-blocker capacitor after each of the buffers for the oscillators. This eliminates the DC-offset in each of the signals, and leaves the pure oscillating AC signal of 10.5 kHz and one of the four frequencies of the communication signals mentioned previously. This
Figure 2.3: Summer circuit to add the sonar and one of four communication oscillators.
CHAPTER 2. TRANSMITTER BOARD

filtration allows for the input signals to the summer not exceed the voltage rails of the op-amp, therefore eliminating any potential clipping.

The scheme for sending two frequencies simultaneously was designed by implementing a summer circuit with 1.5 $M \Omega$ resistors on the output of the sonar and communication buffers and the summer feedback network, shown in Figure 2.3. Using high impedances reduces the load on the buffers. From the output of the summer is a variable voltage divider and another DC blocker capacitor of 1 $nF$ as a cascade to the precious filter after the oscillator buffers. The output of the summer is then gated with the 555-timer, as described below.

2.4 555-Timer Circuit

The combined signal will be gated using a 555-timer’s output signal in astable mode, which is a continuous cycling mode on the output of the timer chip, shown in Figure 2.4. This creates the pinging effect required for the project. The oscillating signals will be allowed to pass through the next stage of the circuit at every high pulse of the 555-timer. With this method, the output of the 555-timer can control an npn $Q2N2222$ transistor switch, that will regulate the passing of the signal. When the 555-timer has a high level pulse, the BJT switch ($Q2N2222$) will turn on and short the oscillating signal to ground, however, when a low level pulse is present, the npn switch is off and the oscillating signal is passed. However, if the signal is connected to the collector and the 555-timer is controlling the base, then the timing for the 555-timer would have to be inverted to function as required. The reason for this being that when a high pulse is output from the 555-timer, the transistor will be turned on and the signal will be shorted to ground through the emitter. Therefore, one would need to invert the timing intervals. The requirement for this project calls for the signal to pass at a rate of 4 $Hz$ with a pulse duration of 4 $ms$. The equation
that governs the high and low durations are:

\[
\text{high duration} = .693 (R_1 + R_2) C \\
\text{low duration} = .693 R_2 C
\] (2.2) (2.3)

As one can see from 2.2 and 2.3, the frequency and duty cycle of the 555-timer is dependent upon the RC network in the circuit. The 555-timer operates by the control of two internal comparators, where the capacitor in the RC network charges to \( \frac{2}{3} \) of the voltage supply, and an internal flip-flop switches the output logic. At a point in a low logic period, the external capacitor begins to discharge through \( R_2 \), and so on.

There are two ways to invert the timing intervals to allow the correct switching for our application. One way is to reverse low and high time intervals so the high pulse widths endure for the period for which the signal should not pass, being about 246 ms. However, the 555-timer cannot achieve duty-cycles close to 100%, but a duty cycle much less than 50% is achievable and can aid in the
correct timing to allow for the appropriate switching in this application. Since this is attainable, then the inverse of the above scheme can be used to allow the signal to pass for 4 ms at a rate of 4 Hz. To allow for a very fast charge during the high logic cycle, a diode is placed in parallel with $R_2$ to bypass it during this cycle and quickly charge the external capacitor. This design will allow $R_1$ and $R_2$ to act independently from each other. This method allows for a very small duty cycle, nevertheless, the signal is the inverted waveform required. Now the output of the high pulse width is approximately 4 ms and low pulse width is 246 ms from the output of the 555-timer. This gating scheme inverts the square wave of timer's signal, therefore a BJT inverter is placed between the 555-timer and npn switch to compensate for this inversion effect. The output of the 555-timer is fed into the base of the BJT inverter and the collector is used as the output to feed the base of the oscillator switch. This switching scheme bypasses the 555-timer’s inability to reach high duty cycles and the diode placed in the RC network allows for the necessary low duty cycle not attainable otherwise. The pulse width and frequency of the 555-timer circuit has been constructed to allow for convenient adjustment, through a potentiometer.

2.5 Power Amplifier

The output of the switch that the 555-timer controls to properly gate the summed signal consisting of the sonar and communication oscillator contains a pulse of 4 ms in duration at a rate of 4 Hz. As mentioned previously, the DC-offset has been removed using DC-block capacitors, therefore, this signal will not consume a majority of the voltage rails when buffered or amplified. Since the lamprey will be at a distance from the projector transmitting this signal, a means of amplification is necessary. The PA-60EU linear power amplifier, manufactured by Apex Microtechnology, was selected for this application, due to its
Figure 2.5: Power amplifier and high pass circuit for the transmitter.

main features being high continuous current ratings up to 15 A, a bandwidth of up to 300 kHz, low quiescent current, and wide supply voltage of up to ±20 V. The signal is input into the positive operational amplifier terminal, to avoid signal inversion. A high-pass design was chosen to eliminate residual DC offset and low frequency noise, shown in Figure 2.5. The gain of this setup is given by

\[ G = \frac{V_o}{V_{in}} = 1 + \left( \frac{R_{18}}{R_{27} + \tilde{Z}_{C_{27}}} \right) \]

The impedance of the capacitor increases as the frequency is decreased, therefore the series combination of \( C_{27} \) and \( R_{27} \) increases in impedance as the frequency decreases and the resulting gain decreases due to the fraction \( \frac{R_{18}}{R_{27} + \tilde{Z}_{C_{27}}} \) decreasing in magnitude. As the frequency becomes larger, the capacitor’s impedance begins to decrease and this fraction’s magnitude increases. At a high enough frequency, the capacitor acts like a short circuit and the gain is
now given $1 + R_{18}/R_{27}$. Upon examination of the output signal of the transmitter circuit, during the off pulse times, slight traces of the oscillator signal were apparent and a polypropylene capacitor (add capacitance value here) was placed across the voltage sources powering the transmitter circuit board to cancel this effect. Upon experimentation, it was further realized that this power amp did not deliver sufficient power to the projector and an 800 W car audio amplifier was used, shown in Figure 2.6. The reason for its use is further explained in the projector section of this chapter. The completed transmitter circuit that produces the sonar and communication signals is shown Figure 2.7.
Figure 2.7: This figure shows the actual transmitter board used to provide the sonar signals.
Figure 2.8: Simulated output of the projector drive circuit. A 10.5 kHz sonar signal and 8.25 kHz communication signal are summed and gated with a pulse width of 4 ms, and a pulse repetition rate of 4 Hz.

2.6 Circuit Simulation

The transmitter circuit was simulated using Orcad PSpice®. The simulated output signal of a single pulse from the power amplifier is shown in Figure 2.8 with a pulse width of 4 ms and two combined carrier frequencies of 10.5 kHz and the communication signal at 8.25 kHz. The FFT (Fast Fourier Transform) of this signal is shown in Figure 2.9, and as one can see a majority of the signal’s frequency components is concentrated at frequencies being 10.5 kHz and 8.25 kHz. This shows the correct summation of the two sinusoidal signals and proper gating through the 555-timer.
Figure 2.9: Spectral density for Figure 2.8
2.7 Projector

The projector is a transducer used to transmit the signal containing the sonar and communication sinusoidal wave combined. It was characterized by experimentation to construct an appropriate circuit model for the projector. In Figure 2.10, the equivalent projector circuit model is shown. The actual projector is shown in Figure 2.13. To maximize the power transfer driving the projector from the power amplifier, one must match the impedance of the output of the amplifier to the equivalent impedance of the projector. Using the equation, $2\pi f_0 = 1/\sqrt{L_s C_p}$, we want to maximize the power delivered, $P_d = \Re\{\frac{1}{2}V_o I_o^*\}$ at the frequency $f_0$. During oscillation, the inductor $L_s$ and the capacitor $C_p$, cancel each other and the equivalent circuit model becomes the circuit depicted in Figure 2.10, with $L_s$ and $C_p$ removed. Through some algebra and circuit analysis, it is further realized that

$$P_d = \frac{V_{RMS}^2 R_m}{(R_m + R_o)^2}$$
And during matched impedance conditions,

\[ P_d = \frac{V_{RMS}^2}{4R_m} \]  

The experiment setup to characterize the projector impedance consists of a resistor, \( R_o \), which is selected and placed in series with the projector to allow for half the voltage drop across the resistor. The unknown projector impedance is given by:

\[ Z_l = R_o \left( \frac{V_i}{V_o} - 1 \right) \]

Where the ratio \( V_i/V_o = \exp(j2\pi\Delta f) \). Using a function generator as the source to the projector, the frequency was swept, and a two-channel oscilloscope was used to monitor the voltage on the resistor and the source. If the reactive of the projector was negative, then the projector has an equivalent capacitance, otherwise inductive. Upon data analysis, it was further realized that the projector is capacitive, and a 5.6 \( mH \) inductor will be set in series with the projector to help match impedances and null the capacitive nature of the projector. Upon this design, the output resistance of the transmitter circuit will be matched with the real part of the projector impedance for further impedance matching, thus allowing for even more power transfer to the projector and improve the propagation distance of the pinger signal.

