Design and Analysis of a Quad-ferential Amplifier

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The design and analysis of a monolithic quad-ferential amplifier is presented. A quad-ferential amplifier consists of four inputs, four outputs, and a $V_{ocm}$ pin which controls the output common-mode voltage. It is similar to a differential amplifier in that it amplifies differences and rejects overall input common-mode. An overview of a quad-ferential amplifier is offered which includes a symbol, a description of each pin, a block diagram and a brief circuit level discussion. The transfer function derivation demonstrates that symmetry of feedback and gain resistors is required for the quad-ferential amplifier to approach its ideal behavior.

The core of a quad-ferential amplifier, a quad-ferential pair, is qualitatively and quantitatively examined. An implementation of the input stage, gain stage, output stage and bias circuitry is offered. Detailed analysis of the open loop gain and loop gain is examined. Feedback is explored qualitatively confirming a quad-ferential amplifier achieves negative feedback. The unit current in the bias cell is generated by a Widlar bandgap reference designed to operate from $-55^\circ C$ to $125^\circ C$ with a temperature coefficient of $9.3nA/^\circ C$. Using a graphical approach the amplifier is compensated to drive a capacitive load of 50pF. The output current drive is designed for a minimum load resistance of 150Ω.

Transient simulations verify two fundamental aspects of a quad-ferential amplifier. First, it amplifies differences. Second, the output common-mode voltage is controlled by $V_{ocm}$. Bode plots for one, two, three, and four modulating inputs are included. Statistical models are utilized in performing Monte Carlo simulations to evaluate offset voltage and common-mode rejection. Simulations show a mean offset voltage of 265μV and a common-mode rejection of 126dB.
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Chapter 1

Introduction

1.1 Objective

Differential amplifiers have been in existence for years. A recent extension of a differential amplifier is a trifferential amplifier. Its main advantage is its ability to reject common-mode signals while amplifying difference signals. Building on the concept of a trifferential amplifier, the goal of this thesis is the design and analysis of a quad-ferential amplifier.

1.2 Background

The trifferential amplifier, designed and patented by Stefano D’Aquino, is the first multi-ferential amplifier. This three input and three output amplifier is designed on ADI’s proprietary XFCB2 process. XFCB2 is a fully dielectrically isolated complementary bipolar process that utilizes silicon-on-insulator bonded wafers. The trifferential amplifier is a sub-circuit in a CAT5 video crosspoint switch matrix. A crosspoint switch matrix is utilized in environments where multiple video inputs are routed to multiple locations. The presence of unwanted common-mode signals in CAT5 video applications corrupts the information and therefore, need to be removed. The trifferential amplifier removes them effectively and efficiently.

1.3 Challenges in video quality and crosspoints

The human eye sees by its ability to distinguish subtle differences in brightness. Its sensitivity varies with brightness, being substantially more sensitive at lower brightness levels. Video is an attempt to recreate visual reality as accurately as possible through
electronic means. Traditional video mixes three primary colors, red, green and blue (RGB) in proportions to match both the color and the brightness of a scene. The presence of unwanted common-mode on the signal lines results in undesirable changes in brightness, reducing image quality. Slight differences in the common-mode DC between the outputs from crosspoint switches can further exacerbate overall image quality due to the brightness variations between outputs.

The common-mode level needs to be controlled for two reasons. First, to retain the dynamic range of the video signal, the overall common-mode must be kept constant. Second, to maximize the amount of information sent over a CAT5 cable. Some video processing architectures superimpose a synchronization (sync) signal upon a twisted pair in the form of a common-mode level or pulse. This sync determines whether or not the horizontal or vertical lines are active. Unwanted common-mode levels will send erroneous sync’s resulting in video errors.

The most efficient method of controlling the common-mode signal is to remove it from all inputs. With a simple two input and one output switch, adding clamping circuitry to the inputs to remove common-mode is feasible. Crosspoints consist of many inputs. Implementing additional circuitry to each input is prohibitive financially with regard to required space on the printed circuit board (PCB) or die area on an integrated circuit (IC). One trifferential amplifier mitigates the need for multiple video clamping circuitry. It removes unwanted input common-mode and allows for the common-mode of the crosspoint outputs to be controlled.

### 1.4 Motivation

Traditional three color display devices provide limited brightness dynamic range. As a result of recent developments in display technology, display devices with four primary colors are becoming commercially available. The fourth color, yellow, is changing the signal processing from the traditional 3-channel RGB video to 4-channel RGBY. The motivation of this thesis is to design and analyze a four channel expanded version of the trifferential amplifier.
1.5 Thesis outline

The behavior of a discrete four input and four output quad-ferential amplifier is investigated.

In chapter 2 the general behavior of a quad-ferential amplifier is established. The definition of such an amplifier is presented and the block diagram is shown followed by a brief circuit level overview. A typical configuration is offered and the non-ideal transfer function is presented with the derivation demonstrating the behavior of a quad-ferential amplifier in a balanced system.

Analysis of a quad-ferential amplifier is presented in chapter 3. A quad-ferential pair, the fundamental building block of such an amplifier, is thoroughly examined. Large signal and direct small signal behavior is thoroughly explored. The input stage, comprised of two quad-ferential pairs, is investigated and its behavior established. A concise study of the remaining sub-circuits such as the gain stage, output stage, bias cells, stability, compensation and feedback is offered.

Simulations are presented in Chapter 4. The primary function of a quad-ferential amplifier is to remove overall common-mode and amplify differences. Simulation results for the differential behavior, common-mode behavior, $V_{ocm}$ functionality, offset voltage and common-mode rejection are presented.

The behavior of the quad-ferential amplifier is summarized in chapter 5. Its characteristics are reviewed and suggestions for future work are offered.
Chapter 2

Quad-ferential Amplifier
Overview

The general behavior of a quad-ferential amplifier is introduced in this chapter. The definition of such an amplifier is presented and the block diagram is shown followed by a brief circuit level overview. The non-ideal transfer function is presented with the derivation demonstrating the behavior of a quad-ferential amplifier in a balanced system.

2.1 Definition of a quad-ferential amplifier

A quad-ferential amplifier is similar in function to a differential amplifier in that it responds to difference signals. The primary function of a quad-ferential amplifier is to remove overall common-mode and amplify differences. By design, it will accept four input voltages and produces four output voltages such that:

- the voltage difference between any pair of output voltages is proportional to the difference between the corresponding pair of input voltages through the same proportionality constant[1],
- the average value of the 4 output voltages is constant and unrelated to the input voltages[1]

A symbol of a quad-ferential amplifier is shown in figure 2.1(a). It has 11 pins. Two supplies, $V_s^+$ and $V_s^-$ (omitted for simplicity), four non-inverting inputs, $V_w$, $V_x$, $V_y$, and $V_z$, four outputs, $V_{out1}$, $V_{out2}$, $V_{out3}$, and $V_{out4}$ (polarity depends upon the levels of the input voltages), and output common-mode control, $V_{ocm}$. 
A quad-ferential amplifier is similar in appearance to a differential amplifier. There are three visual differences between the two architectures, as illustrated in figure 2.1. First, no inverting input terminals exist, second, there are two additional inputs, and third, there are two additional outputs.

Similar to a differential amplifier, a quad-ferential amplifier has a $V_{ocm}$ pin. The $V_{ocm}$ pin sets the output common-mode voltage to a level inside the output voltage swing of the amplifier. As with a differential amplifier, it is desirable for a quad-ferential amplifier to reject input common-mode signals and allow the output common-mode to be controlled by the $V_{ocm}$ pin.

Signal nomenclature will adhere to the IEEE standards as referenced by Hurst and Lewis. For example, $v_{odi}$ is an ac voltage, $V_{odi}$ is a dc voltage, and $v_{odi}$ is a total (ac + dc) voltage. Subsequently, pin references will be designated as an upper case with a lower case subscript, for example, $V_{ocm}$ designates the output common-mode pin.

Although a quad-ferential amplifier possesses four inputs and four outputs, it amplifies six pairs of difference voltages (which may or may not be differential voltages) to produce the following six unique pairs of difference voltages at the outputs (which may or may not be differential voltages):

\[
\begin{align*}
\v_{odi(1,2)} &= v_{o1} - v_{o2} = (v_{w} - v_{x})A(s) \quad \text{(2.1)} \\
\v_{odi(2,3)} &= v_{o2} - v_{o3} = (v_{x} - v_{y})A(s) \quad \text{(2.2)} \\
\v_{odi(3,4)} &= v_{o3} - v_{o4} = (v_{y} - v_{z})A(s) \quad \text{(2.3)} \\
\v_{odi(4,1)} &= v_{o4} - v_{o1} = (v_{z} - v_{w})A(s) \quad \text{(2.4)} \\
\v_{odi(1,3)} &= v_{o1} - v_{o3} = (v_{w} - v_{y})A(s) \quad \text{(2.5)} \\
\v_{odi(4,2)} &= v_{o4} - v_{o2} = (v_{z} - v_{x})A(s) \quad \text{(2.6)}
\end{align*}
\]
Correspondingly, the six unique input difference voltages a quad-ferential pair responds to are

\[ v_{ID(1,2)} = v_W - v_X \]  \hspace{1cm} (2.7)
\[ v_{ID(2,3)} = v_X - v_Y \]  \hspace{1cm} (2.8)
\[ v_{ID(3,4)} = v_Y - v_Z \]  \hspace{1cm} (2.9)
\[ v_{ID(4,1)} = v_Z - v_W \]  \hspace{1cm} (2.10)
\[ v_{ID(1,3)} = v_W - v_Y \]  \hspace{1cm} (2.11)
\[ v_{ID(4,2)} = v_Z - v_X \]  \hspace{1cm} (2.12)

The input common-mode voltage is defined as

\[ v_{IC} = \frac{v_W + v_X + v_Y + v_Z}{4} \]  \hspace{1cm} (2.13)

The output common-mode voltage (which is controlled by \( V_{ocm} \)) is

\[ V_{ocm} = v_{OC} = \frac{v_{O1} + v_{O2} + v_{O3} + v_{O4}}{4} \]  \hspace{1cm} (2.14)
2.2 Block diagram

A block diagram for a quad-ferential amplifier is presented in figure 2.2. For simplicity, bus wire is used to indicate parallel signal flows. The number of parallel signals are designated above each bus wire with a forward slash followed by a number. For example, the input stage accepts four input voltages and outputs eight currents. Additionally, a number in parenthesis under the label of a circuit block, indicates multiple copies, as shown with the gain and buffer stages.

The input stage is the core of a quad-ferential amplifier. It accepts four voltages, outputs eight currents, and consists of two quad-ferential pairs. Similar to a differential pair a quad-ferential pair may be thought of as a $g_m$ cell, a circuit block that accepts voltages and outputs currents.

An illustrative comparison between a differential pair and two quad-ferential pairs is shown in figure 2.3. One quad-ferential pair may be viewed as a single transistor from
a differential pair. Therefore, two quad-ferential pairs are necessary for it to behave comparably to a differential pair.

The currents generated by the input stage proceed to the gain stage. From the gain node, the signals are buffered and continue to the output. The output is fed into two nodes. The input stage via an $R_F$ and $R_G$ network (not shown) to establish negative feedback and the common-mode circuit block for control of the overall output common-mode voltage. The common-mode circuit block samples the output voltages, compares the average value to $V_{ocm}$ and sends the appropriate common-mode bias level to the gain stage.

From an architectural standpoint, a quad-ferential amplifier is an extension of a differential amplifier. Correspondingly, a quad-ferential amplifier processes its signals with three stages and two feedback loops: an input stage, a gain stage, an output stage, a quad-ferential feedback loop to establish negative feedback and a common-mode loop to control the output common-mode voltage.

2.3 Simplified circuit

A simplified circuit of a quad-ferential amplifier is presented in figure 2.4. Although each channel consist of a set of current source transistors, output buffers, and differential input currents, all channels share transistors $Q_9$ through $Q_{12}$, as shown in figure 2.4. The input stage receives its bias from $I_{TAIL1}$. Cascode transistor $Q_5$ ($Q_6$) receives a bias current of $I_{TAIL}/2$ from differential pair $Q_9$ and $Q_{10}$ via the diode connection of $Q_{11}$ ($Q_{12}$) and current source transistor $Q_7$ ($Q_8$). The Wilson current mirror, transistors $Q_1$ through $Q_4$, is biased by $Q_8$ such that $I_{c1} = I_{c5}$.

The input stage accepts four input voltages and outputs four differential currents, $i_{px}$ and $i_{nx}$. Transistors $Q_1$ through $Q_4$ form the Wilson current mirror which translates the transconductance of the input stage to the high impedance node. The high impedance node is formed by the output impedance of the Wilson current mirror in parallel with the output impedance of the cascode current source (transistors $Q_5$ and $Q_7$). From the gain node, the signal is buffered to the output.

From a feedback perspective, the common-mode level is sensed across the 4 resistors ($R_1$, $R_2$, $R_3$ and $R_4$) connected to the base of $Q_{10}$. The average of the output voltage is sampled and compared to the value on the $V_{ocm}$ pin. The differential pair, $Q_9$ and $Q_{10}$, control the output common-mode level of the amplifier by setting the appropriate bias level (as determined by the user’s input level on the $V_{ocm}$ pin) at the output by modulating the currents in $Q_{11}$ and $Q_{12}$. $I_{TAIL2}$ provides the bias level for $Q_9$ and $Q_{10}$. 
Figure 2.4: Simplified circuit of a quad-ferential amplifier.
For simplicity, the quad-ferential feedback was omitted and is discussed in detail in section 2.4

2.4 Transfer function

A typical configuration of a quad-ferential amplifier is shown in figure 2.5. There are four inputs, four outputs, and four feedback paths. Each feedback path contains a summing node, a gain resistor, and three feedback resistors. As shown, the summing nodes are $v_W$, $v_X$, $v_Y$, and $v_Z$ and they are associated with $v_{I1}$, $v_{I2}$, $v_{I3}$, and $v_{I4}$ respectively.

The resistors follow a different nomenclature than the summing nodes. The resistor reference designators indicate the function of the resistor, the input signal summing node the resistor is associated with, and in the case of the feedback resistors, the output voltage it is feeding back to the summing node. For example, $RF_{12}$. The “F” identifies it as a feedback resistor, the number “1” indicates this resistor is associated with the summing node associated with $v_{I1}$, $v_W$, and the number “2” designates $v_{O2}$ is being fed back across this resistor.

---

For the nomenclature in the analysis, the inputs will be abbreviated as $v_{I1}$, $v_{I2}$, $v_{I3}$, and $v_{I4}$ and the outputs will be abbreviated as $v_{O1}$, $v_{O2}$, $v_{O3}$, and $v_{O4}$.
Negative feedback for a quad-ferential amplifier is implemented in a similar manner to that of a differential amplifier. A differential amplifier establishes negative feedback by feeding one output back to its complementary input. The quad-ferential amplifier establishes negative feedback by taking the average of three outputs and feeding it back to the complementary summing junction, as shown in figure 2.5.\(^2\) For example, \(v_{O2}, v_{O3},\) and \(v_{O4}\) are fed back to channel 1’s summing node, \(v_w,\) via feedback resistors \(R_{F12}, R_{F13},\) and \(R_{F14}.\)

To derive the non-ideal transfer function, analysis is performed at each summing node. Applying Millman’s theorem\(^3\) and superposition at the summing nodes, \(v_w, v_x, v_y,\) and \(v_z\) results in

\[
v_w = \frac{v_{I1}}{\beta_w} + \frac{v_{O2}}{R_{F12}} + \frac{v_{O3}}{R_{F13}} + \frac{v_{O4}}{R_{F14}} \tag{2.15}
\]

\[
v_x = \frac{v_{I2}}{\beta_x} + \frac{v_{O1}}{R_{F21}} + \frac{v_{O3}}{R_{F23}} + \frac{v_{O4}}{R_{F24}} \tag{2.16}
\]

\[
v_y = \frac{v_{I3}}{\beta_y} + \frac{v_{O1}}{R_{F31}} + \frac{v_{O2}}{R_{F32}} + \frac{v_{O4}}{R_{F34}} \tag{2.17}
\]

\[
v_z = \frac{v_{I4}}{\beta_z} + \frac{v_{O1}}{R_{F41}} + \frac{v_{O2}}{R_{F42}} + \frac{v_{O3}}{R_{F43}} \tag{2.18}
\]

where

\[
\beta_w = \frac{1}{R_{G1}} + \frac{1}{R_{F12}} + \frac{1}{R_{F13}} + \frac{1}{R_{F14}}
\]

\[
\beta_x = \frac{1}{R_{G2}} + \frac{1}{R_{F21}} + \frac{1}{R_{F23}} + \frac{1}{R_{F24}}
\]

\[
\beta_y = \frac{1}{R_{G3}} + \frac{1}{R_{F31}} + \frac{1}{R_{F32}} + \frac{1}{R_{F34}}
\]

\[
\beta_z = \frac{1}{R_{G4}} + \frac{1}{R_{F41}} + \frac{1}{R_{F42}} + \frac{1}{R_{F43}}
\]

To present a succinct derivation of the transfer function, a concise version for \(\frac{v_{OD(1,2)}}{v_{ID(1,2)}}\) is offered and the detailed derivation for all unique difference voltages is in Appendix A. Substituting \(v_w, (2.15),\) and \(v_x, (2.16),\) into the definition which governs the behavior

\(^{2}\)Negative feedback is discussed in section 3.7.

\(^{3}\)Further information on Millman’s theorem is found in Appendix A.
of a quad-ferential amplifier, (2.1), results in

\[
v_{o1} - v_{o2} = \left[ \frac{v_{i1}}{R_{G1}} + \frac{v_{o2}}{R_{F12}} + \frac{v_{o1}}{R_{G1}} + \frac{v_{o4}}{R_{F14}} \right] - \left[ \frac{v_{i2}}{R_{G2}} + \frac{v_{o1}}{R_{F21}} + \frac{v_{o3}}{R_{F23}} + \frac{v_{o4}}{R_{F24}} \right] \beta_w \beta_x A(s) \tag{2.19}
\]

Although (2.19) describes the behavior between the outputs of \(v_{o1}\) and \(v_{o2}\), it does not include the common-mode component of the signal. Using the definition for \(v_{oc}\), (2.14), and solving it with respect to each output voltage, \(v_{o1}\) and \(v_{o2}\), yields

\[
v_{o1} = 4v_{oc} - v_{o2} - v_{o3} - v_{o4} \tag{2.20}
\]

\[
v_{o2} = 4v_{oc} - v_{o1} - v_{o3} - v_{o4} \tag{2.21}
\]

Substitution of (2.20) and (2.21) into (2.19), algebraic manipulation, and factoring results in

\[
v_{o1} - v_{o2} = \frac{v_{oc}}{R_{F12}}\beta_w - \frac{v_{o2}}{R_{F21}}\beta_x + \frac{v_{i1}}{R_{G1}}\beta_w - \frac{v_{i2}}{R_{G2}}\beta_x + \frac{4A(s)}{R_{F12}}\beta_w - \frac{4A(s)}{R_{F21}}\beta_x + \frac{2A(s)}{R_{F12}}\beta_w + \frac{2A(s)}{R_{F21}}\beta_x + \frac{2A(s)}{R_{F12}}\beta_w + \frac{2A(s)}{R_{F21}}\beta_x + \frac{A(s)}{R_{F12}}\beta_w + \frac{A(s)}{R_{F21}}\beta_x + \frac{A(s)}{R_{F12}}\beta_w + \frac{A(s)}{R_{F21}}\beta_x \tag{2.22}
\]

Applying symmetry to (2.22) by setting all feedback resistors, \(R_{Fxx}\) equal to each other and all gain resistors, \(R_{Gx}\), equal to each other allows for \(\beta_w = \beta_x = \beta_y = \beta_z = \beta'\) such that

\[
\beta' = \frac{1}{R_G} + \frac{3}{R_F} \tag{2.23}
\]

This symmetry allows for the common-mode term, \(v_{oc}\), to cancel and the two undesirable outputs with respect to \(\frac{v_{OD(1,2)}}{v_{OD(1,2)}}\), \(v_{o3}\) and \(v_{o4}\), to cancel. Thus, with symmetry, factoring, and isolating the input and output signals from (2.22), the non-ideal transfer function
Chapter 2. Overview of a Quad-ferential Amplifier

is

\[ A_{CL(1,2)}(s) = \frac{a(s) \frac{R_F}{R_F + 3R_G}}{1 + a(s) \frac{R_F}{R_F + 3R_G} \frac{R_G}{R_F}} \] (2.24)

which is of the canonical form

\[ A_{CL}(s) = \frac{A(s)}{1 + A\beta} \] (2.25)

where

\[ A(s) = \frac{a(s)R_F}{R_F + 3R_G} \] (2.26)

\[ \beta = \frac{R_G}{R_F} \] (2.27)

\[ a(s) = \frac{1}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2})} \] (2.28)

Poles \( p_1 \) and \( p_2 \) are set by the amplifier. The loop gain, \( T(s) \), is

\[ T(s) = A(s)\beta = \frac{R_G}{R_F + 3R_G} \frac{1}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2})} \] (2.29)

The “\( 3R_G \)” term inside the loop gain, (2.29) implies there are three outputs being fed back to a summing node. This is consistent with figure 2.5 which shows three outputs fed back to a summing node.\(^4\)

If \( A(s) \), the open loop gain of the system, is sufficiently high, then (2.24) reduces to \( \frac{1}{\beta} \) and the ideal closed loop gain is

\[ A_{CL,\text{IDEAL}(1,2)} = \frac{R_F}{R_G} \]

Due to the inherent architecture of this amplifier, utilizing it in a balanced system is necessary. As a result of a balanced system the non-ideal and ideal transfer functions remain true for any desired difference voltage. The symmetry allows the output common-mode voltage to be set by \( V_{ocm} \) and it allows for traditional differential signal processing to occur between any two signals.

\(^4\)A multi-input transfer function is discussed in Appendix B.
2.5 Summary

An overview of a quad-ferential amplifier was presented. A definition was introduced that included a symbol and detailed information for each pin. A block diagram was put forward followed by a brief circuit level discussion. The transfer function was derived, that demonstrated symmetry is required with respect to feedback and gain resistors for the quad-ferential amplifier to approach its ideal behavior.
Chapter 3

Analysis

This chapter presents analysis of a quad-ferential amplifier. The core of a quad-ferential amplifier is its input stage. The fundamental building block of its input stage is the quad-ferential pair. A quad-ferential pair is thoroughly examined and its behavior established. The large and small signal behavior for a quad-ferential input stage is analyzed followed by a concise study of the gain stage, output stage, bias cells, stability, compensation, and feedback. The analysis for this chapter is based on the following assumptions:

1. All components are ideal and perfectly matched.
2. All transistors are operating in the active region, unless noted otherwise.
3. For all transistors, $r_b = 0$, $r_o = \infty$, and $r_\mu = \infty$, unless noted otherwise.
4. $V_{CC}$ and $V_{EE}$, the positive and negative supply rails respectively, are equal in magnitude and opposite in polarity.

The following conventions are used to distinguish signal types:

- Signal nomenclature will adhere to IEEE standards, as referenced by Hurst and Lewis.\textsuperscript{2} For example, $v_{o1}$ is an ac voltage, $V_{o1}$ is a dc voltage, and $v_{o1}$ is a total (ac + dc) voltage.

