Second-Hand Market as an Alternative in Reverse Logistics

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ABSTRACT

Collectors of discarded products seldom know when those products were bought and why they are discarded. Also, the products do not indicate their remaining life periods. So, it is difficult to decide if it is “sensible” to repair (if necessary) a particular product for subsequent sale on the second-hand market or to disassemble it partially or completely for subsequent remanufacture and/or recycle. To this end, we build an expert system using Bayesian updating process and fuzzy set theory, to aid such decision-making. A numerical example demonstrates the building approach.

Keywords: Second-hand Market, Expert System, Bayesian Updating, Fuzzy Sets, Uncertainty.

1. MOTIVATION

Reverse logistics encompasses the transfer of discarded products from consumers to producers within a distribution network. The most important driver for companies interested in collecting discarded products is recoverable value through reprocessing (remanufacture/recycle) [3], [5]. However, collectors of discarded products seldom know when those products were bought and why were they discarded. Also, the products do not indicate their remaining life periods. Hence, they often undergo partial or complete disassembly for subsequent remanufacture or recycle. We believe that for some discarded products, it might make more “sense” to make necessary repairs to the products and sell them on the second-hand market than to disassemble them for subsequent remanufacture and/or resale. To this end, we build an expert system using Bayesian updating process and fuzzy set theory, to decide if it is “sensible” to repair the product of interest for subsequent sale on the second-hand market.

In the next section, a brief introduction of expert systems is provided. In Section 3, some important formulae used in the Bayesian updating process are detailed. Section 4 introduces fuzzy set theory that we use to implement the Bayesian updating process. Finally, in Section 5, a numerical example demonstrates how an expert system can be built to decide if it is “sensible” to repair the product of interest for subsequent sale on the second-hand market. Section 6 gives some conclusions.

2. EXPERT SYSTEM

Expert systems are computer programs that can represent human expertise (knowledge) in a particular domain (area of expertise) and then use a reasoning mechanism (applying logical deduction and induction processes) to manipulate this knowledge in order to provide advice in this domain. Although conventional programs like C and C++ also contain knowledge, their main function is to retrieve information, and carry out statistical analysis and numerical calculations [10]. They do not reason with this knowledge or make inferences as to what actions to take or conclusions to reach. So, what mainly distinguishes expert systems from conventional programs is the capability to reason with knowledge. The main components of an expert system are the following:

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• A knowledge base: This is where the knowledge is stored. Typically, this consists of a set of rules of the form: if EVIDENCE then HYPOTHESIS. The knowledge is written in the knowledge base using the syntax of what is termed the knowledge representation language (examples: Lisp and Prolog) of the system.

• An inference engine: This reasons with the knowledge resident in the knowledge base using certain mechanisms.

• A reasoning mechanism: This traces the path or the knowledge steps used to arrive at a conclusion and can relay it back to user as the justification for this conclusion. Examples of these mechanisms are deduction (cause + rule $\rightarrow$ effect), abduction (effect + rule $\rightarrow$ cause), and induction (cause + effect $\rightarrow$ rule).

• An uncertainty modeling process: This aids the inference engine when dealing with uncertainty. The uncertainty modeling process that we use in our approach to decide whether it is “sensible” or not to send a discarded product for any necessary repairs and subsequent sale on the second-hand market is called Bayesian updating.

A shell is an expert system that is complete except for the knowledge base [6]. Thus, a shell includes an inference engine, a user interface for programming, and a user interface for running the system. Typically, the programming interface comprises a specialized editor for creating rules in a pre-determined format, and some debugging tools. The user of the shell enters rules in a declarative fashion (If $X$, then $Y$), and ideally should not need to be concerned with the working of the inference engine. Expert system shells are easy to use and allow a simple expert system to be constructed quickly. In this paper, we use the FLEX expert system shell [8] whose inference engine is programmed in Prolog.

3. BAYESIAN UPDATING

Bayesian updating is an uncertainty modeling technique that assumes that it is possible for an expert in a domain to guess a probability to every hypothesis or assertion in that domain, and that this probability can be updated in light of evidence for or against the hypothesis or assertion. In our approach to decide if it is “sensible” to repair a particular discarded product for subsequent sale on the second-hand market, we use fuzzy set theory [11] to calculate certain probabilities that are difficult to guess (for example, it is difficult to guess how often a component, say $C$, is observed needing repair when it is decided that repairing the product is “sensible”). We introduce the fuzzy set theory to the reader in Section 4 and demonstrate its usage in Section 5.