The matching inductor of 5.6 \( mH \) was required to cancel the projectors equivalent capacitance. However, upon experimentation it was realized that the inductor, at the operating frequencies between 8 – 11 kHz, was approximately 1 \( mH \). This induces the realization that the projector's impedance seen by the amplifier was not purely composed of a real component as designed, and therefore a majority of the transmit power was consumed by reactance.
Therefore through means of a transformer with a suited turns ratio, a matched impedance environment was established. Upon experimentation it was realized that the inductor, at the operating frequencies between $8 - 11kHz$, was approximately $1mH$. This induces the realization that the projector's impedance was not purely composed of a real component, and therefore a majority of the transmit power was consumed by reactance and the $5.6mH$ inductor was not solving the impedance matching problem. Observing the equation for power transfer to the projector with the reactance cancelled by an appropriate inductor, 2.4, one can see that the power can be increased by increasing the voltage rails on the final stage amplifier so the gain can be increased or by reducing the resistance seen by the amplifier, $R_m$. To use both methods of improvement for power, it is advantageous to use car audio amplifiers that can supply an abundant source of power and expect a low output impedance of $4\Omega$. However, since the projector impedance was found to be roughly $50\Omega$, we used a power transformer that will change the impedance seen by the amplifier output when connected to the projector to $4\Omega$. Using the equation, $R_{eq} = (N_p/N_s)^2R$, it was calculated that a turns ratio of $N_p/N_s = 1/3.5$ was needed for the transformer. The equivalent circuit diagram of a non-ideal transformer is given in Figure 2.11, where $L_M$ is the magnetizing inductance which accounts for the fine permeability of the magnetic core, $R_M$ accounts for the magnetic core loss, $L_{eq}$ accounts for the flux leakage into the air, and $R_{eq}$ accounts for winding loss, where $R_L$ and $C_L$ represent the model of the projector, given to be $50\Omega$ and $41nF$ respectively.

The transformer specs were submitted to Sowter Transformers and an LM65 transformer was designed for our application with the following specifications: $L_M = 1\ mH$, $R_M = .01\Omega$, $L_s = 8\ mH$, $R_s = .05\Omega$ and $L_{eq} = .001\ mH$ shown in Figure 2.12. A car audio amplifier by Visonik V308XT (800 Watts Peak Power) was ordered which will expect from $2\ mV$ to $100\ mV$ input signal level.
and a load of 4Ω, which the LM65 transformer matches and provides as the input impedance of the projector. This transformer has solved the matching impedance problem.

2.8 Summary

The topics presented discuss the design and construction of the transmitter board. Colpitts oscillators were used to generate a sinusoidal signal operating at 10.5 kHz, and 4 communication oscillators to provide a 4-FSK downlink to the AUV for command and control. These signals were buffered and combined before being gated. Gating was accomplished by means of a 555-timer and switching circuitry to achieve the proper timing scheme. The gated signal was then amplified by two power amplifiers, due to more transmission power needed as discussed previously. Impedance matching and the use of a transformer was necessary to maximize power transfer to the projector. The signal is then transmitted and will propagate through the medium, specifically ocean water. The signal will be received by the AUV using externally mounted hydrophones. The received signal is then processed and analyzed in the embedded receiver board to calculate the elevation and azimuthal angles and therefore proceed in the correct direction.
Figure 2.12: This figure shows the Sowter custom made transformer.
Figure 2.13: This figure shows the projector used to transmit the sonar and communications signals.
Chapter 3

Receiver Board

This chapter describes the receiver board of the lamprey which will process the incoming signals transmitted by the projector. The essential stages of the receiver board consist of buffering and amplification, filtration, and further appropriate gain selection. The voltage or potential difference from the hydrophone is input into an instrumentation amplifier with a proper gain. This output is followed by a Chebyshev filter to reduce noise and unwanted frequency components outside of the communication and sonar band (8 – 11 kHz). Upon filtration, this signal is passed through a programmable gain amplifier (PGA) to adjust the gain as the AUV approaches the projector as the signal amplitude is increasing to protect the onboard receiver electronics. The signal is then offset by 1.5 V before inputting the DSP processor. This offset is needed because the DSP chip requires an input voltage between 0 – 3 V and therefore a 1.5 V offset is appropriate in this stage. Power regulators to provide the proper voltage from the 12 V battery string to the receiver electronics are also discussed.
A hydrophone is a two lead device piezoelectric transducer that will create an electrical signal when introduced to pressure waves. The hydrophones are used to listen to the transmitter’s pinging consisting of the sonar and communication signal. The hydrophones that will be utilized as the ears for the lamprey were researched and a sensitivity of about $-201 \text{ dB re } 1\mu\text{Pa @ } 20^\circ\text{C}$ will be employed. The Teledyne Benthos AQ-2000 hydrophone, shown in Figure 3.1, was selected for this application due to its specifications in sensitivity and physical features, since size must also be acknowledged. The main equation that is realized for the hydrophone receive level is given by: $SPL = 20\log V_{RMS} - OCV$, where $SPL$ is the sound pressure level at the hydrophone, $OCV$ is the hydrophone sensitivity, and $V_{RMS}$ is the voltage produced by the hydrophone from incoming signal. In our case we select a desired $V_{RMS}$ on the hydrophone and reverse engineer the equation to realize how much power the projector should transmit to receive a certain voltage level at the hydrophone end. The calculation should
also accommodate for spreading loss of about 59dB @ 1 km and absorption loss of 1dB @ 1 km which will be subtracted from OCV.

Syntactic foam from CMT materials will be used as a mounting base for the three AQ-2000 hydrophones for acoustic dampening of the hydrophones’ sensitivity behind the base. This base material will help reduce confusion for the lamprey of the signal source location. Foam core will be utilized for trial fabrication of the hydrophone mounting base, as syntactic foam is expensive, even in small dimensions. Since the impedance of the hydrophones is very high (500 MΩ), the open-circuit voltage produced from them must be buffered using a differential amplification setup, as to not load the hydrophones.

### 3.2 Instrumentation Amplifier Design

As stated previously the hydrophones will produce a potential difference across the two leads in response to pressure waves transmitted by the projector. Since the differential voltage of the hydrophone is desired and the impedance of the hydrophones is very high (500 MΩ), the use of an op-amp is appropriate for this application. The op-amp in general has a very high input impedance, which will prevent load on the hydrophones, and can be configured in such a way to have a high common mode rejection (CMR). Therefore, the voltage output from the leads of the hydrophone were first fed into a two op-amp buffer differential circuit design shown in Figure 3.2, where each lead of the hydrophone is input into a separate op-amp and the voltage difference is amplified by the following equation: \[ \frac{V_{out}}{V_{in}} = \frac{V_1 - V_2}{V_{in}} = 1 + \frac{R_2}{R_1} + 2\frac{R_2}{R_G} \]

Nevertheless, upon inputting a sinusoidal test voltage using the function generator, inconsistent results occurred. The reason being that \( V_1 \) propagates through two op-amps and \( V_2 \) travels through only one op-amp, and when one of the input signals contains frequencies greater than the flat portion of the op-amp gain curve, the \( V_1 \) signal attenuates more
than the $V_2$ signal. This undesired effect causes signal unbalance and the CMR is sacrificed [21].