- A “p” is added to the end of a device name or a signal unique to a positive quad-ferential pair and an “n” is added to the end of a device name or a signal unique to a negative quad-ferential pair. For example, $v_{o1p}$ is a signal for a positive quad-ferential pair and $v_{o1n}$ is a signal for a negative quad-ferential pair.

- A “s” is added to the end of a single-ended output voltage defined in terms of single-ended input voltages and a “d” is added to a single-ended output voltage...
defined in terms of input difference voltages. For example, \( v_{o1p,s} \) is a single-ended output voltage from channel 1 for a positive quad-ferential pair defined in terms of single-ended input voltages \( (v_{i1}, v_{i2}, v_{i3}, v_{i4}) \) and \( v_{o1p,d} \) is a single-ended output voltage from channel 1 for a positive quad-ferential pair defined in terms of input difference voltages \( (v_{id(1,2)}, v_{id(2,3)}, v_{id(3,4)}, v_{id(4,1)}, v_{id(1,3)}, \text{and } v_{id(4,2)}) \).

### 3.1 Quad-ferential pair

A quad-ferential pair is a \( g_m \) circuit block consisting of four inputs and four outputs. This section presents a quad-ferential pair, discusses its abstract configuration and analyzes the large and small signal behavior.

#### 3.1.1 Overview

A quad-ferential pair amplifies differences between any two input signals while rejecting common-mode components. For any circuit block to perform this task, it must fundamentally recognize and amplify differences. A differential pair is one such circuit block. However, a differential pair does not accept or output four signals.

To determine how to arrange differential pairs into a four port circuit block, the four inputs must be examined. Arranging the four inputs into a square results in figure 3.1(a). Input 1, In1, needs to interact with inputs In2, In3, and In4. Drawing a line from In1 to In2, In3, and In4 results in three lines as shown in figure 3.1(a). Continuing for all inputs results in 6 lines (not shown). Each line represents equal and independent interactions for all inputs.

Additionally, each line represents a function, a \( g_m \) cell as shown in figure 3.1(b). Configuring all \( g_m \) cells such that the input voltage connections do not cross and connecting the resulting output currents accordingly yields a quad-ferential pair as shown in figure 3.2. If the polarity on the inputs cross, the circuit will not be balanced and will function adversely. Recognizing each \( g_m \) cell represents a differential pair, each connection at an input correlates to one base connection. For example, three branches convene at In1. This correlates to three base connections. For the outputs, each branch at an output node, for example Out1, represents a collector. Therefore, Out1 consists of three collector connections.

A fundamental trait of a differential pair is its ability to keep the net change in output current zero. Thus, when a relative change (to each input) is applied to the inputs in

---

1Difference voltages may or may not be differential voltages. In most cases the six unique difference voltages are a combination of differential and non-differential difference voltages.
Chapter 3. Analysis: Quad-ferential Pair

(a) Configuring inputs.

(b) Configuring $g_m$ cells.

Figure 3.1: Configuring inputs for a quad-ferential pair.

Figure 3.2: A configuration of a quad-ferential pair.
the form of a “Δ”, the net change at the output must be zero Δ. To verify, figure 3.3 needs consideration. From figure 3.3, if three inputs are held at ground and In1 moves up +1Δ, Io1 changes by −1.5Δ and Io2, Io3, and Io4 each change by +0.5Δ as shown, resulting in a net change of 0Δ. The converse is also true. If In1 moves down a −1Δ, Io1 changes by +1.5Δ and Io2, Io3, and Io4 each change by −0.5Δ. Therefore, a quad-

![Figure 3.3](image_url)

**Figure 3.3:** Verifying the configuration of a quad-ferential pair. The modulating outputs sum to zero Δ.

ferential pair produces a net Δ of zero when a difference is applied to the inputs. A summary of this behavior is listed in table 3.1. The abstract configuration presented in figure 3.2 is valid and behaves as designed. By the principle of superposition, the above proof holds true for 2, 3, and 4 modulating inputs. A verification for multiple modulating inputs is offered in Appendix A.

**Table 3.1:** Output Δ response with one modulating input.

<table>
<thead>
<tr>
<th>Change in Inputs</th>
<th>Resulting Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{i1}$</td>
<td>$v_{i2}$</td>
</tr>
<tr>
<td>$±1Δ$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>$±1Δ$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
3.1.2 Description

A quad-ferential pair refers to a circuit block that accepts four input voltages and outputs four currents. It is comprised of six differential pairs strategically connected to form four output currents. There are at least two options for summing the twelve collectors of a quad-ferential pair. To distinguish between the two options, one is labeled a positive quad-ferential pair and the other a negative quad-ferential pair. A negative quad-ferential pair is created by cross coupling the collectors of a positive quad-ferential pair. Otherwise, the circuit blocks are identical.

A simplified schematic of a positive quad-ferential pair is shown in figure 3.4. Load resistors, $R_c$, are included to translate the output currents to voltages.
Figure 3.4: Simplified schematic of a positive quad-ferential pair.
Chapter 3. Analysis: Quad-ferential Pair

The schematic indicates $i_{O1P}$ is the summation of currents from collectors $Q_{1P}$, $Q_{8P}$, and $Q_{9P}$, $i_{O2P}$ is the summation of currents from collectors $Q_{2P}$, $Q_{3P}$, and $Q_{12P}$, $i_{O3P}$ is the summation of currents from collectors $Q_{4P}$, $Q_{5P}$, and $Q_{10P}$, and $i_{O4P}$ is the summation of currents from collectors $Q_{6P}$, $Q_{7P}$, and $Q_{11P}$. Therefore,

$$i_{O1P} = i_{C1P} + i_{C8P} + i_{C9P} \quad (3.1)$$
$$i_{O2P} = i_{C2P} + i_{C3P} + i_{C12P} \quad (3.2)$$
$$i_{O3P} = i_{C4P} + i_{C5P} + i_{C10P} \quad (3.3)$$
$$i_{O4P} = i_{C6P} + i_{C7P} + i_{C11P} \quad (3.4)$$

Each input voltage controls the base of three transistors. $v_{I1}$ controls the bases of $Q_{1P}$, $Q_{8P}$, and $Q_{9P}$, $v_{I2}$ controls the bases of $Q_{2P}$, $Q_{3P}$, and $Q_{12P}$, $v_{I3}$ controls the bases of $Q_{4P}$, $Q_{5P}$, and $Q_{10P}$, and $v_{I4}$ controls the bases of $Q_{6P}$, $Q_{7P}$, and $Q_{11P}$. There is one tail current, $I_{TAIL}$, for every differential pair and each tail current is identical, as the nomenclature in figure 3.4 suggests.

To create a negative quad-ferential pair, collectors from an additional set of six differential pairs are cross-coupled with respect to the positive quad-ferential pair, as illustrated in figure 3.5. For example, in a positive quad-ferential pair, $i_{O1P}$ is the summation of collector currents from transistors $Q_{1P}$, $Q_{8P}$, and $Q_{9P}$. In a negative quad-ferential pair, $i_{O1N}$ obtains its currents from transistors $Q_{2N}$, $Q_{7N}$, and $Q_{10N}$. Thus, upon inspection of the negative quad-ferential pair in figure 3.5 the output currents are defined as

$$i_{O1N} = i_{C2N} + i_{C7N} + i_{C10N} \quad (3.5)$$
$$i_{O2N} = i_{C1N} + i_{C4N} + i_{C11N} \quad (3.6)$$
$$i_{O3N} = i_{C3N} + i_{C6N} + i_{C9N} \quad (3.7)$$
$$i_{O4N} = i_{C5N} + i_{C8N} + i_{C12N} \quad (3.8)$$

and the input voltages are applied to comparable bases. For instance, in a positive quad-ferential pair $v_{I1}$ controls $Q_{1P}$, $Q_{8P}$, and $Q_{9P}$ and in a negative quad-ferential pair, $v_{II}$ controls $Q_{1N}$, $Q_{8N}$, and $Q_{9N}$.

Because a quad-ferential pair consists of four input and output terminals, it produces six unique difference signals which may or may not be differential voltages. In most cases the six unique difference voltages are a combination of differential and difference
Figure 3.5: Simplified schematic of a negative quad-ferential pair.
voltages. The input difference voltages to which a quad-ferential pair is sensitive to are

\[ v_{ID(1,2)} = v_{I1} - v_{I2} \] (3.9)

\[ v_{ID(2,3)} = v_{I2} - v_{I3} \] (3.10)

\[ v_{ID(3,4)} = v_{I3} - v_{I4} \] (3.11)

\[ v_{ID(4,1)} = v_{I4} - v_{I1} \] (3.12)

\[ v_{ID(1,3)} = v_{I1} - v_{I3} \] (3.13)

\[ v_{ID(4,2)} = v_{I4} - v_{I2} \] (3.14)

The corresponding output difference voltages are

\[ v_{OD(1,2)} = v_{O1} - v_{O2} \] (3.15)

\[ v_{OD(2,3)} = v_{O2} - v_{O3} \] (3.16)

\[ v_{OD(3,4)} = v_{O3} - v_{O4} \] (3.17)

\[ v_{OD(4,1)} = v_{O4} - v_{O1} \] (3.18)

\[ v_{OD(1,3)} = v_{O1} - v_{O3} \] (3.19)

\[ v_{OD(4,2)} = v_{O4} - v_{O2} \] (3.20)

and the input common-mode voltage is

\[ v_{IC} = \frac{v_{I1} + v_{I2} + v_{I3} + v_{I4}}{4} \] (3.21)

3.1.3 Large signal behavior

Large signal behavior gives insight into the available DC linear operating range. It illustrates the limited range of input voltages over which the circuit behaves almost linearly.[3] The large signal behavior presented assumes \( R_{EE} \), the Norton equivalent impedance of the current source[4], is infinite because it does not strongly affect the low-frequency, large-signal behavior of the circuit.[5] To understand the large signal behavior of a quad-ferential pair the following steps are required:

1. Define individual collector currents and manipulate them in terms of their input difference voltages. Result is (3.28) – (3.39).

2. Apply nodal analysis at the outputs, (3.40) – (3.47), and substitute appropriate collector currents, (3.28) – (3.39), to obtain an expression for the individual output voltages. Result is (3.48) – (3.55).
3. Substitute the individual voltages into the output difference expressions for a quad-ferential pair, (3.15) – (3.20), to obtain difference equations for a positive and a negative quad-ferential pair. Result is (3.56) – (3.67).

Because this architecture is symmetrical and the collector currents for a positive and a negative quad-ferential pair are identical, applying KVL to two transistors of a positive quad-ferential pair (figure 3.4) is sufficient for describing how a single collector behaves to a difference input. The nodal equations for the remaining ten transistors is found in Appendix A. Thus, applying KVL to figure 3.4 from \( V_{I1} \) to \( V_{I2} \) in a clockwise direction yields

\[
V_{I1} + V_{EB1} - V_{EB2} - V_{I2} = 0 \quad (3.22)
\]

Recalling

\[
V_{EB} = V_t \ln \frac{I_C}{I_s}
\]

where \( I_C \) is the collector current, \( I_s \) is the saturation current, and \( V_t \) is the thermal voltage,\(^2\) (3.22) becomes

\[
V_{I1} + V_t \ln \frac{I_{C1}}{I_s} - V_t \ln \frac{I_{C2}}{I_s} - V_{I2} = 0 \quad (3.23)
\]

Factoring, collecting like terms, recognizing \( I_s \) is a constant for a given process, and recalling the definition for \( V_{ID(1,2)} \), (3.9), simplifies (3.23) to

\[
\frac{I_{C1}}{I_{C2}} = e^{-\frac{V_{ID(1,2)}}{V_t}} \quad (3.24)
\]

Although (3.24) expresses \( I_{C1} \) and \( I_{C2} \) in terms of a ratio responding exponentially to their input differences, it is necessary to include the constraint \( I_{TAIL} \) imposes upon the emitter currents. Performing mesh analysis\(^3\) at the emitters of \( Q_{IP} \) and \( Q_{IP} \) of figure 3.4 and recalling a collector current is proportional to an emitter current by alpha results in

\[
I_{TAIL} = \frac{I_{C1} + I_{C2}}{\alpha} \quad (3.25)
\]

\(^2\)At ambient temperature \( V_t \) is approximately 25.9mV.

\(^3\)Mesh analysis for all emitters may be found in Appendix A.
Solving (3.25) for $I_{c1}$, substituting it into (3.24), and factoring like terms with respect to the input difference voltage results in an expression for $I_{c2}$

$$I_{c2} = \frac{\alpha I_{TAIL}}{1 + e^{-\frac{V_{ID(1,2)}}{V_t}}}$$

To obtain a relationship for $I_{c1}$, a similar methodology may be applied. Solving (3.25) for $I_{c2}$, substituting it into (3.24), then factoring and solving for $I_{c1}$ yields

$$I_{c1} = \frac{\alpha I_{TAIL} e^{-\frac{V_{ID(1,2)}}{V_t}}}{1 + e^{-\frac{V_{ID(1,2)}}{V_t}}}$$

(3.26)

To manipulate $I_{c1}$ into a form similar to $I_{c2}$ it is necessary to factor out a one as follows

$$I_{c1} = \frac{\alpha I_{TAIL}}{e^{-\frac{V_{ID(1,2)}}{V_t}} + 1} \left( e^{-\frac{V_{ID(1,2)}}{V_t}} \right)$$

(3.27)

Recalling the algebraic property, $\frac{1}{x^{-1}} = x$, and applying it to (3.27) results in

$$I_{c1} = \frac{\alpha I_{TAIL}}{1 + e^{-\frac{V_{ID(1,2)}}{V_t}}}$$

With $I_{c1}$ in this form, it is clear $I_{c1}$ and $I_{c2}$ behave as complements respect to $I_{TAIL}$.

Applying the above methodology to the remaining collector currents results in comparable expressions with respect to the five remaining input difference voltages, (3.9) – (3.14). The even numbered collector currents contain an exponential with a negative input difference voltage (i.e. $I_{c2}$) and the odd numbered collector currents do not (i.e.
$I_{C1}$ as shown in (3.28) – (3.39).

\[
I_{C1} = \frac{\alpha I_{TAIL}}{1 + e^{\frac{-V_{ID(1,2)}}{V_{T}}}} (3.28)
\]

\[
I_{C2} = \frac{\alpha I_{TAIL}}{1 + e^{\frac{V_{ID(1,2)}}{V_{T}}}} (3.29)
\]

\[
I_{C3} = \frac{\alpha I_{TAIL}}{1 + e^{\frac{V_{ID(2,3)}}{V_{T}}}} (3.30)
\]

\[
I_{C4} = \frac{\alpha I_{TAIL}}{1 + e^{-\frac{V_{ID(2,3)}}{V_{T}}}} (3.31)
\]

\[
I_{C5} = \frac{\alpha I_{TAIL}}{1 + e^{\frac{V_{ID(3,4)}}{V_{T}}}} (3.32)
\]

\[
I_{C6} = \frac{\alpha I_{TAIL}}{1 + e^{-\frac{V_{ID(3,4)}}{V_{T}}}} (3.33)
\]

\[
I_{C7} = \frac{\alpha I_{TAIL}}{1 + e^{\frac{V_{ID(4,1)}}{V_{T}}}} (3.34)
\]

\[
I_{C8} = \frac{\alpha I_{TAIL}}{1 + e^{-\frac{V_{ID(4,1)}}{V_{T}}}} (3.35)
\]

\[
I_{C9} = \frac{\alpha I_{TAIL}}{1 + e^{\frac{V_{ID(1,3)}}{V_{T}}}} (3.36)
\]

\[
I_{C10} = \frac{\alpha I_{TAIL}}{1 + e^{-\frac{V_{ID(1,3)}}{V_{T}}}} (3.37)
\]

\[
I_{C11} = \frac{\alpha I_{TAIL}}{1 + e^{\frac{V_{ID(2,4)}}{V_{T}}}} (3.38)
\]

\[
I_{C12} = \frac{\alpha I_{TAIL}}{1 + e^{-\frac{V_{ID(2,4)}}{V_{T}}}} (3.39)
\]

In summarizing (3.28) through (3.39), three important observations surface. First, the collector currents are limited by $I_{TAIL}$, second, they are dependent upon the exponential relationship of their respective input difference voltage, and third, each collector current has a complement. For example $I_{C7}$ (eqn. 3.34) and $I_{CS}$ (eqn. 3.34) are limited by $I_{TAIL}$, they are dependent upon $V_{ID(4,1)}$, and they are complements of each other. Furthermore, small changes in $V_{ID(4,1)}$ will steer the tail current between $I_{C7}$ or $I_{CS}$. With collector currents defined, single-ended and differential output voltages can be examined.

### 3.1.3.1 Single-ended behavior

Single-ended output voltage behavior yields insight into how each output responds to differences at its input, a primary application for an amplifier. Additionally, investigation
into single-ended behavior illustrates the balance and symmetry of this architecture.

Applying KVL to the output nodes of a positive quad-ferential pair (figure 3.4) yields

\[ V_{OIP} = (I_c + I_{CS} + I_{CS}) R_C + V_{EE} \]  
(3.40)

\[ V_{OIP} = (I_c + I_{CS} + I_{CS}) R_C + V_{EE} \]  
(3.41)

\[ V_{OIP} = (I_c + I_{CS} + I_{CS}) R_C + V_{EE} \]  
(3.42)

\[ V_{OIP} = (I_{CS} + I_{MT} + I_{GT}) R_C + V_{EE} \]  
(3.43)

Comparable analysis to the outputs of a negative quad-ferential pair (figure 3.5), yields

\[ V_{OIN} = (I_c + I_{CS} + I_{CS}) R_C + V_{EE} \]  
(3.44)

\[ V_{OIN} = (I_c + I_{CS} + I_{CS}) R_C + V_{EE} \]  
(3.45)

\[ V_{OIN} = (I_c + I_{CS} + I_{CS}) R_C + V_{EE} \]  
(3.46)

\[ V_{OIN} = (I_{CS} + I_{MT} + I_{GT}) R_C + V_{EE} \]  
(3.47)

Substitution of collector currents, \( (3.28) - (3.39) \), into \( (3.40) - (3.47) \) results in single-ended output expressions responding to input differences.
Three observations are prevalent from the large signal single-ended output voltage expressions responding to input differences, (3.48) through (3.55). First, each channel (#1, #2, #3, #4) influences three output difference voltages, (for instance channel one contributes to $V_{OD(1,2)}$, $V_{OD(4,1)}$, $V_{ID(0,3)}$). Second, a single-ended output channel contains three terms and is proportional to three difference inputs. Using $V_{O1}$ as an example, the following generalization is true

$$V_{O1} = f(V_{ID(1,2)}, V_{ID(4,1)}, V_{ID(1,3)})$$

Third, the arguments for the exponential terms, the input difference voltages, adhere to a pattern which reveals the three arguments sum. All arguments with the first number in the input difference voltage corresponding to the single-ended output channel, will have the same sign. The other arguments will be opposite in sign. For example, $V_{O_{1P,D}}$ ( (3.48)). The input difference voltage for the exponential argument in the first and third terms are $V_{ID(1,2)}$ and $V_{ID(1,3)}$. Accordingly, the first and third input difference voltages are of the same sign, in this case positive, while the input difference voltage of the second term is opposite in sign, in this case negative. This observation is important because it describes how the collector currents sum rather than oppose each other.

Before presenting simulation results of how a quad-ferential pair responds to large signal input differences, a discussion of the variable for the independent axis is necessary. An unconventional variable is required of the x-axis because one single-ended input voltage influences three difference voltages. To sweep difference voltages independently, a transient approach is necessary. Time is therefore the independent variable which allows independent movement of the input difference voltages.

The input difference voltages stimulating $I_{OIP}$, $V_{ID(1,2)}$, $V_{ID(4,1)}$ and $V_{ID(1,3)}$ are shown in figure 3.6(a). The resulting output current, $I_{OIP}$ with its individual elements, are shown in figure 3.6(b). The plot shows the individual collector currents summing. With a tail current of 100μA, each collector is capable of sourcing 100μA theoretically making the maximum current through $I_{OIP}$ 300μA. The simulation indicates $I_{OIP}$ is not 300μA. This discrepancy is due to $I_{CP}$ not reaching its final value of 100μA.

An additional trait is evident from the single-ended large signal expressions responding to input difference voltages, (3.48) through (3.55). The same channel between a positive and a negative quad-ferential pair are opposite in sign. For example, $V_{O{IP}}$ and $V_{O{IN}}$, respectively (3.48) and (3.52), describe this behavior which is illustrated in figure 3.7. Its corresponding input difference voltages are shown in figure 3.6(a). According to (3.48) and (3.52), if $I_{TAIL}=100μA$, $R_C=1kΩ$, and supplies are set to ±2.5V, the outputs should
Chapter 3. Analysis: Quad-ferential Pair

(a) Input difference voltages $I_{O1P}$ responds to, $V_{ID(1,2)}$, $V_{ID(4,1)}$ and $V_{ID(1,3)}$.

(b) Simulation of $I_{O1P}$ and its individual collector currents, $I_{C1}$, $I_{C8}$, and $I_{C9}$. $I_{TAIL}=100\mu A$.

Figure 3.6: Simulation of $I_{O1P}$ and its individual collector currents.
swing between $-2.2\text{V}$ and $-2.5\text{V}$. The simulation in figure 3.7 reflects this behavior and as expected, illustrates the complementary relationship between $V_{\text{OIP}}$ and $V_{\text{OIN}}$.

With the behavior of the individual voltages confirmed it is important to prove correlation between derivation and simulation results. For simplicity, one output voltage, $V_{\text{OIP}}$, (3.48), is used for comparison. To superimpose (3.48) onto simulation, the independent axis is bounded such that one variable, a single-ended input voltage ($V_{\text{I1}}$), is swept. Consequently, $V_{\text{I2}}$ is opposite in magnitude to $V_{\text{I1}}$, $V_{\text{I3}}$ is proportional and 50% of $V_{\text{I1}}$, and $V_{\text{I4}}$ is opposite in magnitude and proportional to $V_{\text{I1}}$ by 50% as shown in figure 3.8(a). The relevant input difference voltages are shown in figure 3.8(b).

A plot of the theoretical expression, (3.48), and simulation result for $V_{\text{OIP}}$ is presented in figure 3.9(a). The difference between the two is is shown in figure 3.9(b). As figure 3.9(b) illustrates, the maximum delta between simulation and derivation is approximately 2.7mV, which translates to a 10% difference. Thus, simulation and derivation correlate.

When a circuit block consists of multiple inputs and outputs and is capable of amplifying differences while rejecting common-mode it is typically implemented in a differential signal chain. Thus, another trait of interest is how the four single-ended outputs of a quad-ferential pair responds to two unique differential inputs. To implement two unique differential signals, the individual inputs are paired. $V_{\text{I1}}$ is differential with respect to $V_{\text{I2}}$ and $V_{\text{I3}}$ is differential with respect to $V_{\text{I4}}$. Furthermore, inherent in using two different
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Figure 3.8: Input signals used for correlating theoretical expression, (3.48), with simulation result of $V_{OP}$. 