Suppose the probability of a hypothesis is $P(H)$. Then, the formula for the odds $O(H)$ of the hypothesis $H$ is given by:

$$O(H) = \frac{P(H)}{1-P(H)}$$

(1)

A hypothesis that is absolutely certain, i.e., has a probability of 1, has infinite odds. In practice, limits are often set on odds values so that, for example, if $O(H) > 1000$ then $H$ is true, and if $O(H) < 0.01$ then $H$ is false.

3.1. Updating probabilities with supporting evidence

The standard formula for updating the odds of hypothesis $H$, given that evidence $E$ is observed, is:

$$O(H|E) = (A).O(H)$$

(2)

where $O(H|E)$ is the odds of $H$, given the presence of evidence $E$, and $A$ is the affirms weight of $E$. The definition of $A$ is:

$$A = \frac{P(E|H)}{P(E|\neg H)}$$

(3)

where $P(E|H)$ is the probability of $E$, given that $H$ is true and $P(E|\neg H)$ is the probability of $E$, not given that $H$ is true.

3.2. Updating probabilities with opposing evidence

Bayesian updating assumes that the absence of supporting evidence is equivalent to the presence of opposing evidence. The standard formula for updating the odds of a hypothesis $H$, given that the evidence $E$ is absent, is:

$$O(H|\neg E) = (D).O(H)$$

(4)
where \( O(H|~E) \) is the odds of \( H \), given the absence of evidence \( E \), and \( D \) is the \emph{denies} weight of \( E \). The definition of \( D \) is:

\[
D = \frac{P(\neg E|H)}{P(\neg E|\neg H)} = \frac{1-P(E|H)}{1-P(E|\neg H)}
\]

If a given piece of evidence \( E \) has an \emph{affirms} weight \( A \) which is greater than 1, then its \emph{denies} weight must be less than 1, and vice versa. Also, if \( A > 1 \) and \( D < 1 \), then the presence of evidence \( E \) is supportive of hypothesis \( H \). Similarly, if \( A < 1 \) and \( D > 1 \), then the absence of \( E \) is supportive of \( H \).

For example, while controlling a power station boiler, a rule - “IF (temperature is high) and NOT (water level is low) THEN (pressure is high)” can also be written as “IF (temperature is high - AFFIRMS \( A_1 \), DENIES \( D_1 \)) AND (water level is low - AFFIRMS \( A_2 \), DENIES \( D_2 \)) THEN (pressure is high)”. Here,

\[
A_1 = \frac{P(\text{Temperature is high} | \text{Pressure is high})}{P(\text{Temperature is high} | \neg \text{Pressure is high})}; \quad D_1 = \frac{P(\neg \text{Temperature is high} | \text{Pressure is high})}{P(\neg \text{Temperature is high} | \neg \text{Pressure is high})};
\]

\[
A_2 = \frac{P(\text{Water level is low} | \text{Pressure is high})}{P(\text{Water level is low} | \neg \text{Pressure is high})}; \quad D_2 = \frac{P(\neg \text{Temperature is high} | \text{Pressure is high})}{P(\neg \text{Temperature is high} | \neg \text{Pressure is high})};
\]

### 3.3. Dealing with uncertain evidence

Sometimes, an evidence is neither definitely present nor definitely absent. For example, if one is diagnosing a TV set that is not functioning properly, it is not definite if this is due to a malfunctioning picture tube or not. In such a case, depending upon the value of the probability of the evidence \( P(E) \), the affirms and denies weights are modified using the following formulae [4]:

\[
A' = [2.(A-1).P(E)]+2-A
\]

\[
D' = [2.(1-D).P(E)]+D
\]

When \( P(E) > 0.5 \), the \emph{affirms} weight is used to calculate \( P(O|H) \), and when \( P(E) < 0.5 \), the \emph{denies} weight is used.

### 3.4. Combining evidence

If, \( n \) statistically independent pieces of evidence are found that support or oppose a hypothesis \( H \), then the updating equations are given by [4]:

\[
O(H|E_1 \& E_2 \& E_3 \& \ldots \& E_n) = (A_1).&(A_2).&(A_3)\ldots&(A_n).O(H)
\]

and

\[
O(H|\neg E_1 \& \neg E_2 \& \neg E_3 \& \ldots \& \neg E_n) = (D_1).&(D_2).&(D_3)\ldots&(D_n).O(H)
\]

\( A_i \) and \( D_i \) are given by Equations 10 and 11 respectively.

\[
A_i = \frac{P(E_i|H)}{P(E_i|\neg H)}
\]

\[
D_i = \frac{P(\neg E_i|H)}{P(\neg E_i|\neg H)}
\]