The instrumentation amplifier was redesigned to use three op-amps, shown in Figure 3.3. This design consists of two stages. The front-end stage consists of a buffer amplifier stage that equally amplifies both leads of the signal, where there are three resistors linking the two buffers. The final stage provides differential amplification of this buffered signal. When the hydrophone produces a voltage potential across its leads, the negative feedback of op-amp $U_1$ causes the voltage at the top of $R_{gain}$ to be equivalent to input $V_2$, and similarly the negative feedback of op-amp $U_2$ causes the voltage at the bottom of $R_{gain}$ to be equal to input $V_1$. This design produces a voltage drop across $R_{gain}$ that is equivalent to the difference of the input, being $V_1 - V_2$. This causes a current to pass through $R_{gain}$ and an ideal op-amp allows no current to flow into the feedback loops of the two inputs of the op-amps, therefore the same current flows through the resistors $R_2 = R_3$. As expected, the current will cause a voltage drop between
CHAPTER 3. RECEIVER BOARD

Figure 3.3: Three op-amp instrumentation amplifier design.

the outputs of the two op-amps $U_1$ and $U_2$ equivalent to:

$$V_{3-4} = (V_2 - V_1) \left( 1 + \frac{2R_1}{R_{gain}} \right)$$

The 3rd stage of the instrumentation amplifier is another differential amplifier that amplifies the difference in voltage across the output of op-amp $U_1$ and $U_2$ by a gain of $\frac{R_3}{R_4} = \frac{R_5}{R_6}$. Therefore the final gain of this three op-amp design instrumentation amplifier is given by:

$$\frac{V_{out}}{V_2 - V_1} = \left( 1 + \frac{2R_2}{R_{gain}} \right) \frac{R_6}{R_4}$$

The resistors $R_{gain}$, $R_2 = R_3$, $R_4 = R_5$ and $R_6 = R_7$ were selected to be $1\,k\Omega$, $6\,k\Omega$, $1\,k\Omega$, and $6\,k\Omega$ respectively. This selection produced a final gain of $78\,V$ for the instrumentation amplifier. The three op-amp design also allows for very convenient adjustment of the gain for both inputs simultaneously by just
modifying the value for $R_{\text{gain}}$, therefore the use of a potentiometer would be appropriate. The common mode rejection using the three op-amp design proved to be very effective. A PSpice simulation with 1 mV input for the common mode output is shown in Figure 3.4 and proved to be low and suitable for the front-end buffering and minor amplification in the receiver board.

### 3.3 Chebyshev Filter

In order to better process the signal being transmitted, it is imperative that unwanted frequencies are filtered out. These detrimental frequencies are commonly on the low-end of the frequency band that exist below sea level and some actually caused by mechanic vibration due to the movement of the lamprey. Therefore the output of the instrumentation amplifier for each hydrophone will pass through an LC network to filter these non-fundamental frequencies. A third order active Chebyshev bandpass filter was chosen whose pass-band was
designed to relay frequencies $8 - 11\, kHz$, with a pass-band ripple of $.2\, dB$, center frequency of $9.5\, kHz$, and approximately $30\, dB$ attenuation $1\, kHz$ outside the pass-band, shown in Figure 3.5. The equations used for the inductor and capacitor pairs of the filter are given as follows:

\[
L_1 = \frac{c_1 R_1}{\Delta \omega} \quad (3.1)
\]
\[
C_1 = \frac{1}{\omega_0^2 L_1} \quad (3.2)
\]
\[
C_2 = \frac{1}{\Delta \omega c_2 R_1} \quad (3.3)
\]
\[
L_2 = \frac{1}{\omega_0^2 C_2} \quad (3.4)
\]

where $c_1$ and $c_2$ are the component values for a third order ladder filter.

Since the filter design called for a loss of $30\, dB$ in the stop-band, a third order Chebyshev filter design was chosen. The Chebyshev filter has a loss factor $6\,(n - 1)$ larger than a Buttersworth filter, where $n$ is the filter order and the loss factor of a Buttersworth filter is $6n$. Therefore, a third order Chebyshev filter should provide a $30\, dB$ loss in the stop-band. The use of a Chebyshev filter results in a ripple loss in the pass-band, which can range from $.01\, dB$ to $1\, dB$. This loss is related to the ripple given by: $1 + \alpha = 10^{L_r/10}$, where $\alpha$ is the ripple loss in $dB$. An additional quantity $\beta$ is also calculated where $\beta = sinh\left(\tanh^{-1}(1/\sqrt{n})\right)$. The component values for a third order ladder Chebyshev filter are first calculated so that the inductor and capacitor values can then be equated from 3.1-3.4, where $c_i = \frac{a_i a_{i-1}}{c_{i-1}(\beta^2 + \sin^2[(i-1)\pi/n])}$ and $a_i = 2\sin\left(\frac{(2i-1)\pi}{2n}\right)$, which are the normalized susceptances and reactances of a Buttersworth filter at the center frequency $f_c$ [22]. The frequency response of the filter depicted in Figure 3.5 was simulated and is presented in Figure 3.6.

However, upon testing of the Chebyshev filter, its inoperability was realized.
CHAPTER 3. RECEIVER BOARD

Figure 3.5: Active band-pass filter with center frequency of $9.5 \text{kHz}$

Figure 3.6: Simulated frequency response of the active filter in Figure 3.5
The culprit was recognized to be the inductors being utilized. The feedback network was fed into the inverting terminal of the op-amp, the inductor values were reduced to $2.2\, mH$ all around, and the capacitors were adjusted accordingly until an appropriate frequency response resulted from the output of the filter, shown in Figure 3.7. The final filter will be laid out on a PCB board, and therefore the filter response must not be dependent upon positioning of the filter $LC$ components. Nevertheless, this was the case, and the frequency response was dependent upon inductor location. There was mutual inductance between the components, since they were positioned close together. Also, since the inductor and capacitor values were only adjusted and not based off Chebyshev calculations, the true Chebyshev characteristics were sacrificed. Therefore, this design could not be implemented. Since most of the unwanted frequencies underwater are on the low end of the frequency band, a high pass filter was designed, however, with only $12\, dB$ attenuation, and this approach was also not utilized. It was further realized that using an active filter design was causing the
mutual inductance problem, and a passive Chebyshev filter design was pursued, shown in Figure 3.8. This design was tested and proven to be a stable filter independent of $LC$ positioning and location; its frequency response is shown in Figure 3.8.

The laid out version of the Chebyshev filter on printed circuit board (PCB) is shown in Figures 3.9-3.11.

Figure 3.11 shows the connection method between the modular components of the receiver, specifically, the instrumentation amplifiers for the three hydrophones, the Chebyshev filters for the three channels and the power regulators
Figure 3.9: The instrumentation amplifiers with a gain of $78 \frac{V}{V}$.

Figure 3.10: This figure shows the Chebyshev filter designed with a center frequency of $9.5kHz$. 
that power the electronics. The entire receiver board will connect as shown by bus pins in a stacked fashion.

### 3.4 Programmable Gain Amplifier

The output of each Chebyshev filter will pass through a programmable gain amplifier (PGA), shown in Figure 3.12. The PGA will be used to step down the gain as the lamprey’s location becomes closer to the transmitter or step up the gain as the distance from the transmitter increases. The MAXIM532 digital to analog converter (DAC) was selected for this application, where the internal op-amp and feedback resistors are utilized for the PGA. The DAC code is controlled through SPI interface and sets the feedback resistor value to control the gain, given by:

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-4096}{\text{DAC Code}}
\]

Where the $\text{DAC Code}$ can range from 1 to $2^{12} - 1$. However, there was extensive noise in the output of the DAC from negative power regulators and from
Figure 3.12: Programmable gain amplifier, whose gain is set by the DAC bits, controlled through SPI interface.

the positive supply dropping below the minimum required supply voltage during high current drain from lamprey operation. Therefore, a 4-channel digital potentiometer from Analog Devices AD5204 was used in place of the MAX532, depicted by the squared resistor set by DAC code in Figure 3.12. This provides the same means of operation as the MAX532, however only requires a positive supply, therefore eliminating any noise in the output signal of the digital potentiometer. The resistance of the digital potentiometer is given by:

\[ R_{WB}(DAC\ CODE) = \frac{DAC\ CODE}{256} R_{AB} + R_W \]

where \( R_{AB} \) is the nominal resistance of the digital potentiometer of 100 kΩ and \( R_W \) is the wiper contact resistance of 45 Ω.

The output of the PGA is then fed into a summer circuit shown in Figure 3.12, to provide a +1.5 VDC offset of the received signal for the input require-
ments of the feedback for the DSP chip, being an input voltage between 0 and 3 V. The layed out version on PCB of the PGA is shown in Figure 3.13.