(a) Single-ended input voltages.

(b) Relevant input difference voltages affecting $V_{OP}$, $V_{ID(1,2)}$, $V_{ID(4,1)}$ and $V_{ID(1,3)}$. 
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(a) Theoretical expression (3.48) and simulation result of $V_{O1P}$.

(b) Correlation to within 10%.

**Figure 3.9:** Comparison of theoretical expression, (3.48), and simulation result for $V_{O1P}$. 
input differential voltages is one differential voltage being larger in magnitude than the
other. Thus, $V_{I1}$ and $V_{I4}$ are chosen to be fifty percent less in magnitude with respect
to $V_{I1}$ and $V_{I2}$. To maintain the highest degree of freedom with fully independent input
voltages, simulations are executed with respect to time. The rate of change ($\frac{dV}{dt}$) between
single-ended input voltages is determined by setting the magnitude and rise/fall time of
the input transients.

This configuration of the individual inputs is seen in figure 3.10(a) with the corresponding
difference inputs in figure 3.10(b). For simplicity, transient behavior is identical except
for polarity and magnitude as shown in figure 3.10(a). This input configuration is shown
in figure 3.10(b). Figure 3.10(b) reveals four unique difference voltages instead of six
when two unique differential signals are applied to the inputs. Within the four distinct
difference voltages, $V_{ID(1,2)}$ and $V_{ID(3,4)}$ exhibit unique behavior, $V_{ID(2,3)}$ and $V_{ID(4,1)}$ display
identical differences as does $V_{ID(1,3)}$ and $V_{ID(4,2)}$.

With the input voltages established, figure 3.11(a) is a simulation of the single-ended
output voltages, $V_{0IP}$ through $V_{04P}$, responding to the two unique differential signals from
figure 3.10(a). There is mild peaking on $V_{0IP}$ and $V_{04P}$ due to the presence of two unique
differential input voltages. When two unique differential signals are applied, one of the
collector currents is opposite in magnitude compared to the other two. Instead of three
collector currents summing together, a difference is seen during the transition, and the
subtraction manifests itself through peaking. Using $V_{0IP}$ as an example, its individual
collector currents are $I_{CIP}$, $I_{C3P}$, and $I_{C10P}$ and are shown in figure 3.11(b). The simulation
in figure 3.11(b) illustrates $I_{CIP}$ and $I_{C3P}$ summing while $I_{C10P}$ subtracts from $I_{OIP}$.

To examine this behavior from a qualitative point of view, it is necessary to refer to
the simplified schematic of a positive quad-ferential pair (figure 3.4). From figure 3.4 it
can be seen that transistors $Q_{9P}$ and $Q_{10P}$ are responsible for $I_{OIP}$ and $I_{C10P}$. $V_{II}$ controls
the base of $Q_{9P}$ while $V_{I3}$ controls the base of $Q_{10P}$. Because $V_{II}$ has a faster rate of
change with respect to $V_{I3}$, $I_{C10P}$ must decrease. A comparable argument can be made
for $V_{04P}$, however, the transistors and input voltages involved are $Q_{11P}$, $Q_{12P}$, $V_{I4}$ and
$V_{I2}$. Expanding upon that explanation, $V_{0IP}$ and $V_{03P}$ do not have peaking because the
$\frac{dV}{dt}$ of $V_{I1}$ and $V_{I2}$ are faster compared to $V_{I3}$ and $V_{I4}$ and thus, $V_{0IP}$ and $V_{03P}$, behave as
expected. Although two channels exhibit peaking, it is unimportant because it occurs
outside the linear region. The large signal DC linear region is important because it is
the region where small signal behavior can be modeled with linear approximations.
Figure 3.10: Input voltages for evaluating single-ended outputs of a positive quad-ferential pair.
Figure 3.11: Single-ended output voltages of a positive quad-ferential pair.
3.1.3.2 Differential behavior

Differential behavior describes how a specific difference across two inputs are processed in the signal chain. The primary function of a quad-ferential pair is to amplify differences at its inputs. To evaluate its differential behavior, the difference voltages of a quad-ferential pair are restated as follows:

\[
V_{ID(1,2)} = V_{I1} - V_{I2}
\]
\[
V_{ID(2,3)} = V_{I2} - V_{I3}
\]
\[
V_{ID(3,4)} = V_{I3} - V_{I4}
\]
\[
V_{ID(4,1)} = V_{I4} - V_{I1}
\]
\[
V_{ID(1,3)} = V_{I1} - V_{I3}
\]
\[
V_{ID(4,2)} = V_{I4} - V_{I2}
\]

Due to the symmetrical nature of a quad-ferential pair, two channels of a positive quad-ferential pair are used for analysis and the remaining channels, along with the behavior of a negative quad-ferential pair, follow. Thus, to determine \(V_{OD(1,2)P}, V_{ODP,D}, (3.49)\),

\[
V_{ODP,D} = \alpha I_{TAIL} R_C \left( \frac{1}{1 + e^{-\frac{-V_{ID(1,2)}}{V_t}}} + \frac{1}{1 + e^{-\frac{-V_{ID(2,3)}}{V_t}}} + \frac{1}{1 + e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right) + V_{EE}
\]

is subtracted from \(V_{OIP}, (3.48)\),

\[
V_{OIP} = \alpha I_{TAIL} R_C \left( \frac{1}{1 + e^{-\frac{-V_{ID(1,2)}}{V_t}}} + \frac{1}{1 + e^{-\frac{-V_{ID(4,1)}}{V_t}}} + \frac{1}{1 + e^{-\frac{-V_{ID(1,3)}}{V_t}}} \right) + V_{EE}
\]

yielding

\[
V_{OD(1,2)P} = A \left( \frac{1}{1 + e^{-\frac{-V_{ID(1,2)}}{V_t}}} - \frac{1}{1 + e^{-\frac{-V_{ID(1,2)}}{V_t}}} - \frac{1}{1 + e^{-\frac{-V_{ID(2,3)}}{V_t}}} \right) + A \left( \frac{1}{1 + e^{-\frac{-V_{ID(4,1)}}{V_t}}} + \frac{1}{1 + e^{-\frac{-V_{ID(4,1)}}{V_t}}} - \frac{1}{1 + e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right)
\]

where

\[
A = \alpha I_{TAIL} R_C
\]
Applying comparable analysis to a negative quad-ferential pair (figure 3.5) results in

\[ V_{OD(1,2)} = A \left( \tanh \frac{-V_{ID(1,2)}}{2V_t} + \frac{1}{1+e^{-\frac{-V_{ID(2,3)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(3,4)}}{V_t}}} + \frac{1}{1+e^{-\frac{-V_{ID(4,1)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right) \] (3.56)

Multiplying the first and second term by a form of 1, \( \frac{e^{-x}-e^{x}}{e^{x}+e^{-x}} \), and recalling the tanh identity, \( \tanh x = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}} \), results in

\[ V_{OD(1,2)} = A \left( \tanh \frac{-V_{ID(1,2)}}{2V_t} + \frac{1}{1+e^{-\frac{-V_{ID(2,3)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(3,4)}}{V_t}}} + \frac{1}{1+e^{-\frac{-V_{ID(4,1)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right) \] (3.56)

Applying the same methodology to the remaining difference voltages for a positive quad-ferential pair yields

\[ V_{OD(2,3)} = A \left( \tanh \frac{-V_{ID(2,3)}}{2V_t} + \frac{1}{1+e^{-\frac{-V_{ID(1,2)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(3,4)}}{V_t}}} + \frac{1}{1+e^{-\frac{-V_{ID(4,1)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right) \] (3.57)

\[ V_{OD(3,4)} = A \left( \tanh \frac{-V_{ID(3,4)}}{2V_t} + \frac{1}{1+e^{-\frac{-V_{ID(1,2)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(2,3)}}{V_t}}} + \frac{1}{1+e^{-\frac{-V_{ID(4,1)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right) \] (3.58)

\[ V_{OD(4,1)} = A \left( \tanh \frac{-V_{ID(4,1)}}{2V_t} + \frac{1}{1+e^{-\frac{-V_{ID(1,2)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(2,3)}}{V_t}}} + \frac{1}{1+e^{-\frac{-V_{ID(3,4)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right) \] (3.59)

\[ V_{OD(1,3)} = A \left( \tanh \frac{-V_{ID(1,3)}}{2V_t} + \frac{1}{1+e^{-\frac{-V_{ID(1,2)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(2,3)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(3,4)}}{V_t}}} + \frac{1}{1+e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right) \] (3.60)

\[ V_{OD(2,4)} = A \left( \tanh \frac{-V_{ID(2,4)}}{2V_t} + \frac{1}{1+e^{-\frac{-V_{ID(1,2)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(2,3)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(3,4)}}{V_t}}} + \frac{1}{1+e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right) \] (3.61)

Applying comparable analysis to a negative quad-ferential pair (figure 3.5) results in

\[ V_{OD(1,2)} = A \left( \tanh \frac{-V_{ID(1,2)}}{2V_t} + \frac{1}{1+e^{-\frac{-V_{ID(2,3)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(3,4)}}{V_t}}} + \frac{1}{1+e^{-\frac{-V_{ID(4,1)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right) \] (3.62)

\[ V_{OD(2,3)} = A \left( \tanh \frac{-V_{ID(2,3)}}{2V_t} + \frac{1}{1+e^{-\frac{-V_{ID(1,2)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(3,4)}}{V_t}}} + \frac{1}{1+e^{-\frac{-V_{ID(4,1)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right) \] (3.63)

\[ V_{OD(3,4)} = A \left( \tanh \frac{-V_{ID(3,4)}}{2V_t} + \frac{1}{1+e^{-\frac{-V_{ID(1,2)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(2,3)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(4,1)}}{V_t}}} + \frac{1}{1+e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right) \] (3.64)

\[ V_{OD(4,1)} = A \left( \tanh \frac{-V_{ID(4,1)}}{2V_t} + \frac{1}{1+e^{-\frac{-V_{ID(1,2)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(2,3)}}{V_t}}} + \frac{1}{1+e^{-\frac{-V_{ID(3,4)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right) \] (3.65)

\[ V_{OD(1,3)} = A \left( \tanh \frac{-V_{ID(1,3)}}{2V_t} + \frac{1}{1+e^{-\frac{-V_{ID(1,2)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(2,3)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(3,4)}}{V_t}}} + \frac{1}{1+e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right) \] (3.66)

\[ V_{OD(2,4)} = A \left( \tanh \frac{-V_{ID(2,4)}}{2V_t} + \frac{1}{1+e^{-\frac{-V_{ID(1,2)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(2,3)}}{V_t}}} - \frac{1}{1+e^{-\frac{-V_{ID(3,4)}}{V_t}}} + \frac{1}{1+e^{-\frac{-V_{ID(4,2)}}{V_t}}} \right) \] (3.67)
As expected from this balanced and symmetrical architecture, a difference output voltage from a negative quad-ferential pair is opposite in magnitude of a difference output voltage from a positive quad-ferential pair (with respect to the same input difference voltage) as reflected in $V_{OD(4,2)P}$, (3.56), and $V_{OD(4,2)N}$, (3.62). This complementary trait between a positive and a negative quad-ferential pair creates six pairs of output differential voltages. To further support this large signal behavior, simulation results of the output difference voltages for a positive and negative quad-ferential pair, responding to the inputs of figure 3.10(b), are presented in figure 3.12. This result is expected because it is consistent with the single-ended behavior from subsubsection 3.1.3.1.

Furthermore, two observations are visible with respect to the linear DC operating range of a quad-ferential pair from (3.56) – (3.67) and figure 3.12. First, when the outputs are taken as differences, the linear region responds in a tanh manner with respect to its primary input differential voltage. The knee in the curve in figure 3.12 is determined by the exponential component which is a result of the non-differential input voltages. Second, when two unique input differential voltages are applied, as shown in figure 3.10(a), the input difference range where all transistors remain linear is determined by the input difference pairs which are non-differential and possess the same polarity. From figure 3.10(b), two such difference signals are $V_{ID(1,3)}$ and $V_{ID(4,2)}$. Identifying the linear region for $V_{OD(1,3)}$ and $V_{OD(4,2)}$ as 0.97μs to 0.4μs from figure 3.12 and using that information in conjunction with figure 3.10(b) results in a linear region of operation of approximately 60mV ($\approx 2V_t$) or less. However, if the outputs of interest are the respective differential outputs (in this example $V_{OD(1,2)}$ and $V_{OD(3,4)}$), the input linear region of operation is approximately 200mV ($\approx 7.7V_t$) for $V_{OD(1,2)}$ and approximately 100mV ($\approx 3.8V_t$) for $V_{OD(3,4)}$. 
3.12b sim_ls_all_vodns

(a) Output difference voltages for a positive quad-ferential pair. Input difference voltages are shown in figure 3.10(b).

(b) Output difference voltages for a negative quad-ferential pair. Input difference voltages are shown in figure 3.10(b).

**Figure 3.12:** Large signal output difference voltages for a positive and a negative quad-ferential pair.
3.1.3.3 Large signal behavior summary

The derivation of the individual collector currents revealed each collector is limited by $I_{\text{TAIL}}$, responds exponentially to its respective input difference voltage, and is a complement to the collector of which it shares an emitter node.

The single-ended output behavior showed that each channel is affected by three pairs of input difference voltages. Furthermore, the arguments of the three terms within a single-ended output expression adhered to a pattern such that the collector currents add. Additionally, simulation results illustrated that the same channel between a positive and a negative quad-ferential pair are symmetrical. Correlation between theoretical expressions and simulation was 10%.

The differential behavior built on the analysis from the single-ended output expressions. It was shown that a negative and a positive quad-ferential pair are symmetrical with respect to the same difference input voltage. An investigation was made into the DC linear operating range. If two unique differential inputs are applied the linear operating range is dependent upon the non-differential inputs. If the outputs of interest are the respective input differential voltages the linear operating range increases.

The behavior pertinent for a quad-ferential amplifier is that a positive and a negative quad-ferential pair are symmetrical such that the same channels between the two behave differentially.

3.1.4 Small signal behavior

Many electronic circuits, such as radio receivers, communications, and signal processing circuits generally carry small signals on top of a bias.[6] Small signal analysis defines the necessary parameters to describe a circuit’s AC behavior. To make the small signal analysis realistic to within first order approximations and to be able to take advantage of linearization techniques, the following assumptions are necessary:

1. To maintain operation within the linear region, the DC input voltage difference is assumed to be zero.

2. Norton equivalent resistance of tail currents is finite and is represented by element, $R_{\text{EE}}$. This is important because $R_{\text{EE}}$ has a considerable effect on small signal behavior. For example, if $R_{\text{EE}} = \infty$ then the common-mode rejection would be infinite. Because infinite common-mode rejection is not realistic, $R_{\text{EE}}$ is included in the small signal analysis.
3.1.4.1 Single-ended behavior

A quad-ferential pair is a multi-port circuit block. It has four input terms \(v_{i1}, v_{i2}, v_{i3}, v_{i4}\) and four output terms \(v_{o1}, v_{o2}, v_{o3}, v_{o4}\) which may or may not be referenced to ground. Applying direct small signal analysis in conjunction with superposition results in the following four equations:

\[
\begin{align*}
v_{o1,s} &= A_{11}v_{i1} + A_{12}v_{i2} + A_{13}v_{i3} + A_{14}v_{i4} \quad (3.68) \\
v_{o2,s} &= A_{21}v_{i1} + A_{22}v_{i2} + A_{23}v_{i3} + A_{24}v_{i4} \quad (3.69) \\
v_{o3,s} &= A_{31}v_{i1} + A_{32}v_{i2} + A_{33}v_{i3} + A_{34}v_{i4} \quad (3.70) \\
v_{o4,s} &= A_{41}v_{i1} + A_{42}v_{i2} + A_{43}v_{i3} + A_{44}v_{i4} \quad (3.71)
\end{align*}
\]

The sixteen voltage gains, \(A_{11}\) through \(A_{44}\), from (3.68) through (3.71), specify small signal operation of the circuit where each gain may be described as a response to the following loading conditions:

\[
A_{xy} = \left. \frac{v_{ox}}{v_{iy}} \right|_{v_{iw} = v_{ix} = v_{iz} = 0}
\]

\(A_{xy}\) is the gain constant where \(x\) and \(y\) are independent integers ranging from 1 to 4. The \(x\) represents the output term and \(y\) represents the input term. Terms \(v_{iw}, v_{ix},\) and \(v_{iz}\) represent the independent input voltages. For example, \(A_{21}\) \(^4\) is:

\[
A_{21} = \left. \frac{v_{o2}}{v_{i1}} \right|_{v_{i2} = v_{i3} = v_{i4} = 0}
\]

Furthermore, (3.68) through (3.71) show four gain constants of the form \(A_{xx}\) and twelve gain constants of the form \(A_{xy}\).

Given the fact this circuit is a balanced and symmetrical architecture it will be shown that gain constants of the form \(A_{xx}\) have identical definitions as well as gain constants of the form \(A_{xy}\). Additionally, since there are two different configurations for a quad-ferential pair, a positive and negative (cross-coupled) option, there are four unique gain

\(^4\)An explicit list of the sixteen gain constants with specific input and output conditions is in Appendix A.
constants, $A_{xxp}$, $A_{xyp}$, $A_{xxn}$, and $A_{xyn}$. The gain constants are defined as

$$A_{xxp} = -\frac{g_m R_C}{2} \left( \frac{3}{1 + \frac{1}{2B}} \right)$$

$$A_{xyp} = \frac{\frac{g_m R_C}{2}}{\left( \frac{1}{1 + \frac{1}{2B}} \right)}$$

$$A_{xxn} = \frac{\frac{g_m R_C}{2}}{\left( \frac{3}{1 + \frac{1}{2B}} \right)}$$

$$A_{xyn} = -\frac{g_m R_C}{2} \left( \frac{1 + \frac{1}{B}}{1 + \frac{1}{2B}} \right)$$

where

$$B = g_m R_{EE} \left( 1 + \frac{1}{\beta_0} \right)$$

To derive the small signal gain constants, the following steps are necessary:

1. Establish small signal models. Result is figure 3.13 and figure 3.14.
2. Perform nodal analysis at the outputs. Result is (3.72) – (3.75) and (3.88) – (3.91).
3. Evaluate emitters with mesh analysis. Result is (3.76) – (3.81).
4. Substitute emitter voltages into nodal equations. Result is (3.72) – (3.75) and (3.88) – (3.91).
5. Use fully defined output voltages, (3.82) through (3.85) and (3.92) through (3.85), to extract gain constants. Result is (3.86) – (3.87) and (3.96) – (3.97).

For simplicity, individual output voltages for a positive quad-ferential pair are examined followed by a succinct analysis of the individual output voltages for a negative quad-ferential pair. To perform small signal analysis, a full small signal model is required to illustrate how simultaneously modulating inputs affect each output.\footnote{Because a quad-ferential amplifier responds to six difference voltages and employs six emitter nodes, (3.76) through (3.81), a half-circuit approach is not applicable.} Figure 3.13 is a small signal model for a positive quad-ferential pair.

Inspection of figure 3.13 reveals this is a balanced system. As such, the symmetry will be evident in the derivation for the small signal gain constants. Applying KVL to the...
Figure 3.13: Small signal model for a positive quad-ferential pair.
output nodes of figure 3.13

\[ v_{o1p} = -g_m R_C (v_{\pi 1} + v_{\pi 8} + v_{\pi 9}) \]  
(3.72)

\[ v_{o2p} = -g_m R_C (v_{\pi 2} + v_{\pi 3} + v_{\pi 12}) \]  
(3.73)

\[ v_{o3p} = -g_m R_C (v_{\pi 4} + v_{\pi 5} + v_{\pi 10}) \]  
(3.74)

\[ v_{o4p} = -g_m R_C (v_{\pi 6} + v_{\pi 7} + v_{\pi 11}) \]  
(3.75)

Recognizing \( v_{\pi 1} \) through \( v_{\pi 12} \) are a function of their respective emitters and inputs, applying mesh analysis at the emitters, and solving for the emitter voltages yields

\[ v_{e1} = \frac{v_{i1} + v_{i2}}{2 + \frac{1}{g_m R_{EE} \left( 1 + \frac{1}{\beta C} \right)}} \]  
(3.76)

\[ v_{e2} = \frac{v_{i2} + v_{i3}}{2 + \frac{1}{g_m R_{EE} \left( 1 + \frac{1}{\beta C} \right)}} \]  
(3.77)

\[ v_{e3} = \frac{v_{i3} + v_{i4}}{2 + \frac{1}{g_m R_{EE} \left( 1 + \frac{1}{\beta C} \right)}} \]  
(3.78)

\[ v_{e4} = \frac{v_{i4} + v_{i1}}{2 + \frac{1}{g_m R_{EE} \left( 1 + \frac{1}{\beta C} \right)}} \]  
(3.79)

\[ v_{e5} = \frac{v_{i1} + v_{i3}}{2 + \frac{1}{g_m R_{EE} \left( 1 + \frac{1}{\beta C} \right)}} \]  
(3.80)

\[ v_{e6} = \frac{v_{i4} + v_{i2}}{2 + \frac{1}{g_m R_{EE} \left( 1 + \frac{1}{\beta C} \right)}} \]  
(3.81)
Substitution of emitter voltages into \( v_{o1p} \) through \( v_{o4p} \), (3.72) through (3.75), factoring, and allowing for \( B = g_m R_{EE} \left( 1 + \frac{1}{\beta_0} \right) \) results in

\[
v_{o1p,s} = -\frac{g_m R_C}{2} \left( \frac{3 \left( 1 + \frac{1}{B} \right)}{1 + \frac{1}{2B}} v_{i1} - \frac{1}{1 + \frac{1}{2B}} v_{i2} - \frac{1}{1 + \frac{1}{2B}} v_{i3} - \frac{1}{1 + \frac{1}{2B}} v_{i4} \right)
\]

(3.82)

\[
v_{o2p,s} = -\frac{g_m R_C}{2} \left( \frac{-1}{1 + \frac{1}{2B}} v_{i1} + \frac{3 \left( 1 + \frac{1}{B} \right)}{1 + \frac{1}{2B}} v_{i2} - \frac{1}{1 + \frac{1}{2B}} v_{i3} - \frac{1}{1 + \frac{1}{2B}} v_{i4} \right)
\]

(3.83)

\[
v_{o3p,s} = -\frac{g_m R_C}{2} \left( \frac{-1}{1 + \frac{1}{2B}} v_{i1} - \frac{1}{1 + \frac{1}{2B}} v_{i2} + \frac{3 \left( 1 + \frac{1}{B} \right)}{1 + \frac{1}{2B}} v_{i3} - \frac{1}{1 + \frac{1}{2B}} v_{i4} \right)
\]

(3.84)

\[
v_{o4p,s} = -\frac{g_m R_C}{2} \left( \frac{-1}{1 + \frac{1}{2B}} v_{i1} - \frac{1}{1 + \frac{1}{2B}} v_{i2} - \frac{1}{1 + \frac{1}{2B}} v_{i3} + \frac{3 \left( 1 + \frac{1}{B} \right)}{1 + \frac{1}{2B}} v_{i4} \right)
\]

