ADAPTIVE NEAR-OPTIMAL COMPENSATION IN LOSSY POLYPHASE POWER SYSTEMS

A thesis presented

by

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The role of compensation in power system efficiency optimization is to reduce the power consumption of the source (line) impedance, so that most of the source power is delivered to the load. A classical result by Fryze is used in the case when the voltage drop across the line is negligible in comparison with the load voltage. However, when the source impedance becomes significant the traditional Fryze current is no longer the smallest (by rms) line current that supplies the same real power to the load as the original load current.

An optimal solution considering significant line impedance has been already obtained in recent works. Unfortunately, it relies on network and load parameters that are not easy to determine during operation. This motivates our approach in searching for a sub-optimal easy-to-implement solution.

Our basic objective in this thesis is the construction of an adaptive near-optimal compensator that relies only on measurements of the load voltage and current so as to allow precise control of the compensated load as well as the real power flowing out of the compensator, while reducing line losses to within a few percent of its theoretical minimum. Properties of the solution are illustrated for an asymmetrical three phase induction motor supplied with unbalanced non-sinusoidal voltages.
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Dedicated to my dear family, specially my parents Enrique and Orencia...
Chapter 1

Introduction

The role of compensation in power system efficiency optimization is to reduce the power consumption of the (Thevenin equivalent) source (or “line”) impedance, so that most of the source power is delivered to the load (Fig. 1.1). A classical result by Fryze [1] states that when the voltage drop across the (equivalent) line is negligible in comparison with the load voltage, i.e., when $\|v_s(t) - v(t)\| \ll \|v(t)\|$ then the smallest possible source (line) current $i_s(t)$ (in rms) is

$$i_F(t) = \frac{P_{load}^{(u)}}{\|v\|^2} v(t)$$

(1.1)

where $P_{load}^{(u)}$ is the original real (average) power delivered to the load without compensation (uncompensated load), and $\|v\|$ denotes the rms value of a waveform $v(t)$. However, as stated in [2, 3, 4], when the source impedance becomes significant (namely, a few percent of the load impedance, or higher) the traditional Fryze current is no longer the smallest (by rms) line current that supplies the same real power to the load as the original load current. Thus, when impedance is negligible, all we need is Fryze, and no adaptation is needed. Otherwise, when impedance is non-negligible, the optimum is not achieved by Fryze.

An important issue to be mentioned here is that optimality should not be measured by means of rms line current, but rather by line losses represented by $P_{line}$ (the power dissipated in the source impedance), which allows for source impedances that are frequency dependent, with coupling and possibly unbalanced conditions between phases.
An optimal solution considering significant source (line) impedance has been obtained in [2, 3]. Unfortunately, it relies on network and load parameters that are not easy to determine during operation. Moreover, the network parameters change frequently, making it impractical to use a fixed compensator. This motivates our approach in searching for a sub-optimal easy-to-implement solution.

Our basic objective in this thesis is the construction of an adaptive near-optimal compensator that relies only on measurements of the load voltage and current (or, equivalently, the phasor description of these waveforms) so as to allow precise control of $P_{\text{cloud}}$, the real power delivered to the compensated load, as well as $P_{\text{comp}}$, the real power flowing out of the compensator, while reducing $P_{\text{line}}$ to within a few percent of its theoretical minimum.

In particular, results in [3] suggest that a quadrature Fryze compensator ($\text{quad-Fryze}$ for short) may be a good choice in providing a very close approximation of the optimal solution given in [2]. The objective of a quad-Fryze compensator is to adjust $i_{\text{comp}}(t)$ so that the compensated load is linear and time-invariant with a current-voltage characteristic given by

$$i_s(t) = \alpha v(t) - \beta \mathcal{H}\{v(t)\}, \quad (1.2)$$

where $\alpha, \beta$ are real-valued scaling coefficients and $\mathcal{H}\{\cdot\}$ represents the Hilbert transform of a signal (i.e., the compensated line current is a linear combination of the load voltage

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**Figure 1.1:** Load compensation in a power delivery system.
Chapter 1. Introduction

$v(t)$ and its Hilbert transform $\mathcal{H}\{v(t)\}$. Observe that the term “quadrature Fryze” was adopted because the Hilbert transform imparts a $90^\circ$ phase delay to $v(t)$.

We shall demonstrate the near-optimal performance of quad-Fryze in minimizing $P_{\text{line}}$, while allowing us to control (i.e., specify) $P_{\text{comp}}$ (normally we set $P_{\text{comp}} = 0$, but other “target” values are possible) and $P_{\text{cloud}}$ by means of adjusting the design parameters $\alpha$ and $\beta$. Applying the quad-Fryze compensator (or any compensator) in the configuration of Fig. 1.1 results in a readjustment of all currents and voltages. As a result, the desired relation $(1.2)$ is no longer satisfied, so we need to readjust $i_{\text{comp}}(t)$ again and repeat these steps (iterative procedure) until it converges to steady state conditions that satisfy the near-optimal desired solution. Thus the objective of this process is to enforce a certain steady-state relation (i.e., $i_s(t) = \alpha v(t) - \beta \mathcal{H}\{v(t)\}$) on the source current. Furthermore, we developed an adaptive-iterative implementation of this quad-Fryze compensator for adjusting the values of $\alpha$, $\beta$ to achieve the prescribed $P_{\text{cloud}}$, $P_{\text{comp}}$ values (see Chapter 4).

The remainder of the thesis is organized as follows. Chapter 2 synopsizes the solution of the generalized optimal compensation (solution to Prob. 2.3 in [2]), where we use explicit information about the line and the load to obtain a closed-form expression for the optimized source (line) current. It leads us to a fundamental performance bound, as well as an optimal performance surface (OPS), which in conjunction with special cross-section curves represent a theoretical reference point for the adaptive near-optimal compensator.

Before developing our compensator we need first to consider the generalization of the Fryze compensation, that is, the so-called “quad-Fryze” compensation [3] which has been explained in Chapter 3. Next, the development of this adaptive implementation is carried out in Chapter 4 where we have constructed an adaptive estimator that track changes in line (and load) characteristics, and converges, in steady state, to a near-optimal solution by obtaining very good results. Finally, conclusions and future directions are given in Chapter 5.
1.1 Fundamentals of compensation in lossy polyphase systems

Consider an \( n \)-phase system, i.e., a system with \( n + 1 \) conductors (“wires”) in which the first \( n \) are referenced either to a common ground, or to the \( (n + 1) \)-st (“neutral”) conductor. Then we can define the \( n \)-dimensional voltage and current (row) vectors \( v(t), i(t) \), where all currents have reference directions “toward” the load (Fig. 1.1).

The Hilbert space terminology of [5] is implemented to formulate our objectives and derive our results. Thus \( v(t) \) and \( i(t) \) are row vectors representing load voltage and current, respectively, which we view as elements in a Hilbert space of \( n \)-phase, \( T \)-periodic, square-integrable waveforms, with the inner product represented by

\[
\langle x, y \rangle \overset{\text{def}}{=} \frac{1}{T} \int_T x(t)y(t) \overline{\mathbf{t}} dt
\]

where the superscript \( \overline{\mathbf{t}} \) denotes transposition.

Such waveforms can be represented by a summable Fourier series, viz.,

\[
x(t) = \sum_{l=-\infty}^{\infty} X_l e^{jlt}
\]

where the row-vector Fourier coefficients \( X_l \) are given by

\[
X_l = \frac{1}{T} \int_T x(t) e^{-jlt} dt.
\]

One-sided phasor arrays are used by stacking the vector Fourier coefficients into a long row vector:

\[
\mathbf{X} \overset{\text{def}}{=} [X_0 \sqrt{2}X_1 \sqrt{2}X_2 \ldots]
\]

so that

\[
\langle x, y \rangle = \Re\{\mathbf{X}^H \mathbf{Y}\} = \langle \mathbf{X}, \mathbf{Y} \rangle
\]

where the subscript \( ^H \) indicates conjugate transpose, and \( \mathbf{X}, \mathbf{Y} \) are the phasor arrays representing the waveforms \( x(t), y(t) \), respectively.
Thus, we have a complete isometry between the Hilbert space of periodic square-integrable real waveforms and that of square-summable complex vectors (of infinite length).

The basic circuit equations for the power delivery system of Fig. 1.1 in the frequency domain are given by:

\[
\mathcal{V} = \mathcal{V}_s - \mathcal{I}_s \mathcal{Z}_s \tag{1.8a}
\]

\[
\mathcal{I} = \mathcal{I}_s + \mathcal{I}_{comp} \tag{1.8b}
\]

where \( \mathcal{Z}_s \) is a diagonal matrix (complex-valued) representing the linear time-invariant source impedance. Actually, for our analysis the assumption of diagonality is not needed—it represents the assumption that the source impedance is linear and time-invariant, without coupling between phases.

An important issue we would like to point out is that from (1.8) and for a linear time-invariant load, where

\[
\mathcal{I} = \mathcal{V} \mathcal{Y}, \tag{1.9}
\]

with the matrix \( \mathcal{Y} \) representing the load admittance, we can get

\[
\mathcal{I}_s = (\mathcal{V}_s - \mathcal{I}_s \mathcal{Z}_s) \mathcal{Y} - \mathcal{I}_{comp} \tag{1.10a}
\]

so that

\[
\mathcal{I}_s = (\mathcal{V}_s \mathcal{Y} - \mathcal{I}_{comp})(I + \mathcal{Z}_s \mathcal{Y})^{-1}. \tag{1.10b}
\]

Under these conditions, we have a one-to-one correspondence relation between \( \mathcal{I}_s \) and \( \mathcal{I}_{comp} \) (when we consider a fixed \( \mathcal{V}_s \), \( \mathcal{Z}_s \) and \( \mathcal{Y} \)) so that we shall use \( \mathcal{I}_s \) instead of \( \mathcal{I}_{comp} \) as the independent variable for the optimization problem.

In the linear case, nonsingularity of the matrix \( (I + \mathcal{Z}_s \mathcal{Y}) \) is the only condition needed to ensure one-to-one correspondence between \( \mathcal{I}_s \) and \( \mathcal{I}_{comp} \). This condition is almost always met, either because \( \|\mathcal{Z}_s \mathcal{Y}\| \) is small, or because the eigenvalues of \( \mathcal{Z}_s \mathcal{Y} \) have a positive real part (see Appendix B for further details).
We can now express the various power flows in the system. Thus, the corresponding power dissipated in the source impedance is

\[ P_{\text{line}} = \langle i_s, v_s - v \rangle = \Re\{I_s(V_s - V)^H\} \]  

so that

\[ P_{\text{line}} = \Re\{I_sZ_s^HI_s^H\}. \]  

This means we can replace \( Z_s^H \) in the previous expression by its Hermitian component (see Appendix A), viz.,

\[ P_{\text{line}} = \Re\{I_sR_sI_s^H\} = I_sR_sI_s^H \]  

where

\[ R_s \equiv \frac{1}{2}\{Z_s + Z_s^H\}. \]  

An important observation is that when \( Z_s \) is diagonal, then \( R_s = \Re\{Z_s\} \), a real-valued diagonal matrix.