### 4. FUZZY SET THEORY

Expressions such as “not very clear”, “probably so” and “very likely” can be heard very often in daily life. The commonality in such terms is that they are all tainted with imprecision. This imprecision or vagueness of human decision-making is called “fuzziness” in the literature. With different decision-making problems of diverse intensity, the results can be misleading if fuzziness is not taken into account. However, since Zadeh [11] first proposed fuzzy set theory, an increasing number of studies have dealt with imprecision (fuzziness) in problems by applying the fuzzy set theory. Our paper too makes use of this theory in building an expert system that can decide if it is “sensible” to repair (if necessary) a discarded product of interest and subsequently sell it on the second-hand market. The concepts of the fuzzy set theory, which we utilize in this paper, are as follows:
4.1. Linguistic values and fuzzy sets

When dealing with imprecision, decision-makers may be provided with information characterized by vague language such as: high risk, low profit and good customer service. By using linguistic values like “high”, “low”, “good”, “medium”, “cheap”, etc., people are usually attempting to describe factors with uncertain or imprecise values. For example, “weight” of an object may be a factor with an uncertain or imprecise value and so, its linguistic value can be “very low”, “low”, “medium”, “high”, “very high”, etc. The fuzzy set theory is primarily concerned with quantifying the vagueness in human thoughts and perceptions.

To deal with quantifying vagueness, Zadeh proposed a membership function which associates with each quantified linguistic value a grade of membership belonging to the interval [0, 1]. Thus, a fuzzy set is defined as:

$$\mu_A(x) = \begin{cases} 0, & x < a \\ \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ \frac{(c-x)}{(c-b)}, & b \leq x \leq c \\ 0, & x \geq c. \end{cases}$$

For each quantity $x$ increasing from $a$ to $b$, its corresponding degree of membership linearly increases from 0 to 1. While $x$ increases from $b$ to $c$, its corresponding degree of membership linearly decreases from 1 to 0. The membership function is a mapping from any given $x$ to its corresponding degree of membership.

The TFN is mathematically easy to implement, and more importantly, it represents the rational basis for quantifying the vague knowledge in most decision-making problems.

The basic operations on triangular fuzzy numbers are as follows [1], [2], [9]:

For example, $P_1 = (a, b, c)$ and $P_2 = (d, e, f)$. 

\[ P_1 + P_2 = (a+d, b+e, c+f) \] addition; \hspace{1cm} (16)
\[ P_1 - P_2 = (a-f, b-e, c-d) \] subtraction; \hspace{1cm} (17)
\[ P_1 \times P_2 = (a\times d, b\times e, c\times f) \text{ where } a \geq 0 \text{ and } d \geq 0 \] multiplication; \hspace{1cm} (18)
\[ P_1 / P_2 = (a/f, b/e, c/d) \text{ where } a \geq 0 \text{ and } d > 0 \] division. \hspace{1cm} (19)

4.3. Defuzzification

Defuzzification is a technique to convert a fuzzy number into a crisp real number. There are several methods to serve this purpose [7]. For example, the Centre-of-Area method [12] converts a fuzzy number \( P = (a, b, c) \) into a crisp real number \( Q \) where

\[ Q = \frac{(c-a) + (b-a)}{3} + a \]

(20)

5. NUMERICAL EXAMPLE

Consider the discarded product shown in Figure 2. Given that this product is not functioning properly, we shall decide whether it is “sensible” to repair it for subsequent sale on the second-hand market or not. Table 1 shows the probability values we use to implement the Bayesian updating process.

![Figure 2. Discarded product](image)

5.1. Usage of fuzzy set theory

Since it is difficult for an expert to guess the probabilities shown in bold (unlike the rest) in Table 1, we calculate them using the fuzzy set theory as follows:

I. Ask the expert to assign a linguistic rating to \( P(E|H) \) for each component with respect to each of the following factors (see Table 2):
   a. Is it economical to repair/replace the component? (more economical implies higher rating)
   b. If disposed, will the component be harmful to the environment? (more harmful implies lower rating)
   c. What is the remaining life period of the component? (longer life implies higher rating)
   d. Is the raw material used to make the component depleting fast? (faster depletion implies lower rating)
   e. Is it difficult to repair the component? (more difficult implies lower rating)