(3.85)

where

\[ B = g_m R_{EE} \left( 1 + \frac{1}{\beta_0} \right) \]

Inspection of the coefficients in (3.82) through (3.85) reveals the positive small signal gain constants as

\[
A_{xpx} = -\frac{g_m R_C}{2} \left( \frac{3 \left( 1 + \frac{1}{B} \right)}{1 + \frac{1}{2B}} \right)
\]

(3.86)

\[
A_{xpy} = \frac{g_m R_C}{2} \left( \frac{1}{1 + \frac{1}{2B}} \right)
\]

(3.87)

Although a quad-ferential pair is balanced and symmetric, the gain constants for a positive and a negative quad-ferential pair are not identical because the overall small signal output currents (i.e. \( i_{o1p} \)) are summed from different emitter nodes as shown in figure 3.13 and figure 3.14. Performing a parallel analysis on the negative quad-ferential pair by applying KVL to the output nodes of figure 3.14 results in
Chapter 3. Analysis: Quad-ferential Pair

Figure 3.14: Small signal model for a negative quad-ferential pair.
\[ v_{o1n,s} = -g_m R_C (v_{\pi 2} + v_{\pi 7} + v_{\pi 10}) \]  
\[ v_{o2n,s} = -g_m R_C (v_{\pi 1} + v_{\pi 4} + v_{\pi 11}) \]  
\[ v_{o3n,s} = -g_m R_C (v_{\pi 3} + v_{\pi 6} + v_{\pi 9}) \]  
\[ v_{o4n,s} = -g_m R_C (v_{\pi 5} + v_{\pi 8} + v_{\pi 12}) \]

Substitution of emitter voltages, (3.76) through (3.81), into \( v_{o1n} \) through \( v_{o4n} \) yields

\[ v_{o1n,s} = -g_m R_C \left( \frac{-3}{1 + \frac{1}{2B}} v_{i1} + \frac{1 + \frac{1}{B}}{1 + \frac{1}{2B}} v_{i2} + \frac{1 + \frac{1}{B}}{1 + \frac{1}{2B}} v_{i3} + \frac{1 + \frac{1}{B}}{1 + \frac{1}{2B}} v_{i4} \right) \]  
\[ v_{o2n,s} = -g_m R_C \left( \frac{1 + \frac{1}{B}}{1 + \frac{1}{2B}} v_{i1} + \frac{-3}{1 + \frac{1}{2B}} v_{i2} + \frac{1 + \frac{1}{B}}{1 + \frac{1}{2B}} v_{i3} + \frac{1 + \frac{1}{B}}{1 + \frac{1}{2B}} v_{i4} \right) \]  
\[ v_{o3n,s} = -g_m R_C \left( \frac{1 + \frac{1}{B}}{1 + \frac{1}{2B}} v_{i1} + \frac{1 + \frac{1}{B}}{1 + \frac{1}{2B}} v_{i2} + \frac{-3}{1 + \frac{1}{2B}} v_{i3} + \frac{1 + \frac{1}{B}}{1 + \frac{1}{2B}} v_{i4} \right) \]  
\[ v_{o4n,s} = -g_m R_C \left( \frac{1 + \frac{1}{B}}{1 + \frac{1}{2B}} v_{i1} + \frac{1 + \frac{1}{B}}{1 + \frac{1}{2B}} v_{i2} + \frac{1 + \frac{1}{B}}{1 + \frac{1}{2B}} v_{i3} + \frac{-3}{1 + \frac{1}{2B}} v_{i4} \right) \]  

where

\[ B = g_m R_{EE} \left( 1 + \frac{1}{B_0} \right) \]

Inspection of (3.92) through (3.95) reveals the negative small signal gain constants as

\[ A_{xxn} = \frac{g_m R_C}{2} \left( \frac{3}{1 + \frac{1}{2B}} \right) \]  
\[ A_{xyn} = -\frac{g_m R_C}{2} \left( \frac{1 + \frac{1}{B}}{1 + \frac{1}{2B}} \right) \]

Within the small signal gain constants of \( A_{xxp}, A_{xyp}, A_{xxn}, \) and \( A_{xyn}, \) the \( g_m R_C \) term dominates. However, if the impedance of the tail current, \( R_{EE}, \) is not sufficiently large it will adversely affect a quad-ferential pair by reducing the overall gain. If \( R_{EE} \) is sufficiently large, \( A_{xx} \) realizes an absolute \( \frac{3}{2} g_m R_C \) and \( A_{xy} \) realizes an absolute \( \frac{1}{2} g_m R_C. \)

A small signal transient simulation of \( v_{olp} \) and \( v_{oln} \) is compared with respective derivation
results, (3.82) and (3.92), in figure 3.15(b). The applied input voltages are shown in figure 3.15(a). For an $I_{TAIL}=100\mu A$, $R_C=1k\Omega$, $R_{EE}=1M\Omega$ simulation of $v_{op}$ yields 72.4mV whereas the expected value from (3.82) is 77.2mV. Thus, simulation and (3.82) correlate to within 7%. The 7% difference is shown in figure 3.16.

Although the gain constants, (3.86) – (3.87) and (3.96) – (3.97), and the output voltages, (3.82) – (3.85), (3.92) – (3.95), are accurate, they do not yield insight into how a quad-ferential pair reacts to common-mode and differential components of a signal. To illustrate this behavior, it is necessary to define the individual input voltages in terms

of their contribution to each difference term. Recalling the definition of the input difference voltages, (3.9) through (3.14), and the definition of the input common mode voltage (3.21),

\[
\begin{align*}
    v_{id(1,2)} &= v_{i1} - v_{i2} \\
    v_{id(2,3)} &= v_{i2} - v_{i3} \\
    v_{id(3,4)} &= v_{i3} - v_{i4} \\
    v_{id(4,1)} &= v_{i4} - v_{i1} \\
    v_{id(1,3)} &= v_{i1} - v_{i3} \\
    v_{id(3,2)} &= v_{i3} - v_{i2} \\
    v_{ic} &= \frac{v_{i1} + v_{i2} + v_{i3} + v_{i4}}{4}
\end{align*}
\]

and solving \( v_{id(1,2)} \), \( v_{id(4,1)} \), and \( v_{id(1,3)} \) for \( v_{i2} \), \( v_{i3} \), \( v_{i4} \) respectively, then substitution into \( v_{ic} \) and solving for \( v_{i1} \) results in

\[
v_{i1} = v_{ic} + \frac{v_{id(1,2)} - v_{id(4,1)} + v_{id(1,3)}}{4} \quad (3.98)
\]

Applying the same methodology to determine the other input voltages with respect to their contribution to each difference term results in

\[
\begin{align*}
    v_{i2} &= v_{ic} - \frac{v_{ID(1,2)} + v_{ID(2,3)} - v_{ID(4,2)}}{4} \quad (3.99) \\
    v_{i3} &= v_{ic} - \frac{v_{ID(2,3)} + v_{ID(3,4)} - v_{ID(1,3)}}{4} \quad (3.100) \\
    v_{i4} &= v_{ic} - \frac{v_{ID(3,4)} + v_{ID(4,1)} + v_{ID(4,2)}}{4} \quad (3.101)
\end{align*}
\]

Substitution of (3.98) through (3.101) into the four output voltages that were obtained
via direct small signal analysis, (3.68) through (3.71), and factoring results in the following single-ended output voltages in terms of its common-mode and differential components yields

\[ v_{o1,d} = (A_{11} + A_{12} + A_{13} + A_{14})v_{ic} + \frac{A_{11} - A_{12}}{4}v_{id(1,2)} + \frac{A_{14} - A_{11}}{4}v_{id(4,1)} + \frac{A_{11} - A_{13}}{4}v_{id(1,3)} \]  
(3.102)

\[ v_{o2,d} = (A_{21} + A_{22} + A_{23} + A_{24})v_{ic} + \frac{A_{21} - A_{22}}{4}v_{id(1,2)} + \frac{A_{22} - A_{23}}{4}v_{id(2,3)} + \frac{A_{24} - A_{22}}{4}v_{id(4,2)} \]  
(3.103)

\[ v_{o3,d} = (A_{31} + A_{32} + A_{33} + A_{34})v_{ic} + \frac{A_{32} - A_{33}}{4}v_{id(2,3)} + \frac{A_{33} - A_{34}}{4}v_{id(3,4)} + \frac{A_{31} - A_{33}}{4}v_{id(1,3)} \]  
(3.104)

\[ v_{o4,d} = (A_{41} + A_{42} + A_{43} + A_{44})v_{ic} + \frac{A_{43} - A_{44}}{4}v_{id(3,4)} + \frac{A_{44} - A_{41}}{4}v_{id(4,1)} + \frac{A_{44} - A_{42}}{4}v_{id(4,2)} \]  
(3.105)

Substitution of the gain constants into (3.102) through (3.105) results in the following single-ended output voltages for a positive and negative quad-ferential pair responding to input difference voltages

\[ v_{01p,d} = -\frac{g_mR_c}{2} \left( \frac{3}{B} \right) v_{ic} - Dv_{id(1,2)} + Dv_{id(4,1)} - Dv_{id(1,3)} \]  
(3.106)

\[ v_{02p,d} = -\frac{g_mR_c}{2} \left( \frac{3}{B} \right) v_{ic} + Dv_{id(1,2)} - Dv_{id(2,3)} + Dv_{id(4,2)} \]  
(3.107)

\[ v_{03p,d} = -\frac{g_mR_c}{2} \left( \frac{3}{B} \right) v_{ic} + Dv_{id(2,3)} - Dv_{id(3,4)} + Dv_{id(1,3)} \]  
(3.108)

\[ v_{04p,d} = -\frac{g_mR_c}{2} \left( \frac{3}{B} \right) v_{ic} + Dv_{id(3,4)} - Dv_{id(4,1)} - Dv_{id(4,2)} \]  
(3.109)

\[ v_{01n,d} = -\frac{g_mR_c}{2} \left( \frac{3}{B} \right) v_{ic} + E v_{id(1,2)} - Ev_{id(4,1)} + Ev_{id(1,3)} \]  
(3.110)

\[ v_{02n,d} = -\frac{g_mR_c}{2} \left( \frac{3}{B} \right) v_{ic} - Ev_{id(1,2)} + Ev_{id(2,3)} - Ev_{id(4,2)} \]  
(3.111)

\[ v_{03n,d} = -\frac{g_mR_c}{2} \left( \frac{3}{B} \right) v_{ic} - Ev_{id(2,3)} + Ev_{id(3,4)} - Ev_{id(1,3)} \]  
(3.112)

\[ v_{04n,d} = -\frac{g_mR_c}{2} \left( \frac{3}{B} \right) v_{ic} - Ev_{id(3,4)} + Ev_{id(4,1)} + Ev_{id(4,2)} \]  
(3.113)
where

\[ B = g_m R_{EE} \left( 1 + \frac{1}{\beta_0} \right) \]

and

\[
D = \frac{g_m R_C}{8} \left( \frac{4 + \frac{3}{g_m R_{EE} \left( 1 + \frac{1}{\beta_0} \right)}}{1 + \frac{1}{2g_m R_{EE} \left( 1 + \frac{1}{\beta_0} \right)}} \right) \\
E = \frac{g_m R_C}{8} \left( \frac{4 + \frac{1}{g_m R_{EE} \left( 1 + \frac{1}{\beta_0} \right)}}{1 + \frac{1}{2g_m R_{EE} \left( 1 + \frac{1}{\beta_0} \right)}} \right)
\]

In summary, the single-ended output expressions in terms of input difference voltages, (3.106) through (3.113) further supports the impact of the impedance from the tail current if it is not sufficiently large. Additionally, these expressions show the presence of an input common-mode component \(v_{ic}\) terms, if the “B” term cannot be ignored.

**Qualitative perspective of small signal single-ended gain**

From a qualitative perspective, the expected relative single-ended gain (relative with respect to each output) is determined by evaluating a positive quad-ferential pair from a delta perspective as shown in figure 3.17. When one input is modulating, \(\pm 1\Delta\), the maximum number of deltas one output is capable of, while the other inputs are held constant, is \(\mp 1.5\Delta\). For example, if \(v_{i1}\) increases by \(+1\Delta\), as shown in figure 3.17, while \(v_{i2}, v_{i3}\) and \(v_{i4}\) are held constant about ground, then the expected delta change for \(i_{o1}\) is \(-1.5\Delta\). The individual change for \(i_{o2}, i_{o3},\) and \(i_{o4}\) is \(+0.5\Delta\). An overview of this functionality is summarized in table 3.2.

**Table 3.2:** Small signal single-ended output response of one modulating input for a positive quad-ferential pair.

<table>
<thead>
<tr>
<th>Change in Inputs</th>
<th>Resulting Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v_{i1}) (v_{i2}) (v_{i3}) (v_{i4})</td>
<td>(i_{o1}) (i_{o2}) (i_{o3}) (i_{o4})</td>
</tr>
<tr>
<td>(\pm 1\Delta) (0) (0) (0)</td>
<td>(\mp 1.5\Delta) (\pm 0.5\Delta) (\pm 0.5\Delta) (\pm 0.5\Delta)</td>
</tr>
<tr>
<td>(0) (\pm 1\Delta) (0) (0)</td>
<td>(\pm 0.5\Delta) (\mp 1.5\Delta) (\pm 0.5\Delta) (\pm 0.5\Delta)</td>
</tr>
<tr>
<td>(0) (0) (\pm 1\Delta) (0)</td>
<td>(\pm 0.5\Delta) (\pm 0.5\Delta) (\mp 1.5\Delta) (\pm 0.5\Delta)</td>
</tr>
<tr>
<td>(0) (0) (0) (\pm 1\Delta)</td>
<td>(\pm 0.5\Delta) (\pm 0.5\Delta) (\pm 0.5\Delta) (\mp 1.5\Delta)</td>
</tr>
</tbody>
</table>
Furthermore, examination of the outputs with respect to each output reveals a fundamental characteristic of a quad-ferential pair. When the outputs are normalized to one channel (for this example, channel 1), as listed in table 3.3, a 33% difference is realized such that

$$\text{percentage of change} = \left( \frac{0.5\Delta}{1.5\Delta} \right) \times 100 = 33\% \quad (3.114)$$

This one-third difference (or 9.54dB) is indicative of how a quad-ferential pair functions. As one input increases by $1\Delta$, its output decreases by “three-halves” while the other three outputs each increase by one-half resulting in a net change of zero delta. A simulation of the four outputs of a positive quad-ferential pair responding to a modulating input is offered in figure 3.18. The bode plots in figure 3.18 are consistent with the qualitative $\Delta$ analysis.\(^6\) Furthermore, due to the symmetrical nature of a positive and negative quad-ferential pair (as established in subsection 3.1.3) generalizing this 1:3 behavior to a negative quad-ferential pair is valid.

\(^6\)Bode plots and a table reflecting one, two, three and four modulating inputs are located in Appendix A.
### Table 3.3: Expected single-ended Δ change for one modulating input of a positive quad-ferential pair.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Input</th>
<th>Output</th>
<th>Relative magnitude normalized to $i_{o1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$+1\Delta$</td>
<td>$-1.5\Delta$</td>
<td>0 dB</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$+0.5\Delta$</td>
<td>$-9.54$ dB</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$+0.5\Delta$</td>
<td>$-9.54$ dB</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>$+0.5\Delta$</td>
<td>$-9.54$ dB</td>
</tr>
</tbody>
</table>

Figure 3.18: Single-ended bode plot of a positive quad-ferential pair with one modulating input, $v_{i1}$. 

---

Table 3.3: Expected single-ended Δ change for one modulating input of a positive quad-ferential pair.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Input</th>
<th>Output</th>
<th>Relative magnitude normalized to $i_{o1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$+1\Delta$</td>
<td>$-1.5\Delta$</td>
<td>0 dB</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$+0.5\Delta$</td>
<td>$-9.54$ dB</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$+0.5\Delta$</td>
<td>$-9.54$ dB</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>$+0.5\Delta$</td>
<td>$-9.54$ dB</td>
</tr>
</tbody>
</table>

Figure 3.18: Single-ended bode plot of a positive quad-ferential pair with one modulating input, $v_{i1}$. 

3.1.4.2 Differential behavior

Typically, a differential circuit acts upon the differences at its inputs and ignores the part of the signal that is common to each. [3] A quad-ferential pair is no different. To examine this behavior, the difference voltages a quad-ferential pair responds to in terms of generic gain constants, from (3.102) through (3.105), are

\[
v_{od(1,2)} = \frac{A_{11} - A_{12}}{2} v_{id(1,2)} + \frac{A_{23} - A_{22}}{4} v_{id(2,3)} + \frac{A_{14} - A_{11}}{4} v_{id(4,1)} + \frac{A_{11} - A_{13}}{4} v_{id(1,3)} + \frac{A_{22} - A_{24}}{4} v_{id(4,2)}
\]

(3.115)

\[
v_{od(2,3)} = \frac{A_{21} - A_{22}}{4} v_{id(1,2)} + \frac{A_{22} - A_{23}}{2} v_{id(2,3)} + \frac{A_{34} - A_{33}}{4} v_{id(3,4)} + \frac{A_{31} - A_{33}}{4} v_{id(3,1)} + \frac{A_{24} - A_{22}}{4} v_{id(4,2)}
\]

(3.116)

\[
v_{od(3,4)} = \frac{A_{32} - A_{33}}{4} v_{id(2,3)} + \frac{A_{33} - A_{34}}{2} v_{id(3,4)} + \frac{A_{41} - A_{44}}{4} v_{id(4,1)} + \frac{A_{31} - A_{34}}{4} v_{id(3,1)} + \frac{A_{42} - A_{44}}{4} v_{id(4,2)}
\]

(3.117)

\[
v_{od(4,1)} = \frac{A_{12} - A_{11}}{4} v_{id(1,2)} + \frac{A_{43} - A_{44}}{4} v_{id(3,4)} + \frac{A_{44} - A_{41}}{4} v_{id(4,1)} + \frac{A_{11} - A_{14}}{4} v_{id(1,3)} + \frac{A_{44} - A_{42}}{4} v_{id(4,2)}
\]

(3.118)

\[
v_{od(1,3)} = \frac{A_{11} - A_{12}}{4} v_{id(1,2)} + \frac{A_{33} - A_{32}}{4} v_{id(2,3)} + \frac{A_{34} - A_{33}}{4} v_{id(3,4)} + \frac{A_{14} - A_{11}}{4} v_{id(4,1)} + \frac{A_{11} - A_{13}}{4} v_{id(1,3)}
\]

(3.119)

\[
v_{od(2,4)} = \frac{A_{22} - A_{21}}{4} v_{id(1,2)} + \frac{A_{23} - A_{22}}{4} v_{id(2,3)} + \frac{A_{43} - A_{44}}{4} v_{id(3,4)} + \frac{A_{44} - A_{41}}{4} v_{id(4,1)} + \frac{A_{44} - A_{42}}{4} v_{id(4,2)}
\]

(3.120)

Inspection of (3.115) through (3.120) reveals the common-mode component cancels, which is the desired result when using differential signals. When the specific small signal gain constants, (3.86) through (3.87) and (3.96) through (3.97), are substituted into (3.115) through (3.120), the differential output voltages for a positive and negative
quad-ferential pair are

\[
\begin{align*}
    v_{od(1,2)p} &= -2Dv_{id(1,2)} + Dv_{id(2,3)} + Dv_{id(4,1)} - Dv_{id(1,3)} - Dv_{id(4,2)} \tag{3.121} \\
    v_{od(2,3)p} &= -2Dv_{id(2,3)} + Dv_{id(1,2)} + Dv_{id(3,4)} - Dv_{id(1,3)} + Dv_{id(4,2)} \tag{3.122} \\
    v_{od(3,4)p} &= -2Dv_{id(3,4)} + Dv_{id(1,2)} + Dv_{id(4,1)} + Dv_{id(1,3)} + Dv_{id(4,2)} \tag{3.123} \\
    v_{od(4,1)p} &= -2Dv_{id(4,1)} + Dv_{id(1,2)} + Dv_{id(3,4)} + Dv_{id(1,3)} - Dv_{id(4,2)} \tag{3.124} \\
    v_{od(1,3)p} &= -2Dv_{id(1,3)} - Dv_{id(1,2)} - Dv_{id(2,3)} + Dv_{id(3,4)} + Dv_{id(4,1)} \tag{3.125} \\
    v_{od(4,2)p} &= -2Dv_{id(4,2)} - Dv_{id(1,2)} + Dv_{id(2,3)} + Dv_{id(3,4)} - Dv_{id(4,1)} \tag{3.126} \\
    v_{od(1,2)m} &= 2Ev_{id(1,2)} - Ev_{id(2,3)} - Ev_{id(4,1)} + Ev_{id(1,3)} + Ev_{id(4,2)} \tag{3.127} \\
    v_{od(2,3)m} &= 2Ev_{id(2,3)} - Ev_{id(1,2)} - Ev_{id(3,4)} + Ev_{id(1,3)} - Ev_{id(4,2)} \tag{3.128} \\
    v_{od(3,4)m} &= 2Ev_{id(3,4)} - Ev_{id(1,2)} - Ev_{id(4,1)} - Ev_{id(1,3)} - Ev_{id(4,2)} \tag{3.129} \\
    v_{od(4,1)m} &= 2Ev_{id(4,1)} - Ev_{id(1,2)} - Ev_{id(3,4)} - Ev_{id(1,3)} + Ev_{id(4,2)} \tag{3.130} \\
    v_{od(1,3)m} &= 2Ev_{id(1,3)} + Ev_{id(1,2)} + Ev_{id(2,3)} - Ev_{id(3,4)} - Ev_{id(4,1)} \tag{3.131} \\
    v_{od(4,2)m} &= 2Ev_{id(4,2)} + Ev_{id(1,2)} - Ev_{id(2,3)} - Ev_{id(3,4)} + Ev_{id(4,1)} \tag{3.132}
\end{align*}
\]

where

\[
D = \frac{g_m R_C}{8}, \quad E = \frac{g_m R_C}{8} \left(\frac{1}{1 + 2g_m R_{EE} \left(1 + \frac{1}{\beta C}\right)} \right)
\]

Three observations arise when evaluating (3.121) through (3.132). First, the arguments inside the brackets for the constants D and E are a result of the finite impedance from \(I_{\text{TAIL}}\), \(R_{EE}\). As can be seen from D and E, if \(R_{EE}\) is large, its effect is minimal. However, if \(R_{EE}\) is small, it will reduce the gain. Second, (3.121) through (3.132), reveal that the corresponding input difference term of the desired output difference, contributes twice as much when compared to the other input terms. For example, \(v_{od(1,2)m}\), (3.132). The input term, \(v_{id(4,2)}\), contributes twice as much to \(v_{od(4,2)m}\) in comparison to the other terms. Third, although there are six possible difference voltages as defined by (3.9) through (3.14), (3.121) through (3.132) contain only five difference voltages. The “missing” difference voltage is absent because its channels do not impact the output difference voltage of interest. For example, \(v_{od(4,2)m}\), (3.132). The term, \(v_{od(4,2)m}\), involves channels two and four therefore, the difference between channels one and three, \(v_{id(1,3)}\), is missing, as required. A simulation of the small signal behavior for a positive and negative quad-ferential pair responding to two unique input differential voltages is shown in figure 3.19.
3.19a sim \_ss \_vodps

Difference Outputs for Positive Quad-ferential Pair (mV)

3.19b sim \_ss \_vodns

Difference Outputs for Negative Quad-ferential Pair (mV)

(a) Transient simulation of output difference voltages for a positive quad-ferential pair responding to two unique differential input voltages (shown in figure 3.15(a)).