Similarly, the real power flowing into the compensated load is given by

\[ P_{\text{load}} = \langle i_s, v \rangle = \Re\{I_sV_s^H\} \]
\[ = \Re\{I_s(V_s - I_sZ_s)^H\} \]
\[ = \Re\{I_sV_s^H\} - P_{\text{line}} \]  

and the power \( P_{\text{comp}} \) flowing out of the compensator corresponds to

\[ P_{\text{comp}} \equiv \Re\{I_{\text{comp}}V^H\} \]
\[ = \Re\{IV^H\} - P_{\text{load}}. \]  

### 1.2 Iterative compensation

Fryze compensation works well in the case of an ideal line (negligible impedance which is balanced and no-coupling) for setting \( I_{\text{comp}} \) once achieves optimal performance in a single step by using only load current and voltage information. However, as the source impedance \( Z_s \) enters the “non-negligible” range, setting \( I_{\text{comp}} \) according to (1.1) will result in a change in voltage load and current. This suggests a readjustment of \( I_{\text{comp}} \).
namely,
\[ I_{\text{comp}}(k + 1) = I(k) - \frac{P_{\text{cloud}}(k)}{\|V(k)\|^2} V(k) \]  
(1.14)

where \( P_{\text{cloud}} \) has been used instead of \( P_{\text{load}} \) (recall \( P_{\text{cloud}} + P_{\text{comp}} = P_{\text{load}} \)) because we consider the possibility of \( P_{\text{comp}} \neq 0 \), at least during the iteration process, so that this process will eventually converge, i.e., network will settle to a new steady state (see further discussion in Section 4.1).

When the iteration process has converged we will have
\[ I_s = \Re\{I_s V_H^H\} \]  
(1.15)

with no control over the value of \( P_{\text{cloud}} \) and no guarantee that, for instance, \( P_{\text{comp}} = 0 \).

Our objectives, in order of priority, are:

(a) minimize \( P_{\text{line}} \),

(b) achieve \( P_{\text{comp}} = 0 \) in steady state and \( P_{\text{comp}} \approx 0 \) during the iteration process, and

(c) control the steady-state value of \( P_{\text{cloud}} \) (equivalently \( P_{\text{load}} \)).

In order to control both \( P_{\text{comp}} \) and \( P_{\text{cloud}} \), we need at least two control parameters. Thus, we propose to modify our iterative scheme, so that it will converge in steady state to
\[ I_s = (\alpha + j\beta)V. \]  
We study the effects of \( \alpha, \beta \) in Chapter 3 and show that there is a one-to-one relation between \( (\alpha, \beta) \) and \( (P_{\text{comp}}, P_{\text{cloud}}) \). This means we can achieve our objectives by controlling the values of \( \alpha, \beta \).

The remaining challenge is to find the values of \( \alpha, \beta \) that achieve the desired values for \( P_{\text{comp}}, P_{\text{cloud}} \), both in steady-state and during the iteration process. We propose in Chapter 4 an iterative scheme of the form
\[ I_{\text{comp}}(k + 1) = I(k) - [\alpha(k) + j\beta]V(k) \]  
(1.16)
in which the values of \( \alpha(k), \beta \) depend only on \( I(k), V(k) \), the load current and voltage, and the target values \( P_{\text{comp,desired}}, P_{\text{cloud,desired}} \).
1.3 Optimal performance bounds

In order to evaluate the performance of our adaptive compensator we need to compare it with the theoretical optimum by using the results from [2]. Basically, this theoretical optimum consists of the universal performance bound, along with several cross-section curves (Fig. 1.2) obtained by imposing a constraint on the value of $P_{\text{comp}}$. Hence, the theoretical performance bounds constitute a benchmark for performance evaluation in steady state, as well as an indicator for performance tradeoffs, such as increased $P_{\text{line}}$ when we attempt to increase $P_{\text{cloud}}$.

![Figure 1.2: Optimal performance bounds.](image)

Each cross-section curve describes the theoretical minimum for $P_{\text{line}}$ as a function of $P_{\text{cloud}}$ for a prescribed level of $P_{\text{comp}}$. Naturally, our primary interest in steady-state is the cross-section curve for $P_{\text{comp}} = 0$ (the dotted line in Fig. 1.2). Furthermore, the compensator can be adaptively steered to adjust $P_{\text{cloud}}$ to any desired value up to $P_{\text{nom}}$, the nominal power that would be delivered to the load in the absence of any line/source impedance. Notice that prescribing a $P_{\text{cloud}}$ value near $P_{\text{nom}}$ (which in our case of study corresponds to 22,750W) must come at the cost of increased $P_{\text{line}}$, as is evident from the cross-section curves in Fig. 1.2. In addition, the cross-section curves play a role in analyzing the transient behavior of our adaptive algorithm (see Chapter 4 for details). On the other hand, the universal performance bound (solid line in
Fig. 1.2) represents the theoretical minimum for $P_{\text{line}}$ as a function of $P_{\text{cloud}}$, with no constraints on $P_{\text{comp}}$. Thus this bound is the lower boundary for all cross-section curves: points below this boundary are not feasible for the given (polyphase) source voltage and impedance.

## 1.4 Summary of contributions

Keeping in mind the central goals stated previously in this chapter, our contributions can be summarized as follows:

- We have developed an adaptive near-optimal compensator —one that achieves the smallest possible near-optimal $P_{\text{line}}$ for prescribed $P_{\text{cloud}}$ and $P_{\text{comp}}$ values (most typically, $P_{\text{comp}} = 0$).

- Our adaptive compensation scheme relies only on measurements of the voltage and current load without requiring prior knowledge of any other system parameter (e.g., source voltage, line impedance) which makes possible an on-site application of the compensator, nearby the load.

- With the implementation of the quad-Fryze compensation we have been able to obtain a mapping of the scaling coefficients $\alpha$, $\beta$ into the $P_{\text{comp}}-P_{\text{cloud}}$ plane which represents a key tool in the control analysis of the desired $P_{\text{comp}}$, $P_{\text{cloud}}$ values, leading with this to a better understanding of the problem.

- Despite the non-negligibility condition of the source impedance (which implies a certain voltage drop across the line) it is still possible to obtain very good results in the performance of the quad-Fryze compensator as compared with the optimal solution.

- The algorithm developed is able to carry out an adaptive adjustment of the parameters $\alpha$ and $\beta$ so as to satisfy the constraints imposed on $P_{\text{comp}}$ and $P_{\text{cloud}}$, respectively. In particular, in the adaptive adjustment of $\beta$ it is applied a new method by exploiting the existent monotone increasing behavior between $P_{\text{cloud}}$ and $\beta$ which resembles in some aspects the so-called Newton method.
Chapter 2

Optimal Compensation with Significant Source Impedance

In order to evaluate the performance of our adaptive compensation scheme in the presence of non-negligible source impedance, we shall use the concept of optimal performance curves, originally introduced in [2]. The first step towards obtaining such curves is to derive explicit expressions for the three criteria of interest — $P_{\text{line}}$, $P_{\text{cloud}}$ and $P_{\text{comp}}$ — as a function of the source current $I_s$. After completing this fundamental step we can proceed to use such expressions in the analysis of the optimization problem involved here so that the final result allows us to construct an optimal performance surface and its related cross section curves which are necessary in carrying out the evaluation performance process. The following sections will lead us to achieve the aforementioned objectives.

2.1 Universal performance bound

The best possible tradeoff between $P_{\text{cloud}}$ and $P_{\text{line}}$ can be determined by means of a characterization of the ideal load in terms of a universal performance bound. This universal bound will depend only on the polyphase source voltage and impedance with no assumptions about the load characteristics.
The power dissipated in the source impedance \( P_{\text{line}} \) and the power delivered to the compensated load \( P_{\text{cloud}} \) can be related by means of a modified Cauchy-Schwarz inequality. To do that, first of all let us manipulate the inner product \( \langle i_s, v_s \rangle = \Re\{I_s V_s^H\} \) as follows:

\[
\Re\{I_s V_s^H\} = \Re\{(I_s R_{1/2} s)(V_s R_{s}^{-H/2} s)^H\} \tag{2.1}
\]

where \( R_{1/2} s \) is any (square) matrix such that \( (R_{1/2} s)(R_{1/2} s)^H = R_s \) (for instance, a Cholesky factor).

By applying the Cauchy-Schwarz inequality to (2.1) we have

\[
\Re\{I_s V_s^H\} \leq \|V_s R_{s}^{-H/2} s\| \|I_s R_{1/2} s\| \tag{2.2}
\]

so that from (1.11c) and (1.12) we recall that \( P_{\text{line}} = \Re\{I_s R_{s} I_s^H\} = \|I_s R_{1/2} s\|^2 \) and \( P_{\text{cloud}} = \Re\{I_s V_s^H\} - P_{\text{line}} \), respectively. Thus we can rewrite (2.2) in the following form

\[
P_{\text{cloud}} \leq \sqrt{V_s R_{s}^{-1} V_s^H} \cdot \sqrt{P_{\text{line}} - P_{\text{line}}} \tag{2.3}
\]

As explained in [2], the inequality (2.3) defines a region of feasible \( (P_{\text{line}}, P_{\text{cloud}}) \) pairs (see solid line in Fig. 2.5) where the smallest possible value of \( P_{\text{line}} \) for a given \( P_{\text{cloud}} \) is given by the boundary point corresponding to \( P_{\text{cloud}} \), as well as the highest possible \( P_{\text{cloud}} \) for a given \( P_{\text{line}} \) is given by the boundary point corresponding to \( P_{\text{line}} \), where both conditions have no constraints imposed on \( P_{\text{comp}} \).

### 2.2 Optimal performance surface

The basic objective of compensation is to reduce the power loss in the line. However, as we vary \( i_{\text{comp}}(t) \) to achieve this effect, both the power delivered to the load and the power supplied by the compensator vary as well. In addition, we may wish to impose various constraints on the currents \( i_{\text{comp}}(t) \) and \( i_s(t) \) (e.g., bandwidth), and possibly on the harmonics of the instantaneous compensator power. To these we should also add the cost of storage (i.e., capacitor cost) and possibly the separate cost of losses in the neutral wire.
Here we shall focus on the three quantities, $P_{\text{line}}$, $P_{\text{comp}}$, and $P_{\text{cloud}}$, and we shall formulate our single-criterion/two-constraint compensation problem [2] as follows: “Given a polyphase source voltage $v_s(t)$, a linear time-invariant (polyphase) source impedance $Z_s$ and a load current-voltage characteristic, determine the smallest possible $P_{\text{line}}$ and the corresponding optimized $i_s(t)$, for prescribed values of $P_{\text{cloud}}$ and $P_{\text{comp}}$.”