3.5 DC Voltage Regulators

The power source for the receive board electronics will be powered through a string of batteries producing a total of 12 V. The most efficient way to produce the required voltage rails for the onboard electronics with respect to practicality was to use DC voltage regulators outputting the required voltage for the various components from a single voltage input. The voltages needed were originally ±5 V for the OP467 quad op-amps used for the instrumentation amplifiers and summer stage, ±12 V for the PGA, −1.5 V for the summer DC offset, and finally 3.3 V to power the DSP chip. Since the digital potentiometer was used in place of the DAC for the PGA, only +5 V was required for the PGA. However, since
the motor on the lamprey will need $3 - 6\,A$ of current during operation, the voltage levels drop from $12\,V$ to $8\,V$, and since the minimum voltage rails the op-amps can function properly are $\pm 5\,V$, they will be powered on the $\pm 5\,V$ regulated lines, to ensure proper regulation and stable operation during the current surges created by the lamprey on the $12\,V$ source. The input voltage to the $+5\,V$ regulators needs to be at least higher than the desired output voltage and the $-5\,V$ regulator needs in input of at least $+3.5\,V$ to maintain the output voltage, therefore the fluctuation of the $+12\,V$ supply voltage will not pose a problem on the active receiver components. Appropriate voltage regulators that can supply the necessary current to the receive board components have been selected and constructed, shown in Figure 3.14. The current requirements of the receive board are as follows: $40\,mA$ max for the 4 op-amps operating at $\pm 5\,V$, $180\,\mu A$ for the three PGAs operating at $+5\,V$, $89\,mA$ for the DSP chip operating at $3.3\,V$, and finally minimal current for the summer circuit at $-1.5\,V$. Upon testing of the receive board, the final schematic for layout will be completed for actual PCB layout construction. The circuit design for the DC regulators are shown in Figure 3.14. The layed out version on PCB of the power regulators is shown in Figure 3.15.

\section{3.6 Summary}

As one can see, the receiver electronics were designed to specification, constructed and tested. Two different instrumentation amplifiers were tested for buffering and amplifying the differential voltage of the hydrophones. The three op-amp design was chosen as this provided the best CMR and stability. The output of the instrumentation amplifiers were fed into 3rd-order passive Chebyshev ladder filters with a center frequency of $9.5\,k\,Hz$ and bandwidth of $3\,k\,Hz$. Active filters were experimented with but due to instability, passive filters were
Figure 3.14: DC regulators' circuit design for the receive board.
Figure 3.15: This figure shows the power regulators that power the receiver electronics.

used as discussed previously. The output of each filter was fed into PGA’s that controlled the gain of the received signal. The PGA was designed by using a digital potentiometer in the feedback loop of an op-amp. This component was necessary because the received signal amplitude increases as the AUV approaches the projector which can lead to damaged components on the receiver board. The output of the PGA’s is further offset by 1.5 V because the input level expected by the next stage, the DSP processor, expects an input level between 0 − 3 V. Power regulators are used to step down the 12 V power supply from a string of batteries to power the onboard receiver electronics. Negative power supply sources were also needed to power certain components. This negative power source was achieved by inverting regulators. Now that the received signal has been amplified, filtered, and processed; it can be analyzed by the DSP chip containing the estimation algorithm to provide an estimate of the bearing angles.
Chapter 4

ML Estimation & Gradient

Decent Search Algorithm

This chapter presents the algorithm used to provide a bearing estimation for azimuthal and elevational angles. The geometrical configuration of the receiver hydrophones is given, where a new basis derivation and maximum log-likelihood equation for angle estimation are derived. This derivation of the maximum likelihood estimator is given in detail. The derivatives with respect to θ and φ are calculated to find the global minimum of the log-likelihood function. Also, the method of Newton-Raphson iterations is introduced as a means to the gradient decent search when solving for the MLE. The second-order derivatives with respect to θ, φ, and θφ are found to construct the inverse Hessian matrix. This matrix is used as the variable step size in the Newton-Raphson search. A course grid search is also utilized to find the first step in estimation. This search helps avoid local minima. Numerical results are simulated in Matlab, where plots of the estimation route and accuracy based off the SNR is shown at the end of the chapter. For mathematical notation in this chapter, vectors are underlined.
4.1 Sonar Model Setup

The geometrical setup of the angle of arrivals with the hydrophones is shown in Figure 4.1. The distance given by $\lambda$ is one wavelength of the transmitted sinusoidal signal from the origin. Where the range of the elevation and azimuthal angles range from $-\frac{\pi}{2} \leq \theta, \phi \leq \frac{\pi}{2}$. In order to find a specific point in this system, specifically $(x_0, y_0, z_0)$, one can simply use the Pythagorean theorem on the triangle in the plane with the red dashed line as the hypotenuse, which in this case is given by:

$$x_0^2 + y_0^2 + z_0^2 = \lambda^2. \quad (4.1)$$
The next steps are to find $z_0$ and $x_0$ in terms of $\lambda, \theta, \phi$ and from the vertical triangle shown in 4.1

$$\sqrt{x_0^2 + y_0^2} = \lambda \cos \phi \quad (4.2)$$

and combining 4.1 and 4.2:

$$\lambda^2 \cos \phi + z_0^2 = \lambda^2 \rightarrow z_0 = \lambda \sin \phi, \quad (4.3)$$

also from the triangle on the plane including the point $y_0$,

$$\sqrt{x_0^2 + y_0^2 \sin \theta} = x_0 \quad (4.4)$$

and combining 4.2 and 4.4 results in:

$$x_0 = \lambda \cos \phi \sin \theta. \quad (4.5)$$

So in this case, one wavelength corresponds to $\lambda \sin \phi$ on the z-axis, and $\lambda \cos \phi \sin \theta$ on the x-axis. Also, $y_0 = x_0 / \tan \theta = \lambda \cos \phi \cos \theta$.

Rotation by $\theta$ in the clockwise direction when looking at the origin about the z-axis is achieved by the matrix

$$R_{z\theta} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ 

For example, $[1 \ 0 \ 0]^T$ is now rotated to $[\cos \theta \ -\sin \theta \ 0]^T$.

Now one must find a new basis to relate the wavelength to its span on the axes. Elevational rotation is in the plane containing the z-axis and $[\sin \theta \ \cos \theta \ 0]^T$. These vectors are orthogonal. A third vector, $[a \ b \ c]^T$, orthogonal to the
above two vectors satisfies:

\[ [a \ b \ c]^T \mathbf{z} = 0 \implies c = 0, \]

\[ [a \ b \ c]^T [\begin{array}{rrr} \sin \theta & \cos \theta & 0 \\ 0 & 1 & 0 \end{array}]^T \implies asin \theta + bsin \theta = 0 \implies b = -atan \theta \]

and since \( a^2 + b^2 = a^2(1 + tan^2) \) we choose \( a = 1/\sqrt{1 + tan^2} = |cos| = cos \) for unit norm. The matrix

\[
\begin{bmatrix}
\sin \theta & 0 & \cos \theta \\
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0
\end{bmatrix}^T = [ v_1 \ v_2 \ v_3 ]^T
\]

converts the standard basis to that composed by the columns of this matrix. In this new basis, one wishes to rotate around \( v_3 \), in a counter-clockwise fashion, when looking at the origin. This is done by the rotation matrix

\[
\begin{bmatrix}
cos \phi & -sin \phi & 0 \\
sin \phi & cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

in the new basis. The rotation expressed in the old, (original, standard) basis,
Consider the 2-D subspace containing all the vectors orthogonal to $[0 \ 1 \ 0]^T$. These are expressed as $[K_x \ 0 \ K_z]^T$ for some $K_x, K_z$. This subspace may be rotated in the azimuthal plane, yielding vectors of the form $[K_x \ 0 \ K_z]^T \rightarrow [K_x \cos \theta \ -K_x \sin \theta \ K_z]^T = K_x v_3 + K_z v_2$. In the new basis, the rotated vector is $[0 \ K_z \ K_x]^T$. The rotation of this vector in the elevational plane yields a vector in the new basis $[-K_z \sin \phi \ K_z \cos \phi \ K_x]^T$. In the old basis, this vector is

$$
\begin{bmatrix}
\sin \theta & 0 & \cos \theta \\
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\cos \phi & -\sin \phi & 0 \\
\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\sin \theta & 0 & \cos \theta \\
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0
\end{bmatrix}^T.
$$

For example, consider the rotation of the vector $[0 \ 0 \ 1]^T$: Then $K_x = 0$, $K_z = 1$, and the rotated vector (in the old basis) is $[-\sin \phi \sin \theta \ -\sin \phi \cos \theta \ \cos \phi]^T$.