(b) Transient simulation of output difference voltages for a negative quad-ferential pair responding to two unique differential input voltages (shown in figure 3.15(a)).

Figure 3.19: Transient simulations of a positive and negative quad-ferential pair responding to two unique input differential voltages. Inputs are shown in figure 3.15(a).
Qualitative perspective on difference signaling for a quad-ferential pair

From a qualitative perspective, two conditions require consideration when evaluating output differences of a quad-ferential pair. The first condition is with one input modulating as shown in figure 3.17 and the second condition is with two inputs modulating as in figure 3.20.

With the first condition, of one input modulating, the expected differences are summarized in table 3.4. The difference between a modulating channel and a non-modulating channel is 2.49dB. From a ratio standpoint, the difference from a modulating and non-modulating channel is 1.33 times larger than the single-ended output of the modulating channel.

The second condition, with 2 inputs modulating, is shown in figure 3.20. In figure 3.20, a differential Δ is applied to \( v_{i_1} \) and \( v_{i_2} \) while \( v_{i_3} \) and \( v_{i_4} \) are held constant about ground. The resulting single-ended outputs are tabulated in table 3.6 with the expected output differences (normalized to channel 1) listed in table 3.6. A Bode plot confirming the results in table 3.6 is shown in figure 3.21. As table 3.6 indicates, the output difference of two modulating channels is 6dB.

### Table 3.4: Expected output difference of a positive quad-ferential pair when a \(+1\Delta\) is applied to \( v_{i_1} \) and the remaining inputs are held constant.

<table>
<thead>
<tr>
<th>Output Difference</th>
<th>Relative magnitude normalized to Output1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i_{o_1} - i_{o_2} )</td>
<td>+2.49 dB</td>
</tr>
<tr>
<td>( i_{o_2} - i_{o_3} )</td>
<td>No Change (0 Δ)</td>
</tr>
<tr>
<td>( i_{o_3} - i_{o_4} )</td>
<td>No Change (0 Δ)</td>
</tr>
<tr>
<td>( i_{o_4} - i_{o_1} )</td>
<td>+2.49 dB</td>
</tr>
<tr>
<td>( i_{o_1} - i_{o_3} )</td>
<td>+2.49 dB</td>
</tr>
<tr>
<td>( i_{o_4} - i_{o_2} )</td>
<td>No Change (0 Δ)</td>
</tr>
</tbody>
</table>
Figure 3.20: Positive quad-ferential pair with two modulating inputs.

Table 3.5: Expected single-ended output results for an input differential $\Delta$.

<table>
<thead>
<tr>
<th>$\Delta$ Applied to Inputs</th>
<th>Resulting Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{i1}$</td>
<td>$v_{i2}$</td>
</tr>
<tr>
<td>$\pm 1\Delta$</td>
<td>$\mp 1\Delta$</td>
</tr>
<tr>
<td>0</td>
<td>$\pm 1\Delta$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\mp 1\Delta$</td>
<td>0</td>
</tr>
<tr>
<td>$\pm 1\Delta$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>$\pm 1\Delta$</td>
</tr>
</tbody>
</table>
Table 3.6: Expected relative magnitude in response to an input differential Δ applied to \( v_{i1} \) and \( v_{i2} \).

<table>
<thead>
<tr>
<th>Output Difference</th>
<th>Relative magnitude with respect to ( i_{o1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i_{o1} - i_{o2} )</td>
<td>+6.02 dB</td>
</tr>
<tr>
<td>( i_{o2} - i_{o3} )</td>
<td>+6.02 dB</td>
</tr>
<tr>
<td>( i_{o3} - i_{o4} )</td>
<td>No change (0 Δ)</td>
</tr>
<tr>
<td>( i_{o4} - i_{o1} )</td>
<td>0 dB</td>
</tr>
<tr>
<td>( i_{o1} - i_{o3} )</td>
<td>0 dB</td>
</tr>
<tr>
<td>( i_{o4} - i_{o2} )</td>
<td>0 dB</td>
</tr>
</tbody>
</table>

Figure 3.21: Differential Bode plot of a positive quad-ferential pair in response to an input differential Δ. Differential input Δ applied to \( v_{i1} \) and \( v_{i2} \).
3.1.4.3 Small signal behavior summary

Small signal gain constants were defined using direct small signal analysis. The gain constants revealed the impact of the impedance from the tail current. If $R_{ee}$ is not sufficiently large the gain is reduced. If sufficiently large, terms of the form $A_{xx}$ will realize an absolute $\frac{3}{2}g_mR_c$ and terms of the form $A_{xy}$ will realize an absolute $\frac{1}{2}g_mR_c$. Correlation between single-ended theoretical expressions predicated on the small signal gain constants and simulation was 7%.

Expressions were derived with common-mode and differential components separated. Consistent with the single-ended expressions the differential expressions showed the potential affect of $R_{ee}$ on gain. In addition, the differential expressions revealed the presence of an input common-mode component due to an insufficient $R_{ee}$. If however, $R_{ee}$ is large enough, the input common-mode component reduces to zero and the full single-ended gain is achieved.

The differential small signal expressions showed the common-mode to differential gain components canceling. The possible impact of $R_{ee}$, if not adequate, in regards to the gain remained.

From a qualitative perspective, the small signal single-ended gain discussion revealed a fundamental trait of a quad-ferential pair. If one input moves up a unit, its output will change by 1.5 and the other other outputs will move in the opposite direction by a third resulting in a net output change of zero. This important behavior supports the quad-ferential amplifier in realizing negative feedback.

3.1.5 Quad-ferential pair summary

The large and small signal behavior of a quad-ferential pair was thoroughly investigated. The large signal analysis showed a positive and negative quad-ferential pair as symmetrical with respect to the same channels, such that the same channels are differential. The small signal behavior provided insight into the impact on gain with an insufficient tail current impedance. Transient simulations confirmed the ability of a quad-ferential pair to amplify differences and remove input common-mode (figure 3.19). Additionally, small signal transient simulations confirmed symmetry between a positive and negative quad-ferential pair. Qualitative analysis was consistent with Bode simulations such that the single-ended outputs of the non-modulating channels changed by a third (9.54dB) with respect to the modulating channel.

Two key traits of a quad-ferential pair are relevant when implementing two quad-ferential pairs as an input stage for a quad-ferential amplifier. First, the same channel outputs
of a positive and negative quad-ferential pair are differential. Second, the fundamental response of three outputs individually responding in the opposite direction by a third with respect to the output whose channel is modulating, is necessary for establishing negative feedback. Furthermore, this behavior confirms the ability of a quad-ferential pair to keep the net change in output current zero.
3.2 Quad-ferential input stage

A block diagram of the input stage for a quad-ferential amplifier is presented in figure 3.22.

![Block diagram of a quad-ferential input stage](image)

Figure 3.22: Block diagram of a quad-ferential input stage

It has four input voltages \(v_{I1}, v_{I2}, v_{I3}, \) and \(v_{I4}\) and it outputs eight currents \(i_{OP1}, i_{OP2}, i_{OP3}, i_{OP4}, i_{ON1}, i_{ON2}, i_{ON3}, \) and \(i_{ON4}\) \). The output currents \(i_{OP1}, i_{OP2}, i_{OP3}, i_{OP4}, i_{ON1}, i_{ON2}, i_{ON3}, \) and \(i_{ON4}\) are proportional to the input voltages \(v_{I1}, v_{I2}, v_{I3}, \) and \(v_{I4}\) and a discussion of the overall \(G_m\) is presented in this section.

As figure 3.22 implies, a quad-ferential input stage is comprised of two quad-ferential pairs. The positive quad-ferential pair is responsible for positive currents \(i_{OP1}, i_{OP2}, i_{OP3}, \) and \(i_{OP4}\) \) and the negative quad-ferential pair provides the complement to the positive currents \(i_{ON1}, i_{ON2}, i_{ON3}, \) and \(i_{ON4}\) \). Therefore, the outputs of a positive and a negative quad-ferential pair are differential to each other with respect to the same channel. The outputs of a quad-ferential input stage are taken differentially (for example \(i_{OP1}\) and \(i_{ON1}\) are paired to form the signal chain for channel 1). If, a resistive load is present, then the input stage forms the following differential voltages

\[
\begin{align*}
v_{OD1} &= v_{OP1} - v_{ON1} \\
v_{OD2} &= v_{OP2} - v_{ON2} \\
v_{OD3} &= v_{OP3} - v_{ON3} \\
v_{OD4} &= v_{OP4} - v_{ON4}
\end{align*}
\]
3.2.1 Large signal behavior

A quad-ferential input stage is comprised of two quad-ferential pairs, a positive and a negative quad-ferential pair. Therefore, from a component count perspective, a quad-ferential input stage consists of twenty-four transistors and twelve identical current sources. When a positive and a negative quad-ferential pair are joined, differential output voltages are formed. The large signal differential output voltages are

\[ V_{OD1} = V_{OIP,D} - V_{OIN,D} \] (3.133)
\[ V_{OD2} = V_{OCP,D} - V_{OQN,D} \] (3.134)
\[ V_{OD3} = V_{OIP,D} - V_{OIN,D} \] (3.135)
\[ V_{OD4} = V_{OCP,D} - V_{OQN,D} \] (3.136)

The output differential voltages respond to the following input differences

\[ V_{ID(1,2)} = V_{I1} - V_{I2} \]
\[ V_{ID(2,3)} = V_{I2} - V_{I3} \]
\[ V_{ID(3,4)} = V_{I3} - V_{I4} \]
\[ V_{ID(4,1)} = V_{I4} - V_{I1} \]
\[ V_{ID(1,3)} = V_{I1} - V_{I3} \]
\[ V_{ID(4,2)} = V_{I4} - V_{I2} \]

Thus, subtracting the same single-ended channel \( V_{OIN,D} \) through \( V_{OQN,D} \), (3.52) – (3.55), from \( V_{OIP,D} \) through \( V_{OCP,D} \), (3.40) – (3.43), as defined by (3.133) through (3.136), to obtain the output differential voltages a quad-ferential pair reacts to, results in

\[ V_{OD1} = a I_{TAIL} R_C \left( \tanh \frac{V_{ID(1,2)}}{2V_I} + \tanh \frac{V_{ID(3,4)}}{2V_I} + \tanh \frac{V_{ID(4,1)}}{2V_I} \right) \] (3.137)
\[ V_{OD2} = a I_{TAIL} R_C \left( \tanh \frac{V_{ID(1,2)}}{2V_I} + \tanh \frac{V_{ID(2,3)}}{2V_I} + \tanh \frac{V_{ID(4,2)}}{2V_I} \right) \] (3.138)
\[ V_{OD3} = a I_{TAIL} R_C \left( \tanh \frac{V_{ID(2,3)}}{2V_I} + \tanh \frac{V_{ID(3,4)}}{2V_I} + \tanh \frac{V_{ID(1,3)}}{2V_I} \right) \] (3.139)
\[ V_{OD4} = a I_{TAIL} R_C \left( \tanh \frac{V_{ID(3,4)}}{2V_I} + \tanh \frac{V_{ID(4,1)}}{2V_I} + \tanh \frac{V_{ID(4,2)}}{2V_I} \right) \] (3.140)

As (3.137) through (3.140) reveal, subtracting two individual voltages of the same channel (i.e. \( V_{OIP,D} \) and \( V_{OIN,D} \)) results in the summation of three \( \tanh \) functions.

Most often, when a circuit block consists of multiple inputs and outputs it is typically implemented in a differential signal chain. Thus, the operation of a quad-ferential input...
stage with two unique differential inputs is of interest. To maintain the highest degree of freedom with fully independent input voltages, a transient approach is utilized in the forthcoming simulations.

To implement two unique differential signals, the individual inputs are paired. \( V_{i1} \) is differential with respect to \( V_{i2} \) and \( V_{i3} \) is differential with respect to \( V_{i4} \). Furthermore, inherent in using two different input differential voltages is one differential voltage is larger in magnitude than the other. Thus, \( V_{i2} \) and \( V_{i4} \) are chosen as fifty percent less in magnitude with respect to \( V_{i1} \) and \( V_{i3} \). The rate of change (\( \frac{dV}{dt} \)) between individual voltages is determined by setting the magnitude and rise/fall time of the input transients. The single-ended input voltages are shown in figure 3.23. The resulting difference voltages are shown in figure 3.24(a). Inspection of figure 3.24(a) reveals there are four unique difference voltages. This is a result of using two unique differential voltages. A simulation of the differential output voltages for a quad-ferential input stage, with two unique input differential signals, is shown in figure 3.24(b).

The behavior illustrated in figure 3.24(b) is consistent with the large signal single-ended analysis of a quad-ferential pair responding to a differential input (subsubsection 3.1.3.1). The peaking in figure 3.24(b) is a result of different rate of changes at the inputs, \( \frac{dV}{dt} \), resulting from two unique differential voltages.

Furthermore, inspection of figure 3.24(b) confirms the tanh terms in (3.137) through (3.140). Additionally, tanh behavior is possible because the tail currents are identical. If the tail
3.24a sim_ls_VODS1_4_VIDS
‐600
‐500
‐400
‐300
‐200
‐100
0
100 200 300
400
500 600
Input Difference Voltage (mV)
.7 .75 .8 .85 .9 .95 1 1.05 1.1 1.15 1.2 1.25 1.3
Time (μs)
Vid(1,2) Vid(3,4)Vid(2,3) & Vid(4,1) Vid(1,3) & Vid(4,2)
(a) Input difference voltages for a quad-ferential input stage when two unique differential signals are applied to the inputs.

3.24b sim_ls_VODS1_4
‐400
‐300
‐200
‐100
0
100 200
300 400
.7
.75 .8 .85 .9 .95 1 1.05 1.1 1.15 1.2 1.25 1.3
Time (μs)
Differential Output of Quad‐ferential Input Stage (mV)
VOD1 VOD2 VOD3 VOD4
(b) Simulation of differential output voltages for a quad-ferential input stage.

Figure 3.24: Simulation of differential output voltages for a quad-ferential input stage.
currents were different the curves would exhibit more of an inverted log behavior resulting in a smaller linear region of operation. In summary, (3.137) through (3.140) and figure 3.24(b) describe the large signal behavior of a quad-ferential input stage.

3.2.2 Small signal behavior

The small signal differential output voltages of a quad-ferential pair may be found by subtracting the same channel single-ended voltages from each other. Because the goal is to use the quad-ferential amplifier with differential signaling, the single-ended output voltages defined with respect to differential input voltages (3.106) – (3.109) are used and stated for reference.

\[
\begin{align*}
    v_{o1,p,d} &= \frac{-g_m R_C}{2} \left( \frac{3 \beta}{1 + \frac{1}{2\beta}} \right) v_{ic} - Dv_{id(1,2)} + Dv_{id(4,1)} - Dv_{id(1,3)} \\
    v_{o2,p,d} &= \frac{-g_m R_C}{2} \left( \frac{3 \beta}{1 + \frac{1}{2\beta}} \right) v_{ic} + Dv_{id(1,2)} - Dv_{id(2,3)} + Dv_{id(4,2)} \\
    v_{o3,p,d} &= \frac{-g_m R_C}{2} \left( \frac{3 \beta}{1 + \frac{1}{2\beta}} \right) v_{ic} + Dv_{id(2,3)} - Dv_{id(3,4)} + Dv_{id(1,3)} \\
    v_{o4,p,d} &= \frac{-g_m R_C}{2} \left( \frac{3 \beta}{1 + \frac{1}{2\beta}} \right) v_{ic} + Dv_{id(3,4)} - Dv_{id(4,1)} - Dv_{id(4,2)} \\
    v_{o1,n,d} &= \frac{-g_m R_C}{2} \left( \frac{3 \beta}{1 + \frac{1}{2\beta}} \right) v_{ic} + Ev_{id(1,2)} - Ev_{id(4,1)} + Ev_{id(1,3)} \\
    v_{o2,n,d} &= \frac{-g_m R_C}{2} \left( \frac{3 \beta}{1 + \frac{1}{2\beta}} \right) v_{ic} - Ev_{id(1,2)} + Ev_{id(2,3)} - Ev_{id(4,2)} \\
    v_{o3,n,d} &= \frac{-g_m R_C}{2} \left( \frac{3 \beta}{1 + \frac{1}{2\beta}} \right) v_{ic} - Ev_{id(2,3)} + Ev_{id(3,4)} - Ev_{id(1,3)} \\
    v_{o4,n,d} &= \frac{-g_m R_C}{2} \left( \frac{3 \beta}{1 + \frac{1}{2\beta}} \right) v_{ic} - Ev_{id(3,4)} + Ev_{id(4,1)} + Ev_{id(4,2)}
\end{align*}
\]

where\(^7\)

\[
B = g_m R_{EE} \left( 1 + \frac{1}{\beta_0} \right)
\]

\(^7\beta=85, R_{EE}=1\Omega, g_m=3.8\text{m}\Omega^{-1}.\)
and

\[
D = \frac{g_m R_C}{8} \left( \frac{4 + \frac{3}{g_m R_{EE} (1 + \frac{1}{\beta_c})}}{1 + \frac{2g_m R_{EE} (1 + \frac{1}{\beta_c})}{1 + \text{\textsuperscript{\circ}}}} \right)
\]

\[
E = \frac{g_m R_C}{8} \left( \frac{4 + \frac{1}{g_m R_{EE} (1 + \frac{1}{\beta_c})}}{1 + \frac{2g_m R_{EE} (1 + \frac{1}{\beta_c})}{1 + \text{\textsuperscript{\circ}}}} \right)
\]

Thus, subtracting the negative single-ended voltages from the positive single-ended voltages yields

\[
v_{od1} = -\frac{g_m R_C}{2} F v_{id(1,2)} + \frac{g_m R_C}{2} F v_{id(4,1)} - \frac{g_m R_C}{2} F v_{id(1,3)} \tag{3.141}
\]

\[
v_{od2} = \frac{g_m R_C}{2} F v_{id(1,2)} - \frac{g_m R_C}{2} F v_{id(2,3)} + \frac{g_m R_C}{2} F v_{id(4,2)} \tag{3.142}
\]

\[
v_{od3} = \frac{g_m R_C}{2} F v_{id(2,3)} - \frac{g_m R_C}{2} F v_{id(3,4)} + \frac{g_m R_C}{2} F v_{id(1,3)} \tag{3.143}
\]

\[
v_{od4} = \frac{g_m R_C}{2} F v_{id(3,4)} - \frac{g_m R_C}{2} F v_{id(4,1)} - \frac{g_m R_C}{2} F v_{id(4,2)} \tag{3.144}
\]

where

\[
F = \frac{2 + \frac{1}{\text{\textsuperscript{\circ}}}}{1 + \frac{1}{2g_m R_{EE} (1 + \frac{1}{\beta_c})}}
\]

As seen in (3.141) through (3.144), the common-mode terms cancel when differential signaling is utilized, however the effect of the impedance from the tail current, \( R_{EE} \), remains as shown in variable \( F \). If \( R_{EE} \) is not sufficiently large for \( F \) to reduce to 2, then the full \( g_m \) will not be seen.

Applying two unique differential input voltages, by using the comparable single-ended technique employed from subsection 3.2.1 (as shown in figure 3.23), to perform small signal transient simulations, results in the input difference voltages of figure 3.25(a). The differential output voltages for each channel is shown in figure 3.25(b).

Simulation result in figure 3.25(b) confirms the behavior derived in (3.141) through (3.144). For example, with a \( g_m = 3.8 \text{ms}^8 \), \( R_L = 1\text{k}\Omega \), \( \beta = 85 \), \( R_{EE} = 1\text{M}\Omega \), \( v_{od1} \), (3.141), is 304mV. Simulation reveals \( v_{od1} \) as 290mV thus simulation and (3.141) correlate to within 5%. Relevant differential Bode plots are shown in figure 3.26 and figure 3.27.

---

\(^8\) Determined by an \( I_c = 100\mu\text{A} \)
expected, inspection of figure 3.26 reveals $v_{od2}$, $v_{od3}$, and $v_{od4}$ behave identically as represented by the dashed line. Furthermore, figure 3.27 illustrates $v_{od3}$ and $v_{od4}$ responding identically when a differential signal is applied to channels 1 and 2.

(a) Input difference voltages resulting from two unique input differential voltages.

(b) Simulation of the small signal output voltages for a quad-ferential input stage.

**Figure 3.25:** Small signal transient differential output voltages of a quad-ferential input stage. $R_L = 1k\Omega$, $g_m = 3.8ms$, $\beta = 85$, $R_{ee} = 1M\Omega$
Chapter 3. Analysis: Quad-ferential Input Stage

**Figure 3.26:** Differential Bode plot with one modulating input, $v_{i1}$.

**Figure 3.27:** Differential Bode plot with two modulating inputs, $v_{i1}$ and $v_{i2}$. 
Qualitative perspective on small signal behavior of quad-ferential input stage

From a qualitative perspective, the overall $G_m$ for each channel of the input stage is determined by considering the fundamental behavior of a quad-ferential pair, as discussed in subsection 3.1.4.