II. Use the data in Table 3 to convert the linguistic ratings into TFNs.

III. Calculate the average fuzzy \( P(E|H) \) value for each component.

IV. Defuzzify the average \( P(E|H) \) for each component.

Apply steps I, II, III, and IV to calculate \( P(E|\neg H) \) values for each component. In order to save ourselves from the tedious calculations, we assume here that the values shown in bold in Table 1 are the defuzzified average probabilities obtained after performing steps I, II, III and IV.
Table 1. Probability values used in Bayesian updating

| H                  | E                  | P(H) | O(H) | P(E|H) | P(E|~H) | A  | D  |
|-------------------|--------------------|------|------|-------|--------|-----|----|
| S1 needs repair   | Product needs repair | 0.60 | 1.50 | 1.00  | 0.60   | 1.67| 0.00|
| S2 needs repair   | Product needs repair | 0.70 | 2.33 | 1.00  | 0.40   | 2.50| 0.00|
| C1 needs repair   | S1 needs repair     | 0.45 | 0.82 | 1.00  | 0.45   | 2.22| 0.00|
| C2 needs repair   | S1 needs repair     | 0.55 | 1.22 | 1.00  | 0.30   | 3.33| 0.00|
| C3 needs repair   | S1 needs repair     | 0.30 | 0.43 | 1.00  | 0.55   | 1.82| 0.00|
| C4 needs repair   | S2 needs repair     | 0.32 | 0.47 | 1.00  | 0.70   | 1.43| 0.00|
| C5 needs repair   | S2 needs repair     | 0.10 | 0.11 | 1.00  | 0.80   | 1.25| 0.00|
| Sensible to repair product | C1 needs repair | 0.60 | 1.50 | 0.70  | 0.20   | 3.50| 0.38|
| Sensible to repair product | C2 needs repair | 0.60 | 1.50 | 0.60  | 0.30   | 2.00| 0.57|
| Sensible to repair product | C3 needs repair | 0.60 | 1.50 | 0.45  | 0.60   | 0.75| 1.38|
| Sensible to repair product | C4 needs repair | 0.60 | 1.50 | 0.10  | 0.75   | 0.13| 3.60|
| Sensible to repair product | C5 needs repair | 0.60 | 1.50 | 0.85  | 0.40   | 2.13| 0.25|

5.2. Rules used in Bayesian updating

Rule 1: IF product needs repair (AFFIRMS: 1.67; DENIES: 0.00) THEN S1 needs repair.

Rule 2: IF product needs repair (AFFIRMS: 2.50; DENIES: 0.00) THEN S2 needs repair.

Rule 3: IF S1 needs repair (AFFIRMS: 2.22; DENIES: 0.00) THEN C1 needs repair.

Rule 4: IF S1 needs repair (AFFIRMS: 3.33; DENIES: 0.00) THEN C2 needs repair.

Rule 5: IF S1 needs repair (AFFIRMS: 1.82; DENIES: 0.00) THEN C3 needs repair.
Rule 6: IF S2 needs repair (AFFIRMS: 1.43; DENIES: 0.00) THEN C4 needs repair.

Rule 7: IF S2 needs repair (AFFIRMS: 1.25; DENIES: 0.00) THEN C5 needs repair.

Rule 8: IF C1 needs repair (AFFIRMS: 3.50; DENIES 0.38) AND C2 needs repair (AFFIRMS: 2.00; DENIES 0.57) AND C3 needs repair (AFFIRMS: 0.75; DENIES 1.38) AND C4 needs repair (AFFIRMS: 0.13; DENIES 3.60) AND C5 needs repair (AFFIRMS: 2.13; DENIES 0.25) THEN it is sensible to repair the product.

Table 2. Linguistic P(E|H) ratings

<table>
<thead>
<tr>
<th>Component</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>High</td>
<td></td>
<td>Medium</td>
<td></td>
<td>High</td>
</tr>
<tr>
<td>C2</td>
<td>Very High</td>
<td>High</td>
<td>Very High</td>
<td>Medium</td>
<td>Low</td>
</tr>
<tr>
<td>C3</td>
<td>Low</td>
<td>Low</td>
<td>Very Low</td>
<td>Very High</td>
<td>Medium</td>
</tr>
<tr>
<td>C4</td>
<td>High</td>
<td>High</td>
<td>High</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C5</td>
<td>Medium</td>
<td>High</td>
<td>High</td>
<td></td>
<td>Medium</td>
</tr>
</tbody>
</table>

Table 3. Linguistic value conversion table

<table>
<thead>
<tr>
<th>Linguistic rating</th>
<th>Triangular fuzzy number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very high (VH)</td>
<td>(0.7, 0.9, 1.0)</td>
</tr>
<tr>
<td>High (H)</td>
<td>(0.5, 0.7, 0.9)</td>
</tr>
<tr>
<td>Medium (M)</td>
<td>(0.3, 0.5, 0.7)</td>
</tr>
<tr>
<td>Low (L)</td>
<td>(0.1, 0.3, 0.5)</td>
</tr>
<tr>
<td>Very low (VL)</td>
<td>(0.0, 0.1, 0.3)</td>
</tr>
</tbody>
</table>

5.3. Bayesian updating

Rule 1: H = S1 needs repair; O(H) = 1.50; E = Product needs repair; A = 1.67; O(H|E) = O(H).(A) = 2.51.