Next let’s translate this vector by $[x_0 \ y_0 \ z_0]^T$ given above. This yields

$$
\begin{bmatrix}
\lambda \cos \phi \sin \theta - K_z \sin \phi \sin \theta + K_x \cos \theta \\
\lambda \cos \phi \cos \theta - K_z \sin \phi \cos \theta - K_x \sin \theta \\
\lambda \sin \phi + K_z \cos \phi
\end{bmatrix}.
$$

Now, we seek vectors of this form on the z-axis to find the wavelength span on
the axis. The solution comes from the following:

$$\lambda \cos \phi \sin \theta - K_z \sin \phi \sin \theta + K_x \cos \theta = 0$$

$$\implies K_x = K_z \frac{\sin \phi \sin \theta}{\cos \theta} - \lambda \frac{\cos \phi \sin \theta}{\cos \theta} \text{ and}$$

$$\lambda \cos \phi \cos \theta - K_z \sin \phi \cos \theta - K_x \sin \theta = 0$$

$$\lambda \cos \phi \cos \theta - K_z \sin \phi \cos \theta - K_x \frac{\sin \phi \sin^2 \theta}{\cos \theta} + \lambda \frac{\cos \phi \sin^2 \theta}{\cos \theta} = 0$$

$$\implies K_z = \frac{\lambda \cos \phi \left( \cos \theta + \frac{\sin^2 \theta}{\cos \theta} \right)}{\sin \phi \cos \theta + \frac{\sin \phi \sin^2 \theta}{\cos \theta}}$$

The z-component is

$$\lambda \sin \phi + K_z \cos \phi = \lambda \sin \phi + \frac{\lambda \cos^2 \phi \left( \cos \theta + \frac{\sin^2 \theta}{\cos \theta} \right)}{\sin \phi \cos \theta + \frac{\sin \phi \sin^2 \theta}{\cos \theta}}$$

$$= \lambda \sin \phi + \frac{\lambda \cos^2 \phi \left( \cos^2 \theta + \sin^2 \theta \right)}{\sin \phi \cos^2 \theta + \sin \phi \sin^2 \theta}$$

$$= \lambda \sin \phi + \frac{\lambda \cos^2 \phi}{\sin \phi} = \frac{\lambda}{\sin \phi}.$$

From this calculation one can now say that $\lambda / \sin \phi$ spans one full wavelength on the z-axis, and a length of $d$, where $d$ is the spacing between each hydrophone, on the z-axis spans $d \sin \phi / \lambda$ wavelengths; not including the sign.

Now let’s find the vector in the translated, rotated hyperplane, on the x-axis.

The solution comes from:

$$\lambda \cos \phi \cos \theta - K_z \sin \phi \cos \theta - K_x \sin \theta = 0$$

$$\lambda \sin \phi + K_z \cos \phi = 0$$

$$\implies K_z = -\frac{\lambda \sin \phi}{\cos \phi} \text{ where}$$

$$K_x = \frac{\lambda \cos \phi \cos \theta}{\sin \theta} + \frac{\lambda \sin^2 \phi \cos \theta}{\cos \phi \sin \theta}$$
and the value on the x-axis is
\[
\lambda \cos \phi \sin \theta - K_z \sin \phi \sin \theta + K_x \cos \theta = \lambda \cos \phi \sin \theta + \frac{\lambda \sin \phi \sin \theta}{\cos \phi} + \frac{\lambda \cos \phi \cos \theta}{\sin \theta} + \frac{\lambda \sin^2 \phi \cos \theta}{\cos \phi \sin \theta}
\]
\[
= \frac{\lambda}{\cos \phi \sin \theta} (\sin^2 \theta + \cos^2 \theta)
\]
\[
= \frac{\lambda}{\cos \phi \sin \theta}
\]

So one wavelength spans \(\lambda / \cos \phi \sin \theta\) on the x-axis. A length \(d\) on the x-axis includes \(d \cos \phi \sin \theta / \lambda\) wavelengths; not including the sign.

For example, let the phase be 0. If the phase rotates clockwise, then the phase at point \((0, 0, d)\) is \(-2\pi \frac{d}{\lambda} \sin \phi\) and the phase at \((d, 0, 0)\) is \(-2\pi \cos \phi \sin \theta\). This assumes that, at any point \(\xi\), the phase rotates in the clockwise direction; and that the traveling wave behaves like \(s(\omega t - \beta \xi)\).

Let \(\mathbf{a}_\zeta\) be the vector of time samples for the noiseless signal collected at the origin. Here \(a\) is an unknown complex constant. If \(d < \lambda\), then the corresponding vector at \((0, 0, d)\) is \(\mathbf{a}_\zeta \exp(-j2\pi d \sin \phi / \lambda)\). Also, the vector at \((d, 0, 0)\) is \(\mathbf{a}_\zeta \exp(-j2\pi d \cos \phi \sin \theta / \lambda)\). Arranging the sampled signal vectors in order, we have
\[
\mathbf{a}_\zeta = \begin{bmatrix}
\mathbf{a}_\zeta e^{-j2\pi \frac{d \cos \phi \sin \theta}{\lambda}}
\mathbf{a}_\zeta
\mathbf{a}_\zeta e^{-j2\pi \frac{d \sin \phi}{\lambda}}
\end{bmatrix}.
\]
\[(4.6)\]

Now let \(\mathbf{n}\) denote the corresponding noise vector. Assume that \(\mathbf{n}\) is zero mean, complex circular, with \(\text{Re}(\mathbf{n}) \sim N(0, \mathbf{R})\). The log-likelihood function (ignoring constants) is
\[
-\frac{1}{2} \mathbf{H}^H \mathbf{R}^{-1} \mathbf{n}.
\]

Let \(r = \mathbf{a}_\zeta + \mathbf{n}\) denote the complex observation. Here \(\zeta\) depends on \(\theta\) and \(\phi\), which we are trying to estimate. The maximum likelihood estimators for \(a, \theta,\)
and $\phi$ satisfy

$$
\begin{bmatrix}
\hat{a} \\
\hat{\theta} \\
\hat{\phi}
\end{bmatrix} = \arg\min_{a, \theta, \phi} \left[ r - a \right]^H R^{-1} \left[ r - a \right].
$$

Define $J = \left[ r - a \right]^H R^{-1} \left[ r - a \right]$, and $a = a_r + j a_q$. Then

$$
\frac{\partial}{\partial a_r} J = -2 \text{Re} \left[ z^H R^{-1} \left[ r - \hat{a} \right] \right],
$$

$$
\frac{\partial}{\partial a_q} J = 2 \text{Re} \left[ j z^H R^{-1} \left[ r - \hat{a} \right] \right] = -2 \text{Im} \left[ z^H R^{-1} \left[ r - \hat{a} \right] \right].
$$

To satisfy $\frac{\partial}{\partial a_r} J = \frac{\partial}{\partial a_q} J = 0$, we require $0 = z^H R^{-1} \left[ r - \hat{a} \right]$, or

$$
\hat{a} = \frac{z^H R^{-1} r}{z^H R^{-1} z}.
$$

Substituting $\hat{a}$ into $J$, we get

$$
\begin{bmatrix}
\hat{\theta} \\
\hat{\phi}
\end{bmatrix} = \arg\min_{\theta, \phi} \left[ r - \frac{z^H R^{-1} r}{z^H R^{-1} z} \right]^H R^{-1} \left[ r - \frac{z^H R^{-1} r}{z^H R^{-1} z} \right].
$$

We will define $\hat{J}$ as the compressed log-likelihood given by the argument of $\arg\min$ above, where $z$ depends on $\theta$ and $\phi$. Proceeding as before, we find

$$
\frac{\partial \hat{J}}{\partial \theta} = -2 \text{Re} \left[ \frac{\partial}{\partial \theta} \left[ \frac{z^H R^{-1} r}{z^H R^{-1} z} \right] R^{-1} \left[ r - \frac{z^H R^{-1} r}{z^H R^{-1} z} \right] \right],
$$

(4.7)

and a similar expression for $\frac{\partial \hat{J}}{\partial \theta}$. 
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Now we will simplify 4.7. Consider \( \frac{\partial}{\partial \theta} \mathbf{H} \mathbf{R}^{-1} \mathbf{H} \), since

\[
\mathbf{R}^{-1} = \begin{bmatrix}
\mathbf{R}_1^{-1} & 0 & 0 \\
0 & \mathbf{R}_2^{-1} & 0 \\
0 & 0 & \mathbf{R}_3^{-1}
\end{bmatrix}
\]

and using 4.6 yields

\[
\frac{\partial}{\partial \theta} \mathbf{H} \mathbf{R}^{-1} \mathbf{H} = \frac{\partial}{\partial \theta} \left[ \begin{bmatrix}
\mathbf{S} e^{-j2\pi \frac{\text{dcos} \phi}{\lambda}} & \\
\mathbf{S} e^{-j2\pi \frac{\text{dsin} \phi}{\lambda}} & \\
0 & \mathbf{S} e^{-j2\pi \frac{\text{dth} \phi}{\lambda}}
\end{bmatrix}
\right] \mathbf{H} \left[ \begin{bmatrix}
\mathbf{R}_1^{-1} & 0 & 0 \\
0 & \mathbf{R}_2^{-1} & 0 \\
0 & 0 & \mathbf{R}_3^{-1}
\end{bmatrix}
\right] = 0
\]