Table 3.7: Expected single-ended $\Delta$ change of a positive quad-ferential pair for one modulating input. Replicated from subsubsection 3.1.4.1 for reference.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$+1\Delta$</td>
<td>$-1.5\Delta$</td>
</tr>
<tr>
<td>2</td>
<td>$0$</td>
<td>$+0.5\Delta$</td>
</tr>
<tr>
<td>3</td>
<td>$0$</td>
<td>$+0.5\Delta$</td>
</tr>
<tr>
<td>4</td>
<td>$0$</td>
<td>$+0.5\Delta$</td>
</tr>
</tbody>
</table>

The behavior for one modulating input, as listed in table 3.7, illustrates the effective $G_m$ for each channel. If one input, for example channel 1, moves up by $1\Delta$, while the other inputs are held constant, the $G_m$ for channel 1 is

$$G_{m,\text{CHNL1}} = \frac{3}{2}I_{\text{TAIL}}$$

and the $G_m$ for channels, 2, 3, and 4 is

$$G_{m,\text{CHNL2}} = G_{m,\text{CHNL3}} = G_{m,\text{CHNL4}} = \frac{1}{2}I_{\text{TAIL}}$$

Quantitatively, if $I_{\text{TAIL}} = 200\mu\text{A}$, the resulting transconductances are

$$G_{m,\text{CHNL1}} = 11.7 m\Omega$$

$$G_{m,\text{CHNL2}} = G_{m,\text{CHNL3}} = G_{m,\text{CHNL4}} = 3.8 m\Omega$$
3.3 Gain stage

A gain stage for a single channel of a quad-ferential amplifier is shown in figure 3.28. Although each channel consists of a set of current source transistors, output buffers, and differential input currents, all channels share differential pair $Q_{N9}$ and $Q_{N10}$ and $Q_{P11}$ and $Q_{P12}$. The cascode current source transistors, $Q_{P5}$ through $Q_{P8}$, receive their bias from $I_{TAIL}$ via the differential pair $Q_{N9}$ and $Q_{N10}$. $Q_{N9}$ and $Q_{N10}$ control the output common-mode level of the amplifier by setting the appropriate bias level (as determined by the user’s input level on the $V_{ocm}$ pin) at the output by modulating the currents in $Q_{P11}$ and $Q_{P12}$. Cascode transistor $Q_5$ ($Q_6$) receives a bias current of $I_{TAIL}/2$ from differential pair $Q_9$ and $Q_{10}$ via the diode connection of $Q_{11}$ ($Q_{12}$) and current source transistor $Q_7$ ($Q_8$). The Wilson current mirror, transistors $Q_1$ through $Q_4$, is biased by $Q_8$ such that $I_{CS1} = I_{CS5}$. It translates the transconductance of the input stage, $G_{m1}$, to the high impedance node in the second stage. Theoretically, the transconductance of the input stage is equal to the second stage, $G_{m2}$, such that

$$G_{m1} = G_{m2}$$  \hspace{1cm} (3.149)

---

*An overview of the four channels is shown in figure 2.4 located in section 2.3.*
however, due to base current losses in the second stage, \( G_{m2} \neq G_{m1} \).\(^{10}\) Furthermore, due to the emitter degeneration of \( Q_{N1} \), only 96\% of \( G_{m1} \) is translated by the Wilson current mirror to the high impedance node. The single-ended gain at the high impedance node is

\[
A_{V,SE} = A_{V1}A_{V2}
\]

where

\[
A_{V1} \approx G_{m1}R_{O1}
\]

\[
R_{O1} \approx \frac{1}{g_{m1}}//R_{E1}
\]

\[
A_{V2} \approx G_{m2}R_{O2}
\]

\[
R_{O2} \approx R_{O5}//R_{O3}
\]

The high impedance node, \( R_{O2} \), consists of the output impedance of the cascode current source, \( R_{O5} \), in parallel with the output impedance of the Wilson current source, \( R_{O3} \), such that

\[
R_{O5} \approx \frac{r_{o5}\beta}{2}
\]

\[
R_{O3} \approx \frac{\beta3r_{o3}}{2}
\]

From a qualitative perspective, if channel 1 is modulating while channels 2, 3, and 4, are equal and constant, channel 1 will be 9.54dB larger in magnitude than the other channels as listed in table 3.8. When the absolute value of 9.54dB (from table 3.8) is normalized to channel 1, a 33\% difference is realized between channels 1 and channels,

\[\text{Table 3.8: Table of expected single-ended } \Delta \text{ change for one modulating input, specifically channel 1 (as discussed in subsubsection 3.1.4.1).}\]

<table>
<thead>
<tr>
<th>Channel</th>
<th>Input</th>
<th>Output</th>
<th>Magnitude normalized to Channel 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+1(\Delta)</td>
<td>-1.5 (\Delta)</td>
<td>0dB</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>+0.5 (\Delta)</td>
<td>-9.54 dB</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>+0.5 (\Delta)</td>
<td>-9.54 dB</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>+0.5 (\Delta)</td>
<td>-9.54 dB</td>
</tr>
</tbody>
</table>

\(^{10}\)A discussion of \( G_{m1} \), the transconductance of the inputs stage, is found in section 3.2.
2, 3, and 4 such that

\[
\text{percentage of change} = \left(\frac{0.5}{1.5}\right) \times 100 = 33\% \tag{3.157}
\]

This one-third difference is confirmed in figure 3.29.

![Figure 3.29: Single-ended open loop gain of all channels.](image)

Evaluation of channel 1, from a quantitative perspective, by substituting \(G_{m1,\text{CHNL1}} = 11.7\text{mS}, \ R_{O1} \approx 83.3\Omega, \ G_{m2} = 10\text{mS}, \) and \(R_{O2} \approx 2.4\text{M}\Omega, \)\(^{11}\) into (3.150), yields the approximate single-ended gain at the high impedance gain node

\[
A_{V,\text{SE,CHNL1}} \approx 87.4\text{dB} \tag{3.158}
\]

Simulation reveals a gain of 86.3dB, as shown in figure 3.29, which deviates from theory by 12%.

The difference between \(A_{V,\text{SE,CHNL1}}\) and \(A_{V,\text{SE,CHNL2}}\) is attributed to the transconductance from the input stage. Where a \(G_{m1,\text{CHNL1}} = 11.7\text{mS}\) results in 87.4dB, a \(G_{m1,\text{CHNL2}} = 3.8\text{mS}\)\(^{12}\) yields

\[
A_{V,\text{SE,CHNL2}} \approx 77.5\text{dB} \tag{3.159}
\]

\(^{11}\)\(R_{O1} \approx 3.5\text{M}\Omega\) and \(R_{O2} \approx 8.1\text{M}\Omega.\)

\(^{12}\)A qualitative discussion is found in section 3.2.
Simulation reveals a gain of 76.7dB, as shown in figure 3.29, which deviates from theory by 10%. As expected, figure 3.29 confirms channels 2, 3, and 4 respond identically.

Furthermore, the difference behavior can be examined from a qualitative perspective. With 1 input modulating (as analyzed in section 3.2) the output difference between a modulating and non-modulating channel is 2.5dB larger when normalized to a modulating channel, channel 1, as listed in table 3.9. From a ratio perspective, when 1 input is modulating, the output difference between a modulating and non-modulating channel is 1.33 times greater when compared with the single-ended output of the modulating channel. For example, if channel 1 modulates, the output difference, $i_{o1} - i_{o2}$, will be 1.33 larger in magnitude when compared with the single-ended measurement of $i_{o1}$.

Table 3.9: Table of expected output difference when a $+1\Delta$ is applied to $v_{i1}$ and the remaining inputs are held constant. Replicated from subsubsection 3.1.4.1 for reference.

<table>
<thead>
<tr>
<th>Output Difference</th>
<th>Magnitude normalized to Channel 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_{o1} - i_{o2}$</td>
<td>+2.49 dB</td>
</tr>
<tr>
<td>$i_{o2} - i_{o3}$</td>
<td>No Change (0 $\Delta$)</td>
</tr>
<tr>
<td>$i_{o3} - i_{o4}$</td>
<td>No Change (0 $\Delta$)</td>
</tr>
<tr>
<td>$i_{o4} - i_{o1}$</td>
<td>+2.49 dB</td>
</tr>
<tr>
<td>$i_{o1} - i_{o3}$</td>
<td>+2.49 dB</td>
</tr>
<tr>
<td>$i_{o4} - i_{o2}$</td>
<td>No Change (0 $\Delta$)</td>
</tr>
</tbody>
</table>
3.4 Output stage

The output stage provides isolation between the output and the gain node such that an amplifier can drive a wide variety of loads. The output stage of a quad-ferential amplifier is comprised of four voltage buffers arranged in a parallel configuration. Each buffer is comprised of a diamond voltage buffer and is shown in figure 3.30. For simplicity, ideal current sources, $I_{BIAS,P}$ and $I_{BIAS,N}$ are used for this analysis. When implemented in the quad-ferential amplifier, $I_{BIAS,P}$ and $I_{BIAS,N}$ are generated in the Bias block. One advantage in using a diamond buffer is the approximate equality between the input voltage, $V_{in}$, and the output voltage, $V_{out}$. Thus, the gain from the input to output is approximately unity.

To determine the quiescent value for $I_{BIAS,P}$ and $I_{BIAS,N}$ two things must be considered. First, the relationship between $I_{IN}$ and $I_{OUT}$ needs to be understood and secondly, the output current requirements need to be defined. The relationship between $I_{IN}$ and $I_{OUT}$ can be determined by performing KVL from $V_{in}$ to $V_{out}$ in a clockwise direction which results in

$$V_{BE1} - V_{BE4} - V_{BE3} + V_{BE2} = 0$$

(3.160)
Recalling

\[ V_{BE} = V_t \ln \left( \frac{J_C}{J_S} \right) \] (3.161)

where \( J_S \) is the saturation current density,\(^{13}\) which is constant for a given process, and \( J_1 \) is the total current density of the device. Substituting (3.161) into (3.160) and factoring results in

\[ \frac{I_{C1} A_4}{A_1 I_{C4}} = \frac{I_{C3} A_2}{A_3 I_{C2}} \] (3.162)

Setting \( I_{BIAS,P} = I_{BIAS,N} \) and thus recognizing \( I_{C1} = I_{C2} = I_{IN}, I_{C3} = I_{C4} = I_{OUT}, A_1 = A_2, \) and \( A_3 = A_4, \) (3.162) reduces to

\[ I_{OUT} = I_{IN} \frac{A_3}{A_1} \] (3.163)

Therefore, the input and output currents are dependent upon the ratio of \( Q_4 \) to \( Q_i \) and \( Q_3 \) to \( Q_2 \) when \( I_{BIAS,P} = I_{BIAS,N}. \)

To determine the necessary output drive capability, a market needs to be identified. Because this amplifier consists of four inputs and four outputs it could easily be used within a video type application. Typically, video applications require an amplifier be able to drive 150Ω loads. A load impedance of 150Ω results from 75Ω from CAT5 cabling and 75Ω from termination networks. Being conservative and designing to the supply levels rather than output swing, then for a ±2.5V supply, the current driving requirement is \( I_{OUT} = ±16.6\text{mA}. \)

The maximum output current this output stage is capable of sinking and sourcing is limited by the base current of the PNP (\( Q_3 \)) and NPN (\( Q_4 \)) output devices such that

\[ I_{BIAS,N} \geq \frac{\alpha I_{ELMAX}}{\beta_{Q3}} \] (3.164)

\[ I_{BIAS,P} \geq \frac{\alpha I_{ELMAX}}{\beta_{Q4}} \] (3.165)

Since \( \beta_n \) for \( Q_4 \) is \( \approx 111 \) and \( \beta_p \) for \( Q_3 \) is \( \approx 85 \), then \( Q_3 \) will be the limiting factor in determining the DC output current drive.\(^{14}\) Thus, the requirement of driving ±16.6 mA in conjunction with \( \beta_p \approx 85 \) results in a minimum \( I_{bias} \approx 200\mu\text{A}. \)

Applying KVL to the output to determine the input and output voltage range for the output stage (assuming \( I_{BIAS,P} \) and \( I_{BIAS,N} \) are implemented with an emitter degenerated

\(^{13}\)Current density is the amount of charge crossing a unit area per unit time\(^{[7]}\); stated differently \( J = \frac{1}{T} \).

\(^{14}\)The differences in \( \beta \) results in an inequality in base currents between \( Q_i \) and \( Q \), such that the base current for \( Q_4 \) is larger in magnitude than \( Q_i \).
transistor) yields

\[ V_{\text{IN,MAX}} = V_{\text{OUT,MAX}} = V_{\text{CC}} - V_{\text{RE}} - V_{\text{CESAT}} - V_{\text{BE}} \tag{3.166} \]

\[ V_{\text{IN,MAX}} = V_{\text{OUT,MAX}} = V_{\text{EE}} + V_{\text{RE}} + V_{\text{CESAT}} + V_{\text{BE}} \tag{3.167} \]

Thus, this buffer allows for a signal swing to \( \approx 1 \) V of the rails with \( V_{\text{BE}} \approx 0.8 \) V and \( V_{\text{CESAT}} \approx 0.2 \) V.

The output impedance, \( R_O \), is determined by the bias level of transistors \( Q_3 \) and \( Q_4 \) such that

\[ R_O = \frac{r_{e3}}{r_{e4}} \tag{3.168} \]

where

\[ r_e \approx \frac{1}{g_m} \tag{3.169} \]

and

\[ g_m = \frac{I_C}{V_t} \tag{3.170} \]

With an \( I_C \) set to 200 \( \mu \)A to satisfy output current drive and \( V_t \approx 25.9 \) mV at ambient results in

\[ R_O \approx 86.5 \Omega \tag{3.171} \]

Additionally, \( Q_5 \) and \( Q_6 \) were added to the buffer to assist in base current compensation in another section of the amplifier.
3.5 Bias cell

The bias cell block for the quad-ferential amplifier is shown in figure 3.31. It supplies eleven references for the quad-ferential amplifier: ten bias currents and one voltage reference. Bias currents $I_{BP1} - I_{BP4}$ and $I_{BN1} - I_{BN4}$ supply the positive and negative bias currents for the output stage. $I_{BD}$ provides the bias for the input stage where the quad-ferential pairs are located and $I_{CM}$ supplies the bias reference for the common-mode circuitry. The transistor sizing for those currents vary according to the necessary bias level. Voltage $V_{CP}$ supplies the reference to the cascode circuitry at the gain node.

The core of this bias cell is the Widlar bandgap circuit located to the left in figure 3.31. A magnified version of the bandgap is shown in figure 3.32. This architecture is a common and proven bandgap circuit used in analog circuit design. The principle behind any bandgap is to produce two voltages with opposite temperature coefficients (TC's) such that the reference is stable over temperature. Typically, a stable current reference is created by summing a Complementary to Absolute Temperature (CTAT) current with a Proportional to Absolute Temperature (PTAT) current.

From inspection of figure 3.32, a unit current, $I_{C5}$ (mirrored from $I_{C6}$), feeds $I_1$. Neglecting losses due to base currents and assuming $\alpha = 1$, then $I_{C6} = I_{C5} = I_1$. $I_1$ determines and sets $V_{BE1}$ which then establishes $I_7$. $I_7$ is a CTAT current because it is determined by a $V_{BE}$ which possesses a negative TC, approximately $-2\text{mV/}^\circ\text{C}$. Hence, $I_7$ decreases with temperature. The formation of a $\Delta V_{BE}$ between $V_{BE1}$ and $V_{BE2}$ creates the PTAT current. For equal area ratio, a $\Delta V_{BE}$ possesses a TC approximately
equal to +86\,\mu\text{V/°C}. Therefore, \( I_2 \) is PTAT and \( I_7 \) is CTAT. \( I_2 \) and \( I_7 \) sum at the collectors of \( Q_3 \) and \( Q_4 \) establishing a stable current over temperature. \( Q_8 \) is a \( \beta \) helper and therefore reduces the non-ideal loss of \( I_B \) between \( I_{C6} \) and \( I_{C5} \).

Quantitatively, applying KVL from \( Q_1 \) to \( Q_2 \) in a clockwise direction in figure 3.32, neglecting \( I_B \), and assuming \( \alpha = 1 \), results in an expression for \( R_2 \):

\[
R_2 = \frac{V_t \ln \left( \frac{I_1}{A_2} \frac{A_4}{I_2} \right)}{I_2}
\]  

(3.172)

Applying KVL from \( Q_1 \) and across \( R_7 \) in a clockwise direction in figure 3.32 yields an expression for \( R_7 \):

\[
R_7 = \frac{|V_{EB1}|}{I_7}
\]  

(3.173)

The unit current for this bandgap is 100\,\mu\text{A}, therefore \( I_1 = 100\,\mu\text{A} \). Splitting the current of 100\,\mu\text{A} equally in \( I_{C3} \) and \( I_{C4} \) results in \( I_2 = I_7 = 50\,\mu\text{A} \). Accordingly, the initial area ratio between \( Q_1 \) and \( Q_2 \) is 1:2. With \( V_{EB1} \approx 650\text{mV} \), \( I_1 = 100\,\mu\text{A} \), and \( I_7 = I_2 = 50\,\mu\text{A} \), the initial values from (3.172) and (3.173) for \( R_2 \) and \( R_7 \) are \( R_2 = 1.44\text{kΩ} \) and \( R_7 = 13\text{kΩ} \).

Simulations with a 1:2 area ratio (for \( Q_1:Q_2 \)) results in a temperature curve exhibiting CTAT behavior. Increasing the ratio of the areas and introducing more PTAT current than CTAT current yields a better current reference over temperature due to the unit

\textsuperscript{15} Algebra is in Appendix A.

\textsuperscript{16} At ambient.
TC’s associated between a $V_{BE}$ and a $\Delta V_{BE}$. A $V_{BE}$ possesses a stronger unit TC than a $\Delta V_{BE}$, thus, needing more PTAT current than CTAT current is valid. Simulation shows the best area ratio is 1:8. Varying resistor values to optimize overall temperature variation, results in an $R_2 = 1.6k\Omega$ from 1.44k$\Omega$ and an $R_7 = 14k\Omega$ from 13k$\Omega$. The difference in resistor values between hand calculations and simulation results for an optimal curve over temperature is approximately 10%. As figure 3.33 shows, the change in current over a 180° range is 1.67 µA. At approximately 55°C the TC is zero. Optimizing for an ambient temperature range of 50–55°C is usually good practice due to other integrated circuits within an application which produce power and increase the operating temperature above ambient.

![Figure 3.33: Simulation of Widlar bandgap over temperature.](image)

The voltage reference, $V_{CP}$, biases the cascode transistors at the gain node such that there is zero $V_{CB}$ at the current source transistor at the gain node. The current branches, $I_{CM}$, $I_{BN1}$-$I_{BN4}$, and $I_{BP1}$-$I_{BP4}$ require transistor scaling. For example, $I_{BN1}$ needs to be 200 µA which is twice larger than the unit current of 100 µA. Thus, the transistor area for $I_{BN1}$ needs to double and the emitter resistor needs to be scaled down by a half to ensure it will have the same potential as the other emitters daisy chained off of the unit current source. If there is an emitter voltage mismatch, then temperature variances will affect the bias circuit adversely.

Emitter degeneration resistors are included with amount of degeneration set to 300mV. This value allows the mismatch in current between different branches to be dependent upon resistor mismatch and not $V_{BE}$ mismatch.
Chapter 3. *Analysis: Stability and Compensation*

### 3.6 Stability and compensation

The closed loop transfer function\(^{17}\) of a quad-ferential amplifier is of the canonical form

\[
A_{CL}(s) = \frac{A(s)}{1 + A(s)\beta}
\]  

(3.174)

where

\[
A(s) = a(s)R_F / (R_F + 3R_G)
\]  

(3.175)

\[
\beta = \frac{R_G}{R_F}
\]  

(3.176)

\[
a(s) = \frac{1}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2})}
\]  

(3.177)

The loop gain, \(T(s)\), is

\[
T(s) = A(s)\beta = \frac{R_G}{R_F + 3R_G} \frac{1}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2})}
\]  

(3.178)

Poles \(p_1\) and \(p_2\) are determined by the open loop transfer function of the amplifier, \(a(s)\). Examination of the uncompensated loop gain Bode plot in figure 3.34 reveals the crossover frequency to be 1.5GHz, with \(p_1\) located at 1MHz and \(p_2\) located at 358MHz. The phase margin is \(-30^\circ\), therefore the system is unstable and requires compensation.

In practical applications the amplifier will drive loads of up to 50pF. Inclusion of the load capacitance creates an additional pole, \(p_L\). This pole is determined by \(C_{LOAD}\) in conjunction with the output impedance of the amplifier and results in a \(T(s)\) of

\[
T(s) = \frac{R_G}{R_F + 3R_G} \frac{1}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2})(1 - \frac{s}{p_L})}
\]  

(3.179)

To compensate the quad-ferential amplifier a graphical linear approximation is utilized with Bode plots. A dominant pole is introduced such that the loop gain crosses 0dB at the location of \(p_L\) resulting in a minimum phase margin of \(45^\circ\) for \(C_{LOAD} \leq 50pF\).\(^{18}\)

To ensure the loop gain crosses 0dB with a phase margin of \(45^\circ\), the calculation for the dominant pole, \(p_1\), must consider characteristics from \(p_L\). Designing for a cap load drive,

\(^{17}\)The transfer function was examined in section 2.4.

\(^{18}\)Optimizing settling time for a second order system results in a critically damped system with a phase margin of \(45^\circ\).
Chapter 3. Analysis: Stability and Compensation

Figure 3.34: Uncompensated Bode plot of system loop gain, $|A(s)|$ and $\angle A(s)$.

$C_{LOAD} = 50\mu F$ and with an amplifier output impedance of $86\Omega$, $p_L$ is

$$p_L = \frac{1}{2\pi R_OC_{LOAD}}$$

$$= 37MHz \tag{3.181}$$

Using direct proportionality$[5]$ and acknowledging poles $p_L$ and $p_2$ are separated by approximately a decade, results in the following relationship between $p_1$ and $p_L$

$$p_1 = \frac{p_L}{|T_0|} \tag{3.182}$$

where $|T_0|$ is the low frequency magnitude of the loop gain and $p_1$ is set by the high impedance gain node of the amplifier in conjunction with the compensation capacitor, $C_C$. $^{20}$ The dominant pole, $p_1$, is determined by calculating the compensation capacitor such that the location of the second pole, $p_L$, occurs where the loop gain crosses 0dB.

Substituting the result from (3.181) into (3.182) and converting the value of 72.7dB to 4.3kV/V from figure 3.34 yields the location of the dominant pole, $p_1$

$$p_1 = \frac{37MHz}{4.3kV/V} = 8.6kHz \tag{3.183}$$

$^{19}$Output impedance was examined in section 3.4.

$^{20}$In the absence of $C_C$ the parasitic capacitance sets this pole. However, $C_C$ is significantly larger than the parasitic capacitance.
Using the traditional definition for a pole

\[ p = \frac{1}{2\pi RC} \]  

(3.184)

to solve and calculate for the appropriate compensation capacitor, results in \( C_C = 7.7 \text{pF} \). Iterating \( C_C \) empirically with the simulator yields 45° of phase margin with a \( C_C = 7 \text{pF} \). The iterative \( C_C = 7 \text{pF} \) is approximately 10% different from the graphical linear approximation technique used. The compensated loop gain is shown in figure 3.35.

With the stability and compensation capacitor of the differential loop established, the common-mode loop requires investigation. A \( C_C = 7 \text{pF} \) results in the Bode plot in figure 3.36. The phase margin of 48° for the common-mode loop is comparable to the differential loop. Therefore, the compensation capacitor of \( C_C = 7 \text{pF} \) satisfies the stability needs for both differential and common-mode loops.

\(^{21}\)The impedance of the high gain node of \( R = 2.4 \text{MHz} \) was discussed in section 3.3
Figure 3.36: Compensated Bode plot for common-mode loop. $C_{LOAD}=7pF$. 

**Figure 3.36:** Compensated Bode plot for common-mode loop. $C_{LOAD}=7pF$. 

**Table 3.36:** Compensated Bode plot for common-mode loop. 

- **Magnitude**: 71.6 dB
- **$\omega_o$**: 32.6 MHz
- **Phase margin**: 48°
- **$p_1$**: 10.8 kHz
- **$p_L$**: 35.8 MHz

---

### Figure

- **Magnitude**: 71.6 dB
- **$\omega_o$**: 32.6 MHz
- **Phase margin**: 48°
- **$p_1$**: 10.8 kHz
- **$p_L$**: 35.8 MHz
3.7 Feedback

To achieve negative feedback, from a qualitative perspective, the average of three outputs are fed back to the complementary summing junction. The quad-ferential amplifier executes such a function via its input stage in combination with its feedback and gain resistors. The input stage consist of two quad-ferential pairs (as discussed in section 3.1) such that the output of the negative quad-ferential pair is connected to the gain node. If one input, for example channel 1, moves by $+1\Delta$ and the other three inputs are held constant, channel one will yield a $+1.5\Delta$ while channels 2, 3, and 4 result in a $-0.5\Delta$, as listed in table 3.10. Table 3.10 in conjunction with figure 3.37 provide an example of negative feedback for channel 1. Negative feedback for channel 1 is achieved because the outputs for channels 2, 3, and 4 sum together to form a net “negative” and are fed back to a net “positive” channel 1.