For our purpose, it is useful to define the quantity $P_{\text{nom}}$ as the nominal power, i.e., the power that would be delivered to the load if the source was connected directly to it (without a line impedance).

It will be convenient to use dimensionless variables obtained by suitable normalization. Thus we shall use $P_{\text{nom}}$ as a normalizing factor for power, so that

$$\frac{P_{\text{line}}}{P_{\text{nom}}} = \frac{I_s R_s T_{\text{I}}^H}{P_{\text{nom}}} = \mathcal{X} \mathcal{X}^H \quad (2.4a)$$

where we have introduced the normalized current phasor array, viz.,

$$\mathcal{X} \triangleq \frac{1}{\sqrt{P_{\text{nom}}}} I_s R_s^{1/2} \quad (2.4b)$$

and, similarly, from (1.12) we have

$$\frac{P_{\text{cloud}}}{P_{\text{nom}}} = \mathcal{X} \xi^H + \xi \mathcal{X}^H - \mathcal{X} \mathcal{X}^H \quad (2.4c)$$

where

$$\xi \triangleq \frac{1}{2\sqrt{P_{\text{nom}}}} V_s R_s^{-H/2} \quad (2.4d)$$

Upon examining (2.4a) and (2.4c), it is evident that the cost and the constraint on $P_{\text{cloud}}$ are both quadratic in $\mathcal{X}$.

Now we turn to express the constraint on $P_{\text{comp}}$. To do that, we need first to characterize the current-voltage relation of the load. When $Z_s = 0$ this is not needed, since the introduction of a compensator does not alter the voltage on the load (and therefore, $i, v$, and the power delivered to the load all remain unchanged). However, once the voltage drop across the line impedance is not negligible we need to specify the load to allow us to solve the circuit equations (1.8).
It is important to note that when the load is nonlinear (i.e., the relation between $V$ and $I$ is nonlinear), the constraint on $P_{\text{comp}}$ is also nonlinear, as observed in (1.13). This means the optimal compensator cannot be determined analytically, in general, though it can still be determined by numerical calculation.

Taking into account the linear characteristic of the load (unless otherwise stated, this will be the assumption in the rest of the thesis) given by (1.9) we can define $P_{\text{nom}}$ as

$$P_{\text{nom}} \overset{\text{def}}{=} V_s G V_s^H$$

(2.5a)

where

$$G \overset{\text{def}}{=} \frac{1}{2} (\mathcal{Y} + \mathcal{Y}^H).$$

(2.5b)

Notice that the matrix $G$ is complex valued, except when $\mathcal{Y}$ is diagonal, which occurs for a linear time-invariant load without (magnetic) phase coupling. When phase currents are coupled, as is the case of an electric motor, the matrix $G$ is block diagonal, with each block representing a single harmonic (as in the case study analyzed in this thesis). Then, from (1.13) and (2.5) we can deduce that

$$\frac{P_{\text{comp}}}{P_{\text{nom}}} = \mathcal{X} \mathcal{A} \mathcal{X}^H - \mathcal{X} \eta^H - \eta \mathcal{X}^H + 1$$

(2.6a)

where

$$A \overset{\text{def}}{=} I + (R_s^{-1/2} Z_s) G (R_s^{-1/2} Z_s)^H$$

(2.6b)

and

$$\eta \overset{\text{def}}{=} \frac{1}{\sqrt{P_{\text{nom}}}} V_s \left[ \frac{1}{2} I + G Z_s^H \right] R_s^{-H/2}.$$

(2.6c)

Thus we have a quadratically-constrained, quadratic-cost optimization problem, viz.,

$$\begin{align*}
\min \mathcal{X} \mathcal{X}^H \\
\text{subject to} \\
\frac{P_{\text{load}}}{P_{\text{nom}}} - \mathcal{X} \xi^H - \xi \mathcal{X}^H + \mathcal{X} \mathcal{X}^H = 0 \\
\mathcal{X} \mathcal{A} \mathcal{X}^H - \mathcal{X} \eta^H - \eta \mathcal{X}^H + 1 - \frac{P_{\text{comp}}}{P_{\text{nom}}} = 0,
\end{align*}$$

(2.7)
which can be solved by using the Lagrange multiplier method as follows

$$\frac{\partial}{\partial \chi^H} \left\{ \chi \chi^H + \nu \left[ \frac{P_{\text{cloud}}}{P_{\text{nom}}} - \chi \xi^H - \xi \chi^H + \chi \chi^H \right] + \lambda \left[ \chi A \chi^H - \chi \eta^H - \eta \chi^H + 1 - \frac{P_{\text{comp}}}{P_{\text{nom}}} \right] \right\} = 0$$

(2.10)

resulting in

$$\chi_{\text{opt}} - \nu_{\text{opt}} \xi + \nu_{\text{opt}} \chi_{\text{opt}} + \lambda_{\text{opt}} \chi_{\text{opt}} A - \lambda_{\text{opt}} \eta = 0,$$

(2.11)

so that

$$\chi_{\text{opt}} = \left( \nu_{\text{opt}} \xi + \lambda_{\text{opt}} \eta \right) \left[ (1 + \nu_{\text{opt}}) I + \lambda_{\text{opt}} A \right]^{-1}.$$  

(2.12)

In order to implement the optimal compensator (for prescribed $P_{\text{cloud}}$ and $P_{\text{comp}}$) we need to solve for the values of the Lagrange multipliers $\nu_{\text{opt}}$ and $\lambda_{\text{opt}}$. This involves solving a nonlinear set of equations, which can only be done numerically. Observe that $\chi_{\text{opt}}$ relies on two degrees of freedom, which makes it possible to satisfy the two desired constraints.

As mentioned in [2], the relationship between the pre-specified values of $P_{\text{comp}}$, $P_{\text{cloud}}$ and the corresponding optimal value of $P_{\text{line}}$ delimits an optimal performance surface (OPS) in the three-dimensional criterion space. A segment of the corresponding OPS is presented in Fig. 2.1. It is observed that there are two special performance curves that lie on the OPS. One of them (black dotted curve) is obtained by ignoring the constraint on $P_{\text{comp}}$, i.e., by setting $\lambda = 0$ in (2.12). Thus the projection of this curve on the $P_{\text{line}}-P_{\text{cloud}}$ plane corresponds to the universal performance bound (see solid line in Fig. 2.5). It is explained because upon setting $\lambda = 0$ in (2.12) we arrive to

$$\chi_{\text{opt}} = \frac{\nu_{\text{opt}} \xi}{(1 + \nu_{\text{opt}})},$$

(2.13)

so that by combining (2.13) with (2.4b) and (2.4d) results in

$$\frac{\nu_{\text{opt}} \xi}{(1 + \nu_{\text{opt}})} = \frac{1}{\sqrt{P_{\text{nom}}}} T_s R_s^{1/2}$$

(2.14a)

$$\left[ \frac{\nu_{\text{opt}}}{(1 + \nu_{\text{opt}})} \right] \left[ \frac{1}{2 \sqrt{P_{\text{nom}}}} V_s R_s^{-H/2} \right] = \frac{1}{\sqrt{P_{\text{nom}}}} T_s R_s^{1/2}$$

(2.14b)
Chapter 2. Optimal Compensation with Significant Source Impedance

\[
\left[ \frac{\nu_{opt}}{2(1 + \nu_{opt})} \right] = \alpha V_s R_s^{-1} = I_s \tag{2.14c}
\]

\[
\alpha V_s R_s^{-1} = I_s. \tag{2.14d}
\]

Then it is observed from (2.14d) that there exist a collinearity relation between \(I_s R_s^{1/2}\) and \(V_s R_s^{-H/2}\) (see also [2], eq. 9) which happens when the points on the universal performance bound are achieved (recall (2.2)).

![Figure 2.1: Optimal performance surface (OPS).](image)

Also, upon removing the constraint on the value of \(P_{cloud}\) (i.e., setting \(\nu = 0\)) we can control the power flow through the compensator by using a single design parameter which has already been studied in detail in [3]. The resultant curve corresponds to the green dotted curve that we can observe in Fig. 2.1.

Both curves, \(\lambda = 0\) and \(\nu = 0\), can be distinguished in a better way from the top view of the OPS as shown in Fig. 2.2 where we can also appreciate the relationship between \(P_{comp}\) and \(P_{cloud}\).
2.3 Cross-section curves

Our interest here is to use the optimal compensator solution as a benchmark for comparison with the proposed adaptive scheme. Therefore, this comparison should be made over a whole range of \((P_{\text{cloud}}, P_{\text{comp}})\) values. In such a way, our main objective is to determine cross-section curves corresponding to fixed values of \(P_{\text{comp}}\) which motivates our preference for 2-D curves (easier way to view all 3 measures of power in a single 2-D plot), as in Fig. 2.5, instead of using the 3-D OPS. A cross-section curve defines the smallest possible value of \(P_{\text{line}}\), as a function of \(P_{\text{cloud}}\), for all compensators that achieve a prescribed value of \(P_{\text{comp}}\).

The cross-section curve is obtained by finding all the \((\nu, \lambda)\) pairs that satisfy the constraint

\[
X_{\text{opt}} A X_{\text{opt}}^H - 2 \Re \{X_{\text{opt}} \eta_{\text{opt}}^H\} + 1 = \frac{P_{\text{comp}}}{P_{\text{nom}}}.
\]  

(2.15)

One way to accomplish that is to determine \(P_{\text{line, opt}}\) as a function of \(P_{\text{cloud}}\), with \(P_{\text{comp}}\)
held constant (i.e., treated as a parameter). We achieved that by using a polar representation of the Lagrange multipliers (see Fig. 2.3), viz.,

\[ \nu = \rho \sin \phi \]  
\[ \lambda = \rho \cos \phi \]  

and searching for the prescribed value of \( P_{\text{comp}} \) by varying \( \rho \) for a fixed value of \( \phi \) (see Fig. 2.4). This results in a set of \((\nu_{\text{opt}}(\phi), \lambda_{\text{opt}}(\phi))\) pairs representing a given constant value for \( P_{\text{comp}} \), for example, say \( P_{\text{comp}} = 0 \). When \( P_{\text{line}} \) and \( P_{\text{cloud}} \) are evaluated using these \( (\nu_{\text{opt}}(\phi), \lambda_{\text{opt}}(\phi)) \) pairs of values we get a curve representing \( P_{\text{line,opt}} \) as a function of \( P_{\text{cloud}} \) for a fixed \( P_{\text{comp}} \) (See Fig. 2.5). This can be repeated for several useful values of \( P_{\text{comp}} \).