(4.8)

and a by similar calculation

\[
\frac{\partial}{\partial \phi} \mathbf{H} \mathbf{R}^{-1} \mathbf{H} = 0.
\]

(4.9)

Applying 4.8 into 4.7 we get

\[
\frac{\partial \hat{J}}{\partial \theta} = -2 \text{Re} \left[ \left( \frac{\frac{\partial}{\partial \theta} \mathbf{H} \mathbf{R}^{-1} \mathbf{H}}{\mathbf{S} \mathbf{H} \mathbf{R}^{-1} \mathbf{H}} \right) \mathbf{R}^{-1} \left[ \mathbf{r} - \frac{\mathbf{H} \mathbf{R}^{-1} \mathbf{r}}{\mathbf{H} \mathbf{R}^{-1} \mathbf{H}} \right] \right].
\]

(4.10)

Now

\[
\frac{\partial}{\partial \theta} \left( \frac{\mathbf{H} \mathbf{R}^{-1} \mathbf{H}}{\mathbf{S} \mathbf{H} \mathbf{R}^{-1} \mathbf{H}} \right) = \left( \frac{\partial}{\partial \theta} \mathbf{H} \mathbf{R}^{-1} \mathbf{H} \right) + \mathbf{H} \mathbf{R}^{-1} \mathbf{H} \left( \frac{\partial}{\partial \theta} \mathbf{H} \right),
\]

(4.11)

\[
\frac{\partial}{\partial \theta} = \begin{bmatrix}
\mathbf{S} e^{-j2\pi \frac{\text{dcos} \phi}{\lambda}} & \\
\mathbf{S} e^{-j2\pi \frac{\text{dsin} \phi}{\lambda}} & \\
0 & 0
\end{bmatrix}
\]

(4.12)
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also,
\[
\frac{\partial}{\partial \phi}\xi = \begin{bmatrix}
s\left(-j2\pi \frac{\sin \phi \sin \theta}{\lambda}\right) e^{-j2\pi \frac{d \sin \phi}{\lambda}} \\
0 \\
s\left(-j2\pi \frac{\cos \phi}{\lambda}\right) e^{-j2\pi \frac{\sin \phi}{\lambda}}
\end{bmatrix}.
\tag{4.13}
\]

Applying 4.12 into 4.11 yields,
\[
\frac{\partial}{\partial \theta}(\xi^H \mathbf{R}^{-1} \xi) = \left[ \xi^H \left(j2\pi \frac{\cos \theta \cos \phi}{\lambda}\right) e^{-j2\pi \frac{\sin \theta}{\lambda}} \right] \begin{bmatrix} 0^T & 0^T \end{bmatrix} \mathbf{R}^H \mathbf{R}^{-1} \xi + ...
\tag{4.14}
\]

where \(\xi^T = [r_1^T \ r_2^T \ r_3^T]\). So \(\frac{\partial j}{\partial \phi}\) may be computed by using 4.14 in 4.10.

The evaluation of \(\frac{\partial j}{\partial \phi}\) follows similarly. Due to 4.9, for example we have
\[
\frac{\partial j}{\partial \phi} = -2Re \left[ \frac{\left( \frac{\partial}{\partial \phi} \xi^H \mathbf{R}^{-1} \xi \right) \mathbf{R}^{-1} \left[ r - \frac{\xi^H \mathbf{R}^{-1} \xi}{\xi^H \xi \mathbf{R}^{-1} \xi} \right]}{\xi^H \mathbf{R}^{-1} \xi} \right].
\tag{4.15}
\]

Also,
\[
\frac{\partial}{\partial \phi}(\xi^H \mathbf{R}^{-1} \xi) = \left( \frac{\partial}{\partial \phi} \xi \right)^H \mathbf{R}^{-1} \xi + \xi^H \mathbf{R}^{-1} \left( \frac{\partial}{\partial \phi} \xi \right)
\tag{4.16}
\]

and applying 4.13 to 4.16 we get
\[
\frac{\partial}{\partial \phi}(\xi^H \mathbf{R}^{-1} \xi) = \left[ \xi^H \left(-j2\pi \frac{\sin \phi \sin \theta}{\lambda}\right) e^{-j2\pi \frac{d \sin \phi}{\lambda}} \right] 0^T \left[ \xi^H \left(j2\pi \frac{\cos \phi}{\lambda}\right) e^{-j2\pi \frac{\sin \phi}{\lambda}} \right] \mathbf{R}^H \mathbf{R}^{-1} \xi + ...
\]
\[ \cdots \left[ \begin{array}{c} s^{H} e^{j2\pi \frac{dcos\phi sin\theta}{\lambda}} \\ s^{H} e^{j2\pi \frac{dsin\phi}{\lambda}} \end{array} \right] \left( \begin{array}{c} H_{1}^{-1} s \begin{pmatrix} j2\pi \frac{dsin\phi}{\lambda} \end{pmatrix} e^{-j2\pi \frac{dcos\phi sin\theta}{\lambda}} \\ H_{3}^{-1} s \begin{pmatrix} j2\pi \frac{dcos\phi}{\lambda} \end{pmatrix} e^{-j2\pi \frac{dsin\phi}{\lambda}} \end{pmatrix} \right) \cdots \]

where when 4.17 is applied to 4.15 yields \( \frac{\partial J}{\partial \phi} \).

### 4.2 Newton Raphson Iterations

When the hydrophones receive the sonar signal and the calculation for the angle of arrival is computed, the scheme for this calculation will use Newton Raphson iterations. The stochastic Fisher information matrix plays an important role for solving the maximum likelihood equation \( s(\hat{\theta}, x) = 0 \). Consider the log-likelihood function \( L(\theta, x) = \ln f_\theta(x) \) and its Taylor series expansion about some estimate of \( \theta \) given by \( \theta_n \):

\[
L(\theta, x) = L(\theta_n, x) + [\theta - \theta_n]^T \frac{\partial}{\partial \theta} L(\theta_n, x) + \frac{1}{2} [\theta - \theta_n]^T \frac{\partial^2}{\partial \theta^2} L(\theta_n, x) [\theta - \theta_n] + \text{higher order terms}
\]

\[
\cong L(\theta_n, x) + [\theta - \theta_n]^T s(\theta_n, x) - \frac{1}{2} [\theta - \theta_n]^T J(\theta_n, x) [\theta - \theta_n].
\]

where \( J(\theta, x) \) is the stochastic Fisher information matrix. This approximation is valid in the neighborhood of the MLE where \( s(\hat{\theta}, x) = 0 \). The above shows that the log-likelihood is quadratic in the area of the maximum likelihood estimate \( \theta_n = \hat{\theta} \), where \( s(\hat{\theta}, x) = 0 \), which brings forth the use of Newton-Raphson iteration for finding the zero of \( s(\theta, x) = \frac{\partial}{\partial \theta} L(\theta_n, x) \). In our processing algorithm, the maximum likelihood equation for \( \theta \) and \( \phi \) is given by their respective...
gradient.

The Taylor series for the log likelihood produces the following linear model for \( s(\theta, x) \):

\[
s(\theta, x) = s(\theta_n, x) - J(\theta_n, x)[\theta - \theta_n].
\]

The Newton-Raphson iteration equates \( s(\theta_{n+1}, x) \) to zero and results in the following equation for the next estimate of \( \theta \):

\[
J(\theta_n, x)[\theta_{n+1} - \theta_n] = -s(\theta_n, x)
\]

or

\[
\theta_{n+1} = \theta_n - J^{-1}(\theta_n, x)s(\theta_n, x).
\] (4.18)

Therefore in summary the inverse of the stochastic Fisher information matrix \( J^{-1}(\theta, x) \) plays the role of the inverse Hessian in the Newton-Raphson iteration or as a variable step size in the gradient decent search [23].

In order to calculate the inverse Hessian of this iteration, the second-order derivative with respect to \( \theta, \phi \) of 4.10 and 4.15 respectively, including the mixed partial need to be calculated.