Table 3.10: Expected single-ended $\Delta$ change for a negative quad-ferential pair.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$+1\Delta$</td>
<td>$+1.5\Delta$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$-0.5\Delta$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$-0.5\Delta$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>$-0.5\Delta$</td>
</tr>
</tbody>
</table>

Figure 3.37: Typical configuration of a quad-ferential amplifier.
3.8 Summary

Analysis of a quad-ferential amplifier was presented. The building block for such an amplifier, a quad-ferential pair, was investigated thoroughly. Its small and large signal behavior, when implemented as a quad-ferential input stage, was examined. A concise study of the additional functional blocks that comprise a quad-ferential amplifier were analyzed. A discussion of the open loop gain and the loop gain was presented. The current drive capability and a quantitative investigation of output impedance for the output stage was offered. An examination of the bias cell was presented. Necessary current scaling and a band gap reference was designed. Stability analysis demonstrated the necessity for compensation. Stability was obtained using a graphical approach with bode plots. A dominant pole was introduced such that the loop gain crossed 0dB at the location of $p_L$. This pole is determined by the load capacitance and the output impedance of the amplifier. Feedback was explored qualitatively, which confirmed that a quad-ferential amplifier achieves negative feedback.
Chapter 4

Simulations

This chapter presents simulations of a quad-ferential amplifier. Results of differential and common-mode behavior with respect to time and frequency are presented. Additionally, simulations of the $V_{\text{ocm}}$ functionality and offset voltage are shown. For a gain of 1 the conditions are $V_s^+ = 2.5\text{V}$, $V_s^- = -2.5\text{V}$, $R_F = R_G = 1\text{k}\Omega$, $R_L = 1\text{k}\Omega$, $C_C = 7\text{pF}$ and $C_L = 50\text{pF}$. For a gain of 2 the conditions are identical except $R_G = 500\Omega$.

At a frequency of 2MHz, section 4.1 shows small signal transient response in a gain configuration of 1 and 2, large signal transient response in a gain of 1, and functionality of the $V_{\text{ocm}}$ pin. Selected small signal transient responses for a gain of 1 and 2 are magnified to further illustrate the response of the amplifier. Section 4.2 are Bode plots of selected output differences responding to one, two, three, and four modulating inputs. The plots show a differential bandwidth of 89MHz. Section 4.3 is a simulation of the offset voltage from a statistical perspective.
4.1 Transient response

4.1.1 Small signal

Figure 4.1: Small signal transient response for one modulating input, $v_{i_1}$.
Figure 4.2: Magnified version of difference outputs from figure 4.1(b) responding to one modulating input, $v_i$. 
Figure 4.3: Small signal inputs for transient simulations.
Figure 4.4: Small signal output transient response for $G=+1$. $R_F=R_G=1\,\text{k}\Omega$, $R_L=1\,\text{k}\Omega$, $C_L=50\,\text{pF}$, $C_C=7\,\text{pF}$. 
Figure 4.5: Small signal output transient response for $G=+2$. $R_F=1\,\Omega$, $R_G=500\,\Omega$, $R_L=1\,\Omega$, $C_L=50\,pF$, $C_C=7\,pF$. 

(a) Small signal outputs $v_{OD(1,2)}$, $v_{OD(2,3)}$ and $v_{OD(1,3)}$. $G=+2.$

(b) Small signal outputs $v_{OD(3,4)}$, $v_{OD(4,1)}$ and $v_{OD(4,2)}$. $G=+2.$
Figure 4.6: Magnified transient response of $v_{OD(1,2)}$ from figure 4.4 and figure 4.5.
Figure 4.7: Magnified transient response of $v_{OD(2,3)}$ from figure 4.4 and figure 4.5.
Figure 4.8: Magnified transient response of $v_{OD(1,3)}$ from figure 4.4 and figure 4.5.
4.1.2 Large signal

![Large signal transient inputs.](image1)

![Large signal transient outputs.](image2)

**Figure 4.9:** Large signal transient output response.
4.1.3 $V_{ocm}$ functionality

Figure 4.10: Functionality of $V_{ocm}$. 

(a) Single-ended inputs and $V_{ocm}$.

(b) Single-ended output response to figure 4.9(a).
4.11a  

Inputs (mV)  

<table>
<thead>
<tr>
<th>100</th>
<th>300</th>
<th>500</th>
<th>700</th>
<th>(-700)</th>
<th>(-500)</th>
<th>(-300)</th>
<th>(-100)</th>
</tr>
</thead>
</table>

V_{ocm} (mV)  

| 0   | 40  | 80  | 120 | 160 | 200 | 240 | 280 | 320 | 360 | 400 | 440 | 480 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|

Vi1 \& Vi2 \& Vi3 \& Vi4  

Time (ns)  

4.11b  

 Outputs (mV)  

<table>
<thead>
<tr>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>(-100)</th>
<th>(-200)</th>
<th>(-300)</th>
<th>(-400)</th>
</tr>
</thead>
</table>

Vo1 \& Vo2 \& Vo3 \& Vo4  

Time (ns)  

<table>
<thead>
<tr>
<th>0</th>
<th>40</th>
<th>80</th>
<th>120</th>
<th>160</th>
<th>200</th>
<th>240</th>
<th>280</th>
<th>320</th>
<th>360</th>
<th>400</th>
<th>440</th>
<th>480</th>
</tr>
</thead>
</table>

---

Figure 4.11: Output response with a pulse applied to $V_{ocm}$. 

(a) Inputs constant. Pulse applied to $V_{ocm}$.

(b) Output response to figure 4.11(a).
4.2 Frequency plots

**Figure 4.12:** Bode plots for one and two modulating inputs.
Figure 4.13: Bode plots with three and four inputs modulating inputs.
4.3 Offset

Using statistical models for the transistors of the quad-ferential input stage and running Monte Carlo simulations, figure 4.14 shows the mean offset voltage of a quad-ferential amplifier is $265 \mu$V with a standard deviation of $430 \mu$V.

![Graph showing offset voltage distribution](image)

**Figure 4.14:** Simulation of offset voltage for a quad-ferential amplifier.
4.4 Common-mode rejection

Using statistical models for the resistors of the tail currents\(^1\) feeding the quad-inferential input stage and running Monte Carlo simulations, figure 4.15 shows the mean common-mode rejection of a quad-ferential amplifier is 126dB.

\[\text{Figure 4.15: Simulation of common-mode rejection for a quad-ferential amplifier.}\]

\(^1\)Assuming values of \(R_F\) and \(R_G\) are matched to within 1ppm.
4.5 Summary

Transient simulations confirmed two fundamental aspects of a quad-ferential amplifier. First, it amplifies differences. Second, the output common-mode voltage is controlled by $V_{ocm}$. Magnified small signal transient responses further described the response of the amplifier. Bode plots for one, two, three, and four modulating inputs were included. The small signal differential bandwidth was 89MHz. Statistical models were utilized in performing Monte Carlo simulations to evaluate offset voltage and common-mode rejection. Offset voltage showed a mean of $265\mu V$ with a standard deviation of $430\mu V$. Common-mode rejection showed a mean of 126dB.
Chapter 5

Conclusions

5.1 Design summary

The design and analysis of a monolithic four input and four output quad-ferential amplifier was presented. An overview was offered which included a symbol, a description of each pin, a block diagram and a brief circuit level discussion. The derivation of the transfer function demonstrated that symmetry is required with respect to feedback and gain resistors for the quad-ferential amplifier to approach its ideal behavior.

Analysis of a quad-ferential amplifier was then presented. The building block for such an amplifier, a quad-ferential pair, was investigated thoroughly. Its small and large signal behavior, when implemented as a quad-ferential input stage, was examined. A concise study of the additional functional blocks that comprise a quad-ferential amplifier were analyzed. A discussion of the open loop gain and loop gain was presented. The current drive capability and a quantitative investigation of output impedance for the output stage was offered. An examination of the bias cell was presented and a band gap reference was designed. Using a graphical approach with bode plots the circuit was stabilized to achieve a phase margin of 45° driving a 50pF capacitive load. Feedback was explored qualitatively, which confirmed that a quad-ferential amplifier achieves negative feedback.

Transient simulations verified two fundamental aspects of a quad-ferential amplifier. First, it amplifies differences. Second, the output common-mode voltage is controlled with the \( V_{ocm} \) input. Bode plots for one, two, three, and four modulating inputs were included. The small signal differential bandwidth was 89MHz. Statistical models were utilized in performing Monte Carlo simulations to evaluate offset voltage and common-mode rejection. Offset voltage showed a mean of 265\( \mu \)V with a standard deviation of 430\( \mu \)V. Common-mode rejection showed a mean of 126dB.
5.1.1 Future work

This paper focused on the design and analysis of a quad-ferential amplifier. The fabrication of a quad-ferential amplifier is an area for future work. Furthermore, the bandwidth for a quad-ferential amplifier is 89MHz. A potential option for increasing the bandwidth is the inclusion of an additional gain stage.

Furthermore, market considerations is an area for future work. This amplifier has four inputs and four outputs. A feasibility study as to its application within an RGBY video chain or its application as two differential attenuators in a signal chain are possibilities.
Appendix A

Supplementary Material

A.1 Millman’s theorem

In electrical engineering, Millman’s theorem (or the parallel generator theorem) is a method to simplify the solution of a circuit. Specifically, Millman’s theorem is used to compute the voltage at the ends of a circuit made up of only branches in parallel.[8] For further information, http://www.allaboutcircuits.com/vol_1/chpt_10/11.html

A.2 Transfer function

Analysis for transfer function uses figure 2.5. Difference voltages a quad-ferential amplifier is sensitive to are restated here for reference:

\[
\begin{align*}
v_{ID(1,2)} &= v_w - v_x \\
v_{ID(2,3)} &= v_x - v_y \\
v_{ID(3,4)} &= v_y - v_z \\
v_{ID(4,1)} &= v_z - v_w \\
v_{ID(1,3)} &= v_w - v_y \\
v_{ID(4,2)} &= v_z - v_x
\end{align*}
\]
Appendix A. *Supplementary Material*

Summing node voltages and definition of \( \beta \)'s restated for reference

\[
v_W = \frac{v_{I1} R_{G1} + v_{O2} R_{F12} + v_{O3} R_{F13} + v_{O4} R_{F14}}{\beta_W} \tag{A.7}
\]

\[
v_X = \frac{v_{I2} R_{G2} + v_{O1} R_{F21} + v_{O3} R_{F23} + v_{O4} R_{F24}}{\beta_X} \tag{A.8}
\]

\[
v_Y = \frac{v_{I3} R_{G3} + v_{O1} R_{F31} + v_{O2} R_{F32} + v_{O4} R_{F34}}{\beta_Y} \tag{A.9}
\]

\[
v_Z = \frac{v_{I4} R_{G4} + v_{O1} R_{F41} + v_{O2} R_{F42} + v_{O3} R_{F43}}{\beta_Z} \tag{A.10}
\]

where

\[
\beta_W = \frac{1}{R_{G1}} + \frac{1}{R_{F12}} + \frac{1}{R_{F13}} + \frac{1}{R_{F14}} \tag{A.11}
\]

\[
\beta_X = \frac{1}{R_{G2}} + \frac{1}{R_{F21}} + \frac{1}{R_{F23}} + \frac{1}{R_{F24}} \tag{A.12}
\]

\[
\beta_Y = \frac{1}{R_{G3}} + \frac{1}{R_{F31}} + \frac{1}{R_{F32}} + \frac{1}{R_{F34}} \tag{A.13}
\]

\[
\beta_Z = \frac{1}{R_{G4}} + \frac{1}{R_{F41}} + \frac{1}{R_{F42}} + \frac{1}{R_{F43}} \tag{A.14}
\]

Solving

\[
v_{OC} = \frac{v_W + v_X + v_Y + v_Z}{4} \tag{A.15}
\]

in terms of individual output voltages yields

\[
v_{O1} = 4v_{OC} - v_{O2} - v_{O3} - v_{O4} \tag{A.16}
\]

\[
v_{O2} = 4v_{OC} - v_{O1} - v_{O3} - v_{O4} \tag{A.17}
\]

\[
v_{O3} = 4v_{OC} - v_{O1} - v_{O2} - v_{O4} \tag{A.18}
\]

\[
v_{O4} = 4v_{OC} - v_{O1} - v_{O2} - v_{O3} \tag{A.19}
\]

**Find** \( A_{CL(1,2)}, \frac{v_{OD(1,2)}}{v_{ID(1,2)}} \)

Substituting \( v_W \), (A.7), and \( v_X \), (A.8), into the definition that governs the behavior of a quad-ferential amplifier (A.1), results in

\[
v_{O1} - v_{O2} = \left[ \left( \frac{v_{I1}}{R_{G1}} + \frac{v_{O2}}{R_{F12}} + \frac{v_{O3}}{R_{F13}} + \frac{v_{O4}}{R_{F14}} \right) \frac{1}{\beta_W} \right] - \left[ \left( \frac{v_{I2}}{R_{G2}} + \frac{v_{O1}}{R_{F21}} + \frac{v_{O3}}{R_{F23}} + \frac{v_{O4}}{R_{F24}} \right) \frac{1}{\beta_X} \right] A(s) \tag{A.20}
\]
Substitution of (A.16) and (A.17) into (A.20), algebraic manipulation, and factoring results in

\[ v_{o1} - v_{o2} = \frac{v_{oc}}{R_{F12}R_{G1}} \left( \frac{4A(s)}{R_{F12}R_{G1} \beta_x} \right) + v_{n1} \left( \frac{2A(s)}{R_{G1} \beta_x} \right) - v_{n2} \left( \frac{2A(s)}{R_{G2} \beta_x} \right) + \]

\[ + \frac{2 + A(s)}{R_{F12}R_{G1} \beta_x} + \frac{A(s)}{R_{F21} \beta_x} \]

\[ v_{o1} \left( \frac{2A(s)}{R_{F13}R_{G1} \beta_x} - \frac{A(s)}{R_{F12}R_{G1} \beta_x} - \frac{2A(s)}{R_{F23}R_{G1} \beta_x} + \frac{A(s)}{R_{F21} \beta_x} \right) + \]

\[ + \frac{2 + A(s)}{R_{F12}R_{G1} \beta_x} + \frac{A(s)}{R_{F21} \beta_x} \]

\[ v_{o1} \left( \frac{2A(s)}{R_{F14}R_{G1} \beta_x} - \frac{A(s)}{R_{F12}R_{G1} \beta_x} - \frac{2A(s)}{R_{F24}R_{G1} \beta_x} + \frac{A(s)}{R_{F21} \beta_x} \right) + \]

\[ + \frac{2 + A(s)}{R_{F12}R_{G1} \beta_x} + \frac{A(s)}{R_{F21} \beta_x} \]

(A.21)

Applying symmetry to (A.21) by setting all feedback resistors, \( R_{Fxx} \) equal to each other and all gain resistors, \( R_{Gx} \), equal to each other results in

\[ A_{CL(1,2)}(s) = \frac{R_F}{R_G} \frac{1}{1 + A(s)\beta} \]  

(A.22)

where

\[ \beta = \frac{R_F + 3R_G}{R_G} \]

Find \( A_{CL(2,3)}, \frac{v_{o0/2,3}}{v_{ID(2,3)}} \)

Substituting \( v_x, (A.8), \) and \( v_y, (A.9), \) into the definition that governs the behavior of a quad-ferential amplifier (A.2), results in

\[ v_{o2} - v_{o3} = \left[ \left( \frac{v_{n2}}{R_{G2}} + \frac{v_{o1}}{R_{F21}} + \frac{v_{o3}}{R_{F23}} + \frac{v_{o4}}{R_{F24}} \right) - \left( \frac{v_{n3}}{R_{G3}} + \frac{v_{o1}}{R_{F31}} + \frac{v_{o2}}{R_{F32}} + \frac{v_{o4}}{R_{F34}} \right) \right] A(s) \]

(A.23)
Substitution of (A.17) and (A.18) into (A.23), algebraic manipulation, and factoring results in

\[
v_{o2} - v_{o3} = v_{oc} \left( \frac{4A(s)}{R_{F23}\beta_x} - \frac{4A(s)}{R_{F32}\beta_y} \right) + \frac{v_{i2} \left( \frac{2A(s)}{R_{G2}\beta_x} \right) - v_{i3} \left( \frac{2A(s)}{R_{G3}\beta_y} \right)}{2 + \frac{A(s)}{R_{F23}\beta_x} + \frac{A(s)}{R_{F32}\beta_y}} + \\
v_{o1} \left( \frac{2A(s)}{R_{F21}\beta_x} - \frac{A(s)}{R_{F23}\beta_x} + \frac{A(s)}{R_{F32}\beta_y} - \frac{2A(s)}{R_{F31}\beta_y} \right) + \\
v_{o4} \left( \frac{2A(s)}{R_{F24}\beta_x} - \frac{A(s)}{R_{F23}\beta_x} + \frac{A(s)}{R_{F32}\beta_y} - \frac{2A(s)}{R_{F34}\beta_y} \right) + \\
\frac{2 + \frac{A(s)}{R_{F23}\beta_x} + \frac{A(s)}{R_{F32}\beta_y}}{2 + \frac{A(s)}{R_{F23}\beta_x} + \frac{A(s)}{R_{F32}\beta_y}} \right] (A.24)
\]

Applying symmetry to (A.24) by setting all feedback resistors, \( R_{Fxx} \) equal to each other and all gain resistors, \( R_{Gx} \), equal to each other results in

\[
A_{CL(2,3)}(s) = \frac{R_{F}}{R_{G}} \frac{1}{1 + A(s)\beta} \quad (A.25)
\]

where

\[
\beta = \frac{R_{F} + 3R_{G}}{R_{G}}
\]

Find \( A_{CL(3,4)} \), \( \frac{v_{OD(3,4)}}{v_{ID(3,4)}} \)

Substituting \( v_{y} \), (A.9), and \( v_{z} \), (A.10), into the definition that governs the behavior of a quad-ferential amplifier (A.3), results in

\[
v_{o3} - v_{o4} = \frac{v_{i1} + v_{i2} + v_{i3} + v_{i4} + v_{o1} + v_{o2} + v_{o3} + v_{o4}}{R_{G3} + R_{F31} + R_{F32} + R_{F34}} \left( \frac{v_{i4} + v_{o1} + v_{o2} + v_{o3} + v_{o4}}{R_{G4} + R_{F41} + R_{F42} + R_{F43}} \right) \beta_{z} \quad (A.26)
\]
Substitution of (A.18) and (A.19) into (A.26), algebraic manipulation, and factoring results in

\[
v_{o4} - v_{o1} = v_{oC} \left( \frac{4A(s)}{R_{F34}\beta_y} - \frac{4A(s)}{R_{F43}\beta_z} \right) + \frac{v_{i1}}{2} \left( \frac{2A(s)}{R_{G3}\beta_y} \right) - \frac{v_{i4}}{2} \left( \frac{2A(s)}{R_{G4}\beta_z} \right) + 2 \left( A(s) \right) + \frac{A(s)}{R_{F34}\beta_y} + \frac{A(s)}{R_{F43}\beta_z}
\]

\[
v_{o1} \left( \frac{2A(s)}{R_{F31}\beta_y} - \frac{A(s)}{R_{F34}\beta_y} + \frac{A(s)}{R_{F43}\beta_z} - \frac{2A(s)}{R_{F41}\beta_z} \right) + 2 \left( A(s) \right) + \frac{A(s)}{R_{F34}\beta_y} + \frac{A(s)}{R_{F43}\beta_z}
\]

\[
v_{o2} \left( \frac{2A(s)}{R_{F32}\beta_y} - \frac{A(s)}{R_{F34}\beta_y} + \frac{A(s)}{R_{F43}\beta_z} - \frac{2A(s)}{R_{F42}\beta_z} \right) + 2 \left( A(s) \right) + \frac{A(s)}{R_{F34}\beta_y} + \frac{A(s)}{R_{F43}\beta_z}
\]

(A.27)

Applying symmetry to (A.27) by setting all feedback resistors, \(R_{Fxx}\) equal to each other and all gain resistors, \(R_{Gx}\), equal to each other results in

\[
A_{CL(3,4)}(s) = \frac{R_F}{R_G} \frac{1}{1 + A(s)\beta}
\]

(A.28)

where

\[
\beta = \frac{R_F + 3R_G}{R_G}
\]

Find \(A_{CL(4,1)}\), \(\frac{v_{OD(4,1)}}{v_{ID(4,1)}}\)

Substituting \(v_z\), (A.10), and \(v_w\), (A.7), into the definition that governs the behavior of a quad-ferential amplifier (A.4), results in

\[
v_{o4} - v_{o1} = \left[ \frac{v_{i1} + v_{o1}}{R_{G4}} + \frac{v_{o2}}{R_{F42}} + \frac{v_{o3}}{R_{F43}} }{\beta_z} \right] - \left[ \frac{v_{i1} + v_{o2} + v_{o3} + v_{o4}}{R_{F12}} + \frac{v_{o4}}{R_{F14}} }{\beta_w} \right] A(s)
\]

(A.29)
Substitution of (A.19) and (A.16) into (A.29), algebraic manipulation, and factoring results in

\[ v_{oi} - v_{o1} = v_{oc} \left( \frac{4A(s)}{RF_{14}^2} + \frac{4A(s)}{RF_{14}^2} \right) + v_{11} \left( \frac{2A(s)}{RG_{14}^2} \right) - v_{11} \left( \frac{2A(s)}{RG_{14}^2} \right) + 2 + \frac{A(s)}{RF_{14}^2} \]

\[ v_{o2} \left( \frac{2A(s)}{RF_{14}^2} - \frac{A(s)}{RF_{14}^2} + \frac{A(s)}{RF_{14}^2} - \frac{2A(s)}{RF_{14}^2} \right) + 2 + \frac{A(s)}{RF_{14}^2} \]

\[ v_{o1} \left( \frac{2A(s)}{RF_{14}^2} + \frac{A(s)}{RF_{14}^2} - \frac{2A(s)}{RF_{14}^2} - \frac{A(s)}{RF_{14}^2} \right) + 2 + \frac{A(s)}{RF_{14}^2} \]

(A.30)

Applying symmetry to (A.30) by setting all feedback resistors, \( R_{F_{xx}} \), equal to each other and all gain resistors, \( R_{G_{xx}} \), equal to each other results in

\[ A_{CL(4,1)}(s) = \frac{RF}{RG} \frac{1}{1 + A(s)\beta} \]  

(A.31)

where

\[ \beta = \frac{RF + 3RG}{RG} \]

Find \( A_{CL(1,3)} \), \( \frac{v_{OD(1,3)}}{v_{ID(1,3)}} \)