![Polar representation of the Lagrange multipliers.](image)

To illustrate, we have considered the case of a three-phase induction machine rated at 25kW, supplied from an unbalanced source (variations of roughly 10% in magnitude and phase, with no zero sequence) that contains the fundamental and the fifth harmonic. For simplicity, we assume that phases are not coupled in the line. The numerical values for this example are the same as in [3] (see Appendix C), where the fundamental
harmonic values are taken from [6], the rotor-related parameters are estimated for the 5-th harmonic using [7] and the induction motor admittance matrix is computed as in [8].
Notice that the cross-section curves in Fig. 2.5 define a lower bound that separates feasible \( (P_{\text{line}}, P_{\text{cloud}}) \) values from those that cannot be achieved. This boundary curve (solid line in Fig. 2.5) corresponds to the smallest \( P_{\text{line}} \) that can be achieved for a prescribed \( P_{\text{cloud}} \), with no constraints imposed to the value of \( P_{\text{comp}} \), as mentioned before in Section 2.1. Another important observation is that the curve representing \( \nu = 0 \) corresponds to the collection of minimum points of the cross-section curves as shown in Fig. 2.5 (dashed line).

Furthermore, Fig. 2.5 allows us to obtain the best trade-off between \( P_{\text{line}} \) and \( P_{\text{cloud}} \) achievable for the compensator whether it is supplying power, consuming power, or working under lossless conditions. For instance, observe that the smallest \( P_{\text{line}} \) achievable with a lossless compensator (dotted line in Fig. 2.5) is 1,418 W and the corresponding \( P_{\text{cloud}} \) is 18,660 W. If we attempt to supply the nominal power \( (P_{\text{nom}}) \) to the load, which in our example corresponds to \( P_{\text{cloud}} = 22,750 \text{W} \), then it is necessary to compromise on the power loss in the source impedance, i.e., \( P_{\text{line}} \) has to be increased to 2,376 W. Moreover, observe that for all cross-section curves shown in Fig. 2.5 their corresponding smallest \( P_{\text{line}} \) are achieved with \( P_{\text{cloud}} < P_{\text{nom}} \), so that increasing \( P_{\text{comp}} \) will give us a lower smallest \( P_{\text{line}} \), but push us even further from \( P_{\text{nom}} \), e.g., 2,000 W of \( P_{\text{comp}} \) give us a reduction of 250 W in the smallest \( P_{\text{line}} \) with respect to the lossless compensator and push \( P_{\text{cloud}} \) down by about 1,600 W. If we now bring \( P_{\text{cloud}} \) up to 19,000 W (intersection point with the \( P_{\text{comp}} = 0 \) curve) we get the same working point achieved with \( P_{\text{comp}} = 0 \) but no gain have been obtained because we invested 2,000 W in \( P_{\text{comp}} \).

Finally, it is important to indicate the fact that evaluation of cross-section curves requires complete characterization of the load and is computationally-intensive even more for a nonlinear load. We will show in Chapter 3 how to achieve any point on a determined cross-section curve by varying a single control parameter \( (\beta) \).
Chapter 3

Quad-Fryze Compensation

Our proposed adaptive compensation scheme is based on the concept of quad-Fryze compensation [3] which is an extension of the classical Fryze in (1.1) that works well when $Z_s$ is very small. Here we shall demonstrate two basic characteristics of the quad-Fryze compensation: (1) It has 2 degrees of freedom which allow us to control $P_{comp}$, $P_{cloud}$, and (2) It is nearly optimal when $Z_s$ is not too large.

3.1 Fundamental relations

A quad-Fryze compensator is one that achieves

$$i_s(t) = \alpha v(t) - \beta \mathcal{H}\{v(t)\},$$

for some constants $\alpha$, $\beta$.

In the frequency domain this translates into

$$I_s = \gamma V$$

(3.2a)

where

$$\gamma = \alpha + j\beta.$$  

(3.2b)
Chapter 3. Quad-Fryze Compensation

This means that the compensated polyphase load is linear, time-invariant and balanced with no coupling between phases, and with a star-connected frequency-independent admittance equal to $\gamma$. When $\beta = 0$, this is simply a star-connected balanced, resistive load which is indeed the ideal load in situations where $\|V_s - V\| \ll \|V_s\|$. On the other hand, allowing for $\beta \neq 0$ makes it possible to “fine-tune” the quad-Fryze compensator in the presence of a non-negligible (but not excessively large) line impedance.

The steady-state phasor relation (3.2) implies, in conjunction with (1.8a), that the voltage across the load is given by

$$\mathcal{V} = V_s - \mathcal{I}_sZ_s$$

$$= V_s - \gamma \mathcal{V}Z_s$$

$$= V_s(I + \gamma Z_s)^{-1}$$

for the most general load (including nonlinear ones). Notice from (3.2) and (3.3) that $\mathcal{I}_s$, and consequently $\mathcal{V}$, relies on two degrees of freedom ($\alpha$ and $\beta$), which makes it possible to satisfy the two desired constraints on $P_{comp}$ and $P_{cloud}$.

The values of $P_{line}$, $P_{cloud}$ and $P_{comp}$ associated with a quad-Fryze compensator can be evaluated via (3.2) – (3.3) and the expressions (1.11) – (1.13). Thus

$$P_{line} = \Re\{\mathcal{I}_s \mathcal{R}_s \mathcal{I}_s^H\} = |\gamma|^2 \mathcal{V}_R \mathcal{V}^H,$$

$$P_{cloud} = \Re\{\mathcal{I}_s \mathcal{V}^H\} = \alpha \|\mathcal{V}\|^2$$

for any load. Also, for a linear load,

$$P_{comp} = \Re\{\mathcal{I}_{comp} \mathcal{V}^H\} = \Re\{(I - \mathcal{I}_s) \mathcal{V}^H\}$$

$$= \mathcal{V}_G \mathcal{V}^H - \alpha \|\mathcal{V}\|^2.$$

It is observed that all three powers are functions of $\alpha$, $\beta$, in part because of the dependence of $\mathcal{V}$ on these two parameters.
3.2 Relation between \((P_{\text{comp}}, P_{\text{cloud}})\) and \((\alpha, \beta)\)

There is a connection between constants \(\alpha, \beta\) and prescribed \(P_{\text{comp}}, P_{\text{cloud}}\) values that allows us to understand how these desired values can be controlled by using those constants. This relation can be viewed in Fig. 3.1 where the steady state values of \(P_{\text{comp}}\) and \(P_{\text{cloud}}\) have been mapped from different combinations of constants \(\alpha, \beta\), by keeping one of them fixed while varying the other one. Clearly it is observed that upon increasing \(\beta\) for a fixed value of \(\alpha\) then \(P_{\text{cloud}}\) also increases. However, there is a particular value of \(\alpha\) (around 0.38) which functions as a reference in determining whether \(P_{\text{comp}}\) is going to increase or decrease with the variation of \(\beta\) (actually the line \(\alpha = 0.38\) very nearly corresponds to \(P_{\text{comp}} = 0\)). Indeed, the effect of \(\beta\) on \(P_{\text{comp}}\) (for fixed \(\alpha\)) is minimal as long as the desired \(P_{\text{comp}}\) is close to zero. On the other hand, we can note that for a fixed \(\beta\) if \(\alpha\) increases then \(P_{\text{cloud}}\) increases while \(P_{\text{comp}}\) decreases until a value of zero (lossless compensator); after that, the compensator turns to consume power from the system.

\[
\text{Figure 3.1: Mapping of } \alpha, \beta \text{ into } P_{\text{comp}}-P_{\text{cloud}} \text{ plane.}
\]

In particular, in a single-phase system with sinusoidal excitation and a linear load we find that \(P_{\text{comp}} = (g - \alpha)|V|^2\), where "\(g\)" is the load conductance and "\(V\)" is the phasor associated with the load voltage (\(G\) is now a real scalar "\(g\)" and \(V\) is a complex scalar "\(V\)"), so that \(P_{\text{comp}} = 0\) is achieved by setting \(\alpha = g\), regardless of the value of \(\beta\). This
Chapter 3. Quad-Fryze Compensation

is no longer exactly true for the polyphase and non-sinusoidal case, but our example shows that we still have $\alpha \approx \text{constant}$ for $P_{\text{comp}} = 0$ ($\alpha = 0.38$ in Fig. 3.1). Another relation is given by $P_{\text{cloud}} = \alpha|V|^2$ so $\frac{P_{\text{comp}}}{P_{\text{cloud}}} = \left(\frac{2}{\alpha} - 1\right)$, and lines of “constant $\alpha$” should be all intersecting at the origin $P_{\text{comp}} = 0 = P_{\text{cloud}}$. This is indeed very nearly true in our polyphase non-sinusoidal example (Fig. 3.1) where lines of constant $\alpha$ are almost straight lines, all appearing to emanate from the origin.

These results suggest that we should use $\alpha$ to control $P_{\text{comp}}$, at least for $P_{\text{comp}} = 0$ or near zero. Once the constraint of $P_{\text{comp}}$ has been satisfied, we can use $\beta$ to control $P_{\text{cloud}}$, again as is evident from Fig. 3.1. In particular, we notice that it is possible to adjust $\beta$ so that $P_{\text{cloud}} = P_{\text{nom}} = 22,750\,\text{W}$ while $P_{\text{comp}} = 0$.

### 3.3 Near optimality of quad-Fryze

The quad-Fryze compensator becomes optimal (with $\beta = 0$) when $\frac{\|V_s - V\|}{\|V_s\|} \downarrow 0$. However, we shall demonstrate here that it is very nearly optimal even in the presence of a non-negligible line impedance.

The expressions (3.4) allow us to quad-Fryze equivalent of the cross-section curves of Fig. 2.5, i.e., the $P_{\text{line}}$-$P_{\text{cloud}}$ tradeoff achieved by a quad-Fryze compensator with a prescribed value of $P_{\text{comp}}$. The results for our induction machine example indicate that the performance of the quad-Fryze compensator is almost indistinguishable from the theoretical optimum (see Fig. 3.2, dashed and solid curves represent quad-Fryze approaches while dotted and dash-dot curves represent optimal approaches), i.e., less than three percent of difference between quad-Fryze and optimal $P_{\text{line}}$ values when $0.60P_{\text{nom}} \leq P_{\text{cloud}} \leq P_{\text{nom}}$ for all $P_{\text{comp}}$. Notice that the best match between quad-Fryze and optimal curves is around the minimum point of the cross-section curves.

The quality of the “match” between quad-Fryze and optimal performance depends on how “non-negligible” $Z_s$ is. This negligibility is evaluated through the voltage drop index $\frac{\|V_s - V\|}{\|V_s\|}$, which in our example is around 16%. It is important to indicate that despite this significant value of voltage drop (reasonable values are less than 5%) we have obtained very good results in the performance of the quad-Fryze compensation.
Figure 3.2: Comparison between a quad-Fryze compensator solution versus an optimal solution.