Rule 2: H = S2 needs repair; O(H) = 2.33; E = Product needs repair; A = 2.50; O(H|E) = O(H).(A) = 5.83.

Rule 3: H = C1 needs repair; O(H) = 0.82; E = S1 needs repair; O(E) = 2.51; P(E) = 0.72; A = 2.22; A’ = [2(A-1)*P(E)] + 2 – A = 1.54; O(H|E) = O(H).(A’) = (0.82).(1.54) = 1.26.

Rule 4: H = C2 needs repair; O(H) = 1.22; E = S1 needs repair; O(E) = 2.51; P(E) = 0.72; A = 3.33; A’ = [2(A-1)*P(E)] + 2 – A = 2.03; O(H|E) = O(H).(A’) = (1.22).(2.03) = 2.48.

Rule 5: H = C3 needs repair; O(H) = 0.43; E = S1 needs repair; O(E) = 2.51; P(E) = 0.72; A = 1.82; A’ = [2(A-1)*P(E)] + 2 – A = 1.36; O(H|E) = O(H).(A’) = (0.43).(1.36) = 0.58.

Rule 6: H = C4 needs repair; O(H) = 0.47; E = S2 needs repair; O(E) = 5.83; P(E) = 0.85; A = 1.43; A’ = [2(A-1)*P(E)] + 2 – A = 1.30; O(H|E) = O(H).(A’) = (0.47).(1.30) = 0.61.

Rule 7: H = C5 needs repair; O(H) = 0.11; E = S2 needs repair; O(E) = 5.83; P(E) = 0.85; A = 1.25; A’ = [2(A-1)*P(E)] + 2 – A = 1.18; O(H|E) = O(H).(A’) = (0.11).(1.18) = 0.13.

Rule 8: H = Sensible to repair product; O(H) = 1.50; E1 = C1 needs repair; O(E1) = 1.26; P(E1) = 0.56; A1 = 3.50; A1’ = [2(A1-1).P(E1)] + 2 – A1 = 1.30;

E2 = C2 needs repair; O(E2) = 2.48; P(E2) = 0.71; A2 = 2.00; A2’ = [2(A2-1).P(E2)] + 2 – A2 = 1.42;

E3 = C3 needs repair; O(E3) = 0.58; P(E3) = 0.37; D3 = 1.38; D3’ = [2(1-D3).P(E3)] +D3 = 1.09;

E4 = C4 needs repair; O(E4) = 0.61; P(E4) = 0.38; D4 = 3.60; D4’ = [2(1-D4).P(E4)] +D4 = 1.88;

E5 = C5 needs repair; O(E5) = 0.13; P(E5) = 0.12; D5 = 0.25; D5’ = [2(1-D5).P(E5)] +D5 = 0.43;
O(H|E1&E2&E3&E4&E5) = O(H). (A1’). (A2’). (D3’). (D4’). (D5’) = (1.50). (1.30). (1.42). (1.09). (1.88). (0.43) = 2.44;
P(H|E1&E2&E3&E4&E5) = (2.44)/(3.44) = 0.71.

P(sensible to repair the product) = 0.71.

If the cut-off value as decided by the decision-maker is say, 0.55, he will decide to send the discarded product for repair and subsequent sale on the second-hand market.

5.4. FLEX-based expert system

We employ FLEX shell to build an expert system that can decide if it is “sensible” to repair a particular discarded product for subsequent sale on the second-hand market. Figure 3 shows the user-interface for building the expert system and Figure 4 shows the user-interface for running the expert system.
The probability that it is “sensible” to repair the product is calculated by the FLEX-based expert system as 0.61. The difference in the probability obtained manually in Section 4 and the one obtained by the expert system in this section is most likely due to the difference in the formulae used to calculate A’ and D’ (they are interpolated values). The user of an expert system shell cannot know how exactly the inference engine of the shell works. When there is a significant difference in the probability values, it is advisable to build the expert system using a knowledge representation language like Lisp or Prolog, rather than using an expert system shell.

Figure 4. FLEX user-interface for running the expert system
6. CONCLUSIONS

It is difficult to decide if it is “sensible” to repair (if necessary) a particular discarded product for subsequent sale on the second-hand market or to disassemble it partially or completely for subsequent remanufacture and/or recycle. To aid making such a decision, in this paper, we built an expert system using Bayesian updating process and fuzzy set theory. We illustrated the building approach using a numerical example.

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