Let us start with the second-order partial with respect to \( \theta \). From 4.10:

\[
\frac{\partial^2 J}{\partial \theta^2} = -2Re \left\{ \left( \frac{\partial^2}{\partial \theta^2} \frac{H}{\Sigma} R^{-1} \right) \frac{1}{\Sigma} \left[ \Sigma - \frac{H R^{-1} \Sigma}{\Sigma} \right] - \left( \frac{\partial}{\partial \theta} \frac{H}{\Sigma} R^{-1} \right) \frac{1}{\Sigma} \right\},
\]

where

\[
\frac{\partial^2}{\partial \theta^2} \frac{H}{\Sigma} R^{-1} = \frac{\partial}{\partial \theta} \left( \frac{\partial}{\partial \theta} \frac{H}{\Sigma} R^{-1} \right) = \frac{\partial}{\partial \theta} \left( \left( \frac{\partial}{\partial \theta} \frac{H}{\Sigma} R^{-1} \right) + \frac{H}{\Sigma} R^{-1} \left( \frac{\partial^2}{\partial \theta^2} \frac{\Sigma}{\Sigma} \right) \right)
\]

\[
= \left( \frac{\partial^2}{\partial \theta^2} \frac{H}{\Sigma} R^{-1} \right) + 2 \left( \frac{\partial}{\partial \theta} \frac{H}{\Sigma} R^{-1} \right) + \frac{H}{\Sigma} R^{-1} \left( \frac{\partial^2}{\partial \theta^2} \Sigma \right).
\]
and

\[
\frac{\partial^2 \mathbf{J}}{\partial \phi^2} = \frac{\partial}{\partial \theta} \begin{bmatrix}
S \left( -j2\pi \frac{\cos \theta \cos \phi}{\lambda} \right) e^{\left[ -j2\pi \frac{\cos \phi \sin \theta}{\lambda} \right]}
0
0
\end{bmatrix}
= \begin{bmatrix}
S \left( \left( j2\pi \frac{\cos \phi \sin \theta}{\lambda} \right) e^{\left[ -j2\pi \frac{\cos \phi \sin \theta}{\lambda} \right]} + \left( -j2\pi \frac{\cos \phi \sin \theta}{\lambda} \right)^2 e^{\left[ -j2\pi \frac{\cos \phi \sin \theta}{\lambda} \right]} \right)
0
0
\end{bmatrix}.
\]

Let us now proceed with the second-order partial with respect to \( \phi \). From 4.15:

\[
\frac{\partial^2 \mathbf{J}}{\partial \phi^2} = -2 \text{Re} \left\{ \left( \frac{\partial}{\partial \phi} \frac{\partial}{\partial \phi} \frac{H^H R^{-1} \Sigma}{\xi^H R^{-1} \Sigma} \right)^H R^{-1} \left[ \xi - \frac{H^H R^{-1} \Sigma}{\xi^H R^{-1} \Sigma} \right] \right. \\
- \left. \left( \frac{\partial}{\partial \phi} \frac{H^H R^{-1} \Sigma}{\xi^H R^{-1} \Sigma} \right)^H R^{-1} \left( \frac{\partial}{\partial \phi} \frac{H^H R^{-1} \Sigma}{\xi^H R^{-1} \Sigma} \right) \right\},
\]

where

\[
\frac{\partial^2}{\partial \phi^2} \frac{H^H R^{-1} \Sigma}{\xi^H R^{-1} \Sigma} = \frac{\partial}{\partial \phi} \left( \frac{\partial}{\partial \phi} \frac{H^H R^{-1} \Sigma}{\xi^H R^{-1} \Sigma} \right) = \frac{\partial}{\partial \phi} \left( \left( \frac{\partial}{\partial \phi} \frac{H^H R^{-1} \Sigma}{\xi^H R^{-1} \Sigma} \right)^H R^{-1} + \frac{H^H R^{-1}}{\xi^H R^{-1}} \left( \frac{\partial}{\partial \phi} \frac{H^H R^{-1}}{\xi^H R^{-1}} \right) \right)
= \left( \frac{\partial^2}{\partial \phi^2} \frac{H^H R^{-1} \Sigma}{\xi^H R^{-1} \Sigma} \right)^H + 2 \left( \frac{\partial}{\partial \phi} \frac{H^H R^{-1} \Sigma}{\xi^H R^{-1} \Sigma} \right)^H \frac{H^H R^{-1}}{\xi^H R^{-1}} \left( \frac{\partial}{\partial \phi} \frac{H^H R^{-1} \Sigma}{\xi^H R^{-1} \Sigma} \right) + \frac{H^H R^{-1}}{\xi^H R^{-1}} \left( \frac{\partial^2}{\partial \phi^2} \frac{H^H R^{-1} \Sigma}{\xi^H R^{-1} \Sigma} \right)
\]

and

\[
\frac{\partial^2}{\partial \phi^2} = \begin{bmatrix}
S e^{\left[ -j2\pi \frac{\cos \phi \sin \theta}{\lambda} \right]} \left( j2\pi \frac{\cos \phi \sin \theta}{\lambda} + \left( -j2\pi \frac{\cos \phi \sin \theta}{\lambda} \right)^2 \right)
0
0
\end{bmatrix}.
\]
Finally, we will find the second-order mixed partial of $\hat{J}$:

$$
\frac{\partial^2 \hat{J}}{\partial \theta \partial \phi} = -2\text{Re}\left\{ \frac{\partial^2 S^H L^H R^{-1} S}{\partial \theta \partial \phi} \right\} - \frac{\partial S^H R^{-1} S}{\partial \theta} R^{-1} \left( \frac{\partial S^H L^H R^{-1} S}{\partial \phi} \right) - \frac{\partial S^H R^{-1} S}{\partial \phi} R^{-1} \left( \frac{\partial S^H L^H R^{-1} S}{\partial \theta} \right),
$$

where

$$
\frac{\partial^2 S^H L^H R^{-1} S}{\partial \theta \partial \phi} = \frac{\partial}{\partial \phi} \left( \frac{\partial S^H L^H R^{-1} S}{\partial \theta} \right) = \frac{\partial}{\partial \phi} \left( \frac{\partial S^H L^H R^{-1} S}{\partial \theta} \right),
$$

and

$$
\frac{\partial^2 S^H L^H R^{-1} S}{\partial \theta \partial \phi} = \frac{\partial}{\partial \phi} \left( \frac{\partial S^H L^H R^{-1} S}{\partial \theta} \right) = \frac{\partial}{\partial \phi} \left( \frac{\partial S^H L^H R^{-1} S}{\partial \theta} \right).
$$

To calculate the variable step size or inverse Hessian needed in 4.18, we construct the matrix

$$
H^{-1} = J^{-1}(\theta_n, x) = \frac{1}{\left( \frac{\partial^2 S}{\partial \theta^2} \right) \left( \frac{\partial^2 S}{\partial \phi^2} \right) - \left( \frac{\partial^2 S}{\partial \theta \partial \phi} \right)^2} \begin{bmatrix}
\frac{\partial^2 J}{\partial \theta^2} & \frac{\partial^2 J}{\partial \theta \partial \phi} \\
\frac{\partial^2 J}{\partial \theta \partial \phi} & \frac{\partial^2 J}{\partial \phi^2}
\end{bmatrix},
$$

where the complete iteration for $\theta$ and $\phi$ is given by:

$$
\begin{bmatrix}
\theta_{n+1} \\
\phi_{n+1}
\end{bmatrix} =
\begin{bmatrix}
\theta_n \\
\phi_n
\end{bmatrix} - H^{-1} \begin{bmatrix}
\nabla_{\theta} J \\
\nabla_{\phi} J
\end{bmatrix}.
4.3 Results