The plotting of Fig. 3.1, and the choices for $\alpha$ and $\beta$ values rely on specific information about network and load parameters, which makes a direct implementation of the quad-Fryze compensator rather impractical. The basic challenge is to implement the quad-Fryze compensator, i.e., to find $I_{\text{comp}}$ such that $I_s = \gamma V$ is achieved, without knowing $Z_s$ or the load characteristics. We can rely however on our ability to measure $I$, $V$ whenever needed. In the following section we present an adaptive version of the quad-Fryze compensator that uses only $I$, $V$. 
Chapter 4

Adaptive Near-Optimal Compensation with Significant Source Impedance

In this chapter we identify and discuss four fundamental issues to be considered for constructing an adaptive implementation of the quad-Fryze compensator that converges, in steady state, to a near-minimal $P_{\text{line}}$ for prescribed values of $P_{\text{comp}}$ and $P_{\text{load}}$.

Our adaptive algorithm relies on repeated application of the optimized solution for the negligible impedance case, i.e., the ideal case in which changes in $I_{\text{comp}}$ do not change load conditions ($I$, $V$ remain unchanged). This means we select $I_{\text{comp}} = I - \gamma V$ so that, with negligible voltage drop in the line

$$I_s = I - I_{\text{comp}} = I - (I - \gamma V) = \gamma V$$  \hspace{1cm} (4.1)

and quad-Fryze is achieved in a single step. However, under non-ideal conditions, the voltage drop in the line causes the load voltage and current to change in response to the introduction of $I_{\text{comp}}$. As a result, the new source current is

$$I_s(1) = I(1) - I_{\text{comp}}(1) = I(1) - [I(0) - \gamma V(0)]$$

$$\neq \gamma V(1).$$  \hspace{1cm} (4.2)
Chapter 4. *Adaptive Near-Optimal Compensation*

We now treat the partially compensated load as a target for the same procedure, i.e., we add another compensator, with the incremental current

\[
\Delta I_{\text{comp}}(2) = I_s(1) - \gamma V(1) \\
= [I(1) - I_{\text{comp}}(1)] - \gamma V(1).
\]

Notice that this is equivalent to setting the total compensator current to

\[
I_{\text{comp}}(2) = I_{\text{comp}}(1) + \Delta I_{\text{comp}}(2) \\
= I(1) - \gamma V(1)
\]

and we can repeat the process again and again. Thus, our procedure consists of repeated iteration of the following two steps:

- Given the current and voltage of the load, \( \{I(k), V(k)\} \), set the compensator current to
  \[
  I_{\text{comp}}(k + 1) = I(k) - \gamma V(k).
  \]
- Wait until the network transients have died off, and the load has stabilized at a new current-voltage pair, \( \{I(k + 1), V(k + 1)\} \).

The iteration (4.5) generates a sequence \( (I(k), V(k)) \) of load currents and voltages, that converges (under mild technical constraints) to an equilibrium point of (4.5). This equilibrium is

\[
I_s(\infty) = I(\infty) - I_{\text{comp}}(\infty) = \gamma V(\infty).
\]

so that our objective of implementing quad-Fryze compensation, for a prescribed value of \( \gamma \), is achieved.

In order for our procedure to work properly we need to address the following questions:

- What is the range of \( \gamma \)-values for which the iteration (4.5) converges ?
- How to select values for \( \alpha = \Re(\gamma) \) and \( \beta = \Im(\gamma) \), so that the desired values of \( P_{\text{comp}} \) and \( P_{\text{cloud}} \) are achieved in steady state ?
- How to avoid excessive fluctuations of \( P_{\text{comp}} \) and \( P_{\text{cloud}} \) during the adaptation process ?
4.1 Equilibrium and convergence

The significance of (4.6) is that the map

\[ V(k) \rightarrow I_{\text{comp}}(k + 1) \rightarrow V(k + 1) \]

which is defined by (4.5) and the natural response of the network, has a single equilibrium point (given by (4.6)). We know this equilibrium is unique because, in equilibrium \( I_s = \gamma V \) and there is only one choice of \( V \), given by (3.3), that satisfies \( I_s = \gamma V \). Recall that according to Fig. 3.2, the resulting \( I_s \) is very nearly optimal.

We now turn to discuss the convergence of our adaptive compensator. For simplicity, we shall focus here on the linear load case, \( I = VY \). As mentioned before, setting \( I_{\text{comp}} \) results in a change of the voltage load and current which is governed by the circuit equations (1.8) in conjunction with (1.9), so that once the transients have died off we obtain

\[
V = V_s - (I - I_{\text{comp}})Z_s = V_s - (VY - I_{\text{comp}})Z_s = (V_s + I_{\text{comp}}Z_s)(I + YZ_s)^{-1};
\]

(4.7)

where it is evident that the natural response of the network maps every given \( V_s \) and \( I_{\text{comp}} \) pair into a load voltage \( V \).

Using the linear load relation \( I = VY \), we can deduce from the iteration (4.5) that the compensator current during the adaptation process is given by

\[
I_{\text{comp}}(k + 1) = V(k)[Y - \gamma I].
\]

(4.8)

Combining the compensator current setting (4.8) with the network response (4.7) results in a new load voltage

\[
V(k + 1) = V_s(I + YZ_s)^{-1} + V(k)A,
\]

(4.9a)
where $\mathcal{A}$ is a shorthand notation, viz.,

$$
\mathcal{A} \overset{\text{def}}{=} (\mathcal{Y} - \gamma I)\mathcal{Z}_s (I + \mathcal{Y}\mathcal{Z}_s)^{-1}.
$$

(4.9b)

The equilibrium point is given by (recall (3.3))

$$
\mathcal{V}(\infty) = \mathcal{V}_s (I + \gamma \mathcal{Z}_s)^{-1}
$$

(4.10)

which allows us to rewrite (4.9a) in the form

$$
\mathcal{V}(k + 1) = \mathcal{V}(\infty) - \mathcal{V}(\infty)\mathcal{A} + \mathcal{V}(k)\mathcal{A},
$$

(4.11)

where we used the identity

$$
\mathcal{V}(\infty)[I - \mathcal{A}] = \mathcal{V}_s (I + \mathcal{Y}\mathcal{Z}_s)^{-1}
$$

(4.12)

which follows from (4.10) and the observation

$$
I - \mathcal{A} = [(I + \mathcal{Y}\mathcal{Z}_s) - (\mathcal{Y} - \gamma I)\mathcal{Z}_s][I + \mathcal{Y}\mathcal{Z}_s]^{-1} = (I + \gamma \mathcal{Z}_s)(I + \mathcal{Y}\mathcal{Z}_s)^{-1}.
$$

(4.13)

Equivalently, we define the load voltage error as

$$
\epsilon(k) = \mathcal{V}(k) - \mathcal{V}(\infty),
$$

(4.14)

so that (4.11) reduces to the difference equation

$$
\epsilon(k + 1) = \epsilon(k)\mathcal{A}.
$$

(4.15)

Now from (4.15) it is observed that $||\epsilon(k)|| \to 0$ if, and only if, all the eigenvalues of $\mathcal{A}$ are within the unit circle, namely, $\max \{ |\lambda_i(\mathcal{A})| \} < 1$. Thus, for the case when $\gamma$ is fixed, our convergence analysis reduces to the examination of the eigenvalues of the constant matrix $\mathcal{A}$. In particular, according to the values of $\mathcal{Y}$ and $\mathcal{Z}_s$ from our example we can explore this “convergence region” in terms of $\alpha$, $\beta$ as shown in Fig. 4.1. It is also observed that for $0.30 \leq \alpha \leq 0.46$ and $-0.50 \leq \beta \leq 0.30$, which are the range of
values used to determine the mapping of Fig. 3.1, we have guaranteed convergence of the recursion as expected.

![Diagram](image)

**Figure 4.1:** Convergence region in terms of $\alpha$, $\beta$ for $\max \{|\lambda_i(A)|\} < 1$.

The value of $\gamma$ determines the steady state of $I$, $V$, $I_s$, etc., so that it also determines $P_{\text{clad}}$, $P_{\text{comp}}$ (and $P_{\text{line}}$) — recall (3.4). On the other hand, from the analysis in Chapter 3 (see Fig. 3.1) it appears that we can achieve $P_{\text{comp}} = 0$ by adjusting $\alpha$ alone which is also our first priority. So our next step is to propose an adaptive adjustment of $\alpha$ that will produce a prescribed value of $P_{\text{comp}}$ (most often $P_{\text{comp}} = 0$). Once $\alpha$ has been determined we can turn to obtain a desired value of $P_{\text{clad}}$ by adjusting $\beta$. During this process we can rely only on relations that involve $I$, $V$ (and $\gamma$), but nothing else.

### 4.2 Adaptive adjustment of $\alpha$

We observe from (1.13) and (3.4b) that

$$P_{\text{clad}} = \alpha \| V \|^2 = \Re \{ T V^H \} - P_{\text{comp}}$$  \hspace{1cm} (4.16)

which can be used either to enforce a prescribed value of $P_{\text{clad}}$ or of $P_{\text{comp}}$. However, as we mentioned before, our first priority is to enforce $P_{\text{comp}}$, so it motivates us to use,
in our iteration (4.5), \( \gamma(k) = \alpha(k) + j \beta \), where

\[
\alpha(k) = \Re\{\gamma(k)\} = \frac{\Re\{I(k)V^H(k)\} - P_{\text{comp, desired}}}{\|V(k)\|^2}.
\] (4.17)

This means that our adaptive algorithm is, in fact,

\[
I_{\text{comp}}(k + 1) = I(k) - (\alpha(k) + j \beta)V(k),
\] (4.18)

with \( \alpha(k) \) given by (4.17).

The equilibrium point for the iteration (4.18) is the same as for (4.5), namely a quad-Fryze compensator with \( I_s(\infty) = (\alpha(\infty) + j \beta)V(\infty) \). This is so because in equilibrium \( I(k) = I(\infty), V(k) = V(\infty), \alpha(k) = \alpha(\infty) \) remain unchanged by the iteration, so that

\[
I_{\text{comp}}(\infty) = I(\infty) - (\alpha(\infty) + j \beta)V(\infty)
\] (4.19a)

and

\[
I_s(\infty) = (\alpha(\infty) + j \beta)V(\infty),
\] (4.19b)

as stated.

Furthermore, in equilibrium we have (recall(4.17))

\[
\alpha(\infty) = \frac{\Re\{I(\infty)V^H(\infty)\} - P_{\text{comp, desired}}}{\|V(\infty)\|^2}.
\] (4.20)

Also from (1.13) and (3.4b) we deduce that

\[
P_{\text{comp}}(\infty) = \Re\{I(\infty)V^H(\infty)\} - \alpha(\infty)\|V(\infty)\|^2.
\] (4.21)

It follows from the last two equations that

\[
P_{\text{comp}}(\infty) = P_{\text{comp, desired}}.
\] (4.22)
The analysis of convergence for the iteration (4.18) is very similar to our analysis of (4.5). Instead of (4.9) we now obtain

\[ V(k + 1) = V_s(I + YZ_s)^{-1} + V(k)A(k), \]  
(4.23)

where

\[ A(k) \overset{\text{def}}{=} [Y - \gamma(k)I]Z_s(I + YZ_s)^{-1}. \]  
(4.24)

Next, we notice that the equilibrium point of this iteration is (compare with (4.10))

\[ V(\infty) = V_s(I + \gamma(\infty)Z_s)^{-1}, \]  
(4.25)

where we have assumed that \( \gamma(k) \to \gamma(\infty) \).