Substituting \( v_{w} \), (A.7), and \( v_{y} \), (A.9), into the definition that governs the behavior of a quad-ferential amplifier (A.5), results in

\[ v_{oi} - v_{o3} = \left[ \left( \frac{v_{11}}{RG_{11}} + \frac{v_{02}}{RF_{12}} + \frac{v_{01}}{RF_{13}} + \frac{v_{04}}{RF_{14}} \right) - \left( \frac{v_{11}}{RG_{11}} + \frac{v_{01}}{RF_{13}} + \frac{v_{02}}{RF_{12}} + \frac{v_{04}}{RF_{14}} \right) \right] A(s) \]  

(A.32)
Appendix A. Supplementary Material

Substitution of (A.16) and (A.18) into (A.32), algebraic manipulation, and factoring results in

\[
v_{o1} - v_{os} = \frac{v_{oc}}{2 + A(s)R_{F13} \beta_w + A(s)R_{F31} \beta_y} + \frac{v_{in}(2A(s)R_{G1} \beta_w) - v_{in}(2A(s)R_{G3} \beta_y)}{2} + \frac{v_{o2}}{2 + A(s)R_{F13} \beta_w + A(s)R_{F31} \beta_y} + \frac{A(s)}{2 + A(s)R_{F13} \beta_w + A(s)R_{F31} \beta_y}
\]

(A.33)

Applying symmetry to (A.33) by setting all feedback resistors, \(R_{Fxx}\), equal to each other and all gain resistors, \(R_{Gx}\), equal to each other results in

\[
A_{CL(1,3)}(s) = \frac{R_F}{R_G} \frac{1}{1 + A(s) \beta}
\]

(A.34)

where

\[
\beta = \frac{R_F + 3R_G}{R_G}
\]

Find \(A_{CL(4,2)}\), \(\frac{v_{OD(4,2)}}{v_{ID(4,2)}}\)

Substituting \(v_z\), (A.10), and \(v_w\), (A.7), into the definition that governs the behavior of a quad-ferential amplifier (A.6), results in

\[
v_{o4} - v_{o2} = \left[ \left( \frac{v_{i2}}{R_{G2}} + \frac{v_{o1}}{R_{F21}} + \frac{v_{o2}}{R_{F22}} + \frac{v_{o1}}{R_{F23}} + \frac{v_{o4}}{R_{F24}} \right) \beta_x \right] - \left[ \left( \frac{v_{i2}}{R_{G2}} + \frac{v_{o1}}{R_{F21}} + \frac{v_{o2}}{R_{F22}} + \frac{v_{o4}}{R_{F24}} \right) \beta_x \right] A(s)
\]

(A.35)
Substitution of (A.19) and (A.17) into (A.35), algebraic manipulation, and factoring results in

\[
v_{04} - v_{02} = v_{04} \left( \frac{4A(s)}{R_{F42}\beta_z} - \frac{4A(s)}{R_{F24}\beta_x} \right) + v_{14} \left( \frac{2A(s)}{R_{G4}\beta_w} - \frac{2A(s)}{R_{G2}\beta_x} \right) +
\]

\[
v_{04} \left( \frac{2A(s)}{R_{F41}\beta_z} - \frac{A(s)}{R_{F42}\beta_z} + \frac{A(s)}{R_{F24}\beta_x} - \frac{2A(s)}{R_{F21}\beta_x} \right) +
\]

\[
v_{04} \left( \frac{2A(s)}{R_{F43}\beta_z} + \frac{A(s)}{R_{F42}\beta_z} - \frac{2A(s)}{R_{F23}\beta_x} - \frac{A(s)}{R_{F42}\beta_x} \right)
\]

(A.36)

Applying symmetry to (A.36) by setting all feedback resistors, \( R_{Fxx} \) equal to each other and all gain resistors, \( R_{Gx} \), equal to each other results in

\[ A_{CL(4,2)}(s) = \frac{R_F}{R_G} \frac{1}{1 + A(s)\beta} \]  

(A.37)

where

\[ \beta = \frac{R_F + 3R_G}{R_G} \]

### A.3 Modulating inputs

Table A.1: Single-ended output response for one, two, three, and four modulating inputs for a positive quad-ferential pair.

<table>
<thead>
<tr>
<th>Change in Inputs</th>
<th>Change in Outputs</th>
<th>Sum Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>In1   In2 In3 In4</td>
<td>Io1 Io2 Io3 Io4</td>
<td></td>
</tr>
<tr>
<td>±Δ    0  0  0</td>
<td>±1.5Δ ±0.5Δ ±0.5Δ ±0.5Δ</td>
<td>0Δ</td>
</tr>
<tr>
<td>±Δ    ±Δ  0  0</td>
<td>±2Δ ±2Δ  0  0</td>
<td>0Δ</td>
</tr>
<tr>
<td>±Δ    ±Δ ±Δ  0</td>
<td>±1.5Δ ±2.5Δ ±1.5Δ ±0.5Δ</td>
<td>0Δ</td>
</tr>
<tr>
<td>±Δ    ±Δ ±Δ ±Δ</td>
<td>±2Δ ±2Δ ±2Δ ±2Δ</td>
<td>0Δ</td>
</tr>
</tbody>
</table>
Figure A.1: Positive quad-ferential pair with one modulating input.

Figure A.2: Positive quad-ferential pair with two inputs modulating.
Figure A.3: Positive quad-ferential pair with three inputs modulating.

Figure A.4: Positive quad-ferential pair with four inputs modulating.
A.4 Large signal analysis of quad-ferential pair

Applying KVL to figure 3.4

\[ V_{t1} + V_{EB1P} - V_{EB2P} - V_{i2} = 0 \]
\[ V_{i1} \ln \frac{I_{C1P}}{I_s} - V_{i1} \ln \frac{I_{C2P}}{I_s} = V_{i2} - V_{i1} \]
\[ \frac{I_{C1P}}{I_{C2P}} = \frac{e^{-V_{ID(1,2)}}}{V_{i1}} \]

\[ V_{i2} + V_{EB2P} - V_{EB1P} - V_{i3} = 0 \]
\[ V_{i2} \ln \frac{I_{C1P}}{I_s} - V_{i2} \ln \frac{I_{C2P}}{I_s} = V_{i3} - V_{i2} \]
\[ \frac{I_{C1P}}{I_{C2P}} = \frac{e^{-V_{ID(2,3)}}}{V_{i2}} \]

\[ V_{i3} + V_{EB3P} - V_{EB4P} - V_{i4} = 0 \]
\[ V_{i3} \ln \frac{I_{C1P}}{I_s} - V_{i3} \ln \frac{I_{C2P}}{I_s} = V_{i4} - V_{i3} \]
\[ \frac{I_{C1P}}{I_{C2P}} = \frac{e^{-V_{ID(3,4)}}}{V_{i3}} \]

\[ V_{i4} + V_{EB4P} - V_{EB3P} - V_{i1} = 0 \]
\[ V_{i4} \ln \frac{I_{C1P}}{I_s} - V_{i4} \ln \frac{I_{C2P}}{I_s} = V_{i1} - V_{i4} \]
\[ \frac{I_{C1P}}{I_{C2P}} = \frac{e^{-V_{ID(4,1)}}}{V_{i4}} \]

\[ V_{i1} + V_{EB1P} - V_{EB10P} - V_{i3} = 0 \]
\[ V_{i1} \ln \frac{I_{C1P}}{I_s} - V_{i1} \ln \frac{I_{C2P}}{I_s} = V_{i3} - V_{i1} \]
\[ \frac{I_{C1P}}{I_{C2P}} = \frac{e^{-V_{ID(1,3)}}}{V_{i1}} \]

\[ V_{i4} + V_{EB11P} - V_{EB12P} - V_{i2} = 0 \]
\[ V_{i4} \ln \frac{I_{C1P}}{I_s} - V_{i4} \ln \frac{I_{C2P}}{I_s} = V_{i2} - V_{i4} \]
\[ \frac{I_{C1P}}{I_{C2P}} = \frac{e^{-V_{ID(4,2)}}}{V_{i4}} \]
Applying KVL to figure 3.5

\[ V_{i1} + V_{EB1N} - V_{EB2N} - V_{i2} = 0 \]
\[ V_i \ln \frac{I_{C1N}}{I_s} - V_i \ln \frac{I_{C2N}}{I_s} = V_{i2} - V_{i1} \]
\[ I_{C1N} = e^{-V_{UD(1,2)}} \]
\[ V_{i2} + V_{EB1N} - V_{EB2N} - V_{i3} = 0 \]
\[ V_i \ln \frac{I_{C3N}}{I_s} - V_i \ln \frac{I_{C4N}}{I_s} = V_{i3} - V_{i2} \]
\[ I_{C3N} = e^{-V_{UD(2,3)}} \]
\[ V_{i3} + V_{EB1N} - V_{EB2N} - V_{i4} = 0 \]
\[ V_i \ln \frac{I_{C5N}}{I_s} - V_i \ln \frac{I_{C6N}}{I_s} = V_{i4} - V_{i3} \]
\[ I_{C5N} = e^{-V_{UD(3,4)}} \]
\[ V_{i4} + V_{EB1N} - V_{EB2N} - V_{i1} = 0 \]
\[ V_i \ln \frac{I_{C7N}}{I_s} - V_i \ln \frac{I_{C8N}}{I_s} = V_{i1} - V_{i4} \]
\[ I_{C7N} = e^{-V_{UD(4,1)}} \]
\[ V_{i1} + V_{EB1N} - V_{EB2N} - V_{i3} = 0 \]
\[ V_i \ln \frac{I_{C9N}}{I_s} - V_i \ln \frac{I_{C10N}}{I_s} = V_{i3} - V_{i1} \]
\[ I_{C9N} = e^{-V_{UD(1,3)}} \]
\[ V_{i3} + V_{EB1N} - V_{EB2N} - V_{i2} = 0 \]
\[ V_i \ln \frac{I_{C11N}}{I_s} - V_i \ln \frac{I_{C12N}}{I_s} = V_{i2} - V_{i3} \]
\[ I_{C11N} = e^{-V_{UD(4,2)}} \]

Mesh analysis at the emitters

\[ I_{TAIL} = \frac{(I_{C1} + I_{C2})}{\alpha} \]
\[ I_{TAIL} = \frac{(I_{C3} + I_{C4})}{\alpha} \]
\[ I_{TAIL} = \frac{(I_{C5} + I_{C6})}{\alpha} \]
\[ I_{TAIL} = \frac{(I_{C7} + I_{C8})}{\alpha} \]
\[ I_{TAIL} = \frac{(I_{C9} + I_{C10})}{\alpha} \]
\[ I_{TAIL} = \frac{(I_{C11} + I_{C12})}{\alpha} \]
A.5 Small signal analysis

Explicit list of the sixteen small signal gain constants with loading conditions specified

\[ A_{11} = \frac{v_{o1}}{v_{i1}} \quad v_{i2} = v_{i3} = v_{i4} = 0 \]  \hspace{1cm} (A.38)
\[ A_{12} = \frac{v_{o1}}{v_{i2}} \quad v_{i1} = v_{i3} = v_{i4} = 0 \]  \hspace{1cm} (A.39)
\[ A_{13} = \frac{v_{o1}}{v_{i3}} \quad v_{i1} = v_{i2} = v_{i4} = 0 \]  \hspace{1cm} (A.40)
\[ A_{14} = \frac{v_{o1}}{v_{i4}} \quad v_{i1} = v_{i2} = v_{i3} = 0 \]  \hspace{1cm} (A.41)
\[ A_{21} = \frac{v_{o2}}{v_{i1}} \quad v_{i2} = v_{i3} = v_{i4} = 0 \]  \hspace{1cm} (A.42)
\[ A_{22} = \frac{v_{o2}}{v_{i2}} \quad v_{i1} = v_{i3} = v_{i4} = 0 \]  \hspace{1cm} (A.43)
\[ A_{23} = \frac{v_{o2}}{v_{i3}} \quad v_{i1} = v_{i2} = v_{i4} = 0 \]  \hspace{1cm} (A.44)
\[ A_{24} = \frac{v_{o2}}{v_{i4}} \quad v_{i1} = v_{i2} = v_{i3} = 0 \]  \hspace{1cm} (A.45)
\[ A_{31} = \frac{v_{o3}}{v_{i1}} \quad v_{i2} = v_{i3} = v_{i4} = 0 \]  \hspace{1cm} (A.46)
\[ A_{32} = \frac{v_{o3}}{v_{i2}} \quad v_{i1} = v_{i3} = v_{i4} = 0 \]  \hspace{1cm} (A.47)
\[ A_{33} = \frac{v_{o3}}{v_{i3}} \quad v_{i1} = v_{i2} = v_{i4} = 0 \]  \hspace{1cm} (A.48)
\[ A_{34} = \frac{v_{o3}}{v_{i4}} \quad v_{i1} = v_{i2} = v_{i3} = 0 \]  \hspace{1cm} (A.49)
\[ A_{41} = \frac{v_{o4}}{v_{i1}} \quad v_{i2} = v_{i3} = v_{i4} = 0 \]  \hspace{1cm} (A.50)
\[ A_{42} = \frac{v_{o4}}{v_{i2}} \quad v_{i1} = v_{i3} = v_{i4} = 0 \]  \hspace{1cm} (A.51)
\[ A_{43} = \frac{v_{o4}}{v_{i3}} \quad v_{i1} = v_{i2} = v_{i4} = 0 \]  \hspace{1cm} (A.52)
\[ A_{44} = \frac{v_{o4}}{v_{i4}} \quad v_{i1} = v_{i2} = v_{i3} = 0 \]  \hspace{1cm} (A.53)
Bode plots for a positive quad-ferential pair

Figure A.5: Positive quad-ferential pair Bode plot for one modulating input, $v_{i1}$.

Figure A.6: Positive quad-ferential pair Bode plot for two modulating inputs, $v_{i1}$ and $v_{i2}$.
Figure A.7: Positive quad-ferential pair Bode plot for three modulating inputs, \(v_{i1}, v_{i2}\) and \(v_{i3}\).

Figure A.8: Positive quad-ferential pair Bode plot for four modulating inputs, \(v_{i1}, v_{i2}, v_{i3}\) and \(v_{i4}\).
A.6 Bias cell analysis

Detailed nodal analysis performed on figure 3.32. KVL from $Q_1$ to $Q_2$ in a clockwise direction

$$-V_{be1} + V_{be2} + I_2R_2 = 0 \quad (A.54)$$

solving for $\Delta V_{BE(1,2)}$

$$\Delta V_{BE(1,2)} = I_2R_2 \quad (A.55)$$

where $\Delta V_{BE(1,2)}$ is

$$\Delta V_{BE(1,2)} = V_t \left( \ln \frac{I_{C1}}{A_1J_S} - \ln \frac{I_{C2}}{A_2J_S} \right) \quad (A.56)$$

Substitution of (A.56) into (A.55) results in

$$I_2R_2 = V_t \ln \left( \frac{I_1}{A_1} \frac{A_2}{I_2} \right) \quad (A.57)$$

thus

$$R_2 = \frac{V_t \ln \left( \frac{I_1}{A_1} \frac{A_2}{J_S} \right)}{I_2} \quad (A.58)$$

KVL loop from $Q_1$ to $R_7$ in a clockwise direction

$$-V_{be1} + I_7R_7 = 0 \quad (A.59)$$

Solving for $R_7$ yields

$$R_7 = \frac{\frac{BE_1}{I_7}} \quad (A.60)$$
Appendix B

Multi-input amplification

A multi-input amplifier (also referred to as an “M”-input amplifier where \( M \geq 3 \)) accepts “M” input voltages and produces “M” output voltages such that:

- the voltage difference between any pair of output voltages is proportional to the difference between the corresponding pair of input voltages through the same proportionality constant\[1\],
- the average value of the “M” output voltages is constant and unrelated to the input voltages\[1\]

While a typical operational amplifier and a typical differential amplifier utilize a differential pair for its input stage, a multi-input amplifier utilizes two “M”-ferential pairs for its input stage. A “M”-ferential input stage is the core circuit block for a multi-input amplifier. In its most general form, an “M”-ferential pair is a \( g_m \) cell that accepts “M” input voltages and generates “M” output currents.

For an “M”-ferential pair to behave comparable to a differential pair, two “M”-ferential pairs are necessary. Where each “M”-ferential pair represents each transistor in a differential pair, as seen in figure B.1.

With “M” number of inputs and outputs exists a multiple number of unique difference combinations. Understanding the number of unique “difference” combinations (such as \( V_1 - V_2, V_2 - V_3, \) etc.) is important because it guarantees that any two outputs will produce a quantity proportional to the difference of their respective inputs. The number of independent difference pairs generated by an “M”-ferential pair is

\[
\text{Number of independent difference pairs} = 3(M - 2) + \sum_{k=1}^{M-4} k, \text{for } M \equiv Z \geq 3
\]
An “M”-ferential pair requires the following number of transistors

\[
\text{Number of transistors for M inputs} = M(M - 1) \tag{B.1}
\]

The number of base terminals connected at a single node is

\[
\text{Number of bases or collectors at the same node} = M - 1 \tag{B.2}
\]

The number of collector terminals connected at a single node is also described by (B.2).

**Block Diagram**

A block diagram of a multi-input amplifier is shown in figure B.2. For simplicity, the block diagram (fig. B.2) employs bus wire to describe the signal flow, where the number of signals is designated above each bus wire in terms of “M”. For example, a four input amplifier will have four output traces being fed back to the four inputs. Examination of the common-mode circuit block indicates that it does not increase by “M”. The number of signals it accepts may increase, but the circuit block does not get duplicated “M” times like the gain or buffer stages. The input stage is the core of a multi-input amplifier. It accepts “M” input voltages, outputs 2M the output currents, and consists of two “M”-ferential pairs. Similar to a differential pair am “M”-ferential pair can be thought of as a \( g_m \) cell. The currents generated by the input stage proceed to the gain stage. From the gain node, the signals are buffered and continue to the output. The output is fed into two nodes. The input stage via an \( R_F \) and \( R_G \) network (not shown) to establish negative feedback and the common-mode circuit block for control of the overall output.
common-mode voltage. The common-mode circuit block samples the output voltages, compares the average value to $V_{ocm}$ and sends the appropriate common-mode bias level to the gain node.

From an architectural standpoint, a multi-input amplifier is an extension of a differential amplifier. Correspondingly, a multi-input amplifier processes its signal with three stages and two feedback loops: an input stage, a gain stage, an output stage, an “M”-ferential feedback loop to establish negative feedback and a common-mode loop to control the output common-mode voltage.

**Simplified Circuit**

A simplified circuit of a multi-input amplifier is presented in figure B.3. For as many “M” inputs required of a “M”-ferential amplifier, results in “M” number of gain and “M” number of buffers as seen in figure B.3. Although each channel consist of a set of current source transistors, output buffers, and differential input currents, all channels share transistors $Q_9$ through $Q_{12}$, as shown in figure B.3. The input stage receives its bias from $I_{TAIL}$. Cascode transistor $Q_5$ ($Q_6$) receives a bias current of $I_{TAIL}/2$ from differential pair $Q_9$ and $Q_{10}$ via the diode connection of $Q_{11}$ ($Q_{12}$) and current source transistor $Q_7$ ($Q_8$). The Wilson current mirror is biased by $Q_8$ such that $I_{C3} = I_{C5}$.

The input stage accepts “M” input voltages and outputs “M” differential currents, $i_{px}$ and $i_{nx}$. Transistors $Q_1$ through $Q_4$ form the Wilson current mirror which translates the transconductance of the input stage to the high impedance node. The high impedance node is formed by the output impedance of the Wilson current mirror in parallel with the output impedance of the cascode current source (transistors $Q_5$ and $Q_7$). From the gain node, the signal is buffered to the output.

From a feedback perspective, the common-mode level is sensed across the resistors ($R_1$–$R_{M''}$) connected to the base of $Q_{10}$. The average of the output voltage is sampled...
Figure B.3: Simplified circuit of a multi-input amplifier.

and compared to the value on the $V_{ocm}$ pin. The differential pair, $Q_9$ and $Q_{10}$, control the output common-mode level of the amplifier by setting the appropriate bias level (as determined by the user’s input level on the $V_{ocm}$ pin) at the output by modulating the currents in $Q_{11}$ and $Q_{12}$. $I_{TAIL2}$ provides the bias level for $Q_9$ and $Q_{10}$. For simplicity, the “M”-ferential feedback was omitted and is discussed in detail in section B.

Transfer Function

The simplest form of a multi-input amplifier is a trifferential amplifier. A trifferential amplifier is comprised of 3 inputs and 3 outputs. Figure B.4 is a symbol for a trifferential amplifier. It has 9 pins. Three non-inverting inputs, $V_x$, $V_y$, and $V_z$, three outputs, $V_{out1}$, $V_{out2}$, and $V_{out3}$, output common-mode control, $V_{ocm}$, and two supplies, $V^+$ and $V^-$. The supply pins have been omitted for simplicity and the polarity of the output
voltages will be dependent on the input voltages. Figure B.5 shows a typical circuit configuration for a trifferential amplifier. There are three inputs, three outputs, and three feedback paths. Each feedback path contains a summing node, a gain resistor, and two feedback resistors. As shown, the summing nodes are $v_x$, $v_y$ and $v_z$. They are associated with $v_{I1}$, $v_{I2}$ and $v_{I3}$.

The resistors follow a different nomenclature than the summing nodes. The resistor reference designators indicate the function of the resistor, the input signal summing node the resistor is associated with, and in the case of the feedback resistors, the output voltage it is feeding back to the summing node. For example, $R_{F12}$. The “F” identifies it as a feedback resistor, the number “1” indicates this resistor is associated with the summing node associated with $v_{I1}$, $v_x$, and the number “2” designates $v_{O2}$ is being fed back across this resistor.

Negative feedback for a “M”-ferential amplifier is implemented in a similar manner to that of a differential amplifier. A differential amplifier establishes negative feedback by feeding one output back to its complimentary input. The “M”-ferential amplifier establishes negative feedback by taking the average of M-1 outputs (in this example two outputs for a trifferential amplifier) and feeding it back to the complimentary summing junction, as shown in figure B.5.

![Figure B.5: Trifferential configuration.](image-url)
Applying nodal analysis to figure B.5 and expanding the transfer function to an “M”-input amplifier results in the following non-ideal transfer function

\[ A_{CL}(s) = \frac{a(s) R_F}{1 + a(s) \frac{R_F}{R_F + (M-1)R_G}} \]

which is of the canonical form

\[ A_{CL}(s) = \frac{A(s)}{1 + A\beta} \]

where

\[ A(s) = \frac{a(s) R_F}{R_F + (M-1)R_G} \]

\[ \beta = \frac{R_G}{R_F} \]

\[ a(s) = \frac{1}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2})} \]

Poles \( p_1 \) and \( p_2 \) are set by the amplifier. The loop gain, \( T(s) \), is

\[ T(s) = A(s)\beta = \frac{R_G}{R_F + (M-1)R_G} \frac{1}{(1 - \frac{s}{p_1})(1 - \frac{s}{p_2})} \]

The “\((M - 1)R_G\)” term inside the loop gain, (2.29), implies there are “M-1” outputs being fed back to one summing node. This is consistent with the trifferential example in figure B.5 which shows two outputs fed back to a summing node.
Bibliography