This section presents the results of a simulation in MATLAB where the actual bearing angles of $\theta$ and $\phi$ are 56.65°, 73.04° respectively and were generated randomly. As seen in Figure 4.2, each iteration of estimation is shown in the contour in the dB scale. The solution to the ML equation is given by the blue regions which represent the bottom of the parabolic bowl. There is more than one blue region because they represent solutions when the hydrophones are hearing the signal from behind where there is a 180° offset. When the complex observation is received, the sonar algorithm first tries a set of test points for $\theta$ and $\phi$ and computes the cost for each one and chooses the test point at which the cost is minimum. In this manner, the algorithm starts from a point that is closest to the neighborhood where $s(\hat{\theta}, x) = 0$ and therefore solving for the MLE in fewer iterations. Therefore, upon processing the received signal, the DSP processor will keep iterating the new estimates for the azimuthal angle $\theta$ and elevation angle $\phi$ until a the difference between newest and prior estimates is less than .001%. The lamprey will continue adjusting its heading in the direction of the sonar pinger until it has been reached. Also notice the actual bearing is marked with an “X” in the contour and gradient figures. With an SNR of 30 dB, the bearing estimates of $\theta$ and $\phi$ were 57.08°, 72.8° respectively. The gradient with respect to $\theta$ and $\phi$ is also shown in Figure 4.3. The output mean-squared error in dB degrees vs. each iteration of estimation is shown in Figure 4.5. The cost function per iteration of estimation is also shown in Figure 4.6. As one can see, the error and cost is greatly reduced from the first iteration because of the script that scans the test points and chooses the point with the lowest cost. Also seen in Figure 4.5, the error increases on the 8th iteration, this is due to the step size being too large at the point and slightly overshooting the actual bearing. The lowest SNR capable of estimation close to the actual
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Figure 4.2: This figure depicts the contour of the MLE and the tracking of steps per iteration of estimation.

bearing is $-3\, dB$ and results in an estimation of $20.34^\circ$, $59.24^\circ$ with respect to $\theta$ and $\phi$. The results for this SNR are shown in Figures 4.7-4.9. As one can see, the results are still reasonable considering the SNR and still comes close to the target bearing. Shown in Figure 4.10, is the MSE output error in dBdegrees vs. SNR. As one can see, the ML estimator provides good results for bearing estimation even in a low SNR environment.

4.4 Summary

A sonar model setup was given for the L-shape array configuration of the hydrophone elements. By establishing the trigonometric relationships between the hydrophones, the received signal model is created. Using this model, the equations for the maximum log-likelihood function could be derived. To find the global minimum, the derivative with respect to $\theta$ and $\phi$ of the compressed log-likelihood function are needed. A maximum likelihood estimator is used because
Figure 4.3: This figure shows the gradient with respect to $\theta$ and $\phi$.

Figure 4.4: Shown is a zoomed in version of Figure 4.2.
Figure 4.5: This figure shows the output MSE vs each iteration.

Figure 4.6: This figure shows the cost function vs each iteration.
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Figure 4.7: This figure shows the contour of the MLE and iterations when a low SNR is present.

Figure 4.8: This Figure shows the MSE output error vs iteration when a low SNR is present.
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Figure 4.9: This figure shows the cost history when a low SNR is present.

Figure 4.10: This figure shows the MSE output error vs. SNR in dB.
it is efficient and unbiased for long time samples. A Newton-Raphson search is then used to estimate these angles. The starting point of the Newton-Raphson search is provided by means of a course grid search that takes a series of test points and evaluates the cost at each point and chooses the point that yields the minimum cost. Numerical results are simulated in Matlab where each location of each iteration of the search is shown on a contour plot of the log-likelihood function. The mean-squared error vs. iterations also shows how efficient the MLE performs. A low received SNR simulation is also executed to show the performance of the MLE in these conditions. Approximately 7-11 iterations are needed to compute the bearing estimates. Also, an accuracy of less than 1 degree of standard deviation is achievable with a received SNR of $6 - 10 \, dB$. 
Chapter 5

Conclusion and Future Work

5.1 Conclusion

Underwater mine detection has become of great interest for military and commercial fleets. Dolphins are currently being used to mark the mines where later Navy divers apply satchel charges for mine elimination. AUVs can be used to replace the Navy divers to avoid personal risk, where beacon tracking plays a vital role. A passive ultra short baseline array sonar system was designed and constructed. The system consisted of 3 major components: transmitter, receiver, and ML estimator. The transmitter operated at $10.5\, kHz$ where the sinusoidal signal was generated by Colpitts oscillators. Four communication oscillators are also used to provide a 4-FSK downlink to the AUV for command and control. The signals' DC offsets were removed by capacitors and were summed. The gating scheme to this signal was achieved by means of a 555-timer and transistor switching. This signal was then amplified by two high power amplifiers and delivered to the projector. Impedance matching by means of a custom power transformer played a vital role in power delivery to the projector. The
sonar signal was received by 3 hydrophones in an L-shape configuration. The differential signal of the hydrophones is amplified and filtered to remove any unwanted frequency components heard in the ocean water. A PGA is used to adjust the gain relative to the position of the projector to avoid damaging the receiver components. The signal is then DC offset to meet the specifications of the DSP processor containing the estimator algorithm. Using the sonar array L-shape configuration, a system model was set up. From the trigonometric relationships between the hydrophones a received signal model could be derived. From this model the maximum log-likelihood function was determined, where the derivatives with respect to the azimuthal and elevational angles were needed to find the global minimum for bearing estimation. A Newton-Raphson search was used to produce the next estimates for the angles using the old estimates, the derivative of the log-likelihood function, and the inverse Hessian matrix. The inverse Hessian matrix was also used as a variable step size for the Newton-Raphson search. A course grid search was utilized to provide a starting point for the Newton-Raphson search, which also helps avoid finding local minima when solving for the MLE. From the Matlab simulation results it can be seen that accuracy of less than 1 degree of standard deviation can be achieved with a received SNR of $6 - 10\, dB$.

5.2 Future Work

Beacon tracking can play a vital role in many applications excluding the application discussed in this thesis. For example, establishing an underwater two channel telemetry system would benefit from beacon tracking. If either of the directional projectors move underwater whether intentional or inadvertently, angle adjustment of the projectors will help maintain a stable and reliable means of a communications link. Tow ship docking can also benefit from beacon track-
ing, where estimation of angles can be used to help dock a large ship in space limited areas. Any applications where a tracking system could be used to aid in a process would prove a successful role for beacon tracking.
Chapter 6

Bibliography
Bibliography


formly But Spar sely Spaced V ector Hydrophones,” IEEE Journal of Oceanic

tional invariance techniques,” IEEE Trans. Acoust., Speech, Signal Pro-

[9] R. O. Schmidt, “Multiple emitter location and signal parameter estima-

[10] Xiaodong Liu, Weiqing Zhu, Changle Fang, Wen Xu, Fangsheng Zhang,
Yujia Sun, “Shallow water High Resolution Bathymetric Side Scan Sonar,”

curacy Super Short Base Line (SSBL) System,” Underwater Technology,

Base Line Directional Acoustic Transponder,” OCEANS 2007 - Europe 18-
21 June 2007 pgs. 1 - 5.

T., “High-resolution beamforming techniques for circular sonar arrays,”
OCEANS ’94. ’Oceans Engineering for Today’s Technology and Tomor-
row’s Preservation.’ Proceedings Volume 1, 13-16 Sept. 1994 pgs. I/223 -
I/228 vol.1.


Appendix

Transmitter board parts list

(1) Underwater Projector
(11) 2N2222 NPN Transistors
(4) OP-27 Operational Amplifiers
(1) 1N4148T Diode
(1) TLC555CP 555-timer
(2) 50 Ω resistor
(1) 300 Ω resistor
(2) 1 kΩ resistor
(13) 2.2 kΩ resistor
(1) 2.7 kΩ resistor
(1) 4 kΩ resistor
(1) 5 kΩ resistor
(11) 10 kΩ resistor
(3) 50 kΩ resistor
(4) 100 kΩ resistor
(1) 357 kΩ resistor
(3) 1.5 MΩ resistor
(1) 5.1 mH inductor
(4) 5.6 mH inductor
(10) .1 µF capacitor
(1) 1 nF capacitor
(1) 80 nF capacitor
(1) 100 nF capacitor
(1) 102 nF capacitor
(1) 150 nF capacitor
(1) 200 nF capacitor
(1) 1 µF capacitor
(1) .01 µF capacitor

Receiver board parts list

(3) Teledyne AQ-2000 Hydrophone
(4) OP-467 Operational-Amplifier
(1) Analog Devices AD5204 Digital Potentiometer
(1) LM2937 5V DC regulator
(1) L4931ABV33 3.3V regulator
(2) Maxim765 DC regulators
(2) 1N5817 High speed diode
(6) 100 Ω resistor
(9) 1 kΩ resistor
(12) 6 kΩ resistor
(9) 500 kΩ resistor
(2) 47 µH inductor
(3) 470 µH inductor
(6) 6.8 mH inductor
(6) 10 nF capacitor
(6) 33 nF capacitor
(3) 56 nF capacitor
(3) 560 nF capacitor
(6) .1 µF capacitor
(1) .22 µF capacitor
(1) 2.2 µF capacitor
(4) 100 µF electrolytic capacitor

IN5817 High speed diode
Maxim765 DC regulators