It follows that

\[ V(\infty)[I - A(\infty)] = V_s(I + YZ_s)^{-1} \]  
(4.26)

and thus

\[ V(k + 1) = V(\infty) - V(\infty)A(\infty) + V(k)A(k). \]  
(4.27)

Finally, we can conclude that

\[ \epsilon(k + 1) = \epsilon(k)A(k) + V(\infty)[A(k) - A(\infty)], \]  
(4.28)

where \( \epsilon(k) = V(k) - V(\infty) \) is the load voltage deviation from equilibrium, as before. When \( A(k) = A \) for all \( k \), this recursion reduces to (4.15).

Upon examining (4.28) we observe that this is a non-linear difference equation in \( \epsilon(k) \) because \( A(k) \) depends on \( \gamma(k) \), which in turn depends on \( V(k) \). It is much harder to analyze the convergence in this case because of the presence of a time-varying driving term, \( V(\infty)[A(k) - A(\infty)] \) in (4.28). Clearly a necessary condition for convergence is that \( A(k) \to A(\infty) \), or equivalently, that \( \gamma(k) \to \gamma(\infty) \). This is obviously also a necessary condition for the existence of an equilibrium point for the recursion (4.18). It appears that this condition is also sufficient.

However, it is not easy to verify if the condition \( \gamma(k) \to \gamma(\infty) \) holds, because of the nonlinear dependence of \( \alpha(k) = \Re \{ \gamma(k) \} \) on \( V(k) \). As a result, we do not have yet a convergence condition that can be verified apriori. We can, nevertheless, track
the difference $\gamma(k+1) - \gamma(k) \equiv \alpha(k+1) - \alpha(k)$ during the operation of our adaptive algorithm, and use it as a numerical indicator of convergence (or lack thereof).

### 4.3 Adjustment of $\beta$

When our adaptive algorithm (4.17) and (4.18) converges, $P_{\text{comp}}(\infty) = P_{\text{comp, desired}}$, as we have demonstrated in the previous section. On the other hand, $P_{\text{cloud}}(\infty)$ cannot be set within this recursion, and it clearly depends on the value of $\beta$ (recall Fig. 3.1). Thus a desired value of $P_{\text{cloud}}$ can be achieved by an appropriate selection of the value of $\beta$ used in (4.18). In this section we present an iterative search procedure that gradually modifies the parameter $\beta$ so as to achieve a prescribed value of $P_{\text{cloud}}$.

When the line and load parameters $Z_s$, $Y$ are known, we can determine the explicit relation between $P_{\text{cloud}}$ and $\beta$ (Fig. 4.2). Because this relation is smooth and monotone increasing, one can use a variety of search procedures to determine the correct $\beta$ for a desired $P_{\text{cloud}}$ value. Such procedures must, of course, rely only on measurable quantities such as $I_s$, $I$, and $V$.

From a mathematical standpoint, our problem is to solve the (nonlinear) equation $f(\beta) = P_{\text{cloud, desired}}$, where $f(\cdot)$ is the monotone increasing function shown in Fig. 4.3 (also in Fig. 4.2). This could be done, for instance, via a Newton-Raphson type procedure (http://en.wikipedia.org/wiki/Newton-Raphson). However, since we do not have a closed-form expression for this function, we need to rely on our ability to set $\beta$ at will, and to measure the resulting $P_{\text{cloud}}$, obtained when the adaptive procedure (4.18) has converged. As explained later in this section, we only need to keep track of changes in $P_{\text{cloud}} = \Re\{I_sV^H\}$, which is measurable.

We propose a procedure that relies on two such input-output pairs, say $\{\beta_0, P_{\text{cloud},0}\}$ and $\{\beta_1, P_{\text{cloud},1}\}$ to determine a new value for $\beta$, say $\beta_2$, so that the resulting $P_{\text{cloud},2}$ is closer $P_{\text{cloud,desired}}$ than either $P_{\text{cloud,0}}$ or $P_{\text{cloud,1}}$. To this end we use the two given input-output pairs (described by points 1 and 3 in Fig. 4.3) to determine a straight line connecting these points. This approximates the tangent line used in a Newton-Raphson type procedure. Next, we extend this line until it intersects (at point 4 in Fig. 4.3) the level $P_{\text{cloud,desired}}$ and determines the corresponding $\beta_2$ value. In summary,
we determine $\beta_k$ via the expression

$$\beta_{i+1} = \beta_i - \frac{\beta_i - \beta_{i-1}}{P_{\text{cloud},i} - P_{\text{cloud},i-1}} \left[ \bar{P}_{\text{cloud},i} - \bar{P}_{\text{cloud,desired}} \right],$$

(4.29)

where for computational convenience, we have replaced all $P_{\text{cloud}}$ values by their normalized equivalents, viz.,

$$\bar{P}_{\text{cloud}} = \frac{P_{\text{cloud}}}{P_{\text{nom}}}. \tag{4.30}$$

As a result, typical values for $\bar{P}_{\text{cloud}}$ are in the range $0.6 \leq \bar{P}_{\text{cloud}} \leq 1$. The expression given in (4.29) is well-known method, known as the “secant method” (http://en.wikipedia.org/wiki/Secant_method).

The recursion (4.29) is initialized with a choice of $\beta_0$ and $\beta_1$. A reasonable choice for $\beta_0$ would be $\beta_0 = 0$, since it tends to generate a near-optimal value for the line loss $P_{\text{line}}$ (see Fig. 4.4). The selection of $\beta_1$ has to be consistent with the monotone increasing nature of the curves in Fig. 4.2, so that $\beta_1 > \beta_0$ when $P_{\text{cloud,desired}} > P_{\text{cloud,0}}$, which is usually the case. One possible choice is to use a predetermined slope $m_0$ for the segment $(\beta_0, \beta_1)$ — we use $m_0 = 0.56$ — so that

$$\beta_1 = \beta_0 + \frac{\bar{P}_{\text{cloud,desired}} - \bar{P}_{\text{cloud,0}}}{m_0}. \tag{4.31}$$
Chapter 4. Adaptive Near-Optimal Compensation

Figure 4.3: Iterative adjustment of $\beta$ to get desired $\overline{P}_{\text{cloud}}$.

Now our procedure is completely specified: for each $\beta_i$ we run the adaptive algorithm (4.18) until convergence is achieved. The corresponding steady state $P_{\text{cloud}}$ value, viz., $P_{\text{cloud},i}(\infty) = \Re \{ \mathcal{I}_s(\infty) \mathcal{V}^H(\infty) \}$ is then used to determine $\beta_{i+1}$ via (4.29) for $i \geq 2$, or to determine $\beta_1$ via (4.31).

Table 4.1 shows the sequence of steady-state $\alpha$, $P_{\text{line}}$, $P_{\text{cloud}}$, $P_{\text{comp}}$ values associated with the $\beta_i$ for $i = 0, 1, 2, 3$. Notice that the steady state $\alpha_i(\infty)$ values are practically independent of the $\beta_i$ values: this is consistent with our observations in Section 3.2 about Fig. 3.1. The convergence trajectory starts under conditions of an uncompensated load (black point in Fig. 4.4): in the absence of the compensator, the power loss in the source impedance of our example is 1,944 W, and the power delivered to the load is 16,080 W.

Table 4.1: Values at the end of each segment of the trajectory (dotted line) in Fig. 4.4

<table>
<thead>
<tr>
<th>Equilibrium point</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$P_{\text{line}}$ (W)</th>
<th>$P_{\text{cloud}}$ (W)</th>
<th>$P_{\text{comp}}$ (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>uncompensated</td>
<td>—</td>
<td>—</td>
<td>1,944</td>
<td>16,080</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.383443</td>
<td>1,442</td>
<td>19,286</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.269715</td>
<td>0.382797</td>
<td>2,591</td>
<td>23,199</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.238770</td>
<td>0.382872</td>
<td>2,354</td>
<td>22,704</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.241666</td>
<td>0.382859</td>
<td>2,375</td>
<td>22,750</td>
<td>0</td>
</tr>
</tbody>
</table>
It should be understood that for every segment of the trajectory the $\beta$ value remains constant until the algorithm converges to the corresponding $P_{\text{cloud}}$ value as governed by the relations shown in Fig. 4.2. We then change $\beta$ (starting point for the next segment of the trajectory) to get closer to $P_{\text{cloud,desired}}$ according to the procedure explained before in this section. During this process, the value of $\alpha$ changes a little so as to get the exact $P_{\text{comp}}(\infty)$ value as seen in the second column of Table 4.1. It is also important to point out that $P_{\text{line}}$ achieves its near-optimal value in each segment only when $\alpha(k)$ has converged. During the transient stage of $\alpha(k)$ convergence, $P_{\text{line}}$ can be quite different from its desired value.

Plots in Fig. 4.4 provide an example for the performance of our adaptive near-optimal compensation method. Actually, the information shown in Table 4.1 can be extracted from Fig. 4.4 (dotted line). Here the dotted line simply connects the points obtained by the iterative algorithm. Notice that in steady-state $P_{\text{comp}} = 0$, so that all steady-state points lie on the quad-Fryze equivalent cross section curve corresponding to $P_{\text{comp}} = 0$ (see solid line in the zoomed-in view shown in Fig. 4.5). Recall we use the optimal solution (dash-dot line in Fig. 4.4 and Fig. 4.5) as a benchmark for comparison and due to the near-optimal characteristic of our quad-Fryze compensation it is observed that the steady-state points lie very nearly on the optimum cross section curve. Also notice that $\beta$ (and $P_{\text{cloud}}$) keep increasing during the initial $\beta$-search steps, but eventually oscillate up and down until convergence is achieved with $P_{\text{cloud}}(\infty) = P_{\text{cloud,desired}} = 22,750\, \text{W}$, where it is clear that our ability to enforce a desired value for $P_{\text{cloud}}$ (in this example $P_{\text{cloud,desired}} = P_{\text{nom}}$) comes at the expense of an increase in $P_{\text{line}}$.

Practical implementations of our adaptive algorithm may need to place a cap in the increase of $P_{\text{line}}$ above its minimal value.

### 4.4 Transient behavior

Our discussion so far focused on the steady-state achieved when the adaptive algorithm (4.18) converges, and on how to adjust the value of $\beta$ to control the steady state value of $P_{\text{cloud}}$. However, during the adaptation process for $\alpha(k)$, the value of $P_{\text{comp}}$ can be quite different from its desired (steady-state) value. In fact, all three power quantities — $P_{\text{line}}, P_{\text{comp}}, P_{\text{cloud}}$ — undergo a transient while $\alpha(k)$ is converging.
Figure 4.4: Convergence trajectory of the adaptive near-optimal compensator for $P_{\text{comp}}(\infty) = 0$, $P_{\text{cloud}}(\infty) = 22,750\text{W}$.

Figure 4.5: Zoomed-in view of the final convergence point for $P_{\text{comp}}(\infty) = 0$, $P_{\text{cloud}}(\infty) = 22,750\text{W}$. 
The transient in $P_{\text{comp}}$ and $P_{\text{cloud}}$ is shown in Fig. 4.6 (dashed line), for all the steps of the $\beta$-search process. Notice that while $P_{\text{cloud}}$ tends to increase (at least during the $\beta_0$ and $\beta_1$ steps), $P_{\text{comp}}$ experiences a dramatic overshoot as soon as the adaptation is turned on, and subsequently right after the value of $\beta$ is changed (from $\beta_0$ to $\beta_1$, from $\beta_1$ to $\beta_2$, etc.). This behavior is unacceptable because every $P_{\text{comp}}(k)$ value of the trajectory is not desired to be quite different from the prescribed $P_{\text{comp,desired}}$. This inconvenience is of significant importance in situations when $P_{\text{comp,desired}} = 0$ because, as we can observe in this particular example, there is a certain instant where the compensator is required to supply more than 1kW while we expect $P_{\text{comp}}$ values to be as close as possible to zero. This problem can be overcome by including additional temporary scaling for the value of $\gamma(k)$, but maintaining the recursion formula (4.18) as before, so that

$$
\gamma(k) = [1 + \epsilon(k)][\alpha(k) + j\beta],
$$

(4.32)

where $\epsilon(k) \to 0$ as $k \to \infty$.

**Figure 4.6:** Comparison of the trajectory with significant overshoots versus the effect of $\epsilon_0$ alone with $\epsilon_1 = 0$ in the trajectory

When the values of $Z_s$ and $Y$ are known in advance, the sequence $\epsilon(k)$ can be chosen to optimize the performance, i.e., reduce the “overshoots” and “undershoots” in terms of the difference between $P_{\text{comp}}(k)$ and $P_{\text{comp,desired}}$. 
From Fig. 3.1, it is intuitively plausible to think that to get $P_{\text{comp}} \downarrow$ we need to increase $\alpha$, hence $\epsilon(k) > 0$, especially at the beginning of the iteration process. We have found empirically that a reasonable choice of $\epsilon(k)$ is given by

$$
\epsilon(k) = \begin{cases} 
\epsilon_0, & k = 0 \\
\epsilon_1 \eta_1^{k-1}, & k \geq 1.
\end{cases}
$$

Here we take care of the overshoot in the first step of the trajectory with $\epsilon_0$. The effect of $\epsilon_0$ alone with $\epsilon_1 = 0$ can be appreciated in Fig. 4.6 (solid line). The remaining overshoots during the first segment of the trajectory are taken care of with $\epsilon_1$ and $\eta_1$. Since $0 < \eta_1 < 1$, the contribution of $\epsilon(k)$ in the scaling effect of $\gamma$ turns to be insignificant as $k$ increases beyond certain value. The final effect in using all the three quantities $\epsilon_0$, $\epsilon_1$ and $\eta_1$ is presented in Fig. 4.7 (solid line).

We found by trial and error that the values $\epsilon_0 = 0.037$, $\epsilon_1 = 0.0095$, and $\eta_1 = 0.67$ appear to work well in our example across a variety of $P_{\text{cloud,desired}}$ values for $P_{\text{comp,desired}} = 0$ and they may not work well in other kind of examples. Notice that in our example the first overshoot to be overcome will be the same no matter what is the desired $P_{\text{cloud}}$ (taking into account that our interest is focused on situations where $P_{\text{comp,desired}} = 0$) since our adaptive algorithm is starting with $\beta = 0$. 

**Figure 4.7:** Comparison of the trajectory with significant overshoots versus the trajectory with minimal overshoots.
We have found that the technique (4.32) and (4.33) works well during the first adaptation stage (with $\beta_0 = 0$). To overcome the overshoot in subsequent $\beta$-search steps we chose instead to implement the change from $\beta_i$ to $\beta_i + 1$, as given by (4.29), not in a single step but as a sequence of smaller changes, each one of size $\frac{\beta_{i+1} - \beta_i}{100}$. The final effect of this operation can be appreciated in Figures 4.6 and 4.7 (solid lines) where we can observe that the overshoots in subsequent $\beta$-search steps have been significantly reduced.

Further research is needed to evaluate the utility of our overshoot reduction scheme (4.33) across a wide range of network and load conditions. A practical overshoot reduction scheme should be either universal (requiring no adjustment at all) or adaptive, adjusting its parameters in response to load current and voltage measurements.

Figure 4.8 presents the performance of the convergence trajectory where it is evident that a smoother trajectory can be achieved with the minimization of the overshoots in comparison with that one shown in Fig. 4.4. In addition, observe that after the first segment of the trajectory the adaptive algorithm is very nearly on the theoretical optimum (dash-dot line).

Finally, Fig. 4.9 shows the convergence trajectory of the adaptive algorithm for desired $P_{comp,desired} = 0$, $P_{cloud,desired} = 14,000W$ values. Once again, the smooth path of the trajectory is clearly appreciated along with the near-optimal effect. Also observe that
the initial trajectory segment until $P_{cloud} = 19,286 W$ is the same as in Fig. 4.8 due to the convention in using $\beta = 0$ as the initial value of the coefficient $\beta$.

![Figure 4.9: Convergence trajectory of the adaptive near-optimal compensator for $P_{comp}(\infty) = 0$, $P_{cloud}(\infty) = 14,000 W$.](image-url)
Chapter 5

Conclusions and Future Directions

The concept of compensation in power systems should imply an optimized reduction of the power dissipated in the source impedance ($P_{\text{line}}$), so that most of the source power is efficiently delivered to the load. The classical solution given by Fryze works well under negligible voltage drop conditions across the line. However, when the source impedance becomes significant the traditional Fryze current is no longer the smallest line current that supplies the same real power to the load as the original load current. Given this situation, a theoretically-optimal compensator considering significant source impedance was developed in [2, 3] as a suitable solution to the problem. Unfortunately, since it depends on both the equivalent source impedance and the current-voltage characteristic of the load, such information is usually difficult to obtain, especially in view of the time-varying nature of network parameters, so that an application of such as optimal compensator results rather impractical.

It is the existent restriction on the explicit knowledge of several system parameters which motivates us to explore an alternative to achieve our objective, i.e., the so-called adaptive near-optimal compensator that relies only on knowledge of load voltage and current which can be measured during operation. Thus the optimal compensation approach studied in previous works would function as a benchmark for comparison in the evaluation of the near-optimal compensator.
Near-optimal compensation is intended to get the smallest possible $P_{\text{line}}$ for prescribed $P_{\text{comp}}$, $P_{\text{cloud}}$, where in practical terms is of more interest the implementation of a compensator under lossless conditions ($P_{\text{comp}} = 0$).

The thesis begins with a complete formulation of the problem related to the near-optimal compensation in lossy polyphase systems where the objective in determining an adaptive near-optimal compensator based on the concept of quad-Fryze has been clearly achieved.

It is important to indicate that the two (real-valued) degrees of freedom $\alpha$, $\beta$ employed in the derivation of the near-optimal compensator have been used to satisfy two independent constraints, i.e., setting the values of $P_{\text{comp}}$ and $P_{\text{cloud}}$. In such a way, we have two control parameters ($\alpha$, $\beta$) that can be used to adjust the two variables of interest ($P_{\text{comp}}$, $P_{\text{cloud}}$).

It was found that the iteration process with “fixed $\gamma$” is stable (converges to unique equilibrium). We obtained a characterization of the stability region for $\alpha$, $\beta$ values, at least for a linear load. However, the value of $\gamma$ is not known a priori, and has to be adaptively adjusted during iteration. Thus the challenge of this iteration consists on how to select $\alpha(k)$ and $\beta$.

The adaptive algorithm method based on the quad-Fryze compensation was used in several experiments in order to obtain a good understanding of how the variation of the coefficients $\alpha$ and $\beta$ affects the values of $P_{\text{comp}}$, $P_{\text{cloud}}$ and $P_{\text{line}}$ by using the example of an asymmetrical three-phase induction motor supplied with unbalanced non-sinusoidal voltages. The final result is an adaptive compensator which can track changes in load and network conditions and that is very nearly optimal — we have compared it to the theoretical optimum — at least for a linear load.

In addition, the performance of the convergence trajectory of the method was improved greatly by doing additional adjustments for the minimization of the overshoots. As a result, we obtained a working algorithm with the desired steady state and a manageable transient behavior, essentially independent of network information.

Future directions in this topic should consider the operation of our adaptive compensator in the case of a nonlinear load and possibly of a nonlinear source impedance as well. Moreover, the convergence region of our algorithm obtained experimentally in
Section 4.1 requires further research for determining this analytically, not only for the linear case but also for the nonlinear load, so that an a priori test for convergence can be implemented. On the other hand, despite the resulting iterative algorithm has worked well in several examples, we have not still provided an analytical proof of its stability which is another future direction to be addressed.

The proposed methodology to adjust $\beta$ relies on the slope of the $P_{\text{load}}$-$\beta$ curve. However, this slope is not known a priori in the initial step of the method and needs to be estimated appropriately. It is perhaps needed to find other methods of adjusting $\beta$ maybe before full convergence of the adjustment of $\alpha$. Another possibility is to develop some local estimates based on what can be measured in the load.

On the other hand, the need to find analytical (approximate) results that let us figure out completely the situation of why there are overshoots (mostly above $P_{\text{comp}}(\infty)$), has also to be considered in further research. Certainly, some open questions still left to be answered in this way, such as: is the overshoot always up?, how does it depend on either (or both) $P_{\text{comp}}$ value or the parameters of the load and line impedance?. In such a way, it will be required to create new strategies in minimizing these overshoots. Among them we can mention the study of what works better between using $[1+\epsilon(k)][\alpha(k)+j\beta]$ or $[1+\epsilon(k)]\alpha(k)+j\beta$; the necessity to find a way in selecting $\epsilon(k)$ when $Z_s$, $Y$ are not known, i.e., select it a priori, without experimentation; or, alternatively, the idea of adjusting $\epsilon(k)$ adaptively based only on the transient $P_{\text{comp}}$ values could be a good choice.

Another important future direction to be considered is related to the need to demonstrate the ability to track a jump change in the source impedance $Z_s$. Here it is convenient to establish whether the $\epsilon(k)$ correction is indeed necessary or we can dispense with it by using a different approach in the reduction of the overshoots.

Finally, of particular interest is the application of a coordinated system of adaptive near-optimal compensators which constitutes an ambitious challenge to be met in our following studies so that it can open new possibilities for compensators to collaborate across the network, sending each other information that can be used to estimate relevant network parameters and improve compensation in general.
Appendix A

The Hermitian Component of a Matrix

Consider the quadratic form $XAX^H$, where $X$ is a complex valued row vector and $A$ is a complex valued matrix. Decompose $A$ as follows:

$$A = \frac{1}{2} (A + A^H) + \frac{j}{2j} (A - A^H)$$  \hspace{1cm} (A.1)

Notice that $A_1 = A_1^H$, and $A_2 = A_2^H$. Thus, both $A_1$ and $A_2$ are Hermitian matrixes.

Now, $XA_1X^H$ is real-valued, and so is $XA_2X^H$. Therefore

$$\Re\{XAX^H\} = \Re\{X \frac{A + A^H}{2} X^H\}$$
$$= X \frac{A + A^H}{2} X^H$$  \hspace{1cm} (A.2)
Appendix B

Relation between $\mathcal{I}_S$ and $\mathcal{I}_{comp}$

In order to determine the polyphase load voltage $v(t)$ and load current $i(t)$ we need to consider the circuit equations

$$i(t) = i_s(t) + i_{comp}(t), \quad v_s(t) = v(t) + Z_s\{i_s\}(t)$$  \hspace{1cm} (B.1)

and the load current-voltage relation

$$i(t) = \mathcal{Y}_\ell\{v\}(t).$$  \hspace{1cm} (B.2)

Here $Z_s\{\cdot\}$ is an operator that describes the voltage-current relation of the source impedance, while $\mathcal{Y}_\ell\{\cdot\}$ denotes an operator that maps the voltage applied across the load into the resulting current flow through the load. While $\mathcal{Y}_\ell\{\cdot\}$ can, in general, be nonlinear, it is reasonable to assume that $Z_s\{\cdot\}$ is a linear time-invariant operator. In the absence of a compensator, the load voltage and current satisfy the nonlinear equation

$$v_s(t) = v(t) + Z_s\{\mathcal{Y}_\ell\{v\}\}(t),$$ \hspace{1cm} (B.3)
which involves the *composition* of the operators $\mathcal{Y}_t \{ \cdot \}$ and $\mathcal{Z}_s \{ \cdot \}$. When a compensating current $i_{\text{comp}}(t)$ is present, the fundamental nonlinear equation, obtained by eliminating both $i_s(\cdot)$ and $i(\cdot)$ from (B.1)-(B.2) becomes

$$v_{se}(t) = v(t) + \mathcal{Z}_s \{ \mathcal{Y}_t \{ v \} \}(t), \quad \text{(B.4)}$$

where $v_{se}(t) \triangleq v_s(t) + \mathcal{Z}_s \{ i_{\text{comp}} \}(t)$ acts as an equivalent source voltage, due to the presence of both $v_s(t)$ and $i_{\text{comp}}(t)$.

Equation (B.4) is a non-linear operator equation due to non-linearity of the load. In general, such equations have unique solutions subject to restrictions on the operators $\mathcal{Z}_s$, $\mathcal{Y}_t$ and the driving term $v_s(t) + \mathcal{Z}_s \{ i_{\text{comp}} \}(t)$. For example, when the load is linear and $v_s(t)$, $i_{\text{comp}}(t)$ are periodic square integrable waveforms, this equation can be written in terms of phasor arrays as

$$\mathcal{V}_s + \mathcal{I}_{\text{comp}} \mathcal{Z}_s = \mathcal{V}(I + \mathcal{Y}_t \mathcal{Z}_s), \quad \text{(B.5)}$$

so that a unique solution exists for any choice of $\mathcal{V}_s$ and $\mathcal{I}_{\text{comp}}$ as long as the matrix $(I + \mathcal{Y}_t \mathcal{Z}_s)$ is invertible.

From a practical perspective we know that the loads connected to the power grid always have a unique voltage developed across the load for given network conditions. In other words, the non-linear equation (B.4) always has a unique solution in the regime of normal power grid operation. Further research is needed to determine the precise mathematical conditions that need to be imposed on (B.4) to ensure the existence of a unique solution. For now, we shall adopt the working hypothesis that such conditions are indeed met in cases of practical interest.

One obvious case when uniqueness of solution is guaranteed is when the load is linear and $\| \mathcal{Y}_t \mathcal{Z}_s \| \ll 1$. In this case (B.5), the linear version of (B.4) under periodic conditions, has a unique solution because $(I + \mathcal{Y}_t \mathcal{Z}_s)$ is invertible. The condition $\| \mathcal{Y}_t \mathcal{Z}_s \| \ll 1$ expresses our assumption that the load impedance is much larger than the source impedance, resulting in a negligible voltage drop across the source impedance. However, it is entirely possible to have a relatively small voltage drop even when the norm $\| \mathcal{Y}_t \mathcal{Z}_s \|$ is larger.
Appendix B. *Relation between $I_s$ and $I_{\text{comp}}$*

than one. For instance, according to the numerical values of the example in Appendix C we have $\|Y_t Z_s\| = 2.18$ while $|V_s - V| / |V_s| \approx 0.16$ and $\min \{|\lambda_i (I + Y_t Z_s)|\} = 1.17$.

It turns out that while the relative voltage drop $|V_s - V| / |V_s|$ and the impedance-ratio $\|Y_t Z_s\|$ are very closely related in the sinusoidal single-phase case, this relation is much more flexible in the poly-harmonic, polyphase case.

Another interesting example of existence and uniqueness of a solution for the nonlinear circuit equations is analyzed in [9] in the context of a nonlinear resistive load. It is shown that in such a case, when the load is both upper and lower bounded by linear resistive loads, a unique solution for (B.4) exists subject to the following mild constraints:

- The source impedance is linear, time-invariant and BIBO stable.
- The polyphase waveform $v_s(t)$ (and $i_{\text{comp}}(t)$) is periodic and square integrable.

However, these conditions may not be sufficient when the nonlinear load is dynamic (i.e., non-resistive).

We now turn to analyze the effect of varying $i_{\text{comp}}(t)$ on the load conditions, assuming that $v_s(t)$ remains unchanged. When (B.4) has a unique solution $v(t)$, it defines a map $i_{\text{comp}}(t) \rightarrow v(t)$, which induces a map $i_{\text{comp}}(t) \rightarrow i_s(t)$, where we used the relations $i(t) = Y_t \{v\} (t)$ and $i_s(t) = i(t) - i_{\text{comp}}(t) = Y_t \{v\} (t) - i_{\text{comp}}(t)$. In fact, this map can be described by the implicit non-linear relation

$$i_{\text{comp}}(t) + i_s(t) = Y_t \{v_s - Z_s \{i_s\}\} (t), \quad (B.6)$$

which determines $i_s(t)$ uniquely in terms of $\{v_s, i_{\text{comp}}\}$ whenever (B.4) admits a unique solution.

Moreover, whenever (B.6) admits a unique solution $i_s(t)$ it also establishes a one-to-one correspondence between $i_s(t)$ and $i_{\text{comp}}(t)$, for a predetermined $v_s(t)$. Indeed, for a given $i_s(t)$ in the range of solutions of (B.6) there can be only one $i_{\text{comp}}(t)$, given via (B.6) by

$$i_{\text{comp}}(t) = Y_t \{v_s - Z_s \{i_s\}\} (t) - i_s(t). \quad (B.7)$$
Appendix B. *Relation between $I_s$ and $I_{\text{comp}}$*

The one-to-one relationship $I_{\text{comp}} \leftrightarrow I_s$ allows us to use $I_s$ as the independent optimization variable in Chapter 2.

However, it follows from the preceding discussion that choices of $I_s$ must be constrained to the range of solutions of (B.6).

In the linear case we know that (B.5) has a unique solution if, and only if, the matrix $(I + \mathcal{Y}_t \mathcal{Z}_s)$ is nonsingular. In this case (B.6) reduces to

$$I_{\text{comp}} = -I_s [I + \mathcal{Z}_s \mathcal{Y}_t] + \mathcal{V}_s \mathcal{Y}_t ,$$

which has a unique solution for $I_s$ under the same conditions, since

$$\det (I + \mathcal{Z}_s \mathcal{Y}_t) = \det (I + \mathcal{Y}_t \mathcal{Z}_s) .$$

We conclude that in the linear case the correspondence $I_{\text{comp}} \leftrightarrow I_s$ is well defined for all square summable $I_{\text{comp}}$ and $I_s$, as long as $(I + \mathcal{Z}_s \mathcal{Y}_t)$ is invertible.
Appendix C

Numerical values of the load compensation example

The numerical values of the load compensation circuit analyzed in this thesis are the following:

\[ V_s = \begin{bmatrix} 138.6 & -38.54 - 118.6j & -100 + 118.6j & 13.86 & -6.928 - 12j & -6.928 + 12j \end{bmatrix} \]  \hspace{1cm} (C.1)

\[ Z_s = 0.195 \begin{bmatrix} 1 + 2j & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 + 2j & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 + 2j & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 + 10j & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 + 10j & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 + 10j \end{bmatrix} \]  \hspace{1cm} (C.2)

\[ Y = \begin{bmatrix} Y_1 & 0 \\ 0 & Y_5 \end{bmatrix} \]  \hspace{1cm} (C.3)
Appendix C. Numerical values of the load compensation example

where

\[ Y_1 = \begin{bmatrix}
1.047 - 2.447j & 1.093 - 0.9531j & -0.2024 - 1.212j \\
-0.2024 - 1.212j & 1.047 - 2.447j & 1.093 - 0.9531j \\
1.093 - 0.9531j & -0.2024 - 1.212j & 1.047 - 2.447j
\end{bmatrix} \] (C.4)

and

\[ Y_5 = \begin{bmatrix}
0.1464 - 0.8491j & -0.0212 - 0.1271j & -0.0347 - 0.1014j \\
-0.0347 - 0.1014j & 0.1464 - 0.8491j & -0.0212 - 0.1271j \\
-0.0212 - 0.1271j & -0.0347 - 0.1014j & 0.1464 - 0.8491j
\end{bmatrix} \] (C.5)

Notice that it has been included only the first and fifth harmonic data in \( Y \) (and, similarly, in \( V_s, Z_s \)), since the remaining diagonal blocks of \( Y \) are all zero.

Upon evaluating the eigenvalues for \( (Z_s Y) \) we obtain

\[ \lambda_i (Z_s Y) = \begin{bmatrix}
2.1766 - 0.1437j \\
0.1682 + 0.0999j \\
1.1307 - 0.1627j \\
2.1190 - 0.0337j \\
1.4485 + 0.2424j \\
1.4854 + 0.1510j
\end{bmatrix} \] (C.6)

where it is observed that \( \Re \{ \lambda_i \} > 0 \).

Finally, the corresponding magnitudes of the eigenvalues result in

\[ |\lambda_i (Z_s Y)| = \begin{bmatrix}
2.1813 \\
0.1957 \\
1.1424 \\
2.1192 \\
1.4686 \\
1.4931
\end{bmatrix} \] (C.7)
Bibliography


